University of Exeter Business School

Assignment Cover Sheet	
Candidate Number	024706
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Assignment Title	Fraud Detection

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1. Introduction:

Accurate and reliable financial statements are essential to the effective functioning of capital markets since when financial information is accurate, it enables efficient resource allocation and supports effective contracting practices (Bushman and Smith, 2003). However, financial reporting misconduct, including fraud, misestimations, bias, and manipulation, poses a serious threat to this system. When financial reports are distorted, they undermine the confidence that market participants and gatekeepers rely on to engage in commerce. As a result, detecting and measuring errors in financial reports is a key concern for a range of stakeholders, including investors, analysts, auditors, regulators, and academic researchers.

Although previous studies have introduced methods to evaluate error-related constructs like accruals quality and earnings quality, these approaches often suffer from notable limitations. Specifically, many of the existing measures are influenced by firm-specific characteristics or depend on time-series data, cross-sectional comparisons, or forward-looking estimates (Dechow et al., 2010; Owens et al., 2013). The first paper to address these challenges is Amiram et al. (2015), in which Benford's Law, a statistical approach, was introduced. It is the or law of first digits. Benford's Law, also known as the law of first digits, offers several key advantages over traditional accounting quality measures. Unlike many alternatives, it does not rely on time-series, cross-sectional, or forward-looking data. It can be applied to virtually any firm with available financial statements and is not inherently linked to a company's performance or business model (Amiram et al., 2015).

Research in mathematics, statistics, and economics shows that analysing the distribution of leading digits in a dataset can help detect errors in the underlying data. This approach is grounded in a theorem by Hill (1995), which demonstrates that when random samples are drawn from various randomly selected distributions, the resulting leading digits tend to follow a logarithmic pattern known as Benford's Law. The theoretical basis for using Benford's Law to assess data quality stems from Hill's (1995) theorem, which shows that when random samples are drawn from randomly selected distributions, the leading digits of the combined data tend to follow a logarithmic distribution, known as Benford's Law (BL). This method has been widely applied to detect anomalies in various contexts, including scientific publications, election results, macroeconomic statistics, tax filings, internal accounting records, and even firms' annual financial statements.

The reason many real-world datasets follow Benford's Law can be explained using two key mathematical insights. First, the leading digit of a number can be determined by taking its base-10 logarithm and examining the fractional part. For instance, if the mantissa lies between 0 and 0.301, the number starts with 1; if it's between 0.301 and 0.477, the number starts with 2, and so on. These intervals align precisely with the probabilities defined by Benford’s Law. Second, if the logarithm of the data is distributed in a smooth and symmetric way, a common property of natural data due to the Central Limit Theorem, then the likelihood of a number falling within these intervals corresponds to Benford’s predicted digit frequencies. Deviations from Benford’s Law typically occur when the underlying data is distorted in a way that disrupts the smoothness or symmetry of the logarithmic distribution (Pimbley, 2014).

BL applies to many real-world datasets because the distribution of leading digits aligns with the structure of base-10 logarithms (Amiram et al, 2015). Specifically, if the logarithm of a number has a uniformly distributed fractional part, the leading digit follows Benford’s predicted probabilities. This often occurs when data is naturally distributed, and log-transformed values are smooth and symmetric Law (Pimbley, 2014). These rationales make BL particularly relevant for detecting irregularities in financial statement data. Specifically, financial statement figures represent estimates of unobservable actual cash flows from various economic activities, each likely stemming from distinct underlying distributions. As such, the overall data in financial statements can be seen as a mixture of various distributions. According to Hill’s (1995) theorem, when multiple distributions are randomly selected and combined, the leading digits of the resulting dataset tend to follow BL. However, because actual outcomes are unknown at reporting time, preparers rely on estimates, introducing errors from mistakes, bias, or manipulation, which can cause deviations from the expected digit distribution.

Focusing on the application and suitability of Benford’s Law to annual financial statements, this research aims to assess its effectiveness in detecting accounting irregularities among firms with confirmed cases of fraud. Specifically, the study examines a sample of companies that classified as fraudulent between January 2015 and December 2024. These cases have been identified by Audit Analytics as involving not just misstatements, but deliberate and fraudulent misrepresentation of financial reports. For companies not identified as fraudulent by Audit Analytics, this study will assume they are not having fraudulent behaviour and aims to compare their conformity to Benford’s Law before and after financial restatements. This comparison will help assess whether restatements improve the statistical integrity of reported figures and whether deviations from Benford’s Law can serve as a signal for underlying reporting issues, even in non-fraudulent cases.

In line with Amiram et al.'s (2015) proposition that misstatements are more prevalent among smaller, younger, more volatile, and high-growth firms, this study further investigates the relationship through a multivariate analysis. Specifically, Benford's Law conformity metrics are used as the dependent variables, while financial indicators representing firm size, age, volatility, and growth serve as the independent variables.

2. Literature Review:

2.1. Benford's Law & Applications:

Benford's Law (BL) describes a theoretical distribution governing the frequency of leading digits in naturally occurring numerical datasets. The origins of the law trace back to Newcomb (1881), who noticed that the earlier pages of logarithmic tables tended to be more worn than the later ones. Based on this observation, he proposed that the first significant digit in real-world data appears more frequently as 1 than as any higher digit, with the frequency gradually declining up to 9. The occurrence probability of the leading first digit, d, is then determined as being:

$$P_d = \log_b(d+1) - \log_b(d) = \log_b\left(\frac{d+1}{d}\right)$$

Where P is the probability of the occurrence of first digit d, and b is the logarithmic base. The expected probability of occurrence for leading digits 1 through 9, results in the theoretical distribution which today is referred to as BL and a shown below:

1	2	3	4	5	6	7	8	9
0.3010	0.1761	0.1249	0.0969	0.0792	0.0669	0.0580	0.0512	0.0458

In 1938, physicist Frank Benford examined Newcomb's earlier observation across a wide range of datasets such as river surface areas, molecular weights, death rates, and even numbers from an issue of Reader's Digest, and found that the digit distribution consistently followed Benford's Law (Benford, 1938). Although Newcomb (1881) originally identified the pattern, his work had largely gone unnoticed, and the law came to bear Benford's name following his systematic documentation. Subsequent research further established the law's mathematical properties. Pinkham (1961) demonstrated that Benford's Law is scale-invariant, meaning the distribution remains unchanged under unit conversions or multiplicative transformations. Boyle (1994) extended this finding by showing that data conforming to Benford's Law retain this conformity even when subjected to repeated multiplication, division, or exponentiation by integers. A rigorous theoretical foundation for the law was later provided by Hill (1995), who proved that under general conditions involving randomly selected distributions, the aggregate leading digits converge to the Benford distribution. Hill (1995)'s theorem states that, if distributions are selected at random and random samples are then taken from each of these distributions, the first digits of the combined mixture distribution will converge to the logarithmic or Benford distribution.

Numerous studies have found that Benford's Law (BL) holds across a wide spectrum of datasets, ranging from everyday numerical records to highly structured financial and scientific information. For instance, BL has been observed in physical constants such as the speed of light and gravitational force (Knuth, 1969; Burke & Kincanon, 1991), online content including web data (Leibon, 2008) and internet traffic patterns (Arshadi & Jahangir, 2014). Social science data, such as survey responses and demographic records, also display conformity (Schäfer et al., 2003; Schräpler & Wagner, 2004), as do transactional data like eBay bids (Giles, 2007). In financial contexts, stock prices, returns, and various accounting figures have shown alignment with BL predictions (Ley & Varian, 1994; Ley, 1996; Pietronero et al., 2001; Clippe & Ausloos, 2012; Nigrini, 2012; Amiram et al., 2015). Overall, BL has been showed to have various application, in accounting and finance, particularly for detecting anomalies and potential manipulation in financial statements.

2.2. Benford's Law as Financial Fraud Detection:

In auditing, previous research highlights the application of analytical techniques including BL, by auditors to detect manipulations and fraudulent activity in financial data (Bierstaker et al., 2006). Digit analysis is regarded as a highly cost-effective method for identifying datasets with a high likelihood of being manipulated (Carslaw, 1988; Berton, 1995; Nigrini, 1996; Wright & Ashton, 1989; Quick & Wolz, 2003; Hales et al., 2009; Bhattacharya et al., 2011).

Significant progress in applying BL to accounting and tax-related fraud detection is largely credited to the work of Mark Nigrini and his collaborators. Through a series of influential studies (Nigrini, 1996, 1997, 1999), BL has been established as a core tool in forensic accounting, illustrating its effectiveness in identifying irregularities such as tax evasion and broader financial misstatements. These contributions have led to the adoption of BL-based detection techniques by tax authorities across various countries. In a later collaboration, Nigrini and Miller (2009) outlined detailed, practitioner-oriented procedures for applying BL in audit contexts to uncover anomalies in transactional data. Furthering the application of BL, Nigrini (2012) demonstrated its usefulness in identifying discrepancies in a firm's accounts receivable, especially when detailed invoice-level data is available. Complementing this, Durtschi et al. (2004) developed a practical guide aimed at helping auditors apply BL to internal transaction-level data to detect fraud more effectively. In addition to traditional auditing, researchers have leveraged computational tools to enhance BL's capabilities. For instance, Busta and Weinberg (1998), Huang et al. (2008), and Bhattacharya et al. (2011) introduced algorithmic methods that significantly strengthened the reliability of BL as a fraud detection technique. Beyond financial domains, BL has also proven effective in identifying fabricated data in non-financial contexts, such as fake responses in surveys, particularly when the data is manipulated by interviewers rather than collected from actual participants (Schäfer et al., 2005; Schräpler & Wagner, 2005).

Narrow down specifically on annual financial statements, Amiram et al. (2015) were the first to examine whether firms' financial statements follow Benford's Law. They found that firms diverging from the expected distribution tend to be smaller, younger, more volatile, and in high-growth stages. The study introduced the Financial Statement Deviation (FSD) Score, which is associated with accrual-based earnings management, but also captures unique information beyond traditional manipulation proxies. Firms reporting losses or just-above-zero earnings showed weaker conformity to Benford's Law, while restated figures aligned more closely with the expected distribution, suggesting misreporting as a key driver of divergence. Greater deviation was also linked to lower earnings persistence. Finally, higher FSD Scores predicted future regulatory enforcement actions, indicating that the measure can serve as a useful early warning signal for auditors, investors, and regulators.

In sum, BL has proven to be a powerful and cost-effective tool in financial statement analysis, particularly for detecting manipulation and anomalies. Its widespread adoption in auditing and forensic accounting reflects its practical value in both univariate and multivariate frameworks. Whether through direct digit analysis or integrated into more complex models, as demonstrated in Amiram et al.'s (2015) FSD Score, BL offers a unique lens for identifying red flags in financial reporting.

3. Methodology:

3.1. Univariate - Measuring conformity from Benford's Law:

This study applies three commonly used measures to evaluate conformity: Kolmogorov–Smirnov (KS) Statistic, Mean Absolute Deviation (MAD) Value and Chi-Square (CS) Statistic.

The KS test calculates the largest difference between the cumulative distribution of the observed first-digit frequencies and Benford's expected distribution, capturing the maximum deviation across digits 1 through 9. The KS statistic is calculated as follows:

$$\mathrm{KS} = \mathrm{Max}(\,|\mathrm{AD}^1 - \mathrm{ED}^1|, |(\mathrm{AD}^1 + \mathrm{AD}^2) - (\mathrm{ED}^1 + \mathrm{ED}^2)|, \ldots, |(\mathrm{AD}^1 + \mathrm{AD}^2 + \cdots + \mathrm{AD}^9) - (\mathrm{ED}^1 + \mathrm{ED}^2 + \cdots + \mathrm{ED}^9)|) \quad (1)$$

- Where AD (Actual Distribution) refers to the observed frequency of each leading digit in the dataset, and ED (Expected Distribution) represents the theoretical frequency as predicted by BL.
- The KS statistic is particularly useful for evaluating conformity at the firm level, as it provides a clear critical value for hypothesis testing. At the 5% significance level, the critical value is calculated as:

$$Critical\,Value_{5\%} = \frac{1.36}{\sqrt{P}} \quad (2)$$

- where P is the total number of digit observations in the dataset. This threshold allows researchers to determine whether the deviation from Benford's Law is statistically significant.
- KS becomes less reliable as the number of digit observations PPP increases. Specifically, to maintain the null hypothesis of conformity to Benford's Law at the 5% significance level, the empirical distribution must align almost perfectly with the theoretical distribution when PPP is large (Nigrini, 2012). Consequently, the KS test tends to over-reject the null hypothesis in large samples, making it overly sensitive and potentially misclassifying compliant data as non-conforming.

The MAD score is defined as the mean of the absolute value of the difference between the frequency of each first digit within the sample, and the frequency as determined by BL. The MAD statistic is calculated as follows:

$$MAD = \frac{\sum_{i=1}^{K} |AD_i - ED_i|}{K} \quad (3)$$

- where K is the number of leading digits being analysed.
- Because the denominator is fixed, the MAD is scale-insensitive, making it a robust measure even as the size of the digit pool increases. Unlike the KS, which becomes overly sensitive in large samples, the MAD becomes more informative and reliable with larger datasets, making it particularly well-suited for large-scale financial data analysis.

The CS test compares the counts for each digit to the expected count derived from Benford Law and is calculated as follows:

$$\chi^2 = \sum_1^9 \frac{(AC - EC)^2}{EC} \tag{4}$$

- where AC is the actual count and EC is the expected count. When the Chi-square test statistic is greater than the critical value then we can infer that the distribution does not follow Benford Law.

3.2. Multivariate:

In line with Amiram et al. (2015) research's proposition that misstatements are more likely to occur in smaller, younger, more volatile, growing firm; the research includes a comprehensive set of explanatory variables in the multivariate Benford analysis to control for key firm characteristics that are empirically linked to such firms, specifically:

- **Fraud** (Fraud)**:** 1 for fraud-identified firm, 0 for non-fraudulent firm.
- **Firm size** (log_assets)**:** Measured as the natural logarithm of total assets, ($log\ (Assets)$).
- **Profitability** (profitability)**:** Captured by return on assets $ROA = \frac{Total\ Assets}{Net\ Income}$, reflecting overall financial performance.
- **Loss indicator** (loss)**:** Loss =1 if Net Income < 0; 0 otherwise.
- Firm age (firm_age)**:** Defined as the number of years since the firm's first appearance in the dataset, $Firm\ Age = Year_{obs} - Year_{first\ obs}$.
- Asset Growth = $\frac{Assets_t - Assets_{t-1}}{Assets_{t-1}}$ (asset_growth) and its standard deviation (asset_growth_vol).
- Revenue Growth = $\frac{Revenue_t - Revenue_{t-1}}{Revenue_{t-1}}$ (revenue_growth) and its standard deviation (revenue_growth_vol).
- Dividend Paid (dividend_paid): 1 if common dividends > 0 in that year, 0 otherwise.

- Industry fixed effects (industry)**:** Categorical controls for industry (via GICS sub-industry dummies) and for the type of financial statement. These absorb unobserved heterogeneity in misstatement risk across industries and reporting formats, ensuring that results are not confounded by systematic differences unrelated to firm fundamentals.

Using conformity metrics as dependent variable and above predictors as independent variables, the following Ordinary Least Squares (OLS) regression, using heteroskedasticity-consistent standard errors (specifically HC3 to adjust for potential non-constant variance in the residuals), is tested:

$$\begin{aligned} Y_i = \beta_0 + \beta_1 Fraud_i + \beta_2 \mathrm{log\_assets}_i + \beta_3 profitability_i + \beta_4 loss_i + \beta_5 asset\_growth_i \\ + \beta_6 asset\_growth\_vol_i + \beta_7 revenue\_growth_i + \beta_8 revenue\_growth\_vol_i \\ + \beta_9 dividend\_paid_i + \beta_{10} firm\_age_i + \sum_{k=1}^{K-1} \gamma_k industry_{ik} + \varepsilon_{i,t} \end{aligned} \tag{5}$$

Where:

- Y_i: One of MAD Value, KS or CS for $Firm_i$
- $\varepsilon_{i,t}$: Error term.

4. Data:

This study begins by collecting firm-level data from Compustat – North America – Financial Annual, including both original financial statements (datafmt = STD) and restated versions (datafmt = SUMM_STD) from 01/2015 to 12/2024. The dataset comprises figures from the Balance Sheet, Income Statement, and Cash Flow Statement. After removing observations with missing or incomplete values, the remaining firms are classified into fraudulent and non-fraudulent groups based on a pre-identified list of fraudulent tickers (Table 4-1). This classification is applied separately to both the original and restated datasets, resulting in four distinct groups: fraud (fraudulent firms with original data), clean (non-fraudulent firms with original data), fraud-restated (non-fraudulent firms with restated data), and clean-restated (fraudulent firms with restated data). For each group, the three financial statements are aggregated into a combined dataset to facilitate analysis. he clean (non-fraudulent) group (both in original and restated form)is further filtered based on total assets (within ±50%) and industry classification, aligning closely with the size and sector of the corresponding fraudulent firms. Specifically, for each fraudulent company, two comparable clean firms are selected, creating a matched sample that enhances the validity of the analysis.

Table 4-1. Fraudulent Companies identified by Audit Analytics.

Firms	Tickers
SONOCO PRODUCTS CO	SON
BARRETT BUSINESS SERVICES INC	BBSI
Bausch Health Companies Inc.	BHC
HARVARD BIOSCIENCE INC	HBIO
KOPIN CORP	KOPN
COGNIZANT TECHNOLOGY SOLUTIONS CORP	CTSH
Roadrunner Transportation Systems, Inc.	RRTS
BRAZILIAN ELECTRIC POWER CO	EBR
BRASKEM SA	BAK
WHITE MOUNTAINS INSURANCE GROUP LTD	WTM
SINOVAC BIOTECH LTD	SVA
SOMNIGROUP INTERNATIONAL INC.	SGI
KULICKE & SOFFA INDUSTRIES INC	KLIC
PPG INDUSTRIES INC	PPG
Clear Channel Outdoor Holdings, Inc.	CCO
ESTRE AMBIENTAL, INC.	ESTRF
iHeartMedia, Inc.	IHRT
FTE Networks, Inc.	FTNW
Kraft Heinz Co	KHC
MATTEL INC /DE/	MAT
Commercial Vehicle Group, Inc.	CVGI
NN INC	NNBR
Luckin Coffee Inc.	LKNCY
TCTM Kids IT Education Inc.	TCTM
Hamilton Beach Brands Holding Co	HBB
BGC Group, Inc.	BGC
VASO Corp	VASO
DENTSPLY SIRONA Inc.	XRAY
Root, Inc.	ROOT
TUPPERWARE BRANDS CORP	TUPBQ
Planet 13 Holdings Inc.	PLNH
Primis Financial Corp.	FRST
HCW Biologics Inc.	HCWB
NCR Voyix Corp	VYX
Tingo Group, Inc.	TIOG

5. Empirical Results:

5.1. Univariate By Financial Statements:

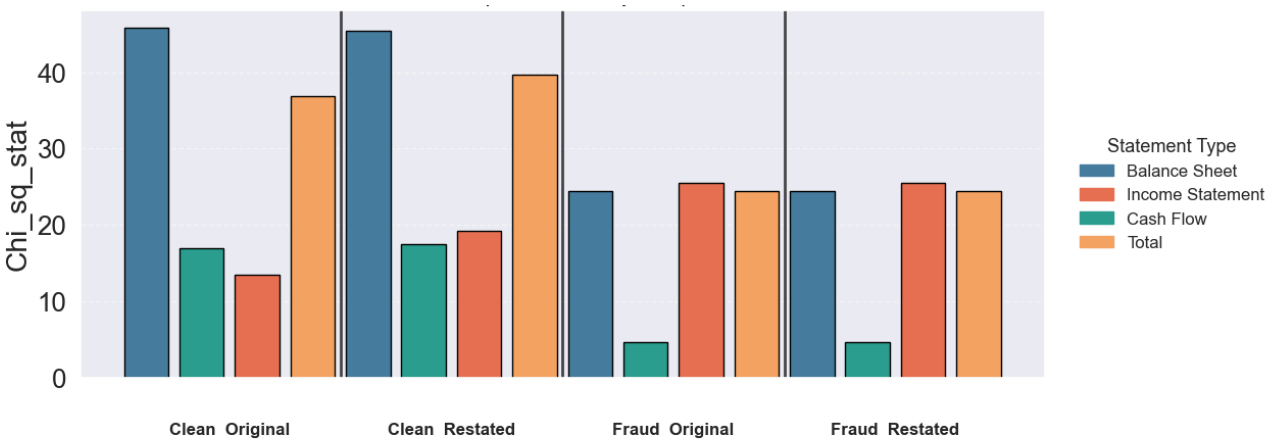

Figure 5-1. CS Statistic by Group.

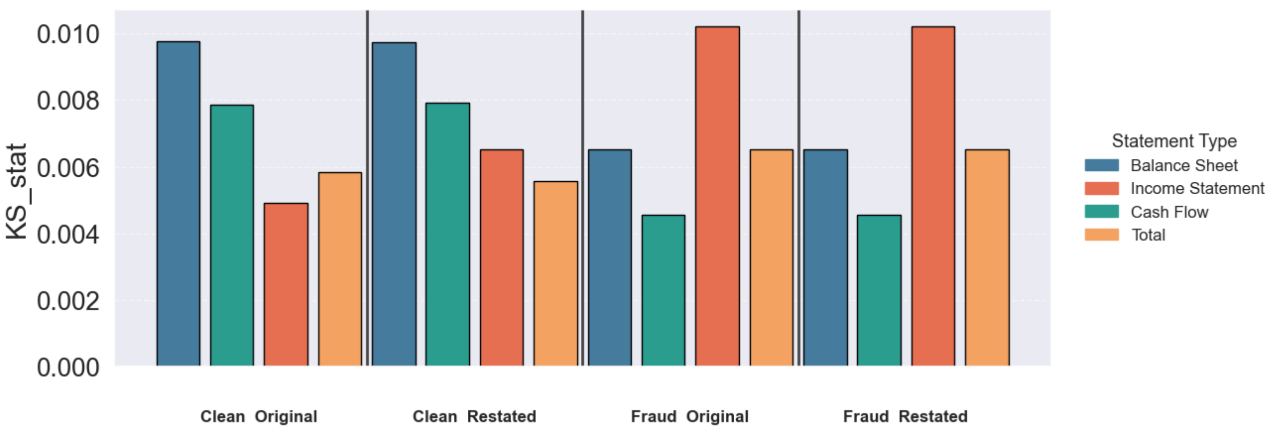

Figure 5-2. KS Statistic by Group.

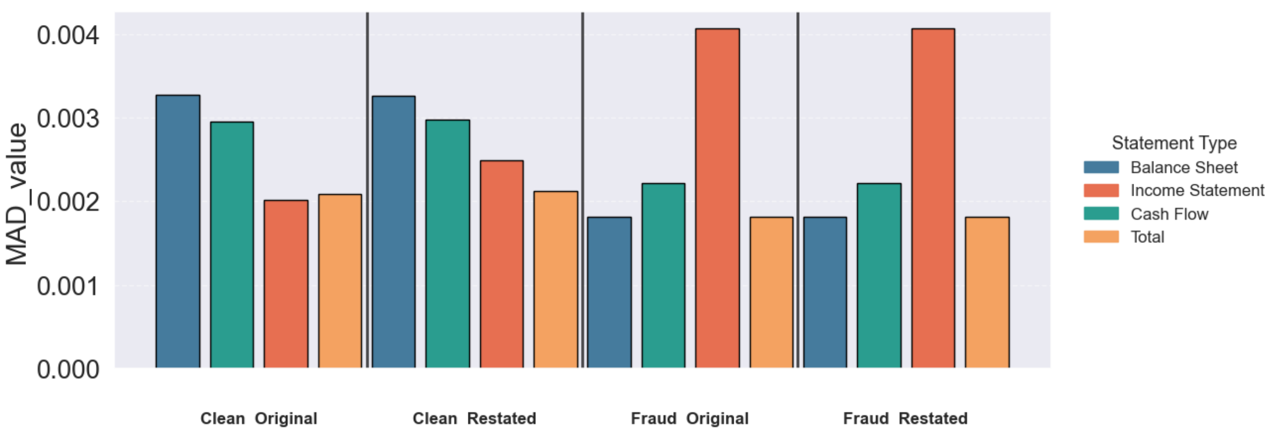

Figure 5-3. MAD Statistic by Group.

BL conformity values across financial statements for both fraudulent and clean firms, before and after restatements, show only minor differences, with all metrics falling within the "Close" conformity range according to the MAD test. The three diagnostic charts: CS (Figure 5-1), KS (Figure 5-2), and MAD (Figure 5-3), collectively provide insight into the conformity of financial statement data with BL across clean and fraudulent firms, both before and after restatements. Interestingly, clean firms consistently exhibit higher CS values, particularly in the balance sheet, indicating greater deviation and pattern remains even after restatement. In contrast, fraudulent firms display notably lower CS statistics, especially in the balance sheet and cash flow sections, hinting at potential strategic efforts to make the figures appear compliant. This trend is reinforced by the KS statistics, which show higher values for clean firms, again led by the balance sheet, and lower values for fraudulent ones, suggesting artificially smoother distributions in manipulated data. The MAD values add another layer to the interpretation: the income statements of fraudulent firms, both original and restated, exhibit the highest MAD scores, indicating significant irregularities in digit distribution. Meanwhile, the balance sheet and total MAD values for fraudulent firms remain low, implying a deliberate attempt to reduce anomalies. Overall, these findings reveal that fraudulent data may be manipulated to mimic Benford's Law more closely, particularly in the balance sheet, while natural, unmanipulated data tends to deviate more.

Table 5-1. Test of Different by Statements.

Test of different	Welch's t-test	KS test	Mann–Whitney U
Original	t=0.9164, p=0.4005	stat=0.5000, p=0.7714	U=10.0000, p=0.6631
Restated	t=1.2277, p=0.2703	stat=0.5000, p=0.7714	U=10.0000, p=0.6631

Statistical tests indicate no statistically significant difference in Benford conformity between fraudulent and clean firms, suggesting that fraudulent firms may not necessarily deviate from BL more than clean firms (Table 5-1).

5.2. Univariate By Firms:

Table 5-2. Statistic of Conformity by Statements - Original.

Metric	n		Mean		Median		Q1		Q3	
	clean	fraud	clean	fraud	clean	fraud	clean	fraud	clean	fraud
Chi-square	539	287	17.4130	16.8115	15.7574	14.550	10.5617	10.4953	21.9925	21.3258
KS	539	287	0.0805	0.07752	0.0717	0.0709	0.0529	0.0523	0.1005	0.0958
MAD Value	539	287	0.0281	0.02700	0.0266	0.0258	0.0213	0.0208	0.0338	0.0321

Table 5-3. Statistic of Conformity by Statements - Restated.

Metric	n		Mean		Median		Q1		Q3	
	clean	fraud	clean	fraud	clean	fraud	clean	fraud	clean	fraud
Chi-square	543	287	17.2425	16.8115	15.7894	14.5498	10.2892	10.4953	21.5379	21.3258
KS	543	287	0.0808	0.0775	0.07120	0.0709	0.0541	0.0522	0.0998	0.0958
MAD Value	543	287	0.0283	0.0270	0.02740	0.0258	0.02094	0.0208	0.0338	0.0321

As seen from above table, clean firms consistently show higher mean and median values than fraudulent firms for all three metrics. For instance, in the original data (Table 5-2), the mean CS value for clean firms is 17.41 versus 16.81 for fraud firms, while MAD and KS statistics follow a similar pattern. Fraudulent firms, by contrast, show slightly lower deviation, potentially due to manipulation intended to disguise irregularities and appear statistically normal. After restatement (Table 5-3), the differences remain largely unchanged, with only minor reductions in CS and KS values for clean firms, while fraudulent firms' statistics remain virtually constant. The stability of fraud firms' conformity metrics pre- and post-restatement could indicate that the restated values still carry over manipulated traits or were not extensively corrected.

Table 5-4. Test of Different by Firms - Original.

Metric	Welch_t	Welch_p	MW_U	MW_p	KS_stat	KS_p	Cohens_d	Cliffs_delta
CS	0.849188	0.396122	80708.0	0.303306	0.066474	0.359615	0.061836	0.043460
KS	1.096744	0.273186	79960.5	0.423460	0.056337	0.567369	0.078916	0.033796
MAD Value	1.551365	0.121359	83057.5	0.080300	0.077153	0.201582	0.113564	0.073837

Table 5-5. Test of Different by Firms – Restated.

Metric	Welch_t	Welch_p	MW_U	MW_p	KS_stat	KS_p	Cohens_d	Cliffs_delta
CS	0.603589	0.546345	80113.0	0.504611	0.066125	0.364059	0.043546	0.028138
KS	1.169842	0.242511	80332.5	0.462907	0.054171	0.614855	0.083265	0.030955
MAD Value	1.706754	0.088354	83390.0	0.095956	0.081955	0.149710	0.120535	0.070193

In both the original and restated datasets (Table 5-4, Table 5-5), none of the p-values fall below the conventional 5% significance threshold, indicating no statistically significant difference between clean and fraud groups across metrics. Specifically, Welch's t-tests yield p-values above 0.10 for all metrics, and the non-parametric Mann-Whitney U tests and KS tests similarly suggest weak evidence of group differences. Effect size measures further reinforce this conclusion: Cohen's d values are consistently small (below 0.12), and Cliff's delta values are close to zero, indicating negligible practical significance. Notably, the MAD statistic comes closest to significance in the restated data ($p = 0.088$), with a slightly higher effect size, possibly reflecting some improved differentiation after correction

5.3. Multivariate:

5.3.1. Multicollinearity:

Table 5-6. VIF Test of predictors.

Variable	VIF
log_assets	7.874768
asset_growth_vol	4.951459
revenue_growth_vol	4.014755
firm_age	3.786718
dividend_paid	3.461767
loss	2.846286
profitability	2.098180
Fraud	1.658277
asset_growth	1.251486
revenue_growth	1.189669

According to Table 5-6, none of the variables exhibit VIF values exceeding the conventional threshold of 10, suggesting that multicollinearity is not severe. However, a few predictors exhibit moderate VIF levels, notably log_assets, asset_growth_vol, and revenue_growth_vol, indicating a potential concern for redundancy that should be monitored in regression modelling.

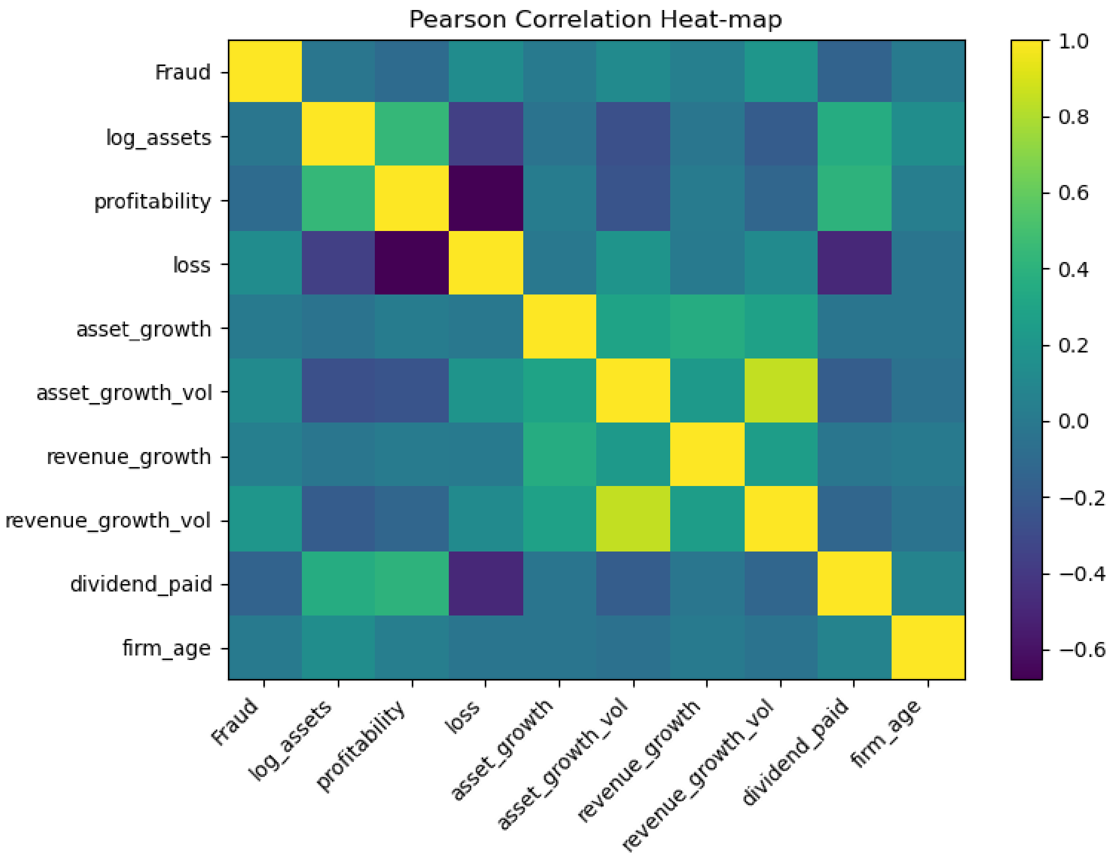

Figure 5-4. Pearson Correlation Heat-map.

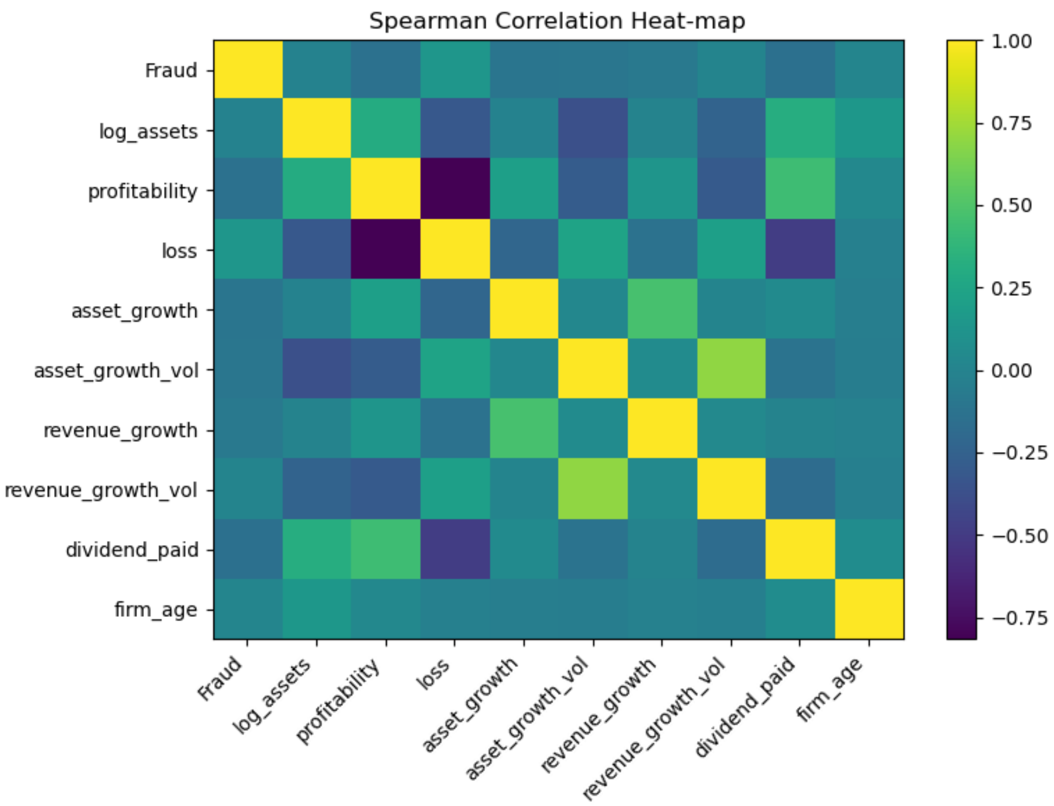

Figure 5-5. Spearman Correlation Heat-map.

The Pearson and Spearman correlation heatmaps (Figure 5-4 and Figure 5-5) provide further insight into pairwise associations. In both matrices, profitability and loss show a strong negative correlation, as expected due to their opposite economic meanings. Similarly, dividend_paid and profitability are positively correlated, while dividend_paid is negatively correlated with loss, consistent with financial theory.

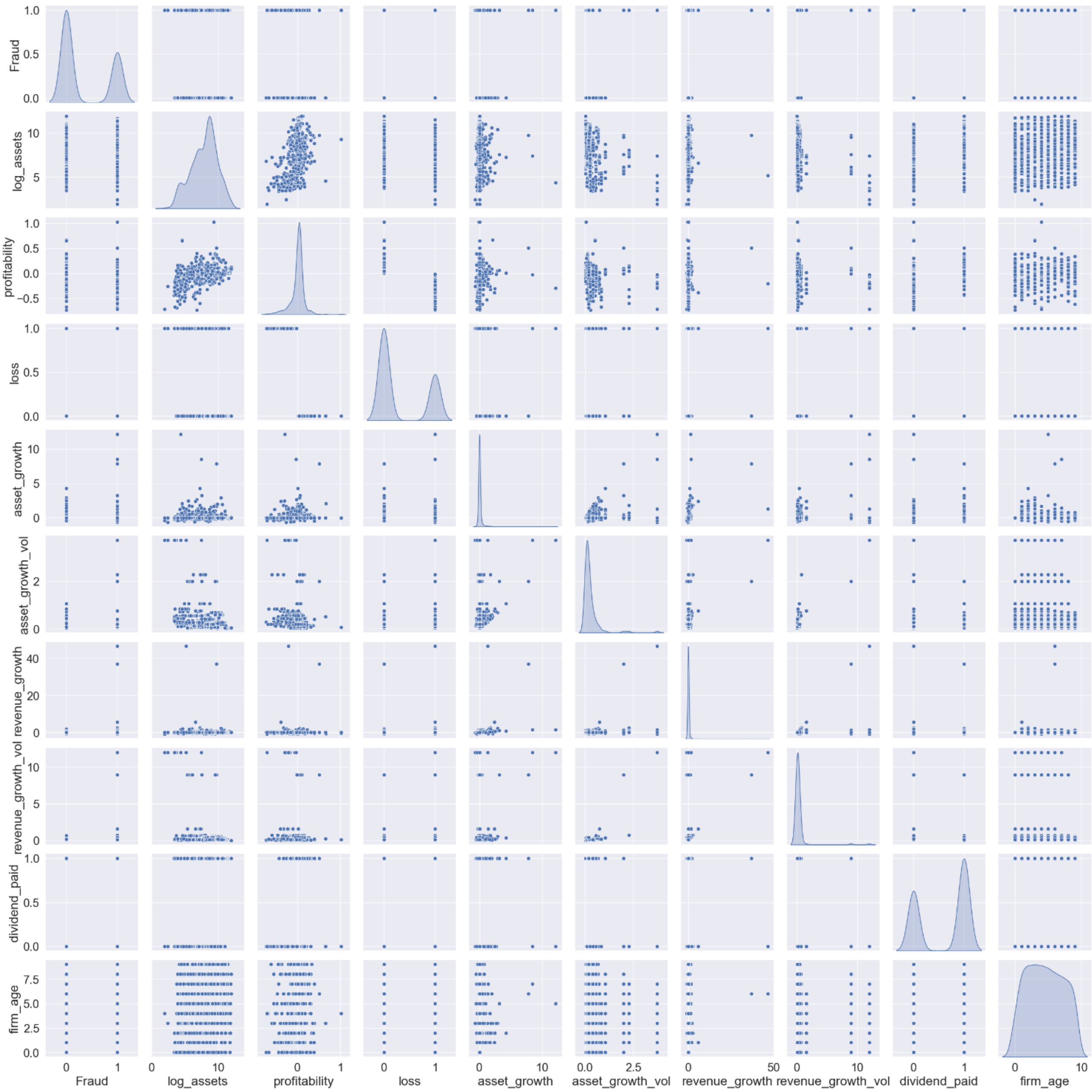

Figure 5-6. Pairwise Scatterplot of Predictors.

The scatterplot matrix (Figure 5-6) confirms these relationships visually and reveals some non-linear patterns and potential clustering. Overall, while some moderate correlations exist among variables, particularly between asset- or revenue-based growth measures and firm size, the diagnostics suggest that multicollinearity is manageable and should not severely distort model estimates.

5.3.2. MAD:

Table 5-7. OLS of MAD Conformity – Original.

	coef	se(HC3)	z	p>|z|	95% CI <	95% CI >
β_0	0.0367	0.0076	4.7996	0.0000	0.0217	0.0517
Fraud	-0.0007	0.0015	-0.4419	0.6585	-0.0036	0.0023
log_assets	-0.0006	0.0006	-0.9911	0.3217	-0.0018	0.0006
profitability	-0.0010	0.0075	-0.1285	0.8978	-0.0156	0.0137
loss	-0.0017	0.0024	-0.6993	0.4844	-0.0065	0.0031
asset_growth	-0.0022	0.0103	-0.2172	0.8280	-0.0223	0.0179
asset_growth_vol	0.0017	0.0051	0.3324	0.7396	-0.0083	0.0118
revenue_growth	0.0023	0.0046	0.5009	0.6164	-0.0067	0.0112
revenue_growth_vol	0.0004	0.0037	0.0985	0.9215	-0.0070	0.0077
dividend_paid	-0.0023	0.0018	-1.2890	0.1974	-0.0057	0.0012
firm_age	-0.0006	0.0006	-0.9482	0.3430	-0.0018	0.0006
industry_2010.0	0.0024	0.0034	0.7045	0.4811	-0.0043	0.0091
industry_2020.0	0.0039	0.0035	1.1042	0.2695	-0.0030	0.0108
industry_2030.0	0.0058	0.0070	0.8288	0.4072	-0.0079	0.0195
industry_2520.0	0.0013	0.0024	0.5645	0.5724	-0.0033	0.0059
industry_2530.0	0.0051	0.0077	0.6651	0.5060	-0.0100	0.0203
industry_3020.0	-0.0008	0.0023	-0.3678	0.7130	-0.0053	0.0036
industry_3510.0	-0.0003	0.0031	-0.0915	0.9271	-0.0063	0.0058
industry_3520.0	0.0028	0.0036	0.7588	0.4480	-0.0044	0.0099
industry_4010.0	0.0128	0.0020	6.3179	0.0000	0.0088	0.0167
industry_4020.0	-0.0007	0.0035	-0.1915	0.8481	-0.0075	0.0062
industry_4030.0	0.0050	0.0030	1.6864	0.0917	-0.0008	0.0108
industry_4510.0	-0.0020	0.0031	-0.6579	0.5106	-0.0081	0.0040
industry_4530.0	0.0004	0.0037	0.1016	0.9191	-0.0068	0.0076
industry_5010.0	0.0070	0.0582	0.1198	0.9047	-0.1072	0.1211
industry_5020.0	0.0014	0.0031	0.4570	0.6476	-0.0047	0.0076
industry_5510.0	0.0056	0.0033	1.7031	0.0885	-0.0008	0.0120

Table 5-8. OLS of MAD Conformity – Restated.

	coef	se(HC3)	z	p>|z|	95% CI <	95% CI >
β_0	0.0453	0.0108	4.2117	0.0000	0.0242	0.0664
Fraud	-0.0009	0.0016	-0.5192	0.6036	-0.0041	0.0024
log_assets	-0.0010	0.0008	-1.2093	0.2265	-0.0025	0.0006
profitability	0.0032	0.0096	0.3358	0.7370	-0.0156	0.0220
loss	-0.0010	0.0027	-0.3723	0.7096	-0.0063	0.0043
asset_growth	-0.0020	0.0061	-0.3293	0.7419	-0.0140	0.0100
asset_growth_vol	0.0011	0.0057	0.1933	0.8467	-0.0101	0.0123
revenue_growth	0.0020	0.0050	0.4000	0.6891	-0.0077	0.0117
revenue_growth_vol	0.0003	0.0027	0.1265	0.8993	-0.0050	0.0056
dividend_paid	-0.0023	0.0016	-1.3867	0.1655	-0.0055	0.0009
firm_age	-0.0012	0.0007	-1.6568	0.0976	-0.0027	0.0002
industry_2010.0	0.0014	0.0039	0.3492	0.7269	-0.0063	0.0091
industry_2020.0	0.0019	0.0036	0.5097	0.6103	-0.0053	0.0090
industry_2030.0	0.0045	0.0071	0.6324	0.5271	-0.0094	0.0184
industry_2520.0	0.0008	0.0025	0.3348	0.7378	-0.0041	0.0058
industry_2530.0	0.0040	0.0078	0.5089	0.6108	-0.0114	0.0194
industry_3020.0	-0.0004	0.0026	-0.1560	0.8761	-0.0054	0.0046
industry_3510.0	-0.0011	0.0032	-0.3366	0.7364	-0.0074	0.0052
industry_3520.0	0.0038	0.0035	1.0802	0.2800	-0.0031	0.0106
industry_4010.0	0.0126	0.0022	5.8089	0.0000	0.0083	0.0168
industry_4020.0	-0.0012	0.0038	-0.3160	0.7520	-0.0087	0.0063
industry_4030.0	0.0041	0.0033	1.2677	0.2049	-0.0023	0.0105
industry_4510.0	-0.0028	0.0030	-0.9306	0.3521	-0.0086	0.0031
industry_4530.0	-0.0005	0.0039	-0.1178	0.9063	-0.0082	0.0072
industry_5010.0	0.0058	0.0332	0.1749	0.8612	-0.0593	0.0709
industry_5020.0	0.0010	0.0035	0.2840	0.7764	-0.0059	0.0079
industry_5510.0	0.0063	0.0034	1.8750	0.0608	-0.0003	0.0130

The regression results from Table 5-7 and Table 5-8, which examine the determinants of MAD conformity with BL, show that the baseline level of deviation is statistically significant across both original and restated financial statements. The intercept term is positive and highly significant in both models ($p < 0.001$), indicating a consistent degree of non-conformity with BL even after accounting for other firm characteristics. This suggests that, on average, firms' reported figures exhibit some degree of deviation from the expected distribution regardless of fraud status or financial characteristics.

Notably, the presence of fraud does not emerge as a significant predictor of MAD deviation in either model. The coefficient on the fraud indicator is negative but statistically insignificant in both the original ($p = 0.6585$) and restated ($p = 0.6036$) models. This aligns with prior non-parametric test results and implies that fraudulent firms do not consistently differ from clean firms in terms of MAD conformity. Such findings suggest that if manipulation occurs, it may not necessarily disturb the first-digit distributions enough to be detected by this metric, or that fraudulent activities may be nuanced and offsetting.

Similarly, log of total assets, firm age, profitability, revenue and asset growth, and their volatilities, also do not show significant explanatory power. Most coefficients are insignificant and close to zero, indicating weak or no relationship with MAD conformity. Although Amiram et al. (2015) proposed that smaller, younger, and faster-growing firms are more likely to misstate financials, this study provides limited support for that claim as shown by insignificant power of MAD above. One potential exception is firm age, which is marginally significant in the restated model (p = 0.0976), hinting that younger firms may show slightly higher deviation from BL.

However, industry fixed effects reveal more meaningful patterns. Firms within the energy sector (GICS industry 4010) show a strong and positive association with MAD deviation in both models, with highly significant coefficients. This may reflect sector-specific accounting practices or reporting complexity that affect numerical patterns. Other industry dummies, while included, generally fail to show statistical significance and lack consistency across models.

Table 5-9. MAD OLS Model Stats.

Model stats	**R-squared**	**Adj. R-squared**	**F-statistic**	**Prob(F-stat)**	**Log-Likelihood**	**AIC**	**BIC**
Original	0.5806	0.3958	15.3775	0.0	363.5925	-673.185	-606.9176
Restated	0.5687	0.3786	20.9867	0.0	352.2379	-650.4757	-584.2084

The model statistics for the OLS regressions of MAD conformity (The OLS regression results for the CS conformity metric reveal weak explanatory power in both the original and restated datasets (Table 5-9). The table indicates moderate explanatory power in both the original and restated specifications. The R-squared values are 0.5806 for the original model and 0.5687 for the restated model, suggesting that approximately 58% and 57% of the variation in Benford conformity of MAD values is explained by the included predictors and industry dummies. However, the adjusted R-squared is notably lower in both cases: 0.3958 (original) and 0.3786 (restated). This decline implies that while the full model explains a fair portion of the variation, a significant share may be due to noise or overfitting, particularly because of the large number of industry dummy variables. The F-statistics (15.38 and 20.99, respectively) and their associated p-values (0.0) indicate that the overall models are statistically significant, meaning the included variables collectively have explanatory power over the dependent variable. In terms of model fit, the log-likelihood, AIC, and BIC values are slightly better in the original data model than the restated one. This suggests that the original data may be marginally more efficient in terms of balancing fit and complexity.

5.3.3. CS:

Table 5-10. OLS of CS Conformity – Original.

	coef	se(HC3)	z	p>|z|	95% CI <	95% CI >
β_0	18.5668	5.9426	3.1244	0.0018	6.9196	30.2140
Fraud	-0.3697	1.2782	-0.2893	0.7724	-2.8749	2.1354
log_assets	-0.3221	0.5069	-0.6355	0.5251	-1.3157	0.6714
profitability	0.8029	6.7069	0.1197	0.9047	-12.3423	13.9482
loss	-0.9470	2.1342	-0.4437	0.6572	-5.1300	3.2360
asset_growth	-2.2186	6.1433	-0.3611	0.7180	-14.2593	9.8221
asset_growth_vol	2.8227	4.4371	0.6362	0.5247	-5.8738	11.5192
revenue_growth	1.6578	2.9263	0.5665	0.5710	-4.0777	7.3933
revenue_growth_vol	0.1210	2.6992	0.0448	0.9643	-5.1694	5.4113
dividend_paid	-1.8636	1.4768	-1.2619	0.2070	-4.7582	1.0310
firm_age	0.0302	0.4926	0.0613	0.9511	-0.9353	0.9957
industry_2010.0	2.1218	3.2636	0.6502	0.5156	-4.2747	8.5184
industry_2020.0	3.9834	4.1061	0.9701	0.3320	-4.0644	12.0312
industry_2030.0	9.0727	6.6422	1.3659	0.1720	-3.9459	22.0912
industry_2520.0	1.0877	2.0896	0.5205	0.6027	-3.0079	5.1832
industry_2530.0	4.3921	7.3694	0.5960	0.5512	-10.0517	18.8359
industry_3020.0	-1.6024	2.2842	-0.7015	0.4830	-6.0794	2.8746
industry_3510.0	-0.8929	2.5359	-0.3521	0.7248	-5.8631	4.0773
industry_3520.0	1.8862	2.9830	0.6323	0.5272	-3.9604	7.7327
industry_4010.0	5.7664	2.1561	2.6745	0.0075	1.5406	9.9923
industry_4020.0	-2.8674	2.8881	-0.9928	0.3208	-8.5279	2.7932
industry_4030.0	3.0527	2.4197	1.2616	0.2071	-1.6899	7.7952
industry_4510.0	-1.5776	2.5791	-0.6117	0.5407	-6.6325	3.4773
industry_4530.0	0.6197	2.9830	0.2077	0.8354	-5.2270	6.4664
industry_5010.0	5.7406	33.5155	0.1713	0.8640	-59.9487	71.4298
industry_5020.0	1.2590	2.8780	0.4375	0.6618	-4.3817	6.8998
industry_5510.0	1.7294	2.0780	0.8323	0.4053	-2.3434	5.8023

Table 5-11. OLS of CS Conformity – Restated.

	coef	se(HC3)	z	p>|z|	95% CI <	95% CI >
β_0	18.6337	6.3604	2.9296	0.0034	6.1675	31.0999
Fraud	-0.0946	1.2969	-0.0730	0.9418	-2.6366	2.4473
log_assets	-0.2459	0.5287	-0.4652	0.6418	-1.2821	0.7902
profitability	-0.6442	7.0278	-0.0917	0.9270	-14.4184	13.1300
loss	-1.6179	2.2447	-0.7207	0.4711	-6.0175	2.7818
asset_growth	-2.1399	6.9027	-0.3100	0.7566	-15.6689	11.3891
asset_growth_vol	2.7102	4.4235	0.6127	0.5401	-5.9597	11.3800
revenue_growth	1.3028	3.4843	0.3739	0.7085	-5.5264	8.1320
revenue_growth_vol	0.1609	3.0749	0.0523	0.9583	-5.8658	6.1875
dividend_paid	-1.9416	1.5144	-1.2821	0.1998	-4.9098	1.0265
firm_age	-0.0632	0.5867	-0.1078	0.9142	-1.2132	1.0868
industry_2010.0	2.4034	3.3651	0.7142	0.4751	-4.1921	8.9988
industry_2020.0	4.0929	3.7771	1.0836	0.2785	-3.3100	11.4958
industry_2030.0	9.2926	6.8181	1.3629	0.1729	-4.0706	22.6559
industry_2520.0	1.2412	2.1267	0.5836	0.5595	-2.9271	5.4095
industry_2530.0	4.7021	7.4353	0.6324	0.5271	-9.8709	19.2750
industry_3020.0	-1.7560	2.2893	-0.7670	0.4431	-6.2430	2.7310
industry_3510.0	-1.0629	2.5697	-0.4136	0.6792	-6.0993	3.9736
industry_3520.0	2.0406	3.0811	0.6623	0.5078	-3.9983	8.0796
industry_4010.0	6.1359	2.2883	2.6814	0.0073	1.6508	10.6209
industry_4020.0	-2.4443	2.9721	-0.8224	0.4108	-8.2696	3.3810
industry_4030.0	3.9558	2.8226	1.4015	0.1611	-1.5764	9.4880
industry_4510.0	-1.6352	2.5210	-0.6487	0.5166	-6.5762	3.3057
industry_4530.0	0.6488	3.1010	0.2092	0.8343	-5.4291	6.7267
industry_5010.0	6.0107	37.2876	0.1612	0.8719	-67.0717	79.0931
industry_5020.0	1.6428	2.9078	0.5650	0.5721	-4.0563	7.3419
industry_5510.0	1.6710	2.0833	0.8021	0.4225	-2.4121	5.7542

The OLS regression results for the CS conformity metric reveal weak explanatory power in both the original and restated datasets (Table 5-10 and Table 5-11). Notably, the variable Fraud is not statistically significant in either model, suggesting that the presence of fraud alone does not meaningfully influence the degree of deviation from BLas measured by the CS test. Similarly, variables associated with Amiram et al.'s (2015) fraud-prone firm profile, such as smaller size (log_assets), younger firms (firm_age), volatility (growth_vol), and growth characteristics (asset_growth and revenue_growth), also fail to reach statistical significance across both models. Most coefficients are small and have large standard errors, and the confidence intervals are wide, often crossing zero, which reinforces the lack of robustness in these predictors.

The only variable that consistently shows significance is industry_4010.0 (Electric Utilities), which exhibits a strong positive and statistically significant coefficient in both the original (β = 5.77, p = 0.0075) and restated models (β = 6.14, p = 0.0073). This may reflect industry-specific reporting patterns rather than firm-level predictors of manipulation.

Table 5-12. CS OLS Model Stats.

Model stats	R-squared	Adj. R-squared	F-statistic	Prob(F-stat)	Log-Likelihood	AIC	BIC
Original	0.5035	0.2847	2.2781	0.0046	-220.0144	494.0287	560.2961
Restated	0.5082	0.2915	2.2703	0.0048	-221.3696	496.7392	563.0066

Despite the lack of significant individual predictors in the CS OLS regressions, the model statistics suggest modest overall explanatory power (Table 5-12). The R-squared values for the original and restated models are 0.5035 and 0.5082, respectively, with corresponding adjusted R-squared values indicating that the models explain around 28-29% of the variance in CS conformity when accounting for the number of predictors, which is relatively substantial in the context of financial accounting research. The F-statistics for both models are significant ($p < 0.005$), suggesting that the set of predictors jointly contribute meaningfully to explaining conformity deviations, even if individual coefficients are not statistically significant. However, the relatively low log-likelihoods and high AIC/BIC values reflect limited model fit and potential overparameterization due to the inclusion of multiple industry dummies. These findings reinforce the idea that while the model captures some general patterns, it lacks precision in identifying specific firm-level determinants of Benford non-conformity. Therefore, the CS test may detect broader structural or sectoral irregularities but remains limited as a targeted fraud detection tool when used in isolation.

5.3.4. KS:

Table 5-13. OLS of KS Conformity – Original.

	coef	se(HC3)	z	p>|z|	95% CI <	95% CI >
β_0	0.0925	0.0325	2.8447	0.0044	0.0288	0.1562
Fraud	-0.0026	0.0079	-0.3325	0.7395	-0.0182	0.0129
log_assets	-0.0023	0.0029	-0.7746	0.4386	-0.0080	0.0035
profitability	0.0138	0.0441	0.3118	0.7552	-0.0727	0.1002
loss	-0.0026	0.0138	-0.1921	0.8477	-0.0296	0.0243
asset_growth	-0.0043	0.0455	-0.0945	0.9247	-0.0935	0.0849
asset_growth_vol	0.0120	0.0302	0.3967	0.6916	-0.0473	0.0713
revenue_growth	0.0075	0.0139	0.5433	0.5869	-0.0196	0.0347
revenue_growth_vol	-0.0003	0.0149	-0.0180	0.9856	-0.0295	0.0290
dividend_paid	-0.0102	0.0073	-1.4080	0.1591	-0.0245	0.0040
firm_age	0.0002	0.0031	0.0540	0.9569	-0.0058	0.0062
industry_2010.0	0.0077	0.0145	0.5271	0.5982	-0.0208	0.0361
industry_2020.0	0.0121	0.0204	0.5918	0.5540	-0.0279	0.0520
industry_2030.0	0.0331	0.0433	0.7644	0.4446	-0.0518	0.1179
industry_2520.0	0.0056	0.0094	0.5966	0.5508	-0.0128	0.0240
industry_2530.0	0.0157	0.0288	0.5440	0.5864	-0.0408	0.0721
industry_3020.0	0.0050	0.0101	0.4980	0.6185	-0.0147	0.0247
industry_3510.0	-0.0032	0.0122	-0.2645	0.7914	-0.0270	0.0206
industry_3520.0	0.0047	0.0136	0.3458	0.7295	-0.0220	0.0314
industry_4010.0	0.0491	0.0156	3.1433	0.0017	0.0185	0.0797
industry_4020.0	-0.0079	0.0195	-0.4034	0.6867	-0.0461	0.0304
industry_4030.0	0.0194	0.0111	1.7552	0.0792	-0.0023	0.0411
industry_4510.0	-0.0040	0.0111	-0.3580	0.7203	-0.0257	0.0178
industry_4530.0	-0.0013	0.0152	-0.0891	0.9290	-0.0310	0.0283
industry_5010.0	0.0077	0.2662	0.0291	0.9768	-0.5139	0.5294
industry_5020.0	0.0054	0.0093	0.5801	0.5618	-0.0128	0.0235
industry_5510.0	0.0116	0.0121	0.9653	0.3344	-0.0120	0.0353

Table 5-14. OLS of KS Conformity – Restated.

	coef	se(HC3)	z	p>|z|	95% CI <	95% CI >
β_0	0.1075	0.0390	2.7573	0.0058	0.0311	0.1838
Fraud	-0.0027	0.0082	-0.3228	0.7469	-0.0188	0.0135
log_assets	-0.0027	0.0033	-0.8233	0.4103	-0.0091	0.0037
profitability	0.0190	0.0479	0.3973	0.6911	-0.0748	0.1128
loss	-0.0022	0.0142	-0.1546	0.8771	-0.0300	0.0256
asset_growth	-0.0042	0.0357	-0.1191	0.9052	-0.0742	0.0657
asset_growth_vol	0.0102	0.0317	0.3228	0.7469	-0.0519	0.0723
revenue_growth	0.0086	0.0140	0.6161	0.5378	-0.0188	0.0361
revenue_growth_vol	-0.0002	0.0126	-0.0174	0.9861	-0.0249	0.0245
dividend_paid	-0.0103	0.0070	-1.4761	0.1399	-0.0240	0.0034
firm_age	-0.0011	0.0031	-0.3548	0.7227	-0.0073	0.0050
industry_2010.0	0.0072	0.0159	0.4563	0.6482	-0.0238	0.0383
industry_2020.0	0.0088	0.0180	0.4855	0.6273	-0.0266	0.0441
industry_2030.0	0.0306	0.0438	0.6999	0.4840	-0.0552	0.1164
industry_2520.0	0.0056	0.0098	0.5721	0.5673	-0.0136	0.0249
industry_2530.0	0.0146	0.0304	0.4791	0.6319	-0.0451	0.0742
industry_3020.0	0.0053	0.0107	0.4953	0.6204	-0.0156	0.0262
industry_3510.0	-0.0032	0.0124	-0.2567	0.7974	-0.0274	0.0211
industry_3520.0	0.0078	0.0130	0.6011	0.5478	-0.0176	0.0332
industry_4010.0	0.0495	0.0158	3.1437	0.0017	0.0187	0.0804
industry_4020.0	-0.0080	0.0201	-0.3965	0.6917	-0.0474	0.0314
industry_4030.0	0.0154	0.0127	1.2094	0.2265	-0.0096	0.0404
industry_4510.0	-0.0054	0.0112	-0.4833	0.6289	-0.0273	0.0165
industry_4530.0	-0.0020	0.0154	-0.1277	0.8984	-0.0323	0.0283
industry_5010.0	0.0077	0.2080	0.0371	0.9704	-0.4000	0.4154
industry_5020.0	0.0058	0.0100	0.5823	0.5603	-0.0137	0.0253
industry_5510.0	0.0133	0.0124	1.0680	0.2855	-0.0111	0.0377

Table 5-13 and Table 5-14 show that the variable Fraud is negative in both the original and restated models but remains statistically insignificant with p-values. This indicates that, when controlling for firm-specific characteristics and industry effects, fraudulent firms do not systematically exhibit greater deviations from BL compared to non-fraudulent firms. These findings cast doubt on the ability of KS-based conformity measures to directly differentiate fraudulent reporting once other explanatory variables are considered.

The regression results fail to provide empirical support for the claim by Amiram et al. (2015), Key proxies for these characteristics such as log_assets, firm_age, asset_growth, revenue_growth, asset_growth_vol and revenue_growth_vol, are all statistically insignificant across both models. Similarly, profitability and the presence of losses show no meaningful relationship with the KS conformity measure. These findings suggest that Benford-based deviations may not be sensitive enough to capture the underlying firm-level risk factors associated with financial manipulation as theorized in prior literature. Interestingly, industry classification appears to be a more important determinant of KS conformity. In both models, the dummy variable for industry 4010 (utilities or energy) exhibits a strong and statistically significant positive coefficient ($p = 0.0017$). This implies that firms within this industry display a higher degree of deviation from BL than firms in other sectors. Other industry dummies, however, remain largely insignificant, suggesting that sectoral effects may be limited to specific cases.

Table 5-15. KS OLS Model Stats.

Model stats	R-squared	Adj. R-squared	F-statistic	Prob(F-stat)	Log-Likelihood	AIC	BIC
Original	0.4581	0.2193	1.3893	0.1483	234.9213	-415.8427	-349.5753
Restated	0.5035	0.2847	2.2781	0.0046	-220.0144	494.0287	560.2961

The model statistics presented in Table 5-15 indicate relatively weak explanatory power for both the original and restated OLS models using the KS statistic as the dependent variable. In the original data model, the R-squared value suggests that approximately 45.8% of the variation in the KS conformity metric is explained by the included predictors. However, the adjusted R-squared drops significantly, indicating a substantial penalty for model complexity and potential overfitting. More critically, the F-statistic of 1.3893 fails to reach statistical significance, implying that the overall model is not statistically different from a model with no predictors. This raises concerns about the joint relevance of the explanatory variables when applied to the original dataset.

In contrast, the restated model shows a modest improvement in explanatory power. The adjusted R-squared increases to 0.2847, and the F-statistic is statistically significant at $p = 0.0046$. This suggests that, collectively, the independent variables have more explanatory value when applied to the restated financial statements. While the improvement is still moderate, it points to the possibility that restated data offer a clearer signal of the structural relationships influencing conformity to BL. However, the overall fit remains limited.

6. Conclusion:

This study assessed the effectiveness of BL in identifying financial statement irregularities among firms flagged as fraudulent by Audit Analytics (2015–2024). By analysing different financial statement types, firm-level patterns, and multivariate regressions, the results offer a nuanced view of BL's diagnostic utility.

At the statement level, fraudulent firms' Income Statements showed the highest MAD values, suggesting greater manipulation or estimation uncertainty. In contrast, their Balance Sheets often displayed *lower* CS and KS statistics than clean firms, both before and after restatements, implying that fraudulent firms may intentionally conform to BL to avoid detection. Cash Flow Statements showed minimal deviations, likely due to their mechanical nature.

At the firm level, clean firms exhibited slightly greater deviations from BL, particularly in Balance Sheets and overall metrics. Yet, these differences were statistically insignificant. This supports the hypothesis that fraudsters may smooth figures to mimic natural distributions, weakening univariate BL tests' effectiveness and suggesting that *conformity*, rather than deviation, could signal manipulation.

Multivariate regressions using MAD, KS, and CS as outcomes found no significant relationship between fraud and BL conformity after controlling for size, age, growth, volatility, and industry. Amiram et al.'s (2015) claim received limited empirical support from the finding of this study. Only industry effects, especially in the energy sector (GICS 4010), consistently predicted deviations from BL.

Model fit statistics were modest, with few models, especially using restated data, yielding significant F-statistics. This suggests restatements may clarify conformity patterns, but not enough to reliably detect fraud through regression alone.

Overall, the findings offer only partial support for Amiram et al.'s framework. While the theory is conceptually strong, this study finds weak empirical links between firm traits and BL deviations. Moreover, the observation that fraudulent firms often show *higher* conformity, particularly in Balance Sheets, raises concerns that Benford-based tools may be manipulated rather than detect manipulation.

Another potential explanation for the weak differentiation between clean and fraudulent firms lies in the classification of the "clean" group. It is plausible that some firms identified as clean in this study may have engaged in misreporting but have not yet been detected or disclosed through audit or regulatory action. This potential misclassification bias could dilute the contrast between the two groups and thereby reduce the explanatory power of the Benford conformity metrics in distinguishing fraudulent behaviour in this study.

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