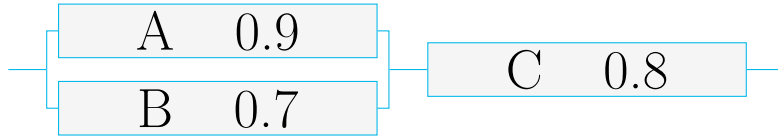1. An electrical system consists of three components as illustrated in the figure. The reliability (probability of working) of each component is also shown in the figure.

| A | 0.9 |
| B | 0.7 |

| C | 0.8 |

Find the probability that

(a) the entire system works.

(b) the component A works, given that the entire system works. Assume that the components work independently.

(c) What if there are more components involved?

2. A certain federal agency employs three consulting firms (A, B, and C) with probabilities 0.350, 0.40, and 0.25, respectively. From past experience it is known that the probabilities of cost overruns for the firms are 0.05, 0.25, and 0.15, respectively.

(a) What is the probability of cost overrun?

(b) Suppose a cost overrun is experienced by the agency. What is the probability that the consulting firm involved is company C?

3. An exciting computer game is released. Seventy percent of players complete all the levels. Twenty percent of them, who complete all the levels, will then buy an advanced version of the game.

(a) Among 15 users, what is the expected number of people who will buy the advanced version?

(b) What is the probability that at least two people, among 15 users, will buy it?

4. Suppose that a large conference room at a certain company can be reserved for no more than 4 hours. It can be assumed that the length X of a conference has a uniform distribution on the interval $[0, 4]$.

(a) What is the probability that any given conference lasts at least 3 hours?

(b) Describe the center and the variation of the distribution of X

5. A certain area of the eastern United States is, on average, hit by 5 hurricanes a year.

(a) Find the probability that in a given year that area will be hit by fewer than 4 hurricanes.

(b) What is the variance of the number hurricanes in a given year that area will be hit?

(c) Find the probability that the area will be hit by fewer than 4 hurricanes in 2 years.

6. Assume that $X \sim N(1, 3)$, $Y \sim N(2, 4)$, and they are independent. What are the distributions of $X + 2Y$ and $X - 2Y$?

7. The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce.

(a) What is the probability that a shoe weighs more than 13 ounces?

(b) What must the standard deviation of weight be in order for the company to state that 99% of its shoes weighs less than 13 ounces?

(c) Select 25 shoes randomly. What is the probability that the average weight of these shoes is less than 13 ounces?

8. Lifetime (in years) of a certain hardware is a continuous random variable with density $f(u) = K - x/50$ for $0 < x < 10$

(a) Find K.

(b) What is the probability of a failure within the first 6 years?

(c) What is the expectation of the lifetime?

9. Let X be a random variable with probability distribution as follows:

u	0	1	2	3
f(u)	1/3	1/2	0	c

Determine c. Then find the variance of X and the expected value of $Y = (X - 1)^2$.

10. Let X and Y be independent uniform random variables defined on $(0, 1)$.

(a) Compute $E(X + Y)$ and $V(3X)$.

(b) Find the conditional probability $P(0.3 < X < 0.7|Y > 0.5)$.

11. Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability mass function over the points with $x = 1, 2$ and $y = 0, 1$. Determine the following

(a) $E(X)$ and $V(X)$

(b) $P(Y = 1 | X = 2)$

(c) $E(Y | X = 2)$.

(d) $E(XY)$

(e) $V(-Y - 2)$

12. Let X be a random variable with pdf

$$f(x) = \begin{cases} -cx, & -2 < x < 0 \\ cx, & 0 \leq x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Determine constant c, the mean of X, and the conditional probability $P(X > 1/2 \mid X > 0)$.

13. Listed below are the ages (years) of randomly selected race car drivers (based on data reported in USA Today): 32 32 33 33 41 29.

   (a) Construct a 95% confidence interval estimate of the mean age of all race car drivers.

   (b) Suppose that we wanted the error in estimating the mean age from the two-sided confidence interval to be 1 year at 95% confidence. What sample size should be used?

14. The yield of a chemical process is being studied. From previous experience, yield is known to be normally distributed and $\sigma = 3$. The past five days of plant operation have resulted in the following percent yields: 92.6, 90.3, 88.0, 90.5, and 88.3.

(a) Find a 99% two-sided confidence interval on the true mean yield.

(b) Suppose that we wanted the error in estimating the mean yield from the two-sided confidence interval to be 1 unit at 99% confidence. What sample size should be used?