Tutorial 4

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COMP210
Discrete Structure

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Problem 1.4

The statement is true.

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LHS:

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Problem 7 direct proof

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$$\begin{aligned}
\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} &= \sum_{i=1}^{n} \left(\frac{1}{2i-1} - \frac{1}{2i+1}\right) \cdot \frac{1}{2} \\
&= \frac{1}{2} \cdot \left(1 - \frac{1}{2n+1}\right) \\
&= \frac{n}{2n+1}
\end{aligned}$$

Problem 7 induction

Basis step: for $n = 1$ $\quad \frac{1}{1 \cdot 3} = \frac{1}{3} = \frac{1}{2 \cdot 1 + 1}$

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$$\begin{aligned}
\sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} &= \sum_{i=1}^{k} \left(\frac{1}{2i-1} - \frac{1}{2i+1}\right) - \left(\frac{1}{2k+1} - \frac{1}{2k+3}\right) \\
&= \frac{k}{2k+1} + \left(\frac{1}{2k+1} - \frac{1}{2k+3}\right) \\
&= \frac{2k^2+3k+1}{(2k+1)(2k+3)} \\
&= \frac{k+1}{2k+3} = \frac{(k+1)}{2(k+1)+1}
\end{aligned}$$

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$$\sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} = \sum_{i=1}^{k} \left(\frac{1}{2i-1} - \frac{1}{2i+1} \right) - \left(\frac{1}{2k+1} - \frac{1}{2k+3} \right)$$
$$= \frac{k}{2k+1} + \left(\frac{1}{2k+1} - \frac{1}{2k+3} \right)$$
$$= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$$
$$= \frac{k+1}{2k+3} = \frac{(k+1)}{2(k+1)+1}$$

By the principle of math induction, the statement is true.

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Q & A

Questions about the problems?