# Kinematic Analysis of the Raven-II$^{TM}$ Research Surgical Robot Platform (REV: 9-Mar-2015)

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**Abstract**

This document describes the forward and inverse kinematics equations for the Raven-II$^{TM}$ surgical robotics research platform.

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## 1 Definitions and Frames

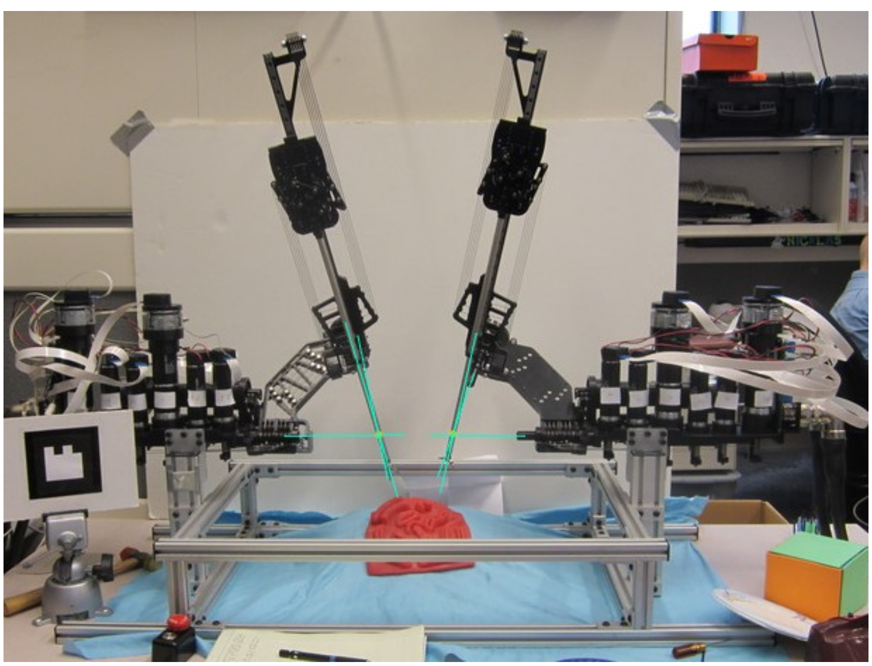

Figure 1: Photo of Left and Right Raven-IIs$^{TM}$ in the reference pose. Axis lines are superimposed in blue showing their intersection at the remote motion center.

- **Reference Pose** The Reference Pose (Figure 1) has the two Raven-IIs$^{TM}$ facing each other and viewed such that the row of vertical motors are directly visible on each arm.
- **Right Arm** The arm which appears to the right in the reference pose.
- **Left Arm** The arm which appears to the left in the reference pose.
- **Green Arm** The Right Arm.
- **Gold Arm** The Left Arm.
- **Base Frame** A frame centered on the bolt pattern on the surface of the Raven-II$^{TM}$ base. There is a separate Base frame for each of the Left and Right arms. The origin of the frame lies between the two bottom bolt holes, directly below the center bolt of the top row, and coincident with the mounting surface on the robot (See Figure 2).

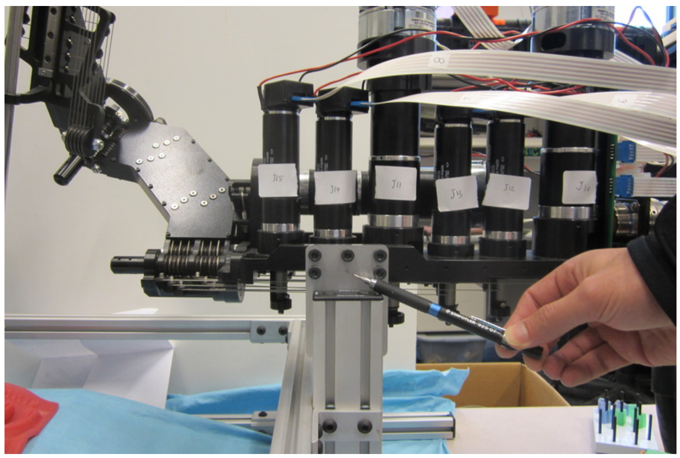

Figure 2: Indicating the location of the base frame for the right arm. Note that the base frame origin is concealed and offset (in the $Y_{BL}$ or $Y_{BR}$ direction) by the thickness of the silver colored mounting plate.

The following diagram shows the orientation of the two Base Frames when facing the bolt patterns and the arms are in the Reference Pose.

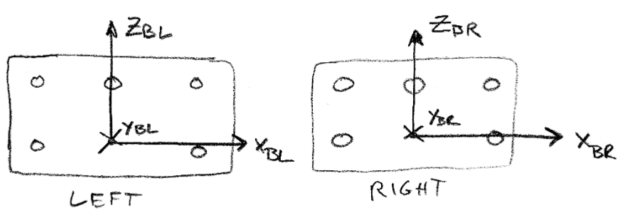


Figure 3: Illustration of base frame position and orientation for left and right arms.

- **Frame Zero** This frame is the origin of the serial kinematic chain for kinematic analysis of the robot. Its origin is the center of motion of the spherical mechanism, i.e. the intersection of axes 1, 2, and 3.

# 2 Robot Description

## 2.1 Base Frame

The Base Frame is a convenient reference frame to identify the origin of the robot with respect to the environment. There is a fixed relationship (Figure 4) between the Base Frame and Frame Zero for each arm. The transforms from

Base Frame to Frame Zero for Left (Gold) Arm are given in Equation 1 and for the Right (Green) Arm in Equation 2.

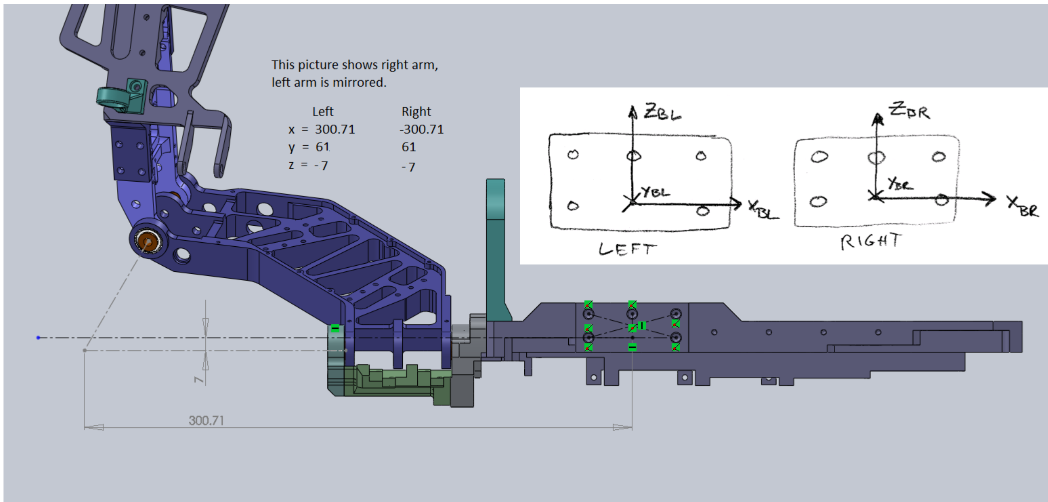


Figure 4: Illustration of offset between base frame and the motion center.

$$ {}^{B}_{0}T_L = \begin{bmatrix} 0 & 0 & 1 & 300.71 \\ 0 & -1 & 0 & 61 \\ 1 & 0 & 0 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

$$ {}^{B}_{0}T_R = \begin{bmatrix} 0 & 0 & -1 & -300.71 \\ 0 & 1 & 0 & 61 \\ 1 & 0 & 0 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

(corrected July 2013) All displacements are given in millimeters.

## 2.2 Robot Constants

Robot dimensions crucial to the kinematics are given in Table 1

| Symbol | Value | Meaning |
|---|---|---|
| $La_{12}$ | $75°$ | Link angle between links 1 and 2 |
| $La_{23}$ | $52°$ | Link angle between links 2 and 3 |
| $d_4$ | $-470mm$ | Tool length, Raven “Diamond” tool. |
| $d_4$ | $-458.69mm$ | Tool length, Raven “Square tool”. |

Table 1: Kinematics constants (Corrected March 2015).

$d_4$ must be negative according to the $Z_4$ direction specified in Figure 8. Raven “Diamond” tool is the default shipped with Raven II’s in 2012 and 2013. Raven “Square” tool is only at UW and UCSC.

## 2.3 Axes of Motion

The axes of motion are identified by $Z_i$ vectors whose direction indicates the convention for positive joint rotation as given by the Right Hand Rule.

## 2.4 Link Frame Assignments

Frames are assigned to each link according to Craig's version of the Denavit Hartenberg convention [1]. In all cases, the direction of the $Z_N$ axis is chosen such that positive changes in $\theta_N$ are consistent with the right-hand-rule. When consecutive axes intersect, the common normal, $X_N$ is chosen such that

$$X_N = Z_N \times Z_{N+1}$$

where $\times$ indicates the vector cross product.

### 2.4.1 Positioning Joints

Link frames for the positioning joints (Frames 1-3) are related to the motion center and are shown in Figure 5. There are differences between the left and right arm geometries and link frames are assigned slightly differently in left vs. right arm.

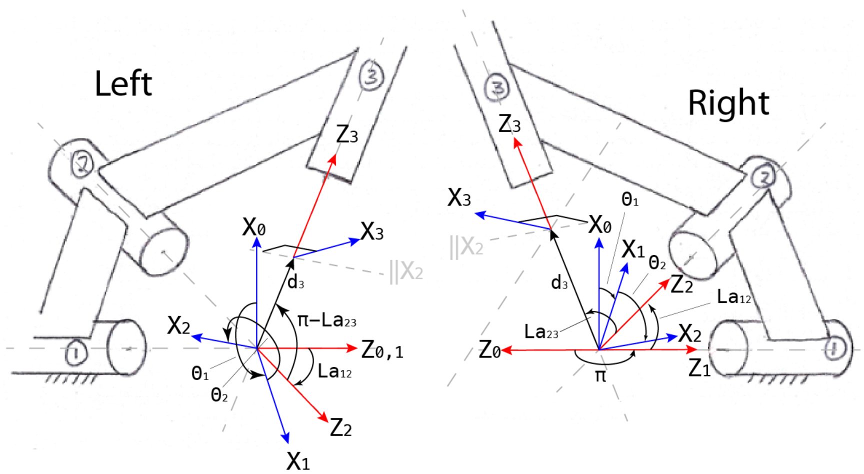


Figure 5: Frame assignments for Frame 0, 1, 2, 3.

In the Figure 5 , $La_{12}$ and $La_{23}$ are the angles of the fixed mechansim links. For the Raven-II$^{TM}$ $La_{12} = La_{12} = 75°$ and $La_{23} = 52°$ (Figure 7).

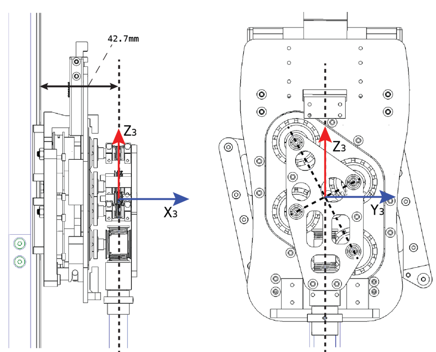


Figure 6: Position of Frame 3 relative to mechanism.

Frame 3 has its origin a the center of a cross formed between the four spindles, and 42.7 mm out from the surface of the sliding rail. Elevation above the rail sets the axis of motion $Z_3$ coincident with the motion center. The location of frame 3 is shown in Figure 6.

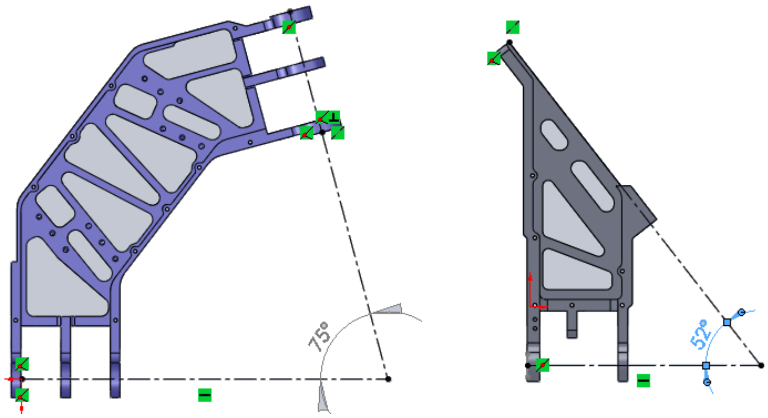


Figure 7: Link angles designated $La_{12}$ and $La_{23}$.

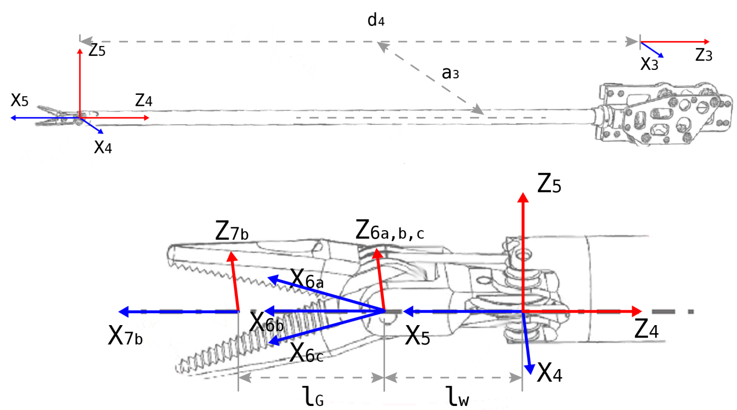


Figure 8: Frame assignments for Frames 4,5,6$_{abc}$. For default Raven II tools, $d_4 = -458.69mm$.

### 2.4.2 Instrument Joints

The link frames and DH parameters for instruments (Axes 4-7) are specific to each instrument, but are the same for left and right arms if the instruments are identical (as in default RII tools). The link frames are shown in Figure 8. In this figure, the instrument is shown from the top as if resting on a table with the spindles down and viewed from above.

Instrument frames 4-7 are offset from the link 3 frame by a distance, $d_4$, which represents the length of the tool. For the default Raven II tools, $d_4 = 458.69mm$.

The surgical instrument can be thought of as three superimposed manipulators. The three manipulators are the same through joint 6. Then we have three "versions" of frame 6.

- **Frame 6a** This frame is aligned with the lower jaw (when $Z_4$ is set so that $Z_5$ points up relative to the instrument base).
- **Frame 6b** This frame represents a virtual link which is oriented between the jaws. $X_{6b}$ should point to the grasp reference point, the origin of the grasping frame $G$.
- **Frame 6c** This frame is aligned with the upper jaw.

Finally, we create a "Grasping Frame," $7_b$ which is oriented in alignment with Frame $6_b$ but is translated out about 1/2 the length of the jaws ($l_G$). This is a useful reference for aligning the grasper with an object to be captured by a subsequent grasping operation.

## 2.5 DH Parameters

### 2.5.1 Positioning Joints

The Denavit Harteberg parameters for the two arms are given in Tables 2 and 3.

In all cases, $d_3$ is a variable which represents the prismatic joint motion, and $d_4$ is a fixed offset representing the tool length- the distance from the center point of the four spindles to the origin of frame 4; which is nominally the wrist.

| $i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\theta_1$ |
| 2 | $La_{12}$ | 0 | 0 | $\theta_2$ |
| 3 | $\pi - La_{23}$ | 0 | $d_3$ | $\pi/2$ |

Table 2: Left (Gold) Arm parameters for the positioning joints.

| $i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | $\pi$ | 0 | 0 | $\theta_1$ |
| 2 | $La_{12}$ | 0 | 0 | $\theta_2$ |
| 3 | $La_{23}$ | 0 | $d_3$ | $-\pi/2$ |

Table 3: Right (Green) Arm parameters for the positioning joints.

### 2.5.2 Instrument Joints

Frame assignment and DH parameters are the same for the left and right default instruments and are given in 4.

Note that $a_3$ may be zero in the case of standard Raven-II$^{TM}$ instruments. When other instruments are used, there may be an offset between the tool roll axis and the motion center and $a_3 \neq 0$. The length of various tools is represented by parameter $d_4$.

| $i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 4 | 0 | $a_3$ | $d_4$ | $\theta_4$ |
| 5 | $\pi/2$ | 0 | 0 | $\theta_5$ |
| $6_{a,b,c}$ | $\pi/2$ | $l_w$ | 0 | $\theta_{6a,b,c}$ |
| $7_b$ | 0 | $l_G$ | 0 | 0 |

Table 4: Instrument parameters.

## 2.6 Kinematic Zero Pose

The pose which is achieved when $\theta_i = 0$ is shown schematically in Figure 9 (pose shown in Figures 5 and 8 is not the zero pose). *This pose is not physically achievable by the robot mechanism.*

In the zero pose:

- $X_{0L} = X_{1L} = X_{2L}$,
- $X_{0R} = X_{1R} = X_{2R}$,
- The origin of frame 3 is coincident with the center of rotation,
- The origin of frame 4 is at distance $|d_4|$ from the center of rotation,
- $X_3||X_4$ ($X_3$ and $X_4$ point in same direction).
- $X_4 = X_5$,

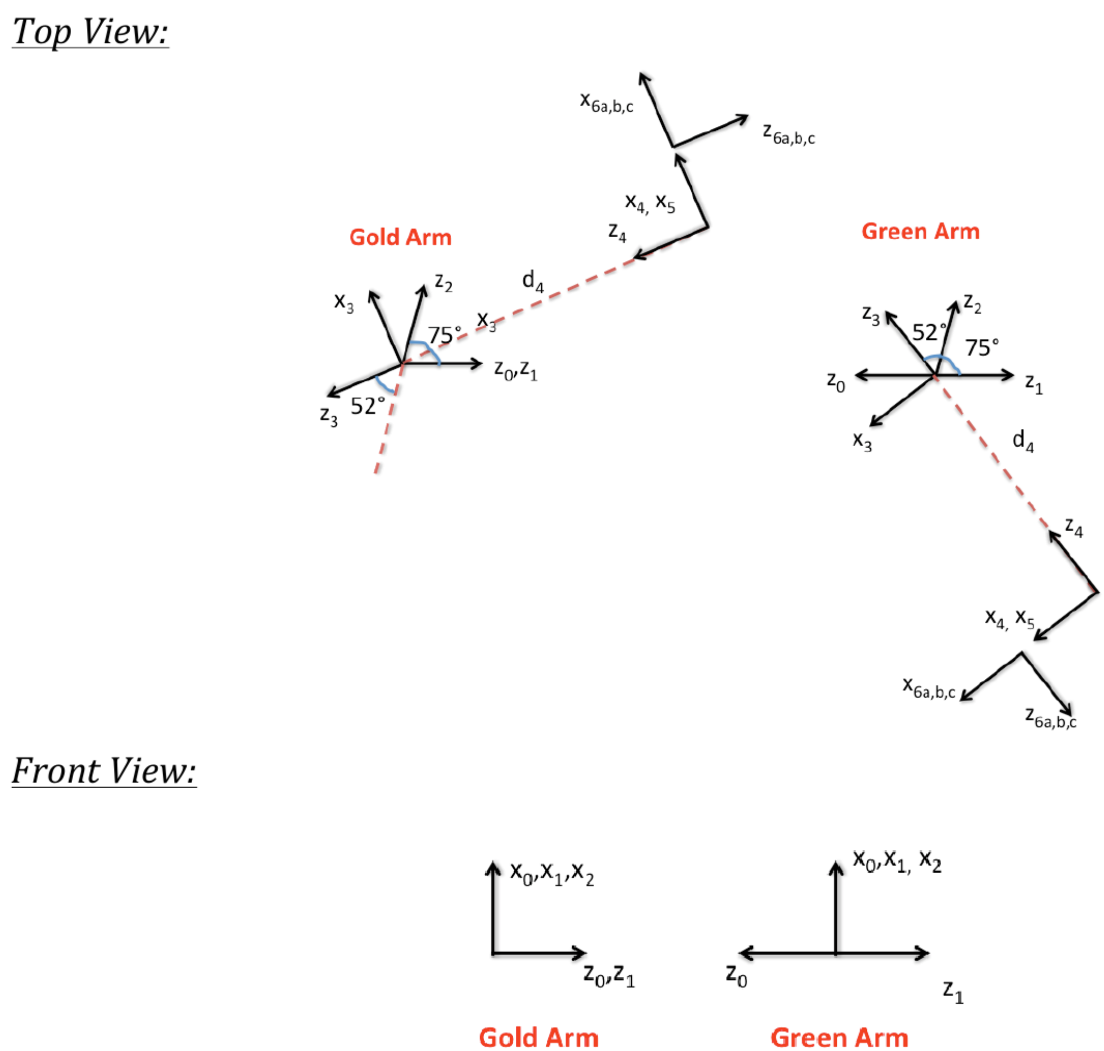


Figure 9: Schematic view of Raven Motion Axes in the zero pose for kinematic analysis. “Gold Arm” is the LEFT arm and “Green Arm” is the RIGHT arm.

- $X_5, X_{6a,b,c}, X_{7b}$ are colinear and point in same direction.
- $X_{6a} = X_{6b} = X_{6c}$.

# 3 Forward Kinematics Analysis

## Link Transform

The following transform is obtained for any link which has DH parameters:

$$
{}^{N-1}_{N}T = \begin{bmatrix} c\theta_N & -s\theta_N & 0 & a_{N-1} \\ s\theta_N c\alpha_{N-1} & c\theta_N c\alpha_{N-1} & -s\alpha_{N-1} & -s\alpha_{N-1} d_N \\ s\theta_N s\alpha_{N-1} & c\theta_N s\alpha_{N-1} & c\alpha_{N-1} & c\alpha_{N-1} d_N \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
$$

## 3.1 Forward Kinematic Equations: Left Arm (HK Convention)

In the matrices below let $k_1 = cos(La_{12}), k_2 = sin(La_{12}), k_3 = cos(La_{23})$, and $k_4 = sin(La_{23})$.

$$
{}^{0}_{1}T_{L/R} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1/-s_1 & c1/-c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 k_1 & c_2 k_1 & -k_2 & 0 \\ s_2 k_2 & c_2 k_2 & k_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

$$
{}^{2}_{3}T_{L/R} = \begin{bmatrix} 0 & -1/1 & 0 & 0 \\ k_3 & 0 & -k_4 & -k_4 d_3 \\ (-1/1)k_4 & 0 & (-1/1)k_3 & (-1/1)k_3 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}
$$

$$
{}^{5}_{6a}T = \begin{bmatrix} c_{6a} & -s_{6a} & 0 & l_w \\ 0 & 0 & -1 & 0 \\ s_{6a} & c_{6_a} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}
$$

$$
{}^{5}_{6b}T = \begin{bmatrix} c_{6b} & -s_{6b} & 0 & l_w \\ 0 & 0 & -1 & 0 \\ s_{6b} & c_{6b} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}
$$

$$
{}^{5}_{6c}T = \begin{bmatrix} c_{6c} & -s_{6c} & 0 & l_w \\ 0 & 0 & -1 & 0 \\ s_{6a} & c_{6c} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}
$$

$$ {}^{6b}_{7b}T = \begin{bmatrix} 1 & 0 & 0 & l_G \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12} $$

## 3.2 Forward Kinematic Equations

The forward kinematics model is obtained by

$$ {}^{0}_{6}T = {}^{0}_{1}T {}^{1}_{2}T {}^{2}_{3}T {}^{3}_{4}T {}^{4}_{5}T {}^{5}_{6x}T \tag{13} $$

where $6x$ designates any of the three final frames $(6a, 6b, 6c)$.

In the following, we will drop the $_x$ subscript. Starting from the end effector, we will multply the matrices and accumulate intermediate products which will be useful later for inverse kinematics.

In the following equations the sign terms in parentheses refer to the Left and Right arms respectively (L/R).

$$ {}^{4}_{6}T = {}^{4}_{5}T {}^{5}_{6}T = \tag{14} $$

$$ \begin{pmatrix} c_5c_6 & -c_5s_6 & s_5 & c_5l_w \\ -s_6 & -c_6 & 0 & 0 \\ c_6s_5 & -s_5s_6 & -c_5 & l_ws_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{3}_{6}T = {}^{3}_{4}T {}^{4}_{6}T = \tag{15} $$

$$ \begin{pmatrix} c_4c_5c_6 + s_4s_6 & c_6s_4 - c_4c_5s_6 & c_4s_5 & a_3 + c_4c_5l_w \\ c_5c_6s_4 - c_4s_6 & -c_4c_6 - c_5s_4s_6 & s_4s_5 & c_5l_ws_4 \\ c_6s_5 & -s_5s_6 & -c_5 & d_4 + l_ws_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}_{6}T_R = {}^{2}_{3}T {}^{3}_{6}T = \tag{16} $$

$$ \begin{pmatrix} c_5c_6s_4 - c_4s_6 & -c_4c_6 - c_5s_4s_6 \\ -c\alpha_{23}\left(c_4c_5c_6 + s_4s_6\right) - c_6s_5s\alpha_{23} & -c\alpha_{23}\left(c_6s_4 - c_4c_5s_6\right) + s_5s_6s\alpha_{23} \\ c_6c\alpha_{23}s_5 - \left(c_4c_5c_6 + s_4s_6\right)s\alpha_{23} & -c\alpha_{23}s_5s_6 - \left(c_6s_4 - c_4c_5s_6\right)s\alpha_{23} \\ 0 & 0 \end{pmatrix. $$

$$ \begin{matrix} s_4s_5 & c_5l_ws_4 \\ -c_4c\alpha_{23}s_5 + c_5s\alpha_{23} & -c\alpha_{23}\left(a_3 + c_4c_5l_w\right) - d_{\mathrm{R3}}s\alpha_{23} - \left(d_4 + l_ws_5\right)s\alpha_{23} \\ -c_5c\alpha_{23} - c_4s_5s\alpha_{23} & c\alpha_{23}d_{\mathrm{R3}} + c\alpha_{23}\left(d_4 + l_ws_5\right) - \left(a_3 + c_4c_5l_w\right)s\alpha_{23} \\ 0 & 1 \end{matrix}\right) $$

$$ {}^{1}_{6}T_R = {}^{1}_{2}T {}^{2}_{6}T = \tag{17} $$

$$ \left(\begin{matrix} c_2\left(c_5c_6s_4 - c_4s_6\right) - s_{\mathrm{R2}}\left(-c\alpha_{23}\left(c_4c_5c_6 + s_4s_6\right) - c_6s_5s\alpha_{23}\right) \\ c\alpha_{12}\left(c_5c_6s_4 - c_4s_6\right)s_{\mathrm{R2}} + c_2c\alpha_{12}\left(-c\alpha_{23}\left(c_4c_5c_6 + s_4s_6\right) - c_6s_5s\alpha_{23}\right) - s\alpha_{12}\left(c_6c\alpha_{23}s_5 - \left(c_4c_5c_6 + s_4s_6\right)s\alpha_{23}\right) \\ \left(c_5c_6s_4 - c_4s_6\right)s_{\mathrm{R2}}s\alpha_{12} + c_2s\alpha_{12}\left(-c\alpha_{23}\left(c_4c_5c_6 + s_4s_6\right) - c_6s_5s\alpha_{23}\right) + c\alpha_{12}\left(c_6c\alpha_{23}s_5 - \left(c_4c_5c_6 + s_4s_6\right)s\alpha_{23}\right) \\ 0 \end{matrix}\right. $$

$$ \begin{matrix} c_2\left(-c_4c_6 - c_5s_4s_6\right) - s_{\mathrm{R2}}\left(-c\alpha_{23}\left(c_6s_4 - c_4c_5s_6\right) + s_5s_6s\alpha_{23}\right) \\ c\alpha_{12}\left(-c_4c_6 - c_5s_4s_6\right)s_{\mathrm{R2}} + c_2c\alpha_{12}\left(-c\alpha_{23}\left(c_6s_4 - c_4c_5s_6\right) + s_5s_6s\alpha_{23}\right) - s\alpha_{12}\left(-c\alpha_{23}s_5s_6 - \left(c_6s_4 - c_4c_5s_6\right)s\alpha_{23}\right) \\ \left(-c_4c_6 - c_5s_4s_6\right)s_{\mathrm{R2}}s\alpha_{12} + c_2s\alpha_{12}\left(-c\alpha_{23}\left(c_6s_4 - c_4c_5s_6\right) + s_5s_6s\alpha_{23}\right) + c\alpha_{12}\left(-c\alpha_{23}s_5s_6 - \left(c_6s_4 - c_4c_5s_6\right)s\alpha_{23}\right) \\ 0 \end{matrix} $$

$$ \begin{matrix} c_2s_4s_5 - s_{\mathrm{R2}}\left(-c_4c\alpha_{23}s_5 + c_5s\alpha_{23}\right) \\ c\alpha_{12}s_4s_5s_{\mathrm{R2}} + c_2c\alpha_{12}\left(-c_4c\alpha_{23}s_5 + c_5s\alpha_{23}\right) - s\alpha_{12}\left(-c_5c\alpha_{23} - c_4s_5s\alpha_{23}\right) \\ s_4s_5s_{\mathrm{R2}}s\alpha_{12} + c_2s\alpha_{12}\left(-c_4c\alpha_{23}s_5 + c_5s\alpha_{23}\right) + c\alpha_{12}\left(-c_5c\alpha_{23} - c_4s_5s\alpha_{23}\right) \\ 0 \end{matrix} $$

$$\left.\begin{matrix} c_2 c_5 l_w s_4 - s_{\mathrm{R2}}\beta_1 \\ c_5 \mathrm{c}\alpha_{12} l_w s_4 s_{\mathrm{R2}} + c_2 \mathrm{c}\alpha_{12}\beta_1 - \mathrm{s}\alpha_{12}\beta_2 \\ c_5 l_w s_4 s_{\mathrm{R2}} \mathrm{s}\alpha_{12} + c_2 \mathrm{s}\alpha_{12}\beta_1 + \mathrm{c}\alpha_{12}\beta_2 \\ 1 \end{matrix}\right)$$

Where,

$$\beta_1 = -\mathrm{c}\alpha_{23}\left(a_3 + c_4 c_5 l_w\right) - d_{\mathrm{R3}}\mathrm{s}\alpha_{23} - \left(d_4 + l_w s_5\right)\mathrm{s}\alpha_{23}$$
$$\beta_2 = \mathrm{c}\alpha_{23} d_{\mathrm{R3}} + \mathrm{c}\alpha_{23}\left(d_4 + l_w s_5\right) - \left(a_3 + c_4 c_5 l_w\right)\mathrm{s}\alpha_{23}$$

$$ {}^{0}_{6}T_R = {}^{0}_{1}T\,{}^{1}_{6}T = \tag{18}$$

$$\begin{pmatrix}
-s_{\mathrm{R1}}\left(\mathrm{c}\alpha_{12} s_{\mathrm{R2}}\tau_{21} - \mathrm{s}\alpha_{12}\left(-\mathrm{s}\alpha_{23}\tau_{11} + \mathrm{c}\alpha_{23}\tau_{31}\right) + c_2\mathrm{c}\alpha_{12}\left(-\mathrm{c}\alpha_{23}\tau_{11} - \mathrm{s}\alpha_{23}\tau_{31}\right)\right) + c_{\mathrm{R1}}\left(c_2\tau_{21} - s_{\mathrm{R2}}\left(-\mathrm{c}\alpha_{23}\tau_{11} - \mathrm{s}\alpha_{23}\tau_{31}\right)\right) \\
c_{\mathrm{R1}}\left(\mathrm{c}\alpha_{12} s_{\mathrm{R2}}\tau_{21} - \mathrm{s}\alpha_{12}\left(-\mathrm{s}\alpha_{23}\tau_{11} + \mathrm{c}\alpha_{23}\tau_{31}\right) + c_2\mathrm{c}\alpha_{12}\left(-\mathrm{c}\alpha_{23}\tau_{11} - \mathrm{s}\alpha_{23}\tau_{31}\right)\right) + s_{\mathrm{R1}}\left(c_2\tau_{21} - s_{\mathrm{R2}}\left(-\mathrm{c}\alpha_{23}\tau_{11} - \mathrm{s}\alpha_{23}\tau_{31}\right)\right) \\
s_{\mathrm{R2}}\mathrm{s}\alpha_{12}\tau_{21} + \mathrm{c}\alpha_{12}\left(-\mathrm{s}\alpha_{23}\tau_{11} + \mathrm{c}\alpha_{23}\tau_{31}\right) + c_2\mathrm{s}\alpha_{12}\left(-\mathrm{c}\alpha_{23}\tau_{11} - \mathrm{s}\alpha_{23}\tau_{31}\right) \\
0
\end{pmatrix.$$

$$\begin{matrix}
-s_{\mathrm{R1}}\left(\mathrm{c}\alpha_{12} s_{\mathrm{R2}}\tau_{22} - \mathrm{s}\alpha_{12}\left(-\mathrm{s}\alpha_{23}\tau_{12} - \mathrm{c}\alpha_{23}\tau_{32}\right) + c_2\mathrm{c}\alpha_{12}\left(-\mathrm{c}\alpha_{23}\tau_{12} + \mathrm{s}\alpha_{23}\tau_{32}\right)\right) + c_{\mathrm{R1}}\left(c_2\tau_{22} - s_{\mathrm{R2}}\left(-\mathrm{c}\alpha_{23}\tau_{12} + \mathrm{s}\alpha_{23}\tau_{32}\right)\right) \\
c_{\mathrm{R1}}\left(\mathrm{c}\alpha_{12} s_{\mathrm{R2}}\tau_{22} - \mathrm{s}\alpha_{12}\left(-\mathrm{s}\alpha_{23}\tau_{12} - \mathrm{c}\alpha_{23}\tau_{32}\right) + c_2\mathrm{c}\alpha_{12}\left(-\mathrm{c}\alpha_{23}\tau_{12} + \mathrm{s}\alpha_{23}\tau_{32}\right)\right) + s_{\mathrm{R1}}\left(c_2\tau_{22} - s_{\mathrm{R2}}\left(-\mathrm{c}\alpha_{23}\tau_{12} + \mathrm{s}\alpha_{23}\tau_{32}\right)\right) \\
s_{\mathrm{R2}}\mathrm{s}\alpha_{12}\tau_{22} + \mathrm{c}\alpha_{12}\left(-\mathrm{s}\alpha_{23}\tau_{12} - \mathrm{c}\alpha_{23}\tau_{32}\right) + c_2\mathrm{s}\alpha_{12}\left(-\mathrm{c}\alpha_{23}\tau_{12} + \mathrm{s}\alpha_{23}\tau_{32}\right) \\
0
\end{matrix}$$

$$\begin{matrix}
c_{\mathrm{R1}}\left(-s_{\mathrm{R2}}\left(c_5\mathrm{s}\alpha_{23} - \mathrm{c}\alpha_{23}\tau_{13}\right) + c_2\tau_{23}\right) - s_{\mathrm{R1}}\left(c_2\mathrm{c}\alpha_{12}\left(c_5\mathrm{s}\alpha_{23} - \mathrm{c}\alpha_{23}\tau_{13}\right) - \mathrm{s}\alpha_{12}\left(-c_5\mathrm{c}\alpha_{23} - \mathrm{s}\alpha_{23}\tau_{13}\right) + \mathrm{c}\alpha_{12} s_{\mathrm{R2}}\tau_{23}\right) \\
s_{\mathrm{R1}}\left(-s_{\mathrm{R2}}\left(c_5\mathrm{s}\alpha_{23} - \mathrm{c}\alpha_{23}\tau_{13}\right) + c_2\tau_{23}\right) + c_{\mathrm{R1}}\left(c_2\mathrm{c}\alpha_{12}\left(c_5\mathrm{s}\alpha_{23} - \mathrm{c}\alpha_{23}\tau_{13}\right) - \mathrm{s}\alpha_{12}\left(-c_5\mathrm{c}\alpha_{23} - \mathrm{s}\alpha_{23}\tau_{13}\right) + \mathrm{c}\alpha_{12} s_{\mathrm{R2}}\tau_{23}\right) \\
c_2\mathrm{s}\alpha_{12}\left(c_5\mathrm{s}\alpha_{23} - \mathrm{c}\alpha_{23}\tau_{13}\right) + \mathrm{c}\alpha_{12}\left(-c_5\mathrm{c}\alpha_{23} - \mathrm{s}\alpha_{23}\tau_{13}\right) + s_{\mathrm{R2}}\mathrm{s}\alpha_{12}\tau_{23} \\
0
\end{matrix}$$

$$\left.\begin{matrix}
c_{\mathrm{R1}}\left(c_2 c_5 l_w s_4 - s_{\mathrm{R2}}\beta_1\right) - s_{\mathrm{R1}}\left(c_5\mathrm{c}\alpha_{12} l_w s_4 s_{\mathrm{R2}} + c_2\mathrm{c}\alpha_{12}\beta_1 - \mathrm{s}\alpha_{12}\beta_2\right) \\
s_{\mathrm{R1}}\left(c_2 c_5 l_w s_4 - s_{\mathrm{R2}}\beta_1\right) + c_{\mathrm{R1}}\left(c_5\mathrm{c}\alpha_{12} l_w s_4 s_{\mathrm{R2}} + c_2\mathrm{c}\alpha_{12}\beta_1 - \mathrm{s}\alpha_{12}\beta_2\right) \\
c_5 l_w s_4 s_{\mathrm{R2}}\mathrm{s}\alpha_{12} + c_2\mathrm{s}\alpha_{12}\beta_1 + \mathrm{c}\alpha_{12}\beta_2 \\
1
\end{matrix}\right)$$

Where

$$\tau_{11} = c_4 c_5 c_6 + s_4 s_6 \tag{19}$$
$$\tau_{21} = c_5 c_6 s_4 - c_4 s_6 \tag{20}$$
$$\tau_{31} = c_6 s_5 \tag{21}$$
$$\tau_{12} = c_6 s_4 - c_4 c_5 s_6 \tag{22}$$
$$\tau_{22} = -c_4 c_6 - c_5 s_4 s_6 \tag{23}$$
$$\tau_{32} = s_5 s_6 \tag{24}$$
$$\tau_{13} = c_4 s_5 \tag{25}$$
$$\tau_{23} = s_4 s_5 \tag{26}$$
$$\tau_{14} = a_3 + c_4 c_5 l_w \tag{27}$$
$$\tau_{24} = c_5 l_w s_4 \tag{28}$$
$$\tau_{34} = d_4 + l_w s_5 \tag{29}$$

# 4 Inverse Kinematics Analysis

Kinematic analysis for the default 7-DoF Raven II tool is described below. The solution assumes that DH parameter $a_3 = 0$; that the tool shaft runs through the RCM. While this assumption is true for the default Raven II tool, it is not necessarily satisfied by other tools, e.g., adapted daVinci tools. The case $a_3 \neq 0$ requires additional steps, since point $P_5$ cannot be determined by the method below.

### 4.1 Notation

Regarding notation:

- Let ${}^{0}_{6}T_D$ be the desired homogeneous transform of the end effector with position $P_6$.
- Let ${}^{A}P_B = \begin{bmatrix} {}^{A}X_{PB} \\ {}^{A}Y_{PB} \\ {}^{A}Z_{PB} \end{bmatrix}$ denote the origin of frame {B} represented in frame {A}.
- Let $X_B$ designate the $X$ axis of frame {B}. Similar for X,Y and Z axes.
- Let $RCM$ designate the robot's remote center of motion.

### 4.2 Strategy and overview

The following solution was obtained through the following strategy:

1. Assume $a_3 = 0$.
2. Start with the desired position and orientation of the end effector: ${}^{0}_{6}T_D$, with position $P_6$.
3. Compute the position of the wrist (origin of frame 5, designated ${}^{0}P_5$).
   - Let $\Omega_1$ denote the X-Y plane of frame {6} formed by $X_6, Y_6, P_6$, and let $P_{RCM}$ be the perpendicular projection of $RCM$ onto $\Omega_1$.
   - Then point ${}^{0}P_5 \in \Omega_1$ at a distance $l_w$ from point $P_6$ in the direction of $\pm P_{RCM}$
4. Using ${}^{0}P_5$, the length of the prismatic joint, $d_3 + d_4$ can be found.
5. Now compute the intermediate forward kinematic result: ${}^{0}_{4}T$
6. $\theta_2$ can be computed by setting
$${}^{0}Z_{P5} = {}^{0}_{4}T(3,4)$$
7. Compute the known value of ${}^{0}_{4}T$ , invert it, and combine with ${}^{0}_{6}T_D$ to solve for $\theta_5$, $\theta_6$

### 4.3 Compute Origin of wrist frame, ${}^{0}P_5$

We designate the end point of the manipulator (actually the center of the jaw hinge which is the origin of frame 6) point $P_6$. A useful point for the inverse kinematics is the origin of frame 5, which we will call $P_5$.

Let $\Omega_1$ denote the X-Y plane of frame {6} formed by $X_6, Y_6, P_6$, and let $P_{RCM}$ be the perpendicular projection of $RCM$ onto $\Omega_1$.

**Assertion:** $P_5 \in \Omega_1$ along $\overleftrightarrow{P_6 P_{RCM}}$

**Proof:**

- By construction of the robot's DH parameters, $X_5 \in \Omega_1 \Rightarrow P_5 \in \Omega_1$.
- Let $\Omega_2$ denote a plane formed by $P_6, Z_6, RCM$
- By construction of the robot and DH parameters, $P_5 \in \Omega_2$.
- $\Omega_2 \perp \Omega_1$ with line of intersection $\Omega_2 \cap \Omega_1 = L$.
- Note that $P_5 \in L$.
- $\forall$ points $K, \{K \in \Omega_2\}$, the perpendicular projection of $K$ onto $\Omega_1$, $P_K \in L$
- $RCM \in \Omega_2 \Rightarrow P_{RCM} \in L$.
- $L = \overleftrightarrow{P_6 P_{RCM}}$.

- Therefore $P_5 \in \overleftrightarrow{P_6 P_{RCM}}$

Furthermore, we note that ${}^6P_{RCM}$ is the $X - Y$ coordinates of ${}^6RCM$:

$$
{}^6RCM = {}^6_0T_D RCM = {}^0_6T_D^{-1} RCM = \begin{bmatrix} {}^6X_{RCM} \\ {}^6Y_{RCM} \\ {}^6Z_{RCM} \end{bmatrix} \tag{30}
$$

$$
{}^6P_{RCM} = \begin{bmatrix} {}^6X_{RCM} \\ {}^6Y_{RCM} \\ 0 \end{bmatrix} \tag{31}
$$

$$
{}^6P_{RCM} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} {}^0_6R_D^T \end{bmatrix} \begin{bmatrix} {}^0P_6 \end{bmatrix} \\ 1 \end{bmatrix} \tag{32}
$$

Knowing that $P_5$ lies at distance $l_w$ from the origin of $\{6\}$ we can compute:

$$
{}^6P_5 = \pm l_w \frac{{}^6P_{RCM}}{|{}^6P_RCM|} \tag{33}
$$

This has two solutions corresponding to distance $l_w$ along L. To find the appropriate solution, the inverse kinematics can be carried out for both possibilities, and the result nearest to the current robot configuration can be selected.

## 4.4 Compute length of prismatic joint, $d_3 + d_4$

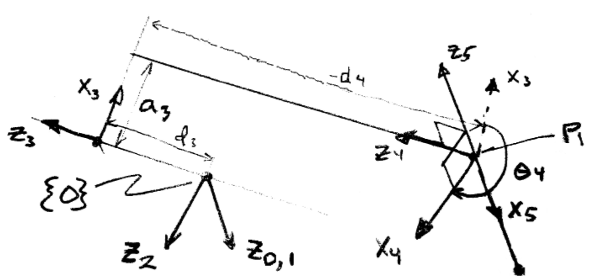


Figure 10: Schematic of Raven kinematics from frame $\{0\}$ to the point ${}^0P_5$.

With reference to Figure 10, and assuming $a_3 = 0$, the distance from the origin to $P_5$ is readily found using the Euclidian norm of ${}^0P_5$:

$$
(d_3 + d_4)^2 = ||{}^0P_5||_2 \tag{34}
$$

$$
d_3 = -d_4 \pm \sqrt{{}^0X_{P5}{}^2 + {}^0Y_{P5}{}^2 + {}^0Z_{P5}{}^2} \tag{35}
$$

Since $||{}^OP_5||_2$ is always positive, there are only two solutions to $d_3$.

## 4.5 Compute $\theta_2$

Next we compute $\theta_2$ using ${}^0P_5$, $d_3$ and $d_4$.

Let $\sin[\alpha_{12}] = \gamma_1, \quad \cos[\alpha_{12}] = \gamma_2, \quad \sin[\alpha_{23}] = \gamma_3, \quad \cos[\alpha_{23}] = \gamma_4, \quad d_3 + d_4 = d$

Then note:

$$ {}^0_5T = \begin{bmatrix} [{}^0_5R] & [{}^0P_5] \\ [0] & 1 \end{bmatrix} \tag{36} $$

$$ {}^0P_5 = \begin{bmatrix} {}^0X_{P5} \\ {}^0Y_{P5} \\ {}^0Z_{P5} \end{bmatrix} \tag{37} $$

$$ {}^0P_{5L} = \begin{bmatrix} d\left(c_1 s_2 \gamma_3 + s_1\left(c_2\gamma_2\gamma_3 - \gamma_1\gamma_4\right)\right) \\ d\left(s_1 s_2 \gamma_3 - c_1\left(c_2\gamma_2\gamma_3 - \gamma_1\gamma_4\right)\right) \\ -d\left(c_2\gamma_1\gamma_3 + \gamma_2\gamma_4\right) \\ 1 \end{bmatrix} \tag{38} $$

$$ {}^0P_{5R} = \begin{bmatrix} d\left(c_1 s_2 \gamma_3 + s_1\left(c_2\gamma_2\gamma_3 + \gamma_1\gamma_4\right)\right) \\ -d\left(s_1 s_2 \gamma_3 - c_1\left(c_2\gamma_2\gamma_3 + \gamma_1\gamma_4\right)\right) \\ d\left(c_2\gamma_1\gamma_3 - \gamma_2\gamma_4\right) \\ 1 \end{bmatrix} \tag{39} $$

Where, ${}^0P_5$ is known and,

$$ {}^0Z_{P5L} = -d\left(c_2\gamma_1\gamma_3 + \gamma_2\gamma_4\right) \tag{40} $$

$$ {}^0Z_{P5R} = d(c_2\gamma_1\gamma_3 - \gamma_2\gamma_4) \tag{41} $$

Solving this for $c_2$ yields[1]:

$$ \theta_{2L} = \pm cos^{-1}\left(\left(\frac{1}{\gamma_1\gamma_3}\right)\left(-\frac{{}^0Z_{P5L}}{d} - \gamma_2\gamma_4\right)\right) \tag{42} $$

$$ \theta_{2R} = \pm cos^{-1}\left(\left(\frac{1}{\gamma_1\gamma_3}\right)\left(\frac{{}^0Z_{P5R}}{d} + \gamma_2\gamma_4\right)\right) \tag{43} $$

Two solutions stem from the arccosine function.

### 4.6 Compute $\theta_1$

Next we compute $\theta_1$ using $d_3$, $\theta_2$, ${}^0P_5$.

Recall Equation 38 and note that for the left side:

$$ {}^0X_{P5L} = d(c_1 s_2\gamma_3 + s_1(c_2\gamma_2\gamma_3 - \gamma_1\gamma_4)) \tag{44} $$

$$ {}^0Y_{P5L} = d(s_1 s_2\gamma_3 - c_1(c_2\gamma_2\gamma_3 - \gamma_1\gamma_4)) \tag{45} $$

Let $\beta_{1L} = s_2\gamma_3$, $\beta_{2L} = c_2\gamma_2\gamma_3 - \gamma_1\gamma_4$.

$$ \begin{bmatrix} {}^0X_{P5L} \\ {}^0Y_{P5L} \end{bmatrix} = d\begin{bmatrix} \beta_{1L} & \beta_{2L} \\ -\beta_{2L} & \beta_{1L} \end{bmatrix}\begin{bmatrix} c_{1L} \\ s_{1L} \end{bmatrix} \tag{46} $$

And, for the right side:

$$ {}^0X_{P5R} = d(c_1 s_2\gamma_3 + s_1(c_2\gamma_2\gamma_3 + \gamma_1\gamma_4)) \tag{47} $$

$$ {}^0Y_{P5R} = d(-s_1 s_2\gamma_3 + c_1(c_2\gamma_2\gamma_3 + \gamma_1\gamma_4)) \tag{48} $$

Let $\beta_{1R} = s_2\gamma_3$, $\beta_{2R} = c_2\gamma_2\gamma_3 + \gamma_1\gamma_4$. Then,

$$ \begin{bmatrix} {}^0X_{P5R} \\ {}^0Y_{P5R} \end{bmatrix} = d\begin{bmatrix} \beta_{1R} & \beta_{2R} \\ \beta_{2R} & -\beta_{1R} \end{bmatrix}\begin{bmatrix} c_{1R} \\ s_{1R} \end{bmatrix} \tag{49} $$

$$ \begin{bmatrix} c_1 \\ s_1 \end{bmatrix} = \frac{1}{d}\boldsymbol{\beta}^{-1}\begin{bmatrix} {}^0X_{P5} \\ {}^0Y_{P5} \end{bmatrix} \tag{50} $$

And,

$$ \theta_1 = atan2(s_1, c_1) \tag{51} $$

This is a unique solution.

[1] It's interesting to note that when $\theta_2 = 0$, trig identities show that $\frac{{}^0Z_{P5R}}{d} = sin(La_{12} + La_{23} + 90°)$. Similar for left arm.

## 4.7 Compute $\theta_4$

Using known values of $\theta_1$, $\theta_2$, and $d_3$ we calculate:

$$ {}^0_6T_D = {}^0_3T\,{}^3_6T \tag{52} $$

$$ {}^3_6T_D = {}^0_3T^{-1}\,{}^0_6T_D \tag{53} $$

$$ {}^3_6T_D = \begin{bmatrix} c_4c_5c_6 + s_4s_6 & c_6s_4 - c_4c_5s_6 & c_4s_5 & c_4c_5l_w \\ c_5c_6s_4 - c_4s_6 & -c_4c_6 - c_5s_4s_6 & s_4s_5 & c_5l_ws_4 \\ c_6s_5 & -s_5s_6 & -c_5 & d_4 + l_ws_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{54} $$

With this result we compute $\theta_4$. In the following steps, we need to avoid division by zero and so some terms must be compared to zero. In a computational solution a more practical test is

```
if(abs(theta[i]) < epslion)
```

where `epsilon` is a small numerical constant (typically $\epsilon < \sin(1°)$).

$$ c_5 = {}^3_6T(3,3) \tag{55} $$

If $c_5 \neq 0$:

$$ c_4 = \frac{{}^3_6T(1,4)}{c_5l_w}, \qquad s_4 = \frac{{}^3_6T(2,4)}{c_5l_w} \tag{56} $$

If $c_5 = 0$

$$ c_4 = \frac{{}^3_6T(1,3)}{c_5l_w}, \qquad s_4 = \frac{{}^3_6T(2,3)}{c_5l_w} \tag{57} $$

And then,

$$ \theta_4 = \operatorname{atan2}(s_4, c_4) \tag{58} $$

## 4.8 Compute $\theta_5$

Next we compute $\theta_5$.

If $\sin(\theta_4) \neq 0$,

$$ s_5 = \frac{{}^3_6T(2,3)}{s_4}, \tag{59} $$

Or if $\sin(\theta_4) = 0$,

$$ s_5 = \frac{{}^3_6T(1,3)}{c_4}, \tag{60} $$

And then using equation 55,

$$ \theta_5 = \operatorname{atan2}(s5, c5). \tag{61} $$

## 4.9 Compute $\theta_6$

Finally, we compute $\theta_6$.

If $s_5 \neq 0$,

$$ s_6 = \frac{-{}^3_6T(3,2)}{s_5}, \qquad c_6 = \frac{{}^3_6T(3,1)}{s_5} \tag{62} $$

Or, if $s_5 = 0$,

$$ {}^0_6T_D = {}^0_3T\,{}^3_5T\,{}^5_6T \tag{63} $$

$$ {}^5_6T_D = {}^3_5T^{-1}\,{}^0_3T^{-1}\,{}^0_6T_D \tag{64} $$

$$ {}^5_6T_D = \begin{bmatrix} c_6 & -s_6 & 0 & l_w \\ 0 & 0 & -1 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{65} $$

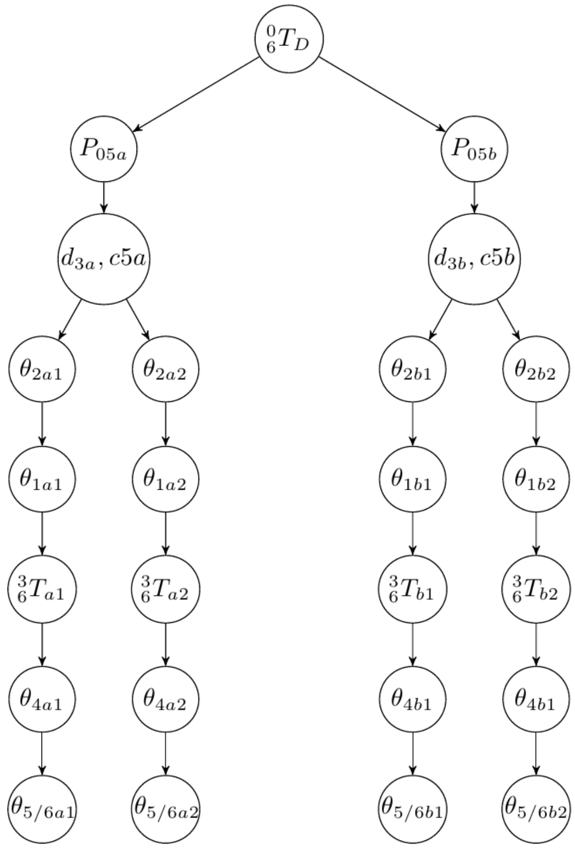


Figure 11: The four solutions to the RavenII inverse kinematics (for $a_3 = 0$). Not all of the solutions will fall inside the mechanism joint limits.

$$s_6 = {}^5_6T(3,1) \quad c_6 = {}^5_6T(3,2) \tag{66}$$

And so,

$$\theta_6 = \mathrm{atan2}(s_6, c_6). \tag{67}$$

**Solution Tree** There are four solutions to the inverse kinematics. They are grouped according to the multiplicity of solutions of various steps above. A graphical illustration of the various solutions is shown in Figure 11.

## 4.10 Kinematics, Joint Angles, and Motor Angles

As a practical matter, the $\theta_i$ used in the kinematics analysis are offset to mechanical joint angles by the Raven II software and further converted to Motor angles. The mechanism joint angles $J_i$, are different from the angles of the motors (which are where position sensors are located) because of mechanical advantage of the cables/pulleys, and gears. Conversion steps must therefore be performed among Motor angles (Mpos), Joint angles ($J_i$), and Kinematics angles ($\theta_i$) (Figure 12). Also note that the Joint variables, $J_i$ are indexed using the C convention (first element is $J_0$) whereas the Kinematics angles, $\theta_i$ are indexed starting with 1 in the traditional Denavit Hartenberg manner. Also note that computational implementations of inverse kinematics may yield $\theta_i$ values in radians. If so, each $\theta_i$ value may need to be multiplied by $180°/\pi$ as part of the following steps.

### 4.10.1 Left arm

Mechanical shoulder ($J_0$) zero angle is elevated 25 degrees from parallel with the base. Also, it varies from kinematic, $\theta_1$ by an additional 180° rotation, so:

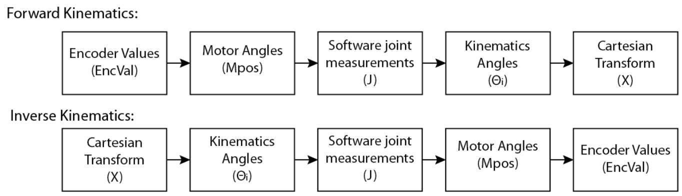


Figure 12: The relationship between kinematics angles $\Theta$, Software joint angles, and motor angles ("Cartesian Transform (X)" is denoted by ${}^{A}_{B}T$ in this document).

$$J_0 = \theta_1 + 180^\circ + 25^\circ = \theta_1 - 205^\circ \tag{68}$$

Mechanical elbow joint ($J_1$) zero is at full extension of joint 2. This is opposite to the joint 2 kinematics result which has $\theta_2 = 0$ when the elbow is folded back, so:

$$J_1 = \theta_2 + 180^\circ \tag{69}$$

Insertion axis ($J_2$) cannot physically achieve d3=0, since it would require the tool carriage to be at the RCM. However, this is not a problem, so long as the robot is not commanded to this pose. Simply set:

$$J_2 = d_3 \tag{70}$$

and initialize the tool offset to 559.40 mm when the tool is fully un-inserted (i.e., retracted and up against its hard stop).

Tool roll ($J_3$) zero corresponds with kinematics $\theta_4$:

$$J_3 = \theta_4 \tag{71}$$

Tool wrist ($J_4$) zero is corresponds to a straight-out pose in which $X_5$ is aligned $180^\circ$ from $Z_4$. Achieving this angle requires $\theta_5 = -90^\circ$, so:

$$J_4 = \theta_5 + 90^\circ \tag{72}$$

The actual tool has two "link 6" structures corresponding to the two fingers of the grasper. The position of these are denoted $J_5$ and $J_6$. Mechanism angles $J_5 = 0$ and $J_6 = 0$ correspond to:

$$X_{6a}||X_{6b}||X_{6c}||X_5. \tag{73}$$

Furthermore, two control modes are possible: control of each finger, or control of grasper center-point. The former requires two iterations of the forward kinematics with the orientation of each finger specified in a desired transform and thus:

$$J_5 = \theta_{6a} \qquad J_6 = \theta_{6c} \tag{74}$$

The latter requires an additional variable: "grasp angle", $g$.

$$J_5 = \theta_{6b} + g/2 \tag{75}$$
$$J_6 = \theta_{6b} - g/2 \tag{76}$$

Conversion of kinematics angles to joint angles can be accomplished by the following:

$$\bar{J}_L = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \\ \theta_4 \\ \theta_5 \\ \theta_{6a} \\ \theta_{6b} \end{bmatrix} + \begin{bmatrix} -205^\circ \\ 180^\circ \\ 0 \\ 0^\circ \\ 90^\circ \\ 0^\circ \\ 0^\circ \end{bmatrix} \tag{77}$$

**DANYING WILL CHECK** $J_4$

### 4.10.2 Right Arm

Mechanical shoulder joint of the left arm ($J_0$) zero angle is elevated 25 degrees elevation from parallel with the base, so:

$$J_0 = \theta_1 - 25^\circ \tag{78}$$

Unlike the right arm, the left mechanical "elbow" joint ($J_1$) has no offset from $\theta_2$, so:

$$J_1 = \theta_2 \tag{79}$$

$J_1$ — $J_7$ are treated the same as in the Left arm, and so:

$$\bar{J}_R = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \\ \theta_4 \\ \theta_5 \\ \theta_{6a} \\ \theta_{6b} \end{bmatrix} + \begin{bmatrix} -25^\circ \\ 0^\circ \\ 0 \\ 0^\circ \\ 90^\circ \\ 0^\circ \\ 0^\circ \end{bmatrix} \tag{80}$$

## References


[1] J. Craig. *Introduction to Robotics: Mechanics and Control*. Addison Wesley, 1986.