$X \sim \text{Uniform}(0,1)$

$$f(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & \text{o.w.} \end{cases}$$

$$E(X) = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\Rightarrow \quad V(X) = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

Central Limit Theorem

$X_1, \ldots, X_n$ iid sample from a distribution with

$E(X_i) = \mu \qquad V(X_i) = \sigma^2$

Take $\bar{X}_n = \sum_{i=1}^{n} X_i / n$

Then, when $n \to \infty$

the distribution of $\bar{X}_n$ goes to a $N\left(\mu, \frac{\sigma^2}{n}\right)$ distribution

χ^2 goodness of fit test

H_0 : Observed frequency distribution fits (conforms with) a given claimed distribution

O : Observed frequency of an outcome

E : Expected frequency of an outcome

n : total # of trials

k : # of categories

Assumptions

- Data randomly selected
- Sample data are frequency counts
- Sample data arises from a multinomial experiment:
  - n is fixed
  - trials are independent
  - probs. for the different categories remain constant over time
  - expected frequency for each category is ≥ 5

Test statistic:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Under H_0 $X^2 \sim \chi^2_{K-1}$