Statistics for the Biological, Environmental and Health Sciences

STAT 007

Hypothesis Testing

Chapter 8

Basics of Hypothesis Testing and Tests for proportions

Sections 8-1 and 8-2

Hypothesis Testing

- We use Hypothesis Testing to make claims about population parameters:
  - "The proportion of students that passes a class is greater than 0.8."
  - "The mean score of the test is greater than 22."
  - "The standard deviation of of the test is smaller than 0.7."
- The structure of the claims can be of three types:
  - The population parameter is greater than a specific value.
  - The population parameter is smaller than a specific value.
  - The population parameter is equal to a specific value.
- Estimates (point and interval) and hypothesis tests are both methods of inferential statistics, but they have different objectives.
- In what follows, we will discuss the general aspects of hypothesis test, but will center our attention to the details of hypothesis tests for a proportion.

Hypothesis Testing

- The **first step** in a Hypothesis Test is to identify the claim of interest and its symbolic form:
  - Claim of interest: The proportion of girls born to parents using the XSORT method of gender selection is greater that 0.5
    Symbolic form: $p > 0.5$.

- The **second step** is to provide a symbolic form for the claim that must be true, when the claim of interest is false:
  - If $p > 0.5$ is false, then $p \leq 0.5$ must be true. Symbolic form: $p \leq 0.5$.

- The **third step** is to identify the formal *null hypothesis (H_0)* and *alternative hypothesis (H_1)*:
  - **null hypothesis (H_0)** is a symbolic statement that the value of a population parameter is *equal* to some claimed value. This hypothesis indicates *no change, no effect,* or *no difference* and it always uses the symbol: $=$.
  - **alternative hypothesis (H_1)** is a statement that the parameter has a value that somehow differs from H_0. In general, the scientist's claim must be worded so that it becomes H_1 and its symbolic form uses the symbols: $>, <, \neq$.

Hypothesis Testing

- Continuation of **third step**:
  — $H_0 : p = 0.5 \qquad H_1 : p > 0.5$

- The **fourth step** is to select the significance level α. For a hypothesis test, the **significance level, α,** is the probability value used as the cutoff for determining when the evidence in the sample data provides significant evidence against the null hypothesis H_0.
  The significant level α is the probability of rejecting the null hypothesis H_0 when it is true:

  $$\text{significance level } \alpha = P(\textit{rejecting } H_0 \textit{ given that } H_0 \textit{ is true}).$$

  Usually α will take the value $0.01, 0.05$, or 0.1.

Hypothesis Testing

- The **fifth step** is to identify the *Test Statistic* relevant for the test of interest and its distribution.
  The **Test Statistic** is a value used to make a decision about H_0 and it is found by converting the sample statistic ($\hat{p}$) to a score (z^{stat}) with the assumption that the null hypothesis is true.
  - For a claim regarding a proportion p:
    we assume that there is a fixed number of trials, the trials are independent, outcomes can be of one of two categories, the probabilities of success remain constant for each trial, $np \geq 5$ and $n(1-p) \geq 5$ under the assumption that H_0 is true (so the normal distribution is a suitable approximation to the binomial distribution).
    sample statistic: $\hat{p}$. Test Statistic $z^{stat} = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$ follows (approximately) a standard normal distribution under the assumption that H_0 is true.

Hypothesis Testing

- The **sixth step** is to compute the *Test Statistic* and then find either the p-value or the critical value(s).
  - Before finding the p-value or the critical value(s) we need to define the **critical (or rejection) region**, which is the area corresponding to all values of the test statistic that causes us to reject the null hypothesis.
  - A **critical value(s)** separates the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis. Critical values depend on H_0, the sampling distribution, and the significance level.
  - A **p-value** is the probability of getting a value of the test statistic that is at least as extreme as the test statistic computed from the data, assuming that the null hypothesis is true. p-values depend on H_0, the sampling distribution, and the computed test statistic.

Hypothesis Testing

- The proportion of girls born to parents using the XSORT method of gender selection is greater that 0.5: $p > 0.5$.

$$H_0 : p = 0.5 \qquad H_1 : p > 0.5$$

From a sample of 14 babies, 13 were girls. Plot the critical region. For a significance level of 0.05, find the critical value and p-value.

Hypothesis Testing

- The proportion of girls born to parents using the XSORT method of gender selection is greater than 0.5: $p > 0.5$. $H_0 : p = 0.5$ $\quad H_1 : p > 0.5$. From a sample of 14 babies, 13 were girls and $z^{stat} = 3.21$. For a significance level of $\alpha = 0.05$, *critical value* $= 1.65$, critical region is values z^{stat} such that $z^{stat} > 1.65$, and $p - value = 0.0007$.

- The **seventh step** is to make a decision to either reject H_0 or to fail to reject H_0.
  — Decision Criteria for a Critical Value
    * If the test statistic is in the critical region, then reject H_0.
    * If the test statistic is not in the critical region, then fail to reject H_0.
  — Decision Criteria for a p-value
    * If the p-value is equal or smaller than α, then reject H_0.
    * If the p-value is larger than α, then fail to reject H_0.

Hypothesis Testing

- The proportion of girls born to parents using the XSORT method of gender selection is greater than 0.5: $p > 0.5$. $H_0 : p = 0.5$ $H_1 : p > 0.5$. From a sample of 14 babies, 13 were girls and $z^{stat} = 3.21$. For a significance level of $\alpha = 0.05$, $critical\ value = 1.65$, critical region is values z^{stat} such that $z^{stat} > 1.65$, and $p - value = 0.0007$.

- The **eighth step** is to restate the decision Using Simple and Nontechnical Terms

Condition	Conclusion
Original claim does not include equality, and you reject H_0.	"There is sufficient evidence to support the claim that... (original claim)."
Original claim does not include equality, and you fail to reject H_0.	"There is not sufficient evidence to support the claim that... (original claim)."
Original claim includes equality, and you reject H_0.	"There is sufficient evidence to warrant rejection of the claim that... (original claim)."
Original claim includes equality, and you fail to reject H_0.	"There is not sufficient evidence to warrant rejection of the claim that... (original claim)."

Example

A study of sleepwalking or “nocturnal wandering” was described in the journal Neurology. From the study it was observed that 29.2% of 19,136 American adults have sleepwalked and a reporter stated that “fewer than 30% of adults have sleepwalked”.

Use a 0.05 significance level to test the reporter’s claim.

Practice

Look at the exercises at the end of Section 8-1 in page 351

Specially, look at exercises: 1, 2, 3, 4, 5-8, 17-20, 25-28, 29-32.

Look at the exercises at the end of Section 8-2 in page 362

Specially, look at exercises: 5-8, 9, 10, 13, 14, 16, 17, 18, 21, 22, 25.

Testing a Claim about a Mean

Section 8-3

Hypothesis Testing

- We will introduce hypothesis tests for claims about the mean of a population when the standard deviation of the population, σ, is unknown.
- When σ is unknown we use the data to estimate it as the sample standard deviation, s.
- When we develop a hypothesis test for the mean when σ is unknown, then the sampling distribution of the test statistic will be a Student t distribution with $(n-1)$ degrees of freedom. Critical values and p-values will be computed using this distribution.

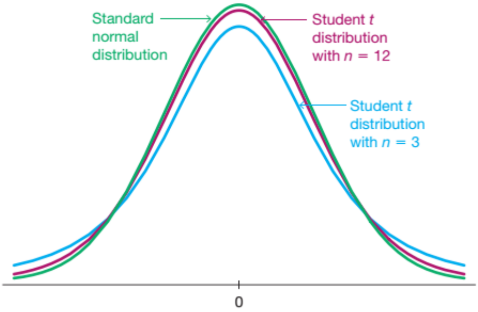
Hypothesis Testing for the population mean, μ

- **Step 1**: identify the claim of interest and its symbolic form.
- **Step 2**: identify the claim that is true when the one of interest is false.
- **Step 3**: write the null and alternative hypothesis.
- **Step 4**: select the significance level $\alpha = P(\textit{Type I error})$.
- **Step 5**: use the test statistic $t^{stat} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$, which under the assumption that H_0 is true follows a Student t distribution with $(n-1)$ degrees of freedom.
- **Step 6**:
  - find the critical region,
  - find the critical value(s) ($t_\alpha, -t_\alpha$, or $t_{\alpha/2}, -t_{\alpha/2}$),
  - find the p-value (area on the right of t^{stat}, area on the left of t^{stat}, or twice the area on the right of t^{stat}, when $t^{stat} > 0$ or twice the area on the left of t^{stat}, when $t^{stat} < 0$).
- **Step 7**: make a decision based on the critical value and/or the p-value.
- **Step 8**: restate your decision in simple nontechnical terms.

- **Requirement for step 5**: the population is normally distributed and/or $n > 30$. If $n \leq 30$ look at histograms, outliers, and quantile plots to check the normality assumption.

Properties of the Student t distribution

- It is different for different sample sizes.
- It has the same general bell shape as the standard normal distribution. The wider shape reflects the greater variability that is expected when σ is estimated by s.
- It has a mean equal to 0.
- It has a standard deviation that varies with the sample size and is larger than 1.
- As the sample size increases, it gets closer to the standard normal distribution.



Hypothesis Testing for μ

Example

A common recommendation is that adults should sleep between 7 hours and 9 hours each night.

From the National Health and Nutrition Examination Study the times of sleep (in hours) for 12 randomly selected adult subjects were obtained. Here are some of the statistics for this sample: $\overline{x} = 6.83$ hours and $s = 1.99$ hours.

a) Use 0.05 significance level to test the claim that the mean amount of sleep for adults is less than 7 hours. Base your conclusions on the critical value.
b) Identify the type of error that you could be making.

Hypothesis Testing and Confidence Intervals

- Hypothesis Testing and Confidence Interval are both methods used in inferential statistics.
- Hypothesis testing is used when claims about the population parameters are made.
- Confidence Intervals are interval estimates for population parameters.
- When making inference regarding the population mean, μ, the confidence interval and the test of hypothesis will always agree:
  - Test statistic: $t^{stat} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$
  - Interval estimate: $\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$
- When making inference regarding the population proportion, p, the confidence interval and the test of hypothesis might disagree.
  - Test statistic: $z^{stat} = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$
  - Interval estimate: $\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Hypothesis Testing and Confidence Intervals

Example

Evaluations Data from student course evaluations were obtained from the University of Texas at Austin. The summary statistics are $n = 436$, $\overline{x} = 3.97$, $s = 0.55$.

a) Use a 0.05 significance level to test the claim that the population of student course evaluations has a mean equal to 4.00. Base your conclusions on the critical value.
b) Identify the type of error that you could be making.
c) Do the results apply to the population of all students?
d) Compute a 95% confidence interval for the mean value of the course evaluations.

Practice

Look at the exercises at the end of Section 8-3 in page 374.

Specially, look at exercises: 15-24. Base your conclusions on the critical value

Example

Before its clinical trials were discontinued, the Genetics & IVF Institute conducted a clinical trial of the *XSORT* method designed to increase the probability of conceiving a girl and, among the 945 randomly selected babies born to parents using the *XSORT* method, there were 879 girls.

a) Find a point estimate for the proportion of borne girls.

b) Find a 95% confidence interval estimate for the proportion of borne girls.

c) Make an interpretation of the confidence interval estimate and of its confidence level.

d) Use a 0.05 significance level to test the claim that the proportion of girls borne to parents using the *XSORT* method is equal to 0.9. Base your conclusion using the critical value and p-value.

e) Identify the type of error that you could be making.

f) Are the requirements to compute a confidence interval and to perform a hypothesis testing satisfied?