HCMC University of Technology
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# Probability and Statistics

## Random Variables and Random Vectors

## Random Variable and its Notation

A variable that associates a number $X(u)$ with the outcome $u$ of a random experiment is called a random variable.

$$X : \Omega \to \mathbb{R}$$
$$u \to X(u)$$

- Uppercase letters $(X, Y, Z)$: Random variables.
- Lowercase letters $(x, y, z)$: Measured values of random variables (after the experiment is conducted). Eg. $x = 70$ milliamperes.



## Discrete Random Variables

A discrete random variable is a random variable with a finite or countably infinite range. Its values are obtained by counting.

- Number of scratches on a surface.
- Number of defective parts among 100 tested.
- Number of transmitted bits received in error.
- Number of common stock shares traded per day.



## Continuous Random Variables

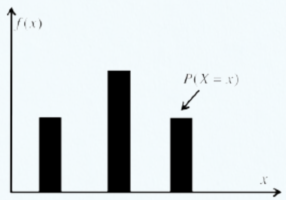
A continuous random variable is a random variable with an interval (either finite or infinite) of real numbers for its range. Its values are obtained by measuring.

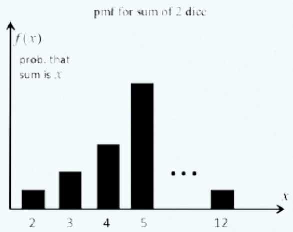
- Electrical current and voltage.
- Physical measurements, e.g., length, weight, time, temperature, pressure.

## Discrete Distribution

The probability mass function of $X$ is defined by

$$f_X(u) = P(X = u)$$




For any event $A$: $P(X \in A) = \sum_{u \in A} f(u)$.

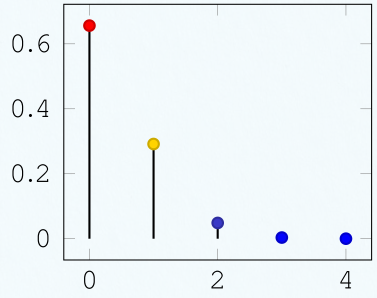




## Example 1 – Digital Channel

- There is a chance that a bit transmitted through a digital transmission channel is received in error.
- $X$:the number of bits received in error in 4 bits transmitted.

| | | |
|---|---|---|
| $P(X=0)$ | $=$ | 0.6561 |
| $P(X=1)$ | $=$ | 0.2916 |
| $P(X=2)$ | $=$ | 0.0486 |
| $P(X=3)$ | $=$ | 0.0036 |
| $P(X=4)$ | $=$ | 0.0001 |
| **Total** | | 1.0000 |







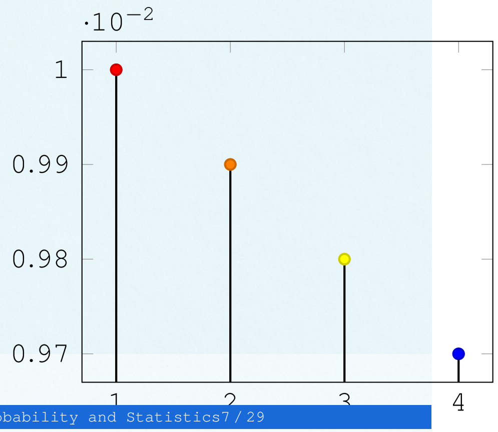

## Example 2 – Wafer Contamination

$X$: the number of wafers that need to be analyzed to detect a large particle of contamination. Assume that the probability that a wafer contains a large particle is 0.01, and that the wafers are independent.

- $\Omega = \{p, ap, aap, aaap, \ldots\}$.
- The range of $X$: $\{1, 2, 3, 4, \ldots\}$.

Probability Distribution

| | | |
|---|---|---|
| $P(X=1) =$ | $0.01$ | 0.01 |
| $P(X=2) =$ | $(0.99)(0.01)$ | 0.0099 |
| $P(X=3) =$ | $(0.99)^2(0.01)$ | 0.0098 |
| $P(X=4) =$ | $(0.99)^3(0.01)$ | 0.0097 |
| ... | ... | ... |
| **Total** | | 1 |








**Proposition (Characteristic properties)**

*A discrete function $f$ is a probability mass function iff*

1. $f(u) \geq 0$ *for all* $u$.
2. $\sum_u f(u) = 1$.

## Example

3. Uniform distribution: $\Omega = \{1, 2, 3, \cdots, n\}$

$$f(k) = P(X = k) = \frac{1}{n}$$

each outcome has equal probability

4. Is $f(k) = P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$ $(k = 0, 1, 2 \ldots)$ a probability mass function?

5. Suppose that a random variable $X$ has a discrete distribution with the following p.d.f.

$$f(u) = \begin{cases} cu, & u = 1, \cdots, 5 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of the constant $c$.

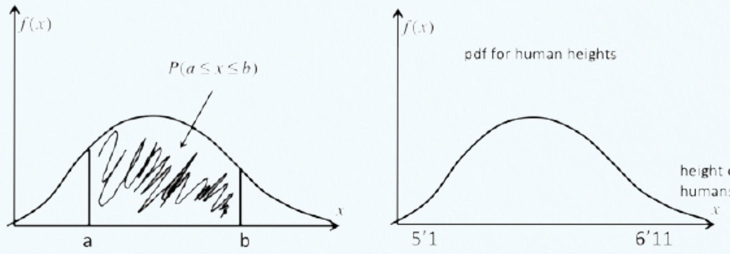




## Continuous Distribution

A random variable $X$ is continuous if $\exists f \geq 0$ such that for any $[a, b]$

$$P(a < X < b) = \int_a^b f(u)du$$








**Proposition (Characteristic properties)**

*If $f$ is a (probability) density function then*

1. $f(u) \geq 0$
2. $\displaystyle\int_{-\infty}^{\infty} f(u)du = 1$





## Example 6 – Current

Let $X$ denote the current measured in a thin copper wire in milliamperes (mA). Assume that the range of $X$ is $4.9 \leq x \leq 5.1$ and $f(x) = 5$. What is the probability that a current is

a. between 4.95mA and 5.1mA?
b. less than 5mA?

**Solution**

$$P(4.95 < X < 5.1) = \int_{4.95}^{5.1} f(x)dx = \int_{4.95}^{5.1} 5\,dx = 0.75$$

$$P(X < 5) = \int_{4.9}^{5} f(x)dx = \int_{4.9}^{5} 5\,dx = 0.5$$

## Example

7 Uniform distribution on $[a,b]$

$$f(u)=\begin{cases}\frac{1}{b-a}, & u\in[a,b]\\ 0, & u\notin[a,b]\end{cases}$$

8 Suppose that the p.d.f. of $X$ is as

$$f(u)=\begin{cases}cu, & 0<u<4\\ 0, & \text{otherwise}\end{cases}$$

Find $c$. Then determine $P(1\le X\le 2)$ and $P(X>2)$.

9 Suppose that $X$ is a continuous random variable whose probability density function is given by

$$f(u)=\begin{cases}c(4u-2u^2), & 0<u<2\\ 0, & \text{otherwise}\end{cases}$$



Find $c$ and $P(X>1)$.

10 The amount of time in hours that a computer



## Example 11 – Digital channel

Consider the probability distribution for the digital channel example.

| $x$ | $P(X=x)$ |
|---|---|
| 0 | 0.6561 |
| 1 | 0.2916 |
| 2 | 0.0486 |
| 3 | 0.0036 |
| 4 | 0.0001 |
| | 1.0000 |

Find the probability of three or fewer bits in error.

- The event $(X\le 3)$ is the total of the events: $(X=0)$, $(X=1)$, $(X=2)$, and $(X=3)$.
- From the table:

$$P(X\le 3)=P(X=0)+P(X=1)+P(X=2)+P(X=3)=0.9999$$





## Cumulative Distribution Function

$$f(u)=\begin{cases}\lambda e^{-u/100}, & u\ge 0\\ 0, & \text{otherwise}\end{cases}$$

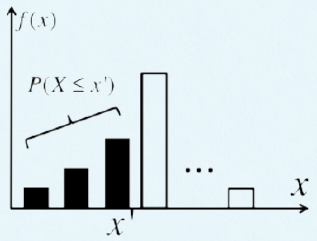
Compute the probability that a computer will function

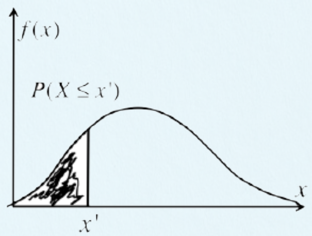
(a) between 50 and 150 hours before breaking down?

(b) for fewer than 100 hours?

$$F(u)=P(X\le u)=\sum_{k\le u} f(k) \quad \text{(discrete distribution)}$$

$$=\int_{-\infty}^{u} f(t)\,dt \quad \text{(continuous distribution)}$$








## Example 12 – Defective parts

A day's production of 850 parts contains 50 defective parts. Two parts are selected at random without replacement. Let the random variable $X$ equal the number of defective parts in the sample. Find the cumulative distribution function of $X$. The probability mass function is calculated as follows:

$$P(X=0)=\frac{800}{850}\cdot\frac{799}{849}=0.886$$

$$P(X=1)=2\cdot\frac{800}{850}\cdot\frac{50}{849}=0.111$$

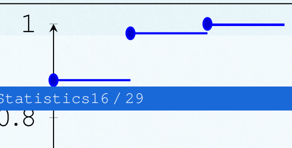
$$P(X=2)=\frac{50}{850}\cdot\frac{49}{849}=0.003$$

$$P(X=0)=0.886, P(X=1)=0.111, P(X=2)=0.003.$$

$$F(0)=P(X\le 0)=0.886$$

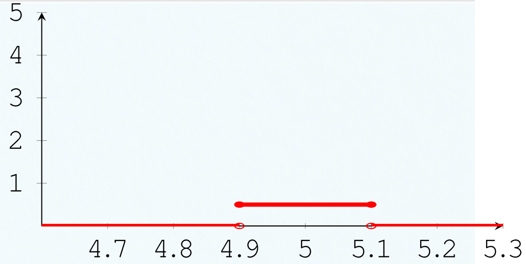
$$F(1)=P(X\le 1)=0.997$$

$$F(2)=P(X\le 2)=1.000.$$



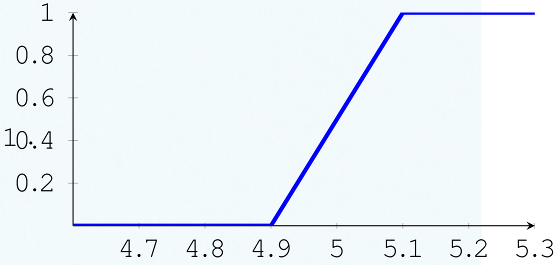
## Example 13 – Electric Current

Consider the current measured in a thin copper wire in milliamperes (mA). Recall that the range of $X$ is $4.9 \le x \le 5.1$ and $f(x) = 5$.




The cdf

$$F(x) = \begin{cases} 0, & x < 4.9 \\ 5(x - 4.9), & 4.9 \le x \le 5.1 \\ 1, & 5.1 \le x \end{cases}$$







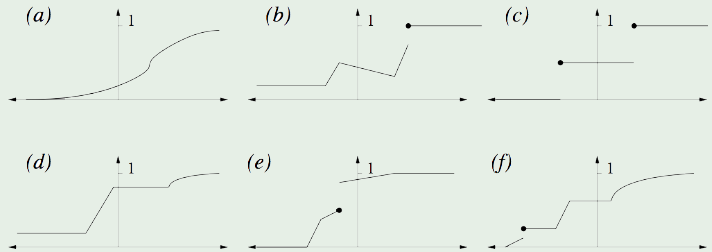

## Properties

**Proposition (Characteristic properties)**

1. $F(u)$ *is nondecreasing*
2. $F(-\infty) = 0$ *and* $F(\infty) = 1$
3. $F$ *is right continuous:* $\lim_{u \to a^+} F(u) = F(a)$

**Example (14)**

Which of the six functions shown are valid CDFs?








## Other properties

**Proposition ($F \implies$ Probability)**

1. $P(X < u) = F(u^-)$ *and* $P(X = u) = F(u) - F(u^-)$.
2. *Probability of random variable occurring within an interval*

$$P(a < X \le b) = F(b) - F(a)$$

**Proposition ($F \implies f$)**

1. *If $X$ has a discrete distribution then*

$$f(u) = F(u) - F(u^-)$$

2. *If $X$ has a continuous distribution, then $F$ is continuous at every $u$ and $F'(u) = f(u)$, i.e.*

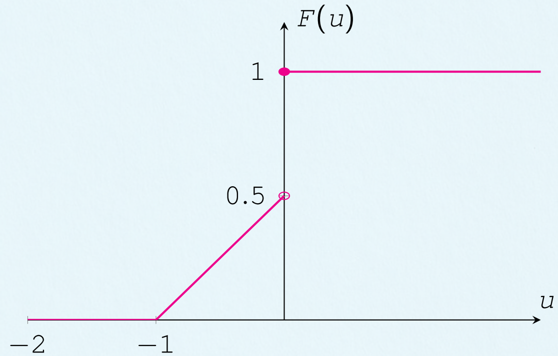
$$f(u) = F'(u)$$





## Example

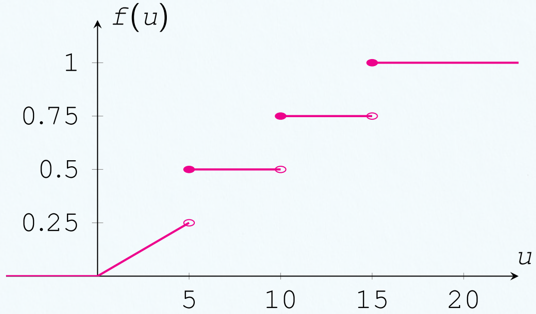
**15** Let $X$ have the CDF




(a) Determine all values of $u$ such that $P(X = u) > 0$.
(b) Find $P(X \le 0)$
(c) Find $P(X < 0)$.

## Example

**16** Let $X$ have the CDF




Find the numerical values of the following quantities

ⓐ $P(X \leq 1)$

ⓑ $P(X \leq 10)$

ⓒ $P(X \geq 10)$

ⓓ $P(X = 10)$

ⓔ $P(|X - 5| \leq 0.1)$.





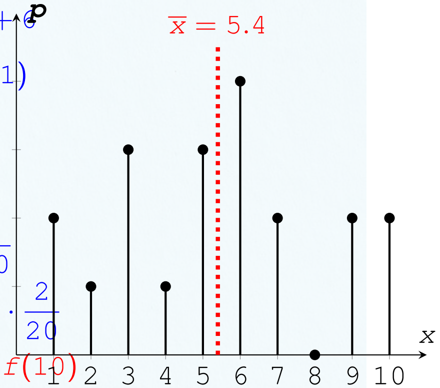
## Example 17 – Introduction to expectation

Random numbers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 9 | 4 | 3 | 3 | 10 | 5 | 7 | 6 | 6 |
| 10 | 6 | 5 | 6 | 5 | 7 | 1 | 3 | 9 | 1 |

The average value

$$\begin{aligned}\overline{x} &= \frac{1}{20}(2+9+4+3+3+10+5+7+6+6\\&\quad+10+6+5+6+5+7+1+3+9+1)\\&= \frac{1}{20}(1\cdot 2+2\cdot 1+3\cdot 3+4\cdot 1+5\cdot 3\\&\quad+6\cdot 4+7\cdot 2+8\cdot 0+9\cdot 2+10\cdot 2)\\&= 1\cdot\frac{2}{20}+2\cdot\frac{1}{20}+3\cdot\frac{3}{20}+4\cdot\frac{1}{20}+5\cdot\frac{3}{20}\\&\quad+6\cdot\frac{4}{20}+7\cdot\frac{2}{20}+8\cdot\frac{0}{20}+9\cdot\frac{2}{20}+10\cdot\frac{2}{20}\\&= 1\cdot f(1)+2\cdot f(2)+3\cdot f(3)+\cdots+10\cdot f(10)\\&= 5.4.\end{aligned}$$








## Example 18 – Introduction to expectation

Play a game

| $u$ | $-1$ | $1$ |
|---|---|---|
| $P(X=u)$ | $\frac{2}{3}$ | $\frac{1}{3}$ |

Then

$$\begin{aligned}\mathrm{E}(X) &= \frac{(-1)\cdot 2+(1)\cdot 1}{3}\\&= (-1)\cdot\frac{2}{3}+(1)\cdot\frac{1}{3} \quad (= -\frac{1}{3})\\&= (-1)f(-1)+(1)f(1).\end{aligned}$$





## Expectation

**Definition**

*The expected value (mean) of a random variable $X$ is*

$$E(X) = \sum_u uf(u) \quad \textit{(discrete)}$$

$$E(X) = \int_{-\infty}^{\infty} uf(u)du \quad \textit{(continuous)}.$$

Other names: Expected value, Mean, Mean value, Average value.

**Proposition (Properties)**

*Expectation is linear:*

$$\mathrm{E}(aX+b) = a\,\mathrm{E}(X)+b$$

*and*

$$\mathrm{E}(X+Y) = \mathrm{E}(X)+\mathrm{E}(Y)$$

## Example

19 A roulette wheel has the numbers 1 through 36, as well as 0 and 00. If you bet \$1 that an odd number comes up, you win or lose \$1 according to whether that event occurs. If $X$ denotes your net gain, find $\mathsf{E}(X)$.

20 A school class of 120 students is driven in 3 buses to a symphonic performance. There are 36



44 in the third bus. When the buses arrive, one of the 120 students is randomly chosen. Let $X$



## The second moment of random variables

The expected value of a random variable $X^2$ is

$$\mathsf{E}(X^2) = \sum_u u^2 f(u) \quad \text{(discrete)}$$

$$\mathsf{E}(X^2) = \int_{-\infty}^{\infty} u^2 f(u)du \quad \text{(continuous).}$$





## Example

22 A roulette wheel has the numbers 1 through 36, as well as 0 and 00. If you bet \$1 that an odd number comes up, you win or lose \$1 according to whether that event occurs. If $X$ denotes your net gain, find $\mathsf{E}(X^2)$.

21 Find $\mathsf{E}[X]$ when the density function of $X$ is

$$f(u) = \begin{cases} 2u, & 0 \le u \le 1 \\ 0, & \text{otherwise} \end{cases}$$

23 Let $X$ denote a random variable that takes on any of the values −1, 0, and 1 with respective probabilities

$$P(X=-1)=0.2, \quad P(X=0)=0.5, \quad P(X=1)=0.3$$

Compute $\mathsf{E}(X^2)$.

24 Let $X$ be the current measured in mA. The PDF is $f(x) = 0.05$ for $0 \le x \le 20$. What is the expected



Use the result that power in watts $P = 10^{-6}RI^2$,



## Expectation of function of a random variable

**Proposition**

*In general, for any function $g(u)$:*

$$\mathsf{E}\, g(X) = \sum_u g(u)f(u) \quad \text{(discrete)}$$

$$\mathsf{E}\, g(X) = \int_{-\infty}^{\infty} g(u)f(u)du \quad \text{(continuous).}$$

## Example 25 – Digital Channel

There is a chance that a bit transmitted through a digital transmission channel is received in error. $X$ is the number of bits received in error of the next 4 transmitted. The probabilities are

$$P(X=0)=0.6561,\quad P(X=2)=0.0486,\quad P(X=4)=0.0001,$$
$$P(X=1)=0.2916,\quad P(X=3)=0.0036,$$

What is the expected value of the cube of the number of bits in error?

**Solution**

*Put* $g(u)=u^3$.

$$\mathsf{E}(X^3)=\mathsf{E}(g(X))=\sum_{u=0}^{4} g(u)f(u)=\sum_{u=0}^{4} u^3 f(u)$$
$$=0^3(0.6561)+1^3(0.2916)+2^3(0.0486)+3^3(0.036)+4^3(0.0001)$$
$$=1.6588.$$





## Example 26 – Expected cost

The time, in hours, it takes to locate and repair an electrical breakdown in a certain factory is a random variable $X$ whose density function is given by

$$f(u)=\begin{cases}1, & \text{if } 0<u<1\\ 0, & \text{otherwise.}\end{cases}$$

If the cost involved in a breakdown of duration $X$ is $X^3$, what is the expected cost of such a breakdown?

**Solution**

*Put* $h(u)=u^3$. *Then*

$$\mathsf{E}(X^3)=\mathsf{E}(h(X))=\int_0^1 u^3 f(u)du=\int_0^1 u^3 du=\frac{u^4}{4}\Big|_0^1=\frac{1}{4}.$$





## Example

**27** Suppose $X$ is a random variable taking values in $\{-2,-1,0,1,2,3,4,5\}$, each with probability 1/8. Let $Y=X^2$. Find $\mathsf{E}[Y]$.

**28** Find $\mathsf{E}[e^X]$ when the density function of $X$ is

$$f(u)=\begin{cases}1, & 0\le u\le 1\\ 0, & \text{otherwise}\end{cases}$$

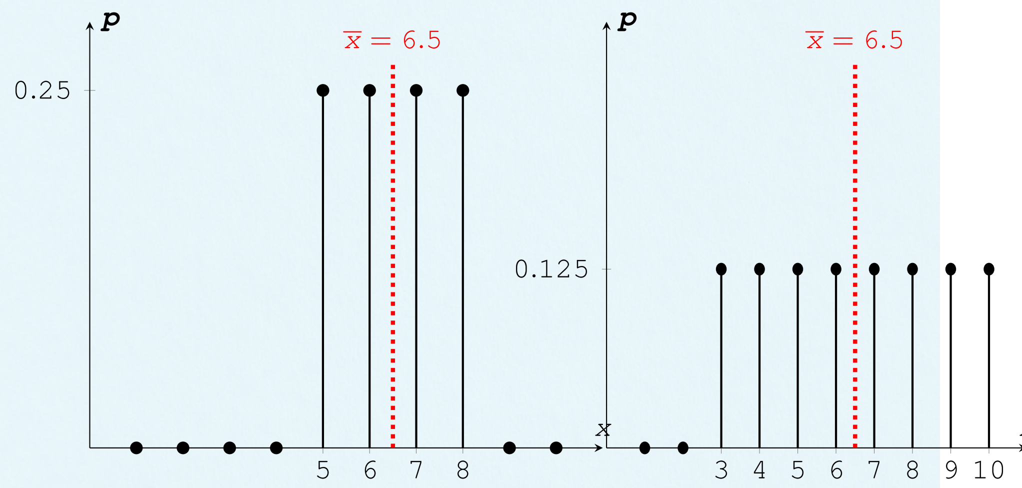
**29** Linda is a sales associate at a large auto dealership. At her commission rate of 25% of gross profit on each vehicle she sells, Linda expects to earn $350 for each car sold and $400 for each truck or SUV sold. Linda motivates herself by using probability estimates of her sales. She estimates her car sales in one day as



| Car sales | 0 | 1 | 2 | 3 |
|---|---|---|---|---|



## Example 30 – Pass or First-Class Honours?

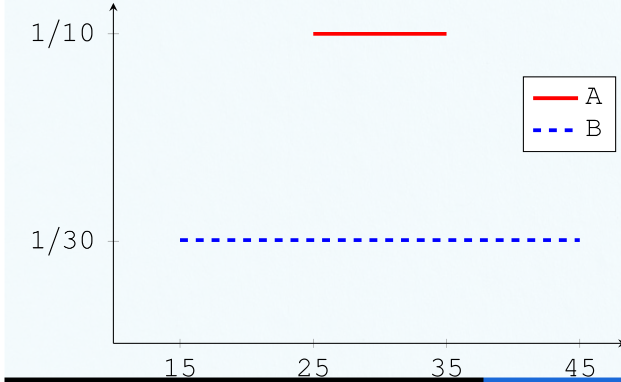



(a) Class 1 (b) Class 2

## Example 31 – Stock Price Changes

Consider the prices $A$ and $B$ of two stocks at a time one month in the future. Assume that

- $A$ has the uniform distribution on the interval $[25, 35]$
- $B$ has the uniform distribution on the interval $[15, 45]$.




The distributions: the mean = 30 but different.

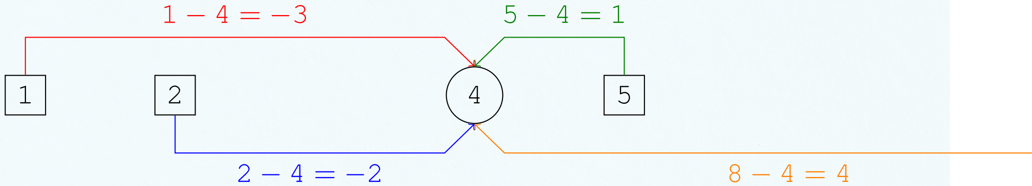
A will surely be worth at least 25 while $P(B < 25) = 1/3$.





## Deviations

Observations: $1, 2, 5, 8 \implies M = \dfrac{1+2+5+8}{4} = 4.$




Sum of squares: $(1-4)^2 + (2-4)^2 + (5-4)^2 + (8-4)^2.$

Average sum of squares: $\dfrac{(1-4)^2 + (2-4)^2 + (5-4)^2 + (8-4)^2}{4}.$ Sample variance: $\dfrac{(1-4)^2 + (2-4)^2 + (5-4)^2 + (8-4)^2}{4-1}.$

Observations: $x_1, x_2, \ldots, x_n \implies \overline{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}.$



$$s^2 = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}.$$



## Variance and Standard Deviation

- The variance of an r.v. $X$ is
$$\mathsf{V}(X) = \mathsf{E}(X-\mu)^2.$$
Variance measures dispersion around the mean.
  - $X$ is discrete
$$\mathsf{V}(X) = \sum_u (u-\mu)^2 f(u).$$
  - $X$ is continuous
$$\mathsf{V}(X) = \int_{-\infty}^{\infty} (u-\mu)^2 f(u) du.$$
- The standard deviation is
$$\mathsf{SD}(X) = \sqrt{\mathsf{Var}(X)}.$$





## Properties

1. Computational formula
$$\mathsf{V}(X) = \mathsf{E}(X^2) - (\mathsf{E}\,X)^2$$
2. Variance and standard deviation are not linear
$$\mathsf{V}(aX+b) = a^2\,\mathsf{V}(X) \quad \text{and} \quad \mathsf{SD}(aX+b) = a\,\mathsf{SD}(X)$$
3. If $X$ and $X$ are independent then
$$\mathsf{V}(X+Y) = \mathsf{V}(X) + \mathsf{V}(Y)$$

## Example 32 – Digital Channel

There is a chance that a bit transmitted through a digital transmission channel is received in error. $X$ is the number of bits received in error of the next 4 transmitted. The probabilities are

$$P(X=0)=0.6561,\quad P(X=2)=0.0486,\quad P(X=4)=0.0001,$$
$$P(X=1)=0.2916,\quad P(X=3)=0.0036$$

Calculate the mean and variance.

| $x$ | $f(x)$ | $xf(x)$ | $(x-0.4)^2$ | $(x-0.4)^2 f(x)$ | $x^2 f(x)$ |
|---|---|---|---|---|---|
| 0 | 0.6561 | 0.0000 | 0.160 | 0.1050 | 0.0000 |
| 1 | 0.2916 | 0.2916 | 0.360 | 0.1050 | 0.2916 |
| 2 | 0.0486 | 0.0972 | 2.560 | 0.1244 | 0.1944 |
| 3 | 0.0036 | 0.0108 | 6.760 | 0.0243 | 0.0324 |
| 4 | 0.0001 | 0.0004 | 12.960 | 0.0013 | 0.0016 |
| | *Total* | 0.4000 | | 0.3600 | 0.5200 |





## Example 33 – Electric Current

For the copper wire current measurement, the PDF is $f(u)=0.05$ for $0\le u\le 20$. Find the mean and variance.

**Solution**

$$\mathrm{E}(X)=\int_{-\infty}^{\infty} uf(u)du=\int_0^{20} u\times(0.05)du=\frac{0.05u^2}{2}\Big|_0^{20}=10$$

$$\mathrm{V}(X)=\int_{-\infty}^{\infty}(u-10)^2 f(u)du=\int_0^{20}(u-10)^2(0.05)du$$
$$=\frac{0.05(u-10)^3}{3}\Big|_0^{20}=\frac{100}{3}.$$

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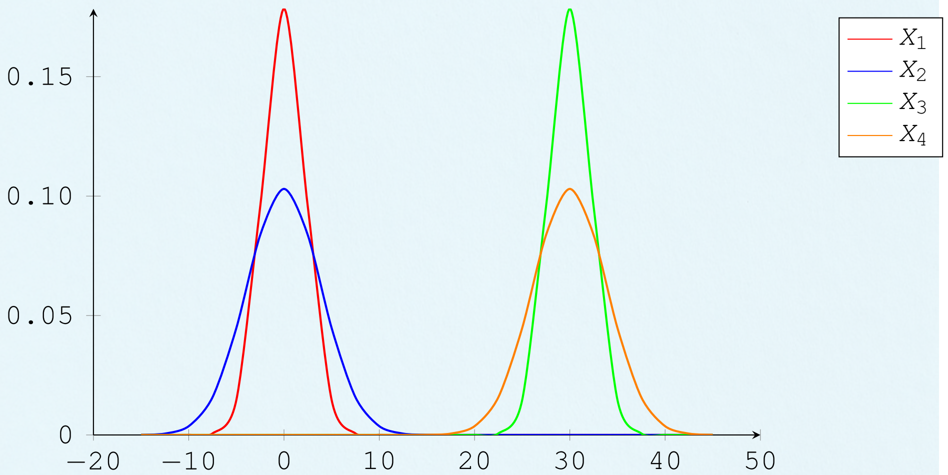
$$\mathrm{E}(X^2)=\int_{-\infty}^{\infty} u^2 f(u)du=\int_0^{20} u^2\times(0.05)du=\frac{0.05u^3}{3}\Big|_0^{20}=\frac{400}{3}$$

$$\mathrm{V}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2=\frac{400}{3}-10^2=\frac{100}{3}.$$





## Example 34 – Comparison




- $\mathrm{E}(X_1)=\mathrm{E}(X_2)<\mathrm{E}(X_3)=\mathrm{E}(X_4)$.
- $\mathrm{V}(X_1)=\mathrm{E}(X_3)<\mathrm{E}(X_2)=\mathrm{E}(X_4)$.





## Example

**35** Suppose that $X$ can take each of the five values $-2,0,1,3,4$ with equal probability. Determine the variance and standard deviation of $X$ and $Y=4X-7$.

**36** Suppose $X$ has the following pdf, where $c$ is a constant to be determined

$$f(u)=\begin{cases} c(1-u^2), & -1\le u\le 1\\ 0, & \text{otherwise}\end{cases}$$

Compute $\mathrm{E}(X),\mathrm{V}(X)$.

In many random experiments, more than one quantity is measured, meaning that there is more than one random variable.

**Example (Cell phone flash unit)**

A flash unit is chosen randomly from a production line; its recharge time X (seconds) and flash intensity Y (watt-seconds) are measured.

To make probability statements about several random variables, we need their joint probability distribution.





## Joint Probability Mass Function

The joint probability mass function of the discrete random variables $X$ and $Y$ denoted as $f_{XY}(u,v)$ satisfies

$$f_{XY}(u,v) = P(X = u, Y = v).$$

**Proposition (Characteristic properties)**

1. $f_{XY}(u,v) \geq 0$ for all $u, v$.
2. $\sum_u \sum_v f_{XY}(u,v) = 1$





## Example 37 - Signal Strength

A mobile web site is accessed from a smart phone; $X$ is the signal strength, in number of bars, and $Y$ is response time, to the nearest second.

| $y =$ Response time (nearest second) | $x =$ Number of Bars of Signal Strength | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | **Total** |
| 1 | 0.01 | 0.02 | 0.25 | 0.28 |
| 2 | 0.02 | 0.03 | 0.20 | 0.25 |
| 3 | 0.02 | 0.10 | 0.05 | 0.17 |
| 4 | 0.15 | 0.10 | 0.05 | 0.30 |
| **Total** | 0.20 | 0.25 | 0.55 | 1.00 |

Determine

a. $P(X < 3, Y \leq 2)$.
b. $P(X < 3 | Y \leq 2)$.
c. $P(Y \leq 2 | X < 3)$.





## Joint Probability Density Function

The joint probability density function for the continuous random variables $X$ and $Y$, denotes as $f_{XY}(u,v)$, satisfies the following properties

$$P((X,Y) \in A) = \iint_A f_{XY}(u,v)dudv.$$

**Proposition (Characteristic properties)**

1. $f_{XY}(u,v) \geq 0$.
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(u,v)dudv = 1$.

## Example 38 – Server Access Time

Let the random variable $X$ denote the time until a computer server connects to your machine (in milliseconds), and let $Y$ denote the time until the server authorizes you as a valid user (in milliseconds). $X$ and $Y$ measure the wait from a common starting point ($u < v$). The joint probability density function for $X$ and $Y$ is

$$f_{XY}(u, v) = k\,e^{-0.001u-0.002v},$$

for $0 < u < v < \infty$.

ⓐ Identify $k$.

ⓑ Calculate $P(X \le 1000, Y \le 2000)$.

**Solution**

$k = 6 \times 10^{-6}$, $P(X \le 1000, Y \le 2000) = 0.915$





## Marginal Probability Distributions (discrete)

Since $X$ is a random variable, it also has its own probability distribution, ignoring the value of $Y$, called its marginal probability distribution.

The marginal probability distribution for $X$

$$\begin{aligned} f_X(u) &= P(X = u) \\ &= \sum_v P(X = u, Y = v) \\ &= \sum_v f_{XY}(u, v) \end{aligned}$$

$$f_X(u) = \sum_v f_{XY}(u, v).$$

The marginal probability distribution for $Y$

$$f_Y(v) = \sum_u f_{XY}(u, v).$$





## Example 39 – Signal Strength

A mobile web site is accessed from a smart phone; $X$ is the signal strength, in number of bars, and $Y$ is response time, to the nearest second.

| $y$ = Response time (nearest second) | $x$ = Number of Bars of Signal Strength | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | **Marginal** $f_Y(y)$ |
| 1 | 0.01 | 0.02 | 0.25 | 0.28 |
| 2 | 0.02 | 0.03 | 0.20 | 0.25 |
| 3 | 0.02 | 0.10 | 0.05 | 0.17 |
| 4 | 0.15 | 0.10 | 0.05 | 0.30 |
| **Marginal** $f_X(x)$ | 0.20 | 0.25 | 0.55 | 1.00 |





## Marginal Probability Distributions (continuous)

If the joint probability density function of random variables $X$ and $Y$ is $f_{XY}(u, v)$, then

The marginal probability density functions of $X$:

$$f_X(u) = \int_{-\infty}^{\infty} f_{XY}(u, v)dv,$$

The marginal probability density functions of $Y$:

$$f_Y(v) = \int_{-\infty}^{\infty} f_{XY}(u, v)du.$$

# Example 40 – Signal Strength

A mobile web site is accessed from a smart phone; $X$ is the signal strength, in number of bars, and $Y$ is response time, to the nearest second.

| $y =$ Response time (nearest second) | $x =$ Number of Bars of Signal Strength | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | **Marginal** $f_Y(y)$ |
| 1 | 0.01 | 0.02 | 0.25 | 0.28 |
| 2 | 0.02 | 0.03 | 0.20 | 0.25 |
| 3 | 0.02 | 0.10 | 0.05 | 0.17 |
| 4 | 0.15 | 0.10 | 0.05 | 0.30 |
| **Marginal** $f_X(x)$ | 0.20 | 0.25 | 0.55 | 1.00 |

Compute the mean and the variance of $X$ and $Y$.