Chapter 13: Nonparametric Tests

13-2: Sign Test

1. a. The only requirement for the matched pairs is that they constitute a simple random sample.
   b. There is no requirement of a normal distribution or any other specific distribution.
   c. The sign test is "distribution free" in the sense that it does not require a normal distribution or any other specific distribution.

2. There is 1 positive sign, 4 negative signs, 1 tie, $n = 6,$ and the test statistic is $x = 1$ (the smaller of 1 and 4).

3. H_0: There is no difference between the populations of 8 AM temperatures and the matching 12 AM temperatures. H_1: There is a difference between the populations of 8 AM temperatures and the matching 12 AM temperatures. The sample data do not contradict H_1 because the numbers of positive signs (1) and negative signs (4) are not exactly the same.

4. The efficiency of 0.63 indicates that with all other things being equal, the sign test requires 100 sample observations to achieve the same results as 63 sample observations analyzed through a parametric test.

5. The test statistic of $x = 2$ is not less than or equal to the critical value of 0 (from Table A-7). There is not sufficient evidence to reject the claim of no difference in heights between mothers and their first daughters.

6. The test statistic of $x = 2$ is not less than or equal to the critical value of 1 (from Table A-7). There is not sufficient evidence to warrant rejection of the claim of no difference. There is not sufficient evidence to support the claim that there is a difference in heights between fathers and their first sons.

7. The test statistic of $x = 1$ is not less than or equal to the critical value of 0 (from Table A-7). There is not sufficient evidence to warrant rejection of the claim that when the 13th day of a month falls on a Friday, the numbers of hospital admissions from motor vehicle crashes are not affected. Hospital admissions do not appear to be affected.

8. The test statistic of $x = 0$ is less than or equal to the critical value of 1 (from Table A-7). There is sufficient evidence to warrant rejection of the claim that Captopril affects blood pressure. It appears that Captopril lowers blood pressure.

9. The test statistic of $z = \dfrac{(401 + 0.5) - \frac{882}{2}}{\sqrt{882}/2} = -2.66$ results in a P-value of 0.0078, and it is in the critical region bounded by $z = -2.575$ and 2.575. There is sufficient evidence to warrant rejection of the claim that there is no difference between the proportions of those opposed and those in favor.

10. The test statistic of $z = \dfrac{(372 + 0.5) - \frac{1228}{2}}{\sqrt{1228}/2} = -13.78$ results in a P-value of 0.0000, and it is in the critical region bounded by $z = -2.575$ and 2.575. There is sufficient evidence to support the claim that there is a difference between the rate of medical malpractice lawsuits that go to trial and the rate of such lawsuits that are dropped or dismissed.

11. The test statistic of $z = \dfrac{(426 + 0.5) - \frac{860}{2}}{\sqrt{860}/2} = -0.24$ results in a P-value of 0.8103, and it is not in the critical region bounded by $z = -1.96$ and 1.96. There is not sufficient evidence to reject the claim that boys and girls are equally likely.

12. The test statistic of $z = \dfrac{(123 + 0.5) - \frac{280}{2}}{\sqrt{280}/2} = -1.97$ results in a P-value of 0.0486, and it is not in the critical region bounded by $z = -2.575$ and 2.575. There is not sufficient evidence to warrant rejection of the claim that that the therapists do better than random guessing. It appears that therapists are not effective at identifying the correct hand.

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13. The test statistic of $x = 9$ is not less than or equal to the critical value of 5. There is not sufficient evidence to warrant rejection of the claim that the sample is from a population with a median IQ score of 100.

14. The test statistic of $z = \frac{(21+0.5)-\frac{77}{2}}{\sqrt{77}/2} = -3.87$ results in a P-value of 0.0000, and it is in the critical region bounded by $z = -2.575$ and 2.575. There is sufficient evidence to warrant rejection of the claim that the median full IQ score of the population with low lead exposure is equal to 100.

15. The test statistic of $z = \frac{(117+0.5)-\frac{273}{2}}{\sqrt{273}/2} = -2.30$ results in a P-value of 0.0215 and it is not in the critical region bounded by $z = -1.96$ and 1.96. There is not sufficient evidence to warrant rejection of the claim that the median is equal to 72 mm Hg.

16. The test statistic of $z = \frac{(123+0.5)-\frac{300}{2}}{\sqrt{300}/2} = -3.06$ results in a P-value of 0.0022, and it is in the critical region bounded by $z = -1.96$ and 1.96. There is sufficient evidence to warrant rejection of the claim that the systolic blood pressure of the population is equal to 125 mm Hg.

17. Second approach: The test statistic of $z = \frac{(30+0.5)-\frac{105}{2}}{\sqrt{105}/2} = -4.29$ is in the critical region bounded by $z = -1.645$, so the conclusions are the same as in Example 4.

    Third approach: The test statistic of $z = \frac{(38+0.5)-\frac{106}{2}}{\sqrt{106}/2} = -2.82$ is in the critical region bounded by $z = -1.645$, so the conclusions are the same as in Example 4. The different approaches can lead to very different results; as seen in the test statistics of –4.21, –4.29, and –2.82. The conclusions are the same in this case, but they could be different in other cases.

18. The column entries are *, *, *, *, *, 0, 0, 0.

13-3: Wilcoxon Signed-Ranks Test for Matched Pairs

1. a. The only requirements are that the matched pairs be a simple random sample and the population of differences be approximately symmetric.

   b. There is no requirement of a normal distribution or any other specific distribution.

   c. The Wilcoxon signed-ranks test is “distribution free” in the sense that it does not require a normal distribution or any other specific distribution.

2. a. 1.0, –1.2, –0.4, 0, –0.7, –0.8, 0.6

   b. 5, 6, 1, 3, 4, 2

   c. 5, –6, –1, –3, –4, 2

   d. 7, 14

   e. $T = 7$

   f. Critical value of T is 1.

3. The sign test uses only the signs of the differences, but the Wilcoxon signed-ranks test uses ranks that are affected by the magnitudes of the differences.

4. The efficiency of 0.95 indicates that with all other things being equal, the Wilcoxon signed-ranks test requires 100 sample observations to achieve the same results as 95 sample observations analyzed through a parametric test.

5. Test statistic: $T = 8.5$; Critical value: $T = 4$; Fail to reject the null hypothesis that the population of differences has a median of 0. There is not sufficient evidence to reject the claim of no difference in heights between mothers and their first daughters. There does not appear to be a difference in heights between mothers and their first daughters.

6. Test statistic: $T = 17$; Critical value: $T = 4$; Fail to reject the null hypothesis that the population of differences has a median of 0. There is not sufficient evidence to reject the claim of no difference in heights between fathers and their first sons. There does not appear to be a difference in heights between fathers and their first sons.

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7. Test statistic: $T = 1.5$; Critical value: $T = 1$; Fail to reject the null hypothesis that the population of differences has a median of 0. There is not sufficient evidence to warrant rejection of the claim that when the 13th day of a month falls on a Friday, the numbers of hospital admissions from motor vehicle crashes are not affected. Hospital admissions do not appear to be affected.

8. Test statistic: $T = 0$; Critical value: $T = 10$; There is sufficient evidence to warrant rejection of the claim that the before and after measurements are the same. there sufficient evidence to support the claim that captopril has an effect on systolic blood pressure

9. Test statistic: $T = 98$; Critical value: $T = 52$; There is not sufficient evidence to warrant rejection of the claim that the sample is from a population with a median IQ score of 100.

10. Convert $T = 725.5$ to the test statistic $z = -3.94$. P-value: 0.0000. Critical values: $z = \pm 2.575$; There is sufficient evidence to warrant rejection of the claim that the median IQ of the low lead group is 100.

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{725.5 - \frac{77(77+1)}{4}}{\sqrt{\frac{77(77+1)(2\cdot 77+1)}{24}}} = -3.94$$

11. Convert $T = 16,236$ to the test statistic $z = -1.89$. P-value: 0.0588. Critical values: $z = \pm 1.96$; There is not sufficient evidence to warrant rejection of the claim that the sample is from a population with a median diastolic blood pressure level of 72 mm Hg.

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{16,236 - \frac{273(273+1)}{4}}{\sqrt{\frac{273(273+1)(2\cdot 273+1)}{24}}} = -1.89$$

12. Convert $T = 17,826$ to the test statistic $z = -3.16$. P-value: 0.0016. Critical values: $z = \pm 1.96$; There is sufficient evidence to warrant rejection of the claim that the times are from a population with a median of 125 mm Hg.

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{17,826 - \frac{300(300+1)}{4}}{\sqrt{\frac{300(300+1)(2\cdot 300+1)}{24}}} = -3.16$$

13. a. Smallest: $T = 0$; Largest: $T = 1+2+3+\cdots+298+299+300 = \frac{300(300+1)}{2} = 45,150$

b. $\frac{45,150}{2} = 22,575$

c. $45,150 - 12,345 = 32,805$

d. $\frac{n(n+1)}{2} - k$

13-4: Wilcoxon Rank-Sum Test for Two Independent Samples

1. The two samples are from populations with the same median.

2. $R = 19.5+2+5+26+12.5+4+9+15.5+7.5+6+24.5+12.5+10.5+17.5 = 172$

Girl	3500	800	2400	4200	3100	2000	2900
Rank	19.5	2	5	26	12.5	4	9
Girl	3300	2800	2500	4000	3100	3000	3400
Rank	15.5	7.5	6	24.5	12.5	10.5	17.5

3. Yes. The samples are independent and both samples have more than 10 values.

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4. The efficiency rating of 0.95 indicates that with all other factors being the same, the Wilcoxon rank-sum test requires 100 sample observations to achieve the same results as 95 observations with the parametric t test of Section 9-2, assuming that the stricter requirements of the parametric t -test are satisfied.

5. $R_1 = 172$; $R_2 = 179$; $\mu_R = 189$; $\sigma_R = 19.4422$; Test statistic: $z = -0.87$; P-value $= 0.3843$; Critical values: $z = \pm 1.96$; Fail to reject the null hypothesis that the populations have the same median. There is not sufficient evidence to warrant rejection of the claim that girls and boys have the same median birth weight.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{14(14+12+1)}{2} = 189$$
$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{14\cdot 12(14+12+1)}{12}} = 19.4422$$
$$z = \frac{R-\mu_R}{\sigma_R} = \frac{172-189}{19.4422} = -0.87$$

6. $R_1 = 194.5$; $R_2 = 105.5$; $\mu_R = 150$; $\sigma_R = 17.321$; Test statistic: $z = 2.57$; P-value $= 0.0102$; Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to reject the claim that the median amount of strontium-90 from Pennsylvania residents is the same as the median from New York residents.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{12(12+12+1)}{2} = 150$$
$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{12\cdot 12(12+12+1)}{12}} = 17.321$$
$$z = \frac{R-\mu_R}{\sigma_R} = \frac{194.5-150}{17.321} = 2.57$$

7. $R_1 = 253.5$; $R_2 = 124.5$; $\mu_R = 182$; $\sigma_R = 20.607$; Test statistic: $z = 3.47$; P-value $= 0.0005$; Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to reject the claim that for those treated with 20 mg of Lipitor and those treated with 80 mg of Lipitor, changes in LDL cholesterol have the same median. It appears that the dosage amount does have an effect on the change in LDL cholesterol.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{13(13+14+1)}{2} = 182$$
$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{13\cdot 14(13+14+1)}{12}} = 20.607$$
$$z = \frac{R-\mu_R}{\sigma_R} = \frac{253.5-182}{20.607} = 3.47$$

8. $R_1 = 437$; $R_2 = 424$; $\mu_R = 525$; $\sigma_R = 37.4166$; Test statistic: $z = -2.35$; P-value $= 0.0188$; Critical values: $z = \pm 2.575$; Fail to reject the null hypothesis of no difference between the two population medians. There is not sufficient evidence to support the claim that the arrangement of the test items has an effect on the score.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{25(25+16+1)}{2} = 525$$
$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{25\cdot 16(25+16+1)}{12}} = 37.4166$$
$$z = \frac{R-\mu_R}{\sigma_R} = \frac{437-525}{37.4166} = -2.35$$

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9. $R_1 = 36,531.5$; $R_2 = 46,668.5$; $\mu_R = 41,102.5$; $\sigma_R = 1155.782$; Test statistic: $z = -3.95$; P-value $= 0.0001$ (Table: 0.0002); Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to warrant rejection of the claim that girls and boys have the same median birth weight.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{205(205+195+1)}{2} = 41,102.5$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{205 \cdot 195(205+195+1)}{12}} = 1155.782$$

$$z = \frac{R-\mu_R}{\sigma_R} = \frac{36,531.5-41,102.5}{1155.782} = -3.95$$

10. $R_1 = 863$; $R_2 = 412$; $\mu_R = 637.5$; $\sigma_R = 2656.25$; Test statistic: $z = 4.38$; P-value $= 0.0001$ (Table: 0.0002): Critical value: $z = 2.326$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to support the claim that the median amount of nicotine in the nonfiltered king-size cigarettes is greater than the median amount of nicotine in the 100-mm filtered cigarettes.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{25(25+25+1)}{2} = 637.5$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{25 \cdot 25(25+25+1)}{12}} = 51.54$$

$$z = \frac{R-\mu_R}{\sigma_R} = \frac{863-637.5}{51.54} = 4.38$$

11. $R_1 = 501$; $R_2 = 445$; $\mu_R = 484$; $\sigma_R = 41.15823$; Test statistic: $z = 0.41$; P-value $= 0.3409$; Critical value: $z = 1.645$; Fail to reject the null hypothesis that the populations have the same median. There is not sufficient evidence to support the claim that subjects with medium lead levels have a higher median of the full IQ scores than subjects with high lead levels. Based on these data, it does not appear that lead level affects full IQ scores.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{22(22+21+1)}{2} = 484$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{22 \cdot 21(22+21+1)}{12}} = 41.158$$

$$z = \frac{R-\mu_R}{\sigma_R} = \frac{501-484}{41.15823} = 0.41$$

12. $R_1 = 4178$; $R_2 = 772$; $\mu_R = 3900$; $\sigma_R = 116.833$; Test statistic: $z = 2.38$; P-value $= 0.0087$; Critical value: $z = 1.645$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to support the claim that subjects with low lead levels have performance IQ scores with a higher median than subjects with high lead levels. It appears that exposure to lead does have an adverse effect.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{78(78+21+1)}{2} = 3900$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{78 \cdot 21(78+21+1)}{12}} = 116.833$$

$$z = \frac{R-\mu_R}{\sigma_R} = \frac{4178-3900}{116.833} = 2.38$$

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13. The test statistic is the same value with opposite sign.

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R = 12\cdot 11 + \frac{12(12+1)}{2} - 123.5 = 86.5$$

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}} = \frac{86.5 - \frac{12\cdot 11}{2}}{\sqrt{\frac{12\cdot 11\cdot(12+11+1)}{12}}} = 1.26$$

14. a.

Rank				**Rank Sum for Treatment A**
1	2	3	4	
A	A	B	B	3
A	B	A	B	4
A	B	B	A	5
B	B	A	A	7
B	A	A	B	5
B	A	B	A	6

b. The R values of 3, 4, 5, 6, 7 have probabilities of $1/6$, $1/6$, $2/6$, $1/6$, and $1/6$, respectively

c. No, none of the probabilities for the values of R would be less than 0.10.

13-5: Kruskal-Wallis Test for Three or More Samples

1. $R_1 = 164.5$; $R_2 = 52.5$; $R_3 = 47$

Low Lead Level	Medium Lead Level	High Lead Level
70 (1.0)	72 (3.0)	82 (7.0)
85 (10.0)	90 (14.0)	93 (16.0)
86 (12.5)	92 (15.0)	85 (10.0)
76 (5.0)	71 (2.0)	75 (4.0)
84 (8.0)	86 (12.5)	85 (10.0)
	79 (6.0)	

(Ranks for each value shown in parentheses.)

2. Yes, the samples are independent simple random samples, and each sample has at least five data values.

3. $n_1 = 5$, $n_2 = 6$, $n_3 = 5$, and $N = 5+6+5 = 16$

4. The efficiency rating of 0.95 indicates that with all other factors being the same, the Kruskal-Wallis test requires 100 sample observations to achieve the same results as 95 observations with the parametric one-way analysis of variance test, assuming that the stricter requirements of the parametric test are satisfied.

5. Test statistic: $H = 4.9054$; Critical value: $\chi^2 = 5.991$; (Tech: P-value $= 0.086$); Fail to reject the null hypothesis of equal medians. The data do not suggest that larger cars are safer.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{21(21+1)}\left(\frac{86^2}{7} + \frac{97^2}{7} + \frac{48^2}{7}\right) - 3(21+1)$$
$$= 4.9054$$

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6. Test statistic: $H = 2.3503$; Critical value: $\chi^2 = 5.991$; (Tech: P-value = 0.309); Fail to reject the null hypothesis of equal medians. The data do not suggest that larger cars are safer.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{30(30+1)}\left(\frac{142^2}{10}+\frac{133.5^2}{10}+\frac{189.5^2}{10}\right)-3(30+1)$$
$$= 2.3503$$

7. Test statistic: $H = 22.8157$; Critical value: $\chi^2 = 9.210$; (Tech: P-value = 0.000); Reject the null hypothesis of equal medians. It appears that the three states have median amounts of arsenic that are not all the same.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{36(36+1)}\left(\frac{192^2}{12}+\frac{116.5^2}{12}+\frac{357.5^2}{12}\right)-3(36+1)$$
$$= 22.8157$$

8. Test statistic: $H = 2.54$; Critical value: $\chi^2 = 5.991$; (Tech: P-value = 0.281); Fail to reject the null hypothesis of equal medians. The times appear to increase for later laps.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{15(15+1)}\left(\frac{34^2}{5}+\frac{33^2}{5}+\frac{53^2}{5}\right)-3(15+1)$$
$$= 2.54$$

9. Test statistic: $H = 59.1546$; Critical value: $\chi^2 = 9.210$; (Tech: P-value = 0.000); Reject the null hypothesis of equal medians. The data suggest that the amounts of nicotine absorbed by smokers are different from the amounts absorbed by people who don't smoke.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{120(120+1)}\left(\frac{3629.5^2}{40}+\frac{2393.5^2}{40}+\frac{1237^2}{40}\right)-3(120+1)$$
$$= 59.1546$$

10. Test statistic: $H = 8.0115$; Critical value: $\chi^2 = 9.210$; (Tech: P-value = 0.018); Reject the null hypothesis of equal medians. It appears that the three categories of blood lead level have different median performance IQ scores.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{121(121+1)}\left(\frac{5277.5^2}{78}+\frac{1112^2}{22}+\frac{991.5^2}{21}\right)-3(121+1)$$
$$= 8.0115$$

11. Test statistic: $H = 27.9098$; Critical value: $\chi^2 = 5.991$; (Tech: P-value = 0.000); Reject the null hypothesis of equal medians. There is sufficient evidence to warrant rejection of the claim that the three different types of cigarettes have the same median amount of nicotine. It appears that the filters do make a difference.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{75(75+1)}\left(\frac{1413.5^2}{25}+\frac{650.5^2}{25}+\frac{786^2}{25}\right)-3(75+1)$$
$$= 27.9098$$

12. Test statistic: $H = 43.3240$; Critical value: $\chi^2 = 5.991$; (Tech: P-value = 0.000); Reject the null hypothesis of equal medians. There is sufficient evidence to warrant rejection of the claim that the three different types of cigarettes have the same median amount of nicotine. It appears that the filters do make a difference.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{75(75+1)}\left(\frac{1534^2}{25}+\frac{620^2}{25}+\frac{696^2}{25}\right)-3(75+1)$$
$$= 32.3240$$

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13. There are 10 zeros and 2 ones, so the values of t are 10 and 2. The values of T are 990 and 6, $\Sigma T = 996$, and $N = 5+6+7 = 18$. The corrected value of H is $6.724 \Big/ \left(1 - \frac{996}{18^3 - 18}\right) = 8.114$, which is quite different from the value of 6.724 found in Example 1. In this case, the large numbers of ties do appear to have a considerable effect on the test statistic H.

Cotinine Level	Rank	t	$t^3 - t$
0	5.5	10	990
1	11.5	2	6
		SUM	996

13-6: Rank Correlation

1. The methods of Section 10-2 should not be used for predictions. The regression equation is based on a linear correlation between the two variables, but the methods of this section do not require a linear relationship. The methods of this section could suggest that there is a correlation with paired data associated by some nonlinear relationship, so the regression equation would not be a suitable model for making predictions.

2. Data at the nominal level of measurement have no ordering that enables them to be converted to ranks, so data at the nominal level of measurement cannot be used with the methods of rank correlation.

3. r represents the linear correlation coefficient computed from sample paired data; ρ represents the parameter of the linear correlation coefficient computed from a population of paired data; r_s denotes the rank correlation coefficient computed from sample paired data; ρ_s represents the rank correlation coefficient computed from a population of paired data. The subscript s is used so that the rank correlation coefficient can be distinguished from the linear correlation coefficient r. The subscript does not represent the standard deviation s. It is used in recognition of Charles Spearman, who introduced the rank correlation method.

4. The efficiency rating of 0.91 indicates that with all other factors being the same, rank correlation requires 100 pairs of sample observations to achieve the same results as 91 pairs of observations with the parametric test using linear correlation, assuming that the stricter requirements for using linear correlation are met.

5. $r_s = 1.000$; Critical values are $r_s = \pm 0.648$. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support a claim of a correlation between height and age.

6. $r_s = -1.000$; Critical values are $r_s = \pm 0.648$. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support a claim of a correlation between service time and number of waiting patients.

7. $r_s = 0.857$; Critical values: $r_s = \pm 0.738$. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to conclude that there is a correlation between the number of chirps in 1 min and the temperature.

8. $r_s = 1.000$; Critical values: $r_s = \pm 0.886$. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to conclude that there is a correlation between overhead widths of seals from photographs and the weights of the seals.

9. $r_s = 0.624$; Critical values: $r_s = \pm 0.587$. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support a conclusion that there is a correlation between the number of cigarettes smoked and the cotinine level.

10. $r_s = 0.797$; Critical values: $r_s = \pm 0.538$. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between the circumference and height of a tree.

11. $r_s = 0.360$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{153-1}} = \pm 0.159$; Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to conclude that there is a correlation between the systolic and diastolic blood pressure levels in males.

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12. $r_s = 0.106$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{20-1}} = \pm 0.450$; Fail to reject the null hypothesis of $\rho_s = 0$. There is not sufficient evidence to support the claim of a correlation between brain volumes and IQ scores.

13. $r_s = 0.984$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{54-1}} = \pm 0.269$; Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between chest sizes and weights of bears.

14. $r_s = 0.924$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{54-1}} = \pm 0.269$; Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between head lengths and neck sizes of bears.

15. $r_s = \pm\sqrt{\frac{1.975799^2}{1.975799^2 + 153 - 2}} = \pm\sqrt{\frac{1.978^2}{1.978^2 + 153 - 2}} = \pm 0.159$; (Use either $t = 1.975799$ from technology or use interpolation in Table A-3 with 151 degrees of freedom, so the critical value of t is approximately halfway between 1.984 and 1.972, which is 1.978.) The critical values are the same as those found by using Formula 13-1.

Chapter Quick Quiz

1.

Value	1.18	1.41	1.49	1.04	1.45	0.74	0.89	1.42	1.45	0.51	1.38
Rank	5	7	11	4	$\frac{9+10}{2} = 9.5$	2	3	8	$\frac{9+10}{2} = 9.5$	1	6

2. The efficiency rating of 0.91 indicates that with all other factors being the same, rank correlation requires 100 pairs of sample observations to achieve the same results as 91 pairs of observations with the parametric test for linear correlation, assuming that the stricter requirements for using linear correlation are met.

3. a. distribution-free test

   b. The term "distribution-free test" suggests correctly that the test does not require that a population must have a particular distribution, such as a normal distribution. The term "nonparametric test" incorrectly suggests that the test is not based on a parameter, but some nonparametric tests are based on the median, which is a parameter; the term "distribution-free test" is better because it does not make that incorrect suggestion.

4. Rank correlation should be used. The rank correlation test is used to investigate whether there is a correlation between foot length and height.

5. No, the P-values are almost always different, and the conclusions may or may not be the same.

6. Rank correlation can be used in a wider variety of circumstances than linear correlation. Rank correlation does not require a normal distribution for any population. Rank correlation can be used to detect some (not all) relationships that are not linear.

7. Because the sign test uses only signs of differences while the Wilcoxon signed-ranks test uses ranks of the differences, the Wilcoxon signed-ranks test uses more information about the data and tends to yield conclusions that better reflect the true nature of the data.

8. Kruskal-Wallis test

9. two independent samples

10. matched pairs

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Review Exercises

1. The test statistic of $z=\frac{(907+0.5)-\frac{2015}{2}}{\sqrt{2015}/2}=-4.46$ results in a P-value of 0.0000 (Table: 0.0001) and it is less than or equal to the critical value of $z=-1.645$. Fail to reject the null hypothesis of $p=0.5$. There is sufficient evidence to support the claim that the majority of adults obtain medical information more often from the Internet than a doctor.

2. The test statistic of $x=3$ is less than or equal to the critical value of 5 (from Table A-7). There is sufficient evidence to warrant rejection of the claim that the sample is from a population with a median equal to 5 min.

3. The test statistic $T=21$ is less than or equal to the critical value of 59. There is sufficient evidence to warrant rejection of the claim that the sample is from a population with a median equal to 5 min.

4. Test statistic: $H=2.5288$; (Tech: P-value $=0.2824$); Critical value: $\chi^2=5.991$; Fail to reject the null hypothesis of equal medians. It appears that times of longevity after inauguration for presidents, popes, and British monarchs have the same median.

$$H=\frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1)=\frac{12}{76(76+1)}\left(\frac{1485.5^2}{38}+\frac{806.5^2}{24}+\frac{635^2}{14}\right)-3(76+1)$$
$$=2.5289$$

5. $r_s=0.888$; The critical values are $r_s=\pm0.618$. Reject the null hypothesis of $\rho_s=0$. There is sufficient evidence to support the claim of a correlation between chocolate consumption and the rate of Nobel Laureates. It does not make sense to think that there is a cause/effect relationship, so the correlation could be the result of a coincidence or other factors that affect the variables the same way.

6. Test statistic: $H=6.6305$; (Tech: P-value $=0.0363$); Critical value: $\chi^2=5.991$; Reject the null hypothesis of equal medians. Interbreeding of cultures is suggested by the data.

$$H=\frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1)=\frac{12}{27(27+1)}\left(\frac{91^2}{9}+\frac{112.5^2}{9}+\frac{174.5^2}{9}\right)-3(27+1)$$
$$=6.6305$$

7. $R_1=60$; $R_2=111$; $\mu_R=85.5$; $\sigma_R=11.3248$; Test statistic: $z=-2.25$; (Tech: P-value $=0.0363$); Critical values: $z=\pm1.96$; Reject the null hypothesis that the populations have the same median. Skull breadths from 4000 B.C. appear to have a different median than those from A.D. 150.

$$\mu_R=\frac{n_1(n_1+n_2+1)}{2}=\frac{9(9+9+1)}{2}=85.5$$
$$\sigma_R=\sqrt{\frac{n_1n_2(n_1+n_2+1)}{12}}=\sqrt{\frac{9\cdot9(9+9+1)}{12}}=11.3248$$
$$z=\frac{R-\mu_R}{\sigma_R}=\frac{60-85.5}{11.3248}=-2.25$$

8. $r_s=0.714$. Critical values: $r_s=\pm0.738$; Fail to reject the null hypothesis of $\rho_s=0$. There is not sufficient evidence to support the claim that there is a correlation between the student ranks and the magazine ranks. When ranking colleges, students and the magazine do not appear to agree.

Cumulative Review Exercises

1. $\bar{x}=14.6$ credit hours, $Q_2=15.0$ credit hours, $s=1.7$ credit hours, $s^2=2.9$ (credit hours)^2, range $=6.0$ credit hours

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2. a. convenience sample

   b. Because the sample is from one class of statistics students, it is not likely to be representative of the population of all full-time college students.

   c. discrete

   d. ratio

3. The data appear do not appear to follow the loose definition for a normal distribution and $n < 30$, proceed with caution.
H_0: $\mu = 14$ credit hours; H_1: $\mu > 14$ credit hours;
Test statistic: $t = \dfrac{14.6 - 14}{1.7/\sqrt{20}} = 1.446$; Critical value $(\alpha = 0.05)$: $t = 1.729$;
P-value $= 0.0822$ (Table: P-value > 0.05);
Fail to reject H_0. There is not sufficient evidence to support the claim that the mean is greater than 14 credit hours.

4. The test statistic of $x = 5$ is not less than or equal to the critical value of 4. There is not sufficient evidence to support the claim that the sample is from a population with a median greater than 14 credit hours.

5. 95% CI: $\bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}} = 14.6 \pm 2.093 \cdot \dfrac{1.7}{\sqrt{20}} \Rightarrow 13.8$ credit hours $< \mu < 15.3$ credit hours; We have 95% confidence that the limits of 13.8 credit hours and 15.3 credit hours contain the true value of the population mean.

6. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.039 \pm 1.96\sqrt{\dfrac{(0.039)(0.961)}{2000}} \Rightarrow 0.031 < p < 0.047$, or $3.1\% < p < 4.7\%$; We have 95% confidence that the limits of 3.1% and 4.7% actually contain the true percentage of the population of workers who test positive for drugs.

7. H_0: $p = 0.03$; H_1: $p > 0.03$; Test statistic: $z = \dfrac{0.039 - 0.03}{\sqrt{\frac{(0.03)(0.97)}{2000}}} = 2.36$;
P-value $= P(z > 2.36) = 0.0091$ (Tech: 0.0092); Critical value: $z = 1.645$;
Reject H_0. There is sufficient evidence to support the claim that the rate of positive drug test results among workers in the United States is greater than 3.0%.

8. $n = \dfrac{[z_{\alpha/2}]^2\,\hat{p}\hat{q}}{E^2} = \dfrac{[1.96]^2 \cdot 0.25}{0.02^2} = 2401$

9. H_0: $p = 0.5$; H_1: $p > 0.5$; Test statistic: $z = \dfrac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{285}}} = 1.36$;
P-value $= P(z > 1.36) = 0.0869$ (Tech: 0.0865); Critical value: $z = 1.645$;
Fail to reject H_0. There is not sufficient evidence to support the claim that the majority of the population is not afraid of heights in tall buildings. Because respondents themselves chose to reply, the sample is a voluntary response sample, not a random sample, so the results might not be valid.

10. There must be an error, because the rates of 13.7% and 10.6% are not possible with samples of size 100.

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