(b) $L(\theta|y) = \prod_{i=1}^{I} \prod_{j=1}^{n_i} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{\left(y_{ij} - \frac{\beta_{0i}}{1+\beta_1 \exp(\beta_2 t_{ij})}\right)^2}{2\sigma^2} \right) \right)$

$\pi(\beta_{0i}|\bar{\beta}_0) = \frac{1}{\sqrt{2\pi\tau^2}} \exp\left(- \frac{(\beta_{0i} - \bar{\beta}_0)^2}{2\tau^2}\right)$

$\pi(\beta_p|\bar{\beta}_p) = \frac{1}{\sqrt{2\pi u_p^2}} \exp\left(- \frac{(\beta_p - \bar{\beta}_p)^2}{2u_p^2}\right)$

$\pi(\sigma^2) = \frac{b_\sigma^{a_\sigma}}{\Gamma(a_\sigma)} (\sigma^2)^{-a_\sigma - 1} \exp\left(- \frac{b_\sigma}{\sigma^2}\right)$

$\pi(\bar{\beta}_0) = \frac{1}{\sqrt{2\pi v^2}} \exp\left(- \frac{(\bar{\beta}_0 - M_0)^2}{2v^2}\right)$

$\pi(\tau^2) = \frac{b_\tau^{a_\tau}}{\Gamma(a_\tau)} (\tau^2)^{-a_\tau - 1} \exp\left(- \frac{b_\tau}{\tau^2}\right)$

$$\pi(\theta|y) \propto (\sigma^2)^{-\frac{1}{2}N} \exp\left(- \frac{\sum_{i=1}^{I}\sum_{j=1}^{n_i} \left(y_{ij} - \frac{\beta_{0i}}{1+\beta_1 \exp(\beta_2 t_{ij})}\right)^2}{2\sigma^2}\right)$$

$$(\tau^2)^{-\frac{1}{2}I} \exp\left(- \frac{\sum_{i=1}^{I} (\beta_{0i} - \bar{\beta}_0)^2}{2\tau^2}\right)$$

$$\exp\left(- \frac{(\beta_1 - \bar{\beta}_1)^2}{2u_p^2}\right) \exp\left(- \frac{(\beta_2 - \bar{\beta}_2)^2}{2u_p^2}\right)$$

$$(\sigma^2)^{-a_\sigma - 1} \exp\left(- \frac{b_\sigma}{\sigma^2}\right) \exp\left(- \frac{(\bar{\beta}_0 - M_0)^2}{2v^2}\right)$$

$$(\tau^2)^{-a_\tau - 1} \exp\left(- \frac{b_\tau}{\tau^2}\right)$$

Full conditional conditions:

$$\pi(\beta_{0i}|y, \text{others}) \propto \exp\left(- \frac{\sum_{j=1}^{n_i} \left(y_{ij} - \frac{\beta_{0i}}{1+\beta_1 \exp(\beta_2 t_{ij})}\right)^2}{2\sigma^2} - \frac{(\beta_{0i} - \bar{\beta}_0)^2}{2\tau^2}\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\beta_{0i}^2\left(\frac{\sum_{j=1}^{n_i}\left(\frac{1}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2}{\sigma^2} + \frac{1}{\tau^2}\right) - 2\beta_{0i}\left(\frac{\sum_{j=1}^{n_i}\left(\frac{y_{ij}}{1+\beta_1\exp(\beta_2 t_{ij})}\right)}{\sigma^2} + \frac{\bar{\beta}_0}{\tau^2}\right)\right]\right)$$

$$\beta_{0i}|y, \text{others} \sim N\left(\frac{\frac{\sum_{j=1}^{n_i} \frac{y_{ij}}{1+\beta_1\exp(\beta_2 t_{ij})}}{\sigma^2} + \frac{\bar{\beta}_0}{\tau^2}}{\frac{\sum_{j=1}^{n_i}\left(\frac{1}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2}{\sigma^2} + 1/\tau^2}, \left[\frac{\sum_{j=1}^{n_i}\left(\frac{1}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2}{\sigma^2} + 1/\tau^2\right]^{-1} \right)$$

$$\pi(\beta_1|y,\text{others}) \propto \exp\left(-\frac{(\beta_1-\bar{\beta}_1)^2}{2u_1^2} - \frac{\sum_{i=1}^{2}\sum_{j=1}^{n_i}\left(y_{ij}-\frac{\beta_{0i}}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2}{2\sigma^2}\right)$$

It's not a closed form, we should use MCMC.

$$\pi(\beta_2|y,\text{others}) \propto \exp\left(-\frac{(\beta_2-\bar{\beta}_2)^2}{2u_2^2} - \frac{\sum_{i=1}^{2}\sum_{j=1}^{n_i}\left(y_{ij}-\frac{\beta_{0i}}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2}{2\sigma^2}\right)$$

Still, should be a MCMC approach.

$$\pi(\sigma^2|\text{others}) \propto (\sigma^2)^{-\frac{N}{2}-a_\sigma-1}\exp\left(-\frac{1}{\sigma^2}\left[\frac{\sum_{i=1}^{2}\sum_{j=1}^{n_i}\left(y_{ij}-\frac{\beta_{0i}}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2}{2}+b_\sigma\right]\right)$$

It's an inverse gamma $IG\left(\frac{N}{2}+a_\sigma, \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{n_i}\left(y_{ij}-\frac{\beta_{0i}}{1+\beta_1\exp(\beta_2 t_{ij})}\right)^2+b_\sigma\right)$

$$\pi(\bar{\beta}_0|\text{others}) \propto \exp\left(-\frac{\sum_{i=1}^{2}(\beta_{0i}-\bar{\beta}_0)^2}{2\tau^2} - \frac{(\bar{\beta}_0-M_0)^2}{2V^2}\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\bar{\beta}_0^2\left(\frac{2}{\tau^2}+\frac{1}{V^2}\right)-2\bar{\beta}_0\left(\frac{\sum_{i=1}^{2}\beta_{0i}}{\tau^2}+\frac{M_0}{V^2}\right)\right]\right)$$

$$\bar{\beta}_0|\text{others} \sim N\left(\frac{\frac{\sum_{i=1}^{2}\beta_{0i}}{\tau^2}+\frac{M_0}{V^2}}{\frac{2}{\tau^2}+\frac{1}{V^2}}, \left(\frac{2}{\tau^2}+\frac{1}{V^2}\right)^{-1}\right)$$

$$\pi(\tau^2|\text{others},y) \propto (\tau^2)^{-a_\tau-\frac{2}{2}-1}\exp\left(-\frac{1}{\tau^2}\left(\frac{\sum_{i=1}^{2}(\beta_{0i}-\bar{\beta}_0)^2}{2}+b_\tau\right)\right)$$

It's an inverse gamma $IG\left(a_\tau+\frac{2}{2}, \frac{\sum_{i=1}^{2}(\beta_{0i}-\bar{\beta}_0)^2}{2}+b_\tau\right)$