Math Science 800, Fall 2010
In Class Test II

NAME:

Directions: Show all work on the test to receive possible partial credit. Unsupported guesses will meet a red pen. You are allowed to use a calculator and an 8×11.5 formula sheet of your own construction.

1. Short Answer (20 points total, 2 points for each part)

a) If ψ is a characteristic function of a random variable X, what is $\psi(0)$?

b) True or False: The moment generating function always exists (is finite).

c) True or False: If $X > 0$, the Laplace transform $E[e^{-sX}]$ exists (is finite) for any $s \geq 0$.

d) Yes or No: Is it possible to have a random variable X with $E[e^{tX}] = (2 + 3t)^3$?

e) Yes or No: Is it possible to have a random variable X with $E[e^{tX}] = e^{t^2/5}$?

f) True or False: Joint normality implies marginal normality.

g) Which of the following is/are necessarily continuous in its argument: The cumulative distribution function, the characteristic function, the probability density function.

h) True or False: The determinant of a covariance matrix could be zero.

i) True or False: The sample mean and variance from an IID normally distributed sample are independent.

j) If $\vec{X}$ is a n-dimensional multivariate normal random vector with mean $\vec{\mu}$ and invertible covariance matrix Λ, what is the distribution type of $(\vec{X} - \vec{\mu})'\Lambda^{-1}(\vec{X} - \vec{\mu})$?

2. (20 points) The probability generating function of a random variable $X \in \{0, 1, \ldots\}$ has the form

$$E[t^X] = \frac{1}{5}e^{5(t-1)} + \frac{4}{5}\frac{t(1+t)}{2}.$$

a) What is $P(X = 0)$?

b) What is $P(X = 1)$?

c) What is $E[X]$?

d) What is $\mathrm{Var}(X)$?

3. (10 points) Sampson the dog survived the death threats of Test 1. Unfortunately for Sampson, a teenager next door has been given a new pellet gun by his parents. The teenager shoots at Sampson a Poisson number of times with mean λ. The teenager is a very good shot and will hit Sampson whenever he fires at the dog. Let W_i denote the weight of the ith pellet shot at Sampson and assume that $\{W_i\}_{i=1}^{\infty}$ is IID with mean μ_W and variance σ_W^2. Write down an expression for the total amount of pellet weight shot into Sampson and calculate its mean and variance.

4. (10 points) Suppose that X_1, X_2, and X_3 are jointly normally distributed with

$$\vec{\mu} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad \mathbf{\Lambda} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

What is the joint distribution of $X_1 + X_2$ and $X_1 - X_3$?

5. (10 points) Derive the Laplace transform of a uniformly distributed random variable over the interval $[0, \beta]$, where $\beta > 0$ is a parameter.

6. (10 points) What is $E[X^4]$ when X has a Poisson distribution with mean $\lambda > 0$?

7. (10 points) Sampson the dog has survived the pellet gun attack in Problem 3. However, Sampson's owners are throwing a party from 6 PM to midnight. Ten guests show up to the party independently and uniformly distributed from 6 PM to midnight. It is known that the last party guest tried to poison Sampson and police are reviewing videotape surveillance of Sampson's food bowl during the party. At what time do you expect the poisoning attempt to have taken place?

8. (10 points) Suppose that X_1 and X_2 are independent standard normal random variables. What is the moment generating function of $X_1 X_2$?

Bonus (10 points) For an integer-valued random variable $X \geq 0$, derive the tail probability generating function relation

$$E[t^X] = 1 + (t-1)\sum_{j=0}^{\infty} t^j P(X > j), \quad 0 \leq t < 1.$$

Bonus. (Trivia, 1 point) What are the only three land animals on Earth that will **consistently both track and kill** a full grown human for food?