1. True. A graph with no links is a trivial D-Map because the graph has no link, of course no conditional independency that is satisfied by a trivial D-Map.
   1. Because links from A to B and C are head-to-tail connections. Hence, the flow is blocked at B and C. Hence, A is conditionally independent from D given B and C.
   2. E<-B->F is a tail-to-tail connection. The connection shows that E and E is not independent. Hence, E is not marginally independent of F.
   3. Removing the AC connection is necessary. Because the AC-connection removal will lead to all head-to-head connections that C is independent of A.

Table P(a)

|  |  |
| --- | --- |
| a | P(a) |
| 1 | 0.4 |
| 0 | 0.6 |

Table P(c)

|  |  |
| --- | --- |
| c | P(c) |
| 1 | 0.52 |
| 0 | 0.48 |

Table P(b|c)

|  |  |  |  |
| --- | --- | --- | --- |
| b | c | P(b,c) | P(b|c) |
| 0 | 0 | 0.384 | 0.8 |
| 0 | 1 | 0.208 | 0.4 |
| 1 | 0 | 0.096 | 0.2 |
| 1 | 1 | 0.312 | 0.6 |

Table P(c|a)

|  |  |  |  |
| --- | --- | --- | --- |
| c | a | P(c,a) | P(c|a) |
| 0 | 0 | 0.24 | 0.4 |
| 0 | 1 | 0.24 | 0.6 |
| 1 | 0 | 0.36 | 0.6 |
| 1 | 1 | 0.16 | 0.4 |

|  |  |  |  |
| --- | --- | --- | --- |
| a | b | c | P(a,b,c) = p(a)P(c|a)P(b|c) |
| 0 | 0 | 0 | P(a=0)P(c=0|a=0)P(b=0|c=0)  = 0.6 \* 0.4 \* 0.8 = 0.192 |
| 0 | 0 | 1 | P(a=0)P(c=1|a=0)P(b=0|c=1)  = 0.6\*0.6\*0.4 = 0.144 |
| 0 | 1 | 1 | P(a=0)P(c=1|a=0)P(b=1|c=1)  = 0.6\*0.6\*0.6 = 0.216 |
| 1 | 0 | 0 | P(a=1)P(c=0|a=1)P(b=0|c=0)  = 0.4\*0.6\*0.8 = 0.192 |
| 1 | 0 | 1 | P(a=1)P(c=1|a=1)P(b=0|c=1)  = 0.4\*0.4\*0.4 = 0.064 |
| 1 | 1 | 0 | P(a=1)P(c=0|a=1)P(b=1|c=0)  = 0.4\*0.6\*0.2 = 0.048 |
| 1 | 1 | 1 | P(a=1)P(c=1|a=1)P(b=1|c=1)  = 0.4 \* 0.4 \* 0.6 = 0.096 |

* 1. Write down the expression for P(S=1|V=1)

P(S=1|V=1) = P(S=1,V=1)/P(V=1)

P(V=1) = P(V=1)\*P(R|V=1,G)\*P(G)\*P(S|G)

P(S=1,V=1) = P(V=1)\*P(R|V=1,G)\*P(G)\*P(S=1|G)

* P(S=1|V=1) = P(S=1|G) / P(S|G) = (2 – Y – B) / 2
  1. Since S is marginally independent of V, hence P(S=1|V=1) = P(S=1|V=0) = (2 – Y – B) / 2

P(G=0) = 1/3 and P(G=1) = 2/3 => α = 1/3

P(S =0|G=0) = 1, P(S=1|G=0) = 0, P(S =0|G=1) = 0, P(S = 1|G = 1) = 1

* P(S=0|G=0) = 1= Υ
* P(S = 0 | G= 1) = 0 = β

P(V = 1) = 1, P(V = 0) = 0

* 𝝏 = 0
  1. Each top node (e.g. X1, X2, X3) has 1 parameter. Hence, the node H has 2^3 = 8 parameters (binary nodes). Each bottom node (X4, X5, X6) has 2 parameters that are independent from other nodes. Totally, the model has 3 + 8 + 6 = 17 parameters
  2. Each top node has 1 parameter because the 3 nodes are independent from each other. X4 has 2^3 parameters because it is dependent on X1, X2, X3. Then, X5 has 16 parameters because it is dependent on X1, X2, X4, and X4. Finally, X6 has 32 parameters because it is dependent on the other nodes. Totally, there is 59 parameters

A picture containing text, document

Description automatically generated

Diagram, text, letter

Description automatically generated

Text, letter

Description automatically generated

Diagram

Description automatically generated

Text, letter

Description automatically generated