

# FRE 6711 Quantitative Portfolio Management

## Report of Final Project

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### 1. Introduction

This report tries different models to build the best factor-based Long/Short Macro Global Strategy by using 13 ETFs . After the research, we find that by setting rational risk preference level and portfolio  $\beta$  fixed, we can simulate a portfolio that follows the same trend but with higher return of S&P500 index.

Our report is divided into three parts.

In the first part, we test three different optimization models and express the strength and weakness of each model. The original model is setting target  $\beta$  fixed and maximizing the expected return. The second model is setting target  $\beta$  fixed and minimizing portfolio volatility. And the third one is setting target  $\beta$  and a personal preference level  $\lambda$  for volatility and turnover. Based on the risk and return preference of our group, we choose the last model to build our Macro Global Strategy.

In the second part, we use the chosen model to back test strategy performance during the period 2007-03-23 to 2016-03-01, based on different estimators (60 days, 90 days, 120days). By setting estimator fixed, we can get different performances with different target  $\beta$ s of the optimal portfolios. Also we compare the return of the portfolios with different estimators, by setting target  $\beta$  fixed. The result of our back testing shows that 60-day portfolio has better returns during the crisis and 120-day portfolio has better returns after the crisis. Meanwhile, 120-day

portfolio has higher return from 2007 to 2016 in total. So we set our optimal portfolio estimator as 120 days, which we use in the next part.

In the third part, we analyze the result of the optimal portfolio and basic 13 ETFs. Also we set 2 benchmark portfolios, S&P500 Index and the portfolio that sets target return as 15% and minimizes the portfolio volatility. By comparing the returns with these portfolios, we find out that our strategy works well.

## 2.Best Optimization Model

In this section, we have tried 3 different optimization models to make sure our optimization efficient and attractive. As an original portfolio, we consider it as the form:

$$\left\{ \begin{array}{l} \max_{\omega \in R^n} \rho^T \omega - \lambda (\omega - \omega_p)^T Q (\omega - \omega_p) \\ \sum_{i=1}^n \beta_i^m \omega_i = \beta_T^M \\ \sum_{i=1}^n \omega_i = 1 \\ -2 \leq \omega_i \leq 2 \end{array} \right.$$

where

- $Q$  is the Identity Matrix,  $\omega_p$  is the composition of the reference portfolio, and  $\lambda$  is a small regularization parameter to limit the turnover which we use 0.01.
- $\beta_i^m = \frac{\text{cov}(r_i, r_M)}{\sigma^2(r_M)}$  is the  $\beta$  of security  $S_i$  as defined in the CAPM Model. So that  $\sum_{i=1}^n \beta_i^m \omega_i = \beta_p^m$  is the  $\beta$  of the portfolio.
- $\beta_T^M$  is the Portfolio Target  $\beta$ , in our test,  $\beta_T^M = 0.5, 1, 1.5$ .
- We use French-FAMA 3 factor model to estimate the expected return of 13 basic ETFs. Under that factor model, the return of a security is given by the formula

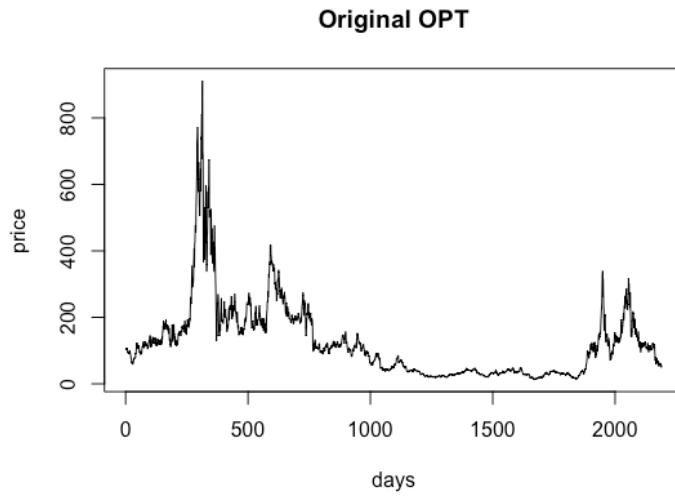
$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{SMB} + b_i^v r_{HML} + \alpha_i + \varepsilon_i$$

with  $E(\varepsilon_i) = 0$  in such a way that we have in terms of Expected Returns:

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{SMB} + b_i^v \rho_{HML} + \alpha_i$$

- Data sources are from Yahoo Finance and Quandl database. The time is from 3/25/2007 to 3/1/2016.

After the implementing strategy, we assume investing \$100 in this portfolio. And the return curve of the “Target Beta=1” portfolio is shown as Figure 1.



*Figure 1*

We find this optimization problem cannot limit the risk as we expected before. Also it cannot get stable and positive return in most of the time. After analyzing the model, we find some reasons. First, if we use CAPM to estimate the expected return of these 13 ETFs, setting Target Beta fixed is equivalent to setting expected return of the portfolio fixed.

$$\begin{aligned} \rho^T \omega &= \sum_{i=1}^n \rho_i^m \omega_i = \sum_{i=1}^n (r_f + \beta_i^m (\rho_M - r_f)) \omega_i = \sum_{i=1}^n r_f \omega_i + \sum_{i=1}^n \beta_i^m (\rho_M - r_f) \omega_i \\ &= r_f + \beta_T^M (\rho_M - r_f) \end{aligned}$$

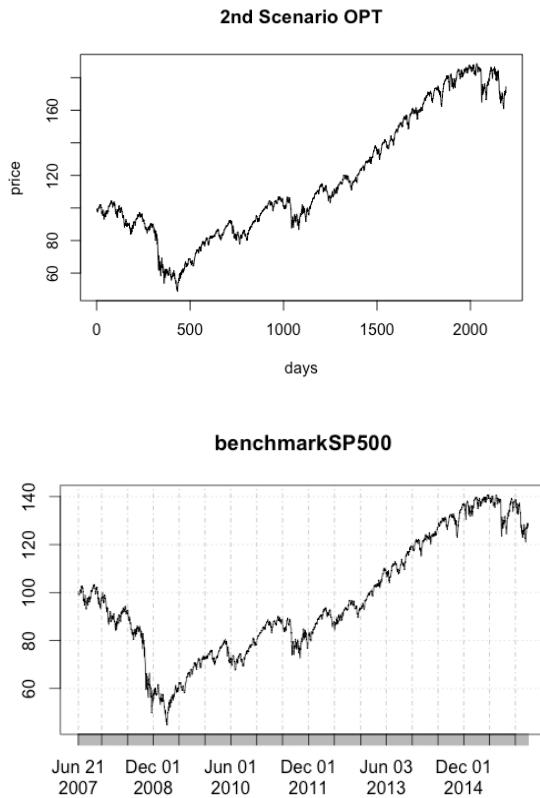
So if we change CAPM to French-FAMA 3 factor model, the expected return can be maximized. But it will depend on the turnover between CAPM and French-FAMA.

Usually when we want to get an efficient portfolio, we need to set fixed return and minimize risk level, or set risk level and maximize the return. The original optimization seems to use the 2<sup>nd</sup> method. However, by setting Portfolio Beta, it sets expected return of the portfolio and only regulates risk level relative to the market risk. Since market risk is still not certain, the risk level of portfolio is volatile.

To make our reason more convincing, we make another optimization problem that is shown below.

$$\left\{ \begin{array}{l} \min_{\omega \in R^n} \omega^T \Sigma \omega + \lambda (\omega - \omega_p)^T Q (\omega - \omega_p) \\ \sum_{i=1}^n \beta_i^m \omega_i = \beta_T^M \\ \sum_{i=1}^n \omega_i = 1 \\ -2 \leq \omega_i \leq 2 \end{array} \right.$$

In this model, we regulate the absolute volatility of the portfolio, instead of simply regulating relative risk level. The comparison of this model and S&P500 is listed in Figure 2.



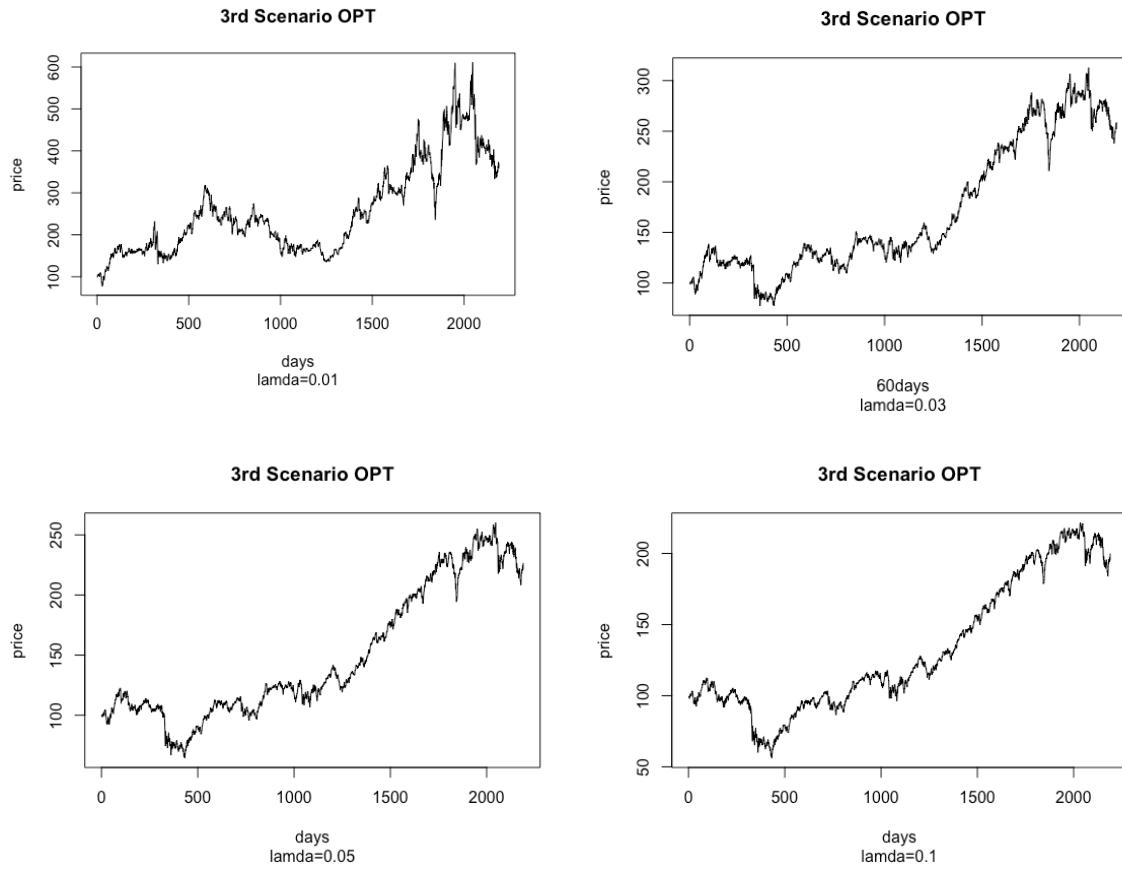
*Figure 2*

As we see, if we regulate the volatility of portfolio and set target Beta, it will simulate the movement of S&P and even a little higher return. So it can prove that for regulating risk level, fixed absolute volatility is much more efficient than fixed target Beta. Also, this strategy can be used as Global Macro Strategy. Because it simulates the movement of S&P, but has a better return. If we use S&P as a hedging asset, this strategy gets actually no risk profit from 2007 to 2016.

However, we want to make it further. As we know, asset managers have their own return-risk preference and love more profits during most of time. So we want to use  $\lambda$  to set both turnover of reference portfolio and risk preference.

$$\left\{ \begin{array}{l} \max_{\omega \in K^n} \rho^T \omega - \lambda \omega^T \Sigma \omega - \lambda (\omega - \omega_p)^T Q (\omega - \omega_p) \\ \sum_{i=1}^n \beta_i^m \omega_i = \beta_T^M \\ \sum_{i=1}^n \omega_i = 1 \\ -2 \leq \omega_i \leq 2 \end{array} \right.$$

If  $\lambda$  is small, for example, 0.01 or 0.05, it can be used in this function. If  $\lambda$  is too big, the reference portfolio will make big impact on the strategy. And we make several experiments by changing  $\lambda$ , in order to find the best optimization model for our preference. The performances of different  $\lambda$  are listed in Figure 3.



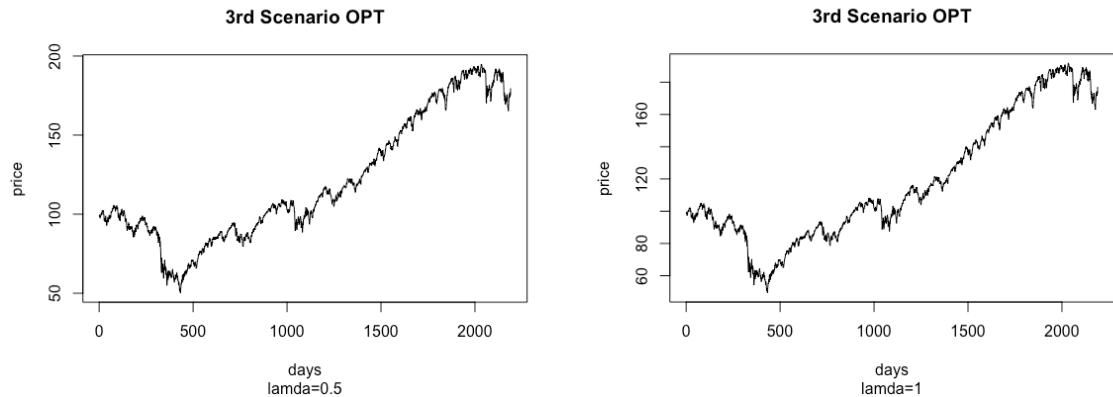


Figure 3

We can find as lambda becomes larger, the performance is more similar to S&P500 Index. From the point of us, we want to choose  $\lambda = 0.03$ , which is small enough to regulate the reference composition turnover. Also, this portfolio resists the crash during the crisis and comparing to other portfolio, for example,  $\lambda = 0.05$ , it has obviously higher profits. As a result, our ultimate best portfolio is shown in the form:

$$\left\{ \begin{array}{l} \max_{\omega \in R^n} \rho^T \omega - 0.03 \omega^T \Sigma \omega - 0.03 (\omega - \omega_p)^T Q (\omega - \omega_p) \\ \sum_{i=1}^n \beta_i^m \omega_i = \beta_T^M \\ \sum_{i=1}^n \omega_i = 1 \\ -2 \leq \omega_i \leq 2 \end{array} \right.$$

where

- We use CAPM to estimate  $\beta_i^m$  and French-FAMA 3 factor model to estimate  $\rho$ .
- $\Sigma$  is the covariance matrix of the 13 ETFs, based on the returns of 60,90,120 days.

### 3. Optimal Portfolio Analysis

Now we have structured our model of optimal portfolio, however, the behavior of the optimal portfolio built using a given estimator (long term -LT, mid term- MT or short term - ST) may change with the market environment and with the target Beta.

To understand the sensitivity of the behavior of our optimal portfolio model with the market environment, we testing our optimal portfolio model using a combination of different return/risk estimators and target Beta during three historical periods: before the subprime (2008) crisis, during the subprime crisis and after the crisis:

| Different Estimators:             | Different Target Beta:                                                  | Different Historical Period:                            |
|-----------------------------------|-------------------------------------------------------------------------|---------------------------------------------------------|
| Short-term estimators:<br>60 days | Reduced risk related to Market:<br>Beta=0.5                             | Before the subprime crisis:<br>2007-9-14 to 2007-11-30  |
| Mid-term estimators:<br>90 days   | Magnitude of Portfolio variance<br>equals to Market variance:<br>Beta=1 | During the subprime crisis:<br>2007-12-01 to 2009-05-31 |
| Long-term estimators:<br>120days. | Augmented risk related to<br>Market:<br>Beta=1.5                        | After the subprime crisis:<br>2009-06-01 to 2016-03-01  |

#### 3.1 Market Analysis

Before we analyze our portfolios, let's take a look at the market performance using the daily return of S&P500 to gain a statistical view of the market and the economic environment.

The statistical property table of the market before, during and after crises is as following:

|                                    | <b>Before Crises</b> | <b>During Crises</b> | <b>After Crises</b> |
|------------------------------------|----------------------|----------------------|---------------------|
| <b>Cumulated PnL</b>               | 0.998106448          | 0.620562544          | 2.152392393         |
| <b>Daily Mean Geometric Return</b> | 3.66E-06             | -0.001120315         | 0.000477171         |
| <b>Daily Min Return</b>            | -0.029369799         | -0.090349778         | -0.066634464        |
| <b>Max 10 days Drawdown</b>        | -0.071125192         | -0.250026813         | -0.159526697        |
| <b>Volatility</b>                  | 0.197013692          | 0.38931395           | 0.161472822         |
| <b>Sharpe Ratio</b>                | -0.073322055         | -0.710921368         | 0.698021167         |
| <b>Skewness</b>                    | 0.006738781          | 0.167185585          | -0.329269782        |
| <b>Kurtosis</b>                    | 0.588717247          | 3.345107122          | 3.396740245         |
| <b>Modified VaR</b>                | -0.020095982         | -0.038573936         | -0.016524712        |
| <b>CVaR</b>                        | -0.026363797         | -0.053770331         | -0.027687049        |

From this table we can clearly capture some properties of the market during these three periods.

- Return of the market

The geometric mean daily return after crises and before crises period is both positive. However after crises the returns are much higher than before crises. The geometric mean return during crises is negative and the absolute value of this number is the biggest one among those three.

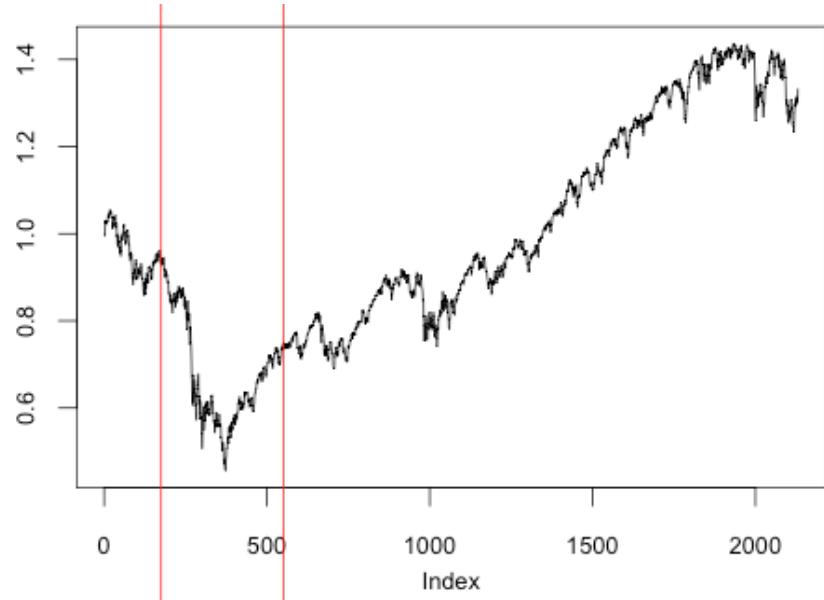
- Variance of the market

During crises, the market has the highest volatility, which shows the market is most turmoil during this period.

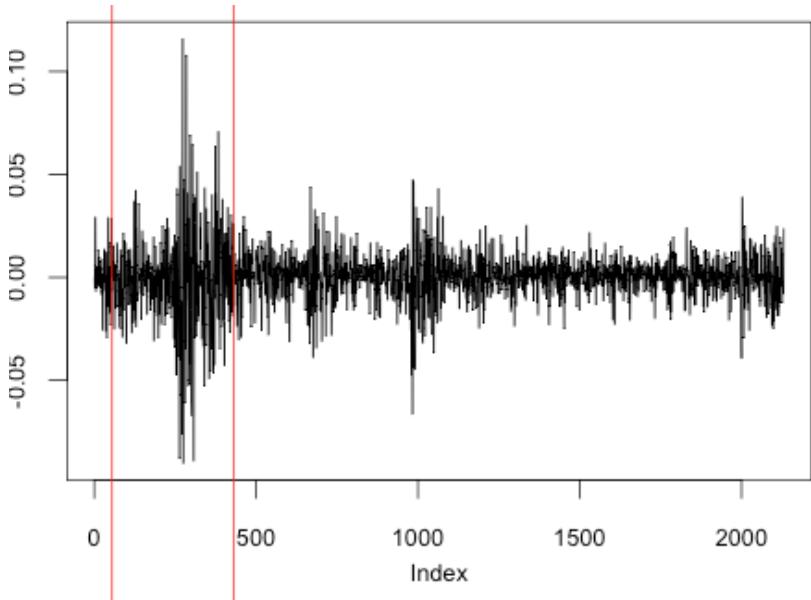
Another way to view the return and variance of the market is to look at the cumulative return plot and daily return plot as following.

The two red lines in the graphs indicate the beginning and end of the subprime crises.

The cumulative return plot:



The daily return plot:



## 3.2 Optimal Portfolio Analysis during Different Historical Period

### 3.2.1 Before the subprime crises:

As we run our code, we can get a summary table for the different performance of our model using different estimators and target Beta as following:

|                                    | 60days<br>Beta=0.5 | 60days<br>Beta=1 | 60days<br>Beta=1.5 | 90days<br>Beta=0.5 | 90days<br>Beta=1 | 90days<br>Beta=1.5 | 120days<br>Beta=0.5 | 120days<br>Beta=1 | 120days<br>Beta=1.5 |
|------------------------------------|--------------------|------------------|--------------------|--------------------|------------------|--------------------|---------------------|-------------------|---------------------|
| <b>Cumulated PnL</b>               | 1.2050246          | 1.218534418      | 1.220721563        | 1.162875529        | 1.18536745       | 1.192654766        | 1.157626136         | 1.156578749       | 1.152219968         |
| <b>Daily Mean Geometric Return</b> | 0.003475092        | 0.003704156      | 0.003782698        | 0.002822818        | 0.003205832      | 0.003369504        | 0.002731876         | 0.002745787       | 0.002726654         |
| <b>Daily Min Return</b>            | -                  | -                | -                  | -                  | -                | -                  | -                   | -                 | -                   |
| <b>Max 10 days Drawdown</b>        | 0.040166051        | 0.042321505      | 0.051164809        | 0.053266046        | 0.063092435      | 0.071347043        | 0.052449099         | 0.063425518       | 0.072841899         |
| <b>Volatility</b>                  | 0.106360311        | 0.127260804      | 0.150144027        | -                  | 0.129709335      | 0.155037373        | 0.103207308         | 0.131046988       | 0.157142984         |
| <b>Sharpe Ratio</b>                | 0.28199747         | 0.325550535      | 0.391591329        | 0.276080833        | 0.332651766      | 0.405038384        | 0.260031717         | 0.313932988       | 0.386666621         |
| <b>Skewness</b>                    | -                  | -                | -                  | -                  | -                | -                  | -                   | -                 | -0.21269805         |
|                                    | 0.524929158        | 0.251254727      | 0.013339552        | 0.710485052        | 0.419709398      | 0.148928052        | 0.803101374         | 0.515902907       |                     |
| <b>Kurtosis</b>                    | -                  | -                | -0.282239502       | 0.781272411        | 0.530562702      | 0.067725644        | 1.286690917         | 1.014061086       | 0.478366742         |
|                                    | 0.324255564        | 0.311908253      |                    |                    |                  |                    |                     |                   |                     |
| <b>Modified VaR</b>                | -                  | -0.03131075      | -                  | -                  | -                | -                  | -                   | -                 | -                   |
| <b>CVaR</b>                        | 0.028176048        | -                | 0.036664909        | 0.028617825        | 0.033169915      | 0.039249963        | 0.027105044         | 0.031901485       | 0.038194475         |
|                                    | 0.034691678        | -0.03944233      | -                  | -                  | -                | -                  | -                   | -                 | -                   |
|                                    |                    |                  | 0.045803536        | 0.039594367        | 0.045715877      | 0.051097205        | 0.039368383         | 0.046074734       | 0.051797297         |

If we look closely into these data we can find out best performer under different criteria:

```
>#Max cumulative return
> which(performanceBC[1]==max(performanceBC[1]))
```

```
60days Beta=1.5
> #min volatility
> which(performanceBC[5]==min(performanceBC[5,]))
120days Beta=0.5
```

```
> #max daily min return
> which(performanceBC[3]==max(performanceBC[3,]))
```

60days Beta=0.5

```
> #min max drawdown  
> which(performanceBC[4]==max(performanceBC[4]))  
120days Beta=0.5
```

```
> #max Sharpe ratio  
> which(performanceBC[6]==max(performanceBC[6]))  
60days Beta=0.5
```

- Return Analysis:

We have the portfolios with 60 days estimators and Beta=1.5 combination gains the highest return. The shortest estimator with the largest Beta gives us the max return. During the period just before crises hit, the economy tends to be overheated so that the shortest estimators combined with largest appetite of risk will contribute to a better performance.

**Result:** Best return before crises is with the shortest estimator =60 days and largest Beta=1.5.

- Risk Analysis

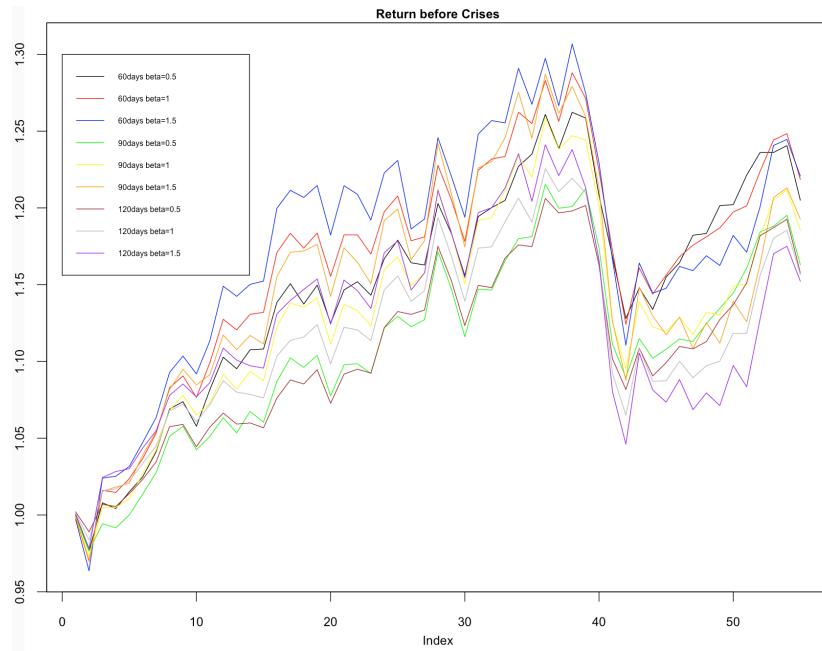
We include 3 different methods to measure the risk: Volatility, daily min return and max 10 days drawdown. As the result shows that 120 days Beta=0.5 portfolio suggests the min risk considering the volatility and max 10 days drawdown. It is not surprised that that Beta=0.5 portfolio suggests the min risk. It also suggests that during a period that the economy is about to recess, longer estimator and smaller Beta be more conservative and reduce the risk.

**Result:** The way to reduce the risk before crises is to use the longest estimator =120 days and smallest Beta=0.5.

- Sharpe Ratio

During this period, portfolio with 60-day Beta=0.5 has the highest Sharpe Ratio which suggests the best-balanced return with risk.

The different portfolios cumulative return plot as following:



### 3.2.2 During the subprime crises:

The different performance of our model using different estimators and target Beta is as following:

|                             | 60days<br>Beta=0.5 | 60days<br>Beta=1 | 60days<br>Beta=1.5 | 90days<br>Beta=0.5 | 90days Beta=1 | 90days<br>Beta=1.5 | 120days<br>Beta=0.5 | 120days<br>Beta=1 | 120days<br>Beta=1.5 |
|-----------------------------|--------------------|------------------|--------------------|--------------------|---------------|--------------------|---------------------|-------------------|---------------------|
| Cumulated PnL               | 1.008596465        | 0.804295107      | 0.612186691        | 0.839450602        | 0.669127067   | 0.505023302        | 0.823872695         | 0.651786022       | 0.488387419         |
| Daily Mean Geometric Return | 7.70E-05           | -0.000428965     | -0.0009877         | -0.000424858       | -0.000932873  | -0.001513451       | -0.000477075        | -0.001000457      | -0.001596012        |
| Daily Min Return            | -0.089019673       | -0.120713952     | -0.15023092        | -0.084836239       | -0.116654256  | -0.148472273       | -0.072123662        | -0.103831363      | -0.143477552        |
| Max 10 days Drawdown        | -0.200748037       | -0.285414118     | -0.374430914       | -0.167528237       | -0.261912759  | -0.359584374       | -0.145803543        | -0.252376561      | -0.353763876        |
| Volatility                  | 0.232718081        | 0.391209028      | 0.569524254        | 0.203613374        | 0.371836542   | 0.555197881        | 0.19845441          | 0.375569896       | 0.561745001         |
| Sharpe Ratio                | -0.00098372        | -0.358546268     | -0.497425044       | -0.566718049       | -0.644136973  | -0.665688691       | -0.636769338        | -0.672872215      | -0.682713041        |
| Skewness                    | -0.128229007       | 0.496309753      | 0.608558114        | -0.546176585       | 0.198842071   | 0.370644237        | -0.161908857        | 0.323611789       | 0.430007645         |
| Kurtosis                    | 8.52122299         | 8.026654477      | 7.37079196         | 6.829402453        | 6.052391957   | 5.963475222        | 5.791875145         | 5.762323458       | 5.8198163           |
| Modified VaR                | -0.022049749       | -0.033317079     | -0.048010724       | -0.021689098       | -0.035212004  | -0.050881491       | -0.020160975        | -0.034890997      | -0.051015259        |
| CVaR                        | -0.035172216       | -0.033317079     | -0.048010724       | -0.042797814       | -0.046792542  | -0.059240824       | -0.03358026         | -0.042651685      | -0.057111705        |

If we look closely into these data we can find out best performer under different criteria:

```
> #max cumulative return
> which(performanceDC[1]==max(performanceDC[1,]))
60days Beta=0.5

> #min volatility
> which(performanceDC[5]==min(performanceDC[5,]))
120days Beta=0.5
```

```
> #max daily min return
> which(performanceDC[3]==max(performanceDC[3,]))
120days Beta=0.5
```

```
> #min max drawdown
> which(performanceDC[4]==max(performanceDC[4,]))
120days Beta=0.5
> #max Sharpe ratio
> which(performanceDC[6]==max(performanceDC[6,]))
60days Beta=0.5
```

- Return Analysis

We have the portfolios with 60 days estimators and Beta=0.5 combination gains the highest return. The shortest estimator with the smallest Beta gives us the maximum return. This outcome shows us that normally higher Beta can contribute to higher return yet during financial crises, this relationship is no longer relevant. During crises, smallest Beta can de-correlate our portfolio to the downward market and reduce the losses. Meanwhile, during the financial crises, the market turns out to be very turmoil, which can be indicated by increasing volatility. The shortest estimators can capture the market variance most efficiently thus contribute to the best estimation of return and risk.

**Result:** Best return during crises is with the shortest estimator =60 days and smallest Beta=0.5.

- Risk Analysis

As the result shows that 120 days Beta=0.5 portfolio suggests the minimum risk. Beta=0.5 suggests the min risk related to the market thus reduced our overall risk of the portfolio. It also suggests that during a period that the economy recesses, longer estimator can reduce the risk.

**Result:** To reduce the risk during crises, we can structure our portfolios with the longest estimator =120 days and smallest Beta=0.5.

- Sharpe Ratio

During this period, portfolio with 60 days Beta=0.5 has the highest Sharpe Ratio which suggests the best-balanced return with risk.

The different portfolios cumulative return plot as following:



### 3.2.3 After the subprime crises:

The summary table of the different performance of our model using different estimators and target Beta is as following:

|                             | 60days Beta=0.5 | 60days Beta=1 | 60days Beta=1.5 | 90days Beta=0.5 | 90days Beta=1 | 90days Beta=1.5 | 120days Beta=0.5 | 120days Beta=1 | 120days Beta=1.5 |
|-----------------------------|-----------------|---------------|-----------------|-----------------|---------------|-----------------|------------------|----------------|------------------|
| Cumulated PnL               | 1.580251223     | 2.457408533   | 3.636462963     | 2.0236741       | 3.197406968   | 4.697381989     | 2.605492823      | 4.079827967    | 5.928334261      |
| Daily Mean Geometric Return | 0.000298217     | 0.000578468   | 0.000843428     | 0.000434399     | 0.000723816   | 0.000984415     | 0.000579648      | 0.000864045    | 0.0011184        |
| Daily Min Return            | -0.064790753    | -0.069205743  | -0.081198517    | -0.046774923    | -0.066867459  | -0.092205822    | -0.043568446     | -0.066074837   | -0.091445758     |
| Max 10 days Drawdown        | -0.133504198    | -0.152537863  | -0.225997803    | -0.085197151    | -0.141281686  | -0.21929309     | -0.090075776     | -0.171417959   | -0.247295922     |
| Volatility                  | 0.169774563     | 0.221675772   | 0.288381315     | 0.140050987     | 0.199300902   | 0.271338774     | 0.127122725      | 0.191058219    | 0.265688361      |
| Sharpe Ratio                | 0.372727016     | 0.607239799   | 0.700196284     | 0.733064291     | 0.900118013   | 0.914115843     | 1.136040543      | 1.164210533    | 1.097116298      |
| Skewness                    | -0.658983787    | -0.489605193  | -0.404288645    | -0.34828804     | -0.349402568  | -0.349446006    | -0.316755987     | -0.355508384   | -0.363367245     |
| Kurtosis                    | 3.497678584     | 2.093999809   | 2.004470607     | 2.817177069     | 2.184318632   | 2.318184835     | 2.629056239      | 2.384416919    | 2.672486416      |
| Modified VaR                | -0.018494827    | -0.023721642  | -0.03036654     | -0.014464224    | -0.020630732  | -0.028023057    | -0.012906952     | -0.019580111   | -0.027229752     |
| CVaR                        | -0.032353022    | -0.037149515  | -0.047023749    | -0.023542555    | -0.0322683    | -0.04425724     | -0.020767286     | -0.031181107   | -0.044220717     |

The best performer under different criteria:

```
> #max cumulative return  
> which(performanceAC[1]==max(performanceAC[1]))  
120days Beta=1.5
```

```
> #min volatility  
> which(performanceAC[5]==min(performanceAC[5]))  
120days Beta=0.5
```

```
> #max daily min return  
> which(performanceAC[3]==max(performanceAC[3]))  
120days Beta=0.5
```

```
> #min max drawdown  
> which(performanceAC[4]==max(performanceAC[4]))  
90days Beta=0.5
```

```
> #max Sharpe ratio  
> which(performanceAC[6]==max(performanceAC[6]))  
120days Beta=1
```

- Return Analysis

We have the portfolios with 120 days estimators and Beta=1.5 combination gains the highest return. This outcome shows us that under a normal economics environment (after the crises), the relationship that higher Beta contributes to higher return is hold. Meanwhile, during this period which market is not that turmoil, the longest estimator that holds the most data can contribute to the best estimation of return and risk.

**Result:** Best return after crises is with the longest estimator 120 day and largest Beta=1.5.

- Risk Analysis

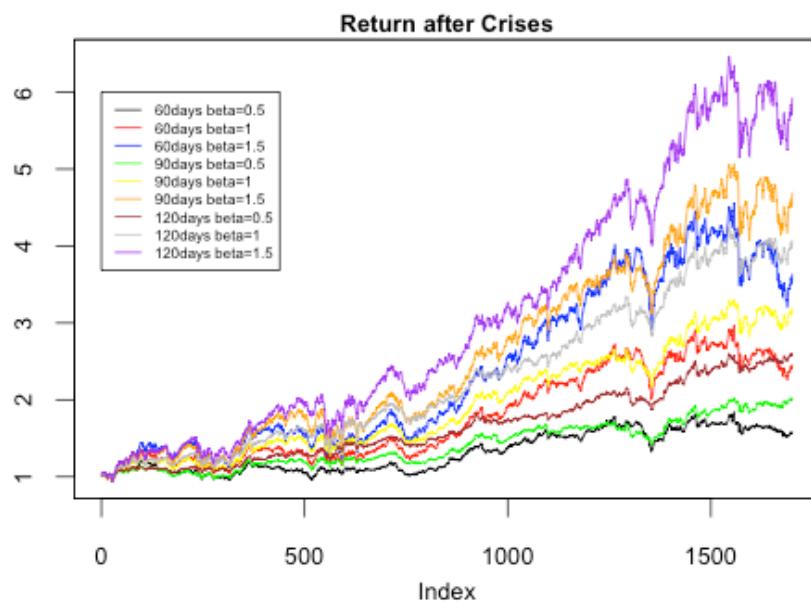
Among the three different methods to measure the risk: Volatility, daily min return and max 10 days drawdown, as the result shows, 120 days Beta=0.5 portfolio suggests the min risk considering the volatility and daily min return. It suggests that during the normal economy environment, longer estimator and smaller Beta be reduce the risk.

**Result:** To reduce the risk after crises, we can structure our portfolios with the longest estimator 120 day and smallest Beta=0.5.

- Sharpe Ratio

During this period, portfolio with 120 days Beta=1 has the highest Sharpe Ratio which suggests the best-balanced return with risk.

The different portfolios cumulative return plot as following:



### 3.3 Summary of the Historical Periods Analysis

The table of optimal portfolio under different market condition is as following:

|                          | <b>Before Crises</b> | <b>During Crises</b> | <b>After Crises</b> |
|--------------------------|----------------------|----------------------|---------------------|
| <b>Highest Return</b>    | 60 days Beta=1.5     | 60 days Beta=0.5     | 120 days Beta=1.5   |
| <b>Minimum Risk</b>      | 120 days Beta=0.5    | 120 days Beta=0.5    | 120 days Beta=0.5   |
| <b>Best Sharpe Ratio</b> | 60 days Beta=0.5     | 60 days Beta=0.5     | 120 days Beta=1     |

From this table, we can clearly observe these following conclusions:

- 1) When economic environment is normal (after crises), longer estimator will include more data and can contribute to finer estimations of risk and return and thus turns out a better performance.

In the linear regression we use to get the Beta of our asset to estimate the return, the more data we use, the smaller analytic standard errors will be during a relatively constant market environment.

The Standard Error of our estimation of  $\beta$ :

$$SE(\beta_i) = \frac{\sigma_{\varepsilon,i}}{\sqrt{T * \sigma_M^2}}$$

$SE(\beta_i)$  will come close to 0 as the estimator T comes larger.

- 2) During a normal economic environment, higher Beta can lead to higher return and lower Beta can reduce market related risk.

When setting Beta, we assume this following model for each security we invest:

$$R_{it} = \alpha_i + \beta_i * R_{Mt} + \varepsilon_{it}$$

$\varepsilon_{it}$  is identical independent random variable and will be diversified away while the number of the securities  $i$  become larger.  $\beta_i$  is the market risk that we cannot be diversified away. Thus lower Beta can reduce risk meanwhile higher Beta usually related to higher return.

- 3) During a bad economic environment (financial crises), the relationship between higher Beta and higher return is not hold.
- 4) Longer estimator can contribute to lower risk no matter what economic environment is involved.
- 5) During some special periods (before crises and during crises), the market is not as constant as usual and is more turmoil thus shorter estimator can lead to better estimation and higher return consequently.

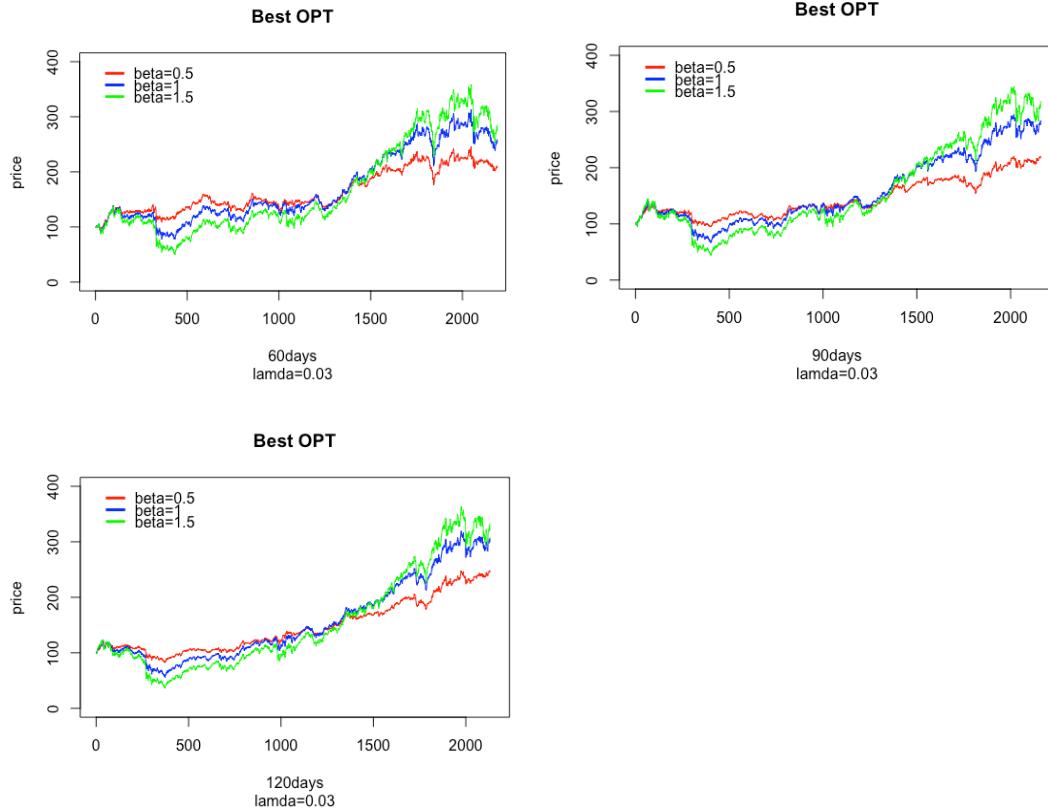
As financial crises are both uncommon and not easy to be predicted, so during the normal economic environment as now, we should choose the estimator =120 days as our optimal portfolios estimator and tailor the Beta with the preference of risk we want.

## 4. Over All Period Analysis

To gain a more global view of different performance with different estimators and Betas, we compare the 3 different Beta using same estimator and three different estimators using same Beta over the whole invest horizon of 9 years.

### 4.1 Different Beta comparison

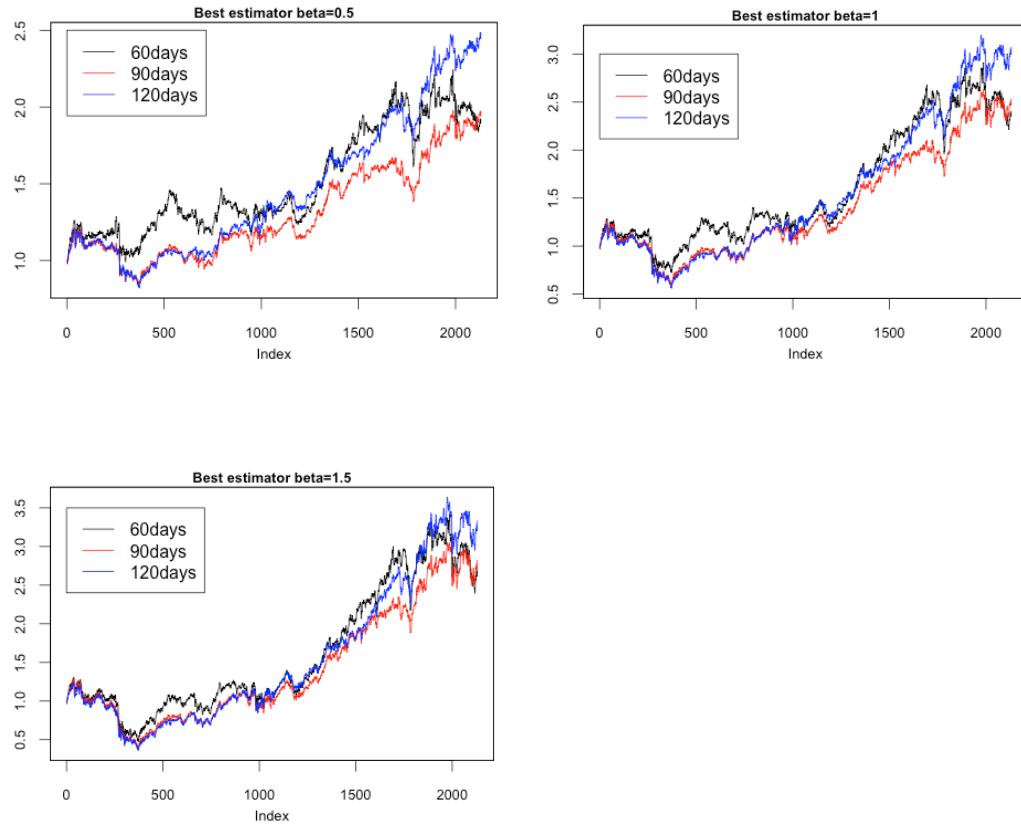
We plot the different Beta cumulative return under the same estimator as following:



From the graphs above we can clearly observe that over a very long term, higher Beta can lead to higher cumulative.

#### 4.2 Different estimator comparison

We plot the different estimator cumulative return under the same Beta as following:



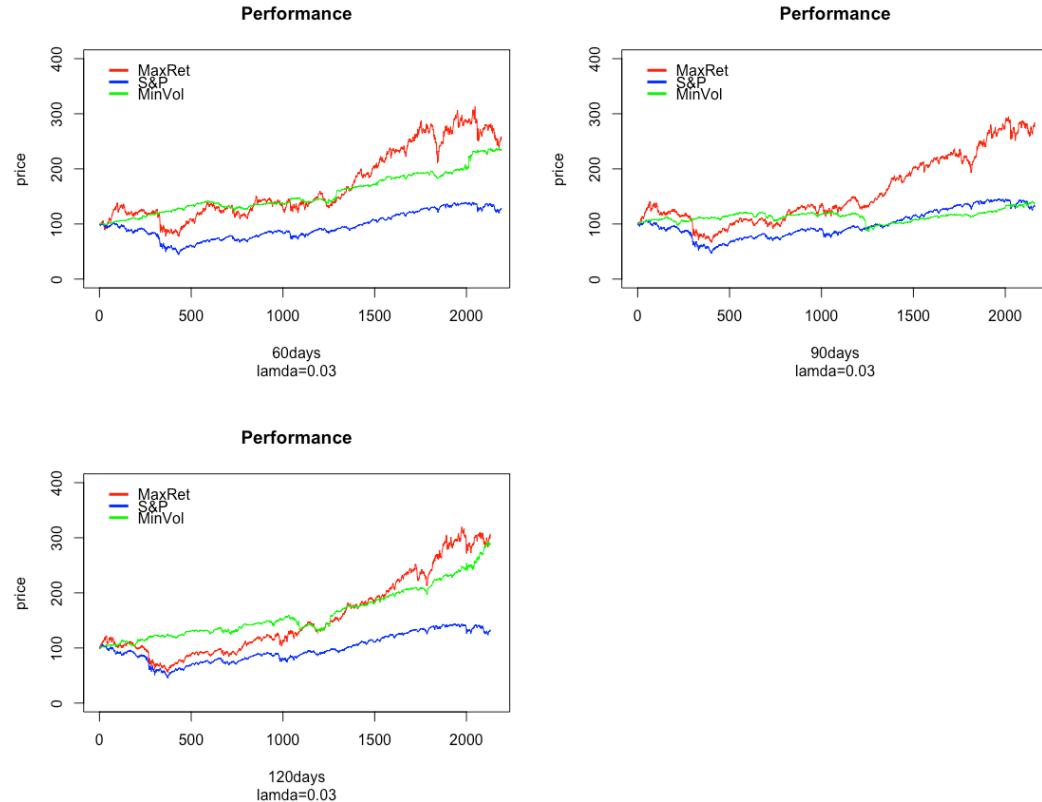
From the graphs above we conclude that over all three historical periods, longer estimator leads to higher cumulative return.

Following is the Pnl summary table of those nine portfolios:

|                | Beta=0.5    | Beta=1      | Beta=1.5    |
|----------------|-------------|-------------|-------------|
| <b>60days</b>  | 2.104158245 | 2.589493298 | 2.855983818 |
| <b>90days</b>  | 2.206133584 | 2.846838312 | 3.188393885 |
| <b>120days</b> | 2.484953774 | 3.07554511  | 3.336049975 |

### 4.3 Comparison with the Benchmarks

Optimal portfolio with a moderate risk preference (Beta=1) performance comparing with the two benchmarks is as following:



### 4.4 Comparison Table of the Investment Universe, Betas and Benchmarks

As we analyze above that under the most scenario, a portfolio with a 120 days estimator performs better than others. So we structured a summary table contains all the 13 securities we use in our portfolio, different Beta with the 120 day estimator and two benchmarks.

The table is as following:

|                             | FXE          | EWJ          | GLD          | QQQ          | SPY          | SHV          |
|-----------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Cumulated PnL               | 0.801868064  | 0.938340991  | 1.6805079    | 2.326883622  | 1.590552508  | 1.047298517  |
| Daily Mean Geometric Return | -9.22E-05    | 2.78E-05     | 0.000285099  | 0.000447717  | 0.000265979  | 2.17E-05     |
| Daily Min Return            | -0.030749545 | -0.104077268 | -0.087808262 | -0.089556676 | -0.098447689 | -0.00361992  |
| Max 10 days Drawdown        | -0.081441365 | -0.241088122 | -0.202335958 | -0.232716665 | -0.249482847 | -0.00361992  |
| Volatility                  | 0.107319577  | 0.240627413  | 0.203243971  | 0.226268104  | 0.219452708  | 0.003996787  |
| Sharpe Ratio                | -0.292123234 | -0.055338835 | 0.278153394  | 0.431657465  | 0.226620939  | -0.127725633 |
| Skewness                    | 0.119462684  | 0.440540783  | -0.081139035 | 0.103882045  | 0.23917286   | -0.447531909 |
| Kurtosis                    | 2.012186879  | 11.36177353  | 6.174578376  | 7.148362865  | 13.10239926  | 44.04478844  |
| Modified VaR                | -0.010734633 | -0.019491557 | -0.01950549  | -0.020544899 | -0.017882782 | -0.000200525 |
| CVaR                        | -0.014798724 | -0.019491557 | -0.031622381 | -0.028562412 | -0.017882782 | -0.000200525 |

|                             | DBA          | USO          | XBI          | ILF          | GAF          | EPP          |
|-----------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Cumulated PnL               | 0.7580365    | 0.152069142  | 2.759641032  | 0.631476191  | 0.982369788  | 1.021541642  |
| Daily Mean Geometric Return | -9.09E-05    | -0.000754319 | 0.000567748  | -8.06E-05    | 9.17E-05     | 0.000100533  |
| Daily Min Return            | -0.086059744 | -0.106845638 | -0.101011637 | -0.194666661 | -0.136226286 | -0.112218109 |
| Max 10 days Drawdown        | -0.228228205 | -0.248964784 | -0.247020492 | -0.38334154  | -0.273112964 | -0.304142602 |
| Volatility                  | 0.197476409  | 0.359365582  | 0.301609092  | 0.368215886  | 0.316604007  | 0.301307105  |
| Sharpe Ratio                | -0.19101445  | -0.565105706 | 0.397425585  | -0.157881155 | -0.025242244 | -0.011378893 |
| Skewness                    | -0.203300462 | -0.012892855 | -0.045819024 | 0.441735429  | 0.216504328  | 0.299636894  |
| Kurtosis                    | 5.319241709  | 2.327001575  | 2.638036407  | 14.58891838  | 8.511341066  | 9.364189774  |
| Modified VaR                | -0.019962006 | -0.037017174 | -0.029942584 | -0.02837825  | -0.028048397 | -0.025891178 |
| CVaR                        | -0.033929037 | -0.053540584 | -0.044857693 | -0.02837825  | -0.03125758  | -0.025891178 |

|                             | FEZ          | Beta=0.5     | Beta=1       | Beta=1.5     | Benchmark 2  | S&P 500      |
|-----------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Cumulated PnL               | 0.741536054  | 2.484953774  | 3.07554511   | 3.336049975  | 2.893516669  | 1.333164892  |
| Daily Mean Geometric Return | -3.82E-05    | 0.000449104  | 0.000584221  | 0.000681745  | 0.000511595  | 0.000183604  |
| Daily Min Return            | -0.114309172 | -0.072123662 | -0.103831363 | -0.143477552 | -0.033499861 | -0.090349778 |
| Max 10 days Drawdown        | -0.269914391 | -0.145803543 | -0.252376561 | -0.353763876 | -0.129708854 | -0.250026813 |
| Volatility                  | 0.320082801  | 0.146984164  | 0.23780857   | 0.340305554  | 0.112841749  | 0.220217336  |
| Sharpe Ratio                | -0.125605161 | 0.722330843  | 0.564338869  | 0.426321145  | 1.117602988  | 0.128084567  |
| Skewness                    | 0.257258657  | -0.355150137 | -0.038428034 | 0.090671188  | 0.736483464  | -0.059343461 |
| Kurtosis                    | 6.984223623  | 5.63615782   | 8.56422526   | 10.18989701  | 10.35559006  | 9.357795835  |
| Modified VaR                | -0.02886838  | -0.014675539 | -0.021657572 | -0.029613279 | -0.008154304 | -0.020276026 |
| CVaR                        | -0.034963273 | -0.027026814 | -0.032124054 | -0.032705023 | -0.008154304 | -0.029223729 |

## **5. Conclusion**

After analyzing and back testing the strategy through the past nine years, we find our best portfolio, which has relatively higher return than S&P and another benchmark portfolio. But as for the Sharpe Ratio, VaR, CVaR, our portfolio is not as good as the second benchmark portfolio, which is minimizing the volatility with fixed expected return. So we conclude that our portfolio has higher risk than second benchmark.

Also the length of the estimators makes impact on the performances in different periods. If we expect the market is stable in a period of time, 120-day strategy is the best choice. However, if we cannot make sure the market trend, which is volatile during this period, 60-day strategy is much better than others.

The portfolio managers and investors always try to find a balance between the return and risk. The best portfolio depends on the personal risk preference. Since our portfolio has a satisfied combination of risk and profit for us, we consider it a suited strategy. If more profit is wanted, setting Beta=1.5 is a good choice. If limited risk is preferred, Beta=1 or Beta=0.5 might be better target to set.