Chapter 11: Goodness-of-Fit and Contingency Tables

Section 11-1: Goodness-of-Fit

1. a. Observed values are represented by O and expected values are represented by E.

   b. For the leading digit of 2, $O = 62$ and $E = (317)(0.176) = 55.792$.

   c. For the leading digit of 2, $(O - E)^2/E = 0.691$.

2. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_9 = 0.046$;
   H_1: At least one of the proportions is not equal to the given claimed value.

3. There is sufficient evidence to warrant rejection of the claim that the leading digits have a distribution that fits well with Benford's law.

4. Because the leading digits of inter-arrival traffic times do not fit the distribution described by Benford's law, it appears that those times are not typical. The anomaly could be due to other factors, but there is a good chance that the computer has been hacked. Computer experts should be used to correct the incursion and secure the computer against further incursions.

5. H_0: $p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = 1/5$;
   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 0.523$; P-value $= 0.9712$ (Table: P-value > 0.95); Critical value: $\chi^2 = 9.488$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that injuries and illnesses occur with equal frequency on the different days of the week.

$$\chi^2 = \frac{(23-21.4)^2}{107/5} + \frac{(23-21.4)^2}{107/5} + \frac{(21-21.4)^2}{107/5} + \frac{(21-21.4)^2}{107/5} + \frac{(19-21.4)^2}{107/5} = 0.523 \text{ (df} = 4)$$

6. H_0: $p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;
   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 30.017$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 12.592$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the different days of the week have the same frequencies of car crash fatalities. This result is due to the higher numbers of fatalities on Fridays, Saturdays, and Sundays. The number of car crash fatalities on Sunday probably result from people returning after a weekend trip.

$$\chi^2 = \frac{(132-117)^2}{819/7} + \frac{(98-117)^2}{819/7} + \cdots + \frac{(133-117)^2}{819/7} + \frac{(159-117)^2}{819/7} = 30.017 \text{ (df} = 6)$$

7. H_0: $p_{\text{Jan}} = p_{\text{Feb}} = p_{\text{Mar}} = p_{\text{Apr}} = p_{\text{May}} = p_{\text{Jun}} = p_{\text{Jul}} = p_{\text{Aug}} = p_{\text{Sep}} = p_{\text{Oct}} = p_{\text{Nov}} = p_{\text{Dec}} = 1/12$;
   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 47.200$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 19.675$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that motorcycle fatalities occur with equal frequencies in the different months. Fatalities might be lower in winter months when colder weather is associated with substantially less use of motorcycles.

$$\chi^2 = \frac{(6-16.67)^2}{200/12} + \frac{(8-16.67)^2}{200/12} + \cdots + \frac{(10-16.67)^2}{200/12} + \frac{(8-16.67)^2}{200/12} = 47.200 \text{ (df} = 11)$$

8. H_0: $p_1 = p_2 = p_3 = p_4 = 0.25$;
H_1: At least one of the proportions is not equal to the given claimed value.
Test statistic: $\chi^2 = 4.600$; P-value = 0.204 (Table: P-value > 0.10); Critical value: $\chi^2 = 7.815$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the tires selected by the students are equally likely. It appears that when four students identify a tire, it is not likely that they would all select the same tire.
$$\chi^2 = \frac{(11-10.25)^2}{0.25\cdot 41} + \frac{(15-10.25)^2}{0.25\cdot 41} + \frac{(18-10.25)^2}{0.25\cdot 41} + \frac{(6-10.25)^2}{0.25\cdot 41} = 4.600 \text{ (df = 3)}$$

9. H_0: $p_{\text{Jan}} = p_{\text{Feb}} = p_{\text{Mar}} = p_{\text{Apr}} = p_{\text{May}} = p_{\text{Jun}} = p_{\text{Jul}} = p_{\text{Aug}} = p_{\text{Sep}} = p_{\text{Oct}} = p_{\text{Nov}} = p_{\text{Dec}} = 1/12$;
H_1: At least one of the proportions is not equal to the given claimed value.
Test statistic: $\chi^2 = 93.072$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 19.675$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that American born Major League Baseball players are born in different months with the same frequency. The sample data appear to support Gladwell's claim.
$$\chi^2 = \frac{(387-376.25)^2}{4515/12} + \frac{(329-376.25)^2}{4515/12} + \cdots + \frac{(398-376.25)^2}{4515/12} + \frac{(371-376.25)^2}{4515/12} = 93.072 \text{ (df = 11)}$$

10. H_0: $p_{\text{RE/NW}} = 9/16$, $p_{\text{SE/NW}} = 3/16$, $p_{\text{RE/VW}} = 3/16$, $p_{\text{SE/VW}} = 1/16$;
H_1: At least one of the proportions is not equal to the given claimed value.
Test statistic: $\chi^2 = 15.822$; P-value = 0.0012 (Table: P-value < 0.005); Critical value: $\chi^2 = 7.815$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the actual numbers of genetic results fit the distribution indicated by the proportions listed in the given table.
$$\chi^2 = \frac{(59-45)^2}{(9/16)\cdot 80} + \frac{(15-15)^2}{(3/16)\cdot 80} + \frac{(2-15)^2}{(3/16)\cdot 80} + \frac{(4-5)^2}{(1/16)\cdot 80} = 15.822 \text{ (df = 3)}$$

11. H_0: $p_{\text{brown}} = 0.87$, $p_{\text{blue}} = 0.08$, $p_{\text{green}} = 0.05$;
H_1: At least one of the proportions is not equal to the given claimed value.
Test statistic: $\chi^2 = 9.658$; P-value = 0.0080 (Table: P-value < 0.005); Critical value: $\chi^2 = 5.991$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the actual frequencies correspond to the predicted distribution.
$$\chi^2 = \frac{(132-1129.63)^2}{0.87\cdot 149} + \frac{(17-11.92)^2}{0.08\cdot 149} + \frac{(0-7.45)^2}{0.05\cdot 149} = 9.658 \text{ (df = 2)}$$

12. H_0: $p_{\text{AA}} = 0.25$, $p_{\text{Aa}} = 0.50$, $p_{\text{aa}} = 0.25$;
H_1: At least one of the proportions is not equal to the given claimed value.
Test statistic: $\chi^2 = 11.552$; P-value = 0.0031 (Table: P-value < 0.005); Critical value: $\chi^2 = 5.991$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the actual frequencies correspond to the predicted distribution.
$$\chi^2 = \frac{(20-36.25)^2}{0.25\cdot 145} + \frac{(90-72.5)^2}{0.50\cdot 145} + \frac{(35-36.25)^2}{0.25\cdot 145} = 11.552 \text{ (df = 2)}$$

13. H_0: $p_1 = 0.756, p_2 = 0.091, p_3 = 0.108, p_4 = 0.038, p_5 = 0.007$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 524.713$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 13.277$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the distribution of clinical trial participants fits well with the population distribution. Hispanics have an observed frequency of 60 and an expected frequency of 391.027, so they are very underrepresented. Also, the Asian/Pacific Islander subjects have an observed frequency of 54 and an expected frequency of 163.286, so they are also underrepresented.

$$\chi^2 = \frac{(3855-3248.532)^2}{0.756\cdot 4297} + \frac{(60-391.027)^2}{0.091\cdot 4297} + \cdots + \frac{(54-163.286)^2}{0.038\cdot 4297} + \frac{(12-30.079)^2}{0.007\cdot 4297} = 524.713 \text{ (df = 4)}$$

14. H_0: $p_{\text{YS}} = 9/16, p_{\text{GS}} = 3/16, p_{\text{YW}} = 3/16, p_{\text{GW}} = 1/16$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 11.161$; P-value = 0.011 (Table: P-value < 0.025); Critical value: $\chi^2 = 7.815$; Reject H_0. There is sufficient evidence to support the claim that the results contradict Mendel's theory.

$$\chi^2 = \frac{(307-281.25)^2}{0.5625\cdot 500} + \frac{(77-93.75)^2}{0.1875\cdot 500} + \frac{(98-93.75)^2}{0.1875\cdot 500} + \frac{(18-32.25)^2}{0.0625\cdot 500} = 11.161 \text{ (df = 3)}$$

15. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_7 = 0.058, p_8 = 0.051, p_9 = 0.046$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 3650.251$; P-value = 0.000 (Table: P-value < 0.005); Critical value: $\chi^2 = 20.090$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law. It appears that the image has been corrupted.

$$\chi^2 = \frac{(0-235.98)^2}{0.301\cdot 784} + \frac{(15-137.98)^2}{0.176\cdot 784} + \cdots + \frac{(23-39.98)^2}{0.051\cdot 784} + \frac{(0-36.06)^2}{0.046\cdot 784} = 3650.251 \text{ (df = 8)}$$

16. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_7 = 0.058, p_8 = 0.051, p_9 = 0.046$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 18.955$; P-value = 0.015 (Table: P-value > 0.01); Critical value: $\chi^2 = 20.090$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law. It does not appear that the image has been corrupted.

$$\chi^2 = \frac{(83-75.25)^2}{0.301\cdot 250} + \frac{(58-44.00)^2}{0.176\cdot 250} + \cdots + \frac{(4-12.75)^2}{0.051\cdot 250} + \frac{(9-11.50)^2}{0.046\cdot 250} = 18.955 \text{ (df = 8)}$$

17. H_0: $p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 9.500$; P-value = 0.147 (Table: P-value > 0.10); Critical value: $\chi^2 = 16.812$; Fail to reject H_0. There is not sufficient evidence to support the claim that births do not occur on the seven different days of the week with equal frequency.

Day	**Sun**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
Expected	57.14	57.14	57.14	57.14	57.14	57.14	57.14
Observed	53	52	66	72	57	57	43

$$\chi^2 = \frac{(53-57.14)^2}{400/7} + \frac{(52-57.14)^2}{400/7} + \cdots + \frac{(57-57.14)^2}{400/7} + \frac{(43-57.14)^2}{400/7} = 9.500 \text{ (df = 6)}$$

18. H_0: $p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 10.760$; P-value = 0.096 (Table: P-value > 0.05); Critical value: $\chi^2 = 16.812$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that discharges occur on the seven different days of the week with equal frequency.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Expected	57.14	57.14	57.14	57.14	57.14	57.14	57.14
Observed	65	50	47	48	73	64	53

$$\chi^2 = \frac{(65-57.14)^2}{400/7} + \frac{(50-57.14)^2}{400/7} + \cdots + \frac{(64-57.14)^2}{400/7} + \frac{(53-57.14)^2}{400/7} = 10.760 \text{ (df = 6)}$$

19. H_0: Heights selected from a normal distribution.
H_1: Heights not selected from a normal distribution.

	Height (cm)	Less than 155.45	155.45 – 162.05	162.05–168.65	Greater than 168.65
a.	**Frequency**	26	46	49	26
b.	**Tech:**	0.2023	0.3171	0.3046	0.1761
	Table:	0.2033	0.3166	0.3039	0.1762
c.	**Tech:**	0.2023(247) = 29.7381	0.3171(247) = 46.6137	0.3046(247) = 44.7762	0.1761(247) = 25.8867
	Table:	0.2033(247) = 29.8851	0.3166(247) = 46.5402	0.3039(247) = 44.6733	0.1762(247) = 25.9014

d. Test statistic: $\chi^2 = 0.877$ (Table: $\chi^2 = 0.831$); P-value = 0.831 (Table: P-value > 0.10); Critical value: $\chi^2 = 11.345$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that heights were randomly selected from a normally distributed population. The test suggests that we cannot rule out the possibility that the data are from a normally distributed population.

$$\chi^2 = \frac{(26-29.7381)^2}{29.7381} + \frac{(46-46.6137)^2}{46.6137} + \frac{(49-44.7762)^2}{44.7762} + \frac{(26-25.8867)^2}{25.8867} = 0.877 \text{ (df = 3)}$$

Section 11-2: Contingency Tables

1. a. $E = \dfrac{(16+50+3)(40+3)}{436+166+40+16+50+3} = \dfrac{(69)(43)}{711} = 4.173$

b. Because the expected frequency of a cell is less than 5, the requirements for the hypothesis test are not satisfied.

2. H_0: Whether a subject is right-handed or left-handed is independent of ear preference for cell phone use.
H_1: Right-left-handedness and ear preference for cell phone use are dependent.

3. Test statistic: $\chi^2 = 64.517$; P-value = 0.0000; Reject the null hypothesis of independence between handedness and cell phone ear preference.

4. The test is right-tailed. The test statistic is based on differences between observed frequencies and the frequencies expected with the assumption of independence between the row and column variables. Only large values of the test statistic correspond to substantial differences between the observed and expected values, and such large values are located in the right tail of the distribution.

5. H_0: Success is independent of type of treatment.
H_1: Success depends on type of treatment.

Test statistic: $\chi^2 = 9.750$; P-value = 0.002 (Table: P-value < 0.005); Critical value: $\chi^2 = 6.635$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that success is independent of the type of treatment. The results suggest that the surgery treatment is better.

$$\chi^2 = \frac{(60-67.6)^2}{67.6} + \frac{(23-15.4)^2}{15.4} + \frac{(67-59.4)^2}{59.4} + \frac{(6-13.6)^2}{13.6} = 9.750$$

6. H_0: Texting while driving is independent of driving when drinking alcohol.
H_1: Texting while driving depends on driving when drinking alcohol.

Test statistic: $\chi^2 = 576.224$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that success is independent of the type of treatment. The results suggest that the surgery treatment is better.

$$\chi^2 = \frac{(731-394.7)^2}{394.7} + \frac{(3054-3390.3)^2}{3390.3} + \frac{(156-492.3)^2}{492.3} + \frac{(4564-4227.7)^2}{4227.7} = 576.224$$

7. H_0: Adverse health condition is independent of restoration type.
H_1: Adverse health condition depends on restoration type.

Test statistic: $\chi^2 = 0.751$; P-value = 0.3862 (Table: P-value > 0.10); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim of independence between the type of restoration and adverse health conditions. Amalgam restorations do not appear to affect health conditions.

$$\chi^2 = \frac{(135-140)^2}{140} + \frac{(145-140)^2}{140} + \frac{(132-127)^2}{127} + \frac{(122-127)^2}{127} = 0.751$$

8. H_0: Sensory disorder is independent of restoration type.
H_1: Sensory disorder depends on restoration type.

Test statistic: $\chi^2 = 1.136$; P-value = 0.2864 (Table: P-value > 0.10); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim of independence between the type of restoration and sensory disorder. Amalgam restorations do not appear to affect sensory disorder.

$$\chi^2 = \frac{(36-32)^2}{32} + \frac{(28-32)^2}{32} + \frac{(231-235)^2}{235} + \frac{(239-235)^2}{235} = 1.136$$

9. H_0: Dog's selection is independent of source of sample.
H_1: Dog's selection depends on source of sample.

Test statistic: $\chi^2 = 34.445$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 6.635$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the source of the sample is independent of the dog’s selections. The results suggest that the dogs have some ability to detect bladder cancer, but they did not do well enough for accurate diagnoses.

$$\chi^2 = \frac{(22-7.9)^2}{7.9} + \frac{(32-46.1)^2}{46.1} + \frac{(32-46.1)^2}{46.1} + \frac{(282-267.9)^2}{267.9} = 34.445$$

10. H_0: Polygraph result is independent of truthfulness of subject.
H_1: Polygraph result depends on truthfulness of subject.
Test statistic: $\chi^2 = 25.571$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df $= (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the results of a polygraph test is independent of the subjects' truthfulness. The results suggest that polygraphs are effective in distinguishing between truths and lies.
$$\chi^2 = \frac{(15-27.3)^2}{27.3} + \frac{(42-29.7)^2}{29.7} + \frac{(32-19.7)^2}{19.7} + \frac{(9-21.3)^2}{21.3} = 25.571$$

11. H_0: Nausea is independent of treatment or placebo.
H_1: Nausea depends on treatment or placebo.
Test statistic: $\chi^2 = 9.854$; P-value = 0.0017 (Table: P-value < 0.005); Critical value: $\chi^2 = 6.635$; df $= (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that nausea is independent of whether the subject took a placebo or Chantix. It appears that nausea is more likely to occur among those who use Chantix, so nausea is a concern. However, the rate of nausea among Chantix users is only about 3.7%, so it is not much of a concern.
$$\chi^2 = \frac{(10-19.8)^2}{19.8} + \frac{(30-20.2)^2}{20.2} + \frac{(795-785.2)^2}{785.2} + \frac{(791-800.8)^2}{800.8} = 9.854$$

12. H_0: Gender is independent of whether call is overturned.
H_1: Call is overturned depends on gender.
Test statistic: $\chi^2 = 153.462$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df $= (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that developing the flu is independent of whether the subject took a placebo or the vaccine. The rate of developing the flu among the vaccine recipients (1.3%) is much lower than the rate for placebo recipients (17.9%), so it appears the vaccine is effective.
$$\chi^2 = \frac{(14-72.8)^2}{72.8} + \frac{(1056-997.2)^2}{997.2} + \frac{(95-36.2)^2}{36.2} + \frac{(437-495.8)^2}{495.8} = 153.462$$

13. H_0: Texting while driving is independent of irregular seat belt use.
H_1: Texting while driving depends on irregular seat belt use.
Test statistic: $\chi^2 = 18.773$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df $= (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim of independence between texting while driving and irregular seat belt use. Those two risky behaviors appear to be somehow related.
$$\chi^2 = \frac{(1737-1638.6)^2}{1638.6} + \frac{(2048-2146.4)^2}{2146.4} + \frac{(1945-2043.4)^2}{2043.4} + \frac{(2775-2676.6)^2}{2676.6} = 18.773$$

14. H_0: Deaths on a shift is independent of whether Gilbert was working.
H_1: Deaths on a shift depends on whether Gilbert was working.
Test statistic: $\chi^2 = 86.481$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 6.635$; df $= (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that deaths on shifts are independent of whether Gilbert was working. The results favor the guilt of Gilbert.
$$\chi^2 = \frac{(40-11.6)^2}{11.6} + \frac{(217-245.4)^2}{245.4} + \frac{(34-62.4)^2}{62.4} + \frac{(1350-1321.6)^2}{1321.6} = 86.481$$

15. H_0: Adverse effect is independent of treatment.
H_1: Adverse effect depends on treatment.

Test statistic: $\chi^2 = 42.568$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 9.210$; df $= (2-1)(3-1) = 2$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that experiencing an adverse reaction in the digestive system is independent of the treatment group. Treatments with 1332 mg doses of Campral appear to be associated with an increase in adverse effects of the digestive system.

$$\chi^2 = \frac{(344-284.1)^2}{284.1} + \frac{(89-143.7)^2}{143.7} + \frac{(8-13.2)^2}{13.2} + \frac{(1362-1421.9)^2}{1421.9} + \frac{(774-719.3)^2}{719.3} + \frac{(71-65.8)^2}{65.8} = 42.568$$

16. H_0: Infection is independent of treatment.
H_1: Infection depends on treatment.

Test statistic: $\chi^2 = 0.773$; P-value $= 0.8560$ (Table: P-value > 0.10); Critical value: $\chi^2 = 7.815$; df $= (2-1)(4-1) = 3$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that infections are independent of treatment. Based on the given sample data, the atorvastatin treatment does not appear to have an effect on infections.

$$\chi^2 = \frac{(27-27.1)^2}{27.1} + \frac{(89-86.6)^2}{86.6} + \frac{(8-7.9)^2}{7.9} + \frac{(7-9.4)^2}{9.4} + \frac{(243-242.9)^2}{242.9} + \frac{(774-776.4)^2}{776.4} + \frac{(71-71.1)^2}{71.1} + \frac{(87-84.6)^2}{84.6} = 0.773$$

17. H_0: Amount of smoking is independent of seat belt use.
H_1: Amount of smoking depends on seat belt use.

Test statistic: $\chi^2 = 1.358$; P-value $= 0.715$ (Table: P-value > 0.10); Critical value $(\alpha = 0.05)$: $\chi^2 = 7.815$; df $= (2-1)(4-1) = 3$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the amount of smoking is independent of seat belt use. The theory is not supported by the given data.

$$\chi^2 = \frac{(175-171.5)^2}{171.5} + \cdots + \frac{(6-7.9)^2}{7.9} + \frac{(149-152.5)^2}{152.5} + \cdots + \frac{(9-7.1)^2}{7.1} = 1.358$$

18. H_0: Getting a cold is independent of treatment.
H_1: Getting a cold depends on treatment.

Test statistic: $\chi^2 = 2.925$; P-value $= 0.232$ (Table: P-value > 0.10); Critical value: $\chi^2 = 5.991$; df $= (2-1)(3-1) = 2$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that getting a cold is independent of the treatment group. The results suggest that echinacea is not effective for preventing colds.

$$\chi^2 = \frac{(88-88.6)^2}{88.6} + \frac{(48-44.7)^2}{44.7} + \frac{(42-44.7)^2}{44.7} + \frac{(15-14.4)^2}{14.4} + \frac{(4-7.3)^2}{7.3} + \frac{(10-7.3)^2}{7.3} = 2.925$$

19. H_0: Injuries are independent of helmet color.
H_1: Injuries depend on helmet color.

Test statistic: $\chi^2 = 9.971$; P-value $= 0.041$ (Table: P-value < 0.05); Critical value $(\alpha = 0.05)$: $\chi^2 = 9.488$; df $= (2-1)(5-1) = 4$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that injuries are independent of helmet color. It appears that motorcycle drivers should use yellow or orange helmets.

$$\chi^2 = \frac{(491-509.5)^2}{509.5} + \cdots + \frac{(55-58.6)^2}{58.6} + \frac{(213-194.5)^2}{194.5} + \cdots + \frac{(26-22.4)^2}{22.4} = 9.971$$

20. H_0: Months of birth of baseball players are independent of being born in America.
H_1: Months of birth of baseball players depends on being born in America.
Test statistic: $\chi^2 = 20.054$; P-value = 0.0446 (Table: P-value < 0.05); Critical value: $\chi^2 = 19.675$; df = $(2-1)(12-1) = 11$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that months of births of baseball players are independent of whether they are born in America. The data do appear to support Gladwell's claim.

$$\chi^2 = \frac{(387-397.2)^2}{397.2} + \cdots + \frac{(371-368.7)^2}{368.7} + \frac{(101-90.8)^2}{90.8} + \cdots + \frac{(82-84.3)^2}{84.3} = 20.054$$

21. From Exercise 5, $\chi^2 = 9.7504$ and $z = -3.122560496$, so $z^2 = \chi^2$. The critical values are : $\chi^2 = 6.635$ and $z^2 = \pm 2.57583$ (Table: ± 2.575), so $z^2 = \chi^2$ (approximately).

22. Without Yates's correction, the test statistic is $\chi^2 = 9.750$. With Yates's correction, the test statistic is $\chi^2 = 8.574$. Yates's correction decreases the test statistic so that sample data must be more extreme in order to reject the null hypothesis of independence.
Without Yates's correction:

$$\chi^2 = \frac{(60-67.6)^2}{67.6} + \frac{(23-15.4)^2}{15.4} + \frac{(67-59.4)^2}{59.4} + \frac{(6-13.6)^2}{13.6} = 9.750$$

With Yates's Correction:

$$\chi^2 = \frac{(|60-67.6|-0.5)^2}{67.6} + \frac{(|23-15.4|-0.5)^2}{15.4} + \frac{(|67-59.4|-0.5)^2}{59.4} + \frac{(|6-13.6|-0.5)^2}{13.6} = 8.574$$

Chapter Quick Quiz

1. H_0: $p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = 0.1$;
H_1: At least one of the probabilities is different from the others.

2. $O = 27$ and $E = 0.1 \cdot 300 = 30$

3. right-tailed

4. df $= n-1 = 10-1 = 9$

5. There is not sufficient evidence to warrant rejection of the claim that the last digits are equally likely. Because reported heights would likely include more last digits of 0 and 5, it appears that the heights were measured instead of reported. (Also, most U.S. residents would have difficulty reporting heights in centimeters, because the United States, Liberia, and Myanmar are the only countries that continue to use the Imperial system of measurement.)

6. H_0: Surviving the sinking is independent of whether the person is a man, woman, boy, or girl.
H_1: Surviving the sinking and whether the person is a man, woman, boy, or girl are somehow related.

7. chi-squared distribution

8. right-tailed

9. df $= (r-1)(c-1) = (2-1)(4-1) = 3$

10. There is sufficient evidence to warrant rejection of the claim that surviving the sinking is independent of whether the person is a man, woman, boy, or girl. Most of the women survived, 45% of the boys survived, and most girls survived, but only about 20% of the men survived, so it appears that the rule was followed quite well.

Review Exercises

1. H_0: $p_{\text{Jan}} = p_{\text{Feb}} = p_{\text{Mar}} = p_{\text{Apr}} = p_{\text{May}} = p_{\text{Jun}} = p_{\text{Jul}} = p_{\text{Aug}} = p_{\text{Sep}} = p_{\text{Oct}} = p_{\text{Nov}} = p_{\text{Dec}} = 1/12$;
   H_1: At least one of the proportions is not equal to the given claimed value.
   Test statistic: $\chi^2 = 269.147$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 24.725$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that weather related deaths occur in the different months with the same frequency. The months of May, June, and July appear to have disproportionately more weather-related deaths, and that is probably due to the fact that vacations and outdoor activities are much greater during those months.
   $$\chi^2 = \frac{(28-37.5)^2}{450/12} + \frac{(17-37.5)^2}{450/12} + \cdots + \frac{(26-37.5)^2}{450/12} + \frac{(25-37.5)^2}{450/12} = 269.147 \text{ (df} = 11)$$

2. H_0: Norovirus is independent of ship.
   H_1: Norovirus depends on ship.
   Test statistic: $\chi^2 = 71.679$ P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df $= (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that getting norovirus infection is independent of the ship. It appears that an outbreak of norovirus infection has a different effect on different ships.
   $$\chi^2 = \frac{(276-185.3)^2}{185.3} + \frac{(1376-1466.7)^2}{1466.7} + \frac{(338-428.7)^2}{428.7} + \frac{(3485-3394.3)^2}{3394.3} = 71.679$$

3. H_0: $p_{\text{Jan}} = p_{\text{Feb}} = p_{\text{Mar}} = p_{\text{Apr}} = p_{\text{May}} = p_{\text{Jun}} = p_{\text{Jul}} = p_{\text{Aug}} = p_{\text{Sep}} = p_{\text{Oct}} = p_{\text{Nov}} = p_{\text{Dec}} = 1/12$;
   H_1: At least one of the proportions is not equal to the given claimed value.
   Test statistic: $\chi^2 = 10.375$; P-value $= 0.4970$ (Table: P-value > 0.10); Critical value: $\chi^2 = 19.675$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that homicides in New York City are equally likely for each of the 12 months. There is not sufficient evidence to support the police commissioner's claim that homicides occur more often in the summer when the weather is warmer.
   $$\chi^2 = \frac{(38-42.667)^2}{512/12} + \frac{(30-42.667)^2}{512/12} + \cdots + \frac{(37-42.667)^2}{512/12} + \frac{(37-42.667)^2}{512/12} = 10.375 \text{ (df} = 11)$$

4. H_0: Left-handedness is independent of parental handedness.
   H_1: Left-handedness dependents on parental handedness.
   Test statistic: $\chi^2 = 784.647$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 11.345$; df $= (4-1)(2-1) = 3$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that left-handedness is independent of parental handedness. It appears that handedness of the parents has an effect on handedness of the offspring, so left-handedness appears to be an inherited trait.
   $$\chi^2 = \frac{(5360-6067.9)^2}{6067.9} + \cdots + \frac{(2736-3125.4)^2}{3125.4} + \frac{(741-475.2)^2}{475.2} + \cdots + \frac{(289-341.7)^2}{341.7} = 784.647$$

5. H_0: $p_{\text{Under 25}} = 0.16$, $p_{25\text{–}44} = 0.44$, $p_{45\text{–}64} = 0.27$, $p_{\text{Over 64}} = 0.13$;
   H_1: At least one of the proportions is not equal to the given claimed value.
   Test statistic: $\chi^2 = 53.051$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 7.815$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the distribution of crashes is the same as the distribution of ages. Drivers under 25 appear to have disproportionately more crashes.
   $$\chi^2 = \frac{(36-14.08)^2}{0.16 \cdot 88} + \frac{(5985-38.72)^2}{0.44 \cdot 88} + \frac{(5985-23.76)^2}{0.27 \cdot 88} \frac{(5985-11.44)^2}{0.13 \cdot 88} = 53.051 \text{ (df} = 3)$$

Cumulative Review Exercises

1. $\bar{x} = \dfrac{29+35+38+42+58+62+64+67+68+74}{10} = 53.7$ years; $Q_2 = \dfrac{58+62}{2} = 60.0$ years;

$s = \sqrt{\dfrac{(29-53.7)^2+(35-53.7)^2+\cdots+(68-53.7)^2+(74-53.7)^2}{10-1}} = 16.1$ years; $s^2 = 16.1^2 = 259.2$ years^2 (258.9 using unrounded values). Because an age of 16 is $\dfrac{16-53.7}{16.1} = -2.34$, or more than 2 standard deviations below the mean of 53.7 years, it is significantly low.

2. The sample appears to meet the loose requirement for being normally distributed.

95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 53.7 \pm 2.626 \cdot \dfrac{16.1}{\sqrt{10}} \Rightarrow 42.2 \text{ years} < \mu < 65.2 \text{ years}$; Yes, the confidence interval limits do contain the value of 65.0 years that was found from a sample of 9269 ICU patients.

3. H_0: Facial injuries received is independent of wearing helmet.

H_1: Facial injuries received depends on wearing helmet.

Test statistic: $\chi^2 = 10.708$; P-value $= 0.0011$ (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; $\text{df} = (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that wearing a helmet has no effect on whether facial injuries are received. It does appear that a helmet is helpful in preventing facial injuries in a crash.

$$\chi^2 = \frac{(30-45.1)^2}{45.1} + \frac{(182-166.9)^2}{166.9} + \frac{(83-67.9)^2}{67.9} + \frac{(236-251.1)^2}{251.1} = 10.708$$

4. a. $\dfrac{30+83+236}{531} = \dfrac{349}{531}$, or 0.657

b. $\dfrac{113}{531} \cdot \dfrac{112}{530} = \dfrac{6328}{93{,}015}$, or 0.0450

c. $\dfrac{182+236}{531} = \dfrac{418}{531}$, or 0.787

5. a. The z score for the bottom 5% is –1.645, which correspond a forward grip reach of $-1.645 \cdot 34 + 686 = 630$ mm.

b. $z_{x=650} = \dfrac{650-686}{34} = -1.06$; which has a probability of 0.1448, or 14.48% (Table: 14.46%) to the left. That percentage is too high, because too many women would not be accommodated.

c. $z_{x=680} = \dfrac{680-686}{34/\sqrt{16}} = -0.71$; which has a probability of $1 - 0.2401 = 0.7599$ (Table: 0.7611) to the right. Groups of 16 women do not occupy a driver's seat or cockpit; because individual women occupy the driver's seat/cockpit, this result has no effect on the design.

6. Determine whether there is a linear correlation between diastolic blood pressure and height. $r = -0.146$; P-value $= 0.7824$ (Table: P-value > 0.05); Critical values ($\alpha = 0.05$): $r = \pm 0.8114$; There is not sufficient evidence to support the claim that for males, there is a linear correlation between diastolic blood pressure and height.