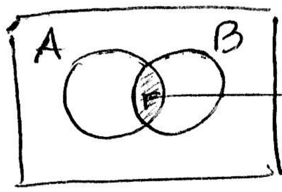
Summary class 6.

A , B

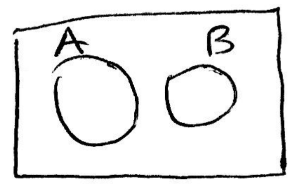
Addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(B \text{ and } A)$$



P(A and B)

A and B are disjoint



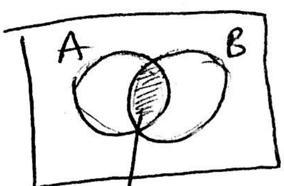
$$P(A \text{ and } B) = 0.$$

$$P(A \text{ or } B) = P(A) + P(B)$$

Multiplication rule

$$P(A \text{ and } B) = P(A)\,P(B \mid A)$$

given



P(B|A)

A and B are independent

$$P(A \text{ and } B) = P(A)\,P(B)$$

- two events that are always disjoint are A and $\bar{A}$

$$P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) = 1.$$

$$P(A) = 1 - P(\bar{A})$$

$$P(\bar{A}) = 1 - P(A)$$

conditional probability:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} \qquad /P(A)$$

$$P(A)\,P(B \mid A) = P(A \text{ and } B)$$

Class 7.

Slide 5.

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let B : subject uses drugs

A : test result is positive.

P(subject uses drug | test result is positive)

$$= P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{45/555}{\frac{(45+25)}{555}}$$

$$= \frac{45}{\cancel{555}} \cdot \frac{\cancel{555}}{(45+25)} = \frac{45}{70} = 0.6428$$

$$P(B) = \frac{45+5}{555} = \frac{50}{555} = 0.09009$$

slide 6.

A = subject uses drugs

B = test is positive

P (A | B) = P (uses drugs given positive test)

information we have available:

- P (positive test) = $\frac{70}{555}$ = P(B)

- P (using drugs) = $\frac{50}{555}$ = P(A)

- P (positive test given uses drugs) = $\frac{45}{50}$ = P(B | A)

use bayes theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} = \frac{\frac{45}{\cancel{50}} \cdot \frac{\cancel{50}}{555}}{\frac{70}{555}} = \frac{45}{70} = 0.6428$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(A \text{ and } B) = P(B|A)\,P(A)$$

Slide 13

Let x be the random variable that describes the number of girls born in two births.

x = number of girls in two births

Probability of number of girls being 0 is 0.25
probability of number of girls being 1 is 0.5.

a) x can take 1, 2, 0.
x is a discrete random variable (finite random variable)

b) probability distribution: $P(x)$

x	$P(x)$	
0	0.25	$(P(0))$
1	0.5	$(P(1))$
2	0.25	$(P(2))$

$\Sigma P(x) = 1$

0.75
0.25.

$\Sigma_i P(x) = 1$ is $P(0) + P(1) + P(2) = 1$.

c) is it a probability distribution?

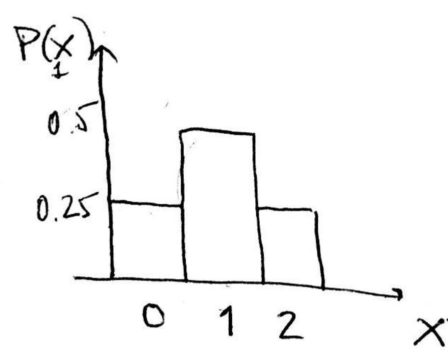
$\Sigma P(x) = 1$
$0 \leq P(x) \leq 1$

x	$P(x)$
0	0.1
1	1.3
2	0.2

d) Find prob. of number of girls being 3.

$x = 3 \qquad P(3) = 0.$

x	$P(x)$
0	0.1
1	0.1
2	0.1

e) 

probability histogram.

slide 15

X = number of girls in two births.

$$P(x) = \frac{1}{2(2-x)!\,x!}$$

X can be 0, 1, or 2.

$\boxed{0! = 1 \qquad 1! = 1 \quad , \quad 2! = 1\cdot 2}$ $\quad 3! = 1\cdot 2\cdot 3$

$$n! = 1\cdot 2\cdot 3\cdot \ldots \cdot (n-1)\cdot n$$

x	P(x)
0	0.25
1	0.5
2	0.25

$$P(0) = \frac{1}{2(2-0)!\,0!} = \frac{1}{2\cdot 2!\cdot 1} = \frac{1}{4} = 0.25$$

$$P(1) = \frac{1}{2(2-1)!\,1!} = \frac{1}{2\cdot 1\cdot 1} = \frac{1}{2} = 0.5$$

$$P(2) = \frac{1}{2(2-2)!\,2!} = \frac{1}{2\cdot 1\cdot 2} = \frac{1}{4} = 0.25$$

a) mean of $\underbrace{\text{number of girls in two births}}_{X}$: μ

$$\mu = \sum x\cdot P(x) = 0\cdot P(0) + 1\cdot P(1) + 2\cdot P(2)$$
$$= 0\cdot 0.25 + 1\cdot 0.5 + 2\cdot 0.25$$
$$= 0.5 + 0.5 = 1$$

$\mu = 1$

Variance of number of girls in two births : σ^2

$$\sigma^2 = \sum (x-\mu)^2 P(x) \quad , \quad \mu = 1$$

$\mu = 1$

X	$(X-1)$	$(X-1)^2$	$P(X)$	$(X-1)^2 P(X)$	$\Sigma (X-1)^2 P(X$
0	-1	1	0.25	0.25	
1	0	0	0.5	0	
2	1	1	0.25	0.25	

$$\sigma^2 = \Sigma (X-1)^2 P(X) = 0.5$$