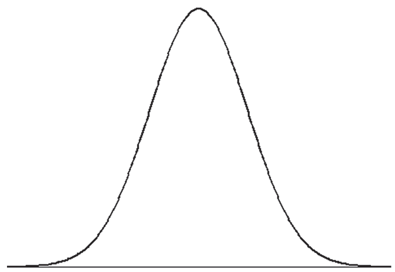
Chapter 6: Normal Probability Distributions

Section 6-1: The Standard Normal Distribution

1. The word "normal" has a special meaning in statistics. It refers to a specific bell-shaped distribution that can be described by Formula 6-1. The lottery digits do not have a normal distribution.

2.



3. The mean and standard deviation have values of $\mu = 0$ and $\sigma = 1$, respectively.

4. The notation z_α represents the z score that has an area of α to its right.

5. $P(x > 3) = 0.2(5-3) = 0.4$

6. $P(x < 4) = 0.2(4-0) = 0.8$

7. $P(2 < x < 3) = 0.2(3-2) = 0.2$

8. $P(2.5 < x < 4.5) = 0.2(4.5-2.5) = 0.4$

9. $P(z < 0.44) = 0.6700$

10. $P(z > -1.04) = 1 - P(z < -1.04) = 1 - 0.1492 = 0.8508$

11. $P(-0.84 < z < 1.28) = P(z < 1.28) - P(z < -0.84) = 0.8997 - 0.2005 = 0.6992$ (Tech: 0.6993)

12. $P(-1.07 < z < 0.67) = P(z < 0.67) - P(z < -1.07) = 0.7486 - 0.1423 = 0.6063$

13. $z = 1.23$

14. $z = -0.51$

15. $z = -1.45$

16. $z = 0.82$

17. $P(z < -1.23) = 0.1093$

18. $P(z < -1.96) = 0.0250$

19. $P(z < 1.28) = 0.8997$

20. $P(z < 2.56) = 0.9948$

21. $P(z > 0.25) = 1 - P(z < 0.25) = 1 - 0.5987 = 0.4013$

22. $P(z > 0.18) = 1 - P(z < 0.18) = 1 - 0.5704 = 0.4286$

23. $P(z > -2.00) = 1 - P(z < -2.00) = 1 - 0.0228 = 0.9772$

24. $P(z > -3.05) = 1 - P(z < -3.05) = 1 - 0.0011 = 0.9989$

25. $P(2.00 < z < 3.00) = P(z < 3.00) - P(z < 2.00) = 0.9986 - 0.9772 = 0.0214$ (Tech: 0.0215)

26. $P(1.50 < z < 2.50) = P(z < 2.50) - P(z < 1.50) = 0.9938 - 0.9332 = 0.0606$

27. $P(-2.55 < z < -2.00) = P(z < -2.00) - P(z < -2.55) = 0.0228 - 0.0054 = 0.0174$

28. $P(-2.75 < z < -0.75) = P(z < -0.75) - P(z < -2.75) = 0.2266 - 0.0030 = 0.2236$

29. $P(-2.00 < z < 2.00) = P(z < 2.00) - P(z < -2.00) = 0.0228 - 0.9772 = 0.9544$ (Tech: 0.9545)

30. $P(-3.00 < z < 3.00) = P(z < 3.00) - P(z < -3.00) = 0.9987 - 0.0013 = 0.9974$ (Tech: 0.9973)

31. $P(-1.00 < z < 5.00) = P(z < 5.00) - P(z < -1.00) = 0.9999 - 0.1587 = 0.8412$ (Tech: 0.8413)

32. $P(-4.27 < z < 2.34) = P(z < 2.34) - P(z < -4.27) = 0.9904 - 0.0001 = 0.9903$

33. $P(z < 4.55) = 0.9999$ (Tech: 0.999997)

34. $P(z > -3.75) = 0.9999$

35. $P(z > 0) = 0.5000$

36. $P(z < 0) = 0.5000$

37. $P_{99} = 2.33$

38. $P_{10} = -1.28$

39. $P_{2.0} = -2.05$ and $P_{98.0} = 2.05$

40. $P_{3.0} = -1.88$ and $P_{97.0} = 1.88$

41. $z_{0.10} = 1.28$

42. $z_{0.02} = 2.05$

43. $z_{0.04} = 1.75$

44. $z_{0.15} = 1.04$

45. $P(-1 < z < 1) = P(z < 1) - P(z < -1) = 0.8413 - 0.1587 = 0.6826 = 68.26\%$ (Tech: 68.27%)

46. $P(-2 < z < 2) = P(z < 2) - P(z < -2) = 0.9772 - 0.0228 = 0.9544 = 95.44\%$ (Tech: 95.45%)

47. $P(-3 < z < 3) = P(z < 3) - P(z < -3) = 0.9987 - 0.0013 = 0.9974 = 99.74\%$ (Tech: 99.73%)

48. $P(-3.5 < z < 3.5) = P(z < 3.5) - P(z < -3.5) = 0.9999 - 0.0001 = 0.9998 = 99.98\%$ (Tech: 99.95%)

49. a. $P(z > 2) = 1 - P(z < 2) = 1 - 0.9772 = 0.0228 = 2.28\%$

    b. $P(z < -2) = 0.0228 = 2.28\%$

    c. $P(-2 < z < 2) = P(z < 2) - P(z < -2) = 0.9772 - 0.0228 = 0.9544 = 95.44\%$ (Tech: 95.45%)

50. a. $\mu = 2.5$ min. and $\sigma = 5/\sqrt{12} = 1.4$ min

    b. The probability is $1/\sqrt{3}$ or 0.5774, and it is very different from the probability of 0.6827 that would be obtained by incorrectly using the standard normal distribution. The distribution does affect the results very much.

Section 6-2: Real Applications of Normal Distributions

1. a. $\mu = 0$ and $\sigma = 1$

   b. The z scores are numbers without units of measurements.

2. a. The area equals the maximum probability value of 1.

   b. The median is the middle value and for normally distributed scores that is also the mean, which is 3152.0 g.

   c. The mode is also 3152.0 g.

   d. The variance is the square of the standard deviation which is 480,803.6 g^2.

3. The standard normal distribution has a mean of 0 and a standard deviation of 1, but a nonstandard normal distribution has a different value for one or both of those parameters.

4. No. Randomly generated digits have a uniform distribution, but not a normal distribution. The probability of a digit less than 3 is $3/10 = 0.3$.

5. $z_{x=118} = \dfrac{118-100}{15} = 1.2$; which has an area of 0.8849 to the left.

6. $z_{x=90} = \dfrac{91-100}{15} = -0.6$; which has an area of 0.7257 to the right.

7. $z_{x=133} = \dfrac{133-100}{15} = 2.2$; which has an area of 0.9861 to the left. $z_{x=79} = \dfrac{79-100}{15} = -1.4$; which has an area of 0.0808 to the left. The area between the two scores is $0.9861-0.0808 = 0.9053$.

8. $z_{x=124} = \dfrac{124-100}{15} = 1.6$; which has an area of 0.9452 to the left. $z_{x=112} = \dfrac{112-100}{15} = 0.8$; which has an area of 0.7881 to the left. The area between the two scores is $0.9452-0.7881 = 0.1571$.

9. $z = 2.44$; so $x = 2.44 \cdot 15 + 100 = 136$

10. $z = 1$; so $x = 1 \cdot 15 + 100 = 115$

11. $z = -2.07$; so $x = -2.07 \cdot 15 + 100 = 69$

12. $z = 1.33$; so $x = 1.33 \cdot 15 + 100 = 120$

13. $z_{x=100} = \dfrac{100-74.0}{12.5} = 2.08$; which has an area of 0.9812 to the left.

14. $z_{x=80} = \dfrac{80-74.0}{12.5} = 0.48$; which has an area of 0.3156 to the right.

15. $z_{x=60} = \dfrac{60-74.0}{12.5} = -1.12$; which has an area of 0.1314 to the left and $z_{x=70} = \dfrac{70-74.0}{12.5} = -0.32$; which has an area of 0.3745 to the left. The area between the two scores is $0.3745-0.1314 = 0.2431$.

16. $z_{x=70} = \dfrac{70-74.0}{12.5} = -0.32$; which has an area of 0.3745 to the left and $z_{x=90} = \dfrac{90-74.0}{12.5} = 1.28$; which has an area of 0.8997 to the left. The area between the two scores is $0.8997-0.3745 = 0.5252$.

17. $P_{90} = 1.28$; so the pulse is $1.28 \cdot 12.5 + 74.0 = 90.0$ BPM.

18. $Q_1 = -0.67$; so the length is $-0.67 \cdot 12.5 + 74.0 = 65.6$ BPM.

19. $P_1 = -2.33$; so the lower length is $-2.33 \cdot 12.5 + 74.0 = 44.9$ BPM and $P_{99} = 2.33$; so the upper length is $2.33 \cdot 12.5 + 74.0 = 103.1$ BPM, so a pulse rate of 102 BPM is not significantly high.

20. $P_{2.5} = -1.96$; so the lower length is $x = -1.96 \cdot 12.5 + 74.0 = 49.5$ BPM and $P_{97.5} = 1.96$; so the upper length is $x = 1.96 \cdot 12.5 + 74.0 = 98.5$ BPM, so a pulse rate of 48 BPM is significantly low.

21. The z score for eye contact of 230.0 seconds is $\dfrac{230.0-184.0}{55.0} = 0.84$; so 0.2005, or 20.05% (Tech: 20.15%) of people would make eye contact longer than 230 seconds. No, the proportion of schizophrenics is not at all likely to be as high as 0.2005, or about 20%.

22. a. The z score for a temperature of 100.4 is $\dfrac{100.4-98.2}{0.62} = 3.87$; which corresponds to a an area of $1-0.9999 = 0.0001 = 0.01\%$ (Tech: 0.02%) to the right, which suggests a cutoff of 100.4°F is appropriate.

b. The z score for a probability of 2% is 2.05 which corresponds to a temperature of $2.05 \cdot 0.62 + 98.2 = 99.47^\circ$F.

23. a. The z score for the minimum weight is $\frac{2495.0 - 3152.0}{693.4} = -0.95$; which has an area to the left of 0.1711 (Tech: 0.1717).

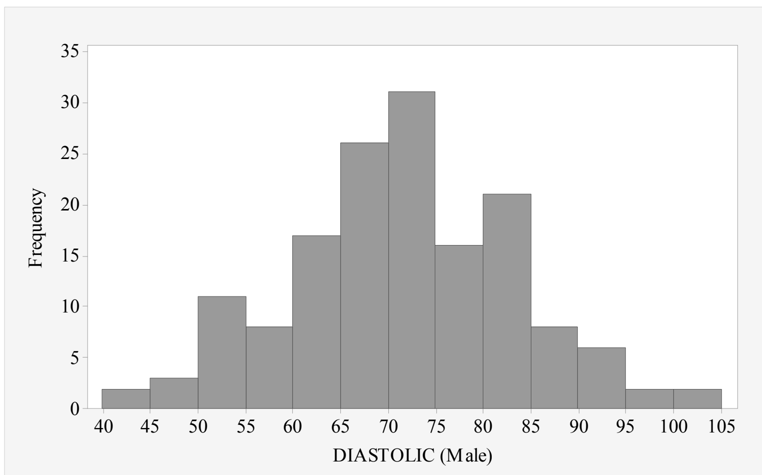
b. The z score for 5% to the left is –1.645, which corresponds to a weight of $-1.645 \cdot 693.4 + 3152.0 = 2011.4$ g (Tech: 2011.5 g). This is about one pound lower that the criterion of 2495 g.

c. Birth weights are significantly low if they are 2011.4 g or less, and they are "low birth weights" if they are 2495 g or less. Birth weights between 2011.4 g and 2495 g are "low birth weights" but they are not significantly low.

24. a. The z score for a 308 day pregnancy is $\frac{308 - 268}{15} = 2.67$; which corresponds to a probability of 0.0038 or 0.38%. Either a very rare event occurred or the husband is not the father.

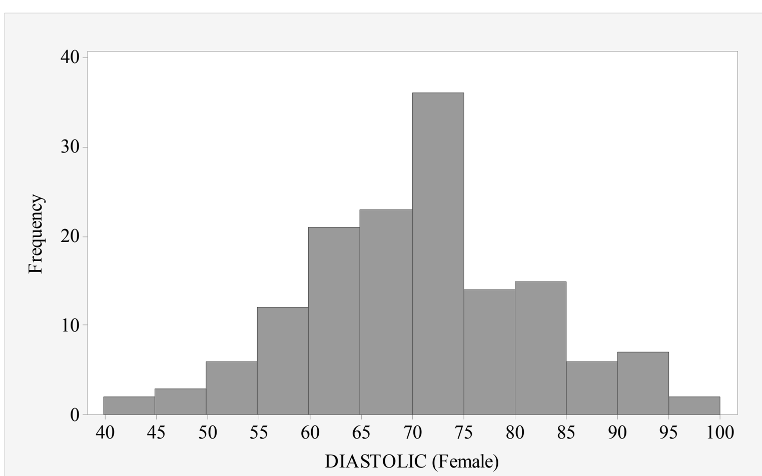
b. The z score corresponding to 3% is –1.87 which corresponds to a pregnancy of $-1.87 \cdot 15 + 268 = 240$ days.

25. a. The mean is 71.320 mm Hg and the standard deviation is 11.994 mm Hg. The histogram for the data confirms that the distribution is roughly normal.



b. The z score for the bottom 2.5% is –1.96, which corresponds to a blood pressure of $-1.96 \cdot 11.994 + 71.320 = 47.8$ mm Hg. The z score for the top 2.5% is 1.96, which corresponds to a blood pressure of $1.96 \cdot 11.994 + 73.320 = 94.8$ mm Hg.

26. a. The mean is 70.163 mm Hg and the standard deviation is 11.220 mm Hg. The histogram for the data confirms that the distribution is roughly normal.



26. (continued)

b. The z score for the bottom 2.5% is –1.96, which corresponds to a blood pressure of $-1.96 \cdot 11.220 + 70.163 = 48.2$ mm Hg. The z score for the top 2.5% is 1.95, which corresponds to a blood pressure of $1.96 \cdot 11.220 + 70.163 = 92.2$ mm Hg.

27. The z score for Q_1 is –0.67, and the z score for Q_3 is 0.67. The IQR is $0.67-(-0.67)=1.34$. $1.5 \cdot IQR = 2.01$, so $Q_1 - 1.5 \cdot IQR = -0.67 - 2.01 = -2.68$ and $Q_3 + 1.5 \cdot IQR = 0.67 + 2.01 = 2.68$.
The percentage to the left of –2.68 is 0.0037 and the percentage to the right of 2.68 is 0.0037. Therefore, the percentage of an outlier is 0.0074 (Tech: 0.0070).

Section 6-3: Sampling Distributions and Estimators

1. a. In the long run, the sample proportions will have a mean of 0.512.
   b. The sample proportions will tend to have a distribution that is approximately normal.
2. a. without replacement
   b. (1) When selecting a relatively small sample from a large population, it makes no significant difference whether we sample with replacement or without replacement.
   (2) Sampling with replacement results in independent events that are unaffected by previous outcomes, and independent events are easier to analyze and they result in simpler calculations and formulas.
3. sample mean, sample variance, sample proportion
4. No. The data set is only one sample, but the sampling distribution of the mean is the distribution of the means from all samples, not the one sample mean obtained from the one sample in Data Set 4.
5. No, the sample is not a simple random sample from the population of all births worldwide. The proportion of boys born in China is substantially higher than in other countries.
6. a. The distribution of the sample means is approximately normal.
   b. The sample means target the mean annual income of all physicians.
7. a. The population mean is $\mu = \dfrac{4+5+9}{3} = 6$, and the population variance is

$$\sigma^2 = \frac{(4-6)^2 + (5-6)^2 + (9-6)^2}{3} = 4.7.$$

   b. The possible sample of size 2 are {(4, 4), (4, 5), (4, 9), (5, 4), (5, 5), (5, 9), (9, 4), (9, 5), (9, 9)} which have the following variances {0, 0.5, 12.5, 0.5, 0, 8, 12.5, 8, 0} respectively.

Sample Variance	Probability
0.0	3/9
0.5	2/9
8	2/9
12.5	2/9

   c. The sample variance's mean is $\dfrac{3 \cdot 0 + 2 \cdot 0.5 + 2 \cdot 8 + 2 \cdot 12.5}{9} = 4.666\ldots \approx 4.7.$

   d. Yes. The mean of the sampling distribution of the sample variances (4.7) is equal to the value of the population variance (4.7) so the sample variances target the value of the population variance.

8. a. The population standard deviation (using the result from the previous problem) is $\sigma = \sqrt{4.666} = 2.160$.

b. By taking the square root of the sample variances from the previous problem we get

Sample Standard Deviation	Probability
0.000	3/9
0.707	2/9
2.828	2/9
3.536	2/9

c. The mean of the sample standard deviations is $\frac{3 \cdot 0 + 2 \cdot 0.707 + 2 \cdot 2.828 + 2 \cdot 3.536}{9} = 1.571$.

d. No. The mean of the sampling distribution of the sample standard deviations is 1.571, and it is not equal to the value of the population standard deviation (2.160), so the sample standard deviations do not target the value of the population standard deviation.

9. a. The population median is 5.

b. The possible sample of size 2 are {(4, 4), (4, 5), (4, 9), (5, 4), (5, 5), (5, 9), (9, 4), (9, 5), (9, 9)}, which have the following medians {4, 4.5, 6.5, 4.5, 5, 7, 6.5, 7, 9} with the following associated probabilities.

Sample Median	Probability
4	1/9
4.5	2/9
5	1/9
6.5	2/9
7	2/9
9	1/9

c. The mean of the sampling distribution of the sampling median is $\frac{4 + 2 \cdot 4.5 + 5 + 2 \cdot 6.52 \cdot 7 + 9}{9} = 6.0$.

d. No. The mean of the sampling distribution of the sample medians is 6.0, and it is not equal to the value of the population median of 5.0, so the sample medians do not target the value of the population median.

10. a. The proportion of odd numbers is 2/3, or 0.7. (There are two odd numbers from the population of 4, 5, and 9.)

b. The possible sample of size 2 are {(4, 4), (4, 5), (4, 9), (5, 4), (5, 5), (5, 9), (9, 4), (9, 5), (9, 9)}, which have the following proportion of odd numbers {0, 0.5, 0.5, 0.5, 1, 1, 0.5, 1, 1.

Sample Proportion	Probability
0.0	1/9
0.5	4/9
1.0	4/9

c. The mean of the sampling distribution of sample proportions is $\frac{0 + 4 \cdot 0.5 + 4 \cdot 1}{9} = \frac{2}{3}$, or 0.7.

d. Yes. The mean of the sampling distribution of the sample proportion of odd numbers is 2/3, and it is equal to the value of the population proportion of odd numbers of 2/3, so the sample proportions target the value of the population proportion

11. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following ranges and associated probabilities.

Sample Range	**Probability**
34	1/16
35	2/16
36	1/16
37.5	2/16
38.5	2/16
41	2/16
42.5	2/16
43.5	1/16
46	2/16
51	1/16

b. The mean of the population is $\frac{34+36+41+51}{4}=40.5$ and the mean of the sample means is

$$\frac{34+2\cdot 35+36+2\cdot 37.5+2\cdot 38.5+2\cdot 41+2\cdot 42.5+43.5+2\cdot 46+51}{16}=40.5 \text{ as well.}$$

c. The sample means target the population mean. Sample means make good estimators of population means because they target the value of the population mean instead of systematically underestimating or overestimating it.

12. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following medians and associated probabilities.

Sample Median	**Probability**
34	1/16
35	2/16
36	1/16
37.5	2/16
38.5	2/16
41	2/16
42.5	2/16
43.5	1/16
46	2/16
51	1/16

b. The median of the population is $\frac{36+41}{2}=38.5$, but the median of the sample medians is $\frac{38.5+41.0}{2}=40.5$.

The two values are not equal.

c. The sample medians do not target the population median of 38.5, so the sample medians do not make good estimators of the population medians.

13. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following ranges and associated probabilities.

Sample Range	**Probability**
0	4/16
2	2/16
5	2/16
7	2/16
10	2/16
15	2/16
17	2/16

b. The range of the population is $51 - 34 = 17.0$, but the mean of the sample ranges is $\frac{4 \cdot 0 + 2 \cdot 2 + 2 \cdot 5 + 2 \cdot 7 + 2 \cdot 10 + 2 \cdot 15 + 2 \cdot 17}{16} = 7.0$. The values are not equal.

c. The sample ranges do not target the population range of 17, so sample ranges do not make good estimators of the population range.

14. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following variances and associated probabilities.

Sample Variance (s^2)	**Probability**
0.0	4/16
2.0	2/16
12.5	2/16
24.5	2/16
50.0	2/16
112.5	2/16
144.5	2/16

b. The mean of the sample variances is $\frac{4 \cdot 0 + 2 \cdot 2 + 2 \cdot 12.5 + 2 \cdot 24.5 + 2 \cdot 50 + 2 \cdot 112.5 + 2 \cdot 144.5}{16} = 43.25$.

The two values are equal.

c. The sample variances do target the population variance of 43.25, so sample variances do make good estimators of the population variance.

15. The possible birth samples are {(b, b), (b, g), (g, b), (g, g)}.

Proportion of Girls	**Probability**
0	0.25
1 / 2	0.5
2/2	0.25

Yes. The proportion of girls in 2 births is 0.5, and the mean of the sample proportions is 0.5. The result suggests that a sample proportion is an unbiased estimator of the population proportion.

16. The possible birth samples are {bbb, bbg, bgb, gbb, ggg, ggb, gbg, bgg}.

Proportion of Girls	**Probability**
0	1/3
1/3	3/8
2/3	3/8
3/3	1/8

16. (continued)

Yes. The proportion of girls in 3 births is 0.5 and the mean of the sample proportions is 0.5. The result suggests that a sample proportion is an unbiased estimator of the population proportion.

17. The possibilities are: both questions incorrect, one question correct (two choices), both questions correct.

a.

Proportion Correct	Probability
$\frac{0}{2}=0$	$\frac{4}{5}\cdot\frac{4}{5}=\frac{16}{25}$
$\frac{1}{2}=0.5$	$2\cdot\left(\frac{1}{5}\cdot\frac{4}{5}\right)=\frac{8}{25}$
$\frac{2}{2}=1$	$\left(\frac{1}{5}\cdot\frac{1}{5}\right)=\frac{1}{25}$

b. The mean is $\frac{16\cdot 0+8\cdot 0.5+1\cdot 1}{25}=0.2$.

c. Yes, the sampling distribution of the sample proportions has a mean of 0.2 and the population proportion is also 0.2 (because there is 1 correct answer among 5 choices). Yes, the mean of the sampling distribution of the sample proportions is always equal to the population proportion.

18. a. The proportions of 0, 0.5, and 1 have the following probabilities.

Proportion with Yellow Pods	Probability
0	1/25
0.5	8/25
1	16/25

b. The mean is $\frac{1\cdot 0+8\cdot 0.5+16\cdot 1}{25}=0.8$.

c. Yes, the population proportion is 0.8 and the mean of the sampling proportions is also 0.8. The mean of the sampling distribution of proportions is always equal to the population proportion.

19. The formula yields $P(0)=\frac{1}{2(2-2\cdot 0)!(2\cdot 0)!}=0.25$, $P(0.5)=\frac{1}{2(2-2\cdot 0.5)!(2\cdot 0.5)!}=0.5$, and $P(1)=\frac{1}{2(2-2\cdot 1)!(2\cdot 1)!}=0.25$, which describes the sampling distribution of the sample proportions. The formula is just a different way of presenting the same information in the table that describes the sampling distribution.

20. Sample values of the mean absolute deviation (MAD) do not usually target the value of the population MAD, so a MAD statistic is not good for estimating a population MAD. If the population of {4, 5, 9} from Example 5 is used, the sample MAD values of 0, 0.5, 2, and 2.5 have corresponding probabilities of 3/9, 2/9, 2/9, and 2/9. For these values, the population MAD is 2, but the sample MAD values have a mean of 1.1, so the mean of the sample MAD values is not equal to the population MAD.

Section 6-4: The Central Limit Theorem

1. The sample must have more than 30 values, or there must be evidence that the population of grade-point averages from statistics students has a normal distribution.

2. No. because the original population is normally distributed, the sample means will be normally distributed for any sample size, not just for $n > 30$.

3. $\mu_{\bar{x}}$ represents the mean of all sample means and $\sigma_{\bar{x}}$ which represents the standard deviation of all sample means. For samples of 64 IQ scores, $\mu_{\bar{x}}=100$, and $\sigma_{\bar{x}}=15/\sqrt{64}=1.875$.

4. No. The sample of annual incomes will tend to have a distribution that is skewed to the right, no matter how large the sample is. If we compute the sample mean, we can consider that value to be one value in a normally distributed population.

5. a. $z_{x=80} = \dfrac{80.0-74.0}{12.5} = 0.48;$ which has a probability of 0.6844 to the left.

   b. $z_{x=80} = \dfrac{80.0-74.0}{12.5/\sqrt{16}} = 1.92;$ which has a probability of 0.9726 to the left (Tech: 0.8889).

   c. Because the original population has a normal distribution, the distribution of sample means is a normal distribution for any sample size.

6. a. $z_{x=70} = \dfrac{70.0-74.0}{12.5} = -0.32;$ which has a probability of $1-0.3745 = 0.6255$ to the right.

   b. $z_{x=70} = \dfrac{70.0-74.0}{12.5/\sqrt{25}} = -1.6;$ which has a probability of $1-0.0548 = 0.9452$ to the right.

   c. Because the original population has a normal distribution, the distribution of sample means is a normal distribution for any sample size.

7. a. $z_{x=72} = \dfrac{72.0-74.0}{12.5} = -0.16$ and $z_{x=76} = \dfrac{76.0-74.0}{12.5} = 0.16;$ which have a probability of $0.5636-0.4364 = 0.1272$ between them (Tech: 0.1271).

   b. $z_{x=72} = \dfrac{72.0-74.0}{12.5/\sqrt{4}} = -0.32$ and $z_{x=76} = \dfrac{76.0-74.0}{12.5/\sqrt{4}} = 0.32;$ which have a probability of $0.6255-0.3745 = 0.2510$ between them.

   c. Because the original population has a normal distribution, the distribution of sample means is normal for any sample size.

8. a. $z_{x=90} = \dfrac{90.0-74.0}{12.5} = 1.28$ and $z_{x=78} = \dfrac{78.0-74.0}{12.5} = 0.32;$ which have a probability of $0.8997-0.6255 = 0.2742$ between them.

   b. $z_{x=90} = \dfrac{90.0-74.0}{12.5/\sqrt{16}} = 5.12$ and $z_{x=78} = \dfrac{78.0-74.0}{12.5/\sqrt{16}} = 1.28;$ which have a probability of $0.9999-0.8997 = 0.1002$ between them (Tech: 0.0.1003).

   c. Because the original population has a normal distribution, the distribution of sample means is normal for any sample size.

9. a. $z_{x=17.7} = \dfrac{17.7-14.7}{1.3} = 2.31$ and $z_{x=13.6} = \dfrac{13.6-14.7}{1.3} = -0.85$ which have a probability of $0.9896-0.1977 = 0.7919$ (Tech: 0.7908) between them.

   b. $z_{x=17.7} = \dfrac{17.7-14.7}{1.3/\sqrt{9}} = 20.8$ and $z_{x=13.6} = \dfrac{13.6-14.7}{1.3/\sqrt{9}} = -7.62$ which have a probability of $0.9999-0.0001 = 0.9998$ (Tech: 0.9944) between them.

10. a. $z_{x=15.1} = \dfrac{15.1-13.0}{1.3} = 1.62$ and $z_{x=12.1} = \dfrac{12.1-13.0}{1.3} = -0.69$ which have a probability of $0.9474-0.2451 = 0.7023$ (Tech: 0.7025) between them.

    b. $z_{x=15.1} = \dfrac{15.1-13.0}{1.3/\sqrt{9}} = 4.85$ and $z_{x=12.1} = \dfrac{12.1-13.0}{1.3/\sqrt{9}} = -2.08$ which have a probability of $0.9999-0.0188 = 0.9811$ between them.

11. a. $z_{x=90} = \frac{90-70.2}{11.2} = 1.77$; which has a probability of $1-0.9616 = 0.0384$ (Tech: 0.0385) to the right.

b. $z_{x=90} = \frac{90-70.2}{11.2/\sqrt{4}} = 7.07$; which has a probability of $1-0.9999 = 0.0001$ (Tech: 0.0000) to the right.

12. a. $z_{x=90} = \frac{90-71.3}{12.0} = 1.56$; which has a probability of $1-0.9406 = 0.0594$ (Tech: 0.0596) to the right.

b. $z_{x=90} = \frac{90-71.3}{12.0/\sqrt{4}} = 3.12$; which has a probability of $1-0.9991 = 0.0009$ to the right.

13. a. The z score for the top 2% is 2.05, which corresponds to an IQ score of $2.05 \cdot 15 + 100 = 131$.

b. $z_{x=131} = \frac{131-100}{15/\sqrt{4}} = 4.13$; which has a probability of $1-0.9999 = 0.0001$ (Tech: 0.0000179) to the right.

c. No, it is possible that the 4 subjects have a mean of 132 while some of them have scores below the Mensa requirement of 131.

14. a. $z_{x=9} = \frac{9-6.8}{1.4} = 1.57$ and $z_{x=7} = \frac{7-6.8}{1.4} = 0.14$; which have a probability of $0.9418 - 0.5557 = 0.3861$ (Tech: 0.3851) between them.

b. $z_{x=9} = \frac{9-6.8}{1.4/\sqrt{5}} = 3.51$ and $z_{x=7} = \frac{7-6.8}{1.4/\sqrt{5}} = 0.32$ which have a probability of $0.9999 - 0.6255 = 0.3744$ (Tech: 0.3745) between them.

15. a. The mean weight of passengers is $3500/25 = 140$ lb.

b. $z_{x=140} = \frac{140-189}{39/\sqrt{25}} = -6.28$; which has a probability of $1-0.0001 = 0.9999$ (Tech: 0.999999998) to the right.

c. $z_{x=175} = \frac{175-189}{39/\sqrt{20}} = -1.61$; which has a probability of $1-0.0537 = 0.9463$ (Tech: 0.9458) to the right.

d. The new capacity of 20 passengers does not appear to be safe enough because the probability of overloading is too high.

16. a. $z_{x=22} = \frac{22-18.2}{1} = 3.8$; which has a probability of 0.9999. So the percentage is 99.99%.

b. $z_{x=18.5} = \frac{18.5-18.2}{1/\sqrt{36}} = 1.8$ which has a probability of 0.9641. No, when considering the diameters of manholes, we should use a design based on individual men, not samples of 36 men.

17. a. $z_{x=17} = \frac{17.0-14.4}{1.0} = 2.60$; which has a probability of $1-0.9953 = 0.0047$ to the right.

b. $z_{x=17} = \frac{17.0-14.4}{1.0/\sqrt{122}} = 28.7$; which has a probability of $1-0.9999 = 0.0001$ (Tech: 0.0000) to the right.

c. The result from part (a) is relevant because the seats are occupied by individuals.

18. a. $z_{x=211} = \frac{211-171}{46} = 0.87$ and $z_{x=140} = \frac{140-171}{46} = -0.67$; which have a probability of $0.8078 - 0.2514 = 0.5564$ between them (Tech: 0.5575).

18. (continued)

b. $z_{x=211} = \dfrac{211-171}{46/\sqrt{25}} = 4.35$ and $z_{x=140} = \dfrac{140-171}{46/\sqrt{25}} = -3.37$; which have a probability of $0.9999-0.0004$ $= 0.9995$ between them (Tech: 0.9996).

c. Part (a) because the ejection seats will be occupied by individual women, not groups of women.

19. a. $z_{x=72} = \dfrac{72-68.6}{2.8} = 1.21$; which has a probability of 0.8869 (Tech: 0.8877).

b. $z_{x=72} = \dfrac{72-68.6}{2.8/\sqrt{100}} = 12.1$; which has a probability of 0.9999. (Tech: 1.0000 when rounded to four decimal places.)

c. The probability of part (a) is more relevant because it shows that about 89% of male passengers will not need to bend. The result from part (b) gives us useful information about the comfort and safety of individual male passengers.

d. Because men are generally taller than women, a design that accommodates a suitable proportion of men will necessarily accommodate a greater proportion of women.

20. $z_{x=167.6} = \dfrac{167.6-189.0}{39/\sqrt{37}} = -3.34$; which has a probability of $1-0.0004 = 0.9996$ to the right. There is a 0.9996 probability that the aircraft is overloaded. Because that probability is so high, the pilot should take action, such as removing excess fuel and/or requiring that some passengers disembark and take a later flight.

21. a. Yes, the sampling is without replacement and the sample size of 50 is greater than 5% of the finite population size of 275. $\sigma_{\bar{x}} = \dfrac{16}{\sqrt{50}}\sqrt{\dfrac{275-50}{275-1}} = 2.0504584$

b. $z_{x=105} = \dfrac{105-95.5}{2.0504584} = 4.63$ and $z_{x=95.5} = \dfrac{95-95.5}{2.0504584} = -0.24$; which have a probability of $1-0.4053 = 0.5947$ between them (Tech: 0.5963).

Section 6-5: Assessing Normality

1. The histogram should be approximately bell-shaped, and the normal quantile plot should have points that approximate a straight-line pattern.

2. Either the points are not reasonably close to a straight-line pattern, or there is some systematic pattern that is not a straight-line pattern.

3. We must verify that the sample is from a population having a normal distribution. We can check for normality using a histogram, identifying the number of outliers, and constructing a normal quantile plot.

4. Because the histogram is roughly bell-shaped, we can conclude that the data are from a population having a normal distribution.

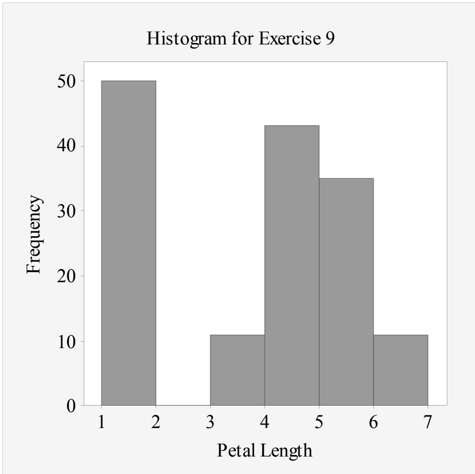
5. Normal; the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern.

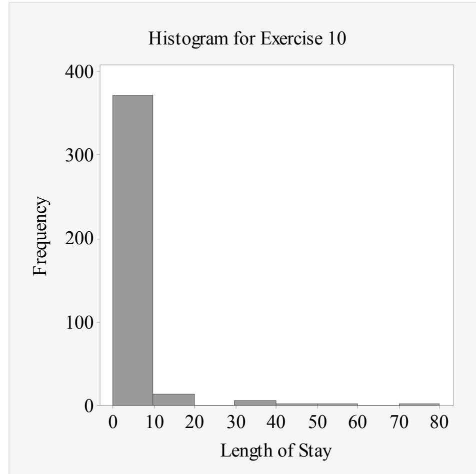
6. Normal; the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern.

7. Not normal; the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.

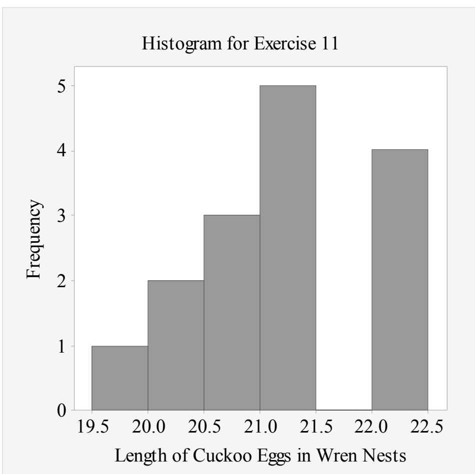
8. Not normal; the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.

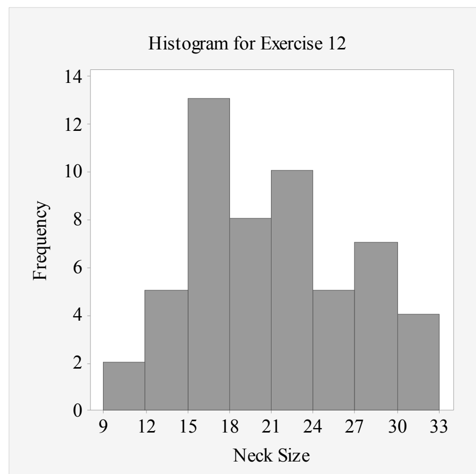
9. not normal



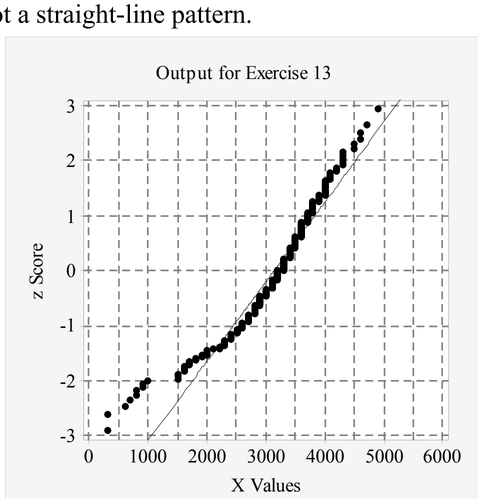


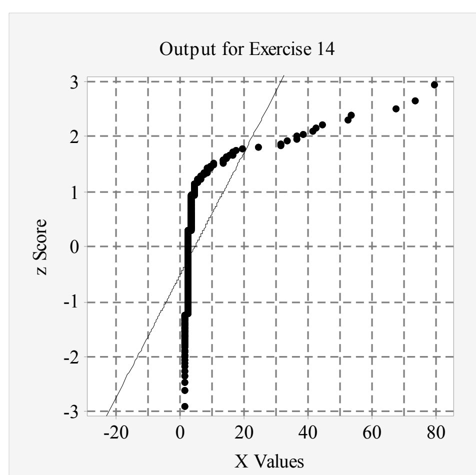
10. not normal
11. not normal





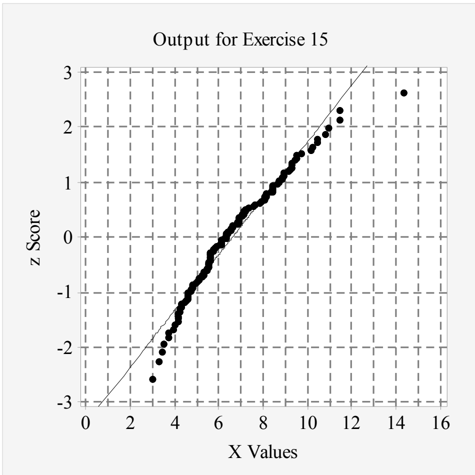
12. normal
13. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.

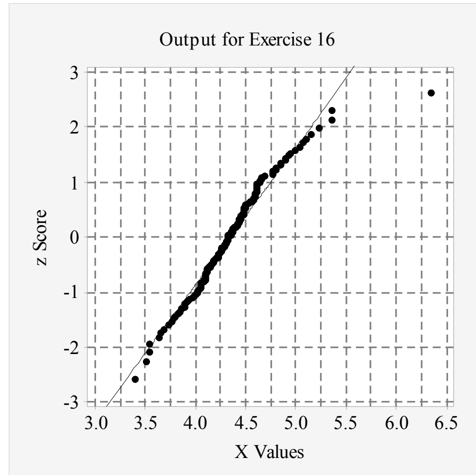




14. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.

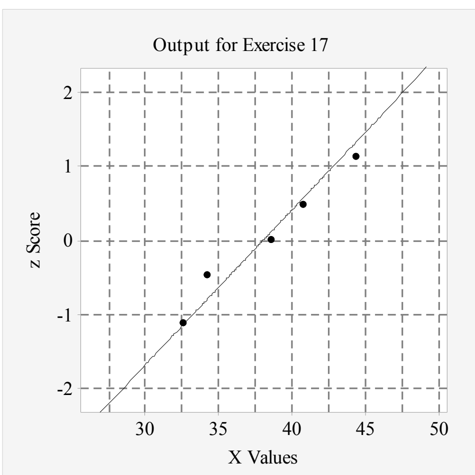
15. normal

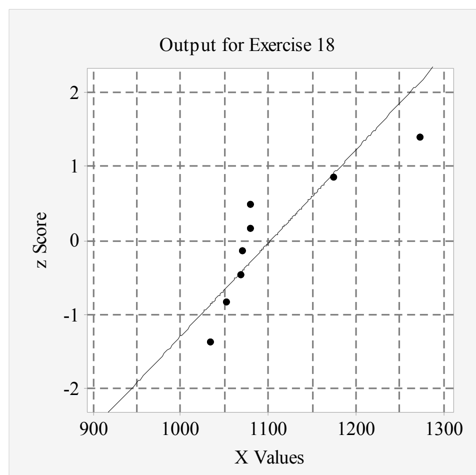




16. Normal, but there is a possible outlier.

17. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern. The points have coordinates $(32.5,-1.28)$, $(34.2,-0.52)$, $(38.5,0)$, $(40.7,0.52)$, and $(44.3,1.28)$.

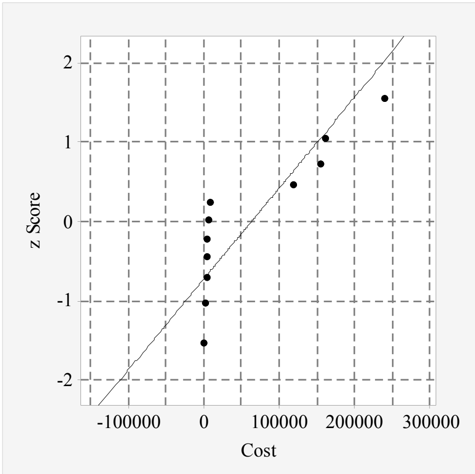


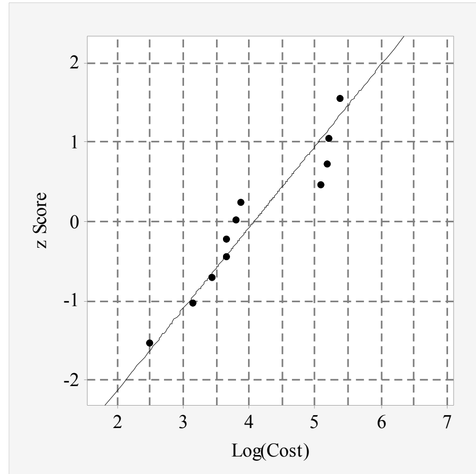


18. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern. The points have coordinates $(1034,-1.53)$, $(1051,-0.89)$, $(1067,-0.49)$, $(1070,-0.16)$, $(1079,0.16)$, $(1079,0.49)$, $(1173,0.89)$, and $(1272,1.53)$.

19. a. Yes, the histogram or normal quantile plot will remain unchanged.
    b. Yes, the histogram or normal quantile plot will remain unchanged.
    c. No, the histogram and normal quantile plot will not indicate a normal distribution.

20. The original values are not from a normally distributed population.





After taking the logarithm of each value, the values appear to be from a normally distributed population. The original values are from a population with a lognormal distribution.

Section 6-6: Normal as Approximation to Binomial

1. a. the area below (to the left of) 502.5
   b. the area between 501.5 and 502.5
   c. the area above (to the right of) 502.5

2. Yes, the circumstances correspond to 100 independent trials of a binomial experiment in which the probability of success is 0.2. Also, with $n = 100$, $p = 0.2$, and $q = 0.8$, the requirements of $np = 100 \cdot 0.2 = 20 \geq 5$ and $nq = 100 \cdot 0.8 = 80 \geq 5$ are both satisfied.

3. $p = 0.2$, $q = 0.8$, $\mu = 20$, $\sigma = 4$; The value of $\mu = 20$ shows that for people who make random guesses for the 100 questions, the mean number of correct answers is 20. For people who make 100 random guesses, the standard deviation of $\sigma = 4$ is a measure of how much the numbers of correct responses vary.

4. The histogram should be approximately normal or bell-shaped, because sample proportions tend to approximate a normal distribution.

5. The requirements for the normal approximation are satisfied with $np = 20 \cdot 0.512 = 10.2 \geq 5$ and $nq = 20 \cdot 0.488 = 9.8 \geq 5$. $z_{x=7.5} = \dfrac{7.5 - 20 \cdot 0.512}{\sqrt{20 \cdot 0.512 \cdot 0.488}} = -1.23$; which has a probability of 0.1093 (Tech: 0.1102) to the left.

6. The requirement of $np = 8 \cdot 0.512 = 4.1 \geq 5$ is not satisfied. The normal approximation should not be used.

7. The requirement of $np = 20 \cdot 0.2 = 4 \geq 5$ is not satisfied. The normal approximation should not be used.

8. The requirements for the normal approximation are satisfied with $np = 50 \cdot 0.2 = 10 \geq 5$ and $nq = 50 \cdot 0.8 = 40 \geq 5$. $z_{x=11.5} = \dfrac{11.5 - 50 \cdot 0.2}{\sqrt{50 \cdot 0.2 \cdot 0.8}} = 0.53$ and $z_{x=12.5} = \dfrac{12.5 - 50 \cdot 0.2}{\sqrt{50 \cdot 0.2 \cdot 0.8}} = 0.88$; which have a probability of $0.8106 - 0.7019 = 0.1087$ (Tech: 0.1096) between them.

9. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.35 = 35 \geq 5$ and $nq = 100 \cdot 0.65 = 65 \geq 5$. $z_{x=39.5} = \dfrac{39.5 - 100 \cdot 0.35}{\sqrt{100 \cdot 0.35 \cdot 0.65}} = 0.94$; which has a probability of $1 - 0.8264 = 0.1736$ (Tech: 0.1727) to the to the right. No, 40 people with blue eyes is not significantly high. (Tech: Using the binomial distribution: 0.1724.)

10. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.35 = 35 \geq 5$ and $nq = 100 \cdot 0.65 = 65 \geq 5$. $z_{x=49.5} = \dfrac{49.5 - 100 \cdot 0.35}{\sqrt{100 \cdot 0.35 \cdot 0.65}} = 3.04$; which has a probability of $1 - 0.9988 = 0.0012$ to the to the right. Yes, 49 people with blue eyes is significantly high. (Tech: Using the binomial distribution: 0.0027.)

11. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.12 = 12 \geq 5$ and $nq = 100 \cdot 0.88 = 88 \geq 5$. $z_{x=4.5} = \dfrac{4.5 - 100 \cdot 0.12}{\sqrt{100 \cdot 0.12 \cdot 0.88}} = -2.31$; which has a probability of 0.0104 (Tech: 0.0105) to the left. Yes, 4 people with green eyes is significantly low. (Tech: Using the binomial distribution: 0.0053.)

12. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.40 = 40 \geq 5$ and $nq = 100 \cdot 0.60 = 60 \geq 5$. $z_{x=33.5} = \dfrac{33.5 - 100 \cdot 0.40}{\sqrt{100 \cdot 0.40 \cdot 0.60}} = -1.33$; which has a probability of 0.0918 (Tech: 0.0923) to the left. No, 33 people with brown eyes is not significantly low. (Tech: Using the binomial distribution: 0.0913.)

13. a. The requirements for the normal approximation are satisfied with $np = 250 \cdot 0.10 = 25 \geq 5$ and $nq = 250 \cdot 0.90 = 225 \geq 5$. $z_{x=17.5} = \dfrac{17.5 - 250 \cdot 0.1}{\sqrt{250 \cdot 0.1 \cdot 0.9}} = -1.58$ and $z_{x=16.5} = \dfrac{16.5 - 250 \cdot 0.1}{\sqrt{250 \cdot 0.1 \cdot 0.9}} = -1.79 \ = 0.92$; which have a probability of $0.0571 - 0.0367 = 0.0204$ between them. (Tech: Using the binomial distribution: 0.0205.)

    b. The requirements for the normal approximation are satisfied with $np = 250 \cdot 0.10 = 25 \geq 5$ and $nq = 250 \cdot 0.90 = 225 \geq 5$. $z_{x=17.5} = \dfrac{17.5 - 250 \cdot 0.1}{\sqrt{250 \cdot 0.1 \cdot 0.9}} = -1.58$; which has a probability of 0.0571 to the left. (Tech: Using the binomial distribution: 0.0513.)

    c. The result of 17 cases of nausea is significantly low.

14. a. The requirements for the normal approximation are satisfied with $np = 929 \cdot 0.75 = 696.75 \geq 5$ and $nq = 929 \cdot 0.25 = 232.25 \geq 5$. $z_{x=704.5} = \dfrac{704.5 - 929 \cdot 0.75}{\sqrt{929 \cdot 0.75 \cdot 0.25}} = 0.59$; which has a probability of $1 - 0.7224 = 0.2776$ (Tech: 0.2785) to the right. (Tech: Using the binomial distribution: 0.2799.)

    b. The result of 705 peas with red flowers is not significantly high.

    c. The result of 705 peas with red flowers is not strong evidence against Mendel's assumption that $3/4$ of peas will have red flowers.

15. a. The requirements for the normal approximation are satisfied with $np = 1480 \cdot 0.292 = 432.16 \geq 5$ and $nq = 1480 \cdot 0.708 = 1047.84 \geq 5$. $z_{x=454.5} = \dfrac{454.5 - 1480 \cdot 0.292}{\sqrt{1480 \cdot 0.292 \cdot 0.708}} = 1.28$; which has a probability of $1 - 0.8997 = 0.1003$ (Tech: 0.1008) to the right. (Tech: Using the binomial distribution: 0.1012.)

    b. The result of 455 who have sleepwalked is not significantly high.

    c. The result of 455 does not provide strong evidence against the rate of 29.2%.

16. a. Using normal approximation: $\mu = 420{,}095 \cdot 0.00034 = 142.83$, $\sigma = \sqrt{420{,}095 \cdot 0.000344 \cdot 0.999656} = 11.9492$, and $z_{x=135.5} = \dfrac{135.5 - 142.83}{\sqrt{420{,}095 \cdot 0.000344 \cdot 0.999656}} = -0.61$; which has a probability of 0.2709 (Tech: 0.2697) to the left. (Tech: Using the binomial distribution: 0.2726.)

    b. Media reports appear to be wrong.

17. (1) The requirements for the normal approximation are satisfied with $np = 11 \cdot 0.512 = 5.632 \geq 5$ and $nq = 11 \cdot 0.488 = 5.368 \geq 5$. $z_{x=6.5} = \frac{6.5 - 11 \cdot 0.512}{\sqrt{11 \cdot 0.512 \cdot 0.488}} = 0.52$ and $z_{x=7.5} = \frac{7.5 - 11 \cdot 0.512}{\sqrt{11 \cdot 0.512 \cdot 0.488}} = 1.13$; which have a probability of $0.8708 - 0.6985 = 0.1723$ between them.

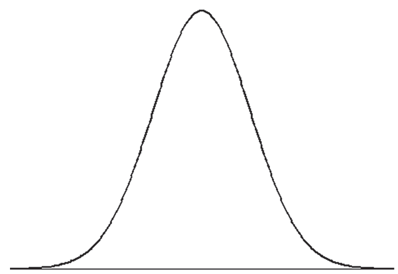
    (2) 0.1704

    (3) 0.1726

    No, the approximations are not off by very much.

Chapter Quick Quiz

1.



2. $z = P_9 = -1.34$

3. $P(z > -2.93) = 1 - P(z < -2.93) = 1 - 0.0017 = 0.9983$

4. $P(0.87 < z < 1.78) = P(z < 1.78) - P(z < 0.87) = 0.9625 - 0.8078 = 0.1547$ (Tech: 0.1546)

5. a. $\mu = 0$ and $\sigma = 1$

   b. $\mu_{\bar{x}}$ represents the mean of all sample means, and $\sigma_{\bar{x}}$ represents the standard deviation of all sample means.

6 $z_{x=80} = \frac{80.0 - 70.2}{11.2} = 0.88$; which has a probability of 0.8106 (Tech: 0.8092) to the left.

7. $z_{x=60} = \frac{60.0 - 70.2}{11.2} = -0.91$ and $z_{x=80} = \frac{80.0 - 70.2}{11.2} = 0.88$; which have a probability of $0.8106 - 0.1814 = 0.6292$ (Tech: 0.6280) between them.

8. The z score for P_{90} is 1.28, which corresponds to a diastolic blood pressure of $1.28 \cdot 11.2 + 70.2 = 84.5$ mm Hg (Tech: 84.6 mm Hg).

9. $z_{x=75} = \frac{75.0 - 70.2}{11.2/\sqrt{16}} = 1.71$; which has a probability of 0.9564 (Tech: 0.9568) to the left.

10. The normal quantile plot suggests that diastolic blood pressure levels of women are normally distributed.

Review Exercises

1. a. The probability to the left of a z score of 1.54 is 0.9382.

   b. The probability to the right of a z score of –1.54 is $1 - 0.0618 = 0.9382$.

   c. The probability between z scores –1.33 and 2.33 is $0.9901 - 0.0918 = 0.8983$.

   d. The z score for Q_1 is –0.67.

   e. $z_{x=0.50} = \frac{0.50 - 0.0}{1/\sqrt{9}} = 1.50$; which has a probability of $1 - 0.9332 = 0.0668$ to the right.

2. a. $z_{x=54} = \dfrac{54-59.7}{2.5} = -2.28;$ which has a probability of 0.0113 to the left, or 1.13%.

   b. The z score for the lowest 95% is 1.645 which corresponds to a standing eye height of $1.645 \cdot 2.5 + 59.7 = 63.8$ in.

3. a. $z_{x=70} = \dfrac{70-64.3}{2.6} = 2.19;$ which has a probability of $1-0.9857 = 0.0143$ to the right, or 1.43%. (Tech: 1.42%)

   b. The z score for the lowest 2% is –2.05 which corresponds to a standing eye height of $-20.05 \cdot 2.6 + 64.3 = 59.0$ in.

4. a. The distribution of samples means is normal.

   b. $\mu_{\bar{x}} = 100$

   c. $\sigma_{\bar{x}} = 15/\sqrt{64} = 1.875$

5. a. An unbiased estimator is a statistic that targets the value of the population parameter in the sense that the sampling distribution of the statistic has a mean that is equal to the mean of the corresponding parameter.

   b. mean, variance and proportion

   c. true

6. a. $z_{x=72} = \dfrac{72-68.6}{2.8} = 1.21;$ which has a probability of 0.8869, or 88.69% (Tech: 88.77%) to the left. With about 11% of all men needing to bend, the design does not appear to be adequate, but the Mark VI monorail appears to be working quite well in practice.

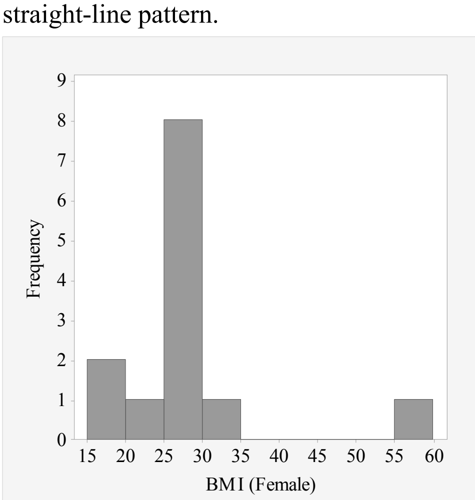
   b. The z score for 99% is 2.33 which corresponds to a doorway height of $2.33 \cdot 2.8 + 68.6 = 75.1$ in.

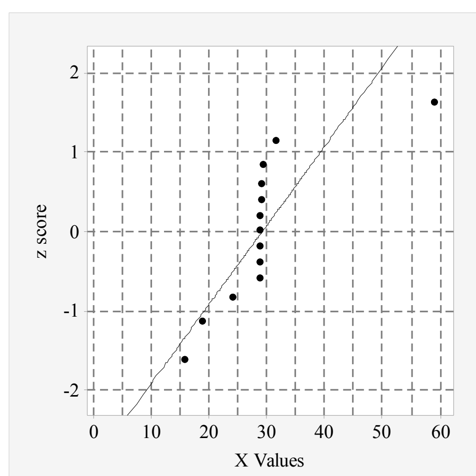
7. a. Because women are generally a little shorter than men, a doorway height that accommodates men will also accommodate women.

   b. $z_{x=72} = \dfrac{72-68.6}{2.8/\sqrt{60}} = 9.41;$ which has a probability of 0.9999, or 1 when rounded.

   c. Because the mean height of 60 men is less than 72 in., it does not follow that the 60 individual men all have heights less than 72 in. In determining the suitability of the door height for men, the mean of 60 heights is irrelevant, but the heights of individual men are relevant.

8. a. No, a histogram is far from bell shaped and a normal quantile plot reveals a pattern of points that is far from a straight-line pattern.





8. (continued)

b. No, the sample size $(n = 13)$ does not satisfy the condition of $n > 30$ and the values do not appear to be from a population having a normal distribution.

9. The requirements for the normal approximation are satisfied with $np = 1064 \cdot 0.75 = 798 \ge 5$ and $nq = 1064 \cdot 0.25 = 266 \ge 5$. $z_{z=787.5} = \dfrac{787.5 - 1064 \cdot 0.75}{\sqrt{11064 \cdot 0.75 \cdot 0.25}} = -0.74$; which has a probability of 0.2296 (Tech: 0.2286) to the left. The occurrence of 787 offspring plants with long stem is not unusually low because its probability is not small. The results are consistent with Mendel's claimed proportion of $3/4$. (Tech: Using the binomial distribution: 0.2278.)

10. a. $z_{x=70} = \dfrac{70.0 - 63.7}{2.9} = 2.17$; which has a probability of $1 - 0.9850 = 0.0150$, or 1.50% (Tech: 1.49%) to the right.

b. The z score for the upper 2.5% is 1.96 which corresponds to a doorway height of $1.96 \cdot 2.9 + 63.7 = 69.4$ in.

Cumulative Review Exercises

1. a. The mean is $\bar{x} = \dfrac{15.9 + 18.7 + 24.2 + 28.7 + 28.8 + 28.9 + 28.9 + 28.9 + 29.0 + 29.1 + 29.3 + 31.4}{12} = 26.82.$

b. The median is $\dfrac{28.9 + 28.9}{2} = 29.90.$

c. The standard deviation is $s = \sqrt{\dfrac{(15.9 - 26.82)^2 + \ldots + (31.4 - 26.82)^2}{12 - 1}} = 4.76.$

d. $z_{x=31.4} = \dfrac{31.4 - 26.82}{4.76} = 0.96$

e. ratio

f. continuous

2. a. Q_1: $L = \dfrac{25 \cdot 12}{100} = 3$, so $Q_1 = \dfrac{24.2 + 28.7}{2} = 26.45$

Q_2: $L = \dfrac{50 \cdot 12}{100} = 6$, so $Q_2 = \dfrac{28.9 + 28.9}{2} = 28.90$

Q_3: $L = \dfrac{75 \cdot 12}{100} = 9$, so $Q_3 = \dfrac{29.0 + 29.1}{2} = 29.05$

b.

c. The sample does not appear to be from a population having a normal distribution.

3. a. $0.10 \cdot 0.10 \cdot 0.10 = 0.001$

   b. $P(X \geq 1) = 1 - P(X = 0) = 1 - {}_3C_0 \cdot 0.10^0 \cdot 0.90^3 = 0.271$

   c. The requirement that $np > 5$ is not satisfied, indicating that the normal approximation would result in errors that are too large.

   d. $\mu = 50(0.10) = 5.0$ people

   e. $\sigma = \sqrt{50 \cdot 0.10 \cdot 0.90} = 2.1$ people

   f. Since 8 is not greater than $\mu + 2\sigma = 5.0 + 2(2.1) = 9.2$, it is not significantly high.

4. a. $\overline{B}$ is the event of selecting someone who does not have blue eyes.

   b. $P(\overline{B}) = 1 - 0.35 = 0.65$

   c. $0.35 \cdot 0.35 \cdot 0.35 = 0.0429$

   d. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.35 = 35 \geq 5$ and $nq = 100 \cdot 0.65 = 65 \geq 5$. $z_{x=39.5} = \dfrac{39.5 - 100 \cdot 0.35}{\sqrt{100 \cdot 0.35 \cdot 0.65}} = 0.94$; which has a probability of $1 - 0.8264 = 0.1736$ (Tech: 0.1727) to the right. (Tech: Using the binomial distribution: 0.1724.)

   e. No, 40 people with blue eyes is not significantly high.

5. a. $z_{x=10} = \dfrac{10 - 9.6}{0.5} = 0.80$; which has a probability of 0.7881 to the left.

   b. $z_{x=8.0} = \dfrac{8.0 - 9.6}{0.5} = -3.20$ and $z_{x=11.0} = \dfrac{11.0 - 9.6}{0.5} = 2.80$; which have a probability of $0.9974 - 0.0007 = 0.9967$ (Tech: 0.9968) between them.

   c. The z score for the top 5% is 1.645, which correspond to the length $1.645 \cdot 0.5 + 9.6 = 10.4$ in.

   d. $z_{x=9.8} = \dfrac{9.8 - 9.6}{0.5/\sqrt{25}} = 2.00$; which has a probability of $1 - 0.9772 = 0.0228$ to the right.