Chapter 4: Probability

Section 4-1: Basic Concepts of Probability

1. The probability of selecting someone with blue eyes is 0.35.
2. The probability of a baby being born a boy is 0.50, or 50%.
3. $P(A) = 0.512$, so $P(\bar{A}) = 1 - 0.512 = 0.488$
4. The answers vary, but a high answer in the neighborhood of 0.999 is reasonable.
5. $0, 3/5, 1, 0.135$
6. $1/5$, or 0.2
7. $1/9$, or 0.111
8. {bb, bg, gb, gg}
9. 47 girls is significantly high.
10. 26 girls is neither significantly low nor significantly high.
11. 23 girls is neither significantly low nor significantly high.
12. 5 girls is significantly low.
13. $1/2$, or 0.5
14. $1/5$, or 0.2
15. $1/4$, or 0.25
16. 0.292
17. $1/10$, or 0.1
18. $1/2$, or 0.5
19. 0
20. 1
21. $\frac{5}{555} = \frac{1}{111}$, or 0.00901; The employer would suffer because it would be at a risk by hiring someone who uses drugs.
22. $\frac{25}{555} = \frac{5}{111}$, or 0.0450; The person tested would suffer because he or she would be suspected of using drugs when in reality he or she does not use drugs.
23. $\frac{50}{555} = \frac{10}{111},$ or 0.0901; This result does appear to be a reasonable estimate of the prevalence rate.
24. $\frac{25+480}{555} = \frac{505}{555} = \frac{101}{111}$, or 0.910; The result does appear to be reasonable as an estimate of the proportion of the adult population that does not use drugs.
25. $\frac{879}{945}$, or 0.93; Yes, the technique appears to be effective.
26. $\frac{239}{291}$, or 0.821; Yes, the technique appears to be effective.
27. $\frac{428}{580}$ = 0.738; Yes, it is reasonable.
28. a. $1/365$
    b. yes
    c. He already knew.
    d. 0
29. $\frac{139}{933+139} = \frac{139}{1072}$, or 0.130. No, it is not unlikely for someone to never seek medical information online. Because the responses are from a voluntary response survey, it is very possible that the results are not very good.

30. $\frac{83,600}{83,600+5,127,400}=\frac{418}{26,055}$, or 0.0160; Yes, in a passenger car crash, a rollover is unlikely.

31. a. brown /brown, brown/blue, blue/brown, blue/blue
   b. $1/4$
   c. $3/4$

32. In the following, the first letter represents the chromosome contributed by the father and the second letter represents the chromosome contributed by the mother. Let X_1 and X_2 represent the possible X chromosomes contributed by the mother.
   a. 0; The possible outcomes are {xX_1, xX_2, YX_1, YX_2}, neither son will have the disease.
   b. 0; The possible outcomes are {xX_1, xX_2, YX_1, YX_2}, neither daughter will have the disease.
   c. $1/2$, or 0.5; The possible outcomes are {Xx_1, XX_2, Yx_1, YX_2}, one of the two sons will have the disease.
   d. 0; The possible outcomes are {Xx_1, XX_2, Yx_1, YX_2}, neither daughter will have the disease.

33. $3/8$, or 0.375

34. $3/8$, or 0.375

35. {bbbb, bbbg, bbgb, bbgg, bgbb, bgbg, bggb, bggg, gbbg, gbbb, gbgb, gbgg, ggbb, ggbg, gggb, gggg}; $4/16=1/4$, or 0.25

36. $2/16=1/8$, or 0.125

37. The high probability of 0.327 shows that the sample results could have easily occurred by chance. It appears that there is not sufficient evidence to conclude that pregnant women can correctly predict the gender of their baby.

38. The low probability of less than 0.0001 shows that the sample results are not likely to occur by chance. It appears that Tamiflu does have an effect on nausea.

39. The low probability of less than 0.001 shows that the sample results could not have easily occurred by chance. It appears that OxyContin does have an effect on sleepiness.

40. The high probability of 0.512 shows that the sample results could have easily occurred by chance. It appears that there is not sufficient evidence to conclude that cell phones have an effect on cancer of the brain or nervous system.

Section 4-2: Addition Rule and Multiplication Rule

1. $P(A)$ represents the probability of selecting an adult with blue eyes, and $P(\overline{A})$ represents the probability of selecting an adult who does not have blue eyes.

2. $P(M \mid B)$ represents the probability of getting a male, given that someone with blue eyes has been selected. $P(M \mid B)$ is not the same as $P(B \mid M)$.

3. Because the selections are made without replacement, the events are dependent. Because the sample size of 1068 is less than 5% of the population size of 15,524,971, the selections can be treated as being independent (based on the 5% guideline for cumbersome calculations).

4. It is certain that the selected adult has type B blood or does not have type B blood.

5. $1-0.26=0.74$

6. $1-0.0025=0.9975$

7. $P(\overline{H})=1-\frac{617}{617+117}=1-\frac{617}{734}=\frac{117}{734}$, or 0.159, where $P(\overline{H})$ is the probability of randomly selecting someone who did not get a headache.

8. $P(\overline{I})$ denotes the probability of screening a driver and finding that he or she is not intoxicated, and $P(\overline{I}) = 0.99112$, or 0.991 when rounded.

Use the following table for Exercises 9–20

	O	A	B	AB	**Total**
Rh^+	59	53	12	6	**130**
Rh^-	9	8	3	2	**22**
Total	**68**	**61**	**15**	**8**	**152**

9. $\frac{152-61}{152} = \frac{91}{152}$, or 0.599

10. $\frac{22}{152} = \frac{11}{76}$, or 0.145

11. $\frac{61}{152} + \frac{130}{152} - \frac{53}{152} = \frac{138}{152} = \frac{69}{76}$, or 0.908; The two events are not disjoint.

12. $\frac{8}{152} + \frac{22}{152} - \frac{2}{152} = \frac{28}{152} = \frac{7}{38}$, or 0.184; The two events are not disjoint.

13. a. $\frac{15}{152} \cdot \frac{15}{152} = 0.00974$; Yes, the events are independent.

    b. $\frac{15}{152} \cdot \frac{14}{151} = 0.00915$; The events are dependent, not independent.

14. a. $\frac{22}{152} \cdot \frac{22}{152} = 0.0209$; Yes, the events are independent.

    b. $\frac{22}{152} \cdot \frac{21}{151} = 0.0201$; The events are dependent, not independent.

15. a. $\frac{130}{152} \cdot \frac{130}{152} = 0.731$; Yes, the events are independent.

    b. $\frac{130}{152} \cdot \frac{129}{151} = 0.731$; The events are dependent, not independent.

16. a. $\frac{8}{152} \cdot \frac{8}{152} = 0.00277$; Yes, the events are independent.

    b. $\frac{8}{152} \cdot \frac{7}{151} = 0.00242$; The events are dependent, not independent.

17. $\frac{61+15}{152} + \frac{22}{152} - \frac{8+3}{152} = \frac{87}{152}$, or 0.572

18. $\frac{68+8}{152} + \frac{130}{152} - \frac{59+12}{152} = \frac{135}{152}$, or 0.888

19. $\frac{61}{152} \cdot \frac{60}{151} \cdot \frac{59}{150} = 0.0627$

20. $\frac{22}{152} \cdot \frac{21}{151} \cdot \frac{20}{150} = 0.00268$

21. Use the following table for parts (a) and (b).

	Positive Test Result	Negative Test Result	**Total**
Subject Used Marijuana	True Positive 119	False Negative 3	**122**
Subject Did Not Use Marijuana	False Positive 24	True Negative 154	**178**
Total	**143**	**157**	**300**

a. There were a total of 300 subjects in the study.

b. 154 subjects had a true negative result.

c. $\frac{154}{300}=\frac{77}{150}=0.513$

22. $\frac{119+3+154}{300}=\frac{23}{25}=0.92$

23. $\frac{119+24+154}{300}=\frac{99}{100}=0.990$

24. $\frac{178}{300}=\frac{89}{150}=0.593$; No, in the general population, the rate of subjects who do not use marijuana is probably much greater than 0.593 or 59.3%.

25. a. 0.03

b. $0.03\cdot 0.03=0.0009$

c. $0.03\cdot 0.03\cdot 0.03=0.000027$

d. By using one drive without a backup, the probability of total failure is 0.03, and with three independent disk drives, the probability drops to 0.000027. By changing from one drive to three, the probability of total failure drops from 0.03 to 0.000027, and that is a very substantial improvement in reliability. Back up your data!

26. a. $0.22\cdot 0.22=0.0484$

b. $1-0.0484=0.9516,$ or 0.952; This probability seems high, but both generators fail about 5% of the time that they are needed. Given the importance of the hospital's needs, the reliability should be improved.

27. $8834-504=8330,$ $\frac{8330}{8834}\cdot\frac{8329}{8833}\cdot\frac{8328}{8832}=0.838$; The probability of 0.838 is high, so it is likely that the entire batch will be accepted, even though it includes many firmware defects.

28. $676-269=407,$ $\frac{407}{676}\cdot\frac{406}{675}\cdot\frac{405}{674}=0.218$; It is somewhat likely that the entire lot would be accepted. With about 40% of the transducers not meeting requirements, the probability of 0.218 seems high.

29. a. $\frac{47,637}{47,637+111,874}=\frac{47,637}{159,511}=0.299$

b. Using the 5% guideline for cumbersome calculations, $(0.299)^5=0.00239$. Using exact probabilities,

$$\frac{47,637}{159,511}\cdot\frac{47,636}{159,510}\cdot\frac{47,635}{159,509}\cdot\frac{47,634}{159,508}\cdot\frac{47,633}{159,507}=0.00238.$$

30. Using the 5% guideline for cumbersome calculations, $\left(\frac{47,637-188}{47,637}\right)^{40}=0.854.$

31. a. $0.985\cdot 0.985+0.985\cdot 0.015+0.015\cdot 0.985=0.999775$

b. $0.985\cdot 0.985=0.970225$

c. The series arrangement provides better protection.

32. $\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\ldots\cdot\frac{341}{365}=0.431$

33. a. $P(A \text{ or } B) = P(A) + P(B) - 2P(A \text{ and } B)$

b. $\dfrac{61}{152} + \dfrac{130}{152} - 2\left(\dfrac{53}{152}\right) = \dfrac{85}{152}$, or 0.559

34. $P(\overline{A \text{ or } B}) = \dfrac{59+9+6+2}{152} = \dfrac{76}{152} = \dfrac{1}{2}$, or 0.5, $P(\bar{A} \text{ or } \bar{B}) = \dfrac{152}{152}$, or 1; The results are different. In general, $P(\overline{A \text{ or } B})$ is not the same as $P(\bar{A} \text{ or } \bar{B})$.

Section 4-3: Complements, Conditional Probability, and Bayes' Theorem

1. $\bar{A}$ is the event of not getting at least 1 defect among the 3 pacemaker batteries, which means that all 3 pacemaker batteries are good.
2. Parts (a) and (b) are correct.
3. The probability that the test indicates that the subject has glaucoma given that the subject actually does have glaucoma.
4. Confusion of the inverse is to think that $P(Y \mid G) = P(G \mid Y)$ or to switch one of those values for the other. That is, confusion of the inverse is to think that the following two probabilities are equal or to incorrectly use one of them for the other: (1) the probability that the test indicates that the subject has glaucoma given that the subject actually does have glaucoma; (2) probability that the subject actually does have glaucoma given that the test indicates that the subject has glaucoma.
5. $1 - \left(\dfrac{1}{2}\right)^3 = \dfrac{7}{8}$, or 0.875
6. $1/2$, or 0.5
7. $1 - (0.512)^6 = 0.982$
8. $1 - (0.545)^6 = 0.974$; The system cannot continue indefinitely because eventually there would be no women to give birth.
9. $1 - (1 - 0.10)^4 = 0.344$
10. $1 - (1 - 0.20)^{10} = 0.893$; There is a good chance of continuing.
11. $1 - (1 - 0.398)^{10} = 0.994$; The probability is high enough so that she can be reasonably sure of getting a defective transducer for her work.
12. $1 - (1 - 0.25)^5 = 0.763$; It is very possible that the researchers will get what they need.
13. a. $P(\text{identical twins} \mid \text{having twins}) = \dfrac{5+5}{30} = \dfrac{1}{3}$, or 0.333

    b. $P(\text{identical twins} \mid \text{twin boys}) = \dfrac{5}{5+5} = \dfrac{1}{2}$, or 0.5
14. a. $P(\text{fraternal twins} \mid \text{having twins}) = \dfrac{20}{30} = \dfrac{2}{3}$, or 0.667

    b. $P(\text{fraternal twins} \mid \text{one boy and one girl}) = \dfrac{10}{10}$, or 1
15. $P(\text{one boy and one girl} \mid \text{fraternal twins}) = \dfrac{10}{20} = \dfrac{1}{2}$, or 0.5
16. $P(\text{two girls} \mid \text{fraternal twins}) = \dfrac{5}{20} = \dfrac{1}{4}$, or 0.25

Use the following table for Exercises 17–20.

	Positive Test Result	Negative Test Result	**Total**
Hepatitis C	335	10	**345**
No Hepatitis C	2	1153	**1155**
Total	**337**	**1163**	**1500**

17. $P(\text{positive result} \mid \text{no hepatitis C}) = \frac{2}{1155}$, or 0.00173; This is the probability of the test making it appear that the subject has hepatitis C when the subject does not have it, so the subject is likely to experience needless stress and additional testing.

18. $P(\text{negative result} \mid \text{hepatitis C}) = \frac{10}{345}$, or 0.0290; The subject gets a test result showing that hepatitis C is not present, but it actually is present, so the subject might delay or forego helpful treatment.

19. $P(\text{hepatitis C} \mid \text{positive result}) = \frac{335}{337}$, or 0.994; The very high result makes the test appear to be effective in identifying hepatitis C.

20. $P(\text{no hepatitis C} \mid \text{negative result}) = \frac{1153}{1163}$, or 0.991; The very high result makes the test appear to be effective in identifying subjects not having hepatitis C.

21. a. $1-(0.03)^2 = 0.9991$

    b. $1-(0.03)^3 = 0.999973$; The usual round-off rule for probabilities would result in a probability of 1.00, which would incorrectly indicate that we are certain to have at least one working hard drive.

22. $1-(0.22)^3 = 0.989$; The result is quite high, but there is roughly a 1% chance that in the event of a power failure, none of the backup generators will work. With the possibility of an outage affecting a hospital and all its patients, it would be wise for manufacturers to improve the reliability of the backup generators so that the 22% failure rate is lowered and the probability of 0.989 is increased.

23. $1-(1-0.126)^5 = 0.490$; The probability is not low, so further testing of the individual samples will be necessary for about 49% of the combined samples.

24. $1-(1-0.005)^{10} = 0.0489$; The probability is quite low, indicating that further testing of the individual samples will be necessary for about 5% of the combined samples.

25. $1-\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\ldots\cdot\frac{341}{365} = 1-0.431 = 0.569$

Section 4-4: Risks and Odds

1. p_t is the proportion of the characteristic in the treatment group, and p_c is the proportion of the characteristic in the control group.

2. With small differences between the rates, the relative risk may be large, implying a bad situation where the very low incidence rates suggest that there is not much risk in either group.

3. We need to treat 37 subjects with the influenza vaccine in order to prevent one case of influenza. The result applies to a large number of subjects, not every particular group of 37 subjects.

4. In a retrospective (or case-control) study, data are collected from a past time period by going back in time (through examination of records, interviews, and so on). In a prospective (or longitudinal or cohort) study, data are collected in the future from groups that share common factors (such groups are called cohorts).

5. prospective

6. $P(\text{headache} \mid \text{viagra treatment}) = \dfrac{117}{117+617} = \dfrac{117}{734} = 0.159$

7. $P(\text{headache} \mid \text{viagra treatment}) = \dfrac{117}{117+617} = \dfrac{117}{734} = 0.159$; $P(\text{headache} \mid \text{placebo}) = \dfrac{29}{29+696} = \dfrac{1}{25} = 0.040$;

The risk of a headache appears to be higher in the $\dfrac{117}{117+617}$ treatment group.

8.Absolute risk reduction $= \left| \dfrac{117}{117+617} - \dfrac{29}{29+696} \right| = 0.119$

9. $\dfrac{1}{\left| \dfrac{117}{117+617} - \dfrac{29}{29+696} \right|} = 8.4$; 9 when rounded up.

10. Odds in favor of headache: $\dfrac{\dfrac{117}{117+617}}{\dfrac{617}{117+617}} = \dfrac{117}{617}$, or 117:617 (roughly 1:5.2)

Odds against headache: $\dfrac{\dfrac{617}{117+617}}{\dfrac{117}{117+617}} = \dfrac{617}{117}$, or 617:117 (roughly 5.2:1)

11. RR $= \dfrac{p_t}{p_c} = \dfrac{\dfrac{117}{117+617}}{\dfrac{29}{29+696}} = \dfrac{2925}{734} = 3.99$, 3.99 or roughly 4. The risk of headaches among Viagra users is roughly 4 times the risk of headaches for those who take a placebo.

12. OR $= \dfrac{ad}{bc} = \dfrac{117 \cdot 696}{617 \cdot 29} = 4.551$; The odds of having a headache in the treatment group are 4.551 times those of the placebo group.

13. a. $P(\text{infection} \mid \text{atorvastatin treatment}) = \dfrac{89}{89+774} = 0.103$

b. $P(\text{infection} \mid \text{placebo}) = \dfrac{27}{27+243} = 0.100$

c. Absolute risk reduction $= \left| \dfrac{89}{89+774} - \dfrac{27}{27+243} \right| = 0.00313$; The chance of infection in the atorvastatin treatment group is slightly higher than for the placebo group. For those in the placebo group, there is a 0.313% reduced chance of infection when compared to the atorvastatin treatment group.

14. $\dfrac{1}{\left| \dfrac{89}{89+774} - \dfrac{27}{27+243} \right|} = 319.6$, or 320 when rounded up. We would need to treat 320 patients with atorvastatin (instead of a placebo) to prevent one of the patients from getting an infection.

15. Atorvastatin: 89:774 or roughly 1:9. Placebo: 27:243 or 1:9. There is not much of a difference between these two results.

16. OR $= \dfrac{ad}{bc} = \dfrac{89 \cdot 243}{27 \cdot 774} = 1.035$; RR $= \dfrac{p_t}{p_c} = \dfrac{\frac{89}{89+774}}{\frac{27}{27+243}} = \dfrac{890}{863} = 1.031$; Since both values are close to 1, it appears that atorvastatin does not increase the risk of infection.

17. RR $= \dfrac{p_t}{p_c} = \dfrac{\frac{26}{2103}}{\frac{22}{1671}} = \dfrac{7241}{7711} = 0.939$; OR $= \dfrac{ad}{bc} = \dfrac{26 \cdot (1671-22)}{(2103-26) \cdot 22} = 0.938$; 0.938; The risk of a headache with Nasonex treatment is slightly less than the risk of a headache with a placebo.

18. a. It would be unethical to assign subjects to the group that does not use seat belts.
    b. Answers will vary. One possible answer: The number of subjects that do not use seat belts may be too small.

Section 4-5: Rates of Mortality, Fertility, and Morbidity

1. During a year in China, there are 12.3 births for every 1000 people in the population.

2. One important advantage of the mortality rate of 12.3 people (per 1000 people in the population) is that it uses fewer decimal places and is generally easier to use and understand.

3. About $0.0123 \cdot 1{,}360{,}762{,}587 = 16{,}737{,}380$ births are expected in a year.

4. A disease incidence rate describes the growth of the number of cases while the disease prevalence rate describes the current number of cases.

5. The neonatal mortality rate is $\left(\dfrac{15{,}973}{3{,}953{,}590}\right)1000 = 4.0$, or 4.0 per 1000.

6. The fetal mortality rate is $\left(\dfrac{26{,}148}{3{,}953{,}590+26{,}148}\right)1000 = 6.6$, or 6.6 per 1000.

7. The perinatal mortality rate is $\left(\dfrac{26{,}148+15{,}973}{3{,}953{,}590+26{,}148}\right)1000 = 10.6$, or 10.6 per 1000.

8. The crude birth rate is $\left(\dfrac{3{,}953{,}590}{312{,}799{,}495}\right)1000 = 12.6$, or 12.6 per 1000.

9. The general fertility rate is $\left(\dfrac{3{,}953{,}590}{61{,}488{,}227}\right)1000 = 64.3$, or 64.3 per 1000 women aged 15–44.

10. The motor vehicle death incidence rate is $\left(\dfrac{33{,}783}{312{,}799{,}495}\right)10{,}000 = 1.1$, or 1.1 deaths per 10,000.

11. The HIV infection prevalence rate is $\left(\dfrac{1{,}155{,}792}{312{,}799{,}495}\right)1000 = 3.7$, or 3.7 per 1000.

12. The HIV infection prevalence rate is $\left(\dfrac{7683}{1{,}155{,}792}\right)1000 = 6.6$, or 6.6 per 1000.

13. $\dfrac{8}{1000} = 0.008$; The rate uses fewer decimal places and is easier to understand.

14. a. $\frac{7.4}{1000} = 0.0074$

b. $(0.0074)(0.0074) = 0.0000548$

c. $(1-0.0074)(1-0.0074) = 0.985$

15. a. $\frac{8.3}{1000} = 0.0083$

b. $(0.0083)(0.0083) = 0.00006889$

c. $1-(0.0083)(0.0083) = 0.999931$; Using three significant digits would result in a probability of 1.00, which would be misleading because it would incorrectly suggest that it is certain that at least one survives the year.

16. a. $\left(\frac{787650}{312799495}\right)1000 = 2.5$, or 2.5 per 1000

b. $\frac{787650}{312799495} = 0.00252$

c. $(1-0.00252)(1-0.00252)(1-0.00252) = 0.992$

17. a. $\left(\frac{787,650}{2,515,458}\right)100 = 31.3$

b. $(1-0.313)(1-0.313)(1-0.313) = 0.324$

18. a. The crude mortality rate for Florida is $\left(\frac{3625+39,820+129,395}{5,716,861+9,842,031+3,375,303}\right)1000 = 9.1$, or 9.1 per 1000.

The crude mortality rate for the United States is $\left(\frac{63,208+619,982+1,832,268}{103,542,603+163,960,163+45,296,729}\right)1000 = 8.0$, or 8.0 per 1000. Florida has a slightly higher mortality rate, but the two rates are close.

b. The crude mortality rate for Florida (65 and older) is $\left(\frac{129395}{18,934,195}\right)1000 = 6.8$, or 6.8 per 1000.

The crude mortality rate for the United States (65 and older) is $\left(\frac{18,32,268}{312,799,495}\right)1000 = 5.8$ or 5.8 per 1000.

Florida now has a slightly higher mortality rate, but the two rates are still close.

c. $\frac{3375303}{18934195} = 0.178$, or 17.8% of Florida's is 65 and older and $\frac{45,296,729}{312,799,495} = 0.14$, or 14.5% of the United States is 65 and older. Since Florida has a higher percentage of those in the population of 65 years or older than the U.S., the difference in crude mortality rate (6.8 for Florida and 5.8 for the U.S.) would be expected.

19. No, the health of the nation is not necessarily declining. The increasing number of deaths each year is probably due to the growing population.

20. The major problem in comparing these rates is that the age distributions of the two nations could be different and that, among other reasons, could be a major factor in the difference.

21. The United States has a population distribution of 33.10190862%, 52.41701653%, and 14.48107485% for the three age categories. If the 18,934,195 Florida residents have that same distribution, the three age groups would have these numbers of people: 6,267,580, 9,924,740, and 2,741,875, respectively. Using the same Florida mortality rates for the three individual age groups and using the new adjusted population sizes for the three Florida age categories, we get these numbers of Florida deaths: 3974, 40,155, and 105,112, respectively. Using the adjusted numbers of deaths and the adjusted population sizes for the different categories, the crude mortality rate for Florida becomes 7.9 per 1000, which is much closer to the U.S. mortality rate of 8.0 per 1000 than the mortality rate of 9.1 per 1000 found for Florida before the adjustments.

Section 4-6: Counting

1. The symbol "!" is the factorial symbol that represents the product of decreasing whole numbers, as in $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$. Six people can be scheduled in 720 different ways.

2. For permutations, the order of the results matter. For combinations, the order of the results do not matter.

3. ${}_9C_4 = \dfrac{9!}{(9-4)!4!} = 126$; The result of 126 is the number of different combinations that are possible when 4 items are selected without replacement from 9 different items that are available.

4. ${}_9P_4 = \dfrac{9!}{(9-4)!} = 3024$; The result of 3024 is the number of different permutations that are possible when 4 items are selected without replacement from 9 different items that are available.

5. $\dfrac{1}{10} \cdot \dfrac{1}{10} \cdot \dfrac{1}{10} \cdot \dfrac{1}{10} = \dfrac{1}{10{,}000}$

6. $\dfrac{1}{10} \cdot \dfrac{1}{10} \cdot \dfrac{1}{10} \cdot \dfrac{1}{10} \cdot \dfrac{1}{10} = \dfrac{1}{100{,}000}$

7. There are ${}_{19}C_2 = \dfrac{19!}{(19-2)!2!} = 171$ ways to choose a team of physicians. The probability is $1/171$, or 0.00585.

8. ${}_{11}C_3 = \dfrac{11!}{(11-3)!3!} = 165, \quad 3! = 6$

9. $8! = 40{,}320$; The probability is $1/40{,}320$.

10. For three additional letters, there are $2 \cdot 26 \cdot 26 \cdot 26 = 35{,}152$ possibilities, for two additional letters, there are $2 \cdot 26 \cdot 26 = 1352$ possibilities, for a total of $35{,}152 + 1352 = 36{,}504$ possibilities.

11. There are ${}_{50}P_5 = \dfrac{50!}{(50-5)!} = 254{,}251{,}200$ possible routes. The probability is $1/254{,}251{,}200$.

12. $\dfrac{12!}{3!4!} = 3{,}326{,}400$

13. $\dfrac{1}{100 \cdot 100 \cdot 100 \cdot 100} = \dfrac{1}{100{,}000{,}000}$; No, there are too many different possibilities.

14. ${}_{10}C_2 = \dfrac{10!}{(10-2)!2!} = 10$

15. The groups of three must be chosen in order, so there are 168,168,000 ways to form the groups.
$$\left({}_{15}C_3\right) \cdot \left({}_{12}C_3\right) \cdot \left({}_9C_3\right) \cdot \left({}_6C_3\right) \cdot \left({}_3C_3\right) = \frac{15!}{(15-3)!3!} \cdot \frac{12!}{(12-3)!3!} \cdot \frac{9!}{(9-3)!3!} \cdot \frac{6!}{(6-3)!3!} \cdot \frac{3!}{(6-3)!3!} = 168{,}168{,}000$$

16. $7! = 5040$

17. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$; The probability is $1/100{,}000$, or 0.00001.

18. ${}_9P_6 = \dfrac{9!}{(9-6)!} = 60,480$

19. There are $8 \cdot 10 \cdot 10 = 800$ possible area codes. There are $8 \cdot 10 \cdot 10 \cdot 8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6,400,000,000$ possible area codes. Yes. (With a total population of about 400,000,000, there would be about 16 phone numbers for every adult and child.)

20. $\dfrac{11!}{4!4!2!} = 34,650$

21. a. ${}_{10}P_4 = \dfrac{10!}{(10-4)!} = 5040$

b. ${}_{10}C_4 = \dfrac{10!}{(10-4)!4!} = 210$

c. The probability is $1/210$.

22. a. $1/4$, or 0.25

b. $\dfrac{3}{4} \cdot \dfrac{1}{4} = \dfrac{3}{16}$, or 0.188

c. Trick question. There is no finite number of attempts, because you could continue to get the wrong position every time.

23. ${}_8P_5 = \dfrac{8!}{(8-5)!} = 6720$

24. a. $\dfrac{1}{10^{16}} = \dfrac{1}{10,000,000,000,000,000}$

b. $\dfrac{1}{10^{12}} = \dfrac{1}{1,000,000,000,000}$

c. $\dfrac{1}{10^8} = \dfrac{1}{100,000,000}$; The number of possibilities (100,000,000) is still quite large, so there is no reason to worry(unless the information is taken from your credit card or is hacked from the Internet).

25. $\dfrac{16!}{2!2!2!2!2!} = 653,837,184,000$

26. a. ${}_{16}P_{14} \dfrac{16!}{(16-14)!} = 10,461,394,944,000$; (Most calculators will give a result in scientific notation, so an answer such as 10,461,395,000,000 is OK.)

b. ${}_{16}C_{14} \dfrac{16!}{(16-14)!14!} = 120$

c. The probability is $1/120$.

27. $\dfrac{1}{{}_{75}C_5 \cdot 15} = \dfrac{1}{258,890,850}$; There is a *much* better chance of being struck by lightning.

28. $\dfrac{{}_2C_1}{{}_{12}C_6} = \dfrac{2}{924} = \dfrac{1}{462}$; Yes, if everyone treated is of one gender while everyone in the placebo group is of the opposite gender, you would not know if different reactions are due to the treatment or gender.

29. There are $2+2\cdot2+2\cdot2\cdot2+2\cdot2\cdot2\cdot2+2\cdot2\cdot2\cdot2\cdot2=2+4+8+16+32=62$ different possible characters. The alphabet requires 26 characters and there are 10 digits, so the Morse code system is more than adequate.

30. $\frac{{}_2C_1}{{}_8C_4}=\frac{2}{70}=\frac{1}{35}$

31. $26+26\cdot36+26\cdot36^2+26\cdot36^3+26\cdot36^4+26\cdot36^5+26\cdot36^6+26\cdot36^7=2{,}095{,}681{,}645{,}538$ or about 2 trillion.

32. a. ${}_5C_2=10$
    b. ${}_nC_2=\frac{n(n-1)}{2}$
    c. $4!=24$
    d. $(n-1)!$

Chapter Quick Quiz

1. $\frac{4}{5}$, or 0.8
2. $1-0.30=0.70$
3. $\frac{4}{12}=\frac{1}{3}$
4. $0.677^2=0.458$
5. Answers will vary, but the probability should be low, such as 0.01.

Use the following table for Exercises 6–10.

	Developed Flu	Did Not Develop Flu	**Total**
Vaccine Treatment	14	1056	**1070**
Placebo	95	437	**532**
Total	**109**	**1493**	**1602**

6. $\frac{109}{1602}$, or 0.0680
7. $\frac{14+1056+95}{1602}=\frac{1165}{1602}$, or $\frac{109}{1602}+\frac{1070}{1602}-\frac{14}{1602}=\frac{1165}{1602}$, or 0.727
8. $\frac{14}{1602}=\frac{7}{801}=0.00874$
9. $\frac{109}{1602}\cdot\frac{108}{1601}=0.00459$
10. $P(\text{developed flu}\mid\text{given vaccine})=\frac{14}{1070}=\frac{7}{535}$, or 0.0131

Review Exercises

Use the following table for Exercises 1–10.

	Successful Treatment	Unsuccessful Treatment	**Total**
Splint Treatment	60	23	**83**
Surgery Treatment	67	6	**73**
Total	**127**	**29**	**156**

1. $\frac{127}{156}$, or 0.814
2. $P(\text{successful treatment}\mid\text{splinting})=\frac{60}{83}$, or 0.723
3. $P(\text{successful treatment}\mid\text{surgery})=\frac{67}{73}$, or 0.918
4. $\frac{127}{156}+\frac{73}{156}-\frac{67}{156}=\frac{133}{156}$, or 0.853
5. $\frac{29}{156}+\frac{83}{156}-\frac{23}{156}=\frac{89}{156}$, or 0.571

6. $\frac{127}{156}\cdot\frac{126}{155}=0.662$ (not 0.663)

7. $\frac{127}{156}\cdot\frac{127}{156}=0.663$

8. $\overline{A}$ is the event of selecting a patient and getting someone who was not treated with surgery. $P(\overline{A})=1-\frac{73}{156}$ $=\frac{83}{156}=0.532$

9. $\overline{A}$ is the event of selecting a patient and getting someone who did not have a successful treatment. $P(\overline{A})=1-\frac{127}{156}=\frac{29}{156}=0.186$

10. $\frac{127}{156}\cdot\frac{126}{155}\cdot\frac{125}{154}=0.537$

11. a. $1-0.75=0.25$, or 25%
    b. $0.75\cdot 0.75\cdot 0.75\cdot 0.75=0.316$
    c. No, it is not unlikely because the probability of 0.316 shows that the event occurs quite often.

12. a. $1/365$
    b. $31/365$
    c. Answers will vary, but it is probably quite small, such as 0.01 or less.
    d. yes

13. $1-\left(1-\frac{34}{10{,}000}\right)^{10}=0.0335$; No, it is not likely.

14. a. $1-\frac{1}{1000}=\frac{999}{1000}$, or 0.999
    b. $1-\left(\frac{1}{1000}\right)^2=\frac{999{,}999}{1{,}000{,}000}$, or 0.999999

Cumulative Review Exercises

1. a. The mean is $\bar{x}=\frac{0.09+0.11+\cdots+0.15+0.17+\cdots+0.23+0.35}{12}=0.165$ g/dL.
   b. The median is $\frac{0.15+0.17}{2}=0.160$ g/dL.
   c. The midrange is $\frac{0.09+0.35}{2}=0.220$ g/dL.
   d. The range is $0.35-0.09=0.260$ g/dL.
   e. $s=\sqrt{\frac{(0.09-0.165)^2+(0.11-0.165)^2+\ldots+(0.23-0.165)^2+(0.35-0.165)^2}{12-1}}=0.069$ g/dL
   f. $s^2=(0.069)^2=0.005\ (\text{g/dL})^2$

2. a. The five number summary is 0.090 g/dL, 0.120 g/dL, 0.160 g/dL, 0.180 g/dL, 0.350 g/dL. The value of 0.350 g/dL is an outlier.

   b.

   c.

0	9
1	1 1 3 4 5 7 7 8 8
2	3
3	5

3. a. $\dfrac{2346}{5100} = 0.46 = 46\%$

   b. 0.460

   c. stratified sample

4. a. a convenience sample

   b. If the students at the college are mostly from a surrounding region that includes a large proportion of one ethnic group, the results will not reflect the general population of the United States.

   c. $0.35 + 0.4 = 0.75$

   d. $1 - (0.6)^2 = 0.64$

5. The lack of any pattern of the points in the scatterplot suggests that there does not appear to be an association between systolic blood pressure and blood platelet count.