hw2

Qi Wang

2022/3/29

Question 1: Hierarchical Model - School Treatment Effect

Question 1a: Hierarchical Model - School Treatment Effect - Prior on Tau

Table 1: Probability of Best School - Prior on Tau

A	B	C	D	E	F	G	H
0.255	0.093	0.083	0.109	0.058	0.07	0.186	0.147

Table 2: Pairwise Comparison - Prior on Tau (Row Index larger than Column Index)

	A	B	C	D	E	F	G	H
A	0.000	0.647	0.689	0.635	0.716	0.686	0.506	0.607
B	0.353	0.000	0.554	0.504	0.620	0.574	0.356	0.490
C	0.311	0.446	0.000	0.428	0.542	0.504	0.335	0.404
D	0.365	0.496	0.572	0.000	0.626	0.591	0.386	0.491
E	0.284	0.380	0.458	0.374	0.000	0.461	0.269	0.379
F	0.314	0.426	0.496	0.409	0.539	0.000	0.297	0.415
G	0.494	0.644	0.665	0.614	0.731	0.703	0.000	0.584
H	0.393	0.510	0.596	0.509	0.621	0.585	0.416	0.000

Question 1b: Hierarchical Model - School Treatment Effect - Very Large Tau

Table 3: Probability of Best School - Tau Very Big

A	B	C	D	E	F	G	H
0.545	0.038	0.018	0.029	0.001	0.013	0.168	0.188

Table 4: Pairwise Comparison - Tau Very Big (Row Index larger than Column Index)

	A	B	C	D	E	F	G	H
A	0.000	0.863	0.928	0.874	0.954	0.922	0.712	0.738
B	0.137	0.000	0.717	0.540	0.768	0.694	0.248	0.429
C	0.072	0.283	0.000	0.319	0.461	0.413	0.129	0.255
D	0.126	0.460	0.681	0.000	0.704	0.626	0.224	0.386
E	0.046	0.232	0.539	0.296	0.000	0.420	0.077	0.259
F	0.078	0.306	0.587	0.374	0.580	0.000	0.130	0.313
G	0.288	0.752	0.871	0.776	0.923	0.870	0.000	0.614
H	0.262	0.571	0.745	0.614	0.741	0.687	0.386	0.000

Question 1b: Hierarchical Model - School Treatment Effect - Very Small Tau

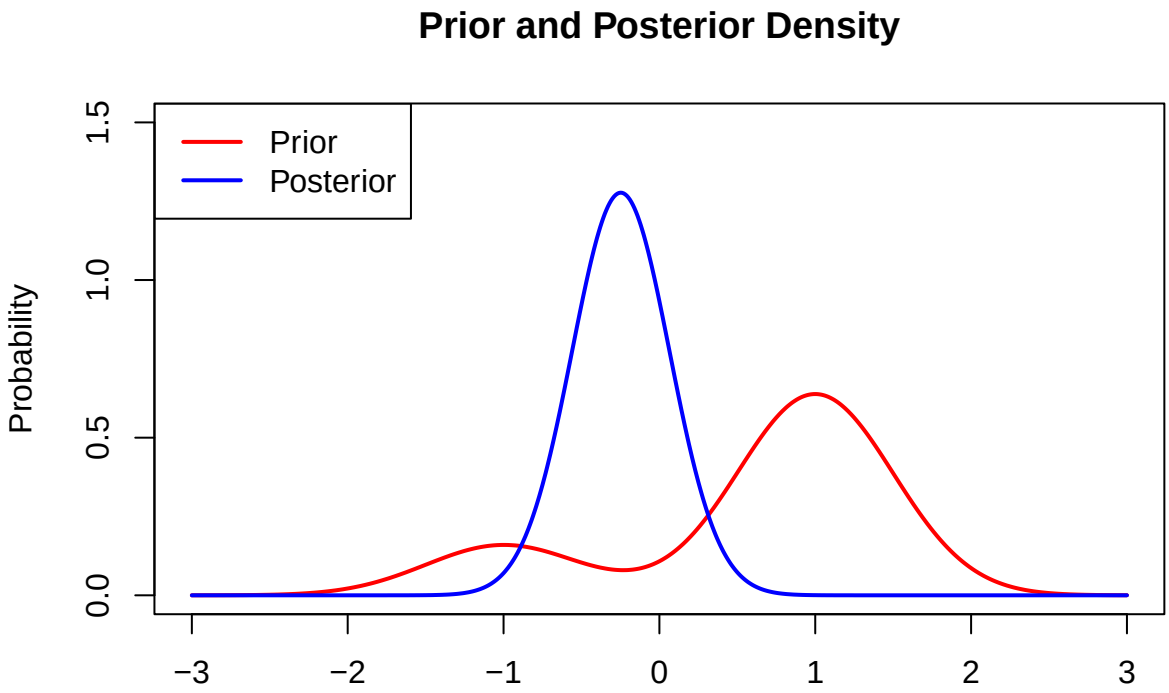
Table 5: Probability of Best School - Tau Very Small

A	B	C	D	E	F	G	H
0.125	0.121	0.121	0.143	0.122	0.127	0.117	0.125

Table 6: Pairwise Comparison - Tau Very Small (Row Index larger than Column Index)

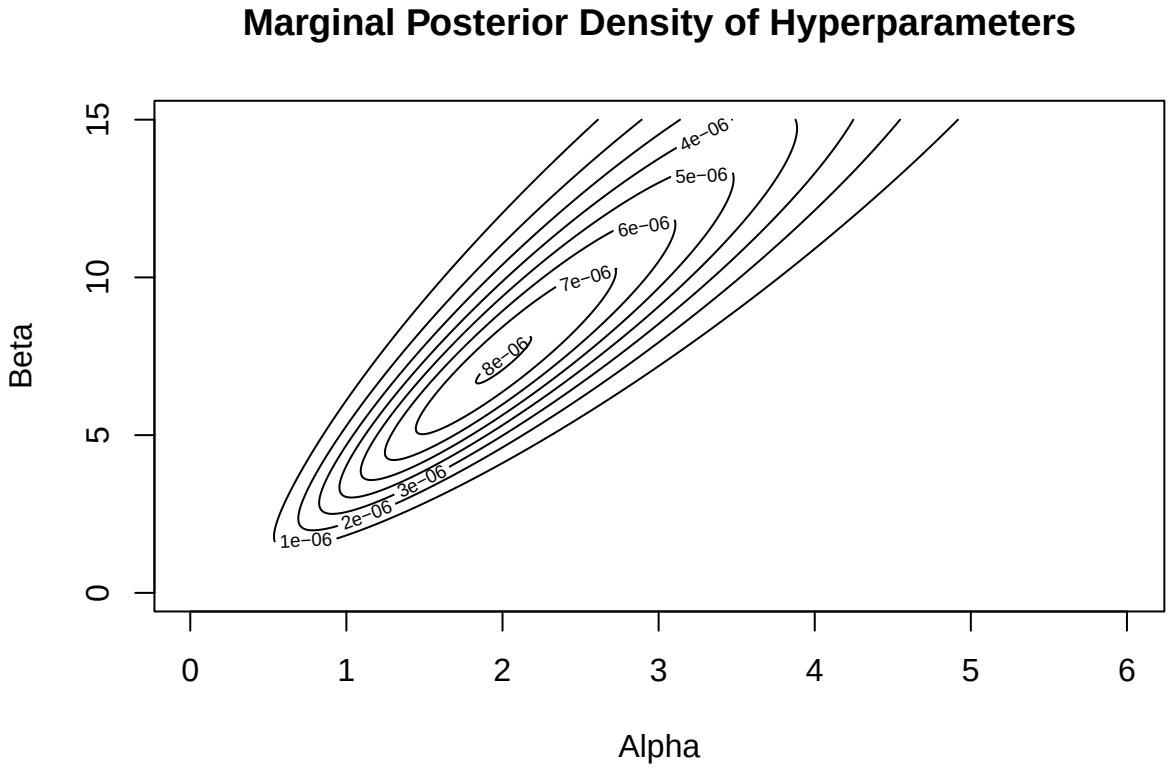
	A	B	C	D	E	F	G	H
A	0.000	0.504	0.523	0.496	0.509	0.515	0.518	0.507
B	0.496	0.000	0.528	0.469	0.491	0.501	0.498	0.513
C	0.477	0.472	0.000	0.481	0.482	0.505	0.503	0.513
D	0.504	0.531	0.519	0.000	0.508	0.511	0.514	0.533
E	0.491	0.509	0.518	0.492	0.000	0.501	0.504	0.509
F	0.485	0.499	0.495	0.489	0.499	0.000	0.500	0.529
G	0.482	0.502	0.497	0.486	0.496	0.500	0.000	0.511
H	0.493	0.487	0.487	0.467	0.491	0.471	0.489	0.000

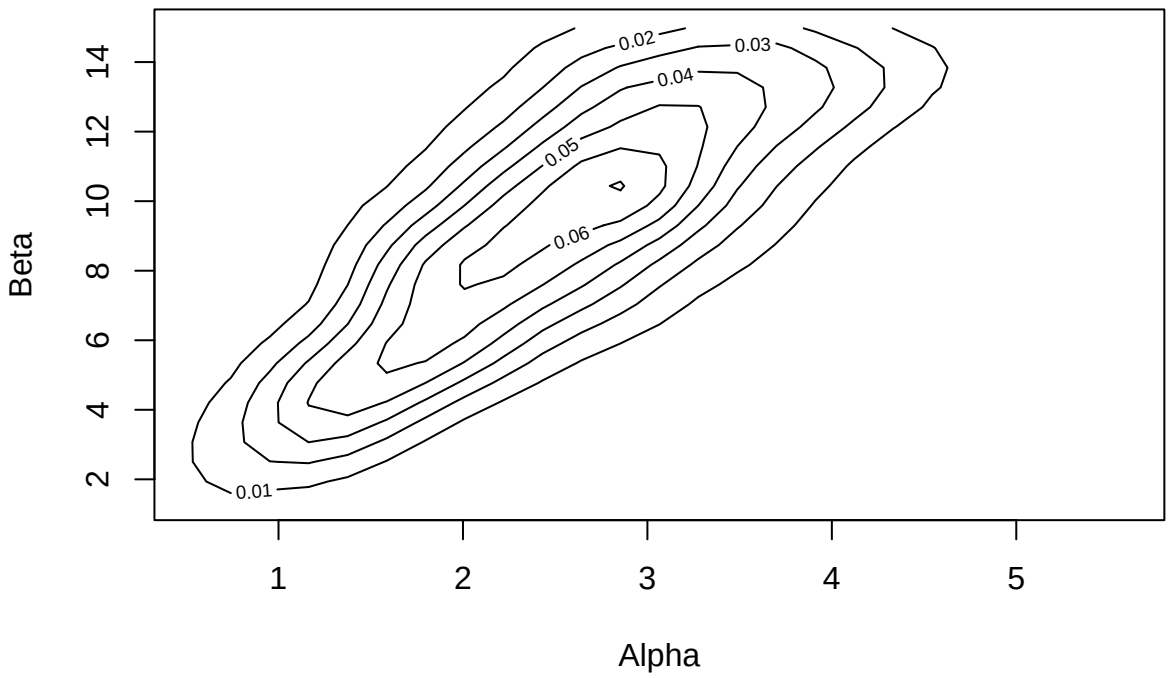
Question 2: Mixture Model



Question 3: Hierachical Beta Distribution

Question 3.b: Hierachical Beta Distribution - Marginal Posterior for Hyperparameter




Sampling Distribution of Hyperparameters
Beta
Alpha
2
4
6
8
10
12
14
1
2
3
4
5
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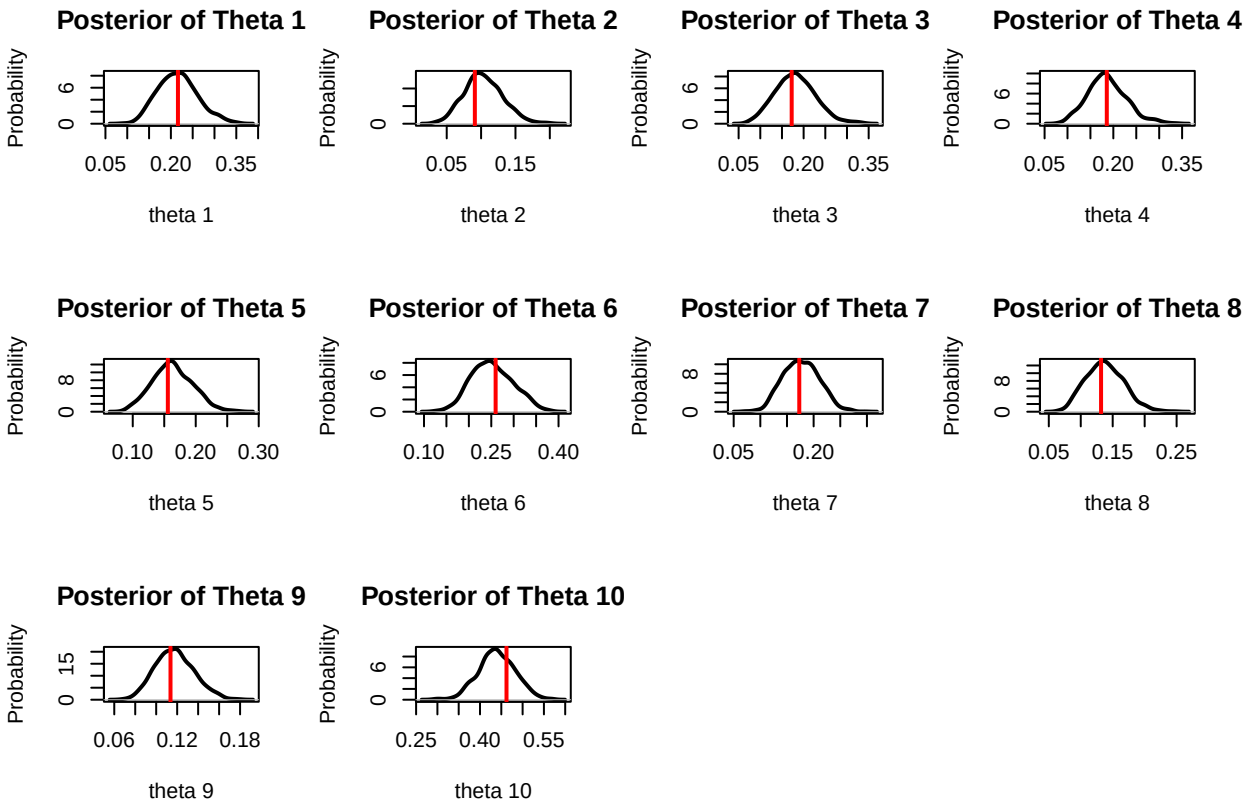
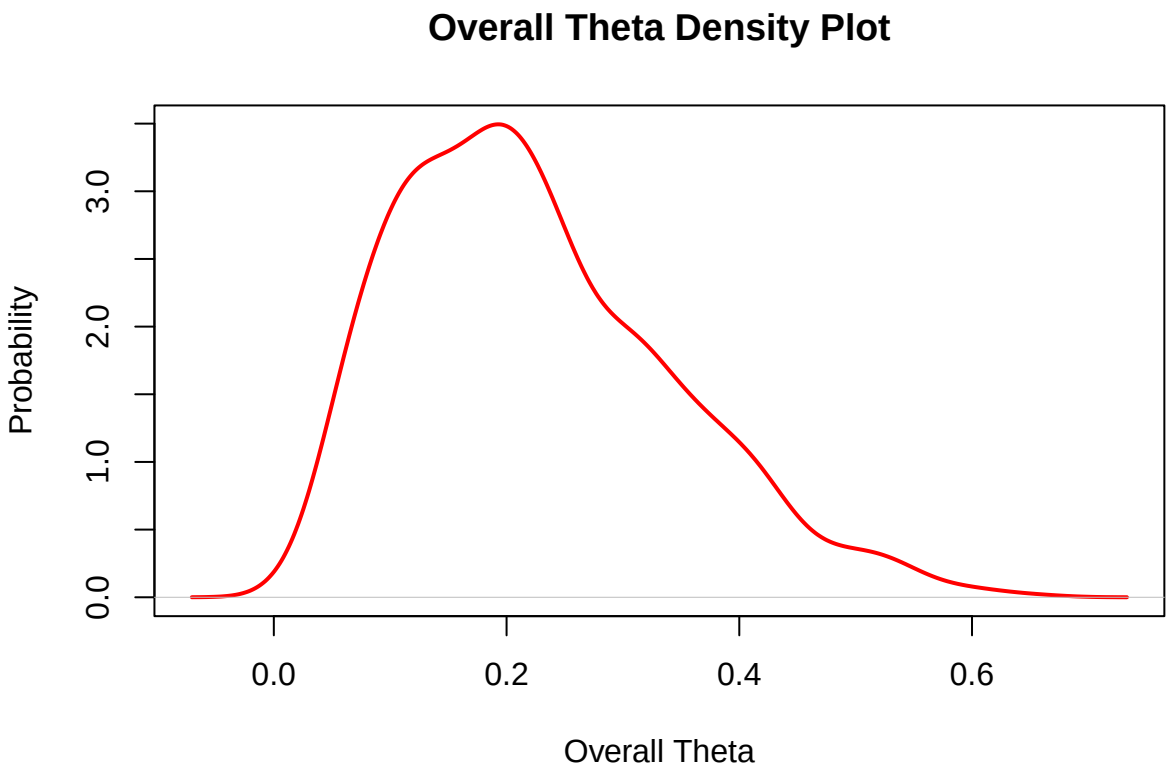
0.02
0.03
0.04
0.05
0.06


Posterior of Theta 1
Probability
0 6
0.05 0.20 0.35
theta 1
Posterior of Theta 2
Probability
0
0.05 0.15
theta 2
Posterior of Theta 3
Probability
0 6
0.05 0.20 0.35
theta 3
Posterior of Theta 4
Probability
0 6
0.05 0.20 0.35
theta 4
Posterior of Theta 5
Probability
0 8
0.10 0.20 0.30
theta 5
Posterior of Theta 6
Probability
0 6
0.10 0.25 0.40
theta 6
Posterior of Theta 7
Probability
0 8
0.05 0.20
theta 7
Posterior of Theta 8
Probability
0 8
0.05 0.15 0.25
theta 8
Posterior of Theta 9
Probability
0 15
0.06 0.12 0.18
theta 9
Posterior of Theta 10
Probability
0 6
0.25 0.40 0.55
theta 10




```
##  2.5% 97.5%
## 0.053 0.502
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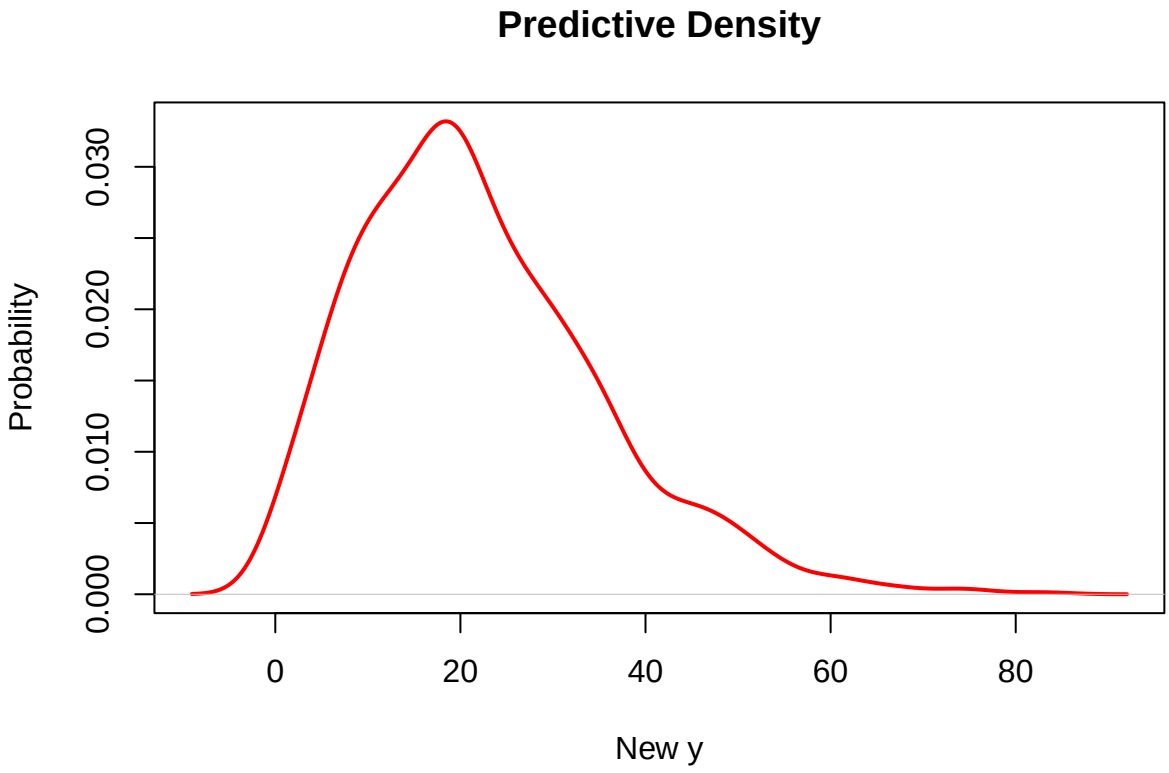


```
##  2.5% 97.5%
##     2    53
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Question 5: NIW Distribution

K = 2

Table 7: V Matrix

	1	2
Movie 1	-0.0197	0.0001
Movie 2	-0.0516	-0.0232
Movie 3	0.1937	0.0203
Movie 4	-0.0831	-0.0053
Movie 5	0.0101	-0.0019
Movie 6	-0.0354	0.0203
Movie 7	-0.0340	-0.2116
Movie 8	-0.0183	-0.2544
Movie 9	0.0075	0.2068
Movie 10	-0.0077	-0.1757
Movie 11	-0.0051	0.0430
Movie 12	0.0248	0.0059
Movie 13	0.0373	-0.0033

	1	2
Movie 14	-0.0066	0.0192
Movie 15	-0.0913	0.0583
Movie 16	0.0149	0.0589
Movie 17	-0.0953	-0.1008
Movie 18	-0.0093	0.0050
Movie 19	0.0696	0.0304
Movie 20	0.0645	0.0742
Movie 21	0.1027	-0.0088
Movie 22	0.0549	-0.0117
Movie 23	-0.0719	0.0101
Movie 24	0.0120	0.0097
Movie 25	0.0473	-0.0025
Movie 26	-0.1420	0.0292
Movie 27	0.0920	-0.0143
Movie 28	0.0453	-0.0185
Movie 29	-0.0990	0.0232
Movie 30	0.0460	0.0372

K = 4

Table 8: V Matrix

	1	2	3	4
Movie 1	-0.0152	0.0170	0.0069	-0.0072
Movie 2	-0.0701	0.0192	0.0363	-0.0541
Movie 3	0.1861	-0.1315	-0.0627	0.1160
Movie 4	-0.0896	0.0405	0.0353	-0.0649
Movie 5	0.0171	0.0028	-0.0044	0.0133
Movie 6	-0.0355	-0.0074	0.0041	-0.0339
Movie 7	-0.1066	0.1758	0.1250	-0.0337
Movie 8	-0.0643	0.2660	0.1203	0.0301
Movie 9	0.0756	-0.1684	-0.1142	0.0126
Movie 10	-0.0381	0.1866	0.0797	0.0290
Movie 11	0.0805	0.0727	-0.0689	0.0990
Movie 12	0.0457	0.0155	-0.0250	0.0445
Movie 13	-0.0088	-0.0849	0.0183	-0.0351
Movie 14	0.0017	-0.0040	-0.0086	0.0042
Movie 15	0.0138	0.1242	-0.0564	0.0614
Movie 16	0.1294	0.0896	-0.0901	0.1414
Movie 17	-0.3234	-0.1909	0.2024	-0.3385
Movie 18	-0.0301	-0.0406	0.0126	-0.0373
Movie 19	0.1781	0.1044	-0.0997	0.1838
Movie 20	0.1631	0.0222	-0.1047	0.1451
Movie 21	0.0767	-0.0897	-0.0132	0.0260
Movie 22	0.0118	-0.0758	0.0189	-0.0199
Movie 23	-0.0367	0.0735	-0.0012	-0.0033
Movie 24	0.0040	-0.0335	-0.0009	-0.0062
Movie 25	0.0114	-0.0802	0.0094	-0.0213
Movie 26	0.1076	0.4424	-0.1328	0.2559
Movie 27	-0.0478	-0.2514	0.0648	-0.1303

	1	2	3	4
Movie 28	-0.0135	-0.0906	0.0298	-0.0412
Movie 29	0.0574	0.2612	-0.0769	0.1397
Movie 30	0.0010	-0.1488	0.0051	-0.0489

K = 6

Table 9: V Matrix

	1	2	3	4	5	6
Movie 1	0.0083	-0.0073	0.0086	-0.0012	-0.0036	-0.0202
Movie 2	0.0029	-0.0210	-0.0017	-0.0248	0.0066	-0.0483
Movie 3	-0.0684	0.0745	-0.0785	0.0391	0.0306	0.2345
Movie 4	0.0276	-0.0196	0.0390	-0.0211	-0.0195	-0.1001
Movie 5	-0.0046	-0.0027	-0.0113	0.0044	0.0040	0.0164
Movie 6	0.0164	0.0222	0.0625	-0.0029	-0.0308	-0.0484
Movie 7	-0.0922	-0.2530	-0.4092	-0.1011	0.2179	0.0645
Movie 8	-0.1120	-0.3371	-0.5257	-0.1013	0.2795	0.1036
Movie 9	0.1031	0.2468	0.4247	0.0940	-0.2214	-0.0955
Movie 10	-0.0732	-0.2328	-0.3688	-0.0671	0.1936	0.0720
Movie 11	0.0437	0.0012	0.0816	0.0451	-0.0299	-0.0392
Movie 12	-0.0015	-0.0037	-0.0086	0.0155	0.0071	0.0216
Movie 13	-0.0282	0.0391	-0.0209	-0.0123	0.0021	0.0579
Movie 14	0.0093	0.0146	0.0354	0.0063	-0.0178	-0.0151
Movie 15	0.0829	-0.0140	0.1443	0.0340	-0.0605	-0.1461
Movie 16	0.0423	-0.0004	0.0780	0.0664	-0.0351	-0.0266
Movie 17	-0.0621	0.0191	-0.0809	-0.1441	0.0270	-0.0260
Movie 18	-0.0004	0.0232	0.0245	-0.0128	-0.0163	-0.0067
Movie 19	0.0170	-0.0249	-0.0094	0.0744	0.0152	0.0484
Movie 20	0.0325	0.0441	0.0893	0.0761	-0.0397	0.0298
Movie 21	-0.0460	0.0465	-0.0629	0.0061	0.0286	0.1397
Movie 22	-0.0371	0.0298	-0.0505	-0.0091	0.0189	0.0790
Movie 23	0.0386	-0.0339	0.0497	-0.0022	-0.0235	-0.1025
Movie 24	-0.0077	0.0238	0.0106	0.0018	-0.0054	0.0131
Movie 25	-0.0254	0.0421	-0.0184	-0.0056	0.0033	0.0664
Movie 26	0.1375	-0.1960	0.0734	0.0882	-0.0076	-0.2554
Movie 27	-0.0803	0.1146	-0.0408	-0.0451	0.0041	0.1482
Movie 28	-0.0394	0.0316	-0.0485	-0.0205	0.0166	0.0786
Movie 29	0.0877	-0.1104	0.0615	0.0513	-0.0134	-0.1660
Movie 30	-0.0146	0.1008	0.0661	-0.0019	-0.0442	0.0536

K = 8

Table 10: V Matrix

	1	2	3	4	5	6	7	8
Movie 1	-0.0052	0.0005	-0.0061	-0.0025	-0.0038	0.0038	-0.0146	0.0054
Movie 2	0.0076	-0.0123	0.0024	0.0161	-0.0068	0.0050	-0.0639	0.0083
Movie 3	0.0530	0.0156	0.0250	0.0090	0.0672	-0.0048	0.1668	-0.0036

	1	2	3	4	5	6	7	8
Movie 4	-0.0281	-0.0120	-0.0123	-0.0012	-0.0183	0.0052	-0.0732	0.0052
Movie 5	0.0062	0.0063	-0.0024	-0.0022	-0.0011	-0.0016	0.0068	0.0007
Movie 6	-0.0335	-0.0059	-0.0153	-0.0065	0.0019	0.0174	-0.0097	-0.0015
Movie 7	0.2409	0.0055	0.1211	0.0947	-0.0661	-0.0990	-0.2230	0.0293
Movie 8	0.3034	0.0293	0.1404	0.0942	-0.1061	-0.1400	-0.2494	0.0323
Movie 9	-0.2502	-0.0085	-0.1262	-0.0934	0.0625	0.1012	0.2054	-0.0253
Movie 10	0.2111	0.0259	0.0982	0.0637	-0.0758	-0.0952	-0.1677	0.0218
Movie 11	-0.0601	0.0310	-0.0411	-0.0548	-0.0372	-0.0063	0.0481	-0.0067
Movie 12	0.0023	0.0093	-0.0056	-0.0062	-0.0035	-0.0096	0.0238	0.0040
Movie 13	0.0168	-0.0173	0.0208	0.0231	0.0422	0.0130	0.0188	0.0082
Movie 14	-0.0200	0.0014	-0.0135	-0.0112	-0.0017	0.0064	0.0131	-0.0052
Movie 15	-0.0969	0.0298	-0.0628	-0.0712	-0.0688	-0.0017	-0.0160	-0.0104
Movie 16	-0.0650	0.0428	-0.0491	-0.0654	-0.0398	-0.0098	0.0799	-0.0038
Movie 17	0.0916	-0.0959	0.0847	0.1303	0.0811	0.0337	-0.1900	0.0067
Movie 18	-0.0100	-0.0117	-0.0004	0.0091	0.0177	0.0116	-0.0049	0.0009
Movie 19	-0.0107	0.0537	-0.0261	-0.0563	-0.0427	-0.0281	0.0967	0.0018
Movie 20	-0.0669	0.0374	-0.0517	-0.0636	-0.0135	0.0060	0.1275	-0.0019
Movie 21	0.0487	0.0022	0.0280	0.0197	0.0509	0.0009	0.0779	0.0173
Movie 22	0.0380	-0.0116	0.0216	0.0262	0.0374	0.0073	0.0282	0.0106
Movie 23	-0.0370	0.0043	-0.0229	-0.0248	-0.0426	0.0017	-0.0422	-0.0130
Movie 24	-0.0020	-0.0026	0.0020	0.0043	0.0123	0.0061	0.0161	-0.0008
Movie 25	0.0193	-0.0106	0.0150	0.0202	0.0404	0.0150	0.0294	0.0090
Movie 26	-0.1006	0.1101	-0.0907	-0.1470	-0.2034	-0.0737	-0.0500	-0.0198
Movie 27	0.0556	-0.0589	0.0477	0.0814	0.1145	0.0385	0.0403	0.0143
Movie 28	0.0400	-0.0182	0.0297	0.0383	0.0430	0.0131	0.0118	0.0115
Movie 29	-0.0697	0.0598	-0.0577	-0.0894	-0.1216	-0.0399	-0.0324	-0.0140
Movie 30	-0.0234	-0.0264	-0.0035	0.0152	0.0629	0.0371	0.0620	0.0012

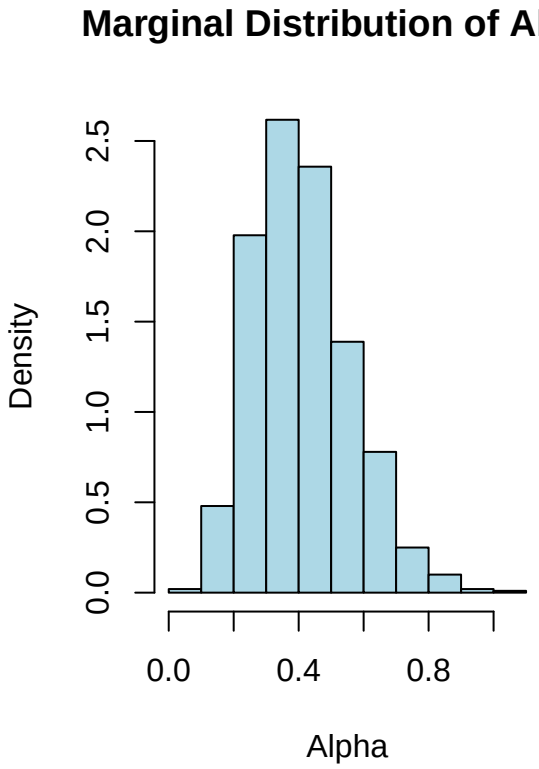
K = 10

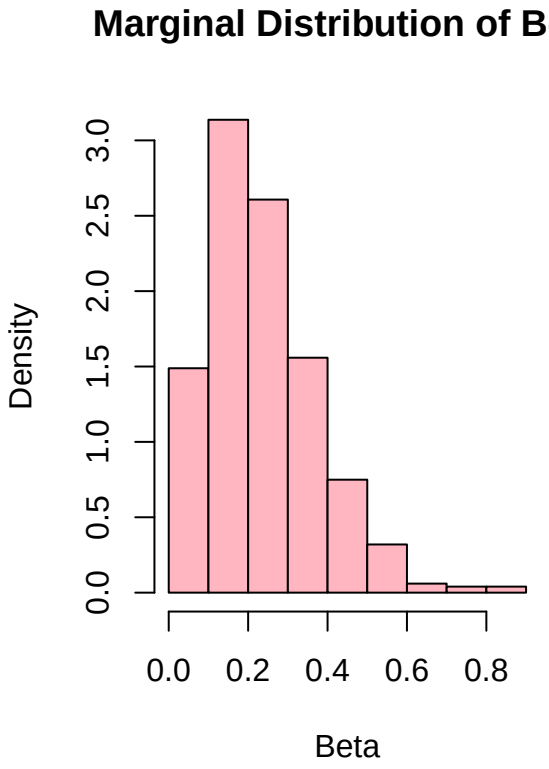
Table 11: V Matrix

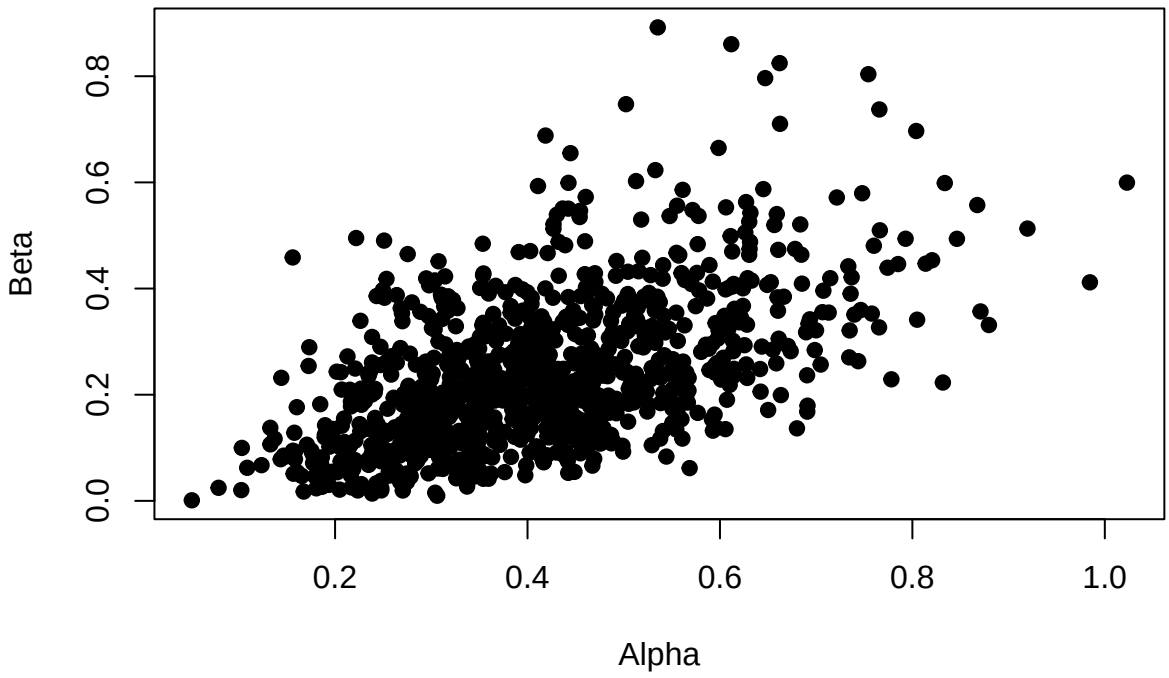
	1	2	3	4	5	6	7	8	9	10
Movie 1	0.0056	0.0162	-0.0059	0.0036	-0.0007	-0.0046	0.0094	0.0025	0.0141	0.0090
Movie 2	0.0121	0.0161	-0.0082	0.0149	-0.0027	0.0130	0.0158	0.0012	0.0446	-0.0026
Movie 3	-0.0178	-0.0981	-0.0600	-0.0904	-0.0079	-0.0313	-0.0663	-0.0347	-0.1596	-0.0198
Movie 4	0.0166	0.0463	0.0285	0.0350	0.0070	0.0170	0.0289	0.0159	0.0679	0.0068
Movie 5	-0.0043	-0.0082	-0.0064	-0.0115	-0.0024	0.0004	-0.0026	-0.0022	-0.0068	-0.0007
Movie 6	0.0140	0.0278	0.0388	0.0250	0.0149	0.0102	0.0101	0.0034	0.0205	-0.0003
Movie 7	-0.0910	-0.1445	-0.1593	-0.0352	-0.0882	0.0383	0.0007	0.0046	0.0943	-0.0255
Movie 8	-0.1398	-0.1940	-0.1960	-0.0511	-0.1132	0.0332	0.0012	0.0142	0.1055	-0.0212
Movie 9	0.0992	0.1635	0.1692	0.0477	0.0863	-0.0326	0.0068	-0.0004	-0.0758	0.0319
Movie 10	-0.0981	-0.1338	-0.1334	-0.0348	-0.0780	0.0183	-0.0019	0.0108	0.0688	-0.0136
Movie 11	-0.0153	0.0432	0.0579	0.0284	0.0117	-0.0239	0.0149	0.0218	-0.0013	0.0287
Movie 12	0.0022	0.0027	-0.0154	-0.0134	-0.0043	-0.0152	0.0002	0.0000	-0.0189	0.0109
Movie 13	0.0266	-0.0160	-0.0477	-0.0373	0.0013	-0.0039	-0.0118	-0.0211	-0.0389	-0.0070

	1	2	3	4	5	6	7	8	9	10
Movie 14	0.0031	0.0119	0.0192	0.0139	0.0089	0.0004	0.0076	0.0012	0.0011	0.0037
Movie 15	-0.0265	0.0800	0.1403	0.0837	0.0184	0.0081	0.0340	0.0458	0.0701	0.0227
Movie 16	-0.0103	0.0521	0.0397	0.0135	0.0086	-0.0450	0.0192	0.0244	-0.0202	0.0480
Movie 17	0.0365	-0.0655	-0.0112	0.0117	0.0007	0.1131	-0.0135	-0.0402	0.0876	-0.0983
Movie 18	0.0130	0.0050	0.0136	0.0034	0.0066	0.0088	0.0000	-0.0029	0.0049	-0.0084
Movie 19	-0.0272	0.0101	-0.0253	-0.0274	-0.0086	-0.0638	0.0019	0.0160	-0.0533	0.0495
Movie 20	0.0070	0.0459	0.0128	-0.0130	0.0121	-0.0571	0.0027	0.0063	-0.0682	0.0439
Movie 21	0.0235	-0.0309	-0.1093	-0.0781	-0.0102	-0.0386	-0.0249	-0.0277	-0.0943	0.0006
Movie 22	0.0206	-0.0284	-0.0643	-0.0421	-0.0014	-0.0079	-0.0181	-0.0239	-0.0442	-0.0079
Movie 23	-0.0267	0.0215	0.0871	0.0589	0.0066	0.0245	0.0202	0.0253	0.0681	0.0024
Movie 24	0.0117	-0.0004	-0.0042	-0.0076	0.0030	-0.0019	-0.0049	-0.0062	-0.0157	-0.0026
Movie 25	0.0312	-0.0120	-0.0536	-0.0395	0.0010	-0.0143	-0.0142	-0.0235	-0.0444	-0.0062
Movie 26	-0.1071	0.1063	0.1544	0.1416	-0.0165	-0.0419	0.0851	0.1118	0.1467	0.0959
Movie 27	0.0623	-0.0637	-0.0879	-0.0847	0.0077	0.0201	-0.0479	-0.0644	-0.0931	-0.0499
Movie 28	0.0199	-0.0350	-0.0564	-0.0419	-0.0031	0.0023	-0.0205	-0.0249	-0.0383	-0.0237
Movie 29	-0.0597	0.0713	0.1049	0.0903	-0.0043	-0.0213	0.0515	0.0683	0.0948	0.0579
Movie 30	0.0464	0.0044	-0.0057	-0.0275	0.0187	-0.0013	-0.0216	-0.0243	-0.0573	-0.0164

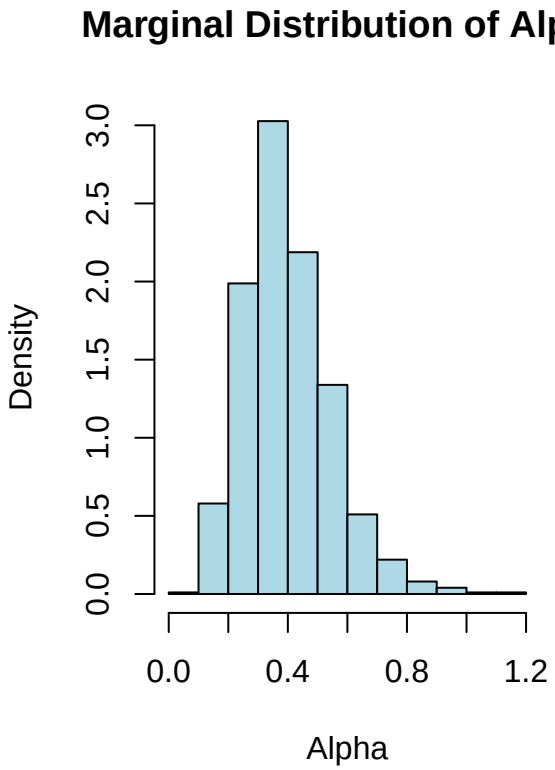
Question 6: Poisson-Gamma Distribution

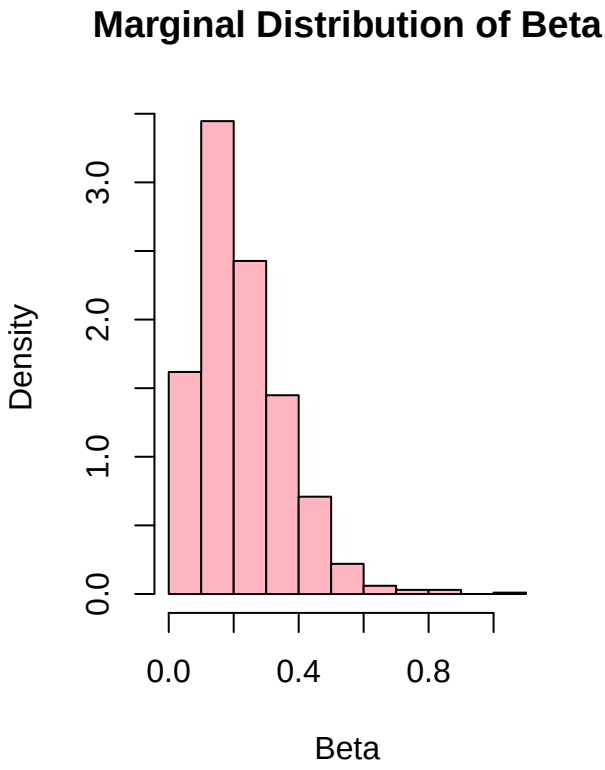


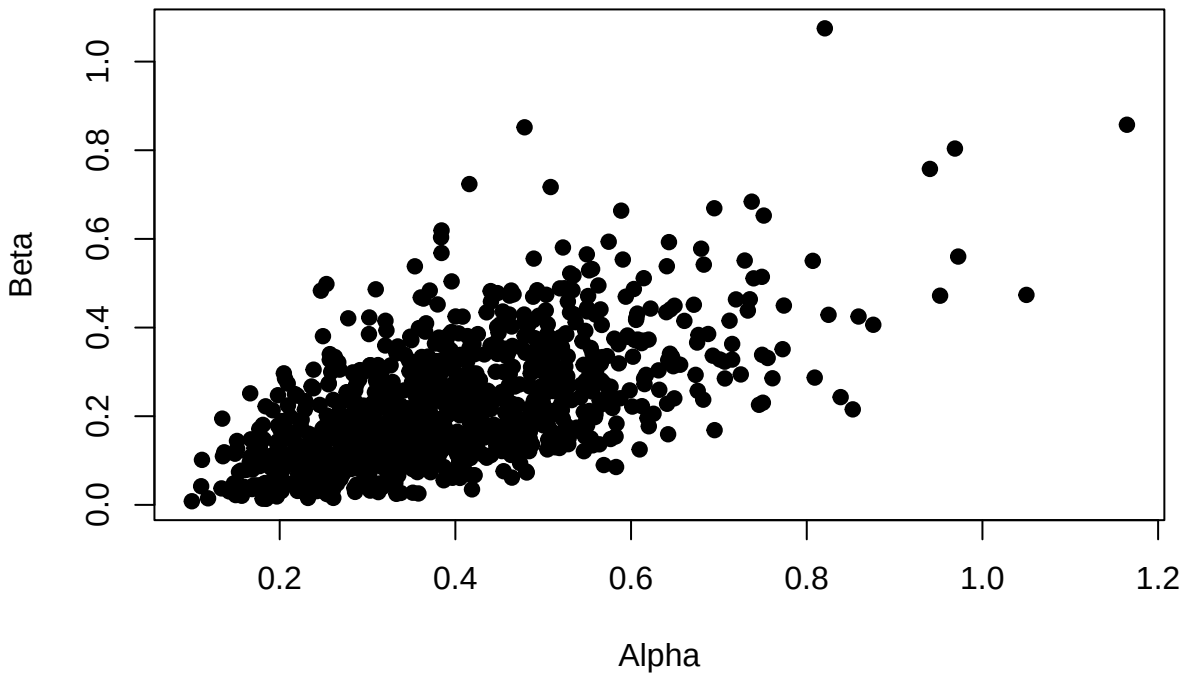




MH to sample Marginal Distribution






Beta
0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8
1.0
1.2
Alpha