Tutorial 2

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COMP435p
Biometrics Authentication

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香港理工大學

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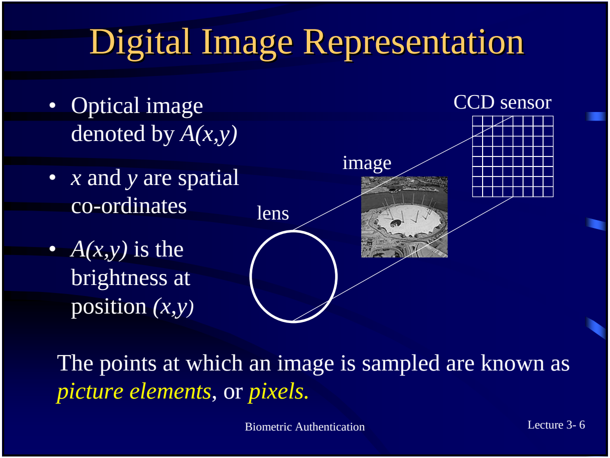
Outline

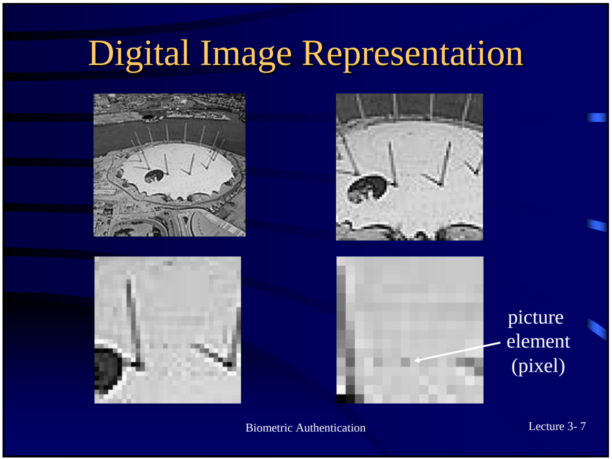
Problem 1.a

Understand some useful definitions in image processing: i) pixel, ii) image, iii) image resolution, d) gray-level image.

Problem 1.a


Digital Image Representation
• Optical image denoted by A(x,y)
• x and y are spatial co-ordinates
• A(x,y) is the brightness at position (x,y)
CCD sensor
image
lens
The points at which an image is sampled are known as picture elements, or pixels.
Biometric Authentication
Lecture 3- 6

Problem 1.a


Digital Image Representation
picture
element
(pixel)
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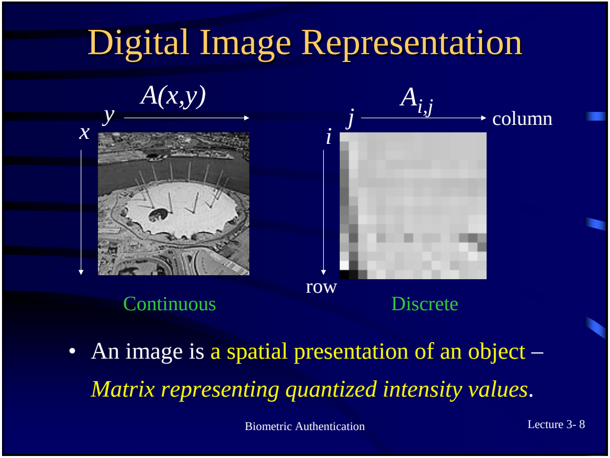
Problem 1.a


Digital Image Representation
A(x,y)
y
x
$A_{i,j}$
j
column
i
row
Continuous
Discrete
• An image is a spatial presentation of an object – Matrix representing quantized intensity values.
Biometric Authentication
Lecture 3- 8

Problem 1.a

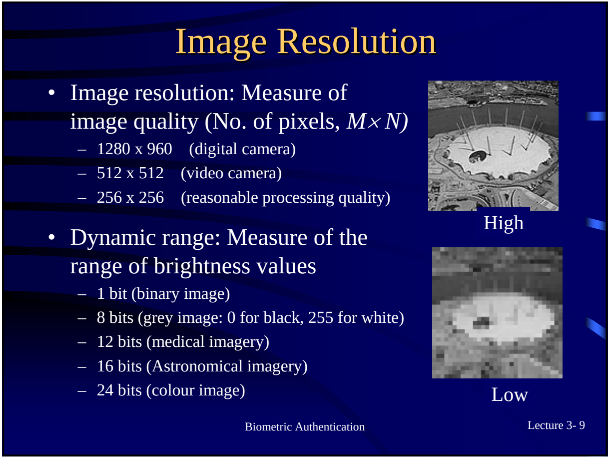


Image Resolution
• Image resolution: Measure of image quality (No. of pixels, $M \times N$)
– 1280 x 960 (digital camera)
– 512 x 512 (video camera)
– 256 x 256 (reasonable processing quality)
• Dynamic range: Measure of the range of brightness values
– 1 bit (binary image)
– 8 bits (grey image: 0 for black, 255 for white)
– 12 bits (medical imagery)
– 16 bits (Astronomical imagery)
– 24 bits (colour image)
High
Low
Biometric Authentication
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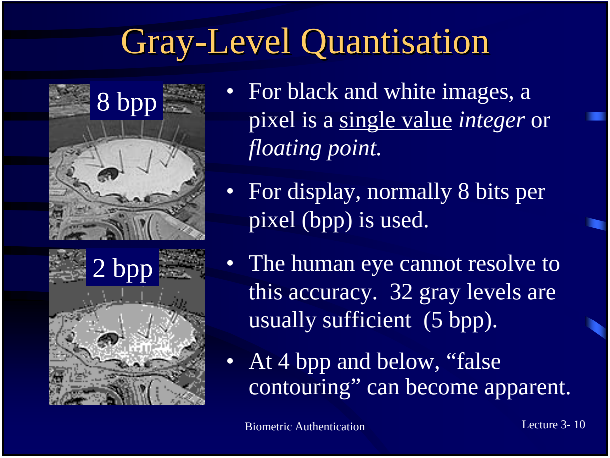
Problem 1.a


Gray-Level Quantisation
8 bpp
2 bpp
• For black and white images, a pixel is a single value *integer* or *floating point.*
• For display, normally 8 bits per pixel (bpp) is used.
• The human eye cannot resolve to this accuracy. 32 gray levels are usually sufficient (5 bpp).
• At 4 bpp and below, “false contouring” can become apparent.
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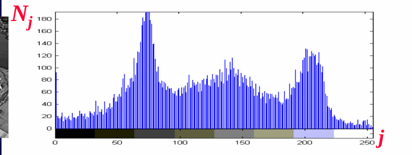
Problem 1.b

What is image histogram? What are peaks and plains in image histogram? Compare the differences between two types of bimodal histograms.

Problem 1.b

Image Histograms



- Plot of N_j versus j: Shows the distribution of image pixels in terms of their gray levels.
- Gray levels $j=0,1,\ldots,255$
- N_j= number of pixels in image with gray level j

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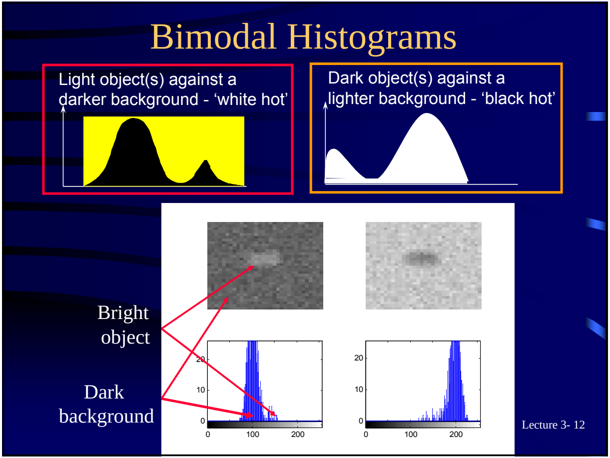
Problem 1.b


Bimodal Histograms
Light object(s) against a darker background - 'white hot'
Dark object(s) against a lighter background - 'black hot'
Bright object
Dark background
0
10
20
0
100
200
Lecture 3- 12

Problem 1.c

Understand the point operation. Contrast enhancement is the main application of pixel operation. How many methods of contrast enhancement do you learn? Please compare them.

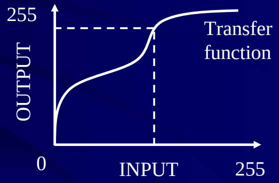
Problem 1.c

Point (Pixel) Operation

- Point operation: A function is applied to every pixel in an image, which operates only on the pixel's current value.
- Thresholding - A mask may be created by setting a pixel value to 1 or 0 depending upon if the current value is above or below a certain threshold value.

Input pixel value, *I*, mapped to output pixel value, *O*, via transfer function *T*.

$$O=T(I)$$



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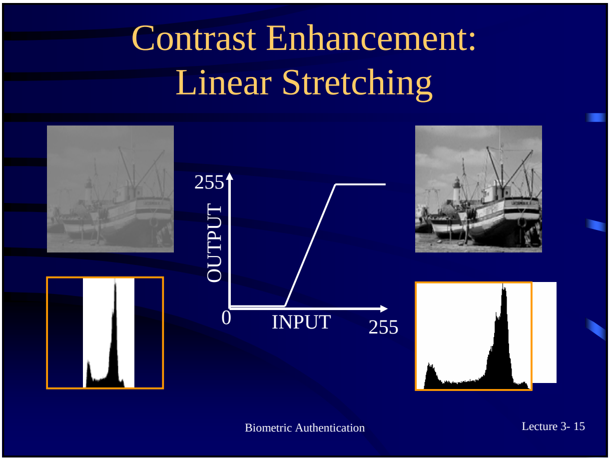
Problem 1.c


Contrast Enhancement:
Linear Stretching
255
OUTPUT
0
INPUT
255
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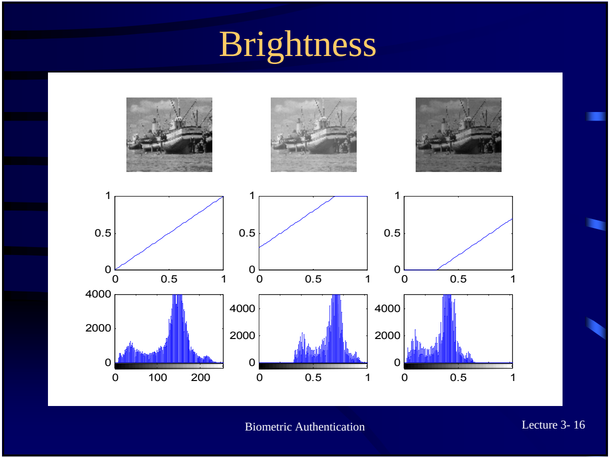
Problem 1.c


Brightness
1
0.5
0
0
0.5
1
4000
2000
0
100
200
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Lecture 3- 16

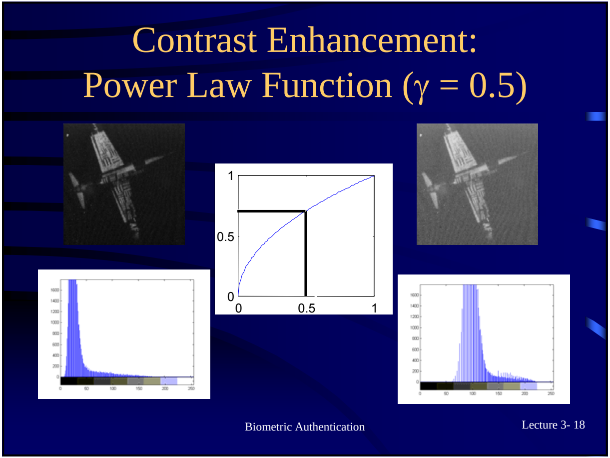
Problem 1.c

Contrast Enhancement: Power Law Function

$$O = I^{\gamma}$$

- $\gamma < 1$ to enhance contrast in dark regions
- $\gamma > 1$ to enhance contrast in bright regions.

Problem 1.c


Contrast Enhancement:
Power Law Function (γ = 0.5)
1
0.5
0
0
0.5
1
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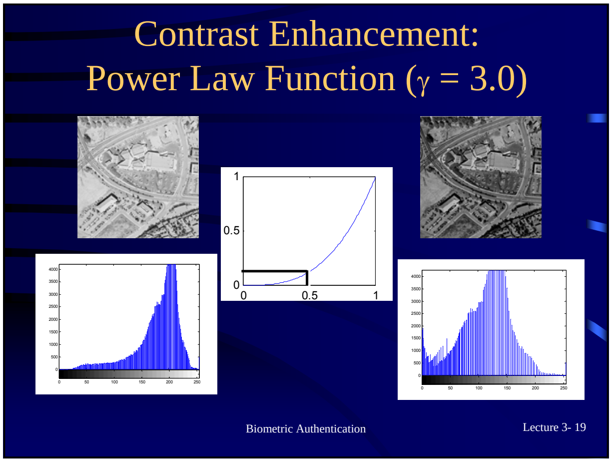
Problem 1.c


Contrast Enhancement:
Power Law Function (γ = 3.0)
1
0.5
0
0
0.5
1
Biometric Authentication
Lecture 3- 19

Problem 1.c

Contrast Enhancement: Power Law Function

- Look-up Table
  - Transfer function implemented as a look-up table (LUT).
  - Implemented in hardware or software.

$\gamma=2$

I	*O*
0	0
0.10	0.01
0.20	0.04
0.30	0.03
0.40	0.16
0.50	0.25
0.60	0.36
0.70	0.49
0.80	0.64
0.90	0.81
1.00	1.00

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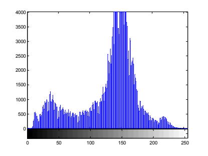
Problem 1.c

Contrast Enhancement: Histogram Equalisation

- Image histograms consisting of peaks and low plains.
- Peaks = many pixels concentrated in a few grey levels
- Plains = small number of pixels distributed over a wider range of grey levels



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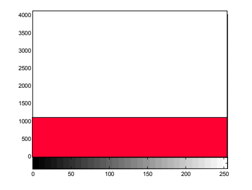
Problem 1.c

Histogram Equalisation

- Expand pixels in peaks over a wider range of gray-levels.
- “Squeeze” low plains pixels into a narrower range of gray levels.
- **Flat** histogram.



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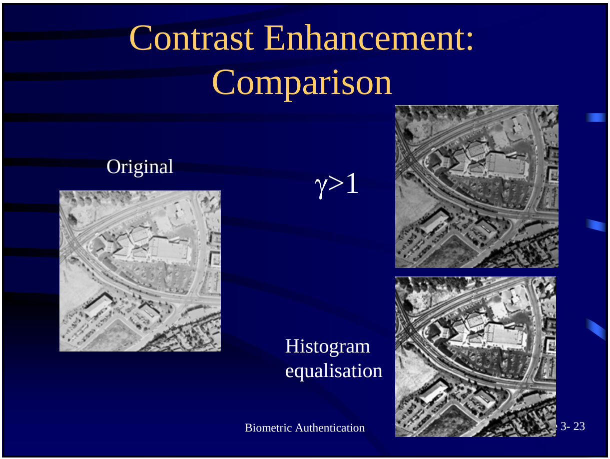
Problem 1.c


Contrast Enhancement: Comparison
Original
γ>1
Histogram equalisation
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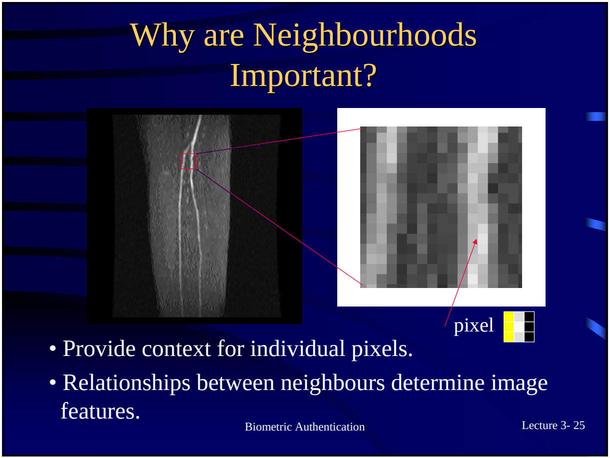
Problem 1.d

(d) Why neighbourhood operations? Give the applications of group operations.

Problem 1.d


Why are Neighbourhoods Important?
pixel
• Provide context for individual pixels.
• Relationships between neighbours determine image features.
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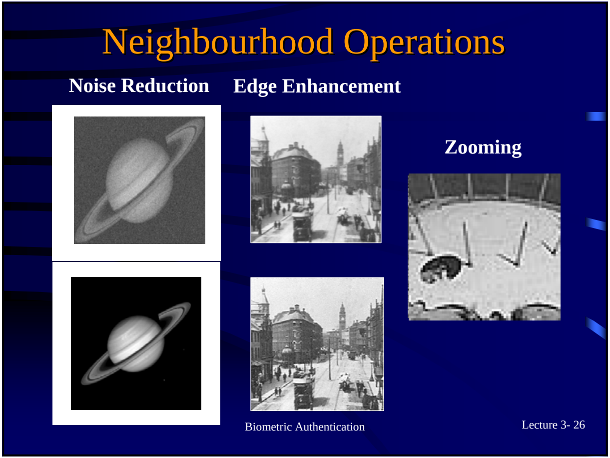
Problem 1.d


Neighbourhood Operations
Noise Reduction
Edge Enhancement
Zooming
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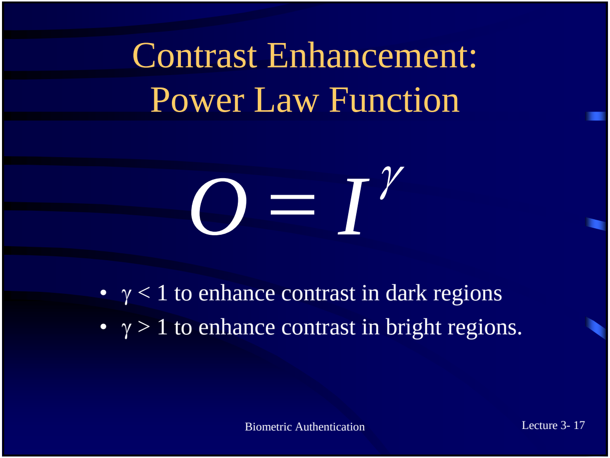
Outline

1. Problems
   - Problem 1: Answer the questions
   - Problem 2: Power law function
   - Problem 3: Convolution
   - Problem 4: Noise reduction
   - Problem 5: Filter operations

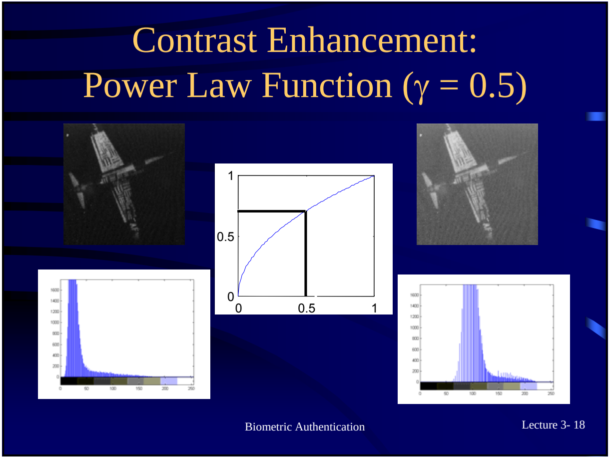
Problem 2: Power law function

What is the power law function? Give the LUT when $\gamma = 0.5$ *in terms of the table in P3-20. Compare the two LUT and understand the impact of the parameter* $\gamma = 0.5$*.*

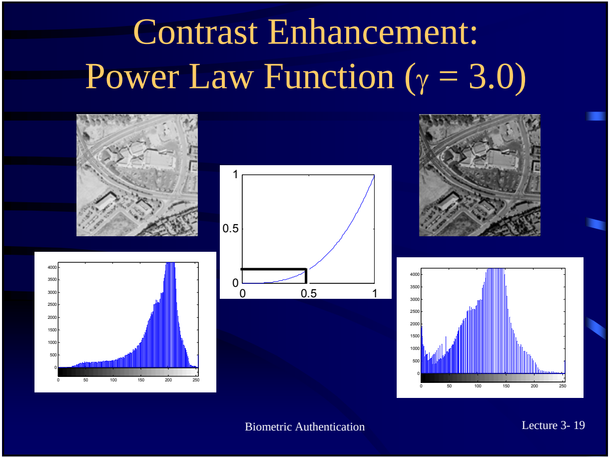
Problem 2: Power law functionterms



Problem 2: Power law functionterms



Problem 2: Power law functionterms



Problem 2: Power law functionterms

Contrast Enhancement: Power Law Function

- Look-up Table
  - Transfer function implemented as a look-up table (LUT).
  - Implemented in hardware or software.

$\gamma=2$

I	*O*
0	0
0.10	0.01
0.20	0.04
0.30	0.03
0.40	0.16
0.50	0.25
0.60	0.36
0.70	0.49
0.80	0.64
0.90	0.81
1.00	1.00

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Problem 2: Power law function

$$
\begin{aligned}
I = & \; [\; 0 \quad 0.10 \quad 0.20 \quad 0.30 \quad 0.40 \quad 0.50 \\
& \quad 0.60 \quad 0.70 \quad 0.80 \quad 0.90 \quad 1.00 \;] \\
O = & \; I^{\gamma} \\
O = & \; [\; 0^{0.5} \quad 0.10^{0.5} \quad 0.20^{0.5} \quad 0.30^{0.5} \quad 0.40^{0.5} \quad 0.50^{0.5} \\
& \quad 0.60^{0.5} \quad 0.70^{0.5} \quad 0.80^{0.5} \quad 0.90^{0.5} \quad 1.00^{0.5} \;] \\
= & \; [\; 0 \quad 0.32 \quad 0.45 \quad 0.55 \quad 0.63 \quad 0.71 \\
& \quad 0.77 \quad 0.84 \quad 0.89 \quad 0.95 \quad 1.00 \;]
\end{aligned}
$$

$\square$

Outline

Problem 3: Convolution

Please check the convolution result in P3-33 and try to answer the question in P3-34. Now there is an input image and a mask as below. Could you give the convolved image?

Problem 3: Convolutionterms

Convolution

- Consists of filtering an image *A* using a filter (mask) *B*.
- Mask is a small image whose pixel values are called weights.
- Weights modify relationships between pixels.

Filter, mask or template **B**

$$\begin{array}{|c|c|} \hline B_{1,1} & B_{1,2} \\ \hline B_{2,1} & B_{2,2} \\ \hline \end{array}$$

2×2

Input image **A**

$$\begin{array}{|c|c|c|c|} \hline A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ \hline A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ \hline A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \\ \hline A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline C_{1,1} & C_{1,2} & C_{1,3} \\ \hline C_{2,1} & C_{2,2} & C_{2,3} \\ \hline C_{3,1} & C_{3,2} & C_{3,3} \\ \hline \end{array}$$

4×4

Convolved Image **C**

3×3

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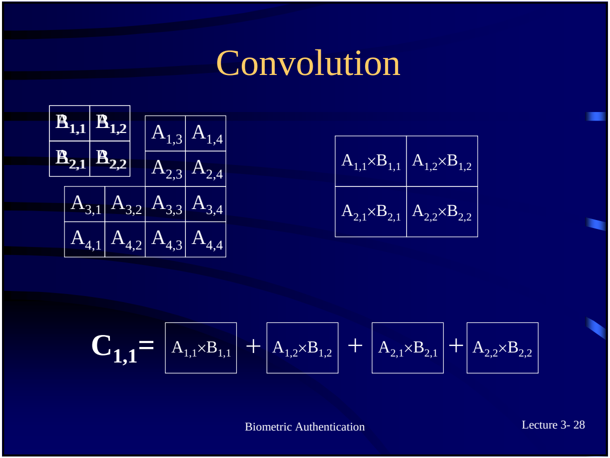
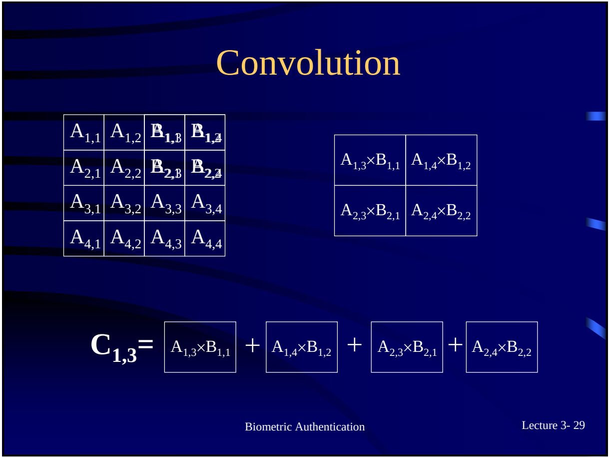
Problem 3: Convolutionterms



Problem 3: Convolutionterms



Problem 3: Convolutionterms

Convolution

$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	$A_{1,4}$
$A_{2,1}$ $B_{1,1}$	$A_{2,2}$ $B_{1,2}$	$A_{2,3}$	$A_{2,4}$
$A_{3,1}$ $B_{2,1}$	$A_{3,2}$ $B_{2,2}$	$A_{3,3}$	$A_{3,4}$
$A_{4,1}$	$A_{4,2}$	$A_{4,3}$	$A_{4,4}$

$A_{2,1}\times B_{1,1}$	$A_{2,2}\times B_{1,2}$
$A_{3,1}\times B_{2,1}$	$A_{3,2}\times B_{2,2}$

$$C_{2,1} = \boxed{A_{2,1}\times B_{1,1}} + \boxed{A_{2,2}\times B_{1,2}} + \boxed{A_{3,1}\times B_{2,1}} + \boxed{A_{3,2}\times B_{2,2}}$$

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Problem 3: Convolutionterms

Mathematical Notation

$$C_{1,1} = \boxed{A_{1,1} \times B_{1,1}} + \boxed{A_{1,2} \times B_{1,2}} + \boxed{A_{2,1} \times B_{2,1}} + \boxed{A_{2,2} \times B_{2,2}}$$

$$B = M \times N$$

$$C_{k,l} = \sum_{i=k}^{k+M-1} \sum_{j=l}^{l+N-1} A_{i,j} \times B_{i-k+1,\, j-l+1}$$

$$\sum_{i=1}^{2} i = 1 + 2$$

$$\sum_{i=1}^{2} A_i = A_1 + A_2$$

$$\sum_{i=1}^{2} \sum_{j=1}^{2} A_{i,j} = \sum_{i=1}^{2} \left(A_{i,1} + A_{i,2} \right) = A_{1,1} + A_{1,2} + A_{2,1} + A_{2,2}$$

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Problem 3: Convolutionterms

Summations

$$C_{k,l} = \sum_{i=k}^{k+M-1} \sum_{j=l}^{l+N-1} A_{i,j} \times B_{i-k+1,\, j-l+1}$$

$$C_{1,1} = \sum_{i=1}^{1+2-1} \sum_{j=1}^{1+2-1} A_{i,j} \times B_{i-1+1,\, j-1+1}$$

M=N=2
k=l=1

$$= \sum_{i=1}^{2} \sum_{j=1}^{2} A_{i,j} \times B_{i,j}$$

$$= \sum_{i=1}^{2} A_{i,1} \times B_{i,1} + A_{i,2} \times B_{i,2}$$

$$= A_{1,1} \times B_{1,1} + A_{1,2} \times B_{1,2} + A_{2,1} \times B_{2,1} + A_{2,2} \times B_{2,2}$$

Problem 3: Convolutionterms

Convolution

Filter, mask or template

B

-1	2
-1	2

2×2

Input image

A

4	4	7	9
4	3	8	9
3	5	9	9
3	6	10	9

4×4

=

Convolved Image

C

6	23	21
9	26	19
16	27	17

3×3

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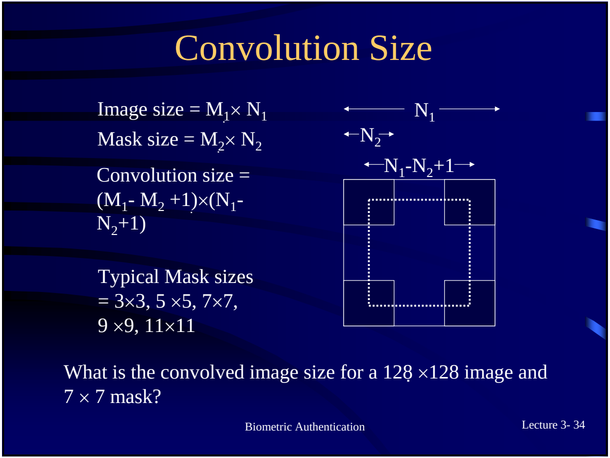
Problem 3: Convolutionterms



Problem 3: Convolution

The convolved image size for a 128×128 image and a 7×7 mask is 122×122.

Problem 3: Convolution

Mask

-1	2	1
-1	2	1

Input image

4	4	7	1	9
4	3	8	2	9
3	5	9	8	9
3	6	9	8	9
3	6	10	9	9

Problem 3: Convolution

Output image

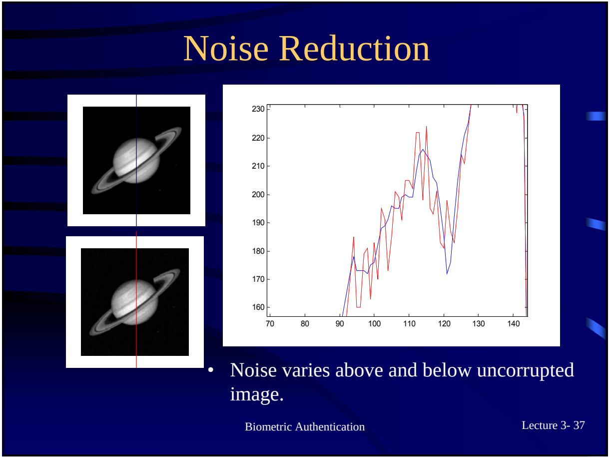
21	26	9
26	37	21
34	41	42
37	43	33

Outline

Problem 4: Noise reduction

There are two methods of noise reduction: low pass filter and median filter (see P3: 37-46). Please give the noise reduction results of the input image in Q3 by using the two methods according to the conditions: a) the mask of low pass filter is $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$*, and b) the mask of median filter is* $\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

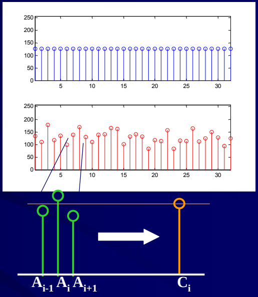
Problem 4: Noise reductionterms



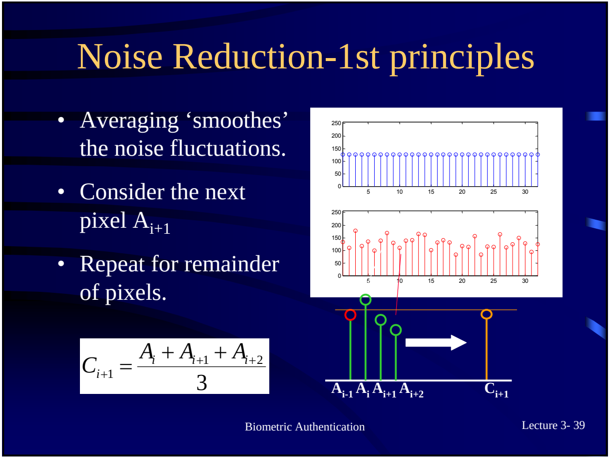
Problem 4: Noise reductionterms

Noise Reduction-1st principles

- How do we reduce noise?
- Consider a uniform 1-d image A and add noise.
- Focus on a pixel neighbourhood.
- Central pixel has been increased and neighbouring pixels have decreased.



Problem 4: Noise reductionterms



Problem 4: Noise reductionterms

Noise Reduction- Neighborhood Operations

- All pixels can be averaged by convolving *1-d* image *A* with mask **B** to give enhanced image *C*.
- Weights of *B* must equal one when added together.

$$\mathbf{C} = \mathbf{A} * \mathbf{B}$$
$$\mathbf{B} = [B_1 \quad B_2 \quad B_3]$$
$$C_i = A_{i-1} \times B_1 + A_i \times B_2 + A_{i+1} \times B_3$$
$$B = \frac{1}{3}[1 \quad 1 \quad 1]$$
$$C_i = \frac{A_{i-1} + A_i + A_{i+1}}{3}$$

$$\mathbf{C} = \mathbf{A} * \mathbf{B}$$
$$\mathbf{B} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Extend to two dimensions.

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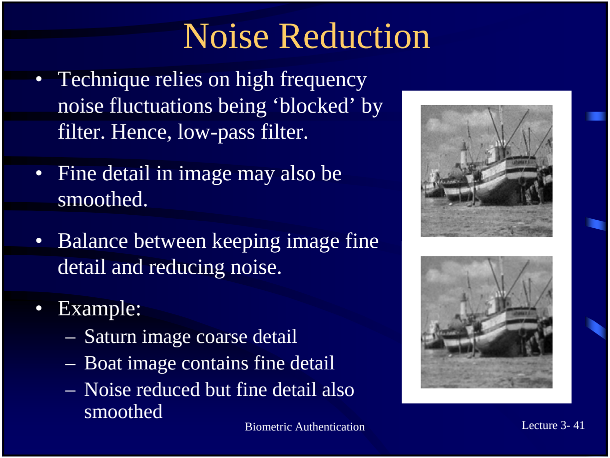
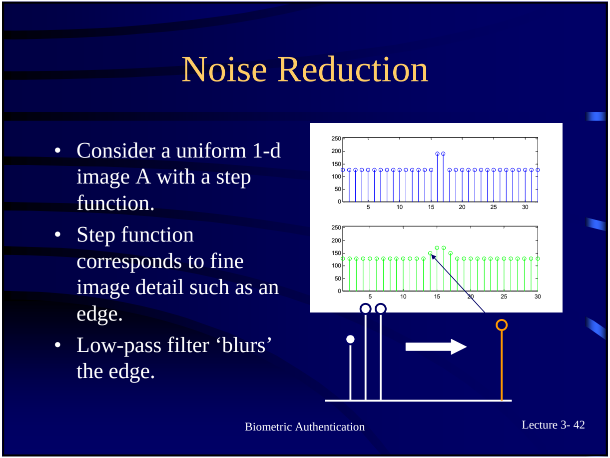
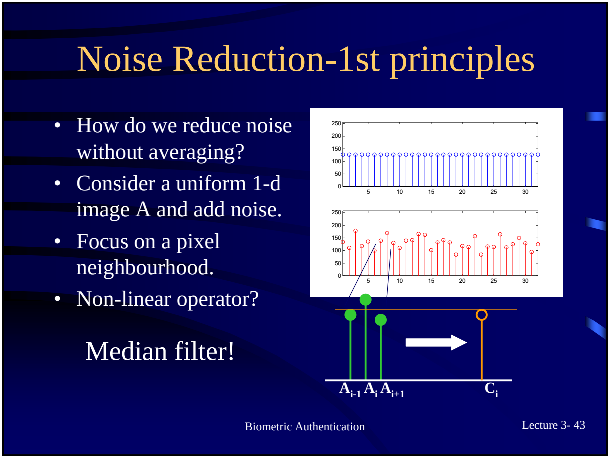
Problem 4: Noise reductionterms



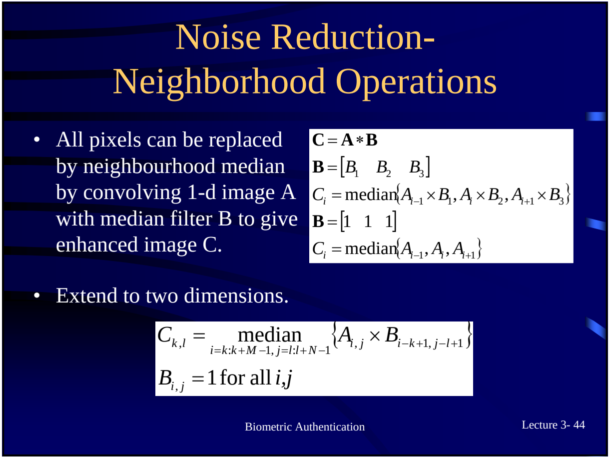
Problem 4: Noise reductionterms



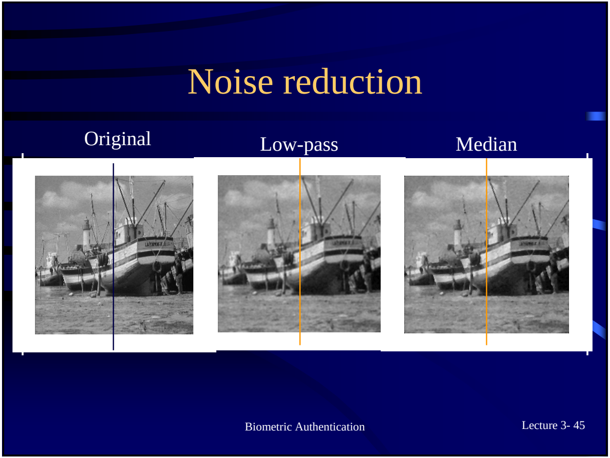
Problem 4: Noise reductionterms



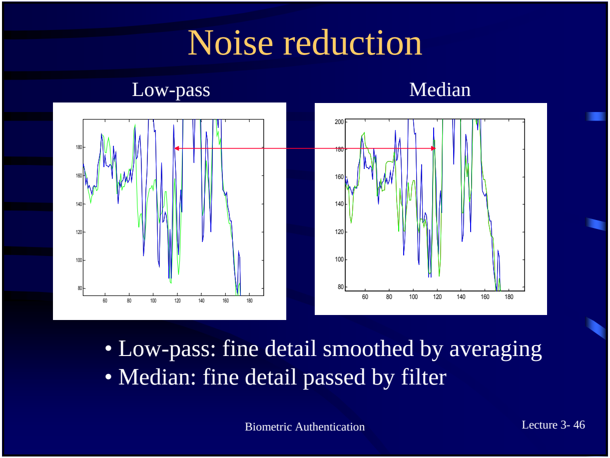
Problem 4: Noise reductionterms



Problem 4: Noise reductionterms



Problem 4: Noise reductionterms



Problem 4: Noise reduction

Input image

4	4	7	1	9
4	3	8	2	9
3	5	9	8	9
3	6	9	8	9
3	6	10	9	9

Low pass filter mask

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Result a)

5	5	7
6	6	8
6	8	9

Problem 4: Noise reduction

Input image

4	4	7	1	9
4	3	8	2	9
3	5	9	8	9
3	6	9	8	9
3	6	10	9	9

Median filter mask

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Result b)

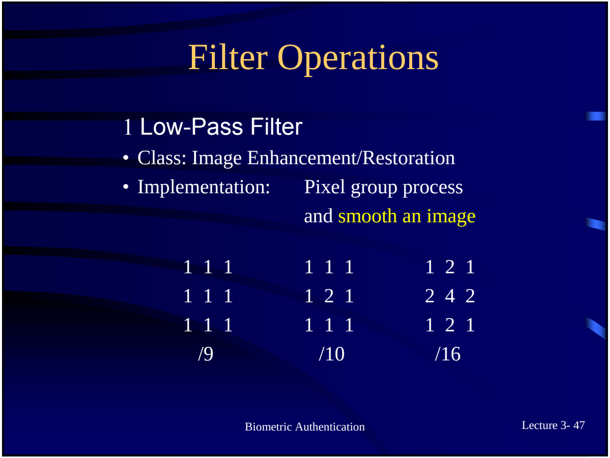
4	5	8
5	8	9
6	8	9

Outline

Problem 5: Filter operations

In P3: 47-49 there are some examples of filter operations. Please point out the difference of Low-pass filters, High-pass filters and Edge enhancement filters. Notice that the ‘’0”-Sum mask for Edge enhancement filters and ‘’1”-Sum for others.

Problem 5: Filter operationsterms



Problem 5: Filter operationsterms

Filter Operations

2 High-Pass Filter

- Implementation: Pixel group process and sharpen an image

$$\begin{matrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{matrix} \qquad \begin{matrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{matrix} \qquad \begin{matrix} 1 & -2 & 1 \\ -2 & 5 & -2 \\ 1 & -2 & 1 \end{matrix}$$

3 Sobel Edge Enhancement

- Implementation: Edge extraction

$$\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix} \qquad \begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}$$

Vertical mask | Horizontal mask

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Problem 5: Filter operationsterms

Filter Operations

4 Shift and Difference Edge Enhancement

- Implementation: Vertical, Horizontal and Diagonal Edge extraction

$$\begin{matrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$$

Vertical Horizontal Diagonal

5 Laplacian Edge Enhancement

- Implementation: All Edge extraction

$$\begin{matrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{matrix} \qquad \begin{matrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{matrix} \qquad \begin{matrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{matrix}$$

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Problems

Any questions?