- Mostly limited only to numerical features (or very limited one-hot encoded otherwise).

 Somehow it turns out it's better to not include the features of the point to predict at all during the actual prediction, but only to use them to retrieve the
- It's still surprising that using an average of the embeddings of the training data fares better than just using a model trained on the training data

TabR: Unlocking the Power of Retrieval-Augmented Tabular Deep Learning

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Abstract

Deep learning (DL) models for tabular data problems are receiving increasingly more attention, while the algorithms based on gradient-boosted decision trees (GBDT) remain a strong go-to solution. Following the recent trends in other domains, such as natural language processing and computer vision, several retrievalaugmented tabular DL models have been recently proposed. For a given target object, a retrieval-based model retrieves other relevant objects, such as the nearest neighbors, from the available (training) data and uses their features or even labels to make a better prediction. However, we show that the existing retrieval-based tabular DL solutions provide only minor, if any, benefits over the properly tuned simple retrieval-free baselines. Thus, it remains unclear whether the retrieval-based approach is a worthy direction for tabular DL.

In this work, we give a strong positive answer to this question. We start by incrementally augmenting a simple feed-forward architecture with an attentionlike retrieval component similar to those of many (tabular) retrieval-based models. Then, we highlight several details of the attention mechanism that turn out to have a massive impact on the performance on tabular data problems, but that were not explored in prior work. As a result, we design TabR - a simple retrievalbased tabular DL model which, on a set of public benchmarks, demonstrates the best average performance among tabular DL models, becomes the new state-ofthe-art on several datasets, and even outperforms GBDT models on the recently proposed "GBDT-friendly" benchmark (Figure 1). The source code is available at https://github.com/yandex-research/tabular-dl-tabr.

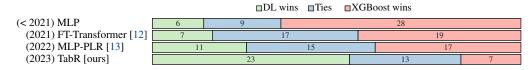


Figure 1: Comparing DL models with XGBoost [7] on the benchmark with middle scale tasks $(\le 50K \text{ objects})$ from "Why do tree-based models still outperform deep learning on typical tabular data?" by Grinsztajn et al. [14]. Our model TabR continues the positive trend for tabular DL.

1 Introduction

Machine learning (ML) problems on tabular data, where objects are described by a set of heterogeneous features, are ubiquitous in industrial applications in medicine, finance, manufacturing, and other fields. Historically, for these tasks, the models based on gradient-boosted decision trees (GBDT) have been a go-to solution for a long time. However, lately, tabular deep learning (DL) models have been receiving increasingly more attention, and the tabular DL models are becoming more competitive [12, 13, 16, 17, 27, 29, 36, 44, 48].

In particular, several attempts to design a retrieval-augmented tabular DL model have been recently made [29, 39, 44]. In these models, for a given input object, one first retrieves relevant data points, such as the object's nearest neighbors, from a database, and then a machine learning model processes the retrieved objects together with the target object to produce a better prediction for this target object. In fact, the retrieval technique is widely popular in other mainstream domains, including natural language processing [9, 19, 49], computer vision [18, 21, 31], CTR prediction [10, 38, 39], protein structure prediction [8] and others. Compared to purely parametric (i.e. retrieval-free) models, the retrieval-based ones can achieve higher performance and also exhibit several practically important properties, such as an ability for incremental learning and better robustness [9, 21].

While multiple retrieval-augmented models applicable to (or directly designed for) tabular data problems exist, in our experiments, we show that they provide only minor benefits at best over the properly tuned multilayer perceptron (MLP) – the simplest parametric model. This is a critical observation, since retrieval components are typically associated with a noticeable overhead, and marginal metric improvements may not be enough to justify the extra costs. Nevertheless, in this work, we show that it is possible to design a retrieval-augmented model for tabular data problems that (1) provides strong average performance with significant improvements over parametric models at least on some datasets (2) while being a mainstream accessible model, that scales to datasets with at least up to several million data points. We summarize our main contributions as follows:

- 1. We revisit the existing retrieval-augmented solutions for tabular data problems and reveal that they provide only minor benefits over a tuned MLP while introducing significant overhead.
- 2. We design TabR a simple retrieval-augmented tabular DL model which, on a set of public benchmarks, demonstrates the best average performance among DL models and achieves the new state-of-the-art on several datasets.
- 3. In particular, TabR outperforms GBDT on the recently proposed benchmark with middle-scale tasks [14], which was originally used to illustrate the superiority of decision-tree-based models over DL models. Tree-based models, in turn, remain a more efficient solution.
- 4. We highlight the important degrees of freedom of the attention mechanism (the often used block in retrieval-based models) that allow designing better retrieval-based tabular models.

2 Related work

Gradient boosted decision trees (GBDT). ML models based on gradient-boosted decision trees (GBDT) are non-DL solutions for supervised problems on tabular data, which are popular within the community due to their strong performance and high efficiency. By employing the modern DL building blocks and, in particular, the retrieval technique, our new model successfully competes with GBDT and, in particular, demonstrates that DL models can be superior on non-big data by outperforming GBDT on the recently proposed benchmark with small-to-middle scale tasks [14].

Parametric deep learning models. Parametric tabular DL is a rapidly developing research direction aimed at bringing the benefits of deep learning to the world of tabular data while achieving competitive performance [12, 13, 16, 17, 27, 36, 48]. The recent studies reveal that MLP-like backbones are still competitive [12, 13, 22], and that embeddings for numerical features [13] significantly reduce the gap between tabular DL and GBDT. In this work, we show that a properly designed retrieval component can boost the performance of tabular DL even further.

Retrieval-augmented models in general. Usually, the retrieval-based models are designed as follows. For a given input object, first, they retrieve relevant samples from available (training) data. Then, they process the input object augmented with features (and potentially labels) of the retrieved instances to produce the final prediction. In some retrieval-based models, the retrieval step is omitted and all training data points serve as the "retrieved" instances [29, 43, 44]. One of

the common motivations for designing retrieval-based schemes is the local learning paradigm [6], and the simplest possible example of such a model is the <u>k</u> nearest neighbors (<u>kNN</u>) algorithm [20]. The promise of retrieval-based approaches was demonstrated across various domains, ranging from natural language processing [5, 9, 15, 19, 25, 30, 49], computer vision [18, 31], CTR prediction [10, 38, 39], reinforcement learning [42], protein structure prediction [8], point cloud processing [51, 52], and others. In addition to better performance, the retrieval-augmented models often have useful properties such as better interpretability [47] and robustness [53].

Retrieval-augmented models for tabular data problems. The classic examples of non-deep retrieval-based tabular models are the "shallow" neighbor-based and kernel methods [20, 34]. There are also deep retrieval-based models applicable to (or directly designed for) tabular data problems [26, 29, 41, 44, 50]. However, we show that they are only marginally better than simple parametric DL models, and that often comes with a cost of using heavy Transformer-like architectures. Compared to prior work, where several layers with multi-head attention between objects and features are often used [29, 41, 44], our model TabR implements its retrieval component with just *one* single-head attention-like module. However, the single attention-like module of TabR is customized in a way that makes it better suited for tabular data problems. As a result, TabR substantially outperforms the existing retrieval-based DL models while being significantly more efficient. In particular, TabR avoids the infamous quadratic complexity of the attention mechanism.

3 TabR

In this section, we design a new retrieval-augmented deep learning model for tabular data problems.

3.1 Preliminaries

Notation. For a given supervised learning problem on tabular data, we denote the dataset as $\{(x_i,y_i)\}_{i=1}^n$ where $x_i \in \mathbb{X}$ represents the i-th object's features and $y_i \in \mathbb{Y}$ represents the i-th object's label. Depending on the context, the i index can be omitted. We consider three types of tasks: binary classification $\mathbb{Y} = \{0,1\}$, multiclass classification $\mathbb{Y} = \{1,...,C\}$ and regression $\mathbb{Y} = \mathbb{R}$. For simplicity, in most places, we will assume that x_i contains only numerical (i.e., continuous) features, and we will give additional comments on binary and categorical features when necessary. The dataset is split into three disjoint parts: $\overline{1,n} = I_{train} \cup I_{val} \cup I_{test}$, where the "train" part is used for training, the "validation" part is used for early stopping and hyperparameter tuning, and the "test" part is used for the final evaluation. An input object for which a given model makes a prediction is referred to as "input object" or "target object".

When the retrieval technique is used for a given target object, the retrieval is performed within the set of "context candidates" or simply "candidates": $I_{cand} \subseteq I_{train}$. The retrieved objects, in turn, are called "context objects" or simply "context". Optionally, the target object can be included in its own context. In this work, we use the same set of candidates for all input objects.

Experiment setup. We extensively describe our tuning and evaluation protocols in subsection D.6. The most important points are that, for any given algorithm, on each dataset, following Gorishniy et al. [13], (1) we perform hyperparameter tuning and early stopping using the *validation* set; (2) for the best hyperparameters, in the main text, we report the metric on the *test* set averaged over 15 random seeds, and provide standard deviations in Appendix E; (3) when comparing any two algorithms, we take the standard deviations into account as described in subsection D.6; (4) to obtain ensembles of models of the same type, we split the 15 random seeds into three disjoint groups (i.e., into three ensembles) each consisting of five models, average predictions within each group, and report the average performance of the obtained three ensembles.

Datasets. In this work, we mostly use the datasets from prior literature and provide their summary in Table 1 (sometimes, we refer to this set of datasets as "our benchmark"). Additionally, in subsection 4.2, we use the recently introduced benchmark with middle-scale tasks ($\leq 50K$ objects) [14] where GBDT was reported to be superior to DL solutions.

	Table 1: Dataset properties.	"RMSE" denotes root-mean-squ	uare error, "Acc." denotes accuracy.
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	СН	CA	НО	AD	DI	OT	HI	BL	WE	СО	MI
#objects	10000	20640	22784	48842	53940	61878	98049	166821	397099	581012	1200192
#num.features	7	8	16	6	6	93	28	4	118	10	131
#bin.features	3	0	0	1	0	0	0	1	1	44	5
#cat.features	1	0	0	7	3	0	0	4	0	0	0
metric	Acc.	RMSE	RMSE	Acc.	RMSE	Acc.	Acc.	RMSE	RMSE	Acc.	RMSE
#classes	2	_	_	2	_	9	2	_	_	7	_
majority class	79%	_	_	76%	_	26%	52%	_	_	48%	-

3.2 Overview of the architecture

Since there are no established architectural blocks for building deep retrieval-based models for tabular data problems, we have to make a subjective choice of the architecture. We choose to keep things simple and to incrementally integrate the retrieval functionality in a conventional feed-forward network as illustrated in Figure 2. In a nutshell, a target object and its context candidates are first passed through an encoder $E: \mathbb{X} \to \mathbb{R}^d$, then the retrieval component R does its job in the residual branch and enriches the target object's representation, and finally, the predictor $P: \mathbb{R}^d \to \hat{\mathbb{Y}}$ makes the prediction. The modules E, P, and R are defined later in the text. Unless otherwise noted, we use the whole training set as the fixed set of candidates for all objects (i.e., $I_{cand} = I_{train}$).

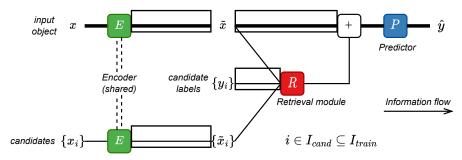


Figure 2: The architecture introduced in subsection 3.2. The model is a feed-forward network with a retrieval component located in the residual branch. The bold path highlights the structure of the feed-forward network before the addition of the retrieval module R.

Encoder and predictor. The encoder E and predictor P modules (Figure 2) are not the focus of this work, so we keep them simple as illustrated in Figure 3.

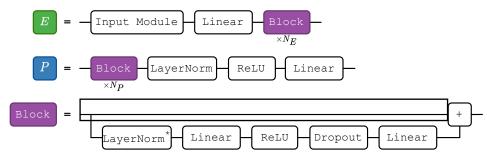


Figure 3: The architecture of the encoder E and predictor P introduced in Figure 2. The Input Module encapsulates the input processing routines (feature normalization, one-hot encoding, etc.) and assembles a vector input for the subsequent linear layer. In particular, Input Module can contain embeddings for numerical features [13]. (* LayerNorm is omitted in the first Block of E.)

Retrieval module. In Figure 4, we provide a *template* of the retrieval module R introduced in Figure 2, which operates over the representation $\tilde{x} \in \mathbb{R}^d$ of the target object and the representations $\{\tilde{x}_i\}_{i \in I_{cand}} \subset \mathbb{R}^d$ and the labels $\{y_i\}_{i \in I_{cand}} \subset \mathbb{Y}$ of the candidates. The specific instances of

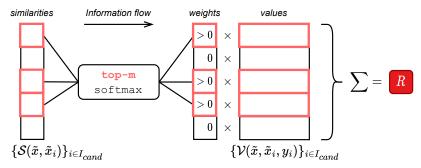


Figure 4: A simplified illustration of the retrieval module R, introduced in Figure 2. For the target object's representation \tilde{x} , the module takes the m nearest neighbors among the candidates $\{\tilde{x}_i\}$ according to the similarity module $S: (\mathbb{R}^d, \mathbb{R}^d) \to \mathbb{R}$ and aggregates their values produced by the value module $\mathcal{V}: (\mathbb{R}^d, \mathbb{R}^d, \mathbb{Y}) \to \mathbb{R}^d$. Some details are omitted for clarity. Best viewed in color.

this template will be explored in subsection 3.3. Overall, the retrieval module R can be seen as a generalized version of the attention mechanism, which is computed only for the target object. Formally, it performs the following steps (the notation follows all the above text and illustrations):

- 1. If the encoder E contains at least one Block (i.e. $N_E > 0$), then \tilde{x} and all \tilde{x}_i are first normalized with a shared layer normalization [2] (this is omitted in Figure 4 for clarity).
- 2. The context objects are defined as the m candidates most similar to the target object according to the similarity module $\mathcal{S}: (\mathbb{R}^d, \mathbb{R}^d) \to \mathbb{R}$. Optionally, the target object itself is added to the set of context objects with the similarity score $\mathcal{S}(\tilde{x}, \tilde{x})$ (this is omitted in Figure 4 for clarity).
- 3. The weights of the context objects are defined as the output of the softmax function over the similarities with the context objects obtained in the previous steps. Then, dropout is applied to the obtained weights (this is omitted in Figure 4 for clarity).
- 4. The values of the context objects are defined as the outputs of the value module $\mathcal{V}: (\mathbb{R}^d, \mathbb{R}^d, \mathbb{Y}) \to \mathbb{R}^d$.
- 5. Output the weighted aggregation using the values and the weights obtained in the previous steps.

From now on, we set the context size to a relatively large value m=96 and allow the softmax function to select the "effective" context size automatically. In the next section, we explore different implementations of the $\mathcal S$ and $\mathcal V$ modules and arrive at the final model.

3.3 Implementing the retrieval module R

After the high-level introduction of the architecture in subsection 3.2, in this section, we explore different implementations of the retrieval module R (Figure 4)), and in particular of the similarity module $\mathcal S$ and the value module $\mathcal V$. During this exploration phase, we do not use embeddings for numerical features [13] in the Input Module of the encoder E and fix $N_E = 0$, $N_P = 1$ (see Figure 3). We obtain the final model in several steps, which we describe in detail in the following paragraphs.

Step-0. Evaluating the similarity and value modules of the vanilla attention. In fact, modulo the top-m operation (see Figure 4), the retrieval module R can be instantiated as the vanilla self-attention [46] as follows:

$$S(\tilde{x}, \tilde{x}_i) = W_Q(\tilde{x})^T W_K(\tilde{x}_i) \cdot d^{-1/2} \qquad \mathcal{V}(\tilde{x}, \tilde{x}_i, y_i) = W_V(\tilde{x}_i) \tag{1}$$

where W_Q , W_K , and W_V are linear layers, and the target object is added as the (m+1)-th object to its own context (i.e., ignoring the top-m operation). We follow the original terminology and call the results of W_Q and W_K queries and keys, respectively. Given the frequent usage of the vanilla attention mechanism in prior work on retrieval-based tabular DL, we see the above configuration as a reasonable baseline implementation of R. As reported in Table 2, the Step-0 configuration performs similarly to MLP, which clearly does not justify the usage of the retrieval component at this point.

Step-1. Adding context labels. A natural attempt to improve the Step-0 configuration is to utilize labels of the context objects, for example, by incorporating them into the value module as follows:

$$S(\tilde{x}, \tilde{x}_i) = W_Q(\tilde{x})^T W_K(\tilde{x}_i) \cdot d^{-1/2} \qquad \mathcal{V}(\tilde{x}, \tilde{x}_i, y_i) = \underline{W_Y(y_i)} + W_V(\tilde{x}_i)$$
(2)

Table 2: The performance of the implementations of the retrieval module R, described in subsection 3.3. If a number is underlined, then it is better than the corresponding number from the previous step, at least by the standard deviation. Noticeable improvements over MLP start at Step-2. Notation: \downarrow corresponds to RMSE, \uparrow corresponds to accuracy.

	$\operatorname{CH}\uparrow$	$CA\downarrow$	$\mathrm{HO}\downarrow$	$AD \uparrow$	DI↓	$OT \uparrow$	$\mathrm{HI} \uparrow$	$BL \downarrow$	$WE\downarrow$	CO↑
MLP	0.854	0.499	3.112	0.853	0.140	0.816	0.719	0.697	1.905	0.963
(Step-0) Transformer-like baseline	0.855	0.484	3.234	0.857	0.142	0.814	0.719	0.699	1.903	0.957
(Step-1) + Context labels	0.855	0.489	3.205	0.857	0.142	0.814	0.719	0.698	1.906	0.960
(Step-2) + New similarity module S	0.860	0.418	3.153	0.858	0.140	0.813	0.720	0.692	1.804	0.972
(Step-3) + New value module V	0.859	0.408	3.158	0.863	0.135	0.810	0.722	0.692	1.814	0.975
(Step-4) + Technical tweaks = TabR	0.860	0.403	3.067	0.865	0.133	$\underline{0.818}$	0.722	$\underline{0.690}$	1.747	0.973

where the difference with Equation 1 is the <u>underlined</u> addition of $W_Y : Y \to \mathbb{R}^d$, which is an <u>embedding table for classification tasks and a linear layer for regression tasks</u>. For the target object, a trainable placeholder vector from \mathbb{R}^d is used instead of the label embedding $W_Y(y)$, since y is not available. Table 2 shows no improvements from using context labels, which is counter-intuitive to say the least. This makes us suspect that the similarity module S of the vanilla attention is the bottleneck which does not allow benefiting from such a valuable signal as labels.

Step-2. Improving the similarity module S. In terms of the local learning paradigm [6], the similarity module S (Figure 4) defines the notion of locality. Currently, however, the S module taken from the vanilla attention is rather an arbitrary choice inherited from other fields where the attention mechanism is used for completely different purposes. First, the motivation for using different representations (query and key) for the target and candidate objects is unclear. Second, in this section, the encoder E is shallow (a single linear layer, in fact), so a similarity measure reasonable in the original features space may remain reasonable after applying the encoder as-well. These two observations motivate us to remove the notion of queries and replace the dot product with the L_2 distance since the latter one is a somewhat reasonable similarity measure in the original feature space of a typical tabular data problem. The obtained "Step-2" configuration is defined as follows:

$$S(\tilde{x}, \tilde{x}_i) = -\|W_K(\tilde{x}) - W_K(\tilde{x}_i)\|^2 \cdot d^{-1/2} \qquad V(\tilde{x}, \tilde{x}_i, y_i) = W_Y(y_i) + W_V(\tilde{x}_i)$$
(3)

where the difference with Equation 2 is <u>underlined</u>. The results in Table 2 show that this is a turning point in our story, which was overlooked in prior work, and the retrieval-based model makes a significant jump in performance on several datasets. Crucially, in subsection B.2, we show that removing any of the three ingredients (context labels, key-only representation, L_2 distance) results in a performance drop back to the level of MLP. So we hypothesize that both things are important: how valuable the additional signal is (Step-1) and how well we measure the distance from the target object to the source of that valuable signal (Step-2). We provide further analysis on the new similarity module in subsection 5.3. Note that the L_2 distance as such is unlikely to be the universally best choice for all domains and problems (even within the tabular domain), but it seems to be a reasonable default choice for tabular data problems.

Step-3. Improving the value module \mathcal{V} . Now, we take inspiration from DNNR [34] – the recently proposed generalization of the kNN algorithm for regression problems. Conceptually, while kNN captures only local label distributions, DNNR also captures local trends (formally, derivatives). Technically, contrary to kNN, in DNNR, a neighbor contributes to the prediction not only its label but also an additional correction term that depends on the difference between the target object and the neighbor in the original feature space. Inspired by DNNR, we make the value module \mathcal{V} more expressive by taking the target object's representation \tilde{x} into account as follows:

$$S(\tilde{x}, \tilde{x}_i) = -\|W_K(\tilde{x}) - W_K(\tilde{x}_i)\|^2 \cdot d^{-1/2} \mathcal{V}(\tilde{x}, \tilde{x}_i, y_i) = W_Y(y_i) + \underline{T(W_K(\tilde{x}) - W_K(\tilde{x}_i))}$$

$$T(\cdot) = \text{LinearWithoutBias}(\text{Dropout}(\text{ReLU}(\text{Linear}(\cdot))))$$
(4)

where the difference with Equation 3 is <u>underlined</u>. Table 2 shows that the new value module further improves the performance on several datasets. Intuitively, the term $W_Y(y_i)$ (the embedding of the context object's label) can be seen as the "raw" contribution of the *i*-th context object. The term $T(W_K(\tilde{x}) - W_K(\tilde{x}_i))$ can be seen as the "correction" term, where the module T translates the differences in the key space into the differences in the label embedding space. We provide further analysis on the new value module in subsection 5.4.

Step-4. TabR. Finally, empirically, we observed that omitting the scaling term $d^{-1/2}$ in the similarity module and not including the target object to its own context (i.e., using cross-attention) leads to better results on average as reported in Table 2. Both aspects can be considered hyperparameters, and the above notes can be seen as our default recommendations. We call the obtained model "TabR" (Tab \sim tabular, R \sim retrieval). The formal complete description of how TabR implements the retrieval module R is as follows:

$$k = W_K(\tilde{x}), \ k_i = W_K(\tilde{x}_i) \quad S(\tilde{x}, \tilde{x}_i) = -\|k - k_i\|^2 \quad V(\tilde{x}, \tilde{x}_i, y_i) = W_Y(y_i) + T(k - k_i)$$
 (5)

Where the notation follows subsection 3.2, W_K is a linear layer, W_Y is an embedding table for classification tasks and a linear layer for regression tasks, (by default) cross-attention is used, (by default) the similarity scores are not scaled, and $T(\cdot) = \mathtt{LinearWithoutBias}(\mathtt{Dropout}(\mathtt{ReLU}(\mathtt{Linear}(\cdot))))$.

3.4 Technical notes

We highlight the following technical aspects of TabR:

- 1. Because of the changes introduced in the Step-3 configuration, the value representations $\mathcal{V}(\tilde{x}, \tilde{x}_i, y_i)$ of the candidates cannot be precomputed for a trained model, since they depend on the target object. This implies roughly twice less memory usage when deploying the model to production (since only the key representations and labels have to be deployed for training objects), but $\mathcal{V}(\tilde{x}, \tilde{x}_i, y_i)$ has to be computed in runtime.
- 2. Despite the attention-like nature of the retrieval module R, contrary to prior work, TabR does not suffer from the quadratic complexity, because it computes attention only for the target object, but not for the context objects.

3.5 Limitations

We highlight two main limitations of TabR. First, as for all retrieval-augmented models, one should evaluate whether the usage of real training objects for making predictions is reasonable and allowed from the application perspective (for example, privacy and ethical aspects must be considered). Second, specifically for TabR, we note that the retrieval component R, while being significantly more efficient than the analogous elements in prior work, still causes noticeable overhead compared to fully parametric models, and may not scale to truly large datasets as-is. We showcase a simple trick to scale TabR to larger datasets in subsection 5.1. We discuss the efficiency aspect in more detail in subsection B.5.

4 Experiments on public benchmarks

In this section, we compare TabR (introduced in section 3) with existing retrieval-augmented solutions and state-of-the-art parametric models. In addition to the fully-fledged configuration of TabR (with all degrees of freedom available for E and P as described in Figure 3), we also use **TabR-S** ("S" stands for "simple") – a simple configuration, which does not use feature embeddings [13], has a linear encoder ($N_E=0$) and a one-block predictor ($N_P=1$) (in fact, in section 3, it was TabR-S all the time). We specify when TabR-S is used only in tables, figures, and captions but not in the text. For other details on TabR, including hyperparameter tuning, see subsection D.8.

4.1 Evaluating retrieval-augmented deep learning models for tabular data

In this section, we compare TabR (section 3) and the existing retrieval-augmented solutions with fully parametric DL models (see Appendix D for implementation details for all algorithms). Table 3 indicates that TabR is the only retrieval-based model that provides a significant performance boost over MLP on many datasets. In particular, the full variation of TabR outperforms MLP-PLR (the modern parametric DL model with the highest average rank from Gorishniy et al. [13]) on several datasets (CA, OT, BL, WE, CO), and performs on par with it on the rest except for the MI dataset. Regarding the prior retrieval-based solutions, we faced various technical limitations, such as incompatibility with classification problems and scaling issues (e.g., as we show in subsection B.5, it takes dramatically less time to train TabR than NPT [29] – the closest retrieval-based competitor from Table 3). Note that the retrieval component is not universally beneficial for all datasets; however, even in those cases, TabR remains competitive with the parametric solutions.

Table 3: Comparing TabR with existing retrieval-augmented tabular models and parametric DL models. The notation follows Table 2. The bold entries are the best-performing algorithms, which are defined with standard deviations taken into account as described in subsection D.6.

	СН↑	CA ↓	НО↓	AD↑	DI↓	OT ↑	НІ↑	BL↓	WE↓	CO↑	MI↓	Avg. Rank
kNN	0.837	0.588	3.744	0.834	0.256	0.774	0.665	0.712	2.296	0.927	0.764	6.0 ± 1.7
DNNR [34]	_	0.430	3.210	_	0.145	_	_	0.704	1.913	_	0.765	4.8 ± 1.9
DKL [50]	_	0.521	3.423	_	0.147	_	_	0.699	_	_	-	6.2 ± 0.5
ANP [26]	_	0.472	3.162	_	0.140	_	_	0.705	1.902	_	-	4.6 ± 2.5
NPT [29]	0.858	0.474	3.175	0.853	0.138	0.815	0.721	0.692	1.947	0.966	0.753	3.6 ± 1.0
SAINT [44]	0.860	0.468	3.242	0.860	0.137	0.812	0.724	0.693	1.933	0.964	0.763	3.8 ± 1.5
MLP	0.854	0.499	3.112	0.853	0.140	0.816	0.719	0.697	1.905	0.963	0.748	3.7 ± 1.3
MLP-PLR	0.860	0.476	3.056	0.870	0.134	0.819	0.729	0.687	1.860	0.970	0.744	2.0 ± 1.0
TabR-S	0.860	0.403	3.067	0.865	0.133	0.818	0.722	0.690	1.747	0.973	0.750	1.9 ± 0.7
TabR	0.862	0.400	3.105	0.870	0.133	0.825	0.729	0.676	1.690	0.976	0.750	1.3 ± 0.6

The obtained results highlight the retrieval technique and embeddings for numerical features [13] (used in MLP-PLR and TabR) as two powerful architectural elements that improve the optimization properties of tabular DL models. Interestingly, the two techniques are not fully orthogonal, but none of them can recover the full power of the other, and it depends on a given dataset whether one should prefer the retrieval, the embeddings, or a combination of both.

The main takeaway. TabR becomes a new strong deep learning solution for tabular data problems and demonstrates a good potential of the retrieval-based approach. TabR demonstrates strong average performance and achieves the new state-of-the-art on several datasets.

4.2 Comparing TabR with gradient-boosted decision trees

In this section, we compare TabR with models based on gradient-boosted decision trees (GBDT): XGBoost [7], LightGBM [23] and CatBoost [37]. Specifically, we compare ensembles of TabR with ensembles of GBDT (e.g., an ensemble of MLPs vs. an ensemble of XGBoosts) for a fair comparison since gradient boosting is already an ensembling technique.

Our benchmark. Table 4 shows that, on our benchmark, tuned TabR provides noticeable improvements over tuned GBDT on several datasets (CH, CA, HO, HI, WE, CO), while being competitive on the rest, except for the MI dataset. The table also demonstrates that TabR has a competitive default configuration (defined in subsection D.8).

The benchmark from Grinsztajn et al. [14]. Now, we go further and use the benchmark with middle-scale tasks, which has been recently introduced in Grinsztajn et al. [14]. Importantly, this benchmark was originally used to illustrate the superiority of GBDT over parametric DL models on datasets with $\leq 50 K$ objects, which makes it an interesting challenge for TabR. We adjust the benchmark to our tuning and evaluation protocols (see subsection C.2 for details) and report the results in Table 5.

Table 4: Comparing ensembles of TabR with ensembles of GBDT models. See subsection D.8 to learn how the "default" TabR-S was obtained. The notation follows Table 3.

	СН↑	$CA\downarrow$	$\mathrm{HO}\downarrow$	$\mathrm{AD}\uparrow$	DI↓	OT ↑	HI ↑	$BL\downarrow$	$WE\downarrow$	CO ↑	$MI\downarrow$	Avg. Rank
					Tuned l	nyperpar	ameters					
XGBoost	0.861	0.432	3.164	0.872	0.136	0.832	0.726	0.680	1.769	0.971	0.741	2.5 ± 0.9
CatBoost	0.859	0.426	3.106	0.872	0.133	0.827	0.727	0.681	1.773	0.969	0.741	2.5 ± 1.1
LightGBM	0.860	0.434	3.167	0.872	0.136	0.832	0.726	0.679	1.761	0.971	0.741	2.4 ± 0.9
TabR	0.865	0.391	3.025	0.872	0.131	0.831	0.733	0.674	1.661	0.977	0.748	1.3 ± 0.9
					Default	hyperpa	rameters					
XGBoost	0.856	0.471	3.368	0.871	0.143	0.817	0.716	0.683	1.920	0.966	0.750	3.4 ± 0.9
CatBoost	0.861	0.432	3.108	0.874	0.132	0.822	0.726	0.684	1.886	0.924	0.744	2.1 ± 0.8
LightGBM	0.856	0.449	3.222	0.869	0.137	0.826	0.720	0.681	1.817	0.899	0.744	2.5 ± 0.9
TabR-S	0.864	0.398	2.971	0.859	0.131	0.824	0.724	0.688	1.721	0.974	0.752	2.0 ± 1.3

Table 5: Comparing ensembles of DL models with ensembles of GBDT models on the benchmark from Grinsztajn et al. [14] (e.g., an ensemble of MLPs vs ensemble of XGBoosts; note that in Figure 1, we compare *single* models, hence the different numbers). See subsection D.8 for the details on the "default" TabR-S. The default configuration of TabR-S is compared against the default configurations of GBDT models. The comparison is performed in a pairwise manner with standard deviations taken into account as described in subsection D.6.

	vs. Y	(GB	oost	VS	s. C	CatBo	oost	vs. I	vs. LightGBM				
	Win/	Tie /	Loss	s Wi	n/	Tie /	Loss	Win /	Tie /	Loss			
				Tuned	hyj	perpa	ramete	ers					
MLP	6	11	26	6	;	8	29	5	11	27			
MLP-PLR	12	17	14	1	0	11	22	14	15	14			
TabR-S	21	13	9	1	7	11	15	21	15	7			
TabR	26	14	3	2	3	13	7	26	14	3			
				Default	hy	perpa	aramet	ers					
TabR-S	28	10	5	1	7	16	10	25	9	9			

While MLP-PLR (one of the best parametric DL models) indeed is slightly inferior to GBDT on this set of tasks, TabR makes a significant step forward and outperforms GBDT on average.

In the appendix, we provide more analysis: in subsection B.4, we try augmenting XGBoost with a retrieval component; in subsection B.5, we compare training times of TabR and GBDT models.

The main takeaway. After the comparison with GBDT, TabR confirms its status of a new strong solution for tabular data problems: it provides strong average performance and can provide a noticeable improvement over GBDT on some datasets.

5 Analysis

5.1 Freezing contexts for faster training of TabR

In the vanilla formulation of TabR (section 3), for each training batch, the most up-to-date contexts are mined by encoding all the candidates and computing similarities with all of them, which can be prohibitively slow on large datasets. For example, it takes more than 18 hours to train a single TabR on the full "Weather prediction" dataset [33] (3M+ objects; with the default hyperparameters from Table 4). However, as we show in Figure 5, for an average training object, its context (i.e. the top-m candidates and the distribution over them according to the similarity module \mathcal{S}) gradually "stabilizes" during the course of training, which gives an opportunity for simple optimization. Namely, after a fixed number of epochs, we can perform "context freeze": i.e., compute the up-to-date contexts for all training (but not validation and test) objects for the one last time and then reuse these contexts for the rest of the training. Table 6 indicates that, on some datasets, this simple technique allows accelerating training of TabR without much loss in metrics, with more noticeable speedups on larger datasets. In particular, on the full "Weather prediction" dataset, we achieve nearly sevenfold speedup (from 18h9min to 3h15min) while maintaining competitive RMSE. See subsection D.2 for implementation details.

5.2 Updating TabR with new training data without retraining (preliminary exploration)

Getting access to new unseen training data *after* training a machine learning model (e.g., after collecting yet another portion of daily logs of an application) is a common practical scenario. Technically, TabR allows utilizing the new data *without retraining* by adding the new data to the set of candidates for retrieval. We test this approach on the full "Weather prediction" dataset [33] (3M+objects). Figure 6 indicates that such "online updates" may be a viable solution for incorporating new data into an already trained TabR. Additionally, this approach can be used to scale TabR to large datasets by training the model on a subset of data and retrieving from the full data. Overall, we consider the conducted experiment as a preliminary exploration, and leave a systematic study of continual updates for future work. See subsection D.3 for implementation details.

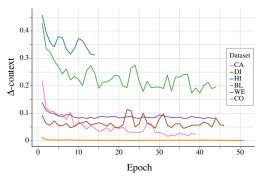


Figure 5: Δ -context (explained below) averaged over training objects until the early stopping while training TabR-S. On a given epoch, for a given object, Δ -context shows the portion of its context (the top-m candidates and their weights) changed compared to the previous epoch (i.e., the lower the value, the smaller the change; see subsection D.2 for formal details). The plot shows that context updates become less intensive during the course of training, which motivates the optimization described in subsection 5.1.

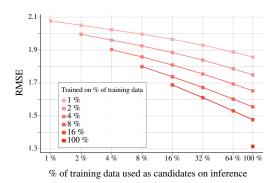


Figure 6: Training TabR-S on various portions of the training data of the full "Weather prediction" dataset and gradually adding the remaining unseen training data to the set of candidates without retraining as described in subsection 5.2. For each curve, the leftmost point corresponds to not adding any new data to the set of candidates after the training, and the rightmost point corresponds to adding all unseen training data to the set of candidates.

Table 6: The performance of TabR-S with the "context freeze" as described in subsection 5.1. The first column shows after how many epochs the contexts are frozen. In parentheses, we provide the fraction of time spent on training compared to the training without freezing (the last row).

	CA ↓	DI↓	НІ↑	BL↓	WE↓	СО↑	WE (full) ↓
1 ep.	0.414 (0.72)	0.137 (0.47)	0.718 (0.80)	0.692 (0.61)	1.770 (0.57)	0.973 (0.49)	1.325 (0.13)
4 ep.	0.409 (0.71)	0.136(0.51)	0.717(0.73)	0.691 (0.62)	1.763(0.56)	$0.973 \ (0.59)$	_
_	0.406 (1.00)	$0.133 \ (1.00)$	0.719 (1.00)	0.691 (1.00)	1.755 (1.00)	0.973 (1.00)	1.315 (1.00)

5.3 Analyzing the similarity module of TabR

In this section, we analyze the similarity module S introduced in Step-2 of subsection 3.3, which greatly improved the performance on several datasets in Table 2 and which is used in TabR.

Diversity of attention patterns. Formally, for a given input object, the similarity module defines a distribution over candidates ("weights" in Figure 4) with exactly m+1 non-zero entries (m is the context size; +1 comes from adding the target object to its own context in Step-2). Intuitively, the less diverse such distributions are on average, the more frequently different input objects are augmented with similar contexts. In Table 7, we demonstrate that such distributions are more diverse on average with the new similarity module compared to the one from the vanilla attention. The implementation details are provided in subsection D.4.

Table 7: Entropy of the average distribution over candidates (the averaging is performed over individual distributions for test objects). The distributions are produced by the similarity module as explained in subsection 5.3. The trained Step-1 and Step-2 models are taken directly from Table 2. The similarity module introduced at Step-2 of subsection 3.3 produces more diverse contexts.

	СН	CA	НО	AD	DI	OT	HI	WE
Step-1								
Step-2	8.4	9.0	9.3	9.7	10.3	10.1	10.5	9.5
Uniform	8.8	9.5	9.6	10.2	10.4	10.6	11.0	12.6

Domain analysis. In subsection B.1, we perform additional analysis, which we now briefly describe in this paragraph. First, we select three datasets where it is possible to define "good neighbors" from

human expert intuition (e.g., to forecast weather in a given region, it is useful to pay attention to geographical neighbors from the training set). Then, we show that the similarity module of TabR allows the model to find and exploit those good neighbors, which is not the case for the vanilla attention. The best part of this story is that one of the three datasets contains a leak (i.e., for a target object, the "good neighbor" is its almost precise copy available in the training data!), and that dataset was used in prior work, but *none* of the considered ML models (including DL and GBDT), except for TabR, could exploit that.

5.4 Analyzing the value module of TabR

In this section, we analyze the value module V of TabR (see Equation 5):

$$\mathcal{V}(\tilde{x}, \tilde{x}_i, y_i) = W_Y(y_i) + T(k - k_i) = W_Y(y_i) + T(\Delta k_i)$$
(6)

Intuitively, for a given context object, its label y_i can be an important part of its contribution to the prediction. Let's consider regression problems, where, in Equation 6, $y_i \in \mathbb{R}$ is embedded by W_Y to $\tilde{\mathbb{Y}} \subset \mathbb{R}^d$. Since W_Y is a linear layer, $\tilde{\mathbb{Y}}$ is just a line, and each point on this line can be mapped back to the corresponding label from \mathbb{R} . Then, the projection of the correction term $T(\Delta k_i)$ on $\tilde{\mathbb{Y}}$ can be translated to the correction of the context label y_i :

$$\mathcal{V}(\tilde{x}, \tilde{x}_i, y_i) = W_Y(y_i) + \operatorname{proj}_{\tilde{\mathbb{V}}^{\perp}} T(\Delta k_i) + \operatorname{proj}_{\tilde{\mathbb{V}}^{\perp}} T(\Delta k_i) = W_Y(y_i + \Delta y_i) + \operatorname{proj}_{\tilde{\mathbb{V}}^{\perp}} T(\Delta k_i)$$
 (7)

To check whether the underlined correction term $\operatorname{proj}_{\tilde{\mathbb{Y}}}T(\Delta k_i)$ (or Δy_i) is important, we take a trained TabR, and reevaluate it without retraining while ignoring this projection (which is equivalent to setting $\Delta y_i = 0$). As a baseline, we also try ignoring the projection of $T(\Delta k_i)$ on a random one-dimensional subspace instead of $\tilde{\mathbb{Y}}$. Table 8 indicates that the correction along $\tilde{\mathbb{Y}}$ plays a vital role for the model. The implementation details are provided in subsection D.5.

Table 8: Evaluating RMSE of trained TabR-S while ignoring projections of $T(\Delta k_i)$ on different one-dimensional subspaces as described in subsection 5.4. The first column shows the projection on which one-dimensional subspace is removed from $T(\Delta k_i)$. The first row corresponds to not removing any projections (i.e., the unmodified TabR-S). Ignoring the projection on $\tilde{\mathbb{Y}}$ (the label embedding space) breaks the model while ignoring a random projection does not have much effect.

	$CA\downarrow$	$HO\downarrow$	$\text{DI}\downarrow$	$BL\downarrow$	$WE\downarrow$
_	0.403	3.067	0.133	0.690	1.747
random	0.403	3.071	0.133	0.690	1.754
$\tilde{\mathbb{Y}}$	0.465	3.649	0.364	0.695	2.003

For classification problems, we tested similar hypotheses but did not obtain any interesting results. Perhaps, the value module $\mathcal V$ and specifically the T module should be designed differently to better model the nature of classification problems.

6 Conclusion & Future work

In this work, we have demonstrated that retrieval-based deep learning models have great potential in supervised machine learning problems on tabular data. Namely, we have designed TabR - a retrieval-augmented tabular DL architecture that provides strong average performance and achieves the new state-of-the-art on several datasets. Importantly, we have highlighted similarity and value modules as the important details of the attention mechanism which have a significant impact on the performance of attention-based retrieval components.

An important direction for future work is improving the efficiency of retrieval-augmented models to make them faster in general and in particular applicable to tens and hundreds of millions of data points. Also, in this paper, we focused more on the aspect of task performance, so some other properties of TabR remain underexplored. For example, the retrieval nature of TabR provides new opportunities for interpreting the model's predictions through the influence of context objects. Also, TabR may enable better support for continual learning (we scratched the surface of this direction in subsection 5.2). Regarding architecture details, possible directions are improving similarity and value modules, as well as performing multiple rounds of retrieval and interactions with the retrieved instances.

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Supplementary material

A Source code

The source code is available at: https://github.com/yandex-research/tabular-dl-tabr

B Additional analysis

B.1 Similarity module of TabR

In this section, we consider three datasets where the transition to the key-only L_2 similarity module from the "vanilla" dot-product-between-queries-and-keys demonstrated the most impressive performance. Formally, this is the transition from "Step-1" to "Step-2" in Table 2. For each of the three datasets, first, we notice that, for a given input object, there is a domain-specific notion of "good neighbors", i.e., such neighbors that, from a human perspective, are very relevant to the input object and provide strong hints for making a better prediction for the input object. Then, we show that the new similarity module allows finding and exploiting those natural hints.

California housing (CA). On this dataset, the transition from the "vanilla" dot-product-between-queries-and-keys similarity module to the key-only L_2 similarity module resulted in a substantial performance boost, as indicated by the difference between "Step-1" and "Step-2" in Table 2. On this dataset, the task is to estimate the prices of houses in California. Intuitively, for a given house from the test set, the prices of the training houses in the geographical neighborhood should be a strong hint for solving the task. Moreover, there are coordinates (longitude and latitude) among the features, which should simplify finding good neighbors. And the "Step-2" model successfully does that, which is not true for the "Step-1" model. Specifically, for an average test object, the "Step-2" model concentrates approximately 7% of the attention mass on the object itself (recall that "Step-2" includes the target object in the context objects) and approximately 77% on the context objects within the 10km radius. The corresponding numbers of the "Step-1" model are 0.07% and 1%.

Weather prediction (WE). Here, the story is seemingly similar to the one with the CA dataset analyzed in the previous paragraph, but in fact has a major difference. Again, here, for a given test data point, the dataset contains natural hints in the form of geographical neighbors from the training set which allow making a better weather forecast for a test query; and the "Step-2" model (Table 2) successfully exploits that, while the "Step-1" model cannot pay any meaningful attention to those hints. Specifically, for an average object, the "Step-2" model concentrates approximately 29% of the attention mass on the object itself (recall that "Step-2" includes the target object in the context objects) and approximately 25% on the context objects within the 200km radius. The corresponding numbers of the "Step-1" model are 0.25% and 0.5%. However, there is a crucial distinction from the CA case: in the version of the dataset WE that we used, the features did not contain the coordinates. In other words, to perform the analysis, after the training, we restored the original coordinates for each row from the original dataset and observed that the model learned the "correct" notion of "good neighbors" from other features.

Facebook comments volume (FB). In this paper, this is the first time when we mention this dataset, which was used in prior work [13] and which we also used for some time in this project. Notably, on this dataset, TabR was demonstrating unthinkable improvements over competitors (including GBDT and the best-in-class parametric DL models). Then we noticed a strange pattern: often, for a given input, TabR concentrated an abnormally high percentage of its attention mass on just one context object (a different one for each input object). This is how we discovered that the dataset split that we inherited from Gorishniy et al. [13] contained a "leak": roughly speaking, for many objects, it was possible to find their almost exact copies in the training set, and the task was dramatically simpler with this kind of hint. In practice, it was dramatically simpler for the TabR, but not for other models. Specifically, for an average object, the "Step-2" model concentrates approximately 20% of the attention mass on the object itself (recall that "Step-2" includes the target object in the context objects) and approximately 35% on its leaked almost-copies. The corresponding numbers of the "Step-1" model are 0.5% and 0.09%.

B.2 Ablation study

Recall that on Step-2 of subsection 3.3, we mentioned that it was crucial that all changes from Step-2 compared to Step-0 (using labels + not using queries + using the L_2 distance instead of the dot product) are important to provide noticeable improvements over MLP on several datasets. Note that not using queries is equivalent to sharing weights of W_Q and W_K : $W_Q = W_K$. Table 9 contains the results of the corresponding experiment and indeed demonstrates that the Step-2 configuration cannot be trivially simplified without loss in metrics (see the CH, CA, BL, WE datasets).

Table 9: The ablation study as described in subsection B.2. $W_Q = W_K$ means using only keys and not using queries. Step-2 is the only variation providing noticeable improvements over MLP on the CH, CA, BL, WE datasets.

	L_2 ,	$W_Q = W_K,$	W_Y	СН	CA	НО	AD	DI	ОТ	HI	BL	WE Avg. Rank
MLP				0.854	0.499	3.112	0.853	0.140	0.816	0.719	0.697	$1.905 \mid 2.4 \pm 1.4$
Step-0	Х	Х	Х	0.855	0.484	3.234	0.857	0.142	0.814	0.719	0.699	$1.903 \mid 2.4 \pm 0.9$
Step-1	Х	×	1	0.855	0.489	3.205	0.857	0.142	0.814	0.719	0.698	$1.906 2.4 \pm 1.2$
-	Х	✓	X	0.853	0.495	3.178	0.857	0.143	0.808	0.719	0.698	$1.903 \mid 2.9 \pm 0.8$
	Х	✓	1	0.857	0.495	3.217	0.857	0.141	0.808	0.717	0.698	1.881 2.7 ± 0.7
	1	×	X	0.855	0.488	3.170	0.857	0.143	0.813	0.719	0.698	$1.901 2.3 \pm 1.0$
	1	×	1	0.856	0.498	3.206	0.858	0.142	0.812	0.721	0.699	$1.900 2.4 \pm 1.1$
	1	✓	X	0.856	0.442	3.154	0.856	0.141	0.811	0.722	0.698	$1.896 \mid 2.0 \pm 0.7$
Step-2	✓	✓	1	0.860	0.418	3.153	0.858	0.140	0.813	0.720	0.692	$1.804 1.2 \pm 0.4$

B.3 Additional results for the "context freeze" technique

We report the extended results for subsection 5.1 in Figure 7, Table 10 and Table 11. For the formal definition of the Δ -context metric, see subsection D.2.

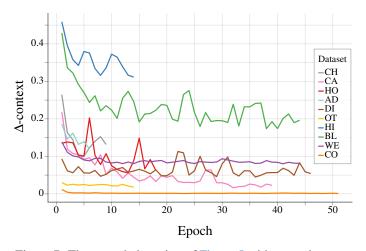


Figure 7: The extended version of Figure 5 with more datasets.

Table 10: The extended version of Table 6. Freezing after 0 epochs means freezing with a randomly initialized model. The speedups are provided in Table 11

	СН↑	$CA\downarrow$	НО↓	AD↑	DI↓	ОТ↑	НІ↑	$BL\downarrow$	$WE\downarrow$	CO↑	WE (full)	↓ Avg. Rank
MLP	0.854	0.499	3.112	0.853	0.140	0.816	0.719	0.697	1.905	0.963	_	2.9 ± 1.5
0 ep.	0.857	0.424	3.075	0.857	0.137	0.816	0.718	0.700	1.787	0.969	1.387	2.3 ± 1.4
1 ep.	0.856	0.414	3.065	0.856	0.137	0.816	0.718	0.692	1.770	0.973	1.325	1.8 ± 1.0
2 ep.	0.856	0.411	3.074	0.856	0.137	0.816	0.718	0.691	1.767	0.973	_	1.7 ± 0.8
4 ep.	0.858	0.409	3.087	0.857	0.136	0.816	0.717	0.691	1.763	0.973	_	1.3 ± 0.5
8 ep.	0.858	0.407	3.118	0.857	0.135	0.817	0.719	0.691	1.761	0.973	_	1.3 ± 0.5
TabR-S	0.859	0.406	3.093	0.858	0.133	0.816	0.719	0.691	1.755	0.973	1.315	1.0 ± 0.0

Table 11: Fraction of time spent on training in Table 10, relative to the training time without the context freeze (the last row; the format is hours:minutes:seconds).

	СН	CA	НО	AD	DI	OT	HI	BL	WE	CO	WE (full)
0 ep.	0.96	0.78	0.79	0.83	0.75	0.87	1.03	0.64	0.53	0.52	0.13
1 ep.	0.88	0.72	0.89	0.89	0.47	0.80	0.80	0.61	0.57	0.49	0.13
2 ep.	0.94	0.65	0.78	0.83	0.47	0.86	0.82	0.63	0.57	0.60	_
4 ep.	1.01	0.71	0.73	0.73	0.51	0.97	0.73	0.62	0.56	0.59	_
8 ep.	1.03	0.76	0.71	0.82	0.61	0.90	0.78	0.67	0.59	0.59	_
TabR-S	0:00:08	0:00:36	0:00:42	0:00:46	0:01:25	0:00:58	0:01:24	0:05:03	0:16:19	0:39:13	18:08:39

B.4 Augmenting XGBoost with a retrieval component

After the successful results of TabR reported in subsection 4.2, we tried augmenting XGBoost with a simple retrieval component to ensure that we do not miss this opportunity to improve the baselines. Namely, for a given input object, we find m=96 (equal to the context size of TabR) nearest training objects in the original feature space, average their features and labels (the label as-is for regression problems, the one-hot encoding representations for classification problems), concatenate the target object's features with the "average neighbor's" features and label, and the obtained vector is used as the input for XGBoost. The results in Table 12 indicate that this strategy does not lead to any noticeable profit for XGBoost. We tried to vary the number of neighbors but did not achieve any significant improvements.

Table 12: Results for ensembles of tuned models. "XGBoost + retrieval" stands for XGBoost augmented with the "average neighbor's" features and label as described in subsection B.4.

	СН↑	CA↓	НО↓	AD↑	DI↓	ОТ↑	HI↑	BL↓	WE↓	CO↑	MI↓	Avg. Rank
XGBoost XGBoost + retrieval TabR	0.855	0.436	3.134	0.871	0.133	0.815	0.724	0.687	1.788	0.962	0.743	1.9 ± 0.7 2.5 ± 0.5 1.2 ± 0.6

B.5 Comparing training times

Among the results published along with the source code, we report tuning and training times for most, if not all, experiments. In this section, for convenience, we summarize some of the most interesting and important figures.

Comparing TabR with retrieval-based tabular DL. In Table 13, we report training times of TabR and NPT [29] (the second best retrieval-based model, after TabR, according to Table 3). The table indicates that, in addition to much better performance, TabR is also significantly faster.

Comparing TabR with parametric DL and GBDT models. In Table 14, we report training times of the tuned configurations of TabR, XGBoost [7] (trained on NVidia A100), LightGBM (trained on 4 threads on CPU), and "MLP-EMB" (MLP with the same embeddings for numerical features [13] as the corresponding TabR on a given dataset). The results are rather expected, with GBDT being

Table 13: Training times of NPT [29] and TabR-S averaged over multiple runs. The format is hh:mm:ss.

	СН	CA	НО	AD	DI	OT	HI	BL	WE	СО	MI
	00:08:44										
TabR	00:00:16	00:00:40	00:00:55	00:01:30	00:01:24	00:01:47	00:06:22	00:04:14	01:03:18	00:37:03	01:46:07

the fastest solution. In practice, however, absolute times can be of more importance than relative overhead.

Table 14: Training times of the tuned configurations of several models. "MLP-EMB" is an MLP with the same embeddings for numerical features as the corresponding TabR on a given dataset. The format is hh:mm:ss.

	СН	CA	НО	AD	DI	OT	HI	BL	WE	CO	MI
XGBoost (GPU)	0:00:01	0:00:20	0:00:05	0:00:05	0:00:02	0:00:35	0:00:15	0:00:08	0:02:02	0:01:55	0:03:43
LightGBM (CPU)	0:00:01	0:00:04	0:00:01	0:00:01	0:00:03	0:00:34	0:00:10	0:00:07	0:06:40	0:06:22	0:06:45
MLP-EMB (GPU)	0:00:02	0:00:18	0:00:09	0:00:17	0:00:15	0:00:31	0:00:24	0:01:38	0:00:29	0:04:01	0:02:09
TabR (GPU)	0:00:16	0:00:40	0:00:55	0:01:30	0:01:24	0:01:47	0:06:22	0:04:14	1:03:18	0:37:03	1:46:07

C Benchmarks

C.1 Our benchmark

In Table 15, we provide more information on the datasets from Table 1. The datasets include:

- Churn Modeling¹
- California Housing (real estate data, [24])
- House 16H²
- Adult (income estimation, [28])
- Diamond³
- Otto Group Product Classification⁴
- Higgs (simulated physical particles, [3]; we use the version with 98K samples available in the OpenML repository [45])
- Black Friday⁵
- Weather (temperature, [33]). We take 10% of the dataset for our experiments due to its large size.
- Weather (full) (temperature, [33]). Original splits from the paper.
- Covertype (forest characteristics, [4])
- Microsoft (search queries, [40]). We follow the pointwise approach to learning to rank and treat this ranking problem as a regression problem.

C.2 The benchmark from Grinsztajn et al. [14]

In this section, we describe how exactly we used the benchmark proposed in Grinsztajn et al. [14].

- We use the same train-val-test splits.
- When there are several splits for one dataset (i.e., when the n-fold-cross-validation was performed in Grinsztajn et al. [14]), we first treat each of them as separate datasets while tuning and evaluating algorithms as described in Appendix D, but then, we average the metrics over the splits to obtain the final numbers for the dataset. For example, if there are five splits for a given dataset, then we tune and evaluate a given algorithm five times, each of the five tuned

¹https://www.kaggle.com/shrutimechlearn/churn-modelling

²https://www.openml.org/d/574

³https://www.openml.org/d/42225

⁴https://www.kaggle.com/c/otto-group-product-classification-challenge/data

⁵https://www.openml.org/d/41540

Table 15: Details on datasets from the main benchmark. "# Num", "# Bin", and "# Cat" denote the number of numerical, binary, and categorical features, respectively. The "Batch size" is the default batch size used to train DL-based models.

Abbr	Name	# Train	# Validation	# Test	# Num	# Bin	# Cat	Task type	Batch size
СН	Churn Modelling	6400	1600	2000	10	3	1	Binclass	128
CA	California Housing	13209	3303	4128	8	0	0	Regression	256
НО	House 16H	14581	3646	4557	16	0	0	Regression	256
AD	Adult	26048	6513	16281	6	1	8	Binclass	256
DI	Diamond	34521	8631	10788	6	0	3	Regression	512
OT	Otto Group Products	39601	9901	12376	93	0	0	Multiclass	512
HI	Higgs Small	62751	15688	19610	28	0	0	Binclass	512
BL	Black Friday	106764	26692	33365	4	1	4	Regression	512
WE	Shifts Weather (subset)	296554	47373	53172	118	1	0	Regression	1024
CO	Covertype	371847	92962	116203	54	44	0	Multiclass	1024
WE (full)	Shifts Weather (full)	2965542	47373	531720	118	1	0	Regression	1024

configurations is evaluated under 15 random seeds on the corresponding splits, and the reported metric value is the average over 5*15=75 runs.

- When there are *multiple* versions of *one* dataset (e.g., the original regression task and the same dataset but converted to the binary classification task or the same dataset, but with the categorical features removed, etc.), we keep only one *original* dataset.
- We removed the "Eye movements" dataset because there is a leak in that dataset.
- We use the tuning and evaluation protocols as described in Appendix D, which was also used in prior works on tabular DL [12, 13]. Crucially, we tune hyperparameters of the GBDT models more extensively than most (if not all) prior work in terms of both budget (20 warmup iterations of random sampling followed by 180 iterations of the tree-structured Parzen estimator algorithm) and hyperparameter spaces (see the corresponding sections in Appendix D).

D Implementation details

D.1 Hardware

We report the used hardware in the results published along with the source code. In a nutshell, the vast majority of experiments on GPU were performed on one NVidia A100 GPU, the remaining small part of GPU experiments was performed on one Nvidia 2080 Ti GPU, and there was also a small portion of runs performed on CPU (e.g. all the experiments on LightGBM).

D.2 Implementation details of subsection 5.1

In subsection 5.1, we used TabR-S with the default hyperparameters (see subsection D.8). To compute Δ -context, we collect context distributions for training objects *between training epochs*. That is, after the *i*-th training epoch, we pause the training, collect the context distributions for all training objects, and then start the next (i+1)-th training epoch.

 Δ -context. Intuitively, this heuristic metric describes in a single number how much, for a given input object, the context attention mass was updated compared to the *previous* epoch. Namely, it is a sum of two terms:

- 1. the novel attention mass, i.e. the attention mass coming from the context objects presented on the current epoch, but not presented on the previous epoch
- the increased attention mass, i.e. we take the intersection of the current and the previous context objects and compute the increase of their total attention mass. We set it to 0.0 if actually decreased.

Now, we formally define this metric. For a given input object, let $a \in \mathbb{R}^{|I_{train}|}$ and $b \in \mathbb{R}^{|I_{train}|}$ denote the two distributions over the candidates from the previous and the current epochs, respectively. Let denote the sets of non-zero entries as $A = \{i : a_i > 0\}$ and $B = \{i : a_i > 0\}$. Note that

|A| = |B| = m = 96. In other words, A and B are the contexts from the two epochs. Then:

$$\Delta$$
-context = novel + increased (8)

$$novel = \sum_{i \in B \setminus A} b_i \tag{9}$$

$$\operatorname{increased} = \max \left(\sum_{i \in B \cap A} b_i - \sum_{i \in B \cap A} a_i, 0.0 \right)$$
 (10)

D.3 Implementation details of subsection 5.2

In subsection 5.2, we used TabR-S with the default hyperparameters (see subsection D.8).

D.4 Implementation details of subsection 5.3

In subsection 5.3, we performed the analysis over exactly the same model checkpoints that we used to assemble the lines "Step-1" and "Step-2" in section 3.

To reiterate, this is how the entropy in Table 7 is computed:

- 1. First, we obtain individual distributions over candidates for all test objects. One such distribution contains exactly (m+1) non-zero entries.
- 2. Then, we average all individual distributions and obtain the average distribution.
- 3. Table 7 reports the entropy of the average distribution.

Note that, when obtaining the distribution over candidates, the top-m operation is taken into account. Without that, if the distribution is always uniform regardless of the input object, then the average distribution will also be uniform and with the highest possible entropy, which would be misleading in the context of the story in subsection 5.3.

Lastly, recall that in the Step-1 and Step-2 models, an input object is added to its own context. Then, the edge case when all input objects pay 100% attention only to themselves would lead to the highest possible entropy, which would be misleading for the story in subsection 5.3. In other words, for the story in subsection 5.3, we should treat the "paying attention to self" behavior similarly for all objects. To achieve that, on the first step of the above recipe, we reassign the attention mass from "self" to a new virtual context object, which is *the same* for all input objects.

D.5 Implementation details of subsection 5.4

To build Table 8, we used TabR-S with the default hyperparameters (see subsection D.8).

D.6 Experiment setup

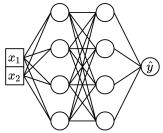
For the most part, we simply follow Gorishniy et al. [13], but we provide all the details for completeness. Note that some of the prior work may differ from the common protocol that we describe below, but we provide the algorithm-specific implementation details further in this section.

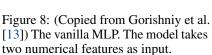
Data preprocessing. For each dataset, for all DL-based solutions, the same preprocessing was used for fair comparison. For numerical features, by default, we used the quantile normalization from the Scikit-learn package [35], with rare exceptions when it turned out to be detrimental (for such datasets, we used the standard normalization or no normalization). For categorical features, we used one-hot encoding. Binary features (i.e. the ones that take only two distinct values) are mapped to $\{0,1\}$ without any further preprocessing.

Training neural networks. For DL-based algorithms, we minimize cross-entropy for classification problems and mean squared error for regression problems. We use the AdamW optimizer [32]. We do not apply learning rate schedules. We do not use data augmentations. For each dataset, we used a predefined dataset-specific batch size. We continue training until there are patience + 1 consecutive epochs without improvements on the validation set; we set patience = 16 for the DL models.

How we compare algorithms. For a given dataset, first, we define the "preliminary best" algorithm as the algorithm with the best mean score. Then, we define a set of the best algorithms (i.e. their results are in bold in tables) as follows: a given algorithm is included in the best algorithms if its mean score differs from the mean score of the preliminary best algorithm by no more than the standard deviation of the preliminary best algorithm.

D.7 Embeddings for numerical features





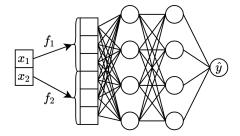


Figure 9: (Copied from Gorishniy et al. [13]) The same MLP as in Figure 8, but now with embeddings for numerical features.

In this work, we actively used embeddings for numerical features from [13] (see Figure 8 and Figure 9), the technique which was reported to universally improve DL models. In a nutshell, for a given scalar numerical feature, an embedding module is a trainable module that maps this scalar feature to a vector. Then, the embeddings of all numerical features are concatenated into one flat vector which is passed to further layers. Following the original paper, when we use embeddings for numerical features, the same embedding architecture is used for all numerical features.

In this work, we used the LR (the combination of a linear layer and ReLU) and PLR (the combination of periodic embeddings, a linear layer, and ReLU) embeddings from the original paper. Also, we introduce the PLR(lite) embedding, a simplified version of the PLR embedding where the linear layer is shared across all features. We observed it to be significantly more lightweight without critical performance loss.

Hyperparameters tuning. For the LR embeddings, we tune the embedding dimension in Uniform[16,96]. For the PLR and PLR(lite) embeddings, we tune the number of frequencies in Uniform[16,96] (in Uniform[8,96] for TabR on the datasets from Grinsztajn et al. [14]), the frequency initialization scale in LogUniform[0.01,100.0] and the embedding dimension in Uniform[16,64] (in Uniform[4,64] for TabR on the datasets from Grinsztajn et al. [14]).

D.8 TabR

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

Embeddings for numerical features. (see subsection D.7) For the non-simple configurations of TabR, on datasets CH, CA, HO, AD, DI, OT, HI, BL, and on all the datasets from Grinsztajn et al. [14], we used the PLR(lite) embeddings as defined in subsection D.7. For other datasets, we used the LR embeddings.

Other details. We observed that initializing the W_Y module properly may be important for good performance. Please, see the source code.

Default TabR-S. The default hyperparameters for TabR-S were obtained at some point in the project by literally averaging the tuned hyperparameters over multiple datasets. The specific set of datasets for averaging included all datasets from Table 15 plus two datasets that used to be a part of our benchmark, but were excluded later. So, in total, 13 datasets contributed to the default hyperparameters.

Formally, this is not 100% fair to evaluate the obtained default TabR-S on the datasets which contributed to this default hyperparameters as in Table 4. However, we tested the fair leave-one-out

approach as well (i.e. for a given dataset, averaging tuned hyperparameters over all datasets except for this one dataset) and did not observe any meaningful changes, so we decided to keep things simple and to have one common set of default hyperparameters for all datasets. Plus, the obtained default TabR-S demonstrates decent performance in Table 5 as well, which illustrates that the obtained default configuration is not strongly "overfitted" to the datasets from Table 15. The specific default hyperparameter values of TabR-S are as follows:

- d = 265
- Attention dropout rate = 0.38920071545944357
- Dropout rate in FFN = 0.38852797479169876
- Learning rate = 0.0003121273641315169
- Weight decay = 0.0000012260352006404615

Hyperparameters. The output size of the first linear layer of FFN and of T is 2d. We performed tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library. The same protocol and hyperparameter spaces were used when tuning models in Table 2 and Table 9.

Table 16: The hyperparameter tuning space for TabR. Here (A) = {CH, CA, HO, AD, DI, OT, HI, BL}, (B) = {WE, CO, MI}. For the datasets from Grinsztajn et al. [14], the tuning space is identical to (A) with the only difference that d is tuned in UniformInt[16, 384].

Parameter	(Datasets) Distribution	Comment
Width d	(A,B) UniformInt[96, 384]	
Attention dropout rate	(A,B) Uniform[0.0, 0.6]	
Dropout rate in FFN	(A,B) Uniform[0.0, 0.6]	
Learning rate	(A,B) LogUniform $[3e-5, 1e-3]$	
Weight decay	$(A,B) \{0, LogUniform[1e-6, 1e-3]\}$	
N_E	(A,B) UniformInt $[0,1]$	Const[0] for TabR-S
N_P	(A,B) UniformInt $[1,2]$	Const[1] for TabR-S
# Tuning iterations	(A) 100 (B) 50	

D.9 MLP

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

We used the implementation from Gorishniy et al. [13].

Hyperparameters. We use the same hidden dimension throughout the whole network. We performed tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library.

Table 17: The hyperparameter tuning space for MLP

Parameter	Distribution
# layers	UniformInt $[1, 6]$
Width (hidden size)	UniformInt[64, 1024]
Dropout rate	$\{0.0, \text{Uniform}[0.0, 0.5]\}$
Learning rate	LogUniform[3e-5, 1e-3]
Weight decay	$\{0, \operatorname{LogUniform}[1e\text{-}6, 1e\text{-}3]\}$
# Tuning iterations	100

D.10 FT-Transformer

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

We used the implementation from the "rtdl" Python package (version 0.0.13).

Hyperparameters. We use the rtdl.FTTransformer.make_baseline method to create FT-Transformer, so most of hyperparameters is inherited from this method's signature, and the rest is tuned as shown in the corresponding table.

Distribution Parameter # blocks UniformInt[1, 4] d_{token} UniformInt[16, 384] Attention dropout rate Uniform[0.0, 0.5]FFN hidden dimension expansion rate Uniform [2/3, 8/3]FFN dropout rate Uniform[0.0, 0.5]Residual dropout rate $\{0.0, \text{Uniform}[0.0, 0.2]\}$ Learning rate LogUniform[1e-5, 1e-3]Weight decay $\{0, \text{LogUniform}[1e-6, 1e-4]\}$ 100 # Tuning iterations

Table 18: The hyperparameter tuning space for FT-Transformer

D.11 kNN

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

The features are preprocessed in the same way as for DL models. The only hyperparameter is the number of neighbors which we tune in UniformInt[1, 128].

D.12 DNNR

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

We've used the official implementation ⁶, but to evaluate DNNR on larger datasets with greater hyperparameters variability, we have rewritten parts of the source code to make it more efficient: enabling GPU usage, batched data processing, multiprocessing, where possible. Crucially, we leave the underlying method unchanged. We provide our efficiency-improved DNNR in the source code. There is no support for classification problems, so we evaluate DNNR only on regression problems.

Hyperparameters. We performed a grid-search over the main DNNR hyperparameters on all datasets, falling back to defaults (suggested by the authors) due to scaling issues on WE and MI.

⁶https://github.com/younader/dnnr

Table 19: The hyperparameter grid used for DNNR. Here (A) = {CA, HO}; (B) = {DI, BL, WE, MI}. Notation: N_f – number of features for the dataset.

Parameter	(Datasets) Parameter grid	Comment
# neighbors k	$(A,B)[1,2,3,\ldots,128]$	
Learned scaling	(A,B) [No scaling, Trained scaling]	
# neighbors used in scaling	(A,B) $[8 \cdot N_f, 2, 3, 4, 8, 16, 32, 64, 128]$	$8 \cdot N_f$ on WE, MI
# epochs used in scaling	10	
Cat. feature encoding	[one-hot, leave-one-out]	
# neighbors for derivative k'	(A) $\operatorname{LinSpace}[2 \cdot N_f, 18 \cdot N_f, 20]$ (B) $\operatorname{LinSpace}[2 \cdot N_f, 12 \cdot N_f, 14]$	

D.13 DKL

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

We used DKL implementation from GPyTorch [11]. We do not evaluate DKL on WE and MI datasets due to scaling issues (tuning alone takes 1 day and 17 hours, compared to 3 hours for TabR on the medium DI dataset, for example). There is no support for classification problems, thus we evaluate DKL only on regression problems.

Hyperparameters. As with MLP we use the same hidden dimension throughout the whole network. And perform tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library.

Table 20: The hyperparameter tuning space for DKL

Parameter	Distribution
Kernel	$\{rbf, sm\}$
# layers	UniformInt[1, 4]
Width (hidden size)	UniformInt[64, 768]
Dropout rate	$\{0.0, \text{Uniform}[0.0, 0.5]\}$
Learning rate	LogUniform[1e-5, 1e-2]
Weight decay	$\{0, \operatorname{LogUniform}[1e\text{-}6, 1e\text{-}3]\}$
# Tuning iterations	100

D.14 ANP

While the original paper introducing ANP did not focus on the tabular data, conceptually, it is very relevant to prior work on retrieval-based tabular DL, so we consider it as one of the baselines.

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

We used the Pytorch implementation from an unofficial repository⁷ and modified it with respect to the official implementation from [26]. Specifically, we reimplemented Decoder class exactly as it was done in [26] and changed a binary cross-entropy loss with a Gaussian negative log-likelihood loss in LatentModel class since it matches with the official implementation.

⁷https://github.com/soobinseo/Attentive-Neural-Process

We do not evaluate ANP on the MI dataset due to scaling issues. Tuning alone on the smaller WE dataset took more than four days for 20(!) iterations (instead of 50-100 used for other algorithms). Also, there is no support for classification problems, thus we evaluate ANP only on regression problems.

We used 100 tuning iterations on CA and HO, 50 on DI, and 20 on BL and WE.

Table 21: The hyperparameter tuning space for ANP

Parameter	Distribution
# decoder layers	UniformInt[1,3]
# cross-attention layers	UniformInt[1, 2]
# self-attention layers	UniformInt[1, 2]
Width (hidden size)	UniformInt[64, 384]
Learning rate	LogUniform[3e-5, 1e-3]
Weight decay	$\{0, \operatorname{LogUniform}[1e\text{-}6, 1e\text{-}4]\}$

D.15 NPT

We use the official NPT [29] implementation ⁸. We leave the model and training code unchanged and only adjust the datasets and their preprocessing according to our protocols.

We evaluate the NPT-Base configuration of the model and follow both NPT-Base architecture and optimization hyperparameters. We train NPT for 2000 epochs on CH, CA, AD, HO, 10000 epochs on OT, WE, MI, 15000 epochs on DI, BL, HI and 30000 epochs on CO. For all datasets that don't fit into the A100 80GB GPU, we use batch size 4096 (as suggested in the NPT paper). We also decrease the hidden dim to 32 on WE and MI to avoid the OOM error.

Note that NPT is conceptually equivalent to other transformer-based non-parametric tabular DL solutions: [43, 44]. All three methods use dot-product-based self-attention modules alternating between self-attention between object features and self-attention between objects (for the whole training dataset or its random subset).

D.16 SAINT

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

We use the official implementation of SAINT 9 with one important fix. Recall that, in SAINT, a target object interacts with its context objects with intersample attention. In the official implementation of SAINT, context objects are taken from the same dataset part, as a target object: for training objects, context objects are taken from the training set, for validation objects – from the validation set, for test objects – from the test set. This is different from the approach described in this paper, where context objects are always taken from the training set. Taking context objects from different dataset parts, as in the official implementation of SAINT, may be unwanted because of the following reasons:

- 1. model can have suboptimal validation and test performance because it is trained to operate when context objects are taken from the training set, but evaluated when context objects are taken from other dataset parts.
- 2. for a given validation/test object, *the prediction depends on other validation/test objects*. This is not in line with other retrieval-based models, which may result in inconsistent comparisons. Also, in many real-world scenarios, during deployment/test time, input objects should be processed independently, which is not the case for the official implementation of SAINT.

⁸https://github.com/OATML/non-parametric-transformers

⁹https://github.com/somepago/saint

For the above reasons, we slightly modify SAINT such that each individual sample attends only to itself and to context samples from the training set, both during training and evaluation. See the source code for details.

On small datasets (CH, CA, HO, AD, DI, OT, HI, BL) we fix the number of attention heads at 8 and performed hyperparameter tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library.

Table 22: The hyperparameter tuning space for SAINT

Parameter	Distribution
Depth	UniformInt $[1, 4]$
Width	UniformInt[4, 32, 4]
Feed forward multiplier	Uniform[2/3, 8/3]
Attention dropout	Uniform[0, 0.5]
Feed forward dropout	Uniform[0, 0.5]
Learning rate	LogUniform[3e-5, 1e-3]
Weight decay	$\{0, \operatorname{LogUniform}[1e\text{-}6, 1e\text{-}4]\}$

On larger datasets (WE, CO, MI) we use slightly modified (for optimizing memory consumption) default configuration from the paper with following fixed hyperparameters:

- depth = 4
- $n_{heads} = 8$
- dim = 32
- $ffn_mult = 4$
- $attn_head_dim = 48$
- attn_dropout = 0.1
- ff_dropout = 0.8
- learning_rate = 0.0001
- weight_decay = 0.01

D.17 XGBoost

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

In this work, we made our best to tune GBDT models as good as possible to make sure that the comparison is fair, and the conclusions are reliable. Compared to prior work [12, 13], where GBDT is already extensively tuned, we doubled the number of tuning iterations, doubled the number of trees, increased the maximum depth and increased the number of early stopping rounds by 4x.

The following hyperparameters are fixed and not tuned:

- booster = "gbtree"
- $n_{estimators} = 4000$
- tree_method = "gpu_hist"
- early_stopping_rounds = 200

We performed tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library.

Table 23: The hyperparameter tuning space for XGBoost

Parameter	Distribution
colsample_bytree	Uniform $[0.5, 1.0]$
gamma	$\{0.0, \text{LogUniform}[0.001, 100.0]\}$
lambda	$\{0.0, \operatorname{LogUniform}[0.1, 10.0]\}$
learning_rate	LogUniform[0.001, 1.0]
max_depth	UniformInt[3, 14]
min_child_weight	LogUniform[0.0001, 100.0]
subsample	Uniform[0.5, 1.0]
# Tuning iterations	200

D.18 LightGBM

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

In this work, we made our best to tune GBDT models as good as possible to make sure that the comparison is fair, and the conclusions are reliable. Compared to prior work [12, 13], where GBDT is already extensively tuned, we doubled the number of tuning iterations, doubled the number of trees, increased the maximum depth and increased the number of early stopping rounds by 4x.

The following hyperparameters are fixed and not tuned:

- $n_{estimators} = 4000$
- early_stopping_rounds = 200

We performed tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library.

Table 24: The hyperparameter tuning space for LightGBM

Parameter	Distribution
feature_fraction	Uniform[0.5, 1.0]
lambda_12	$\{0.0, \operatorname{LogUniform}[0.1, 10.0]\}$
<pre>learning_rate</pre>	LogUniform[0.001, 1.0]
num_leaves	UniformInt[4,768]
min_sum_hessian_in_leaf	LogUniform[0.0001, 100.0]
bagging_fraction	Uniform[0.5, 1.0]
# Tuning iterations	200

D.19 CatBoost

The implementation, tuning hyperparameters, evaluation hyperparameters, metrics, execution times, hardware and other details are available in the source code. Here, we summarize some of the details for convenience.

In this work, we made our best to tune GBDT models as good as possible to make sure that the comparison is fair, and the conclusions are reliable. Compared to prior work [12, 13], where GBDT is already extensively tuned, we doubled the number of tuning iterations, doubled the number of trees, increased the maximum depth and increased the number of early stopping rounds by 4x.

The following hyperparameters are fixed and not tuned:

- $n_{estimators} = 4000$
- early_stopping_rounds = 200
- od_pval = 0.001

We performed tuning using the tree-structured Parzen Estimator algorithm from the Akiba et al. [1] library.

Table 25: The hyperparameter tuning space for CatBoost

Parameter	Distribution
bagging_temperature	Uniform[0.0, 1.0]
depth	UniformInt[3, 14]
12_leaf_reg	$\mathrm{Uniform}[0.1, 10.0]$
<pre>leaf_estimation_iterations</pre>	Uniform[1, 10]
learning_rate	LogUniform[0.001, 1.0]
# Tuning iterations	200

E Extended results with standard deviations

In this section, we provide the extended results with standard deviations for the main results reported in the main text. The results for our benchmark are in the Table 26. The results for the benchmark from Grinsztajn et al. [14] are in the Table 27.

Table 26: Extended results for our benchmark. Results are grouped by datasets and span multiple pages below. Notation: \downarrow corresponds to RMSE, \uparrow corresponds to accuracy.

	СН↑				$CA\downarrow$	
Method	Single model	Ensemble	Metho	od	Single model	Ensemble
	Tuned Hyperparamete	ers			Tuned Hyperparamet	ters
kNN	0.837 ± 0.000	_	kNN		0.588 ± 0.000	_
DNNR	_	_	DNN	R	0.430 ± 0.000	_
DKL	_	_	DKL		0.521 ± 0.055	_
ANP	_	_	ANP		0.472 ± 0.007	_
NPT	0.858 ± 0.003	_	NPT		0.474 ± 0.003	_
SAINT	0.860 ± 0.003	_	SAIN	T	0.468 ± 0.005	_
MLP	0.854 ± 0.003	_	MLP		0.499 ± 0.004	_
MLP-PLR	0.860 ± 0.002	0.860 ± 0.001	MLP-	PLR	0.476 ± 0.004	0.470 ± 0.001
TabR-S	0.860 ± 0.002	0.862 ± 0.002	TabR-	-S	0.403 ± 0.002	0.396 ± 0.001
TabR	0.862 ± 0.002	0.865 ± 0.001	TabR		0.400 ± 0.003	0.391 ± 0.002
CatBoost	0.858 ± 0.002	0.859 ± 0.001	CatBo	oost	0.429 ± 0.001	0.426 ± 0.000
XGBoost	0.861 ± 0.002	0.861 ± 0.001	XGBo	oost	0.433 ± 0.002	0.432 ± 0.001
LightGBM	0.860 ± 0.001	0.860 ± 0.000	Light	GBM	0.435 ± 0.002	0.434 ± 0.001
	Default hyperparamet	ers		Default hyperparameters		
CatBoost	0.860 ± 0.002	0.861 ± 0.001	CatBo		0.433 ± 0.001	0.432 ± 0.001
XGBoost	0.855 ± 0.000	0.856 ± 0.000	XGBo	oost	0.471 ± 0.000	0.471 ± 0.000
LightGBM	0.856 ± 0.000	0.856 ± 0.000	Light	GBM	0.449 ± 0.000	0.449 ± 0.000
TabR-S	0.859 ± 0.003	0.864 ± 0.001	TabR-	-S	0.406 ± 0.003	0.398 ± 0.001
Tun	ned hyperparameters (T	Table 2)		Tun	ed hyperparameters (Table 2)
step-0	0.855 ± 0.003	0.857 ± 0.002	step-0)	0.484 ± 0.006	0.470 ± 0.005
step-1	0.855 ± 0.003	0.858 ± 0.002	step-1		0.489 ± 0.007	0.474 ± 0.005
step-2	0.860 ± 0.002	0.862 ± 0.003	step-2	2	0.418 ± 0.002	0.411 ± 0.000
step-3	0.859 ± 0.002	0.862 ± 0.002	step-3	3	0.408 ± 0.003	0.399 ± 0.002

	НО↓			AD ↑	
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame	ters		Tuned Hyperparame	ters
kNN	3.744 ± 0.000	_	kNN	0.834 ± 0.000	_
DNNR	3.210 ± 0.000	_	DNNR	_	_
DKL	3.423 ± 0.393	_	DKL	_	_
ANP	3.162 ± 0.028	_	ANP	_	_
NPT	3.175 ± 0.032	_	NPT	0.853 ± 0.010	_
SAINT	3.242 ± 0.059	_	SAINT	0.860 ± 0.002	_
MLP	3.112 ± 0.036	_	MLP	0.853 ± 0.001	_
MLP-PLR	3.056 ± 0.021	2.993 ± 0.019	MLP-PLR	0.870 ± 0.002	0.873 ± 0.001
TabR-S	3.067 ± 0.040	2.996 ± 0.027	TabR-S	0.865 ± 0.002	0.868 ± 0.002
TabR	3.105 ± 0.041	3.025 ± 0.010	TabR	0.870 ± 0.001	0.872 ± 0.001
CatBoost	3.117 ± 0.013	3.106 ± 0.002	CatBoost	0.871 ± 0.001	0.872 ± 0.001
XGBoost	3.177 ± 0.010	3.164 ± 0.007	XGBoost	0.872 ± 0.001	0.872 ± 0.000
LightGBM	3.177 ± 0.009	3.167 ± 0.005	LightGBM	0.871 ± 0.001	0.872 ± 0.000
	Default hyperparame	ters		Default hyperparame	ters
CatBoost	3.122 ± 0.011	3.108 ± 0.002	CatBoost	0.873 ± 0.001	0.874 ± 0.001
XGBoost	3.368 ± 0.000	3.368 ± 0.000	XGBoost	0.871 ± 0.000	0.871 ± 0.000
LightGBM	3.222 ± 0.000	3.222 ± 0.000	LightGBM	0.869 ± 0.000	0.869 ± 0.000
TabR-S		2.971 ± 0.017	TabR-S		0.859 ± 0.000
Tun	ed hyperparameters (Table 2)	Tun	ed hyperparameters (Table 2)
step-0	3.234 ± 0.053	3.144 ± 0.034	step-0	0.857 ± 0.002	0.858 ± 0.000
step-1	3.205 ± 0.056	3.104 ± 0.043	step-1	0.857 ± 0.002	0.860 ± 0.000
step-2	3.153 ± 0.031	3.117 ± 0.012	step-2	0.858 ± 0.002	0.862 ± 0.001
step-3		3.117 ± 0.006	step-3		0.866 ± 0.001
			orep c		
	DI		<u> </u>		
Method	DI↓ Single model	Ensemble		OT ↑	
Method	Single model	Ensemble	Method	OT ↑ Single model	Ensemble
	Single model Tuned Hyperparame	ters	Method	OT ↑ Single model Tuned Hyperparameter	Ensemble
kNN	Single model Tuned Hyperparame 0.256 ± 0.000		Method kNN	OT ↑ Single model	Ensemble
kNN DNNR	Single model Tuned Hyperparame 0.256 ± 0.000 0.145 ± 0.000	ters	Method kNN DNNR	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000	Ensemble
kNN DNNR DKL	Single model Tuned Hyperparame 0.256 ± 0.000 0.145 ± 0.000 0.147 ± 0.005	ters	Method kNN DNNR DKL	OT ↑ Single model Tuned Hyperparameter	Ensemble
kNN DNNR DKL ANP	Single model Tuned Hyperparame 0.256 ± 0.000 0.145 ± 0.000 0.147 ± 0.005 0.140 ± 0.001	ters	Method kNN DNNR DKL ANP	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000	Ensemble
kNN DNNR DKL ANP NPT	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \end{array}$	ters — — — — — — — — — — — — — — — — — — —	Method kNN DNNR DKL ANP NPT	OT \uparrow Single model Tuned Hyperparamet 0.774 \pm 0.000 0.815 \pm 0.002	Ensemble
kNN DNNR DKL ANP NPT SAINT	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \end{array}$	ters	Method kNN DNNR DKL ANP NPT SAINT	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \end{array}$	ters — — — — — — — — — — — — — — — — — — —	Method kNN DNNR DKL ANP NPT SAINT MLP	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000 0.815 ± 0.002 0.815 ± 0.002 0.816 ± 0.003	Ensemble ters
kNN DNNR DKL ANP NPT SAINT MLP	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \end{array}$	rters	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \end{array}$	ters 0.133 ± 0.000 0.131 ± 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \end{array}$	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.002	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.134 \pm 0.001 \end{array}$	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 - 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.002 0.825 ± 0.001	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.137 \pm 0.001 \\ 0.137 \pm 0.000 \\ \end{array}$	ters $ -$	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.001 0.830 ± 0.001	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \end{array}$	ters $ -$	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.001 0.830 ± 0.001	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.136 \pm 0.000 \\ \end{array}$	ters $\begin{array}{c} - \\ - \\ - \\ - \\ - \\ - \\ - \\ 0.133 \pm 0.000 \\ 0.131 \pm 0.000 \\ 0.131 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \\ \end{array}$	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000 $ 0.815 \pm 0.002$ 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.001 0.830 ± 0.001 Default hyperparamet	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.136 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.134 \pm 0.000 \\ 0.136 \pm 0.0000 \\ 0.136 \pm 0.0000 \\ 0.136 \pm 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.00000 \\ 0.00000000$	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.133 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.132 \pm 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.001 0.830 ± 0.001 Default hyperparame 0.820 ± 0.001	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 $	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.133 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.132 \pm 0.000 0.132 \pm 0.000 0.132 \pm 0.000 0.134 \pm 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000 $ 0.815 \pm 0.002$ 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.825 ± 0.002 0.825 ± 0.001 0.830 ± 0.001 Default hyperparame 0.820 ± 0.001 0.817 ± 0.000	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.137 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 $	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.133 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.132 \pm 0.000 0.137 \pm 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM	OT ↑ Single model Tuned Hyperparamet 0.774 ± 0.000 0.815 ± 0.002 0.815 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.818 ± 0.002 0.825 ± 0.001 0.830 ± 0.001 Default hyperparame 0.820 ± 0.001 0.817 ± 0.000 0.826 ± 0.000	Ensemble
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost LightGBM TabR-S	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.145 \pm 0.001 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.137 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 $	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.133 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.143 \pm 0.000 0.143 \pm 0.000 0.137 \pm 0.000 0.131 \pm 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	OT ↑ Single model Tuned Hyperparame 0.774 ± 0.000 0.815 ± 0.002 0.812 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.825 ± 0.002 0.825 ± 0.001 0.830 ± 0.001 Default hyperparame 0.820 ± 0.001 0.817 ± 0.000 0.826 ± 0.000 0.816 ± 0.002	Ensemble ters 0.822 \pm 0.002 0.824 \pm 0.001 0.831 \pm 0.001 0.832 \pm 0.001 0.832 \pm 0.001 0.832 \pm 0.001 ters 0.822 \pm 0.001 0.817 \pm 0.000 0.826 \pm 0.000 0.824 \pm 0.000
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	Single model Tuned Hyperparame 0.256 ± 0.000 0.145 ± 0.000 0.145 ± 0.001 0.147 ± 0.005 0.140 ± 0.001 0.138 ± 0.001 0.134 ± 0.001 0.133 ± 0.001 0.134 ± 0.001 0.137 ± 0.000 0.136 ± 0.000 Default hyperparame 0.133 ± 0.000 0.133 ± 0.000 do 0.133 ± 0.000 0.1300 Default hyperparame 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300 0.1300	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.132 \pm 0.000 0.143 \pm 0.000 0.143 \pm 0.000 0.137 \pm 0.000 0.131 \pm 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	OT \uparrow Single model Tuned Hyperparamete 0.774 ± 0.000 0.815 ± 0.002 0.815 ± 0.002 0.816 ± 0.003 0.819 ± 0.002 0.825 ± 0.002 0.825 ± 0.001 0.830 ± 0.001 0.830 ± 0.001 Default hyperparamete 0.820 ± 0.001 0.817 ± 0.000 0.826 ± 0.000	Ensemble ters 0.822 \pm 0.002 0.824 \pm 0.001 0.831 \pm 0.001 0.827 \pm 0.000 0.832 \pm 0.001 0.832 \pm 0.001 0.832 \pm 0.001 0.817 \pm 0.000 0.826 \pm 0.000 0.824 \pm 0.000 Table 2)
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.256 \pm 0.000 \\ 0.145 \pm 0.000 \\ 0.145 \pm 0.001 \\ 0.147 \pm 0.005 \\ 0.140 \pm 0.001 \\ 0.138 \pm 0.001 \\ 0.137 \pm 0.002 \\ 0.140 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.133 \pm 0.001 \\ 0.134 \pm 0.001 \\ 0.137 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.136 \pm 0.000 \\ 0.137 \pm 0.000 \\ 0.133 \pm 0.000 \\ 0.133 \pm 0.001 \\ 0.143 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.143 \pm 0.000 \\ 0.142 \pm 0.001 \\ \end{array}$	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.132 \pm 0.000 0.143 \pm 0.000 0.137 \pm 0.000 0.131 \pm 0.000 Table 2) 0.139 \pm 0.001	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	OT ↑ Single model Tuned Hyperparameter 0.774 \pm 0.000 $-$ 0.815 \pm 0.002 0.815 \pm 0.002 0.816 \pm 0.002 0.816 \pm 0.002 0.818 \pm 0.002 0.825 \pm 0.001 0.830 \pm 0.001 0.830 \pm 0.001 Default hyperparameter 0.820 \pm 0.001 0.817 \pm 0.000 0.816 \pm 0.002 ed hyperparameters (0.814 \pm 0.002	Ensemble ters 0.822 \pm 0.002 0.824 \pm 0.001 0.831 \pm 0.001 0.832 \pm 0.001 0.832 \pm 0.001 0.832 \pm 0.001 0.817 \pm 0.000 0.826 \pm 0.000 0.824 \pm 0.000 Table 2) 0.823 \pm 0.002
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S Tun step-0 step-1	Single model Tuned Hyperparame 0.256 ± 0.000 0.145 ± 0.000 0.145 ± 0.001 0.147 ± 0.005 0.140 ± 0.001 0.138 ± 0.001 0.137 ± 0.002 0.140 ± 0.001 0.133 ± 0.001 0.133 ± 0.001 0.134 ± 0.001 0.137 ± 0.000 0.136 ± 0.000 Default hyperparame 0.133 ± 0.000 0.143 ± 0.000 0.142 ± 0.001 0.142 ± 0.001	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.137 \pm 0.000 0.137 \pm 0.000 0.131 \pm 0.000 Table 2) 0.139 \pm 0.001 0.138 \pm 0.000	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	OT ↑ Single model Tuned Hyperparameter 0.774 \pm 0.000 $-$ 0.815 \pm 0.002 0.815 \pm 0.002 0.816 \pm 0.002 0.816 \pm 0.002 0.818 \pm 0.002 0.825 \pm 0.001 0.830 \pm 0.001 0.830 \pm 0.001 Default hyperparameter 0.820 \pm 0.001 0.817 \pm 0.000 0.816 \pm 0.002 ed hyperparameters (0.814 \pm 0.002 0.814 \pm 0.002	Ensemble ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	Single model Tuned Hyperparame 0.256 ± 0.000 0.145 ± 0.000 0.145 ± 0.001 0.147 ± 0.005 0.140 ± 0.001 0.138 ± 0.001 0.137 ± 0.002 0.140 ± 0.001 0.133 ± 0.001 0.133 ± 0.001 0.134 ± 0.001 0.137 ± 0.000 0.136 ± 0.000 Default hyperparame 0.133 ± 0.000 0.143 ± 0.000 0.142 ± 0.001 0.142 ± 0.001 0.142 ± 0.001	ters 0.133 \pm 0.000 0.131 \pm 0.000 0.131 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.136 \pm 0.000 0.132 \pm 0.000 0.143 \pm 0.000 0.137 \pm 0.000 0.131 \pm 0.000 Table 2) 0.139 \pm 0.001	Method kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	OT ↑ Single model Tuned Hyperparameter 0.774 \pm 0.000 $-$ 0.815 \pm 0.002 0.815 \pm 0.002 0.816 \pm 0.002 0.816 \pm 0.002 0.818 \pm 0.002 0.825 \pm 0.001 0.830 \pm 0.001 0.830 \pm 0.001 Default hyperparameter 0.820 \pm 0.001 0.817 \pm 0.000 0.816 \pm 0.002 ed hyperparameters (0.814 \pm 0.002	Ensemble ters

	HI↑			$\mathrm{BL}\downarrow$	
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame	ters		Tuned Hyperparame	ters
kNN	0.665 ± 0.000	_	kNN	0.712 ± 0.000	_
DNNR	_	_	DNNR	0.704 ± 0.000	_
DKL	_	_	DKL	0.699 ± 0.001	_
ANP	_	_	ANP	0.705 ± 0.005	_
NPT	0.721 ± 0.003	_	NPT	0.692 ± 0.001	_
SAINT	0.724 ± 0.002	_	SAINT	0.693 ± 0.001	_
MLP	0.719 ± 0.002	_	MLP	0.697 ± 0.001	_
MLP-PLR			MLP-PLR		
TabR-S	0.722 ± 0.001		TabR-S		0.688 ± 0.000
TabR	0.729 ± 0.001		TabR	0.676 ± 0.001	
CatBoost	0.726 ± 0.001		CatBoost		0.681 ± 0.000
XGBoost	0.725 ± 0.002		XGBoost		0.680 ± 0.000
LightGBM	0.726 ± 0.001	0.726 ± 0.001	LightGBM	0.680 ± 0.000	0.679 ± 0.000
	Default hyperparame	ters		Default hyperparame	ters
CatBoost	0.725 ± 0.001	0.726 ± 0.001	CatBoost	0.685 ± 0.000	0.684 ± 0.000
XGBoost	0.716 ± 0.000	0.716 ± 0.000	XGBoost	0.683 ± 0.000	0.683 ± 0.000
LightGBM	0.720 ± 0.000	0.720 ± 0.000	LightGBM	0.681 ± 0.000	0.681 ± 0.000
TabR-S	0.719 ± 0.002	0.724 ± 0.000	TabR-S	0.691 ± 0.000	0.688 ± 0.000
Tun	ed hyperparameters (Table 2)	Tur	ed hyperparameters (Table 2)
step-0	0.719 ± 0.002	0.727 ± 0.000	step-0	0.699 ± 0.001	0.694 ± 0.001
step-1	0.719 ± 0.002		step-1	0.698 ± 0.001	0.693 ± 0.001
step-2	0.720 ± 0.002	0.723 ± 0.001	step-2	0.692 ± 0.001	0.690 ± 0.000
step-3		0.724 ± 0.000	step-3		0.688 ± 0.000
	XX75 1			CO. A	
Method	WE↓	Ensemble	Method	CO↑	Ensemble
Method	Single model	Ensemble	Method	Single model	Ensemble
	Single model Tuned Hyperparame			Single model Tuned Hyperparamet	
kNN	Single model Tuned Hyperparame 2.296 ± 0.000		kNN	Single model	
kNN DNNR	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000	ters	kNN DNNR	Single model Tuned Hyperparamet 0.927 ± 0.000 $-$	
kNN DNNR DKL	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $-$	ters	kNN DNNR DKL	Single model Tuned Hyperparamet	
kNN DNNR DKL ANP	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$	ters	kNN DNNR DKL ANP	Single model Tuned Hyperparame 0.927 ± 0.000	
kNN DNNR DKL ANP NPT	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006	ters — — — — — — — — — — — — — — — — — — —	kNN DNNR DKL ANP NPT	Single model Tuned Hyperparame 0.927 ± 0.000 0.966 ± 0.001	
kNN DNNR DKL ANP NPT SAINT	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028	ters	kNN DNNR DKL ANP NPT SAINT	Single model Tuned Hyperparame 0.927 ± 0.000 0.966 ± 0.001 0.964 ± 0.010	
kNN DNNR DKL ANP NPT SAINT MLP	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005		kNN DNNR DKL ANP NPT SAINT MLP	Single model Tuned Hyperparame 0.927 ± 0.000 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001	ters — — — — — — — — — — — — — — — — — — —
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002		kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR	Single model Tuned Hyperparame 0.927 ± 0.000 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001	ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S	Single model Tuned Hyperparame 0.927 ± 0.000 — 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.973 ± 0.000	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR	Single model Tuned Hyperparame 0.927 ± 0.000 — $ 0.966 \pm 0.001$ 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.973 ± 0.000 0.976 ± 0.000	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.002	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.927 \pm 0.000 \\ -\\ -\\ -\\ 0.966 \pm 0.001 \\ 0.964 \pm 0.010 \\ 0.963 \pm 0.001 \\ 0.970 \pm 0.001 \\ 0.973 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.968 \pm 0.000 \end{array}$	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 2.296 \pm 0.000 \\ 1.913 \pm 0.000 \\ - \\ 1.902 \pm 0.009 \\ 1.947 \pm 0.006 \\ 1.933 \pm 0.028 \\ 1.905 \pm 0.005 \\ 1.860 \pm 0.002 \\ 1.747 \pm 0.002 \\ 1.690 \pm 0.003 \\ 1.807 \pm 0.002 \\ 1.784 \pm 0.001 \end{array}$	ters $ -$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.927 \pm 0.000 \\ -\\ -\\ -\\ 0.966 \pm 0.001 \\ 0.964 \pm 0.010 \\ 0.963 \pm 0.001 \\ 0.970 \pm 0.001 \\ 0.973 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.968 \pm 0.000 \\ 0.971 \pm 0.000 \end{array}$	ters $ \begin{array}{ccccccccccccccccccccccccccccccccccc$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.002 1.784 ± 0.001 1.771 ± 0.001	ters $ -$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost	Single model Tuned Hyperparame 0.927 ± 0.000 — 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 0.971 ± 0.000	ters $\begin{array}{c} - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ 0.974 \pm 0.000 \\ 0.974 \pm 0.000 \\ 0.977 \pm 0.000 \\ 0.969 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ \end{array}$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.002 1.771 ± 0.001 Default hyperparame	ters $ -$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM	Single model Tuned Hyperparame 0.927 ± 0.000 - 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.973 ± 0.000 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 Default hyperparame	ters $\begin{array}{c} - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ 0.974 \pm 0.000 \\ 0.974 \pm 0.000 \\ 0.977 \pm 0.000 \\ 0.969 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ \end{array}$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.002 1.784 ± 0.001 Default hyperparame 1.895 ± 0.001	ters $ -$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM	Single model Tuned Hyperparame 0.927 ± 0.000 — 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 0.971 ± 0.000 Default hyperparame 0.923 ± 0.000	ters $ -$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000	ters $ -$	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.927 \pm 0.000 \\ -\\ -\\ -\\ 0.966 \pm 0.001 \\ 0.964 \pm 0.010 \\ 0.963 \pm 0.001 \\ 0.970 \pm 0.001 \\ 0.973 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.964 \pm 0.000 \\ 0.964 \pm 0.000 \\ 0.966 \pm 0.000 \\ 0.966 \pm 0.000 \\ \end{array}$	ters $ -$
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000 1.845 ± 0.003	ters 1.833 \pm 0.002 1.718 \pm 0.001 1.661 \pm 0.002 1.773 \pm 0.001 1.769 \pm 0.001 1.761 \pm 0.001 ters 1.886 \pm 0.000 1.920 \pm 0.000 1.817 \pm 0.001	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.927 \pm 0.000 \\ -\\ -\\ -\\ 0.966 \pm 0.001 \\ 0.964 \pm 0.010 \\ 0.963 \pm 0.001 \\ 0.970 \pm 0.001 \\ 0.973 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.968 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.961 \pm 0.000 \\ 0.966 \pm 0.000 \\ 0.966 \pm 0.000 \\ 0.884 \pm 0.016 \\ \end{array}$	ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost LightGBM TabR-S	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000 1.845 ± 0.003 1.755 ± 0.002	ters 1.833 \pm 0.002 1.718 \pm 0.001 1.661 \pm 0.002 1.773 \pm 0.001 1.769 \pm 0.001 1.761 \pm 0.001 ters 1.886 \pm 0.000 1.920 \pm 0.000 1.817 \pm 0.001 1.721 \pm 0.002	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost LightGBM TabR-S	$\begin{array}{c} \text{Single model} \\ \text{Tuned Hyperparame} \\ 0.927 \pm 0.000 \\ -\\ -\\ -\\ -\\ 0.966 \pm 0.001 \\ 0.963 \pm 0.001 \\ 0.970 \pm 0.001 \\ 0.973 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.976 \pm 0.000 \\ 0.971 \pm 0.000 \\ 0.$	ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.002 1.784 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000 1.845 ± 0.003 1.755 ± 0.002 ed hyperparameters (**edocument of the content of the	ters 1.833 \pm 0.002 1.718 \pm 0.001 1.661 \pm 0.002 1.773 \pm 0.001 1.769 \pm 0.001 1.761 \pm 0.001 ters 1.886 \pm 0.000 1.920 \pm 0.000 1.817 \pm 0.001 1.721 \pm 0.002 Table 2)	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost LightGBM TabR-S	Single model Tuned Hyperparamet 0.927 ± 0.000 0.966 ± 0.001 0.963 ± 0.001 0.963 ± 0.001 0.970 ± 0.001 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 0.971 ± 0.000 Default hyperparame 0.923 ± 0.000 0.984 ± 0.000 0.966 ± 0.000 0.973 ± 0.000	ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000 1.845 ± 0.003 1.755 ± 0.002 ed hyperparameters (*1.903 ± 0.004	ters	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost LightGBM TabR-S	Single model Tuned Hyperparamet 0.927 ± 0.000 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 0.971 ± 0.000 Default hyperparame 0.923 ± 0.000 0.984 ± 0.000 0.966 ± 0.000 0.966 ± 0.000 0.973 ± 0.001 0.973 ± 0.001 0.973 ± 0.001	ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S Tun step-0 step-1	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.002 1.784 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000 1.845 ± 0.003 1.755 ± 0.002 ed hyperparameters (** 1.903 ± 0.004 1.906 ± 0.003	ters	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	Single model Tuned Hyperparamet 0.927 ± 0.000 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 0.971 ± 0.000 Default hyperparame 0.923 ± 0.000 0.966 ± 0.000 0.884 ± 0.016 0.973 ± 0.001 red hyperparameters (*0.957 ± 0.002 0.960 ± 0.002	ters
kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost XGBoost LightGBM TabR-S	Single model Tuned Hyperparame 2.296 ± 0.000 1.913 ± 0.000 $ 1.902 \pm 0.009$ 1.947 ± 0.006 1.933 ± 0.028 1.905 ± 0.005 1.860 ± 0.002 1.747 ± 0.002 1.690 ± 0.003 1.807 ± 0.001 1.771 ± 0.001 Default hyperparame 1.895 ± 0.001 1.920 ± 0.000 1.845 ± 0.003 1.755 ± 0.002 ed hyperparameters (*1.903 ± 0.004) 1.906 ± 0.003 1.804 ± 0.003	ters	kNN DNNR DKL ANP NPT SAINT MLP MLP-PLR TabR-S TabR CatBoost XGBoost LightGBM CatBoost LightGBM TabR-S	Single model Tuned Hyperparamet 0.927 ± 0.000 0.966 ± 0.001 0.964 ± 0.010 0.963 ± 0.001 0.970 ± 0.001 0.976 ± 0.000 0.976 ± 0.000 0.971 ± 0.000 0.971 ± 0.000 Default hyperparamet 0.923 ± 0.000 0.984 ± 0.000 0.966 ± 0.000 0.966 ± 0.000 0.973 ± 0.001 0.966 ± 0.000 0.973 ± 0.001 0.973 ± 0.001 0.973 ± 0.001 0.973 ± 0.001	ters

MI ↓

Method	Single model	Ensemble
	Tuned Hyperparame	ters
kNN	0.764 ± 0.000	_
DNNR	0.765 ± 0.000	_
DKL	_	_
ANP	_	_
NPT	0.753 ± 0.001	_
SAINT	0.763 ± 0.007	_
MLP	0.748 ± 0.000	_
MLP-PLR	0.744 ± 0.000	0.743 ± 0.000
TabR-S	0.750 ± 0.001	0.749 ± 0.000
TabR	0.750 ± 0.001	0.748 ± 0.000
CatBoost	0.741 ± 0.000	0.741 ± 0.000
XGBoost	0.741 ± 0.000	0.741 ± 0.000
LightGBM	0.742 ± 0.000	0.741 ± 0.000
	Default hyperparame	ters
CatBoost	0.745 ± 0.000	0.744 ± 0.000
XGBoost	0.750 ± 0.000	0.750 ± 0.000
LightGBM	0.747 ± 0.000	0.744 ± 0.000
TabR-S	0.757 ± 0.001	0.752 ± 0.001

Table 27: Extended results for Grinsztajn et al. [14] benchmark. Results are grouped by datasets and span multiple pages below. Notation: \downarrow corresponds to RMSE, \uparrow corresponds to accuracy.

Ailerons ↓			Bi	ke Sharing Den	nand ↓	
Method	Single model	Ensemble		Method	Single model	Ensemble
	Tuned Hyperparamet	ers			Tuned Hyperparame	ters
MLP	1.624 ± 0.035	1.620 ± 0.037		MLP	45.702 ± 0.756	43.203 ± 0.132
MLP-PLR	1.591 ± 0.021	1.582 ± 0.019		MLP-PLR	42.615 ± 0.415	41.470 ± 0.324
TabR-S	1.620 ± 0.030	1.595 ± 0.022		TabR-S	43.637 ± 0.681	42.339 ± 0.415
TabR	1.615 ± 0.035	1.585 ± 0.042		TabR	42.649 ± 0.939	41.227 ± 0.615
CatBoost	1.533 ± 0.034	1.527 ± 0.037		CatBoost	40.927 ± 0.232	40.552 ± 0.090
XGBoost	1.571 ± 0.041	1.565 ± 0.040		XGBoost	42.766 ± 0.126	42.606 ± 0.039
LightGBM	1.581 ± 0.038	1.577 ± 0.040		LightGBM	42.503 ± 0.190	42.342 ± 0.149
	Default hyperparamet	ters			Default hyperparame	ters
TabR-S	1.615 ± 0.029	1.599 ± 0.029		TabR-S	43.486 ± 0.573	42.369 ± 0.354
CatBoost	1.542 ± 0.041	1.538 ± 0.043		CatBoost	42.848 ± 0.256	42.626 ± 0.243
XGBoost	1.644 ± 0.046	1.644 ± 0.048		XGBoost	45.100 ± 0.381	45.100 ± 0.410
LightGBM	1.594 ± 0.051	1.594 ± 0.053		LightGBM	43.089 ± 0.103	43.089 ± 0.111

]	Brazilian house	es ↓			Higgs ↑	
Method	Single model	Ensemble		Method	Single model	Ensemble
	Tuned Hyperparame	ters			Tuned Hyperparame	ters
MLP		0.046 ± 0.021		MLP	0.723 ± 0.002	
MLP-PLR		0.040 ± 0.022		MLP-PLR	0.728 ± 0.001	
TabR-S		0.045 ± 0.017		TabR-S	0.725 ± 0.001	
TabR		0.041 ± 0.017		TabR	0.730 ± 0.001	
CatBoost		0.046 ± 0.033		CatBoost	0.729 ± 0.000	
XGBoost		0.053 ± 0.029		XGBoost	0.729 ± 0.001	
LightGBM	0.060 ± 0.025	0.059 ± 0.027		LightGBM	0.727 ± 0.001	
	Default hyperparame	eters			Default hyperparame	ters
TabR-S	0.052 ± 0.016	0.048 ± 0.018		TabR-S	0.722 ± 0.001	0.727 ± 0.001
CatBoost	0.043 ± 0.027	0.042 ± 0.029		CatBoost	0.727 ± 0.001	0.728 ± 0.001
XGBoost		0.052 ± 0.027		XGBoost	0.718 ± 0.000	
	0.071 ± 0.021				0.721 ± 0.000	
KD	DCup09 upsel	lling↑]	MagicTelescop	e ↑
Method	Single model	Ensemble		Method	Single model	Ensemble
	Tuned Hyperparame	ters			Tuned Hyperparame	ters
MLP	0.776 ± 0.011	0.782 ± 0.009		MLP	0.853 ± 0.006	0.857 ± 0.004
MLP-PLR		0.802 ± 0.010		MLP-PLR	0.860 ± 0.007	
TabR-S		0.786 ± 0.017		TabR-S	0.868 ± 0.006	
TabR		0.803 ± 0.008		TabR	0.864 ± 0.005	
CatBoost		0.801 ± 0.012		CatBoost	0.859 ± 0.007	
XGBoost		0.795 ± 0.010		XGBoost	0.855 ± 0.009	
	0.793 ± 0.012				0.855 ± 0.008	
	Default hyperparame				Default hyperparame	
TabR-S		0.781 ± 0.013		TabR-S	0.868 ± 0.006	
CatBoost		0.804 ± 0.006		CatBoost	0.860 ± 0.007	
XGBoost		0.794 ± 0.009		XGBoost	0.856 ± 0.011	
	0.789 ± 0.007				0.859 ± 0.009	
Mercedes E	Benz Greener M	Manufacturing ↓		M	iamiHousing20)16↓
Method	Single model	Ensemble	•	Method	Single model	Ensemble
	Tuned Hyperparame	eters	-		Tuned Hyperparame	ters
MLP		8.336 ± 0.888		MLP	0.161 ± 0.003	0.157 ± 0.003
MLP-PLR	8.383 ± 0.854	8.336 ± 0.888		MLP-PLR	0.150 ± 0.002	
TabR-S	8.351 ± 0.815	8.269 ± 0.840		TabR-S		0.139 ± 0.002
TabR	8.319 ± 0.819	8.244 ± 0.844		TabR		0.136 ± 0.002 0.136 ± 0.002
CatBoost	8.163 ± 0.819	8.155 ± 0.844		CatBoost	0.142 ± 0.002	
XGBoost	8.218 ± 0.817	8.209 ± 0.846		XGBoost	0.144 ± 0.003	
	8.208 ± 0.823	8.162 ± 0.857			0.146 ± 0.002	
- G . C = 1.12	Default hyperparam		-		Default hyperparame	
T 1 D C	Default hyperparalli	0.000 0.00		T 1 D C	Deraun nyperparame	

TabR-S

CatBoost

 $0.141 \pm 0.002 \;\; 0.139 \pm 0.002$

 $0.142 \pm 0.003 \ 0.141 \pm 0.003$

XGBoost 0.160 ± 0.003 0.160 ± 0.003

LightGBM 0.152 ± 0.004 0.152 ± 0.004

TabR-S

 8.290 ± 0.838 8.223 ± 0.865

 $\begin{array}{lll} \textbf{CatBoost} & 8.167 \pm 0.825 & 8.164 \pm 0.848 \\ \textbf{XGBoost} & 8.371 \pm 0.787 & 8.371 \pm 0.810 \\ \end{array}$

LightGBM 8.280 ± 0.845 8.280 ± 0.869

	MiniBooNE	↑		Onl:	ineNewsPopul	arity
Method	Single model	Ensemble	M	/Iethod	Single model	Ensemble
	Tuned Hyperparame	ters	_		Tuned Hyperparamet	ers
MLP		0.948 ± 0.001	M	/ILP	0.862 ± 0.001	
MLP-PLR		0.949 ± 0.000		ILP-PLR	0.862 ± 0.001	
TabR-S		0.950 ± 0.000		abR-S	0.868 ± 0.001	
TabR		0.949 ± 0.000		abR	0.862 ± 0.001	
CatBoost		0.946 ± 0.001		CatBoost	0.853 ± 0.000	
XGBoost		0.945 ± 0.000	X	KGBoost	0.854 ± 0.000	
LightGBM	0.942 ± 0.001	0.943 ± 0.000	Li	ightGBM	0.855 ± 0.000	0.854 ± 0.000
	Default hyperparame	eters]	Default hyperparame	ters
TabR-S	0.947 ± 0.001	0.950 ± 0.001	Ta	abR-S	0.870 ± 0.001	0.864 ± 0.000
CatBoost	0.945 ± 0.001	0.945 ± 0.000	C	CatBoost	0.855 ± 0.000	0.854 ± 0.000
XGBoost	0.942 ± 0.000	0.942 ± 0.000	X	KGBoost	0.874 ± 0.000	0.874 ± 0.000
LightGBM	0.944 ± 0.000	0.944 ± 0.000	Li	ightGBM	0.862 ± 0.000	0.862 ± 0.000
SGEMM	GPU kernel pe	erformance ↓		ana	alcatdata supre	me ↓
Method	Single model	Ensemble	M	/Iethod	Single model	Ensemble
	Tuned Hyperparame				Tuned Hyperparamet	
MLP		0.016 ± 0.000	M	ILP	0.078 ± 0.009	
MLP-PLR		0.015 ± 0.000 0.015 ± 0.000		/ILP-PLR	0.079 ± 0.008	
TabR-S		0.016 ± 0.000		abR-S	0.080 ± 0.007	
TabR		0.015 ± 0.000	Ta	abR	0.081 ± 0.009	
CatBoost	0.017 ± 0.000	0.017 ± 0.000	C	CatBoost	0.078 ± 0.007	0.073 ± 0.002
XGBoost	0.017 ± 0.000	0.017 ± 0.000	X	KGBoost	0.080 ± 0.013	0.077 ± 0.011
LightGBM	0.017 ± 0.000	0.017 ± 0.000	Li	ightGBM	0.078 ± 0.012	0.077 ± 0.011
	Default hyperparame	eters]	Default hyperparame	ters
TabR-S	0.017 ± 0.001	0.016 ± 0.000	Ta	abR-S	0.077 ± 0.007	0.074 ± 0.007
CatBoost	0.017 ± 0.000	0.017 ± 0.000	C	CatBoost	0.071 ± 0.004	0.071 ± 0.004
XGBoost	0.017 ± 0.000	0.017 ± 0.000	X	KGBoost	0.076 ± 0.006	0.076 ± 0.006
LightGBM	0.017 ± 0.000	0.017 ± 0.000	Li	ightGBM	0.073 ± 0.006	0.073 ± 0.006
	bank-marketin	g↑			black friday	ļ
Method	Single model	Ensemble	M	I ethod	Single model	Ensemble
	Tuned Hyperparame	ters			Tuned Hyperparamet	ers
MLP	0.786 ± 0.006	0.790 ± 0.004	M	ILP	0.369 ± 0.000	0.367 ± 0.000
MLP-PLR	0.795 ± 0.005	0.798 ± 0.004	M	/ILP-PLR	0.363 ± 0.000	0.363 ± 0.000
TabR-S	0.800 ± 0.005	0.802 ± 0.004		abR-S	0.364 ± 0.000	0.363 ± 0.000
TabR	0.802 ± 0.009	0.804 ± 0.010	Та	abR	0.362 ± 0.002	0.359 ± 0.001
CatBoost	0.803 ± 0.007	0.806 ± 0.008		CatBoost	0.361 ± 0.000	0.360 ± 0.000
XGBoost		0.803 ± 0.008		KGBoost	0.360 ± 0.000	
LightGBM	0.801 ± 0.008	0.801 ± 0.007	Li	LightGBM	0.360 ± 0.000	0.360 ± 0.000
	Default hyperparame	eters		1	Default hyperparame	ters
TabR-S	0.800 ± 0.006	0.801 ± 0.005	Ta	abR-S	0.364 ± 0.000	0.363 ± 0.000
CatBoost		0.803 ± 0.009		CatBoost	0.361 ± 0.000	
VCDoost	0.000 0.000	0.000 ± 0.000	\mathbf{v}	CDoort	0.269 ± 0.000	0.269 ± 0.000

 $\begin{array}{lll} \textbf{XGBoost} & 0.362 \pm 0.000 & 0.362 \pm 0.000 \\ \textbf{LightGBM} & 0.361 \pm 0.000 & 0.361 \pm 0.000 \end{array}$

 $\begin{array}{lll} \textbf{XGBoost} & 0.800 \pm 0.009 & 0.800 \pm 0.009 \\ \textbf{LightGBM} & 0.803 \pm 0.004 & 0.803 \pm 0.004 \end{array}$

california ↓		compass ↑	
Method Single model Ensemble	Method	Single model	Ensemble
Tuned Hyperparameters		Tuned Hyperparame	ters
MLP $0.149 \pm 0.002 \ 0.146 \pm 0.001$	MLP	0.768 ± 0.005	0.776 ± 0.006
MLP-PLR $0.138 \pm 0.001 \ 0.135 \pm 0.000$	MLP-PLR	0.783 ± 0.007	0.796 ± 0.006
TabR-S $0.124 \pm 0.001 \ 0.121 \pm 0.000$	TabR-S	0.863 ± 0.003	0.870 ± 0.003
TabR $0.122 \pm 0.001 \ 0.120 \pm 0.000$	TabR	0.871 ± 0.003	0.879 ± 0.001
CatBoost $0.129 \pm 0.000 \ 0.128 \pm 0.000$	CatBoost	0.771 ± 0.004	0.775 ± 0.003
XGBoost $0.131 \pm 0.001 \ 0.130 \pm 0.000$	XGBoost	0.819 ± 0.005	0.822 ± 0.003
LightGBM 0.131 ± 0.001 0.130 ± 0.000	LightGBM	0.771 ± 0.003	0.773 ± 0.003
Default hyperparameters		Default hyperparame	ters
TabR-S $0.124 \pm 0.001 \ 0.122 \pm 0.000$	TabR-S	0.865 ± 0.004	0.870 ± 0.001
CatBoost $0.129 \pm 0.000 \ 0.129 \pm 0.000$	CatBoost		0.760 ± 0.001
XGBoost $0.141 \pm 0.000 \ 0.141 \pm 0.000$	XGBoost	0.751 ± 0.000	0.751 ± 0.000
LightGBM $0.135 \pm 0.000 \ 0.135 \pm 0.000$	LightGBM	0.762 ± 0.004	0.762 ± 0.004
covertype ↑		cpu act↓	
Method Single model Ensemble	Method	Single model	Ensemble
Tuned Hyperparameters		Tuned Hyperparame	ters
MLP $0.929 \pm 0.001 \ 0.934 \pm 0.001$	MLP	2.712 ± 0.207	2.544 ± 0.052
MLP-PLR $0.944 \pm 0.002 \ 0.950 \pm 0.001$	MLP-PLR	2.270 ± 0.048	2.214 ± 0.059
TabR-S $0.953 \pm 0.000 \ 0.954 \pm 0.000$	TabR-S	2.298 ± 0.053	2.223 ± 0.050
TabR $0.957 \pm 0.000 \ 0.958 \pm 0.000$	TabR	2.128 ± 0.078	2.063 ± 0.050
CatBoost $0.938 \pm 0.000 \ 0.939 \pm 0.000$	CatBoost	2.124 ± 0.049	2.109 ± 0.050
XGBoost $0.940 \pm 0.000 \ 0.940 \pm 0.000$	XGBoost	2.524 ± 0.353	2.472 ± 0.379
LightGBM 0.939 ± 0.000 0.939 ± 0.000	LightGBM	2.222 ± 0.089	2.207 ± 0.092
Default hyperparameters		Default hyperparame	ters
TabR-S $0.952 \pm 0.000 \ 0.953 \pm 0.000$	TabR-S	2.285 ± 0.045	2.214 ± 0.032
CatBoost $0.912 \pm 0.000 \ 0.913 \pm 0.000$	CatBoost	2.185 ± 0.088	2.162 ± 0.091
XGBoost $0.927 \pm 0.000 \ 0.927 \pm 0.000$	XGBoost	2.910 ± 0.463	2.910 ± 0.486
LightGBM $0.936 \pm 0.000 \ 0.936 \pm 0.000$	LightGBM	2.274 ± 0.128	2.274 ± 0.135
credit ↑		diamonds ↓	
Method Single model Ensemble	Method	Single model	Ensemble
Tuned Hyperparameters		Tuned Hyperparame	ters
MLP $0.772 \pm 0.004 \ 0.774 \pm 0.003$	MLP	0.091 ± 0.002	0.086 ± 0.000
MLP-PLR $0.774 \pm 0.004 \ 0.775 \pm 0.006$	MLP-PLR	0.087 ± 0.001	0.084 ± 0.001
TabR-S $0.773 \pm 0.004 \ 0.774 \pm 0.004$	TabR-S	0.083 ± 0.001	0.082 ± 0.000
TabR $0.772 \pm 0.004 \ 0.775 \pm 0.003$	TabR	0.083 ± 0.001	0.081 ± 0.000
CatBoost $0.773 \pm 0.003 \ 0.775 \pm 0.004$	CatBoost	0.084 ± 0.000	0.083 ± 0.000
XGBoost 0.770 ± 0.003 0.771 ± 0.003	XGBoost	0.085 ± 0.000	0.084 ± 0.000
LightGBM 0.769 ± 0.003 0.773 ± 0.003		0.085 ± 0.000	
Default hyperparameters		Default hyperparame	ters
TabR-S $0.772 \pm 0.005 \ 0.774 \pm 0.005$	TabR-S	0.084 ± 0.001	0.082 ± 0.001
CatBoost $0.771 \pm 0.005 \ 0.773 \pm 0.002$	CatBoost		0.084 ± 0.000

 $\begin{array}{ll} \textbf{XGBoost} & 0.088 \pm 0.000 \;\; 0.088 \pm 0.000 \\ \textbf{LightGBM} & 0.086 \pm 0.000 \;\; 0.086 \pm 0.000 \end{array}$

XGBoost 0.772 ± 0.002 0.772 ± 0.002 LightGBM 0.771 ± 0.003 0.771 ± 0.003

	electricity ↑			elevators↓	
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame			Tuned Hyperparame	
MLP	** *	0.841 ± 0.002	MLP	0.005 ± 0.000	
MLP-PLR		0.849 ± 0.002	MLP-PLR	0.003 ± 0.000 0.002 ± 0.000	
TabR-S		0.929 ± 0.001	TabR-S		0.002 ± 0.000 0.005 ± 0.000
TabR		0.942 ± 0.000	TabR	0.003 ± 0.000 0.002 ± 0.000	
CatBoost		0.882 ± 0.001	CatBoost	0.002 ± 0.000 0.002 ± 0.000	
XGBoost		0.891 ± 0.001	XGBoost	0.002 ± 0.000	
	0.887 ± 0.001			0.002 ± 0.000	
	Default hyperparame	eters		Default hyperparame	ters
TabR-S	0.887 ± 0.004	0.893 ± 0.002	TabR-S	0.005 ± 0.000	0.005 ± 0.000
CatBoost	0.875 ± 0.001	0.877 ± 0.000	CatBoost	0.002 ± 0.000	0.002 ± 0.000
XGBoost	0.882 ± 0.000	0.882 ± 0.000	XGBoost	0.002 ± 0.000	0.002 ± 0.000
LightGBM	0.890 ± 0.000	0.890 ± 0.000		0.002 ± 0.000	0.002 ± 0.000
	€£o			house 16II	
Method	fifa ↓ Single model	Ensemble	Method	house 16H \subseteq Single model	Ensemble
- TVICTIOG	Tuned Hyperparame				
MLP	** *	0.801 ± 0.015	MLP	Tuned Hyperparame	
MLP-PLR		0.801 ± 0.013 0.792 ± 0.012	MLP-PLR	0.598 ± 0.012	
TabR-S		0.792 ± 0.012 0.786 ± 0.012	TabR-S	0.594 ± 0.003	0.589 ± 0.002 0.590 ± 0.006
TabR-S		0.780 ± 0.012 0.787 ± 0.016	TabR	0.629 ± 0.024	
CatBoost		0.787 ± 0.010 0.782 ± 0.011	CatBoost	0.529 ± 0.024 0.599 ± 0.005	
XGBoost		0.782 ± 0.011 0.780 ± 0.011	XGBoost	0.599 ± 0.003 0.591 ± 0.007	
	0.780 ± 0.011 0.781 ± 0.012			0.571 ± 0.007 0.575 ± 0.002	
<u> </u>	Default hyperparame			Default hyperparame	
TabR-S		0.786 ± 0.012	TabR-S	0.603 ± 0.015	
CatBoost		0.780 ± 0.012 0.781 ± 0.013	CatBoost	0.591 ± 0.002	
XGBoost		0.790 ± 0.013 0.790 ± 0.013	XGBoost	0.581 ± 0.002 0.589 ± 0.000	
	0.790 ± 0.012 0.780 ± 0.011			0.593 ± 0.000 0.593 ± 0.000	
<u> </u>	0.700 ± 0.011	0.700 ± 0.011		0.000 ± 0.000	0.000 ± 0.000
	house sales	_		houses ↓	
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame	ters		Tuned Hyperparame	ters
MLP	0.181 ± 0.001	0.178 ± 0.000	MLP	0.233 ± 0.002	0.227 ± 0.001
MLP-PLR	0.169 ± 0.001	0.168 ± 0.000	MLP-PLR	0.228 ± 0.002	0.224 ± 0.000
TabR-S		0.166 ± 0.000	TabR-S		0.196 ± 0.000
TabR	0.164 ± 0.001	0.161 ± 0.000	TabR	0.201 ± 0.002	0.197 ± 0.000
CatBoost	0.167 ± 0.000	0.167 ± 0.000	CatBoost	0.216 ± 0.001	0.214 ± 0.000
XGBoost		0.169 ± 0.000	XGBoost	0.219 ± 0.001	
LightGBM	0.169 ± 0.000	0.169 ± 0.000	LightGBM	0.219 ± 0.001	0.217 ± 0.000
	Default hyperparame	eters		Default hyperparame	ters
TabR-S	0.169 ± 0.001	0.167 ± 0.000	TabR-S	0.200 ± 0.001	0.197 ± 0.001
CatBoost		0.167 ± 0.000	CatBoost	0.216 ± 0.000	
XGBoost		0.179 ± 0.000	XGBoost	0.234 ± 0.000	
	0.173 ± 0.000			0.226 ± 0.000	

	isolet↓			jannis ↑	
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame	ters		Tuned Hyperparame	ters
MLP	2.223 ± 0.189	2.037 ± 0.106	MLP	0.785 ± 0.003	0.787 ± 0.002
MLP-PLR	2.224 ± 0.156	2.030 ± 0.103	MLP-PLR	0.799 ± 0.003	0.804 ± 0.001
TabR-S	1.976 ± 0.174	1.763 ± 0.152	TabR-S	0.798 ± 0.002	0.802 ± 0.002
TabR	1.992 ± 0.181	1.748 ± 0.143	TabR	0.805 ± 0.002	0.811 ± 0.001
CatBoost		2.848 ± 0.002	CatBoost	0.798 ± 0.002	
XGBoost		2.729 ± 0.037	XGBoost	0.797 ± 0.002	
LightGBM	2.701 ± 0.030	2.690 ± 0.029	LightGBM	0.796 ± 0.002	0.797 ± 0.001
	Default hyperparame	ters		Default hyperparame	ters
TabR-S		1.754 ± 0.106	TabR-S	0.795 ± 0.002	
CatBoost		2.863 ± 0.013	CatBoost	0.795 ± 0.001	
XGBoost		3.368 ± 0.011	XGBoost		0.783 ± 0.000
LightGBM	2.953 ± 0.056	2.953 ± 0.058	LightGBM	0.794 ± 0.000	0.794 ± 0.000
kdd	l ipums la 97-si	mall ↑		medical charge	c
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame			Tuned Hyperparame	
MLP		0.880 ± 0.006	MLP		0.081 ± 0.000
MLP-PLR	0.883 ± 0.005		MLP-PLR	0.081 ± 0.000	
TabR-S		0.882 ± 0.008	TabR-S		0.081 ± 0.000
TabR		0.884 ± 0.005	TabR	0.081 ± 0.000	
CatBoost	0.879 ± 0.009		CatBoost	0.082 ± 0.000	
XGBoost		0.883 ± 0.008	XGBoost		0.082 ± 0.000
	0.879 ± 0.007			0.082 ± 0.000	
	Default hyperparame			Default hyperparame	
TabR-S		0.878 ± 0.007	TabR-S		0.081 ± 0.000
CatBoost		0.881 ± 0.007	CatBoost	0.082 ± 0.000	
XGBoost		0.883 ± 0.011	XGBoost		0.084 ± 0.000
LightGBM	0.884 ± 0.005	0.884 ± 0.005	LightGBM	0.083 ± 0.000	0.083 ± 0.000
nyc-	taxi-green-dec-	-2016↓	particu	late-matter-uka	nir-2017↓
Method	Single model	Ensemble	Method	Single model	Ensemble
	Tuned Hyperparame	ters		Tuned Hyperparame	ters
MLP	0.397 ± 0.001	0.391 ± 0.001	MLP	0.377 ± 0.001	0.374 ± 0.000
MLP-PLR	0.368 ± 0.002	0.364 ± 0.000	MLP-PLR	0.367 ± 0.001	0.366 ± 0.000
TabR-S	0.358 ± 0.022	0.338 ± 0.003	TabR-S	0.361 ± 0.000	0.359 ± 0.000
TabR	0.372 ± 0.009	0.350 ± 0.003	TabR	0.360 ± 0.000	0.358 ± 0.000
CatBoost	0.365 ± 0.001	0.363 ± 0.000	CatBoost	0.365 ± 0.000	0.364 ± 0.000
XGBoost	0.379 ± 0.000	0.379 ± 0.000	XGBoost	0.364 ± 0.000	0.364 ± 0.000
LightGBM	0.369 ± 0.000	0.368 ± 0.000	LightGBM	0.364 ± 0.000	0.363 ± 0.000
	Default hyperparame	eters		Default hyperparame	ters
TabR-S	0.389 ± 0.001	0.385 ± 0.000	TabR-S	0.361 ± 0.001	0.359 ± 0.000
CatBoost	0.366 ± 0.000	0.366 ± 0.000	CatBoost	0.366 ± 0.000	0.366 ± 0.000
VCRoost	0.296 ± 0.000	0.296 ± 0.000	YGRoost	0.269 ± 0.000	0.368 ± 0.000

XGBoost $0.368 \pm 0.000 \ 0.368 \pm 0.000$

LightGBM 0.366 ± 0.000 0.366 ± 0.000

XGBoost $0.386 \pm 0.000 \ 0.386 \pm 0.000$

LightGBM 0.372 ± 0.000 0.372 ± 0.000

	phoneme ↑				pol↓	
Method	Single model	Ensemble	N	Method	Single model	Ensemble
	Tuned Hyperparame	ters			Tuned Hyperparame	ters
MLP	0.851 ± 0.014	0.861 ± 0.013	N	ЛLР	5.659 ± 0.543	5.143 ± 0.579
MLP-PLR	0.866 ± 0.012	0.875 ± 0.012	N	//LP-PLR	2.615 ± 0.137	2.445 ± 0.073
TabR-S	0.878 ± 0.010	0.884 ± 0.005	T	TabR-S	6.071 ± 0.537	5.558 ± 0.404
TabR	0.877 ± 0.009	0.885 ± 0.007	T	TabR	2.577 ± 0.169	2.326 ± 0.058
CatBoost	0.883 ± 0.012	0.890 ± 0.005	C	CatBoost	3.632 ± 0.101	3.551 ± 0.090
XGBoost	0.868 ± 0.017	0.877 ± 0.016	X	KGBoost	4.296 ± 0.064	4.255 ± 0.049
LightGBM	0.870 ± 0.013	0.873 ± 0.013	L	LightGBM	4.232 ± 0.337	4.188 ± 0.311
	Default hyperparame	eters	_		Default hyperparame	ters
TabR-S	0.877 ± 0.007	0.880 ± 0.003	T	TabR-S	6.200 ± 0.396	5.804 ± 0.248
CatBoost	0.879 ± 0.011	0.881 ± 0.012	C	CatBoost	4.479 ± 0.051	4.400 ± 0.039
XGBoost	0.870 ± 0.016	0.870 ± 0.016	X	KGBoost	5.249 ± 0.183	5.249 ± 0.197
LightGBM	0.874 ± 0.007	0.874 ± 0.007	L	LightGBM	4.382 ± 0.195	4.382 ± 0.210
			_			
36.1.1	rl↑		_	7 .1 1	road-safety 1	
Method	Single model	Ensemble	<u>N</u>	Method	Single model	Ensemble
	Tuned Hyperparame		_		Tuned Hyperparame	
MLP		0.677 ± 0.013		ЛLР	0.786 ± 0.001	
MLP-PLR		0.767 ± 0.027		MLP-PLR	0.785 ± 0.002	
TabR-S		0.880 ± 0.006		TabR-S	0.840 ± 0.001	
TabR		0.891 ± 0.013		TabR	0.837 ± 0.001	
CatBoost		0.793 ± 0.005		CatBoost	0.801 ± 0.001	
XGBoost		0.799 ± 0.012		KGBoost	0.810 ± 0.002	
LightGBM	0.781 ± 0.010	0.787 ± 0.007	L	1ghtGBM	0.798 ± 0.001	0.800 ± 0.000
	Default hyperparame		_		Default hyperparame	
TabR-S		0.876 ± 0.007		abR-S	0.791 ± 0.003	
CatBoost		0.790 ± 0.004		CatBoost	0.792 ± 0.001	
XGBoost		0.775 ± 0.003		KGBoost	0.796 ± 0.000	
LightGBM	0.778 ± 0.003	0.778 ± 0.003	<u>L</u>	LightGBM	0.803 ± 0.000	0.803 ± 0.000
	sulfur↓				superconduc	t↓
Method	Single model	Ensemble	$\overline{\mathbf{N}}$	Method	Single model	Ensemble
	Tuned Hyperparame	eters	_		Tuned Hyperparam	eters
MLP	0.022 ± 0.002	0.021 ± 0.002	N	ИLР	10.724 ± 0.062	$2\ 10.455 \pm 0.0$
	0.020 ± 0.002			MLP-PLR	10.566 ± 0.058	
TabR-S		0.021 ± 0.002		TabR-S		710.480 ± 0.0
TabR		0.020 ± 0.003		TabR		610.137 ± 0.0
CatBoost		0.019 ± 0.002		CatBoost		$2\ 10.212 \pm 0.0$
XGBoost		0.020 ± 0.002		KGBoost		10.141 ± 0.0
	0.020 ± 0.002			LightGBM		
	Default hyperparame	eters			Default hyperparan	neters
TabR-S	0.021 ± 0.003	0.021 ± 0.002	T	TabR-S	10.812 ± 0.110	10.423 ± 0.0
CatBoost		0.019 ± 0.003		CatBoost	10.263 ± 0.028	
XGBoost		0.022 ± 0.002		KGBoost	10.736 ± 0.000	
	0.021 ± 0.001				10.471 ± 0.000	

LightGBM $10.471 \pm 0.000 \ 10.471 \pm 0.000$

LightGBM 0.021 ± 0.001 0.021 ± 0.001

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visual	11	71n o	SOL	- 1

Method	Single model	Ensemble		
	Tuned Hyperparameters			
MLP	0.138 ± 0.012	0.132 ± 0.010		
MLP-PLR	0.158 ± 0.067	0.144 ± 0.060		
TabR-S	0.398 ± 0.352	0.387 ± 0.375		
TabR	0.227 ± 0.264	0.202 ± 0.147		
CatBoost	0.055 ± 0.006	0.047 ± 0.006		
XGBoost	0.176 ± 0.071	0.154 ± 0.054		
LightGBM	0.062 ± 0.016	0.062 ± 0.017		
Default hyperparameters				
TabR-S	0.327 ± 0.254	0.310 ± 0.257		
CatBoost	0.064 ± 0.005	0.058 ± 0.005		
XGBoost	0.066 ± 0.009	0.066 ± 0.010		
LightGBM	0.061 ± 0.013	0.061 ± 0.014		

wine quality ↓

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Method	Single model	Ensemble	
	Tuned Hyperparame	ters	
MLP	0.672 ± 0.015	0.659 ± 0.016	
MLP-PLR	0.654 ± 0.018	0.634 ± 0.018	
TabR-S	0.632 ± 0.010	0.620 ± 0.010	
TabR	0.641 ± 0.011	0.620 ± 0.007	
CatBoost	0.609 ± 0.013	0.606 ± 0.014	
XGBoost	0.604 ± 0.013	0.602 ± 0.014	
LightGBM	0.613 ± 0.014	0.612 ± 0.014	
Default hyperparameters			
TabR-S	0.628 ± 0.015	0.614 ± 0.015	
CatBoost	0.628 ± 0.012	0.626 ± 0.012	
XGBoost	0.648 ± 0.008	0.648 ± 0.008	
LightGBM	0.641 ± 0.011	0.641 ± 0.012	

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Method	Single model	Ensemble		
	Tuned Hyperparameters			
MLP	0.027 ± 0.001	0.027 ± 0.001		
MLP-PLR	0.027 ± 0.001	0.027 ± 0.001		
TabR-S	0.027 ± 0.000	0.027 ± 0.001		
TabR	0.027 ± 0.000	0.027 ± 0.000		
CatBoost	0.027 ± 0.000	0.027 ± 0.001		
XGBoost	0.027 ± 0.001	0.027 ± 0.001		
LightGBM	0.027 ± 0.000	0.027 ± 0.000		
Default hyperparameters				
TabR-S	0.027 ± 0.001	0.027 ± 0.001		
CatBoost	0.027 ± 0.000	0.027 ± 0.000		
XGBoost	0.027 ± 0.001	0.027 ± 0.001		
LightGBM	0.027 ± 0.000	0.027 ± 0.000		

wine \uparrow

Method	Single model	Ensemble
	Tuned Hyperparame	ters
MLP	0.769 ± 0.015	0.784 ± 0.010
MLP-PLR	0.771 ± 0.016	0.783 ± 0.014
TabR-S	0.794 ± 0.011	0.805 ± 0.006
TabR	0.780 ± 0.015	0.795 ± 0.012
CatBoost	0.799 ± 0.013	0.806 ± 0.010
XGBoost	0.795 ± 0.018	0.801 ± 0.019
LightGBM	0.789 ± 0.016	0.793 ± 0.011
Default hyperparameters		
TabR-S	0.791 ± 0.012	0.800 ± 0.008
CatBoost	0.796 ± 0.010	0.799 ± 0.010
XGBoost	0.796 ± 0.010	0.796 ± 0.010
LightGBM	0.798 ± 0.004	0.798 ± 0.004

year ↓				
Method	Single model	Ensemble		
	Tuned Hyperparameters			
MLP	8.964 ± 0.018	8.901 ± 0.003		
MLP-PLR	8.927 ± 0.013	8.901 ± 0.006		
TabR-S	9.007 ± 0.015	8.913 ± 0.009		
TabR	8.972 ± 0.010	8.917 ± 0.003		
CatBoost	9.037 ± 0.007	9.005 ± 0.003		
XGBoost	9.031 ± 0.003	9.024 ± 0.001		
LightGBM	9.020 ± 0.002	9.013 ± 0.001		
Default hyperparameters				
TabR-S	9.067 ± 0.022	8.893 ± 0.008		
CatBoost	9.073 ± 0.008	9.046 ± 0.001		
XGBoost	9.376 ± 0.000	9.376 ± 0.000		
LightGBM	9.214 ± 0.000	9.214 ± 0.000		