Chapter 7: Estimating Parameters and Determining Sample Sizes

Section 7-1: Estimating a Population Proportion

1. The confidence level (such as 95%) was not provided.

2. When using 14% to estimate the value of the population percentage for those who prefer chocolate pie, the maximum likely difference between 14% and the true population percentage is four percentage points, so the interval from $14\% - 4\% = 10\%$ to $14\% + 4\% = 18\%$ is likely to contain the true population percentage.

3. $\hat{p} = 0.14$ is the sample proportion; $\hat{q} = 0.86$ (found from evaluating $1 - \hat{p}$); $n = 1000$ is the sample size; $E = 0.04$ is the margin of error; p is the population proportion, which is unknown. The value of α is 0.05.

4. The 95% confidence interval will be wider than the 80% confidence interval. A confidence interval must be wider in order for us to be more confident that it captures the true value of the population proportion. (Think of estimating the age of a classmate. You might be 90% confident that she is between 20 and 30, but you might be 99.9% confident that she is between 10 and 40.)

5. $z_{0.05} = 1.645$

6. $z_{0.005} = 2.576$ (Table: 2.575)

7. $z_{0.0025} = 2.81$

8. $z_{0.01} = 2.33$

9. $\hat{p} = \dfrac{0.375 + 0.425}{2} = 0.400$ and $E = \dfrac{0.425 - 0.375}{2} = 0.025$; 0.400 ± 0.025

10. $\hat{p} = \dfrac{0.275 + .0425}{2} = 0.350$ and $E = \dfrac{0.425 - 0.275}{2} = 0.075$; 0.350 ± 0.075

11. $\hat{p} = \dfrac{0.0780 + 0.162}{2} = 0.120$ and $E = \dfrac{0.162 - 0.0780}{2} = 0.042$; $0.120 - 0.042 < p < 0.120 + 0.42$

12. $0.070 - 0.021 < p < 0.070 + 0.021$, or $0.049 < p < 0.091$

13. a. $\hat{p} = 16/227 = 0.0705$

    b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\dfrac{\left(\frac{16}{227}\right)\left(\frac{211}{227}\right)}{227}} = 0.0333$

    c. $\hat{p} - E < p < \hat{p} - E$

    $0.0705 - 0.0333 < p < 0.0705 + 0.0333$

    $0.0371 < p < 0.104$ (Tech: $0.0372 < p < 0.104$)

    d. We have 95% confidence that the interval from 0.0371 to 0.104 actually does contain the true value of the population proportion of subjects treated with OxyContin who experience headaches.

14. a. $\hat{p} = 153/5924 = 0.0258$

    b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 2.58\sqrt{\dfrac{\left(\frac{153}{5924}\right)\left(\frac{771}{5924}\right)}{5924}} = 0.00531$

    c. $\hat{p} - E < p < \hat{p} - E$

    $0.0258 - 0.00531 < p < 0.0258 + 0.00531$

    $0.0205 < p < 0.0311$

    d. We have 99% confidence that the interval from 0.0205 to 0.0311 actually does contain the true value of the population proportion of all Eliquis users who experience nausea.

15. a. $\hat{p} = 717/5000 = 0.143$

b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 1.645\sqrt{\dfrac{\left(\frac{717}{5000}\right)\left(\frac{4283}{5000}\right)}{5000}} = 0.00815$

c.
$$\hat{p} - E < p < \hat{p} - E$$
$$0.143 - 0.00815 < p < 0.143 + 0.00815$$
$$0.135 < p < 0.152$$

d. We have 90% confidence that the interval from 0.135 to 0.152 actually does contain the true value of the population proportion of returned surveys.

16. a. $\hat{p} = 856/1228 = 0.697$

b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\dfrac{\left(\frac{856}{1228}\right)\left(\frac{372}{1228}\right)}{1228}} = 0.0257$

c.
$$\hat{p} - E < p < \hat{p} - E$$
$$0.697 - 0.0257 < p < 0.697 + 0.0257$$
$$0.671 < p < 0.723$$

d. We have 95% confidence that the interval from 0.671 to 0.723 actually does contain the true value of the population proportion of medical malpractice lawsuits that are dropped or dismissed.

17. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{426}{860} \pm 1.96\sqrt{\dfrac{\left(\frac{426}{860}\right)\left(\frac{434}{860}\right)}{860}} \Rightarrow 0.462 < p < 0.529$; Because 0.512 is contained within the confidence interval, there is not strong evidence against 0.512 as the value of the proportion of boys in all births.

18. a. 99% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{428}{580} \pm 2.58\sqrt{\dfrac{\left(\frac{428}{580}\right)\left(\frac{152}{580}\right)}{580}} \Rightarrow 0.691 < p < 0.785$, or $69.1\% < p < 78.5\%$

b. No, the confidence interval includes 75%, so the true percentage could easily equal 75%.

19. a. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{52}{227} \pm 1.96\sqrt{\dfrac{\left(\frac{52}{227}\right)\left(\frac{175}{227}\right)}{227}} \Rightarrow 0.174 < p < 0.284$, or $17.4\% < p < 28.4\%$

b. Because the two confidence intervals overlap, it is possible that the OxyContin treatment group and the placebo group have the same rate of nausea. Nausea does not appear to be an adverse reaction made worse with OxyContin.

20. a. $3005(0.817) = 2455$

b. 90% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.817 \pm 1.645\sqrt{\dfrac{(0.817)(0.183)}{3005}} \Rightarrow 0.805 < p < 0.829$, or $80.5\% < p < 82.9\%$

c. nothing

21. a. 90% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.000321 \pm 1.645\sqrt{\dfrac{(0.000321)(0.999679)}{420,095}} \Rightarrow 0.000276 < p < 0.000366$, or $0.0276\% < p < 0.0366\%$; (Using $x = 135$: $0.0276\% < p < 0.0367\%$)

b. No, because 0.0340% is included in the confidence interval.

22. Placebo group:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{7}{270} \pm 1.96\sqrt{\dfrac{\left(\frac{7}{270}\right)\left(\frac{263}{270}\right)}{270}} \Rightarrow 0.00697 < p < 0.0449$, or $0.697\% < p < 4.49\%$

22. (continued)

Treatment group:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{8}{863} \pm 1.96\sqrt{\dfrac{\left(\frac{7}{863}\right)\left(\frac{855}{863}\right)}{863}} \Rightarrow 0.00288 < p < 0.0157$, or $0.288\% < p < 1.57\%$

Because the two confidence intervals overlap, there does not appear to be a significant difference between the rates of allergic reactions. Allergic reactions do not appear to be a concern for Lipitor users.

23. XSORT: 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{879}{945} \pm 1.96\sqrt{\dfrac{\left(\frac{879}{945}\right)\left(\frac{66}{945}\right)}{945}} \Rightarrow 0.914 < p < 0.946$, or $91.4\% < p < 94.6\%$

YSORT: 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{239}{291} \pm 1.96\sqrt{\dfrac{\left(\frac{239}{291}\right)\left(\frac{52}{291}\right)}{291}} \Rightarrow 0.777 < p < 0.865$, or $77.7\% < p < 86.5\%$

The two confidence intervals do not overlap. It appears that the success rate for the XSORT method is higher than the success rate for the YSORT method, but the big story here is that the XSORT method and the YSORT method both appear to be very effective because the success rates are well above the 50% rates expected with no treatments.

24. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{239}{291} \pm 1.96\sqrt{\dfrac{\left(\frac{239}{291}\right)\left(\frac{52}{291}\right)}{291}} \Rightarrow 0.777 < p < 0.865$, or $77.7\% < p < 86.5\%$

It appears the YSORT method was very effective because the success rate is well above the 50% rates expected with no treatments.

25. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{6062}{12{,}000} \pm 1.96\sqrt{\dfrac{\left(\frac{6062}{12{,}000}\right)\left(\frac{5938}{12{,}000}\right)}{12{,}000}} \Rightarrow 0.496 < p < 0.514$, or $59.6\% < p < 51.4\%$; No, because the proportion could easily equal 0.5. The proportion does not appear to be significantly less than 0.5 the week before Thanksgiving.

26. 99% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{901}{1012} \pm 2.56\sqrt{\dfrac{\left(\frac{901}{1012}\right)\left(\frac{111}{1012}\right)}{1012}} \Rightarrow 0.871 < p < 0.910$ or $87.1\% < p < 91.0\%$; Yes, because the proportion that oppose cloning appears to be greater than 0.5.

27. Sustained care:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.828 \pm 1.96\sqrt{\dfrac{(0.828)(0.172)}{198}} \Rightarrow 0.775 < p < 0.881$, or $77.5\% < p < 88.1\%$; (Using $x = 164$: $77.5\% < p < 88.1\%$)

Standard care:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.628 \pm 1.96\sqrt{\dfrac{(0.628)(0.372)}{199}} \Rightarrow 0.561 < p < 0.695$, or $56.1\% < p < 69.5\%$; The two confidence intervals do not overlap. It appears that the success rate is higher with sustained care.

28. Measured:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.258 \pm 1.96\sqrt{\dfrac{(0.258)(0.742)}{198}} \Rightarrow 0.197 < p < 0.319$, or $19.7\% < p < 31.9\%$;

(Using $x = 51$: $19.7\% < p < 31.8\%$)

28. (continued)

Reported:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.409 \pm 1.96\sqrt{\frac{(0.409)(0.591)}{198}} \Rightarrow 0.341 < p < 0.477$, or $34.1\% < p < 47.7\%$;

(Using $x = 81$: $34.1\% < p < 47.8\%$)

The two confidence intervals do not overlap. It appears that the success rate is higher with sustained care.

29. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 \cdot 0.25}{0.03^2} = 1842$ (Tech: 1844)

b. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 \cdot (0.10)(0.90)}{0.03^2} = 664$

c. They don't change.

30. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.645]^2 \cdot 0.25}{0.02^2} = 1692$ (Tech: 1691)

b. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.645]^2 \cdot (0.95)(0.05)}{0.02^2} = 332$

c. Yes, the added knowledge results in a very substantial decrease in the required sample size.

31. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.96]^2 \cdot 0.25}{0.05^2} = 385$

b. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.96]^2 \cdot (0.40)(0.60)}{0.05^2} = 369$

c. No, the sample size doesn't change much.

32. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 \cdot 0.25}{0.04^2} = 1037$

b. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 \cdot (0.26)(0.74)}{0.04^2} = 798$

33. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.96]^2 \cdot 0.25}{0.025^2} = 1537$

b. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.96]^2 \cdot (0.53)(0.47)}{0.025^2} = 1532$

34. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 \cdot 0.25}{0.02^2} = 4145$ (Tech: 4147)

b. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 \cdot (0.45)(0.55)}{0.02^2} = 4103$ (Tech: 4106)

c. No, the sample size doesn't change much.

35. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.645]^2 \cdot 0.25}{0.03^2} = 752$

35. (continued)

b. $n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[1.645]^2 \cdot (0.53)(0.47)}{0.03^2} = 749$

c. No. A sample of the people you know is a convenience sample, not a simple random sample, so it is very possible that the results would not be representative of the population.

36. a. $n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[2.575]^2 \cdot 0.25}{0.02^2} = 4145$ (Tech: 4147)

b. $n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[2.575]^2 \cdot (0.82)(0.18)}{0.02^2} = 2447$ (Tech: 2449)

c. Randomly selecting adult women would result in an underestimate, because some women will give birth to their first child after the survey was conducted. It will be important to survey women who have completed the time during which they can give birth.

37. $n = \dfrac{N\hat{p}\hat{q}[z_{\alpha/2}]^2}{\hat{p}\hat{q}[z_{\alpha/2}]^2 + (N-1)E^2} = \dfrac{2500(0.82)(0.18)[2.575]^2}{(0.82)(0.18)[2.575]^2 + (2500-1)0.02^2} = 1237$ (Tech: 1238)

38. Because we have 95% confidence that p is less than 0.0991, we can safely conclude that rate of headaches among OxyContin users is less than 10%.

$$\hat{p} + z_\alpha\sqrt{\frac{\hat{p}\hat{q}}{n}} = \tfrac{16}{227} + 1.645\sqrt{\frac{\left(\frac{16}{227}\right)\left(\frac{211}{227}\right)}{227}} \Rightarrow p < 0.0991$$

39. a. The requirement of at least 5 successes and at least 5 failures is not satisfied, so the normal distribution cannot be used.

b. $3/40 = 0.075$

Section 7-2: Estimating a Population Mean

1. a. $12.855 \text{ g/dL} < \mu < 13.391 \text{ g/dL}$

b. The best point estimate of μ is $\bar{x} = \dfrac{12.855 + 13.391}{2} = 13.123$ g/dL. The margin of error is

$E = \dfrac{13.391 - 12.855}{2} = 0.268$ g/dL.

c Because the sample size of 50 is greater than 30, we can consider the sample mean to be from a population with a normal distribution.

2. a. $\text{df} = 100 - 1 = 99$

b. 1.9842 (Table: 1.984)

c. In general, the number of degrees of freedom for a collection of sample data is the number of sample values that can vary after certain restrictions have been imposed on all data values.

3. We have 95% confidence that the limits of 12.855 g/dL and 13.391 g/dL contain the true value of the mean hemoglobin level of the population of all adult females.

4. When we say that the confidence interval methods of this section are robust against departures from normality, we mean that these methods work reasonably well with distributions that are not normal, provided that departures from normality are not too extreme.

5. Neither the normal nor the t distribution applies.

6. $t_{\alpha/2} = 1.671$

7. $z_{\alpha/2} = 2.576$ (Table: 2.575)

8. $t_{\alpha/2} = 1.972$

9. The sample size is greater than 30 and the data appear to be from a population that is normally distributed.
95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 30.4 \pm 1.972 \cdot \frac{7.1}{\sqrt{205}} \Rightarrow 29.4 \text{ hg} < \mu < 31.4 \text{ hg};$ No, the results do not differ by much.

10. The sample size is greater than 30 and the data appear to be from a population that is normally distributed.
95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 32.7 \pm 1.972 \cdot \frac{6.6}{\sqrt{205}} \Rightarrow 31.8 \text{ hg} < \mu < 36.6 \text{ hg};$ Yes, it appears that birth weights of boys are substantially greater than birth weights of girls.

11. The sample size is greater than 30 and the data appear to be from a population that is normally distributed.
95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 98.20 \pm 1.983 \cdot \frac{0.62}{\sqrt{106}} \Rightarrow 98.08°\text{F} < \mu < 98.32°\text{F};$ Because the confidence interval does not contain $98.6°\text{F},$ it appears that the mean body temperature is not $98.6°\text{F},$ as is commonly believed.

12. The sample size is greater than 30. 90% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 2.1 \pm 1.685 \cdot \frac{4.8}{\sqrt{40}} \Rightarrow 0.8 \text{ lb} < \mu < 3.4 \text{ lb};$ Because the confidence interval does not include 0 or negative values, it does appear that the weight loss program is effective, with a positive loss of weight. Because the amount of weight lost is relatively small, the weight loss program does not appear to be very practical.

13. It is assumed that the 16 sample values appear to be from a normally distributed population.
98% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 98.9 \pm 2.602 \cdot \frac{42.3}{\sqrt{16}} \Rightarrow 71.4 \text{ min} < \mu < 126.4 \text{ min};$ The confidence interval includes the mean of 102.8 min that was measured before the treatment, so the mean could be the same after the treatment. This result suggests that the zopiclone treatment does not have a significant effect.

14. The sample size is greater than 30. 98% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 0.4 \pm 2.407 \cdot \frac{21.0}{\sqrt{49}} \Rightarrow -6.8 \text{ mg/dL} < \mu < 7.6 \text{ mg/dL};$
Because the confidence interval includes the value of 0, it is very possible that the mean of the changes in LDL cholesterol is equal to 0, suggesting that the garlic treatment did not affect LDL cholesterol levels. It does not appear that garlic has a significant effect in reducing LDL cholesterol.

15. The sample appears to have a normal distribution. 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 2.6 \pm 2.262 \cdot \frac{1.07}{\sqrt{10}} \Rightarrow 1.8 < \mu < 3.4;$
The given numbers are just substitutes for the four DNA base names, so the numbers don't measure or count anything, and they are at the nominal level of measurement. The confidence interval has no practical use.

16. The sample appears to have a normal distribution. 90% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 6.35 \pm 1.833 \cdot \frac{2.325}{\sqrt{10}}$
$\Rightarrow 5.00\ \mu\text{g} < \mu < 7.70\ \mu\text{g};$ No, the samples obtained from California might be very different from those obtained in Arkansas.

17. The sample data meet the loose requirement of having a normal distribution.
90% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 0.9382 \pm 1.8125 \cdot \frac{0.4229}{\sqrt{11}} \Rightarrow 0.707 \text{ W/kg} < \mu < 1.169 \text{ W/kg};$ Because the confidence interval is entirely below the standard of 1.6 W/kg, it appears that the mean amount of cell phone radiation is less than the FCC standard, but there could be individual cell phones that exceed the standard.

18. The sample data meet the loose requirement of having a normal distribution.
95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 11.05 \pm 2.262 \cdot \frac{6.46}{\sqrt{10}} \Rightarrow 6.43\ \mu\text{g/g} < \mu < 15.7\ \mu\text{g/g};$ No, it does not appear that the lead concentrations are less than $7\ \mu\text{g/g}$ since the upper limit of the confidence exceeds that amount.

19. The sample appears to have a normal distribution. 98% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 0.719 \pm 3.143 \cdot \dfrac{0.366}{\sqrt{7}}$ $\Rightarrow 0.284 \text{ ppm} < \mu < 1.153 \text{ ppm}$; Using the FDA guideline, the confidence interval suggests that there could be too much mercury in fish because it is possible that the mean is greater than 1 ppm. Also, one of the sample values exceeds the FDA guideline of 1 ppm, so at least some of the fish have too much mercury.

20. The data appear to have a distribution that is far from normal, so the confidence interval might not be a good estimate of the population mean. 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 6.5 \pm 1.729 \cdot \dfrac{3.51}{\sqrt{20}}$ $\Rightarrow 5.1 \text{ years} < \mu < 7.9 \text{ years}$ (Tech: $4.9 \text{ years} < \mu < 8.1 \text{ years}$); The confidence interval does not contain the value of 4 years, but six of the twenty values are 4, so it is common to earn a bachelor's degree in four years, but the typical college student uses more than 4 years.

21. The presence of five zeros suggests that the sample is not from a normally distributed population, so the normality requirement is violated and the confidence interval might not be a good estimate of the population mean. 99% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 32.6 \pm 2.861 \cdot \dfrac{20.33}{\sqrt{20}} \Rightarrow 19.5 \text{ mg} < \mu < 45.6 \text{ mg}$; People consume some brands much more often than others, but the 20 brands are all weighted equally in the calculations. This, along with the violation of the normality requirement, means the confidence interval might not be a good estimate of the population mean.

22. It must be assumed the samples have a normal distribution.

    a. Manual: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 175 \pm 2.262 \cdot \dfrac{15}{\sqrt{10}} \Rightarrow 164 \text{ BPM} < \mu < 186 \text{ BPM}$

    b. Electric: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 124 \pm 2.262 \cdot \dfrac{18}{\sqrt{10}} \Rightarrow 111 \text{ BPM} < \mu < 137 \text{ BPM}$

    c. The upper limit, which shows maximum strain on the heart.

    d. Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do not overlap, so there does appear to be a significant difference between the mean pulse rate of manual snow shoveling and using an electric snow thrower, with manual snow shoveling resulting in higher pulse rates.

23. The samples sizes are both large.

    a. Echinacea: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 6.0 \pm 1.967 \cdot \dfrac{2.3}{\sqrt{337}} \Rightarrow 5.8 \text{ days} < \mu < 6.2 \text{ days}$

    b. Placebo: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 6.1 \pm 1.966 \cdot \dfrac{2.4}{\sqrt{370}} \Rightarrow 5.9 \text{ days} < \mu < 6.3 \text{ days}$

    c. The two confidence intervals are very similar. The Echinacea treatment group does not appear to fare any better than the placebo group, so the Echinacea treatment does not appear to be effective.

24. The samples sizes are both large.

    a. Acupuncture: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 1.8 \pm 1.977 \cdot \dfrac{1.4}{\sqrt{142}} \Rightarrow 1.6 < \mu < 2.0$

    b. Sham Treatment: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 1.6 \pm 1.990 \cdot \dfrac{1.2}{\sqrt{80}} \Rightarrow 1.3 < \mu < 1.9$

    c. Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do overlap, so there does not appear to be a significant difference between the acupuncture treatment and the sham treatment.

25. Both samples appear to have a normal distribution.

Males: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 69.58 \pm 1.976 \cdot \dfrac{0.916}{\sqrt{153}} \Rightarrow 67.8 \text{ bpm} < \mu < 71.4 \text{ bpm}$

Females: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 74.0 \pm 1.976 \cdot \dfrac{1.03}{\sqrt{147}} \Rightarrow 72.0 \text{ bpm} < \mu < 76.1 \text{ bpm}$

Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do not overlap, so adult females appear to have a mean pulse rate that is higher than the mean pulse rate of adult males.

26. Neither sample appears to have a normal distribution, so the normality requirement is violated and the confidence intervals might not be good estimates of the population means.

a. King: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 1.26 \pm 2.064 \cdot \dfrac{0.233}{\sqrt{25}} \Rightarrow 1.16 \text{ mg} < \mu < 1.35 \text{ mg}$

b. 100: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 0.9 \pm 2.064 \cdot \dfrac{0.248}{\sqrt{25}} \Rightarrow 0.8 \text{ mg} < \mu < 1.0 \text{ mg}$

c. Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do not overlap, so the 100 mm cigarettes appear to have a mean nicotine level that is lower than king size cigarettes.

27. The sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.575 \cdot 15}{4}\right]^2 = 94$. This does appear to be very practical.

28. The sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.326 \cdot 15}{3}\right]^2 = 136$. This does appear to be very practical.

29. The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{1.96 \cdot 1.0}{0.01}\right]^2 = 38{,}416$ (Tech: 38,415). This does not appear to be very practical.

30. The sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{1.645 \cdot 17.65}{1.5}\right]^2 = 375$. Yes, the assumption seems reasonable.

31. The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{1.96 \cdot 17.7}{0.5}\right]^2 = 4815$ (Tech: 4814). Yes, the assumption seems reasonable.

32. a. $\sigma \approx \dfrac{104-36}{4} = 17.0$; The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.575 \cdot 17.0}{2}\right]^2 = 480$.

b. The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.575 \cdot 12.5}{2}\right]^2 = 260$.

c. The result from part (a) is substantially larger than the result from part (b). The result from part (b) is likely to be better because it uses s instead of the estimated σ obtained from the range rule of thumb.

33. a. $\sigma \approx \dfrac{104-40}{4} = 16.0$; The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.575 \cdot 16.0}{2}\right]^2 = 425$.

b. The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.575 \cdot 11.3}{2}\right]^2 = 212$.

c. The result from part (a) is substantially larger than the result from part (b). The result from part (b) is likely to be better because it uses s instead of the estimated σ obtained from the range rule of thumb.

34. a. $\sigma \approx \dfrac{99.6-96.5}{4} = 0.775$; The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.326 \cdot 0.775}{0.1}\right]^2 = 326$ (Table: 327).

b. The required sample size is $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.326 \cdot 0.62}{0.1}\right]^2 = 209$.

c. The result from part (a) is substantially larger than the result from part (b). The result from part (b) is likely to be better because it uses s instead of the estimated σ obtained from the range rule of thumb.

35. a. Large pop: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 255.1 \pm 2.023 \cdot \dfrac{65.4}{\sqrt{40}}$

$\Rightarrow 234.2\ (1000\ \text{cells}/\mu\text{L}) < \mu < 276.0\ (1000\ \text{cells}/\mu\text{L})$

b. Finite pop: 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}}\sqrt{\dfrac{N-n}{n-1}} = 255.1 \pm 2.26 \cdot \dfrac{65.4}{\sqrt{40}}\sqrt{\dfrac{500-40}{400-1}}$

$\Rightarrow 235.0\ (1000\ \text{cells}/\mu\text{L}) < \mu < 275.2\ (1000\ \text{cells}/\mu\text{L})$

c. The second confidence interval is narrower, indicating that we have a more accurate estimate when the relatively large sample is selected without replacement from a relatively small finite population.

Section 7-3: Estimating a Population Standard Deviation or Variance

1. $\sqrt{9027.8\ \left(\text{cm}^3\right)^2} < \sqrt{\sigma^2} < \sqrt{33299.8\ \left(\text{cm}^3\right)^2} \Rightarrow 95.0\ \text{cm}^3 < \sigma < 182.5\ \text{cm}^3$. We have 95% confidence that the limits of 95.0 cm^3 and 182.5 cm^3 contain the true value of the standard deviation of brain volumes.

2. The format implies that $s = 15.7$, but s is given as 14.3. In general, a confidence interval for σ does not have s at the center.

3. The dotplot does not appear to depict sample data from a normally distributed population. The large sample size does not justify treating the values as being from a normally distributed population. Because the normality requirement is not satisfied, the confidence interval estimate of s should not be constructed using the methods of this section.

4. The normality requirement for a confidence interval estimate of σ has a much stricter normality requirement than the loose normality requirement for a confidence interval estimate of μ. Departures from normality have a much greater effect on confidence interval estimates of σ than on confidence interval estimates of μ.

5. df = 24, $\chi_L^2 = 12.401$, and $\chi_R^2 = 39.364$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(25-1)0.24^2}{39.364}} < \sigma < \sqrt{\frac{(25-1)0.24^2}{12.401}}$$

95% CI: $0.19\ \text{mg} < \sigma < 0.33\ \text{mg}$

6. df = 152, $\chi_L^2 = 119.759$, and $\chi_R^2 = 118.026$

(Table: $\chi_L^2 = 74.222$, and $\chi_R^2 = 129.561$)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(153-1)0.01648^2}{118.026}} < \sigma < \sqrt{\frac{(153-1)0.01648^2}{119.759}}$$

95% CI: $0.0187\ \text{g} < \sigma < 0.0186\ \text{g}$

(Table: $0.0179\ \text{g} < \sigma < 0.0226\ \text{g}$)

7. df $= 146$, $\chi_L^2 = 105.761$, and $\chi_R^2 = 193.761$

(Table: $\chi_L^2 = 67.328$, and $\chi_R^2 = 140.169$)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(147-1)65.4^2}{193.761}} < \sigma < \sqrt{\frac{(147-1)65.4^2}{105.741}}$$

99% CI: $56.8 < \sigma < 76.8$

(Table: $66.7 < \sigma < 96.3$)

8. df $= 152$, $\chi_L^2 = 110.846$, and $\chi_R^2 = 200.657$

(Table: $\chi_L^2 = 67.328$, and $\chi_R^2 = 140.169$)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(153-1)65.4^2}{200.657}} < \sigma < \sqrt{\frac{(153-1)65.4^2}{110.846}}$$

99% CI: 6.18 cm $< \sigma <$ 8.31 cm

(Table: 7.39 cm $< \sigma <$ 10.68 cm)

9. df $= 100$, $\chi_L^2 = 74.222$, and $\chi_R^2 = 129.561$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(106-1)0.62^2}{129.561}} < \sigma < \sqrt{\frac{(106-1)0.62^2}{74.222}}$$

95% CI: 0.56°F $< \sigma < 0.74^\circ$F

(Tech: 0.55°F $< \sigma < 0.72^\circ$F)

10. df $= 40$, $\chi_L^2 = 26.509$, and $\chi_R^2 = 55.758$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(40-1)4.8^2}{55.758}} < \sigma < \sqrt{\frac{(40-1)4.8^2}{26.509}}$$

90% CI: 4.0 lb $< \sigma <$ 5.9 lb

(Tech: 4.1 lb $< \sigma <$ 5.8 lb)

The confidence interval gives us information about the variation among the amounts of lost weight, but it does not give us information about the effectiveness. of the diet.

11. df $= 15$, $\chi_L^2 = 5.229$, and $\chi_R^2 = 30.578$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(16-1)42.3^2}{30.578}} < \sigma < \sqrt{\frac{(16-1)42.3^2}{5.229}}$$

98% CI: 29.6 min $< \sigma <$ 71.6 min

No, the confidence interval does not indicate whether the treatment is effective.

12. df $= 50$, $\chi_L^2 = 29.707$, and $\chi_R^2 = 76.154$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(49-1)21.0^2}{76.154}} < \sigma < \sqrt{\frac{(49-1)21.0^2}{29.707}}$$

98% CI: 16.7 mg/dL $< \sigma <$ 26.7 mg/dL

(Tech: 16.9 mg/dL $< \sigma <$ 27.4 mg/dL)

No, the confidence interval does not indicate whether the treatment is effective.

13. The sample appears to have a normal distribution.

df $= 14$, $\chi_L^2 = 5.629$, and $\chi_R^2 = 26.119$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(15-1)0.25875^2}{26.119}} < \sigma < \sqrt{\frac{(15-1)0.25875^2}{5.629}}$$

95% CI: 0.019 g $< \sigma <$ 0.041 g

14. The samples must be assumed to have normal distributions.

Single Line

$\text{df} = 9,\ \chi_L^2 = 2.700,\ \text{and}\ \chi_R^2 = 19.023$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(10-1)28.6^2}{19.023}} < \sigma < \sqrt{\frac{(10-1)28.6^2}{2.700}}$$

95% CI: 19.7 sec $< \sigma <$ 52.2 sec

Individual Lines

$\text{df} = 9,\ \chi_L^2 = 2.700,\ \text{and}\ \chi_R^2 = 19.023$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(10-1)109.3^2}{19.023}} < \sigma < \sqrt{\frac{(10-1)109.3^2}{2.700}}$$

95% CI: 75.0 sec $< \sigma <$ 200.0 sec

The variation appears to be significantly higher for individual lines, so a single line appears to be better.

15. The samples must be assumed to have normal distributions.

a. Manual Snow Shoveling

$\text{df} = 9,\ \chi_L^2 = 2.700,\ \text{and}\ \chi_R^2 = 19.023$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(10-1)15^2}{19.023}} < \sigma < \sqrt{\frac{(142-1)15^2}{2.700}}$$

95% CI: 10.3 BPM $< \sigma <$ 27.4 BPM

b. Electric Snow Thrower

$\text{df} = 9,\ \chi_L^2 = 2.700,\ \text{and}\ \chi_R^2 = 19.023$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(10-1)18^2}{19.023}} < \sigma < \sqrt{\frac{(142-1)18^2}{2.700}}$$

95% CI: 12.4 BPM $< \sigma <$ 32.9 BPM

c. The variation does not appear to be significantly different.

16. The samples must be assumed to have normal distributions.

a. Acupuncture

$\text{df} = 141,\ \chi_L^2 = 110.020,\ \text{and}\ \chi_R^2 = 175.765$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(142-1)1.4^2}{175.765}} < \sigma < \sqrt{\frac{(142-1)1.4^2}{110.020}}$$

95% CI: $1.3 < \sigma < 1.6$

b. Sham Treatment

$\text{df} = 79,\ \chi_L^2 = 56.309,\ \text{and}\ \chi_R^2 = 105.473$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(80-1)1.2^2}{105.473}} < \sigma < \sqrt{\frac{(80-1)1.2^2}{56.309}}$$

95% CI: $1.0 < \sigma < 1.4$

c. The variation does not appear to be significantly different.

17. a. Girls: The sample appears to be from a population with a distribution that is far from normal, so the confidence interval estimate might not be very good.

$\text{df} = 204,\ \chi_L^2 = 166.338,\ \text{and}\ \chi_R^2 = 245.448$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(205-1)9.7^2}{245.448}9} < \sigma < \sqrt{\frac{(205-1)9.7^2}{166.338}}$$

95% CI: 8.8 days $< \sigma <$ 10.7 days

b. Boys: The sample appears to be from a population with a distribution that is far from normal, so the confidence interval estimate might not be very good.

$\text{df} = 79,\ \chi_L^2 = 0.000,\ \text{and}\ \chi_R^2 = 0.000$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(195-1)8.2^2}{234.465}} < \sigma < \sqrt{\frac{(195-1)8.2^2}{157.321}}$$

95% CI: $7.5 < \sigma < 9.2$

c. The variation does not appear to be significantly different.

18. a. The sample appears to have a normal distribution.

df = 204, $\chi_L^2 = 166.338$, and $\chi_R^2 = 245.448$ (Using technology.)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(205-1)706.3^2}{245.448}} < \sigma < \sqrt{\frac{(205-1)706.3^2}{166.338}}$$

95% CI: 643.9 g $< \sigma <$ 782.1 g

b. The sample appears to have a normal distribution.

df = 194, $\chi_L^2 = 157.321$, and $\chi_R^2 = 234.465$ (Using technology.)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(195-1)660.2^2}{234.465}} < \sigma < \sqrt{\frac{(195-1)660.2^2}{157.321}}$$

95% CI: 600.5 g $< \sigma <$ 733.1 g

c. The amounts of variation are about the same.

19. 19,205 is too large. There are not 19,205 statistics professors in the population, and even if there were, that sample size is too large to be practical.

20. 33,218 is too large for most sample designs.

21. The sample size is 48. No, with many very low incomes and a few high incomes, the distribution is likely to be skewed to the right and will not satisfy the requirement of a normal distribution.

22. The sample size is 336.

23. $\chi_L^2 = \frac{1}{2}\left[-z_{\alpha/2} + \sqrt{2k-1}\right]^2 = \frac{1}{2}\left[-1.96 + \sqrt{2\cdot 151-1}\right]^2 = 109.565$ and

$\chi_R^2 = \frac{1}{2}\left[z_{\alpha/2} + \sqrt{2k-1}\right]^2 = \frac{1}{2}\left[1.96 + \sqrt{2\cdot 151-1}\right]^2 = 175.276$

The approximate values are quite close to the actual critical values.

24. $n = \frac{1}{2}\left(\frac{z_{\alpha/2}}{d}\right)^2 = \frac{1}{2}\left(\frac{2.326}{0.15}\right)^2 = 121$ (Statdisk yields a sample size of 122.)

Section 7-4: Bootstrapping: Using Technology for Estimates

1. Without replacement, every sample would be identical to the original sample, so the proportions or means or standard deviations or variances would all be the same, and there would be no confidence "interval."

2. For the given sample, a bootstrap sample is another sample in which five of the numbers of patients in a day who required medication are randomly selected with replacement.

3. Part (b): (There are only three elements, not five.), Part (d): (14 and 20 are not in the original sample), Part (e): (There are too many elements), are not possible bootstrap samples.

4. There is no universal exact number, but there should be at least 1000 bootstrap samples, and the use of 10,000 or more is common.

5. The proportions from the 10 samples (in ascending order) are: 0, 0, 0, 0, 0, 0.25, 0.25, 0.5, 0.5, and 0.5. $P_5 = 0.000$ and $P_{95} = 0.500$, so the 90% interval is $0.000 < p < 0.500$.

6. The proportions from the 10 samples (in ascending order) are: 0, 0.25, 0.25, 0.5, 0.5, 0.5, 0.5, 1.5, 0.75, and 0.75. $P_{10} = 0.125$ and $P_{90} = 0.750$, so the 80% interval is $0.125 < p < 0.750$.

7. a. The means from the 10 samples (in ascending order) are: –0.25, 0.5, 0.5, 0.75, 3, 3, 3, 5, 8.25, and 9. $P_{10} = 0.125$ and $P_{90} = 8.625$, so the 80% interval is $0.1 \text{ kg} < \mu < 8.6 \text{ kg}$.

   b. The standard deviations from the 10 samples (in ascending order) are: 1.5, 2.363, 2.886751, 2.887, 4, 5.5, 5.715, 5.715, 5.715476, and 6.976. $P_{10} = 1.931$ and $P_{90} = 6.346$, so the 80% interval is $1.9 \text{ kg} < \sigma < 6.3 \text{ kg}$.

8. a. The means from the 10 samples (in ascending order) are: 62, 70.5, 73.5, 77.5, 78.25, 81, 81, 85.25, 93, and 103.5. $P_{10} = 66.25$ and $P_{90} = 98.25$, so the 80% interval is $66.3 \text{ cW/kg} < \mu < 98.3 \text{ cW/kg}$.

   b. The standard deviations from the 10 samples (in ascending order) are: 15.5, 17.898, 27.713, 37.621, 42.429, 45, 47.067, 47.067, 51.772, and 57.449. $P_{10} = 16.699$ and $P_{90} = 54.610$, so the 80% interval is $16.7 \text{ cW/kg} < \sigma < 54.6 \text{ cW/kg}$.

9. Answers will vary, but here are typical answers.
   a. 90% CI: $-0.8 \text{ kg} < \mu < 7.8 \text{ kg}$

   b. 90% CI: $1.2 \text{ kg} < \sigma < 7.0 \text{ kg}$

10. Answers will vary, but here are typical answers.
   a. 90% CI: $50.0 \text{ cW/kg} < \mu < 118.3 \text{ cW/kg}$

   b. 90% CI: $9.8 \text{ cW/kg} < \sigma < 57.3 \text{ cW/kg}$

11. Answers will vary, but here are typical answers.
   a. 99% CI: $55.3 \text{ min} < \mu < 67.3 \text{ min}$; This isn't dramatically different from $51.9 \text{ min} < \mu < 71.4 \text{ min}$.

   b. 95% CI: $6.2 \text{ min} < \sigma < 13.7 \text{ min}$; This isn't dramatically different from $7.7 \text{ min} < \sigma < 18.4 \text{ min}$.

12. Answers will vary, but here are typical answers.
   a. 99% CI: $22.0 \text{ min} < \mu < 41.6 \text{ min}$; This isn't dramatically different from $19.6 \text{ min} < \mu < 43.3 \text{ min}$ using the methods of Section 7-2.
   b. 95% CI: $8.5 \text{ min} < \sigma < 18.9 \text{ min}$; This isn't dramatically different from $10.7 \text{ min} < \sigma < 23.7 \text{ min}$ using the methods of Section 7-3.

13. Answers will vary, but here is a typical result: 95% CI: $0.348\% < p < 0.1.62\%$. This is quite close to the confidence interval of $0.288\% < p < 1.57\%$ found in Exercise 22 from Section 7-1.

14. Answers will vary, but here is a typical result: 99% CI: $0.0208 < p < 0.0317$. This is quite close to the confidence interval of $0.0205 < p < 0.0311$ found in Exercise 14 from Section 7-1.

15. Answers will vary, but here is a typical result: 90% CI: $0.1356 < p < 0.152$. The result is essentially the same as the confidence interval of $0.135 < p < 0.152$ found in Exercise 15 from Section 7-1.

16. Answers will vary, but here is a typical result: 95% CI: $0.671 < p < 0.722$. The result is very close to the confidence interval of $0.671 < p < 0.723$ found in Exercise 16 from Section 7-1.

17. Answers will vary, but here is a typical result: 90% CI: $3.69 < \mu < 4.15$. This result is very close to the confidence interval $3.676 < \mu < 4.17$ found by using the methods from Section 7-2.

18. Answers will vary, but here is a typical result: 99% CI: $21.6 \text{ mg} < \mu < 43.1 \text{ mg}$. This result is reasonably close to the confidence interval of $19.5 \text{ mg} < \mu < 45.6 \text{ mg}$ found in Exercise 21 in Section 7-2.

19. Answers will vary, but here is a typical result: 90% CI: $0.712 \text{ W/kg} < \mu < 1.18 \text{ W/kg}$. This result is very close to the confidence interval of $0.707 \text{ W/kg} < \mu < 1.169 \text{ W/kg}$ found in Exercise 17 of Section 7-2.

20. Answers will vary, but here is a typical result: 90% CI: $0.286 \text{ W/kg} < \mu < 0.492 \text{ W/kg}$. This result would be very close to the confidence interval $0.313 \text{ W/kg} < \mu < 0.674 \text{ W/kg}$ using the methods of Section 7-3.

21. a. Answers will vary, but here is a typical result: 95% CI: $2.5 < \sigma < 3.3$.

    b. 95% CI: $2.4 < \sigma < 3.7$

    c. The confidence interval from the bootstrap method is not very different from the confidence interval found using the methods of Section 7-3. Because a histogram or normal quantile plot shows that the sample appears to be from a population not having a normal distribution, the bootstrap confidence interval of $2.5 < \sigma < 3.3$ would be a better estimate of σ.

22. a. Answers will vary, but here is a typical result: 95% CI: $1.6 < \mu < 3.3$.

    b. 95% CI: $1.6 < \mu < 3.3$

    c. The confidence interval found from the bootstrap method is essentially the same as the confidence interval found using the methods of Section 7-2.

23. Answers will vary, but here is a typical result using 10,000 bootstrap samples: $2.5 < \sigma < 3.3$. This result is the same as the confidence interval found using 1000 bootstrap samples. In this case, increasing the number of bootstrap samples from 1000 to 10,000 does not have much of an effect on the confidence interval.

24. a. No. A histogram or normal quantile plot would show that the distribution of the sample data is far from normal.

    b. Yes. The sample means appear to have a distribution that is approximately normal.

    c. Yes. The sample standard deviations appear to have a distribution that is approximately normal.

Chapter Quick Quiz

1. $\hat{p} = \dfrac{0.110 + 0.150}{2} = 0.130$

2. We have 95% confidence that the limits of 0.110 and 0.150 contain the true value of the proportion of adults in the population who correct their vision by wearing contact lenses.

3. $z_{0.01/2} = 2.576$ (Table: 2.575)

4. $3\% - 1.0\% < p < 3\% + 1.0\% \Rightarrow 2.0\% < p < 4.0\%$

5. $n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[1.96]^2 (0.25)}{0.04^2} = 601$

6. $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.326 \cdot 15}{3}\right]^2 = 136$

7. There is a loose requirement that the sample values are from a normally distributed population.

8. The degrees of freedom is the number of sample values that can vary after restrictions have been imposed on all of the values. For the sample data described in Exercise 7, $\text{df} = 12 - 1 = 11$.

9. $t_{0.05/2} = 2.201$

10. No, the use of the χ^2 distribution has a fairly strict requirement that the data must be from a normal distribution. The bootstrap method could be used to find a 95% confidence interval estimate of σ.

Review Exercises

1. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \frac{723}{223{,}000} \pm 1.96\sqrt{\dfrac{\left(\frac{723}{223{,}000}\right)\left(\frac{222{,}277}{223{,}000}\right)}{223{,}000}} \Rightarrow 0.00301 < p < 0.00348$; We have 95% confidence that the limits of 0.00301 and 0.00348 contain the value of the population proportion.

2. $n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[1.645]^2 (0.25)}{0.04^2} = 423$

3. a. $\bar{x} = 22.083$ mm

   b. 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 22.083 \pm 2.201 \cdot \dfrac{0.077850}{\sqrt{12}} \Rightarrow 22.034 \text{ mm} < \mu < 22.133 \text{ mm}$

   c. We have 95% confidence that the limits of 22.034 mm and 22.133 mm contain the value of the population mean μ.

4. $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.575 \cdot 15}{4}\right]^2 = 94$

5. a. Student t distribution

   b. normal distribution

   c. None of the three distributions is appropriate, but a confidence interval could be constructed by using bootstrap methods.

   d. χ^2 (chi-square distribution)

   e. normal distribution

6. a. $n = \dfrac{\left[z_{\alpha/2}\right]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[1.96]^2(0.25)}{0.03^2} = 1068$

   b. $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{1.96 \cdot 39}{5}\right]^2 = 234$

   c. You must take the larger sample of 1068.

7. The sample meets the loose requirement of being from a normal distribution.

95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 331.8 \pm 2.201 \cdot \dfrac{37.121}{\sqrt{12}} \Rightarrow 308.2 \text{ mg} < \mu < 355.4 \text{ mg}$; The confidence interval limits do contain the desired amount of 325 mg, so the mean is not too bad, but examination of the individual amounts of aspirin shows that some tablets have considerably more than the desired amount of 325 mg, while others have considerable less than that desired amount. These tablets indicate a serious production problem that should be corrected.

8. a. The sample may not be normally distributed, proceed with caution.

   $\text{df} = 11$, $\chi_L^2 = 3.816$, and $\chi_R^2 = 21.920$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(12-1)37.121^2}{21.920}} < \sigma < \sqrt{\frac{(12-1)37.121^2}{3.816}}$$

   95% CI: $26.3 \text{ mg} < \sigma < 63.0 \text{ mg}$

   b. $\sigma \approx \dfrac{335 - 315}{4} = 5$ mg

   c. The desired $\sigma = 5$ mg from part (b) is not contained within the confidence interval from part (a). It appears that the current production method produces aspirin tablets with too much variation, and that should be corrected.

9. Answers will vary, but here is a typical result: $310.6 \text{ mg} < \mu < 350.9 \text{ mg}$. The result is very close to the confidence interval found in Exercise 7.

10. a. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.02 \pm 1.96\sqrt{\frac{(0.02)(0.98)}{1000}} \Rightarrow 0.0113 < p < 0.0287$

b. Answers will vary, but here is a typical result: $0.0120 < p < 0290$.

c. The confidence intervals are quite close.

Cumulative Review Exercises

1. $\bar{x} = \frac{25.4+\cdots+26.7+26.8+27.5+27.9+\cdots+29.2}{10} = 27.27\ \text{cm},\ Q_2 = \frac{26.8+27.5}{2} = 27.15\ \text{cm},$

$s = \sqrt{\frac{(25.4-27.27)^2+\cdots+(29.2-27.27)^2}{10-1}} = 1.24\ \text{cm},$ range $= 29.2-25.4 = 3.80$ cm; These results are statistics.

2. Using the range rule of thumb, significantly low values are $\mu - 2\sigma = 27.27 - 2(1.24) = 24.79$ cm or lower, and significantly high values are $\mu - 2\sigma = 27.27 + 2(1.24) = 29.75$ cm or higher. Because 30 cm exceeds 29.75 cm, a foot length of 30 cm is significantly high (or long).

3. ratio level of measurement; continuous data.

4. A histogram is not very helpful with only 10 data values, but a normal quantile plot shows that the sample data appear to be from a population having a distribution that is approximately normal.

5. The sample meets the loose requirement of being from a normal distribution.

95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 27.27 \pm 2.262 \cdot \frac{1.24}{\sqrt{10}} \Rightarrow 26.38\ \text{cm} < \mu < 28.16\ \text{cm}.$

6. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \cdot 1.12}{0.1}\right]^2 = 482$ adult females

7. a. $z_{x=25.00} = \frac{25.00-24.20}{1.12} = 0.71;$ which has a probability of $1-0.7611 = 0.2389$ (Tech: 0.2375) to the right.

b. $z_{x=25.00} = \frac{25.00-24.20}{1.12/\sqrt{25}} = 3.57;$ which has a probability of $1-0.9999 = 0.0001$ (Tech: 0.0002) to the right.

c. The z score for the lower 95% is 1.645, which correspond to a length of $1.645 \cdot 1.12 + 24.20 = 26.04$ cm (Tech: 15.5 min).

8. $\hat{p} = \frac{1440}{1440+48} = \frac{30}{31}$ 99% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{30}{31} \pm 2.575\sqrt{\frac{\left(\frac{30}{31}\right)\left(\frac{1}{31}\right)}{1488}} \Rightarrow 0.956 < p < 0.980;$ With such high limits on the interval, it does appear that the public is well informed about the risks of tattoos and infectious disease.