Summary

Estimation.

point estimator

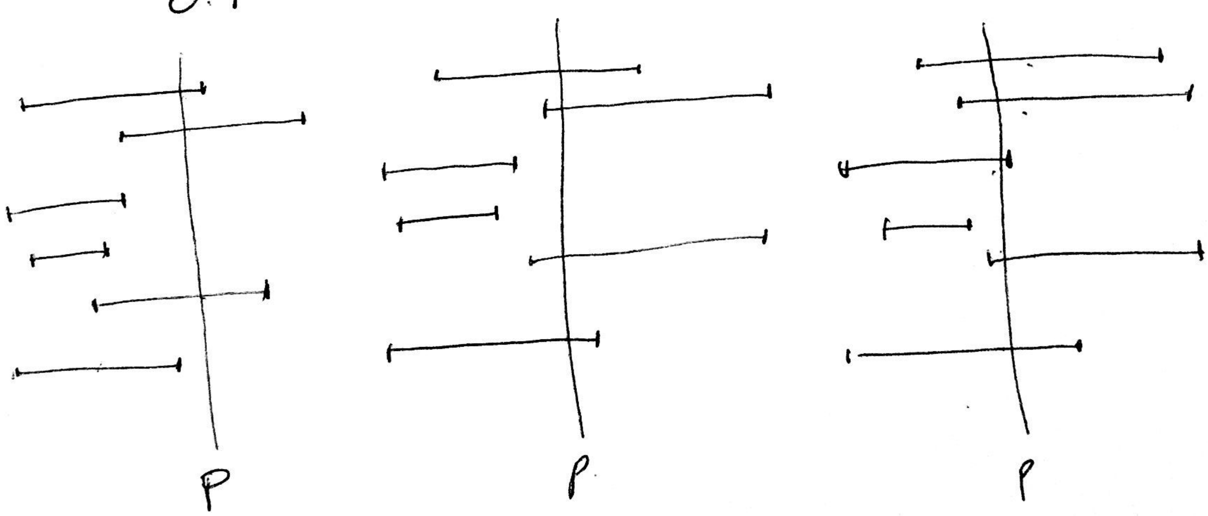
interval estimator:

point estimator $- E <$ population parameter $<$ point estimator $+ E$

E : margin of error.
maximum difference between population parameter and point estimator.

every interval estimator has a level of confidence

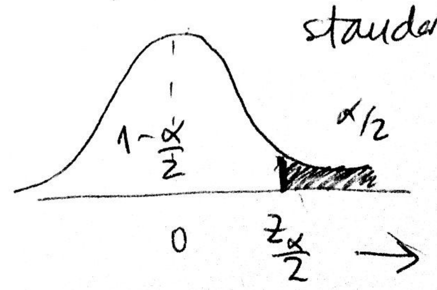
90%	95%	99%
0.9	0.95	0.99



critical value: is the value that separates an upper area of $\alpha/2$ from the lower area $1-\alpha/2$

standard normal

$Z_{\frac{\alpha}{2}}$



in the case in which we focus on estimating a population proportion: p.

point estimator : $\hat{p} = \dfrac{\#\text{ of successes}}{\#\text{ of trials}}$

interval estimator.

$$\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

size of the sample.

- confidence level of the interval.
- desired margin of error : E

if $\hat{p}$ is available $n = \dfrac{[z_{\alpha/2}]^2\, \hat{p}(1-\hat{p})}{E^2}$

if no $\hat{p}$ is available $n = \dfrac{[z_{\alpha/2}]^2\, 0.5(1-0.5)}{E^2} = \dfrac{[z_{\alpha/2}]^2\, 0.25}{E^2}$.

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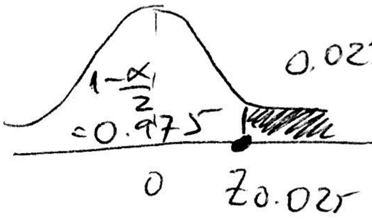
inference (estimation) for proportion of children that have received measles vaccination: p.

95% confidence: $1-\alpha = 0.95 \Rightarrow \alpha = 0.05$

$$Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{0.025}$$



$$Z_{0.025} = 1.96$$

$$E = 3\% = \frac{3}{100} = 0.03$$

a) we are provided $\hat{p} = 0.9$.

$$n = \frac{[Z_{\alpha/2}]^2 \hat{p}(1-\hat{p})}{E^2} = \frac{1.96^2 \cdot 0.9(1-0.9)}{0.03^2}$$

$$= \frac{1.96^2 \cdot 0.9 \cdot 0.1}{0.03^2} = 384.16$$

$n = 385.$

b) $n = \frac{[Z_{\alpha/2}]^2 \, 0.25}{0.03^2} = \frac{1.96^2 \, 0.25}{0.03^2} = 1067.11$

$n = 1068.$

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$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

we are going to assume that we also do not know σ

Slide 8.

$n = 15$, $\bar{X} = 30.9$, $s = 2.9$.

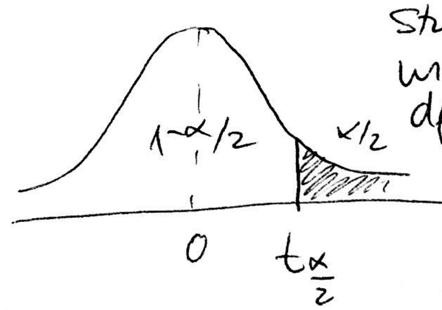
a) point estimate for mean birth weight of girls : μ.

a point estimate for μ is $\bar{X} = 30.9$.

b) the assumptions for the normal distribution seem adequate. because the points in the quantile plot are close to the diagonal line.

Also, the sample is a simple random sample.

Student t with $df = n-1 = 14$



c) $E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

95% confidence interval

so $\alpha = 0.05$

$t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} = 2.145$

$$E = \frac{2.145 \cdot 2.9}{\sqrt{15}} = \cancel{0.8833}$$

1.6061

d) $\bar{X} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

$30.9 - 1.6061 < \mu < 30.9 + 1.6061$

$29.2939 < \mu < 32.5061$

e) interpretation of d

we are 95% confident that the interval from 29.2939 to 32.5061 actually does contain the true value of mean birth weight of girls.

f) interpretation of 95% confidence.

with probability 0.95 the confidence interval actually contains the mean birth weight of girls, assuming that the estimation process is repeated many times.

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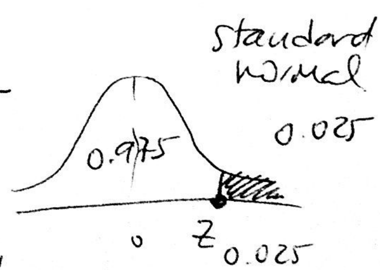
slide 10.

$\sigma = 15$. 95% confident. so $\alpha = 0.05$

$E = 3$.



$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{1.96 \cdot 15}{3} \right)^2 = 96.04$

$n = 97$

you need to randomly select 97 smokers to be 95% confident that the sample mean is within 3 IQ points from the population mean.