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Stationary Josephson current as a tool to detect charge density waves in high-T_c oxides

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ABSTRACT

Nonmonotonic and even sign-changing dependences on the temperature and the doping level were predicted for the stationary Josephson tunnel current I_c between superconductors with d-wave order parameter symmetry and partial gapping by charge density waves (CDWs). The junction electrodes were considered in the framework of the two-dimensional electron spectrum appropriate to high-T_c cuprates. The non-trivial behavior can be observed for certain relative electrode orientations. Hence, I_c-measurements in wide ranges of doping and temperature may serve as an indicator of CDW existence.
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1. Introduction

Since the unexpected discovery of high-T_c superconductivity in cuprates in 1986 [1], experts have been trying to find the origin of superconductivity in them, but in vain. There are several problems that are interconnected and so complex that probably cannot be solved independently. Therefore, researchers are forced to consider them separately in order to find the key concepts and express key ideas explaining the huge totality of experimental data. General discussion and the analysis of high-T_c-oxide superconductivity can be found in a number of recent comprehensive reviews [2–14]. In particular, the main questions to be answered are as follows:

(i) Is superconductivity in cuprates a conventional one based on the Cooper pairing concept?
(ii) If the answer to the first question is positive, what is the mechanism of superconductivity, i.e. what are the virtual bosons that glue electrons in pairs?
(iii) Which is the symmetry of the superconducting order parameter? The majority of the researchers in the field believe that the problem is already resolved. Namely, they consider the symmetry to be the $d_{x^2-y^2}$-wave one (see, e.g., Refs. [15–17]). Adepts of this viewpoint mainly lean upon experiments on coherent tunneling in ab-plane of cuprate-based junctions and already built devices using the d-wave character of the order parameter (see also Refs. [18–20]). On the other hand, a large number of c-axis Josephson tunnel measurements showing a non-zero current testify that at least a conspicuous s-wave component of the order parameter exists, since otherwise the contributions from positive and negative d-wave lobes would have cancelled out [21]. We mean, in particular, c-axis junctions between Pb and $YBa_2Cu_3O_{7-\delta}$ [22–25] or $Bi_2Sr_2CaCuO_{8+\delta}$ [26] oxides, as well as twist junctions between $Bi_2Sr_2CaCuO_{8+\delta}$ crystals [27] and cross-whisker junctions [28–31].
An indirect confirmation of the none–zero-gap superconductivity in $Bi_2\ Sr_2CaCuO_{8+\delta}$ can also be inferred from the observation of the non-stationary Josephson-effect terahertz radiation either from mesas [32] or Au–$Bi_2Sr_2CaCuO_{8+\delta}$–Au structures [33]. Namely, the radiation frequency remains finite testifying that the actual energy gap is nodeless. One sees, that the controversy over the type of the order-parameter symmetry in high-T_c cuprates still remains. This conclusion is strengthened by the fact that structural imperfections and microscopic nonhomogeneities of the samples are unavoidable in the non-stoichiometric oxides, rendering the interpretation of subtle effects rather problematic.
(iv) What is the role of the intrinsic disorder and non-stoichiometry in the superconducting properties [3,34–40]?
(v) What is the origin of the symmetry loss and, specifically, the emerging nematicity [3,36,38,39,41,42]?

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(vi) What is the origin of the so-called pseudogap [3,4,13,43–45]?

(vii) What is the role of spin- and charge- density waves (SDWs and CDWs) both in the normal and superconducting states of cuprates? The role of various electron spectrum instabilities competing with the Cooper pairing below the critical temperature T_c is a part of the more general problem.

(viii) How can certain anomalous high-T_c oxide properties observed above T_c, e.g., the linear behavior of the resistivity [46,47], be explained? In this connection, a quite reasonable viewpoint was expressed that, if one understands the normal state of cuprates, the superconducting state properties will be perceived [13,48]. Here, it is also worth to mention a possible failure [46,49] of the Fermi liquid concept belonging to Landau [50] and the role of strong electron correlations [51–54].

During last decades we have been developing a phenomenological theory to elucidate the influence of CDWs on superconductivity in high-T_c oxides, since CDWs were observed in a number of those materials [55–60]. We identified the CDW energy gap with the pseudogap. Such an identification is based, in particular, on the appearance of CDWs only inside the approximate borders of the pseudogapped area in phase diagrams appropriate to $La_{2-x}Sr_xCuO_4$ [61,62] and $YBa_2Cu_3O_{7-\delta}$ [63,64]. Moreover, the following facts should be taken into account: the symmetry of the pseudogap order parameter (isotropic) differs from that for the superconducting one ($d_{x^2-y^2}$) in $Bi_2Sr_2CaCuO_{8+\delta}$ [65]; superconductivity in $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ emerges with doping when the (nodal) pseudogap disappears [66]; the pseudogap competes with the superconducting gap at antinodes in $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$ [37]; and the interplay of pseudogapping and superconductivity among different members of the oxide family $(Ca_xLa_{1-x})(Ba_{1.75-x}La_{0.25+x})Cu_3O_y$ is not the same for the varying doping x [67]. It is worthy of note that both angle-resolved photoemission spectroscopy (ARPES) and scanning tunnel microscopy (STM) experiments allow one to measure only total energy gaps, whatever their microscopic origin. That is why it is usually difficult to distinguish for sure between superconducting, SDW, and CDW gaps even in the case when they manifest themselves separately in certain momentum ranges each [13,68].

As for direct experiments confirming the existence of other electron spectrum instabilities competing with the superconducting one in cuprates, CDWs have been shown to be a more important factor in this sense than SDWs, the remnants of which survive far from the antiferromagnetic state appropriate to zero-doped samples of superconducting families [69]. It is useful to shortly summarize the main new findings in this area.

X-ray scattering experiments in $YBa_2Cu_3O_{6+x}$ revealed the CDW ordering at temperatures lower than those of the pseudogap formation, giant phonon anomalies, and elastic central peak induced by nanodomain CDWs [64,70–72]. The CDW correlation length increases with the temperature, T, lowering. However, the competing superconducting order parameter that emerges below T_c so depresses CDWs that the true CDW long-range order does not develop, as was shown by Raman scattering [63]. The suppression of CDWs by Cooper pairing was also found in X-ray measurements of $La_{2-x}Sr_xCuO_4$ [61].

The well-known CDW manifestations in $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ were recently confirmed by complex X-ray, ARPES, and STM studies [73]. Those authors associated CDWs with pseudogapping, but argue that the CDW wave vector connects the Fermi arc tips rather than the antinodal Fermi surface (FS) sections, as stems from the Peierls-insulator scenario [74,75]. This conclusion, if being true, makes the whole picture even more enigmatic than in the conventional density-wave approach to pseudogaps either in the mean-field approximation or taking into account fluctuations.

The electron–hole asymmetric CDW ordering was demonstrated by STM and resonant elastic X-ray scattering measurements [76] for $Bi_2Sr_2CaCuO_{8+\delta}$ samples with the pseudogapping in the antinodal momentum region. As was shown in those experiments, CDWs and concomitant periodic crystal lattice distortions (PLDs) can be observed directly, whereas their interplay with superconductivity manifestations can be seen only indirectly, e.g., as anticorrelations between T_c and the structural, T_s, or CDW, T_{CDW}, transition temperature. (There is a viewpoint [77] that the strong interrelation between electronic CDW modulations and PLDs [74], which are inherent, e.g., to the Peierls model of the structural phase transition [75], does not exist, and PLDs can emerge without electronic contributions, which seems strange in the context of indispensable Coulomb forces.) This fact is well known, say, for superconducting transition metal dichalcogenides [78] or pseudoternary systems $(Lu_{1-x}Sc_x)_5Ir_4Si_{10}$ [79]. Recently, the revival of superconductivity after the density-wave state is weakened was observed in $BaTi_2(Sb_{1-x}Bi_x)_2O$ [80]. Here, it is not known for sure whether this density wave has a CDW or an SDW character. It is remarkable that the phase diagram of $BaTi_2(Sb_{1-x}Bi_x)_2O$ includes a superconducting dome as in cuprates [69] or $BaPb_{1-x}Bi_xO_3$ [81]. Relevant phase diagrams show the depression of superconductivity in the overdoped region, where electron–hole correlations do not already exist but the pairing interaction constant becomes smaller. The latter can happen either in the case when the Cooper pairing interaction (being of electron–phonon, spin-fluctuation or whatever else origin) is attenuated or when the parent normal-state electron density of states is suppressed due, e.g., to a metal–insulator transition. A competition between density waves and superconductivity was discovered for other oxides as well. One can indicate, for instance, a family of $Ba_{1-x}Na_xTi_2Sb_2O$ [82], $Ba_{1-x}K_xTi_2Sb_2O$ [83], $BaTi_2(Sb_{1-x}Sn_x)_2O$ [84], $Ba_{1-x}Rb_xTi_2Sb_2O$ [85], and $Ba_2Ti_2Fe_2As_4O$ [86] superconductors.

For any of those materials and especially for cuprates, it seems important to propose such experimental studies of superconducting properties, which would demonstrate manifestations of CDW existence, although the CDW gapping is an insulating rather than a superconducting one. In a number of publications, we suggested that certain measurements of the stationary Josephson critical current, I_c, between quasi-two-dimensional CDW superconductors with the $d_{x^2-y^2}$ order parameter symmetry (probably inherent to cuprates with the reservations discussed above) can conspicuously reveal such dependences that would reflect CDW gapping as well or at least demonstrate that the actual gapping character differs from the pure $d_{x^2-y^2}$ one [60,87–90]. In particular, we considered in detail [89] the angular dependences of I_c, i.e. the dependences on the orientation of one or both electrodes with respect to the junction plane.

In this paper, the results of our further studies are presented. In particular, we suggest experimental configurations of tunnel junctions with CDW d-wave superconductors, for which the dependences of the Josephson current I_c on the CDW parameters and the temperature could be nonmonotonic and even sign-changing. Such a behavior is closely connected to the d-wave character of superconducting pairing. Therefore, by detecting this nontrivial behavior and analyzing how external parameters (doping and temperature) affect it, an experimental confirmation of our theoretical model and a verification of the d-wave character of the superconducting order parameter in cuprates and, possibly, other objects can be obtained.

2. Formulation

2.1. Partially gapped CDW superconductors

Following the dominating idea concerning the electron spectrum of high-T_c oxides identified as partially gapped CDW superconductors (CDWSs)—see our previous publications [60,87–92] and references therein)—we restrict the consideration to the two-dimensional case with the corresponding FS shown in Fig. 1a. The superconducting d-wave order parameter Δ is assumed to span the whole FS, whereas the s-wave mean-field dielectric (CDW) order parameter Σ develops only on the nested (dielectrized, d) FS sections. There are $N = 4$ or 2 of the latter (the checkerboard and unidirectional CDW configurations, respectively), and they are nested in pairs by the CDW-vectors $\mathbf{Q}$'s in the momentum space. The non-nested sections remain non-dielectrized (nd). The orientations of $\mathbf{Q}$'s are assumed to be fixed with respect to the crystal lattice. In particular, they are considered to be directed along the $\mathbf{k}_x$- and $\mathbf{k}_y$-axes in the momentum space (anti-nodal nesting) [58,93,94]. The same orientation along $\mathbf{k}_x$- and $\mathbf{k}_y$-axes is also appropriate to Δ-lobes (the "positive" and "negative" lobes are assumed to be directed along the $\mathbf{k}_x$- and the $\mathbf{k}_y$-axis, respectively), so that we confine ourselves to the $d_{x^2-y^2}$-wave symmetry of the superconducting order parameter as the only one found in the experiments for cuprates). Hence, the profile of the d-wave superconducting order parameter over the FS is written down in the form

$$\bar{\Delta}(T,\theta) = \Delta(T) f_\Delta(\theta). \tag{1}$$

The function $\Delta(T)$ is the T-dependent magnitude of the superconducting gap, and the angular factor $f_\Delta(\theta)$ looks like

$$f_\Delta(\theta) = \cos 2\theta. \tag{2}$$

In the case $N = 4$, the experimentally measured magnitudes of the CDW order parameter Σ in high-T_c oxides are identical in all four CDW sectors, and the corresponding sector-connecting $\mathbf{Q}$ vectors are oriented normally to each other. Therefore, we assume the CDWs to possess the four- (the checkerboard configuration) or the twofold (the unidirectional configuration) symmetry [58,59,89,95–98]. The latter is frequently associated with the electronic nematic, smectic or more complex ordering [38,39,42,99–107]). The opening angle of each CDW sector, where $\Sigma \neq 0$, equals 2α. Such a profile of Σ over the FS can also be described in the factorized form as

$$\bar{\Sigma}(T,\theta) = \Sigma(T) f_\Sigma(\theta), \tag{3}$$

where $\Sigma(T)$ is the T-dependent CDW order parameter, and the angular factor

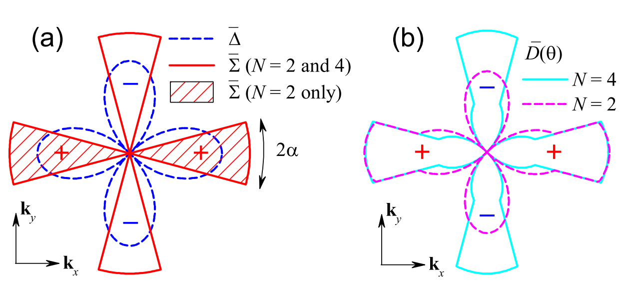

Fig. 1. (a) Superconducting, $\bar{\Delta}(\theta)$, and dielectric, $\bar{\Sigma}(\theta)$, order parameter profiles of the partially gapped d-wave charge-density-wave (CDW) superconductor. N is the number of CDW sectors with the width 2α each. (b) The corresponding energy-gap contours (gap roses).

$$f_\Sigma(\theta) = \begin{cases} 1 & \text{for } |\theta - m\Omega| < \alpha \quad (d \text{ section}), \\ 0 & \text{otherwise} \qquad\qquad (nd \text{ section}). \end{cases} \tag{4}$$

Here, m is an integer number, and the parameter $\Omega = \pi/2$ for $N = 4$ and π for $N = 2$.

The both gapping mechanisms (superconducting and CDW-driven) suppress each other, because they compete for the same quasiparticle states near the FS [60,87–92]. As a result, a combined gap (the gap rose in the momentum space, see Fig. 1b)

$$\bar{D}(T,\theta) = \sqrt{\bar{\Sigma}^2(T,\theta) + \bar{\Delta}^2(T,\theta)}, \tag{5}$$

arises on the FS. The actual $\Delta(T)$- and $\Sigma(T)$-values are determined from a system of self-consistent equations describing mutual suppression between these two order parameters. The relevant initial parameters, besides N and α, include the constants of superconducting and electron–hole couplings recalculated into the pure BCS (no CDWs) and CDW (no superconductivity) limiting cases as the corresponding Δ_0 and Σ_0 order parameters at $T = 0$. Fig. 1b also demonstrates that, in the case of $d_{x^2-y^2}$-wave symmetry, the combined gap rose $\bar{D}(T,\theta)$ consists of four lobes with the same orientation as the Δ-lobes have. Therefore, for the illustrative purpose, it is convenient to symbolically give each $\bar{D}$-lobe the sign of the corresponding Δ-lobe (see also a short discussion in Section 3.2). It should be emphasized that our model is a simplified, generic one, because real CDWs are complex objects, which behave differently on the crystal surfaces and in the bulk [108]. Thus, it is quite natural that they are not identical for various high-T_c oxides [67]. Nevertheless, the presented model allows the main features of the materials concerned to be taken into account. For brevity, we mark the CDW d-wave superconductor with N CDW sectors as S^d_{CDWN}. Similarly, we introduce the notation S^d_{BCS} for the conventional d-wave BCS superconductor [109].

Our model [60,87–92] is a phenomenological one and is a generalization of the previous approach [55,57,110–113] applied to s-wave superconductors with CDWs. Formulated in the mean-field approximation, it does not include any microscopic driving forces of the electron–hole pairing, the constant Σ_0 being the only parameter describing this phenomenon. On the other hand, the actual microscopic picture is very important and should be material-specific even if the two-dimensionality and the nesting of the parent electron spectrum are preserved. Moreover, microscopic scenarios should explain intrinsic inhomogeneity of normal-state and superconducting properties manifested, in particular, for cuprates [11,68,114–119]. Another task is to find the origin of the observed nematicity [41,42,99] taken here for granted ($N = 2$). Therefore, we want to attract attention to the microscopic or semi-microscopic attempts intended to solve the problems concerned (see works [14,120–126] and references therein). The majority of those scenarios implicitly adopt the basic idea [110] of the rivalry between superconductivity and CDWs, whatever the specific CDW structure (see, e.g., Ref. [126]).

2.2. Josephson current

In the tunnel Hamiltonian approximation, the stationary Josephson critical current is given by the formula [127–129]

$$I_c(T) = 4eT \sum_{\mathbf{pq}} \left| \widetilde{T}_{\mathbf{pq}} \right|^2 \sum_{\omega_n} \mathsf{F}^+(\mathbf{p};\omega_n) \mathsf{F}'(\mathbf{q};-\omega_n). \tag{6}$$

Here, $\widetilde{T}_{\mathbf{pq}}$ are the tunnel Hamiltonian matrix elements, $\mathbf{p}$ and $\mathbf{q}$ are the transferred momenta; $e > 0$ is the elementary electrical charge, and $\mathsf{F}(\mathbf{p};\omega_n)$ and $\mathsf{F}'(\mathbf{q};-\omega_n)$ are Gor'kov Green's functions for superconductors to the left and to the right, respectively, from the tunnel barrier (hereafter, all primed quantities are associated with the right hand side electrode). The internal summation is carried out over the discrete fermionic "frequencies" $\omega_n = (2n+1)\pi T$, $n = 0, \pm 1, \pm 2, \ldots$. The relevant anomalous Green's functions (other, "normal", ones appear only in the case of quasiparticle currents) for the *d* and *nd* FS sections are [60,89]

$$\mathsf{F}_{d,nd}(\mathbf{p};\omega_n) = \frac{\bar{\Delta}(T,\theta-\gamma)}{\omega_n^2 + \bar{D}^2(T,\theta-\gamma) + \xi_{d,nd}^2(\mathbf{p})}, \tag{7}$$

where $\xi_{nd,d}(p)$ are the quasiparticle dispersion functions on the corresponding FS sections.

Below, we consider symmetric $S^d_{\mathrm{CDW}N} - I - S^d_{\mathrm{CDW}N}$ tunnel junctions between two identical CDWSs (here, I stands for the insulator). Since CDWS electrodes are anisotropic, their orientations with respect to the junction plane will be characterized by the angles γ and γ', i.e. the deflections of the "positive" Δ- and Δ'-lobes (or $\bar{D}$- and $\bar{D}'$-lobes) from the normal $\mathbf{n}$ to the junction (Fig. 2). Accordingly, the angular dependences $f_\Delta(\theta)$ and $f_\Sigma(\theta)$ of the corresponding order parameters (see formulas (2) and (4), respectively) should be modified by changing θ to $\theta-\gamma$ or $\theta-\gamma'$.

An important factor while calculating the Josephson current is tunnel directionality [130], which should be taken into consideration in the tunnel Hamiltonian $\widetilde{T}_{\mathbf{pq}}$. Indeed, if we calculate I_c between, e.g., pure BCS *d*-wave superconductors, S^d_{BCS}, making no allowance for this factor, formula (6) would produce an exact zero. It is so because, owing to the alternating signs of superconducting lobes, the current contributions from the FS points described by the angles θ and $\theta + \frac{\pi}{2}$ would exactly compensate each other in this case. The same situation also takes place in the case of a junction with $S^d_{\mathrm{CDW}4}$. For a junction with $S^d_{\mathrm{CDW}2}$, it is not so, but, in the framework of the general approach, we have to introduce tunnel directionality in this case as well.

Here, we briefly consider three factors responsible for tunnel directionality (see a more thorough discussion in Ref. [90]). First, the velocity component normal to the junction should be taken into account. This circumstance is reflected by the $\cos\theta$-factor in the integrand and an angle-independent factor that can be incorporated into the junction normal-state resistance R_N [131,132]. Second, superconducting pairs that cross the barrier at different angles penetrate through barriers with different effective widths [133] (the height of the junction barrier is assumed to be much larger than the relevant quasiparticle energies, so that this height may be considered constant). Since the actual θ-dependences of $\widetilde{T}_{\mathbf{pq}}$ for realistic junctions are not known, we simulate the barrier-associated directionality by the phenomenological function

$$w(\theta) = \exp\left[-\left(\frac{\tan\theta}{\tan\theta_0}\right)^2 \ln 2\right]. \tag{8}$$

This means that the effective opening of relevant tunnel angles equals $2\theta_0$ (see Fig. 2). The barrier transparency is normalized by the maximum value obtained for the normal tunneling with respect to the junction plane and included into the junction resistance R_N. Hence, $w(\theta = 0) = 1$. The multiplier $\ln 2$ in (8) was selected to

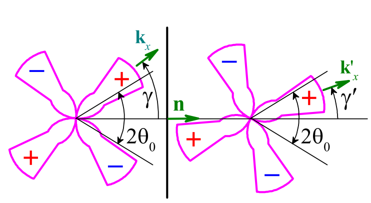

Fig. 2. Experimental configuration of the symmetric Josephson junction between identical $S^d_{\mathrm{CDW}4}$'s. See further explanations in the text.

provide $w(\theta = \theta_0) = \frac{1}{2}$. Third, we use the model of coherent tunneling [21,133,134], when the superconducting pairs are allowed to tunnel between the points on the FSs of different electrodes characterized by the same angle θ.

As a result of the standard calculation procedure [127,128] applied to formula (6) and in the framework of the approximations made above, we obtain the following formula for the stationary Josephson critical current across the tunnel junction:

$$I_c(T,\gamma,\gamma') = \frac{1}{2eR_N} \times \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos\theta\, w(\theta) P(T,\theta,\gamma,\gamma')\mathrm{d}\theta, \tag{9}$$

where [135,136]

$$P(T,\theta,\gamma,\gamma') = \bar{\Delta}\bar{\Delta}' \int_{\min\{\bar{D},\bar{D}'\}}^{\max\{\bar{D},\bar{D}'\}} \frac{\tanh\frac{x}{2T}\,\mathrm{d}x}{\sqrt{(x^2-\bar{D}^2)(\bar{D}'^2-x^2)}}. \tag{10}$$

Here, for brevity, we omitted the arguments in the dependences $\bar{\Delta}(T,\theta-\gamma)$, $\bar{\Delta}'(T,\theta-\gamma')$, $\bar{D}(T,\theta-\gamma)$, and $\bar{D}'(T,\theta-\gamma')$. Integration over θ in Eq. (9) is carried out within the interval $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$, i.e. over the "FS semicircle" turned towards the junction plane. If any directionality and CDW gapping are excluded (so that the integration over θ is reduced to a factor of π) and the angular factors f_Δ and f'_Δ remain preserved, we arrive at the Sigrist–Rice model [137].

Concerning Eq. (10), we would like to make the following remark. Although we cannot assign a definite sign to the combined gap $\bar{D}$ (see Eq. (5), the corresponding Δ with an unambiguous sign enters this expression used for the calculation of I_c. In this sense, the FS of the CDWS "remembers" the specific Δ-sign at every of its points. Hence, the corresponding sign can be attributed to each lobe of the gap rose (see Fig. 3). Then, the expression

$$p(T,\theta,\gamma,\gamma') = \frac{\bar{\Delta}\bar{\Delta}' \tanh\frac{x}{2T}\, dx d\theta}{\sqrt{\left(x^2-\bar{D}^2\right)\left(\bar{D}'^2-x^2\right)}} \tag{11}$$

can be interpreted as an unweighted (i.e. without the account of tunnel directionality) contribution with a definite sign to the superconducting current made by superconducting paired states within the energy interval $(x, x+dx)$ and within the sector $(\theta, \theta+d\theta)$ in the momentum space in each electrode. The latter circumstance is a result of the adopted here coherent tunneling model [21,133,134]. Due to the interplay between the contributions of different signs, the CDWS electrode can serve a differential detector of the current at definite relative electrode orientations (see Fig. 2).

3. Results and their discussion

The influence of various problem parameters on the critical stationary Josephson current in the symmetric, $S^d_{\mathrm{CDW}N} - I - S^d_{\mathrm{CDW}N}$, junctions was analyzed in detail in works [89,90]. Here, we attract attention to the problem of CDW detection in high-T_c oxides.

The number of problem parameters can be diminished by normalizing the "order parameter" quantities by one of them. For such a normalization, we selected the parameter Δ_0 and introduced the dimensionless order parameter $\sigma_0 = \Sigma_0/\Delta_0$ (for the superconducting order parameter of CDWS, $\delta_0 = \Delta_0/\Delta_0 = 1$). With regard to experimental needs, we also introduced the reduced temperature $\tau = T/T_c$. Here T_c is the actual critical temperature of the CDWS. In the framework of our theory, it has to be found from the corresponding system of equations [95]. For the Josephson current amplitude I_c, we introduced the dimensionless combination $i_c = I_c eR_N/\Delta_0$.

One more preliminary remark concerns the parameter of effective tunnel directionality θ_0 (see formula (8)). Our calculations [89,90] showed that its choice is very important. On the one hand,

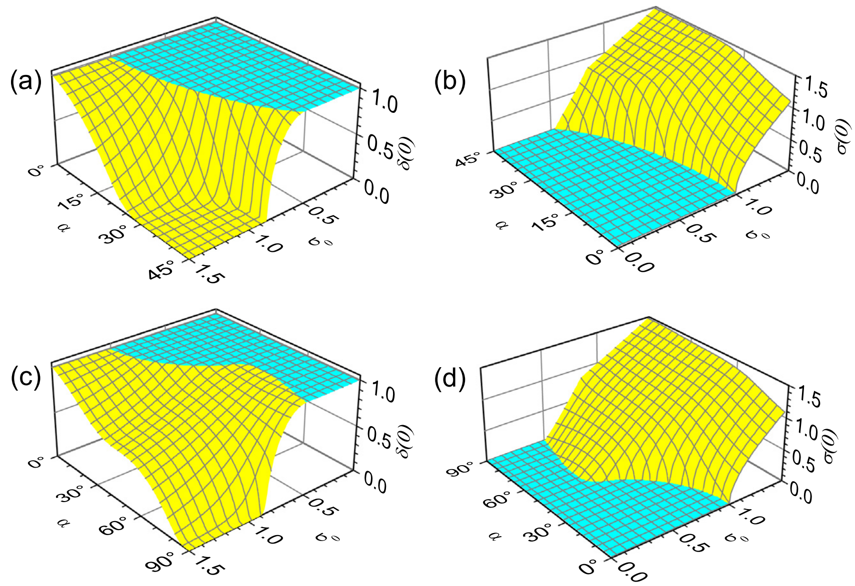

Fig. 3. Dependences of the normalized superconducting, $\delta(0) = \Delta(T=0)/\Delta_0$ (panels a and c), and dielectric, $\sigma(0) = \Sigma(T=0)/\Delta_0$ (panels b and d), order parameter amplitudes at zero temperature T for $N = 4$ (panels a and b) and 2 (panels c and d). Here, Δ_0 is the zero-temperature superconducting order parameter amplitude in the absence of CDWs.

large values of this parameter correspond to thin junctions and large values of the tunnel current, which is beneficial for the experiment. However, in this case, the predicted phenomena become effectively smoothed out up to their disappearance. On the other hand, narrow tunnel cones (small θ_0-values) provide well pronounced effects, but correspond to thick interelectrode layers and, as a result, small tunnel currents. Hence, in the real experiment, a reasonable compromise should be found between those two extremes.

While examining Fig. 2, it becomes clear that the clearest way to prove that electrons in high-T_c oxides undergo an additional pairing of some origin besides the d-wave BCS one is to demonstrate that the gap rose differs from that in the S^d_{BCS} superconductor. The case in question concerns pairing symmetries, which may be different from the d-wave one or/and extend over only certain FS regions. In the framework of the tunnel technique, the most direct way to perform the search is to fix one electrode and rotate the other (e.g., $\gamma' = \mathrm{const}$ and $\gamma = \mathrm{var}$). From the viewpoint of the adopted here tunneling model, this is a trivial result of changing the overlapping between the sectors with different gap signs (see Section 2.2). In the case of $S^d_{\mathrm{BCS}} - I - S^d_{\mathrm{BCS}}$ junction, the corresponding $i_c(\gamma)$ dependences are known to have a cosine profile stemming from dependence (2) for the superconducting order parameter Δ and, since any other gapping is absent, for the corresponding gap rose ($\overline{D}(T,\theta) = |\Delta(T,\theta)|$). Any deviations of the gap rose from this behavior will testify in favor of the existence of additional order parameter(s). Certainly, averaging the current over the FS will smooth the relevant peculiarities and making allowance for tunnel directionality will distort them. Nevertheless, the proposed method will be sufficient to detect the competing pairing without its ultimate identification.

A thorough analysis of $i_c(\gamma)$ dependences and how various problem parameters affect them can be found in Ref. [89]. The results obtained testify that the formulated task is feasible. An attractive feature of this technique is that, instead of the fixed S^d_{CDWN} electrode, we may use the S^s_{BCS} one as well, which might be more convenient from the experimental point of view.

Our further consideration is based on the results of Ref. [138], where we showed that, at some fixed electrode orientations ($\gamma = \mathrm{const}$ and $\gamma' = \mathrm{const}$), the critical Josephson current through the symmetric $S^d_{\mathrm{BCS}} - I - S^d_{\mathrm{BCS}}$ junction may reveal a nonconventional nonmonotonic temperature behavior. In this work devoted to the $S^d_{\mathrm{CDWN}} - I - S^d_{\mathrm{CDWN}}$ junctions, we show that this behavior can be controlled by varying the parameters of S^d_{CDWN} electrodes, e.g., by doping the latter. Besides, we demonstrate that i_c may be a nonmonotonic function of S^d_{CDWN} parameters (doping) at a fixed T. In contrast to the case with rotated electrodes [89,90], now the overlapping of momentum-space sectors with different quasiparticle state signs does not change, so that other reasons shoul be attracted to explain this phenomenon.

To explain the presentation form of the calculation results, the following remark should be made. The adopted here basic model of the CDW partial gapping [110–113], whatever the superconducting competitor, considers the parameter α to be fixed. Therefore, the system of gap equations (Σ and Δ) is a consequence of the Landau-like free energy minimization with a constant α. Of course, it is only an approximation. Actually, some kind of strong coupling should occur, i.e. the emergence of Σ should affect the FS gapping degree. Hence, the reconstructed state should be sought by the free-energy minimization over three quantities, namely, Σ, Δ and α. One more parameter, the phase φ of the dielectric order parameter Σ, completing the full set of parameters for the CDWS, does not affect the thermodynamically driven interdependence between Δ and Σ. It does not reveal itself in the Josephson tunnel current. So, it is not considered here. This very complicated approach has never been applied in the general phenomenological scheme [110–113], although electronic-band calculations for specific materials include this effect to some extent automatically. The necessity of the Σ and α (in our notations) self-consistent calculations was explicitly recognized in the hidden-nesting concept [139–141]. In view of all that, one should understand that the doping dependence cannot be identified with pure σ_0- or α-ones but is rather a function of both arguments in the $\sigma_0 - \alpha$ plane. This consideration pertains equally to the Josephson current case studied here.

In the $i_c(\sigma_0, \alpha)$ dependences plotted below, the light (yellow) color corresponds to positive, and the darker (cyan) one to negative i_c-values. Besides, the finite error of numerical calculations forced us to consider every i_c-value within the interval $[-10^{-7}, 10^{-7}]$ to equal zero. Those values are marked by the dark (magenta) color.

In a number of cases, when the dependence $i_c(\sigma_0, \alpha)$ turned out rather complicated with obscured peculiarities, we made the corresponding animation version of the figure (see Supplementary data).

3.1. Anomalous doping dependence of I_c

Now, let the electrode orientations be fixed by the experimentalist [142,143] and the temperature be zero (for simplicity), but the both parameters α and σ_0 can be varied (by doping). In Fig. 3, the dependences of the dimensionless order parameters $\delta(0) = \Delta(T=0)/\Delta_0$ and $\sigma(0) = \Sigma(T=0)/\Delta_0$ on α and σ_0 are exhibited for both analyzed CDW structures ($N = 4$ and 2). The calculations were carried out by solving the system of two linked gap equations for $\Delta(T)$ and $\Sigma(T)$ self-consistently [95]. One can see that, in every cross-section $\alpha =$ const or $\sigma_0 =$ const, both $\delta(0)$ and $\sigma(0)$ profiles are monotonic. At first glance, the Josephson tunnel current should also demonstrate such a behavior. However, our previous calculations [60,89,90] showed that it is so when the orientations of $S^d_{\mathrm{CDW}N}$ electrodes in the $S^d_{\mathrm{CDW}N} - I - S^d_{\mathrm{CDW}N}$ junction are close or rotated by about 90° with respect to each other, i.e. when the superconducting lobes strongly overlap in the momentum space and make contributions of the same sign to the current. But if they are oriented in such a way that mutually form a kind of differential detector for monitoring the states at the gapped and non-gapped FS sections, contributions with different signs cancel each other and more tiny effects become observable.

This statement is illustrated in Fig. 4, where the $i_c(\sigma_0, \alpha)$ dependences calculated for the "reference" combination ($\sigma_0 = 1.3$, $\alpha = 45^\circ$) in both CDW geometries ($N = 4$ and 2) are shown. While analyzing those figures, the following consideration should be taken into account. Namely, we reasonably suppose that gradual doping monotonically affects both parameters (α and σ_0) characterizing $S^d_{\mathrm{CDW}N}$ superconductors. The results presented in Fig. 4 testify that of each of those parameters differently affects the current i_c. Moreover, underdoping is usually accompanied by the increase of both α and Σ (proportional to the structural phase transition temperature, i.e. the pseudogap appearance temperature, T^*) [13,43,58,144]. Therefore, the situation when the doping-induced simultaneous changes in the values of α and Σ_0 would lead to their mutual compensation seems improbable. Accordingly, we believe that the proposed experiments may be useful in one more, this time indirect, technique to probe CDWs in high-T_c oxides. In particular, the oscillating dependences $i_c(\alpha)$ retrieved from Fig. 4 as cross-sections, if reproduced in the experiment, would unambiguously prove the interplay between the superconducting order parameter and another, competing, one; here, the latter is considered theoretically to be associated with CDWs.

It is clear that the predicted effect originates from the change of relative contributions to the total current made by the negative and positive current components. However, now, when the orientation of both electrodes remain fixed, the elementary contributions (11) to the total current "preserve their signs". When varying only Σ_0, we affect (in the framework of our model with $\alpha =$ const) the gapped spectrum, e.g., in the left hand side electrode, in such a way that for every superconducting paired state participating in tunneling the spectrum parameter $\bar{D}$ and the superconducting order parameter $\bar{\Delta}$ in formula (11) change. A correlation between characteristic features of the $i_c(\sigma_0 = \text{const}, \alpha)$ and $i_c(\sigma_0, \alpha = \text{const})$ isolines in Fig. 4 with those of the $\delta(0)(\sigma_0 = \text{const}, \alpha)$ and $\delta(0)(\sigma_0, \alpha = \text{const})$ isolines, respectively, in Fig. 3 testifies that the dominating role in all that is played by the modification of the actual superconducting order parameter $\delta(0)$ under the influence of the Σ_0 variation. so that we may say that changing σ_0 reveals itself indirectly through affecting $\delta(0)$ (the factor $\bar{\Delta}$ in formula (11)). Nevertheless, the role of varying $\bar{D}$ can be appreciable, and sometimes it prevails. In particular, Fig. 4 shows that, in both CDW geometries ($N = 4$ and 2), by changing only α, we obtain a much stronger effect. This is so, because the overall gap has a jump (see Fig. 2) at the angles $\theta = \gamma \pm \alpha$, so that for a given γ, the states making a substantial contribution to the current, change their spectrum in a jump-like manner. Of course, all those effects are observable when the junction operates in the differential detector mode.

In Fig. 3, the dark (cyan) color marks those areas in the CDWS phase diagram [95] where superconductivity prohibits the development of CDWs at $T = 0$, so that the CDWS reveals itself as a conventional pure d-wave BCS superconductor. In Fig. 4, those areas correspond to i_c-plateaus at low σ_0's. The corresponding i_c values (of course, they are identical for two CDW patterns) equal the Josephson current between two identical parent d-wave BCS superconductors and, together with the zero i_c-values (the dark (magenta) color), may serve in every case as a reference for the evaluation of the $i_c(\sigma_0, \alpha)$ profile in whole. The same remark is valid for all $i_c(\sigma_0, \alpha)$ profiles exhibited below.

Hence, the main result of this section consists in that, in some experimental setups when the positive and negative contributions to the critical Josephson tunnel current are almost mutually compensated and we can observe the interplay between those contributions, we, by doping electrodes or subjecting them to pressure, can not only affect the current amplitude, but also change its sign, i.e. transform the 0-junction into the π-one or vice versa. For some parameters, if their intervals of change are sufficiently wide, there can be several subsequent transformations. This is the more so interesting, because the dependences of both order parameters on σ_0 and α are monotonic (Fig. 3).

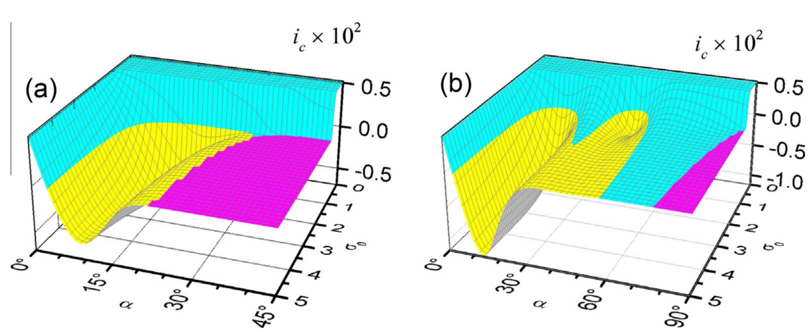

Fig. 4. Dependences of the normalized critical dc Josephson current $i_c = I_c e R_N / \Delta_0$, where e is the elementary charge and R_N the normal-state resistance of the tunnel junction, on α and $\sigma_0 = \Sigma_0/\Delta_0$ for $N = 4$ (a) and 2 (b). Here, Σ_0 is the dielectric order parameter at $T = 0$ in the absence of superconductivity. The electrode orientation angles are $\gamma = 15^\circ$ and $\gamma' = 45^\circ$. See also the animated versions of both panels in Supplementary data.

3.1.1. Effect of electrode orientation

The property called the “differential detector” can be better understood if one tries to vary the electrode orientations. For instance, we calculated the function $i_c(\alpha, \sigma_0)$ for the cases shown in Figs. 5–8 where either the left or right hand side electrode was turned through certain angles. For $N = 4$, one can see that the influence of the γ' variation near an “optimal” value of 45° is stronger than that of the γ variation. Anyway, to see the transition between 0- and π-junctions induced by doping, i.e. the CDW variation, one should properly select geometrical configurations. The main obstacle is working with low Josephson tunnel current amplitudes, which is challenging *per se*. Nevertheless, we believe that the “differential detector” setup will be useful in studying the tiny peculiarities in the cuprate electron spectrum.

The doping behavior of i_c for superconductors with unidirectional CDWs are even more impressive, as stems from Figs. 7 and 8. Depending on the orientation angles γ and γ' changing within relatively narrow intervals, the doping influence on the Josephson current becomes very different. In particular, the boundaries of the regions with the positive i_c (0-junctions) may become rather complicated.

Therefore, electrode doping and crystal orientation in the spirit of Refs. [142,143] are mutually complementary instruments to construct differential detectors suitable to discover the CDW influence on the Josephson currents between CDWSs. It is remarkable that CDWs, being a result of the electron spectrum dielectric instability in the electron–hole channel, affect the superconducting coherent phenomenon.

3.2. Anomalous temperature dependence of I_c

The measurements of the temperature dependences of the critical Josephson tunnel current $I_c(T)$ seem to be the most convenient method to detect CDWs in Josephson junctions. The dependence $I_c(T)$ for symmetric $S^s_{BCS} - I - S^s_{BCS}$ junctions has a monotonic convex shape. Among other things, this fact is associated with the constant sign of order parameter over the whole FS. However, in the case of symmetric $S^d_{BCS} - I - S^d_{BCS}$ junctions, the situation may change. Indeed, for junctions involving $YBa_2Cu_3O_{7-\delta}$, nonmonotonic $I_c(T)$-dependences and even the change of I_c sign, i.e. the transformation of the 0-junction into the π-one or vice versa were

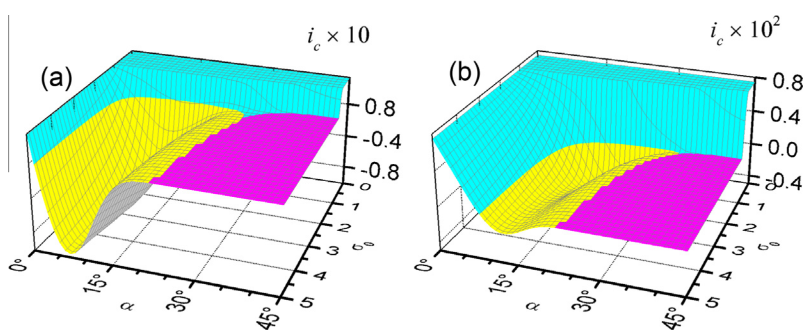

Fig. 5. The same as in Fig. 4a, but for $\gamma = 10°$ (a) and 20° (b).

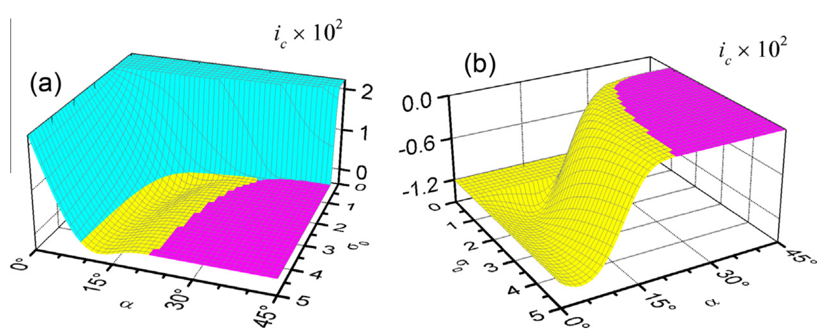

Fig. 6. The same as in Fig. 4a, but for $\gamma' = 43°$ (a) and 47° (b). See also the animated version of panel (b) in Supplementary data.

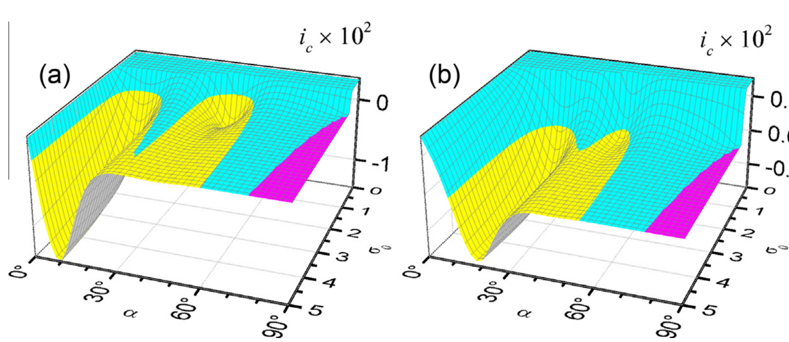

Fig. 7. The same as in Fig. 5, but for $N = 2$.

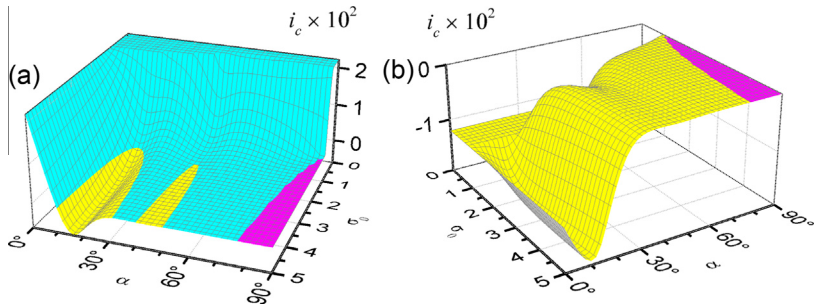

Fig. 8. The same as in Fig. 6, but for $N = 2$. See also the animated version of panel (b) in Supplementary data.

observed [145,146]. Such a phenomenon was not found for other cuprates, because it turned out extremely difficult to produce controllable Josephson junctions made of other materials than $YBa_2Cu_3O_{7-\delta}$. Therefore, further technological breakthrough is needed to make sure that the non-monotonic behavior is a general phenomenon inherent to all high-T_c oxides with the d-wave superconducting order parameter.

It should be noted that, in the measurements concerned, the electrodes remained fixed, so that the peculiar behavior of $I_c(T)$ could not result from the change of overlapping between the superconducting lobes with different signs. There is an explanation based on the existence of the bound states in the junction due to the Andreev–Saint-James effect [131,132]. This theory predicts that the current $I_c(T)$ between d-wave superconductors must exhibit a singularity at $T \to 0$. Nevertheless, the latter has not been observed experimentally until now. Probably, this effect is wiped out by the roughness of the interfaces in the oxide junctions [147,148] and therefore may be of academic interest.

Earlier we suggested a different scenario [138]. Namely, we showed that, at some relative orientations of $S^d_{\rm BCS} - I - S^d_{\rm BCS}$ junction electrodes, one of them can play a role of differential detector, which enables tiny effects connected with the thermally induced repopulation of quasiparticle levels near the FS to be observed. In our approach, no zero-T singularity of the current could arise.

A similar situation takes place for CDWSs. As a result, the dependences $I_c(T)$ both for symmetric $S^d_{\rm CDWN} - I - S^d_{\rm CDWN}$ and nonsymmetric $S^d_{\rm CDWN} - I - S^d_{\rm BCS}$ junctions can also be nonmonotonic and even sign-changing functions. Unlike the $S^d_{\rm BCS} - I - S^d_{\rm BCS}$ junctions, for which the $I_c(T)$-behavior could depend only on the orientation angles of both electrodes (γ and γ'), now the other parameters responsible for the superconducting and combined gaps—these are σ_0 and α—become relevant. In panels a to c of Figs. 9 and 10, the $i_c(\sigma_0, \alpha)$ dependences are shown for various temperatures $T/T_c \neq 0$ for the "checkerboard" and "unidirectional" CDW patterns, respectively (note that the critical temperature T_c is different in each case). Together with the corresponding panel in Fig. 4 plotted for the case $T = 0$, they qualitatively illustrate the temperature evolution of the $i_c(\sigma_0, \alpha)$ dependences. In order to feel it quantitatively, panel d in each of the figures concerned demonstrate the cross-section of $i_c(\sigma_0, \alpha)$ surfaces along the coordinate $\sigma_0 = 1.5$. One can see that, in the case $N = 2$, there can exist a number of doping-driven junction-type transitions at different α's, as is shown in Fig. 10d. This behavior is essentially changed by the variation of T. In contrast to the checkerboard case, the transitions

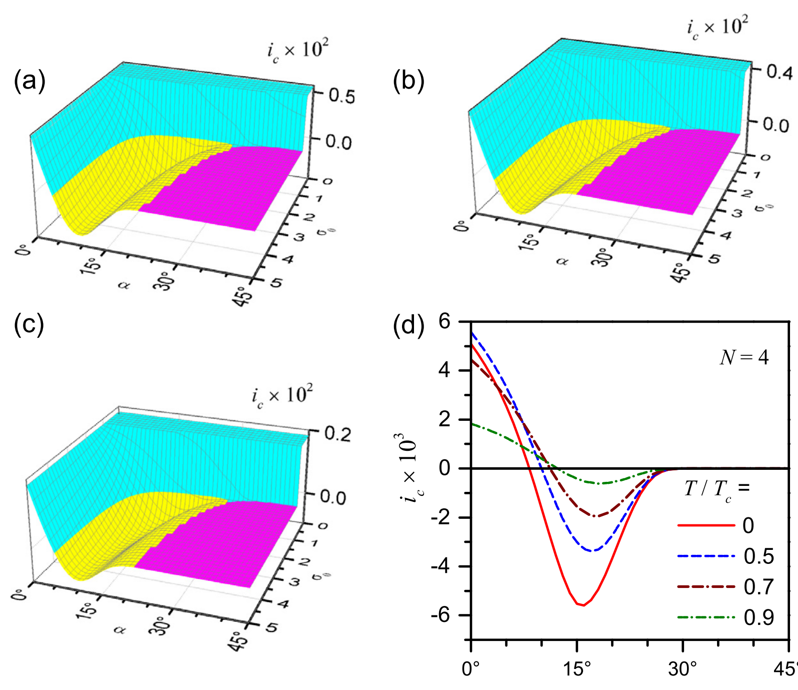

Fig. 9. The same as in Fig. 4a, but for $\tau = T/T_c = 0.5$ (a), 0.7 (b), and 0.9 (c). Panel d illustrates the temperature evolution of cross-sections $i_c(\sigma_0 = 1.3, \alpha)$.

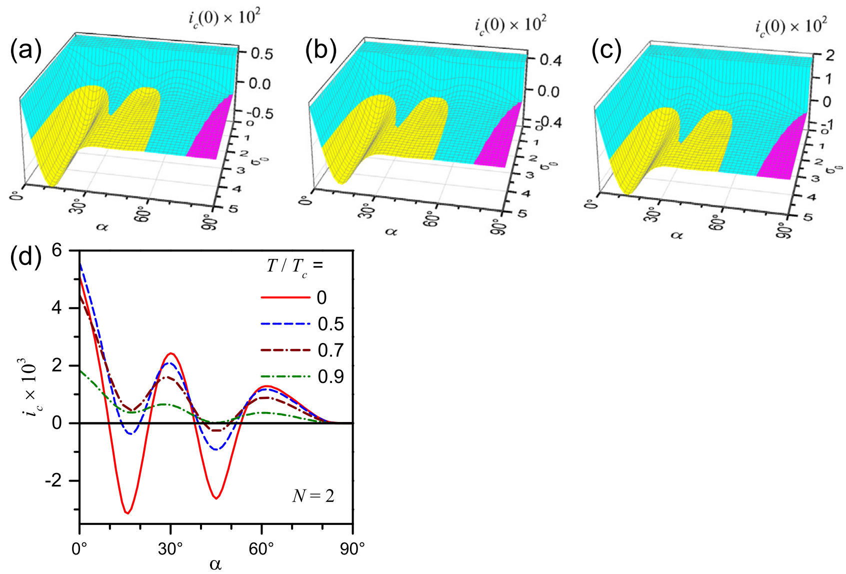

Fig. 10. The same as in Fig. 9, but for $N = 2$.

between 0- and π-junctions are eliminated at large T in CDWSs with unidirectional CDWs. Such a feature can be relatively easily traced in real experiments.

The temperature influence on the relative importance of the positive and negative current components reveals itself by means of the multiplier $\tanh\frac{x}{2T}$ in formula (11). Since the overall current is obtained by the integration over x in Eq. (10), different contributions to $I_c(T)$ (generally speaking of either sign) will be generated by different order parameters $\bar{\Delta}(T, \theta - \gamma)$, $\bar{\Delta}'(T, \theta - \gamma')$, $\overline{D}(T, \theta - \gamma)$, and $\overline{D}'(T, \theta - \gamma')$, which determine the square-root x-dependent factors in the integrand of Eq. (11) multiplied by $\tanh\frac{x}{2T}$. That is why ratios between positive and negative components will inevitably depend on the temperature. The same effect should be observed for the parent d-wave BCS superconductor [138], which, as was explained above, corresponds to the i_c-value of the plateau taking place at small σ_0's.

3.2.1. The interplay between electrode orientation and temperature effects

Similarly to what was done in Section 3.1.1, it is of interest to determine how the electrode rotation affects the temperature-induced transition between the 0- and π-states of the junction. For this purpose, in Fig. 11a, we plotted the dependences $i_c(\tau)$ for the $S^d_{\rm CDW4} - I - S^d_{\rm CDW4}$ junction with the "reference" CDWS parameters ($\sigma_0 = 1.3$ and $\alpha = 15^\circ$) for various values of orientation angle γ and the fixed $\gamma' = 45^\circ$. The latter value was selected, because we assumed the corresponding differential-detecting properties to be the best. The results of calculations showed that the temperature-induced change of the junction type can be observed within rather a narrow interval $17^\circ \lesssim \gamma \lesssim 22^\circ$.

The orientation of the other ("detecting") electrode gives even more spectacular effect on the possibility to observe the change of the junction type. The $i_c(\tau)$ dependences in Fig. 11b were plotted for the left-hand-side electrode orientation $\gamma = 20^\circ$. This value was selected by analyzing the curves in panel a as that, at which the "junction-type transition" temperature is maximally close to $T_c/2$. It is clearly seen that even the change of the "detecting" electrode orientation angle γ' by less than 1° makes the transition temperature unobservable.

The dependences $i_c(\tau)$ for the $S^d_{\rm CDW2} - I - S^d_{\rm CDW2}$ junctions analogous to those found for $S^d_{\rm CDW4} - I - S^d_{\rm CDW4}$ ones and described above are plotted in Fig. 12. One can see that the "unidirectional" CDW pattern provides a weaker sensitivity with respect to the angle γ, whereas the sensitivity to the angle γ' is as high as of its counterpart in the checkerboard case.

The doping- and temperature-induced transitions between the 0- and π-junctions can be measured, in particular, by the Rifkin–Deaver method when the examined junction is included into a

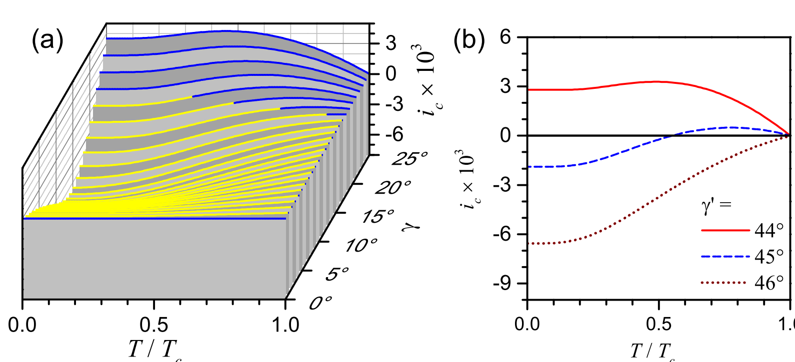

Fig. 11. (a) Influence of the left-hand-side electrode orientation angle γ on the appearance of the temperature-driven $0/\pi$-junction transition in the case $N = 4$ and for $\gamma' = 45^\circ$; (b) the same, but for the right-hand-side electrode orientation angle γ'; $\gamma = 20^\circ$.

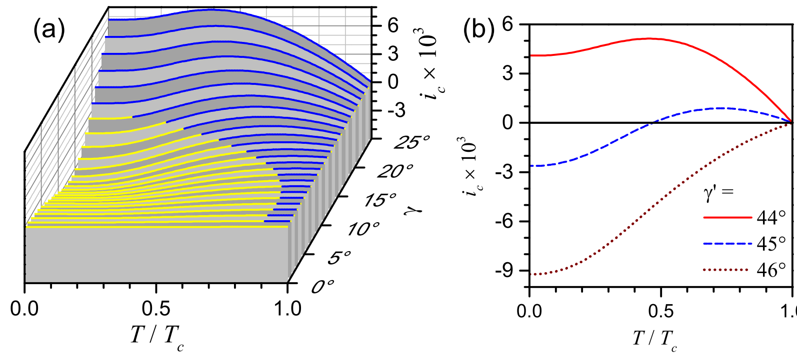

Fig. 12. The same as in Fig. 11, but for $N = 2$. In panel b, $\gamma = 16^{\circ}$.

superconducting circuit penetrated by a magnetic field [149]. The method has been successively applied to detect phase-changing effects in the $YBa_2Cu_3O_{7-\delta}$ grain-boundary Josephson junctions [145]. But even without any monitoring of the current sign, the very passage of I_c through zero while changing T or the oxygen content can serve as an indicator of the $0 - \pi$ transition.

3.3. Additional remarks

In general, the problem concerned includes too many parameters for the analysis to be carried out in detail. In particular, we did not considered the case with such a geometry (in the case $N = 2$) when the left-hand-side electrode is so oriented that the γ-values are close to 90°) and the CDW-ungapped d-lobe (in Fig. 2, those lobes are marked by "–"), owing to the directionality cone (θ_0), makes a main contribution to the total tunnel current. We believe that, since it does not contain CDW-driven gap discontinuities, the corresponding results will preserve the qualitative character of their counterparts shown above, but will be less pronounced. At the same time, such a difference between the relevant results obtained for two electrode orientations in the unidirectional case with its C_2 symmetry may serve as an additional confirmation in favor of the CDW availability.

We totally omitted the analysis concerning the influence of the directionality parameter θ_0 on the final results. It was done, because the variation of the parameters Σ_0 and α (doping) can be carried out *in situ*, so that their certain successive changes could be maintained. On the other hand, the modification of θ_0 would demand the fabrication of new junctions, and both Σ_0 and α could hardly be kept invariant. The *in situ* doping can be achieved, e.g., by postannealing samples in the O_2 atmosphere at various pressures [150]. Another doping method [151] consisting in the *in situ* deposition of potassium atoms on cleaved $YBa_2Cu_3O_{7-\delta}$ is more difficult to be applied to the junctions in question.

The suggested experiments with the *in situ* doping in order to study the junctions with identical crystal arrangements are, of course, rather cumbersome. However, if performed, they can be easily supplemented by Josephson current measurements not only at $T = 0$ but also at finite temperatures. Moreover, at some fixed electron orientations, changing the temperature alone could result in non-trivial $i_c(T)$ dependences; however, as our calculations show, the same phenomenon can be observed for pure d-wave BCS superconductors ae well. Only the complex of suggested experiments will be able to confirm the existence of CDWs in cuprates or other d-wave superconductors with a partial dielectric gapping by carrying out the measurements of the coherent superconducting current.

Another remark concerning the experimental verification of the proposed theory in the case when the variation of the parameter σ_0 or α induced by doping turns out insufficient to force the current crossing the point, at which $i_c = 0$, should be made. From Eq. (11), it is evident that when the orientations of both electrodes coincide, i.e. $\gamma = \gamma'$, all elementary contributions to the total current are positive, because the values of the parameters $\bar{\Delta}$ and $\bar{\Delta}'$ are of the same sign. Therefore, when integrating over the parameter θ, tunnel directionality can affect the total current magnitude but not the sign. This is valid for every point in the $\sigma_0 - \alpha$ plane, so that $i_c(\sigma_0, a) > 0$ over the whole plane. At the same time, if $\gamma = \gamma' + 90^{\circ}$, one obtains, on the same footing, that $i_c(\sigma_0, a) < 0$ over the phase plane (Figs. 6(b) and 8(b) may serve an illustration of this case). It is evident that, for any (σ_0, α) point, when passing from one orientation to the other by varying either γ or γ', one will undoubtedly arrive at such a relative electrode configuration, at which $i_c = 0$. In other words, by varying γ or γ', the contour of the critical-current sign-inversion can be made by the experimentalist to pass as close as required to an arbitrary point of the (σ_0, α) plane. This conclusion is valid for any $T < T_c$ and irrespective of the type of tunnel directionality realization.

4. Conclusions

To summarize, in the context of the two-dimensional picture appropriate for cuprates, we calculated the dependences of the stationary critical Josephson tunnel current I_c in junctions involving d-wave superconductors with CDWs on the temperature and the doping-dependent CDW parameters. However, other factors affecting the CDW parameters were also made allowance for. It was shown that the intertwining of the CDW and superconducting order parameters leads to a peculiar behavior of I_c, which indirectly reflects the existence of CDW gapping. We would like to emphasize that the discussed effects are rather subtle and can be observed only provided certain experimental setups. The proposed electrode configurations play the role of differential detector able to reveal the CDW existence in d-wave superconductors. The idea of differential detector can be very fruitful for studying objects in which the order parameter is sign-alternating in the momentum space. As to the specific results of this work, doping serves here as a control process to reveal the CDW manifestations, whereas the temperature variation is able to tune the differential detector. Such configurations have already been created for $YBa_2Cu_3O_{7-\delta}$ [142,143] and may be used to check the predictions of our theory.

Acknowledgements

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.physc.2015.06.014.

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