Computational Physics Homework Set #3

(Due 10/29, at noon)

1) Download from the course website a data file that contains temperature in degree Celsius (first column) and percentage of relative humidity (second column) for two years. Each row corresponds to a week (there should be 104 rows).

a) Calculate the average temperature and humidity, and their *rms* fluctuations ($\sigma_x \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$). Do this for the first and second years separately, and then the two years together. Compare the different values.

b) Calculate the FFT for the whole (2 years) data stream. Calculate the power spectra of temperature and humidity. Plot them and explain what you see.

c) Taking away seasonal variations (how do you do this?), which quantity (temperature or humidity) has larger fluctuations? (Be careful about this, do not compare quantities with different units!)

2) Download from the course website 3 data files corresponding to different two-dimensional Gaussian random fields. Each file has three columns: $x, y, \Phi(x, y)$, in a 100×100 grid with positions in $[0, 1)$ and periodic boundary conditions.

a) Compute the FFT for each case.

b) Measure the power spectrum for each of them (use bin size in Fourier space equal to the fundamental mode $k_{\rm F} = 2\pi/L$). Make a log-log plot of all of them as a function of wavenumber up to the Nyquist frequency of the grid.

c) Can you tell what power spectrum was used to generate each of them?

d) Resample the fields in a grid two times coarser than the original spacing. Repeat parts a-b). Explain what you see.

3*) Consider the Rayleigh-Lévy random walk in three dimensions where, starting from a seed particle (whose coordinates are drawn at random), the next particle is placed at a randomly chosen direction and distance r drawn from the *cumulative* distribution,

$$C(r) = \int_r^\infty P(r)\, d^3r = \left(\frac{r_0}{r}\right)^\alpha, \tag{1}$$

for $r \geq r_0$ and $C(r) = 1$ otherwise, with $1 < \alpha < 2$. This process is repeated with r and the direction independently chosen each time through a large number of steps N_s.

*This problem counts as extra credit for undergraduate students.

Perform a numerical simulation of this random walk, with the following additional feature. Start from a randomly generated seed particle, random walk for $N_s - 1$ steps (thus generating N_s particles), and repeat this process (each time generating a new random seed particle) N_c times, thus generating N_c clusters with N_s particles each. Assume your particles are inside a cubic box of side $L_{\rm box}$, and use periodic boundary conditions.

a) Write a code that performs the simulation, using a reasonable random number generator, and the following parameters,

$$\alpha = \frac{3}{2}, \qquad r_0 = 1, \qquad N_s = 100, \qquad N_c = 500, \qquad L_{\rm box} = 250, \qquad N_{\rm grid} = 400. \quad (2)$$

b) Obtain the particle density in a grid of size $N_{\rm grid}$ in each direction using CIC interpolation, extending the discussion in class to three dimensions.

c) FFT the particle density to Fourier space, e.g. using `FFTW`[2].

d) Measure the power spectrum, and show $P(k)$ as a function of k in a log-log plot, before and after correction for the CIC interpolation window. You can use linear or log binning in k, with k from the fundamental mode to the Nyquist frequency of the grid.

e) Explain all the different regimes and features you see in the window-corrected plot.

[2]Download from `http://www.fftw.org` and install in your system if not available. Check the manual at the website for compilation flags and examples on how to call the `FFTW` library inside your code.