

# Tutorial 5

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Biometrics Authentication

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- Problem 1: Answer The Questions
- Problem 2: Indeterminate Region
- Problem 3: Classifier
- Problem 4: Syntactic PR
- Problem 5: Frequency Transform
- Problem 6: Frequency Transform



# Outline

## 1 Problems

- Problem 1: Answer The Questions
- Problem 2: Indeterminate Region
- Problem 3: Classifier
- Problem 4: Syntactic PR
- Problem 5: Frequency Transform
- Problem 6: Frequency Transform

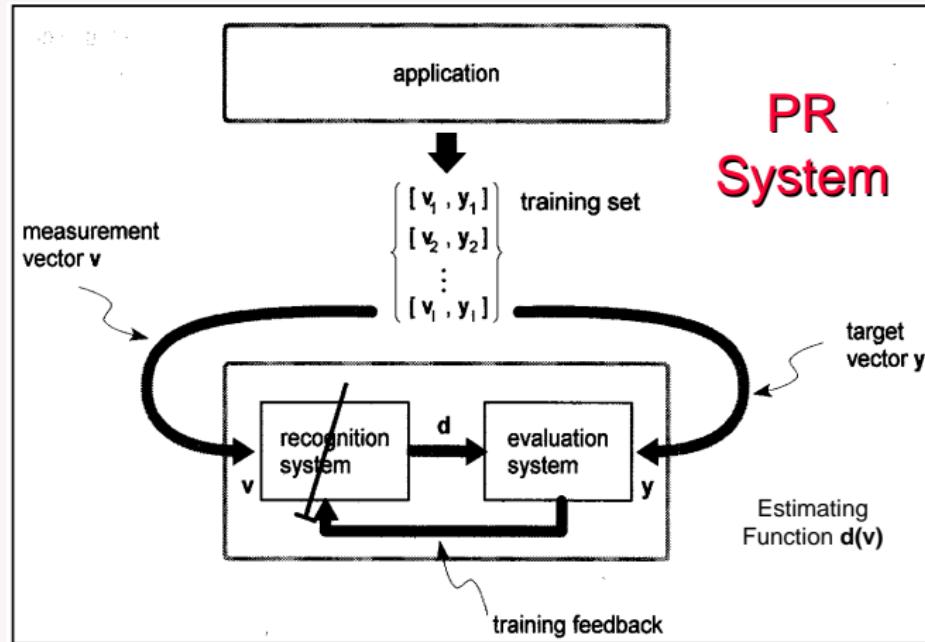


# Problem 1.1 PR system

*Explain about PR system using the figure in P6:4*



# Problem 1.1 PR system



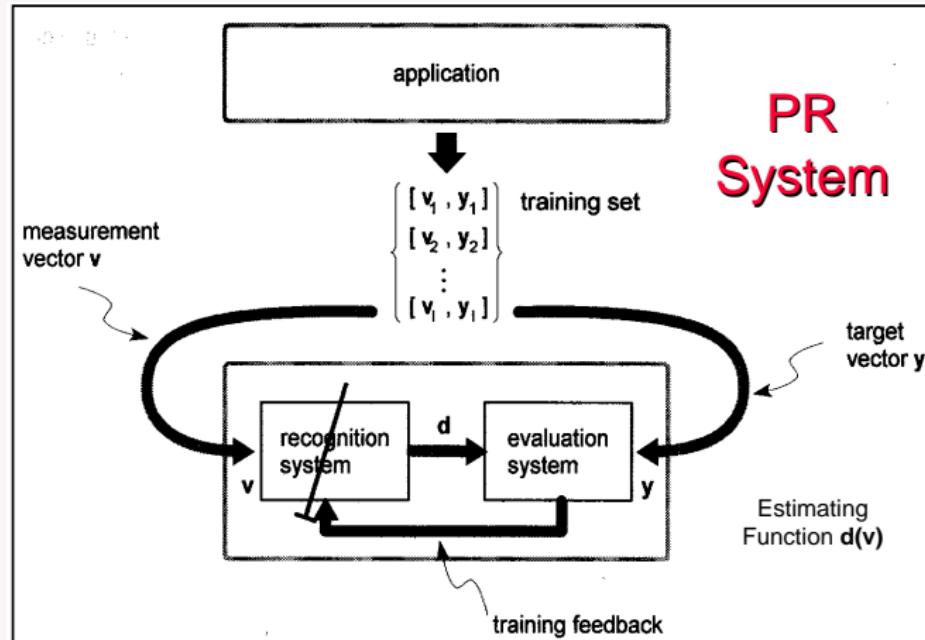


# Problem 1.2 Decision Function

*What is the definition about Decision Function (P6:4-9)*



# Problem 1.2 Decision Function

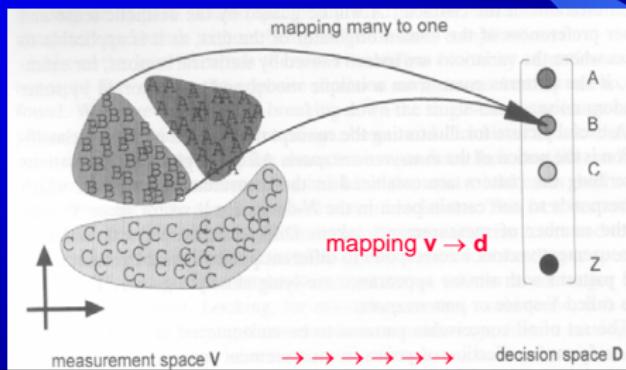




# Problem 1.2 Decision Function

## Pattern Classification

- The principal function of a PR system is to yield decisions *concerning the class membership of the patterns* with which it is confronted.
- In order to accomplish this task, it is necessary to establish some rule upon which to base these decisions. One important approach is the use of **decision functions**.





# Problem 1.2 Decision Function

**Decision Function**

- Example: 2-D decision space
- As a way of introduction to this relatively simple concept, consider two pattern classes, which can be conveniently separated by a line.

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# Problem 1.2 Decision Function

For two pattern classes, they can be conveniently separated by a hyperplane.

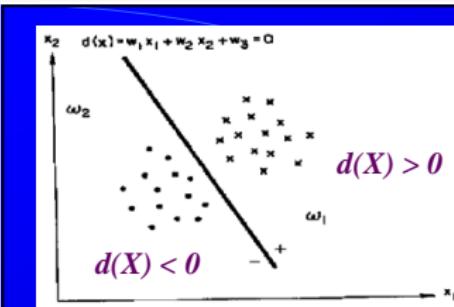
## Decision Space

- Example: 3-D decision space spanned by three target vectors,  $y_1$ ,  $y_2$  and  $y_3$ .

Lecture 6 - 7



# Problem 1.2 Decision Function



## Decision Function

### Example 1: Two pattern classes

- Let  $d(X)=w_1x_1+w_2x_2+w_3=0$  be the equation of a separating line where  $W$ 's are parameters and  $x_1$  and  $x_2$  are the general coordinate variables. It is clear that any pattern  $X$  belonging to  $\omega_1$  will yield a positive quantity when substituted in  $d(X)$ .
- Similarly,  $d(X)$  becomes negative upon substitution of any pattern  $X$  from  $\omega_2$ . Therefore,  $d(X)$  can be used as a **decision (or discriminant) function** since, given a pattern  $X$  of unknown classification, we may say that  $X$  belongs to  $\omega_1$  if  $d(X) > 0$ , or to  $\omega_2$  if  $d(X) < 0$ .

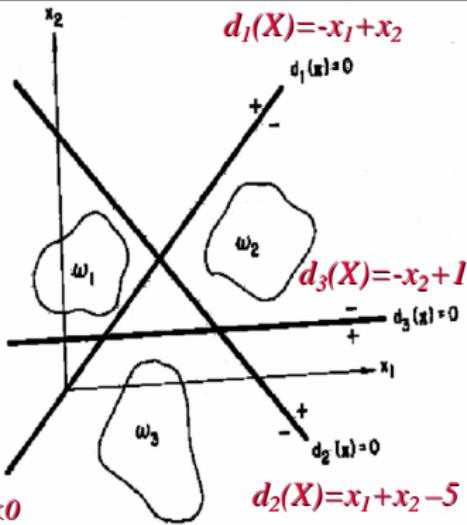


# Problem 1.2 Decision Function

## Linear Decision Function

### Example 2: Simple multi-class case

- It is noted that each class is separable from the rest by a single decision boundary. For instance, if a specific pattern  $X$  belongs to class  $\omega_1$ , it is clear from the geometry that  $d_1(X) > 0$  while  $d_2(X) < 0$  and  $d_3(X) < 0$
- The boundary between class  $\omega_1$  and the other classes is given by the values of  $X$  for which  $d_1(X) = 0$ .



The separating line or the separating hyperplane on different conditions.



# Problem 1.3 time domain and frequency domain

*Why we need the frequency domain for feature extraction (P6:18)? What difference between time(spatial) domain and frequency domain? (P6:20-22)*



# Problem 1.3 time domain and frequency domain

## Feature Extraction in StatPR

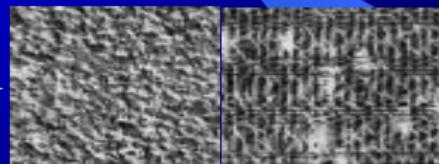
- What features can we extract?



- But, sometimes it is difficult

As an example: Texture feature –

*Surfaces characterized by more or less regular aggregates of similar patterns.*



Irregular Texture Pattern in iris image

→ One solution: Frequency filters

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Figure 6 - 18

To extract features like irregular texture pattern in images



# Problem 1.3 time domain and frequency domain

## Time(Spatial)/Frequency Domain

- **Time domain** is a term used to describe the analysis of mathematical functions, or physical signals, with respect to **time**.
  - In two-dimensional cases, time domain is also called **Spatial domain** which refers to the image plane itself.
  - Approaches in this category are based on direct manipulation of pixels in an image.
- **Frequency domain** is a term used to describe the analysis of mathematical functions or signals with respect to **frequency**.
  - In two-dimensional cases, frequency domain is nothing more than the space defined by values of the **Fourier transform** and its frequency variables.
  - Frequency domain processing techniques are based on modifying the Fourier transform of an image.



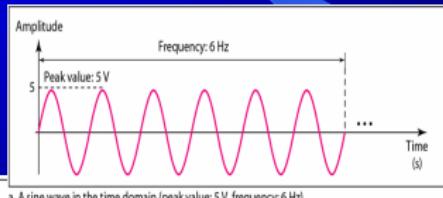
# Problem 1.3 time domain and frequency domain

## Frequency Domain

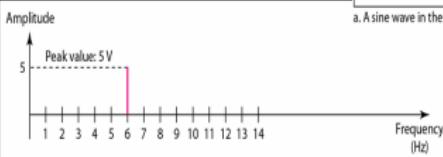
- A time domain graph shows *how a signal changes over time*, whereas a frequency domain graph shows *how much of the signal lies within each given frequency band* over a range of frequencies.
- A complete sine wave in the time domain can be represented by one single spike in the frequency domain.

### Time Domain

(peak value: 5V; frequency: 6 Hz)



a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)



b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

### Frequency Domain

(peak value: 5V; frequency: 6 Hz)

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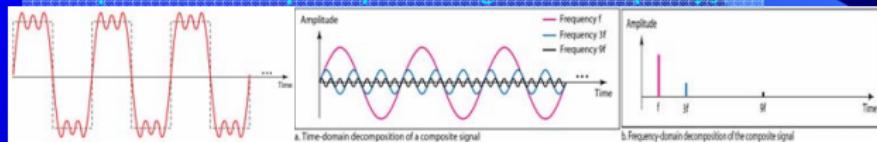


# Problem 1.3 time domain and frequency domain

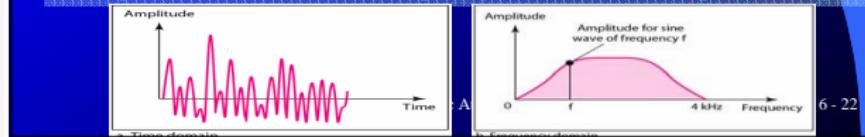
## Function of Frequency Domain

- The **frequency domain** relates to the **Fourier transform** by decomposing a function into an infinite or finite number of frequencies.
  - All signals can be decomposed into pure sinusoidal signals.
  - The frequency domain is *more compact and useful* when we are dealing with **more than one sine wave**.

### Example 1. A composite periodic signal with frequency $f$



### Example 2. The time and frequency domains of a nonperiodic signal





# Problem 1.4 Fourier and Gabor

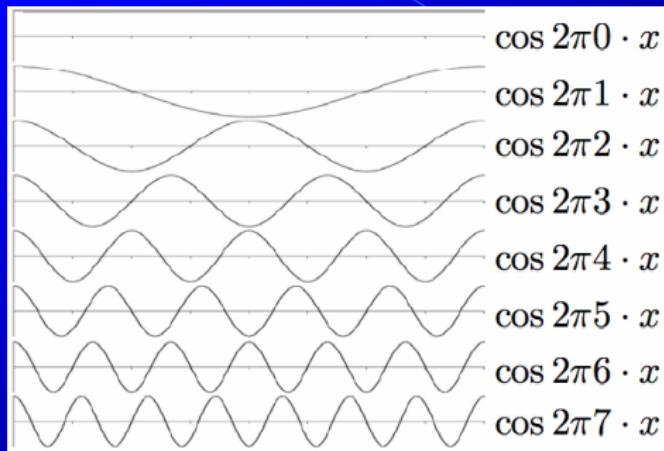
*Understand two frequency transforms, Fourier transform (P6:25-28) and Gabor transform (P6:40)*



# Problem 1.4 Fourier and Gabor

## Fourier Theory

- Expresses any signal as sum of *Sin* and *Cos* functions

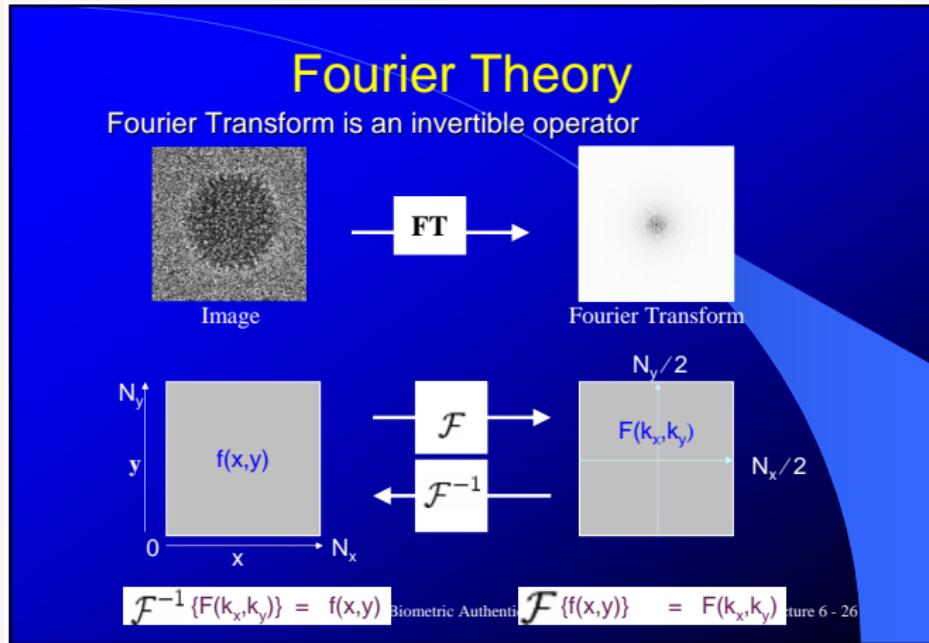


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# Problem 1.4 Fourier and Gabor





# Problem 1.4 Fourier and Gabor

## 1-D Fourier Transform



$$\left\{ f(x) \right\} = F(s)$$

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi s x} dx \quad 1)$$

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{j2\pi s x} ds$$

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-j s x} dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{j s x} ds \quad 2)$$

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-j s x} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{j s x} ds \quad 3)$$

where

$$e^{ix} = \cos x + i \sin x$$

Euler's Formula



# Problem 1.4 Fourier and Gabor

## 2-D Fourier Transform



Two-Dimensional Fourier Transform:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \cdot e^{-i \cdot 2\pi(ux+vy)} dx dy$$

Where in  $f(x,y)$ ,  $x$  and  $y$  are real, not complex variables.

Two-Dimensional Inverse Fourier Transform:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) \cdot e^{+i \cdot 2\pi(ux+vy)} du dv$$

Basis functions

Amplitude and phase of required basis functions



# Problem 1.4 Fourier and Gabor

## Gabor Filters - Definition

- Gabor filters are band-pass filters which are used for feature extraction, and texture analysis.
- They are both directional and frequency-selective filters and has the **optimal space-frequency resolution**, i.e. the best joint space-frequency localization.
- A Gabor filter is basically a Gaussian multiplied by a complex sinusoid. In 2D cases,

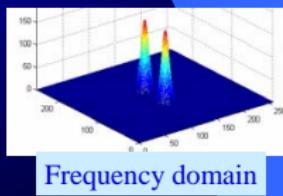
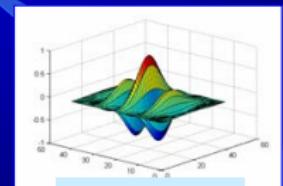
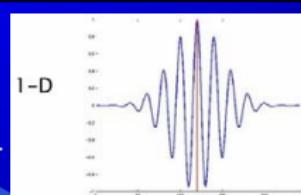
$$h(x, y) = g(x, y) \cdot s(x, y)$$

where,

$$g(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2\right]\right\}$$

$$s(x, y) = \exp[-j2\pi(ux + vy)]$$

$(u, v)$  are the 2D frequencies of the complex sinusoid, and its orientation is given by  $\phi = \arctan(v/u)$





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## Problem 2: Indeterminate Region

*Note that there are some regions in which a decision cannot be reached, which are labeled as IR (Indeterminate Region)(see P6:6). In these regions either more than one decision functions are greater than zero or all decision functions are less than zero. There are three functions which have been defined In P6:9, including  $d1(x)$ ,  $d2(x)$  and  $d3(x)$ . Which regions can be taken as IR?*



# Problem 2: Indeterminate Region

The figure shows a 2D coordinate system with axes labeled  $f_1$  and  $f_2$ . A diagonal line, labeled 'decision surface', separates the space into two regions. The upper-left region is shaded gray and labeled 'Region of class  $C_1$ '. The lower-right region is also shaded gray and labeled 'Region of class  $C_2$ '. A vector  $t$  is shown originating from the decision surface, pointing towards the class  $C_1$  region. The angle between the decision surface and vector  $t$  is indicated by a right-angle symbol.

**Decision Function**

- Example: 2-D decision space
- As a way of introduction to this relatively simple concept, consider two pattern classes, which can be conveniently separated by a line.

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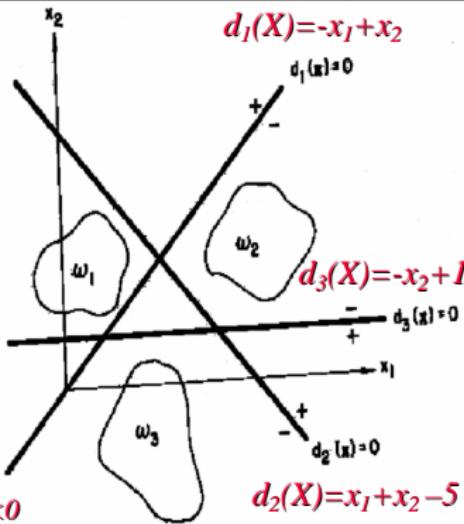


# Problem 2: Indeterminate Region

## Linear Decision Function

### Example 2: Simple multi-class case

- It is noted that each class is separable from the rest by a single decision boundary. For instance, if a specific pattern  $X$  belongs to class  $\omega_1$ , it is clear from the geometry that  $d_1(X) > 0$  while  $d_2(X) < 0$  and  $d_3(X) < 0$
- The boundary between class  $\omega_1$  and the other classes is given by the values of  $X$  for which  $d_1(X) = 0$ .



$d_1(x)$ ,  $d_2(x)$  and  $d_3(x)$ : 000, 011, 101, 110, 111





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## Problem 3: Classifier

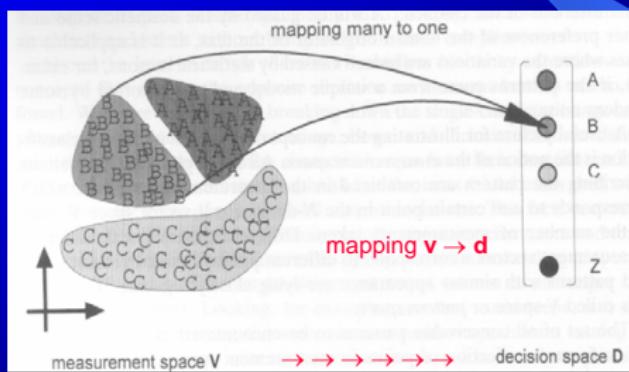
*There are two spaces, Measurement Space and Decision Space, which are given in P6:5. Pattern classification is defined to map measurement features to one of the classes. Could you design a simple classifier (see P4:36 and P4:39) to implement the given function?*



# Problem 3: Classifier

## Pattern Classification

- The principal function of a PR system is to yield decisions *concerning the class membership of the patterns* with which it is confronted.
- In order to accomplish this task, it is necessary to establish some rule upon which to base these decisions. One important approach is the use of **decision functions**.





# Problem 3: Classifier

## Inner Products

We can analyze minimum-distance classifiers easily by using a little **linear algebra**. Let  $\mathbf{x}$  stand for a column vector of  $d$  features,  $x_1, x_2, \dots, x_d$ . By using the **transpose operator**  $'$  we can convert the column vector  $\mathbf{x}$  to the row vector  $\mathbf{x}'$ :

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_d \end{bmatrix} \quad \mathbf{x}' = [x_1 \ x_2 \ \dots \ x_d]$$

The **inner product** of two column vectors  $\mathbf{x}$  and  $\mathbf{y}$  is defined by

$$\mathbf{x}'\mathbf{y} = x_1 y_1 + x_2 y_2 + \dots + x_d y_d = \sum_{k=1}^d x_k y_k$$

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# Problem 3: Classifier

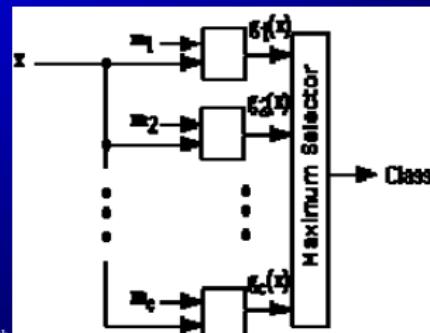
## Linear Discriminants (2)

Let us define the **linear discriminant function**  $g(x)$  by

$$g(x) = \mathbf{m}' \mathbf{x} - 0.5 \|\mathbf{m}\|_2^2.$$

Then we can say that a minimum-Euclidean-distance classifier classifies an input feature vector  $\mathbf{x}$  by computing  $c$  linear discriminant functions  $g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_c(\mathbf{x})$  and assigning  $\mathbf{x}$  to the class corresponding to the maximum discriminant function. We can also think of the linear discriminant functions as measuring the correlation between  $\mathbf{x}$  and  $\mathbf{m}_k$ , with the addition of a correction for the "template energy" represented by  $\|\mathbf{m}_k\|_2$ . With this correction included, a minimum-Euclidean-distance classifier is equivalent to a maximum-correlation classifier.

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# Problem 4: Syntactic PR

An example for Syntactic PR is given in P6:16 by the written (English) language, where a paragraph may be decomposed into sentences → words → letters → strokes. Consider the sentence: “The powerful computer could do many things for us.” produced by using your sequence of “rewriting” rules.



# Problem 4: Syntactic PR

## Syntactic PR

- **Example: Sentence Formation as Productions**

Consider the sentence: '**The boy runs quickly.**' produced by using the following sequence of 'rewriting' rules.

1. <sentence>
2. <noun phrase><verb phrase>
3. <article><noun><verb phrase>
4. the <noun><verb phrase>
5. the boy <verb phrase>
6. the boy <verb><adverb>
7. the boy runs <adverb>
8. the boys runs quickly



# Problem 4: Syntactic PR

- $< sentence >$
- $< noun\ phrase >< verb\ phrase >< preposition\ phrase >$
- $< noun\ phrase >< verb\ phrase >< noun\ phrase >< preposition\ phrase >$
- $< article >< adjective >< noun >< auxiliary\ verb >< main\ verb >< adjective >< noun >< preposition >< pronoun >$
- $the < adjective >< noun >< auxiliary\ verb >< main\ verb >< adjective >< noun >< preposition >< pronoun >$
- $the\ powerful < noun >< auxiliary\ verb >< main\ verb >< adjective >< noun >< preposition >< pronoun >$



## Problem 4: Syntactic PR

- *the powerful computer < auxiliary verb >< main verb >< adjective >< noun >< preposition >< pronoun >*
- *the powerful computer could < main verb >< adjective >< noun >< preposition >< pronoun >*
- *the powerful computer could do < adjective >< noun >< preposition >< pronoun >*
- *the powerful computer could do many < noun >< preposition >< pronoun >*
- *the powerful computer could do many things < preposition >< pronoun >*
- *the powerful computer could do many things for < pronoun >*
- *the powerful computer could do many things for us*





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## Problem 5: Frequency Transform

*Frequency transform can be applied to many different applications (P6:24, 35-38, 41-45). Could you consider a new application using frequency transform in image processing and pattern recognition?*



# Problem 5: Frequency Transform

## *Fourier Transform*

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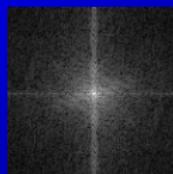
Lecture 6 - 24



# Problem 5: Frequency Transform

## 2D FT: Applications

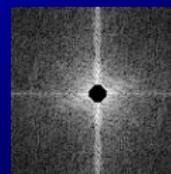
- Fourier transform for the image enhancement and image analysis
  - High-pass filter** (sharpens): restrict data to high frequency components
  - Low-pass filter** (blurs): restrict data to low frequency components



unprocessed



Bio lowpass application



highpass

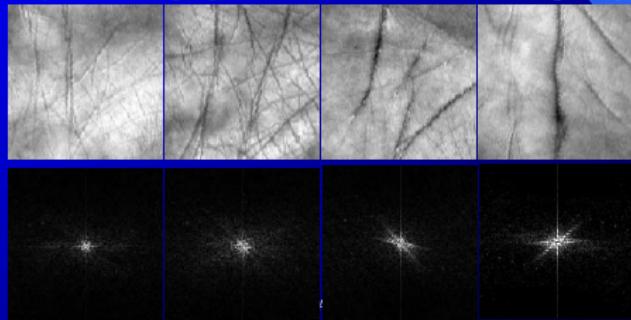
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# Problem 5: Frequency Transform

## 2D FT: Applications (1)

- There exist some correspondences between the features on the spatial domain image and those on the frequency domain image.
  - The **stronger the lines** on the spatial domain image, **the less compact the information** on the frequency domain image;
  - A strong line in the spatial domain will have **more information** along the line's perpendicular direction in the frequency domain.

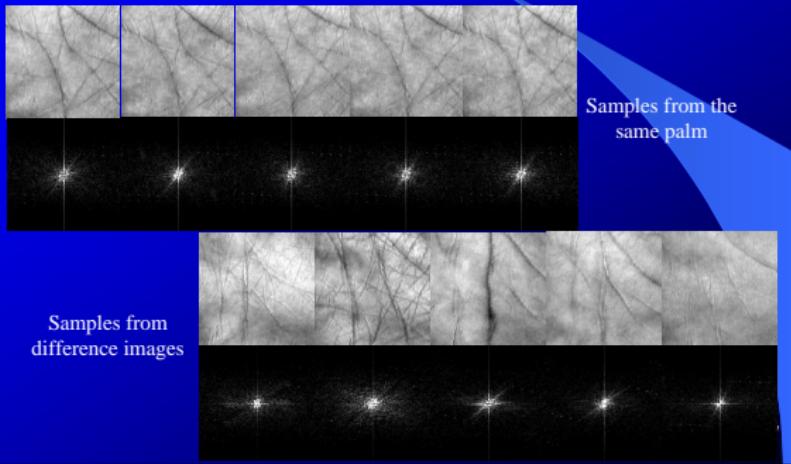




# Problem 5: Frequency Transform

## 2D FT: Applications (2)

- The features in frequency domain should be discriminative.
  - Features extracted from similar images resemble each other
  - Features vary with different appearance images

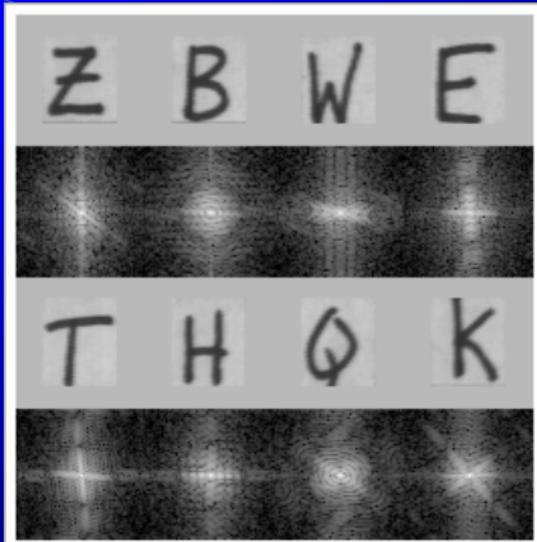




# Problem 5: Frequency Transform

## 2D FT: Applications (3)

- The Letters have quite different FTs, especially at the lower frequencies. The FTs also tend to have bright lines that are perpendicular to lines in the original letter. If the letter has circular segments, then so does the FT.

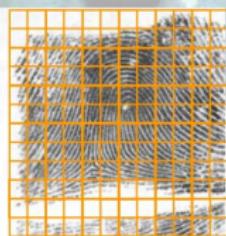




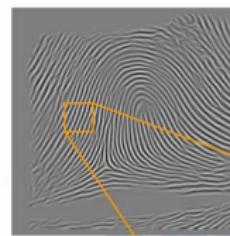
# Problem 5: Frequency Transform

## Gabor Filters: Applications

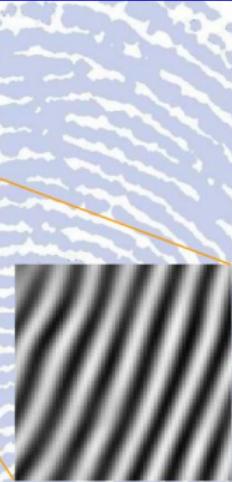
Fingerprint identification: image enhancement



Divide into cells,  
each has an  
orientation angle  
and a frequency



Apply gabor filter  
corresponding to a  
specific orientation  
and frequency,  
to each cell

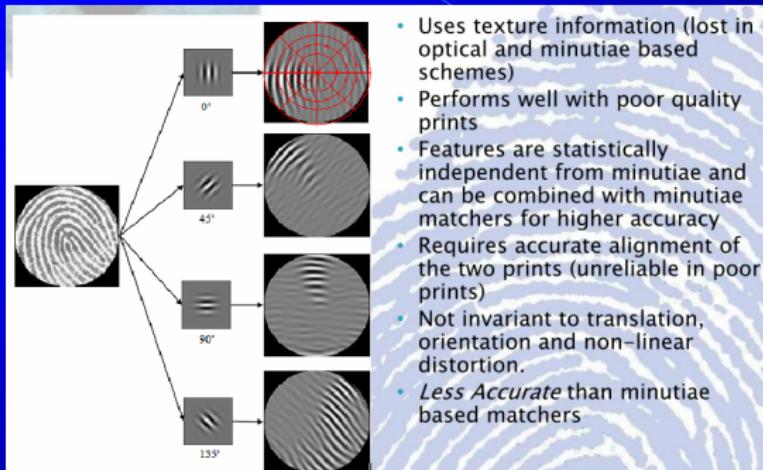




# Problem 5: Frequency Transform

## Gabor Filters: Applications (1)

Fingerprint verification: texture analysis





# Problem 5: Frequency Transform

## Gabor Filters: Applications (2)

Palmprint identification: texture analysis

The diagram illustrates the process of palmprint identification using Gabor filters. It shows a sequence of four panels:

- Original palmprint image:** A grayscale image of a palmprint with a red crosshair indicating the center. A blue rectangular box labeled "Copped Region Of Interest" is overlaid on the image.
- Copped Region Of Interest:** A close-up view of the ROI area, showing the skin texture in grayscale.
- Real parts of the results after filtering with the 4 Gabor filters:** Four separate grayscale images showing the filtered results for different orientations (0°, 45°, 90°, 135°).
- The filters:** Four small grayscale images showing the Gabor filter kernels used for each orientation.

**Gabor filter revisited:**

- a Gabor filter bank consists of 4 complex gabor filter that differ only in its orientation, i.e. of orientation  $0^\circ, 45^\circ, 90^\circ, 135^\circ$
- each filter is applied to the ROI, strengthening texture at a specific orientation, at the same time attenuating noise

A 3D surface plot is also shown, representing the response of a Gabor filter across a 2D spatial domain.

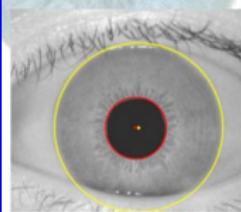


# Problem 5: Frequency Transform

## Gabor Filters: Applications (3)

Iris identification: texture coding

Original image of an eye



"Unrolled" iris



Real Part of Iris Code



Imaginary Part of Iris Code



- Any given iris has a unique texture that is generated through a random process before birth
- Gabor filters turn out, again, to be very good at detecting patterns in this unique texture
- Matching is done again in hamming distance comparison



# Problem 5: Frequency Transform

## Gabor Filters: Applications (4)

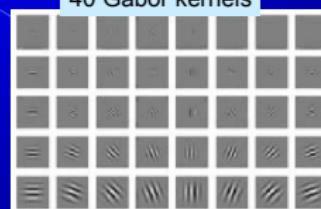
Face recognition: Gaborface

Cropped face image

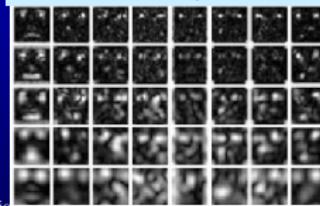


- A Gabor wavelet bank consists of 40 different filters and exhibit desirable characteristics of spatial frequency, spatial locality, and orientation selectivity
- Gaborface, representing one face image, is computed by convoluting it with corresponding Gabor filters

40 Gabor kernels



Gabor wavelet representation





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# Problem 6: Frequency Transform

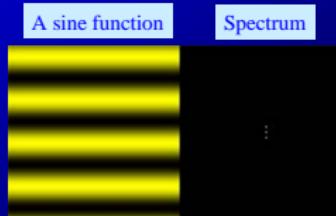
*What results could you obtain from the examples using 2D Fourier Transforms in P6:29-34?*



# Problem 6: Frequency Transform

## 2-D Fourier Transform

- A sine function with a DC component added
  - The center dot is the **DC component**
    - The DC component, standing for Direct Current, is the center of the spectrum (Fourier space) of the image and represents the average value of the image
  - The two others represent the frequency of the sine function
    - The one dot is just a mirrored version of the other one
  - No dots in the x-direction because the image is the same everywhere in that direction



Lecture 6 - 29

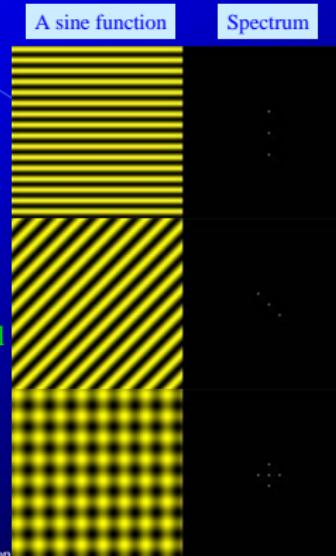
constant frequency - dot



# Problem 6: Frequency Transform

## 2-D Fourier Transform

- A sine function with a higher frequency
  - The two dots are further away from the origin to represent the higher frequency
  - The image contracts, and its spectrum becomes wider
- If the image is rotated, the spectrum will rotate in the same direction
- The sum of 2 sine functions, each in another direction



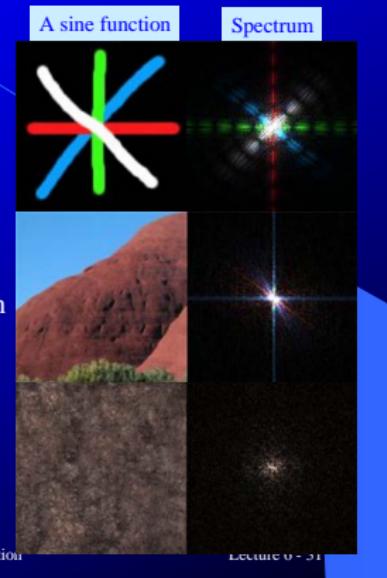
directions of the frequencies



# Problem 6: Frequency Transform

## 2-D Fourier Transform

- Lines in an image often generate perpendicular lines in the spectrum
- The sloped lines in the spectrum here are obviously due to the sharp transition from the sky to the mountain
- Here's the FT of a tillable texture. Since it's tillable there are no abrupt changes on the horizontal and vertical sides, so there are no horizontal and vertical lines in the spectrum



Biometric Authentication

Lecture 6 - 31

lines gets responses in their orthogonal directions



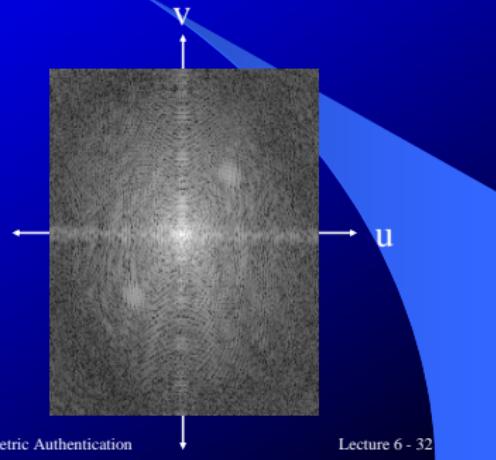
# Problem 6: Frequency Transform

## Information in Fourier Space

Image



Fourier Space

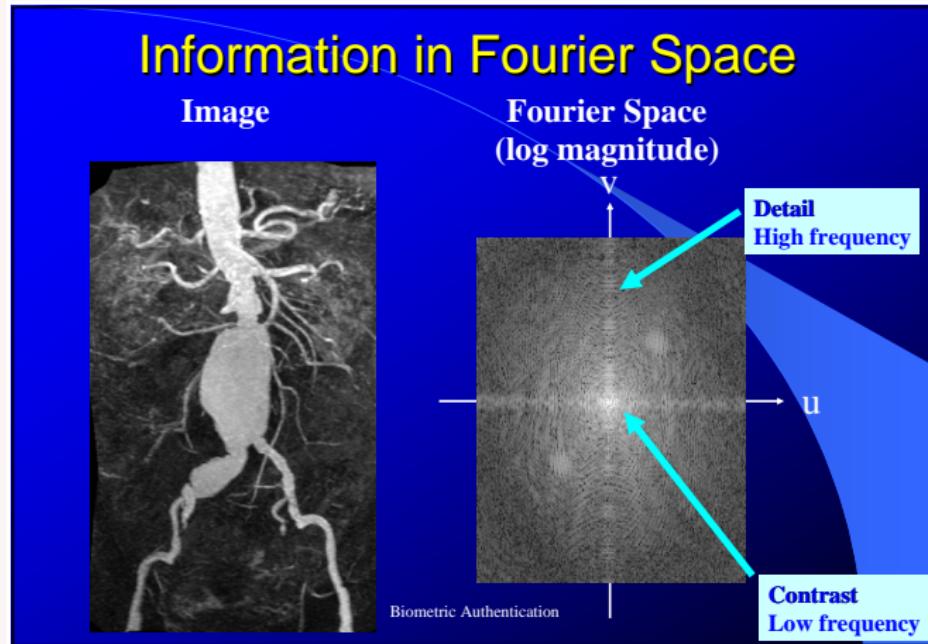


Biometric Authentication

Lecture 6 - 32

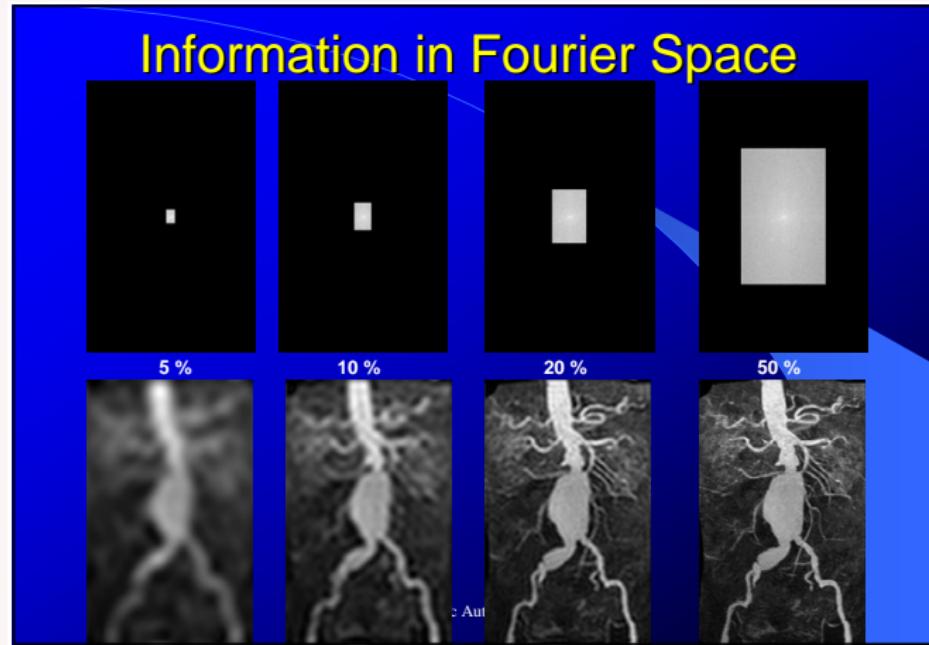


# Problem 6: Frequency Transform





# Problem 6: Frequency Transform



frequency resolution





# Problems

Any questions?