

# Coordinated Path Following for Multiple Underactuated Surface Vehicles with Error Constraints

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**Abstract**—In this paper, the coordinated path following control of multiple underactuated surface vehicles with error constraints is studied. By combining tan-type barrier Lyapunov functions, a novel coordinated guidance law composed by desired surge speed and heading angle for each vehicle is proposed. The coordination task is completed by assigning the same number of parametric paths to vehicles, introducing coordination error variables based on graph theory, and then the desired update law for each parameter of path is proposed. In order to quickly and accurately track the desired guidance signal with high robustness, a neural network controller was designed for each unmanned vehicle using backstepping method, in which the neural network is used to quickly estimate unknown dynamic disturbances. All closed-loop tracking errors are proved to be uniform ultimately bounded by Lyapunov theory. In addition, the coordinated path following errors are bounded in the prescribed boundaries.

**Index Terms**—Coordinated path following, underactuated surface vehicles, error constraints, neural network

## I. INTRODUCTION

In the past two decades, the coordinated control of underactuated surface vehicles (USVs) has attracted more and more attention due to its high efficiency in performing complicated tasks such as environmental monitoring and chart mapping. According to the different guidance signal, the coordinated control can be divided as path-guided coordinated control, trajectory-guided coordinated control and target-guided coordinated control. Compared to the other two methods, the path-guided coordinated control method can provide smoother guidance signals and trajectories. In this paper, we study the coordinated path following (CPF) control of USVs with error constraints.

There are many studies have been made for coordinated path following. In [1], the integral action is added into line-of-sight (LOS) guidance law to compensate for the adverse effects caused by currents, and the vehicles achieve the formation task by assigning different velocities to each USV according to relative inter-vehicle distance. The method proposed in [1] is verified by CybershipII in [2]. In [3], the cyber-attack is modeled as a time-varying state-dependent variable. An adaptive term is incorporated into the coordinated guidance law to compensate the time-varying cyber-attack. In [4], the event-triggering mechanism (ETM) is developed to reduce the communication cost among the USVs. To estimate and cancel the unknown external disturbances, the extended state

observer (ESO) is proposed in [5]. In order to accelerate the convergence of errors, the finite-time CPF controllers based on fast terminal sliding mode control (FTSMC) technique are proposed in [6]. Considering the unmeasurable velocities of USVs, the output-feedback control law is designed in [7]. Above coordinated guidance laws above are based on LOS. Besides LOS, there also exist other guidance methods. In [8], the desired path is expressed in an implicit function, the coordinated guidance law is proposed by the sliding mode control (SMC) technique. The guiding vector field (GVF) is proposed for CPF in [9], and is verified by unmanned aerial vehicle (UAV) in [10]. Compared with GVF, LOS is able to provide smoother guidance signal and achieve global convergence of tracking errors. Therefore, we will use LOS to design the coordinated guidance law in this paper.

Different from the above methods, the constraints of the path following errors will be considered in this paper. The tan-type barrier Lyapunov function (BLF) is introduced to design coordinated guidance law. In [11], the error-constrained LOS (ELOS) is proposed for single USV to realize the constraints of path following errors. In this paper, we will extend this method to CPF for multiple USVs. Different from [12], the unknown kinetic disturbances are considered in this paper. The radial basis function neural network (RBFNN) is introduced to estimate the lumped kinetic disturbances.

The main contributions of this paper are listed as follows:

- 1) The tan-type BLF are constructed to effectively guarantee the transient performance of each USV.
- 2) By assigning a parameterized path to each USV, the coordinated error variable is introduced based on the graph theory, and then the desired update law for each parameter is proposed to accomplish the coordination task.
- 3) It is proven that all errors of the closed-loop control system are uniform ultimately bounded (UUB) by using the proposed control method.

The rest of this paper is organized as follows. Section II formulates the CPF problem of multiple USVs. In Section III, the coordinated control law is designed for CPF by Backstepping method. The stability analysis of the closed-loop system is given in Section IV. Simulation studies is conducted in Section V. Section VI concludes this paper.

## II. PROBLEM FORMULATION

### A. Mathematical Model

The kinematics of  $i$ th USV are given as:

$$\begin{cases} \dot{x}_i = u_i \cos(\psi_i) - v_i \sin(\psi_i) \\ \dot{y}_i = u_i \sin(\psi_i) + v_i \cos(\psi_i) \\ \dot{\psi}_i = r_i \end{cases} \quad (1)$$

where  $x_i$  and  $y_i$  and  $\psi_i$  are north position, east position and heading angle of  $i$ th USV expressed in frame  $\{\mathcal{I}\}$  (see Fig. 1),  $u_i$ ,  $v_i$  and  $r_i$  denote the surge, sway and yaw velocities of  $i$ th USV respectively.

The kinetics of  $i$ th USV are given as [13]:

$$\begin{cases} m_{11}\dot{u}_i - m_{22}v_i r_i + d_{11}u_i = T_{li} + T_{ri} + \delta_u \\ m_{22}\dot{v}_i + m_{11}u_i v_i + d_{22}v_i = \delta_v \\ m_{33}\dot{r}_i - (m_{11} - m_{22})u_i v_i + d_{33}r_i = (T_{li} - T_{ri})d_p + \delta_r \end{cases} \quad (2)$$

where  $m_{ll}$ ,  $l = 1, 2, 3$  denote the inertial masses of  $i$ th USV,  $d_{ll}$  denote the damping terms of  $i$ th USV respectively,  $T_{li}$  and  $T_{ri}$  are thrust generated by the left and right thrusters,  $d_p$  is the lateral distance from the centerline of the USV to the centerline of each thruster,  $\delta_u$ ,  $\delta_v$  and  $\delta_r$  are the unknown external disturbances.

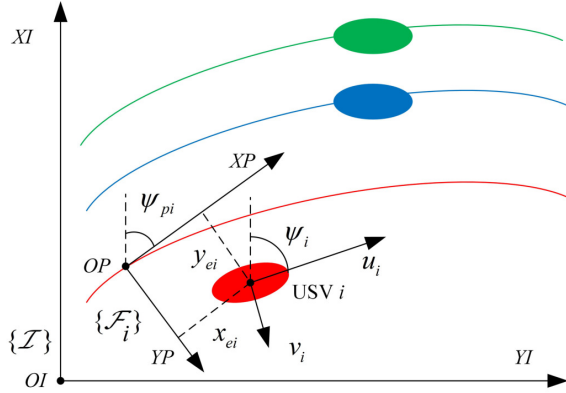


Fig. 1. Schematic Diagram of Coordinated path following.

### B. Control Objective

The path following errors can be represented by the positions of  $i$ th USV in the frame  $\mathcal{F}_i$  denoted by  $x_{ei}$  and  $y_{ei}$ , which are calculated as:

$$\begin{bmatrix} x_{ei} \\ y_{ei} \end{bmatrix} = \begin{bmatrix} \cos(\psi_{pi}) & \sin(\psi_{pi}) \\ -\sin(\psi_{pi}) & \cos(\psi_{pi}) \end{bmatrix} \begin{bmatrix} x_i - x_{pi}(\theta_i) \\ y_i - y_{pi}(\theta_i) \end{bmatrix} \quad (3)$$

where  $\psi_{pi}$  is the angle of the  $i$ th path  $\mathcal{P}_i$  at point  $P$  with respect to the inertial XI-axis (see Fig. 1),  $\psi_{pi} = \arctan 2(y'_{pi}, x'_{pi})$ ,  $\theta_i$  is the parameter of  $i$ th path. Combining with (1) and (3), the derivatives of  $x_{ei}$  and  $y_{ei}$  are calculated as:

$$\begin{cases} \dot{x}_{ei} = u_i \cos(\psi_{ei}) - v_i \sin(\psi_{ei}) + k_{ci} u_{pi} y_{ei} - u_{pi} \\ \dot{y}_{ei} = u_i \sin(\psi_{ei}) + v_i \cos(\psi_{ei}) - k_{ci} u_{pi} x_{ei} \end{cases} \quad (4)$$

where  $u_{pi} = u_{pi}^* \dot{\theta}_i$ ,  $u_{pi}^* = \sqrt{x_{pi}^{\prime 2} + y_{pi}^{\prime 2}}$ .

Considering the virtual leader  $\mathcal{V}_0$  and  $N$  agents, we assign a parameter  $\theta_0$  to  $\mathcal{V}_0$ . Then, the coordinated errors can be defined as:

$$e_{\theta i} = \sum_{j=1}^N a_{ij}(\theta_i - \theta_j - d_{\theta ij}) + b_i(\theta_i - \theta_0 - d_{\theta i0}) \quad (5)$$

The path following errors and coordinated errors are expressed in (3) and (5). In the following, we given the control objectives of coordinated path following.

O1) Heading control: The  $i$ th USV is required to follow the given path  $\mathcal{P}_i$  with the desired heading angle  $\psi_{ci}$ , and the path following errors  $x_{ei}$  and  $y_{ei}$  will be constrained in the prescribed boundaries for the specified initial condition, i.e.,  $\lim_{t \rightarrow \infty} |x_{ei}| < \sigma_{xi}$  and  $\lim_{t \rightarrow \infty} |y_{ei}| < \sigma_{yi}$  for the initial condition  $\lim_{t \rightarrow 0} |x_{ei}(0)| < \sigma_{xi}(0)$  and  $\lim_{t \rightarrow 0} |y_{ei}(0)| < \sigma_{yi}(0)$ , where  $\sigma_{xi}$  and  $\sigma_{yi}$  are prescribed boundaries.

O2) speed control: All the USVs will achieve the desired formation by regulating surge speed, i.e., the desired speed  $u_{pi}$  will be assigned to each USV such that the following relations hold,  $\lim_{t \rightarrow \infty} |e_{\theta i}| < d_1$ , and there is  $\lim_{t \rightarrow \infty} |u_i - u_{pi}| < d_2$ , where  $d_1$  and  $d_2$  are small positive constants.

### C. Graph Theory

Consider a directed graph  $\mathcal{G} = \{\mathcal{N}, \mathcal{E}\}$ , where  $\mathcal{N} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_N\}$  denote the set of  $N$  nodes and  $\mathcal{E} = \{(\mathcal{V}_i, \mathcal{V}_j) \in \mathcal{N} \times \mathcal{N}\}$  is the edge set. The node  $\mathcal{V}_i$  can get the information from node  $\mathcal{V}_j$  if  $(\mathcal{V}_i, \mathcal{V}_j) \in \mathcal{E}$ . The communication topology between  $\mathcal{V}_i$  and  $\mathcal{V}_j$  is represented by the adjacency matrix  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ , where  $a_{ij} = 1$  if  $(\mathcal{V}_i, \mathcal{V}_j) \in \mathcal{E}$  and  $i \neq j$ ,  $a_{ij} = 0$  otherwise. Let  $\mathcal{D} = \text{diag}\{d_1, d_2, \dots, d_N\}$  where  $d_i = \sum_{j=1}^N a_{ij}$ ,  $i = 1, 2, \dots, N$ . The Laplacian matrix of  $\mathcal{G}$  is defined as  $\mathcal{L} = \mathcal{D} - \mathcal{A}$ . The communication between node  $\mathcal{V}_i$  and the virtual leader  $\mathcal{V}_0$  is represented by the leader adjacency matrix  $\mathcal{B} = \text{diag}\{b_1, b_2, \dots, b_N\} \in \mathbb{R}^{n \times n}$ , where  $b_i = 1$  if node  $\mathcal{V}_i$  can obtain information from virtual leader,  $b_i = 0$  otherwise.

*Assumption 1:* The graph  $\mathcal{G}$  has a spanning tree with the root node  $\mathcal{V}_0$ .

*Lemma 1:* Let  $\mathcal{H} = \mathcal{L} + \mathcal{B}$ . All the eigenvalues of  $\mathcal{H}$  have positive real parts if and only if Assumption 1 hold. In addition, there exists  $\mathcal{Q} = \mathcal{Z}\mathcal{H} + \mathcal{H}^T \mathcal{Z}$ , where  $\mathcal{Q} = [q_1, q_2, \dots, q_N]^T = \mathcal{H}^{-1} \mathbf{1}_n$ ,  $\mathcal{Z} = \text{diag}\{1/q_i\}$ . (see [14], [15])

## III. COORDINATED CONTROL LAW DESIGN

In the section, we will design the coordinated guidance law and neural-network (NN) controller for each USV by backstepping method and graph theory.

Step 1. Define tracking errors as  $\tilde{u}_i = u_i - u_{ci}$ ,  $\tilde{\psi}_i = \psi_i - \psi_{ci}$ ,  $\tilde{r}_i = r_i - r_{ci}$ , then, the error dynamics of (3) can be rewritten as:

$$\begin{cases} \dot{x}_{ei} = u_{ci} + \tilde{u}_i - 2u_i \sin^2(\frac{\psi_{ei}}{2}) - v_i \sin(\psi_{ei}) + k_{ci} u_{pi} y_{ei} - u_{pi} \\ \dot{y}_{ei} = U_{ci} \sin(\psi_{ci} - \psi_{pi} + \beta_{ci}) - k_{ci} u_{pi} x_{ei} + U_i \omega_i \tilde{\psi}_i + \tilde{u}_i \sin(\psi_{ei}) \end{cases} \quad (6)$$

where  $\omega_i = \frac{\cos(\psi_{ci} - \psi_{pi} + \beta_i) \sin(\tilde{\psi}_i) + \sin(\psi_{ci} - \psi_{pi} + \beta_i) (\cos(\tilde{\psi}_i) - 1)}{\tilde{\psi}_i}$ ,  $U_{ci} = \sqrt{u_{ci}^2 + v_i^2}$ , and  $\beta_{ci} = \arctan(\frac{v_i}{u_{ci}})$ .

The first Lyapunov function is constructed as the tan-type BLF:

$$V_{1i} = \frac{\sigma_{xi}^2}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + \frac{\sigma_{yi}^2}{\pi} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \quad (7)$$

where  $\sigma_{xi}$  and  $\sigma_{yi}$  are the prescribed boundaries. The derivative of  $V_{1i}$  can be calculated as:

$$\begin{aligned} \dot{V}_{1i} = & \frac{2\sigma_{xi}\dot{\sigma}_{xi}}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + x_{ei}\dot{x}_{ei} \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) - \\ & \frac{\dot{\sigma}_{xi}}{\sigma_{xi}} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + \frac{2\sigma_{yi}\dot{\sigma}_{yi}}{\pi} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) + \\ & y_{ei}\dot{y}_{ei} \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) - \frac{\dot{\sigma}_{yi}}{\sigma_{yi}} y_{ei}^2 \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \end{aligned} \quad (8)$$

To realize the constraints on the path following errors, the desired heading angle and surge speed can be given as:

$$\begin{cases} u_{ci} = u_{pi} + 2u_i \sin^2\left(\frac{\psi_{ei}}{2}\right) + v_i \sin\left(\frac{\psi_{ei}}{2}\right) - \\ \quad k_{ci} u_{pi} y_{ei} (1 - \alpha_i) - \rho_{xi} \\ \psi_{ci} = \psi_{pi} - \beta_i - \arctan\left(\frac{\rho_{yi}}{\Delta_i}\right) \end{cases} \quad (9)$$

where  $\Delta_i > 0$  denotes the look-ahead distance, and

$$\begin{cases} \rho_{xi} = \frac{k_{x1i}\sigma_{xi}^2}{\pi x_{ei}} \sin\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) \cos\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + k_{x2i} x_{ei} \\ \rho_{yi} = \frac{k_{y1i}\sigma_{yi}^2}{\pi y_{ei}} \sin\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \cos\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) + k_{y2i} y_{ei} \\ \alpha_i = u_{pi}^* \left(1 - k_{ci} y_{ei} \left(1 - \cos^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right)\right)\right) \end{cases} \quad (10)$$

where  $k_{x1i}$ ,  $k_{x2i}$ ,  $k_{y1i}$  and  $k_{y2i}$  are positive parameters which will be discussed in Section IV.

The derivative of  $V_{1i}$  in (8) can be further calculated as

$$\begin{aligned} \dot{V}_{1i} = & -\frac{k_{x1i}\sigma_{xi}^2}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) - k_{x2i} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) - \\ & \frac{U_i k_{y1i} \sigma_{yi}^2}{\pi \sqrt{\Delta_i^2 + \rho_{yi}}} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) - \frac{\dot{\sigma}_{xi}}{\sigma_{xi}} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) - \\ & \frac{U_i k_{y2i} y_{ei}^2}{\sqrt{\Delta_i^2 + \rho_{yi}}} \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) + \tilde{u}_i x_{ei} \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + \\ & \tilde{u}_i \sin(\psi_{ei}) \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) + \frac{2\sigma_{xi}\dot{\sigma}_{xi}}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + \\ & U_i \omega_i \tilde{\psi}_i y_{ei} \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) + \frac{2\sigma_{yi}\dot{\sigma}_{yi}}{\pi} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) - \\ & \frac{\dot{\sigma}_{yi}}{\sigma_{yi}} y_{ei}^2 \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \end{aligned} \quad (11)$$

Step 2. The second Lyapunov function is chosen as

$$V_{2i} = 0.5 \tilde{\psi}_i^2 \quad (12)$$

The desired yaw velocity is designed as

$$r_{ci} = \dot{\psi}_{ci} - k_{ri} \tilde{\psi}_i - U_i \rho_i y_{ei} \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \quad (13)$$

The derivative of  $V_{2i}$  is

$$\dot{V}_{2i} = -k_{ri} \tilde{\psi}_i^2 - U_i \rho_i \tilde{\psi}_i y_{ei} \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \quad (14)$$

Step 3. Let  $\xi_i = [u_i, r_i]^T$ ,  $\xi_{ci} = [u_{ci}, r_{ci}]^T$ . The tracking error can be rewritten as  $\tilde{\xi}_i = \xi_i - \xi_{ci}$ . To facilitate control design, the kinetics are rewritten as

$$\dot{\xi}_i = F_{\xi i} + G T_i \quad (15)$$

where  $T_i = [T_{li}, T_{ri}]^T$ ,  $F_{\xi i} = [f_{ui}, f_{ri}]^T$ ,  $F_{\xi i}$  and  $G$  are given as

$$F_{\xi i} = \begin{bmatrix} \frac{m_{22} v_i r_i}{m_{11}} - \frac{d_{11} u_i}{m_{11}} \\ \frac{(m_{11} - m_{22}) u_i v_i}{m_{33}} - \frac{d_{33} r_i}{m_{33}} \end{bmatrix}, G = \begin{bmatrix} \frac{1}{m_{11}} & \frac{1}{d_p} \\ \frac{1}{m_{33}} & -\frac{1}{m_{33}} \end{bmatrix}$$

The derivative of  $\tilde{\xi}_i$  is

$$\dot{\tilde{\xi}}_i = F_{\xi i} + G T_i - \dot{\xi}_{ci} \quad (16)$$

Since the nonlinear term  $F_{\xi i}$  is unknown, the neural network is used to approximate it as follows

$$F_{\xi i} = W_i^T \Phi_i(X_i) + \zeta_i(X_i) \quad (17)$$

where  $W_i$  is the desired weight matrix of neural network, which is unknown but bounded,  $X_i$  is the input vector of NN,  $\Phi_i(X_i)$  is the radial basis function, and  $\zeta_i(X_i)$  is the approximate error.

The third Lyapunov function is chosen as

$$V_{3i} = 0.5 \tilde{\xi}_i^T \tilde{\xi}_i + 0.5 \text{tr}(\tilde{W}_i^T \Gamma_{Wi}^{-1} \tilde{W}_i) \quad (18)$$

Then, we can get the control law as

$$T_i = G^{-1}(\dot{\xi}_{ci} - \hat{W}^T \Phi_i(X_i) - K_{\xi i} \tilde{\xi}_i + \rho_{\xi i}) \quad (19)$$

where  $\hat{W}_i$  is estimated value of  $W_i$ ,  $\rho_{\xi i} = \left[ x_{ei} \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + y_{ei} \sin(\psi_{ei}) \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right), \tilde{\psi}_i \right]^T$ .

The update law is designed as

$$\dot{\tilde{W}}_i = \Gamma_{Wi}(\Phi_i(X_i) \tilde{\xi}_i^T - K_{Wi} \tilde{W}_i) \quad (20)$$

Combined with (19) and (20), the derivative of  $V_{3i}$  can be calculated as

$$\dot{V}_{3i} = -\tilde{\xi}_i^T K_{\xi i} \tilde{\xi}_i + k_{Wi} \text{tr}(\tilde{W}_i^T \dot{\tilde{W}}_i) + \tilde{\xi}_i^T \rho_{\xi i} \quad (21)$$

The control law for path following is completed here. To realize the formation task, the coordinated guidance law is designed as follows.

Step 4. Let  $\mathbf{E}_\theta = [e_{\theta 1}, e_{\theta 2}, \dots, e_{\theta n}]^T$ . There is  $\mathbf{E}_\theta = \mathcal{H}(\theta - \theta_{01n})$ . The derivative of  $\mathbf{E}_\theta$  is

$$\dot{\mathbf{E}}_\theta = \mathcal{H} \begin{bmatrix} \frac{u_{p1}}{u_{p1}^*} - \frac{u_{p0}}{u_{p0}^*} \\ \dots \\ \frac{u_{pn}}{u_{pn}^*} - \frac{u_{p0}}{u_{p0}^*} \end{bmatrix} \quad (22)$$

The fourth Lyapunov function can be chosen as

$$V_4 = 0.5 \mathbf{E}_\theta^T \mathbf{Q} \mathbf{E}_\theta \quad (23)$$

The coordinated guidance law is designed as

$$u_{pi} = u_{pi}^* \left( \frac{u_0}{u_{p0}^*} - k_{\theta i} e_{\theta i} \right) \quad (24)$$

Then, the update law of the parameter of path  $i$  is  $\dot{\theta}_i = \frac{u_{pi}}{u_{pi}^*}$ . Combined with (24) and (22), the derivative of  $V_4$  is calculated as

$$\dot{V}_4 = 0.5 \mathbf{E}_\theta^T (\mathbf{K}_\theta^T \mathcal{H}^T \mathbf{Q} + \mathbf{Q} \mathcal{H} \mathbf{K}_\theta) \mathbf{E}_\theta \quad (25)$$

The coordinated control law for CPF is completed here. The block diagram of the control system for CPF is illustrated in Fig. 2.

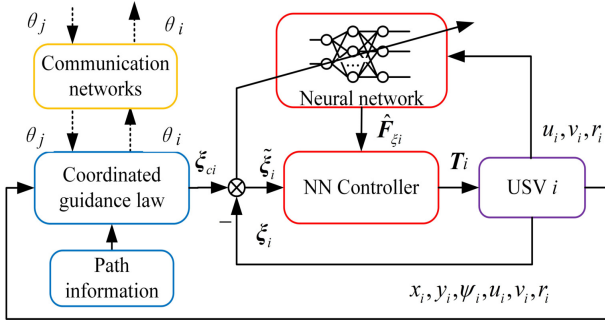


Fig. 2. Block diagram of Coordinated path following control system.

#### IV. STABILITY ANALYSIS

*Theorem 1:* The path following errors  $x_{ei}$ ,  $y_{ei}$ , and tracking errors  $\tilde{u}_i$ ,  $\tilde{v}_i$ ,  $\tilde{r}_i$  will be UUB, if there exist parameters  $k_{x1i}$ ,  $k_{x2i}$ ,  $k_{y1i}$ , and  $k_{y2i}$  such that the equalities given in (26) hold. In addition, the path following errors will be constrained in the prescribed boundaries  $\sigma_{xi}$  and  $\sigma_{yi}$ , i.e.,  $|x_{ei}| \leq \sigma_{xi}$  and  $|y_{ei}| \leq \sigma_{yi}$ , for the given initial value  $|x_{ei}(0)| \leq \sigma_{xi}(0)$  and  $|y_{ei}(0)| \leq \sigma_{yi}(0)$ .

$$\begin{cases} k_{x2i} = \sqrt{\frac{\dot{\sigma}_{xi}^2}{\sigma_{xi}^2}} + k_{x0i} \\ k_{y2i} = \frac{\dot{\sigma}_{yi} \sqrt{\Delta_i (\sigma_{ui}^2 - \dot{\sigma}_{yi}^2 y_{ei}^2)} + k_{y0i}^2 \sigma_{ui}^2 + \dot{\sigma}_{yi}^2 k_{y0i} y_{ei}}{\sigma_{ui}^2 - \dot{\sigma}_{yi}^2} \end{cases} \quad (26)$$

where  $\sigma_{ui}^2 = \sigma_{yi}^2 U_{ci}^2$ ,

*Proof:* Construct the Lyapunov function as

$$V = V_4 + \sum_{i=1}^3 V_{1i} + V_{2i} + V_{3i} \quad (27)$$

Under the condition (26), there are

$$\begin{cases} \frac{U_i k_{y2i} y_{ei}^2}{\sqrt{\Delta_i^2 + \rho_{yi}^2}} \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) = \frac{\dot{\sigma}_{yi}}{\sigma_{yi}} y_{ei}^2 \sec^2\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) \\ \frac{\dot{\sigma}_{xi}}{\sigma_{xi}} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) < k_{x2i} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) \end{cases} \quad (28)$$

Combined with (11), (14), (21), (25), (26), (28) and Lemma 1, the derivative of  $V$  can be calculated as:

$$\begin{aligned} \dot{V} &= \mathbf{E}_\theta^T \mathbf{Z}_\theta \mathbf{E}_\theta + \sum_{i=1}^N -\frac{k_{x1i} \sigma_{xi}^2}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) - \\ & k_{x2i} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) - \frac{U_i k_{y1i} \sigma_{yi}^2}{\pi \sqrt{\Delta_i^2 + \rho_{yi}^2}} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) + \\ & \frac{2\sigma_{xi} \dot{\sigma}_{xi}}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + \frac{2\sigma_{yi} \dot{\sigma}_{yi}}{\pi} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) - \\ & \xi_i^T K_{\xi i} \xi_i + k_{wi} \text{tr}(\tilde{\mathbf{W}}_i^T \hat{\mathbf{W}}_i) - \frac{\dot{\sigma}_{xi}}{\sigma_{xi}} x_{ei}^2 \sec^2\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) \\ & \leq -\underline{\lambda}(\mathbf{K}_\theta \mathbf{Z}_\theta) \mathbf{E}_\theta^T \mathbf{E}_\theta - \sum_{i=1}^N k_{ri} \tilde{\psi}_i^2 + \underline{\lambda}(\mathbf{K}_{\xi i}) \xi_i^T \xi_i + \\ & (k_{x1i} - 2k_{x2i}) \frac{\sigma_{xi}^2}{\pi} \tan\left(\frac{\pi x_{ei}^2}{2\sigma_{xi}^2}\right) + k_{wi} \text{tr}(\tilde{\mathbf{W}}_i^T \hat{\mathbf{W}}_i) + \\ & \frac{(k_{y1i} - 2k_{y2i}) U_i \sigma_{yi}^2}{\pi \sqrt{\Delta_i^2 + \rho_{yi}^2}} \tan\left(\frac{\pi y_{ei}^2}{2\sigma_{yi}^2}\right) - K_{wi} \text{tr}(\tilde{\mathbf{W}}_i^T \mathbf{W}) \end{aligned} \quad (29)$$

$$\text{Let } \kappa_1 = \max\{\bar{\lambda}(\mathbf{Q}), \Gamma_{wi}\}, \quad \kappa_2 = 2 \min\{\underline{\lambda}(\mathbf{K}_\theta \mathbf{Z}_\theta), \underline{\lambda}(\mathbf{K}_{\xi i}), \frac{k_{x1i} - 2k_{x2i}}{2}, \frac{(k_{y1i} - 2k_{y2i}) U_i}{2\sqrt{\Delta_i^2 + \rho_{yi}^2}}\}, \quad \kappa = \frac{\kappa_2}{\kappa_1},$$

where  $\underline{\lambda}(\cdot)$  denotes the minimum eigenvalue of  $\cdot$ , and  $\bar{\lambda}(\cdot)$  denotes the maximum eigenvalue of  $\cdot$ . Combining with (27) and (29), we have

$$\dot{V} \leq -\kappa V + \gamma \quad (30)$$

The solution of (30) is

$$V \leq e^{-\kappa t} V(0) + \frac{\gamma}{\kappa} \quad (31)$$

It can be concluded that  $V$  is bounded. According to (27) and (31), we have

$$\begin{cases} x_{ei}^2 \leq \frac{2\sigma_{xi}^2}{\pi} \arctan\left(\frac{\pi V}{\sigma_{xi}^2}\right) \\ y_{ei}^2 \leq \frac{2\sigma_{yi}^2}{\pi} \arctan\left(\frac{\pi V}{\sigma_{yi}^2}\right) \end{cases} \quad (32)$$

From (32), we have  $x_{ei}^2 \leq \sigma_{xi}^2$  and  $y_{ei}^2 \leq \sigma_{yi}^2$ . ■

## V. SIMULATION RESULTS

In this section, the simulation results on the proposed control method is presented to verify its effectiveness. Consider a fleet compose by three USVs with the communication topology is shown in Fig. 3. The adjacency matrix and Laplacian matrix are given as follows.

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \mathcal{L} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (33)$$

and the leader adjacency matrix is  $\mathcal{B} = \text{diag}\{1, 0, 0\}$ .

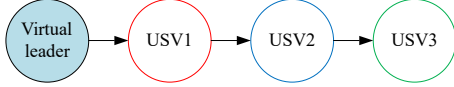


Fig. 3. Communication topology.

The parameters of USV model are given as  $m_{11} = 17.21\text{kg}$ ,  $m_{22} = 84.36\text{kg}$ ,  $m_{33} = 17.21\text{kg}$ ,  $d_{11} = 151.57\text{kg/s}$ ,  $d_{22} = 132.5\text{kg/s}$ ,  $d_{33} = 34.56\text{kg/s}$ ,  $d_p = 0.26\text{kgm}$ . The initial values of USV states are  $x_1(0) = 12\text{m}$ ,  $y_1(0) = -0.5\text{m}$ ,  $\psi_1(0) = 0\text{rad}$ ,  $u_1(0) = v_1(0) = 0\text{m/s}$ ,  $r_1(0) = 12\text{rad/s}$ ,  $x_2(0) = 23\text{m}$ ,  $y_2(0) = -0.1\text{m}$ ,  $\psi_2(0) = 0\text{rad}$ ,  $u_2(0) = v_2(0) = 0\text{m/s}$ ,  $r_2(0) = 12\text{rad/s}$ ,  $x_3(0) = 32\text{m}$ ,  $y_3(0) = -0.3\text{m}$ ,  $\psi_3(0) = 0\text{rad}$ ,  $u_3(0) = v_3(0) = 0\text{m/s}$ ,  $r_3(0) = 12\text{rad/s}$ . The control parameters are  $\sigma_{xi} = \sigma_{yi} = 4\exp(-0.05t) + 1$ ,  $\Gamma_{Wi} = 1$ ,  $k_{Wi} = 0.005$ ,  $K_{Wi} = \text{diag}\{4, 2\}$ ,  $k_{x0i} = 0.05$ ,  $k_{ri} = 5$ ,  $k_{\theta i} = 0.5$ ,  $\Delta_i = 3\text{m}$ ,  $k_{x1i} = 1$ ,  $k_{y1i} = 1$ . The desired speed  $u_{p0} = 0.3\text{m/s}$ . Disturbances are chosen as

$$\begin{cases} \delta_u = 5 \sin(0.08t) \cos(0.15t) + 5 \\ \delta_v = 3 \sin(0.08t) \cos(0.15t) + 3 \\ \delta_r = 5 \sin(0.08t) \cos(0.15t) + 5 \end{cases} \quad (34)$$

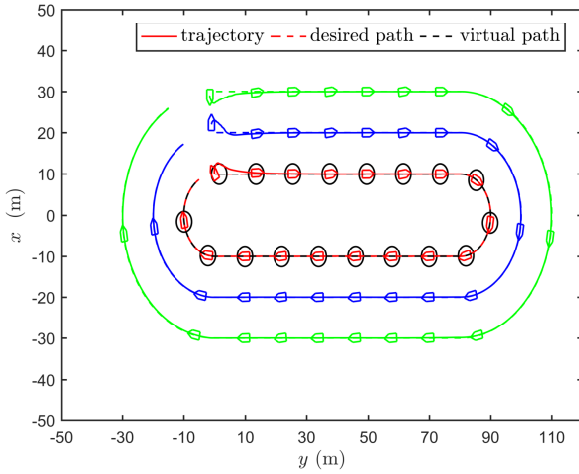


Fig. 4. Coordinated path following performance.

The simulation results are presented in Fig. 4~Fig. 8. As shown in Fig. 4, all the USVs will follow the given paths

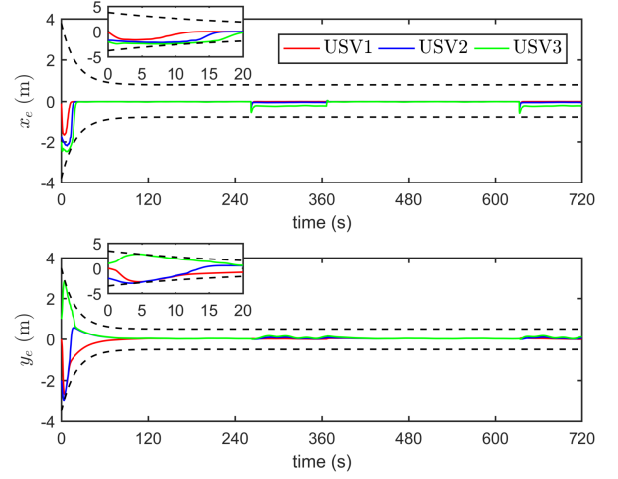


Fig. 5. Path following errors.

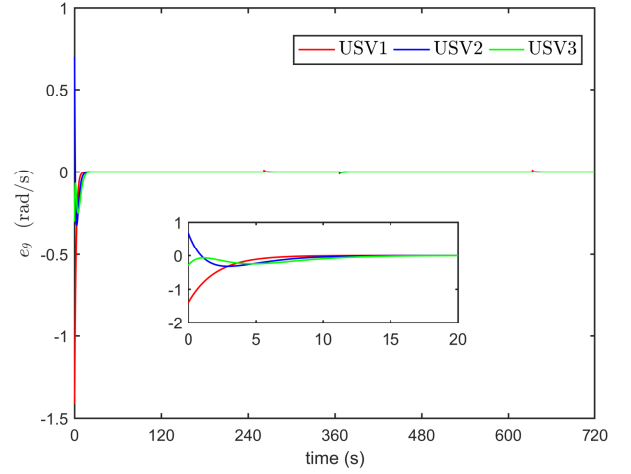


Fig. 6. Coordinated errors.

and achieve the desired formation after a brief transition. The path following errors are presented in Fig. 5. We can easily get all the path following errors are bounded and constrained in the boundaries, which are expressed in black dotted line. As illustrated in Fig. 6, all the coordinated errors  $e_{\theta i}$ ,  $i = 1, 2, 3$ , will converge into the small neighborhood of zero. The velocities of USVs are shown in Fig. 7, the surge speed will reach the desired speed  $0.3\text{m/s}$  if the desired formation is accomplished well, and the sway and yaw velocities are bounded. When the USVs arrive at the circular section, the outer USVs will speed up to maintain the formation. On the straight section, all USVs will maintain a surge speed of  $0.3\text{m/s}$ . The estimation of disturbances of 1th USV shown in Fig. 8, we can see that the RBFNN can estimate the lumped kinetic disturbances accurately and quickly. From the simulation results and the theoretical analysis given in section IV, we can conclude that the control objectives  $O1$  and  $O2$  are achieved by the proposed control method.

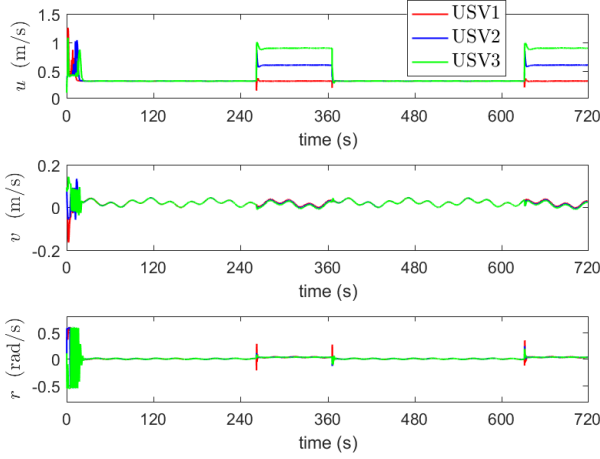


Fig. 7. Velocities of USVs.

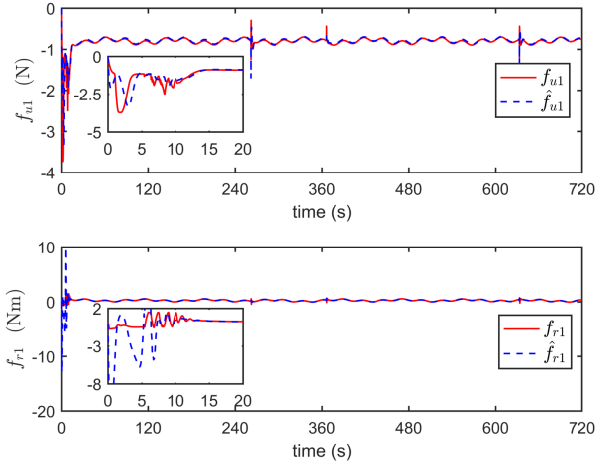


Fig. 8. The estimations of disturbances.

## VI. CONCLUSION

By combining tan-type barrier Lyapunov functions, the graph theory and the backstepping technique, this paper proposes a novel CPF guidance law for multiple USVs to realize collision avoidance in the initial stage of formation. All the closed-loop errors are proved uniformly bounded by Lyapunov stability theory. In addition, the path following errors are bounded in the prescribed boundaries. Finally, the simulations are conducted to verify the effectiveness and robustness of the proposed guidance and control system. It should be noted that the guidance method based on tan-type BLF proposed in this paper requires relatively strict initial conditions, i.e., the initial states of USVs should be selected within the prescribed boundaries. Therefore, in the future, an improved constraint method will be studied to solve collision problems, which will be loose for the initial state selection of USVs. In addition, the constraints on actuators will also be

considered in the future.

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