Risk Calculation System
Model Documentation and Validation Report

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1. Executive Summary

This document provides the model validation and documentation for the developed Risk Calculation system which enables the calculation of VaR and ES for a user-defined portfolio of stocks and options.

For this project, an intelligent risk calculation system is developed, and the models applied are explained in detail. This risk system allows users to customize their investment portfolios within the scope of stock markets and related option products. The output results contain the VaR and ES matrices, which are calculated by three different methods in this system and the supporting data source is Yahoo Finance.

This is a robust risk calculation system with detailed test plan and software design process explained in the second part of this project. An arbitrary set of stocks and options are backtested to examine the performance of the system and the accuracy of the calculated VaR. Detailed graphic demonstration is provided.

2. Introduction

This is the term project for quantitative finance course Financial Risk Management & Regulation and this project aims to develop and provide an intelligent risk calculation system with user-defined inputs. This project derives from the hard work of the weekly homework and can be considered as an assembled and upgraded version of the homework outputs

The risk calculation system in this project is able to take a portfolio of stock and option positions as input from the users and compute the Monte Carlo VaR (Value at Risk) and ES (Expected Shortfall), Historical VaR and ES, Parametric VaR and ES. Also, this system enables the backtests of the computed VaRs against the historical data.

3. Model Documentation

3.1 The Scope of the System Model

This risk calculation system supports stock only portfolios and stock-option mixed portfolios selected from the public stock markets. So far, other major financial products including bonds, foreign currencies, commodities and the derivatives related cannot be selected and inputted into the portfolio. The expected output of this system is the risk related metrices of the input option portfolios, namely VaR and ES based on different calculation methods along with selected weighting methods.

Regarding the stock-option combined portfolio, based on the Black Scholes model, implied volatility is required to input to the model for calculation to calibrate a flat implied volatility surface.

3.2 The Assumptions of the System Model

(a) Long/short position

When users input their selected portfolio, they will have the option to choose the long or short position of their portfolios. The system itself does not assume a default long or short position for the input portfolio.

(b) GBM & Normal Distribution Assumption

The type of stock price distribution is required to better calculate the risk. This risk system enables the users to select between portfolio GBM (Geometric Brownian Motion) and individual underlier GBM to calculate the portfolio VaR/ES. If underliers are assumed to follow GBM, the portfolio is assumed to follow multivariate normal distribution.

Geometric Brownian Motion

When users choose the assumption of portfolio GBM to simulate the stock price, the portfolio price V follows:

$$dV = \mu V dt + \sigma V dW$$

where μ denotes the annualized value drift, σ denotes the annualized value standard deviation and W denotes the standard Wiener process, such that

$$V_T = V_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma W_T}$$

And the users input portfolio value is denoted by:

$$V = \sum_{i=1}^{n} \omega_i * S_i$$

Where ω_i denotes the weights of stock i and S_i denotes the stock price of stock i.

Normal Distribution

When users choose the assumption of underlier GBM as well as portfolio normal distribution to simulate the stock price, the system assumes the input portfolio contains N stocks and the price S_i of each individual stock in the portfolio follows:

$$dS_i = \mu_i S_i dt + \sigma_i S_i dW_i$$

where μ_i denotes the annualized value drift, σ_i denotes the annualized value standard deviation and W_i denotes the standard Wiener process. An arbitrary pair of stocks has correlated movement,

$$dW_i dW_j = \rho_{ij} dt, \quad i \neq j$$

And the users input portfolio value is denoted by:

$$V = \sum_{i=1}^{n} \omega_i * S_i$$

Where ω_i denotes the weights of stock i and S_i denotes the stock price of stock $i,\ i = 1, ..., n.$ Normal distribution assumption leads to the expression for cross-price stochastic process,

$$S_{it} S_{jt} = S_{i0} S_{j0} e^{\left(\mu_i + \mu_j - \frac{1}{2}\left(\sigma_i^2 + \sigma_j^2\right)\right)t + \sigma_i W_{it} + \sigma_j W_{jt}}$$

and

$$\sigma_i W_{it} + \sigma_j W_{jt} \sim N\left(0, \left(\sigma_i^2 + \sigma_j^2 + 2\rho_{ij}\sigma_i\sigma_j\right)t\right)$$

follows normal distribution.

(c) VaR and ES Calculation

This risk system can take different day level horizon of VaR and ES, i.e, 5-day VaR, different calibration window and look-back period (year) as input parameters with the users pre-defined confidence level for VaR and ES. The mathematical derivation of VaR and ES are presented in the later part of the document.

(d) Exponential Weighting Method & Unweighted Window Method Selection

This risk system enables the users to select between Exponential Weighting Method and Unweighted Window Method to calculate the portfolio price based on their preferred weights assignment. Accordingly, μ and σ are calculated differently of these two weighting methods.

EWMA (Exponential Weighting Moving Average Method)

EWMA assigns more weights to the price of the date that is closer to the current date, namely the weights are divided by decaying adjustment factor in beginning periods to account for imbalance in relative weightings. It responds more quickly to price changes than the unweighted simple moving average does. This is particularly helpful to users attempting to trade intraday swing highs and lows.

The EW function is calculated using weights:

$$\omega_i = (1 - \alpha)^i$$

In this risk system, the EW moving average of the stock price within the window period is calculated as:

$$S_T = \frac{S_{T-1} + \omega_1 S_{T-2} + \omega_2 S_{T-3} + ... + \omega_t S_0}{1 + \omega_1 + \omega_2 + ... + \omega_t}$$

Unweighted Window Method

The Unweighted Window Method assigns equal weight to the price of every trading day within the window period. It fairly reflects the historical price change while some may argue that a simple moving average gives too much weight to old data, which are deemed to be less significant.

3.3. Model Introduction

(a) VaR (Value at Risk)

Given the input stocks or options, this risk system calculates the VaR of the portfolio. The p level VaR is X if p of the time our losses are less than or equal to X:

$$VaR(V,\ T,\ p) = G^{-1}(p)$$

where

$$G(x) = P\left[V_0 - V_T \leq X\right] = E^P[1_{V_0 - V_T \leq X}]$$

Often, given as:

$$VaR(V,\ T,\ p) = \inf inf\ \{\, I \mid P\left[V_0 - V_T > I\right] \leq 1 - p\}$$

VaR represents the quantile of loss distribution and it gives a precise percentile loss, which enables VaR estimation actionable. VaR is also elicitable and robust. It essentially same as variance if distribution is normal.
However, VaR does not measure worst case loss. p level VaR means that $1 - p$ of the case he loss is expected to be greater than X. Value at Risk does not say anything about the size of losses within this $1 - p$ of trading days and by no means does it say anything about the maximum possible loss.

(b) ES (Expected Shortfall)

Given the input stocks or options, this risk system also calculates the ES of the portfolio, which is another major risk measure of portfolios. ES is a risk measure sensitive to the shape of the tail of the distribution of returns on a portfolio. It is calculated by averaging all of the returns in the distribution that are worse than the VaR of the portfolio at a given p level of confidence.
Its formula is given by:

$$ES(V, T, p) = -\ E^p\left[V_T - V_0 < -\ VaR\left(V_T, p\right)\right]$$
$$= E^p\left[V_0 - V_T > VaR\left(V_T, p\right)\right]$$

ES is a coherent measure of risk due to that diversification never increases risk and ES is robust by some measures. However, ES is not actionable or elicitable and thus difficult to do backtest.

(c) Parametric Method

Parametric Method measures VaR through making simplifying assumptions so as to yield a formula for the VaR based on approximate mean and variance calculations.
The formula to calculate VaR under long position is:

$$LongVaR(S, T, p) = S_0 - S_0 e^{\sigma\sqrt{T}\Phi^{-1}(1-p)+\left(\mu-\frac{\sigma^2}{2}\right)T}$$

The formula to calculate VaR under short position is:

$$ShortVaR(S, T, p) = S_0 e^{\sigma\sqrt{T}\Phi^{-1}p+\left(\mu-\frac{\sigma^2}{2}\right)T} - S_0$$

The formula to calculate ES under long position is:

$$LongES(S, T, p) = S_0 - \frac{S_0 e^{\mu T}\phi(\phi^{-1}(1-p)-\sigma\sqrt{T})}{1-p}$$

The formula to calculate ES under short position is:

$$ShortES(S, T, p) = \frac{S_0 e^{\mu T}\phi(\phi^{-1}p-\sigma\sqrt{T})}{1-p} - S_0$$

Where the μ and σ represents the stock price return average and standard deviation accordingly and should be calculated under either EWMA or unweighted window method. Under EWMA, this risk system applies an exponential decay factor λ between 0 and 1 to calculate the exponential weights:

$$\omega = \sum \omega_i$$

Where ω denotes the total weight and ω_i denotes the weight of stock price at trading day i. Since

$$LogReturn_i = \log log\left(\frac{S_{i+1}}{S_i}\right)$$

the stock price return average μ is denoted by

$$\mu = \frac{\sum LogReturn_i}{n}$$

where n denoted the number of trading days in total.
Denote the weighted return average by p_i:

$$p_i = \frac{\omega_i}{\omega}\mu$$

The standard deviation σ is calculated as:

$$\sigma = \sqrt{\sum p_i LogReturn_i^2 - (\sum p_i LogReturn_i)^2}$$

Under unweighted window method, the standard deviation σ is calculated as:

$$\sigma = \sqrt{\sum LogReturn_i^2 - (\sum LogReturn_i)^2}$$

With the stock price return average μ is denoted by

$$\mu = \frac{\sum LogReturn_i}{n}$$

Then these daily estimates need to be converted to annually estimates. Normally each year contains 252 trading days. Thus the μ and σ can be scaled as:

$$\mu = 252\bar{\mu} + \frac{\sigma^2}{2}$$
$$\sigma = \bar{\sigma} \times \sqrt{252}$$

Where $\bar{\mu}$ and $\bar{\sigma}$ denote the sample mean and sample standard deviation.

In this risk system, Denote V_0 as the initial portfolio value at day 0 and S_0 as the stock price, $S_0^{'}$ as the option price at day 0:

$$V_0 = S_0\left(1 - \sum_{i=1}^{n} \omega_i\right) + S_0^{'}\left(\sum_{i=1}^{n} \omega_i\right)$$

Where ω_i denotes the weights of option i, and the first part denotes the value of stock and the second part denotes the value of options (if users input options).

In this risk system, Denote V_t as the portfolio value at day t and S_t as the stock price, $S_t^{'}$ as the option price at day t:

$$V_t = S_t\left(1 - \sum_{i=1}^{n} \omega_i\right) + S_t^{'}\left(\sum_{i=1}^{n} \omega_i\right)$$

where under long position:

$$S_{t\_long} = S_0 e^{\sigma\sqrt{t}\Phi^{-1}(1-p)+\left(\mu-\frac{\sigma^2}{2}\right)t}$$

Under short position:

$$S_{t\_short} = 2S_0 - S_0 e^{\sigma\sqrt{t}\Phi^{-1}p+\left(\mu-\frac{\sigma^2}{2}\right)t}$$

Therefore, VaR can be calculated as:

$$VaR = V_0 - V_t$$

(d) Historical Method

Historical VaR is a non-parametric method of VaR calculation. This methodology is based on the approach that the pattern of historical returns is indicative of the pattern of future returns.

Historical Method measures the VaR by assuming risk factors follow actual historical distributions. It does not make assumptions about distribution of historical changes but assume today's distribution of market changes = historical distribution of market changes, and for each day of history, apply that day's change to today. And each percentage change is then calculated with current market values to present the scenarios for future value.
Follow the similar logic in Parametric Method, denote V_0 as the initial portfolio value at day 0 and S_0 as the stock price, $S_0^{'}$ as the option price at day 0:

$$V_0 = S_0\left(1 - \sum_{i=1}^{n} \omega_i\right) + S_0^{'}\left(\sum_{i=1}^{n} \omega_i\right)$$

Where ω_i denotes the weights of option i, and the first part denotes the value of stock and the second part denotes the value of options (if users input options).

Denote V_t as the portfolio value at day t and S_t as the stock price, $S_t^{'}$ as the option price at day t:

$$V_t = S_t\left(1 - \sum_{i=1}^{n} \omega_i\right) + S_t^{'}\left(\sum_{i=1}^{n} \omega_i\right)$$

Where $S_t^{'}$ can be obtained from the Yahoo finance website and calculated by BS model.

By normalizing the historical return:

$$VaR = \frac{V_t - V_0}{V_0}$$

Next, sort the distribution of historical returns in ascending order (basically in order of worst to best returns observed over the period) to interpret the VaR for the portfolio in the time horizon based on the selected confidence level.

(e) Monte Carlo Method

Monte Carlo Method measures the VaR by simulating the risk factors and use the prices to directly compute the VaR.

The basic concept behind the Monte Carlo approach is to repeatedly run a large number of simulations of a random process for the VaR. The repeated matrices are drawn from pre-specified probability distributions that are assumed to be known, including the analytical function and its parameters. Monte Carlo simulations inherently try to recreate the distribution of the return of a position, where Monte Carlo VaR can be computed.

This risk system assumes that the input portfolio follows the Geometric Brownian Motion or the underlying asset follows the Geometric Brownian Motion:

$$S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t}$$

where W_t denotes the Winner Process.

The option price is calculated using Black Scholes Model with the simulated stock price. Follow the similar logic in Parametric Method, denote V_0 as the initial portfolio value at day 0 and S_0 as the stock price, $S_0^{'}$ as the option price at day 0:

$$V_0 = S_0\left(1 - \sum_{i=1}^{n} \omega_i\right) + S_0^{'}\left(\sum_{i=1}^{n} \omega_i\right)$$

Where ω_i denotes the weights of option i, and the first part denotes the value of stock and the second part denotes the value of options (if users input options).

According to the actual return of the selected stocks, next step is to generate the correlation matrix in order to perform the Monte Carlo Simulation:

$$S_{(t+1),i} = S_{ti} e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_{ti}}$$

For at least 10,000 times, then sum the portfolio value each time.

Denote V_t as the portfolio value at day t and S_t as the stock price, $S_t^{'}$ as the option price at day t:

$$V_t = S_t\left(1 - \sum_{i=1}^{n} \omega_i\right) + S_t^{'}\left(\sum_{i=1}^{n} \omega_i\right)$$

where $S_t^{'}$ can be obtained from the Yahoo finance website and calculated by BS model.

By normalizing the historical return:

$$VaR = \frac{V_t - V_0}{V_0}$$

Next, sort the distribution of simulated returns in ascending order (basically in order of worst to best returns observed over the period) to interpret the VaR for the portfolio in the time horizon based on the selected confidence level.

3.4. Major Methods Comparison

Parametric VaR is not really a model or a type, it is an approximation methodology applied to the problem. The measurement is accurate when positions and payoffs are linear, but then no different from a simple variance calculation. Otherwise, it might be inaccurate and require hacks to improve it. The Parametric Method has the minimal computation time compared to the other two methods and it is simple to give a quick reference. It relies on distribution and is less data intense.

Monte Carlo VaR is not really a model or type, it is a numerical method applied to the problem. It is straight forward to compute and correct for nonlinear payoffs. If positions and payoffs are linear and normally distributed, then it is the same as parametric VaR. However, it is very slow to get high accuracy. Besides, Monte Carlo Method is time consuming and relatively complex due to the large number of replications. But it has the advantage of not being affected much in market changing or crisis events.

The Historical Method is a simple and fast method to calculate VaR. For a portfolio, it eliminates the need to estimate the variance-covariance matrix and simplifies the computations especially in cases of portfolios with a large number of assets. This method is also intuitive. VaR corresponds to a large loss sustained over an historical period that is known. Hence users can go back in time and explain the circumstances behind the VaR measure.

But the choice of absolute change or relative change should be carefully thought. It still requires to pick a model. If a risk factor is considered to have constant absolute volatility, then absolute changes would be most representative. Otherwise, if a risk factor is considered to have constant relative volatility, then relative changes would be most representative. Besides, problems will arise if the historical risk factor crosses zero.

In Historical Model, the assumption is that the past represents the immediate future is highly unlikely in the real world. Also, if the horizon window omits important events (like stock market booms and crashes), the distribution will not be well represented. Its calculation is only as strong as the number of correct data points measured that fully represent changing market dynamics even capturing crisis events that may have occurred such as the Covid-19 crisis in 2020 or the financial crisis in 2008. In fact, even if the data does capture all possible historical dynamics, it may not be sufficient because market will never entirely replicate past movements.

4. Software Design Documentation

4.1. Goal

The software is designed to measure the market risk of an arbitrary portfolio which consists of several stocks invested in a certain period. The market risk metrics used include value-at-risk (VaR) and expected shortfall (ES), and three types of models are built in for evaluation, 1) parametric model, 2) historical model and 3) Monte Carlo model. Also, through the backtest, users can see which method is the most suitable for a certain portfolio.

4.2. Assumptions

1. The combine portfolio value or individual underliers follow the geometric Brownian Motion (GBM) based on pre-selected parameters. If underliers are assumed to follow GBM, they are assumed to follow multivariate normal distribution.
2. Cash is assumed to be invested at the start date and fully liquidated at the end of VaR calculation horizon (i.e 5-day VaR). And the results are saved with respect to each start date.
3. This software does not consider transaction costs and positions can always be fulfilled.
4. Risk-free rate is assumed to be 0.5% and compounding dividend rate is assumed to be zero.
5. 1-year ATM options can be invested as a hedge to bring down the VaR level. And only one stock is allowed for stock and option portfolios.
6. Implied volatility surface is assumed to be flat and implied volatility risk is ignored.

4.3. Interface

Input
The input required from the user include two configuration .json files and an optional historical stock data file. Samples are provided in ‘data’ directory and users should modify content without changing filenames. Functions for checking illegal input will be executed before calculation.

- **data_config.json**: a dictionary that configures portfolio information, date range for retrieving data, and weighting schemes for individual stocks, etc.

```
{
"use_history": 0,
"datastart": "2002-01-01",
"dataend": "2022-11-30",
"portfolio_type": 1,
"total_position": 10000,
"option_weight": 0,
"stock_config": {
        "long_tickers": ["BA", "NOC"],
        "long_weight": "custom",
        "long_custom_weight": [0.3,0.7],
        "short_tickers": ["JNJ", "PFE"],
        "short_weight": "equal",
        "short_custom_weight": [0.5,0.5]},
"option_config": {
        "tickers": [],
        "option_type": "put",
        "moneyness": 1.0}
}
```

- use_history: bool
  1: historical data provided; 0: otherwise, data fetched from yahoo finance;
- datestart, dateend: str, "yyyy-mm-dd", range of historical data;
- portfolio_type: int in [1,2,3]
  1: Long only stock portfolio; 2: Short only stock portfolio; 3: Long only stock with atm option;
- total_position: int, initial portfolio total position, default: 10000;
- option_weight: float in [0,1), weight of option position in the portfolio;
- long_tickers, short_tickers, tickers: list of str;
- long_weight, short_weight: str, 'equal' or 'custom', default: 'equal';
- long_custom_weight, short_custom_weight: list of float, must sum to 1;
- option_type: str, "call" or "put";
- moneyness: float, default: 1.0 (at-the-money).

- **param_config.json**: a dictionary that configures all parameters needed for computing VaR and ES, as well as options regarding which model to use, and whether to generate figures and outputs.

```
{
"risk_config": {
        "horizon": 5,
        "start": "2010-01-04",
        "end": "2022-08-30",
        "var_percentile": 0.99,
        "es_percentile": 0.975},
"tradedays": 252,
"calib_window": 5,
"calib_lambda": 0.998,
"calib_weighting": "unweighting",
"param_model": 1,
"param_config": {
        "assumption": "gbm"},
"hist_model": 1,
"hist_window": 5,
"mc_model": 1,
"mc_config": {"n_paths": 10000,
        "assumption": "gbm"},
"plot_figure": 1,
"save_output": 0}
```

- horizon: int, holding period (day) for VaR calculation, default: 5 (days);
- start, end: str, "yyyy-mm-dd", range of VaR calculation;
- var_percentile, es_percentile: float in (0,1), default: 0.99, 0.975;
- tradedays: int, expected trading days in a year, default: 252 (days);
- calib_window: int, look-back no. of years for μ and σ calibration, default: 5;
- calib_lambda: the exponential decay rate for μ and σ calibration, default: 0.9989;
- calib_weighting: str, "unweighting" or "exponential";
- param_model, hist_model, mc_model: bool;
  1: including this model; 0: not including this model;
- assumption: str, "gbm" or "normal", assumption for portfolio movement;
- hist_window: look-back no. of years for historical VaR calculations;
- n_paths: int, no. of paths for Monte Carlo simulation;
- plot_figure: bool;
- save_output: bool.

- For the optional historical stock data file input, acceptable file types are '.csv', '.xlsx' and '.txt', while '.csv' file is highly recommended.

Architecture and directory

- Utils (static): Functions and class objects that define parametric, historical, and Monte Carlo VaR/ES computation are managed under 'utils' directory.
- Data (user control): As in (a), configuration files and historical data are saved here.
- Output (static): Figures are saved under 'figures' in .png format. VaR/ES results and backtest results are saved into excel files categorized by models. Note that to keep the simplicity of

output, only VaR/ES results are exported, while intermediate outputs such as μ and σ of portfolio and each stock, can be retrieved from class attributes.

- Backtest (static): Backtest results are saved under this directory in .png format, which includes the plots of exceptions, binomial test statistics, and Kupiec's test statistics.
- System.py (user control): The main program of the risk management system to be executed for portfolio set-up, calibration, VaR/ES calculation, etc,. Specifically, users are required to run system.py after providing configuration parameters.
- Backtest.py (user control): The main program of the backtest system to be executed, which includes the BacktestCreator class that runs the backtest, computes historical losses, and calculated relevant statistics.

4.4. Data structure and flow

- Dictionary parameters will firstly be loaded to serve later operations according to configuration files provided by the users.
- Pandas dataframe type of historical stock data will be downloaded from yahoo finance using 'yfinance' API or read from provided historical data file. If options are provided in portfolio, pandas series type of at-the-money implied volatility should be imported.

```
Fetching stocks: ['BA', 'NOC']
[*********************100%***********************]  2 of 2 completed
                   BA        NOC
Date
2002-01-02  25.237362  28.558233
2002-01-03  25.667929  28.627817
2002-01-04  26.734404  28.752481
2002-01-07  27.158327  28.540840
2002-01-08  26.714533  28.561138
```

- Individual stock statistics (arranged in the order of adj. close, log return, log return square) and portfolio statistics will be calculated and saved as pandas dataframe respectively for calibration use.

```
                   BA        NOC        BA       NOC        BA           NOC
Date
2002-01-03  25.667929  28.627817  0.016917  0.002434  0.000286  5.922399e-06
2002-01-04  26.734404  28.752481  0.040709  0.004345  0.001657  1.888078e-05
2002-01-07  27.158327  28.540840  0.015732 -0.007388  0.000248  5.458293e-05
2002-01-08  26.714533  28.561138 -0.016476  0.000711  0.000271  5.054339e-07
2002-01-09  26.429688  28.656794 -0.010720  0.003344  0.000115  1.117933e-05
             log_rtn  log_rtn_sq
Date
2002-01-02  0.000000    0.000000
2002-01-03  0.004838    0.000023
2002-01-04  0.011264    0.000127
2002-01-07  0.002086    0.000004
2002-01-08 -0.003941    0.000016
```

- A python class object for calibrations and models will be initialized.

```
system = {varmodel} <utils.VaRModel.varmodel object at 0x0000022B8E4E7610>
    V_0 = {int} 10000
    calib_corr = {DataFrame: (5264, 1)} 0 [2002-01-03 00:00:00: nan], [2002-01-04 00:00:00: nan], [2002-01-07 00:00:00: nan ...View as DataFrame
    calib_cov = {DataFrame: (5264, 1)} 0 [2002-01-03 00:00:00: nan], [2002-01-04 00:00:00: nan], [2002-01-07 00:00:00: nan] ...View as DataFrame
    calib_drift = {DataFrame: (5264, 3)} BA NOC portfolio [2002-01-03 00:00:00: nan nan nan], [2002-01-04 00:00:00: nan nar ...View as DataFrame
    calib_lambda = {float} 0.998
    calib_params = {dict: 0} {}
    calib_vol = {DataFrame: (5264, 3)} BA NOC portfolio [2002-01-03 00:00:00: nan nan nan], [2002-01-04 00:00:00: nan nan ...View as DataFrame
    calib_weight = {int} 1
    calib_win = {int} 5
    dt = {float} 0.003968253968253968
    end = {datetime} 2022-08-30 00:00:00
    hist_result = {DataFrame: (3187, 2)} hist_VaR hist_ES [2010-01-04 00:00:00: 245.5828535401639 253.49325087962333],...View as DataFrame
    horizon = {float} 0.01984126984126984
    mc_result = {DataFrame: (3187, 2)} mc_VaR mc_ES [2010-01-04 00:00:00: 365.6807781271838 376.041933305842], [20 ...View as DataFrame
    param_result = {DataFrame: (3187, 2)} param_VaR param_ES [2010-01-04 00:00:00: 414.44252741943274 416.2689988 ...View as DataFrame
    params = {dict: 13} {'risk_config': {'horizon': 5, 'start': '2010-01-04', 'end': '2022-08-30', 'var_percentile': 0.99, 'es_percentile': 0.975}, 'tra... View
    pes = {float} 0.975
    plot_figure = {int} 1
    pvar = {float} 0.99
    save_output = {int} 0
    start = {datetime} 2010-01-04 00:00:00
    stock_handle = {DataFrame: (5264, 6)} BA NOC BA NOC BA NOC [2002-01-03 00:00:00: 25.66792869567871 28.627817 ...View as DataFrame
    tickers = {list: 2} ['BA', 'NOC']
    version = {str} '1.0'
```

- Pandas dataframe type of μ and σ (and pairwise ρ if more than one stock) will be calibrated using class method based on assumptions.

```
In[4]: system.calib_drift.tail()
Out[4]:
                  BA       NOC  portfolio
Date
2022-11-22  0.052970  0.168635   0.026852
2022-11-23  0.053911  0.165516   0.026306
2022-11-25  0.059306  0.169793   0.028717
2022-11-28  0.051432  0.168212   0.026330
2022-11-29  0.055721  0.168933   0.027576
```

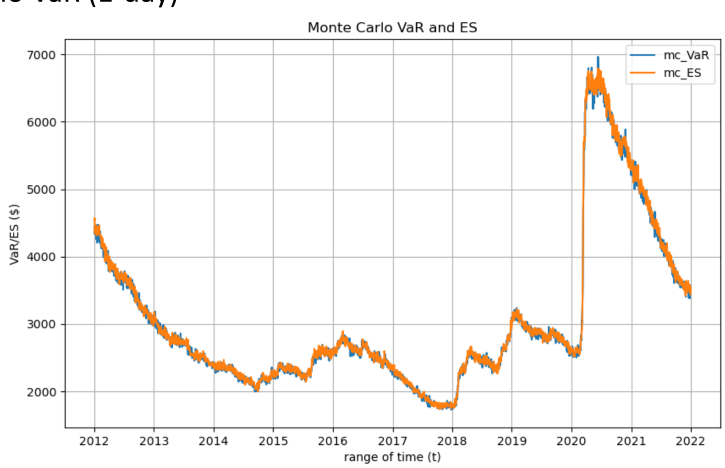
```
In[5]: system.calib_vol.tail()
Out[5]:
                  BA       NOC  portfolio
Date
2022-11-22  0.507794  0.287846   0.167571
2022-11-23  0.507815  0.287815   0.167559
2022-11-25  0.507886  0.287853   0.167593
2022-11-28  0.508154  0.287865   0.167662
2022-11-29  0.508236  0.287876   0.167688
```

```
In[6]: system.calib_corr.tail()
Out[6]:
                   0
Date
2022-11-22  0.371398
2022-11-23  0.371196
2022-11-25  0.371387
2022-11-28  0.371477
2022-11-29  0.371576
```

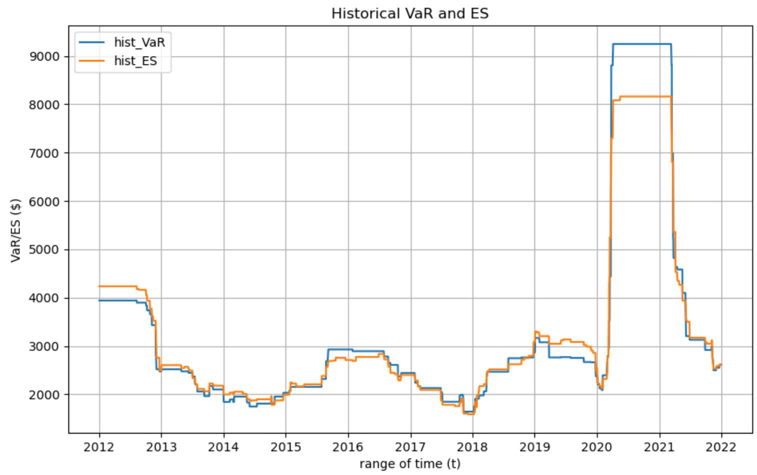
- Based on user selection of models (parametric, historical, Monte Carlo), VaR and ES will be calculated and save as a pandas dataframe for each model.

	A	B	C
1	Date	param_VaR	param_ES
2	2010-01-04 00:00:00	811.7087288	814.8527
3	2010-01-05 00:00:00	810.6498861	813.7945
4	2010-01-06 00:00:00	809.1028224	812.2474
5	2010-01-07 00:00:00	809.1965294	812.3424
6	2010-01-08 00:00:00	809.3522433	812.4982
7	2010-01-11 00:00:00	809.4781682	812.6242
8	2010-01-12 00:00:00	809.7693294	812.9152
9	2010-01-13 00:00:00	809.3719841	812.5194
10	2010-01-14 00:00:00	809.5547411	812.7014
11	2010-01-15 00:00:00	809.0007484	812.1468

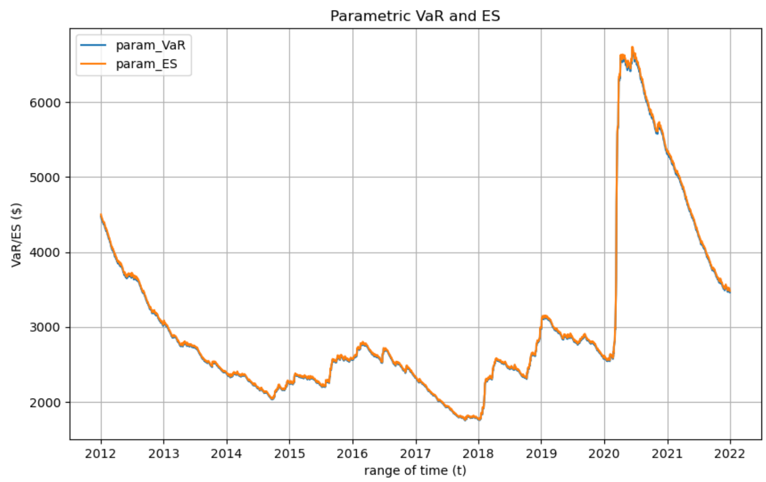
- Based on user preference, figures of .png type of each model can be plotted and result files of .xlsx type can be saved.
  Examples: Short-only portfolio, five stocks, exponential weighting
  - Monte Carlo VaR (1-day)



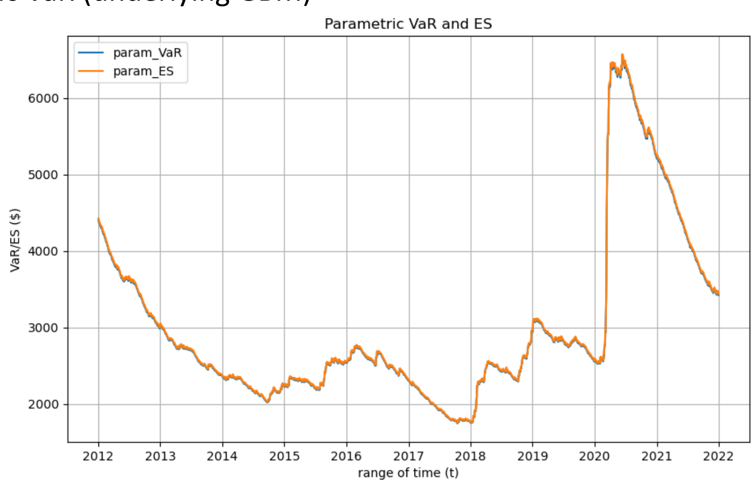
  - Historical VaR (1-day)



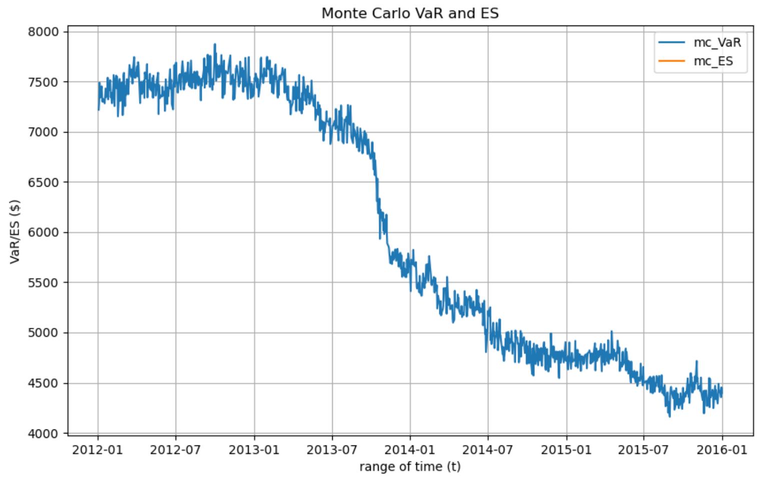
- Parametric VaR (portfolio GBM)



- Parametric VaR (underlying GBM)



- Single stock with a 12 month ATM put option



- The original stock price dataframe will be used to backtest and calculate exceptions.

4.5. Requirement

- Python: 3.6 or above
- pandas: 1.3.3 or above
- numpy: 1.21.2 or above
- matplotlib: 3.4.2 or above
- scipy: 1.7.1 or above
- yfinance: 0.1.87 or above
- json: 2.0.9 or above

5. Test Plan

5.1. Back Testing Design

The purpose of this document is to assess the accuracy and robustness of the Risk Calculation System. In accordance with the current Basel guidelines on VaR backtests, 1-day 99th percentile VaR calibrated against the prior 12-months' data is used. With a 1-day 99th percentile VaR, we expect daily losses to exceed the VaR measure approximately 1% of the time or, equivalently, 2.5 times in a 12-months period (252 trading days).

5.2. Accuracy Test

The first test is to examine the accuracy of the VaR measure. That is to assess whether the frequency of exceptions is consistent with the confidence quantile of loss the VaR measure intends to reflect. The Basel committee provides the following traffic light classifications:

Backtesting Zone	Number of exceptions	Backtesting dependent multiplier

Green	0-4	1.50
Amber	5-9	1.70-1.92
Red	10 or more	2.00

This classification splits the backtesting results into three groups based on the number of instances that the daily loss exceeds the VaR. The green zone suggest no problem with the accuracy of the model with no add-on to the capital requirement while the amber zone and the red zone indicates an indefinitive problem or a definitive problem to the risk model, respectively.

Three straightforward statistical tests for accuracy are also included for completeness, which are the Binomial Test, the Kupiec's proportion of failures (POF) test, and the Kupiec's time until first failure (TUFF) test, which all produces reject-or-accept result where the null hypothesis is that the exception probability is consistent with 1-VaR level. The test statistics are calculated as the following:

$$Z_{bin} = \frac{x-Np}{\sqrt{Np(1-p)}}$$

$$LR_{POF} = -2\,log(\frac{(1-p)^{N-x}p^{x}}{\left(1-\frac{x}{N}\right)^{N-x}\left(\frac{x}{N}\right)^{x}})$$

$$LR_{TUFF} = -2\,log(\frac{(1-p)^{n-1}p}{\left(\frac{1}{n}\right)\left(1-\frac{1}{n}\right)^{n-1}})$$

where x is the number of exceptions, N is the number of observations, n is the number of days until the first exceptions, and p = 1 – VaR level. Specifically, the binomial test is essentially the z-test with the underlying distribution as binomial distributions. The test statistics is distrusted as standard normal. Similarly, for the Kupiec's POF test, x is modelled as following the binomial distribution and the LR_{POF} statistics computes the likelihood ratio between the likelihood of x exceptions given the VaR level implied probability and the likelihood of x exceptions given the empirical probability. LR_{POF} asymptotically follows Chi-squared distribution with one degree of freedom and can be expressed alternatively as $LR_{POF} = 2\,log\{\frac{L(H_A)}{L(H_0)}\}$ where $\hat{p} = \frac{x}{N}$ and $p = 1 - VaR\ level.$ H_A stands for the alternative hypothesis that the probability of exceptions is not the VaR implied probability while H_0 stands for the null hypothesis that the probability of exception is the VaR implied probability. The Kupiec's POF test is again a likelihood ratio value while the POF test models the time to the first exception and models it as following geometric distribution. Again, the statistics is distributed as chi-squared with one degree of freedom.

Both the binomial test and Kupiec's tests suggested that the null hypothesis should be rejected if the test statistics exceed the critical value. For this backtest, a significance level of 0.05 and 0.01 is used. The critical values (one-sided) are summarized as below.

Distribution	$\alpha = 0.05$	$\alpha = 0.01$

Chi-squared distribution with 1 degree of freedoms	3.841	6.635
Standard Normal	1.645	2.326

All test statistics computed for this backtest are unconditional coverage tests which does not capture the clustering of exceptions commonly observed in empirical observations. Conditional coverage tests such as the Christoffersen's test will reflect it. For further backtesting of the risk calculation system, conditional coverage tests will be included.

Three methods of VaR calculation, Monte Carlo method (GBM assumption), Historical method, and Parametric Method (GBM and normal assumptions), are included in the risk system so for each of the method, we will compute the 1-day 99^{th} VaR with 1-year look-back window and the above-mentioned statistics for all three methods year over year for a 10-year window. For both Monte Carlo and Parametric methods, calibration is done by using unweighting moving average and exponentially weighted moving average. Detailed graphic demonstrations will be provided. For the Monte Carlo Method, drifts and volatility will be calculated using both methods, unweighted moving average method and the exponentially weighted moving average method.

5.3. Robustness Test

The next part of the test plan is to assess the robustness of the risk calculation system. For simplicity, only the traffic light test and the Kupiec's POF test will be provided in this section. For the test against different market conditions and specifications, a long-only and short-only portfolios of size 5 will be used, with a total position of 100000.

To test the stability and robustness of the model, we will consider the following time periods to reflect changes in the market and investigate the robustness of the model under different market conditions.

Early 2000s Recession	2001.01 – 2002.12
Economic Expansion post 2000s Recession	2004.01 – 2005.12
2008 Financial Crisis	2008.01 – 2009.12
Economic Expansion post 2008	2014.01 – 2015.12
COVID-19 Recession	2020.01 – 2021.12

All time periods are two-years window so the level of accuracy should be similar. All three methods will be applied to each of the five market periods with 3 representing market down times with high volatility and correlation and 2 representing market up times with low volatility and correlation. The standard 5-day 99^{th} percentile VaR with a 5-year window will be used.

To test the stability and robustness of the model against different specifications, the following sets of parameters will be used over the 10-year testing window.

1. VaR percentile: 95%, 99%
2. VaR frequency: 1-day, 5-days
3. VaR window: 1-year, 5-years

To assess the behavior over a large range of inputs, a long only portfolio of size 1, 5, and 10 will be considered to compare the model's accuracy. The portfolio has a total position of 100000.. Again, the standard 5-day 99^{th} percentile VaR with 5-year look-back window will be used.

6 Test results

For this section, we will present the results from different backtests as described in the test plan. The statistics we used are all computed with a rolling window of 252 trading days (1 year).

- Exceptions: The exception is defined as the event when the portfolio loss is beyond the calculated VaR value. For a window of 252 trading days, the VaR result is compared with the independently computed losses on that day. For example, for a 5 day VaR result, we compute the portfolio loss for a long-only portfolio as: the portfolio value of day 1 – the portfolio value of day 6. We attribute this computed loss as the portfolio loss of day 6 and compare it with the VaR result from day 6. We count the number of exceptions over a rolling window of 252 days on each date.

- Traffic Light Test/ Binomial Test/ Kupiec's POF Test: For these two tests, we use the number of computed exceptions as described above.
- Kupiec's TUFF Test: For this test, for a rolling window of 252 trading days, we compute the number of days until the first exception occurs. Then, we use the corresponding formulas to compute the statistics.

For the sets of backtests, we provided five types of result demonstration. They are:

1. Exception Plot: This plot provides the curves of number of exceptions, VaR, and 1-day change in portfolio value with the right axis representing the value of portfolio and the left axis representing the number of counts for exceptions.
2. Traffic Light Test Table: This table summarizes the average number of exceptions in each traffic light zones across backtesting days with a rolling window of 252 trading days.
3. Binomial Test Plot: This plot plots the binomial test statistics with the 95% and 99% confidence level critical values.
4. Kupiec's POF Test Plot: This plot plots the Kupic's POF test statistics with the 95% and 99% confidence level critical values.
5. Kupic's TUFF Test Table: This table summarizes the percentage (in decimal) that the null hypothesis that the VaR implied level of exceptions is consistent with the observed level of exceptions is rejected using the Kupic's TUFF test statistics with both 95% confidence level and 99% confidence level.

The summary table of the backtests is provided below for easy reference.

	Percentile	Portfolio	No. stock	Stockname	Backtest period	Calib year	Testname (filename)	Param_gbm	Param_nm	Historical	MonteCarlo
Baseline	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2012.01.03 - 2021.12.31	1	long_5_1_win	1	1	1	1
(1d VaR)	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2012.01.03 - 2021.12.31	1	long_5_1_exp	1	1	1	1
	0.99	Short	5	"BA","NOC","INTC","IBM","KWR"	2012.01.03 - 2021.12.31	1	short_5_1_win	1	1	1	1
	0.99	Short	5	"BA","NOC","INTC","IBM","KWR"	2012.01.03 - 2021.12.31	1	short_5_1_exp	1	1	1	1
Other tests	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2000.01.03 – 2002.12.31	5	long_5_5d_5_20002002	1	1	1	1
(5d VaR)	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2004.01.02 – 2005.12.30	5	long_5_5d_5_20042005	1	1	1	1
	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2008.01.02 – 2009.12.31	5	long_5_5d_5_20082009	1	1	1	1
	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2014.01.02 – 2015.12.31	5	long_5_5d_5_20142015	1	1	1	1
	0.99	Long	5	"BA","NOC","INTC","IBM","KWR"	2020.01.02 – 2021.12.31	5	long_5_5d_5_20202021	1	1	1	1
	0.99	Long	1	"BA"	2012.01.03 - 2021.12.31	5	long_1_5d_5_win	1		1	1
	0.99	Short	1	"BA"	2012.01.03 - 2021.12.31	5	short_1_5d_5_exp	1		1	1
	0.99	Long	10	"BA","NOC","INTC","IBM","KWR","WMT","AMZN","GE","XOM","MRK"	2012.01.03 - 2021.12.31	5	long_10_5d_5_win	1	1	1	1
	0.99	Short	10	"BA","NOC","INTC","IBM","KWR","WMT","AMZN","GE","XOM","MRK"	2012.01.03 - 2021.12.31	5	short_10_5d_5_exp	1	1	1	1
	0.95	Long	5	"BA","NOC","INTC","IBM","KWR"	2012.01.03 - 2021.12.31	5	long_0.95_5_5d_5_win	1	1	1	1

6.1 . Baseline Test

For the baseline test, we consider a long-only and short-only equal-weighted portfolios of 5 stocks over the backtesting period of 2012-01-02 to 2021-12-31 with a total position of 100000. The stocks are:

1. Boeing (BA)
2. Northrop Grumman Corp (NOC)
3. Intel Corporation (INTC)
4. IBM Common Stock (IBM)
5. Quaker Chemical Group (KWR)

We calculated the 1-day 99th percentile VaR with 1-year look-back window for this batch of backtests. We consider the following 7 methods of VaR calculation:

1. Historical Method
2. Monte Carlo Method with Unweighted Calibration

3. Monte Carlo Method with Exponentially Weighted Calibration
4. Parametric Method with GBM Assumption and Unweighted Calibration
5. Parametric Method with GBM Assumption and Exponentially Weighted Calibration
6. Parametric Method with Normal Assumption and Unweighted Calibration
7. Parametric Method with Normal Assumption and Exponentially Weighted Calibration

Long-only Portfolio
Monte Carlo Result: Unweighted Calibration

Fig. Exceptions Plot

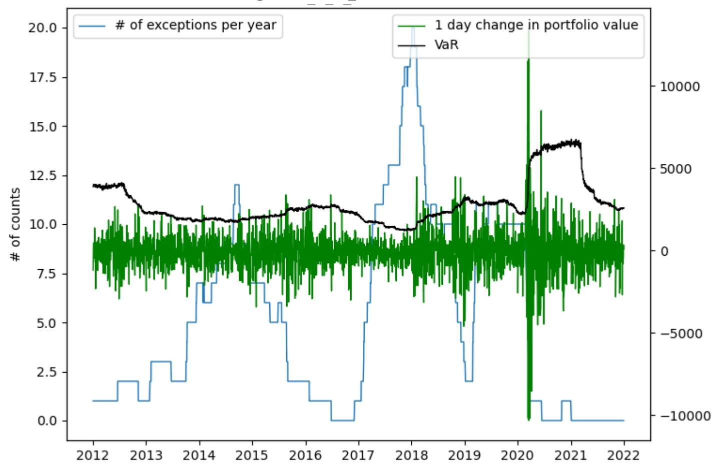


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	56%
% in Amber Zone (5-9 exceptions)	21%
% in Rd Zone (10 or more exceptions)	23%

Fig. Binomial Test Plot

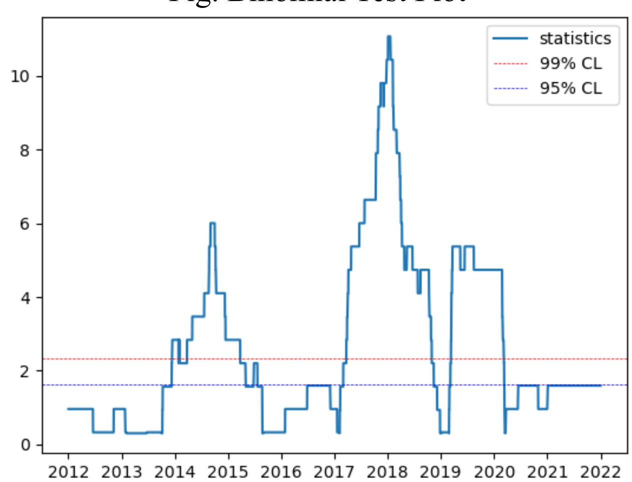


Fig. Kupiec's POF Test Plot

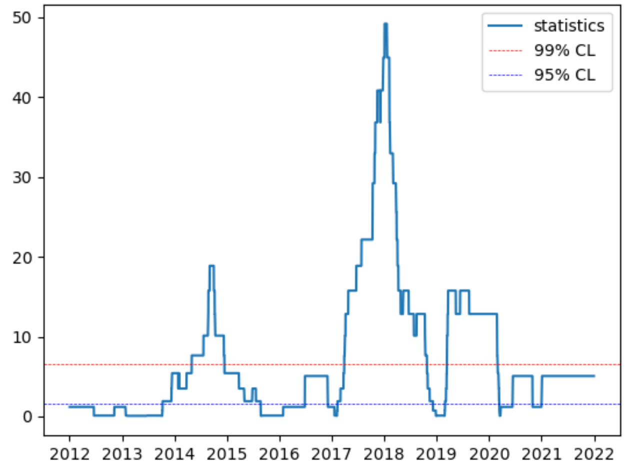


Table. Kupic's TUFF Test Result

95% CL	0.215
99% CL	0.017

Monte Carlo Result: Exponentially Weighted Calibration

Fig. Exceptions Plot

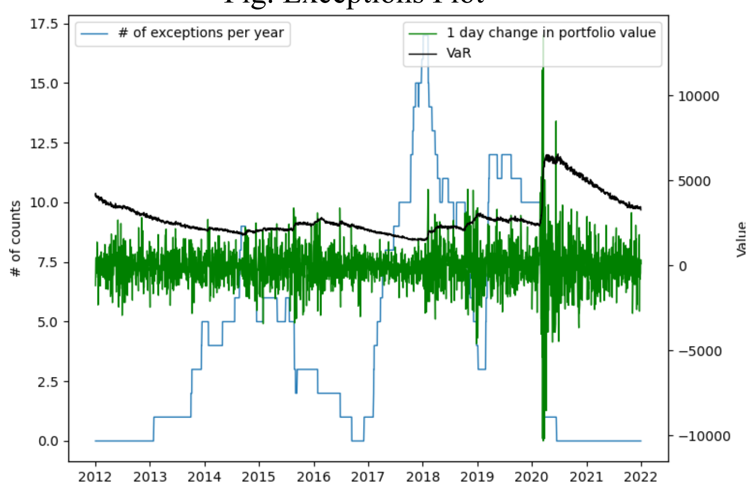


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	57%
% in Amber Zone (5-9 exceptions)	22%
% in Rd Zone (10 or more exceptions)	21%

Fig. Binomial Test Plot

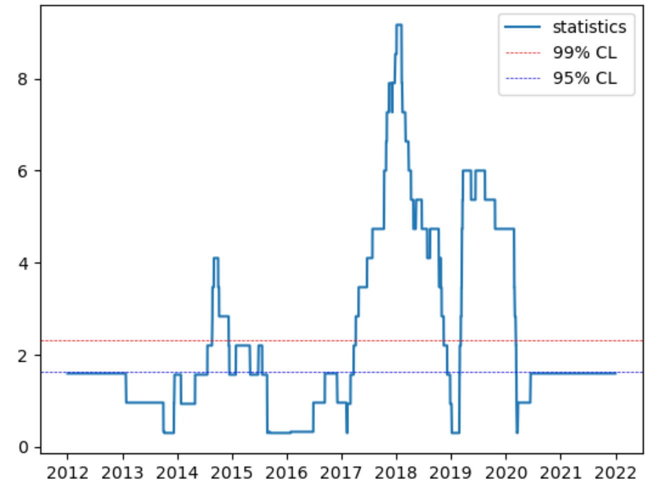


Fig. Kupiec's POF Test Plot

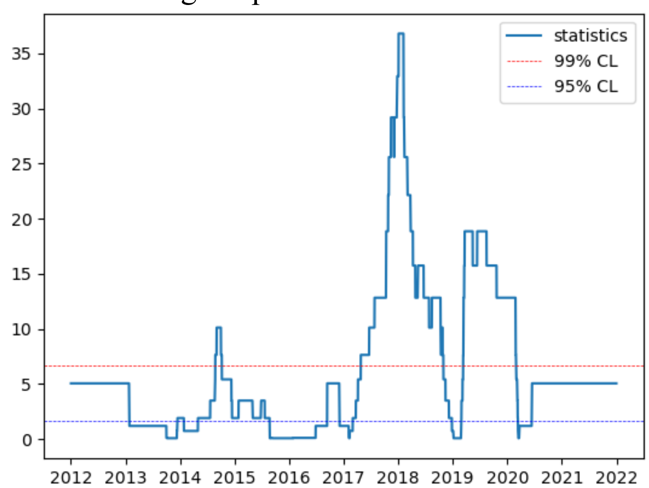


Table. Kupic's TUFF Test Result

95% CL	0.198
99% CL	0.015

GBM Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

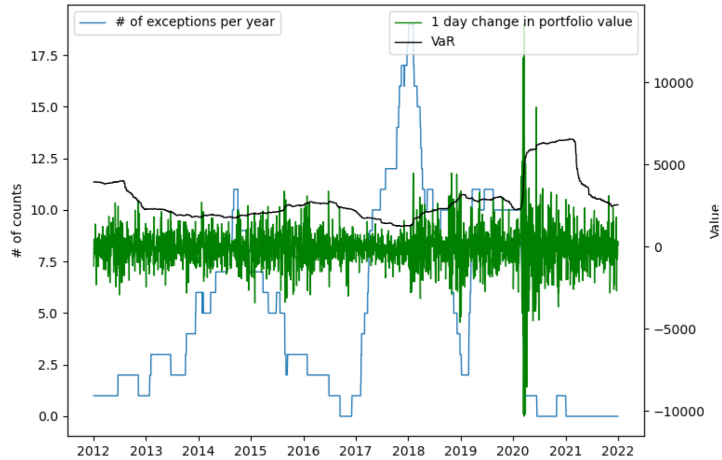


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	55%
% in Amber Zone (5-9 exceptions)	20%
% in Rd Zone (10 or more exceptions)	25%

Fig. Binomial Test Plot

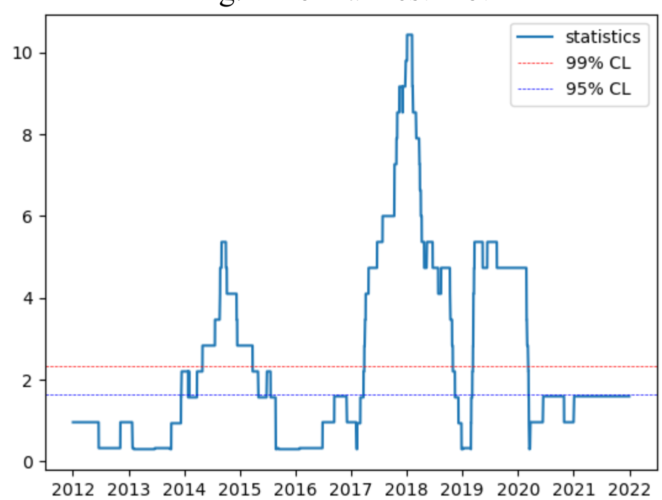


Fig. Kupiec's POF Test Plot

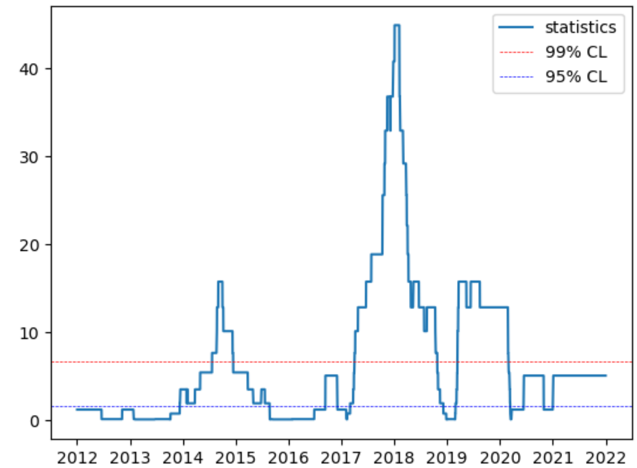


Table. Kupic's TUFF Test Result

95% CL	0.223
99% CL	0.018

GBM Parametric Result: Exponentially Weighted Calibration

Fig. Exceptions Plot

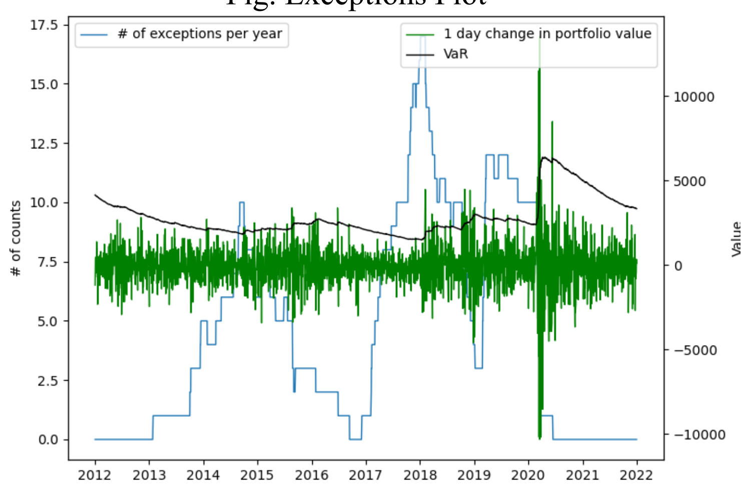


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	56%
% in Amber Zone (5-9 exceptions)	22%
% in Rd Zone (10 or more exceptions)	22%

Fig. Binomial Test Plot

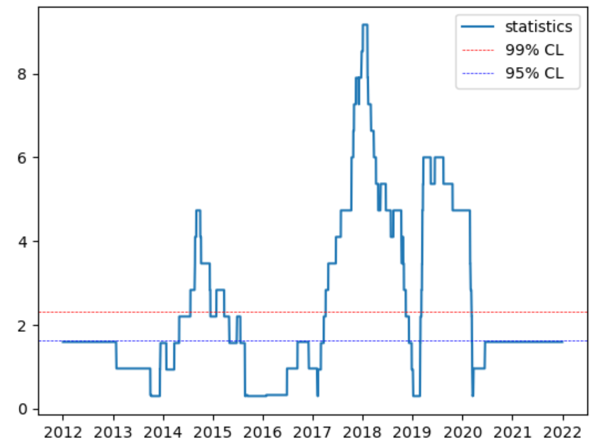


Fig. Kupiec's POF Test Plot

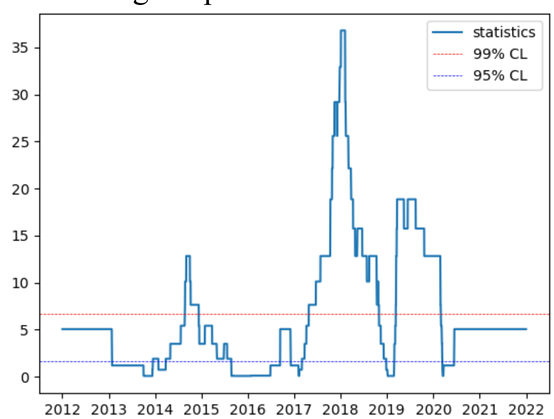


Table. Kupic's TUFF Test Result

95% CL	0.206
99% CL	0.016

Normal Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

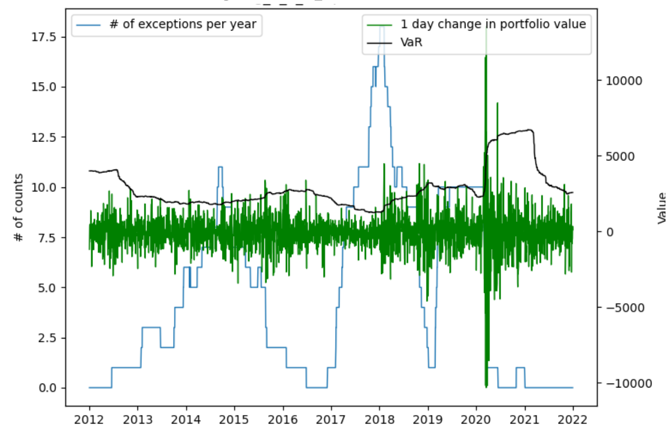


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	56%
% in Amber Zone (5-9 exceptions)	24%
% in Rd Zone (10 or more exceptions)	21%

Fig. Binomial Test Plot

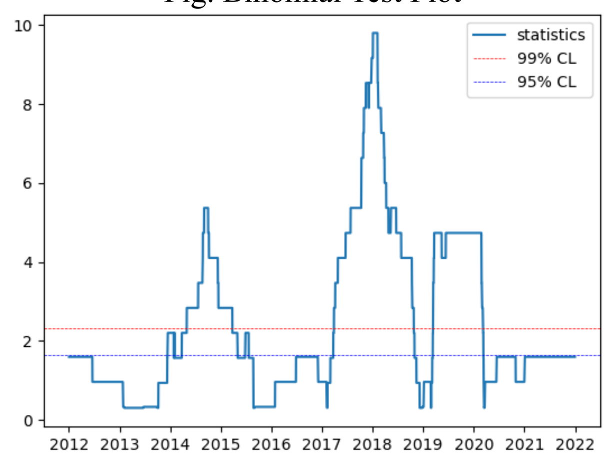


Fig. Kupiec's POF Test Plot

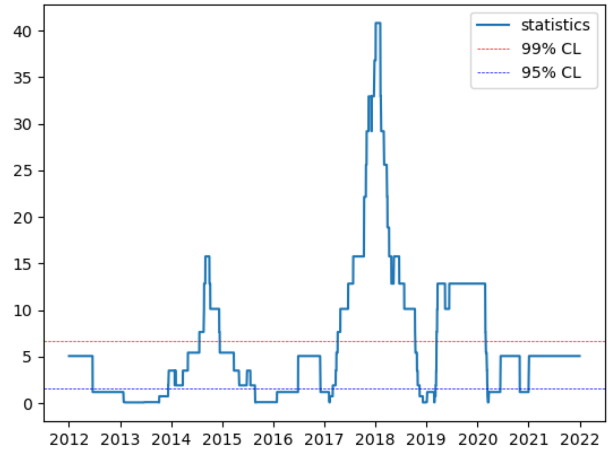


Table. Kupic's TUFF Test Result

95% CL	0.200
99% CL	0.016

Normal Parametric Result: Exponentially Weighted Calibration

Fig. Exceptions Plot

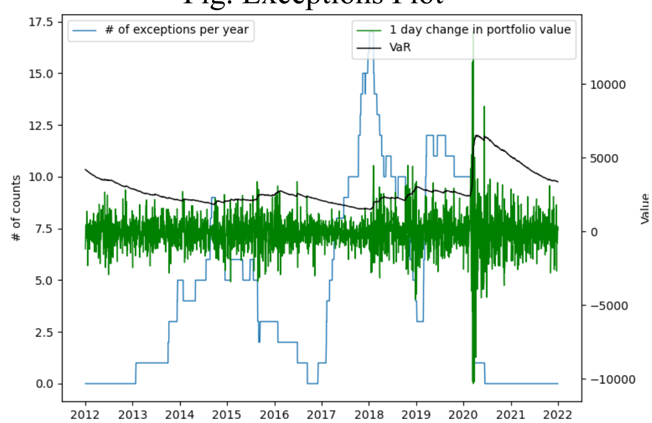


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	57%
% in Amber Zone (5-9 exceptions)	22%
% in Rd Zone (10 or more exceptions)	21%

Fig. Binomial Test Plot

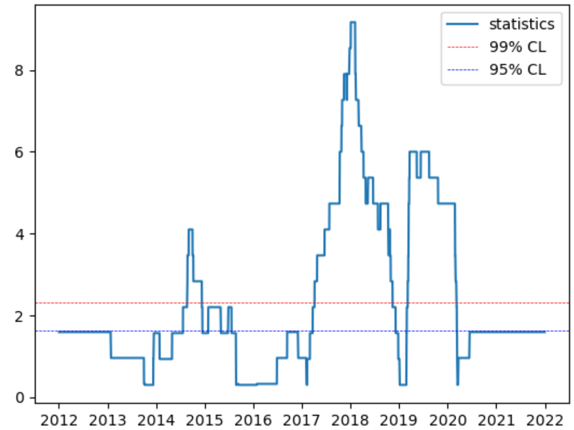


Fig. Kupiec's POF Test Plot

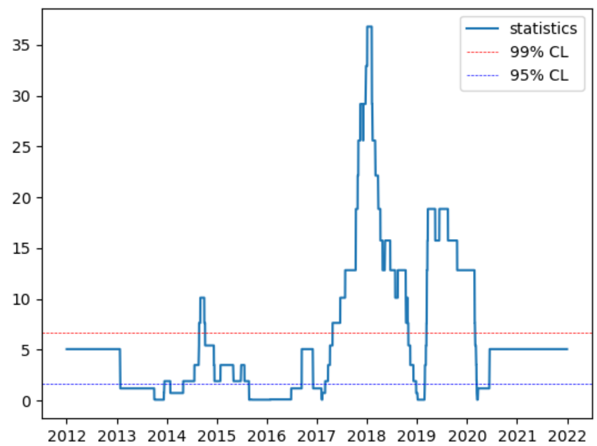


Table. Kupic's TUFF Test Result

95% CL	0.198
99% CL	0.015

Historical Result

Fig. Exceptions Plot

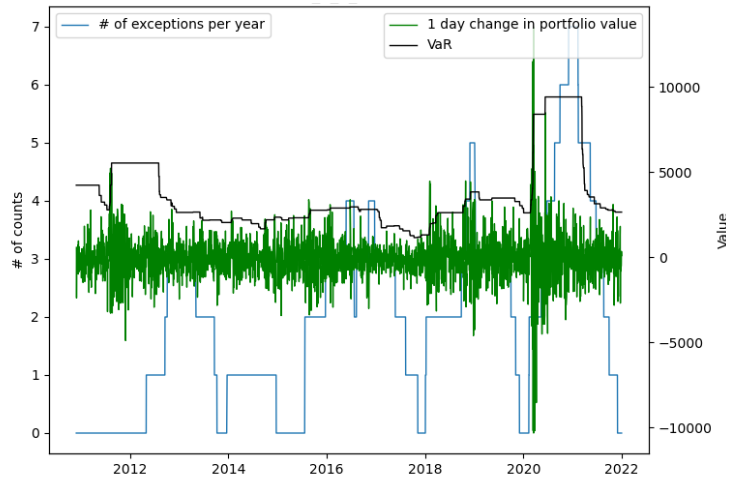


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	98%
% in Amber Zone (5-9 exceptions)	2%
% in Rd Zone (10 or more exceptions)	0%

Fig. Binomial Test Plot

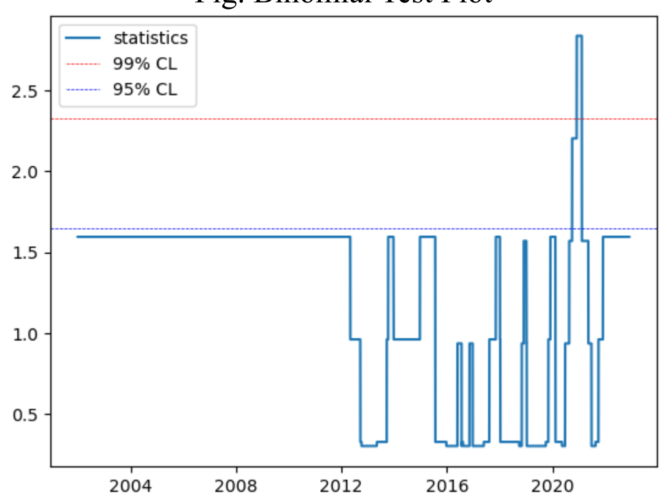


Fig. Kupiec's Test Plot

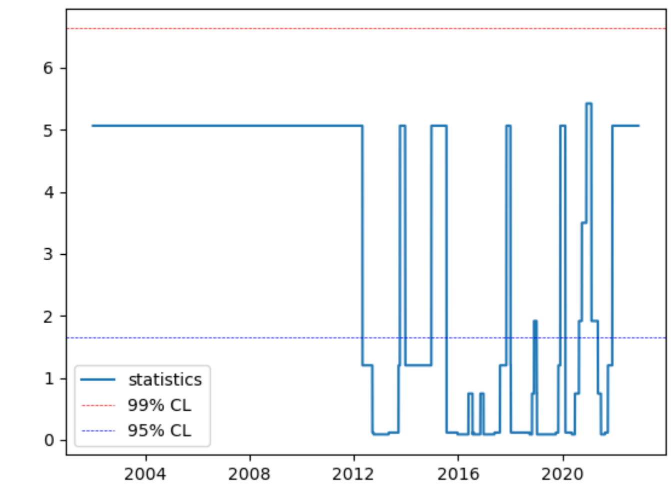


Table. Kupic's TUFF Test Result

95% CL	0.045
99% CL	0.002

Short-only Portfolio:
Monte Carlo Result: Unweighted Calibration

Fig. Exceptions Plot

Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	69%
% in Amber Zone (5-9 exceptions)	23%
% in Rd Zone (10 or more exceptions)	8%

Fig. Binomial Test Plot

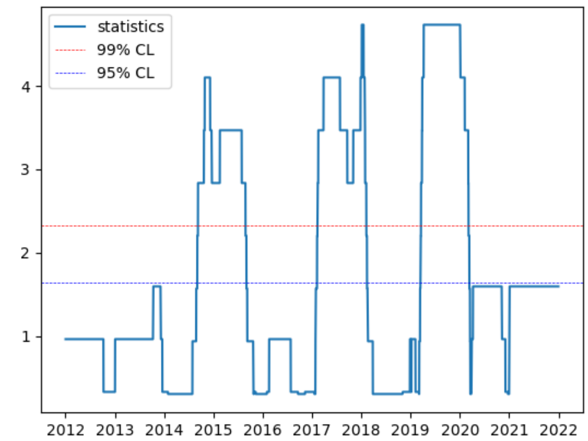


Fig. Kupiec's POF Test Plot

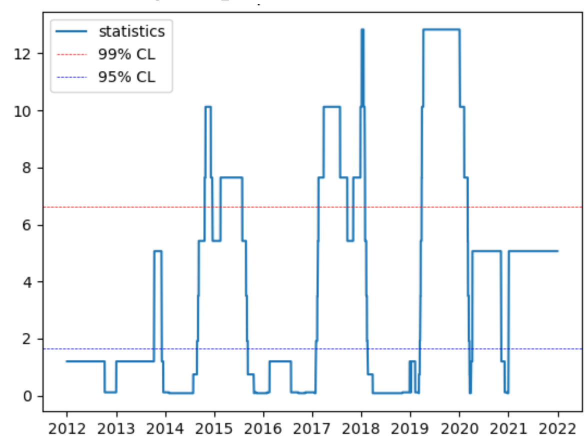


Table. Kupic's TUFF Test Result

95% CL	0.168
99% CL	0.014

Monte Carlo Result: Exponentially Weighted Calibration

Fig. Exceptions Plot

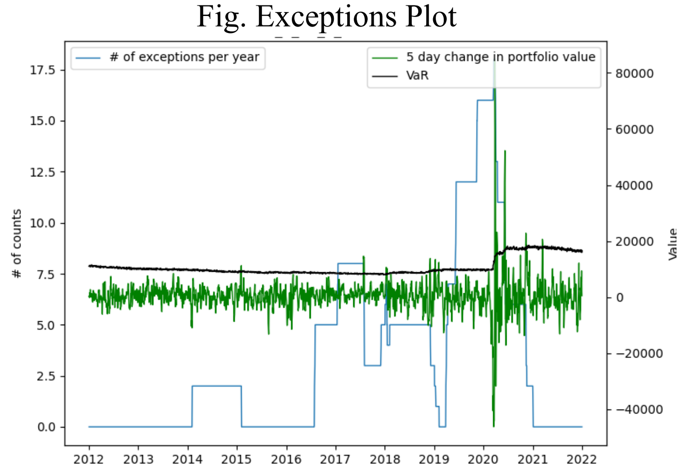


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	69%
% in Amber Zone (5-9 exceptions)	31%
% in Rd Zone (10 or more exceptions)	0%

Fig. Binomial Test Plot

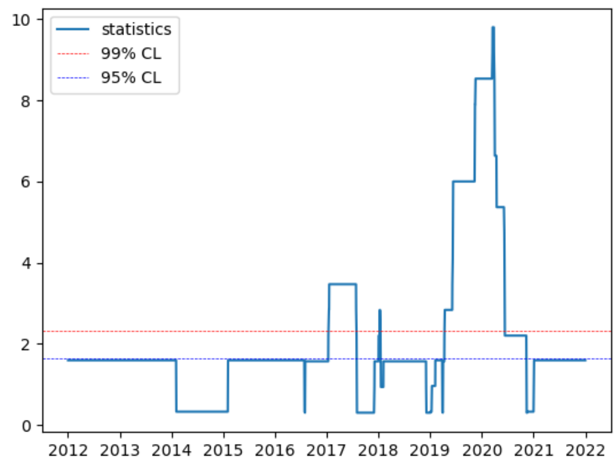


Fig. Kupiec's POF Test Plot

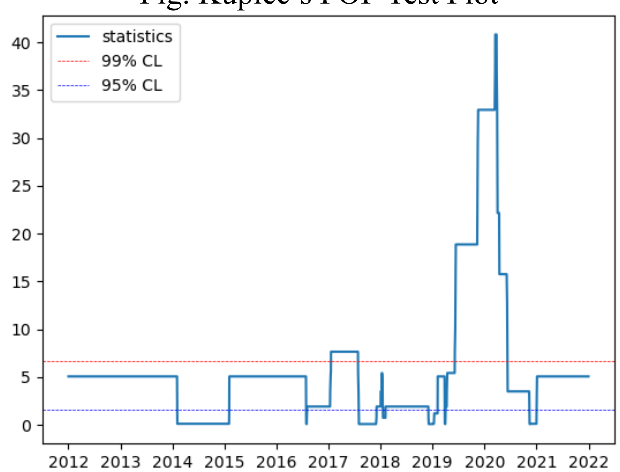


Table. Kupic's TUFF Test Result

95% CL	0.139
99% CL	0.012

GBM Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

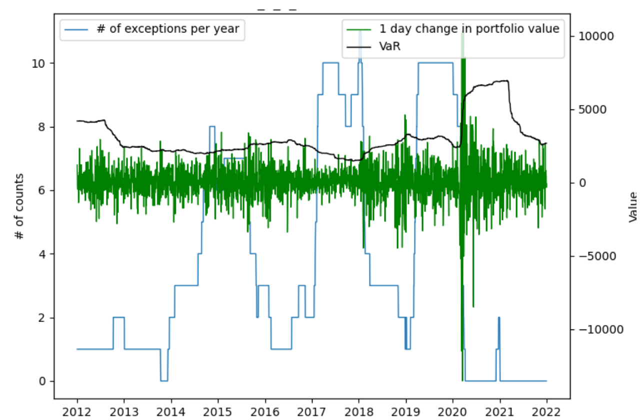


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	69%
% in Amber Zone (5-9 exceptions)	19%
% in Rd Zone (10 or more exceptions)	12%

Fig. Binomial Test Plot

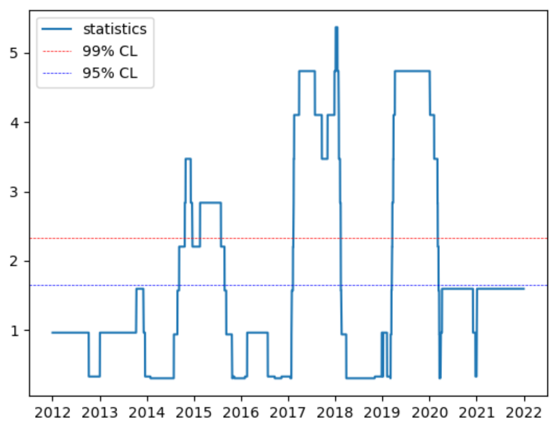


Fig. Kupiec's POF Test Plot

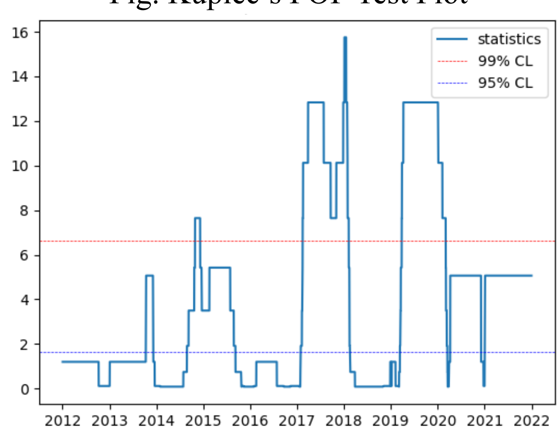


Table. Kupic's TUFF Test Result

95% CL	0.168
99% CL	0.014

GBM Parametric Result: Exponentially Weighted Calibration

Fig. Exceptions Plot

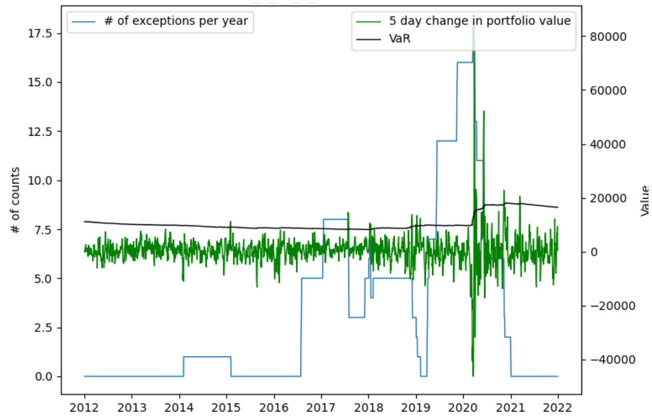


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	69%
% in Amber Zone (5-9 exceptions)	31%
% in Rd Zone (10 or more exceptions)	0%

Fig. Binomial Test Plot

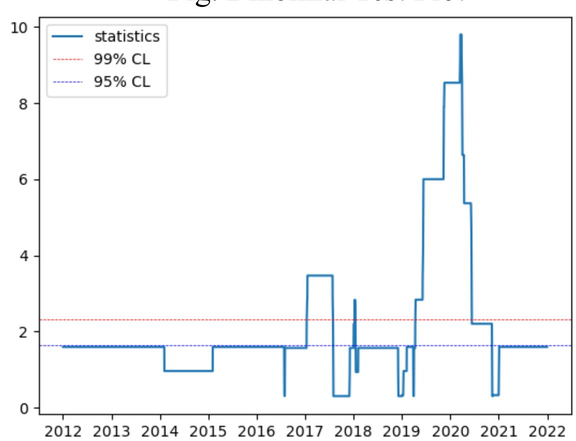


Fig. Kupiec's POF Test Plot

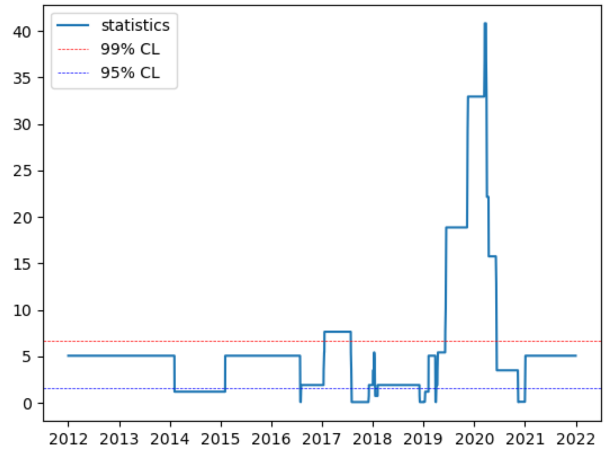


Table. Kupic's TUFF Test Result

95% CL	0.139
99% CL	0.012

Normal Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

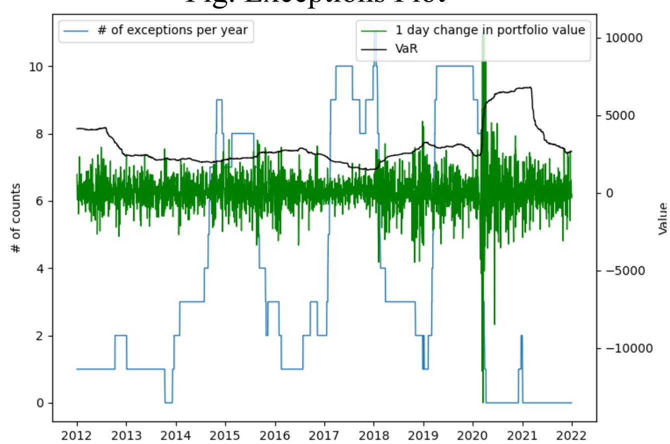


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	69%
% in Amber Zone (5-9 exceptions)	19%
% in Rd Zone (10 or more exceptions)	12%

Fig. Binomial Test Plot

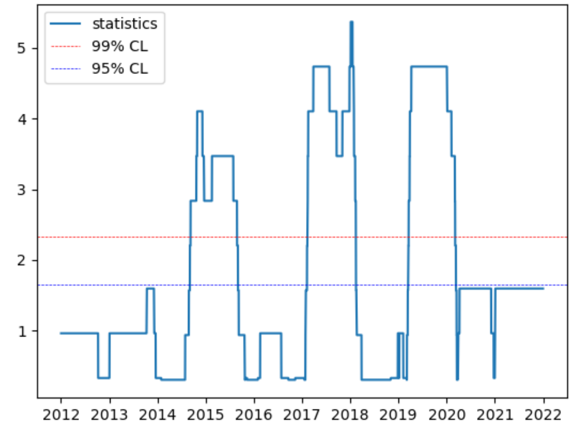


Fig. Kupiec's POF Test Plot

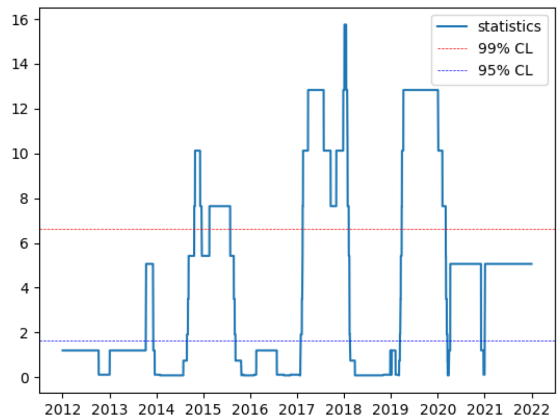


Table. Kupic's TUFF Test Result

95% CL	0.168
99% CL	0.014

Normal Parametric Result: Exponentially Weighted Calibration

Fig. Exceptions Plot

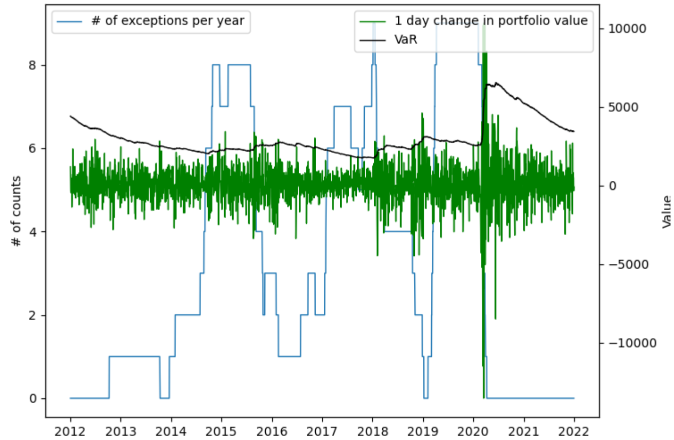


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	68%
% in Amber Zone (5-9 exceptions)	32%
% in Rd Zone (10 or more exceptions)	0%

Fig. Binomial Test Plot

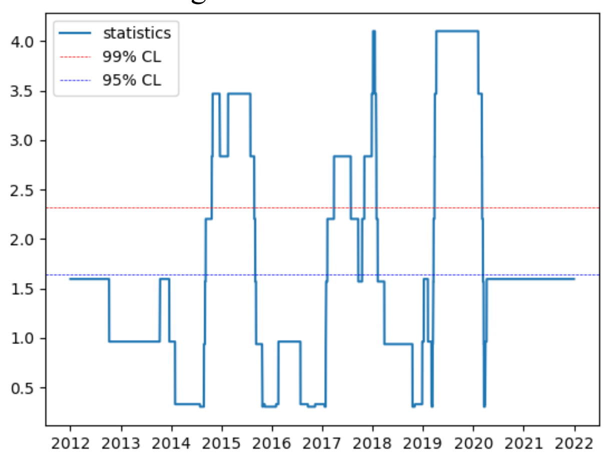


Fig. Kupiec's POF Test Plot

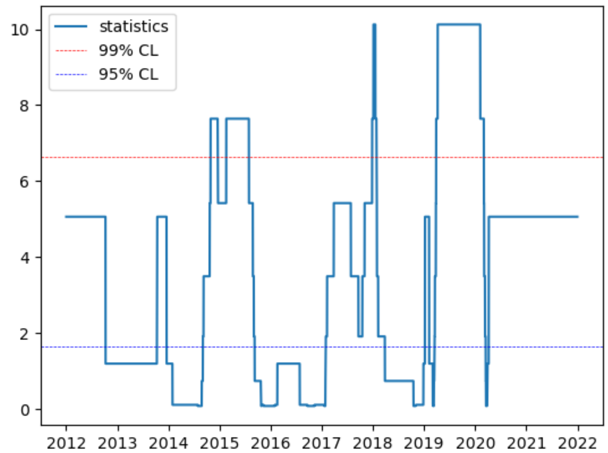


Table. Kupic's TUFF Test Result

95% CL	0.151
99% CL	0.012

Historical Result

Fig. Exceptions Plot

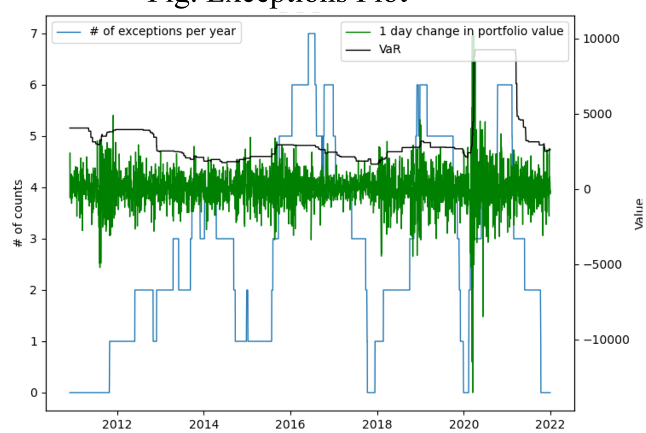


Table. Traffic Light Test Result

Backtest Window (yrs)	9.99
% in Green Zone (0-4 exceptions)	93%
% in Amber Zone (5-9 exceptions)	7%
% in Rd Zone (10 or more exceptions)	0%

Fig. Binomial Test Plot

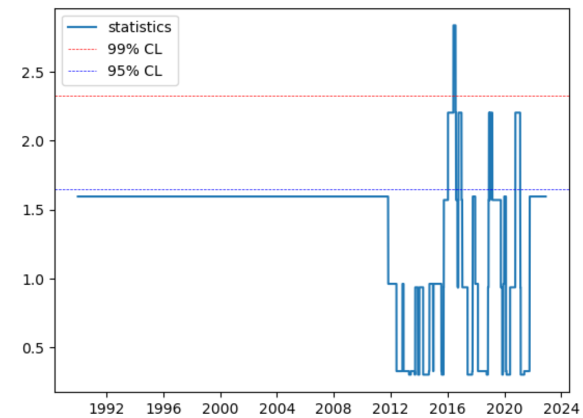


Fig. Kupiec's POF Test Plot

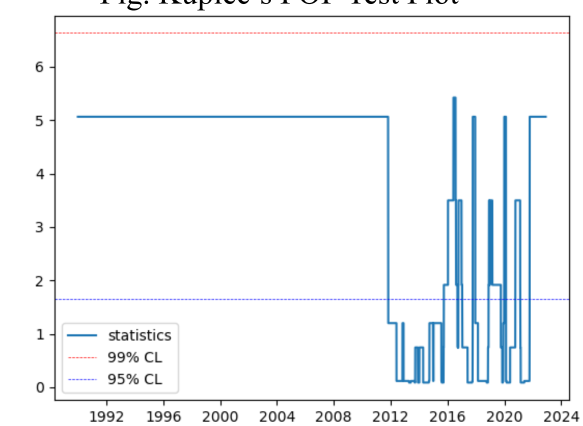


Table. Kupic's TUFF Test Result

95% CL	0.055
99% CL	0.004

This set of backtests provides reasonably satisfactory results, while the short-only portfolio provides more accurate result compared to the long-only portfolio. This is consistent with what we observed in homework 11. We see that for the parametric VaR and the Monte Carlo VaR results, the number of exceptions are below 2.5 most of times with the maximum value around 20. For the traffic light test, we see that the majority of exceptions is in the green zone. For the three types of accuracy statistics, the majority of the statistics support the null hypothesis with 95% and 99% confidence levels. The exponential weighted calibration provides a slightly more accurate result compared to the unweighted calibration for both the Monte Carlo method and the Parametric Method. The Normal Parametric Method seems not to differ very much from the GBM Parametric method.

6.2 Backtest with Different Market Conditions

For the following sets of backtests, only the exception plot, the traffic light test, and the Kupiec's POF Test will be provided

For the backtest with different market conditions, we consider a long-only equal-weighted portfolios of 5 stocks over the different backtesting periods with a total position of 100000. The stocks are:

1. Boeing (BA)
2. Northrop Grumman Corp (NOC)
3. Intel Corporation (INTC)
4. IBM Common Stock (IBM)
5. Quaker Chemical Group (KWR)

We calculated the 5-day 99th percentile VaR with 5-year window for this batch of backtests.

For this test, we consider the following 4 methods of VaR calculation:

1. Historical Method
2. Monte Carlo Method with Unweighted Calibration
3. Parametric Method with GBM Assumption and Unweighted Calibration
4. Parametric Method with Normal Assumption and Unweighted Calibration

Early 2000s Recession (2001.01 – 2002.12)
Monte Carlo Result: Unweighted Calibration

Fig. Exception Plot

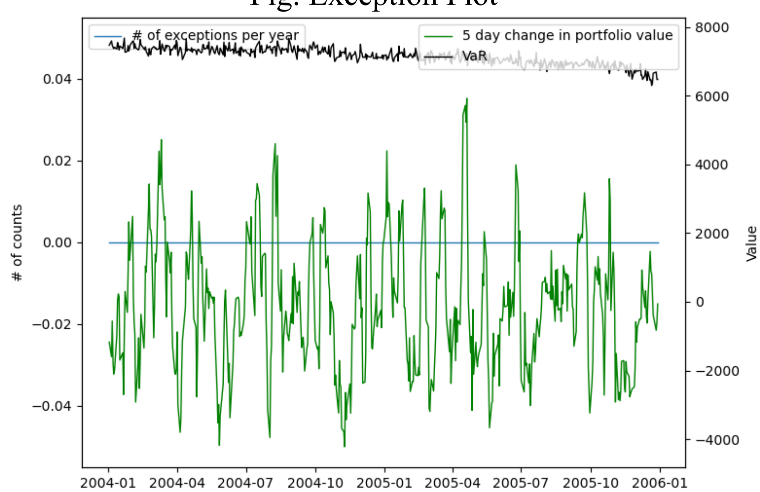
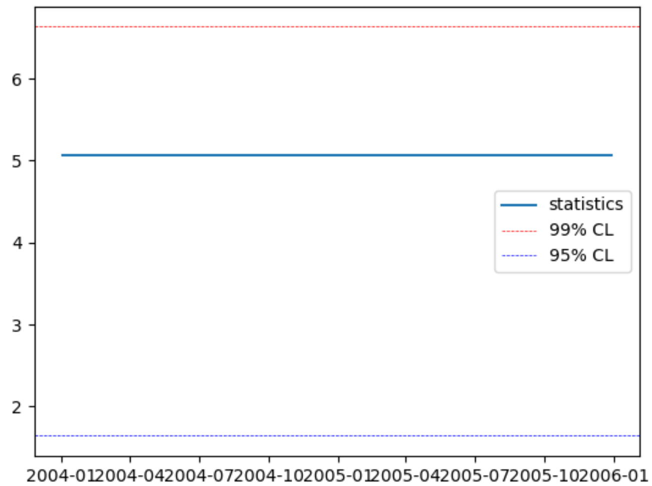


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	60%
% in Amber Zone (5-9 exceptions)	21%
% in Rd Zone (10 or more exceptions)	20%

Fig. Kupiec's POF Test Plot



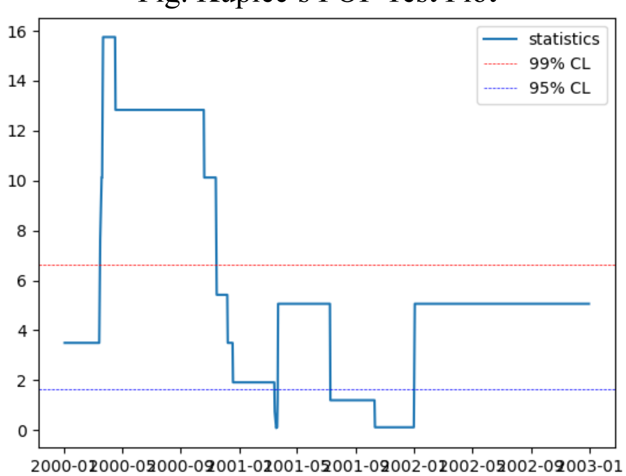
GBM Parametric Result: Unweighted Calibration

Fig. Exception Plot

Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	60%
% in Amber Zone (5-9 exceptions)	21%
% in Rd Zone (10 or more exceptions)	20%

Fig. Kupiec's POF Test Plot



Normal Parametric Result: Unweighted Calibration

Fig. Exception Plot

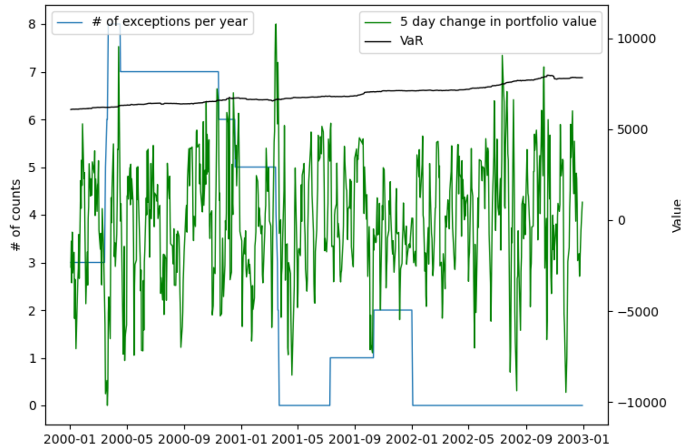
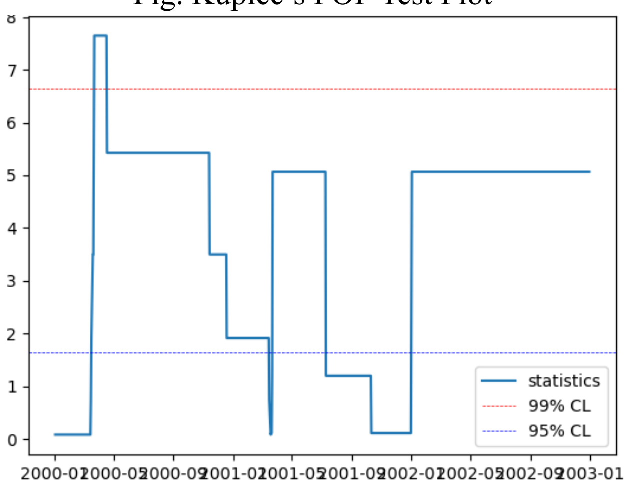


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	67%
% in Amber Zone (5-9 exceptions)	33%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



Historical Result

Fig. Exceptions Plot

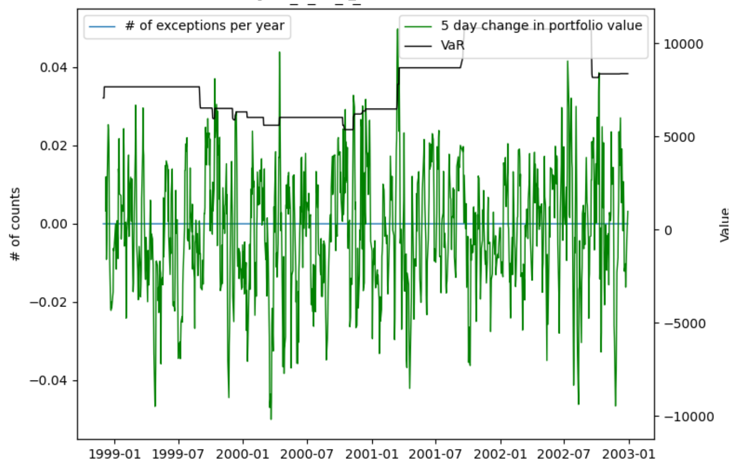
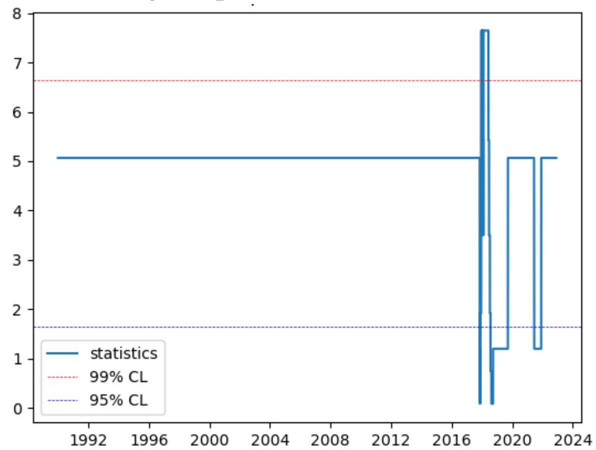


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	98%
% in Amber Zone (5-9 exceptions)	2%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



Economic Expansion post 2000s Recession (2004.01-2005.12)
Monte Carlo Result: Unweighted Calibration

Fig. Exception Plot

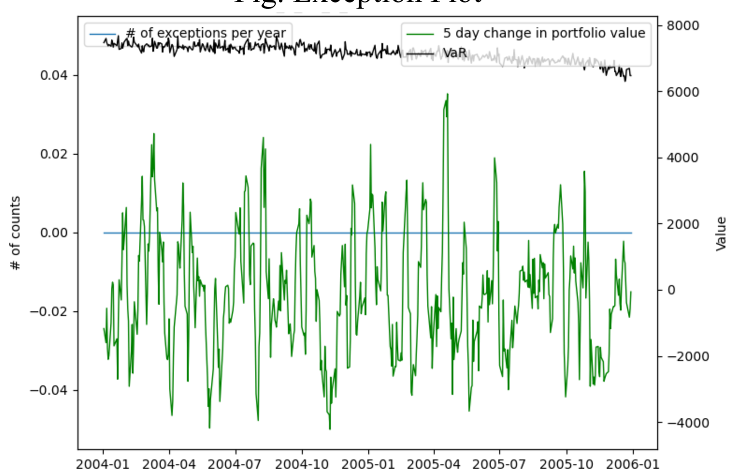
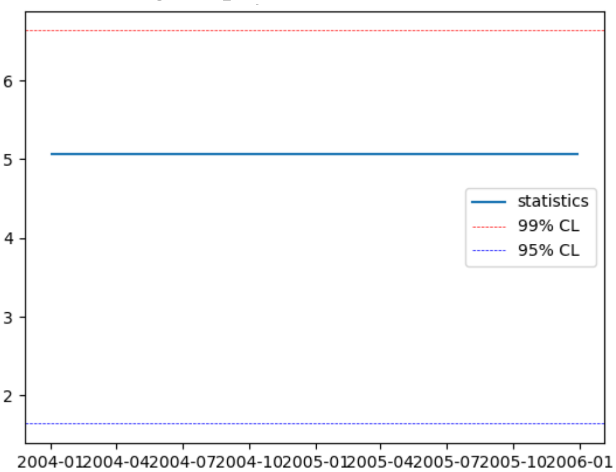


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



GBM Parametric Result: Unweighted Calibration

Fig. Exception Plot

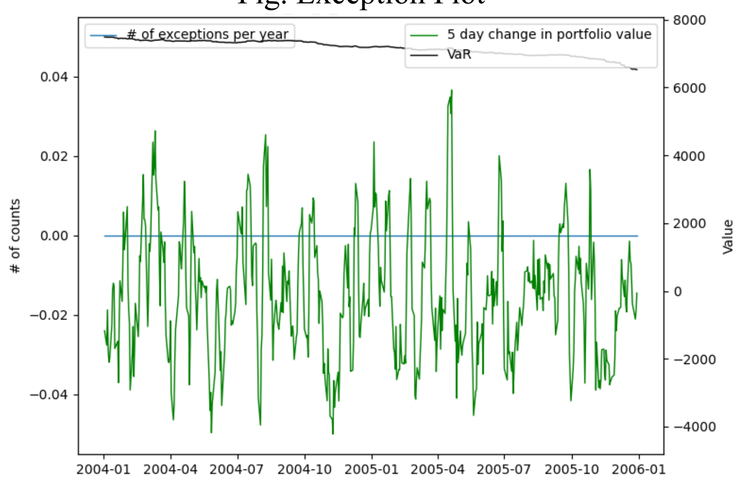
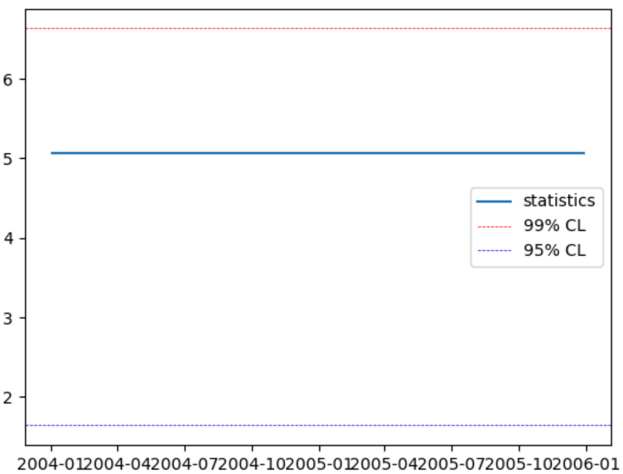


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



Normal Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

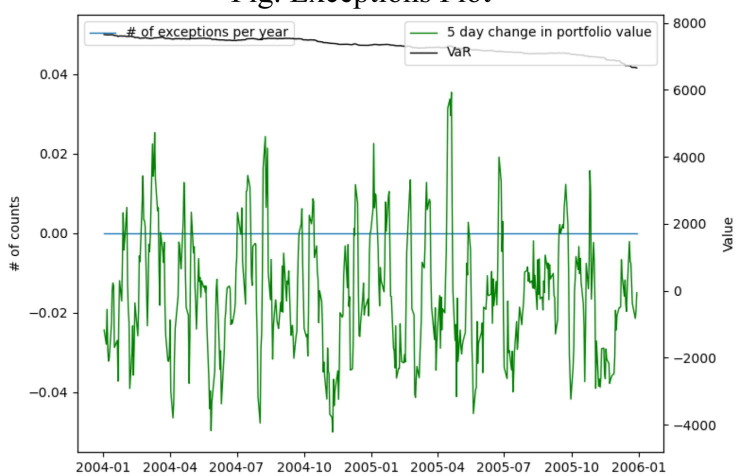
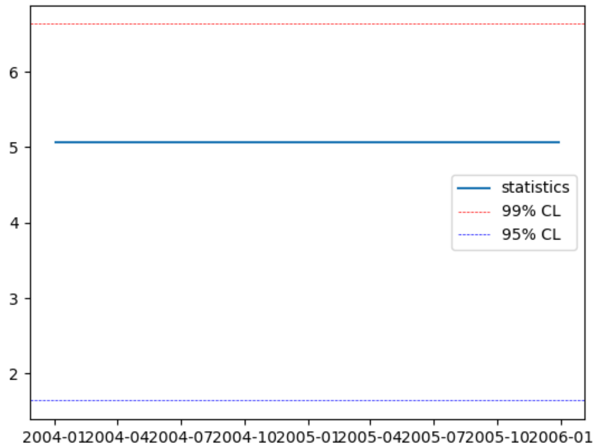


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



Historical Result

Fig. Exception Plot

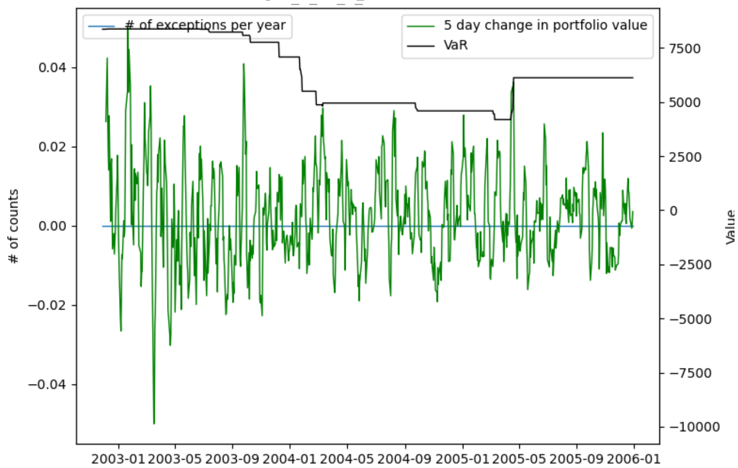
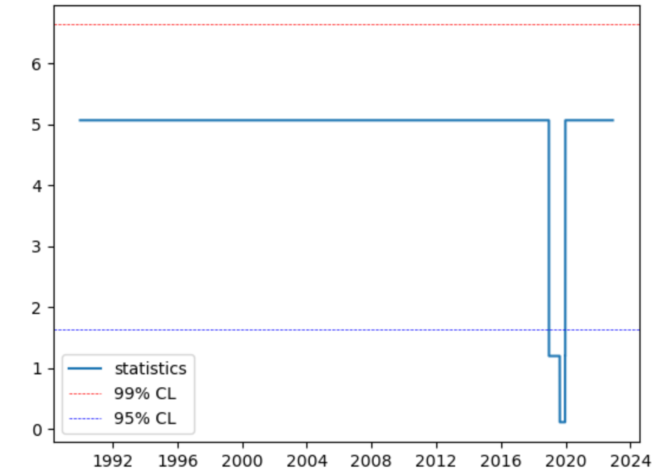


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



2008 Financial Crisis (2008.01 – 2009.12)
Monte Carlo Result: Unweighted Calibration

Fig. Exceptions Plot

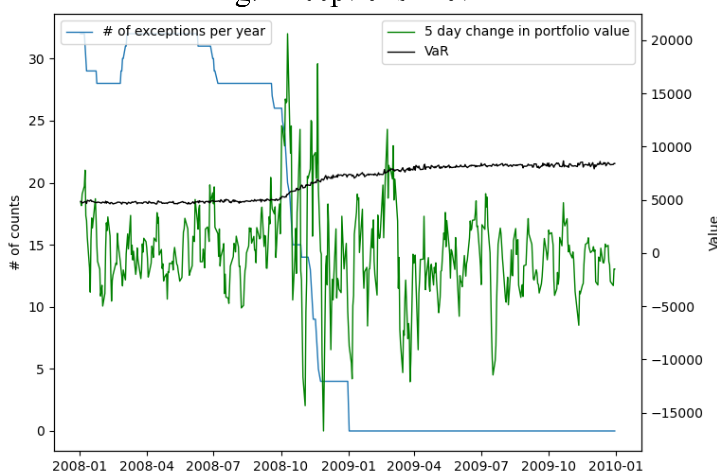
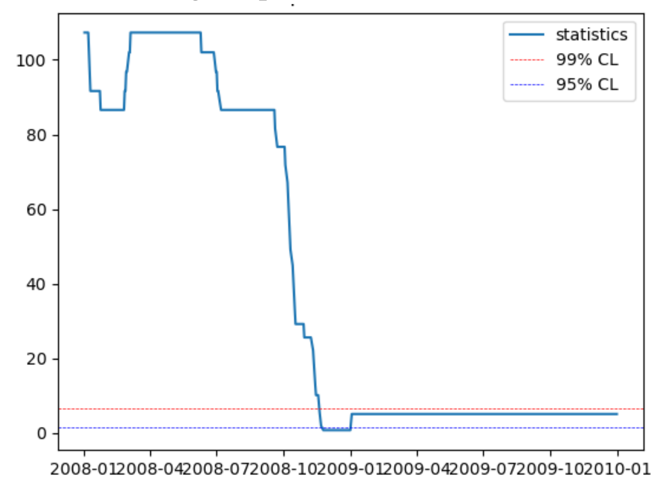


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	55%
% in Amber Zone (5-9 exceptions)	1%
% in Rd Zone (10 or more exceptions)	44%

Fig. Kupiec's POF Test Plot



GBM Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

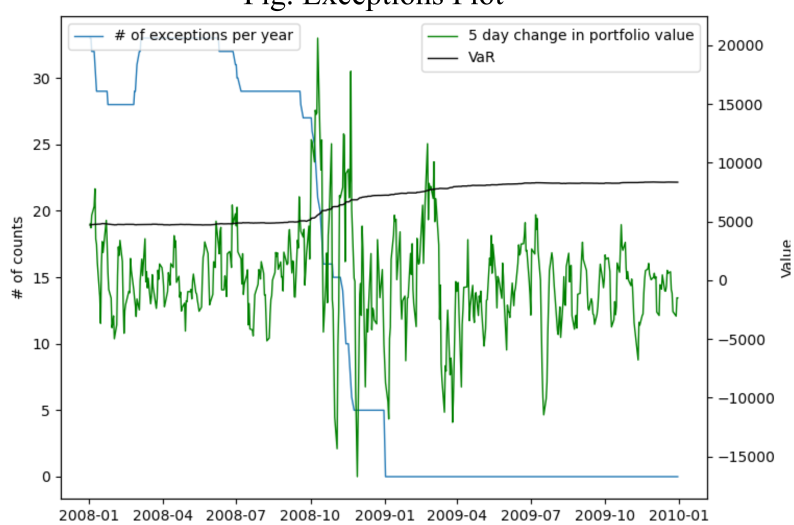
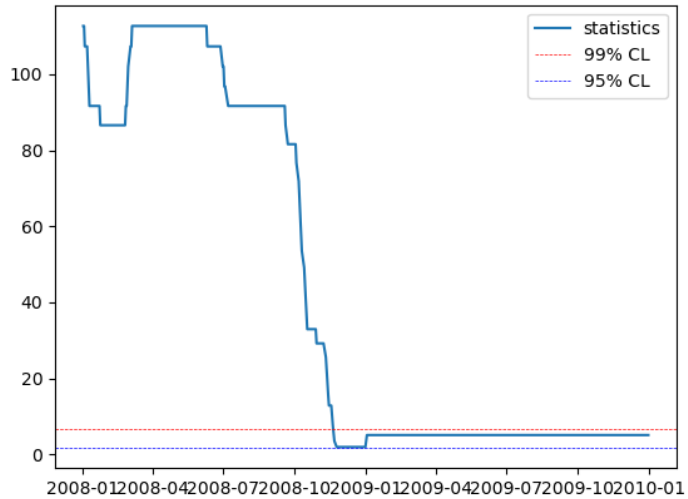


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	50%
% in Amber Zone (5-9 exceptions)	6%
% in Rd Zone (10 or more exceptions)	44%

Fig. Kupiec's POF Test Plot



Normal Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

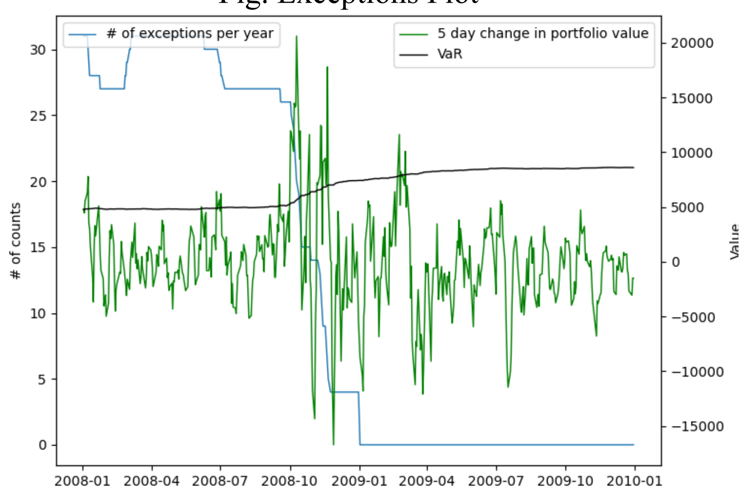
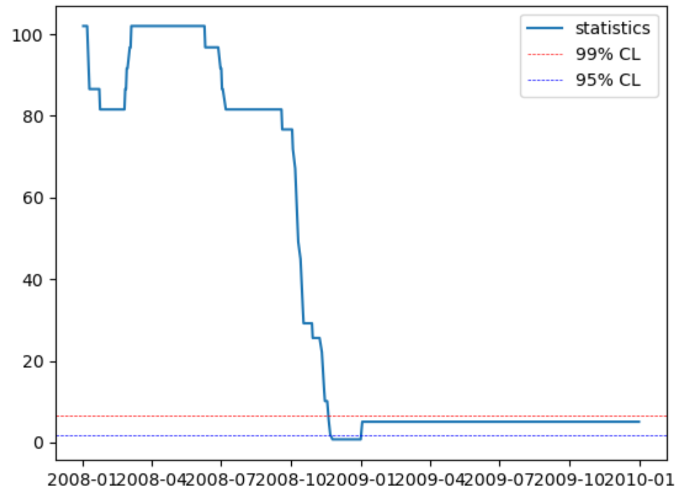


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	55%
% in Amber Zone (5-9 exceptions)	1%
% in Rd Zone (10 or more exceptions)	44%

Fig. Kupiec's POF Test Plot



Historical Result

Fig. Exceptions Plot

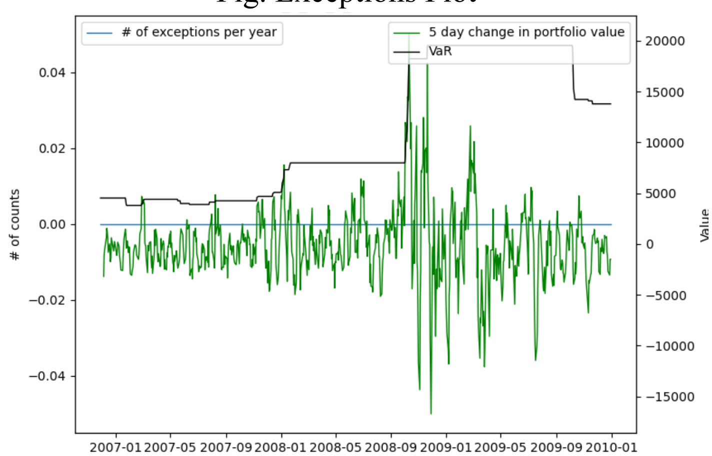
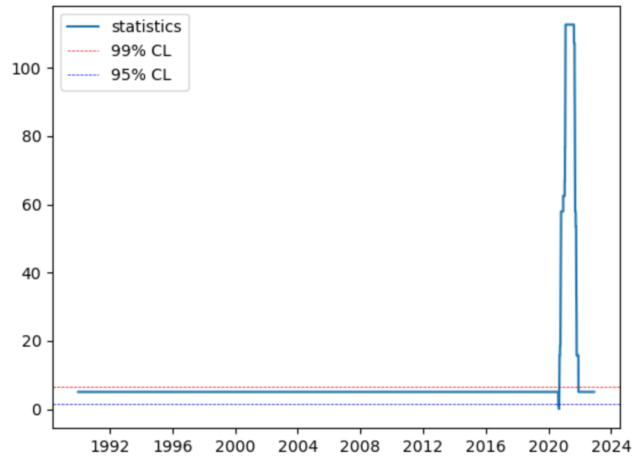


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	96%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	4%

Fig. Kupiec's POF Test Plot



Economic Expansion post 2008 (2014.01-2015.12)
Monte Carlo Result: Unweighted Calibration

Fig. Exceptions. Plot

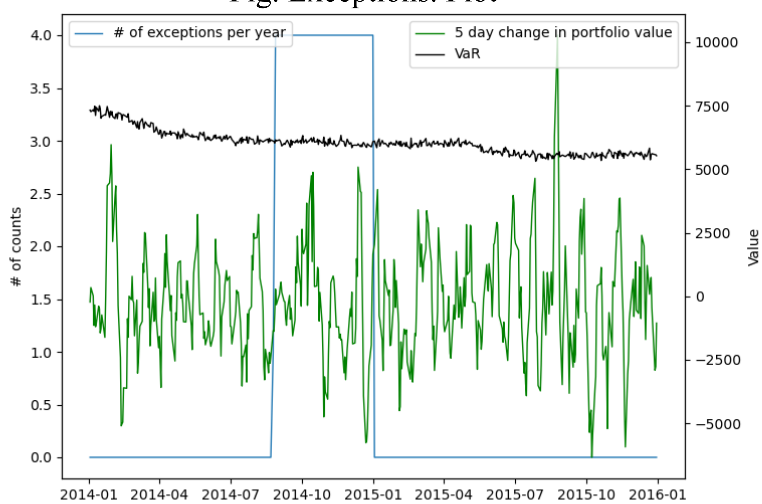
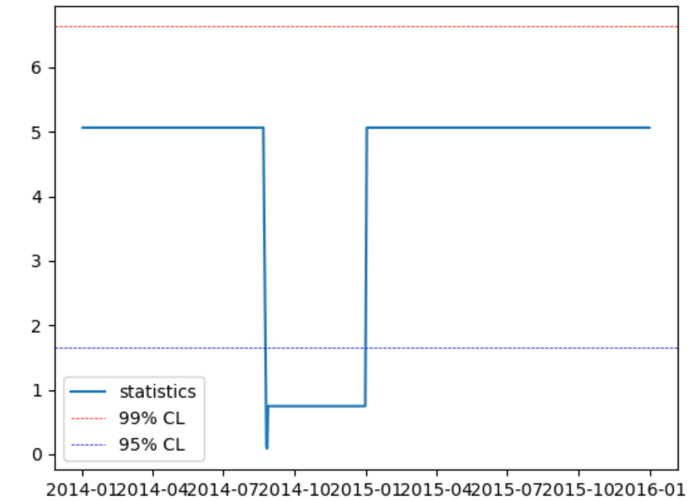


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



GBM Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

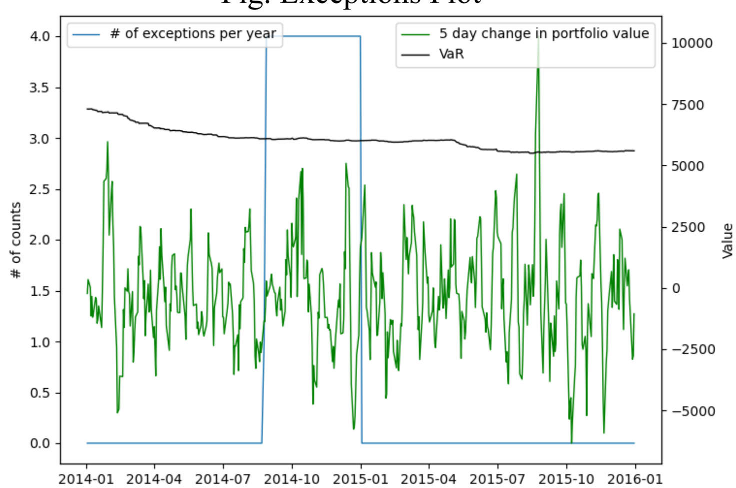
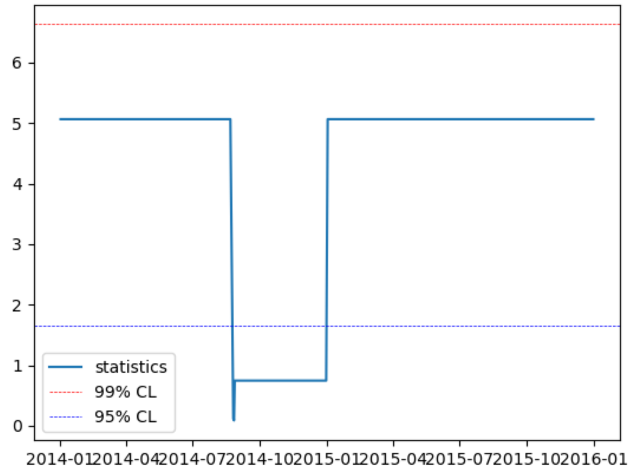


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



Normal Parametric Result: Unweighted Calibration

Fig. Exceptions Plot

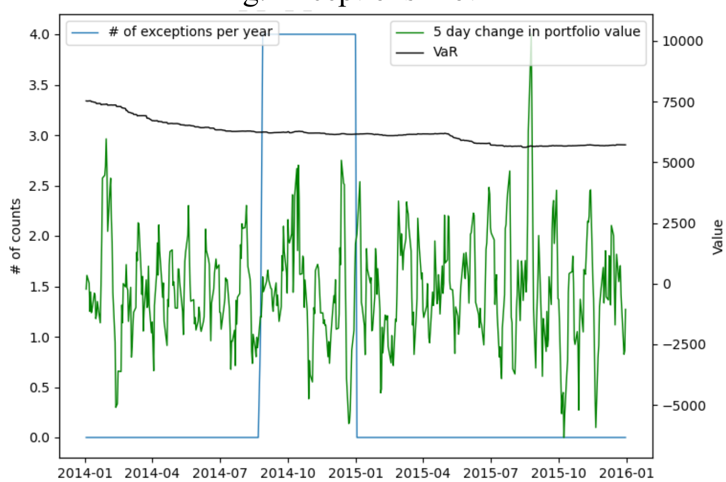
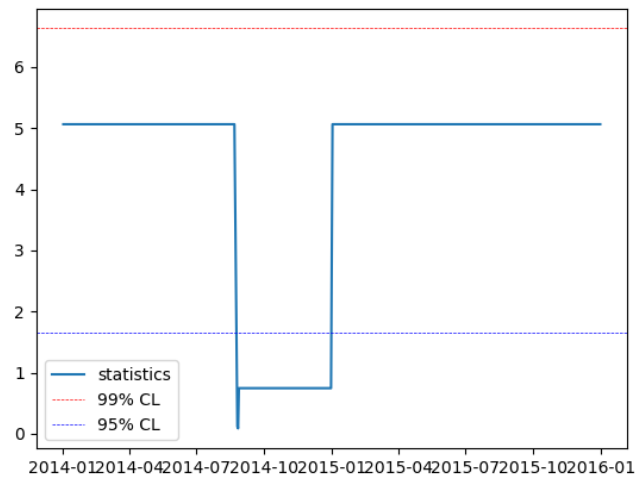


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



Historical Result

Fig. Exceptions Plot

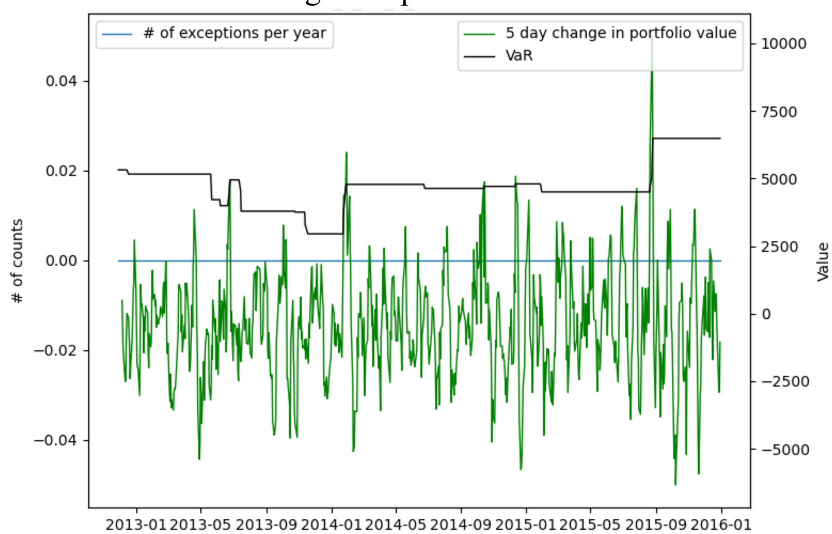
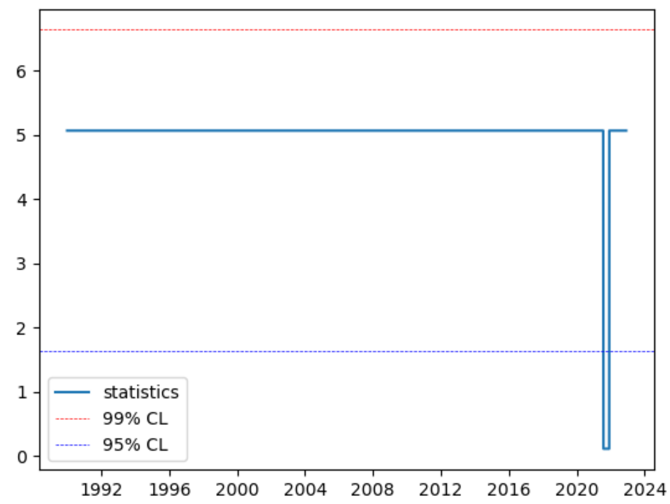


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test Plot



COVID-19 Recession (2020.01 – 2021.12)
Monte Carlo Method: Unweighted Calibration

Fig. Exceptions Plot

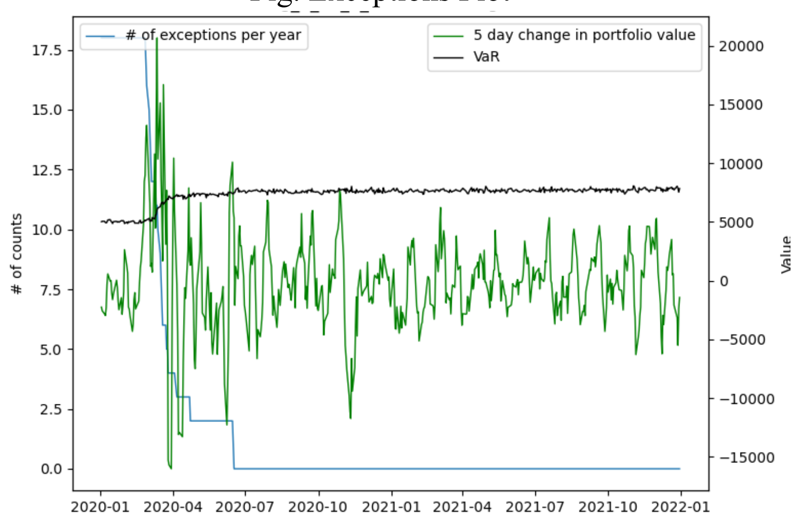
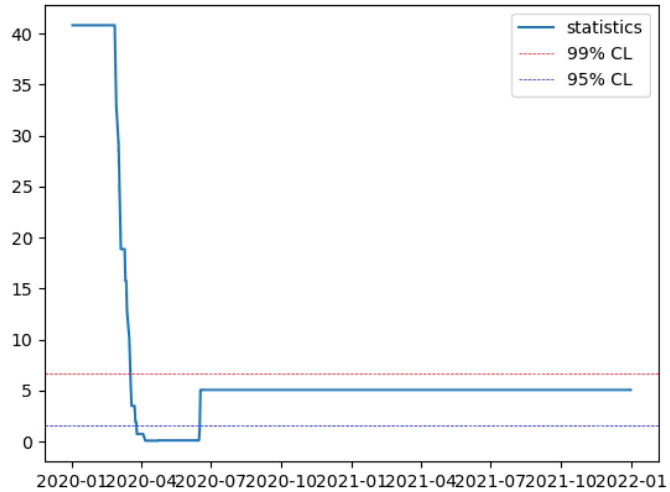


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	89%
% in Amber Zone (5-9 exceptions)	2%
% in Rd Zone (10 or more exceptions)	10%

Fig. Kupiec's POF Test Plot



GBM Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

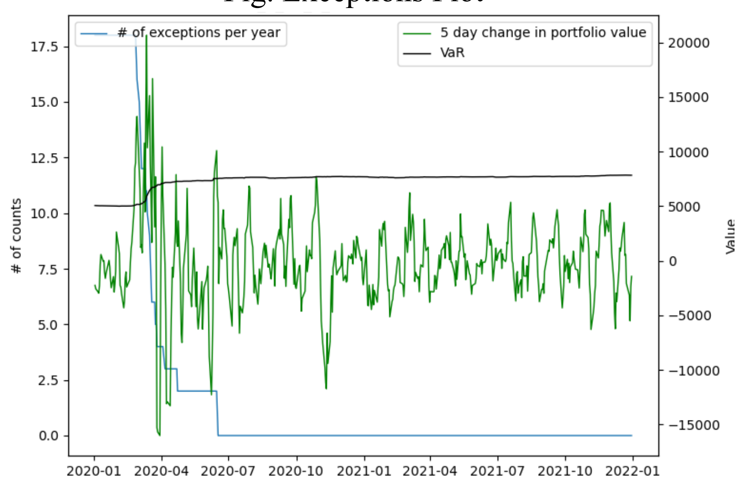
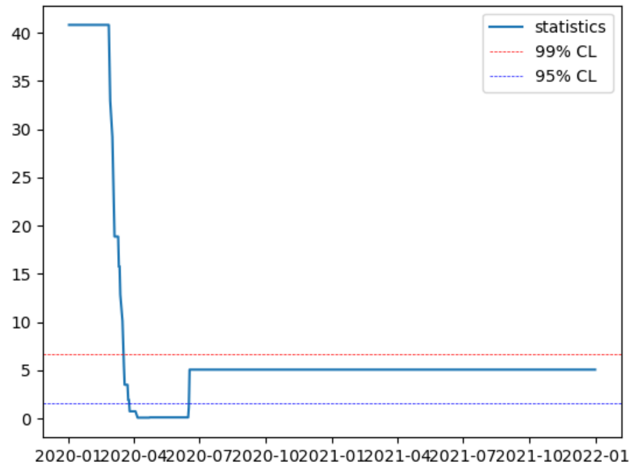


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	89%
% in Amber Zone (5-9 exceptions)	2%
% in Rd Zone (10 or more exceptions)	10%

Fig. Kupiec's POF Test Plot



Normal Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

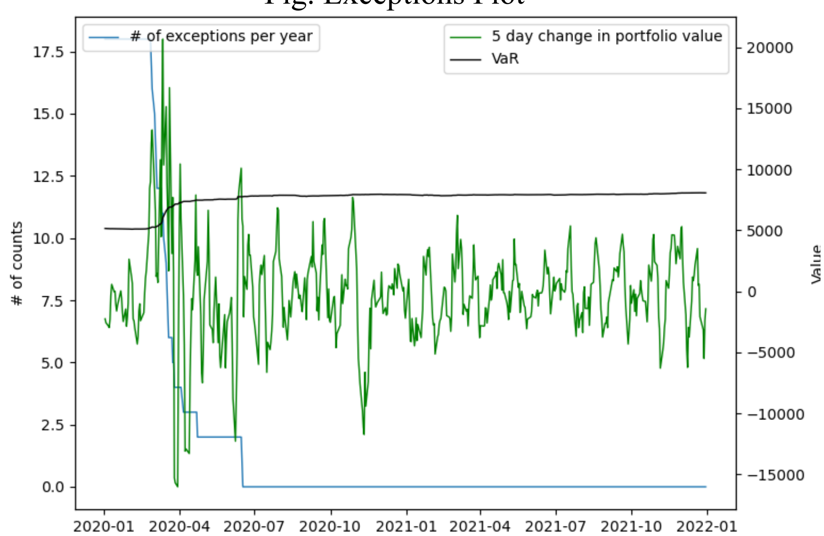
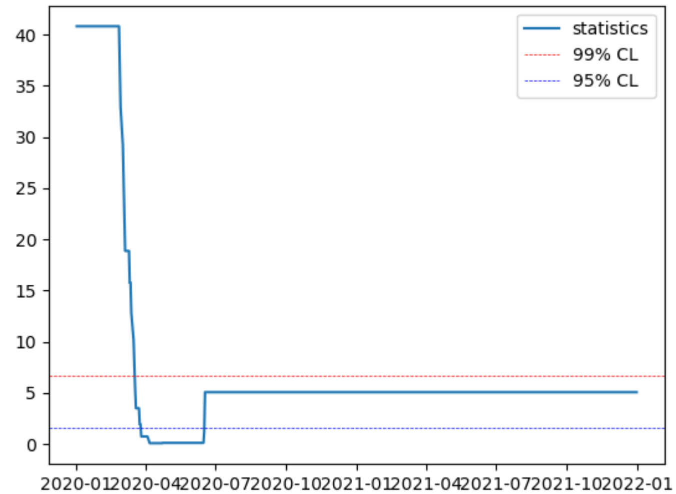


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	89%
% in Amber Zone (5-9 exceptions)	2%
% in Rd Zone (10 or more exceptions)	10%

Fig. Kupiec's POF Test Plot



Historical Method

Fig. Exceptions Plot

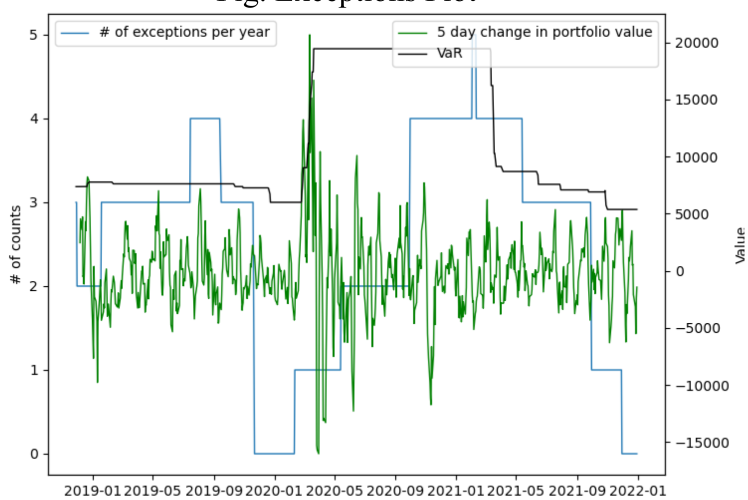
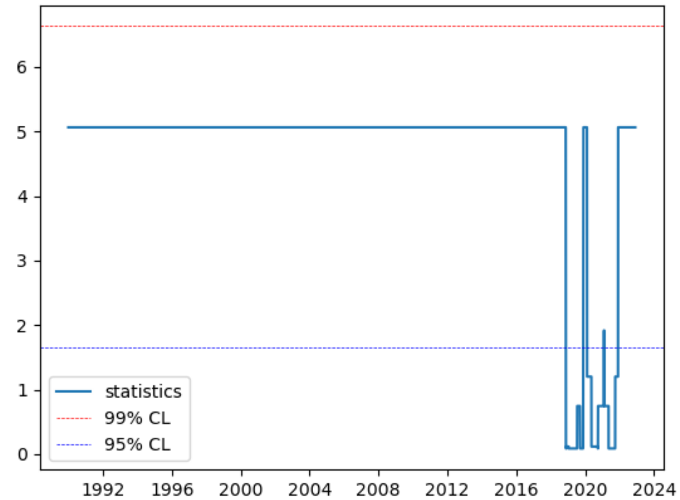


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	100%
% in Amber Zone (5-9 exceptions)	0%
% in Rd Zone (10 or more exceptions)	0%

Fig. Kupiec's POF Test



This set of backtests during two periods of economic expansions provides excellent result with zero exceptions, 100% green zone result, and the Kupiec's POF test statistics well below the 99% confidence level critical values. On the other hand, the result during three periods of economic recessions provides satisfactory result as well with the majority of instances in the green zone. Noticably, the VaR model predicts the losses during the COVID-19 recession well compared to the other recessions with less amber zone and red zone results. The VaR model during the 2008 crisis and the COVID-19 recession have more extreme exceptions compared to the 2000s recession. This indicates the VaR model fails to capture the fat tailed nature of the price distribution and the extreme events. Same as before, we notice that the normal parametric method produces slightly better result compared to the GBM parametric method. The historical method is the best across all charts with almost 0 instances of exceptions under all economic scenarios.

6.3 Backtests with Portfolios of Different Sizes

For the following sets of backtests, only the exception plot, the traffic light test, and the Kupiec's POF Test will be provided. For the backtest with different market conditions, we consider a long-only equal-weighted portfolios of 1, 5, 10 stocks over the different backtesting periods with a total position of 100000 over the backtesting period of 2012-01-02 to 2021-12-31 (same as the baseline test). In the last batch of test, we considered a portfolio of 5 stocks so we won't be backtesting on portfolios of size 5 here anymore.

- For the portfolio of size 1, we use the stock Boeing (BA).
- For the portfolio of size 10, the stocks are
    1. Boeing (BA)
    2. Northrop Grumman Corp (NOC)
    3. Intel Corporation (INTC)
    4. IBM Common Stock (IBM)
    5. Quaker Chemical Group (KWR)
    6. Walmart Inc. (WMT)

7. Amazon.com, Inc. (AMZN)
8. General Electric Company (GE)
9. Exxon Mobil Corp (XOM)
10. Merck & Co Inc (MRK)

We calculated the 5-day 99th percentile VaR with 5-year window for this batch of backtests. For this test, we consider the following 4 methods of VaR calculation:

1. Historical Method
2. Monte Carlo Method with Unweighted Calibration
3. Parametric Method with GBM Assumption and Unweighted Calibration
4. Parametric Method with Normal Assumption and Unweighted Calibration

For the single stock portfolio, the parametric method with normal assumption will not be applicable and thereby omitted.

Portfolio of Size 1
Monte Carlo Method: Unweighted Calibration

Fig. Exceptions Plot

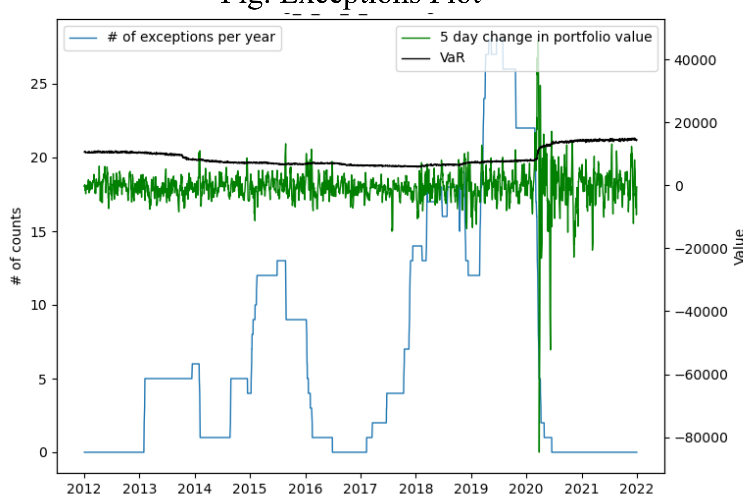
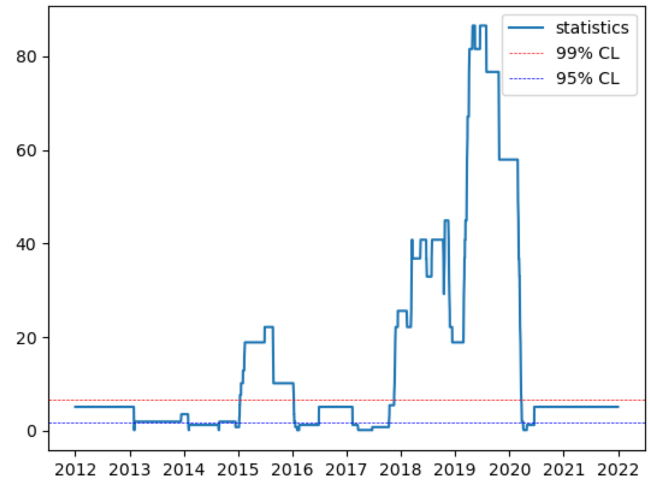


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	52%
% in Amber Zone (5-9 exceptions)	19%
% in Rd Zone (10 or more exceptions)	29%

Fig. Kupiec' POF Test Plot



GBM Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

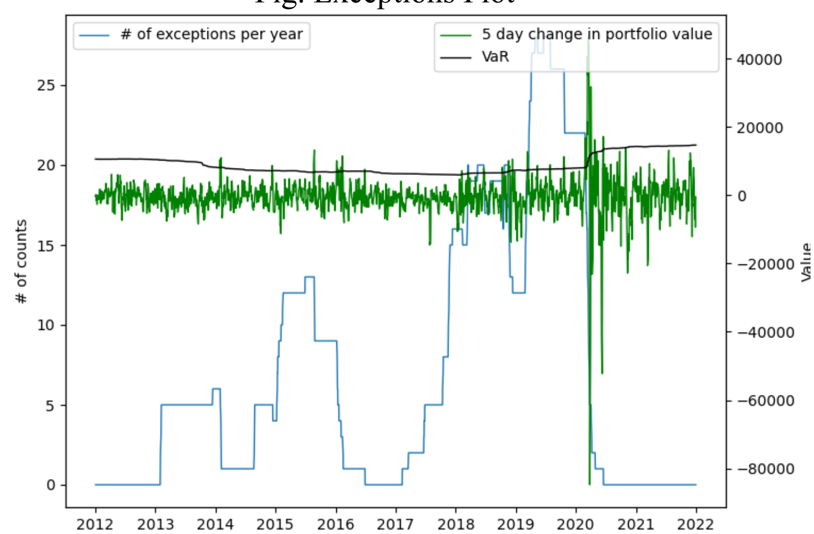
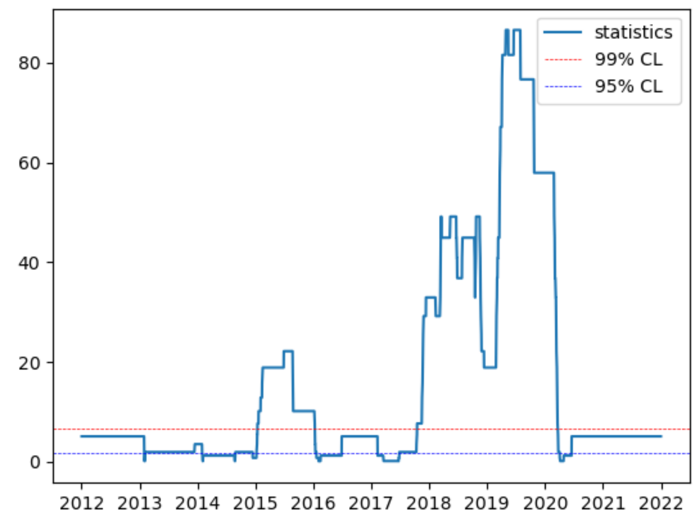


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	49%
% in Amber Zone (5-9 exceptions)	22%
% in Rd Zone (10 or more exceptions)	29%

Fig. Kupiec's POF Test Plot



Historical Method

Fig. Exceptions Plot

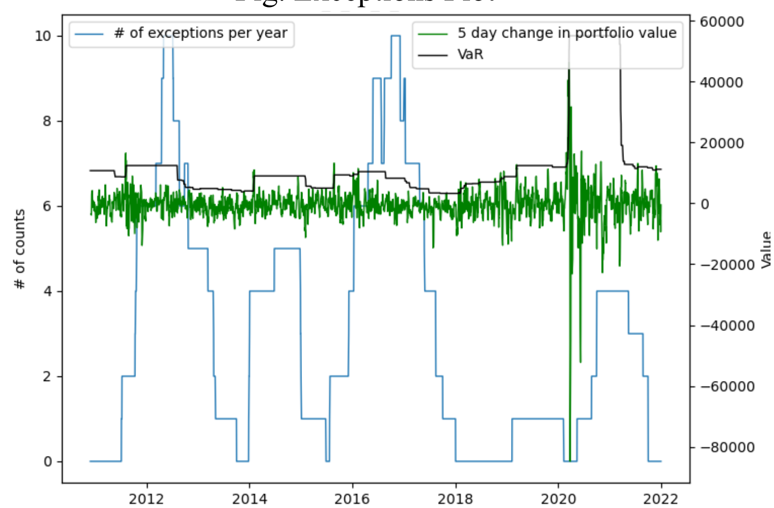
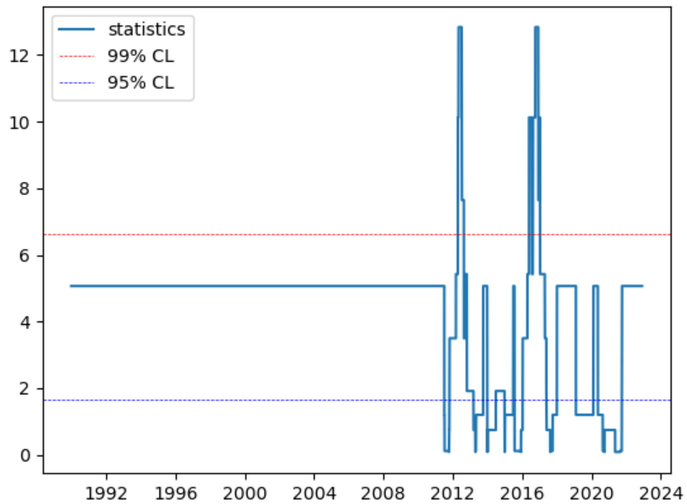


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	90%
% in Amber Zone (5-9 exceptions)	9%
% in Rd Zone (10 or more exceptions)	1%

Fig. Kupiec's POF Test Plot



Portfolio of Size 10
Monte Carlo Method: Unweighted Calibration

Fig. Exceptions Plot

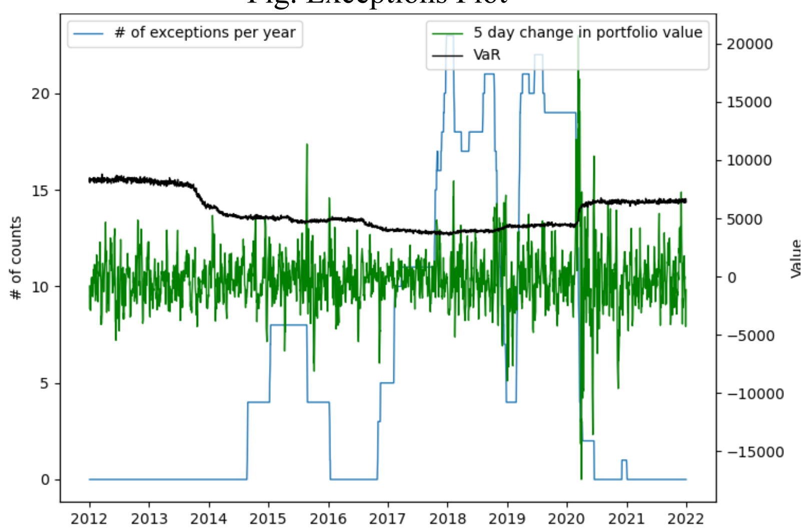
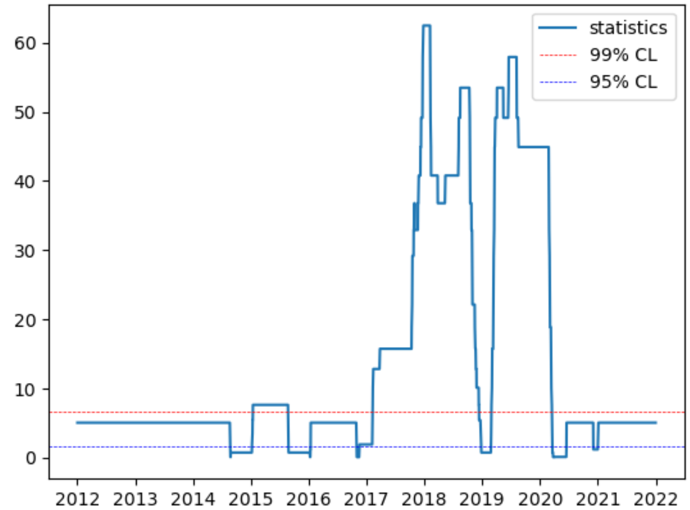


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	62%
% in Amber Zone (5-9 exceptions)	10%
% in Rd Zone (10 or more exceptions)	28%

Fig. Kupiec's POF Test Plot



GBM Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

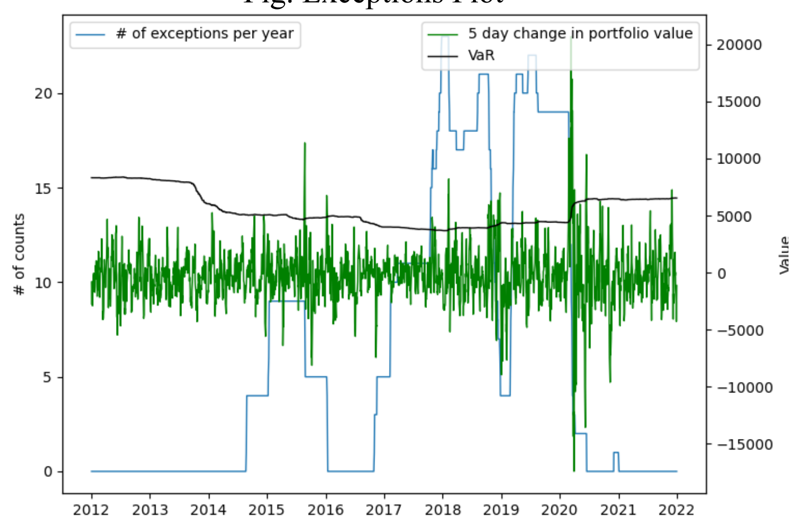
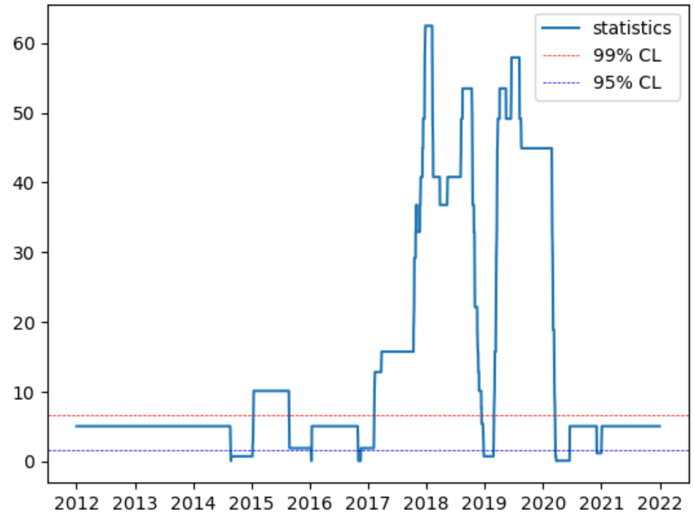


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	58%
% in Amber Zone (5-9 exceptions)	14%
% in Rd Zone (10 or more exceptions)	28%

Fig. Kupiec's POF Test Plot



Normal Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

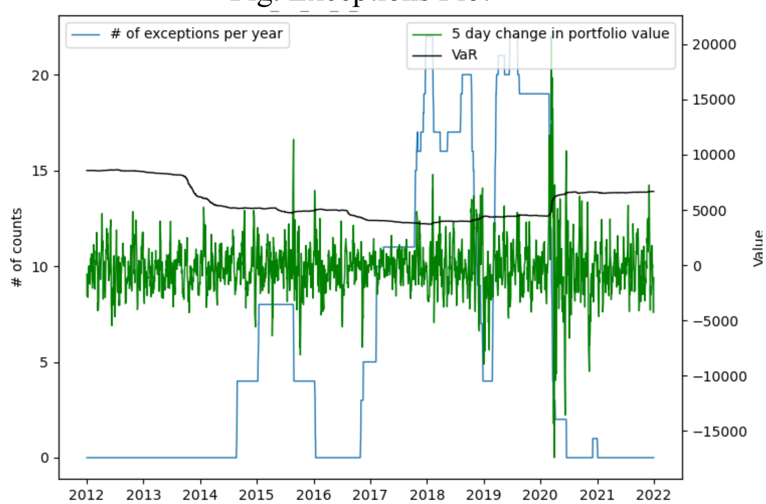
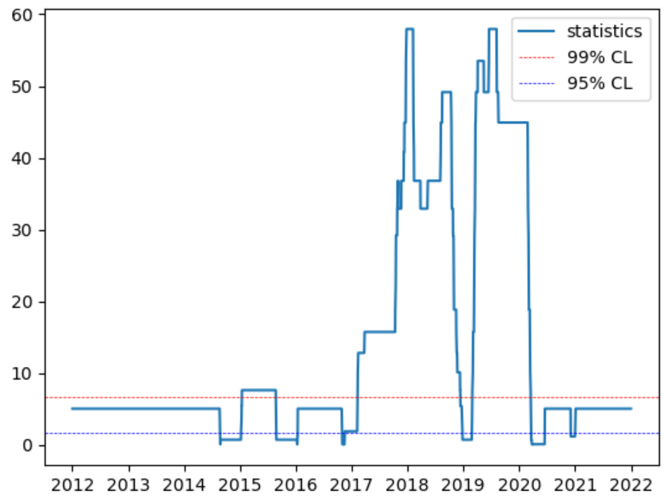


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	62%
% in Amber Zone (5-9 exceptions)	10%
% in Rd Zone (10 or more exceptions)	28%

Fig. Kupiec's POF Test Plot



Historical Method

Fig. Exceptions Plot

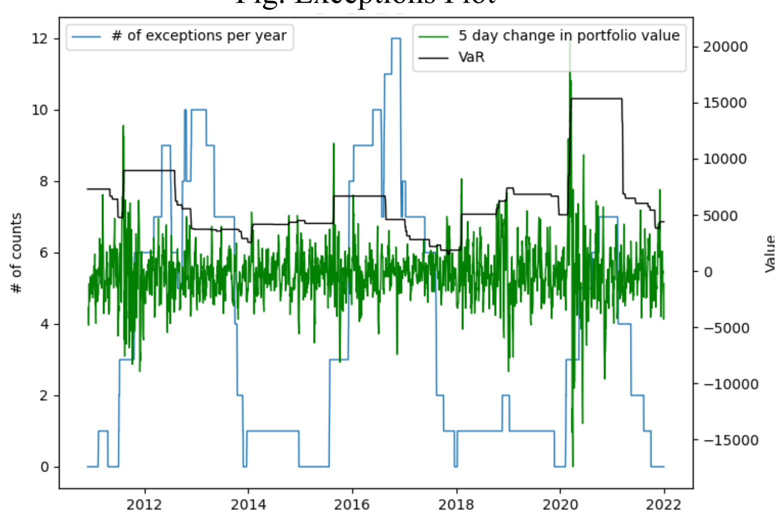
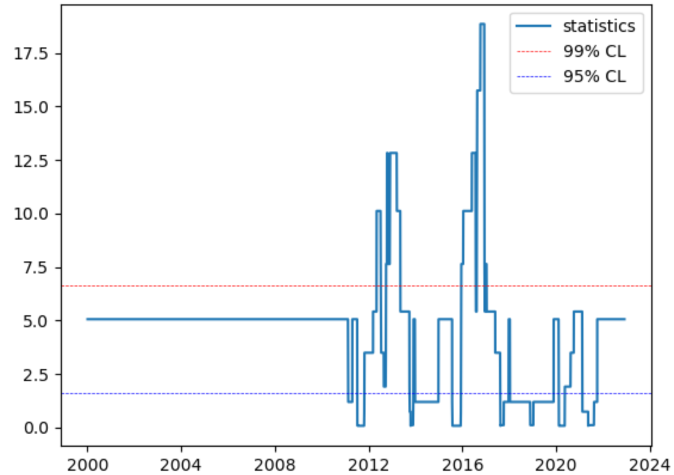


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	81%
% in Amber Zone (5-9 exceptions)	16%
% in Rd Zone (10 or more exceptions)	3%

Fig. Kupiec's POF Test Plot



This set of backtests provide satisfactory result, indicating that our risk calculation system is robust with portfolios of different sizes. Since we backtest over a 10 year periods, the result is not as good as the result during the economic expansions but still, the majority of the instances is in the green zone and below the 99% confidence level for the Kupiec's POF test. We see no difference between the Normal Parametric and the GBM Parametric methods for the portfolio of size 10. This might be due to that the effect of correlations is diluted as the number of stocks increases inside the portforlio.

6.4. Backtests with Different VaR Parameters

For the following sets of backtests, only the exception plot, the traffic light test, and the Kupiec's POF Test will be provided.

For the backtest with VaR parameters, we consider a long-only equal-weighted portfolios of 5 stocks over the different backtesting periods with a total position of 100000.
The stocks are:

1. Boeing (BA)
2. Northrop Grumman Corp (NOC)
3. Intel Corporation (INTC)
4. IBM Common Stock (IBM)
5. Quaker Chemical Group (KWR)

We intend to backtest with this set of parameters. However, we backtested with 1-day VaR frequency in the baseline test and with 5-days VaR frequency in the backtest with different market conditions and portfolio of different sizes, we won't do the frontier analysis on the VaR frequency here. Also, we backtested with 1-year VaR calibration window in the baseline test and with 5-years VaR window in the backtest with different market conditions and portfolio of different sizes, we won't do the frontier analysis on the VaR window here. We have done backtests on 99% VaR so we will only do backtests on 95% VaR here

1. VaR percentile: 95%, 99%

2. VaR frequency: 1-day, 5-days
3. VaR window: 1-year, 5-years

For this test, we consider the following 4 methods of VaR calculation:

1. Historical Method
2. Monte Carlo Method with Unweighted Calibration
3. Parametric Method with GBM Assumption and Unweighted Calibration
4. Parametric Method with Normal Assumption and Unweighted Calibration

95% VaR
Monte Carlo Method: Unweighted Calibration

Fig. Exceptions Plot

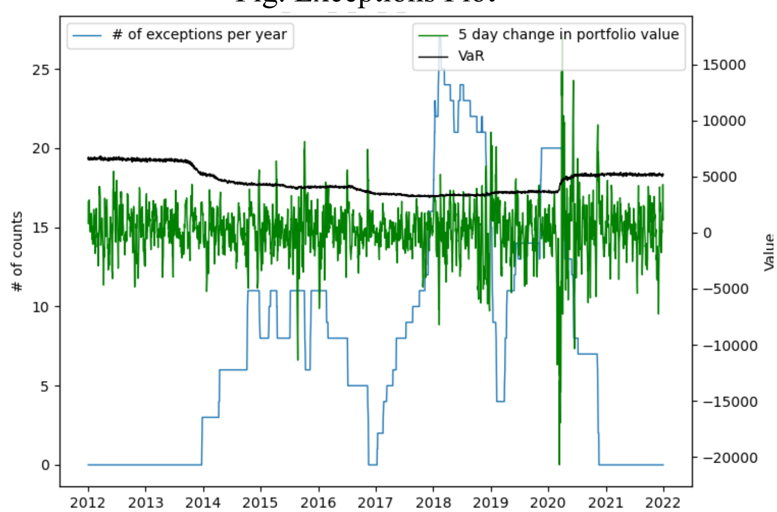
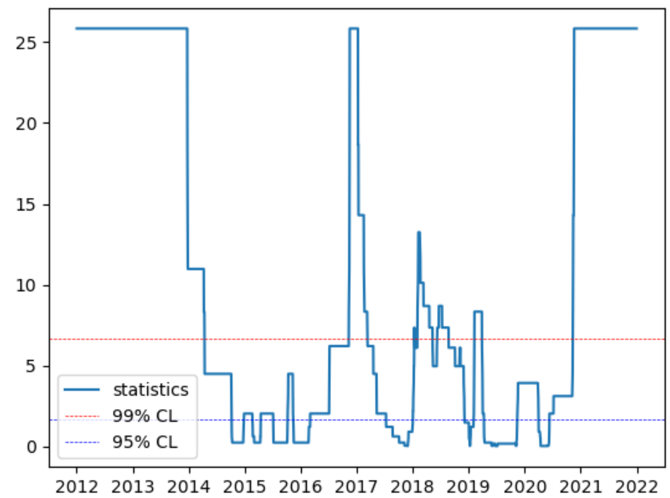


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	39%
% in Amber Zone (5-9 exceptions)	27%
% in Rd Zone (10 or more exceptions)	34%

Fig. Kupiec's POF Test Plot



GBM Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

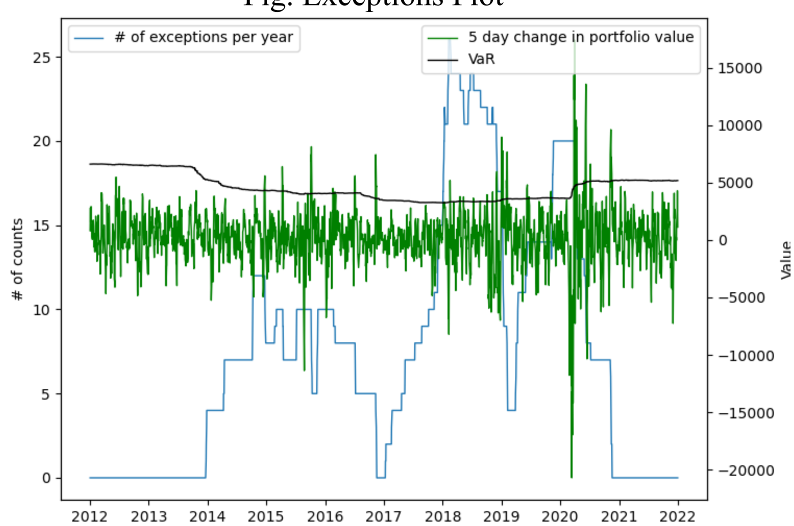
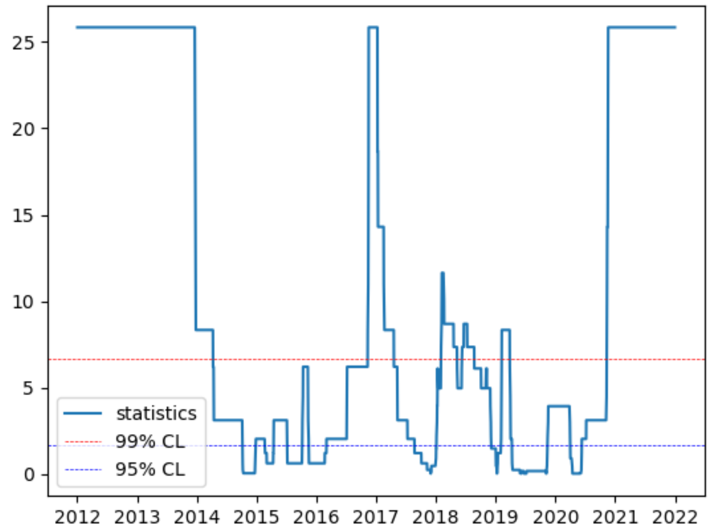


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	40%
% in Amber Zone (5-9 exceptions)	28%
% in Rd Zone (10 or more exceptions)	33%

Fig. Kupiec's POF Test Plot



Normal Parametric Method: Unweighted Calibration

Fig. Exceptions Plot

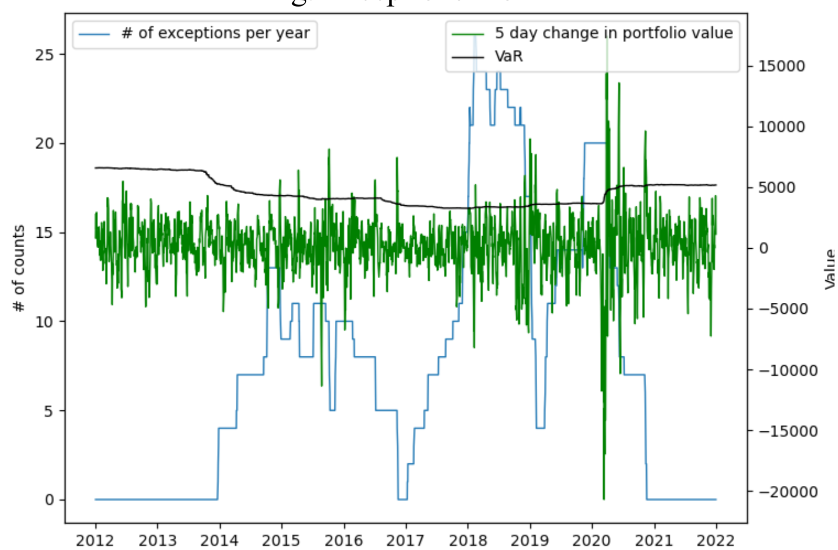
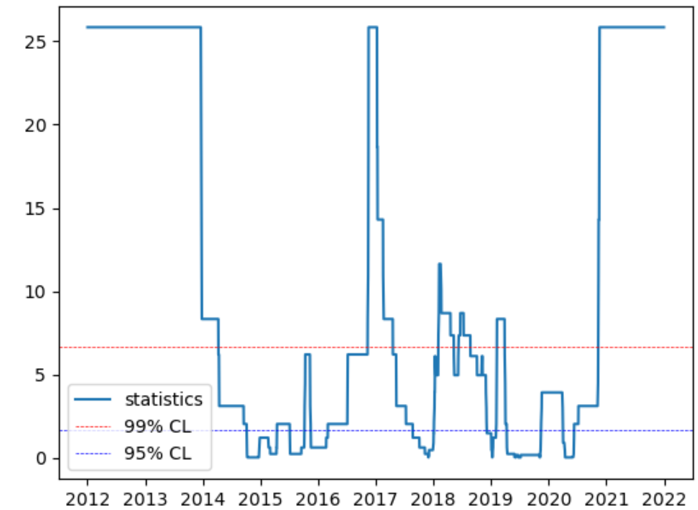


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	40%
% in Amber Zone (5-9 exceptions)	27%
% in Rd Zone (10 or more exceptions)	33%

Fig. Kupiec's POF Test Plot



Historical Method

Fig. Exceptions Plot

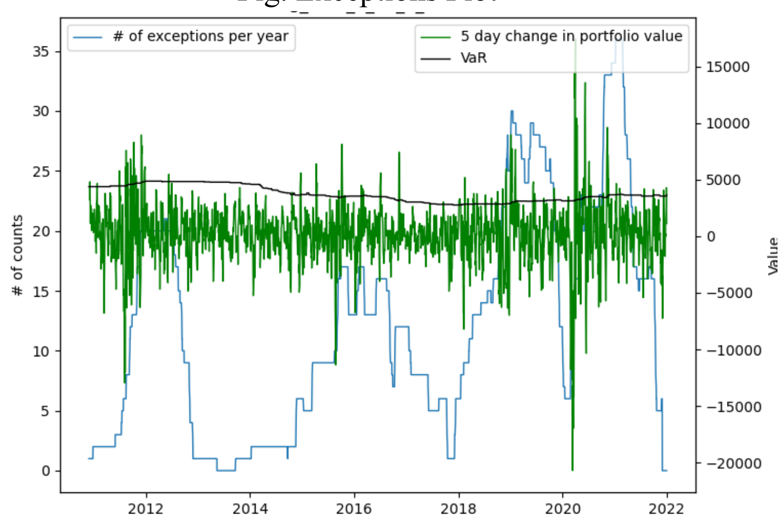
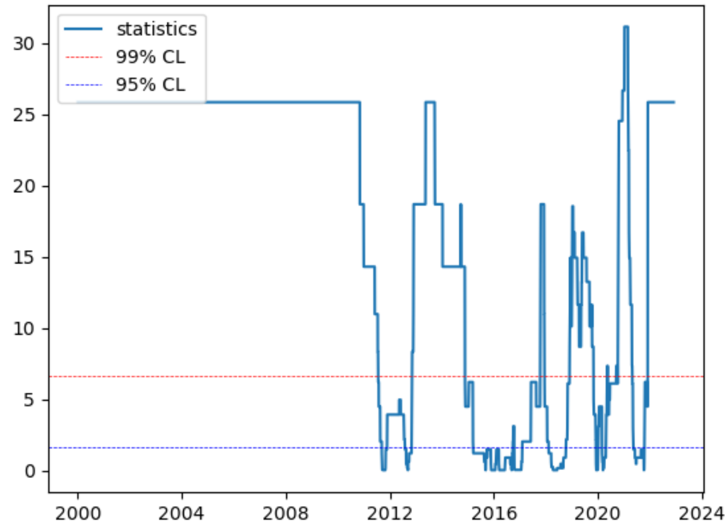


Table. Traffic Light Test Result

Backtest Window (yrs)	2
% in Green Zone (0-4 exceptions)	64%
% in Amber Zone (5-9 exceptions)	10%
% in Rd Zone (10 or more exceptions)	26%

Fig. Kupiec's POF Test Plot



This set of backtests provide satisfactory results, indicating that our risk calculation system is robust with different VaR window, VaR frequency and VaR percentile. While the difference in VaR window and frequency only has a small impact on the accuracy of the result. The lower VaR percentile seems to deteriorate the accuracy of the VaR test noticeably. This makes sense because the 95% VaR will capture less of the tails of the price distribution.

Overall, we observed that our risk calculation system is accurate with reasonable exceptions and robust under different market scenarios, portfolio sizes, portfolio types, and different VaR parameters.