Influence of the spatially inhomogeneous gap distribution on the quasiparticle current in *c*-axis junctions involving *d*-wave superconductors with charge density waves

IOP Publishing Journal of Physics: Condensed Matter
J. Phys.: Condens. Matter **28** (2016) 445701 (10pp) doi:10.1088/0953-8984/28/44/445701

Influence of the spatially inhomogeneous gap distribution on the quasiparticle current in *c*-axis junctions involving *d*-wave superconductors with charge density waves

T Ekino[1], A M Gabovich[2], Mai Suan Li[3], H Szymczak[3] and A I Voitenko[2]

[1] Hiroshima University, Graduate School of Integrated Arts and Sciences, Higashi-Hiroshima, 739-8521, Japan
[2] Institute of Physics, Nauka Ave. 46, Kyiv 03680, Ukraine
[3] Institute of Physics, Al. Lotników 32/46, PL-02-668 Warsaw, Poland

E-mail: gabovich@iop.kiev.ua

Received 6 July 2016, revised 5 August 2016
Accepted for publication 19 August 2016
Published 8 September 2016

Abstract
The quasiparticle tunnel current $J(V)$ between the superconducting *ab*-planes along the *c*-axis and the corresponding conductance $G(V) = \mathrm{d}J/\mathrm{d}V$ were calculated for symmetric junctions composed of disordered *d*-wave layered superconductors partially gapped by charge density waves (CDWs). Here, V is the voltage. Both the checkerboard and unidirectional CDWs were considered. It was shown that the spatial spread of the CDW-pairing strength substantially smears the peculiarities of $G(V)$ appropriate to uniform superconductors. The resulting curves $G(V)$ become very similar to those observed for a number of cuprates in intrinsic junctions, e.g. mesas. In particular, the influence of CDWs may explain the peak-dip-hump structures frequently found for high-T_c oxides.

Keywords: *d*-wave superconductors, charge density waves, spatially inhomogeneous distribution, quasiparticle tunnel current

(Some figures may appear in colour only in the online journal)

1. Introduction

The coexistence of superconductivity and other states of matter arising, in particular, owing to the reconstruction of the Fermi surface (FS) sections by various instabilities has been observed in plenty of materials. Those phenomena are of interest due to both theoretical and practical reasons [1–16]. This issue is extremely significant for high-T_c oxides (oxides with a high critical temperature of the superconducting transition) because of the role played by spin density waves (SDWs) and, especially, charge density waves (CDWs), which lead to a conspicuous T_c reduction [10, 17–28]. The very existence of CDWs in various cuprates (contrary to the case of SDWs) has been considered for a long time as tentative, but now this is a well-established fact due to a large body of experiments, including direct scanning-tunnel-microscopy (STM) and x-ray diffraction one, for a number of high-T_c oxides. Among those, the most popular are hole-doped $La_{1.875}Ba_{0.125}CuO_4$ [29], $YBa_2Cu_3O_{7-\delta}$ [13, 29–38], and $Bi_2Sr_2CaCu_2O_{8+\delta}$ cuprates [13, 36, 39, 40], as well as electron-doped $Nd_{2-x}Ce_xCuO_4$ one [41]. The observations demonstrate either a unidirectional (stripe) or a bidirectional (checkerboard) configuration of two-dimensional CDWs (or even three-dimensional ones [38, 42]). Their attribution [43, 44] and mutual interplay are also a hot issue [45–47]. The violation of the C_4 symmetry and the emergence of the C_2 (nematic or smectic) charge order in cuprates compose a very interesting topic, which is intensively studied concerning not only cuprates but also other materials [37, 48–61].

A real problem consists in the 'soft' character of CDW manifestations, which is a result of the short-range or fluctuation-like CDW appearance and the crucial role of the intrinsic disorder in cuprates [62–76]. All those factors lead to wide distributions of the apparent energy gap values successfully felt by tunnel spectroscopy [68, 77–84]. It was also found that the spread of normal-state gaps (pseudogaps, which we identify as CDW-related gaps) is wider than that of their superconducting counterparts [85]. Therefore, all mean-field-like theories of CDWs in superconductors, whatever their details, should be, strictly speaking, complemented with a proper account of the disorder and fluctuation effects in order to reproduce subtle observed features attributed to CDWs (including pseudogaps, which are most probably just another incarnation of CDW gaps [1–3, 14, 86]).

In this connection, quasiparticle currents J in tunnel junctions and conductance spectra $G(V) \equiv \mathrm{d}J/\mathrm{d}V(V)$, where V is the bias voltage, are of special interest, because they reveal both superconducting and CDW-related gaps. Hence, we have made calculations of $G(V)$ in the symmetric (mesas) and nonsymmetric (STM) set-ups for superconducting electrodes made of superconductors with the d-wave symmetry of the superconducting order parameter and partially gapped by CDWs [87]. The charge ordering was considered in the mean-field approximation, and the calculations of the d-wave superconducting, Δ, and dielectric (CDW), Σ, order parameters were carried out self-consistently [88], our approach being a generalization of the Bilbro–McMillan one [20, 89] developed for s-wave superconductors. The calculated $G(V)$ dependences demonstrated a number of peculiarities reflecting the interplay between the d-wave shaped $\Delta(\mathbf{k}, T)$ and the s-wave four-sector (in the checkerboard case) or two-sector (in the stripe-like case) $\Sigma(\mathbf{k}, T)$ in the two-dimensional momentum space. Here, $\mathbf{k}$ is the wave vector and T is the temperature.

However, since so far only the fixed-gap-value case was considered [87], we did not take into account the intrinsic actual spread of the gap magnitude distributions in cuprates indicated above. In this article, we analysed this circumstance numerically. The necessity of taking into account the statistical scatter of CDW-gap values to fit the quasiparticle tunnel characteristic $G(V)$ was understood earlier for partially-gapped CDW conductors other than cuprates [90, 91].

Both checkerboard and unidirectional CDWs were treated theoretically, the results being qualitatively similar but quantitatively different. We note that the problem of the actual superconducting order parameter symmetry in cuprates remains unsolved [92, 93], so that the phase-insensitive quasiparticle current–voltage characteristics might serve only as an indirect, although important, evidence. On the contrary, the in-plane Josephson tunnelling may not only reveal a true nature of the superconducting order parameter symmetry, but also find the features related to charge-order FS gapping [94].

2. Theoretical description

2.1. Basic equations

For d-wave superconductors with CDWs, we adopt the theoretical model suggested earlier [3, 88, 94, 95]. This model, in agreement with the large body of experimental data for cuprates, treats the superconducting order parameter as a Bardeen–Cooper–Schrieffer-like (BCS-like) $d_{x^2-y^2}$ one, whereas the CDW order parameter is considered as the s-wave one distorting only certain sections of the FS below the critical temperature T_s, which is, as a rule, substantially higher than T_c. This distorted part of the FS may comprise $N = 4$ (bidirectional CDWs) or 2 (unidirectional CDWs) sectors symmetrically arranged in the k-space (figure 1). The emergence of those two variants in cuprates testifies (along with other evidence [3, 14, 96]) that CDWs and superconductivity are of different origins and the CDW (electron–hole) pairing in high-T_c oxides most probably has the s-wave symmetry, rather than the d-wave one assumed in other approaches [58, 97–100].

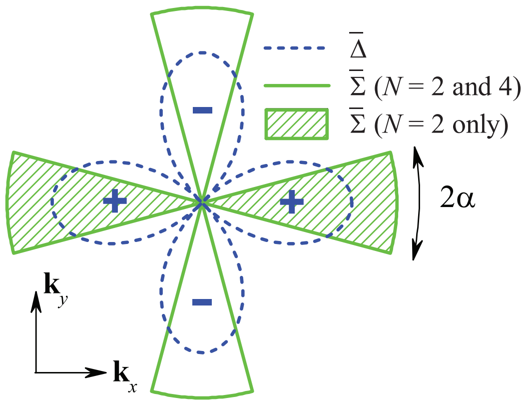

Figure 1. Momentum-space profiles of the parent order parameters of the d-wave superconductor partially gapped by charge-density waves (CDWs). Here, N is the number of CDW sectors describing the CDW configuration ($N = 4$ for the checkerboard and $N = 2$ for the unidirectional one), and 2α is the angular width of each CDW sector. See further explanations in the text.

The mean-field Hamiltonian has the form

$$H = H_{\mathrm{kin}} + H_{\mathrm{BCS}} + H_{\mathrm{CDW}}, \quad (1)$$

where

$$H_{\mathrm{kin}} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} \sum_{i=d,nd} \xi_i(\mathbf{k}) a^{\dagger}_{i,\mathbf{k},\sigma} a_{i,\mathbf{k},\sigma}, \quad (2)$$

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}} \bar{\Delta}(\mathbf{k}) \sum_{i=d,nd} a^{\dagger}_{i,\mathbf{k},\uparrow} a^{\dagger}_{i,-\mathbf{k},\downarrow} + \mathrm{h.c.}, \quad (3)$$

$$H_{\mathrm{CDW}} = \sum_{\mathbf{k},\sigma=\uparrow,\downarrow} \sum_{i=d} \bar{\Sigma}(\mathbf{k}) a^{\dagger}_{i,\mathbf{k},\sigma} a_{i,\mathbf{k}+\mathbf{Q},\sigma} + \mathrm{h.c.} \quad (4)$$

Here, $\xi_i(\mathbf{k})$ is the initial quasiparticle spectrum; the d-wave superconducting momentum-dependent order parameter, $\bar{\Delta}(\mathbf{k})$, and the dielectric one, $\bar{\Sigma}(\mathbf{k})$, are described below; $a^{\dagger}$ and a are the creation and annihilation operators, respectively; σ is the quasiparticle spin projection, $\mathbf{Q}$ is the CDW vector, and the notations 'd' and 'nd' indicate the CDW-gapped and CDW-non-gapped FS sections, respectively (to distinguish from the superconducting gapping, we dub the CDW gapping as dielectrization).

In one of the parent—the partially dielectrized normal metal—state, the CDW complex order parameter $\Sigma_0(T)\mathrm{e}^{\mathrm{i}\varphi}$ (here, φ is the CDW phase) is $\mathbf{k}$-independent inside any of N CDW-gapped sectors, directed along the $\mathbf{k}_x$ and $\mathbf{k}_y$ axes (figure 1). Each of the sectors spans the angle 2α. In the adopted BCS-like theory [2, 20, 89], the order parameter magnitude equals

$\Sigma_0(T=0)=\frac{\pi}{\gamma}T_{s0}$, where $\gamma=e^C=1.78\ldots$, $C=0.577...$ is the Euler constant, e is the base of natural logarithm, the Boltzmann constant $k_B=1$, and T_{s0} is the corresponding critical temperature, because any intertwining influence of the Cooper pairing is absent in the pure CDW state. In our model, the dielectric order parameter has the conventional *s*-wave Mühlschlegel temperature dependence

$$\Sigma_0(T)=\Sigma_0(0)\mathrm{Mü}_s(T/T_{s0}) \tag{5}$$

where $\mathrm{Mü}_s(x)$ is the reduced ($\mathrm{Mü}_s(0)=1$) factor of this dependence. It should be noted that in CDW-gapped materials, the ratio $\Sigma_0(T=0)/T_{s0}$ is, as a rule, unexpectedly larger than the *s*-BCS value $\frac{\pi}{\gamma}$ [101–104], although the relation $\Sigma_0(T)/\Sigma_0(T=0)=\mathrm{Mü}_s(T/T_{s0})$ preserves [101, 102, 105, 106]. This fact has not been explained by the Migdal–Eliashberg-like strong-coupling corrections (more rigorously calculated for superconductors [107]) to the CDW mean-field picture [108, 109]. Although our approach is phenomenological, in order to lean upon basic theoretical concepts, we will assume the weak-coupling *s*-wave BCS value $\frac{\pi}{\gamma}$ for CDWs. The angle θ in the two-dimensional **k**-plane is reckoned from the $\mathbf{k}_x$ axis. The profile $\bar{\Sigma}_0(T,\mathbf{k})$, or $\bar{\Sigma}_0(T,\theta)$, over the whole FS contains the factor $f_\Sigma(\theta)$, which is equal to 1 inside and 0 outside each sector. Then, $\bar{\Sigma}_0(T,\theta)$ can be presented as follows

$$\bar{\Sigma}_0(T,\theta)=\Sigma_0(T)f_\Sigma(\theta). \tag{6}$$

The other parent—BCS $d_{x^2-y^2}$-wave superconductor (dBCS) [110]—state is characterized by the superconducting order parameter $\Delta_0(0)$ at $T=0$ and $T_{c0}=\frac{\gamma\sqrt{e}}{2\pi}\Delta_0(0)$. The order parameter lobes are oriented in the $\mathbf{k}_x$ and $\mathbf{k}_y$ directions, i.e. in the same (antinodal) directions as the bisectrices of CDW sectors (figure 1). The profile $\bar{\Delta}_0(T,\theta)$ in the **k**-space spans the whole FS. It has the form

$$\bar{\Delta}_0(T,\theta)=\Delta_0(T)f_\Delta(\theta), \tag{7}$$

with the angular factor

$$f_\Delta(\theta)=\cos 2\theta. \tag{8}$$

Here,

$$\Delta_0(T)=\Delta_0(0)\mathrm{Mü}_d(T/T_{c0}), \tag{9}$$

where $\mathrm{Mü}_d(x)$ is the reduced ($\mathrm{Mü}_d(0)=1$) *d*-wave superconducting order parameter dependence [110].

In our case, when CDWs and superconductivity coexist and compete (we call this state SCDW), the order parameter dependences $\Sigma(T)$ and $\Delta(T)$ differ from those appropriate to the pure phases, i.e. $\Sigma_0(T)$ and $\Delta_0(T)$, respectively [88, 96]. The resulting set of equations, which determines the order parameters $\Sigma(T)$ and $\Delta(T)$ for the given input model parameters ($\Delta_0(0)$, $\Sigma_0(0)$—for brevity, they are denoted below as Δ_0 and Σ_0, respectively—α, and N) has the form obtained earlier [3, 87, 88, 96]. Those equations, which should be solved self-consistently, are given below for completeness:

$$\int_{-\alpha}^{\alpha} I_M(\sqrt{\Sigma^2+\Delta^2\cos^2 2\theta}, T, \Sigma_0)\mathrm{d}\theta=0, \tag{10}$$

$$\int_{-\alpha}^{\alpha} I_M(\sqrt{\Sigma^2+\Delta^2\cos^2 2\theta}, T, \Delta_0\cos 2\theta)\cos^2 2\theta\mathrm{d}\theta + \int_{\alpha}^{\Omega-\alpha} I_M(\Delta\cos 2\theta, T, \Delta_0\cos 2\theta)\cos^2 2\theta\mathrm{d}\theta=0, \tag{11}$$

Here

$$I_M(\Delta,T,\Delta_0)=\int_0^\infty\left(\frac{1}{\sqrt{\xi^2+\Delta^2}}\tanh\frac{\sqrt{\xi^2+\Delta^2}}{2T} - \frac{1}{\sqrt{\xi^2+\Delta_0^2}}\right)\mathrm{d}\xi \tag{12}$$

is the Mühlschlegel integral of the conventional (*s*-wave) BCS theory. The angle Ω equals π for $N=2$ and $\frac{\pi}{2}$ for $N=4$. Due to the order parameter interplay, the lowest of the parent critical temperatures, T_{c0} or T_{s0}, is suppressed by the competing pairing, so that the actual critical temperatures become $T_c<T_{c0}$ or $T_s<T_{s0}$. As for high-T_c oxides and other existing CDW superconductors, the observed superconducting critical temperatures are lower than their dielectric counterparts: $T_c<T_s$ [2, 3, 91]. Of course, in our calculations we can choose the ratio between the input parameters T_{c0} and T_{s0} arbitrary, so that the relationship between resulting T_c and T_s may vary over a wide range even to the point of the reentrance behaviour within a certain interval $0<T_r<T<T_s$ [3, 88, 96]. We remind that our approach is a mean-field one, whereas a more complicated consideration, involving a certain preemptive CDW order, can ensure the actual (theoretical) inequality $T_c<T_s$, although the mean-field temperatures obey the opposite condition $T_s<T_c$ [57]. In our case, 'mean-field' means 'actual', so that all the input parameters, including N, are phenomenological and should be reconciled with the experiment if one wants to calculate any derivatives, such as the quasiparticle current–voltage characteristics.

As a consequence of the order parameter intertwining, a combined gap of the mixed origin and no definite symmetry,

$$D_d(T,\theta)=\sqrt{\Sigma^2(T)+\bar{\Delta}^2(T,\theta)}, \tag{13}$$

appears on the dielectrized (*d*) FS sections. At the same time, the 'pure superconducting' gap $D_{nd}(T,\theta)=\left|\bar{\Delta}(T,\theta)\right|$ exists on the nd ones. Using the angular factors $f_\Sigma(\theta)$ and $f_\Delta(\theta)$, the resulting gap over the whole FS (the gap rose) can be written in the universal form

$$\bar{D}(T,\theta)=\sqrt{\bar{\Sigma}^2(T,\theta)+\bar{\Delta}^2(T,\theta)}, \tag{14}$$

where

$$\bar{\Delta}(T,\theta)=\Delta(T)f_\Delta(\theta), \tag{15}$$

$$\bar{\Sigma}(T,\theta)=\Sigma(T)f_\Sigma(\theta). \tag{16}$$

2.2. Quasiparticle current. Fixed order parameters

Here, we consider quasiparticle tunnelling along the *c*-axis, i.e. between the superconducting planes of two cuprate crystals suggested to be partially CDW-gapped d-wave SCDWs. Such configurations can be realized in mesas [15, 111], twist-crystal structures made of bicrystals [112], the artificial cross-whiskers [113], or the natural cross-whiskers [114]. High-T_c oxide break junctions can also be intentionally produced to ensure tunnelling along the c-axis [115].

The quasiparticle tunnel current J is a functional of the product $G_1G_2\,G_1G_2$ [116], where G_i means the normal Green function of the ith d-wave SCDW electrode. We assume the strongly incoherent tunnelling in the c-direction between the electrodes, which is supported by the experimental evidence for the Josephson current [92], as was discussed by us earlier for the situation when there is no distribution of gap values [87].

Bearing all the aforesaid in mind, the formulas for the quasiparticle tunnel current can be obtained in the conventional way [116, 117], which was applied to the s-wave and d-wave SCDW cases and was described in more detail elsewhere [2, 118–122]. In line with the previous treatments, the phenomenological tunnel-Hamiltonian approach was adopted [123]. The ultimate formula to calculate CVCs for the quasiparticle incoherent tunnel current through a junction along the c-axis between the a-b facets of two SCDWs (the mesa-like set-up) looks like

$$J(V) = \frac{1}{2(2\pi)^2 eR}\int_{-\pi}^{\pi}\mathrm{d}\theta\int_{-\pi}^{\pi}\mathrm{d}\theta'\int_{-\infty}^{\infty}\mathrm{d}\omega \times K(\omega,V,T)P(\omega,\theta)P'(\omega-eV,\theta'). \quad (17)$$

Here,

$$K(\omega,V,T) = \tanh\frac{\omega}{2T} - \tanh\frac{\omega-eV}{2T}, \quad (18)$$

the P-factors describe SCDW electrodes, and all the barrier properties were incorporated into the single constant R describing the junction resistance in the normal state. The latter parameter includes, in particular, modifications associated with the directionality of tunnelling, This can be done, because this factor is identical for all combinations of FS segments in the counter electrodes [87]. The primed quantities in equation (17) are associated with the electrode that the potential V is applied to (the V-electrode); its counter electrode will be referred to as 0-electrode. In particular, for the 0-electrode,

$$P(\omega,\theta) = \frac{\Theta(|\omega| - \bar{D}(T,\theta))}{\sqrt{\omega^2 - \bar{D}^2(T,\theta)}} \times[|\omega| + \operatorname{sign}\omega\,\cos\varphi\,\bar{\Sigma}(T,\theta)], \quad (19)$$

where $\Theta(x)$ is the Heaviside step-function, the CDW phase φ is usually pinned by the junction interface and acquires the values 0 or π (see discussion in [2, 3]), and $\bar{D}(T,\theta)$ and $\bar{\Sigma}(T,\theta)$ are the gap and CDW-order-parameter profiles on the Fermi surface described by formulas (14) and (16), respectively. For $P'(\omega - eV,\theta')$, ω in formula (19) has to be changed to $\omega - eV$, and all other parameters but T have to be primed, i.e. associated with the V-electrode. The peculiar term in the brackets of equation (19) is generated by the electron–hole-pairing Green function G_{ib}, which is dubbed 'normal' because it is proportional to the product $c_l^\dagger c_r$ [124, 125]. However, this term can be also called 'anomalous', since it contains the CDW order parameter as a factor in the same way as the Gor'kov Green function F is proportional to the factor Δ, i.e. a superconducting order parameter parameter [116, 117]. Here, the subscripts l and r correspond to two different nested FS sections.

2.3. Account of disorder

The clear-cut d-wave superconducting coherent peaks obtained at some feature points of the CVC conductances $G(V)$ calculated according to equation (17) are very strong [87] as compared to the experimental, more subtle dip-hump structures [126–129]. A possible solution of this discrepancy consists in taking into account the existing spatial non-homogeneity of SCDWs [66–75].

Formula (17) was obtained for the tunnel current between two uniform SCDW electrodes. The parameter set (Δ_0, Σ_0, α, and N) for the 0-electrode and the corresponding set for the V-one determine the dependences $\bar{D}(T,\theta)$, $\bar{\Sigma}(T,\theta)$ and $\bar{D}'(T,\theta)$, $\bar{\Sigma}'(T,\theta)$ in the integrand multipliers $P(\omega,\theta)$ and $P'(\omega - eV,\theta')$, respectively (see equation (19)), thus specifying the current value at a given voltage V. Although in this work we consider CVCs for symmetric tunnel junctions, this symmetricity turns out to be only nominal for the calculation procedure. We simulated the spatial inhomogeneity in each electrode as an independent statistical distribution of individual model parameters (except for N; for obvious reasons, this parameter was considered identical for both electrodes). In such a way, each electrode was represented as a statistical ensemble of domains with relevant 'parent' (Δ_0, Σ_0, α) and 'actual' (Δ, Σ, α; see equations (10) and (11)) parameter sets. The total current was calculated as a weighted sum of current cross-contributions between the electrode domains. In effect, almost every contribution was a current through a corresponding non-symmetric junction.

In the general case, the required averaging of the current $J(V)$ (see equation (17)) in the framework of the described model for the spatial electrode inhomogeneity would demand the calculation of a six-fold integral: one integration for each model parameter of each electrode. However, in accordance with the experimental data for $Bi_2Sr_2CuO_{6+x}$ [85], we suggest that the disorder can be modelled as a spread of only the parameter Σ_0, which means that CDW-gap amplitudes Σ and Σ' vary in space against the superconducting pairing strength effectively described by the identical parameter Δ_0 in both electrodes (the junction is supposed to be nominally symmetric), which is selected as the reference value. Indeed, first of all, according to the self-consistent coupling between the 'actual' Δ and Σ parameters expressed by the system of equations (10) and (11), the variation of either of them inevitably results in the variation of the other. Second, no precise information exists so far concerning the spread of the FS dielectrization degree in those objects. Hence, we selected this parameter in both electrodes fixed, identical ($\alpha = \alpha'$), and close to available data. Those assumptions allowed us to analyse the spatial-inhomogeneity effect by varying only a single dimensionless model parameter $\sigma_0 = \Sigma_0/\Delta_0$.

The calculations were carried out similarly to work [120]. However, as was explained above, the current $J(V)$, which is

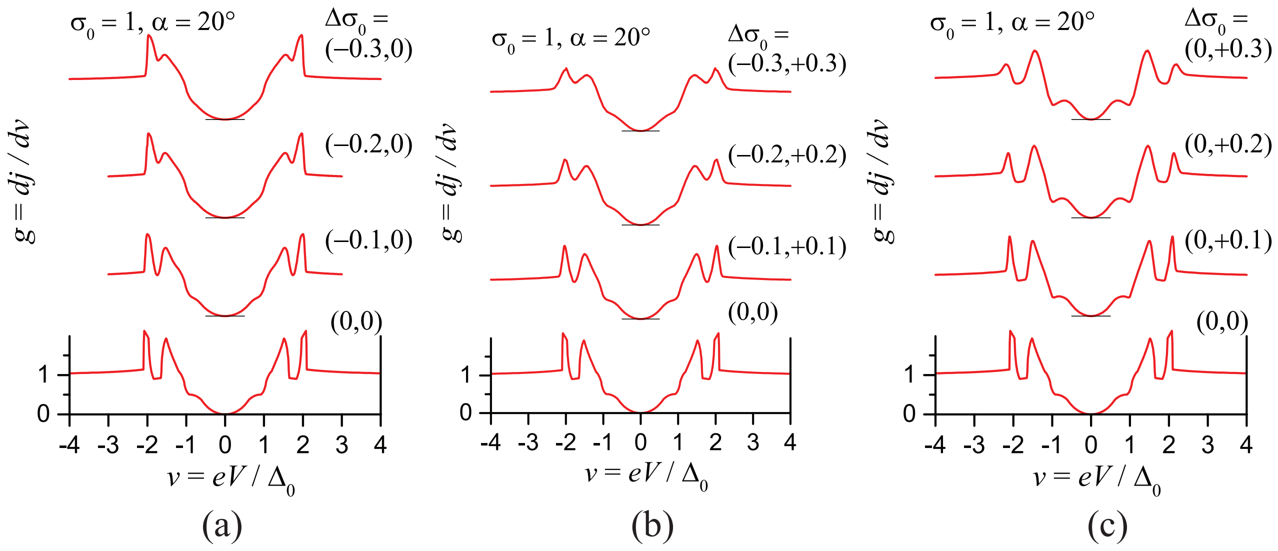

Figure 2. Dimensionless conductance $g(v) = \mathrm{d}j/\mathrm{d}v$, where $v = eV/\Delta_0$ is the dimensionless bias voltage, $j = eRJ/\Delta_0$ dimensionless tunnel current, e the elementary charge, and Δ_0 the superconducting order parameter magnitude for the parent d-wave state, for symmetric tunnel junctions with spatially inhomogeneous CDW superconductors characterized by various spreads $\Delta\sigma_0$ of the peak value σ_0. Here, $\sigma_0 = \Sigma_0/\Delta_0$, Σ_0 is the CDW order parameter magnitude for the parent CDW normal state. The specific calculation parameters are: $N = 4$, $\sigma_0 = 1$, $\alpha = 20°$, and the temperature $T = 0$.

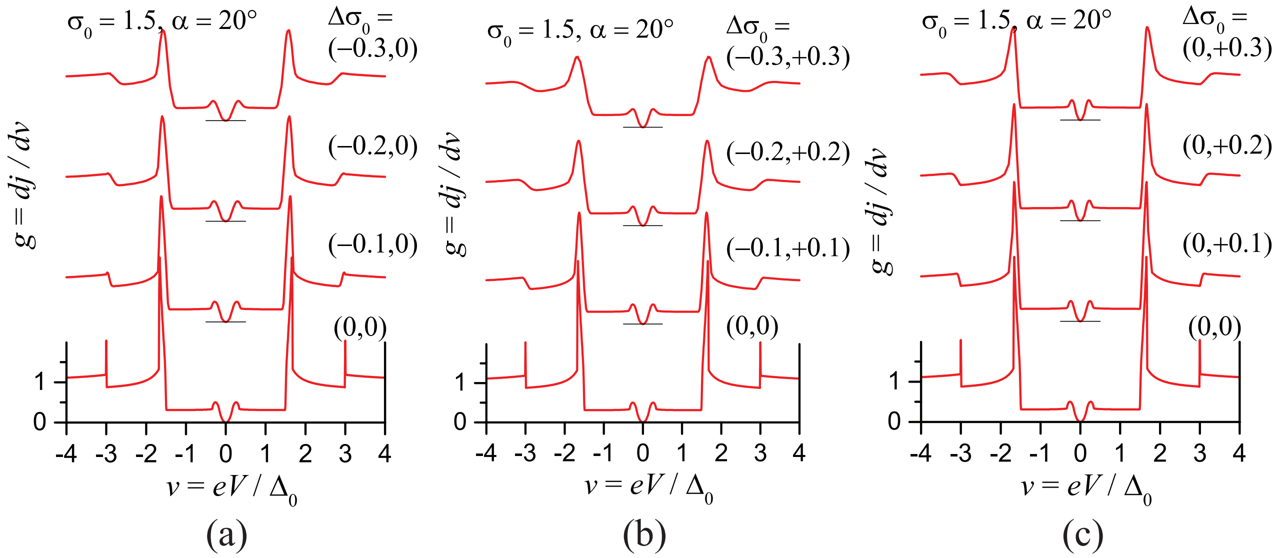

Figure 3. The same as in figure 2, but for $N = 4$, $\sigma_0 = 1.5$, $\alpha = 20°$, and $T = 0$.

now better to be rewritten in the form $J(V, \sigma_0, \sigma_0')$, had to be averaged over the interval $[\sigma_0 - \delta_-, \sigma_0 + \delta_+]$ for the σ_0 parameters in both electrodes,

$$\langle J \rangle = \frac{1}{Z} \int_{\sigma_0-\delta_-}^{\sigma_0+\delta_+} \int_{\sigma_0-\delta_-}^{\sigma_0+\delta_+} \mathrm{d}\sigma_0 \mathrm{d}\sigma_0' \times w(\sigma_0) w(\sigma_0') J(V, \sigma_0, \sigma_0'), \tag{20}$$

where

$$Z = \int_{\sigma_0-\delta_-}^{\sigma_0+\delta_+} \int_{\sigma_0-\delta_-}^{\sigma_0+\delta_+} \mathrm{d}\sigma_0 \mathrm{d}\sigma_0' \, w(\sigma_0) w(\sigma_0'). \tag{21}$$

The non-normalized (this is a reason why the normalizing denominator Z appears in equation (20)) asymmetric bell-shaped distribution weight function

$$w(\sigma) \sim \begin{cases} \left[\left(\dfrac{\sigma - \sigma_0}{\delta_-}\right)^2 - 1\right]^2 & \text{if } \sigma_0 - \delta_- \leqslant \sigma \leqslant \sigma_0, \\ \left[\left(\dfrac{\sigma - \sigma_0}{\delta_+}\right)^2 - 1\right]^2 & \text{if } \sigma_0 < \sigma \leqslant \sigma_0 - \delta_+ \end{cases} \tag{22}$$

was selected to describe the inhomogeneous spatial distribution of the parameter σ_0 in both electrodes. In such a way, we left room for non-symmetric scatter of the parameter σ_0, so that any wing of the distribution, $\delta_\pm$, corresponding to smaller or larger deviations from the mean value can dominate. For instance, asymmetric pseudogap (CDW gap) distributions were found

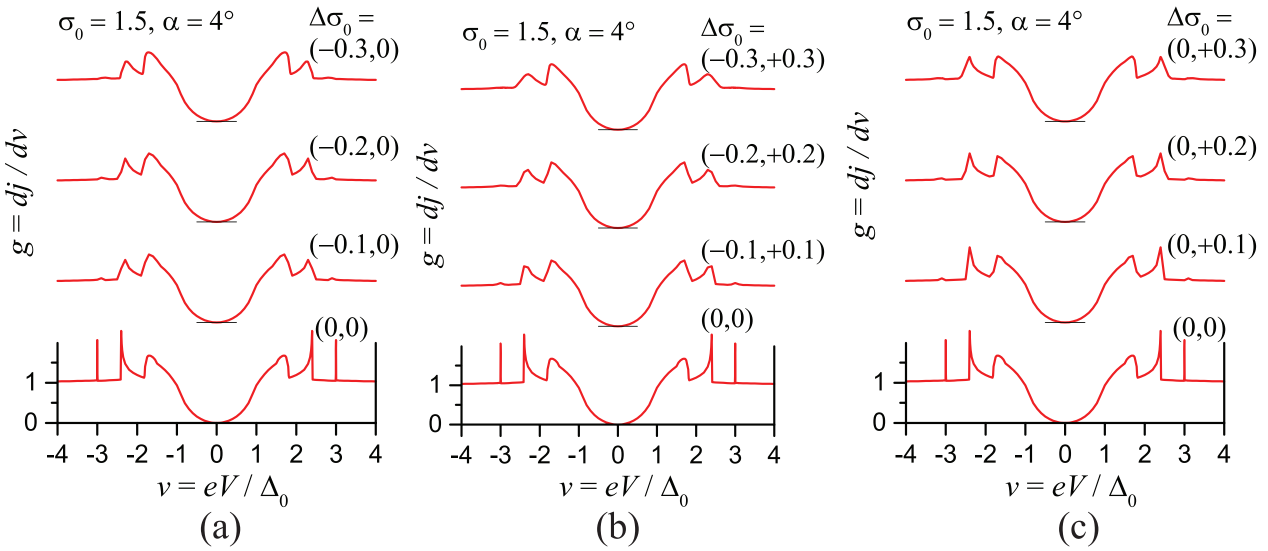

Figure 4. The same as in figure 2, but for $N = 4$, $\sigma_0 = 1.5$, $\alpha = 4°$, and $T = 0$.

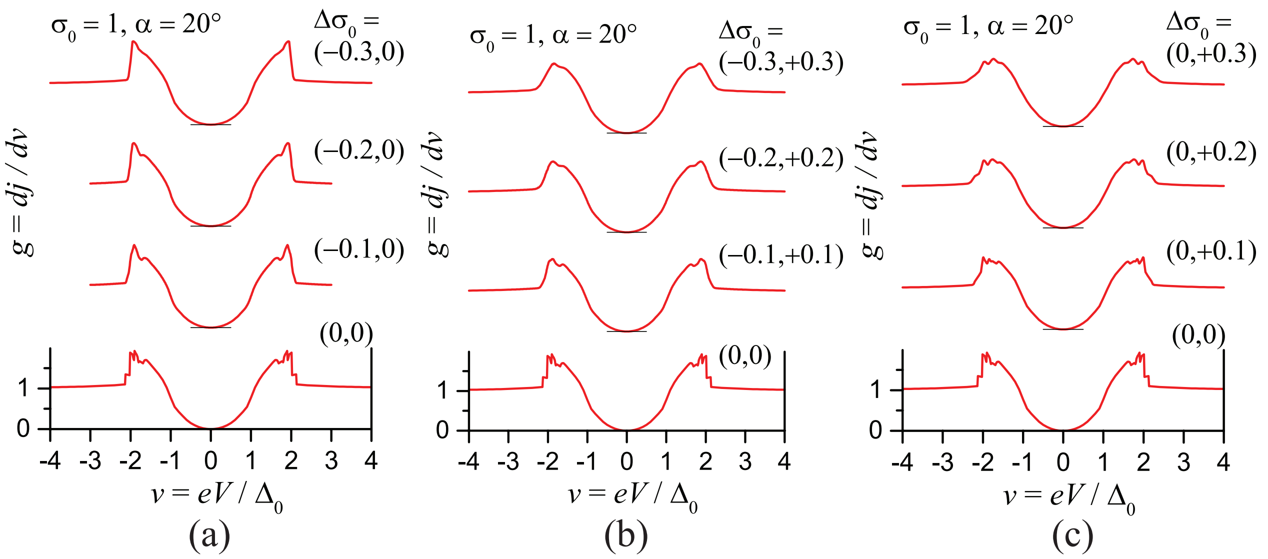

Figure 5. The same as in figure 2, but for $N = 2$, $\sigma_0 = 1$, $\alpha = 20°$, and $T = 0$.

for overdoped [80] and underdoped [79] $Bi_2Sr_2CaCu_2O_{8+\delta}$, as well as overdoped $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ [82]. Strictly speaking, to make our results more illustrative, below we analyse the limiting distributions in which only one of the branches, i.e. $\sigma < \sigma_0$ or $\sigma > \sigma_0$, is taken into consideration.

The both double integrals in equations (20) and (21) were calculated synchronously using the Monte Carlo method.

Hereafter, the temperature is considered to equal zero, so that the spatial averaging was carried out and analysed without any interference of the inevitable thermal smearing, which, nonetheless, becomes negligible for high-T_c cuprates usually studied at $T = 4.2$ K. Only symmetric configurations were treated as the model of intrinsic tunnel junctions, realized, e.g. in mesas [127–132]. A simpler case of non-symmetric junctions was investigated earlier [95]. It should be noted that the disorder intrinsic to cuprates and modelled here is, most probably, connected to the distribution of oxygen atoms [133, 134]. At the same time, the intentional replacement of La by Eu in $Bi_2Sr_{1.7}R_{0.3}CuO_{6+\delta}$ (R-Bi2201, R = La or Eu) [76] leads to an additional out-of-plane disorder, which substantially influences the checkerboard CDW ordering there. This effect, being very interesting *per se*, is not covered by our approach. On the other hand, the pseudogap features in the resistivity are the same in pristine $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals and $Bi_2Sr_2CaCu_2O_{8+\delta}$ intercalated by heavy $HgBr_2$ molecules, although the absolute values of c-axis tunnel resistance grow by a factor of 20 in intercalated samples [135]. Thus, the out-of-plane disorder seems to be not so crucial for CDWs, which lead to pseudogaps according to our interpretation.

3. Results of calculations

In this section, we present the result obtained for the dimensionless conductance $g(v) = \mathrm{d}j/\mathrm{d}v$, where $v = eV/\Delta_0$ is the

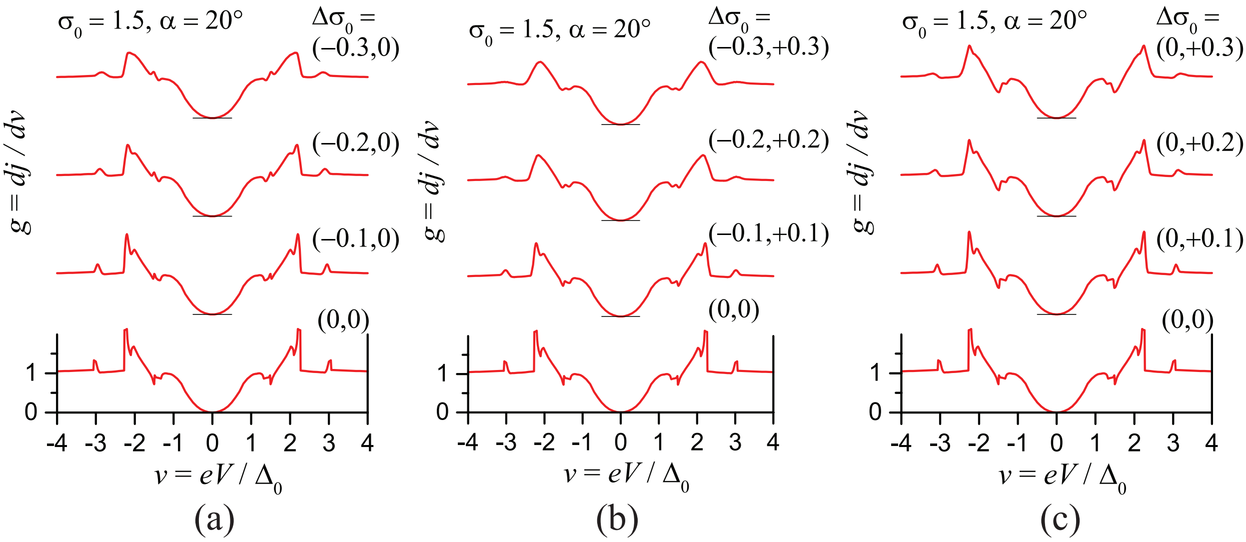

Figure 6. The same as in figure 2, but for $N = 2$, $\sigma_0 = 1.5$, $\alpha = 20°$, and $T = 0$.

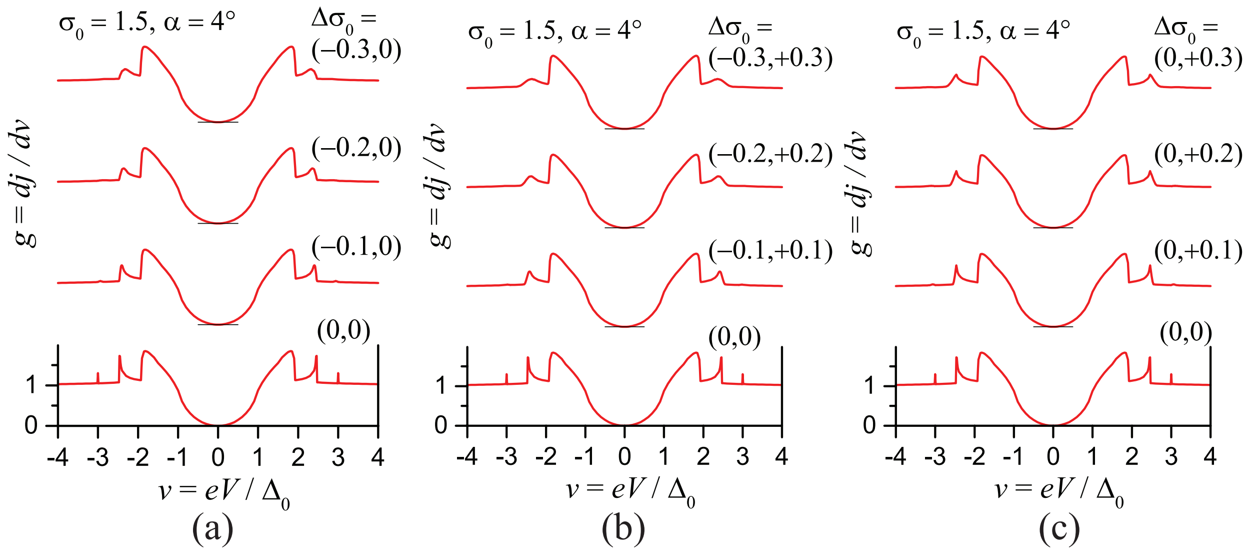

Figure 7. The same as in figure 2, but for $N = 2$, $\sigma_0 = 1.5$, $\alpha = 4°$, and $T = 0$.

dimensionless bias voltage and $j = eRJ/\Delta_0$ the dimensionless tunnel current, using the numerical procedure described in [87].

3.1. Checkerboard structures

Let us start with the checkerboard CDW structures ($N = 4$), where the crystal lattice symmetry is not lost below the dielectric phase transition point. The evolution of CVCs driven by nonhomogeneous spatial distributions of the parameter σ_0 is shown in figure 2 for $\alpha = 20°$, the peak value $\sigma_0 = 1$, and the varying lower, δ_-, and upper, δ_+, limits. Panels (a) and (c) describe the asymmetric distributions, whereas panel (b) corresponds to the symmetric one, when $\delta_- = \delta_+$. One sees that the disorder influences the CVC significantly for all kinds of distributions, but the overall appearance of the CVCs for the chosen parameter sets remains the same and is similar to that observed for $Bi_2Sr_2CaCu_2O_{8+\delta}$ [128, 130, 131]. The main difference between the CVCs for ordered and disordered structures consists in the smearing of Σ-driven peculiarities.

One should pay attention that, in this figure, as well as in figures below, the CVC in panel (b) cannot be obtained as a normalized sum of CVCs in panels (a) and (c). It is so because in the currents shown in panels (a) and (c), the contributions between the 'SCDW domains' with, e.g. $\sigma < \sigma_0$ in the 0-electrode and $\sigma > \sigma_0$ in the V-one, are absent. Nevertheless, we believe that such a selection of the distribution function for σ_0 in panels (a) and (c) will help to compare and better understand the effect of σ_0 spread in either direction.

The next set of parameters includes a more realistic, at first glance, central σ_0 value, namely $\sigma_0 = 1.5$, but the same large intervals of CDW existence. Such a ratio between the parent coupling strengths Σ_0 and Δ_0 reflects the fact that pseudogaps appear in cuprates at temperatures far above the superconducting domes [14, 136]. The results of calculations are presented in figure 3. It is readily seen that in this case superconductivity is substantially suppressed and survives only in a narrow vicinity near $eV = 0$. Of course, such patterns are possible, in principle, but were not found for examined cuprates.

Hence, we made calculations for $\sigma_0 = 1.5$, $\alpha = 4°$ and various disorder values. The results are shown in figure 4. The CVCs seem to describe well experimental curves with an *U*-like behaviour at small *eV*'s and peak-dip-hump structures, which are found in tunnel measurements for a good many cuprates cuprates [126, 137–140].

3.2. Unidirectional CDWs

In high-T_c oxides, there are many observations of the C_4 symmetry breaking, and the appearance of the nematic- or smectic-like CDW structures (stripes) [10, 22]. Hence, we made CVC calculations for $N = 2$ in the framework of the same approach. In figure 5, the dimensionless conductances $g(V)$ are shown for $\alpha = 20°$and the peak value $\sigma_0 = 1$. It comes about that, in the case of stripes, the influence of CDWs is minor as compared to that for $N = 4$ (figure 2), and the spatial averaging over Σ distributions leads to the complete disappearance of the CDW-dominated features, conspicuous for $\delta_- = \delta_+ = 0$.

The CVCs for $\sigma_0 = 1.5$ and $\alpha = 20°$ (figure 6) turn out to be quite realistic and resemble the experimental data [128, 130, 131]. Therefore, a transition from C_4 to C_2 symmetry of the dielectric order parameter qualitatively changes the $G(V)$ patterns. It means that the most typical observed CVCs in SCDW junctions may be interpreted either as a consequence of similar strengths of the electron–hole and Cooper pairings ($\Delta_0 \approx \Sigma_0$) and the development of checkerboard CDWs, or as the electron–hole pairing domination in the case of CDW stripe structures ($N = 2$).

Finally, we calculated CVCs for $\sigma_0 = 1.5$, $\alpha = 4°$, $N = 2$, and various kinds of spatial disorder (figure 7). The results are similar to those for the checkerboard CDW configurations, but with conspicuously smaller peak-dip-hump structures. Similar CVC profiles were observed for certain underdoped samples of $Bi_2Sr_2CaCu_2O_{8+\delta}$ [141].

4. Conclusions

In the model of partial electron spectrum gapping by CDWs, the quasiparticle tunnel currents along the *c*-axis between the conducting planes of two-dimensional *d*-wave superconductors have been considered. The intrinsic spatial disorder of the dielectric order parameter in the electrodes was taken into account. Both the checkerboard and stripe-like configurations of CDWs were examined. It was shown that the influence of disorder substantially smears the peculiarities of the tunnel conductance $G(V)$. The calculated smoothed CVCs reproduce well the experimental results for $Bi_2Sr_2CaCu_2O_{8+\delta}$ with their extra large or small features attributed to pseudogaps (CDW gaps in our interpretation). The results can be applied to different cuprates with their CDW manifestations as well as to other layered conventional and unconventional superconductors [142], such as dichalcogenides [143–147].

For instance, CVCs for electron-doped oxides $Nd_{2-x}Ce_xCuO_4$ [41], in which a charge order was revealed by x-rays, should also be measured in order to find peculiarities at high *eV*'s as a smoking gun of CDWs. At the same time, the superconducting order parameter in $Nd_{2-x}Ce_xCuO_4$ may possess, as comes about from tunnel experiments [148], the *s*-wave symmetry rather than the *d*-wave one assumed in our model. The same conclusion concerning $Nd_{2-x}Ce_xCuO_4$ was made later on the basis of various experimental studies [149]. In this case, our previous theory of CVCs in CDW-gapped isotropic superconductors could be applied with necessary modifications specifying the form and number of CDW sectors in the momentum space [120, 122, 138].

As for superconducting dichalcogenides, they remain a useful testing ground for various models of CDW formation [142, 150–154]. Nevertheless, one may hope that the phenomenological approach adopted here and going back to Bilbro and McMillan [89] (see also [155–157]) will be useful whatever the microscopic background of the CDW state. Therefore, CVCs in the mixed state with the coexisting dielectric and superconducting order parameters can be studied in the same manner as was described here taking into account the inhomogeneity effects smearing CVC peculiarities. The dielectric gapping has specific features in various dichalcogenides. In particular, a node was observed in the CDW gap for 2 H-TaS_2 [143], which may indicate a CDW-pairing symmetry different from the *s*-one appropriate to our model presented here and applied to cuprates. Hence, natural extensions of our approach may be needed to describe other CDW layered superconductors.

Acknowledgments

The work was partially supported by the Project N 24 of the 2015–2017 Scientific Cooperation Agreement between Poland and Ukraine. MSL also acknowledges the Polish NCN grant 2015/19/B/792 ST4/02721.

References

[1] Gabovich A M, Voitenko A I, Annett J F and Ausloos M 2001 *Supercond. Sci. Technol.* **14** R1
[2] Gabovich A M, Voitenko A I and Ausloos M 2002 *Phys. Rep.* **367** 583
[3] Gabovich A M, Voitenko A I, Ekino T, Suan L M, Szymczak H and Pekała M 2010 *Adv. Condens. Matter Phys.* **2010** 681070
[4] Platt C, Hanke W and Thomale R 2013 *Adv. Phys.* **62** 453
[5] Kamusella S, Doan P, Goltz T, Luetkens H, Sarkar R, Guloy A and Klauss H-H 2009 *J. Phys.: Conf. Ser.* **551** 012026
[6] Hashimoto M, Vishik I M, He R-H, Devereaux T P and Shen Z-X 2014 *Nat. Phys.* **10** 483
[7] Gabovich A M and Voitenko A I 2013 *Fiz. Nizk. Temp.* **39** 301
[8] Keimer B, Kivelson S A, Norman M R, Uchida S and Zaanen J 2015 *Nature* **518** 179
[9] Spałek J 2015 *Phil. Mag.* **95** 661
[10] Fradkin E, Kivelson S A and Tranquada J M 2015 *Rev. Mod. Phys.* **87** 457
[11] Wolowiec C T, White B D and Maple M B 2015 *Physica* C **514** 113

[12] Klemm R A 2015 *Physica* C **514** 86
[13] Sebastian S E and Proust C 2015 *Annu. Rev. Condens. Matter Phys.* **6** 411
[14] Kordyuk A A 2015 *Fiz. Nizk. Temp.* **41** 417
[15] Krasnov V M 2015 *Phys. Rev.* B **91** 224508
[16] Martinelli A, Bernardini F and Massidda S 2016 *C. R. Phys.* **17** 5
[17] Gabovich A M *et al* 1987 *Fiz. Nizk. Temp.* **13** 844
[18] Benfatto L, Caprara S and Di Castro C 2000 *Eur. Phys. J.* B **17** 95
[19] Fujita M, Goka H, Yamada K and Matsuda M 2002 *Phys. Rev. Lett.* **88** 167008
[20] Gabovich A M, Suan L M, Szymczak H and Voitenko A I 2003 *J. Phys.: Condens. Matter* **15** 2745
[21] Tranquada J M *et al* 2008 *Phys. Rev.* B **78** 174529
[22] Vojta M 2009 *Adv. Phys.* **58** 699
[23] Ghiringhelli G *et al* 2012 *Science* **337** 821
[24] Scalapino D J 2012 *Rev. Mod. Phys.* **84** 1383
[25] Tranquada J M 2013 *Lectures on the Physics of Strongly Correlated Systems XVII. 17th Training Course in the Physics of Strongly Correlated Systems* ed A Avella and F Mancini (Melville, NY: American Institute of Physics) p 114
[26] Das T, Markiewicz R S and Bansil A 2014 *Adv. Phys.* **63** 151
[27] Basov D N and Chubukov A V 2011 *Nat. Phys.* **7** 272
[28] Comin R and Damascelli A 2016 *Annu. Rev. Condens. Matter Phys.* **7** 369
[29] Achkar A J *et al* 2016 *Nat. Mater.* **15** 616
[30] Chang J *et al* 2012 *Nat. Phys.* **8** 751
[31] LeBoeuf D, Krämer S, Hardy W N, Liang R, Bonn D A and Proust C 2013 *Nat. Phys.* **9** 79
[32] Tabis W *et al* 2014 *Nat. Commun.* **5** 5875
[33] Doiron-Leyraud N *et al* 2015 *Nat. Commun.* **6** 6034
[34] Comin R, Sutarto R, da Silva Neto E H, Chauviere L, Liang R, Hardy W N, Bonn D A, He F, Sawatzky G A and Damascelli A 2015 *Science* **347** 1335
[35] Wu T, Mayaffre H, Krämer S, Horvatić M, Berthier C, Hardy W N, Liang R, Bonn D A and Julien M-H 2015 *Nat. Commun.* **6** 6438
[36] Comin R *et al* 2015 *Nat. Mater.* **14** 796
[37] Cyr-Choinière O, Grissonnanche G, Badoux S, Day J, Bonn D A, Hardy W N, Liang R, Doiron-Leyraud N and Taillefer L 2015 *Phys. Rev.* B **92** 224502
[38] Gerber S *et al* 2015 *Science* **350** 949
[39] Hoffman J E, Hudson E W, Lang K M, Madhavan V, Eisaki H, Uchida S and Davis J C 2002 *Science* **295** 466
[40] Kurosawa T *et al* 2015 *Int. J. Mod. Phys.* B **29** 1542009
[41] da Silva Neto E H, Comin R, He F, Sutarto R, Jiang Y, Greene R L, Sawatzky G A and Damascelli A 2015 *Science* **347** 282
[42] Julien M-H 2015 *Science* 350 914
[43] Fine B V 2016 *Science* **351** 235
[44] Comin R, Sutarto R, da Silva Neto E H, Chauviere L, Liang R, Hardy W N, Bonn D A, He F, Sawatzky G A and Damascelli A 2016 *Science* **351** 235
[45] Wang Y, Chowdhury D and Chubukov A V 2015 *Phys. Rev.* B **92** 161103
[46] Wang Y and Chubukov A V 2015 *Phys. Rev.* B **92** 245140
[47] Nie L, Sierens L E H, Melko R G, Sachdev S and Kivelson S A 2015 *Phys. Rev.* B **92** 174505
[48] Daou R *et al* 2010 *Nature* **463** 519
[49] Lawler M J *et al* 2010 *Nature* **466** 347
[50] Vojta M 2010 *Eur. Phys. J. Spec. Top.* **188** 49
[51] Wang J, Liu G-Z and Kleinert H 2011 *Phys. Rev.* B **83** 214503
[52] Phillabaum B, Carlson E W and Dahmen K A 2012 *Nat. Commun.* **3** 915
[53] da Silva Neto E H, Aynajian P, Baumbach R E, Bauer E D, Mydosh J, Ono S and Yazdani A 2013 *Phys. Rev.* B **87** 161117
[54] Wang J and Liu G-Z 2013 *New J. Phys.* **15** 073039
[55] Sugai S, Takayanagi Y, Hayamizu N, Muroi T, Shiozaki R, Nohara J, Takenaka K and Okazaki K 2013 *J. Phys.: Condens. Matter* **25** 475701
[56] Davis J C S and Lee D-H 2013 *Proc. Natl Acad. Sci. USA* **110** 17623
[57] Wang Y and Chubukov A 2014 *Phys. Rev.* B **90** 035149
[58] Yamakawa Y and Kontani H 2015 *Phys. Rev. Lett.* **114** 257001
[59] Markiewicz R S, Lorenzana J, Seibold G and Bansil A 2016 *Sci. Rep.* **6** 19678
[60] Achkar A J *et al* 2016 *Science* **351** 576
[61] Volkov P A and Efetov K B 2016 *Phys. Rev.* B **93** 085131
[62] Castellani C, Di Castro C and Grilli M 1995 *Phys. Rev. Lett.* **75** 4650
[63] Becca F, Tarquini M, Grilli M and Di Castro C 1996 *Phys. Rev.* B **54** 12443
[64] Seibold G, Becca F, Bucci F, Castellani C, Di Castro C and Grilli M 2000 *Eur. Phys. J.* B **13** 87
[65] Andergassen S, Caprara S, Grilli M and Di Castro C 2001 *Phys. Rev. Lett.* **87** 056401
[66] Phillips J C, Saxena A and Bishop A R 2003 *Rep. Prog. Phys.* **66** 2111
[67] Mostovoy M V, Marchetti F M, Simons B D and Littlewood P B 2005 *Phys. Rev.* B **71** 224502
[68] Pasupathy A N, Pushp A, Gomes K K, Parker C V, Wen J, Xu Z, Gu G, Ono S, Ando Y and Yazdani A 2008 *Science* **320** 196
[69] Achkar A J, Mao X, McMahon C, Sutarto R, He F, Liang R, Bonn D A, Hardy W N and Hawthorn D G 2014 *Phys. Rev. Lett.* **113** 107002
[70] Poccia N, Lankhorst M and Golubov A A 2014 *Physica* C **503** 82
[71] Carlson E W, Liu S, Phillabaum B and Dahmen K A 2015 *J. Supercond.* **28** 1237
[72] Honma T and Hor P H 2015 *Physica* C **509** 11
[73] Shengelaya A and Múller K A 2015 *Europhys. Lett.* **109** 27001
[74] Campi G *et al* 2015 *Nature* **525** 359
[75] Campi G and Bianconi A 2016 *J. Supercond.* **29** 627
[76] Kurosawa T, Takeyama K, Baar S, Shibata Y, Kataoka M, Mizuta S, Yoshida H, Momono N, Oda M and Ido M 2016 *J. Phys. Soc. Japan* **85** 044709
[77] Kirtley J R 1990 *Int. J. Mod. Phys.* B **4** 201
[78] Fang A, Howald C, Kaneko N, Greven M and Kapitulnik A 2004 *Phys. Rev.* B **70** 214514
[79] Gomes K K, Pasupathy A N, Pushp A, Ono S, Ando Y and Yazdani A 2007 *Physica* C **460–2** 212
[80] Gomes K K, Pasupathy A N, Pushp A, Ono S, Ando Y and Yazdani A 2007 *Nature* **447** 569
[81] Yazdani A 2009 *J. Phys.: Condens. Matter* **21** 164214
[82] Kato T, Funahashi H, Nakamura H, Fujimoto M, Machida T, Sakata H, Nakao S and Hasegawa T 2010 *J. Supercond.* **23** 771
[83] Alldredge J W, Fujita K, Eisaki H, Uchida S and McElroy K 2013 *Phys. Rev.* B **87** 104520
[84] Sugimoto A, Ekino T, Tanaka K, Mineta K, Tanabe K and Tokiwa K 2014 *Phys. Proced.* **58** 78
[85] Boyer M C, Wise W D, Chatterjee K, Yi M, Kondo T, Takeuchi T, Ikuta H and Hudson E W 2007 *Nat. Phys.* **3** 802
[86] Yoshida T *et al* 2016 *Phys. Rev.* B **93** 014513
[87] Gabovich A M, Suan L M, Szymczak H and Voitenko A I 2015 *Phys. Rev.* B **92** 054512
[88] Ekino T, Gabovich A M, Suan L M, Pekała M, Szymczak H and Voitenko A I 2011 *J. Phys.: Condens. Matter* **23** 385701
[89] Bilbro G and McMillan W L 1976 *Phys. Rev.* B **14** 1887
[90] Ekino T and Akimitsu J 1994 *Physica* B **194–6** 1221
[91] Monceau P 2012 *Adv. Phys.* **61** 325

[92] Klemm R A 2005 *Phil. Mag.* **85** 801
[93] Kirtley J R 2011 *C. R. Phys.* **12** 436
[94] Gabovich A M, Suan L M, Szymczak H and Voitenko A I 2013 *Phys. Rev.* B **87** 104503
[95] Gabovich A M and Voitenko A I 2014 *Physica* C **503** 7
[96] Ekino T, Gabovich A M, Suan L M, Pekała M, Szymczak H and Voitenko A I 2011 *Symmetry* **3** 699
[97] Eremin M V and Larionov I A 1998 *JETP Lett.* **68** 611
[98] Seo K, Chen H-D and Hu J 2007 *Phys. Rev.* B **76** 020511
[99] Fujita K *et al* 2014 *Proc. Natl Acad. Sci. USA* **111** E3026
[100] Hamidian M H, Edkins S D, Kim C K, Davis J C, Mackenzie A P, Eisaki H, Uchida S, Lawler M J, Kim E-A, Sachdev S and Fujita K 2016 *Nat. Phys.* **12** 150
[101] Ekino T and Akimitsu J 1987 *Japan J. Appl. Phys.* **26** 625
[102] Greenblatt M 1988 *Chem. Rev.* **88** 31
[103] Wang C, Giambattista B, Slough C G, Coleman R V and Subramanian M A 1990 *Phys. Rev.* B **42** 8890
[104] Dai Z, Slough C G, McNairy W W and Coleman R V 1992 *Phys. Rev. Lett.* **69** 1769
[105] Sinchenko A A and Monceau P 2003 *Phys. Rev.* B **67** 125117
[106] Chen P, Chan Y-H, Fang X-Y, Zhang Y, Chou M Y, Mo S-K, Hussain Z, Fedorov A-V and Chiang T-C 2015 *Nat. Commun.* **6** 8943
[107] Carbotte J P 1990 *Rev. Mod. Phys.* **62** 1027
[108] Blawid S and Millis A J 2000 *Phys. Rev.* B **62** 2424
[109] Wang Y and Chubukov A 2015 *Phys. Rev.* B **91** 195113
[110] Won H and Maki K 1994 *Phys. Rev.* B **49** 1397
[111] Yurgens A, Winkler D, Claeson T, Hwang S-J, and Choy J-H 1999 *Int. J. Mod. Phys. B* **13** 3758
[112] Li Q, Tsay Y N, Suenaga M, Klemm R A, Gu G D and Koshizuka N 1999 *Phys. Rev. Lett.* **83** 4160
[113] Takano Y, Hatano T, Fukuyo A, Ishii A, Ohmori M, Arisawa S, Togano K and Tachiki M 2003 *J. Low Temp. Phys.* **131** 533
[114] Latyshev Yu I, Orlov A P, Nikitina A M, Monceau P and Klemm R A 2004 *Phys. Rev.* B **70** 094517
[115] Hartge J, Forr L, Mandrus D, Martin M C, Kendziora C and Mihaly L 1993 *J. Phys. Chem. Solids* **54** 1359
[116] Larkin A I and Ovchinnikov Yu N 1966 *Sov. Phys.—JETP* **24** 1035
[117] Barone A and Paterno G 1982 *The Physics and Applications of the Josephson Effect* (New York: Wiley)
[118] Gabovich A M and Voitenko A I 1997 *Phys. Rev.* B **55** 1081
[119] Gabovich A M and Voitenko A I 1997 *J. Phys.: Condens. Matter* **9** 3901
[120] Gabovich A M and Voitenko A I 2007 *Phys. Rev.* B **75** 064516
[121] Voitenko A I and Gabovich A M 2007 *Fiz. Tverd. Tela* **49** 1356
[122] Ekino T, Gabovich A M, Suan L M, Pekała M, Szymczak H and Voitenko A I 2008 *J. Phys.: Condens. Matter* **20** 425218
[123] Cohen M H, Falicov L M and Phillips J C 1962 *Phys. Rev. Lett.* **8** 316
[124] Kopaev Yu V 1975 *Tr. Fiz. Inst. Akad. Nauk SSSR* **86** 3
[125] Gabovich A M, Pashitskii E A and Shpigel A S 1976 *Sov. Phys. Solid State* **18** 1911
[126] Fischer Ø, Kugler M, Maggio-Aprile I and Berthod C 2007 *Rev. Mod. Phys.* **79** 353
[127] Krasnov V M 2009 *Phys. Rev.* B **79** 214510
[128] Motzkau H, Jacobs T, Katterwe S-O, Rydh A and Krasnov V M 2012 *Phys. Rev.* B **85** 144519
[129] Kambara H, Kakeya I and Suzuki M 2013 *Phys. Rev.* B **87** 214521
[130] Krasnov V M, Yurgens A, Winkler D, Delsing P and Claeson T 2000 *Phys. Rev. Lett.* **84** 5860
[131] Krasnov V M 2002 *Phys. Rev.* B **65** 140504
[132] Yurgens A, Winkler D, Claeson T, Ono S and Ando Y 2003 *Phys. Rev. Lett.* **90** 147005
[133] Karppinen M and Yamauchi H 1999 *Mater. Sci. Eng.* R **26** 51
[134] Alloul H, Bobroff J, Gabay M and Hirschfeld P J 2009 *Rev. Mod. Phys.* **81** 45
[135] Yurgens A, Winkler D, Claeson T, Hwang S-J, and Choy J-H 2001 *Physica C* **362** 286
[136] Damascelli A, Hussain Z and Shen Z-X 2003 *Rev. Mod. Phys.* **75** 473
[137] Ekino T, Sezaki Y and Fujii H 1999 *Phys. Rev.* B **60** 6916
[138] Ekino T, Gabovich A M, Suan L M, Pekała M, Szymczak H and Voitenko A I 2007 *Phys. Rev.* B **76** 180503
[139] Vedeneev S I, Piot B A and Maude D K 2010 *Phys. Rev.* B **81** 054501
[140] Berthod C, Fasano Y, Maggio-Aprile I, Piriou A, Giannini E, Levy de Castro G and Fischer Ø 2013 *Phys. Rev.* B **88** 014528
[141] Katterwe S O, Rydh A and Krasnov V M 2008 *Phys. Rev. Lett.* **101** 087003
[142] Klemm R A 2012 *Layered Superconductors* vol 1 (Oxford: Oxford University Press)
[143] Tonjes W C, Greanya V A, Olson R, Liu C G and Molinié P 2001 *Phys. Rev.* B **63** 235101
[144] Inosov D S, Zabolotnyy V B, Evtushinsky D V, Kordyuk A A, Büchner B, Follath R, Berger H and Borisenko S V 2008 *New J. Phys.* **10** 125027
[145] Borisenko S V *et al* 2009 *Phys. Rev. Lett.* **102** 166402
[146] Inosov D S, Evtushinsky D V, Zabolotnyy V B, Kordyuk A A, Büchner B, Follath R, Berger H and Borisenko S V 2009 *Phys. Rev.* B **79** 125112
[147] Rossnagel K 2011 *J. Phys.: Condens. Matter* **23** 213001
[148] Shan L, Huang Y, Gao H, Wang Y, Li S L, Dai P C, Zhou F, Xiong J W, Ti W X and Wen H H 2005 *Phys. Rev.* B **72** 144506
[149] Zhao G-M 2010 *Phys. Rev. B* **82** 012506
[150] Hellmann S *et al* 2012 *Nat. Commun.* **3** 1069
[151] Dai J, Calleja E, Alldredge J, Zhu X, Li L, Lu W, Sun Y, Wolf T, Berger H and McElroy K 2014 *Phys. Rev.* B **89** 165140
[152] Koley S, Laad M S, Vidhyadhiraja N S and Taraphder A 2014 *Phys. Rev.* B **90** 115146
[153] Ugeda M M *et al* 2016 *Nat. Phys.* **12** 92
[154] Chen C-W, Choe J and Morosan E 2016 *Rep. Prog. Phys.* **79** 084505
[155] Nakayama I 1977 *J. Phys. Soc. Japan* **43** 1533
[156] Balseiro C A and Falicov L M 1979 *Phys. Rev.* B **20** 4457
[157] Gabovich A M, Pashitskii E A and Shpigel A S 1979 *Sov. Phys.—JETP* **50** 583