Summary.

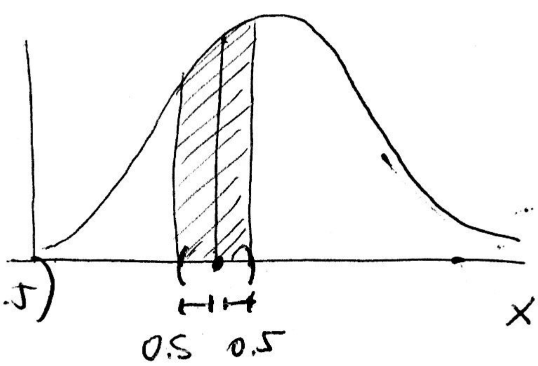
Using the normal distribution to approximate Binomial probabilities.

X that describes number of successes in (n) trials.

n is big.

$np \geq 5$ and $n(1-p) \geq 5$

use continuity correction



$P(\text{exactly } x) = P(x - 0.5 < Y < x + 0.5)$

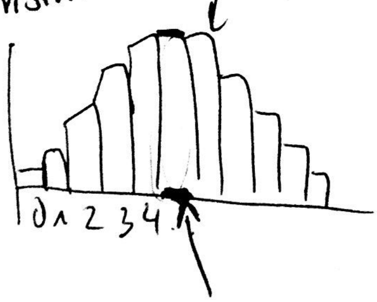
Y has the normal distribution with mean np and standard deviation $\sqrt{np(1-p)}$.

$P(700 \text{ patients have the disease}) = P(699.5 < X < 700.5)$

$1000 = n$. $np \geq 5$ and $n(1-p) \geq 5$.

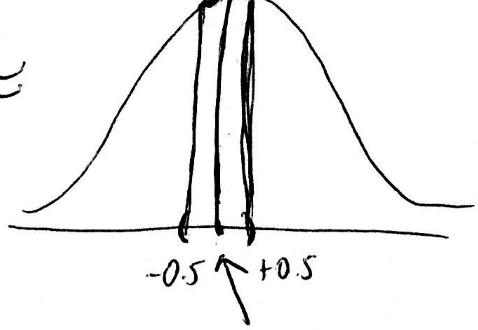
$\mu = np$ $\sigma = \sqrt{np(1-p)}$

Binomial



$\approx$

Normal.



$p = 0.5$

$n = 1$

$np = 0.5$

$n = 10$

$np = 5.$

$p = 0.2$

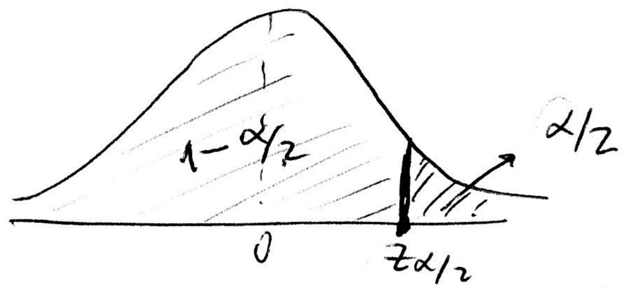
$n = 10$

$np = 2$

Class 12

slide 7.

critical value : $z_{\frac{\alpha}{2}}$



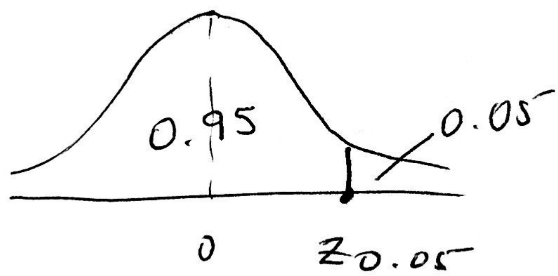
standard normal distribution

$\sigma = 1$

$\alpha = 0.1 \ , \ 0.05 \ , \ 0.01$

if $\alpha = 0.1$

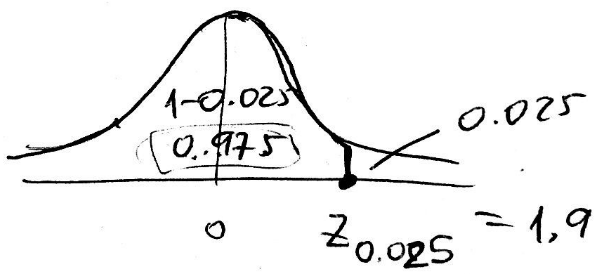
$z_{\frac{\alpha}{2}} = z_{\frac{0.1}{2}} = z_{0.05}$



from the table

$\alpha = 0.05 \qquad \frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

$z_{\frac{\alpha}{2}} = z_{0.025}$



$= 1{,}960.$

confidence level.	critical value
90%.	1.645
95%.	1.960
99%.	2.575

Slide 10.

860 births

426 boys.

make inference for p: population proportion of newborn boys.

a) point estimate for p.

$$\hat{p} = \frac{\text{number of successes}}{\text{number of trials}} = \frac{426}{860} = 0.4953.$$

b) yes: 1) random sample.

2) number of successes $426 > 5$ ✓

number of failures: $860 - 426 = 434 > 5$ ✓.

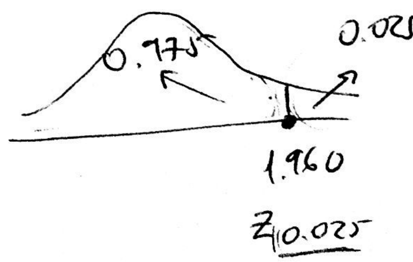
c) Find E for a 95% confidence interval for p.

a 95% confidence interval for p is:

95% confidence $\Rightarrow \alpha = 0.05 \Rightarrow z_{\frac{\alpha}{2}} = 1.960$

$$\hat{p} - \underbrace{1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} < p < \hat{p} + 1.960\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

E : maximum difference between p and $\hat{p}$.



$$E = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 1.96\sqrt{\frac{0.4953(1-0.4953)}{860}}$$

$$= 0.0334.$$

d) 95% confidence interval for p.

$$\underbrace{0.4953 - 0.0334} < p < \underbrace{0.4953 + 0.0334}$$

$$0.4619 < p < 0.5287.$$

e) we are 95% confident that the interval from 0.4619 to 0.5287 actually contains the true value of population proportion of new-born boys.

f) it is believed that $p = 0.512$
the evidence (426 newborn boy out of 860) supports the belief that the proportion of newborn boys is 0.512.

g) interpretation of the confidence level: 95%.
with probability 0.95 the confidence ~~...~~ intervall contains the population proportion of newborn boys, assumming that the estimation process is repeated a large number of times.