Binomial distribution.

X is a random variable that describes the number of "successes" in n trials.

assumptions:

1) number of total trials is fixed.
2) trials are independent
3) in each trial the outcome can only be of two categories.
4) probability of success is the same in each trial

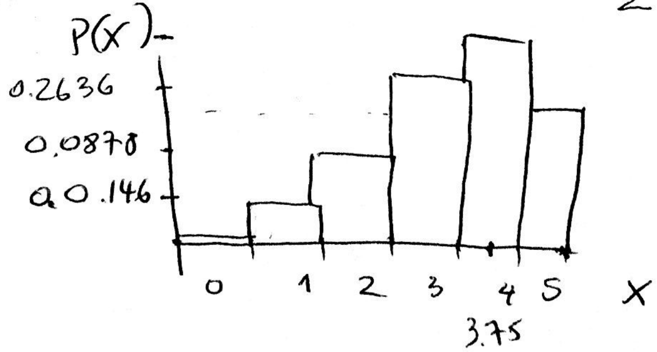
$$P(x) = \frac{n!}{(n-x)!\,x!}\, p^{x}(1-p)^{n-x}$$

p: Probability of success
n: number of trials
x: possible counts. : $0, 1, 2, \ldots, n$.

$0 \leq P(x) \leq 1$
$\sum P(x) = 1$

$n = 5 \qquad p = 0.75$

X	P(x)
0	0.0009 (0+)
1	0.0146
2	0.0878
3	0.2636
4	0.3955
5	0.2373



$$\mu = \sum x P(x) = \sum_{x=0}^{n} x P(x)$$
$$\sigma^2 = \sum (x-\mu)^2 P(x)$$

parameters

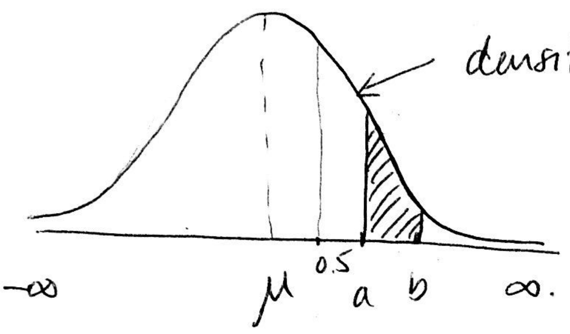
mean: $\mu = np = 3.75$

stand deviation $\sigma = \sqrt{np(1-p)} = 0.968$

variance $\sigma^2 = np(1-p) = 0.937$

normal probability distribution

μ , σ^2



density curve: $f(x)$

- $f(x) > 0$
- $\int_{-\infty}^{\infty} f(x)dx = 1$: total area under the density curve has to be 1

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty.$$

parameters: mean: μ $\left(\int_{-\infty}^{\infty} x f(x)\, dx = \mu\right)$

standard deviation: σ

variance : σ^2

$$P(a < X < b) = \int_a^b f(x)dx. \rightarrow$$ not going to solve this!

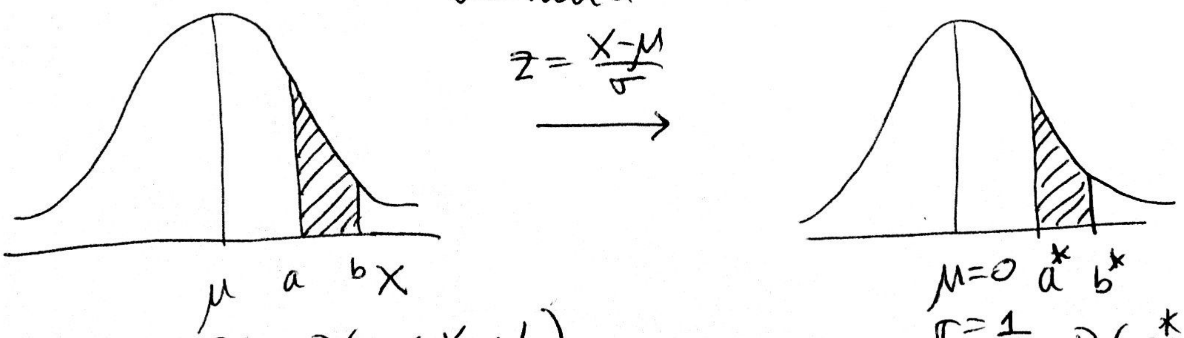
Use the standard normal distribution: Z.

$$Z = \frac{X-\mu}{\sigma} \qquad f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2} \qquad -\infty < z < \infty.$$

parameters: mean: $\mu = 0$.

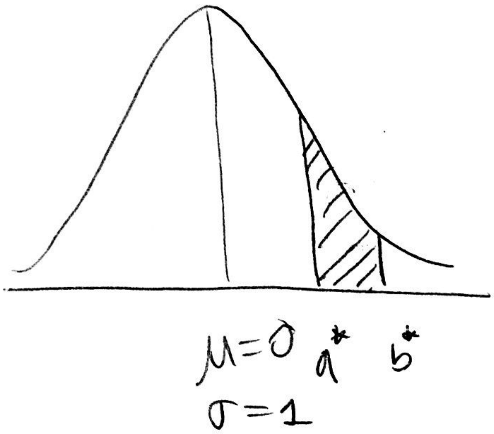
standard deviation $\sigma = 1$

variance $\sigma^2 = 1$

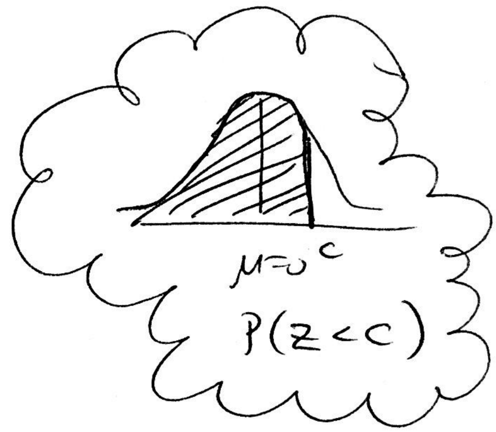


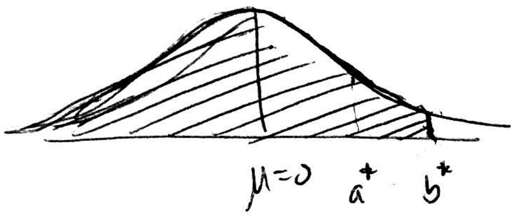
σ $P(a < X < b)$ $\qquad$ $\sigma = 1$ $P(a^* < Z < b^*)$

$f(z)$



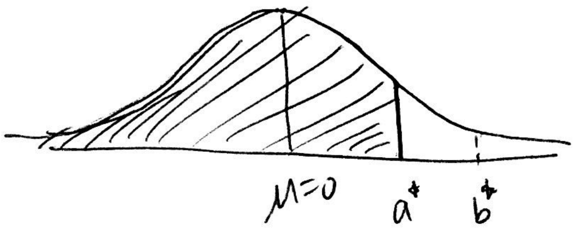
$P(a^* < z < b^*)$



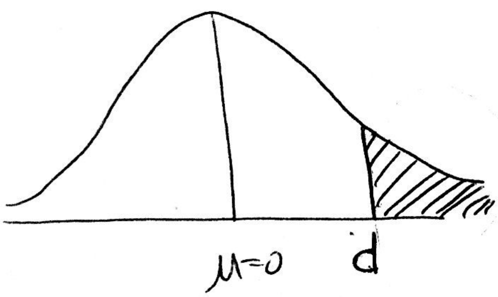


$P(z < b^*)$

$$P(a^* < z < b^*) = P(z < b^*) - P(z < a^*)$$

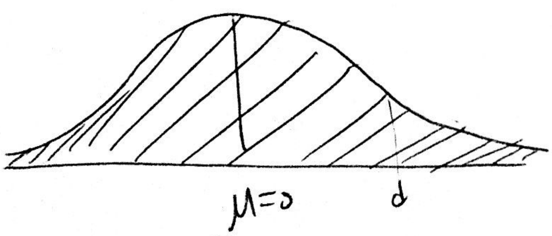


$P(z < a^*)$

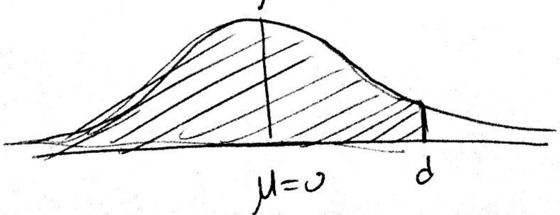


$P(z > d)$

$$P(z > d) = 1 - P(z \leq d)$$



$P(-\infty < z < \infty) = 1$



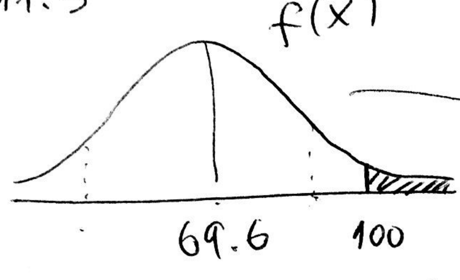
$P(z < d)$

Class 9.

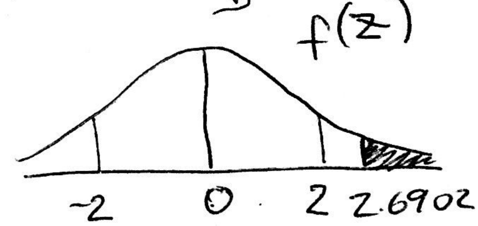
slide 4.

X is the random variable that describes the pulse rate of adult males.
X follows a normal distribution $\mu = 69.6$
$\sigma = 11.3$

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 69.6}{11.3}$$



b) $P(X > 100) = P\left(\frac{X - 69.6}{11.3} > \frac{100 - 69.6}{11.3}\right)$



$= P(Z > 2.6902)$

$= 1 - P(Z \leq 2.6902)$

$= 1 - 0.9964$ from table of positive Z scores

$= 0.0036.$

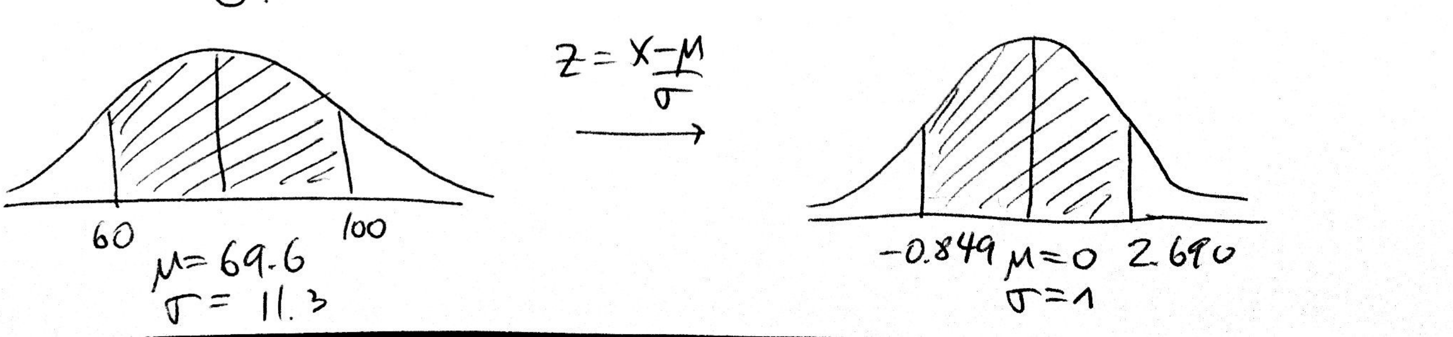
c) $P(60 < X < 100) = P\left(\frac{60 - 69.6}{11.3} < \frac{X - 69.6}{11.3} < \frac{100 - 69.6}{11.3}\right)$

$= P(-0.8495 < Z < 2.6902)$

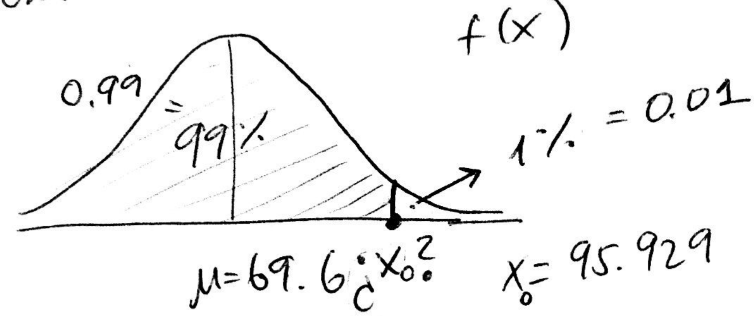
$= P(Z < 2.6902) - P(Z < -0.8495)$

$= 0.9964 - 0.1977$

$= 0.7987.$

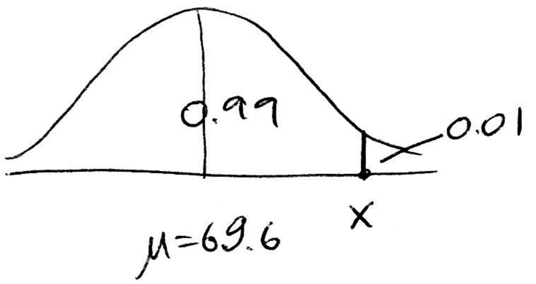


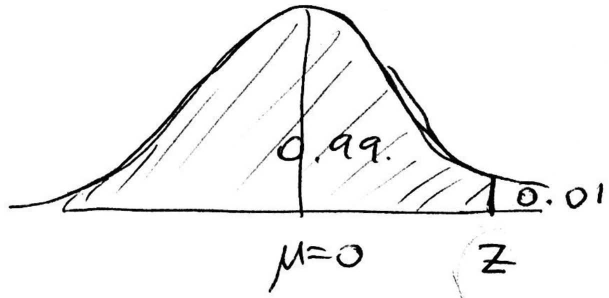
d) Find X so that we separate the highest 1% from the lowest 99%.



$P(X \le X_0) = 0.99$
↓
95.929

$$Z = \frac{X-\mu}{\sigma} = \frac{X-69.6}{11.3}$$





which Z separate the highest 1% from the lowest 99%. $Z = 2.33$

recall that $Z = \frac{X-69.6}{11.3}$

$$2.33 = \frac{X-69.6}{11.3} \qquad / \cdot 11.3$$

$$2.33 \cdot 11.3 = X - 69.6 \qquad / +69.6$$

$$X = 69.6 + 2.33 \cdot 11.3$$
$$= 95.929$$