MCMS Problem Set

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Question 2 Continued

.) First equilibrium point Jacobian Matrix, $\overset{A,\ B}{(N, 0)}$

$$\begin{bmatrix} 0 & \beta - \gamma \\ 0 & -\beta + \gamma \end{bmatrix}$$

→ Finding the eigen values

$$\begin{bmatrix} -\lambda & \beta - \gamma \\ 0 & -\beta + \gamma - \lambda \end{bmatrix}$$

→ So $-\lambda(\gamma - \beta - \lambda) = 0$

$\boxed{\lambda = 0}$ or $\boxed{\lambda = \gamma - \beta}$

So if $\gamma - \beta < 0$ then stability is achieved, since the eigen value will be negative.

.) Second equilibrium point Jacobian Matrix, $\left(\overset{A}{\frac{N}{R_0}}, \overset{B}{N - \frac{N}{R_0}}\right)$

$$\begin{bmatrix} \frac{\beta[N - N/R_0]}{N} & \frac{\beta[N/R_0]}{N} - \gamma \\ -\frac{\beta[N - N/R_0]}{N} & -\frac{\beta[N/R_0]}{N} + \gamma \end{bmatrix}$$

$$\begin{bmatrix} \beta - \beta/R_0 & 0\ \cancel{\beta/R_0 - \gamma} \\ -\beta - \beta/R_0 & 0\ \cancel{-\beta/R_0 + \gamma} \end{bmatrix}$$

→ plug in $R_0 = \frac{\beta}{\gamma}$