| For office use only | Team Control Number | For office use only |
|---|---|---|
| T1 ________________ | **67708** | F1 ________________ |
| T2 ________________ | | F2 ________________ |
| T3 ________________ | | F3 ________________ |
| T4 ________________ | Problem Chosen | F4 ________________ |
| | **A** | |

**2011 Mathematical Contest in Modeling (MCM) Summary Sheet**

(Attach a copy of this page to each copy of your solution paper.)

# Take what rescue you, our Kariba


We have analyzed and modeled three projects respectively. In the first place，we had analyzed the main causes are on the damage of the dam and evaluate the rough cost of this project. In the second place, we had considered a number of factors and had focused on the cost caused by the movement of population. In the end, we replace the Kariba dam with a series of small dam group and validate the feasibility including the degree of protection and the management of water resource. Of course, we also provide a few specific suggestions for reference.

Each project analyzing as follows:

**Project One** Repair the Kariba dam. We had found that the dam is facing the crisis of collapse. Which, out of the crash of the flowing water. We had found that the feasibility of fixing the sluice gate is poor, so we choose the method to rebuilding **the plunge pool**. Then, we calculate the cost and analyse the benefit.

**Project Two** Rebuild the Kariba dam. We find that will costs much time to rebuild the dam. In addition, we should maintain the dam simple before we building. For the cost, we are going to consider the main cost including (destroy/rebuild the dam, build the new flood control works and the moving cost of residents who live around the Zambezi river). Because the engineering cost is assured during it’s rebuilding, we just focus on the residents’ moving cost. We can set up the **GM (1, 1)** model to predict the increase of population, calculate the cost and analysis the benefits.

**Project Three** Replace the original dam, with small group dams. Firstly, our model is based on the water storage. And, add the number of dams, one after another. Then, comparing the results. Thus, we had noticed that the multistage dams are better. And we had discovered two important factors by the AHP model——Topography and hydropower. Based on the results measured by the **google earth**, the two factors and Greedy Algorithm, we can choose the better site to place the small dam. According to the results, we had found the multistage dams are better on the management of water resource and flood control. Of course, we also had calculated the cost and analyse the benefit, basically.

Ultimately, we establish the continuous time model, which based on the physical structure. We proposed remedial strategy uniting with the literature and the results. At last, we get the total cost from the model.



**Key Words:** Rebuild the plunge pool、The AHP model、Google earth、Greedy Algorithm、Continuous time model

# Contents

# I Introduction

## 1.1 Overview

Reservoirs created by dams not only suppress floods but also provide water for meaningful activities. The dam plays an important role in human development.

However, in the southern Africa, the Kariba dam faces a dilemma. Three projects in particular are of interest to Zambezi River Authority:

- Repairing the existing Kariba Dam
- Rebuilding the existing Kariba Dam
- Removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River.

- **Requirement:**

Make a brief assessment of the three projects, Analyze the select of sites where build the small dam to make sure the higher security and the management of water resource. Provide the solution on the situation of flood or low water level.

**To project 1** we should consider the reason of damage. If we repair the dam, how much will it cost and how much the benefits will it brings

**To project 2** we should take the repairment into consideration initially before rebuilding it. Then, we should make a model about many factors such as the rebuilding cost, the influence caused by it (mainly included the moving cost of population) and so on.

**To project 3** we should find what the site depends on, find the factors and choose the site. Then, we should analyze the effect (mainly included the water volume).At last, proposing solutions on flood and the low level.

- **Interpretation of requirement:**

**To project 1** we should consider the reason of damage. If we repair the dam, how much will it cost and how much the benefits will it brings.

**To project 2** we should take the repairment into consideration initially before rebuilding it. Then , we should make a model about many factors such as the rebuilding cost, the influence caused by it(mainly included the moving cost of population)and so on.

**To project 3** we should find what the site depends on, find the factors and choose the site. Then, we should analyze the effect (mainly included the water volume).At last, proposing solutions on flood and the low level

## 1.2 Our work

According to《**The influence of fault of the Kariba Dam**》**[1]**, there is a plunge pool at the bottom of the dam. Starting in 1960, it increases the depth from 10 meters to 90 meters, which has a great influence on the root of the dam. we should deal with it quickly, or 108 billion cubic meter water will pour down, which will destroy the downstream area.

About repairing the dam, 《**MSA REPORT**》 [1]give us three ideas, the first is to adjust the outlet, the second is to rainforce the root of the dam, the third is to rebuild the plunge pool. so we make two models(the repairing spillway model and the rebuilding plunge pool model ).The repairing spillway model focus on the structure and the theory, the rebuilding plunge pool model pay more attention to the influence of the dam root and the introduction on rebuilding.

How to make a model about rebuilding the dam? What is the improtant factors should we consider? As it is mentioned by 《**California Calaveras Dam**》 [2],where includes the functions of rebuilding the dam and the stability. so we make a model on the stability and the engineering cost. On the other hand, we analyse the statistics of population and predict its number. In the end, we get the engineering cost and the moving cost model.

As for removing the dam, we meet the problem on choosing sites. we learn from 《**Multi-Criteria Analysis**》 [3]**.**After we know something, we use muti-standard analysis to make sure the standard of choosing sites. By importing the required the contour map of study area and combining the AHP model and ArcGis 9.3, we get some rough sites.

Based on the sites we've got, we can gain the water volume of each dam. Then, pointing the different area with grey level.

Because of the limit of software and information resource, we simplify the choosing sites and improve it by Greedy Algorithm. At last, we get a good project of choosing sites. After that, we make an outflow's time model. Combining with the sites data, we conduct a quantitative analysis, which helps us make a solution of flood and the low water level. From 《**Ecological effects of dam**》 [9],we get the relation of the dam and ecosystem. Combining the restrictment around the Zambezi River, we describe the influence of ecosystem and the economy using our model.

# II Terminology

## 2.1 Terms

- **Froude number**[11] In continuum mechanics, the Froude number (Fr) is a dimensionless number defined as the ratio of the flow inertia to the external field (the latter in many applications simply due to gravity).Named after William Froude , the Froude number is based on the speed–length ratio
- **Spillway**[7] A spillway is a structure used to provide the controlled release of flows from a dam or levee into a downstream area, typically being the river that was dammed.

- **Greedy algorithm Intelligent Algorithm**[11]Which help people make the best choice in the solution to the problem.Not from the overall optimization to be considered, it made only in a sense of the local optimal solution. Greedy algorithm can not obtain the global optimal solution to all problems, but can give the global optimal solution of the whole optimal solution to many problems with wide range.

## 2.2 Symbols

◆ Symbols for Repair And Reconstruction Model:

| ***Symbols*** | ***Descriptions*** |
|---|---|
| $\Delta\alpha$ | Parameters of the spill hole - angle |
| R | Parameters of the spill hole - Radius |
| $K$ | Relief fund distribution ratio |

◆ Symbols for Location Model:

| ***Symbols*** | ***Descriptions*** |
|---|---|
| $A-F$ | Location of multistage dams |
| $X$ | Distance between adjacent dams - length |
| $Y$ | Water in the reservoir - width |
| $Z$ | Elevation of the site |
| $N$ | Number of multistage dams |

◆ Symbols for Build dams Model:

| ***Symbols*** | ***Descriptions*** |
|---|---|
| $W$ | Parameters of the spill hole - width |
| $H$ | Parameters of the spill hole - height |
| $h$ | Height of the horizontal plane |
| $h_0$ | The distance from the horizontal plane to the bottom of the drain hole |
| $P$ | The pressure of the discharge hole |
| $S$ | Represents the area of the corresponding object |
| $F$ | Pressure of the drain hole |
| $v_0$ | Water flow velocity |
| $\eta$ | Energy conversion efficiency |
| $d$ | Altitude drop |

Note：The remaining symbols in the paper

# III Assumptions and Justifications

- **Assumes that the repair process smoothly, ignoring the influence of artificial factors.** Because the progress of the project investment is closely related with the government, the slower progress, the bigger government investment will be .And Residents satisfaction will increase.
- **We choose the appropriate dam type for the construction of the dam group.** In order to avoid dams of different dam types, it can reduce the complexity of dam location model and simplify the solution of its optimal scheme.
- **The effects of rock types on the dams are not taken into account on both sides of the Zambezi River Basin。** To avoid the terrain to meet the dam, but the surrounding rock structure is not enough to support the case of dam. It can simplify the solving process of the optimal solution of the location model with the greedy algorithm。

# IV The Repair Model

In the past 20 years, due to the large number of spills the Zambezi river flood continued. it has caused the river bedrock cut. The bedrock is below the normal water level (90 meters)[1] now. Bedrock of the pool is called "plunge pool". in order to weaken the regional concentration of turbulence, and prolong the life of The Kariba Dam the survival of the dam, we have discussed a series of solutions, including adjust the drainage hole spacing, expand the stilling basin and dam reinforcement. Local Assumption.

## 4.1 Local Assumption

- **Assumption in the adjustment process of drainage hole, water flow will not change**. To see a single variable (such as the drainage hole position adjustment) influence on experiment, so as to verify the effect of a single variable.
- **Assuming that riverbed composition is the same, and evenly distributed.** In order to maintain consistency of blasting effect.
- **Assuming normal repair process, regardless of other factors of interference**. Because of the delay may exacerbate the project cost, also can affect the quality of construction, resulting in some minor cost.

## 4.2 Establishment of Model

The Kariba Dam is 128 meters high and has a crest of 617 meters. In order to clearly describe the restoration of the Zambezi River, we refer to the **reservoir discharge map [1].**

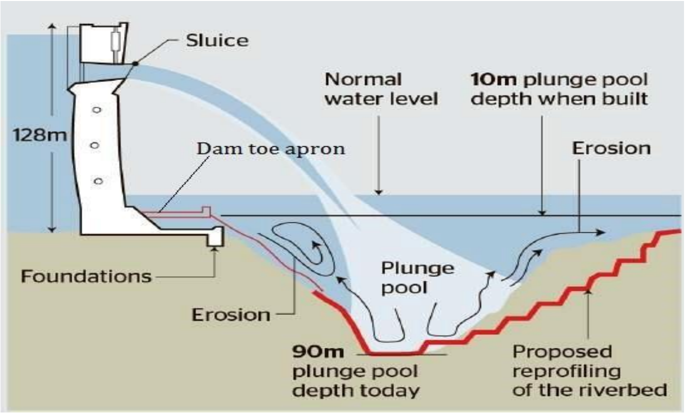



Figure 1 Reservoir discharge diagram[1]

Figure 1 Reservoir discharge diagram Embodying the overall height of the Kariba reservoir, describing the process of forming the "pool" of the spillway ,clearly; And reflects the "diving pool" generated by the role of the dam back to the impact of erosion, thus the object we will repair is clearly——The Plunge pool.

Watching the spatter of flow, we thought of a waterfall, why falls not appear this kind of phenomenon? We think this may be related to the form of water falling, and may also be associated with that "metamorphic apple".（Associated with Plunge pool itself）。After reading the relevant information, we learned the following things：

## ◆ Model One : The adjustment of the drainage hole

We began to consider only adjusting the spacing of the discharge holes, as in the **SUNJian model**[4]. Kariba dam is a double-curved arch dam, a total of six holes discharge hole, the size are $9m*9m$， The maximum discharge capacity is $9500\,m^3/s$。 And the operation of the power plant will reduce the discharge hole flow, so the two middle holes often abandoned water overflow, the formation of certain erosion of the river bed, due to the scope of concentration caused serious cracks, collapse formed " Plunge pool.

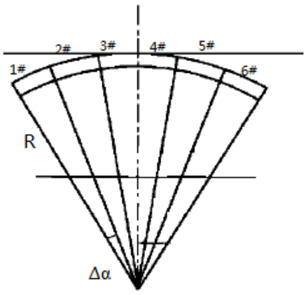



Figure 2Discharge hole

Taking into account the Kariba dam discharge hole size and 6 water height consistent. Under such conditions, the energy distribution of the incoming water depends on $R$ and $\Delta\alpha$。 Figure 2Discharge hole is a top view of the six spill holes.

**Model** give the parameter statistics of the hole in the middle bay of Horizontal Joint , the middle hole in Xiluodu and the middle hole arch dam in Shimen are given. From which we can see $\Delta\alpha$ Of the general range （$4°\sim 4.25°$）， $R$ is a big difference。Therefore, we believe that Kaliba arch dam $\Delta\alpha$ should also be in this range. Take $\Delta\alpha=4°$ analysis.。

Table 1 Arch dam parameters

| parameter | Small bend | River off the crossing | Shimen | Cariba |
|---|---|---|---|---|
| $\Delta\alpha$ | 4.00 | 4.60 | 5.42 | 4.00 |
| $R$/m | 360 | 263 | 120 | 270 |

The longitudinal distance between 4 # and 6 # holes (or 3 # and 1 # holes) is only 4 m along the river direction and only about 2 m in the adjacent holes. The tongs can not be opened longitudinally。Therefore, the six mesopores have the same body shape and the centripetal effect of the arch dam makes the water flow into the river.

We found from the literature to the Kariba arch dam outlet Fr0's number is only 1.84. From the hole elevation, the orifice location in the table below, but Fr0 than the table hole (Froude number is 2) is small, so the hole for the shallow hole. Because of the high orifice height, the degree of underpressure on the water tongue is reduced, and the transverse diffusion of the overflow decreases in the air. Therefore, the mesopores have large orifice, low **Froude number** and shallow hole characteristics. Causing the overflow air diffusion is not large, while almost vertical into the water (into the water angle of 83 °), the fall from the dam is only 62.9 m[4].

Therefore, in this paper, we propose to shrink the jets of the watercourse, and then , shrink gradually along the course and increase the throwing arc length after the overflow are thrown from the orifice. By adjusting the discharge hole, making the water tongue.in the water cushion in the proliferation and overlap, reducing the water tongue erosion of the river bed.

However, since the distance between the holes is fixed, it is not practical to adjust the distance between them. Then we propose another kind of repair measures, remodeling "plunge pool."

◆ **Model Two :Reshape the plunge pool**

First, we consider that when the adjacent gates are opened simultaneously, the overall arrangement of the six drainage holes causes the lateral convergence of the jets. The energy of the discharge is concentrated in the middle of the broad riverbed, where the jet gradually erodes to form a deep steep slough pool. Coupled with the impact of the risk of backflow, leading to dam basement continues to back erosion.

We consider the establishment of dam toe guard to achieve the maintenance shown in Figure 3 bed. In the **S.Z. Melanie[5]** literature model, it was also noted that concrete slabs, which extend from the toe of the dam to a depth of about 30 m and span across 125 m, are built on a solid bedrock with a minimum thickness of 1.07 m. With a deep downstream wall, two continuous layers of steel mesh are arranged at depths of 0.15 m and 0.45 m below the surface, and one 38 mm bolt is arranged every 3 m2.

Maintenance of the dam and the monitoring of the dam need to be carried out in a dry condition, which requires a lot of resources. And in recent years PH gradually biased acid, resulting in concrete cushion corrosion is extremely serious. Therefore, we modeled the establishment of a dam toe-guard, without modeling.

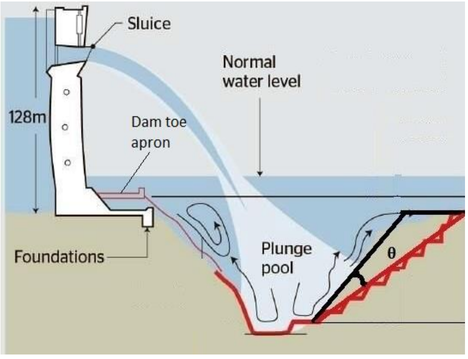


Figure 3 bed[1]

Next, we describe the process of reforming the riverbed. The figure, the discharge hole water has been accumulated in the "plunge pool", can not be discharged from the pool. Therefore, we want to use 10gTNT equivalent water medium explosion containers to achieve the re-arrangement of the river bed. Reduce the angle of the river bed, drain the water.

Figure 3 bed shows that the inclination of the riverbed is reduced when the tank is remoulded with 10gTNT equivalent water medium[6], so as to reduce the scouring of the riverbed to the base of the river bed and realize the protection of the dam base.

Next, we will briefly describe the river bed remodeling process, and the required cost of a simple calculation. In order to ensure the realization of the project, it is implemented and implemented in the non-spill season, considering the compatibility of the project and the necessary spillover. However, the specific construction of the complexity and unpredictability, we only need to calculate the total cost.

At least 90 meters from the water level of the Kaliba Dam, we have to check that at least 300,000 cubic meters[1] of earth and rock must be removed in order to achieve the remodeling of the Kaliba Dam bed, in order to avoid post-explosion Of the stones caused by blockage of the river, we require the size of stones to be less than 25 cm[6].

Table 2 Percentage of mass distribution

| Stone size （cm） | $0.182\, kg/m^3$ | $0.348\, kg/m^3$ | $0.514\, kg/m^3$ | $0.679\, kg/m^3$ |
|---|---|---|---|---|
| 0-5 | 3.5 | 5.4 | 8.6 | 11.8 |
| 5-10 | 10.6 | 10.3 | 11.0 | 7.4 |
| 10-15 | 13.1 | 13.4 | 18.4 | 15.1 |
| 15-20 | 10.2 | 15.1 | 23.0 | 34.4 |
| 20-25 | 10.3 | 17.4 | 38.5 | 29.8 |
| 25-30 | 52.5 | 37.5 | 0 | 0 |

We looked at the amount of explosives required to reconstruct the bed, which is positively correlated with the pressure (ie, the depth of the water). We assume that explosives are placed at a depth of 51 meters and take $0.514 kg/m^3$ calculation (the mass of explosives required per cubic meter of rock). We get the total amount of 10gTNT needed.

$$Count = Stone * 0.514$$

We calculated the number of 10gTNT at least 154,200 kg. (Since 10gTNTs are dangerous goods that can not be traded directly as commodities), we do not give specific costs.

◆ **Model Three : Spillway rehabilitation works**

We found that Kariba dam is also facing another serious problem, due to the construction of age-old, the upstream grooves are in poor condition and need refurbition. to this end，the Zambezi River Authority plans to equip the spillway with an emergency gate.[1]

Here we only consider the reinforcement material and the cost of the emergency gate, plus the dam surface repair and reinforcement costs. Since the specific cost data is not known, we only give the approximate formula:

$$\text{Cos t} = material + emergency + wage$$

# V The Reconstruction Model

Whilst reviewing the impact of the failure of the Kariba Dam it has been assumed that the dam would be re-built, however with climate change and rainfall patterns impacting future dam levels and new technologies coming to the fore, will this necessarily be the solution?[1]

Of course, we should consider rebuilding the dam. Some factors we consider is necessary such as geological conditions of dam foundation, selection of dam type, ravity retaining wall ,remaining part of the original dam deformation potential impact on the water inlet and so on. But we focus on the change of population during the rebuilding time. By GM (1,1) model, we predict the population and calculate the cost.

## 5.1 Local Assumption

- **With 33% of the total population living in the coastal population[1].** The total population is about 30 million people, almost 33% of the total population of the riparian countries lives in the basin.[1]
- **Rebuild and remove the dam needs 8 years**[1]**.** According to the literature, it costs at least 5 years to rebuild and 3 years to remove.
- **Provide the emergency funds according to the national policy.** Different areas develop differently, we should adjust measures to local conditions.
- **The number of people who live along the river is equal to Thirty-three percent of the total. (the total refers to the population of eight countries)** The total population is about 30 million people, almost 33% of the total population of the riparian countries lives in the basin.

## 5.2 Establishment of Model

We only consider the total cost and ignore some details during the rebuilding time, like Forrest[2] model.

Rebuilding a same but new dam in the downstream. According to the structure of the dam, during the design, we apply the previous model (Repair Model) to rain force the dam simply. Then we choose the same dam type to build. At last, we consider the spillway building.

Figure 4 terrain

In the picture, the old dam needs total 1.036 billion cubic meters of concrete[12]. If the cost only includes concrete (Ignore the artificial cost).**Rebuild dam** spend fee will be far more than provided by the maintenance fund.

On the other hand, the reconstruction process will affect the ecological environment along the Zambezi River. The loss of ecological environment is Incalculable.

It also brings us some benefits. It brings us more generated energy, but the benefits are so limited. We do not recommend using this method.

## ◆ Gray GM(1.1)

In addition,we focus on the population,there are some reasons we choose GM(1,1) as follows:

a) **The Logistic model.** We can't distinguish the increase of population between natural increase and sluggish increase, we don't know the local living conditions.so we give it up.

b) **BP network model.** The method needs many trains to get a good result. It has a slow convergence speed and cost long time.so this method is abandoned.

c) **As for time-sequence predictive model.** This model is suitable for medium-term prediction. In consideration of our assumption, the time is short. using the model may affect its accuracy.so we don't consider this way.

At last, we choose the GM(1,1) model. Applying the model to predict the population next 8 years. We collect the population from 1980-2013 years and use the method of relative error to prove the rationality. Then, enter the formula with population, we can get the final costs.

We refer to the population data, which will be placed in the appendix. Here are the amount of population from 1980 to 2013 years.[13]

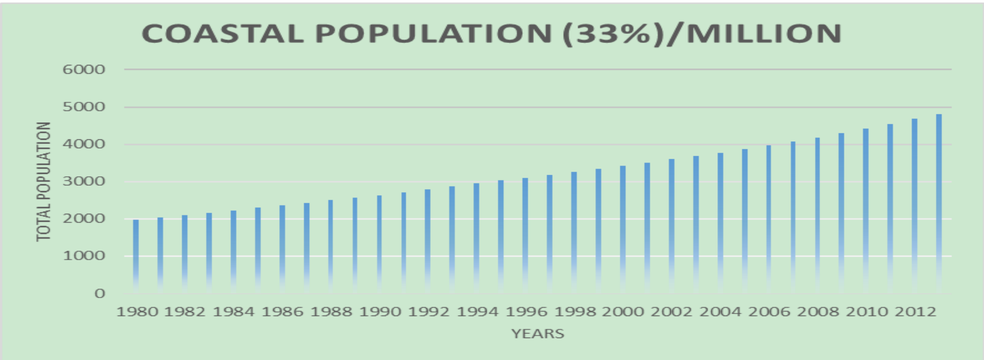



Figure 5 Coastal population (33%)/million

Here are the population data we predict.The stability analysis for GM(1,1) model will be shown in parts of VII Sensitivity Analysis.

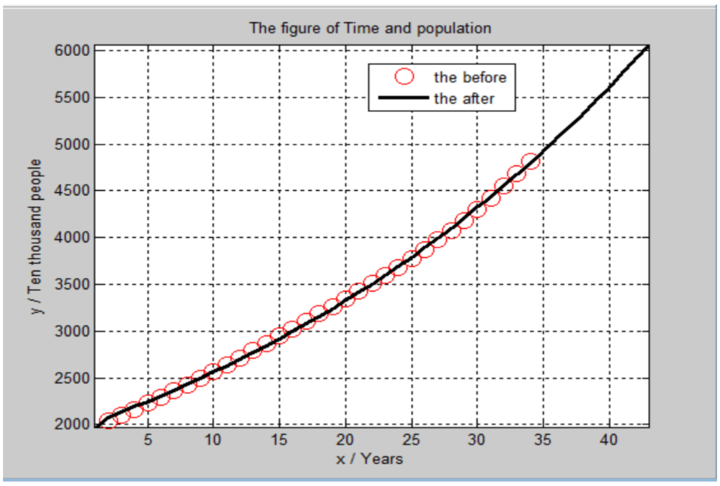



Figure 6 Time and Population

From Figure 6 Time and Populationconclusions could be made that the amount of population is increasing during the removing to rebuilding time. We can calculate the cost by the emergency funds the goverment provided.

Table 3 Expenditure on salvage

| | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|---|---|---|
| Coastal population (33%)/million | 5463.21 | 5607.73 | 5756.07 | 5908.35 | 6064.6 | 6225.08 | 6389.76 | 6558.7 9 |
| Cost (everyday/ million) | $Cost = Population * relief\,money$ | | | | | | | |

According to Table 3, we can calculate the cost. However, the cost is too big, we can try to distribute the funds to each family. Another words, family is as one unit.so the results will be multiplied by a coefficient K(suppose we know it).

# VI Our Model

We start from the Repair Model and the Rebuild Model, finding a mutistage dam structure to replace the Kariba Dam. To the beginning, make a model to describe the reservoir and flood control ability. Applying **Google earth** software to depict the terrain structure along the Zambezi River.

## 6.1 Local Assumption

- **Flood water keeping.** We should make sure a fixed value so that we can go into calculating.
- **Here are many dams. Their base address ranges in same level, the dams are the same except for the difference of outlet entrance’s level.** To keep variables uniquely.

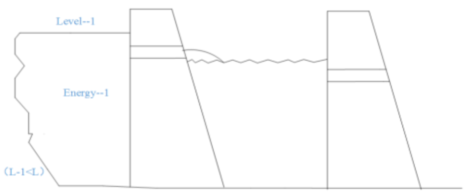
## 6.2 A Simple Example

As for the change in the removing-after time, we can image that taking advantage of multistage dam to describe.。

◆ **Model One**

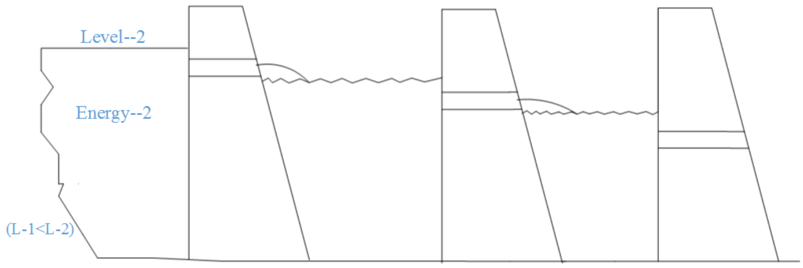
Firstly, we consider the two-stage dam. Because of the level of the dam height determines the reservoir storage capacity, the reservoir storage determination the generated energy and the flood control capacity determination water storage.


(a) One dam




(b) Tow dams

Maybe, two-stage dams isn't obvious, we try to add another stage.




(c) Three dams

Figure 7 Multi - level dam

From Figure 7 Multi - level dam, we can see that the multi-stage dams' water-storge capacity is improved and the total energy decrease $mg\Delta h$ .we know the decreased water level will enter next dam. But where does the lost energy go to? Yes, the reduced hydroenergy can create energy. Otherwise, we will find multi-stage dams can store more water, which will reduce the influence of flood.

- **Model Two**

Maybe the above model can provide many useful conclusion, but it's not practical obviously. Building dams on the same level will waste resources. Thus, referring to the Google Map, we can get the terrain data along the Zambezi River. we can improve the above model to let it become more significant.

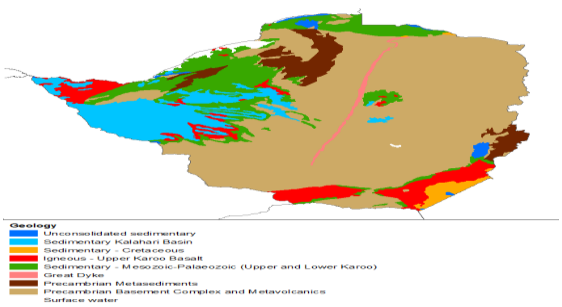



Figure 8 google earth

From figure 8, we cancel the setting that all dams are on the same level. All dams are built on the different levels. When considering the different altitude of different dams, we find that each model has different levels ( $Level_3 < Level_2 < Level_1 < Level$ ) when considering the same outlet. We get a conclusion that the capacity of releasing the flood, the capacity of water storage and generating capacity will be improved further.

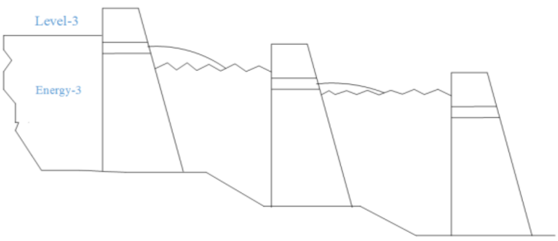



Figure 9 Gradient dam

## 6.3 The Location model

Althrough **a simple model** has certain value, it can't meet the actual terrain along the Zambezi River. we will conduct a practical describe on how to choose sites, like the Hydropower station site selection of **MCA**《Multi-Criteria Analysis Mehthod》**[3]**.By data processing software ArcGis9.3, we can deal with data (like terrain, ISO height, etc). Combineing the AHP Model, rank analytical method and orographic map, we will get the different height of dam, the different generated energy and the different area of reservoir under the different sites.

We will analysis the model in detail as follows:

Firstly, analysising the protential factors of choosing sites. Then, with the help of Google Map to point the terrain along the river bank. On the basis of the assumptions, we can take advantage of Greedy Algorithm to get the better number of dams. At last, By calculating the water storage from removing-before time to removing-after time, we can prove the correctness of choosing sites.

- **The AHP Model**

The requirement (literature) we look up about building the dam as follows:

Table 4Site requirements[3]

| Factor | Terrain | The energy of water | Population density | Land use |
|---|---|---|---|---|
| Limit | $slope > 35°$ | $energy > 0$ | Architectural pixels<40/km$^2$ | $distance > 400m$ |

We build the AHP Model by using the factors of choosing sites. we get four factors by calculation as follows:

$$Contribution = (0.5 \quad 0.3571 \quad 0.0714 \quad 0.0714)$$

We choose the terrain and the water storage as the most important factors.

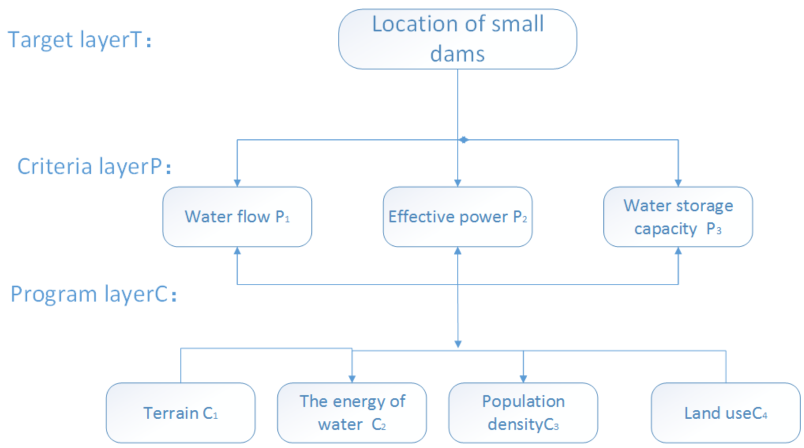



Figure 10 AHP Hierarchy

- **The Location Model**

We analysis the terrain structure by Figure 8 google earth and point some dots. After analysising, the footstone is mainly gneiss and granite. We also get the elevation graph. Because of the residential area in the second part. If the dams are built in the third part, the residents will not be protected and we can't save water. Thus, we choose first pat to build dams.

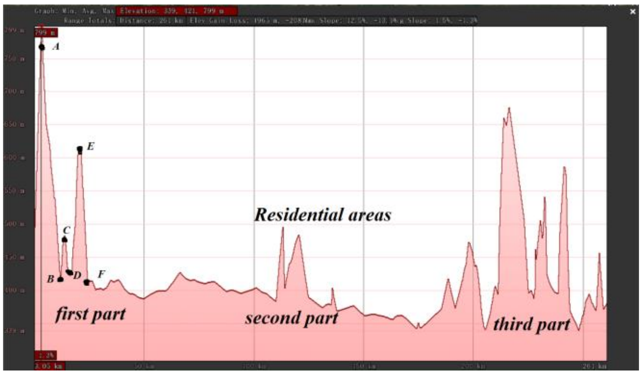



Figure 11 Terrain trend

Figure 11 Terrain trend We use **Google earth** to measure data as follows (point A to F represents sites we choose, x(km) represents the distance of adjacent dam, y(m) represents the flow width, z(m) represents the altitude).Specific data is as follows:

Table 5Addressing coordinates

| point | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| coordinate | (6.38,799) | (11.6,415) | (13.9,475) | (15.5,123) | (20,676) | (24.4,382) |
| Y(m) | $A \leftrightarrow B = 120$ | | $C \leftrightarrow D = 160$ | | $E \leftrightarrow F = 290$ | |

Next, we will calculate the specific water storage by two-stage model. According to assumptions, we use 100 meter as the dam height. The figure of specific water storage as follows:

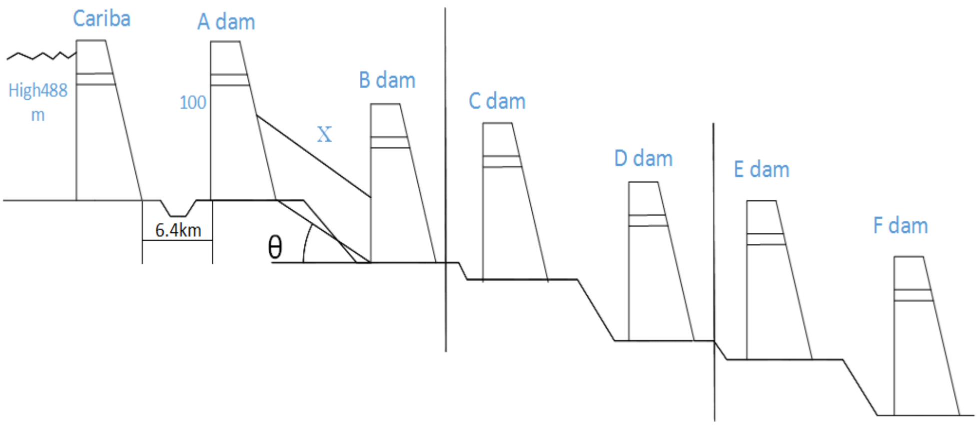



Figure 12 Water storage

Figure 12 Water storage **Calculating the water storage:**

a) calculating the length of water storage(X)(Taking an example of A to B):

$$X = \frac{|X_A - X_B|}{\tan\theta} \qquad 其中, \tan\theta = \left|\frac{Z_A - Z_B}{X_A - X_B}\right|$$

b) Calculating the volume of water storage:

$$Volume = 100 * X * Y$$

c) According to the calculation, the water storage will improve $2.14\%$.

$$\begin{cases} MRL = 1.8*10^{11} m^3 \\ Voleme = 3.9*10^{8} m^3 \\ Voleme/MRL \approx 2.14\% \end{cases}$$

- **The Number of dams Model**

The above calculation indicate that building the two-stage dams will improve the water storage.it also prove the correctness of multi-storage dams. Similarly, If adding the number of dams between A and F (A to F belongs to the two-stage dams), the structure of the dams will be improved greatly. Therefore, we will choose sites from A to F.

We assume that each drop 50 meters to build a dam. Applying Greedy Algorithm, we will get value in weight optimal solution. By calculation, the number of dams instead of the Kariba Dam is 18.The situation of distribution is as follows:

Table 6 Distribution of dams

| Distributed | $A-B$ | $C-D$ | $E-F$ |
|---|---|---|---|
| Quantity | 7 | 7 | 4 |

Next, based on the idea of iteration, canceling the assumption that each drop 50 meters to build a dam. The building of dams needs to care for the terrain factors and the requirement in Table 4, it's so complicated. Here are some steps of arithmetic we just give out.

Taking an example of A to B, to assume the number of dams is N.

**Step 1** adjust the dams from the N the first in turn, write down the value of Volume each time.

**Step 2** reduce the value of N one by one (N=N-1).

**Step 3** judge $N = 0 \begin{cases} fulse & \cdots return\ step1 \\ true & \cdots break \end{cases}$

**Step 4** End.

In this way, through iterative calculation, we will get a good site to provide greater levels of protection and water management.

## 6.4 The Build dams Model

### 6.4.1 Physical model

First, we make a physical model to describe the total structure of dams. To assume the size of the outlet entrance is all for $W*H$ , the length is $L$ and the number of outlet entrance is $n$ .The shape of outlet entrance is as Figure 12.

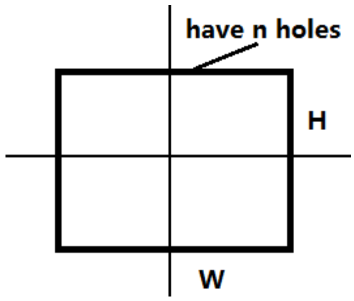



Figure 13Holes

**The effect of outlet entrance** is to drain away water. Therefore, the distance from the horizontal level to the outlet entrance is important factors, which affect the water discharge and effective output.

To assume the distance from the horizontal level to the bottom of river is $h$ .we make a classified discussion about $h_0$ , which represents the distance from the horizontal level to the bottom of outlet entrance. In the following three models, we calculate one dam's the water discharge and effective output, then the results multiply $n$ (the number of outlet entrance) will be the total (both two).The small dams' model is as follows:

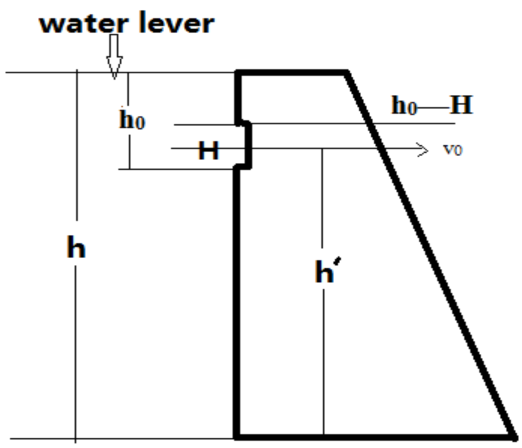



Figure 14Small dam

- **Class One :** When $H < h_0$:

Any outlet entrance of the pressure $P$ :

$$P = \rho * g * h$$

Calculation for the pressure $F$ from water storage to the outlet entrance:

$$\begin{cases} F = P * S \\ S = W * H \\ F = \int_{h_0-9}^{h_0} \rho gh * W * dH \end{cases}$$

Kinetic energy theorem is applied to estimate the flow rate of the water $v_0$

$$F * L_{bore} = \frac{1}{2} * m * v_0^2$$

To simplify the calculation, we assume that the loss of Kinetic energy isn't considered. Calculating the water discharge and effective output as follows:

$$\begin{cases} Flow = n * v_0 * S \\ Power = (\frac{1}{2} m v_0^2 + mgh') * \eta \end{cases}$$

Note: $m$ is the weight of water, $h'$ is the distance from the center of outlet entrance to the riverbed.

◆ **Class Two :** 当 $0 < h_0 < H$ 时

In a similar way, we build kinetic energy equation to calculate the water discharge and effective output as follows:

$$\begin{cases} F = \int_{0}^{h_0} \rho gh * W * dH \\ F * L_{bore} = \frac{1}{2} * m * v_0^2 \end{cases} \xrightarrow{v_0^2 = \sqrt{(2FL_{bore})/m}} \begin{cases} Flow = n * v_0 * S \\ Power = (\frac{1}{2} m v_0^2 + mgh') * \eta \end{cases}$$

◆ **Model Three :** When $h_0 < 0$

When the water level in the reservoir is under the outlet entrance, the water in the reservoir doesn't pressure the outlet entrance. Therefore, as long as taking the data in, we will calculate the water discharge and effective output.

### 6.4.2 The Flood control model

Combineing with the physical model, we try to solve the process of flood. By **Kariba Report**[1], we know that if there is one dam, the flood will destroy downstream area in 8-10 hours.

We assume the dam is an ecosystem. In this way, the largest amount of flood is still needed to consider local rainfall and evaporation. To simplify it, we assume that water yield is constant. To discuss the maximum water discharge's flood process, we assume the shape of reservoir is cylinder. By adding the time of draining away water. We can get the maximum escape time $T$ .Because of the decrease of water yield, the pressure of spillway has been reduced, which leads to a variable named flow velocity.

$$\begin{cases} (h_0 - H) * S_{\text{lake}} = nv_0 S * t_1 \\ h_0 * S_{\text{lake}} = nt_2 \int_0^{Min(v_0)} Sdv \\ t = t_1 + t_2 \end{cases}$$

Note: $t_1$ is the time that the water level has dropped down to $(h_0 - H)$, $t_2$ is the time that the water level has dropped down to $(h_0 - H)$.

In the process of multistage dam flood discharge, to assume the number of dams is N. Each one stage will open the spillway to release the flood, so we can calculate the all time, when all the dams defend the flood as follows:

$$T = \sum_0^N t_i + 8 \; or \; 10 \cdots (i = 0, \cdots, N)$$

Through the model, we only need to bring a different water level, will be able to draw a specific amount of flood discharge and flood discharge time.

### 6.4.3 The Low water maintenance Model

According to the decrease of water level, which will extend the mutagenicity, the Lake Kariba drys up frequently. It will have a great effect on the lives of residents, yielding, irrigation and industrial manufacture etc. we just put forward some emergency measures as follows:

- Reduce sluicing times, maintain the water storage
- Reduce the generated energy, improve the price of electricity

## VII Sensitivity Analysis

### 7.1 Gray GM(1.1)

We Test the stability of Gray GM (1.1) with relative error (Relative error = | measured value - true value | / true value). As a result of data constraints, only the data of Zambezi River inhabitants can be found during the 1980-2013.We apply the Gray GM (1.1) to predict the data and the original data to compare the results are as follows:

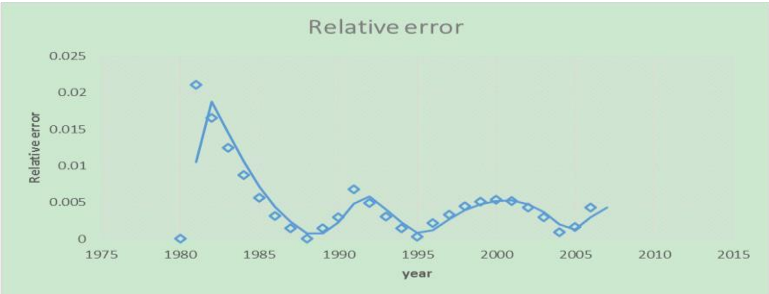



Figure 15 Relative error

Figure 15 Relative error , we can get the relative errors of the Relative errors <0.02, verify the Gray GM (1.1) model is stable.

## 7.2 The Location model

Taking into account the number of dam groups is variable, In addition, considering the altitude, we add the cost of building dams for analysis. Therefore, we converted the relationship between the altitude of the dam group and the cost of the project to the relationship between the altitude and the number of dams. We make an assumption that the average cost per dam is $Money$ ($) .

On the basis of **the Build Dams Model**, we calculated the corresponding bus number and the required cost by changing the altitude difference between the dams. According to the topic, we know the limitation of the number of dams (10-20). Then we select a reasonable elevation difference calculated for each additional 10m after the cost of the rate of decline:

Table 7Location error

| $d$(m) | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|
| $Money$ | 32 $Sn$ | 24 $Sn$ | 18 $Sn$ | 15 $Sn$ | 13 $Sn$ | 11 $Sn$ | 10 $Sn$ | 8 $Sn$ |
| ratio | | | | 16.7% | 13.3% | 15.4% | 9.9% | |
| $\text{ratio} = (Money_N - Money_{N-1})/Money_{N-1}$ | | | | | | | | |

Table 7Location error Calculates the number of dams built at different intervals, we found that as the spacing increased, the number of dams decreased and the cost was reduced. However, the greater pressure of each dam.

# VIII Final Remarks

## 8.1 Strengths And Weeknesses

- **Strengths**

➢ The proving model is modeled using the patterning method, which can clearly analyze the relationship between the various parts of the model.

➢ The gray-forecasting mode of population is less error than the actual data, so this model has good effect in predicting the change of local future population.

➢ The factors affecting site selection are selected by AHP, the optimal site number is obtained by greedy algorithm, and then the optimal or near optimal solution is obtained by iteration. Finally, a continuous time model is established to dynamically simulate the flood discharge process.

- **Weeknesses**

**(a)** Because the data of site selection conditions is difficult to collect in a short time, we select the subjective strong AHP to screen the main factors affecting site selection, and there are some human errors. If time is enough, we can use principal component analysis to eliminate the subjective error.

**(b)** Due to the lack of data on the lake variation in the Kaliba reservoir every year, we can't further validate the constructed flood model and re-optimize the model parameters. In the case of sufficient data, we can adjust the model parameters to build a more accurate model of the flood.

**(c)** In addition, the reliability of the dam site is hindered by the inability to analyze the geographical environment and geological structure of each basin in detail. By careful geographical structure, we can set up the dam in the river can't be built on the obstruction point, when the greedy algorithm iterative optimal location, to avoid these dam points can't be built so that the site of the more practical significance.

## 8.2 Future Model Development

Using **iterative greedy algorithm**, combined with sensing equipment for real-time dam data detection and adjustment, will play a better regulation of water resources management role. In addition, the use of analytic hierarchy process and fuzzy comprehensive evaluation of the results of each basin can be used for water use on-site survey and grade assessment to help the government real-time understanding of the situation near the basin. Our location model, coupled with a different geographical structure, can be extended to wind power stations, nuclear power stations and other energy station location.

# IX Evaluation Report Analysis for ZRA

## Repairing the existing Kariba Dam

| | | |
|---|---|---|
| ***The adjustment of the drainage hole*** | The size of the 6 - hole dam which is double - curvature arch dam is $9m*9m$, The maximum discharge capacity is $9500m^3/s$.[12] Through the water flow model, reduce the influence of THE plunge | |
| Cost and Income | As the location of the scupper hole has been fixed, this option is of worthless | |
| ***Remodeling plunge pool---Option One*** | Construction of the dam site Protector: the establishment of concrete retaining tank, extending from the dam toe about $30m$, the two sides span $125m$, Thickness is $1.07m$. Including a section of downstream key-wall, Below the surface $0.15m$ and deep laying 2 layers of continuous steel mesh, each $3m^2$ arranged a $38mm$ anchor.[5] | |
| | Cost- ***One*** | $30*125*1.07m^3 concrete and$ and $(30*125*1.07)/3\ anchor$ |
| ***Remodeling plunge pool--Option Two*** | A total of $30*10^4\ m^3$ [1] stones need to be cleaned, The size of the reformed stones is less than $25cm$.[6] According to the relationship between the power of explosives and the pressure of water depth, It was found that the hole filled with explosive was $51m$[6] deep, Requires $15.42*10^4 kg$ of $10gTNT$. | |
| | Cost- ***Two*** | $M*15.42*10^4(M-kg/\$)$ |
| **Repair drainage hole** | The installation of an emergency gate on the upstream grooves to repair the surface of the dam to repair and reinforce. | |
| Cost | $\text{Cos t} = material + emergency + wage$ | |
| Income | Extend the service life of the dam, and eliminate security risks | |

## Rebuilding the existing Kariba Dam

| | |
|---|---|
| **The construction of dam** | We apply Gray GM (1.1) to forecast the number of people who need to be diverted into the international minimum living security fund to obtain the required expenditure. |
| cost | $\text{Cos}\,t = Population * relief\,money*K$ |
| **Construction of flood control** | To build flood control works before the reconstruction to cope with possible flooding in the rainy season. |
| cost | Expense much more than remodeling the plunge pool |
| Income | To protect security in the short term, but the cost of the huge amounts of money is almost no way to earn back, and the flood security risks are still |

## Removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River.

| | |
|---|---|
| **Dam group** | It is necessary to build 10-20 small dams, assuming an average cost of F for each small dam. |
| | $N * Money$ |
| **Population Transfer and Resettlement** | The dam group is built on the upper reaches of the Kariba neighborhood. Reduce the downstream residents of the transfer, just transfer upstream residents |
| | $\text{Cos}\,t = Population * relief\,money*K$ |
| Income | Increase water storage capacity, improve flood capacity |

# X Reference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[12] http://www.zaraho.org.zm/hydro-electric-schemes/kariba-hes/technical-data

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# XI appendix

- **AHP Model**

We use AHP (analytic hierarchy process) to analyze the location factors of small dams, classify them according to the objectives of decision-making, the factors (decision criteria) and decision-making objects, draw the hierarchy chart as follows:

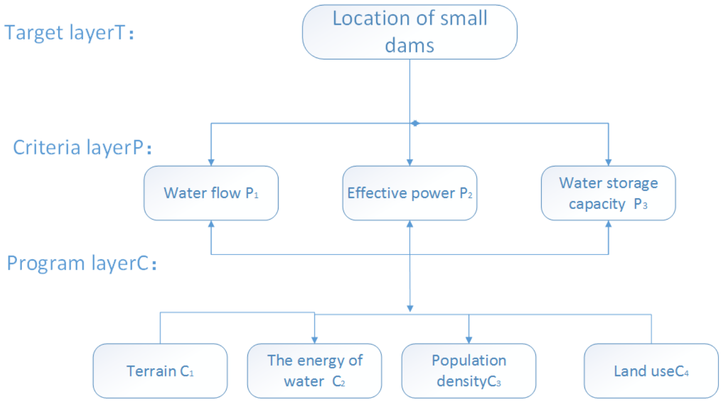


Figure 16 AHP Model

**Construct Consistent Matrix Method**

$$\text{We set the impact weights as : } a_{ij} = \frac{X_i \to T}{X_j \to T}$$

Table 8 Weighting factor

| X to T | Equal | Slightly better | Better | Best |
|---|---|---|---|---|
| $a_{ij}$ | 1 | 3 | 5 | 7 |

Provisions: If the comparison of the impact of small: $a_{ij} = \frac{1}{a_{ij}}$ 。

**The judgment matrix is as follows：**

$$w_1 = \begin{bmatrix} 1 & 7/5 & 7/5 & 7/5 \\ 5/7 & 1 & 5 & 5 \\ 1/7 & 1/5 & 1 & 1 \\ 1/7 & 1/5 & 1 & 1 \end{bmatrix} \quad \text{and} \quad w_1 = w_2 = w_3 \qquad w_4 = \begin{bmatrix} 1 & 3/5 & 3/5 \\ 5/3 & 1 & 1 \\ 5/3 & 1 & 1 \end{bmatrix}$$

**Consistency check of the judgment matrix**

The maximum eigenvalue of the judgment matrix is $\lambda_{\max}$， so that：

$$CI = \frac{|\lambda_{\max} - n|}{n-1}, \qquad CR = \frac{CI}{RI}$$

Note: $CI$ for the consistency test indicators; $CR$ For the consistency ratio.

The consistency ratio of the above-obtained judgment matrix is as follows：

$$CR_1 = 0 \text{，} CR_2 = CR_3 = CR_4 = 6.571*10^{-16}$$

When $CR \le 0.1$， it is considered that the judgment matrix has a satisfactory consistency; otherwise, it must be re-modified judgment matrix until $CR \le 0.1$。 Therefore, the judgment matrix constructed by us is satisfactory。

- **From 1980 to 2013 years, The population data**

| Million people | Malawi | Zambia | Zimbabwe | Mozambique | Angola | Tanzania | Botswana | Namibia | Total number of people | Coastal population (33%) | Forecast data | Relative error |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1980 | 623.7 | 584.7 | 728.9 | 1214 | 763.7 | 1869 | 99.75 | 101.3 | 5985.05 | 1975.0665 | 1975.0665 | 0 |
| 1981 | 640.4 | 603.9 | 757.2 | 1244 | 790.2 | 1928 | 103.4 | 103.4 | 6170.5 | 2036.265 | 2079.149299 | 0.02106 |
| 1982 | 655.9 | 623.5 | 787.7 | 1273 | 819 | 1990 | 107.1 | 105.7 | 6361.9 | 2099.427 | 2134.151187 | 0.01654 |
| 1983 | 673.1 | 643.5 | 819.8 | 1299 | 849 | 2053 | 110.8 | 108.2 | 6556.4 | 2163.612 | 2190.608097 | 0.012477 |
| 1984 | 695.8 | 663.7 | 852.8 | 1320 | 878.5 | 2118 | 114.6 | 111.2 | 6754.6 | 2229.018 | 2248.558519 | 0.008766 |
| 1985 | 726.5 | 683.8 | 886 | 1334 | 906.4 | 2185 | 118.5 | 114.9 | 6955.1 | 2295.183 | 2308.041964 | 0.005603 |
| 1986 | 766.8 | 704.1 | 919.4 | 1339 | 932.1 | 2253 | 122.4 | 119.5 | 7156.3 | 2361.579 | 2369.098987 | 0.003184 |
| 1987 | 815 | 724.4 | 952.7 | 1337 | 956.1 | 2322 | 126.3 | 124.7 | 7358.2 | 2428.206 | 2431.771213 | 0.001468 |
| 1988 | 865.5 | 744.6 | 985.4 | 1335 | 979.8 | 2393 | 130.3 | 130.4 | 7564 | 2496.12 | 2496.101374 | 7.46E-06 |
| 1989 | 910.5 | 764.7 | 1017 | 1339 | 1005 | 2469 | 134.3 | 136.1 | 7775.6 | 2565.948 | 2562.133326 | 0.001487 |
| 1990 | 944.7 | 784.5 | 1046 | 1357 | 1033 | 2548 | 138.4 | 141.5 | 7993.1 | 2637.723 | 2629.91209 | 0.002961 |
| 1991 | 965.8 | 803.8 | 1073 | 1389 | 1065 | 2634 | 142.5 | 146.6 | 8219.7 | 2712.501 | 2699.483876 | 0.004799 |
| 1992 | 975.9 | 822.9 | 1098 | 1435 | 1100 | 2724 | 146.5 | 151.4 | 8453.7 | 2789.721 | 2770.896116 | 0.006748 |
| 1993 | 980.1 | 842.3 | 1121 | 1489 | 1137 | 2816 | 150.5 | 155.9 | 8691.8 | 2868.294 | 2844.197498 | 0.008401 |
| 1994 | 985.2 | 862.5 | 1143 | 1545 | 1174 | 2907 | 154.5 | 160.6 | 8931.8 | 2947.494 | 2919.437997 | 0.009519 |
| 1995 | 996.4 | 884.1 | 1164 | 1598 | 1210 | 2994 | 158.3 | 165.4 | 9170.2 | 3026.166 | 2996.668912 | 0.009747 |
| 1996 | 1015 | 907.3 | 1185 | 1646 | 1245 | 3078 | 162.1 | 170.5 | 9408.9 | 3104.937 | 3075.942895 | 0.009338 |
| 1997 | 1040 | 932 | 1205 | 1691 | 1279 | 3159 | 165.7 | 175.8 | 9647.5 | 3183.675 | 3157.313995 | 0.00828 |
| 1998 | 1070 | 957.7 | 1223 | 1735 | 1314 | 3238 | 169.2 | 181 | 9887.9 | 3263.007 | 3240.837689 | 0.006794 |
| 1999 | 1101 | 983.9 | 1238 | 1780 | 1351 | 3318 | 172.5 | 185.7 | 10130.1 | 3342.933 | 3326.570922 | 0.004895 |
| 2000 | 1132 | 1010 | 1250 | 1828 | 1392 | 3402 | 175.5 | 189.8 | 10379.3 | 3425.169 | 3414.572145 | 0.003094 |
| 2001 | 1162 | 1036 | 1259 | 1879 | 1439 | 3490 | 178.3 | 193.1 | 10636.4 | 3510.012 | 3504.901355 | 0.001456 |
| 2002 | 1193 | 1063 | 1264 | 1932 | 1489 | 3581 | 180.9 | 195.8 | 10898.7 | 3596.571 | 3597.620137 | 0.000292 |
| 2003 | 1224 | 1089 | 1267 | 1987 | 1542 | 3676 | 183.3 | 198.1 | 11166.4 | 3684.912 | 3692.791705 | 0.002138 |
| 2004 | 1257 | 1117 | 1269 | 2044 | 1598 | 3777 | 185.5 | 200.3 | 11447.8 | 3777.774 | 3790.480945 | 0.003364 |
| 2005 | 1292 | 1147 | 1271 | 2101 | 1654 | 3882 | 187.6 | 202.7 | 11737.3 | 3873.309 | 3890.75446 | 0.004504 |
| 2006 | 1331 | 1178 | 1272 | 2159 | 1712 | 3994 | 189.6 | 205.3 | 12040.9 | 3973.497 | 3993.680613 | 0.00508 |
| 2007 | 1371 | 1211 | 1274 | 2217 | 1771 | 4112 | 191.5 | 208.1 | 12355.6 | 4077.348 | 4099.329579 | 0.005391 |
| 2008 | 1414 | 1246 | 1278 | 2276 | 1831 | 4235 | 193.4 | 211.1 | 12684.5 | 4185.885 | 4207.773386 | 0.005229 |
| 2009 | 1457 | 1283 | 1289 | 2336 | 1893 | 4364 | 195.2 | 214.3 | 13031.5 | 4300.395 | 4319.085969 | 0.004346 |
| 2010 | 1501 | 1322 | 1308 | 2397 | 1955 | 4497 | 196.9 | 217.9 | 13394.8 | 4420.284 | 4433.34322 | 0.002954 |
| 2011 | 1546 | 1363 | 1336 | 2458 | 2018 | 4635 | 198.7 | 221.8 | 13776.5 | 4546.245 | 4550.623035 | 0.000963 |
| 2012 | 1591 | 1408 | 1372 | 2520 | 2082 | 4778 | 200.4 | 225.9 | 14177.3 | 4678.509 | 4671.005375 | 0.001604 |
| 2013 | 1636 | 1454 | 1415 | 2583 | 2147 | 4925 | 202.1 | 230.3 | 14592.4 | 4815.492 | 4794.572313 | 0.004344 |
| 2016 | | | | | | | | | | | 5185.234377 | |
| 2017 | | | | | | | | | | | 5322.404747 | |
| 2018 | | | | | | | | | | | 5463.20 | |
| 2019 | | | | | | | | | | | 5607.73 | |
| 2020 | | | | | | | | | | | 5756.07 | |
| 2021 | | | | | | | | | | | 5908.35 | |
| 2022 | | | | | | | | | | | 6064.65 | |
| 2023 | | | | | | | | | | | 6225.08 | |
| 2024 | | | | | | | | | | | 6389.76 | |
| 2025 | | | | | | | | | | | 6558.79 | |

Figure 17 The population data

- **GM（1,1）**

```
clc;
clear all;
x0=[put in the value];
                                        % The value of the test.
n=length(x0);
                                        % Do level than judge whether it is suitable for modeling
lamda=x0(1:n-1)/x0(2:n);                % The comparison of the previous and the next；
range=minmax(lamda);                    % And the maximum value among the ratios is obtained
                                        % Judge
if range(1,1)<exp(-(2/(n+2)))|range(1,2)>exp(2/(n+2))
error('Can't use the GM');
else
disp('');                               % Blank line output
```

```
disp('Can use the GM');
end
                                    % Do AGO cumulative processing
x1=cumsum(x0);
for i=2:n;
z(i)=0.5*(x1(i)+x1(i-1));
end
B=[-z(2:n)',ones(n-1,1)];
Y=x0(2:n)';
u=B\Y;                              % And the value of the least square method is obtained
                                    % Differential equation that with dsolve, D said derivative
x=dsolve('Dx+a*x=b','x(0)=x0');
% Use the subs function for parameter substitution
x=subs(x,{'a','b','x0'},{u(1),u(2),x1(1)});
forecast1=subs(x,'t',[0:n]);        % To predict the next time, then N-1 to N, and so on
     % Use the digits and vpa functions to control the calculated number of significant digits
digits(6);
                                    %y Value or AGO form of (cumulative)
y=vpa(x);
                                    % AGO for the cumulative reduction
     %diff When outputting a dimension vector, it represents the difference of phase elements
exchange=diff(forecast1);
forecast=[x0(1),exchange];
plot(1:n,x0,'ro','markersize',11);
hold on;
plot(1:n+1,forecast,'k-','linewidth',2.5);    % Predict the next year will be n to n +1grid on;
axis tight;
xlabel('x');
ylabel('y');
title('The relation of years and population');
legend('Year','population');
```