

Tutorial 4

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COMP210
Discrete Structure

November 3, 2011



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- Q & A

Outline



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Problem 1.4

The statement is true.

$$X \cap (Y - Z) = (X \cap Y) - (X \cap Z)$$



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RHS:

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LHS:

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Problem 2



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- Let $x_i, i = 1, \dots, 9$ be the numbers of coins in the i^{th} bag.
By hypothesis, we have

$$\left\{ \begin{array}{l} x_i \geq 1, i = 1, \dots, 9 \\ \sum_{i=1}^n x_i = 40 \end{array} \right\}$$



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- contradiction here!
- then the original statement is correct.

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Problem 7 direct proof

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Problem 7 induction

Basis step: for $n = 1$ $\frac{1}{1 \cdot 3} = \frac{1}{3} = \frac{1}{2 \cdot 1 + 1}$



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By the principle of math induction, the statement is true.



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Questions about the problems?