Homework 3 - Spectral Method

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Note: The zip folder include main c++ code in /comp/hw3/ and ROOT plot code in /root/macros. Code is compiled in g++ (clang) in Mac OSX.

1 Problem 1: T and H

1.a Average and RMS

	first year	second year	both
T mean	23.3115	23.4154	23.3635
σ_T	3.47185	3.33382	3.40393
H mean	41.5769	40.6145	41.0962
σ_H	2.78972	2.97524	2.92377

1.b FFT and power spectrum

Before FFT, the given T and H data was subtracted by their means to remove the DC mode in the Fourier transform. Complex one-dimension FFTW3 routine was called to produce an 104×2 array of Fourier coefficients. The power spectrum is calculated by adding absolute value of $f(\hat{k})$ and $f(\hat{-k})$ for each k (efficiently the 1D k shell with bin size $2\Pi/104$. The spectra are shown in Fig. 1.

Explain: The temperature original data (blue plot in Fig. 1) displays a dominant mode with period of one year. That mode gives the highest point the the Fourier transform. The secondary dominant mode is 2 times higher in frequency. This is the main fluctuation on top of the dominant seasonal mode. Other frequencies contribute a lot less power.

The humidity original data (red plot in Fig. 1) is more spread out and contain no clear dominant frequency thus the spectrum is more even out.

1.c Removing seasonal variation

Seasonal variation has period of 1 year. Thus the corresponding frequency is $\pm 2\Pi/104$ x $2 = \Pi/26$. These frequencies were removed from the FFT which subsequently got inverse FFT to get fluctuation in T and H without seasonal variations. See Fig. 2

Comparing fluctuation between H and T: RMS value of re-transformed T and H were optained.

$$T_{rms}/ < T > \approx 0.90914/23.3635 \approx 0.039 \tag{1}$$

$$H_{rms}/ < H > \approx 1.85591/41.0962 \approx 0.045 \tag{2}$$

Thus, without seasonal variation, the humid data fluctuates more. This can be sean from Fig. 2

2 Problem 2: Gaussian Fields

2.a implementation of FFT

Main code in comp/hw3/problem2_inmodule.cc. Plot code /root/macros/hw3_2.cc and hw3_2c.cc The data is ready in row major format and was fft by fftw3.

2.b Power spectrum

In this problem, it is not neccesary to substract the mean before FFT because Gaussian fields have mean zero.

Log-Log plots of all three Gaussian fields power spectra are shown in blue in Fig. (4), (6) and (8). The 3D plots are linear.

2.c Spectra types

Gaussian1 and Gaussian3 field has log-log linear power spectrum in k space with some slope. The power spectrum is a power law of k^{slope}.

Gaussian2 field power spectrum is gaussian.

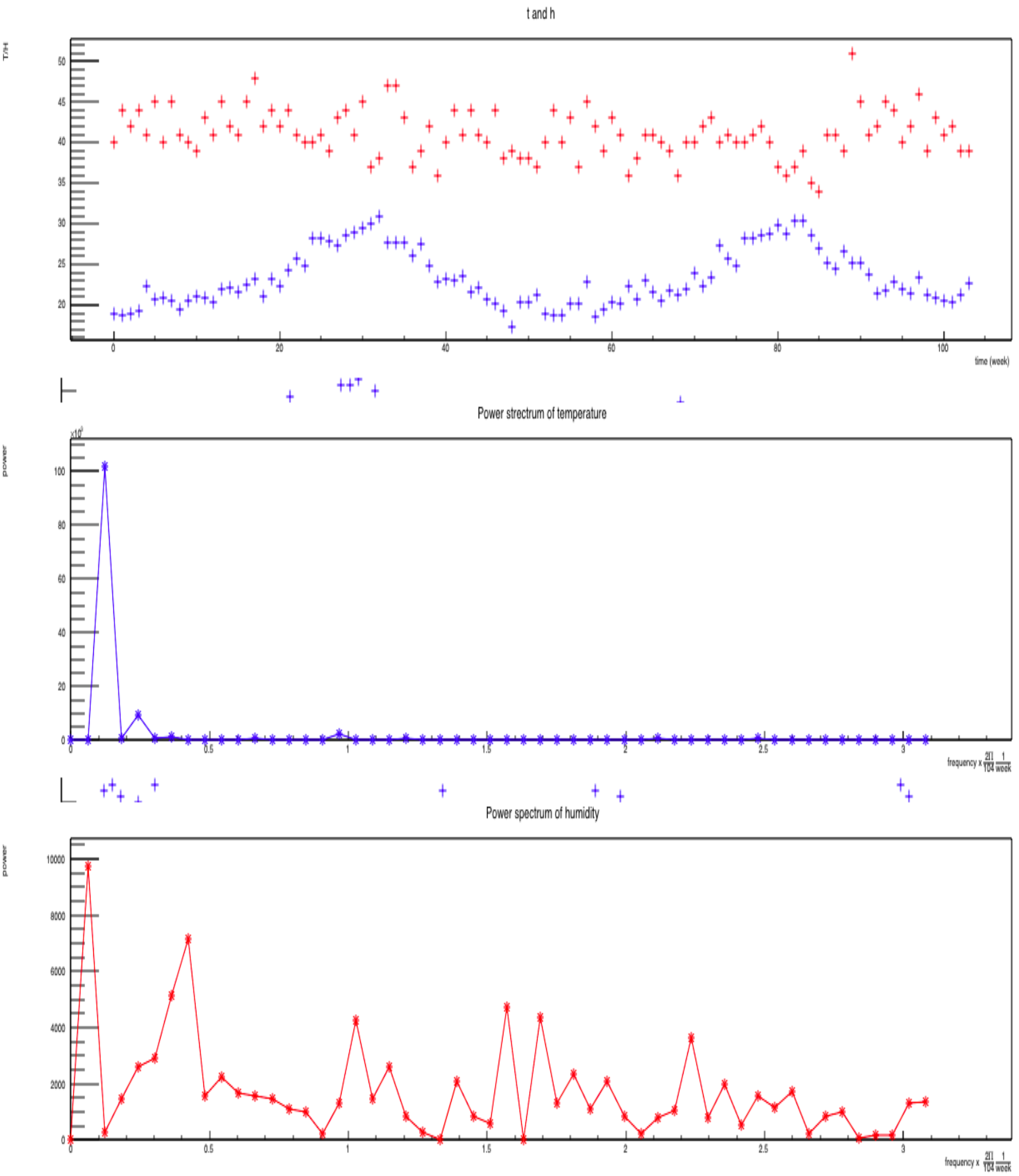

Figure 1: Temperature and humid fluctuation and their power spectra

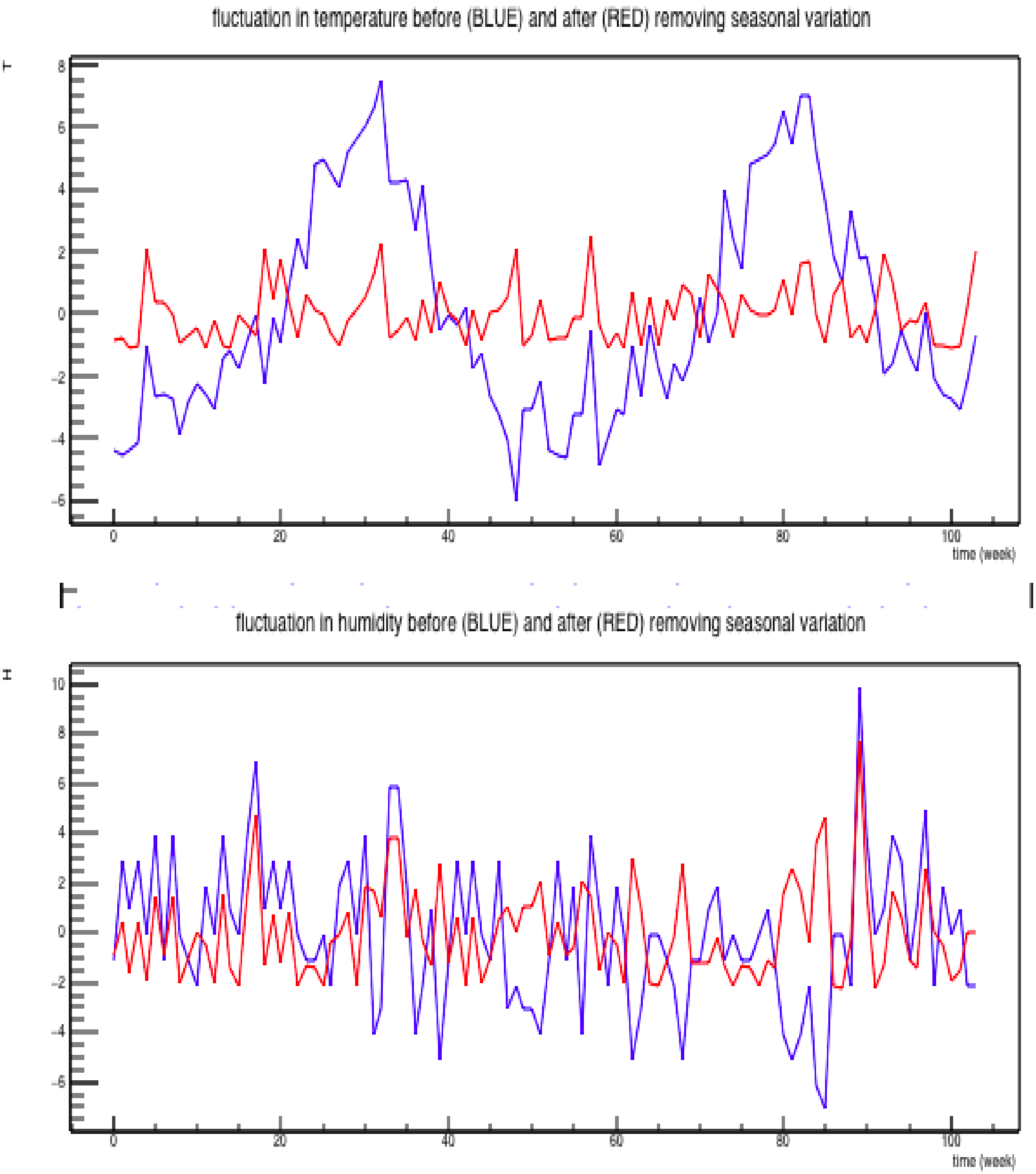

Figure 2: Temperature and humid fluctuation with and without seasonal variation

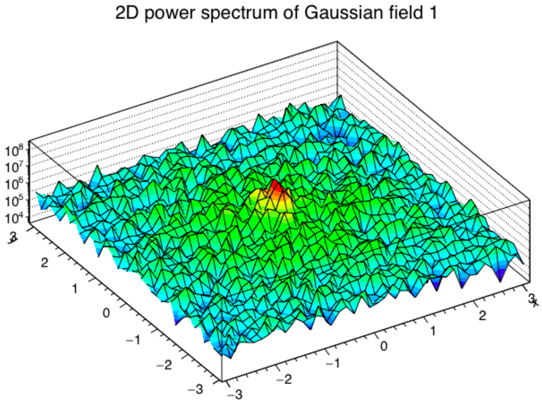

Figure 3: Gaussian1

Figure 4: Gaussian1 before and after resampling. The plot remains linear feature up unil new Nyquist frequency $\Pi/2$

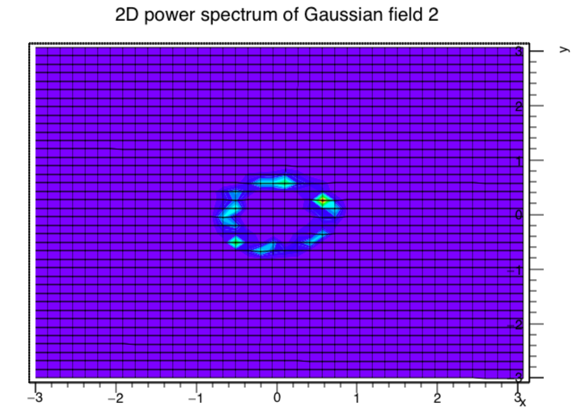

Figure 5: Gaussian2

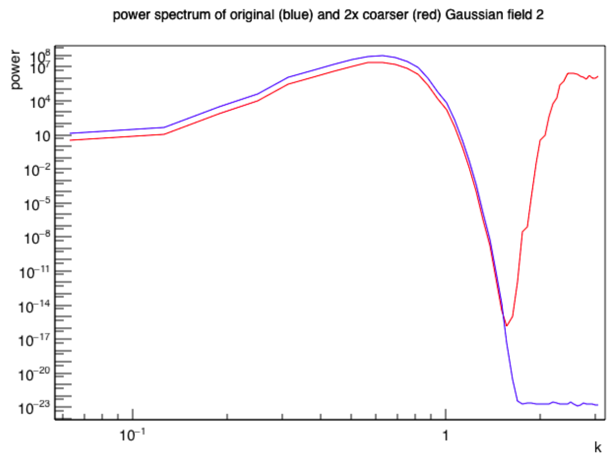

Figure 6: Gaussian2 before and after resampling. Because the original power spectrum is very low beyond new $K_{Nyquist}^{new} = \Pi/2$, aliasing doesn't affect the original lower spectrum very much. The power spectrum beyond $\Pi/2$, $K_{Nyquist}^{new}$ is no longer meaningful

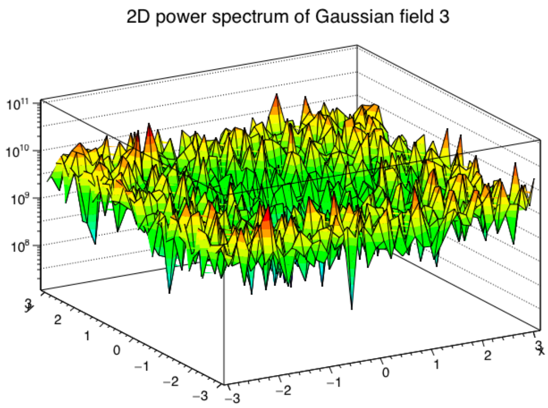

Figure 7: Gaussian3

Figure 8: Gaussian3 before and after resampling. Because the original power spectrum is high beyond new $K_{Nyquist}^{new} = \Pi/2$, a lot of power is aliased back into lower frequencies.

2.d Power spectrum of 2 times coarser re-sampled fields

When iterate through the input grid, every other line was skipped. Effectively, the Nyquist frequency is lower by half. Depends on the power spectrum, aliasing affects differently. See more explaination under figures.

3 Problem 3: Rayleight Levy Random Walk

First, formula for generating random step size r and θ and ϕ from uniform random real number [0,1] were derived. The accumulative distribution

$$C(r) = \int_r^\infty P(r)d^3r = \int_{r_o}^\infty P(r)d^3r - \int_{r_o}^r P(r)d^3r \tag{3}$$

where r_0 is the minimum radial step. The given C(r) and normalization condition gives

$$(\frac{r_o}{r})^\alpha = 1 - \int_{r_o}^r P(r)d^3r \tag{4}$$

The last term on the RHS must be a random uniform number between [0,1], say u_1. Invert this and use α=3/2 one gets:

$$r = \frac{r_o}{(1-u_1)^{2/3}} \tag{5}$$

The solid angle $d\Omega = \sin(\theta)d\theta\ d\phi$ distribution must also be uniformly random, requiring dcos(θ) and θ to be uniformly random in [-1,1] and [0,2Π] respectively. Invert these, one gets:

$$\theta = cos^{-1}(2u_2 - 1) \tag{6}$$

$$\phi = 2\Pi u_3 \tag{7}$$

The starting position (x,y,z) is also randomly generated for each run. Increment in x,y,z are then calculated from spherical coordinate transformation. The boundaries are all period (as in a 3-torus).

3.a Implementation

See file /comp/hw3/problem3.cc for main code and /root/macros/hw3_3.cc for plotting

See Fig. 9 for visualization of the random walk

3.b CIC algorithm

The code go through all 50000 particles and sort them into cubes of size $s = \frac{250}{400}$. A cube is indexed by the coordinate of the corner closest to the origin (x_i, y_i, z_i), i from 0 to 399. For each particle at (x,y,z), its density value δ^3(x,y,z) is distributed to eight points:

$$(x_i, y_i, z_i), (x_{i+1}, y_i, z_i), (i_x, y_{i+1}, i_z), (x_{i+1}, y_{i+1}, z_i) \tag{8}$$

$$(x_i, y_i, z_{i+1}), (x_{i+1}, y_i, z_{i+1}), (i_x, y_{i+1}, z_{i+1}), (x_{i+1}, y_{i+1}, z_{i+1}) \tag{9}$$

where $x_{i+1} - x_i =$ s. Since the the boundary is periodic, if i=399, set i+1 $\rightarrow$ 0 instead. The equation for density distributed to point (x_p, y_p, z_p) from δ^3(x,y,z)

$$f(p) = \frac{s - |x_p - x|}{s} \times \frac{s - |y_p - y|}{s} \times \frac{s - |z_p - z|}{s} \tag{10}$$

each grid point will then receive its density from 8 neighboring random particles.

3.c FFT

Use 3D FFTW3 routine. See Fig. 10 for result.

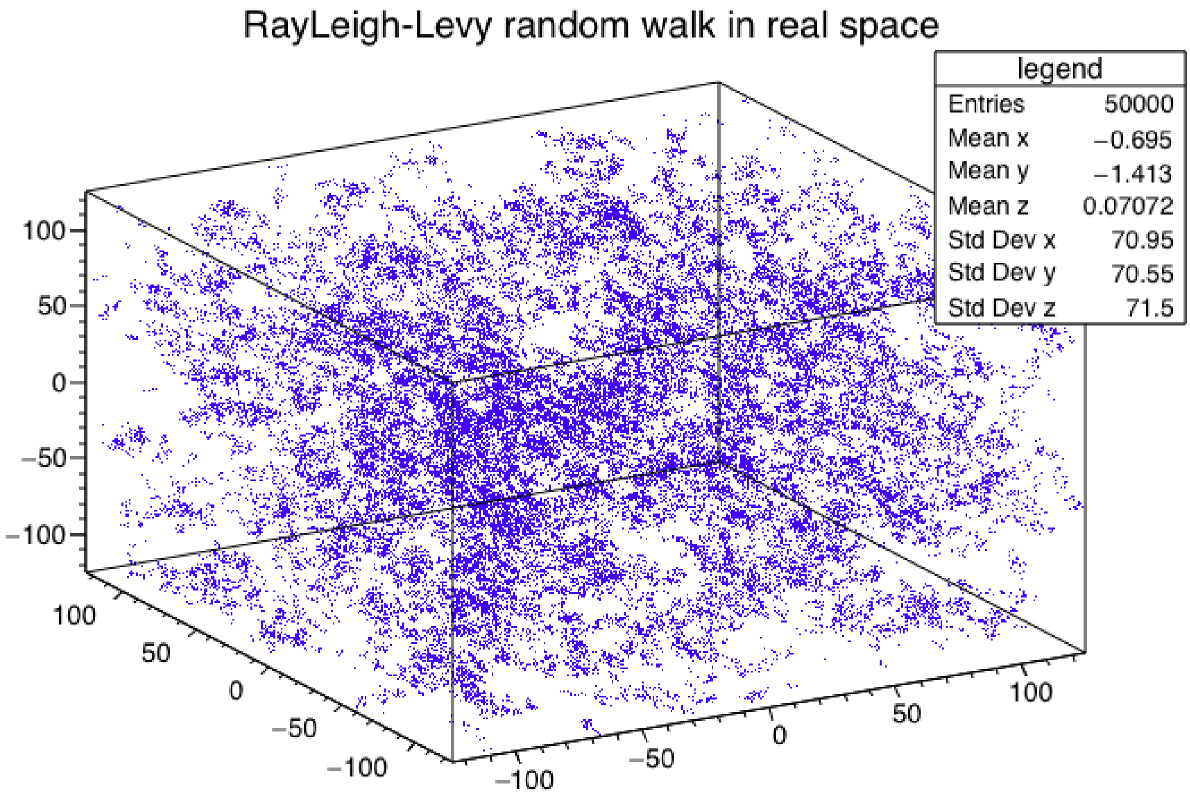

Figure 9: 3D plot of the random walk in a periodical cube

3.d Power Spectrum with and without CIC window correction

Iterate through every one in 400^3 cubes in k space and obtained its associated k value.

In k space, for every spherical shell of radius k and thickness $2\Pi/400$, all cubes (aka volume element) belongs to the shell is counted and their Fourier coefficient is distributed to that shell. The total shell contribution/shell volume gives power of k mode.

The FFT of the Window function is

$$\hat{W} = \hat{W_x}\hat{W_y}\hat{W_z} \tag{11}$$

where for k_{x_i} non zero

$$\hat{W_{x_i}} = \frac{sin^2(k_{x_i}s/2)}{(k_{x_i}s/2)^2}; x_i = x, y, z \tag{12}$$

and for k_{x_i} =0, $\hat{W_{x_i}} = 1$

3.e Explanation

Power spectrum of higher frequencies near Nyquist value (Π) are amplified more after convoluting with the window function. Mathematically the correction $\frac{1}{\hat{W}}$ increases as k approach Nyquist value so the correction for $|\hat{f}|$ is stronger in this regime. As k approaches 0, the correction $\frac{1}{\hat{W}}$ approaches 1 when the original spectrum is unaffected. See Fig. (11)

(More figures below)

References

[1] Roman Scoccimarro, *Lecture notes.* 2015

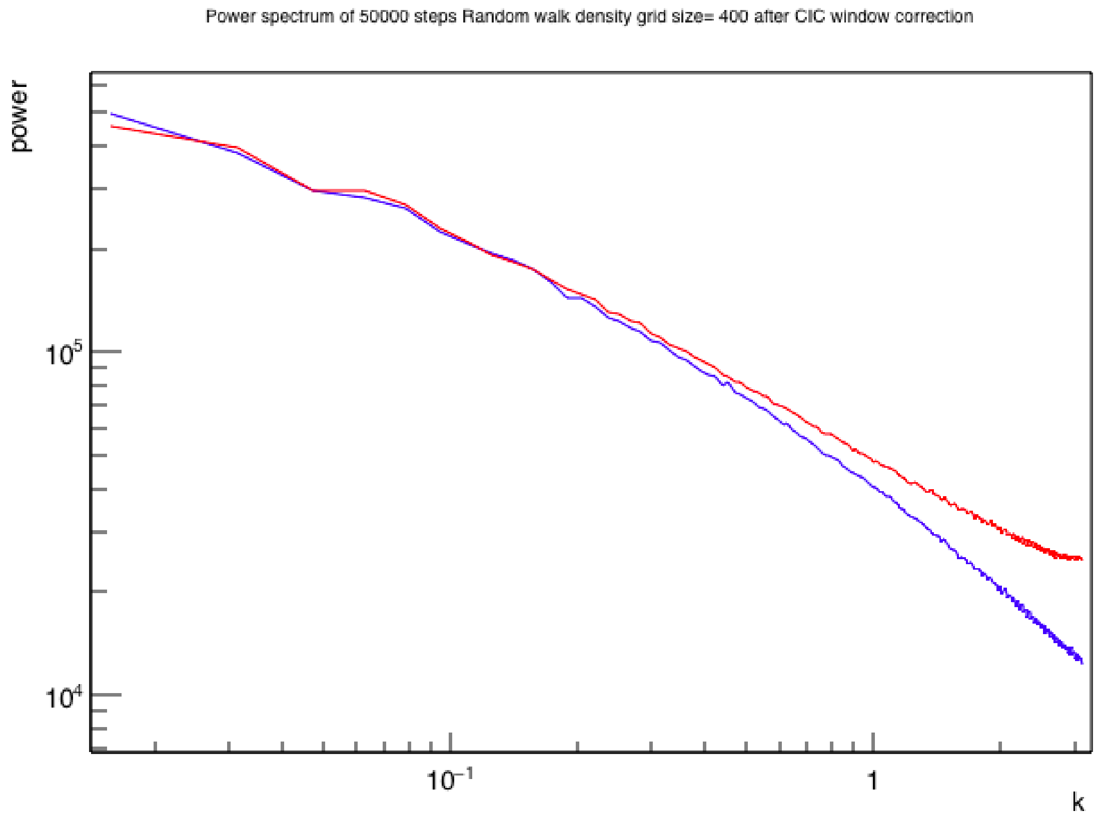

Figure 10: Power spectrum of the random walk before (blue) and after (red) applying CIC window correction. K space 400x400x400

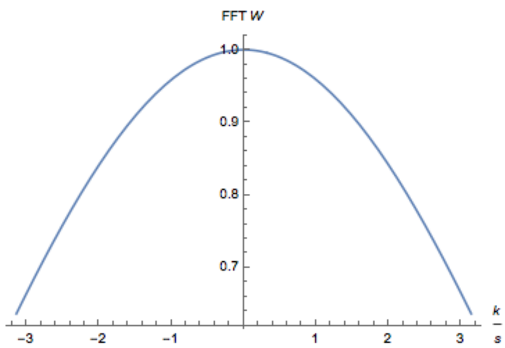

Figure 11: The 1D $\hat{W}$ Function approach 1 as k approach 0 but become small as k approach Nyquist value

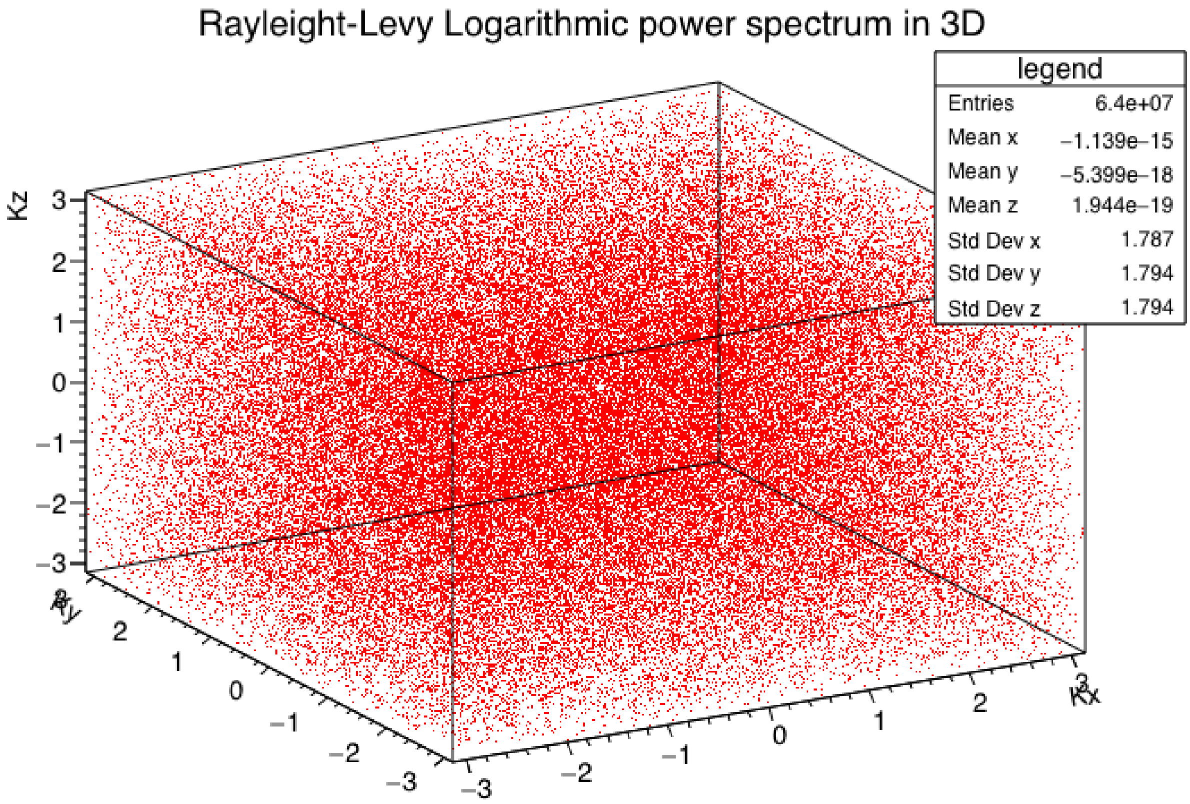

Figure 12: Log of power spectrum of in K space after window correction