$\bar{E}$ ~ where no student has to answer more than 1q

1) $P(\bar{E}) = \dfrac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{15^8} = 0.1012$

no. of students decreases at each pick, because we can't pick the same student twice.

2) $\underbrace{\underline{5}\ \underline{4}}_{2\text{ odd}}\ \underbrace{\underline{7}}_{8\text{ left}}\ \underline{6}\ \underbrace{\underline{5}}_{\text{even choices}} = 4200$

but we take out 1 even no. for the end.

$P(\text{success}) = \dfrac{4200}{100{,}000} = \dfrac{42}{1000} = \dfrac{21}{500}$

$P(\text{fail}) = \dfrac{479}{500}$

X – no. of "successes" in 8 trials.

$X \sim \text{Binom}\left(\overset{n}{8}, \overset{p}{\dfrac{21}{500}}\right)$

$P(x = 5) = \binom{8}{5}\left(\dfrac{21}{500}\right)^5 \cdot \left(\dfrac{479}{500}\right)^3 = 0.000006\,4347$

chance of getting 5 successes with 8 trials

4) Sample space $= 6^3 = 216$

since we want to get P(AT LEAST two die are 4 or bigger)

$P(A) = P(2\text{ dice} \geq 4) + P(\text{All dice} \geq 4)$

$= \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \underbrace{\dfrac{3!}{2!}}_{\text{possible permutations}} + \left(\dfrac{1}{2}\right)^3$

$= \dfrac{1}{2}$

$P(B) = \dfrac{6}{216} \Leftarrow 111, 222, \ldots, 666 = \dfrac{1}{36}$

$P(AB) = \{ 444, 555, 666 = \dfrac{3}{216} = \dfrac{1}{72}$

if $A \perp B$, then $P(AB) = P(A)P(B)$

$\dfrac{1}{72} = \dfrac{1}{2} \cdot \dfrac{1}{36}$

$= \dfrac{1}{72}$, proves that $A, B$ independent.

5) 4 suits, ways of choosing 5 cards from a suit

$P(\text{Flush}) = \dfrac{4\binom{13}{5}}{\binom{52}{5}} = \dfrac{5148}{2{,}598{,}960} = 0.00198$

$\binom{52}{5}$ ⇐ total ways of picking 5 cards from deck

How many hands played til you see a flush

Geometric!

$E[X] = \dfrac{1}{p}$ for geometric $= \dfrac{1}{0.00198}$

$E[X] \approx 505$ trials until first flush

5) $P(W \mid \text{superstar}) = 0.7$ $\quad P(\text{Superstar}) = 0.75$

$P(W \mid \text{No superstar}) = 0.5$ $\quad P(\text{No Superstar}) = 0.25$

Let $E$ = event team wins 4/5 games

We want : $P(\text{Super star plays} \mid E)$

$P(E \mid \text{Superstar}) =$ Binomial with $n = 5$, $K = 4$
$p = 0.7$

$P(E \mid \text{Superstar}) = \binom{5}{4}(0.7)^4(0.3) = 0.36015$

$P(E \mid \text{No Superstar}) = \binom{5}{4}(0.5)^4(0.5) = 0.15625$

So, using Baye's Formula:

$$P(\text{Superstar} \mid E) = \frac{P(E \mid \text{Superstar})\,P(\text{super star})}{P(E \mid \text{Superstar})\,P(\text{super star}) + P(E \mid \text{No Superstar})\,P(\text{No Superstar})}$$

$= 0.8736$, meaning that there was approx 87.36% chance superstar played given team won $\frac{4}{5}$ games.