

ELU 501

Data science, graph theory and social network studies

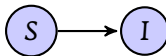
Yannis Haralambous (IMT Atlantique)

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Part VI

Lecture 6 Epidemic models

The SI model



- The *SI model* is the one of *susceptible* and *infected*.
- Each person meets with β persons per unit of time.
- If there are X infected to start with then the average rate of new infections per unit of time is $\beta SX/n$. Hence we have

$$\frac{dX}{dt} = \beta \frac{SX}{n}$$

$$\frac{dS}{dt} = -\beta \frac{SX}{n}$$

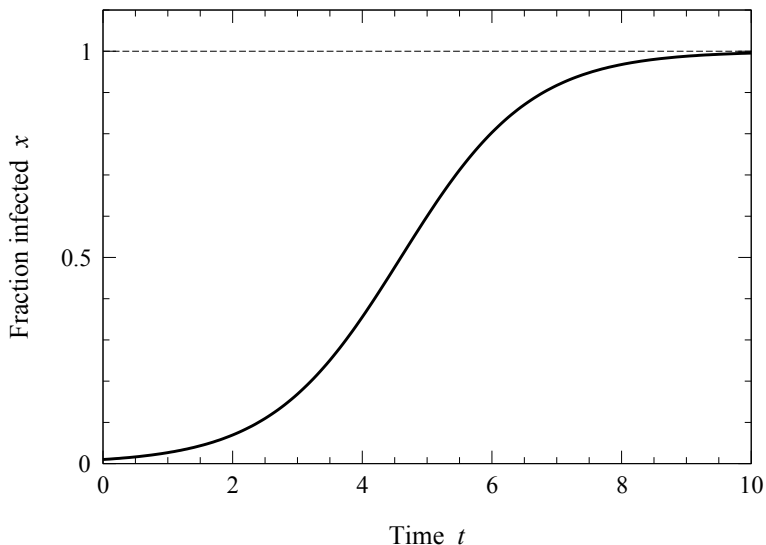
- By using $s = S/n$ and $x = X/n$, we get the solution $x(t) = \frac{x_0 e^{\beta t}}{1 - x_0 + x_0 e^{\beta t}}$ which is a logistic function.

The SI model

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(IMT
Atlantique)

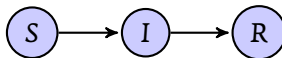


From [Newman, 2010, Fig. 17.1].

The SIR model

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- The *SIR model* is the one of *susceptible*, *infected* and *recovered* (or dead).
- Each person meets with β persons per unit of time and infected recover (or die) at a constant average rate of γ .
- If there are X infected to start with then the average rate of new infections per unit of time is $\beta SX/n$.

- How long does an infected take to start recovering (and hence stop infecting others)? The probability of not recovering at δt is $1 - \gamma \delta t$ and hence the probability of not recovering in a total time t is $\lim_{\delta t \rightarrow 0} (1 - \gamma \delta t)^{t/\delta t} = e^{-\gamma t}$. If we add the condition “recovering at time t ” we get $\gamma e^{-\gamma t}$. Hence we have

$$\begin{aligned}\frac{dX}{dt} &= \beta \frac{SX}{n} \\ \frac{dS}{dt} &= -\beta \frac{SX}{n}\end{aligned}$$

- By using $s = S/n$ and $x = X/n$, we get the solution $x(t) = \frac{x_0 e^{\beta t}}{1 - x_0 + x_0 e^{\beta t}}$ which is a logistic function.

- The differential equations become

$$\frac{ds}{dt} = -\beta sx$$

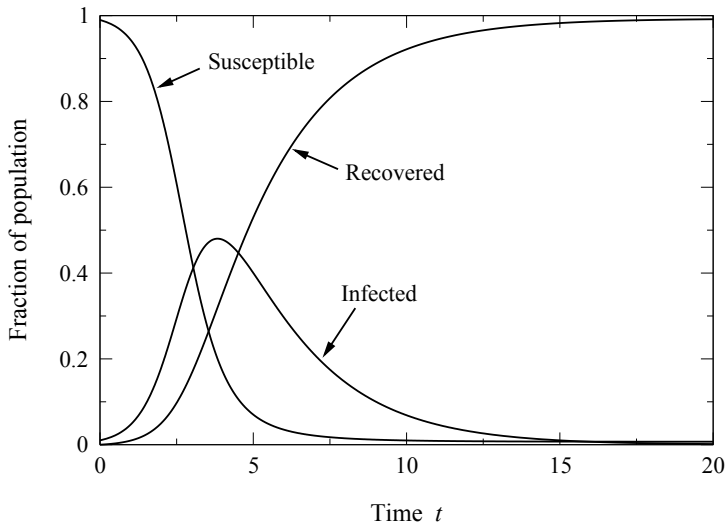
$$\frac{dx}{dt} = \beta sx - \gamma x$$

$$\frac{dr}{dt} = \gamma x.$$

The SIR model

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From [Newman, 2010, Fig. 17.2], here
 $\beta = 1$, $\gamma = 0.4$, $s_0 = 0.99$, $x_0 = 0.01$, $r_0 = 0$.

The basic reproduction number

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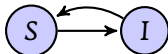
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- The *basic reproduction number* R_0 is the average number of people to which an infected passes the disease before recovering.
- If $R < 1$ the disease will die out. The value $R_0 = 1$ is called *epidemic threshold*.
- In the SIR model,

$$R_0 = \beta \gamma \int_0^{\infty} t e^{-\gamma t} dt = \frac{\beta}{\gamma}.$$

The SIS model



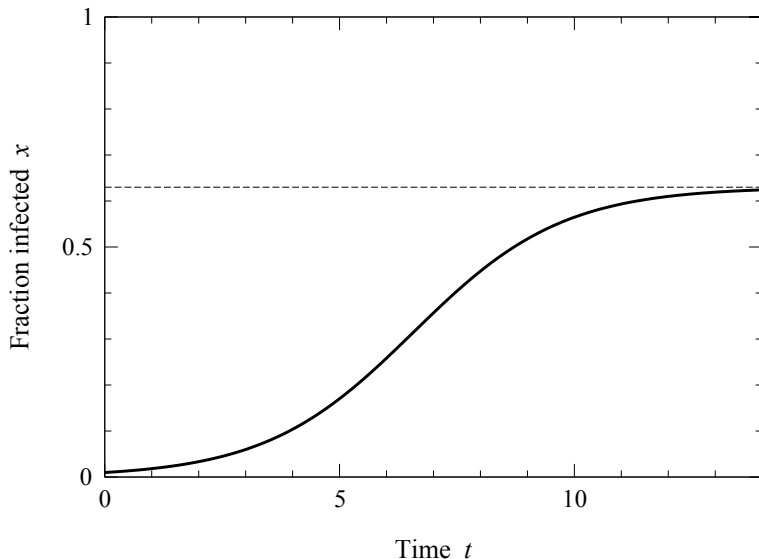
- The *SIS model* is the one of *susceptible*, *infected* which can return to susceptible state, and then get reinfected, etc.
- Each person meets with β persons per unit of time and infected recover at a constant average rate of γ .
- The differential equations are

$$\frac{ds}{dt} = \gamma x - \beta s x$$

$$\frac{dx}{dt} = \beta s x - \gamma x.$$

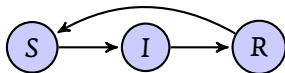
- We get the solution $x(t) = x_0 \frac{(\beta - \gamma)e^{(\beta - \gamma)t}}{\beta - \gamma + \beta x_0 e^{(\beta - \gamma)t}}$.

The SIS model



From [Newman, 2010, Fig. 17.3].

The SIRS model



- The *SIRS model* is the one of *susceptible*, *infected*, *recovered* which can return to susceptible state, and then get reinfected, etc.
- Each person meets with β persons per unit of time and infected recovers at a constant average rate of γ . Let δ the average rate at which people lose immunity.
- The differential equations are

$$\frac{ds}{dt} = \delta r - \beta s x$$

$$\frac{dx}{dt} = \beta s x - \gamma x$$

$$\frac{dr}{dt} = \gamma x - \delta r.$$

- There is no analytic solution for this model.

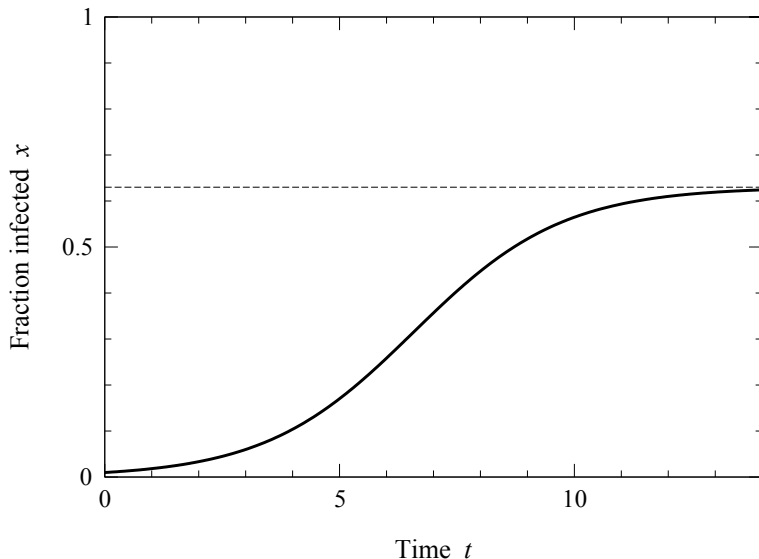
Epidemic models on networks

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- The epidemic models we described assumed full mixing of population.
- On a network we can consider that an infected individual can infect only its susceptible neighbors, with probability β per unit of time.
- M.J. Newman says: *It is difficult to solve a model such as this for a general network, and in many cases the best we can do is to simulate it on a computer.*
- We can study:
 - ① late-time properties: what happens when $t \rightarrow \infty$?
 - ② the relation of epidemics with giant components: if there is an infected individual in a giant component the result will be different than if it is in a small component;
 - ③ we can introduce probabilities of pairs of vertices of having given pairs of states;
 - ④ the time-related properties of a model for different degree distributions.
- For further reading, see [House, 2011].

The SIS model



From [Newman, 2010, Fig. 17.3].



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