EVENT-TRIGGERED COORDINATED PATH FOLLOWING CONTROL FOR UNDERACTUATED SURFACE VEHICLES WITH INPUT SATURATION

ABSTRACT

INTRODUCTION

PRELIMINARIES AND PROBLEM FORMULATION

Graph theory

Vehicle's model

Consider a group of underactuated surface vehicles labeled from 1 to N. The kinematic equation of ith vehicle can be expressed as:

$$\begin{cases} \dot{x}_i = u_i\, cos(\psi_i) - v_i sin(\psi_i) \\ \dot{y}_i = u_i\, sin(\psi_i) + v_i cos(\psi_i) \\ \dot{\psi}_i = r_i \end{cases} \tag{1}$$

The kinetic equation of ith vehicle can be expressed as:

$$\begin{cases} \dot{u}_i = f_{ui} + (T_{1i} + T_{2i})/m_{11} \\ \dot{v}_i = f_{vi} - d_{22} v_i/m_{22} \\ \dot{r}_i = f_{ri} + d_p(T_{1i} - T_{2i})/m_{33} \end{cases} \tag{2}$$

where

$$\begin{cases} \dot{x}_i = U_i cos(\psi_i + \beta_i) \\ \dot{y}_i = U_i sin(\psi_i + \beta_i) \\ \dot{\psi}_i = r_i \\ \dot{w}_i = F_i(u_i, v_i, r_i) + \bar{M}^{-1} B T_i \end{cases} \tag{3}$$

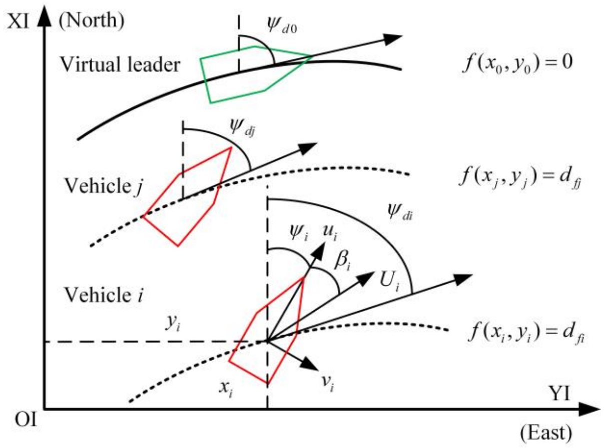


Fig. 1. The geometric illustration of coordinated path following

Problem formulation

The desired path can be represented by an implicit function as follows:

$$\mathcal{P} = \{(x_p, y_p) | f(x_p, y_p) = 0\} \tag{4}$$

Then the path following error of the ith vehicle can be described by $f_i(x_i, y_i)$. The desired orientation of ith vehicle is calculated by:

$$\psi_{di} = \begin{cases} -arctan(f_{xi}/f_{yi}), if\ f_{yi} \neq 0 \\ -arccot(f_{yi}/f_{xi}), if\ f_{xi} \neq 0 \end{cases} \tag{5}$$

The derivative of f_i can be obtained as

$$\dot{f}_i = \|\nabla f_i\| U_i sin(\psi_i + \beta_i - \psi_{di}) \tag{6}$$

The main objective is to force the errors f_i and f_j to satisfy predefined constraints, so that the vehicles will locate on the different level set of the desired path. Simultaneously, we regulated the vehicles' dynamic behavior according to the difference of arc length between two different vehicles which can be calculated by:

$$s_{ij} = \begin{cases} \int_{x_j}^{x_i} \sqrt{1 + f_x^2/f_y^2}\, dx, if\ f_y \neq 0 \\ \int_{y_j}^{y_i} \sqrt{1 + f_y^2/f_x^2}\, dy, if\ f_x \neq 0 \end{cases} \tag{7}$$

The coordinated path following control objective can be concluded as follows:

O1) path-following task: Lead a fleet of surface vehicles to navigate along the different level sets of the predefined geometric path without time constraints which can be described as:

$$\lim_{t \to \infty} |f_i - f_j - d_{fij}| < \epsilon_1 \tag{8}$$

where ϵ_1 is a constant.

O2) dynamic formation task: Drive all the vehicles to form a designated formation by regulating the course speed which can be described as

$$\lim_{t\to\infty}\left|s_{ij}-d_{sij}\right|\leq\epsilon_2 \tag{9}$$

In addition, each vehicle' course speed should satisfy the desired speed assignment

$$\lim_{t\to\infty}\left|U_i-U_d\right|\leq\epsilon_3 \tag{10}$$

where ϵ_2 and ϵ_3 are positive constants.

COORDINATED COTROLLER DESIGEN

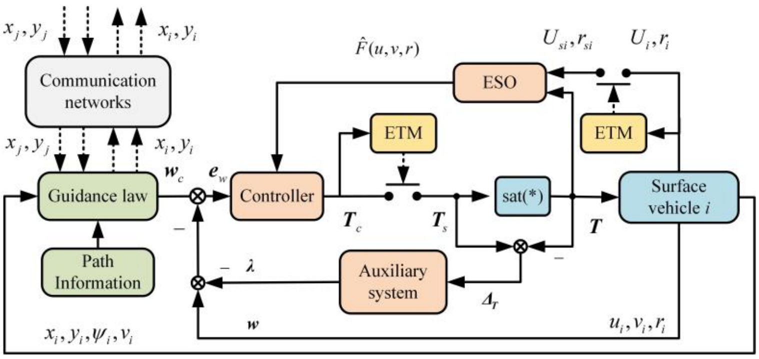


Fig. 2. Block diagram of the coordinated path following control system

Coordinated guidance law design

According to the control objective (O1) and (O2), the design process of the coordinated guidance law can be divided into two steps. In the first step, the coordinated path following guidance law will be designed to lead each vehicle to follow the corresponding level set of the desired path without time constraints. In the second step, the dynamic formation guidance law is designed to force the vehicles to satisfy the expected formation with temporal constraints. Before the design process, a virtual leader on the desired path is introduced as

$$\begin{cases}\dot{x}_0=U_0cos(\psi_{d0})\\ \dot{y}_0=U_0sin(\psi_{d0})\end{cases} \tag{11}$$

where ψ_{d0} is the heading angle of the virtual leader which can be calculated by (5). The virtual leader will always on the desired path if the initial values of x_0 and y_0 locate on the desired path. Then, the design procedure is elaborated in the following two steps.

Step1. Denote the coordinated path following error of ith vehicle as e_{fi}:

$$e_{fi}=\sum_{j\in N_i}a_{ij}\left(f_i-f_j-d_{fij}\right)+b_i(f_i-f_0-d_{fi0}) \tag{12}$$

Let $E_f=\left[e_{f1},e_{f2},\dots,e_{fn}\right]^T$, $F_f=[f_1,f_2,\dots,f_n]^{\mathrm{T}}$. The coordinated path following error can be rewrite as the following compact form

$$E_f = (L+B)F_f \tag{13}$$

The derivative of E_f is calculated as

$$\dot{E}_f = (L+B)\begin{bmatrix} \|\nabla f_1\| U_1 \sin(\psi_1+\beta_1-\psi_{d1}) \\ \dots \\ \|\nabla f_n\| U_n \sin(\psi_n+\beta_n-\psi_{dn}) \end{bmatrix} \tag{14}$$

Then, the desired heading angle of ith vehicle can be designed as

$$\psi_{ci} = \psi_{di} - \beta_{0i} - arctan(k_{1i} e_{fi}) \tag{15}$$

where $\beta_{0i} = \arcsin\left(\frac{v_i}{U_0}\right)$, k_{1i} is a positive constant. Define the tracking errors $e_{\psi i}$ and e_{ri} as $e_{\psi i} = \psi_i - \psi_{ci}$ and $e_{ri} = r_i - r_{ci} - \lambda_{ri}$, where ψ_{cfi} and r_{cfi} are the filtered version of ψ_{ci} and r_{ci}, r_{ci} and λ_r are the virtual control law and auxiliary state which will be designed in the next subsection. The filtered errors are defined as $e_{\psi fi} = \psi_{ci} - \psi_{cfi}$ and $e_{rfi} = r_{ci} - r_{cfi}$. The first order filters are introduced as

$$\dot{\psi}_{cfi} = -(\psi_{cfi} - \psi_{ci})/T_{f1} \tag{16}$$

The errors' dynamics of filters are calculated as

$$\dot{e}_{\psi fi} = -\frac{e_{\psi fi}}{T_{\psi fi}} + \dot{\psi}_{ci} \tag{17}$$

Then, the derivative of $e_{\psi i}$ can be obtained as

$$\dot{e}_{\psi i} = r_{ci} + e_{ri} - \dot{\psi}_{cfi} + \lambda_{ri} - \frac{e_{\psi fi}}{T_{\psi fi}} + \dot{\psi}_{ci} \tag{18}$$

The desired yaw angular velocity is designed as

$$r_{ci} = \dot{\psi}_{cfi} - k_{2i} e_{\psi i} \tag{19}$$

Step2. Define the dynamic formation error of ith vehicle as

$$e_{si} = \sum_{j\in N_i} a_{ij}(s_{ij} - d_{sij}) + b_i(s_{i0} - d_{si0}) \tag{20}$$

where $\mathrm{s_{ij}}$ is the arc length between ith vehicle and jth vehicle, $\mathrm{s_{i0}}$ is the arc length between ith vehicle and virtual leader. s_{ij} can be calculated as

$$\begin{cases} s_{ij} = \int_{x_j}^{x_i} \sqrt{1+\left(\frac{f_x}{f_y}\right)^2}\, dx, f_y \neq 0 \\ s_{ij} = \int_{y_j}^{y_i} \sqrt{1+\left(\frac{f_y}{f_x}\right)^2}\, dy, f_x \neq 0 \end{cases} \tag{21}$$

The derivative of $\mathrm{e_{si}}$ is calculated as

$$\dot{e}_{si} = \sum_{j\in N_i} (a_{ij} + b_i)\left(\sqrt{1+\left(\frac{f_{xi}}{f_{yi}}\right)^2} U_i \cos(\psi_i - \beta_i - \psi_{di} + \psi_{di})\right) -$$

$$\sum_{j\in N_i} a_{ij}\left(\sqrt{1+\left(\frac{f_{xj}}{f_{yj}}\right)^2}\,U_j\,cos(\psi_j-\beta_j-\psi_{dj}+\psi_{dj})\right)-b_iU_0$$

$$=\sum_{j\in N_i}(a_{ij}+b_i)\left(U_i\,cos(\chi_{ei})-\frac{f_{xi}}{f_{yi}}U_i\,sin(\chi_{ei})\right)-$$

$$\sum_{j\in N_i} a_{ij}\left(U_j\,cos(\chi_{ej})-\frac{f_{xj}}{f_{yj}}U_j\,sin(\chi_{ej})\right)-b_iU_0$$

$$=\sum_{j\in N_i}(a_{ij}+b_i)(U_i-\vartheta_{e2i})-\sum_{j\in N_i}a_{ij}(U_j-\vartheta_{e2j})-b_iU_0 \tag{22}$$

where $\chi_{ei}=\psi_i-\beta_i-\psi_{di}$, $\vartheta_{e2i}=2U_i\sin\left(\frac{\chi_{ei}}{2}\right)\left(\sin\left(\frac{\chi_{ei}}{2}\right)+\frac{f_{xi}}{f_{yi}}\cos\left(\frac{\chi_{ei}}{2}\right)\right)$. Let $E_s=[e_{s1},e_{s2},\dots,e_{sn}]^T$, $\Theta_s=[\vartheta_{e1},\vartheta_{e2},\dots,\vartheta_{en}]^T$. Then the error dynamics can be written as the following compact form.

$$\dot{E}_s=(L+B)\begin{bmatrix}U_1-U_0\\ \dots\\ U_n-U_0\end{bmatrix}+(L+B)\Theta_s \tag{23}$$

Then, the desired course speed of ith vehicle can be designed as

$$U_{ci}=U_0(1-tanh(k_{3i}e_{si})) \tag{24}$$

Event triggered controller design

In this subsection, the event triggered controller will be designed to force the course velocity U_i and the yaw angular velocity r_i of ith vehicle to track the desired guidance signals U_{ci} and r_{ci} designed in last subsection. Firstly, the ET-ESO is introduced to estimate the unknown disturbances. Then, based on the estimated information generated by ET-ESO, the proportional feedback controller is developed by backstepping method. Lastly, the event triggered mechanism is designed to reduce the operations of thrusters.

Step1. Define the estimated errors as $\tilde{w}_i=w_i-\hat{w}_i$, $\tilde{F}_i=F_i-\hat{F}_i$. According to (3), ESO can be designed as

$$\begin{cases}\dot{\hat{w}}=\hat{F}_i+k_{o1i}(w_{si}-\hat{w})+BT_i\\ \dot{\hat{F}}_i=k_{o2i}(w_{si}-\hat{w}_i)\end{cases} \tag{25}$$

where $w_{si}=[U_{si},r_{si}]^T$. The ETM for U_i and r_i is designed as:

$$\begin{cases}U_{si}(t)=U_i(t_{Ui,k}),t\in(t_{Ui,k},t_{Ui,k+1})\\ t_{Ui,k+1}=inf\{t>t_{Ui,k}\,||\epsilon_{Ui}|\geq\rho_{Ui}\}\end{cases} \tag{26}$$

$$\begin{cases}r_{si}(t)=r_i(t_{ri,k}),t\in(t_{ri,k},t_{ri,k+1})\\ t_{ri,k+1}=inf\{t>t_{ri,k}\,||\epsilon_{ri}|\geq\rho_{ri}\}\end{cases} \tag{27}$$

Step2. Considering the input saturation, we can rewrite the kinetic equation as

$$\dot{w}_i = F_i(u_i, v_i, r_i) + \bar{M}^{-1}BT_{si} - \bar{M}^{-1}B\Delta_{Tsi} \tag{28}$$

where $\Delta_{\mathrm{Tsi}} = T_{si} - T_i$, $\mathrm{T_{si}}$ is the measured value of $\mathrm{T_{ci}}$. To compensate the input saturation, the dynamic auxiliary system is designed as

$$\dot{\Lambda}_i = -G\Lambda_i - \bar{M}^{-1}B\Delta_{Tsi} \tag{29}$$

Let $w_{ci} = [U_{ci}, r_{ci}]^T$ be the desired guidance signal, $w_{cfi} = [U_{cfi}, r_{cfi}]^T$ be the filtered version of $\mathrm{w_{ci}}$.The first order differentiator is introduced to obtain the derivative of w_{ci} as

$$\dot{w}_{cfi} = -T_{wi}^{-1}(w_{cfi} - w_{ci}) \tag{30}$$

where $\mathrm{T_{wi}} = \mathrm{diag}\{T_{Ufi}, T_{rfi}\}$, $T_{Ufi}, T_{rfi} > 0$. Define the tracking error and filtered error as $e_{wi} = w_i - w_{ci} - \Lambda_i$ and $e_{wfi} = w_{ci} - w_{cfi}$. The error dynamics of $\mathrm{e_{wfi}}$ and $\mathrm{e_{wi}}$ are calculated as

$$\dot{e}_{wfi} = -T_{wfi}^{-1}e_{wfi} + \dot{w}_{ci} \tag{31}$$

$$\dot{e}_{wi} = F_i + \bar{M}^{-1}BT_{si} + G\Lambda_i - \dot{w}_{cfi} + \dot{e}_{wfi} \tag{32}$$

Then, the proportional feedback control law is designed as

$$T_{ci} = B^{-1}\bar{M}(-\hat{F}_i - G\Lambda_i + \dot{w}_{cfi} - K_{wi}e_{wi}) \tag{33}$$

The control law is completed here. To reduce the operations of thrusters, the ETM for controller will be designed in next step.

Step3. Let $\tau_{ci} = BT_{ci}$, $\tau_{ci} = [\tau_{uci}, \tau_{rci}]^T$. The ETM for τ_{ci} can be designed as

$$\begin{cases} \tau_{usi}(t) = \tau_{uci}(t_{uci,k}), t \in (t_{\tau_u i,k}, t_{\tau_u i,k+1}) \\ t_{\tau_u i,k+1} = inf(t \geq t_{\tau_u i,k} \| \epsilon_{\tau_u i}| \geq \mu_{\tau_u i}|\tau_{uci}| + \rho_{\tau_u i}) \\ \tau_{rsi}(t) = \tau_{rci}(t_{uci,k}), t \in (t_{\tau_r i,k}, t_{\tau_r i,k+1}) \\ t_{\tau_r i,k+1} = inf(t \geq t_{\tau_r i,k} \| \epsilon_{\tau_r i}| \geq \mu_{\tau_r i}|\tau_{rci}| + \rho_{\tau_r i}) \end{cases} \tag{34}$$

Let $\tau_{si} = [\tau_{usi}, \tau_{rsi}]^T$.Then, the ETM for T_{ci} is designed as

$$\begin{cases} T_{si} = B^{-1}\tau_{si}, t \in (t_{T_c i,k}, t_{T_c i,k+1}) \\ t_{T_c i,k+1} = min\{t_{\tau_u i,k+1}, t_{\tau_r i,k+1}\} \end{cases} \tag{35}$$

Stability analysis

Step1. We analyze the stability of kinematic tracking errors e_{fi}, $e_{\psi i}$ and e_{si} in this step. Let $E_\psi = [e_{\psi 1}, \dots, e_{\psi n}]^T$, $E_\beta = [\beta_1 - \beta_0, \dots, \beta_n - \beta_0]^T$, $E_r = [e_{r1}, \dots, e_{rn}]^T$, $E_{\lambda r} = [\lambda_{r1}, \dots, \lambda_{rn}]^T$, $E_{\psi f} = [e_{\psi f1}, \dots, e_{\psi fn}]^T$, $\dot{\Psi}_c = [\dot{\psi}_{c1}, \dots, \dot{\psi}_{cn}]^T$. Substituting the virtual control law (15), (19) and (24) into (14), (18) and (23) respectively, we can get

$$\begin{cases}\dot{E}_f = (L+B)Q_f E_f + (L+B)\Theta_{f1} + (L+B)\Theta_{f2} \\ \dot{E}_\psi = -K_2 E_\psi + E_r + E_\lambda - \Gamma_{\psi f}^{-1} E_{\psi f} + \dot{\Psi}_c \\ \dot{E}_{\psi f} = -\Gamma_{\psi f}^{-1} E_{\psi f} + \dot{\Psi}_c \\ \dot{E}_s = -(L+B)tanh(E_s) + (L+B)\Theta_s \end{cases} \tag{36}$$

where $Q_f \in \mathbb{R}^{n\mathrm{xn}}$, $Q_f = \mathrm{diag}\left\{\frac{\left\|\nabla_{f_1}\right\| U_0 k_{11}}{\sqrt{1+(k_{11}f_{11})^2}}, \dots, \frac{\left\|\nabla_{f_n}\right\| U_0 k_{1n}}{\sqrt{1+(k_{1n}f_{1n})^2}}\right\}$, $\Theta_{\mathrm{fi}} \in \mathbb{R}^n$, $\Theta_{\mathrm{f1i}} = 2\left\|\nabla_{f_i}\right\| U_0 \sin\left(\frac{\mathrm{e}_{\psi\mathrm{i}}+\mathrm{e}_{\beta\mathrm{i}}}{2}\right)\left(\cos\left(\frac{\mathrm{e}_{\psi\mathrm{i}}+\mathrm{e}_{\beta\mathrm{i}}}{2}\right)\cos(\psi_{ci}+\beta_{0i}-\psi_{di}) - \sin\left(\frac{e_{\psi i}+e_{\beta i}}{2}\right)\sin(\psi_{ci}+\beta_{0i}-\psi_{di})\right)$, $\Theta_{f2} \in \mathbb{R}^n$, $\Theta_{f2i} = \left\|\nabla_{f_i}\right\|(U_i - U_0)\sin(\psi_1 + \beta_1 - \psi_{d1})$.

Lemma 1. Considering the

Proof: Construct Lyapunov function as $V_1 = E_f^T E_f + E_\psi^T E_\psi + E_{\psi f}^T E_{\psi f} + E_s^T E_s$. Taking the derivative of V_1 along subsystem (36), we can obtain that

$$\dot{V}_1 =$$

Step2. The stability of ESO is analyzed in this step. According to the property of the ETM described as (26) and (27), the w_{si} can be expressed by

$$w_{si} = w_i + \zeta_{wi}\rho_{wi} \tag{37}$$

where $\rho_{\mathrm{wi}} = [\rho_{Ui}, \rho_{ri}]^T$, $\zeta_{\mathrm{wi}} = [\zeta_{\mathrm{Ui}}, \zeta_{ri}]^T$, $-1 \le \zeta_{\mathrm{Ui}}, \zeta_{ri} \le 1$. Combined (3), (25) and (37), the estimated errors' dynamics can be calculated as

$$\begin{cases}\dot{\widetilde{w}}_i = \widetilde{F}_i - k_{o1i}\widetilde{w}_i - k_{o1i}\zeta_{wi}\rho_{wi} \\ \dot{\widetilde{F}}_i = -k_{o2i}\widetilde{w}_i - k_{o2i}\zeta_{wi}\rho_{wi} + \dot{F}_i \end{cases} \tag{38}$$

Let $\mathrm{E}_{\mathrm{oi}} = [\widetilde{w}_i^T, \widetilde{F}_i^T]^T$, $\mathrm{E}_{\mathrm{o}} = [E_1^T, \dots, E_n^T]^T$. The error dynamics (38) can be rewritten as

$$\dot{E}_o = Q_w E_o - K_o \Theta_{o1} + \Theta_{o2} \tag{39}$$

Lemma 2.

Step3. We analyze the stability of kinetic tracking errors e_{fi}, $e_{\psi i}$ and e_{si}. According to the property of the ETM described as (26) and (27), the w_{si} can be expressed by

Lemma 3.

Based on the above lemmas, we give the following stability theorem of the whole closed-loop control system.

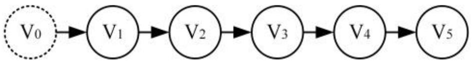
Theorem 1.

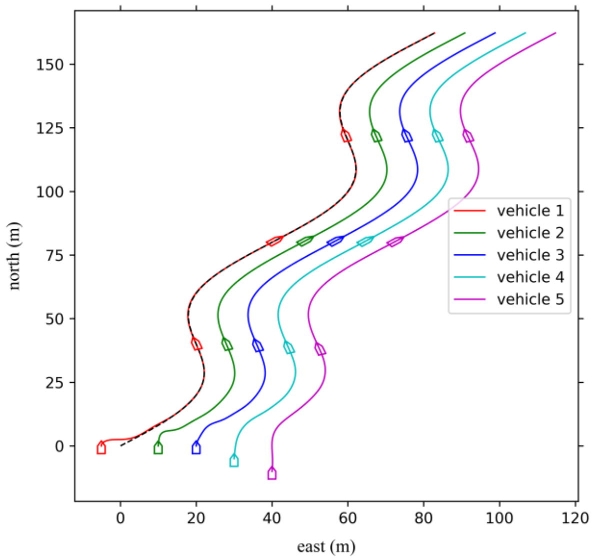
In addition, we will proof the Zeno phenomenon will be avoided by the ETM (26), (27) and (35).

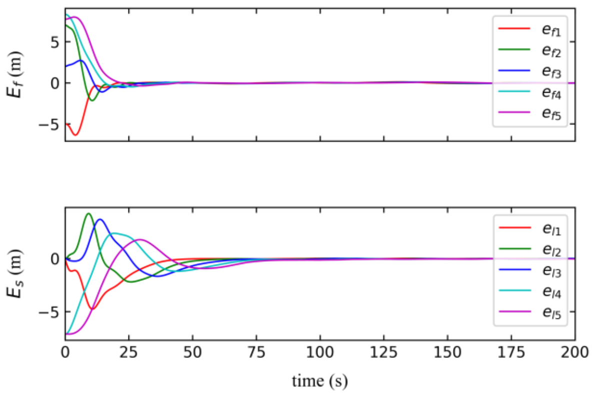
Theorem 2.

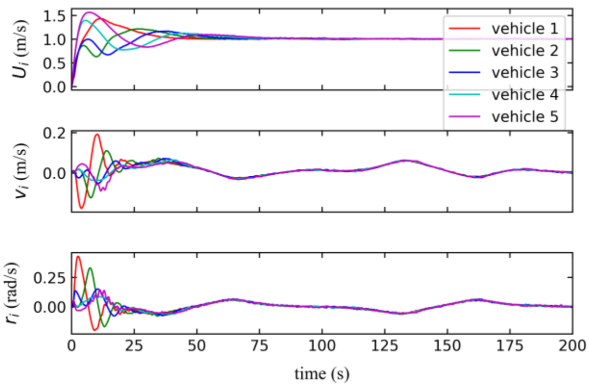
SIMULATION

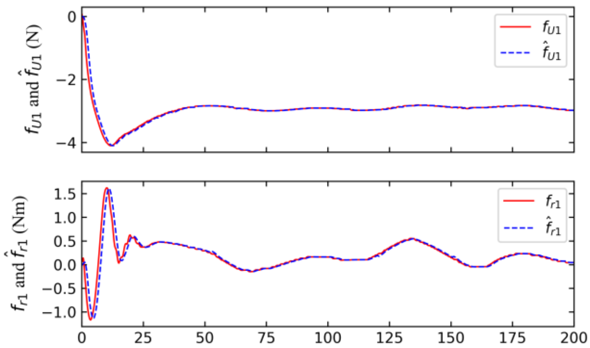
In this section,



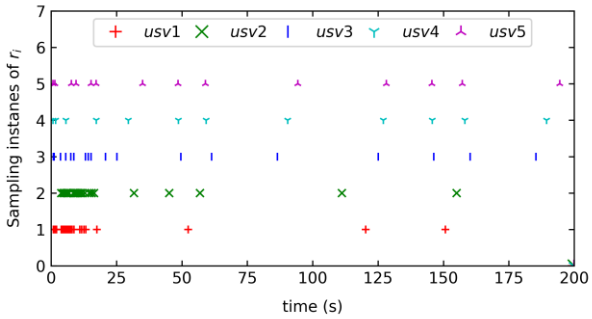




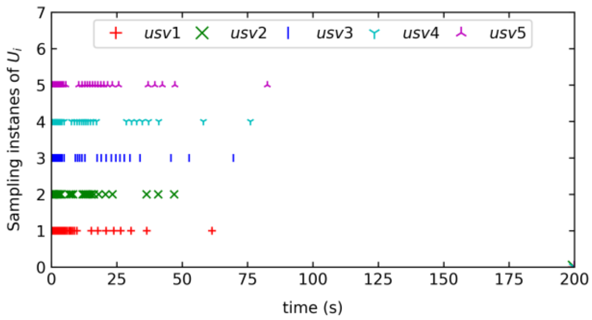




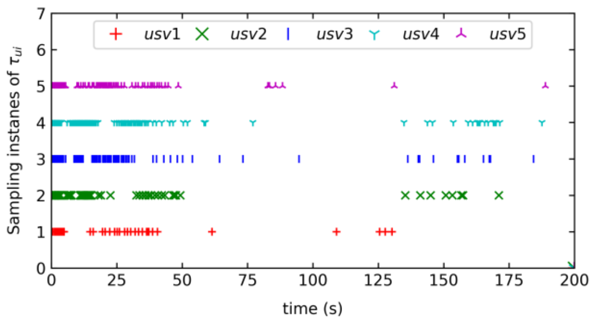
f_{U1} and $\hat{f}_{U1}$ (N)
f_{U1}
$\hat{f}_{U1}$
f_{r1} and $\hat{f}_{r1}$ (Nm)
f_{r1}
$\hat{f}_{r1}$



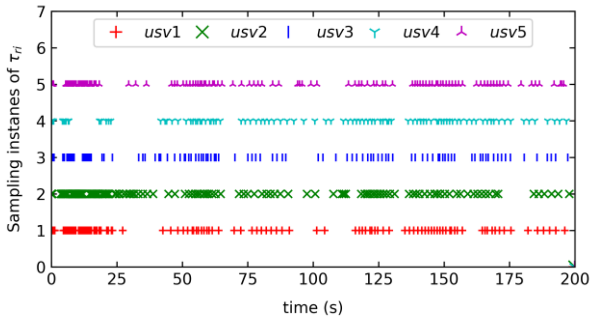
usv1 usv2 usv3 usv4 usv5
Sampling instanes of r_i
time (s)

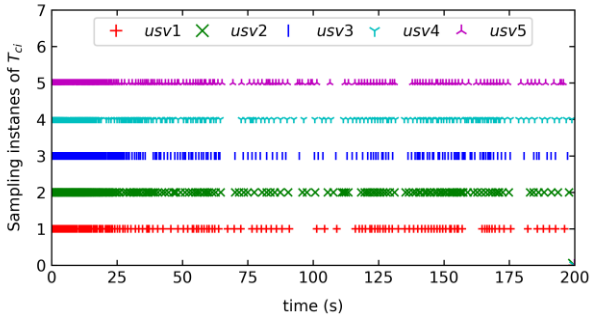


usv1 usv2 usv3 usv4 usv5
Sampling instanes of U_i
time (s)



usv1 usv2 usv3 usv4 usv5
Sampling instanes of τ_{ui}
time (s)





CONCLUSION

REFERENCE