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Statistical Methods for the Biological,
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STAT 007

Answers to Quiz 5

1. There are about 200,000 active mathematicians in the United States and they have annual incomes with a distribution that is skewed to the right instead of being normal. Many different samples of 55 mathematicians are randomly selected and the mean annual income is computed for each sample.

   (a) What is the approximate shape of the distribution of the sample mean (normal, skewed to the left, skewed to the right, or other)?

   The approximate shape of the distribution of the sample mean is normal. ***[3 pts.]*** *The shape of the distribution of the sample mean computed for many samples is bell-shaped (normal).*

   (b) What value do the sample mean target? This is, what is the mean of all such sample means?
   The sample mean targets the population mean. ***[3 pts.]*** *This is, the mean of the sampling distribution of the sample mean is the population mean.*

2. People that have a good performance during their day sleep an amount of time that is in the top 25% of amount of time sleeping. The amount of time that adults sleep are normally distributed with a mean of 6.8 hours and a standard deviation of 1.4 hours.

   (a) Find the minimum time that some needs to sleep in order to have a good performance during the day.
   First we find the z score that separates the lowest 75% from the highest 25%, which is $z = 0.67$ ***[1.5 pts.]****. Then the minimum time that some needs to sleep in order to have a good performance during the day is* $0.67 * 1.4 + 6.8 = 7.738$ *hours* ***[1.5 pts.]****.*

   (b) If we randomly select 5 adults, what is the probability that the sample mean is between 7 and 9 hours of sleep (the typical recommendation of time sleeping)?

   From the central limit theorem, the sample mean follows a normal distribution with mean 6.8 and standard deviation $1.4/\sqrt{5} = 0.6261$***[1.5 pts.]*** *. So,*

$$\begin{aligned} P(7 < \bar{x} < 9)\textbf{[0.5 pts.]} &= P((7-6.8)/0.6261 < z < (9-6.8)/0.6261) \\ &= P(z < 3.51) - P(z < 0.32) = 0.9998 - 0.6255 = 0.3743\textbf{[1 pts.]} \end{aligned}$$