

BUS2 194B - Business Analytics. Spring 2022

Module 3: Discrete Optimization - Case

Note: This case should be solved in teams of 3, 4 or 5 members. You should present your work progress at the end of today's session, and then submit the final work according to the deadline shown at the end of the problem. One submission per team will be enough as long as all members names are included.

Note 2: The cases need to be solved by each team. If you decide to discuss with other teams, please indicate which teams participated in the discussion.

Case - Ymir goes to Graduate School

Ymir just got accepted into a graduate program at the University of Narnia, in the district of Hogwarts. After finishing her undergrad, she is impressed by the fact that she needs to create her own plan of study. In other words, she can take any course that is required to finalize her thesis project, as long as the total number of credits obtained at the end of the program is at least 100¹.

She wants to develop a project in one of the following areas:

- Quality
- Optimization
- Supply Chain

Ymir needs to attend in-person classes two hours a week for each course that she selects. Besides that, she needs to use some time for extra-class activities, such as reading papers, meetings with her advisor, doing homework, among others. She estimates that 10 hours per week per course, in addition to the two hours for in-person lectures, will be sufficient.

¹Source: created by the instructor

If Ymir understands and passes the class, her thesis will be improved. Not all the courses are equally useful for each topic. For example, the “Quality Engineering” course may be very helpful if she wants to focus her thesis in the area of Quality, but it may not be that useful to improve her project if her thesis is focused on Optimization. After reviewing the whole set of offered courses and their syllabi (yes, she did invest a lot of time on it!), she created the following table that includes the name of the course, the number of credits, the semester that the course is going to be offered, and the two areas in which the course is focused. She is very interested in creating a top-level thesis project. She created a scoring system for this, and included it in the same table under the columns “Main focus” and “Secondary focus”. A course that has a certain area under the “Main focus” column would add 5 points to the thesis level, while a course that has a certain area under the “Secondary focus” column would add 3 points to the thesis level. If the area is not in any of these columns, it means that the course is not useful for her thesis project.

In order to keep a good life-school balance, Ymir should sleep 8 hours daily and dedicate at least another 2 hours daily for self-hygiene, making food, among other activities. Also, leisure time is important. For each weekly hour that Ymir does something completely unrelated to the school (e.g., working out or hanging out with friends) her creativity will work in the back-end, and it may improve her thesis level. Ymir considers each weekly hour for this type of activity will give about 0.1 points, and she decides she wants to have at least 6 hours per day for leisure activities.

(Hint: some data are on a daily basis, and others on a weekly basis. I consider that the problem becomes much easier if everything is on the same basis, weekly or daily)

Course	Credits	Semester	Main focus	Secondary focus
Introduction to Grad Methods: Probability and Statistics.	8	Fall 2022	ALL	
Introduction to Grad Methods: Stochastic Processes.	8	Fall 2022	ALL	
Introduction to Grad Methods: Linear Programming.	8	Fall 2022	ALL	
Quality Engineering.	6	Fall 2022	Quality	Supply Chain
Stochastic Modeling I.	6	Fall 2022	Quality	Optimization
Engineering Decision Making Under Uncertainty.	6	Fall 2022	Supply Chain	Optimization
Queuing Theory.	6	Fall 2022	Optimization	Supply Chain
Survey of Optimization Methods.	7	Fall 2022	Optimization	
Linear Programming.	7	Fall 2022	Optimization	
Fundamentals of Optimization.	7	Fall 2022	Optimization	
Engineering Statistics.	6	Spring 2023	Quality	
Simulation Modeling and Analysis.	6	Spring 2023	Quality	Optimization
Experiment Design and Regression.	6	Spring 2023	Supply Chain	Quality
Algorithms, Graphs, and Network.	7	Spring 2023	Supply Chain	Optimization
Computational Issues in Optimization.	7	Spring 2023	Optimization	
Theory of Linear Systems.	7	Spring 2023	Optimization	
Space Systems Engineering.	7	Spring 2023	Quality	Supply Chain
Systems Engineering Process.	6	Fall 2023	Supply Chain	Optimization
Fundamentals of Guidance for Aerospace Systems.	6	Fall 2023	Quality	Optimization
Project Management.	6	Fall 2023	Supply Chain	Quality
Traffic Modeling & Simulation.	6	Fall 2023	Supply Chain	Optimization
Advanced Production Control.	6	Fall 2023	Supply Chain	Quality
Integrated Logistics and Distribution Systems.	6	Fall 2023	Supply Chain	Quality
Cost Estimation.	6	Fall 2023	Optimization	Quality
Financial Modeling for Innovation.	5	Spring 2024	Supply Chain	
Computer Integrated Manufacturing Systems (CIM).	5	Spring 2024	Supply Chain	Quality
Manufacturing Automation.	5	Spring 2024	Quality	Supply Chain
Independent Study.	5	Spring 2024	Supply Chain	Quality
Advanced Quality Engineering.	5	Spring 2024	Quality	
Selected Topics in Probability Modeling.	5	Spring 2024	Quality	
Digital Systems Simulation.	5	Spring 2024	Optimization	
Advanced Experiment Design.	7	Spring 2024	Quality	
Large Scale Optimization.	7	Spring 2024	Optimization	Supply Chain
Integer and Combinatorial Optimization.	7	Spring 2024	Optimization	
Nonlinear Optimization.	7	Spring 2024	Optimization	
Topics of Optimization.	7	Spring 2024	Optimization	
Advance Concepts in Systems Engineering.	7	Spring 2024	Quality	Optimization
Transportation Systems.	7	Spring 2024	Supply Chain	

Figure 1: Courses available for the plan of study

Please do the following

- a) Present your work progress of *parts b)*, and *c)* by the end of today's session (50 points).
- b) First, select the area that Ymir will focus her thesis (pick one of the following: Quality,

Optimization, or Supply Chain). Utilize Mixed BIP to find the optimal plan of study for Ymir, balancing life and school, and creating a thesis of the highest level (25 points).

- c) Ymir has learned that independent study is also important. She likes reading journal papers on her own after classes and networking during conferences. The International Business Analysts (IBA) Annual Conference takes place during the Fall semesters. If she decides to attend that conference, it may boost her thesis level by another 4 points for each conference attended (Fall 2022 and Fall 2023), but will require her to spend two extra hours per week to catch up with other class activities. Should she attend the conference? (Modify the model of part *b*) to answer this question) (25 points)

- d) **(Bonus question)** Use the table of truth to find what would be the (six) constraints that model the following logical expressions (5 points):

x_1	x_2	AND	Not AND	OR	Not OR	Exclusive OR	Not Exclusive OR
0	0	×	✓	×	✓	×	✓
0	1	×	✓	✓	×	✓	×
1	0	×	✓	✓	×	✓	×
1	1	✓	×	✓	×	×	✓

Please fill and submit the following table by including the name of each team member and evaluating her or his participation and commitment as follows: 3 - high level of participation and commitment in the project; 2 - intermediate level of participation and commitment in the project; 1 - poor level or no participation and commitment in the project. The individual grades will be adjusted depending on the evaluation of your team. The table should be filled only once per submission, and it has to be done by team consensus.

Member name	Evaluation
Member A	1 or 2 or 3
Member B	1 or 2 or 3
⋮	⋮

Deadline

The solutions for this case should be submitted to the appropriate section in Canvas at <https://sjsu.instructure.com/>. **Please include all the team members names.** The deadline for the submission is:

→ **Tuesday, March 08, 11:59 PM**

Rubrics

The solutions for this problem homework will be evaluated utilizing the following *rubrics*:

	Assessment		
Points	Modeling	Solution	Writing
100%	The model reflects correctly the problem under study, and it was developed utilizing the six-steps process	The solution is numerically correct, and the interpretation of the solution is appropriate to the problem	Writing is concrete and addresses the problem
85%	The model reflects correctly the problem under study. It was not developed utilizing the six-steps process	The numbers are not correct but the interpretation of the solution is appropriate to the problem	The problem is addressed but the writer repeats some phrases unnecessarily
50%	The model has some flaws. At least half of it reflects the problem under study	The numbers are correct but the interpretation of the solution is not provided	There are contradictions in the writing
0%	Less than half of the problem is represented in the model	The numbers are not correct and the interpretation of the solution is not provided	The problem is not addressed

Case #3

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Solution:

B.

Ymir will focus her thesis on optimization

Step 1: goal: to determine Ymir's optimal study plan to maximize her school-life balance and create a thesis of the highest level

Step 2: parameters: course descriptions can be found in the table, along with the following facts:

- She can take any course that is required to finalize her thesis project, as long as the total number of credits obtained at the end of the program is at least 100
- 10 hours per course + 2 hours for in-person lectures per course (per week) => 12 hours for each class is sufficient (per week)
- "Main focus" column would add 5 points to the thesis level, "secondary focus" column would add 3 points to the thesis level
- She should sleep 8 hours daily, at least 2 hours daily for self-care, at least 6 hours daily for leisure activities (0.1 points for each weekly hour of leisure activities) => 56 hours of sleep per week, at least 14 hours of self-care per week, and at least 42 hours for leisure activities per week

Step 3: decision variables: x_1 corresponds to course 1 with main or secondary focus on optimization, $x_1 = 1$ if the course is selected and $x_1 = 0$ if the course is not selected. X_{24} to X_{35} are continuous variables for sleeping, self-hygiene, and leisure activities hours

Step 4: constraints:

$$8X_1 + 8X_2 + 8X_3 + 6X_4 + 6X_5 + 6X_6 + 7X_7 + 7X_8 + 7X_9 + 6X_{10} + 7X_{11} + 7X_{12} + 7X_{13} + 6X_{14} + 6X_{15} + 6X_{16} + 6X_{17} + 5X_{18} + 7X_{19} + 7X_{20} + 7X_{21} + 7X_{22} + 7X_{23} \geq 100 \text{ (credits are at least 100)}$$

$$12X_1 + 12X_2 + 12X_3 + 12X_4 + 12X_5 + 12X_6 + 12X_7 + 12X_8 + 12X_9 + X_{24} + X_{25} + X_{26} \leq 168 \text{ (classes with optimization focus available for Fall 22, 12 hours each week for each course selected)}$$

$$X_{24} = 56 \text{ (She should sleep 8 hours a day, or 56 hours a week)}$$

$$X_{25} \geq 14 \text{ (She wants to spend time on self-hygiene at least 2 hours a day, or 14 hours a week)}$$

$$X_{26} \geq 42 \text{ (She wants to spend time on leisure activities at least 6 hours a day, or 42 hours a week)}$$

*Note: X_{24} is continuous variable for sleeping, X_{25} is continuous variable for self-hygiene, and X_{26} is continuous variable for leisure activities

$$12X_{10} + 12X_{11} + 12X_{12} + 12X_{13} + X_{27} + X_{28} + X_{29} \leq 168 \text{ (Spring 23)}$$

$$X_{27} = 56$$

$$X_{28} \geq 14$$

$$X_{29} \geq 42$$

$$12X_{14} + 12X_{15} + 12X_{16} + 12X_{17} + X_{30} + X_{31} + X_{32} \leq 168 \text{ (Fall 23)}$$

$$X_{30} = 56$$

$$X_{31} \geq 14$$

$$X_{32} \geq 42$$

$$12X_{18} + 12X_{19} + 12X_{20} + 12X_{21} + 12X_{22} + 12X_{23} + X_{33} + X_{34} + X_{35} \leq 168 \text{ (Spring 24)}$$

$$X_{33} = 56$$

$$X_{34} \geq 14$$

$$X_{35} \geq 42$$

Step 5: objective function:

$$\text{Max } z = 5X_1 + 5X_2 + 5X_3 + 3X_4 + 3X_5 + 5X_6 + 5X_7 + 5X_8 + 5X_9 + 3X_{10} + 3X_{11} + 5X_{12} + 5X_{13} + 3X_{14} + 3X_{15} + 3X_{16} + 5X_{17} + 5X_{18} + 5X_{19} + 5X_{20} + 5X_{21} + 5X_{22} + 3X_{23} + 0.1X_{26} + 0.1X_{29} + 0.1X_{32} + 0.1X_{35}$$

Step 6: model:

We use the same equations from step 4 to step 5 in addition to:

$$x_1, \dots, x_{23} \in \{0,1\}$$

$$x_{24}, \dots, x_{35} \in \{56, 14, 42, \text{etc}\}$$

Excel Solver: the solution can be found in our Excel file, the screenshot could not capture the entire data so we could not paste it here

Interpretation: For Fall 2022-Spring 2024, if Ymir spends 8 hrs/day sleeping, at least 2 hrs/day on self care, and 50 hrs/week on leisure activities, the maximum thesis points Ymir will be able to score is 90.

During this term, they will be able to meet the credit requirement as they will be completing 108 credits.

Moreover, Ymir will have 50 hrs/week of leisure time which is greater than the minimum of 42 hrs/week. This can also translate into more available time for self care.

C.

Attend conference in Fall 2022 and/or Fall 2023: contributes 4 points/conference to thesis

Modify constraint for Fall 2022:

$$12X_1 + 12X_2 + 12X_3 + 12X_4 + 12X_5 + 12X_6 + 12X_7 + 12X_8 + 12X_9 + X_{24} + X_{25} + X_{26} + 2X_{36} \leq 168$$

*Note: X₃₆ is a binary variable, if she attends the conference then she will spend extra 2 hours per week

$$X_{24} = 56$$

$$X_{25} \geq 14$$

$$X_{26} \geq 42$$

Modify constraint for Fall 2023:

$$12X_{14} + 12X_{15} + 12X_{16} + 12X_{17} + X_{30} + X_{31} + X_{32} + 2X_{37} \leq 168$$

*Note: X₃₇ is a binary variable for attending the conference

$$X_{30} = 56$$

$$X_{31} \geq 14$$

$$X_{32} \geq 42$$

Model:

$$\begin{aligned} \text{Max } z = & 5X_1 + 5X_2 + 5X_3 + 3X_4 + 3X_5 + 5X_6 + 5X_7 + 5X_8 + 5X_9 + 3X_{10} + 3X_{11} + 5X_{12} + 5X_{13} + 3X_{14} + \\ & 3X_{15} + 3X_{16} + 5X_{17} + 5X_{18} + 5X_{19} + 5X_{20} + 5X_{21} + 5X_{22} + 3X_{23} + 0.1X_{26} + 0.1X_{29} + 0.1X_{32} + 0.1X_{35} + \\ & 4X_{36} + 4X_{37} \end{aligned}$$

s.t.

$$\begin{aligned} & 8X_1 + 8X_2 + 8X_3 + 6X_4 + 6X_5 + 6X_6 + 7X_7 + 7X_8 + 7X_9 + 6X_{10} + 7X_{11} + 7X_{12} + 7X_{13} + 6X_{14} + 6X_{15} + 6X_{16} \\ & + 6X_{17} + 5X_{18} + 7X_{19} + 7X_{20} + 7X_{21} + 7X_{22} + 7X_{23} \geq 100 \text{ (credits are at least 100)} \end{aligned}$$

$$12X_1 + 12X_2 + 12X_3 + 12X_4 + 12X_5 + 12X_6 + 12X_7 + 12X_8 + 12X_9 + X_{24} + X_{25} + X_{26} + 2X_{36} \leq 168 \text{ (Fall 22)}$$

$$X_{24} = 56$$

$$X_{25} \geq 14$$

$$X_{26} \geq 42$$

$$12X_{10} + 12X_{11} + 12X_{12} + 12X_{13} + X_{27} + X_{28} + X_{29} \leq 168 \text{ (Spring 23)}$$

$$X_{27} = 56$$

$$X_{28} \geq 14$$

$$X_{29} \geq 42$$

$$12X_{14} + 12X_{15} + 12X_{16} + 12X_{17} + X_{30} + X_{31} + X_{32} + 2X_{37} \leq 168 \text{ (Fall 23)}$$

$$X_{30} = 56$$

$$X_{31} \geq 14$$

$$X_{32} \geq 42$$

$$12X_{18} + 12X_{19} + 12X_{20} + 12X_{21} + 12X_{22} + 12X_{23} + X_{33} + X_{34} + X_{35} \leq 168 \text{ (Spring 24)}$$

$$X_{33} = 56$$

$$X_{34} \geq 14$$

$$X_{35} \geq 42$$

$$x_1, \dots, x_{23}, x_{36}, x_{37} \in \{0, 1\}$$

$$x_{24}, \dots, x_{35} \in \{56, 14, 42, \text{etc}\}$$

Excel Solver: the solution can be found in our Excel file, the screenshot could not capture the entire data so we could not paste it here

Interpretation: In every semester, if Ymir can spend 8 hrs/day sleeping, at least 2 hrs/day on self care. In Fall 2022 & Fall 2023, Ymir will have to sacrifice 2 hrs/week from their leisure time to attend a conference which will take away 0.2 creativity points/semester but contribute 4 additional points/semester. Therefore, Ymir can spend 48 hrs/week on leisure activities. Further, in the Spring semesters, Ymir can go back to spending 50 hrs/week on leisure. The combination of the aforementioned actions would result in a thesis score of 97.6 points. During this term, they will be able to meet the credit requirement as they will be completing 106 credits.

D.

Constraint 1:

X1	X2	AND (X1 * X2) >= 1
0	0	0 (No => satisfied)
0	1	0 (No => satisfied)
1	0	0 (No => satisfied)
1	1	1 (Yes => satisfied)

Constraint 2:

X1	X2	NOT AND (X1 * X2) >= 1
0	0	1 (Yes => satisfied)
0	1	1 (Yes => satisfied)
1	0	1 (Yes => satisfied)
1	1	0 (No => satisfied)

Constraint 3:

X1	X2	OR (X1 + X2) >= 1
0	0	0 (No => satisfied)
0	1	1 (Yes => satisfied)
1	0	1 (Yes => satisfied)
1	1	1 (Yes => satisfied)

Constrain 4:

X1	X2	NOT OR (X1 + X2) >= 1
0	0	1 (Yes => satisfied)
0	1	0 (No => satisfied)
1	0	0 (No => satisfied)
1	1	0 (No => satisfied)

Constraint 5:

X1	X2	Exclusive OR (X1 or X2) >= 1
0	0	0 (No => satisfied)

0	1	1 (Yes => satisfied)
1	0	1 (Yes => satisfied)
1	1	0 (No => satisfied)

Constraint 6:

X1	X2	Not Exclusive OR (X1 or X2) >= 1
0	0	1 (Yes => satisfied)
0	1	0 (No => satisfied)
1	0	0 (No => satisfied)
1	1	1 (Yes => satisfied)