2.3 b)

KMeans - All Features (Purity = 0.51708)

| Case (1) | | Control (2) | Unknown (3) | |
|-----------|-------------|-------------|-------------|--|
| Cluster 0 | 14.54918033 | 48.62869198 | 18.02721088 | |
| Cluster 1 | 85.45081967 | 50.84388186 | 78.62811791 | |
| Cluster 2 | 0 | 0.52742616 | 3.344671202 | |
| | 100 | 100 | 100 | |

KMeans - Filtered Features (Purity - 0.89134)

| | Case (1) | Control (2) | Uknown (3) |
|-----------|-------------|-------------|-------------|
| Cluster 0 | 1.536885246 | 100 | 1.230769231 |
| Cluster 1 | 98.46311475 | 0 | 29.53846154 |
| Cluster 2 | 0 | 0 | 69.23076923 |
| | 100 | 100 | 100 |

2.4 b)

GMM - All Features (Purity = 0.50651)

| | Case (1) | Control (2) | Unknown (3) |
|-----------|-------------|-------------|-------------|
| Cluster 0 | 19.15983607 | 43.88185654 | 17.68707483 |
| Cluster 1 | 75.30737705 | 43.24894515 | 65.53287982 |
| Cluster 2 | 5.532786885 | 12.86919831 | 16.78004535 |
| | 100 | 100 | 100 |

GMM – Filtered Features (Purity = 0.65574)

| | Case (1) | Control (2) | Unknown (3) |
|-----------|-------------|-------------|-------------|
| Cluster 0 | 80.22540984 | 0 | 26.56410256 |
| Cluster 1 | 0.204918033 | 97.78481013 | 57.23076923 |
| Cluster 2 | 19.56967213 | 2.215189873 | 16.20512821 |
| | 100 | 100 | 100 |

In the streaming k-means setting, the data comes in batches and in real time. The cluster centers take into account this new batch and readjust accordingly. Exactly by how much they adjust is determined by the half-life. A large half-life means centers adjust slowly whereas a small half life means that they adjust very rapidly (i.e. are forgetful)

The update rule is given by:

$$C_{t+1} = (\alpha C_t N_t + C_t' N_t') / (\alpha N_t + N_t')$$
$$N_{t+1} = N_t + N_t'$$

Where C_t is the cluster center before the new batch, N_t is the number of points in that cluster, C'_t is the center of the new batch, N'_t is the number of points in the new batch, alpha is the weight constant (can be interpreted as the half life described above).

Here is a quick high level step-by-step of the algorithm:

- 1) Assign the new data points to its nearest cluster
- 2) Update the weight alpha by time unit
- 3) Update the clusters
- 4) If a cluster is no longer needed, we separate the largest cluster into two separate

The pros of streaming kmeans are that: It can use live streaming data and capture dynamic changes in the data sources as time passes. I.e. if the distribution of one of the clusters changes after some time, streaming kmeans will do a better job of adapting to this new change by focusing on this newer distribution thanks to its ability to be "forgetful".

Cons of streaming kmeans: Since the algorithm doesn't see all the data at once like typical kmeans, the chance of error is greater. For example, maybe towards the very end, our data source seems to have some noise. Streaming kmeans will fit that noise more aggressively than traditional kmeans since it's weighted to emphasize newer data points.

Forgetfulness in general gives us the ability to assign a weight to how much we want to weigh the old data compared to the new, incoming data. For example, if a data source is changing over time, we want streaming kmeans to be forgetful so it will effectively model the newer data.

Streaming Kmeans – All Features (Purity = 0.47831)

| | Case (1) | Control (2) | Unknown (3) |
|-----------|-------------|-------------|-------------|
| Cluster 0 | 9.323770492 | 28.27004219 | 26.41723356 |
| Cluster 1 | 90.67622951 | 69.51476793 | 70.1814059 |
| Cluster 2 | 0 | 2.215189873 | 3.401360544 |
| | 100 | 100 | 100 |

Streaming Kmeans – Filtered Features (Purity = 0.63781)

| Ŭ | Column2 | Column3 | Column4 |
|-----------|-------------|-------------|-------------|
| Cluster 0 | 4.918032787 | 2.215189873 | 0.717948718 |
| Cluster 1 | 5.532786885 | 97.78481013 | 70.35897436 |
| Cluster 2 | 89.54918033 | 0 | 28.92307692 |
| | 100 | 100 | 100 |

2.6

- a) We see that when we consider all features, the purity is not very good. This is because both models seem to assign all patients to a single cluster. This is occurring because there are simply too many features and they all tend to overlap so the model tends to fit them mostly into a single cluster. When we reduce the dimensionality using PCA, this makes it easier to distinguish between the features and more evenly distribute them among different clusters. We found that using filtered features, K-means performed quite a bit better (purity ~= 0.9) than GMM (purity ~= 0.65). K-means successfully manages to cluster control and case patient features almost completely accurately, it only suffers with the unknown patients where they are mostly in a separate cluster but some features leak into the other clusters and therefore reduce the purity score a little.
- b) In the below table, we see that purity score increases as we increase K. This is becase the model is overfitting the data. Obviously, by allowing the model to fit too many clusters, it will be able to find subtle clusters and that will group the training data very well but will not neccesarily generalize to new examples very well. This can be thought of more clearly in the limit that k = number of features where we can give each feature its own cluster and therefore achieve perfect purity.

| k | K-Means (All) | K-Means (Filt) | GMM (All) | GMM (Filt) |
|----|---------------|----------------|-----------|------------|
| 2 | 0.54121 | 0.65954 | 0.49187 | 0.38565 |
| 5 | 0.53525 | 0.88789 | 0.54474 | 0.69334 |
| 10 | 0.60493 | 0.89031 | 0.5865 | 0.88651 |
| 15 | 0.69875 | 0.89479 | 0.59409 | 0.86651 |