Tuesday, March 30, 2021 12:37 PM

- Entire distribution: pdf / pmf $\left\langle \begin{matrix} \text{range} \\ \text{prob.} \end{matrix} \right)$

- Some characteristics: $\left\langle \begin{matrix} E(X) \\ V(X) \end{matrix} \right)$

- Meaning $X \sim B(n,p)$ $\left\langle \begin{matrix} \text{meaning} \\ \text{pmf} \\ E(X) \\ V(X) \end{matrix} \right.$

$$E(X) = \sum_{k=0}^{n} k f(k) = \sum_{k=0}^{n} k C_n^k p^k (1-p)^{n-k}$$

$$E(X^2) = \sum_{k=0}^{n} k^2 f(k) = \sum_{k=0}^{n} k^2 C_n^k p^k (1-p)^{n-k}$$

n=2: $k C_2^k p^k (1-p)^{2-k}$

$$0 + C_2^1 p(1-p) + 2p^2$$

$$= 0 + 2p(1-p) + 2p^2 = 2p$$

$E(X) = 2p$

$E(X) = np$

$V(X) = E(X^2) - (EX)^2$

$= \underline{np(1-p)}$

$$E(X^2) = 0^2 + C_2^1 p(1-p) + 2^2 p^2$$

$$= 0 + 2p(1-p) + 4p^2 = 2p(1+p)$$

$$V(X) = E(X^2) - (EX)^2 = 2p(1+p) - (2p)^2 = 2p(1-p)$$

If $X \sim B(10, 0.2)$

$V(X) < E(X$

$E(X) = 10(0.2) = 2$

$V(X) = 10(0.2)(0.8) = 1.6$

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant.

(a) Find the probability that, in the next 18 samples, exactly 2 contain the pollutant.

(b) Determine the probability that at least 4 samples contain the pollutant.

(c) Now determine the probability that $3 \leq X < 7$.

X = # of samples containing the pollutant.

$P(3 \leq X < 7) = ?$ pmf of X?

$X \sim B(18, 0.1) \Rightarrow P(3 \leq X < 7) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= C_{18}^{3}\, 0.1^{3}\, 0.9^{15} + C_{18}^{4}\, 0.1^{4}\, 0.9^{14} + C_{18}^{5}\, 0.1^{5}\, 0.9^{13} + C_{18}^{6}\, 0.1^{6}\, 0.9^{12}$

$= 0.2650$

A certain electronic system contains 10 components. Suppose that the probability that each individual component will fail is 0.2 and that the components fail independently of each other. Given that at least one of the components has failed, what is the probability that at least two of the components have failed?

X = # of failures, $X \sim B(10, 0.2)$

$$P(X \geq 2 \mid X \geq 1)$$

$$= \frac{P(X \geq 2, X \geq 1)}{P(X \geq 1)} \qquad \begin{cases} X \geq 2 \\ X \geq 1 \end{cases} \Leftrightarrow X \geq 2$$

$$= \frac{P(X \geq 2)}{P(X \geq 1)} = \frac{1 - P(X=0) - P(X=1)}{1 - P(X=0)}$$

$$P(X=0) = C_{10}^{0}\ 0.2^{0}\ 0.8^{10}$$

$$P(X=1) = C_{10}^{1}\ 0.2^{1}\ 0.8^{9}$$

Three men A, B, and C shoot at a target. Suppose that A shoots three times and the probability that he will hit the target on any given shot is 1/8, B shoots five times and the probability that he will hit the target on any given shot is 1/4, and C shoots twice and the probability that he will hit the target on any given shot is 1/3. What is the expected number of times that the target will be hit?

X = # of times that the target will be hit

X_A = ______________________ A hits the target

$X_B =$ ________ B ________

$X_C =$ ________ C ________

$$E(X) = E(X_A + X_B + X_C)$$
$$= \underline{E(X_A)} + \underline{E(X_B)} + \underline{E(X_C)} = \frac{3}{8} + \frac{5}{4} + \frac{2}{3}$$

$X_A \sim B(3, \frac{1}{8}) \Rightarrow E(X_A) = np = 3 \cdot \frac{1}{8} = 3/8$

$X_B \sim B(5, 1/4) \Rightarrow E(X_B) = 5/4$

$X_C \sim B(2, 1/3) \Rightarrow E(X_C) = 2/3$

$N = 10\,000$, $p = \underbrace{0.1\%}$, $Z = \#$ of tests.

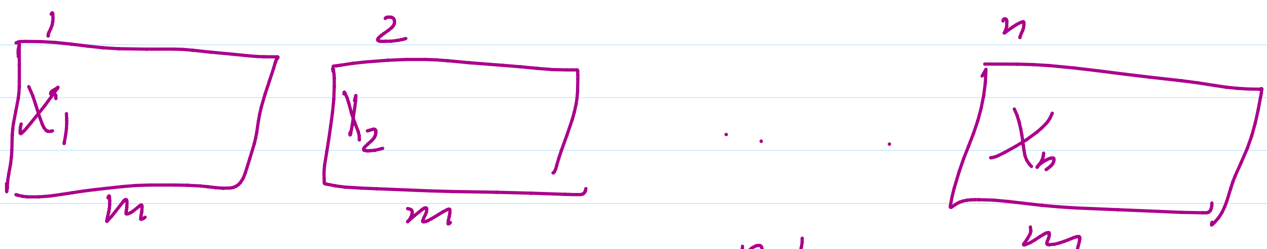
+ Test all: $Z = N$

+ n groups, each group has m people.

  $N = nm$.

  best case: $Z = n$ Optimize m

  worst case: $Z = n + N$



$$Z_k = \begin{cases} 1 & \text{if } X_k > 0 \\ 0 & \text{if } X_k = 0 \end{cases}$$

X_k	1 (retest)	0
P	q	$1-q$

$$Z = n + mZ_1 + mZ_2 + \cdots + mZ_n$$
$$= n + m(Z_1 + \cdots + Z_n)$$
$$E(Z) = n + m\left[E(Z_1) + \cdots + E(Z_n)\right]$$

minimize

$$E(Z_k) = 1q + 0(1-q) = q$$
$$q = \underbrace{P(Z_k = 1)} = P(X_k > 0) = 1 - P(X_k = 0)$$

$$X_k \sim B(m, p)$$
$$\Rightarrow P(X_k = 0) = C_m^0 p^0 (1-p)^m$$
$$\Rightarrow q = 1 - (1-p)^m$$

Thus $E(Z) = n + m \cdot n \cdot \left[1 - (1-p)^m\right]$

$N = nm$

$$= \frac{N}{m} + N\left[1 - (1-p)^m\right]$$
$$= N\Big[\underbrace{\frac{1}{m} + 1 - (1-p)^m}_{f(m)}\Big]$$

$$p = \frac{2600}{96M}$$

$$f(t) = \frac{1}{t} + 1 - (1-p)^{t}$$

$$f'(t) = -\frac{1}{t^2} - (1-p)^t \ln(1-p) = 0$$

t	0	$t_0 = 192.65$	
$f'(t)$	$-$	0	$+$
f	$\searrow$	$\cdot$	$\nearrow$