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Collective dynamics of one-dimensional charge density waves

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The effect of disorder on the static and dynamic behavior of one-dimensional charge density waves at low temperatures is studied by analytical and numerical approaches. In the low-temperature region the spatial behavior of the phase-phase correlation function is dominated by disorder but the roughness exponent remains the same as in the pure case. Contrary to high-dimensional systems the dependence of the creep velocity on the electric field is described by an analytic function.

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I. INTRODUCTION

The collective dynamics of condensed modulated structures like charge (or spin) density waves, 1-3 flux line systems, ^{4,5} and Wigner crystals³ in random environments has been the subject of detailed investigation for more than 20 years. In systems with dimension d>2 the collective creep of these structures is determined by a zero-temperature disorder fixed point resulting in a nonanalytic current-voltage relation with zero linear resistivity.⁶ In d=2 dimensions this fixed point is extended to a fixed line which terminates at the glass transition temperature T_g . The high-temperature phase $T > T_g$ is characterized by a fixed point of zero disorder and a power law decay of the density correlation function. The dynamic behavior is Ohmic. In the low-temperature phase $T < T_g$, correlations decay slightly faster than a power law and the linear resistivity still vanishes (for a recent review, see Ref. 5).

It should be noted that quasi-one-dimensional systems such as TaS_3 (Refs. 8 and 9) and whiskers¹⁰ have been a subject of intensive experimental research. From the theoretical point of view these systems are of special interest because the situation in less than two dimensions (d<2) is entirely different compared to higher dimensions. As follows already from a dimensional continuation of the Cardy-Ostlund flow equations⁷ to dimensions d<2, the glass temperature T< T_g is shifted to T=0. Nevertheless, there remains a residual trace of the disorder which is reflected in the low-temperature behavior of spatial correlations and the dynamics. To be specific we will denote the temperature where the influence of disorder gradually sets in by T*.

The pair correlation function is defined as follows:

$$C(x) = \langle \overline{[\varphi(x) - \varphi(0)]^2} \rangle, \tag{1.1}$$

where $\varphi(x)$ is the phase of the charge density wave (CDW), and the overbar and the brackets denote disorder and thermal averaging, respectively. For the one-dimensional system one can show that the roughness exponent is equal to $\frac{1}{2}$, i.e.,

$$C(x) = A|x|. (1.2)$$

At high temperatures the coefficient A is defined by thermal fluctuations (disorder is irrelevant) and $A \sim T$. At strictly zero

temperature A was calculated (ignoring effects from plasticity) previously both by Feigel'man¹¹ and Villain and Fernandez.¹² Their results at $T\!=\!0$ predict essentially the same decay of the correlation function as at high temperatures but with T replaced by T^* . An expression for A in the crossover region from the high-T to the zero-temperature regime remains unknown.

Therefore it is one of the aims of the present paper to study the influence of disorder in the—so-far not considered—low-temperature region $0 < T \le T^*$ on the static behavior of the CDW's. Using the lowest order of perturbation theory we have shown that at low temperatures the coefficient A is a linear function of T and disorder. This result has also been verified by Monte Carlo simulations. Our numerical results show that the temperature region where the T=0 behavior dominates is narrower than the crossover region.

In this paper the dynamics of one-dimensional CDW's in the creep regime is also studied. According to the scaling theory for manifolds,⁶ in the case of the CDW's the dependence of the creep velocity v on an external electric field E is given by the following expression:

$$v(E) \sim \exp\left[-\frac{T_{\xi}}{T}\left(\frac{E_{\xi}}{E}\right)^{\mu}\right], \quad \mu = \frac{d-2}{2}.$$
 (1.3)

Here T_{ξ} and E_{ξ} are parameters which depend on the effective barrier height and the typical length scale of the problem.⁵ Formula (1.3) is valid for d>2 for which the exponent μ is positive (for d=2 one has a logarithmic behavior and the exponential is replaced by a power law¹³). However, it is not longer valid for d<2 as μ becomes negative. Thus, the problem of creep dynamics of CDW's in one-dimensional (1D) systems remains open.

In this paper we have made an attempt to develop the theory for creep dynamics in one dimension. We have shown that the dependence of the creep velocity on the electric field is given by the hyperbolic sine function. Such behavior is in sharp contrast to the nonanalytic behavior (1.3) for $d \ge 2$. As will be demonstrated below, the analytic dependence is due to the absence of the transverse motion of manifolds. The analytical prediction was confirmed by solving the corresponding Langevin equations numerically. Our results seem

to be in agreemnt with the experimental findings of Zaitsev-Zotov⁸ for the temperature dependence of the current at low values of T (see Fig. 2 in Ref. 8).

II. MODEL AND PHASE-PHASE CORRELATION FUNCTION

The charge density $\rho(x)$ of a 1D CDW can be expressed as $\rho(x) = \rho_0 + \rho_1 \cos[Qx + \varphi(x)]$ where $Q = 2k_F$ denotes the wave vector of the undistorted wave, k_F the Fermi momentum, and $\varphi(x)$ a slowly varying phase variable. The Hamiltonian of the phase field is then given by 14

$$\mathcal{H} = \int dx \left\{ \frac{1}{2} \hbar v_F \left(\frac{\partial}{\partial x} \varphi \right)^2 - \sum_i V_i \delta(x - x_i) \cos(\varphi + Qx) \right\}, \tag{2.1}$$

where $V_i > 0$ and x_i denote the strength and the position of the impurity potential acting on the CDW. The mean impurity distance 1/c is assumed to be large in comparison with the CDW wavelength such that $Q \gg c$.

For the further treatment it is important to separate between the cases of weak and strong disorder, respectively.

For weak disorder $V_i \ll \hbar v_F c$ the Fukuyama-Lee length $L_c = (\hbar v_F)^{2/3} (c \overline{V_i^2})^{-1/3}$ is large compared with the impurity distance. Here $\overline{V_i^2}$ denotes the averaged potential strength of the impurity. In the following we will therefore restrict ourselves to the case

$$L_c \gg c^{-1} \gg Q^{-1}$$
. (2.2)

The length scale L_c sets an energy scale

$$T^* = (\hbar v_F c \overline{V_i^2})^{1/3} = \hbar v_F L_c^{-1}. \tag{2.3}$$

Typically T^*/T if of the order of 10^3-10^7 . ¹⁶

Under condition (2.2) the Hamiltonian can be rewritten in the form of the random field XY model:

$$\mathcal{H} = \int dx \left\{ \frac{1}{2} \hbar v_F \left(\frac{\partial}{\partial x} \varphi - g(x) \right)^2 - V \cos[\varphi - \alpha(x)] \right\}, \quad (2.4)$$

where $V^2 = \overline{V_i^2}c$. Here $\alpha(x)$ is a random phase with zero average and

$$\overline{e^{i[\alpha(x)-\alpha(x')]}} = \delta(x-x'). \tag{2.5}$$

In Eq. (2.4) we added also a linear gradient term which in general will be generated under a renormalization group transformation. Here

$$\overline{g(x)} = 0$$
, $\overline{g(x)g(x')} = \sigma \delta(x - x')$. (2.6)

Model (2.4) exhibits a statistical tilt symmetry, ¹⁷ which excludes a renormalization of the stiffness constant $\hbar v_F$. This can most easily be seen from the replica Hamiltonian corresponding to Eq. (2.4)

$$\mathcal{H}_{n} = \sum_{\alpha,\beta} \int_{0}^{L} dx \left\{ \frac{1}{2} \hbar v_{F} \left(\frac{\partial}{\partial x} \varphi_{\alpha} \right)^{2} \delta_{\alpha\beta} - \frac{(\hbar v_{F})^{2} \sigma}{2T} \left(\frac{\partial}{\partial x} \varphi_{\alpha} \right) \right. \\ \times \left(\frac{\partial}{\partial x} \varphi_{\beta} \right) - \frac{V^{2}}{4T} \cos(\varphi_{\alpha} - \varphi_{\beta}) \right\}. \tag{2.7}$$

Adding a term $-g_0 \int_0^L (\partial \varphi / \partial x) dx$ to \mathcal{H} , the full stiffness constant $\hbar \tilde{v}_F$ follows from the average free energy by

$$(\hbar \tilde{v}_F)^{-1} = -\frac{1}{L} \frac{\partial^2 \bar{F}}{\partial g_0^2} \bigg|_{g_0 = 0}.$$
 (2.8)

Rewriting \mathcal{H}_n in terms of $\widetilde{\varphi}(x) = \varphi(x) - (g_0/\hbar v_F)x$, the only remaining g_0 term is $-\frac{1}{2}(g_0^2/\hbar v_F)$, which proves our statement.

For the further discussion it is convenient to go over to rescaled length and energy units. With $x=L_c y$ and $\varphi(x)=\tilde{\varphi}(y)$,

$$\frac{\mathcal{H}}{T} = \frac{1}{\widetilde{T}} \int dy \left[\frac{1}{2} \left(\frac{\partial \widetilde{\varphi}}{\partial y} - \widetilde{g}(y) \right)^{2} + \cos[\widetilde{\varphi} - \widetilde{\alpha}(y)] \right], \quad (2.9)$$

where $\tilde{T} = T/T^*$ with T^* defined in Eq. (2.3) and

$$\overline{e^{i[\tilde{\alpha}(y)-\tilde{\alpha}(y')]}} = \delta(y-y')\overline{\tilde{g}(y)}\,\overline{\tilde{g}(y')} = \tilde{\sigma}\,\delta(y-y'),$$

$$\tilde{\sigma} = L_{c}\sigma.$$
(2.10)

Since $\sigma = 0$ in the initial Hamiltonian (2.1), the static properties of the model are characterized by L_c and T^* .

Defining

$$\frac{\partial \widetilde{\phi}}{\partial y} = \frac{\partial \widetilde{\varphi}}{\partial y} - \widetilde{g}(y), \tag{2.11}$$

we have

$$\widetilde{\varphi}(y) = \widetilde{\phi}(y) - \widetilde{\phi}(0) + \int_0^y \widetilde{g}(z)dz.$$
 (2.12)

To the lowest order of perturbation theory one can ignore the nonlinear term in Eq. (2.9). Then, using Eqs. (2.12) and (2.6), we obtain for the pair correlation function

$$C(x) = \langle \overline{[\varphi(x) - \varphi(0)]^2} \rangle = \langle \overline{[\varphi(y) - \widetilde{\varphi}(0)]^2} \rangle$$

$$= \langle \overline{[\varphi(y) - \widetilde{\phi}(0)]^2} \rangle + \int_0^y \int_0^y dz \, dz' \, \overline{\widetilde{g}(z)} \, \overline{\widetilde{g}(z')}$$

$$= (\widetilde{T} + \widetilde{\sigma})|y| = \left(\frac{T}{\hbar v_F} + \sigma\right)|x|^{2\zeta}, \qquad (2.13)$$

with the roughness exponent $\zeta = \frac{1}{2}$.

It should be noted that using a transfer matrix method Feigel'man¹¹ found in the zero-temperature limit a value of σ of the order L_c^{-1} ; i.e., $\tilde{\sigma}$ is of the order 1. The same result was obtained by Villain and Fernandez¹² by a real-space renormalization group transformation. Our formula (2.13)

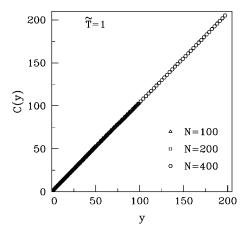


FIG. 1. The distance dependence of the correlation function C(y) for various system sizes. N=100, 200, and 400, $\tilde{T}=1$. The results are averaged over 50–200 samples.

obtained by the renormalization of the replica Hamiltonian is the extension of their result to the finite temperature case.

To check the theoretical prediction (2.13) numerically we have performed a detailed Monte Carlo study using the discretized version of Eq. (2.9) with $\tilde{g}(y) = 0$. The free boundary condition is implemented. The acceptance ratio of Metropolis moves was controlled to be around 0.5 for the whole run. The equilibration is checked by monitoring the stability of data against runs which are at least 3 times longer. The first half of the Monte Carlo steps are not taken into account when averaging.

Figure 1 shows the pair correlation function C(y) as a function of y for different system sizes and for $\tilde{T}=1$. Clearly, C(y) has the same slope for any value of N. In what follows we will take N=200 to calculate C(y)/y.

Figure 2 shows C(y)/y as a function of \widetilde{T} . Above $\widetilde{T} \approx 1.3$ the disorder is irrelevant and the high-temperature behavior sets in. For $\widetilde{T} \leq 0.3$ the disorder dominates and C(y)/y becomes independent of T. Clearly, our numerical results support prediction (2.13) for low and high temperatures.

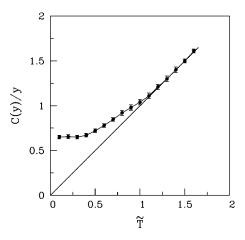


FIG. 2. The temperature dependence the slope C(y)/y. The results are averaged over 50–200 samples. The straight line corresponds to $C(y)/y = \tilde{T}$.

From Fig. 2 we obtain $\tilde{\sigma} \approx 0.65$ and it is of the order of unity as predicted by theory. The crossover region $0.3 \leq \tilde{T} \leq 1.3$ is wider than the region where the $\tilde{T} = 0$ behavior is valid ($\tilde{T} \leq 0.3$).

III. DYNAMICS

Since the roughness exponent is $\zeta = 1/2$, the random field potential creates rugged energy barriers $\hbar v_F L_c^{-1} = T^*$. Barriers on larger length scales are of the same order. In the following we will use these findings to determine the creep motion of the CDW under the influence of an external electric field E. To this aim we have to add to the Hamiltonian (2.4) the following term²:

$$H_{ext} = -\int dx E \varphi(x). \tag{3.1}$$

The equation of the (overdamped) CDW is then given by

$$\frac{\partial \varphi}{\partial t} = -\gamma \left(\frac{\delta \mathcal{H}}{\delta \varphi} - E \right) + \eta(x, t), \tag{3.2}$$

where γ is a kinetic coefficient and $\eta(x,t)$ a Gaussian thermal noise characterized by $\langle \eta \rangle = 0$ and

$$\langle \eta(x,t) \eta(x',t') \rangle = 2T\gamma \delta(x-x') \delta(t-t').$$
 (3.3)

The rescaling in the previous section amended by a rescaling of time according to $\tau = \gamma(T^*/L_c)t$ leads to an equation of motion which includes as the only parameters $\widetilde{T} = T/T^*, \widetilde{\sigma} = \sigma/L_c$, and $\widetilde{E} = E/E^*$, where $E^* = T^*/L_c$ is of the order of the T=0 depinning threshold field E_T .

For temperatures $T \gg T^*$ the energy landscape is essentially flat and the CDW makes a damped motion with $\varphi \approx \gamma E$. In the opposite case of $T \ll T^*$ energy barriers on the scale L are of the order $E(L > L_c)$. In fact, in the absence of the external field the energy barrier E_B is of the order of T^* . From Eq. (3.1) it is clear that due to the external field the energy barrier has an additional term proportional to

$$EL\langle \varphi^2 \rangle^{1/2} \sim EL\left(\frac{L}{L_c}\right)^{1/2}$$
. (3.4)

In obtaining the last equation the roughness exponent $\zeta = 1/2$ was taken into account. So we have the following expression for the energy barrier:

$$E_B^{\pm}(L) \approx c_B T^* \mp c_E E \left(\frac{L}{L_c}\right)^{1/2} L,$$
 (3.5)

where c_B and c_E are constants of order unity and the \pm sign refers to the motion parallel and antiparallel to the external field.

Since the largest energy barrier arise when $L{\approx}L_c$, we find from the Arrhenius law for the creep velocity of the CDW

$$v(E) \approx \frac{\gamma}{2} \frac{T}{L_c} \left[\exp(-E_B^+/T) - \exp(-E_B^-/T) \right]$$
$$= \gamma \frac{T}{L_c} e^{-c_B(T^*/T)} \sinh \frac{c_E E L_c}{T}. \tag{3.6}$$

In Eq. (3.6) we have chosen the prefactor in such a way that for $T \gg T^*$ and $EL_c \ll T$ the linear behavior $v \sim \gamma E$ is recovered. So the creep law shows the conventional Kim-Anderson behavior and is drastically different from the behavior in higher dimensions where a nonanalytic dependence of v on E is found.

We now try to understand the difference between d=1and d>1 systems qualitatively. For the creep motion of a vortex lattice or a CDW in higher than one dimension, strings (or manifolds) must get thermally activated over a potential barrier of a typical length L_z . In the case of a vortex lattice the strings are vortices whereas for a CDW it is a phase manifold of dimensionality d-1. Here two factors should be taken into account. First, due to the disorder and the roughness of manifolds, the height of the barriers scales as a power law in L_7 with the exponent of the free energy fluctuations. Strickly speaking, this is correct for the case without a driving force. However, in the creep regime where the force is much smaller than the depinning threshold such an approximation is still valid. Second, in the presence of a weak driving force the strings try to move a long distance to overcome the barrier. When the manifold overcomes such a barrier, one gains an energy which is proportional to E and is generally a nonanalytic function of L_z . The interplay between the energy gain and the existence of barriers leads to a nontrivial dependence of L_z on E or to the nonanalytic dependence of the creep velocity on the driving force. Our discussion is based on the review of Nattermann and Scheidl.⁵ A similar argument may be also found in the review of Blatter et al.4

In the d=1 case the phase manifolds are points and one has no strings perpendicular to the direction of the creep

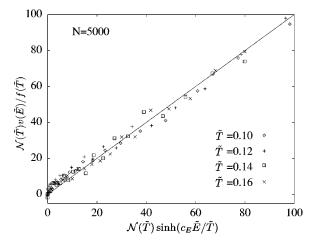


FIG. 3. The driving force dependence of the creep velocity for fixed \tilde{T} . Here $f(\tilde{T}) = \gamma(T/L_c)e^{-c_B/\tilde{T}}$ and $\mathcal{N}(\tilde{T})$ is a scaling factor for a better illustration of the linear behavior with values: $\mathcal{N}(0.10) = 1.0$, $\mathcal{N}(0.12) = 2.5$, $\mathcal{N}(0.14) = 6.5$, and $\mathcal{N}(0.16) = 9.0$.

motion. Due to the lack of the transverse motion of strings, the typical height of barriers is defined by the disorder strength, which is proportional to T^* . The driving force changes the barriers by an amount given by Eq. (3.5). So the crucial difference between d=1 and d>1 systems is that the latter have a transverse motion of strings leading to the non-analytic dependence of v(E) on E.

In the next section we will check the prediction (3.6) for the creep motion by a numerical simulation.

IV. CREEP SIMULATION

In our simulation we used the following discrete and rescaled version of Eq. (3.2):

$$\frac{\Delta \widetilde{\varphi}_{i}}{\Delta \tau} = (\widetilde{\varphi}_{i+1} - 2\widetilde{\varphi}_{i} + \widetilde{\varphi}_{i-1}) + \sin(\widetilde{\varphi}_{i} - \widetilde{\alpha}_{i})
+ \widetilde{E} + \widetilde{\eta}(i, \tau), \quad i = 1, \dots, N.$$
(4.1)

The first term is the one-dimensional lattice Laplacian and $\tilde{\alpha}_i$ is uniformly distributed in the interval $[0,2\pi]$. The thermal noise $\tilde{\eta}(i,\tau)$ is defined by Eq. (3.3).

The creep velocity v is given by

$$v(\widetilde{E}, \widetilde{T}) = \left\langle \frac{1}{N} \sum_{i=1}^{N} \frac{\Delta \widetilde{\varphi}_i}{\Delta \tau} \right\rangle_{\tau}.$$
 (4.2)

It should be noted that $v \propto j_{cdw}$, where j_{cdw} is the CDW current. The equation of motion is integrated by a *modified Runge-Kutta* algorithm suitable for stochastic systems. ¹⁸ Periodic boundary conditions are applied.

We first tested our algorithm by studying the depinning transition at zero temperature. We found a threshold field of $\tilde{E}_T \approx 0.22$ with a critical exponent $\xi = 0.57 \pm 0.07$. This value of ξ is in agreement with previous works. ^{19,20}

For the creep simulations we used a time of $1000\tau_0$ and $\Delta \tau = 0.05\tau_0$. Runs which are 3 times longer do not change the results in any substantial way. In order to check the pre-

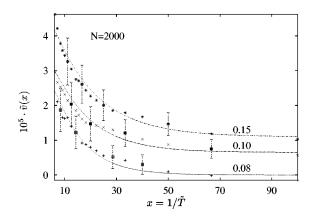


FIG. 4. The temperature dependence of the creep velocity for different values of \tilde{E} which are shown next to the curves. For a better visualization we rescaled the data. For \tilde{E} = 0.08, $3\tilde{v}(x)$ is plotted. For \tilde{E} = 0.10 and 0.15 the data are shifted by 6×10^{-6} and 10^{-5} , respectively. Typical error bars are shown.

dicted behavior (3.6) we first fix \tilde{T} and vary \tilde{E} . In this case we took a system size of 5000 and the results were averaged over 50 disorder realizations (for larger system sizes the results remain almost the same). Our results are shown in Fig. 3. The linear fit by a straight line in this figure indicates that Eq. (3.6) captures the field dependence correctly. By an iterative least-squares fitting, we found $c_E = 2.5 \pm 0.2$.

We now study the temperature dependence of $v(\tilde{E})$ for fixed values of \tilde{E} . We took N=2000 and averaged typically over 500 samples. The results are shown in Fig. 4.

Using $c_E=2.5$, we fitted the function $\tilde{v}(x)=\gamma e^{-c_B x}\times \sinh(Bx)$ with $x=1/\tilde{T}$ and $B=c_E\tilde{E}$ and found $c_B=0.35\pm0.10$. Combining the results shown in Figs. 3 and 4 one can see that our simulation supports the predicted behavior (3.6) for the creep velocity.

It is clear from Fig. 4 that for fixed values of \widetilde{E} the current saturates at low temperatures. This agrees with the experimental data presented in Fig. 2 of Ref. 8. At high temperatures the slopes of the current, when plotted against $1/\widetilde{T}$, were found to depend on \widetilde{E} . 8 On the other hand, as one can

see from Eq. (3.4), the slopes should be independent of \tilde{E} . Such discrepancy is due to the fact that formula (3.4) is valid only for the creep regime.

In conclusion we have shown that although the glass behavior is governed by the T=0 fixed point, the disorder has a dramatic effect on the low temperature properties of 1D CDW's. At low temperatures C(y)/y is determined by the disorder strength. In one dimension the dependence of the creep motion velocity on the driving field was found to be not a nonanalytic function as in higher dimensions but an analytic one. Our theoretical predictions were confirmed by numerical simulations.

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