Central limit theorem.

Let X be a random variable that describes some procedure of interest. It has a mean μ and a standard deviation σ

if you have a ~~ran~~ random sample X_1 X_2 .... X_n

then the central limit theorem states that the sample mean, $\bar{X}$, has approximately a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$

if X has a normal distribution, then replace "approximately" by "exactly". You donot need to have $n > 30$. ($n = 11$)

if X has not a normal distribution, then you need to have at least $n > 30$.

Class 11

Slide 4.

X discusses body temperature. $\mu = 98.6$ F and $\sigma = 0.62$ F. $n = 106$.

a) P(sample mean of 98.2F or lower) =

$$P(\bar{X} \leq 98.2) = 0.0178.$$

1) we used central limit theorem. $\bar{X}$ follows normal distribution with mean 98.6 F and standard deviation $\frac{0.62}{\sqrt{106}}$. $(n > 30)$.

b) because the probability of observing a sample mean body temperature of 98.2 F or less is very small (0.0178), under the above assumptions, we can say that probably our assumption is wrong

Slide 16

580 offspring peas

152 are observed to be yellow

assume that 25% should be yellow.

a) first this is a binomial experiment (the number of trials is fixed, probability of success is the same, trials are independent, each trial is success or failure only)

$np \geq 5$ and $np(1-p) \geq 5$.

$np = 580 \cdot 0.25 = 145$

$np(1-p) = 580 \cdot 0.25 \cdot 0.75 = 108.75$.

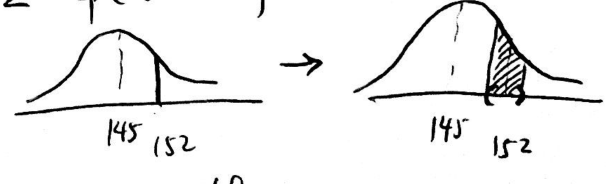
therefore both requirements are satisfied and we can use the normal approximation to compute probabilities from this binomial experiment

b) parameters of the normal distribution:

$\mu = n \cdot p = 580 \cdot 0.25 = 145$

$\sigma = \sqrt{np(1-p)} = \sqrt{580 \cdot 0.25 \cdot 0.75} = \sqrt{108.75} = 10.42833$

c) Find the prob. of exactly 152 yellow peas.



$P(X = 152)$

$= P(152 - 0.5 < X < 152 + 0.5)$ ← using the continuity correction.

$= P(151.5 < X < 152.5)$

$= P\left(\frac{151.5 - 145}{10.42833} < Z < \frac{152.5 - 145}{10.42833}\right)$

$Z = \frac{X - 145}{10.42833}$

$= P(0.6233 < Z < 0.7191)$

$= P(Z < 0.72) - P(Z < 0.62)$

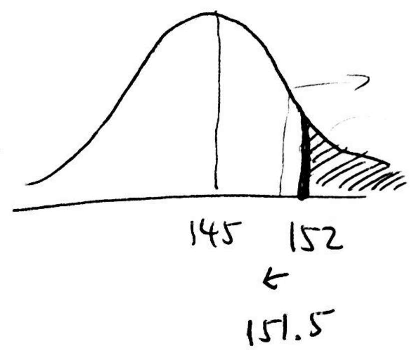
$= 0.7642 - 0.7324$

$= 0.0318.$

d) P(of observing 152 or more yellow peas)

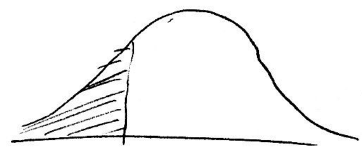
$= P(X \geq 152)$

$= P(X \geq 152 - 0.5)$ continuity correction.



$= P(X \geq 151.5)$

$= P\left(Z \geq \frac{151.5 - 145}{10.42833}\right)$



$= P(Z \geq 0.6233)$

$= 1 - P(Z < 0.6233)$

$= 1 - 0.7324$

$= 0.2676$

So, observing 152 yellow peas is <u>not</u> considered significantly high.

e) rule of thumb. $\mu + 2\sigma = 145 + 2 \cdot 10.42833 = 165.8567$

So, 152 is not significantly high.