Simulating the Schwinger model via matrix product states

Aniruddha Bapat[1,2,*]

[1] *Joint Center for Quantum Information and Computer Science, NIST/University of Maryland, College Park, Maryland 20742, USA*

[2] *Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA*

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The massive Schwinger model describes 1+1D QED with one flavor of fermionic particles of mass m. The theory interacts via a $U(1)$ gauge field with coupling g. In the Kogut-Susskind formulation (and in axial gauge $A_0 = 0$), the model is discretized into a lattice with spacing a consisting of alternating odd and even sites that each hold one component of the fermion field (electron and positron, respectively). We will adopt a serial numbering of sites and links, so that sites get odd numbers while links get even numbers (see Figure 1).

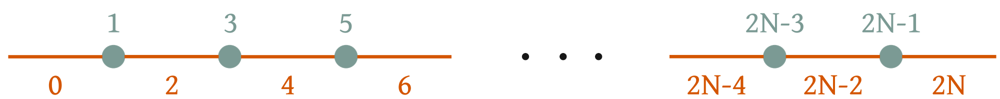

FIG. 1. The lattice

Under a Jordan-Wigner transformation, the Hamiltonian on N matter sites (with open boundary conditions) takes on the form

$$H = \frac{g}{2\sqrt{x}} \left(\sum_{i=1}^{N} E_{2i}^2 + \frac{\mu}{2} \sum_{i=1}^{N} \frac{1 + (-1)^i \sigma_{2i-1}^z}{2} + x \sum_{i=1}^{N-1} \left(\sigma_{2i-1}^+ U_{2i} \sigma_{2i+1}^- + \text{h.c.} \right) \right) , \tag{1}$$

where we introduced dimensionless quantities $x \coloneqq \frac{1}{g^2 a^2}$ and $\mu \coloneqq \frac{2m}{g^2 a}$ (in units of $\hbar = c = 1$). In the above equation, E_{2i} is the electric field at link $2i$, and U_{2i} is the canonically conjugate unitary to E_{2i} that satisfies $[U_{2i}, E_{2i}] = U_{2i}$. To simplify notation, we define the number operator for each matter site as $n_{2i-1} \coloneqq \frac{1+(-1)^i \sigma_{2i-1}^z}{2}$. Extrapolating the theory to continuum is carried out in two steps. The bulk limit involves sending number of sites $N \to \infty$. Next, the continuum limit corresponds to a vanishing lattice spacing, or $x \to \infty$. In order to avoid a singularity, the limits must be carried out in the order prescribed, first N and then x [1, 2].

Due to the lack of the temporal gauge term, the physical Hilbert space is specified by imposing an additional local gauge symmetry, namely, Gauss' law:

$$G_i \coloneqq E_{2i+2} - E_{2i} - (-1)^i n_i \,. \tag{2}$$

In the electric field basis $\{|\epsilon\rangle, \epsilon \in \mathbb{Z}\}$, the gauge operator U can be represented as a "clock" unitary $U = \sum_{\epsilon\in\mathbb{Z}} |\epsilon\rangle\langle\epsilon+1|$. It can be checked that

$$UE - EU = \sum (\epsilon + 1 - \epsilon)|\epsilon\rangle\langle\epsilon+1| = U. \tag{3}$$

We will use the `itensor` package [3] to simulate the ground state of the Schwinger model using matrix product states (MPS).

SETUP

In `itensor`, a local Hilbert space and some in-built operators can be specified by a *site set.* For example, for a lattice of spin-half particles, a site set known as "SpinHalf" creates a two-dimensional local Hilbert space and comes with spin operators tagged as "Sx", "S+", etc. Custom site sets can be created using the given templates.

For the Schwinger model, the matter sites can be modeled as a spin-half site, while the gauge link is modeled as a truncated bosonic site. We call such a site a "ladder", and truncate symmetrically to field eigenvalues $|\epsilon| \leq \epsilon_{max}$. The truncated operator U is then

$$\bar{U} \coloneqq \sum_{\epsilon=-\epsilon_{max}}^{\epsilon_{max}-1} |\epsilon\rangle\langle\epsilon+1|. \tag{4}$$

We impose the global $U(1)$ symmetry $n \coloneqq \sum_{i=0}^{N-1} n_{2i+1}$ on the MPS. It is currently unknown to us whether gauge symmetries can also be used to constrain the block structure in `itensor`. In order to ensure that the final ground state ansatz lies in the physical space (i.e. satisfies Gauss law) we pick an initial state that satisfies Gauss law.

The MPS simulation now has several tunable parameters, which we summarize below:

1. Electric field cutoff ϵ_{max}. This determines the number of dimensions in the link Hilbert space $(2\epsilon_{max} + 1)$ and introduces an error in the ansatz.

2. Number of matter sites N.

3. Lattice spacing parameter x.

4. Mass parameter μ.

5. Maximum bond dimension D.

ELECTRIC FIELD TRUNCATION

We study the effect of electric field truncation on the ground state energy.

Calculations were performed using the ITensor Library [3].

* ani@umd.edu

[1] T. Byrnes, *Density matrix renormalization group: A new approach to lattice gauge theory* (timbyrnes, 2003).

[2] C. Hamer, J. Kogut, D. Crewther, and M. Mazzolini, Nuclear Physics B **208**, 413 (1982).

[3] ITensor Library (version 2.0.11) http://itensor.org.