• Shoes data: Experiment reported by Box, Hunter & Hunter (1978). Amounts of "shoe wear": Two materials (A and B) randomly assigned to left & right shoes (L and R) of 10 boys;

```
> library("BHH2") #reproduces examples in Box, Hunter & Hunter
> data(shoes.data)
> shoes.data
  boy matA sideA matB sideB
    1 13.2 L 14.0
                        R
2
   2 8.2 L 8.8
                        R
3
    3 10.9 R 11.2
                        \mathbf{L}
4
   4 14.3 L 14.2
                        R
5
    5 10.7 R 11.8
                        L
6
    6 6.6 L 6.4
                        R
    7 9.5 L 9.8
                        R
8
    8 10.8 L 11.3
                        R
9
    9 8.8 R 9.3
                        \mathbf{L}
              L 13.6
10
   10 13.3
                        R
```

One sample t-test:

```
> attach(shoes.data)
The following objects are masked from shoes.data (pos = 3):
    boy, matA, matB, sideA, sideB
> t.test(matA, mu=10)
   One Sample t-test
data: matA
t = 0.81272, df = 9, p-value = 0.4373
alternative hypothesis: true mean is not equal to 10
95 percent confidence interval:
  8.876427 12.383573
sample estimates:
mean of x
    10.63
```

Two-sample tests: paired vs. unpaired (equal vs. unequal variances)

- Two-sample unpaired, unequal variances
 - Test for equal variances:

```
> var.test(matA,matB)
   F test to compare two variances
data: matA and matB
F = 0.94739, num df = 9, denom df = 9, p-value = 0.9372
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.2353191 3.8142000
sample estimates:
ratio of variances
         0.9473933
> var(matA)
[1] 6.009
> var(matB)
[1] 6.342667
```

t-test with unequal variances (default in R)

```
> t.test(matA,matB)

Welch Two Sample t-test

data: matA and matB
t = -0.36891, df = 17.987, p-value = 0.7165
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -2.745046    1.925046
sample estimates:
mean of x mean of y
    10.63    11.04
```

t-test with equal variances

```
> t.test(matA,matB,var.equal =T)

Two Sample t-test

data: matA and matB
t = -0.36891, df = 18, p-value = 0.7165
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -2.744924    1.924924
sample estimates:
mean of x mean of y
    10.63    11.04
```

Paired t-test

```
> t.test(matA,matB,paired=T)

Paired t-test

data: matA and matB
t = -3.3489, df = 9, p-value = 0.008539
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -0.6869539 -0.1330461
sample estimates:
mean of the differences
    -0.41
```

Density Estimation

$$\hat{f}(x) = \frac{1}{nb} \sum_{j=1}^{n} K\left(\frac{x - x_j}{b}\right)$$

bandwidth

Kernel

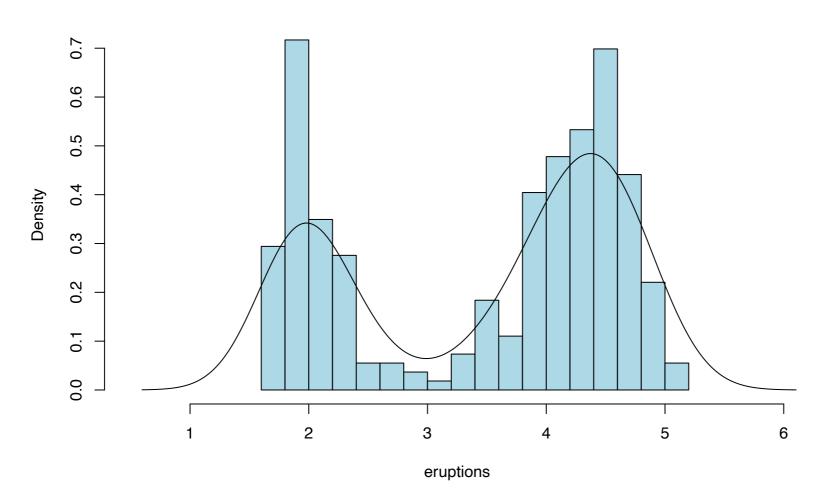
Default bandwidth choice in **R** is

$$\hat{b} = 0.9 \min(\hat{\sigma}, R/1.34) n^{-1/5}$$

with R the IQR.

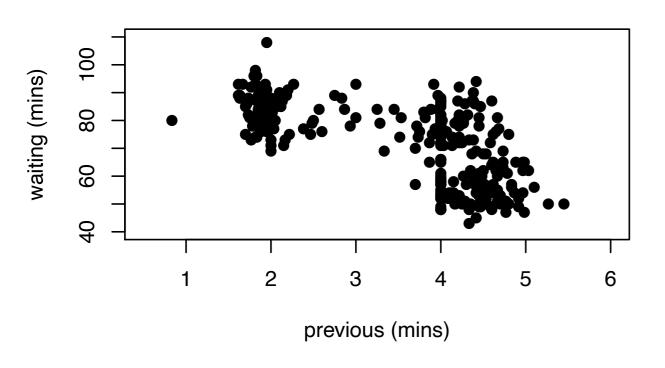
- > attach(faithful)
- > hist(eruptions,breaks=15,xlim=c(0.5,6),col='lightblue',prob=T)
- > lines(density(eruptions))

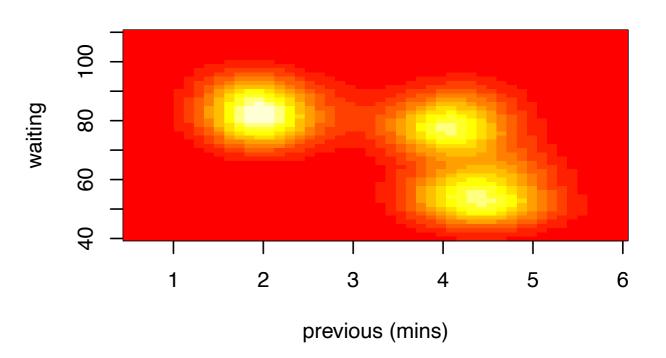
Histogram of eruptions

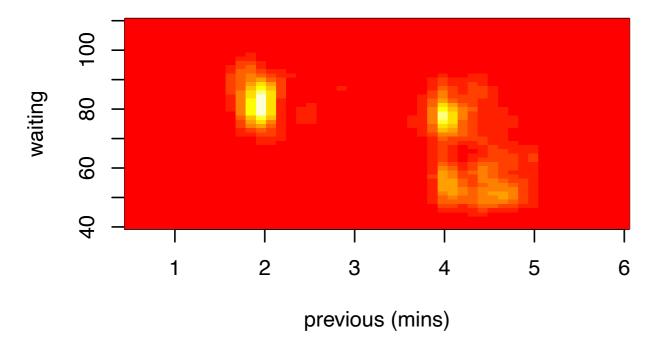


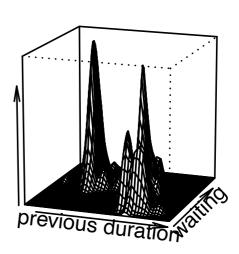
2-dimensional density estimation:

```
> library(MASS)
> attach(geyser)
> par(mfrow=c(2,2))
> plot(duration,
waiting, xlab="previous(mins)", ylim=c(40,110), xlim=c(0.5,6),
     ylab="waiting (mins)",pch=19)
> f1=kde2d(duration, waiting, n=50, lims=c(0.5, 6, 40, 110))
> image(f1,xlab="previous (mins)",ylab="waiting")
> f2=kde2d(duration, waiting, n=50, lims=c(0.5, 6, 40, 110),
    h=c(width.SJ(duration), width.SJ(waiting)))
> image(f2,xlab="previous (mins)",ylab="waiting")
> persp(f2,phi=15,theta=20,d=10,xlab="previous duration",
     ylab="waiting",zlab="")
```









Additional basic inference tools

Test for proportions based on large sample approx.:

$$H_0: p = p_0$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

or equivalently:

$$Z = \frac{y - np_0}{\sqrt{np_0(1 - p_0)}}$$

Additional basic inference tools

- Test for proportions based on large sample approx.:
 - Continuity correction;
 - Exact test;
- Nonparametric methods: read
- Two-sample inference: we have described most of the tests but read about permutation tests.