

## Answers to Quiz 7

1. In a study, the length of 12 cuckoo eggs was recorded. From the study it was observed that the sample mean is 22.08 and the sample standard deviation is 0.08.

- (a) What requirements must be satisfied in order to compute a 95% confidence interval estimate for the mean length of cuckoo eggs and how do you verify if these requirements are satisfied?

*In order to compute a 95% confidence interval estimate, we need to check that the sample data is a simple random sample [0.5 pts.] that comes from the normal distribution [0.5 pts.], because we only have  $n = 12$  data measurements. To check this requirement, we can make a histogram and/or a quantile plot [0.5 pts.]. We want the histogram to have the shape of a bell with no outliers and in the quantile plot we want to see that the pattern of the points lie on the diagonal line [1.5 pts.].*

- (b) Find the critical value used to compute a 95% confidence interval estimate for the mean length of cuckoo eggs.

*The critical value is the one of a Student  $t$  distribution with  $(n - 1) = 11$  degrees of freedom,  $t_{\alpha/2}$ , [1 pts.] that separates an upper area of  $\alpha/2 = 0.05/2 = 0.025$  from a lower area of  $1 - \alpha/2 = 0.975$ ,  $t_{\alpha/2} = t_{0.025}$  [1 pts.]. From the table at the back of the book (row eleven) the critical value is  $t_{0.025} = 2.201$  [1 pts.]*

- (c) Compute a 95% confidence interval estimate for the mean length of cuckoo eggs.

*The 0.95 confidence interval for the mean is  $\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \frac{s}{\sqrt{n}}$ , [1 pts.] where  $\mu$  denotes mean length of cuckoo eggs. This confidence interval is  $22.08 - 2.201 * \frac{0.08}{\sqrt{12}} < \mu < 22.08 + 2.201 * \frac{0.08}{\sqrt{12}}$ , so  $22.0291 < \mu < 22.1308$  [2 pts.]*

- (d) How many cuckoo eggs do you need to sample if you want 95% confidence that the sample mean is in error by no more than 0.01 units from the population mean? Assume that  $\sigma = 0.08$ . Justify your answer.

*To find the size of the sample we note that  $z_{\alpha/2} = z_{0.025} = 1.96$ ,  $E = 0.01$ , and use  $\sigma = 0.08$  [1.5 pts.]. The sample size is  $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.96*0.08}{0.01}\right)^2 = 245.86$ . So the required sample size is  $n = 246$  [1.5 pts.]. we need to sample 246 cuckoo eggs to be 0.95 confidence that the sample mean is in error by no more than 0.01 from the population mean.*