Sampling distribution and estimators.

sample 1. $\longrightarrow$ $\bar{X}$

$X_1\ X_2 \cdots X_n$

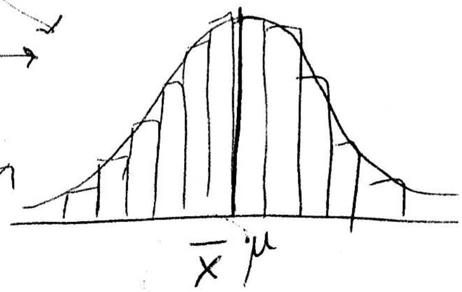
sample 2 $\longrightarrow$ $\bar{X}$

$X_1\ X_2 \cdots X_n$

$\vdots$

sample N $\longrightarrow$ $\bar{X}$

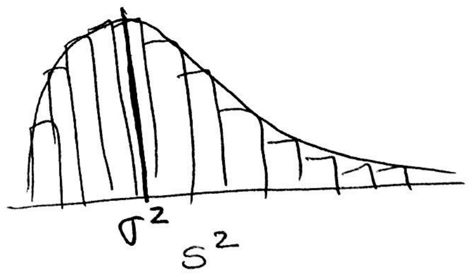
$X_1 \cdot X_2 \cdots X_n$



sample distribution of the sample mean.

$\hat{p}$: sample proportion
the distribution of the sample proportion is centered around p. (population proportion)

S^2 : sample variance.
the distribution of the sample proportion is centered around σ^2 (population variance).



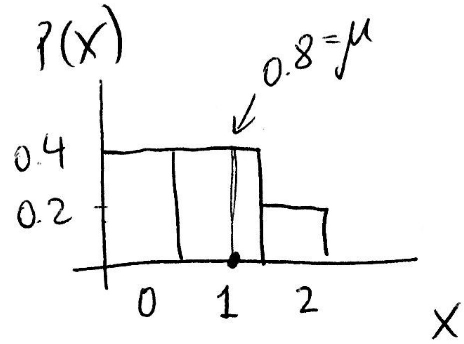
Class 10

Slide 4.

Let X be a random variable that describes the number of kids that a family has.

X	P(X)
0	0.4
1	0.4
2	0.2
	1.

$\longrightarrow$



a) find population mean of kids that families haves.

$$\mu = \sum x P(x) = \sum_{x=0}^{2} x P(x) = \underbrace{0 \cdot 0.4}_{0.} + \underbrace{1 \cdot 0.4}_{0.4} + \underbrace{2 \cdot 0.2}_{0.4}$$

$= 0.8.$

b) $\boxed{\bar{x},\ \hat{p},\ s^2}$ data: 1, 1, 0, 2, 1

$$\bar{x} = \frac{1+1+0+2+1}{5} = 1$$

$\uparrow$

we use the sample mean statistic to describe the mean number of kids that families have.

c) • population parameter: $\mu = 0.8$.

• population mean: $\mu = 0.8$.

• sample mean: $\bar{x} = 1$.

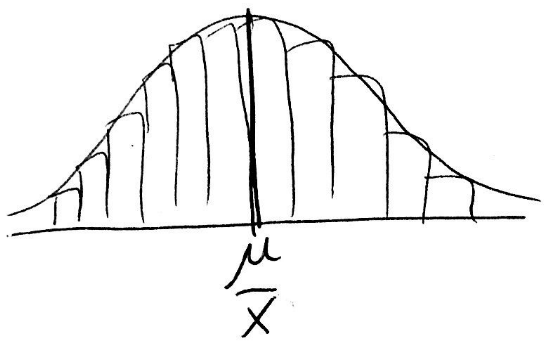
• statistic: $\bar{x} = 1$

• estimator: $\bar{x} = 1$

• sampling distribution: distribution of a statistic

• sampling distribution of the sample mean: distribution of all $\bar{x}$ that you can compute for all samples that you can gather.

- biased/unbiased estimator: an unbiased estimator has a distribution that is centered on the population parameter. In this context the sample mean ~~is an~~ ~~...~~ (is an estimator) has a distribution that is ~~unbiased~~ centered on the population mean.



d) sampling distribution of the sample mean number of kids that a family has:

gather many samples of the number of kids that n families have. For each of those samples compute the mean number of kids they have. ~~plot~~ a histogram of all the sample means shows the distribution of the sample mean of kids that families have.

e) the mean of the sampling distribution of the sample mean is the population mean. Because the sample mean is an unbiased estimator for population mean.

central limit theorem.

let X be a random variable with mean μ and standard deviation σ.

if $n \geq 30$ then

$\overline{X}$ has the normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$

---

slide 9.

let X be the random variable that describes the pulse rate of an adult male.

$\mu = 69.6$ and $\sigma = 11.3$

$$Z = \frac{X - 69.6}{11.3}$$

a) P(pulse rate greater than 76) = $P(X > 76)$

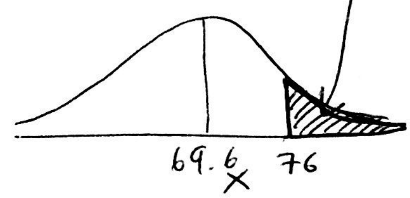
$$= P\left(Z > \frac{76 - 69.6}{11.3}\right)$$

$$= P(Z > 0.5664)$$

$$= 1 - P(Z \leq 0.5664)$$

$$= 1 - 0.7144$$

$$= 0.2856.$$



$$\overline{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

b) P(mean pulse rate greater than 76) = $P(\overline{X} > 76)$

to find the distribution of $\overline{X}$ we use the central limit theorem. $\overline{X}$ has the normal distribution with mean $\mu = 69.6$ and standard deviation $\sigma/\sqrt{n} = 11.3/\sqrt{16} = \frac{11.3}{4} = 2.825$

$$\left(z = \frac{\bar{X} - 69.6}{2.825}\right)$$

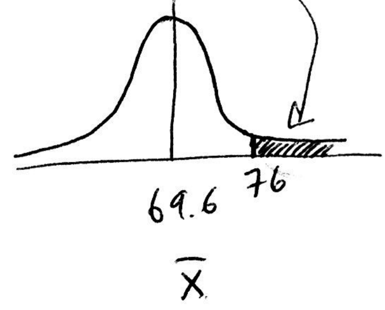
$$P(\bar{X} > 76) = P\left(Z > \frac{76 - 69.6}{2.825}\right)$$

$$= P(Z > 2.2655)$$

$$= 1 - P(Z \leq 2.2655)$$

$$= 1 - 0.9883$$

$$= 0.0117$$



c) we can use the central limit theorem while having $n=16$ (instead of $n>30$) because the pulse rate of an adult male (X) follows a normal distribution.

Slide 10.

let X be the random variable that describes the body temperature of a human.

we assume that $\mu = 98.6$ F

$\sigma = 0.62$ F

a sample of size $n=106$ was collected

a) P(sample mean of 98.2 or lower) = $P(\overline{X} \leq 98.2)$

because $n>30$. we can use the central limit
theorem that states that $\overline{X}$ has the ~~#~~ normal
distribution with mean 98.6 and standard
deviation $\sigma/\sqrt{n} = 0.62/\sqrt{106} = 0.1904$

$$Z = \frac{\overline{X} - 98.6}{0.62}$$

$$P(\overline{X} \leq 98.2) = P\left(Z \leq \frac{98.2 - 98.6}{0.1904}\right)$$
$$= P(Z \leq -2.10084)$$
$$= 0.0178.$$