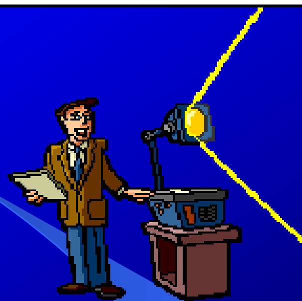
Biometric Authentication

Lecture 6

Biometrics Technology: Pattern Recognition (3)

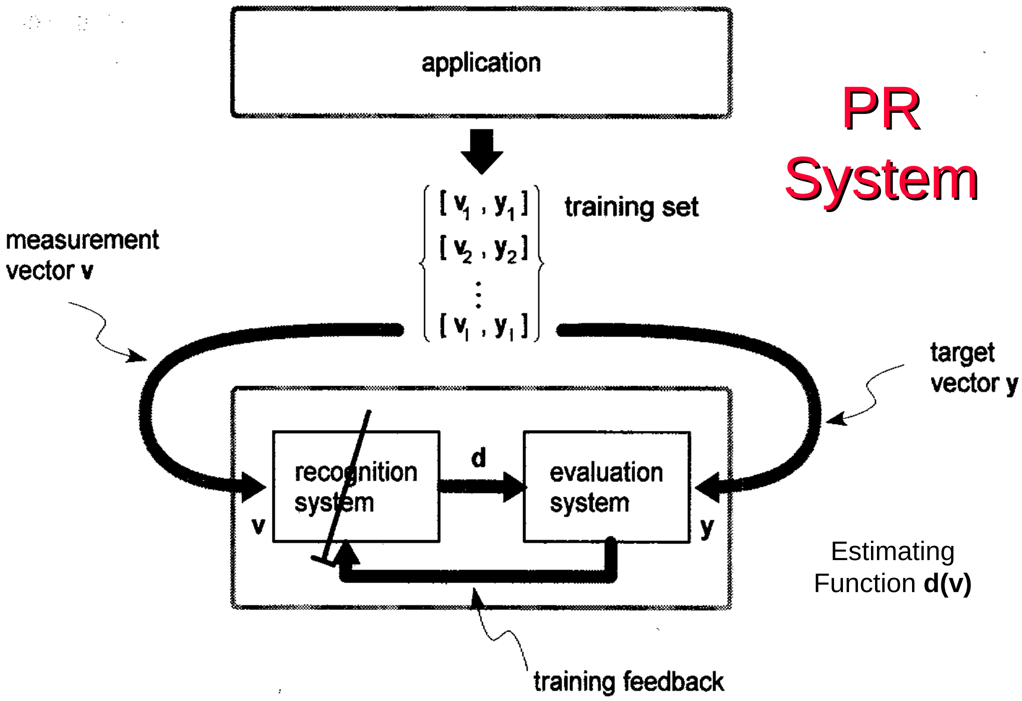


Outline

- Decision Functions
- Syntactic PR
- Frequency Filters
  - *-- Fourier Transform*
  - *-- Gabor Filters*

Decision Function

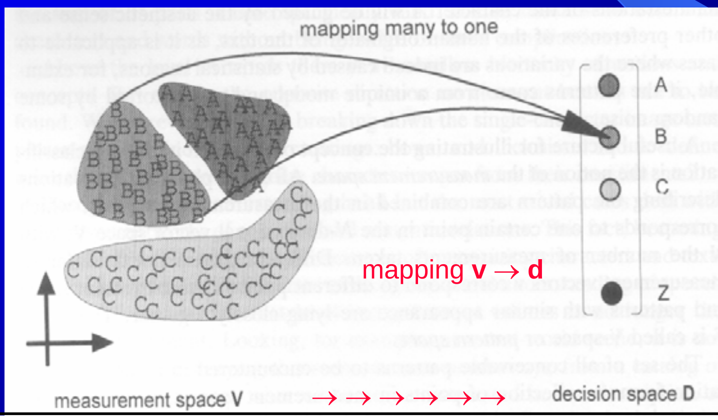
PR System

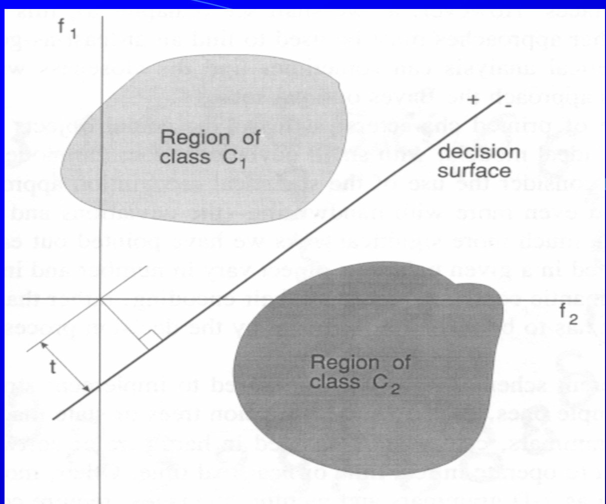


Estimating Function **d(v)**

Pattern Classification

- The principal function of a PR system is to yield decisions *concerning the class membership of the patterns* with which it is confronted.
- In order to accomplish this task, it is necessary to establish some rule upon which to base these decisions. One important approach is the use of decision functions.

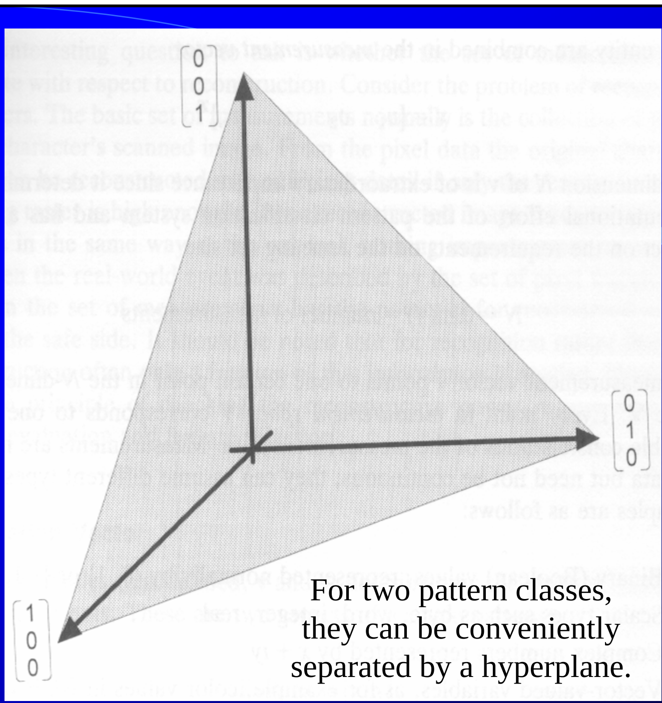




Decision Function

- Example: 2-D decision space

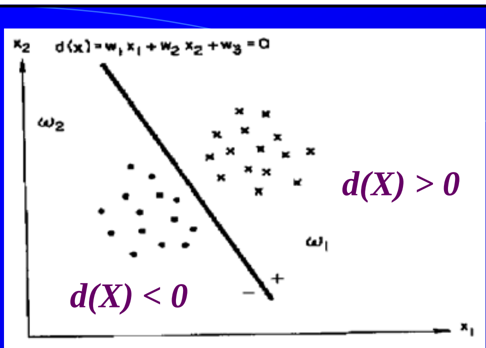
- As a way of introduction to this relatively simple concept, consider two pattern classes, which can be conveniently separated by a line.



For two pattern classes, they can be conveniently separated by a hyperplane.

Decision Space

- Example: 3-D decision space spanned by three target vectors, y1, y2 and y3.



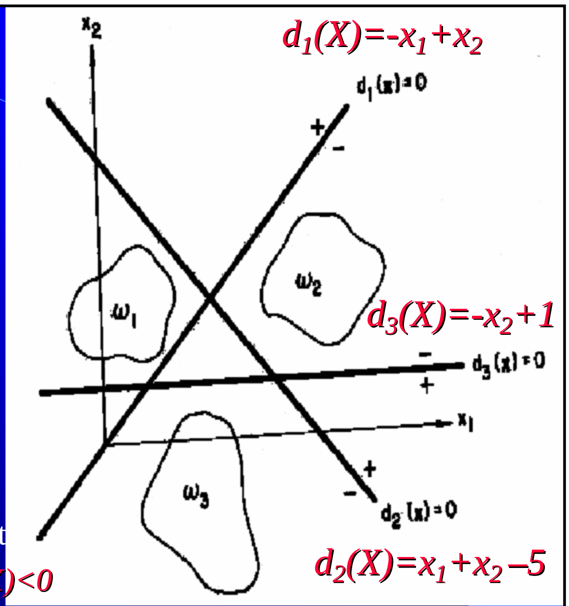
Decision Function

Example 1: Two pattern classes

- Let $d(X)=w_1x_1+w_2x_2+w_3=0$ be the equation of a separating line where *W's* are parameters and x_1 and x_2 are the general coordinate variables. It is clear that any pattern *X* belonging to ϖ_1 will yield a positive quantity when substituted in *d(X)*.
- Similarly, *d(X)* becomes negative upon substitution of any pattern *X* from ϖ_2. Therefore, *d(X)* can be used as a ***decision (or discriminate) function*** since, given a pattern *X* of unknown classification, we may say that *X* belongs to ϖ_1 if $d(X) > 0$, or to ϖ_2 if $d(X) < 0$.

Linear Decision Function

Example 2: Simple multi-class case



- It is noted that each class is separable from the rest by a single decision boundary. For instance, if a specific pattern X belongs to class ϖ_1, it is clear from the geometry that $d_1(X)>0$ while $d_2(X)<0$ and $d_3(X)<0$
- The boundary between class ϖ_1 and the other classes is given by the values of X for which $d_1(X)=0$.

Syntactic PR

PR: Statistical & Syntactic

- Statistical PR attempts to classify patterns (or entities) based on a set of extracted features and an underlying statistical (perhaps ad hoc) model for the generation of these patterns. It might be nice, if all PR problems could be approached by using a single straightforward procedure:

  (1) determine feature vector, x;

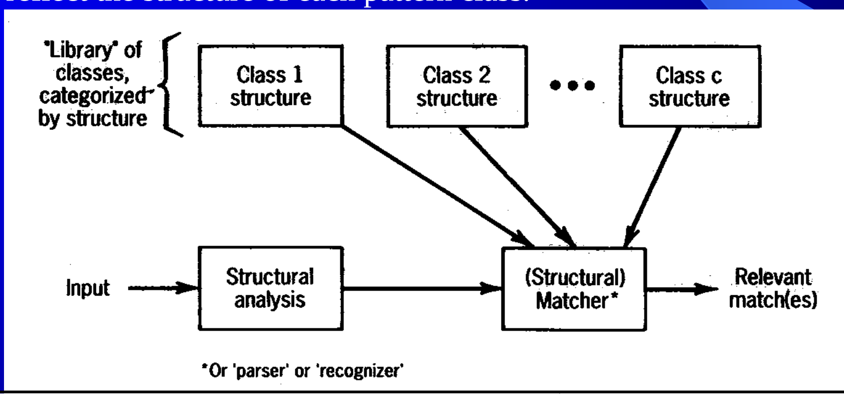
  (2) train system; and

  (3) classify patterns.

⇒ Unfortunately, for many realistic problems, this is not the case.

Syntactic PR

- In fact, many patterns contain structural or relational information, that is difficult or impossible to quantify in feature vector form.
- The premise is that the structure of an entity is paramount, and that it may be used for classification and description. This could be accomplished, e.g., by defining suitable and distinct grammars that reflect the structure of each pattern class.



Syntactic PR

- Classification may be based on measures of pattern structural similarity, e.g, each pattern class could be defined according to a common structural representation or description.
- However, there exist situations where each possible structural description defines a unique class. In this case, a quantitative description of pattern structure is still desirable even though the objective is not classification.

Syntactic PR

- Quantifying Structure in Pattern Description/Recognition

  Structure-based PR assumes pattern ‘structure’ is quantifiable, using two approaches:

  1. Formal grammars;
  2. Relational descriptions (principally graphs).

  They allow structurally quantitative pattern representation, which facilitates recognition, classification, or description.

- A class of procedures for syntactic recognition, including

  — parsing (for formal grammars);

  — relational graph matching (for attributed relational graphs).

Syntactic PR

- Hierarchical Approaches

  Although it is not mandatory, many SyntPR techniques are based on generation and analysis of complex patterns by a hierarchical decomposition into simpler patterns.

- An example is the written (English) language, where a paragraph may be decomposed into sentences → words → letters → strokes.

  Complex patterns are often best treated by successive decomposition into more manageable entities. Continuing the previous example, to generate a paragraph.

Syntactic PR

- **Example: Sentence Formation as Productions**

  Consider the sentence: 'The boy runs quickly.' produced by using the following sequence of 'rewriting' rules.

  1. <sentence>
  2. <noun phrase><verb phrase>
  3. <article><noun><verb phrase>
  4. the <noun><verb phrase>
  5. the boy <verb phrase>
  6. the boy <verb><adverb>
  7. the boy runs <adverb>
  8. the boys runs quickly

Frequency Filters

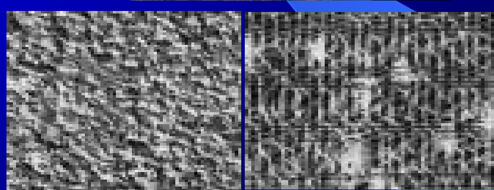
Feature Extraction in StatPR

- What features can we extract?

- But, sometimes it is difficult

  As an example: Texture feature –

  Surfaces characterized by more or less regular aggregates of similar patterns.

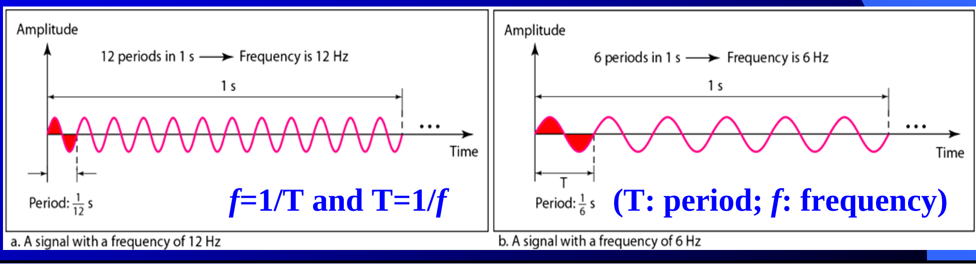


Irregular Texture Pattern in iris image

→ **One solution: Frequency filters**

Frequency

- **Frequency** is a measure of the number of occurrences of a repeating event per unit time. The period is the duration of one cycle in a repeating event. So frequency and period are the *reciprocal* of each other.
  - Frequency is the ***rate of change*** with respect to time.
  - Change in a short span of time means High Frequency, while change over a long span of time means Low Frequency.
  - If a signal does not change at all, its frequency is zero; while if a signal changes instantaneously, its frequency is infinite.



f=1/T and T=1/f **(T: period; f: frequency)**

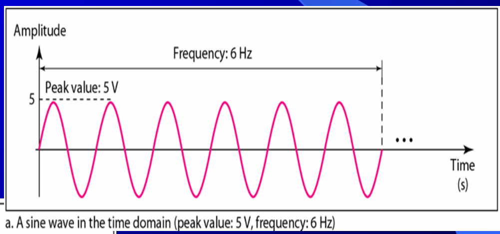
a. A signal with a frequency of 12 Hz b. A signal with a frequency of 6 Hz

Time(Spatial)/Frequency Domain

- **Time domain** is a term used to describe the analysis of mathematical functions, or physical signals, with respect to **time**.
  - In two-dimensional cases, time domain is also called ***Spatial domain*** which refers to the image plane itself.
  - Approaches in this category are based on direct manipulation of pixels in an image.
- **Frequency domain** is a term used to describe the analysis of mathematical functions or signals with respect to **frequency**.
  - In two-dimensional cases, frequency domain is nothing more than the space defined by values of the *Fourier transform* and its frequency variables.
  - Frequency domain processing techniques are based on modifying the Fourier transform of an image.

Frequency Domain

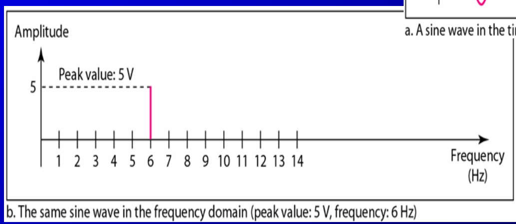
- A time domain graph shows ***how a signal changes over time***, whereas a frequency domain graph shows ***how much of the signal lies within each given frequency band*** over a range of frequencies.
- A complete sine wave in the time domain can be represented by one single spike in the frequency domain.



a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)

Time Domain

(peak value: 5V; frequency: 6 Hz)



b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

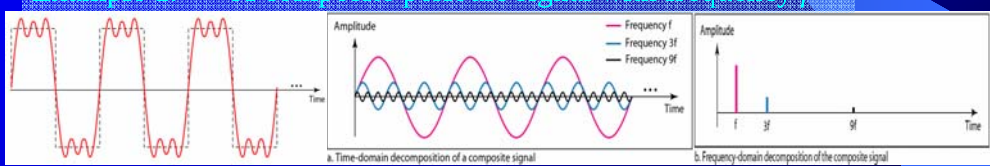
Frequency Domain

(peak value: 5V; frequency: 6 Hz)

Function of Frequency Domain

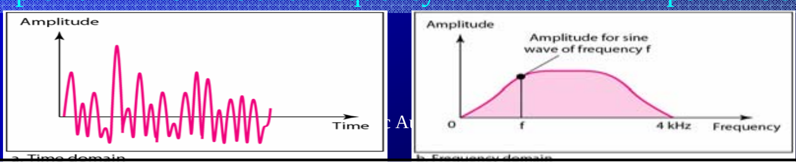
- The **frequency domain** relates to the ***Fourier transform*** by decomposing a function into an infinite or finite number of frequencies.
  - All signals can be decomposed into pure sinusoidal signals.
  - The frequency domain is *more compact and useful* when we are dealing with more than one sine wave.

Example 1. A composite periodic signal with frequency *f*



a. Time-domain decomposition of a composite signal
b. Frequency-domain decomposition of the composite signal

Example 2. The time and frequency domains of a nonperiodic signal



Function of Frequency Domain

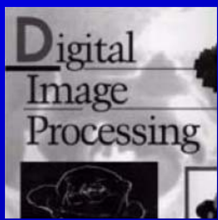
- Image Denoise

Corrupted by sinusoidal noise

Spectrum

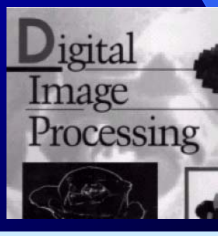
Result of frequency domain filtering

- Image restoration



Original image

Blurred by motion

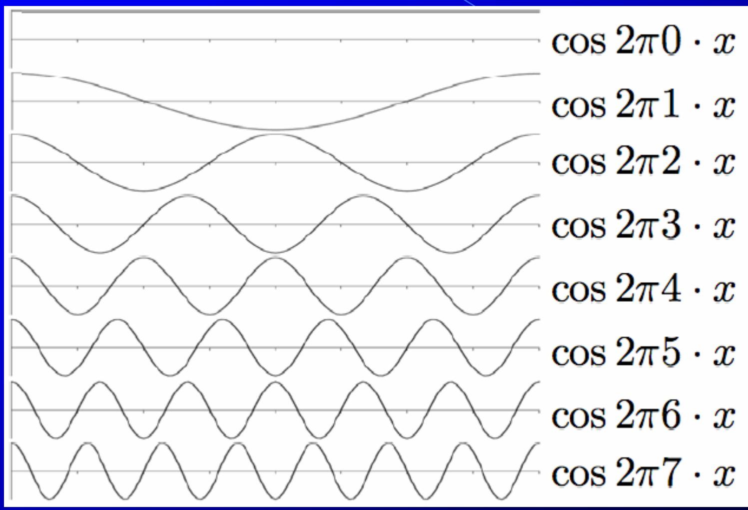


Result of frequency domain filtering

Fourier Transform

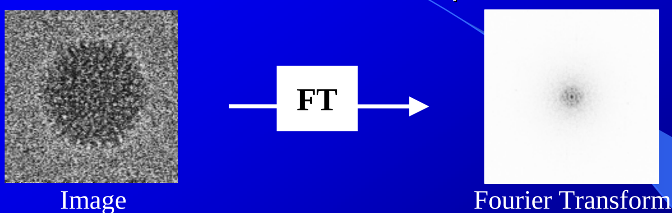
Fourier Theory

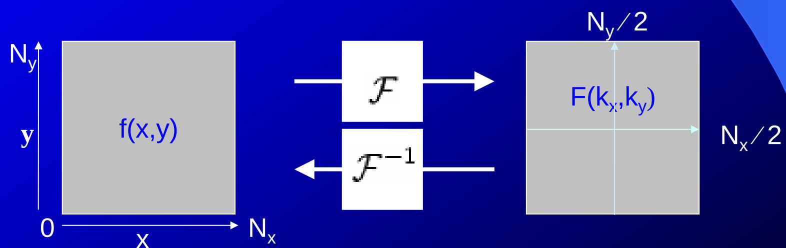
- Expresses any signal as sum of *Sin* and *Cos* functions



Fourier Theory

Fourier Transform is an invertible operator





$\mathcal{F}^{-1}\{F(k_x,k_y)\} = f(x,y)$

$\mathcal{F}\{f(x,y)\} = F(k_x,k_y)$

1-D Fourier Transform

$\mathcal{F}\{f(x)\} = F(s)$

1)
$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xs} dx$$
$$f(x) = \int_{-\infty}^{\infty} F(s) e^{i2\pi xs} ds$$

2)
$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-ixs} dx$$
$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{ixs} ds$$

3)
$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ixs} dx$$
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{ixs} ds$$

The same results but different formats

where

$$e^{ix} = \cos x + i \sin x$$

Euler's Formula

2-D Fourier Transform

Spatial Domain *f(x,y)*	Fourier Transform → ← Inverse Fourier Transform	Frequency Domain *F(u, v)*

Two-Dimensional Fourier Transform:

$$\mathrm{F}(u,v) = \iint_{-\infty}^{\infty} \mathrm{f}(x,y) \cdot \mathrm{e}^{-i\cdot 2\pi(ux+vy)} dxdy$$

Where in f(x,y), x and y are real, not complex variables.

Two-Dimensional Inverse Fourier Transform:

$$f(x,y) = \iint_{-\infty}^{\infty} \mathrm{F}(u,v) \cdot \mathrm{e}^{+i\cdot 2\pi(ux+vy)} dudv$$

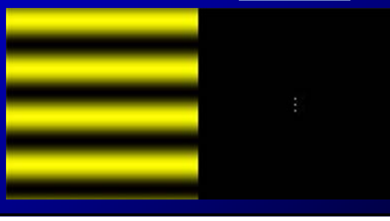
Basis functions

Amplitude and phase of required basis functions

2-D Fourier Transform

- A sine function with a DC component added
  - The center dot is the DC component
    - The DC component, standing for Direct Current, is the center of the spectrum (Fourier space) of the image and represents the average value of the image
  - The two others represent the frequency of the sine function
    - The one dot is just a mirrored version of the other one
  - No dots in the x-direction because the image is the same everywhere in that direction

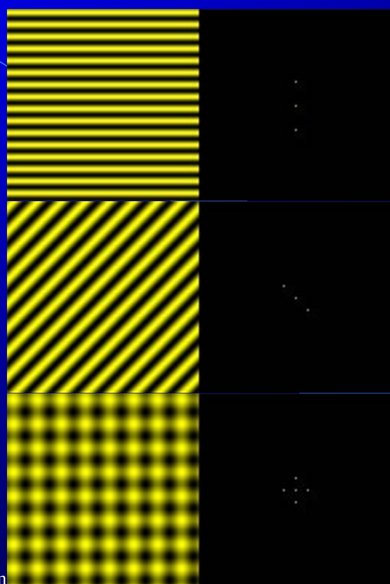
A sine function | Spectrum



2-D Fourier Transform

- A sine function with a higher frequency
  - The two dots are further away from the origin to represent the higher frequency
  - The image contracts, and it's spectrum becomes wider
- If the image is rotated, the spectrum will rotate in the same direction
- The sum of 2 sine functions, each in another direction

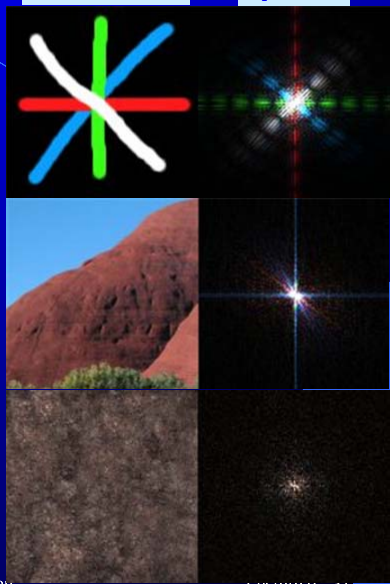
A sine function | Spectrum



2-D Fourier Transform

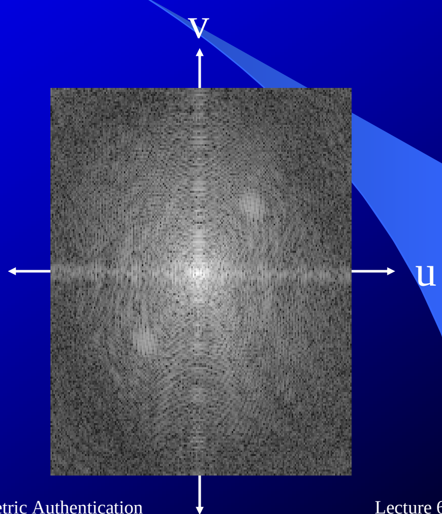
- Lines in an image often generate perpendicular lines in the spectrum
- The sloped lines in the spectrum here are obviously due to the sharp transition from the sky to the mountain
- Here's the FT of a tillable texture. Since it's tillable there are no abrupt changes on the horizontal and vertical sides, so there are no horizontal and vertical lines in the spectrum

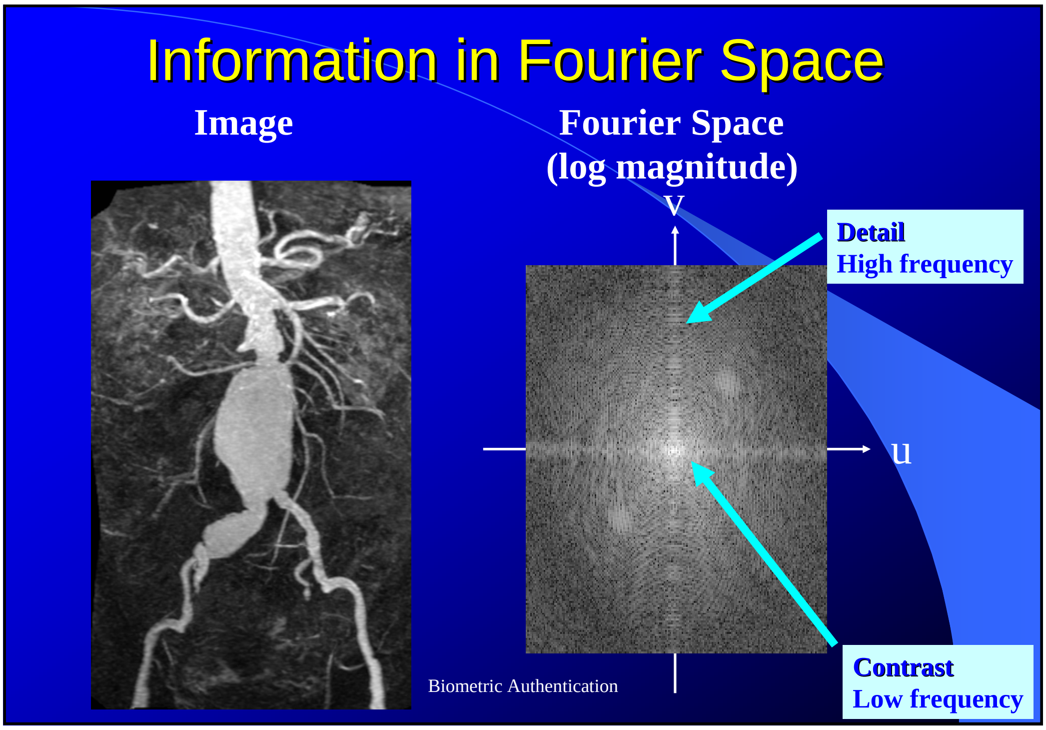
A sine function | Spectrum

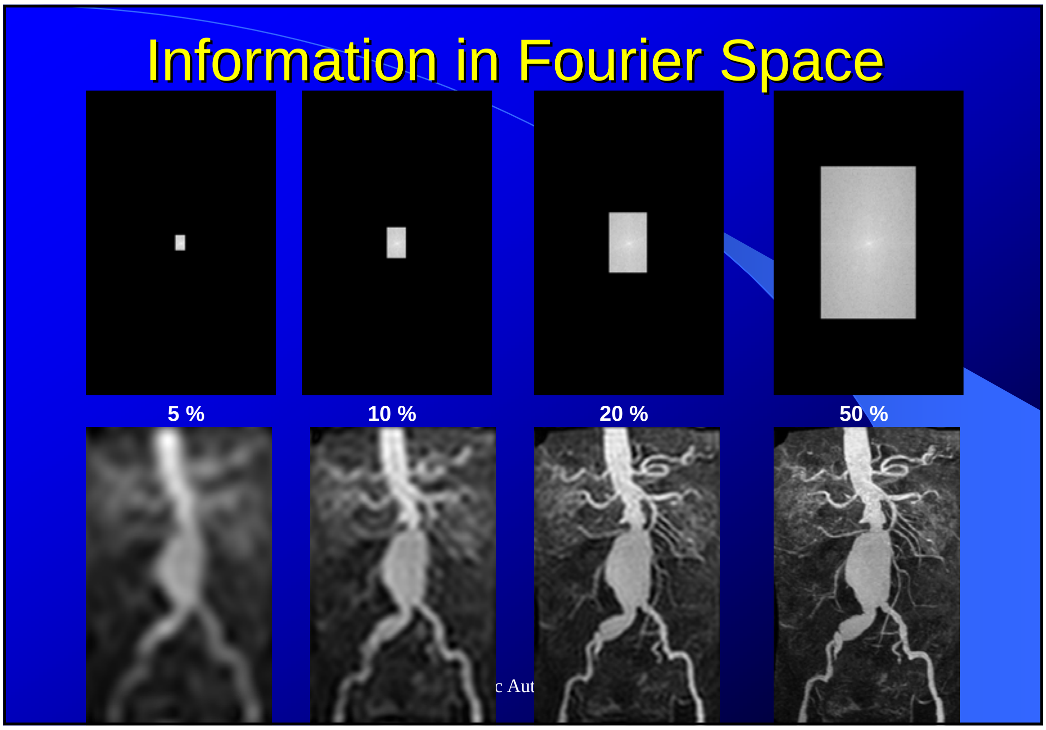


Information in Fourier Space

Image | **Fourier Space**

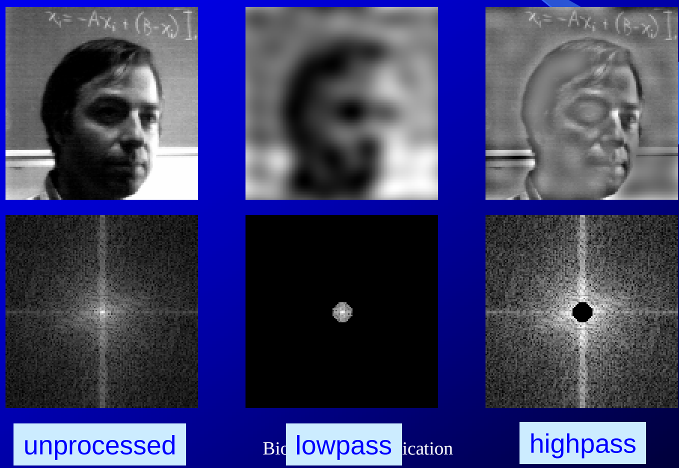



Information in Fourier Space
Image
Fourier Space
(log magnitude)
v
u
Detail
High frequency
Contrast
Low frequency
Biometric Authentication


Information in Fourier Space
5 %
10 %
20 %
50 %

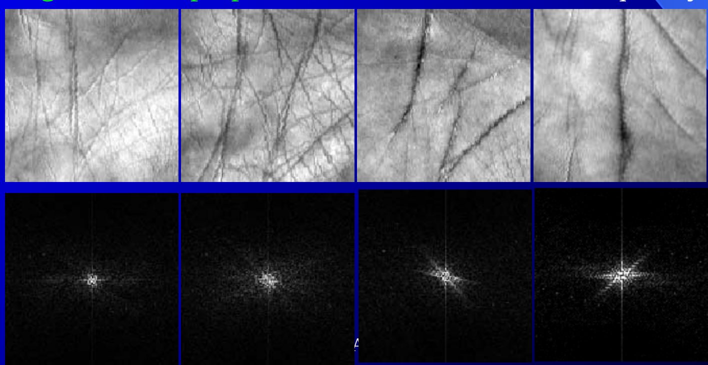
2D FT: Applications

- Fourier transform for the image enhancement and image analysis
  - **High-pass filter** (sharpens): restrict data to high frequency components
  - **Low-pass filter** (blurs): restrict data to low frequency components



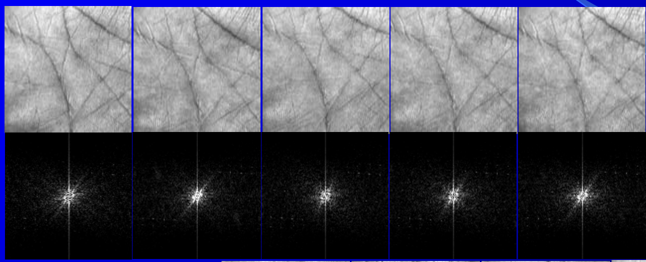
2D FT: Applications (1)

- There exit some correspondences between the features on the spatial domain image and those on the frequency domain image.
  - The stronger the lines on the spatial domain image, the less compact the information on the frequency domain image;
  - A strong line in the spatial domain will have more information along the line’s perpendicular direction in the frequency domain.



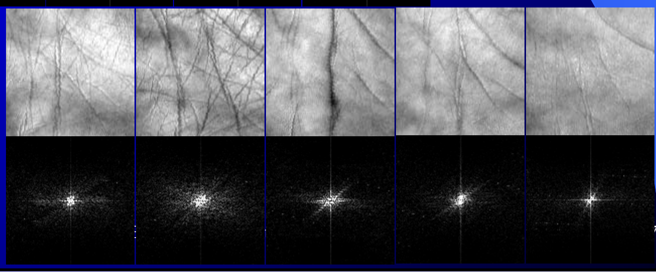
2D FT: Applications (2)

- The features in frequency domain should be discriminative.
  - Features extracted from similar images resemble each other
  - Features vary with different appearance images



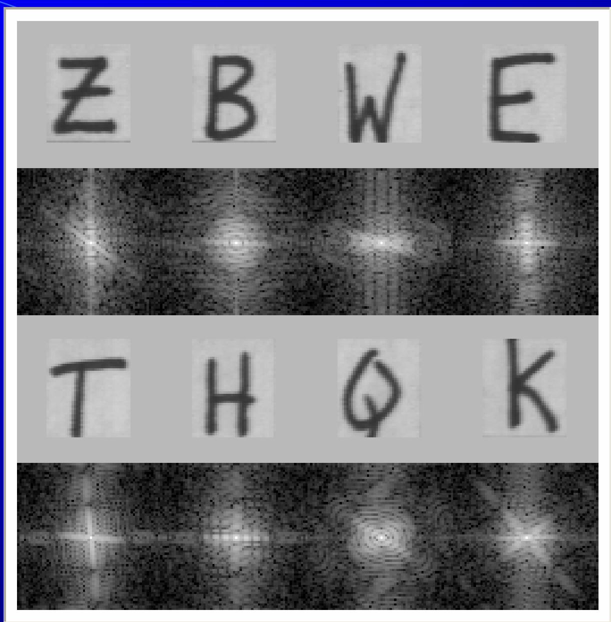
Samples from the same palm

Samples from difference images



2D FT: Applications (3)

- The Letters have quite different FTs, especially at the lower frequencies. The FTs also tend to have bright lines that are perpendicular to lines in the original letter. If the letter has circular segments, then so does the FT.



Gabor Filter

Gabor Filters - Definition

- Gabor filters are band-pass filters which are used for feature extraction, and texture analysis.
- They are both directional and frequency-selective filters and has the optimal space frequency resolution, i.e. the best joint space-frequency localization.
- A Gabor filter is basically a Gaussian multiplied by a complex sinusoid. In 2D cases,

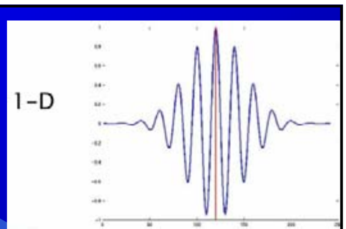
$$h(x,y) = g(x,y) \cdot s(x,y)$$

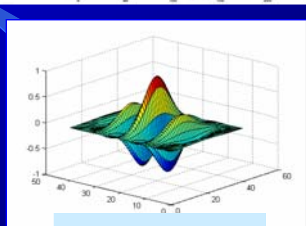
where,

$$g(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\{-\frac{1}{2}[(\frac{x}{\sigma_x})^2 + (\frac{y}{\sigma_y})^2]\}$$

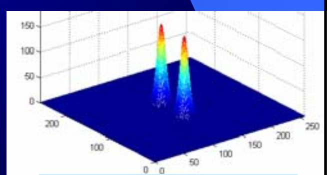
$$s(x,y) = \exp[-j2\pi(ux+vy)]$$

(u,v) are the 2D frequencies of the complex sinusoid, and its orientation is given by $\phi = \arctan(v/u)$





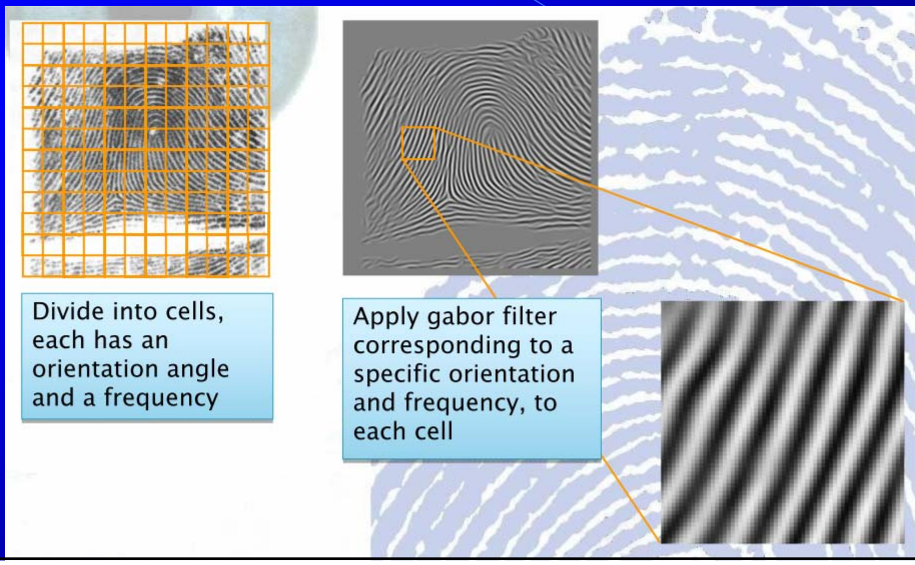
Spatial domain



Frequency domain

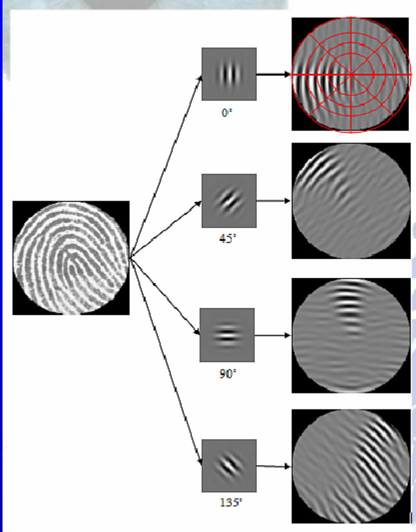
Gabor Filters: Applications

Fingerprint identification: image enhancement



Gabor Filters: Applications (1)

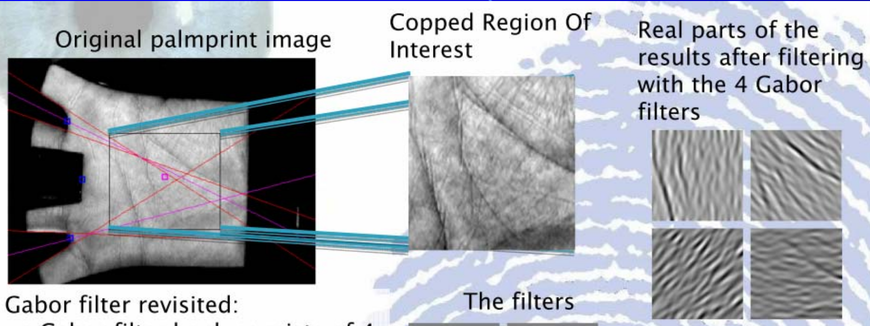
Fingerprint verification: texture analysis



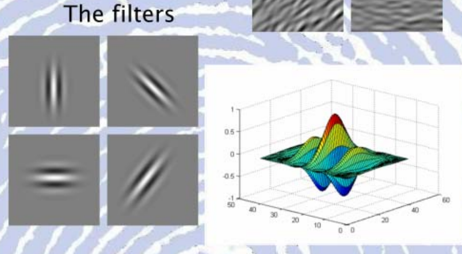
- Uses texture information (lost in optical and minutiae based schemes)
- Performs well with poor quality prints
- Features are statistically independent from minutiae and can be combined with minutiae matchers for higher accuracy
- Requires accurate alignment of the two prints (unreliable in poor prints)
- Not invariant to translation, orientation and non-linear distortion.
- *Less Accurate* than minutiae based matchers

Gabor Filters: Applications (2)

Palmprint identification: texture analysis

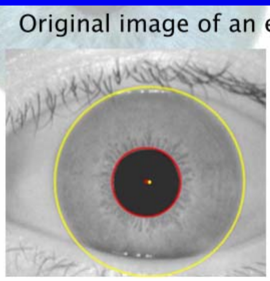


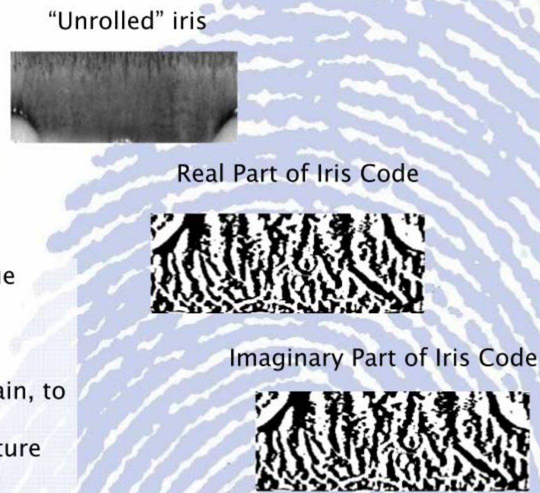
Gabor filter revisited:
• a Gabor filter bank consists of 4 complex gabor filter that differ only in its orientation, i.e. of orientation 0°, 45°, 90°, 135°
• each filter is applied to the ROI, strengthening texture at a specific orientation, at the same time attenuating noise



Gabor Filters: Applications (3)

Iris identification: texture coding





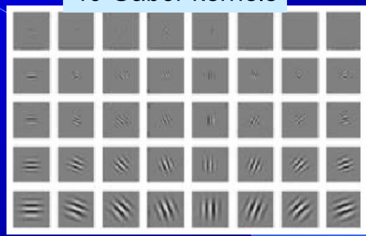
• Any given iris has a unique texture that is generated through a random process before birth
• Gabor filters turn out, again, to be very good at detecting patterns in this unique texture
• Matching is done again in hamming distance comparison

Gabor Filters: Applications (4)

Face recognition: Gaborface

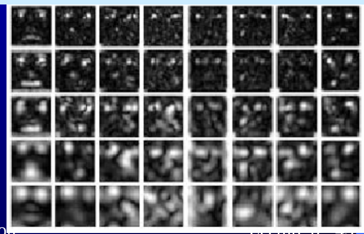
Cropped face image

40 Gabor kernels



- A Gabor wavelet bank consists of 40 different filters and exhibit desirable characteristics of spatial frequency, spatial locality, and orientation selectivity
- Gaborface, representing one face image, is computed by convoluting it with corresponding Gabor filters

Gabor wavelet representation



Questions?

1. **What is the definition about Decision Function?**
2. **Why we need the frequency domain for image processing and feature extraction? What difference between time(spatial) domain and frequency domain?**
3. **Understand two frequency transforms, Fourier transform and Gabor transform. What are the main functions for each?**
4. **Frequency transform can be applied to many different applications. Could you consider a new application using frequency transform in biometrics ?**

So much for today!

Thank you !!!