

# The Statistics Behind Perceptual Decision Making



## 1 Introduction

Perceptual decision making is a term to describe how sensory information is used to guide behavior toward the external world. The drift diffusion model (DDM) [1] has been developed to provide an account of accuracy, response time, and confidence occurring in two-choice perceptual decisions. DDM assumes that decisions are made by extracting sensory evidence from the stimulus and subsequently accumulating the evidence over time. Once the amount of accumulated evidence reaches one of two response thresholds, a response is elicited. When the sensory evidence is stronger, it is accumulated faster and more rapidly hits the decision boundary. It has been also proposed that stronger evidence leads to higher confidence, compared to weak evidence [2].

Dou and Samaha [3] examined how evidence strength influences confidence in observers' perceptual decisions using a two-choice discrimination task with six motion coherence (evidence strength) levels. In our current project, we will explore the behavioral data from their study to look at if observers' actual responses follow the predictions of DDM.

## 2 The Data

The data is from a repeated measured experiment with evidence strength as the factor variable. The response variables include response time, accuracy, and confidence of the decision. Each of the 25 participants completed 180 trials on each strength level, amassing to a total of 1080 theoretical trials. However, the behavioral data has been trimmed by the experimenters due to excluding trials in which the EEG signals in the experiment were noisy. The details of the five variables are listed below:

- **Participant Number (P):** Integer variable, the index for the participants
- **Strength Level (S):** Ordinal variable with 6 levels: 1%, 4.5%, 8%, 12%, 25% and 40%
- **Response Time (R):** Numeric variable
- **Accuracy (A):** Binary variable with 0 and 1, 0 means inaccurate and 1 means accurate

- **Confidence (C):** Ordinal variable with 4 levels: 1, 2, 3 and 4

## 3 The Goal

The central question that we want to address is how the sensory evidence strength statistically influences decision making according to the DDM. To answer this central question, we will regard each trial from participants as an observation, and explore three technical aspects of the data: (1) what are the statistical relationships between response time, evidence strength level, accuracy, and confidence that we can further uncover; (2) whether confidence can be predicted by evidence strength and response time; (3) whether accuracy can be predicted by evidence strength and response time. From these results we will then be able to qualitatively and quantitatively examine how evidence strength influences decision making in this psychological experiment.

## 4 Planned Statistical Analysis

The statistical analysis will be broken up into the following four steps corresponding with the goals:

### 1. Missing Value and Influential Data Analysis

If there are missing values, we need to evaluate if it is necessary to drop them or apply some imputation methods. In addition, we will need to inspect any outliers or influential observations that arise in the data.

### 2. Variable Explorations

To discern the basic relationships of the variables, we will be utilizing side-by-side box plots for the response time versus evidence strength, confidence, and accuracy respectively. The use of box plots can also be helpful in diagnosing the differences of the response time between factor levels. Additionally, we want to adopt mosaic plots to visually examine independence between evidence strength, confidence, and accuracy. Then, to test

for independence, we will consider running a Chi-square test for the categorical variables of interest.

### 3. Model Establishment and Diagnostics

In our modeling framework, we treat participants as random effects considering they are randomly sampled from the population. As for the evidence strength, it is treated as a fixed effect.

To further understand the relationship between evidence strength and response time, we are interested in fitting a randomized block model where response time is the response, evidence strength level is the treatment, and participants are the block effect. This will allow us to understand the mean difference in response time based on evidence strength and participants respectively.

To predict accuracy, a binary variable, we propose a Binomial Generalized Linear Mixed Model(1) with a fixed effect strength level ( $X_S$ ), continuous response time variable ( $X_R$ ), and random effect participants ( $S_P$ ).  $P_A$  is defined as the vector of probability that the Two Choice Decision is accurate.

$$\begin{aligned} \text{logit}(P_A) &= \beta_0 + \beta_1 X_S + \beta_2 X_R + S_P \\ S_P &\sim N(0, \sigma_s^2) \end{aligned} \quad (1)$$

To predict confidence, an ordinal variable treated as a categorical variable, we propose a Multinomial Logistic Regression Model(2) with the same predictors.  $P_{Cj}$  is defined as the vector of probability that confidence is in category level  $j$ .

$$\begin{aligned} \log(P_{Cj}/P_{C4}) &= \beta_{0j} + \beta_{1j} X_S + \beta_{2j} X_R + S_P \\ S_P &\sim N(0, \sigma_s^2), j = 1, 2, 3 \end{aligned} \quad (2)$$

In both prediction models, we aim to explore the addition of the interaction term of response time and evidence strength after the inspection of the appropriate interaction plot. Furthermore, we intend to apply appropriate goodness-of-fit tests and inspect the model residuals to understand model robustness. After, we will inspect the t-tests to check the statistical significance of the predictor variables. If so, we can attempt to drop appropriate variables and see if the model will fit better under the criteria of adjusted  $R^2$  and AIC.

### 4. Parameter Analysis and Comparisons

After obtaining the final model summaries, we intend to analyze the parameters and interpret the variable effects

through the use of t-tests, F-tests, and coefficient estimation. Based on the randomized block model we are interested in performing pairwise comparisons to further understand the variation of response time. As our data set indicates slight difference for the sample size under each factor level, we consider using the Scheffe's method to test all the contrasts at the same time.

### 5 Potential Challenges

A primary challenge is that the predictors variables evidence strength and response time appear to be moderately correlated. Although we prefer to include both evidence strength level and response time in the predictive models simultaneously, we need to further examine the correlations with statistical tests.

A secondary challenge is that evidence strength and confidence are both ordinal variables which require specific statistical techniques beyond the scope of the course. Hence, initially we will be treating them both as categorical variables (unordered). In this case, there are two main objectives for the analysis of the evidence strength variable. One is to check if the evidence strength in general is significant for predicting the accuracy and confidence. And two, is to deliver comparisons between each strength level. For the confidence variable, the information about the ordering of the categories will be ignored as we fit the multinomial logistic regression model, but may be included in the interpretation.

### 6 Prior Data Analysis

Dou and Samaha [3] performed three one-way ANOVAs to participants' means of the behavioral data to examine how evidence strength affects accuracy, response time, and confidence respectively. They found that both accuracy and confidence increases with stronger evidence, and response time decreases with stronger evidence.

### References

- [1] J. I. Gold and M. N. Shadlen, "The neural basis of decision making.," *Annual Review of Neuroscience*, vol. 30, 2007.
- [2] R. Kiani, L. Corthell, and M. Shadlen, "Choice certainty is informed by both evidence and decision time.," *Neuron*, vol. 84, pp. 1329–1342, 2014.
- [3] W. Dou and J. Samaha, "The neural signature of subjective confidence in perceptual decision making.," 2020. Neuromatch Conference 2020.