

## BUS2 194B - Business Analytics. Spring 2022

### Module 2: Network Optimization - Problem homework 2

#### Problem 1

The Heisenberg Motors auto company has three production plants in Waverly, MI, Kenner, LA, and Santa Monica, CA, and two major distribution centers in Greeley, CO and Austin, TX. The capacities of the three plants for the following month are 1,600, 1,200, and 1,100 cars, and the demands at the two distribution centers for the same period are 1,750 and 2,050 cars. The mileage chart between the plants and the distribution centers is given in the following table:

|              | Greeley | Austin |
|--------------|---------|--------|
| Waverly      | 1,174   | 1,343  |
| Kenner       | 1,309   | 497    |
| Santa Monica | 1,091   | 1,391  |

The trucking company in charge of transporting the cars charges 11 cents per mile per car<sup>1</sup>.

Please do the following:

1. Draw a network that depicts the company's distribution network. Identify the supply nodes, transshipment nodes (if any), and demand nodes in this network. Include the data shown in the above table (15 points).
2. Formulate a mathematical model for this problem utilizing the six-step process discussed in class (20 points).
3. Solve this problem utilizing Excel solver and provide the management the appropriate recommendations and expected cost (15 points).

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<sup>1</sup>Modified version of problem 5.1-1 of the textbook: Taha, H. A. (2017). *Operations Research: An Introduction, 10th Edition*. Pearson

## Problem 2

The transportation problem can be used in other situations other than transportation. For example, production-inventory control. The following table summarizes the parallels between the elements of the production-inventory problem and the transportation model:

| Transportation                           | Production-inventory   |
|--|--|
| Supply node $i$                          | Production period $i$  |
| Demand node $j$                          | Demand period $j$  |
| Supply amount at node $i$                | Production capacity of period $i$                                      |
| Demand amount at node $j$                | Demand for period $j$  |
| Unit transportation cost from $i$ to $j$ | Unit cost (production + inventory) in period $i$ for use at period $j$ |

Solve the following production-inventory problem modeling it as a transportation problem.

Soala Co. produces solar powered air conditioning systems for use in households. The periods with major demand are during Spring and Summer (i.e., through March until August). The demand starts slow, peaks in the middle of the season, and tapers off toward the end. The following table provides the production capacities and demands (in hundreds) for the six months under consideration for the Santa Clara county<sup>2</sup>:

| Month           | March | April | May | June | July | August |
|-----------------|-------|-------|-----|------|------|--------|
| <b>Capacity</b> | 170   | 150   | 200 | 180  | 170  | 170    |
| <b>Demand</b>   | 100   | 190   | 200 | 200  | 170  | 150    |

The unit production costs during March and April is \$600, for months May and June is a little bit more expensive at \$900, and it goes down to \$700 during July and August. The holding cost per unit per month is \$50, during any month.

<sup>2</sup>Modified version of problem 13.4-1 of the textbook: Taha, H. A. (2017). *Operations Research: An Introduction*, 10th Edition. Pearson

**For the holding and production cost, consider that what you produce in a month to sell in the same month will only cost you the production cost, but if you produce to sell in future months, you need to add the holding cost (depending on how many months you keep the product) to the production cost.**

Please do the following:

1. Draw a network that depicts production-inventory problem using the same idea of the transportation problem. Identify the supply nodes, transshipment nodes (if there exist), and demand nodes in this network. Include the data shown in the above table (15 points).
2. Formulate a mathematical model for this problem utilizing the six-step process discussed in class (20 points).
3. Solve this problem utilizing Excel solver and provide the management the appropriate recommendations and expected cost (15 points).

## Bonus problem

Please do the following:

1. **(Bonus question)** Briefly discuss the implications of the solution of these problems under a *sustainable operations* approach. Specifically, discuss how these decisions would have the following impact either to the inside or the outside of the organization (5 *bonus* points):
  - a) economical impact
  - b) environmental impact
  - c) social impact

## Deadline

The solutions for this problem homework should be submitted to the appropriate section in Canvas at <https://sjsu.instructure.com/>. The deadline for the submission is:

→ **Sunday, February 13, 11:59 PM**

## Rubrics

The solutions for this problem homework will be evaluated utilizing the following *rubrics*:

|               | <b>Assessment</b>  |   |  |
|---------------|--|---|--|
| <b>Points</b> | <b>Modeling</b>  | <b>Solution</b>   | <b>Writing</b>   |
| 100%          | The model reflects correctly the problem under study, and it was developed utilizing the six-steps process | The solution is numerically correct, and the interpretation of the solution is appropriate to the problem | Writing is concrete and addresses the problem                              |
| 85%           | The model reflects correctly the problem under study. It was not developed utilizing the six-steps process | The numbers are not correct but the interpretation of the solution is appropriate to the problem          | The problem is addressed but the writer repeats some phrases unnecessarily |
| 50%           | The model has some flaws. At least half of it reflects the problem under study                             | The numbers are correct but the interpretation of the solution is not provided                            | There are contradictions in the writing                                    |
| 0%            | Less than half of the problem is represented in the model  | The numbers are not correct and the interpretation of the solution is not provided                        | The problem is not addressed   |

## Problem homework 2

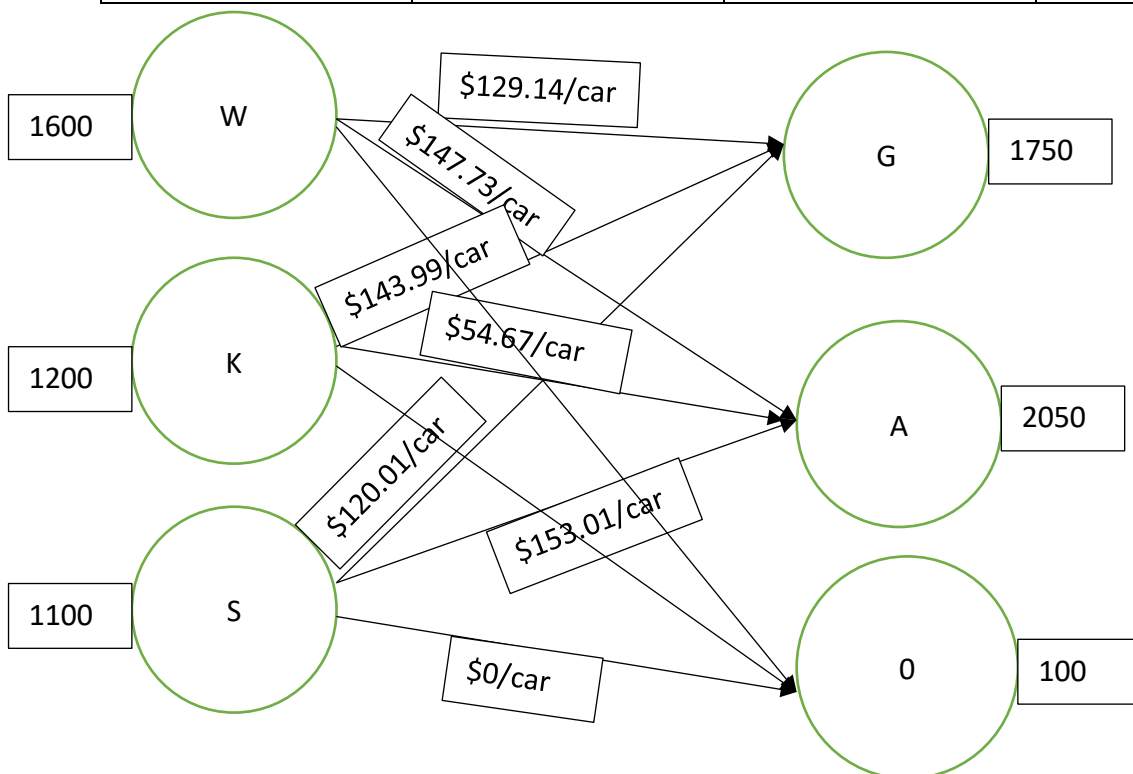
### Problem 1

1.

|              | Greeley | Austin |
|--------------|---------|--------|
| Waverly      | 1,174   | 1,343  |
| Kenner       | 1,309   | 497    |
| Santa Monica | 1,091   | 1,391  |

⇒ New table:

|                  | Greeley (G)           | Austin (A)      | Capacity  |
|------------------|-----------------------|-----------------|-----------|
| Waverly (W)      | $1174(0.11) = 129.14$ | $1343 = 147.73$ | 1600      |
| Kenner (K)       | $1309 = 143.99$       | $497 = 54.67$   | 1200      |
| Santa Monica (S) | $1091 = 120.01$       | $1391 = 153.01$ | 1100      |
| <b>Demand</b>    | 1750                  | 2050            | 3900/3800 |



W, K, S are supply nodes. G, A are demand nodes. O is a dummy demand node.

2.

Step 1: objective: how many cars should each plant ship to each distributor to minimize total shipping cost

Step 2: parameters: the network has 6 nodes (3 supply nodes, 2 demand nodes, and 1 dummy demand node), and 9 arcs

Step 3: decision variables: we have 9 decisions to make to determine how many cars should we ship to each arc. We set up  $x_{i,j}$  to represent number of cars shipped from supply node to demand node

Step 4: we set up constraints

$$x_{WG} + x_{WA} + x_{W0} = 1600 \text{ (node W)}$$

$$x_{KG} + x_{KA} + x_{K0} = 1200 \text{ (node K)}$$

$$x_{SG} + x_{SA} + x_{S0} = 1100 \text{ (node S)}$$

$$x_{WG} + x_{KG} + x_{SG} = 1750 \text{ (node G)}$$

$$x_{WA} + x_{KA} + x_{SA} = 2050 \text{ (node A)}$$

$$x_{W0} + x_{K0} + x_{S0} = 100 \text{ (node 0)}$$

$$\text{all } x \geq 0$$

step 5: we set up objective function

$$\begin{aligned} \min z = & 129.14x_{WG} + 147.73x_{WA} + 143.99x_{KG} + 54.67x_{KA} + 120.01x_{SG} + 153.01x_{SA} + 0x_{W0} + \\ & 0x_{K0} + 0x_{S0} \end{aligned}$$

step 6: we build model

$$\begin{aligned} \min z = & 129.14x_{WG} + 147.73x_{WA} + 143.99x_{KG} + 54.67x_{KA} + 120.01x_{SG} + 153.01x_{SA} + 0x_{W0} + \\ & 0x_{K0} + 0x_{S0} \end{aligned}$$

s.t.

$$x_{WG} + x_{WA} + x_{WO} = 1600$$

$$x_{KG} + x_{KA} + x_{KO} = 1200$$

$$x_{SG} + x_{SA} + x_{SO} = 1100$$

$$x_{WG} + x_{KG} + x_{SG} = 1750$$

$$x_{WA} + x_{KA} + x_{SA} = 2050$$

$$x_{WO} + x_{KO} + x_{SO} = 100$$

all  $x \geq 0$

3.

Excel Solver:

| Decision Variables | From To | W G    | W A    | W O | K G    | K A   | K O | S G    | S A    | S O |          |      |  |
|--------------------|---------|--------|--------|-----|--------|-------|-----|--------|--------|-----|----------|------|--|
|                    |         | 650    | 850    | 100 | 0      | 1200  | 0   | 1100   | 0      | 0   |          |      |  |
| Min                |         | 129.14 | 147.73 | 0   | 143.99 | 54.67 | 0   | 120.01 | 153.01 | 0   | 407126.5 |      |  |
| S.t.               |         | 1      | 1      | 1   |        |       |     |        |        |     | 1600 =   | 1600 |  |
|                    |         |        |        |     | 1      | 1     | 1   |        |        |     | 1200 =   | 1200 |  |
|                    |         |        |        |     |        |       |     | 1      | 1      | 1   | 1100 =   | 1100 |  |
|                    |         | 1      |        |     | 1      |       |     | 1      |        |     | 1750 =   | 1750 |  |
|                    |         |        | 1      |     |        | 1     |     |        | 1      |     | 2050 =   | 2050 |  |
|                    |         |        |        | 1   |        |       | 1   |        |        | 1   | 100 =    | 100  |  |

The total shipping cost is \$407126.5

650 cars should be shipped from W to G

850 cars should be shipped from W to A

100 cars should be shipped from W to O

0 cars should be shipped from K to G

1200 cars should be shipped from K to A

0 cars should be shipped from K to O

1100 cars should be shipped from S to G

0 cars should be shipped from S to A

0 cars should be shipped from S to O

## Problem 2

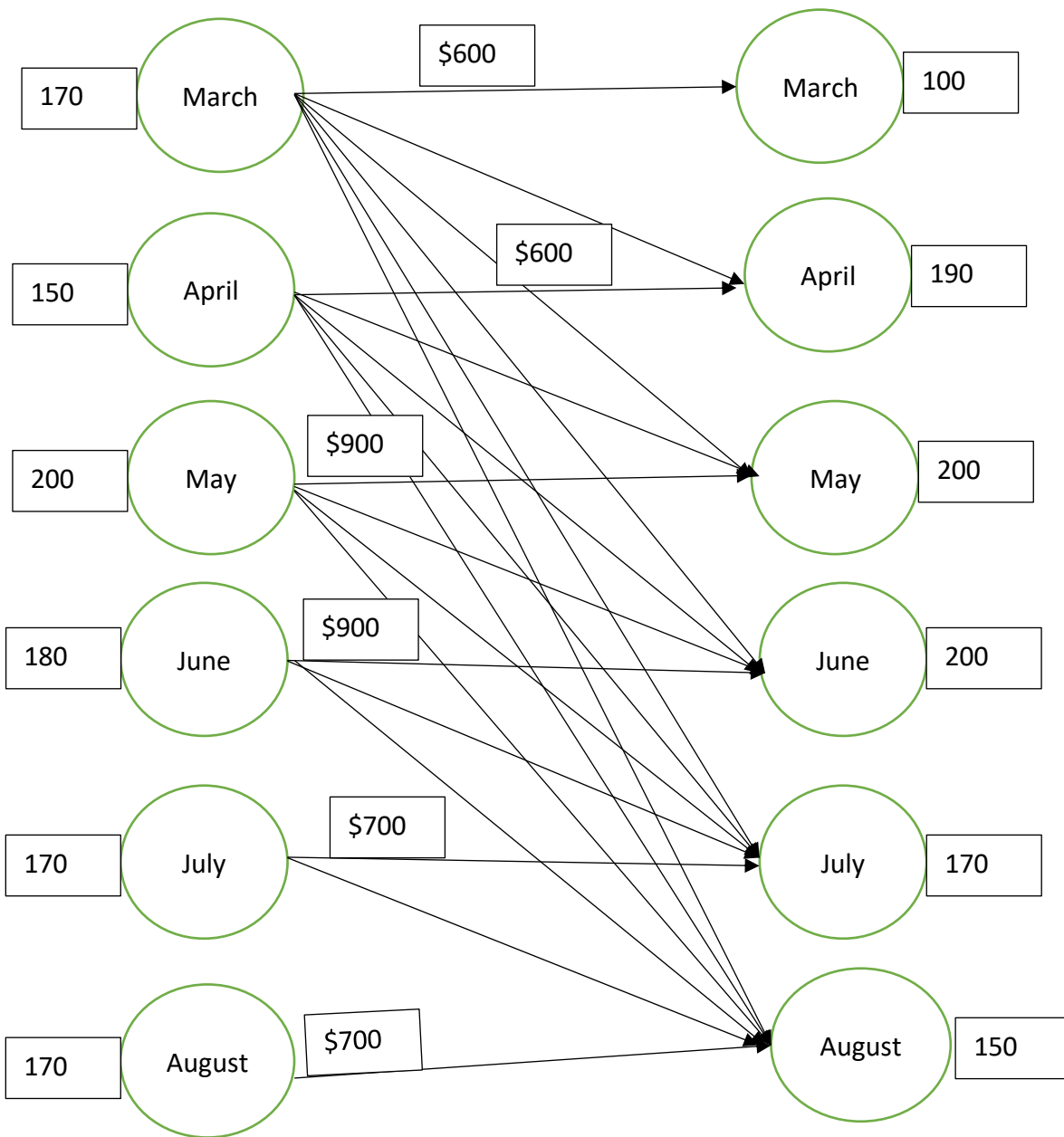
1.

| Month    | March | April | May | June | July | August |
|----------|-------|-------|-----|------|------|--------|
| Capacity | 170   | 150   | 200 | 180  | 170  | 170    |
| Demand   | 100   | 190   | 200 | 200  | 170  | 150    |

⇒ New table:

| Month        | Unit cost (production + holding cost) | Capacity | Demand |
|--------------|---------------------------------------|----------|--------|
| March (M)    | $\$600 + \$50 = \$650$                | 170      | 100    |
| April (A)    | $\$600 + \$50 = \$650$                | 150      | 190    |
| May (Ma)     | $\$900 + \$50 = \$950$                | 200      | 200    |
| June (J)     | $\$900 + \$50 = \$950$                | 180      | 200    |
| July (Ju)    | $\$700 + \$50 = \$750$                | 170      | 170    |
| August (Au)  | $\$700 + \$50 = \$750$                | 170      | 150    |
| <b>Total</b> |                                       | 1040     | 1010   |





On the left are capacity nodes (supply nodes). On the right are demand nodes.

2.

Step 1: objective: how many units should be produced each month to minimize total unit cost

Step 2: parameters: the network has 12 nodes (6 supply nodes, 6 demand nodes) and 21 arcs

Step 3: decision variables: we have 21 decisions to make to determine how many units should we produce for each month. We set up  $x_{i,j}$  to represent units produced from month  $i$  for month  $j$ .

Step 4: we set up constraints

$$x_{MM} + x_{MA} + x_{MMa} + x_{MJ} + x_{MJu} + x_{MAu} \leq 170$$

$$x_{AA} + x_{AMa} + x_{AJ} + x_{AJu} + x_{AAu} \leq 150$$

$$x_{MaMa} + x_{MaJ} + x_{MaJu} + x_{MaAu} \leq 200$$

$$x_{JJ} + x_{JJu} + x_{JJu} \leq 180$$

$$x_{JuJu} + x_{JuAu} \leq 170$$

$$x_{AuAu} \leq 170$$

$$x_{MM} \geq 100$$

$$x_{MA} + x_{AA} \geq 190$$

$$x_{MMa} + x_{AMa} + x_{MaMa} \geq 200$$

$$x_{MJ} + x_{AJ} + x_{MaJ} + x_{JJ} \geq 200$$

$$x_{MJu} + x_{AJu} + x_{MaJu} + x_{JJu} + x_{JuJu} \geq 170$$

$$x_{MAu} + x_{AAu} + x_{MaAu} + x_{JJu} + x_{JuAu} + x_{AuAu} \geq 150$$

step 5: we set up objective function

$$\begin{aligned} \min z = & 600x_{MM} + 650x_{MA} + 700x_{MMa} + 750x_{MJ} + 800x_{MJu} + 850x_{MAu} + 600x_{AA} + 650x_{AMa} \\ & + 700x_{AJ} + 750x_{AJu} + 800x_{AAu} + 900x_{MaMa} + 950x_{MaJ} + 1000x_{MaJu} + 1050x_{MaAu} + 900x_{JJ} + \\ & 950x_{JJu} + 1000x_{JAu} + 700x_{JuJu} + 750x_{JuAu} + 700x_{AuAu} \end{aligned}$$

step 6: we build model

$$\begin{aligned} \min z = & 600x_{MM} + 650x_{MA} + 700x_{MMa} + 750x_{MJ} + 800x_{MJu} + 850x_{MAu} + 600x_{AA} + 650x_{AMa} \\ & + 700x_{AJ} + 750x_{AJu} + 800x_{AAu} + 900x_{MaMa} + 950x_{MaJ} + 1000x_{MaJu} + 1050x_{MaAu} + 900x_{JJ} + \\ & 950x_{JJu} + 1000x_{JAu} + 700x_{JuJu} + 750x_{JuAu} + 700x_{AuAu} \end{aligned}$$

s.t.

$$x_{MM} + x_{MA} + x_{MMa} + x_{MJ} + x_{MJu} + x_{MAu} \leq 170$$

$$x_{AA} + x_{AMa} + x_{AJ} + x_{AJu} + x_{AAu} \leq 150$$

$$x_{MaMa} + x_{MaJ} + x_{MaJu} + x_{MaAu} \leq 200$$

$$x_{JJ} + x_{JJu} + x_{JAu} \leq 180$$

$$x_{JuJu} + x_{JuAu} \leq 170$$

$$x_{AuAu} \leq 170$$

$$x_{MM} \geq 100$$

$$x_{MA} + x_{AA} \geq 190$$

$$x_{MMa} + x_{AMa} + x_{MaMa} \geq 200$$

$$x_{MJ} + x_{AJ} + x_{MaJ} + x_{JJ} \geq 200$$

$$x_{MJu} + x_{AJu} + x_{MaJu} + x_{JJu} + x_{JuJu} \geq 170$$

$$x_{MAu} + x_{AAu} + x_{MaAu} + x_{JAu} + x_{JuAu} + x_{AuAu} \geq 150$$

$$\text{all } x \geq 0$$

3.

Excel Solver:

|                   | A       | B   | C   | D   | E   | F   | G   | H   | I   | J   | K   | L   | M   | N   | O    | P    | Q   | R   | S    | T   | U   | V   | W      | X      | Y   | Z |  |
|-------------------|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|-----|-----|------|-----|-----|-----|--------|--------|-----|---|--|
| Decision Variable | From To | M   | M   | M   | M   | M   | M   | M   | A   | A   | A   | A   | A   | Ma  | Ma   | Ma   | Ma  | J   | J    | J   | Ju  | Ju  | Au     |        |     |   |  |
|                   |         | M   | A   | Ma  | J   | Ju  | Au  | A   | Ma  | J   | Ju  | Au  | Ma  | J   | Ju   | Au   | J   | Ju  | Au   | Ju  | Au  | Au  |        |        |     |   |  |
|                   |         | 100 | 40  | 10  | 20  | 0   | 0   | 0   | 150 | 0   | 0   | 0   | 0   | 190 | 0    | 0    | 0   | 180 | 0    | 0   | 170 | 0   | 150    |        |     |   |  |
| Min               |         | 600 | 650 | 700 | 750 | 800 | 850 | 600 | 650 | 700 | 750 | 800 | 900 | 950 | 1000 | 1050 | 900 | 950 | 1000 | 700 | 750 | 700 | 755000 |        |     |   |  |
| S.t.              |         | 1   | 1   | 1   | 1   | 1   | 1   |     | 1   | 1   | 1   | 1   | 1   |     | 1    | 1    | 1   | 1   |      |     |     |     | 170 <= | 170    |     |   |  |
|                   |         |     |     |     |     |     |     |     |     |     |     |     |     | 1   | 1    | 1    | 1   |     |      |     |     |     | 150 <= | 150    |     |   |  |
|                   |         |     |     |     |     |     |     |     |     |     |     |     |     |     |      |      |     | 1   | 1    | 1   |     |     | 190 <= | 200    |     |   |  |
|                   |         |     |     |     |     |     |     |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 180 <= | 180    |     |   |  |
|                   |         |     |     |     |     |     |     |     |     |     |     |     |     |     |      |      |     |     |      |     | 1   | 1   | 170 <= | 170    |     |   |  |
|                   |         | 1   |     |     |     |     |     |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 150 <= | 170    |     |   |  |
|                   |         |     | 1   |     |     |     |     |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 100 >= | 100    |     |   |  |
|                   |         |     |     | 1   |     |     |     |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 190 >= | 190    |     |   |  |
|                   |         |     |     |     | 1   |     |     |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 200 >= | 200    |     |   |  |
|                   |         |     |     |     |     | 1   |     |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 200 >= | 200    |     |   |  |
|                   |         |     |     |     |     |     | 1   |     |     |     |     |     |     |     |      |      |     |     |      |     |     |     | 170 >= | 170    |     |   |  |
|                   |         |     |     |     |     |     |     | 1   |     |     |     |     |     | 1   |      |      |     | 1   |      | 1   |     | 1   | 1      | 150 >= | 150 |   |  |

The unit cost is \$755000

We should produce 100 units in March for use in March

We should produce 40 units in March for use in April

We should produce 10 units in March for use in May

We should produce 20 units in March for use in June

We should produce 150 units in April for use in April

We should produce 190 units in May for use in May

We should produce 180 units in June for use in June

We should produce 170 units in July for use in July

We should produce 150 units in August for use in August

### Bonus problem

- Economic impact: the objective is to minimize total shipping cost and total unit cost, and by using the suggestion from Excel Solver, we are able to ship and produce units economically wise
- Environmental impact: by minimizing the cost, we live up to our corporate social responsibilities and not use up resources, especially when these resources are scarce

- c. Social impact: we won't waste any resources, cause pollution, and affect the communities that live near our plants if we implement this solution.