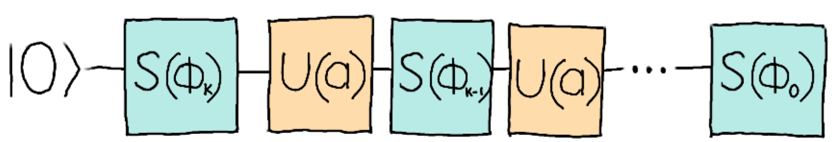
Challenge statement

As you explore deep into the Femto Forest, you notice that the ground is a bit moist and thick. You see a sign with large bold letters telling you to *"Keep out of this swamp"*. Resisting the urge to watch Shrek for the 100th time on your smartphone, you recall that you read about this place in a tourist guide. The Quantum Swamp of Peril is known to absorb any substance and turn it into a toxic version of itself. Leat you become a venomous zombie (and not the superhero kind), make sure to do as the sign says!

While the transformation that the swamp applies on individual quantum states is still unknown, a simplified model suggests that might be a Quantum Singular Value Transform. This seems to be an approximation, and some scholars think that such monstrosities can only be created by non-linear quantum effects, hence the interest in studying this swamp!

The Quantum Singular Value Transform (QSVT) is a quantum algorithm that has become really popular in the research community due to its many applications. In this challenge, we will study a precursor to QSVT, an algorithm known as **Quantum Signal Processing** (QSP). This algorithm works on one qubit and is given by the following circuit



Here, $S(\phi)$ denotes to the phase operator

$$S(\phi) = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix},$$

also known as the quantum signal processing matrix. The operator $U(a)$ is given by the unitary matrix

$$U(a) = \begin{pmatrix} a & \sqrt{1-a^2} \\ \sqrt{1-a^2} & -a \end{pmatrix},$$

where $a \in [-1, 1]$.

If we write down the matrix that represents the whole circuit, we can show that it takes the form

$$S(\phi_0) \prod_{k=1}^{d} U(a) S(\phi_k) = \begin{pmatrix} P(a) & * \\ * & * \end{pmatrix},$$

where $P(a)$ is a complex polynomial of at most degree d and parity $d \bmod 2$. This polynomial depends only on the phase angles ϕ_k.

In this challenge, you are given four phase angles ϕ_k $(k = 1, \ldots, 4)$ which, when fed to the QSP routine, encode a polynomial of the form

$$P(a) = \alpha a^3 + \beta a, \quad \alpha,\ \beta \in \mathbb{C}.$$

Your objective in this challenge is to find the complex coefficients α and β of this polynomial given the phase angles.

Challenge Code

In the challenge template, you must complete the following function.

- `coefficients:` Given an array $[\phi_0, \phi_1, \phi_2, \phi_3]$ (`np.array(float)`) containing the phase angles, this function yields an array $[\alpha, \beta]$ (`np.array(complex)`) with the coefficients of the polynomial P generated by the QSP circuit.

You are also given some space to write some helper functions. It might be useful, for example, to write a routine that returns the matrix associated with the phase angles and a number $a \in [-1, 1]$. Remember that `qml.qsvt` is available for you to use.

Input

As an input to this challenge, you are given an array $[\phi_0, \phi_1, \phi_2, \phi_3]$ (`np.array(float)`) containing the phase angles, in the order shown in the circuit above.

Output

The expected outputs as listed in the test cases below are encoded in a list of the form $[[\mathrm{Re}(\alpha), \mathrm{Im}(\alpha)], [\mathrm{Re}(\beta), \mathrm{Im}(\beta)]]$ (`list`). The output in this format will be built from the `np.array(complex)` output of your `coefficients` function.

Test cases

The following **public test cases** are available for you to check your work. There are also some **hidden test cases** which we will use to check that your solution works in full generality.

```
test_input: [0.5,0.8,1.0,1.0]
expected_ouput: [[-0.1707976, -2.4084889], [-0.8166822, 2.2507432]]

test_input: [-0.20409113, -0.91173829, 0.91173829, 0.20409113]
expected_output: [[2.5,0],[-1.5,0]]
```

If your solution matches the correct one up to a relative tolerance of 1×10^{-4}, the output will be `"Success!"`. Otherwise, you will receive an `"Incorrect"` prompt.

Good luck!