

## Appendix A

### Statistical Tables and Charts

**TABLE • I** Summary of Common Probability Distributions

| Name              | Probability Distribution   | Mean  | Variance   | Section in Book |
|-------------------|--|---|--|-----------------|
| <b>Discrete</b>   |  |   |  |                 |
| Uniform           | $\frac{1}{n}, a \leq b$  | $\frac{(b+a)}{2}$                               | $\frac{(b-a+1)^2-1}{12}$   | 3-5             |
| Binomial          | $\binom{n}{x} p^x (1-p)^{n-x}$<br>$x = 0, 1, \dots, n, 0 \leq p \leq 1$  | $np$  | $np(1-p)$  | 3-6             |
| Geometric         | $(1-p)^{x-1} p$<br>$x = 1, 2, \dots, 0 \leq p \leq 1$  | $1/p$   | $(1-p)/p^2$  | 3-7             |
| Negative binomial | $\binom{x-1}{r-1} (1-p)^{x-r} p^r$<br>$x = r, r+1, r+2, \dots, 0 \leq p \leq 1$  | $r/p$   | $r(1-p)/p^2$   | 3-7             |
| Hypergeometric    | $\frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$<br>$x = \max(0, n-N+K), 1, \dots$<br>$\min(K, n), K \leq N, n \leq N$           | $np$<br>where $p = \frac{K}{N}$                 | $np(1-p) \left( \frac{N-n}{N-1} \right)$   | 3-8             |
| Poisson           | $\frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots, 0 < \lambda$   | $\lambda$                                       | $\lambda$  | 3-9             |
| <b>Continuous</b> |  |   |  |                 |
| Uniform           | $\frac{1}{b-a}, a \leq x \leq b$   | $\frac{(b+a)}{2}$                               | $\frac{(b-a)^2}{12}$   | 4-5             |
| Normal            | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$<br>$-\infty < x < \infty, -\infty < \mu < \infty, 0 < \sigma$ | $\mu$   | $\sigma^2$   | 4-6             |
| Exponential       | $\lambda e^{-\lambda x}, 0 \leq x, 0 < \lambda$  | $1/\lambda$                                     | $1/\lambda^2$  | 4-8             |
| Erlang            | $\frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}, 0 < x, r = 1, 2, \dots$  | $r/\lambda$                                     | $r/\lambda^2$  | 4-9.1           |
| Gamma             | $\frac{\lambda x^{r-1} e^{-\lambda x}}{\Gamma(r)}, 0 < x, 0 < r, 0 < \lambda$  | $r/\lambda$                                     | $r/\lambda^2$  | 4-9.2           |
| Weibull           | $\frac{\beta}{\delta} \left( \frac{x}{\delta} \right)^{\beta-1} e^{-(x/\delta)^\beta}$<br>$0 < x, 0 < \beta, 0 < \delta$             | $\delta \Gamma\left(1 + \frac{1}{\beta}\right)$ | $\delta^2 \Gamma\left(1 + \frac{2}{\beta}\right) - \delta^2 \left[ \Gamma\left(1 + \frac{1}{\beta}\right) \right]^2$ | 4-10            |
| Lognormal         | $\frac{1}{x\omega\sqrt{2\pi}} \exp\left(\frac{-[\ln(x)-\theta]^2}{2\omega^2}\right)$   | $e^{\theta+\omega^2/2}$                         | $e^{2\theta+\omega^2}(e^{\omega^2}-1)$   | 4-11            |
| Beta              | $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$<br>$0 \leq x \leq 1, 0 < \alpha, 0 < \beta$  | $\frac{\alpha}{\alpha+\beta}$                   | $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$   | 4-12            |

**TABLE • II** Cumulative Binomial Probabilities  $P(X \leq x)$ 

| <i>n</i> | <i>x</i> | <i>P</i> |        |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.1      | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    | 0.95   | 0.99   |
| 1        | 0        | 0.9000   | 0.8000 | 0.7000 | 0.6000 | 0.5000 | 0.4000 | 0.3000 | 0.2000 | 0.1000 | 0.0500 | 0.0100 |
| 2        | 0        | 0.8100   | 0.6400 | 0.4900 | 0.3600 | 0.2500 | 0.1600 | 0.0900 | 0.0400 | 0.0100 | 0.0025 | 0.0001 |
|          | 1        | 0.9900   | 0.9600 | 0.9100 | 0.8400 | 0.7500 | 0.6400 | 0.5100 | 0.3600 | 0.1900 | 0.0975 | 0.0199 |
| 3        | 0        | 0.7290   | 0.5120 | 0.3430 | 0.2160 | 0.1250 | 0.0640 | 0.0270 | 0.0080 | 0.0010 | 0.0001 | 0.0000 |
|          | 1        | 0.9720   | 0.8960 | 0.7840 | 0.6480 | 0.5000 | 0.3520 | 0.2160 | 0.1040 | 0.0280 | 0.0073 | 0.0003 |
|          | 2        | 0.9990   | 0.9920 | 0.9730 | 0.9360 | 0.8750 | 0.7840 | 0.6570 | 0.4880 | 0.2710 | 0.1426 | 0.0297 |
| 4        | 0        | 0.6561   | 0.4096 | 0.2401 | 0.1296 | 0.0625 | 0.0256 | 0.0081 | 0.0016 | 0.0001 | 0.0000 | 0.0000 |
|          | 1        | 0.9477   | 0.8192 | 0.6517 | 0.4752 | 0.3125 | 0.1792 | 0.0837 | 0.0272 | 0.0037 | 0.0005 | 0.0000 |
|          | 2        | 0.9963   | 0.9728 | 0.9163 | 0.8208 | 0.6875 | 0.5248 | 0.3483 | 0.1808 | 0.0523 | 0.0140 | 0.0006 |
|          | 3        | 0.9999   | 0.9984 | 0.9919 | 0.9744 | 0.9375 | 0.8704 | 0.7599 | 0.5904 | 0.3439 | 0.1855 | 0.0394 |
| 5        | 0        | 0.5905   | 0.3277 | 0.1681 | 0.0778 | 0.0313 | 0.0102 | 0.0024 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.9185   | 0.7373 | 0.5282 | 0.3370 | 0.1875 | 0.0870 | 0.0308 | 0.0067 | 0.0005 | 0.0000 | 0.0000 |
|          | 2        | 0.9914   | 0.9421 | 0.8369 | 0.6826 | 0.5000 | 0.3174 | 0.1631 | 0.0579 | 0.0086 | 0.0012 | 0.0000 |
|          | 3        | 0.9995   | 0.9933 | 0.9692 | 0.9130 | 0.8125 | 0.6630 | 0.4718 | 0.2627 | 0.0815 | 0.0226 | 0.0010 |
|          | 4        | 1.0000   | 0.9997 | 0.9976 | 0.9898 | 0.6988 | 0.9222 | 0.8319 | 0.6723 | 0.4095 | 0.2262 | 0.0490 |
| 6        | 0        | 0.5314   | 0.2621 | 0.1176 | 0.0467 | 0.0156 | 0.0041 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.8857   | 0.6554 | 0.4202 | 0.2333 | 0.1094 | 0.0410 | 0.0109 | 0.0016 | 0.0001 | 0.0000 | 0.0000 |
|          | 2        | 0.9842   | 0.9011 | 0.7443 | 0.5443 | 0.3438 | 0.1792 | 0.0705 | 0.0170 | 0.0013 | 0.0001 | 0.0000 |
|          | 3        | 0.9987   | 0.9830 | 0.9295 | 0.8208 | 0.6563 | 0.4557 | 0.2557 | 0.0989 | 0.0159 | 0.0022 | 0.0000 |
|          | 4        | 0.9999   | 0.9984 | 0.9891 | 0.9590 | 0.9806 | 0.7667 | 0.5798 | 0.3446 | 0.1143 | 0.0328 | 0.0015 |
|          | 5        | 1.0000   | 0.9999 | 0.9993 | 0.9959 | 0.9844 | 0.9533 | 0.8824 | 0.7379 | 0.4686 | 0.2649 | 0.0585 |
| 7        | 0        | 0.4783   | 0.2097 | 0.0824 | 0.0280 | 0.0078 | 0.0016 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.8503   | 0.5767 | 0.3294 | 0.1586 | 0.0625 | 0.0188 | 0.0038 | 0.0004 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.9743   | 0.8520 | 0.6471 | 0.4199 | 0.2266 | 0.0963 | 0.0288 | 0.0047 | 0.0002 | 0.0000 | 0.0000 |
|          | 3        | 0.9973   | 0.9667 | 0.8740 | 0.7102 | 0.5000 | 0.2898 | 0.1260 | 0.0333 | 0.0027 | 0.0002 | 0.0000 |
|          | 4        | 0.9998   | 0.9953 | 0.9712 | 0.9037 | 0.7734 | 0.5801 | 0.3529 | 0.1480 | 0.0257 | 0.0038 | 0.0000 |
|          | 5        | 1.0000   | 0.9996 | 0.9962 | 0.9812 | 0.9375 | 0.8414 | 0.6706 | 0.4233 | 0.1497 | 0.0444 | 0.0020 |
|          | 6        | 1.0000   | 1.0000 | 0.9998 | 0.9984 | 0.9922 | 0.9720 | 0.9176 | 0.7903 | 0.5217 | 0.3017 | 0.0679 |
| 8        | 0        | 0.4305   | 0.1678 | 0.0576 | 0.0168 | 0.0039 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.8131   | 0.5033 | 0.2553 | 0.1064 | 0.0352 | 0.0085 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.9619   | 0.7969 | 0.5518 | 0.3154 | 0.1445 | 0.0498 | 0.0113 | 0.0012 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9950   | 0.9437 | 0.8059 | 0.5941 | 0.3633 | 0.1737 | 0.0580 | 0.0104 | 0.0004 | 0.0000 | 0.0000 |
|          | 4        | 0.9996   | 0.9896 | 0.9420 | 0.8263 | 0.6367 | 0.4059 | 0.1941 | 0.0563 | 0.0050 | 0.0004 | 0.0000 |
|          | 5        | 1.0000   | 0.9988 | 0.9887 | 0.9502 | 0.8555 | 0.6846 | 0.4482 | 0.2031 | 0.0381 | 0.0058 | 0.0001 |
|          | 6        | 1.0000   | 0.9999 | 0.9987 | 0.9915 | 0.9648 | 0.8936 | 0.7447 | 0.4967 | 0.1869 | 0.0572 | 0.0027 |
|          | 7        | 1.0000   | 1.0000 | 0.9999 | 0.9993 | 0.9961 | 0.9832 | 0.9424 | 0.8322 | 0.5695 | 0.3366 | 0.0773 |
| 9        | 0        | 0.3874   | 0.1342 | 0.0404 | 0.0101 | 0.0020 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.7748   | 0.4362 | 0.1960 | 0.0705 | 0.0195 | 0.0038 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.9470   | 0.7382 | 0.4628 | 0.2318 | 0.0889 | 0.0250 | 0.0043 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9917   | 0.9144 | 0.7297 | 0.4826 | 0.2539 | 0.0994 | 0.0253 | 0.0031 | 0.0001 | 0.0000 | 0.0000 |
|          | 4        | 0.9991   | 0.9804 | 0.9012 | 0.7334 | 0.5000 | 0.2666 | 0.0988 | 0.0196 | 0.0009 | 0.0000 | 0.0000 |
|          | 5        | 0.9999   | 0.9969 | 0.9747 | 0.9006 | 0.7461 | 0.5174 | 0.2703 | 0.0856 | 0.0083 | 0.0006 | 0.0000 |
|          | 6        | 1.0000   | 0.9997 | 0.9957 | 0.9750 | 0.9102 | 0.7682 | 0.5372 | 0.2618 | 0.0530 | 0.0084 | 0.0001 |
|          | 7        | 1.0000   | 1.0000 | 0.9996 | 0.9962 | 0.9805 | 0.9295 | 0.8040 | 0.5638 | 0.2252 | 0.0712 | 0.0034 |
|          | 8        | 1.0000   | 1.0000 | 1.0000 | 0.9997 | 0.9980 | 0.9899 | 0.9596 | 0.8658 | 0.6126 | 0.3698 | 0.0865 |

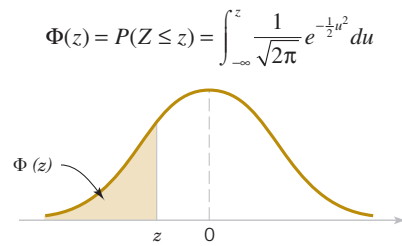
(Continued)

**TABLE • II** Cumulative Binomial Probabilities  $P(X \leq x)$  (*Continued*)

| <i>n</i> | <i>x</i> | <i>P</i> |        |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.1      | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    | 0.95   | 0.99   |
| 10       | 0        | 0.3487   | 0.1074 | 0.0282 | 0.0060 | 0.0010 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.7361   | 0.3758 | 0.1493 | 0.0464 | 0.0107 | 0.0017 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.9298   | 0.6778 | 0.3828 | 0.1673 | 0.0547 | 0.0123 | 0.0016 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9872   | 0.8791 | 0.6496 | 0.3823 | 0.1719 | 0.0548 | 0.0106 | 0.0009 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9984   | 0.9672 | 0.8497 | 0.6331 | 0.3770 | 0.1662 | 0.0473 | 0.0064 | 0.0001 | 0.0000 | 0.0000 |
|          | 5        | 0.9999   | 0.9936 | 0.9527 | 0.8338 | 0.6230 | 0.3669 | 0.1503 | 0.0328 | 0.0016 | 0.0001 | 0.0000 |
|          | 6        | 1.0000   | 0.9991 | 0.9894 | 0.9452 | 0.8281 | 0.6177 | 0.3504 | 0.1209 | 0.0128 | 0.0010 | 0.0000 |
|          | 7        | 1.0000   | 0.9999 | 0.9984 | 0.9877 | 0.9453 | 0.8327 | 0.6172 | 0.3222 | 0.0702 | 0.0115 | 0.0001 |
|          | 8        | 1.0000   | 1.0000 | 0.9999 | 0.9983 | 0.9893 | 0.9536 | 0.8507 | 0.6242 | 0.2639 | 0.0861 | 0.0043 |
|          | 9        | 1.0000   | 1.0000 | 1.0000 | 0.9999 | 0.9990 | 0.9940 | 0.9718 | 0.8926 | 0.6513 | 0.4013 | 0.0956 |
| 11       | 0        | 0.3138   | 0.0859 | 0.0198 | 0.0036 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.6974   | 0.3221 | 0.1130 | 0.0302 | 0.0059 | 0.0007 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.9104   | 0.6174 | 0.3127 | 0.1189 | 0.0327 | 0.0059 | 0.0006 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9815   | 0.8389 | 0.5696 | 0.2963 | 0.1133 | 0.0293 | 0.0043 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9972   | 0.9496 | 0.7897 | 0.5328 | 0.2744 | 0.0994 | 0.0216 | 0.0020 | 0.0000 | 0.0000 | 0.0000 |
|          | 5        | 0.9997   | 0.9883 | 0.9218 | 0.7535 | 0.5000 | 0.2465 | 0.0782 | 0.0117 | 0.0003 | 0.0000 | 0.0000 |
|          | 6        | 1.0000   | 0.9980 | 0.9784 | 0.9006 | 0.7256 | 0.4672 | 0.2103 | 0.0504 | 0.0028 | 0.0001 | 0.0000 |
|          | 7        | 1.0000   | 0.9998 | 0.9957 | 0.9707 | 0.8867 | 0.7037 | 0.4304 | 0.1611 | 0.0185 | 0.0016 | 0.0000 |
|          | 8        | 1.0000   | 1.0000 | 0.9994 | 0.9941 | 0.9673 | 0.8811 | 0.6873 | 0.3826 | 0.0896 | 0.0152 | 0.0002 |
|          | 9        | 1.0000   | 1.0000 | 1.0000 | 0.9993 | 0.9941 | 0.9698 | 0.8870 | 0.6779 | 0.3026 | 0.1019 | 0.0052 |
| 12       | 0        | 0.2824   | 0.0687 | 0.0138 | 0.0022 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.6590   | 0.2749 | 0.0850 | 0.0196 | 0.0032 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.8891   | 0.5583 | 0.2528 | 0.0834 | 0.0193 | 0.0028 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9744   | 0.7946 | 0.4925 | 0.2253 | 0.0730 | 0.0153 | 0.0017 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9957   | 0.9274 | 0.7237 | 0.4382 | 0.1938 | 0.0573 | 0.0095 | 0.0006 | 0.0000 | 0.0000 | 0.0000 |
|          | 5        | 0.9995   | 0.9806 | 0.8822 | 0.6652 | 0.3872 | 0.1582 | 0.0386 | 0.0039 | 0.0001 | 0.0000 | 0.0000 |
|          | 6        | 0.9999   | 0.9961 | 0.9614 | 0.8418 | 0.6128 | 0.3348 | 0.1178 | 0.0194 | 0.0005 | 0.0000 | 0.0000 |
|          | 7        | 1.0000   | 0.9994 | 0.9905 | 0.9427 | 0.8062 | 0.5618 | 0.2763 | 0.0726 | 0.0043 | 0.0002 | 0.0000 |
|          | 8        | 1.0000   | 0.9999 | 0.9983 | 0.9847 | 0.9270 | 0.7747 | 0.5075 | 0.2054 | 0.0256 | 0.0022 | 0.0000 |
|          | 9        | 1.0000   | 1.0000 | 0.9998 | 0.9972 | 0.9807 | 0.9166 | 0.7472 | 0.4417 | 0.1109 | 0.0196 | 0.0002 |
| 13       | 0        | 0.2542   | 0.0550 | 0.0097 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.6213   | 0.2336 | 0.0637 | 0.0126 | 0.0017 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.8661   | 0.5017 | 0.2025 | 0.0579 | 0.0112 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9658   | 0.7473 | 0.4206 | 0.1686 | 0.0461 | 0.0078 | 0.0007 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9935   | 0.9009 | 0.6543 | 0.3530 | 0.1334 | 0.0321 | 0.0040 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
|          | 5        | 0.9991   | 0.9700 | 0.8346 | 0.5744 | 0.2905 | 0.0977 | 0.0182 | 0.0012 | 0.0000 | 0.0000 | 0.0000 |
|          | 6        | 0.9999   | 0.9930 | 0.9376 | 0.7712 | 0.5000 | 0.2288 | 0.0624 | 0.0070 | 0.0001 | 0.0000 | 0.0000 |
|          | 7        | 1.0000   | 0.9988 | 0.9818 | 0.9023 | 0.7095 | 0.4256 | 0.1654 | 0.0300 | 0.0009 | 0.0000 | 0.0000 |
|          | 8        | 1.0000   | 0.9988 | 0.9960 | 0.9679 | 0.8666 | 0.6470 | 0.3457 | 0.0991 | 0.0065 | 0.0003 | 0.0000 |
|          | 9        | 1.0000   | 1.0000 | 0.9993 | 0.9922 | 0.9539 | 0.8314 | 0.5794 | 0.2527 | 0.0342 | 0.0031 | 0.0000 |
| 14       | 10       | 1.0000   | 1.0000 | 0.9999 | 0.9987 | 0.9888 | 0.9421 | 0.7975 | 0.4983 | 0.1339 | 0.0245 | 0.0003 |
|          | 11       | 1.0000   | 1.0000 | 1.0000 | 0.9999 | 0.9983 | 0.9874 | 0.9363 | 0.7664 | 0.3787 | 0.1354 | 0.0072 |
|          | 12       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9987 | 0.9903 | 0.9450 | 0.7458 | 0.4867 | 0.1225 |
|          | 13       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9987 | 0.9903 | 0.9450 | 0.7458 | 0.1225 |
| 14       | 0        | 0.2288   | 0.0440 | 0.0068 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

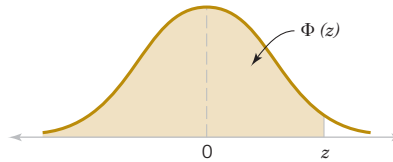
**TABLE • II** Cumulative Binomial Probabilities  $P(X \leq x)$  (Continued)

| <i>n</i> | <i>x</i> | <i>P</i> |        |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.1      | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    | 0.95   | 0.99   |
| 15       | 1        | 0.5846   | 0.1979 | 0.0475 | 0.0081 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.8416   | 0.4481 | 0.1608 | 0.0398 | 0.0065 | 0.0006 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9559   | 0.6982 | 0.3552 | 0.1243 | 0.0287 | 0.0039 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9908   | 0.8702 | 0.5842 | 0.2793 | 0.0898 | 0.0175 | 0.0017 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 5        | 0.9985   | 0.9561 | 0.7805 | 0.4859 | 0.2120 | 0.0583 | 0.0083 | 0.0004 | 0.0000 | 0.0000 | 0.0000 |
|          | 6        | 0.9998   | 0.9884 | 0.9067 | 0.6925 | 0.3953 | 0.1501 | 0.0315 | 0.0024 | 0.0000 | 0.0000 | 0.0000 |
|          | 7        | 1.0000   | 0.9976 | 0.9685 | 0.8499 | 0.6047 | 0.3075 | 0.0933 | 0.0116 | 0.0002 | 0.0000 | 0.0000 |
|          | 8        | 1.0000   | 0.9996 | 0.9917 | 0.9417 | 0.7880 | 0.5141 | 0.2195 | 0.0439 | 0.0015 | 0.0000 | 0.0000 |
|          | 9        | 1.0000   | 1.0000 | 0.9983 | 0.9825 | 0.9102 | 0.7207 | 0.4158 | 0.1298 | 0.0092 | 0.0004 | 0.0000 |
|          | 10       | 1.0000   | 1.0000 | 0.9998 | 0.9961 | 0.9713 | 0.8757 | 0.6448 | 0.3018 | 0.0441 | 0.0042 | 0.0000 |
|          | 11       | 1.0000   | 1.0000 | 1.0000 | 0.9994 | 0.9935 | 0.9602 | 0.8392 | 0.5519 | 0.1584 | 0.0301 | 0.0003 |
|          | 12       | 1.0000   | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9919 | 0.9525 | 0.8021 | 0.4154 | 0.1530 | 0.0084 |
|          | 13       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9932 | 0.9560 | 0.7712 | 0.5123 | 0.1313 |
| 15       | 0        | 0.2059   | 0.0352 | 0.0047 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.5490   | 0.1671 | 0.0353 | 0.0052 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.8159   | 0.3980 | 0.1268 | 0.0271 | 0.0037 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.9444   | 0.6482 | 0.2969 | 0.0905 | 0.0176 | 0.0019 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9873   | 0.8358 | 0.5155 | 0.2173 | 0.0592 | 0.0093 | 0.0007 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 5        | 0.9978   | 0.9389 | 0.7216 | 0.4032 | 0.1509 | 0.0338 | 0.0037 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
|          | 6        | 0.9997   | 0.9819 | 0.8689 | 0.6098 | 0.3036 | 0.0950 | 0.0152 | 0.0008 | 0.0000 | 0.0000 | 0.0000 |
|          | 7        | 1.0000   | 0.9958 | 0.9500 | 0.7869 | 0.5000 | 0.2131 | 0.0500 | 0.0042 | 0.0000 | 0.0000 | 0.0000 |
|          | 8        | 1.0000   | 0.9992 | 0.9848 | 0.9050 | 0.6964 | 0.3902 | 0.1311 | 0.0181 | 0.0003 | 0.0000 | 0.0000 |
|          | 9        | 1.0000   | 0.9999 | 0.9963 | 0.9662 | 0.8491 | 0.5968 | 0.2784 | 0.0611 | 0.0022 | 0.0001 | 0.0000 |
|          | 10       | 1.0000   | 1.0000 | 0.9993 | 0.9907 | 0.9408 | 0.7827 | 0.4845 | 0.1642 | 0.0127 | 0.0006 | 0.0000 |
|          | 11       | 1.0000   | 1.0000 | 0.9999 | 0.9981 | 0.9824 | 0.9095 | 0.7031 | 0.3518 | 0.0556 | 0.0055 | 0.0000 |
|          | 12       | 1.0000   | 1.0000 | 1.0000 | 0.9997 | 0.9963 | 0.9729 | 0.8732 | 0.6020 | 0.1841 | 0.0362 | 0.0004 |
|          | 13       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9948 | 0.9647 | 0.8329 | 0.4510 | 0.1710 | 0.0096 |
| 20       | 14       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9953 | 0.9648 | 0.7941 | 0.5367 | 0.1399 |
|          | 0        | 0.1216   | 0.0115 | 0.0008 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 1        | 0.3917   | 0.0692 | 0.0076 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 2        | 0.6769   | 0.2061 | 0.0355 | 0.0036 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 3        | 0.8670   | 0.4114 | 0.1071 | 0.0160 | 0.0013 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 4        | 0.9568   | 0.6296 | 0.2375 | 0.0510 | 0.0059 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 5        | 0.9887   | 0.8042 | 0.4164 | 0.1256 | 0.0207 | 0.0016 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 6        | 0.9976   | 0.9133 | 0.6080 | 0.2500 | 0.0577 | 0.0065 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 7        | 0.9996   | 0.9679 | 0.7723 | 0.4159 | 0.1316 | 0.0210 | 0.0013 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|          | 8        | 0.9999   | 0.9900 | 0.8867 | 0.5956 | 0.2517 | 0.0565 | 0.0051 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
|          | 9        | 1.0000   | 0.9974 | 0.9520 | 0.7553 | 0.4119 | 0.1275 | 0.0171 | 0.0006 | 0.0000 | 0.0000 | 0.0000 |
|          | 10       | 1.0000   | 0.9994 | 0.9829 | 0.8725 | 0.5881 | 0.2447 | 0.0480 | 0.0026 | 0.0000 | 0.0000 | 0.0000 |
|          | 11       | 1.0000   | 0.9999 | 0.9949 | 0.9435 | 0.7483 | 0.4044 | 0.1133 | 0.0100 | 0.0001 | 0.0000 | 0.0000 |
|          | 12       | 1.0000   | 1.0000 | 0.9987 | 0.9790 | 0.8684 | 0.5841 | 0.2277 | 0.0321 | 0.0004 | 0.0000 | 0.0000 |
|          | 13       | 1.0000   | 1.0000 | 0.9997 | 0.9935 | 0.9423 | 0.7500 | 0.3920 | 0.0867 | 0.0024 | 0.0000 | 0.0000 |
|          | 14       | 1.0000   | 1.0000 | 1.0000 | 0.9984 | 0.9793 | 0.8744 | 0.5836 | 0.1958 | 0.0113 | 0.0003 | 0.0000 |
|          | 15       | 1.0000   | 1.0000 | 1.0000 | 0.9997 | 0.9941 | 0.9490 | 0.7625 | 0.3704 | 0.0432 | 0.0026 | 0.0000 |
|          | 16       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 0.9987 | 0.9840 | 0.8929 | 0.5886 | 0.1330 | 0.0159 | 0.0000 |
|          | 17       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9964 | 0.9645 | 0.7939 | 0.3231 | 0.0755 | 0.0010 |
|          | 18       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9924 | 0.9308 | 0.6083 | 0.2642 | 0.0169 |
|          | 19       | 1.0000   | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9992 | 0.9885 | 0.8784 | 0.6415 | 0.1821 |

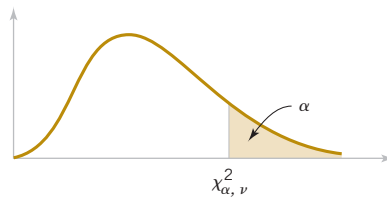
**TABLE • III** Cumulative Standard Normal Distribution

| <i>z</i> | −0.09    | −0.08    | −0.07    | −0.06    | −0.05    | −0.04    | −0.03    | −0.03    | −0.01    | −0.00    |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| −3.9     | 0.000033 | 0.000034 | 0.000036 | 0.000037 | 0.000039 | 0.000041 | 0.000042 | 0.000044 | 0.000046 | 0.000048 |
| −3.8     | 0.000050 | 0.000052 | 0.000054 | 0.000057 | 0.000059 | 0.000062 | 0.000064 | 0.000067 | 0.000069 | 0.000072 |
| −3.7     | 0.000075 | 0.000078 | 0.000082 | 0.000085 | 0.000088 | 0.000092 | 0.000096 | 0.000100 | 0.000104 | 0.000108 |
| −3.6     | 0.000112 | 0.000117 | 0.000121 | 0.000126 | 0.000131 | 0.000136 | 0.000142 | 0.000147 | 0.000153 | 0.000159 |
| −3.5     | 0.000165 | 0.000172 | 0.000179 | 0.000185 | 0.000193 | 0.000200 | 0.000208 | 0.000216 | 0.000224 | 0.000233 |
| −3.4     | 0.000242 | 0.000251 | 0.000260 | 0.000270 | 0.000280 | 0.000291 | 0.000302 | 0.000313 | 0.000325 | 0.000337 |
| −3.3     | 0.000350 | 0.000362 | 0.000376 | 0.000390 | 0.000404 | 0.000419 | 0.000434 | 0.000450 | 0.000467 | 0.000483 |
| −3.2     | 0.000501 | 0.000519 | 0.000538 | 0.000557 | 0.000577 | 0.000598 | 0.000619 | 0.000641 | 0.000664 | 0.000687 |
| −3.1     | 0.000711 | 0.000736 | 0.000762 | 0.000789 | 0.000816 | 0.000845 | 0.000874 | 0.000904 | 0.000935 | 0.000968 |
| −3.0     | 0.001001 | 0.001035 | 0.001070 | 0.001107 | 0.001144 | 0.001183 | 0.001223 | 0.001264 | 0.001306 | 0.001350 |
| −2.9     | 0.001395 | 0.001441 | 0.001489 | 0.001538 | 0.001589 | 0.001641 | 0.001695 | 0.001750 | 0.001807 | 0.001866 |
| −2.8     | 0.001926 | 0.001988 | 0.002052 | 0.002118 | 0.002186 | 0.002256 | 0.002327 | 0.002401 | 0.002477 | 0.002555 |
| −2.7     | 0.002635 | 0.002718 | 0.002803 | 0.002890 | 0.002980 | 0.003072 | 0.003167 | 0.003264 | 0.003364 | 0.003467 |
| −2.6     | 0.003573 | 0.003681 | 0.003793 | 0.003907 | 0.004025 | 0.004145 | 0.004269 | 0.004396 | 0.004527 | 0.004661 |
| −2.5     | 0.004799 | 0.004940 | 0.005085 | 0.005234 | 0.005386 | 0.005543 | 0.005703 | 0.005868 | 0.006037 | 0.006210 |
| −2.4     | 0.006387 | 0.006569 | 0.006756 | 0.006947 | 0.007143 | 0.007344 | 0.007549 | 0.007760 | 0.007976 | 0.008198 |
| −2.3     | 0.008424 | 0.008656 | 0.008894 | 0.009137 | 0.009387 | 0.009642 | 0.009903 | 0.010170 | 0.010444 | 0.010724 |
| −2.2     | 0.011011 | 0.011304 | 0.011604 | 0.011911 | 0.012224 | 0.012545 | 0.012874 | 0.013209 | 0.013553 | 0.013903 |
| −2.1     | 0.014262 | 0.014629 | 0.015003 | 0.015386 | 0.015778 | 0.016177 | 0.016586 | 0.017003 | 0.017429 | 0.017864 |
| −2.0     | 0.018309 | 0.018763 | 0.019226 | 0.019699 | 0.020182 | 0.020675 | 0.021178 | 0.021692 | 0.022216 | 0.022750 |
| −1.9     | 0.023295 | 0.023852 | 0.024419 | 0.024998 | 0.025588 | 0.026190 | 0.026803 | 0.027429 | 0.028067 | 0.028717 |
| −1.8     | 0.029379 | 0.030054 | 0.030742 | 0.031443 | 0.032157 | 0.032884 | 0.033625 | 0.034379 | 0.035148 | 0.035930 |
| −1.7     | 0.036727 | 0.037538 | 0.038364 | 0.039204 | 0.040059 | 0.040929 | 0.041815 | 0.042716 | 0.043633 | 0.044565 |
| −1.6     | 0.045514 | 0.046479 | 0.047460 | 0.048457 | 0.049471 | 0.050503 | 0.051551 | 0.052616 | 0.053699 | 0.054799 |
| −1.5     | 0.055917 | 0.057053 | 0.058208 | 0.059380 | 0.060571 | 0.061780 | 0.063008 | 0.064256 | 0.065522 | 0.066807 |
| −1.4     | 0.068112 | 0.069437 | 0.070781 | 0.072145 | 0.073529 | 0.074934 | 0.076359 | 0.077804 | 0.079270 | 0.080757 |
| −1.3     | 0.082264 | 0.083793 | 0.085343 | 0.086915 | 0.088508 | 0.090123 | 0.091759 | 0.093418 | 0.095098 | 0.096801 |
| −1.2     | 0.098525 | 0.100273 | 0.102042 | 0.103835 | 0.105650 | 0.107488 | 0.109349 | 0.111233 | 0.113140 | 0.115070 |
| −1.1     | 0.117023 | 0.119000 | 0.121001 | 0.123024 | 0.125072 | 0.127143 | 0.129238 | 0.131357 | 0.133500 | 0.135666 |
| −1.0     | 0.137857 | 0.140071 | 0.142310 | 0.144572 | 0.146859 | 0.149170 | 0.151505 | 0.153864 | 0.156248 | 0.158655 |
| −0.9     | 0.161087 | 0.163543 | 0.166023 | 0.168528 | 0.171056 | 0.173609 | 0.176185 | 0.178786 | 0.181411 | 0.184060 |
| −0.8     | 0.186733 | 0.189430 | 0.192150 | 0.194894 | 0.197662 | 0.200454 | 0.203269 | 0.206108 | 0.208970 | 0.211855 |
| −0.7     | 0.214764 | 0.217695 | 0.220650 | 0.223627 | 0.226627 | 0.229650 | 0.232695 | 0.235762 | 0.238852 | 0.241964 |
| −0.6     | 0.245097 | 0.248252 | 0.251429 | 0.254627 | 0.257846 | 0.261086 | 0.264347 | 0.267629 | 0.270931 | 0.274253 |
| −0.5     | 0.277595 | 0.280957 | 0.284339 | 0.287740 | 0.291160 | 0.294599 | 0.298056 | 0.301532 | 0.305026 | 0.308538 |
| −0.4     | 0.312067 | 0.315614 | 0.319178 | 0.322758 | 0.326355 | 0.329969 | 0.333598 | 0.337243 | 0.340903 | 0.344578 |
| −0.3     | 0.348268 | 0.351973 | 0.355691 | 0.359424 | 0.363169 | 0.366928 | 0.370700 | 0.374484 | 0.378281 | 0.382089 |
| −0.2     | 0.385908 | 0.389739 | 0.393580 | 0.397432 | 0.401294 | 0.405165 | 0.409046 | 0.412936 | 0.416834 | 0.420740 |
| −0.1     | 0.424655 | 0.428576 | 0.432505 | 0.436441 | 0.440382 | 0.444330 | 0.448283 | 0.452242 | 0.456205 | 0.460172 |
| 0.0      | 0.464144 | 0.468119 | 0.472097 | 0.476078 | 0.480061 | 0.484047 | 0.488033 | 0.492022 | 0.496011 | 0.500000 |

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$


**TABLE • III** Cumulative Standard Normal Distribution (*Continued*)

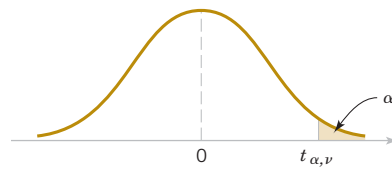
| <i>z</i> | 0.00     | 0.01     | 0.02     | 0.03     | 0.04     | 0.05     | 0.06     | 0.07     | 0.08     | 0.09     |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.0      | 0.500000 | 0.503989 | 0.507978 | 0.511967 | 0.515953 | 0.519939 | 0.523922 | 0.527903 | 0.531881 | 0.535856 |
| 0.1      | 0.539828 | 0.543795 | 0.547758 | 0.551717 | 0.555670 | 0.559618 | 0.563559 | 0.567495 | 0.571424 | 0.575345 |
| 0.2      | 0.579260 | 0.583166 | 0.587064 | 0.590954 | 0.594835 | 0.598706 | 0.602568 | 0.606420 | 0.610261 | 0.614092 |
| 0.3      | 0.617911 | 0.621719 | 0.625516 | 0.629300 | 0.633072 | 0.636831 | 0.640576 | 0.644309 | 0.648027 | 0.651732 |
| 0.4      | 0.655422 | 0.659097 | 0.662757 | 0.666402 | 0.670031 | 0.673645 | 0.677242 | 0.680822 | 0.684386 | 0.687933 |
| 0.5      | 0.691462 | 0.694974 | 0.698468 | 0.701944 | 0.705401 | 0.708840 | 0.712260 | 0.715661 | 0.719043 | 0.722405 |
| 0.6      | 0.725747 | 0.729069 | 0.732371 | 0.735653 | 0.738914 | 0.742154 | 0.745373 | 0.748571 | 0.751748 | 0.754903 |
| 0.7      | 0.758036 | 0.761148 | 0.764238 | 0.767305 | 0.770350 | 0.773373 | 0.776373 | 0.779350 | 0.782305 | 0.785236 |
| 0.8      | 0.788145 | 0.791030 | 0.793892 | 0.796731 | 0.799546 | 0.802338 | 0.805106 | 0.807850 | 0.810570 | 0.813267 |
| 0.9      | 0.815940 | 0.818589 | 0.821214 | 0.823815 | 0.826391 | 0.828944 | 0.831472 | 0.833977 | 0.836457 | 0.838913 |
| 1.0      | 0.841345 | 0.843752 | 0.846136 | 0.848495 | 0.850830 | 0.853141 | 0.855428 | 0.857690 | 0.859929 | 0.862143 |
| 1.1      | 0.864334 | 0.866500 | 0.868643 | 0.870762 | 0.872857 | 0.874928 | 0.876976 | 0.878999 | 0.881000 | 0.882977 |
| 1.2      | 0.884930 | 0.886860 | 0.888767 | 0.890651 | 0.892512 | 0.894350 | 0.896165 | 0.897958 | 0.899727 | 0.901475 |
| 1.3      | 0.903199 | 0.904902 | 0.906582 | 0.908241 | 0.909877 | 0.911492 | 0.913085 | 0.914657 | 0.916207 | 0.917736 |
| 1.4      | 0.919243 | 0.920730 | 0.922196 | 0.923641 | 0.925066 | 0.926471 | 0.927855 | 0.929219 | 0.930563 | 0.931888 |
| 1.5      | 0.933193 | 0.934478 | 0.935744 | 0.936992 | 0.938220 | 0.939429 | 0.940620 | 0.941792 | 0.942947 | 0.944083 |
| 1.6      | 0.945201 | 0.946301 | 0.947384 | 0.948449 | 0.949497 | 0.950529 | 0.951543 | 0.952540 | 0.953521 | 0.954486 |
| 1.7      | 0.955435 | 0.956367 | 0.957284 | 0.958185 | 0.959071 | 0.959941 | 0.960796 | 0.961636 | 0.962462 | 0.963273 |
| 1.8      | 0.964070 | 0.964852 | 0.965621 | 0.966375 | 0.967116 | 0.967843 | 0.968557 | 0.969258 | 0.969946 | 0.970621 |
| 1.9      | 0.971283 | 0.971933 | 0.972571 | 0.973197 | 0.973810 | 0.974412 | 0.975002 | 0.975581 | 0.976148 | 0.976705 |
| 2.0      | 0.977250 | 0.977784 | 0.978308 | 0.978822 | 0.979325 | 0.979818 | 0.980301 | 0.980774 | 0.981237 | 0.981691 |
| 2.1      | 0.982136 | 0.982571 | 0.982997 | 0.983414 | 0.983823 | 0.984222 | 0.984614 | 0.984997 | 0.985371 | 0.985738 |
| 2.2      | 0.986097 | 0.986447 | 0.986791 | 0.987126 | 0.987455 | 0.987776 | 0.988089 | 0.988396 | 0.988696 | 0.988989 |
| 2.3      | 0.989276 | 0.989556 | 0.989830 | 0.990097 | 0.990358 | 0.990613 | 0.990863 | 0.991106 | 0.991344 | 0.991576 |
| 2.4      | 0.991802 | 0.992024 | 0.992240 | 0.992451 | 0.992656 | 0.992857 | 0.993053 | 0.993244 | 0.993431 | 0.993613 |
| 2.5      | 0.993790 | 0.993963 | 0.994132 | 0.994297 | 0.994457 | 0.994614 | 0.994766 | 0.994915 | 0.995060 | 0.995201 |
| 2.6      | 0.995339 | 0.995473 | 0.995604 | 0.995731 | 0.995855 | 0.995975 | 0.996093 | 0.996207 | 0.996319 | 0.996427 |
| 2.7      | 0.996533 | 0.996636 | 0.996736 | 0.996833 | 0.996928 | 0.997020 | 0.997110 | 0.997197 | 0.997282 | 0.997365 |
| 2.8      | 0.997445 | 0.997523 | 0.997599 | 0.997673 | 0.997744 | 0.997814 | 0.997882 | 0.997948 | 0.998012 | 0.998074 |
| 2.9      | 0.998134 | 0.998193 | 0.998250 | 0.998305 | 0.998359 | 0.998411 | 0.998462 | 0.998511 | 0.998559 | 0.998605 |
| 3.0      | 0.998650 | 0.998694 | 0.998736 | 0.998777 | 0.998817 | 0.998856 | 0.998893 | 0.998930 | 0.998965 | 0.998999 |
| 3.1      | 0.999032 | 0.999065 | 0.999096 | 0.999126 | 0.999155 | 0.999184 | 0.999211 | 0.999238 | 0.999264 | 0.999289 |
| 3.2      | 0.999313 | 0.999336 | 0.999359 | 0.999381 | 0.999402 | 0.999423 | 0.999443 | 0.999462 | 0.999481 | 0.999499 |
| 3.3      | 0.999517 | 0.999533 | 0.999550 | 0.999566 | 0.999581 | 0.999596 | 0.999610 | 0.999624 | 0.999638 | 0.999650 |
| 3.4      | 0.999663 | 0.999675 | 0.999687 | 0.999698 | 0.999709 | 0.999720 | 0.999730 | 0.999740 | 0.999749 | 0.999758 |
| 3.5      | 0.999767 | 0.999776 | 0.999784 | 0.999792 | 0.999800 | 0.999807 | 0.999815 | 0.999821 | 0.999828 | 0.999835 |
| 3.6      | 0.999841 | 0.999847 | 0.999853 | 0.999858 | 0.999864 | 0.999869 | 0.999874 | 0.999879 | 0.999883 | 0.999888 |
| 3.7      | 0.999892 | 0.999896 | 0.999900 | 0.999904 | 0.999908 | 0.999912 | 0.999915 | 0.999918 | 0.999922 | 0.999925 |
| 3.8      | 0.999928 | 0.999931 | 0.999933 | 0.999936 | 0.999938 | 0.999941 | 0.999943 | 0.999946 | 0.999948 | 0.999950 |
| 3.9      | 0.999952 | 0.999954 | 0.999956 | 0.999958 | 0.999959 | 0.999961 | 0.999963 | 0.999964 | 0.999966 | 0.999967 |

**TABLE • IV** Percentage Points  $\chi^2_{\alpha, v}$  of the Chi-Squared Distribution

| $\alpha$<br>$v$ | .995  | .990  | .975  | .950  | .900  | .500  | .100   | .050   | .025   | .010   | .005   |
|-----------------|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|--------|
| 1               | .00+  | .00+  | .00+  | .00+  | .02   | .45   | 2.71   | 3.84   | 5.02   | 6.63   | 7.88   |
| 2               | .01   | .02   | .05   | .10   | .21   | 1.39  | 4.61   | 5.99   | 7.38   | 9.21   | 10.60  |
| 3               | .07   | .11   | .22   | .35   | .58   | 2.37  | 6.25   | 7.81   | 9.35   | 11.34  | 12.84  |
| 4               | .21   | .30   | .48   | .71   | 1.06  | 3.36  | 7.78   | 9.49   | 11.14  | 13.28  | 14.86  |
| 5               | .41   | .55   | .83   | 1.15  | 1.61  | 4.35  | 9.24   | 11.07  | 12.83  | 15.09  | 16.75  |
| 6               | .68   | .87   | 1.24  | 1.64  | 2.20  | 5.35  | 10.65  | 12.59  | 14.45  | 16.81  | 18.55  |
| 7               | .99   | 1.24  | 1.69  | 2.17  | 2.83  | 6.35  | 12.02  | 14.07  | 16.01  | 18.48  | 20.28  |
| 8               | 1.34  | 1.65  | 2.18  | 2.73  | 3.49  | 7.34  | 13.36  | 15.51  | 17.53  | 20.09  | 21.96  |
| 9               | 1.73  | 2.09  | 2.70  | 3.33  | 4.17  | 8.34  | 14.68  | 16.92  | 19.02  | 21.67  | 23.59  |
| 10              | 2.16  | 2.56  | 3.25  | 3.94  | 4.87  | 9.34  | 15.99  | 18.31  | 20.48  | 23.21  | 25.19  |
| 11              | 2.60  | 3.05  | 3.82  | 4.57  | 5.58  | 10.34 | 17.28  | 19.68  | 21.92  | 24.72  | 26.76  |
| 12              | 3.07  | 3.57  | 4.40  | 5.23  | 6.30  | 11.34 | 18.55  | 21.03  | 23.34  | 26.22  | 28.30  |
| 13              | 3.57  | 4.11  | 5.01  | 5.89  | 7.04  | 12.34 | 19.81  | 22.36  | 24.74  | 27.69  | 29.82  |
| 14              | 4.07  | 4.66  | 5.63  | 6.57  | 7.79  | 13.34 | 21.06  | 23.68  | 26.12  | 29.14  | 31.32  |
| 15              | 4.60  | 5.23  | 6.27  | 7.26  | 8.55  | 14.34 | 22.31  | 25.00  | 27.49  | 30.58  | 32.80  |
| 16              | 5.14  | 5.81  | 6.91  | 7.96  | 9.31  | 15.34 | 23.54  | 26.30  | 28.85  | 32.00  | 34.27  |
| 17              | 5.70  | 6.41  | 7.56  | 8.67  | 10.09 | 16.34 | 24.77  | 27.59  | 30.19  | 33.41  | 35.72  |
| 18              | 6.26  | 7.01  | 8.23  | 9.39  | 10.87 | 17.34 | 25.99  | 28.87  | 31.53  | 34.81  | 37.16  |
| 19              | 6.84  | 7.63  | 8.91  | 10.12 | 11.65 | 18.34 | 27.20  | 30.14  | 32.85  | 36.19  | 38.58  |
| 20              | 7.43  | 8.26  | 9.59  | 10.85 | 12.44 | 19.34 | 28.41  | 31.41  | 34.17  | 37.57  | 40.00  |
| 21              | 8.03  | 8.90  | 10.28 | 11.59 | 13.24 | 20.34 | 29.62  | 32.67  | 35.48  | 38.93  | 41.40  |
| 22              | 8.64  | 9.54  | 10.98 | 12.34 | 14.04 | 21.34 | 30.81  | 33.92  | 36.78  | 40.29  | 42.80  |
| 23              | 9.26  | 10.20 | 11.69 | 13.09 | 14.85 | 22.34 | 32.01  | 35.17  | 38.08  | 41.64  | 44.18  |
| 24              | 9.89  | 10.86 | 12.40 | 13.85 | 15.66 | 23.34 | 33.20  | 36.42  | 39.36  | 42.98  | 45.56  |
| 25              | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 24.34 | 34.28  | 37.65  | 40.65  | 44.31  | 46.93  |
| 26              | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 25.34 | 35.56  | 38.89  | 41.92  | 45.64  | 48.29  |
| 27              | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 26.34 | 36.74  | 40.11  | 43.19  | 46.96  | 49.65  |
| 28              | 12.46 | 13.57 | 15.31 | 16.93 | 18.94 | 27.34 | 37.92  | 41.34  | 44.46  | 48.28  | 50.99  |
| 29              | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 28.34 | 39.09  | 42.56  | 45.72  | 49.59  | 52.34  |
| 30              | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 29.34 | 40.26  | 43.77  | 46.98  | 50.89  | 53.67  |
| 40              | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 39.34 | 51.81  | 55.76  | 59.34  | 63.69  | 66.77  |
| 50              | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 49.33 | 63.17  | 67.50  | 71.42  | 76.15  | 79.49  |
| 60              | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 59.33 | 74.40  | 79.08  | 83.30  | 88.38  | 91.95  |
| 70              | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 69.33 | 85.53  | 90.53  | 95.02  | 100.42 | 104.22 |
| 80              | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 79.33 | 96.58  | 101.88 | 106.63 | 112.33 | 116.32 |
| 90              | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 89.33 | 107.57 | 113.14 | 118.14 | 124.12 | 128.30 |
| 100             | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 99.33 | 118.50 | 124.34 | 129.56 | 135.81 | 140.17 |

 $v$  = degrees of freedom.



**TABLE • V** Percentage Points  $t_{\alpha, \nu}$  of the  $t$  Distribution

| $\alpha$<br>$\nu$ | .40  | .25   | .10   | .05   | .025   | .01    | .005   | .0025  | .001   | .0005  |
|-------------------|------|-------|-------|-------|--------|--------|--------|--------|--------|--------|
| 1                 | .325 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 127.32 | 318.31 | 636.62 |
| 2                 | .289 | .816  | 1.886 | 2.920 | 4.303  | 6.965  | 9.925  | 14.089 | 23.326 | 31.598 |
| 3                 | .277 | .765  | 1.638 | 2.353 | 3.182  | 4.541  | 5.841  | 7.453  | 10.213 | 12.924 |
| 4                 | .271 | .741  | 1.533 | 2.132 | 2.776  | 3.747  | 4.604  | 5.598  | 7.173  | 8.610  |
| 5                 | .267 | .727  | 1.476 | 2.015 | 2.571  | 3.365  | 4.032  | 4.773  | 5.893  | 6.869  |
| 6                 | .265 | .718  | 1.440 | 1.943 | 2.447  | 3.143  | 3.707  | 4.317  | 5.208  | 5.959  |
| 7                 | .263 | .711  | 1.415 | 1.895 | 2.365  | 2.998  | 3.499  | 4.029  | 4.785  | 5.408  |
| 8                 | .262 | .706  | 1.397 | 1.860 | 2.306  | 2.896  | 3.355  | 3.833  | 4.501  | 5.041  |
| 9                 | .261 | .703  | 1.383 | 1.833 | 2.262  | 2.821  | 3.250  | 3.690  | 4.297  | 4.781  |
| 10                | .260 | .700  | 1.372 | 1.812 | 2.228  | 2.764  | 3.169  | 3.581  | 4.144  | 4.587  |
| 11                | .260 | .697  | 1.363 | 1.796 | 2.201  | 2.718  | 3.106  | 3.497  | 4.025  | 4.437  |
| 12                | .259 | .695  | 1.356 | 1.782 | 2.179  | 2.681  | 3.055  | 3.428  | 3.930  | 4.318  |
| 13                | .259 | .694  | 1.350 | 1.771 | 2.160  | 2.650  | 3.012  | 3.372  | 3.852  | 4.221  |
| 14                | .258 | .692  | 1.345 | 1.761 | 2.145  | 2.624  | 2.977  | 3.326  | 3.787  | 4.140  |
| 15                | .258 | .691  | 1.341 | 1.753 | 2.131  | 2.602  | 2.947  | 3.286  | 3.733  | 4.073  |
| 16                | .258 | .690  | 1.337 | 1.746 | 2.120  | 2.583  | 2.921  | 3.252  | 3.686  | 4.015  |
| 17                | .257 | .689  | 1.333 | 1.740 | 2.110  | 2.567  | 2.898  | 3.222  | 3.646  | 3.965  |
| 18                | .257 | .688  | 1.330 | 1.734 | 2.101  | 2.552  | 2.878  | 3.197  | 3.610  | 3.922  |
| 19                | .257 | .688  | 1.328 | 1.729 | 2.093  | 2.539  | 2.861  | 3.174  | 3.579  | 3.883  |
| 20                | .257 | .687  | 1.325 | 1.725 | 2.086  | 2.528  | 2.845  | 3.153  | 3.552  | 3.850  |
| 21                | .257 | .686  | 1.323 | 1.721 | 2.080  | 2.518  | 2.831  | 3.135  | 3.527  | 3.819  |
| 22                | .256 | .686  | 1.321 | 1.717 | 2.074  | 2.508  | 2.819  | 3.119  | 3.505  | 3.792  |
| 23                | .256 | .685  | 1.319 | 1.714 | 2.069  | 2.500  | 2.807  | 3.104  | 3.485  | 3.767  |
| 24                | .256 | .685  | 1.318 | 1.711 | 2.064  | 2.492  | 2.797  | 3.091  | 3.467  | 3.745  |
| 25                | .256 | .684  | 1.316 | 1.708 | 2.060  | 2.485  | 2.787  | 3.078  | 3.450  | 3.725  |
| 26                | .256 | .684  | 1.315 | 1.706 | 2.056  | 2.479  | 2.779  | 3.067  | 3.435  | 3.707  |
| 27                | .256 | .684  | 1.314 | 1.703 | 2.052  | 2.473  | 2.771  | 3.057  | 3.421  | 3.690  |
| 28                | .256 | .683  | 1.313 | 1.701 | 2.048  | 2.467  | 2.763  | 3.047  | 3.408  | 3.674  |
| 29                | .256 | .683  | 1.311 | 1.699 | 2.045  | 2.462  | 2.756  | 3.038  | 3.396  | 3.659  |
| 30                | .256 | .683  | 1.310 | 1.697 | 2.042  | 2.457  | 2.750  | 3.030  | 3.385  | 3.646  |
| 40                | .255 | .681  | 1.303 | 1.684 | 2.021  | 2.423  | 2.704  | 2.971  | 3.307  | 3.551  |
| 60                | .254 | .679  | 1.296 | 1.671 | 2.000  | 2.390  | 2.660  | 2.915  | 3.232  | 3.460  |
| 120               | .254 | .677  | 1.289 | 1.658 | 1.980  | 2.358  | 2.617  | 2.860  | 3.160  | 3.373  |
| $\infty$          | .253 | .674  | 1.282 | 1.645 | 1.960  | 2.326  | 2.576  | 2.807  | 3.090  | 3.291  |

 $\nu$  = degrees of freedom.

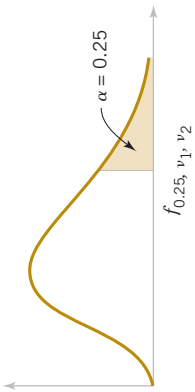


TABLE • VI Percentage Points  $f_{\alpha, v_1, v_2}$  of the F Distribution

| $\nu_2 \backslash \nu_1$ |     | Degrees of freedom for the numerator ( $\nu_1$ ) |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      | Degrees of freedom for the denominator ( $\nu_2$ ) |  |  |  |
|--------------------------|-----|--|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|--|--|--|--|
|                          |     | 1  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 12   | 15   | 20   | 24   | 30   | 40   | 60   | 120  | ∞    |  |  |  |  |
| 1                        | 1   | 5.83   | 7.50 | 8.20 | 8.58 | 8.82 | 8.98 | 9.10 | 9.19 | 9.26 | 9.32 | 9.41 | 9.49 | 9.58 | 9.63 | 9.67 | 9.71 | 9.76 | 9.80 | 9.85 |  |  |  |  |
| 2                        | 2   | 2.57   | 3.00 | 3.15 | 3.23 | 3.28 | 3.31 | 3.34 | 3.35 | 3.37 | 3.38 | 3.39 | 3.41 | 3.43 | 3.43 | 3.44 | 3.45 | 3.46 | 3.47 | 3.48 |  |  |  |  |
| 3                        | 3   | 2.02   | 2.28 | 2.36 | 2.39 | 2.41 | 2.42 | 2.43 | 2.44 | 2.44 | 2.44 | 2.45 | 2.46 | 2.46 | 2.46 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 |  |  |  |  |
| 4                        | 4   | 1.81   | 2.00 | 2.05 | 2.06 | 2.07 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 |  |  |  |  |
| 5                        | 5   | 1.69   | 1.85 | 1.88 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.88 | 1.88 | 1.88 | 1.88 | 1.88 | 1.87 | 1.87 |  |  |  |  |
| 6                        | 6   | 1.62   | 1.76 | 1.78 | 1.79 | 1.79 | 1.78 | 1.78 | 1.78 | 1.77 | 1.77 | 1.77 | 1.76 | 1.76 | 1.76 | 1.75 | 1.75 | 1.75 | 1.74 | 1.74 |  |  |  |  |
| 7                        | 7   | 1.57   | 1.70 | 1.72 | 1.72 | 1.71 | 1.71 | 1.70 | 1.70 | 1.70 | 1.69 | 1.68 | 1.68 | 1.67 | 1.67 | 1.66 | 1.66 | 1.66 | 1.65 | 1.65 |  |  |  |  |
| 8                        | 8   | 1.54   | 1.66 | 1.67 | 1.66 | 1.66 | 1.65 | 1.64 | 1.64 | 1.63 | 1.63 | 1.62 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.58 |  |  |  |  |
| 9                        | 9   | 1.51   | 1.62 | 1.63 | 1.63 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.57 | 1.56 | 1.56 | 1.55 | 1.54 | 1.54 | 1.53 | 1.53 |  |  |  |  |
| 10                       | 10  | 1.49   | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.57 | 1.56 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.52 | 1.51 | 1.51 | 1.50 | 1.49 | 1.48 |  |  |  |  |
| 11                       | 11  | 1.47   | 1.58 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 |  |  |  |  |
| 12                       | 12  | 1.46   | 1.56 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 |  |  |  |  |
| 13                       | 13  | 1.45   | 1.55 | 1.55 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 |  |  |  |  |
| 14                       | 14  | 1.44   | 1.53 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 |  |  |  |  |
| 15                       | 15  | 1.43   | 1.52 | 1.52 | 1.51 | 1.49 | 1.48 | 1.47 | 1.46 | 1.46 | 1.45 | 1.44 | 1.43 | 1.41 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 |  |  |  |  |
| 16                       | 16  | 1.42   | 1.51 | 1.51 | 1.50 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.44 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 |  |  |  |  |
| 17                       | 17  | 1.42   | 1.51 | 1.50 | 1.49 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 |  |  |  |  |
| 18                       | 18  | 1.41   | 1.50 | 1.49 | 1.48 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.42 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 |  |  |  |  |
| 19                       | 19  | 1.41   | 1.49 | 1.49 | 1.47 | 1.46 | 1.44 | 1.43 | 1.42 | 1.41 | 1.41 | 1.40 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 | 1.30 |  |  |  |  |
| 20                       | 20  | 1.40   | 1.49 | 1.48 | 1.47 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.29 |  |  |  |  |
| 21                       | 21  | 1.40   | 1.48 | 1.48 | 1.46 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.28 |  |  |  |  |
| 22                       | 22  | 1.40   | 1.48 | 1.47 | 1.45 | 1.44 | 1.42 | 1.41 | 1.40 | 1.39 | 1.39 | 1.37 | 1.36 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 |  |  |  |  |
| 23                       | 23  | 1.39   | 1.47 | 1.47 | 1.45 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.28 | 1.27 |  |  |  |  |
| 24                       | 24  | 1.39   | 1.47 | 1.46 | 1.44 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.38 | 1.36 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.26 |  |  |  |  |
| 25                       | 25  | 1.39   | 1.47 | 1.46 | 1.44 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.34 | 1.33 | 1.32 | 1.31 | 1.29 | 1.28 | 1.27 | 1.25 |  |  |  |  |
| 26                       | 26  | 1.38   | 1.46 | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.37 | 1.35 | 1.34 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.26 | 1.25 |  |  |  |  |
| 27                       | 27  | 1.38   | 1.46 | 1.45 | 1.43 | 1.42 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.24 |  |  |  |  |
| 28                       | 28  | 1.38   | 1.46 | 1.45 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.34 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.25 | 1.24 |  |  |  |  |
| 29                       | 29  | 1.38   | 1.45 | 1.45 | 1.43 | 1.41 | 1.40 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.32 | 1.31 | 1.30 | 1.29 | 1.27 | 1.26 | 1.25 | 1.23 |  |  |  |  |
| 30                       | 30  | 1.38   | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.32 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 | 1.24 | 1.23 |  |  |  |  |
| 40                       | 40  | 1.36   | 1.44 | 1.42 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.31 | 1.30 | 1.28 | 1.26 | 1.25 | 1.24 | 1.22 | 1.21 | 1.19 |  |  |  |  |
| 60                       | 60  | 1.35   | 1.42 | 1.41 | 1.38 | 1.37 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.27 | 1.25 | 1.24 | 1.22 | 1.21 | 1.19 | 1.17 | 1.15 |  |  |  |  |
| 120                      | 120 | 1.34   | 1.40 | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.26 | 1.24 | 1.22 | 1.21 | 1.19 | 1.18 | 1.16 | 1.13 | 1.10 |  |  |  |  |
| ∞                        | ∞   | 1.32   | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.29 | 1.28 | 1.27 | 1.25 | 1.24 | 1.22 | 1.19 | 1.18 | 1.16 | 1.14 | 1.12 | 1.08 | 1.00 |  |  |  |  |

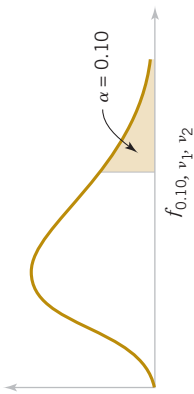


TABLE • VI Percentage Points  $f_{\alpha, v_1, v_2}$  of the  $F$  Distribution (Continued)

| $\begin{matrix} v_1 \\ v_2 \end{matrix}$         |          | Degrees of freedom for the numerator ( $v_1$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          | $f_{0.10, v_1, v_2}$ |  |  |  |
|--|----------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------------------|--|--|--|
|  |          | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |                      |  |  |  |
| Degrees of freedom for the denominator ( $v_2$ ) | 1        | 39.86  | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 | 60.19 | 60.71 | 61.22 | 61.74 | 62.00 | 62.26 | 62.53 | 62.79 | 63.06 | 63.33    |                      |  |  |  |
|  | 2        | 8.53   | 9.00  | 9.16  | 9.24  | 9.29  | 9.33  | 9.35  | 9.37  | 9.38  | 9.39  | 9.41  | 9.42  | 9.44  | 9.45  | 9.46  | 9.47  | 9.47  | 9.48  | 9.49     |                      |  |  |  |
|  | 3        | 5.54   | 5.46  | 5.39  | 5.34  | 5.31  | 5.28  | 5.27  | 5.25  | 5.24  | 5.23  | 5.22  | 5.20  | 5.18  | 5.18  | 5.17  | 5.16  | 5.15  | 5.14  | 5.13     |                      |  |  |  |
|  | 4        | 4.54   | 4.32  | 4.19  | 4.11  | 4.05  | 4.01  | 3.98  | 3.95  | 3.94  | 3.92  | 3.90  | 3.87  | 3.84  | 3.83  | 3.82  | 3.80  | 3.79  | 3.78  | 3.76     |                      |  |  |  |
|  | 5        | 4.06   | 3.78  | 3.62  | 3.52  | 3.45  | 3.40  | 3.37  | 3.34  | 3.32  | 3.30  | 3.27  | 3.24  | 3.21  | 3.19  | 3.17  | 3.16  | 3.14  | 3.12  | 3.10     |                      |  |  |  |
|  | 6        | 3.78   | 3.46  | 3.29  | 3.18  | 3.11  | 3.05  | 3.01  | 2.98  | 2.96  | 2.94  | 2.90  | 2.87  | 2.84  | 2.82  | 2.80  | 2.78  | 2.76  | 2.74  | 2.72     |                      |  |  |  |
|  | 7        | 3.59   | 3.26  | 3.07  | 2.96  | 2.88  | 2.83  | 2.78  | 2.75  | 2.72  | 2.70  | 2.67  | 2.63  | 2.59  | 2.58  | 2.56  | 2.54  | 2.51  | 2.49  | 2.47     |                      |  |  |  |
|  | 8        | 3.46   | 3.11  | 2.92  | 2.81  | 2.73  | 2.67  | 2.62  | 2.59  | 2.56  | 2.54  | 2.50  | 2.46  | 2.42  | 2.40  | 2.38  | 2.36  | 2.34  | 2.32  | 2.29     |                      |  |  |  |
|  | 9        | 3.36   | 3.01  | 2.81  | 2.69  | 2.61  | 2.55  | 2.51  | 2.47  | 2.44  | 2.42  | 2.38  | 2.34  | 2.30  | 2.28  | 2.25  | 2.23  | 2.21  | 2.18  | 2.16     |                      |  |  |  |
|  | 10       | 3.29   | 2.92  | 2.73  | 2.61  | 2.52  | 2.46  | 2.41  | 2.38  | 2.35  | 2.32  | 2.28  | 2.24  | 2.20  | 2.18  | 2.16  | 2.13  | 2.11  | 2.08  | 2.06     |                      |  |  |  |
|  | 11       | 3.23   | 2.86  | 2.66  | 2.54  | 2.45  | 2.39  | 2.34  | 2.30  | 2.27  | 2.25  | 2.21  | 2.17  | 2.12  | 2.10  | 2.08  | 2.05  | 2.03  | 2.00  | 1.97     |                      |  |  |  |
|  | 12       | 3.18   | 2.81  | 2.61  | 2.48  | 2.39  | 2.33  | 2.28  | 2.24  | 2.21  | 2.19  | 2.15  | 2.10  | 2.06  | 2.04  | 2.01  | 1.99  | 1.96  | 1.93  | 1.90     |                      |  |  |  |
|  | 13       | 3.14   | 2.76  | 2.56  | 2.43  | 2.35  | 2.28  | 2.23  | 2.20  | 2.16  | 2.14  | 2.10  | 2.05  | 2.01  | 1.98  | 1.96  | 1.93  | 1.90  | 1.88  | 1.85     |                      |  |  |  |
|  | 14       | 3.10   | 2.73  | 2.52  | 2.39  | 2.31  | 2.24  | 2.19  | 2.15  | 2.12  | 2.09  | 2.06  | 2.02  | 1.97  | 1.92  | 1.90  | 1.87  | 1.85  | 1.82  | 1.79     |                      |  |  |  |
|  | 15       | 3.07   | 2.70  | 2.49  | 2.36  | 2.27  | 2.21  | 2.16  | 2.12  | 2.09  | 2.06  | 2.02  | 1.97  | 1.92  | 1.86  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72     |                      |  |  |  |
|  | 16       | 3.05   | 2.67  | 2.46  | 2.33  | 2.24  | 2.18  | 2.13  | 2.09  | 2.06  | 2.03  | 1.99  | 1.94  | 1.89  | 1.87  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72     |                      |  |  |  |
|  | 17       | 3.03   | 2.64  | 2.44  | 2.31  | 2.22  | 2.15  | 2.10  | 2.06  | 2.03  | 2.00  | 1.96  | 1.91  | 1.86  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72  | 1.69     |                      |  |  |  |
|  | 18       | 3.01   | 2.62  | 2.42  | 2.29  | 2.20  | 2.13  | 2.08  | 2.04  | 2.00  | 1.98  | 1.93  | 1.89  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72  | 1.69  | 1.66     |                      |  |  |  |
|  | 19       | 2.99   | 2.61  | 2.40  | 2.27  | 2.18  | 2.11  | 2.06  | 2.02  | 1.98  | 1.96  | 1.91  | 1.86  | 1.81  | 1.79  | 1.76  | 1.73  | 1.70  | 1.67  | 1.63     |                      |  |  |  |
|  | 20       | 2.97   | 2.59  | 2.38  | 2.25  | 2.16  | 2.09  | 2.04  | 2.00  | 1.96  | 1.94  | 1.89  | 1.84  | 1.79  | 1.77  | 1.74  | 1.71  | 1.68  | 1.64  | 1.61     |                      |  |  |  |
|  | 21       | 2.96   | 2.57  | 2.36  | 2.23  | 2.14  | 2.08  | 2.02  | 1.98  | 1.95  | 1.92  | 1.87  | 1.83  | 1.78  | 1.75  | 1.72  | 1.69  | 1.66  | 1.62  | 1.59     |                      |  |  |  |
|  | 22       | 2.95   | 2.56  | 2.35  | 2.22  | 2.13  | 2.06  | 2.01  | 1.97  | 1.93  | 1.90  | 1.86  | 1.81  | 1.76  | 1.73  | 1.70  | 1.67  | 1.64  | 1.60  | 1.57     |                      |  |  |  |
|  | 23       | 2.94   | 2.55  | 2.34  | 2.21  | 2.11  | 2.05  | 1.99  | 1.95  | 1.92  | 1.89  | 1.84  | 1.80  | 1.74  | 1.72  | 1.69  | 1.66  | 1.62  | 1.59  | 1.55     |                      |  |  |  |
|  | 24       | 2.93   | 2.54  | 2.33  | 2.19  | 2.10  | 2.04  | 1.98  | 1.94  | 1.91  | 1.88  | 1.83  | 1.78  | 1.73  | 1.70  | 1.67  | 1.64  | 1.61  | 1.57  | 1.53     |                      |  |  |  |
|  | 25       | 2.92   | 2.53  | 2.32  | 2.18  | 2.09  | 2.02  | 1.97  | 1.93  | 1.89  | 1.87  | 1.82  | 1.77  | 1.72  | 1.69  | 1.66  | 1.63  | 1.59  | 1.56  | 1.52     |                      |  |  |  |
|  | 26       | 2.91   | 2.52  | 2.31  | 2.17  | 2.08  | 2.01  | 1.96  | 1.92  | 1.88  | 1.86  | 1.81  | 1.76  | 1.71  | 1.68  | 1.65  | 1.61  | 1.58  | 1.54  | 1.50     |                      |  |  |  |
|  | 27       | 2.90   | 2.51  | 2.30  | 2.17  | 2.07  | 2.00  | 1.95  | 1.91  | 1.87  | 1.85  | 1.80  | 1.75  | 1.70  | 1.67  | 1.64  | 1.60  | 1.57  | 1.53  | 1.49     |                      |  |  |  |
|  | 28       | 2.89   | 2.50  | 2.29  | 2.16  | 2.06  | 2.00  | 1.94  | 1.90  | 1.87  | 1.84  | 1.79  | 1.74  | 1.69  | 1.66  | 1.63  | 1.59  | 1.56  | 1.52  | 1.48     |                      |  |  |  |
|  | 29       | 2.89   | 2.50  | 2.28  | 2.15  | 2.06  | 1.99  | 1.93  | 1.89  | 1.86  | 1.83  | 1.78  | 1.73  | 1.68  | 1.65  | 1.62  | 1.58  | 1.55  | 1.51  | 1.47     |                      |  |  |  |
|  | 30       | 2.88   | 2.49  | 2.28  | 2.14  | 2.03  | 1.98  | 1.93  | 1.88  | 1.85  | 1.82  | 1.77  | 1.72  | 1.67  | 1.64  | 1.61  | 1.57  | 1.54  | 1.50  | 1.46     |                      |  |  |  |
|  | 40       | 2.84   | 2.44  | 2.23  | 2.09  | 2.00  | 1.93  | 1.87  | 1.83  | 1.79  | 1.76  | 1.71  | 1.66  | 1.61  | 1.57  | 1.54  | 1.51  | 1.47  | 1.42  | 1.38     |                      |  |  |  |
|  | 60       | 2.79   | 2.39  | 2.18  | 2.04  | 1.95  | 1.87  | 1.82  | 1.77  | 1.74  | 1.71  | 1.66  | 1.60  | 1.54  | 1.51  | 1.48  | 1.44  | 1.40  | 1.35  | 1.29     |                      |  |  |  |
|  | 120      | 2.75   | 2.35  | 2.13  | 1.99  | 1.90  | 1.82  | 1.77  | 1.72  | 1.68  | 1.65  | 1.60  | 1.55  | 1.48  | 1.45  | 1.41  | 1.37  | 1.32  | 1.26  | 1.19     |                      |  |  |  |
|  | $\infty$ | 2.71   | 2.30  | 2.08  | 1.94  | 1.85  | 1.77  | 1.72  | 1.67  | 1.63  | 1.60  | 1.55  | 1.49  | 1.42  | 1.38  | 1.34  | 1.30  | 1.24  | 1.17  | 1.00     |                      |  |  |  |

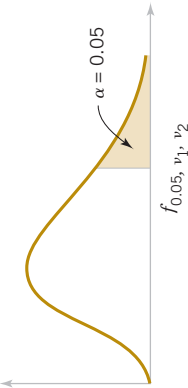
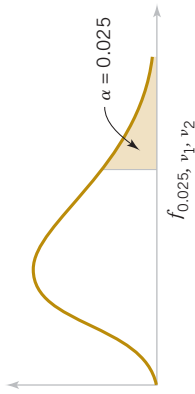


TABLE • VI Percentage Points  $f_{\alpha, v_1, v_2}$  of the F Distribution (Continued)

|          |       | Degrees of freedom for the numerator ( $v_1$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |  |  |
|----------|-------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|--|--|
| $v_2$    | $v_1$ | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |  |  |
| 1        | 1     | 161.4  | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3    |  |  |
| 2        | 1     | 18.51  | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50    |  |  |
| 3        | 1     | 10.13  | 9.55  | 9.28  | 9.12  | 9.01  | 8.94  | 8.89  | 8.85  | 8.81  | 8.79  | 8.74  | 8.70  | 8.66  | 8.64  | 8.62  | 8.59  | 8.57  | 8.55  | 8.53     |  |  |
| 4        | 1     | 7.71   | 6.94  | 6.59  | 6.39  | 6.26  | 6.16  | 6.09  | 6.04  | 6.00  | 5.96  | 5.91  | 5.86  | 5.80  | 5.77  | 5.75  | 5.72  | 5.69  | 5.66  | 5.63     |  |  |
| 5        | 1     | 6.61   | 5.79  | 5.41  | 5.19  | 5.05  | 4.95  | 4.88  | 4.82  | 4.77  | 4.74  | 4.68  | 4.62  | 4.56  | 4.53  | 4.50  | 4.46  | 4.43  | 4.40  | 4.36     |  |  |
| 6        | 1     | 5.99   | 5.14  | 4.76  | 4.53  | 4.39  | 4.28  | 4.21  | 4.15  | 4.10  | 4.06  | 4.00  | 3.94  | 3.87  | 3.84  | 3.81  | 3.77  | 3.74  | 3.70  | 3.67     |  |  |
| 7        | 1     | 5.59   | 4.74  | 4.35  | 4.12  | 3.97  | 3.87  | 3.79  | 3.73  | 3.68  | 3.64  | 3.57  | 3.51  | 3.44  | 3.41  | 3.38  | 3.34  | 3.30  | 3.27  | 3.23     |  |  |
| 8        | 1     | 5.32   | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  | 3.35  | 3.28  | 3.22  | 3.15  | 3.12  | 3.08  | 3.04  | 3.01  | 2.97  | 2.93     |  |  |
| 9        | 1     | 5.12   | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  | 3.14  | 3.07  | 3.01  | 2.94  | 2.90  | 2.86  | 2.83  | 2.79  | 2.75  | 2.71     |  |  |
| 10       | 1     | 4.96   | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  | 2.98  | 2.91  | 2.85  | 2.77  | 2.74  | 2.70  | 2.66  | 2.62  | 2.58  | 2.54     |  |  |
| 11       | 1     | 4.84   | 3.98  | 3.59  | 3.36  | 3.20  | 3.09  | 3.01  | 2.95  | 2.90  | 2.85  | 2.79  | 2.72  | 2.65  | 2.61  | 2.57  | 2.53  | 2.49  | 2.45  | 2.40     |  |  |
| 12       | 1     | 4.75   | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  | 2.75  | 2.69  | 2.62  | 2.54  | 2.51  | 2.47  | 2.43  | 2.38  | 2.34  | 2.30     |  |  |
| 13       | 1     | 4.67   | 3.81  | 3.41  | 3.18  | 3.03  | 2.92  | 2.83  | 2.77  | 2.71  | 2.67  | 2.60  | 2.53  | 2.46  | 2.42  | 2.38  | 2.34  | 2.30  | 2.25  | 2.21     |  |  |
| 14       | 1     | 4.60   | 3.74  | 3.34  | 3.11  | 2.96  | 2.85  | 2.76  | 2.70  | 2.65  | 2.60  | 2.53  | 2.46  | 2.39  | 2.35  | 2.31  | 2.27  | 2.22  | 2.18  | 2.13     |  |  |
| 15       | 1     | 4.54   | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  | 2.54  | 2.48  | 2.40  | 2.33  | 2.29  | 2.25  | 2.20  | 2.16  | 2.11  | 2.07     |  |  |
| 16       | 1     | 4.49   | 3.63  | 3.24  | 3.01  | 2.85  | 2.74  | 2.66  | 2.59  | 2.54  | 2.49  | 2.42  | 2.35  | 2.28  | 2.24  | 2.19  | 2.15  | 2.11  | 2.06  | 2.01     |  |  |
| 17       | 1     | 4.45   | 3.59  | 3.20  | 2.96  | 2.81  | 2.70  | 2.61  | 2.55  | 2.49  | 2.45  | 2.38  | 2.31  | 2.23  | 2.19  | 2.15  | 2.10  | 2.06  | 2.01  | 1.96     |  |  |
| 18       | 1     | 4.41   | 3.55  | 3.16  | 2.93  | 2.77  | 2.66  | 2.58  | 2.51  | 2.46  | 2.41  | 2.34  | 2.27  | 2.19  | 2.15  | 2.11  | 2.06  | 2.02  | 1.97  | 1.92     |  |  |
| 19       | 1     | 4.38   | 3.52  | 3.13  | 2.90  | 2.74  | 2.63  | 2.54  | 2.48  | 2.42  | 2.38  | 2.31  | 2.23  | 2.16  | 2.11  | 2.07  | 2.03  | 1.98  | 1.93  | 1.88     |  |  |
| 20       | 1     | 4.35   | 3.49  | 3.10  | 2.87  | 2.71  | 2.60  | 2.51  | 2.45  | 2.39  | 2.35  | 2.28  | 2.20  | 2.12  | 2.08  | 2.04  | 1.99  | 1.95  | 1.90  | 1.84     |  |  |
| 21       | 1     | 4.32   | 3.47  | 3.07  | 2.84  | 2.68  | 2.57  | 2.49  | 2.42  | 2.37  | 2.32  | 2.25  | 2.18  | 2.10  | 2.05  | 2.01  | 1.96  | 1.92  | 1.87  | 1.81     |  |  |
| 22       | 1     | 4.30   | 3.44  | 3.05  | 2.82  | 2.66  | 2.55  | 2.46  | 2.40  | 2.34  | 2.30  | 2.23  | 2.15  | 2.07  | 2.03  | 1.98  | 1.94  | 1.89  | 1.84  | 1.78     |  |  |
| 23       | 1     | 4.28   | 3.42  | 3.03  | 2.80  | 2.64  | 2.53  | 2.44  | 2.37  | 2.32  | 2.27  | 2.20  | 2.13  | 2.05  | 2.01  | 1.96  | 1.91  | 1.86  | 1.81  | 1.76     |  |  |
| 24       | 1     | 4.26   | 3.40  | 3.01  | 2.78  | 2.62  | 2.51  | 2.42  | 2.36  | 2.30  | 2.25  | 2.18  | 2.11  | 2.03  | 1.98  | 1.94  | 1.89  | 1.84  | 1.79  | 1.73     |  |  |
| 25       | 1     | 4.24   | 3.39  | 2.99  | 2.76  | 2.60  | 2.49  | 2.40  | 2.34  | 2.28  | 2.24  | 2.16  | 2.09  | 2.01  | 1.96  | 1.92  | 1.87  | 1.82  | 1.77  | 1.71     |  |  |
| 26       | 1     | 4.23   | 3.37  | 2.98  | 2.74  | 2.59  | 2.47  | 2.39  | 2.32  | 2.27  | 2.22  | 2.15  | 2.07  | 1.99  | 1.95  | 1.90  | 1.85  | 1.80  | 1.75  | 1.69     |  |  |
| 27       | 1     | 4.21   | 3.35  | 2.96  | 2.73  | 2.57  | 2.45  | 2.37  | 2.31  | 2.25  | 2.20  | 2.13  | 2.06  | 1.97  | 1.93  | 1.88  | 1.84  | 1.79  | 1.73  | 1.67     |  |  |
| 28       | 1     | 4.20   | 3.34  | 2.95  | 2.71  | 2.56  | 2.45  | 2.36  | 2.29  | 2.24  | 2.19  | 2.12  | 2.04  | 1.96  | 1.91  | 1.87  | 1.82  | 1.77  | 1.71  | 1.65     |  |  |
| 29       | 1     | 4.18   | 3.33  | 2.93  | 2.70  | 2.55  | 2.43  | 2.35  | 2.28  | 2.22  | 2.18  | 2.10  | 2.03  | 1.94  | 1.90  | 1.85  | 1.81  | 1.75  | 1.70  | 1.64     |  |  |
| 30       | 1     | 4.17   | 3.32  | 2.92  | 2.69  | 2.53  | 2.42  | 2.33  | 2.27  | 2.21  | 2.16  | 2.09  | 2.01  | 1.93  | 1.89  | 1.84  | 1.79  | 1.74  | 1.68  | 1.62     |  |  |
| 40       | 1     | 4.08   | 3.23  | 2.84  | 2.61  | 2.45  | 2.34  | 2.25  | 2.18  | 2.12  | 2.08  | 2.00  | 1.92  | 1.84  | 1.79  | 1.74  | 1.69  | 1.64  | 1.58  | 1.51     |  |  |
| 60       | 1     | 4.00   | 3.15  | 2.76  | 2.53  | 2.37  | 2.25  | 2.17  | 2.10  | 2.04  | 1.99  | 1.92  | 1.84  | 1.75  | 1.70  | 1.65  | 1.59  | 1.53  | 1.47  | 1.39     |  |  |
| 120      | 1     | 3.92   | 3.07  | 2.68  | 2.45  | 2.29  | 2.17  | 2.09  | 2.02  | 1.96  | 1.91  | 1.83  | 1.75  | 1.66  | 1.61  | 1.55  | 1.50  | 1.43  | 1.35  | 1.25     |  |  |
| $\infty$ | 1     | 3.84   | 3.00  | 2.60  | 2.37  | 2.21  | 2.10  | 2.01  | 1.94  | 1.88  | 1.83  | 1.75  | 1.67  | 1.57  | 1.52  | 1.46  | 1.39  | 1.32  | 1.22  | 1.00     |  |  |

Degrees of freedom for the denominator ( $v_2$ )



**TABLE • VI** Percentage Points  $f_{\alpha, v_1, v_2}$  of the  $F$  Distribution (Continued)

| $\nu_1$  |      | Degrees of freedom for the numerator ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |
|----------|------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|          |      | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |
| $\nu_2$  | 1    | 647.8  | 799.5 | 864.2 | 899.6 | 921.8 | 937.1 | 948.2 | 956.7 | 963.3 | 968.6 | 976.7 | 984.9 | 993.1 | 997.2 | 1001  | 1006  | 1010  | 1014  | 1018     |
|          | 2    | 38.51  | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.41 | 39.43 | 39.45 | 39.46 | 39.46 | 39.47 | 39.48 | 39.49 | 39.50    |
|          | 3    | 17.44  | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.34 | 14.25 | 14.17 | 14.12 | 14.08 | 14.04 | 13.99 | 13.95 | 13.90    |
|          | 4    | 12.22  | 10.65 | 9.98  | 9.60  | 9.36  | 9.20  | 9.07  | 8.98  | 8.90  | 8.84  | 8.75  | 8.66  | 8.56  | 8.51  | 8.46  | 8.41  | 8.36  | 8.31  | 8.26     |
|          | 5    | 10.01  | 8.43  | 7.76  | 7.39  | 7.15  | 6.98  | 6.85  | 6.76  | 6.68  | 6.62  | 6.52  | 6.43  | 6.33  | 6.28  | 6.23  | 6.18  | 6.12  | 6.07  | 6.02     |
|          | 6    | 8.81   | 7.26  | 6.60  | 6.23  | 5.99  | 5.82  | 5.70  | 5.60  | 5.52  | 5.46  | 5.37  | 5.27  | 5.17  | 5.12  | 5.07  | 5.01  | 4.96  | 4.90  | 4.85     |
|          | 7    | 8.07   | 6.54  | 5.89  | 5.52  | 5.29  | 5.12  | 4.99  | 4.90  | 4.82  | 4.76  | 4.67  | 4.57  | 4.47  | 4.42  | 4.36  | 4.31  | 4.25  | 4.20  | 4.14     |
|          | 8    | 7.57   | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  | 4.30  | 4.20  | 4.10  | 4.00  | 3.95  | 3.89  | 3.84  | 3.78  | 3.73  | 3.67     |
|          | 9    | 7.21   | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  | 3.96  | 3.87  | 3.77  | 3.67  | 3.61  | 3.56  | 3.51  | 3.45  | 3.39  | 3.33     |
|          | 10   | 6.94   | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  | 3.72  | 3.62  | 3.52  | 3.42  | 3.37  | 3.31  | 3.26  | 3.20  | 3.14  | 3.08     |
|          | 11   | 6.72   | 5.26  | 4.63  | 4.28  | 4.04  | 3.88  | 3.76  | 3.66  | 3.59  | 3.53  | 3.43  | 3.33  | 3.23  | 3.17  | 3.12  | 3.06  | 3.00  | 2.94  | 2.88     |
|          | 12   | 6.55   | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  | 3.37  | 3.28  | 3.18  | 3.07  | 3.02  | 2.96  | 2.91  | 2.85  | 2.79  | 2.72     |
|          | 13   | 6.41   | 4.97  | 4.35  | 4.00  | 3.77  | 3.60  | 3.48  | 3.39  | 3.31  | 3.25  | 3.15  | 3.05  | 2.95  | 2.89  | 2.84  | 2.78  | 2.72  | 2.66  | 2.60     |
|          | 14   | 6.30   | 4.86  | 4.24  | 3.89  | 3.66  | 3.50  | 3.38  | 3.29  | 3.21  | 3.15  | 3.05  | 2.95  | 2.84  | 2.79  | 2.73  | 2.67  | 2.61  | 2.55  | 2.49     |
|          | 15   | 6.20   | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  | 3.06  | 2.96  | 2.86  | 2.76  | 2.70  | 2.64  | 2.59  | 2.52  | 2.46  | 2.40     |
|          | 16   | 6.12   | 4.69  | 4.08  | 3.73  | 3.50  | 3.34  | 3.22  | 3.12  | 3.05  | 2.99  | 2.89  | 2.79  | 2.68  | 2.63  | 2.57  | 2.51  | 2.45  | 2.38  | 2.32     |
|          | 17   | 6.04   | 4.62  | 4.01  | 3.66  | 3.44  | 3.28  | 3.16  | 3.06  | 2.98  | 2.92  | 2.82  | 2.72  | 2.62  | 2.56  | 2.50  | 2.44  | 2.38  | 2.32  | 2.25     |
|          | 18   | 5.98   | 4.56  | 3.95  | 3.61  | 3.38  | 3.22  | 3.10  | 3.01  | 2.93  | 2.87  | 2.77  | 2.67  | 2.56  | 2.50  | 2.44  | 2.38  | 2.32  | 2.26  | 2.19     |
|          | 19   | 5.92   | 4.51  | 3.90  | 3.56  | 3.33  | 3.17  | 3.05  | 2.96  | 2.88  | 2.82  | 2.72  | 2.62  | 2.51  | 2.45  | 2.39  | 2.33  | 2.27  | 2.20  | 2.13     |
|          | 20   | 5.87   | 4.46  | 3.86  | 3.51  | 3.29  | 3.13  | 3.01  | 2.91  | 2.84  | 2.77  | 2.68  | 2.57  | 2.46  | 2.41  | 2.35  | 2.29  | 2.22  | 2.16  | 2.09     |
|          | 21   | 5.83   | 4.42  | 3.82  | 3.48  | 3.25  | 3.09  | 2.97  | 2.87  | 2.80  | 2.73  | 2.64  | 2.53  | 2.42  | 2.37  | 2.31  | 2.25  | 2.18  | 2.11  | 2.04     |
|          | 22   | 5.79   | 4.38  | 3.78  | 3.44  | 3.22  | 3.05  | 2.93  | 2.84  | 2.76  | 2.70  | 2.60  | 2.50  | 2.39  | 2.33  | 2.27  | 2.21  | 2.14  | 2.08  | 2.00     |
|          | 23   | 5.75   | 4.35  | 3.75  | 3.41  | 3.18  | 3.02  | 2.90  | 2.81  | 2.73  | 2.67  | 2.57  | 2.47  | 2.36  | 2.30  | 2.24  | 2.18  | 2.11  | 2.04  | 1.97     |
|          | 24   | 5.72   | 4.32  | 3.72  | 3.38  | 3.15  | 2.99  | 2.87  | 2.78  | 2.70  | 2.64  | 2.54  | 2.44  | 2.33  | 2.27  | 2.21  | 2.15  | 2.08  | 2.01  | 1.94     |
| 25       | 5.69 | 4.29   | 3.69  | 3.35  | 3.13  | 2.97  | 2.85  | 2.75  | 2.68  | 2.61  | 2.51  | 2.41  | 2.30  | 2.24  | 2.18  | 2.12  | 2.05  | 1.98  | 1.91  |          |
| 26       | 5.66 | 4.27   | 3.67  | 3.33  | 3.10  | 2.94  | 2.82  | 2.73  | 2.65  | 2.59  | 2.49  | 2.39  | 2.28  | 2.22  | 2.16  | 2.09  | 2.03  | 1.95  | 1.88  |          |
| 27       | 5.63 | 4.24   | 3.65  | 3.31  | 3.08  | 2.92  | 2.80  | 2.71  | 2.63  | 2.57  | 2.47  | 2.36  | 2.25  | 2.19  | 2.13  | 2.07  | 2.00  | 1.93  | 1.85  |          |
| 28       | 5.61 | 4.22   | 3.63  | 3.29  | 3.06  | 2.90  | 2.78  | 2.69  | 2.61  | 2.55  | 2.45  | 2.34  | 2.23  | 2.17  | 2.11  | 2.05  | 1.98  | 1.91  | 1.83  |          |
| 29       | 5.59 | 4.20   | 3.61  | 3.27  | 3.04  | 2.88  | 2.76  | 2.67  | 2.59  | 2.53  | 2.43  | 2.32  | 2.21  | 2.15  | 2.09  | 2.03  | 1.96  | 1.89  | 1.81  |          |
| 30       | 5.57 | 4.18   | 3.59  | 3.25  | 3.03  | 2.87  | 2.75  | 2.65  | 2.57  | 2.51  | 2.41  | 2.31  | 2.20  | 2.14  | 2.07  | 2.01  | 1.94  | 1.87  | 1.79  |          |
| 40       | 5.42 | 4.05   | 3.46  | 3.13  | 2.90  | 2.74  | 2.62  | 2.53  | 2.45  | 2.39  | 2.29  | 2.18  | 2.07  | 2.01  | 1.94  | 1.88  | 1.80  | 1.72  | 1.64  |          |
| 60       | 5.29 | 3.93   | 3.34  | 3.01  | 2.79  | 2.63  | 2.51  | 2.41  | 2.33  | 2.27  | 2.17  | 2.06  | 1.94  | 1.88  | 1.82  | 1.74  | 1.67  | 1.58  | 1.48  |          |
| 120      | 5.15 | 3.80   | 3.23  | 2.89  | 2.67  | 2.52  | 2.39  | 2.30  | 2.22  | 2.16  | 2.05  | 1.94  | 1.82  | 1.76  | 1.69  | 1.61  | 1.53  | 1.43  | 1.31  |          |
| $\infty$ | 5.02 | 3.69   | 3.12  | 2.79  | 2.57  | 2.41  | 2.29  | 2.19  | 2.11  | 2.05  | 1.94  | 1.83  | 1.71  | 1.64  | 1.57  | 1.48  | 1.39  | 1.27  | 1.00  |          |

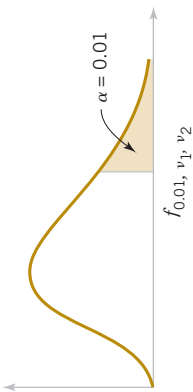
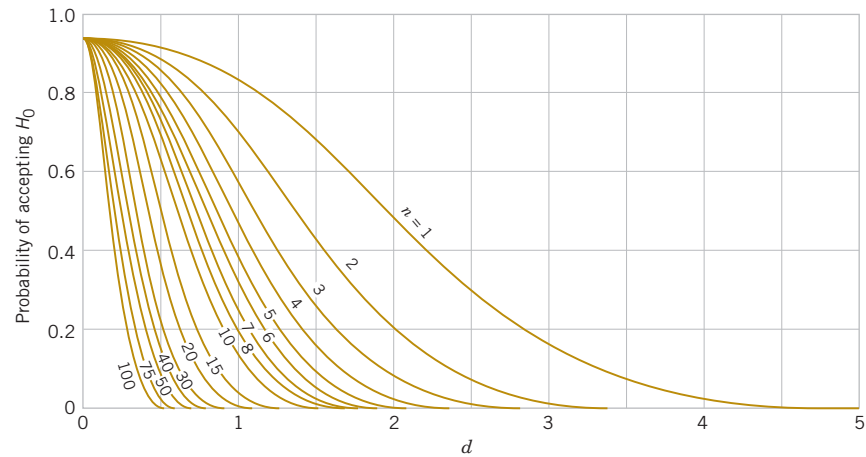
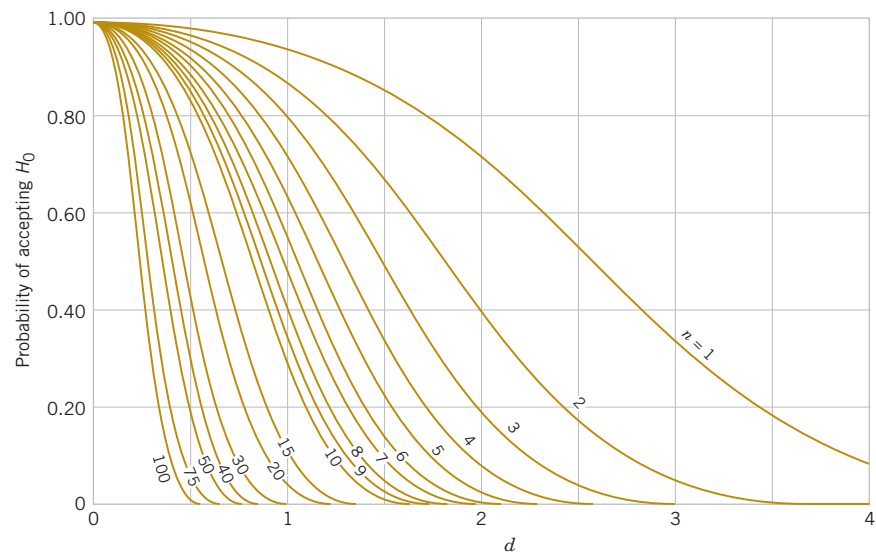


TABLE • VI Percentage Points  $f_{\alpha, v_1, v_2}$  of the F Distribution (Continued)

|          |          | Degrees of freedom for the numerator ( $v_1$ ) |        |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |  |  |
|----------|----------|--|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|--|--|
| $v_2$    | $v_1$    | 1  | 2      | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ | Degrees of freedom for the denominator ( $v_2$ ) |  |
|          |          | 4052   | 4999.5 | 5403  | 5625  | 5764  | 5859  | 5928  | 5982  | 6022  | 6056  | 6106  | 6157  | 6209  | 6235  | 6261  | 6287  | 6313  | 6339  | 6366     |  |  |
| 1        | 2        | 98.50  | 99.00  | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50    | 1  |  |
| 2        | 3        | 34.12  | 30.82  | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.87 | 26.69 | 26.00 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13    | 2  |  |
| 3        | 4        | 21.20  | 18.00  | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46    | 3  |  |
| 4        | 5        | 16.26  | 13.27  | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89  | 9.72  | 9.55  | 9.47  | 9.38  | 9.29  | 9.20  | 9.11  | 9.02     | 4  |  |
| 5        | 6        | 13.75  | 10.92  | 9.78  | 9.15  | 8.75  | 8.47  | 8.26  | 8.10  | 7.98  | 7.87  | 7.72  | 7.56  | 7.40  | 7.31  | 7.23  | 7.14  | 7.06  | 6.97  | 6.88     | 5  |  |
| 6        | 7        | 12.25  | 9.55   | 8.45  | 7.85  | 7.46  | 7.19  | 6.99  | 6.84  | 6.72  | 6.62  | 6.47  | 6.31  | 6.16  | 6.07  | 5.99  | 5.91  | 5.82  | 5.74  | 5.65     | 6  |  |
| 7        | 8        | 11.26  | 8.65   | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  | 5.81  | 5.67  | 5.52  | 5.36  | 5.28  | 5.20  | 5.12  | 5.03  | 4.95  | 4.86     | 7  |  |
| 8        | 9        | 10.56  | 8.02   | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  | 5.26  | 5.11  | 4.96  | 4.81  | 4.73  | 4.65  | 4.57  | 4.48  | 4.40  | 4.31     | 8  |  |
| 9        | 10       | 10.04  | 7.56   | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  | 4.85  | 4.71  | 4.56  | 4.41  | 4.33  | 4.25  | 4.17  | 4.08  | 4.00  | 3.91     | 9  |  |
| 10       | 11       | 9.65   | 7.21   | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  | 4.54  | 4.40  | 4.25  | 4.10  | 4.02  | 3.94  | 3.86  | 3.78  | 3.69  | 3.60     | 10   |  |
| 11       | 12       | 9.33   | 6.93   | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  | 4.30  | 4.16  | 4.01  | 3.86  | 3.78  | 3.70  | 3.62  | 3.54  | 3.45  | 3.36     | 11   |  |
| 12       | 13       | 9.07   | 6.70   | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  | 4.10  | 3.96  | 3.82  | 3.66  | 3.59  | 3.51  | 3.43  | 3.34  | 3.25  | 3.17     | 12   |  |
| 13       | 14       | 8.86   | 6.51   | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  | 3.94  | 3.80  | 3.66  | 3.51  | 3.43  | 3.35  | 3.27  | 3.18  | 3.09  | 3.00     | 13   |  |
| 14       | 15       | 8.68   | 6.36   | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  | 3.80  | 3.67  | 3.52  | 3.37  | 3.29  | 3.21  | 3.13  | 3.05  | 2.96  | 2.87     | 14   |  |
| 15       | 16       | 8.53   | 6.23   | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  | 3.69  | 3.55  | 3.41  | 3.26  | 3.18  | 3.10  | 3.02  | 2.93  | 2.84  | 2.75     | 15   |  |
| 16       | 17       | 8.40   | 6.11   | 5.18  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  | 3.59  | 3.46  | 3.31  | 3.16  | 3.08  | 3.00  | 2.92  | 2.83  | 2.75  | 2.65     | 16   |  |
| 17       | 18       | 8.29   | 6.01   | 5.09  | 4.58  | 4.25  | 4.01  | 3.84  | 3.71  | 3.60  | 3.51  | 3.37  | 3.23  | 3.08  | 3.00  | 2.92  | 2.84  | 2.75  | 2.66  | 2.57     | 17   |  |
| 18       | 19       | 8.18   | 5.93   | 5.01  | 4.50  | 4.17  | 3.94  | 3.77  | 3.63  | 3.52  | 3.43  | 3.30  | 3.15  | 3.00  | 2.92  | 2.84  | 2.76  | 2.67  | 2.58  | 2.50     | 18   |  |
| 19       | 20       | 8.10   | 5.85   | 4.94  | 4.43  | 4.10  | 3.87  | 3.70  | 3.56  | 3.46  | 3.37  | 3.23  | 3.09  | 2.94  | 2.86  | 2.78  | 2.69  | 2.61  | 2.52  | 2.42     | 19   |  |
| 20       | 21       | 8.02   | 5.78   | 4.87  | 4.37  | 4.04  | 3.81  | 3.64  | 3.51  | 3.40  | 3.31  | 3.17  | 3.03  | 2.88  | 2.80  | 2.72  | 2.64  | 2.55  | 2.46  | 2.36     | 20   |  |
| 21       | 22       | 7.95   | 5.72   | 4.82  | 4.31  | 3.99  | 3.76  | 3.59  | 3.45  | 3.34  | 3.25  | 3.12  | 2.98  | 2.83  | 2.75  | 2.67  | 2.58  | 2.50  | 2.40  | 2.31     | 21   |  |
| 22       | 23       | 7.88   | 5.66   | 4.76  | 4.26  | 3.94  | 3.71  | 3.54  | 3.41  | 3.30  | 3.21  | 3.07  | 2.93  | 2.78  | 2.70  | 2.62  | 2.54  | 2.45  | 2.35  | 2.26     | 22   |  |
| 23       | 24       | 7.82   | 5.61   | 4.72  | 4.22  | 3.90  | 3.67  | 3.50  | 3.36  | 3.26  | 3.17  | 3.03  | 2.89  | 2.74  | 2.66  | 2.58  | 2.49  | 2.40  | 2.31  | 2.21     | 23   |  |
| 24       | 25       | 7.77   | 5.57   | 4.68  | 4.18  | 3.85  | 3.63  | 3.46  | 3.32  | 3.22  | 3.13  | 2.99  | 2.85  | 2.70  | 2.62  | 2.54  | 2.45  | 2.36  | 2.27  | 2.17     | 24   |  |
| 25       | 26       | 7.72   | 5.53   | 4.64  | 4.14  | 3.82  | 3.59  | 3.42  | 3.29  | 3.18  | 3.09  | 2.96  | 2.81  | 2.66  | 2.58  | 2.50  | 2.42  | 2.33  | 2.23  | 2.13     | 25   |  |
| 26       | 27       | 7.68   | 5.49   | 4.60  | 4.11  | 3.78  | 3.56  | 3.39  | 3.26  | 3.15  | 3.06  | 2.93  | 2.78  | 2.63  | 2.55  | 2.47  | 2.38  | 2.29  | 2.20  | 2.10     | 26   |  |
| 27       | 28       | 7.64   | 5.45   | 4.57  | 4.07  | 3.75  | 3.53  | 3.36  | 3.23  | 3.12  | 3.03  | 2.90  | 2.75  | 2.60  | 2.52  | 2.44  | 2.35  | 2.26  | 2.17  | 2.06     | 27   |  |
| 28       | 29       | 7.60   | 5.42   | 4.54  | 4.04  | 3.73  | 3.50  | 3.33  | 3.20  | 3.09  | 3.00  | 2.87  | 2.73  | 2.57  | 2.49  | 2.41  | 2.33  | 2.23  | 2.14  | 2.03     | 28   |  |
| 29       | 30       | 7.56   | 5.39   | 4.51  | 4.02  | 3.70  | 3.47  | 3.30  | 3.17  | 3.07  | 2.98  | 2.84  | 2.70  | 2.55  | 2.47  | 2.39  | 2.30  | 2.21  | 2.11  | 2.01     | 29   |  |
| 30       | 40       | 7.31   | 5.18   | 4.31  | 3.83  | 3.51  | 3.29  | 3.12  | 2.99  | 2.89  | 2.80  | 2.66  | 2.52  | 2.37  | 2.29  | 2.20  | 2.11  | 2.02  | 1.92  | 1.80     | 30   |  |
| 60       | 60       | 7.08   | 4.98   | 4.13  | 3.65  | 3.34  | 3.12  | 2.95  | 2.82  | 2.72  | 2.63  | 2.50  | 2.35  | 2.20  | 2.12  | 2.03  | 1.94  | 1.84  | 1.73  | 1.60     | 60   |  |
| 120      | 120      | 6.85   | 4.79   | 3.95  | 3.48  | 3.17  | 2.96  | 2.79  | 2.66  | 2.56  | 2.47  | 2.34  | 2.19  | 2.03  | 1.95  | 1.86  | 1.76  | 1.66  | 1.53  | 1.38     | 120  |  |
| $\infty$ | $\infty$ | 6.63   | 4.61   | 3.78  | 3.32  | 3.02  | 2.80  | 2.64  | 2.51  | 2.41  | 2.32  | 2.18  | 2.04  | 1.88  | 1.79  | 1.70  | 1.59  | 1.47  | 1.32  | 1.00     | $\infty$   |  |

**Chart • VII Operating Characteristic Curves (Continued)**

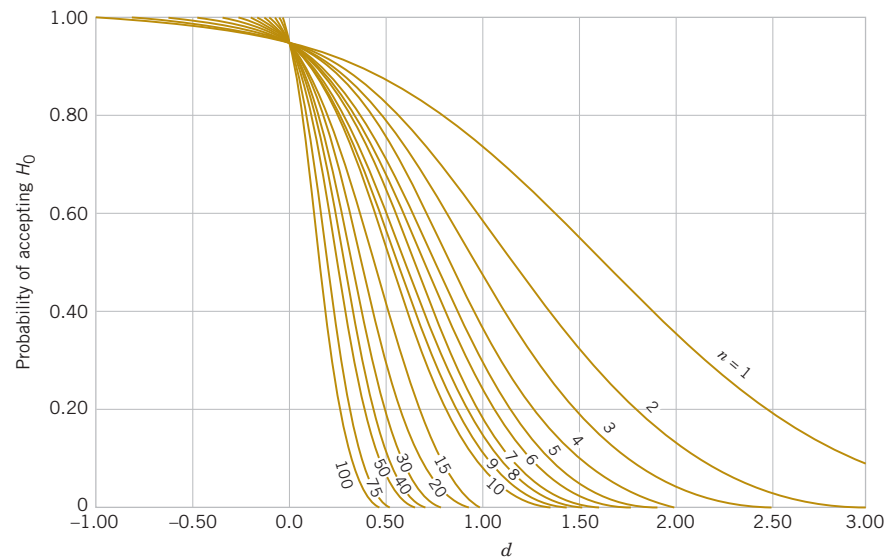
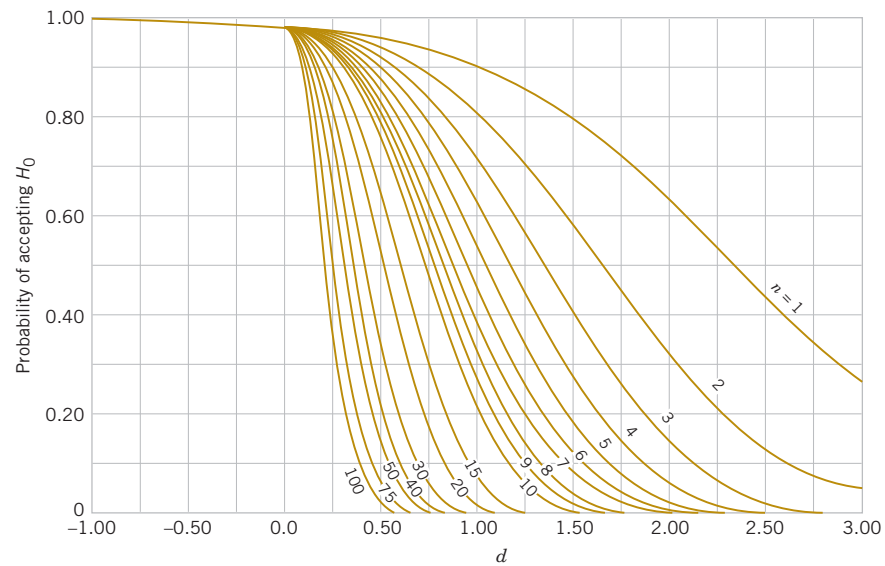
(a) O.C. curves for different values of  $n$  for the two-sided normal test for a level of significance  $\alpha = 0.05$ .



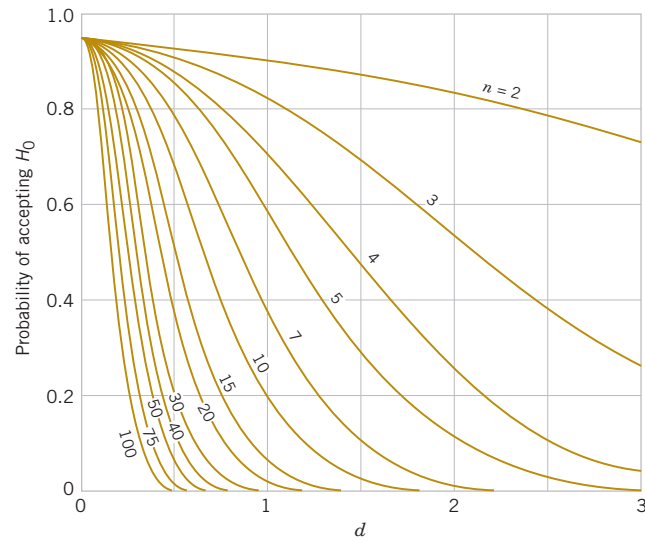
(b) O.C. curves for different values of  $n$  for the two-sided normal test for a level of significance  $\alpha = 0.01$ .

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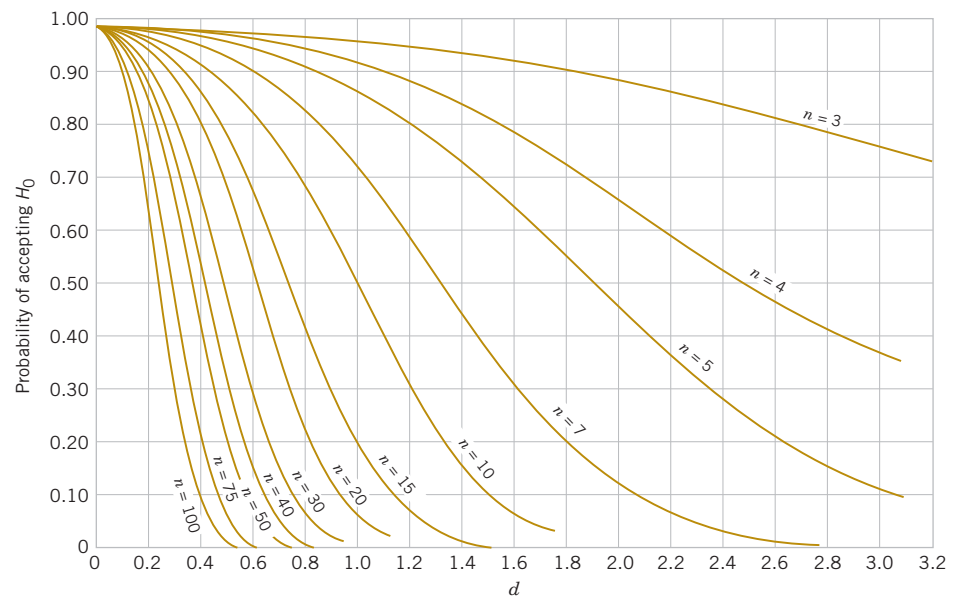
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**Chart • VII** Operating Characteristic Curves (*Continued*)(c) O.C. curves for different values of  $n$  for the one-sided normal test for a level of significance  $\alpha = 0.05$ .(d) O.C. curves for different values of  $n$  for the one-sided normal test for a level of significance  $\alpha = 0.01$ .

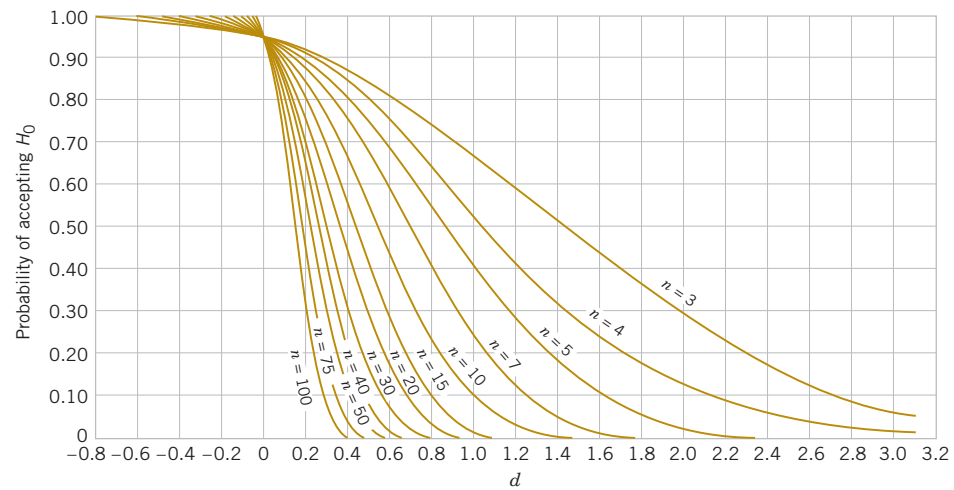


**Chart • VII** Operating Characteristic Curves (*Continued*)

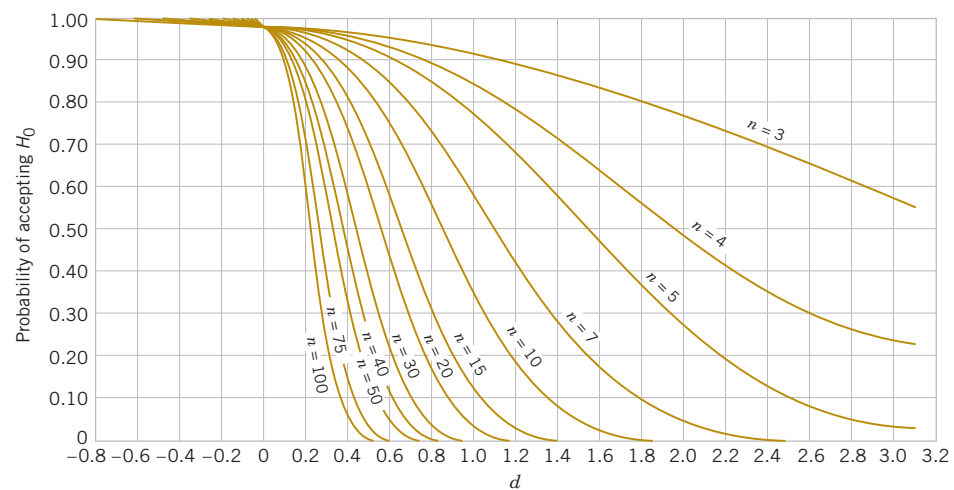
(e) O.C. curves for different values of  $n$  for the two-sided  $t$ -test for a level of significance  $\alpha = 0.05$ .



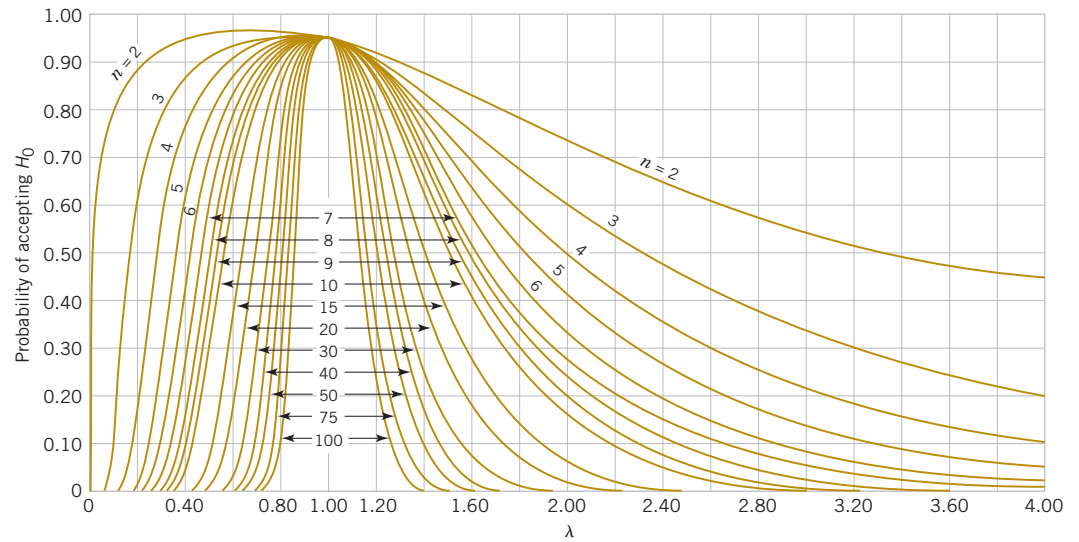
(f) O.C. curves for different values of  $n$  for the two-sided  $t$ -test for a level of significance  $\alpha = 0.01$ .

**Chart • VII** Operating Characteristic Curves (*Continued*)

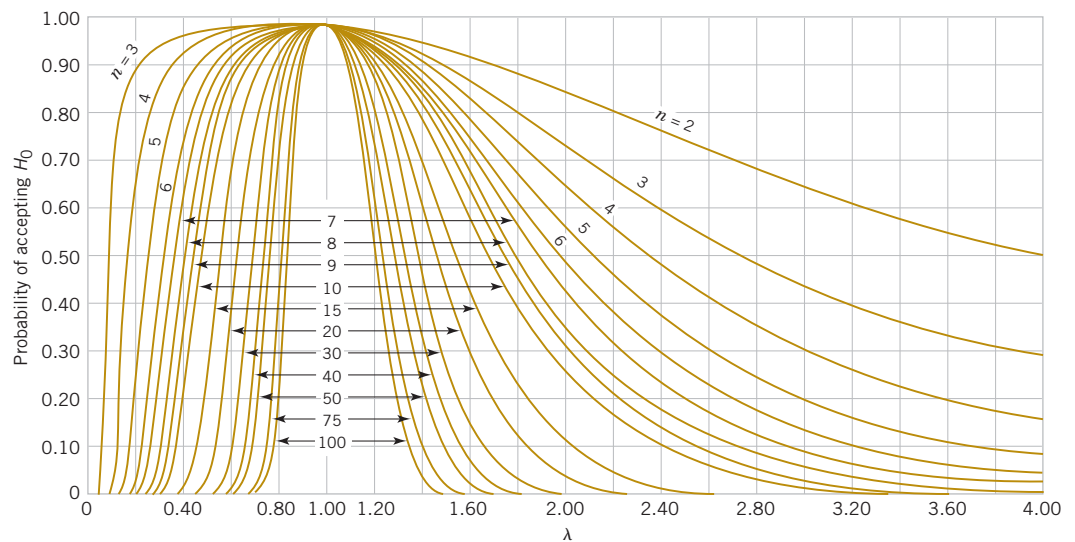
(g) O.C. curves for different values of  $n$  for the one-sided  $t$ -test for a level of significance  $\alpha = 0.05$ .



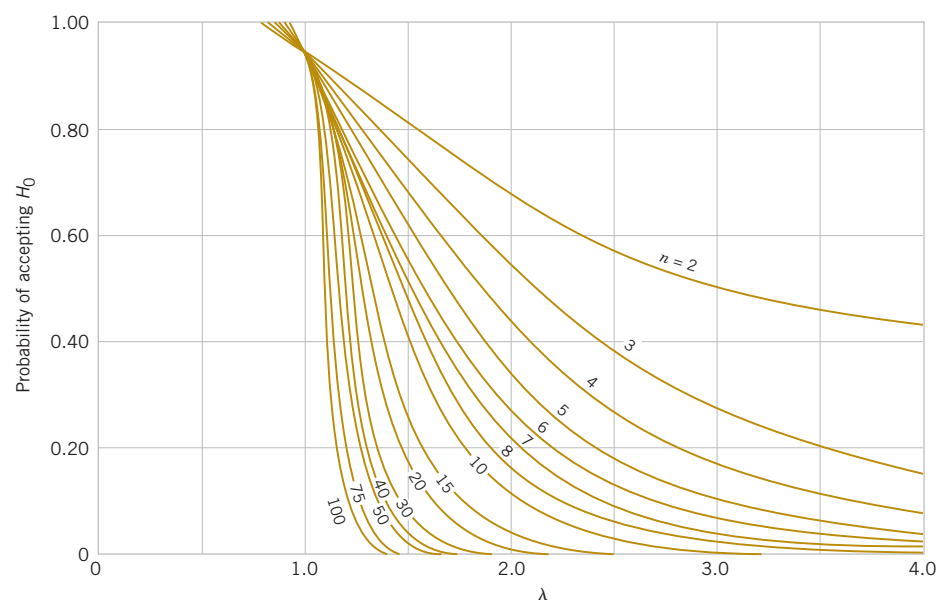
(h) O.C. curves for different values of  $n$  for the one-sided  $t$ -test for a level of significance  $\alpha = 0.01$ .

Chart • VII Operating Characteristic Curves (*Continued*)

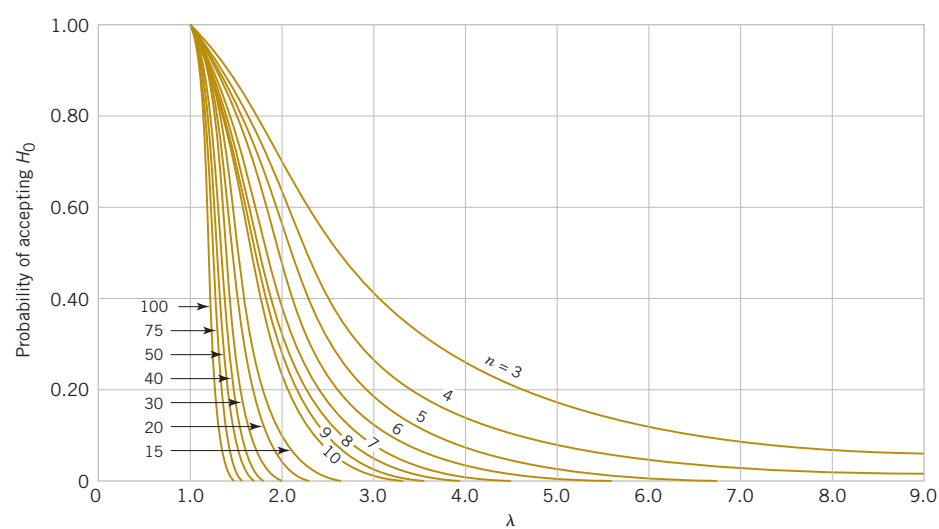
(i) O.C. curves for different values of  $n$  for the two-sided chi-square test for a level of significance  $\alpha = 0.05$ .



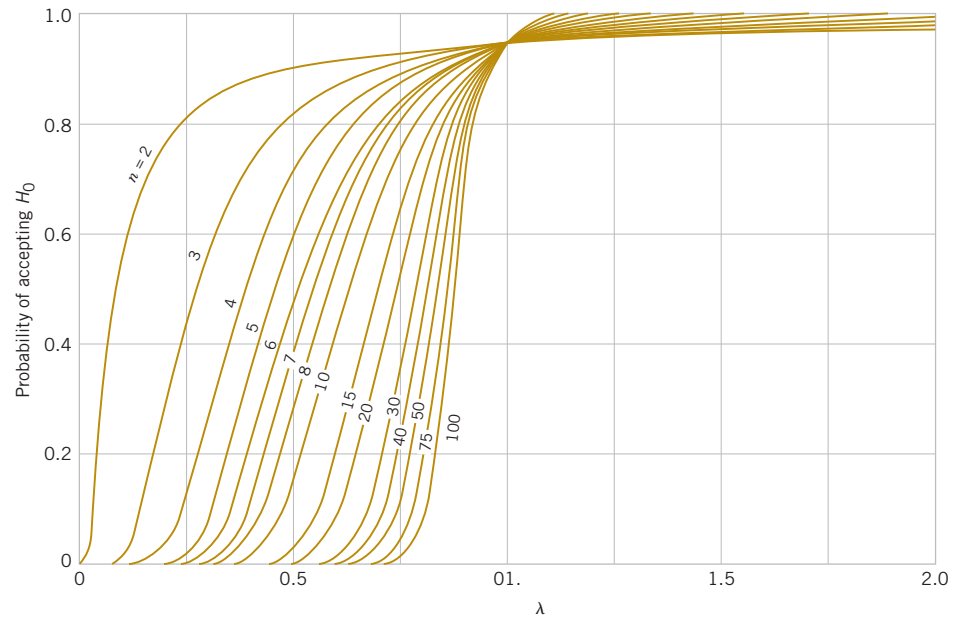
(j) O.C. curves for different values of  $n$  for the two-sided chi-square test for a level of significance  $\alpha = 0.01$ .

**Chart • VII** Operating Characteristic Curves (*Continued*)

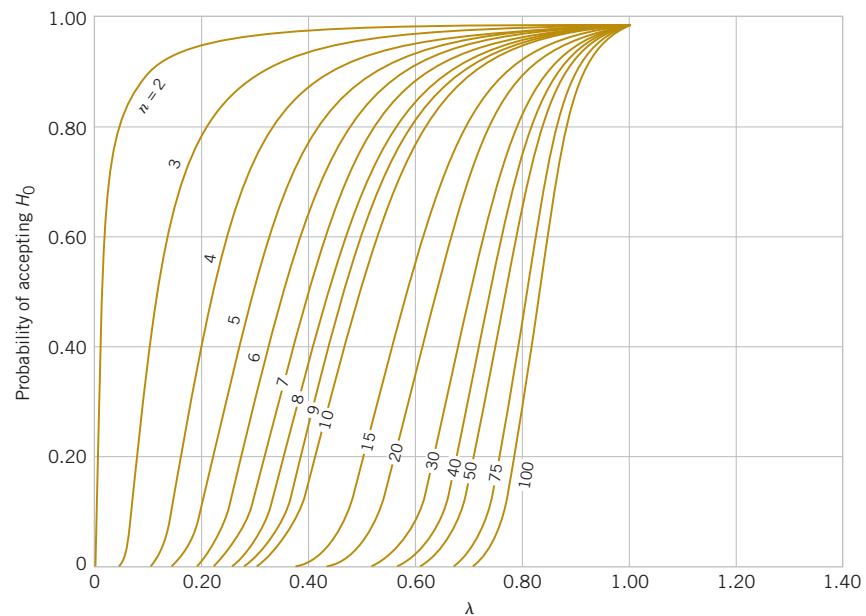
(k) O.C. curves for different values of  $n$  for the one-sided (upper-tail) chi-square test for a level of significance  $\alpha = 0.05$ .



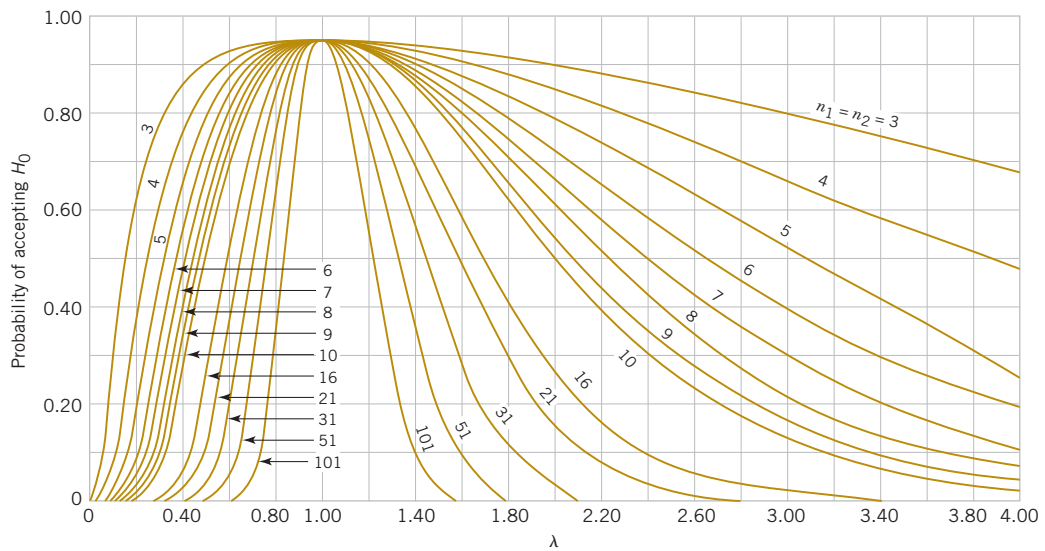
(l) O.C. curves for different values of  $n$  for the one-sided (upper-tail) chi-square test for a level of significance  $\alpha = 0.01$ .

**Chart • VII** Operating Characteristic Curves (*Continued*)

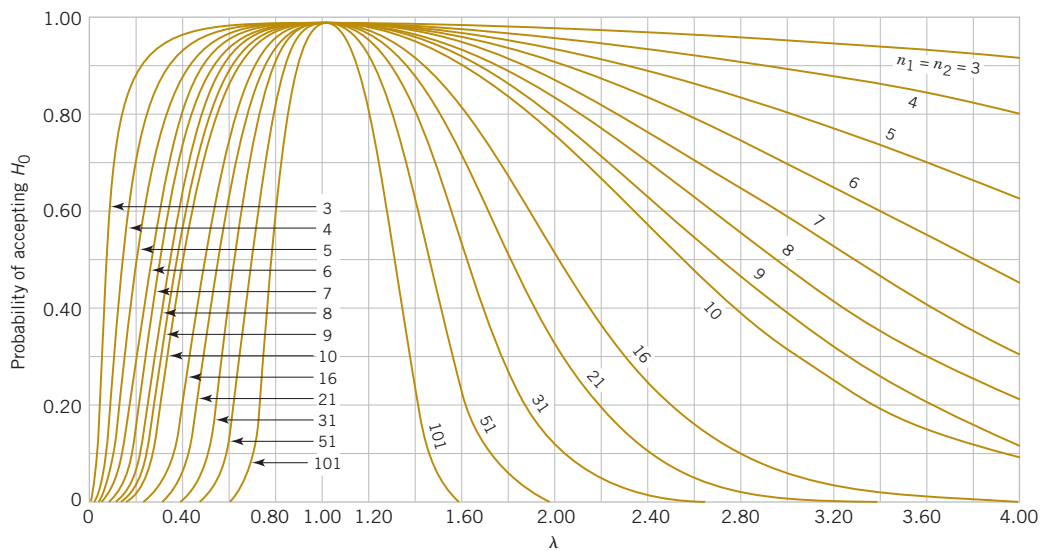
(m) O.C. curves for different values of  $n$  for the one-sided (lower-tail) chi-square test for a level of significance  $\alpha = 0.05$ .



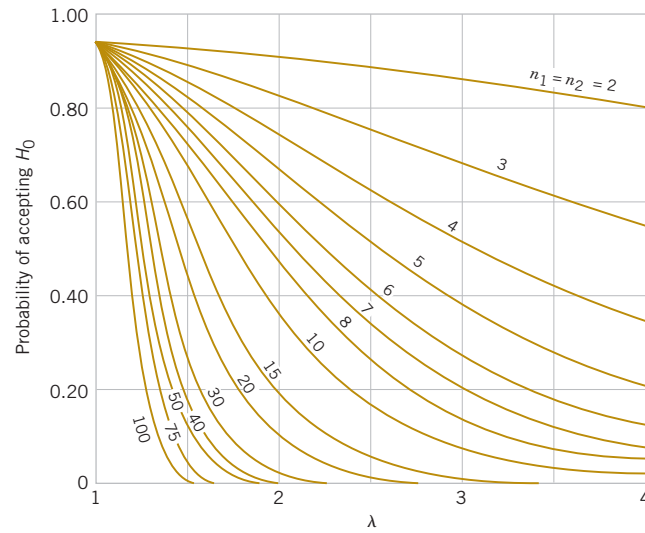
(n) O.C. curves for different values of  $n$  for the one-sided (lower-tail) chi-square test for a level of significance  $\alpha = 0.01$ .

Chart • VII Operating Characteristic Curves (*Continued*)

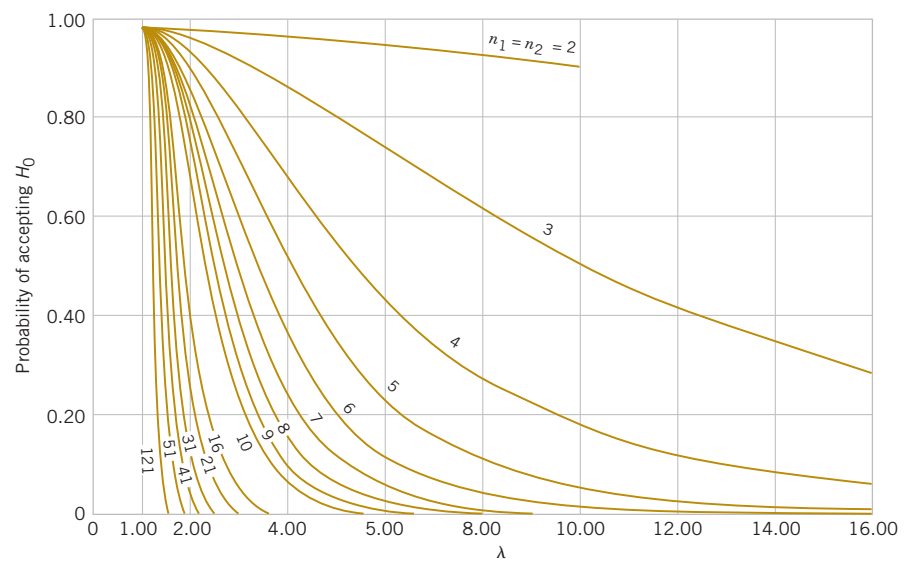
(o) O.C. curves for different values of  $n$  for the two-sided  $F$ -test for a level of significance  $\alpha = 0.05$ .



(p) O.C. curves for different values of  $n$  for the two-sided  $F$ -test for a level of significance  $\alpha = 0.01$ .

**Chart • VII** Operating Characteristic Curves (*Continued*)

(*q*) O.C. curves for different values of  $n$  for the one-sided  $F$ -test for a level of significance  $\alpha = 0.05$ .



(*r*) O.C. curves for different values of  $n$  for the one-sided  $F$ -test for a level of significance  $\alpha = 0.01$ .

**TABLE • VIII** Critical Values for the Sign Test $r_{\alpha}^*$ 

| $n \backslash \alpha$ | 0.10<br>0.05 | 0.05<br>0.025 | 0.01<br>0.005 | Two-sided tests<br>One-sided tests | $n \backslash \alpha$ | 0.10<br>0.05 | 0.05<br>0.025 | 0.01<br>0.005 | Two-sided tests<br>One-sided tests |
|-----------------------|--------------|---------------|---------------|------------------------------------|-----------------------|--------------|---------------|---------------|------------------------------------|
| 5                     | 0            |               |               |                                    | 23                    | 7            | 6             | 4             |                                    |
| 6                     | 0            | 0             |               |                                    | 24                    | 7            | 6             | 5             |                                    |
| 7                     | 0            | 0             |               |                                    | 25                    | 7            | 7             | 5             |                                    |
| 8                     | 1            | 0             | 0             |                                    | 26                    | 8            | 7             | 6             |                                    |
| 9                     | 1            | 1             | 0             |                                    | 27                    | 8            | 7             | 6             |                                    |
| 10                    | 1            | 1             | 0             |                                    | 28                    | 9            | 8             | 6             |                                    |
| 11                    | 2            | 1             | 0             |                                    | 29                    | 9            | 8             | 7             |                                    |
| 12                    | 2            | 2             | 1             |                                    | 30                    | 10           | 9             | 7             |                                    |
| 13                    | 3            | 2             | 1             |                                    | 31                    | 10           | 9             | 7             |                                    |
| 14                    | 3            | 2             | 1             |                                    | 32                    | 10           | 9             | 8             |                                    |
| 15                    | 3            | 3             | 2             |                                    | 33                    | 11           | 10            | 8             |                                    |
| 16                    | 4            | 3             | 2             |                                    | 34                    | 11           | 10            | 9             |                                    |
| 17                    | 4            | 4             | 2             |                                    | 35                    | 12           | 11            | 9             |                                    |
| 18                    | 5            | 4             | 3             |                                    | 36                    | 12           | 11            | 9             |                                    |
| 19                    | 5            | 4             | 3             |                                    | 37                    | 13           | 12            | 10            |                                    |
| 20                    | 5            | 5             | 3             |                                    | 38                    | 13           | 12            | 10            |                                    |
| 21                    | 6            | 5             | 4             |                                    | 39                    | 13           | 12            | 11            |                                    |
| 22                    | 6            | 5             | 4             |                                    | 40                    | 14           | 13            | 11            |                                    |

**TABLE • IX** Critical Values for the Wilcoxon Signed-Rank Test $w_{\alpha}^*$ 

| $n^* \backslash \alpha$ | 0.10<br>0.05 | 0.05<br>0.025 | 0.02<br>0.01 | 0.01<br>0.005 | Two-sided tests<br>One-sided tests |
|-------------------------|--------------|---------------|--------------|---------------|------------------------------------|
| 4                       |              |               |              |               |                                    |
| 5                       | 0            |               |              |               |                                    |
| 6                       | 2            | 0             |              |               |                                    |
| 7                       | 3            | 2             | 0            |               |                                    |
| 8                       | 5            | 3             | 1            | 0             |                                    |
| 9                       | 8            | 5             | 3            | 1             |                                    |
| 10                      | 10           | 8             | 5            | 3             |                                    |
| 11                      | 13           | 10            | 7            | 5             |                                    |
| 12                      | 17           | 13            | 9            | 7             |                                    |
| 13                      | 21           | 17            | 12           | 9             |                                    |
| 14                      | 25           | 21            | 15           | 12            |                                    |
| 15                      | 30           | 25            | 19           | 15            |                                    |
| 16                      | 35           | 29            | 23           | 19            |                                    |
| 17                      | 41           | 34            | 27           | 23            |                                    |
| 18                      | 47           | 40            | 32           | 27            |                                    |
| 19                      | 53           | 46            | 37           | 32            |                                    |
| 20                      | 60           | 52            | 43           | 37            |                                    |
| 21                      | 67           | 58            | 49           | 42            |                                    |
| 22                      | 75           | 65            | 55           | 48            |                                    |
| 23                      | 83           | 73            | 62           | 54            |                                    |
| 24                      | 91           | 81            | 69           | 61            |                                    |
| 25                      | 100          | 89            | 76           | 68            |                                    |

\*If  $n > 25$ ,  $W^-$  (or  $W^+$ ) is approximately normally distributed with mean  $n(n+1)/4$  and variance  $n(n+1)(2n+1)/24$ .





**TABLE • XI** Factors for Constructing Variables Control Charts

| <i>n</i> <sup>*</sup> | Factor for Control Limits |                       |                       |                       |                       |                       | <i>n</i> |
|-----------------------|---------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|----------|
|                       | $\bar{X}$ Chart           |                       |                       | <i>R</i> Chart        |                       | <i>S</i> Chart        |          |
|                       | <i>A</i> <sub>1</sub>     | <i>A</i> <sub>2</sub> | <i>d</i> <sub>2</sub> | <i>D</i> <sub>3</sub> | <i>D</i> <sub>4</sub> | <i>C</i> <sub>4</sub> |          |
| 2                     | 3.760                     | 1.880                 | 1.128                 | 0                     | 3.267                 | 0.7979                | 2        |
| 3                     | 2.394                     | 1.023                 | 1.693                 | 0                     | 2.575                 | 0.8862                | 3        |
| 4                     | 1.880                     | .729                  | 2.059                 | 0                     | 2.282                 | 0.9213                | 4        |
| 5                     | 1.596                     | .577                  | 2.326                 | 0                     | 2.115                 | 0.9400                | 5        |
| 6                     | 1.410                     | .483                  | 2.534                 | 0                     | 2.004                 | 0.9515                | 6        |
| 7                     | 1.277                     | .419                  | 2.704                 | .076                  | 1.924                 | 0.9594                | 7        |
| 8                     | 1.175                     | .373                  | 2.847                 | .136                  | 1.864                 | 0.9650                | 8        |
| 9                     | 1.094                     | .337                  | 2.970                 | .184                  | 1.816                 | 0.9693                | 9        |
| 10                    | 1.028                     | .308                  | 3.078                 | .223                  | 1.777                 | 0.9727                | 10       |
| 11                    | .973                      | .285                  | 3.173                 | .256                  | 1.744                 | 0.9754                | 11       |
| 12                    | .925                      | .266                  | 3.258                 | .284                  | 1.716                 | 0.9776                | 12       |
| 13                    | .884                      | .249                  | 3.336                 | .308                  | 1.692                 | 0.9794                | 13       |
| 14                    | .848                      | .235                  | 3.407                 | .329                  | 1.671                 | 0.9810                | 14       |
| 15                    | .816                      | .223                  | 3.472                 | .348                  | 1.652                 | 0.9823                | 15       |
| 16                    | .788                      | .212                  | 3.532                 | .364                  | 1.636                 | 0.9835                | 16       |
| 17                    | .762                      | .203                  | 3.588                 | .379                  | 1.621                 | 0.9845                | 17       |
| 18                    | .738                      | .194                  | 3.640                 | .392                  | 1.608                 | 0.9854                | 18       |
| 19                    | .717                      | .187                  | 3.689                 | .404                  | 1.596                 | 0.9862                | 19       |
| 20                    | .697                      | .180                  | 3.735                 | .414                  | 1.586                 | 0.9869                | 20       |
| 21                    | .679                      | .173                  | 3.778                 | .425                  | 1.575                 | 0.9876                | 21       |
| 22                    | .662                      | .167                  | 3.819                 | .434                  | 1.566                 | 0.9882                | 22       |
| 23                    | .647                      | .162                  | 3.858                 | .443                  | 1.557                 | 0.9887                | 23       |
| 24                    | .632                      | .157                  | 3.895                 | .452                  | 1.548                 | 0.9892                | 24       |
| 25                    | .619                      | .153                  | 3.931                 | .459                  | 1.541                 | 0.9896                | 25       |

\**n* > 25:  $A_1 = 3/\sqrt{n}$  where *n* = number of observations in sample.