Chapter 5: Discrete Probability Distributions

Section 5-1: Probability Distributions

1. The random variable is *x*, which is the number of girls in four births. The possible values of *x* are 0, 1, 2, 3, and 4. The values of the random variable x are numerical.

2. The random variable is discrete because the number of possible values is 5, and 5 is a finite number. The random variable is discrete if it has a finite number of values or a countable number of values.

3. The table does describe a probability distribution because the three requirements are satisfied. First, the variable *x* is a numerical random variable and its values are associated with probabilities. Second, $\Sigma P(x) = 0.063 + 0.250 + 0.375 + 0.250 + 0.063 = 1.001$, which is not exactly 1 due to round-off error, but is close enough to satisfy the requirement. Third, each of the probabilities is between 0 and 1 inclusive, as required.

4. The probability of 0.136 is relevant; 56 is not significantly high because the probability of 56 or more girls is 0.136, which is not small, such as 0.05 or less. With random chance, it is likely that the outcome could be 56 or more girls.

5. a. continuous random variable
   b. not a random variable
   c. discrete random variable
   d. continuous random variable
   e. discrete random variable

6. a. not a random variable
   b. continuous random variable
   c. discrete random variable
   d. not a random variable
   e. discrete random variable

7. Probability distribution with
$\mu = 0 \cdot 0.031 + 1 \cdot 156 + 2 \cdot 0.313 + 3 \cdot 313 + 4 \cdot 0.156 + 5 \cdot 0.031 = 2.5$
$\sigma = \sqrt{(0-2.5)^2 \cdot 0.031 + (1-2.5)^2 \cdot 0.156 + \cdots + (4-2.5)^2 \cdot 0.156 + (5-2.5)^2 \cdot 0.031} = 1.1$

8. Probability distribution (The sum of the probabilities is 1.001, but that is due to rounding errors.) with
$\mu = 0 \cdot 0.659 + 1 \cdot 0.287 + 2 \cdot 0.050 + 3 \cdot 004 + 4 \cdot 0.001 + 5 \cdot 0 = 0.4$
$\sigma = \sqrt{(0-0.4)^2 \cdot 0.659 + (1-0.4)^2 \cdot 0.287 + \cdots + (4-0.4)^2 \cdot 0.001 + (5-0.4)^2 \cdot 0} = 0.6$

9. Not a probability distribution because the sum of the probabilities is 0.94, which is not 1 as required. Also, Ted clearly needs a new approach.

10. Probability distribution with
$\mu = 0 \cdot 0.0000 + 1 \cdot 0.0001 + 2 \cdot 0.0006 + 3 \cdot 0.0387 + 4 \cdot 0.9606 = 4.0$
$\sigma = \sqrt{(0-4.0)^2 \cdot 0.0000 + (1-4.0)^2 \cdot 0.0001 + (2-4.0)^2 \cdot 0.0006 + (3-4.0)^2 \cdot 0.0387 + (4-4.0)^2 \cdot 0.9606} = 0.2$

11. Probability distribution with
$\mu = 0 \cdot 0.4219 + 1 \cdot 0.4219 + 2 \cdot 0.1406 + 3 \cdot 0.0156 = 0.7$
$\sigma = \sqrt{(0-0.7)^2 \cdot 0.4219 + (1-0.7)^2 \cdot 0.4219 + (2-0.7)^2 \cdot 0.1406 + (3-0.7)^2 \cdot 0.0156} = 0.7$
(The sum of the probabilities is 0.999, but that is due to rounding errors.)

12. Not a probability distribution because the sum of the probabilities is 0.986, which is not 1 as required

13. $\mu = 0 \cdot 0.004 + 1 \cdot 0.031 + 2 \cdot 0.109 + \cdots + 6 \cdot 0.109 + 7 \cdot 0.031 + 8 \cdot 0.004 = 4.0$ girls
$\sigma = \sqrt{(0-4.0)^2 \cdot 0.004 + (1-4.0)^2 \cdot 0.031 + \cdots + (7-4.0)^2 \cdot 0.031 + (8-4.0)^2 \cdot 0.004} = 1.4$ girls

14. The lower limit is $\mu - 2\sigma = 4.0 - 2(1.4) = 1.2$ girls. Because 1 girl is less than or equal to 1.2 girls, it is a significantly low number of girls.

15. The upper limit is $\mu + 2\sigma = 4.0 + 2(1.4) = 6.8$ girls. Because 6 girls is not greater than or equal to 6.8 girls, it is not a significantly high number of girls.

16. a. $P(X = 7) = 0.031$

b. $P(X \geq 7) = 0.031 + 0.004 = 0.035$

c. The probability from part (b), since it is the probability of the given or more extreme result.

d. Yes, because the probability of 7 or more girls is 0.035, which is low (less than or equal to 0.05).

17. a. $P(X = 6) = 0.109$

b. $P(X \geq 6) = 0.109 + 0.031 + 0.004 = 0.144$

c. The result from part (b), since it is the probability of the given or more extreme result.

d. No, because the probability of six or more girls is 0.144, which is not very low (less than or equal to 0.05).

18. a. $P(X = 1) = 0.031$

b. $P(X \leq 1) = 0.004 + 0.031 = 0.035$

c. The result from part (b), since it is the probability of the given or more extreme result.

d. Yes, because the probability of one or fewer girls is 0.035, which is low (less than or equal to 0.05).

19. $\mu = 0 \cdot 0.172 + 1 \cdot 0.363 + 2 \cdot 0.306 + 3 \cdot 0.129 + 4 \cdot 0.027 + 5 \cdot 0.002 = 1.5$ sleepwalkers

$\sigma = \sqrt{(0-1.5)^2 \cdot 0.172 + (1-1.5)^2 \cdot 0.363 + ... + (4-1.5)^2 \cdot 0.027 + (5-1.5)^2 \cdot 0.002} = 1.0$ sleepwalkers

20. Significantly high numbers of sleepwalkers are greater than or equal to $\mu + 2\sigma = 1.5 + 2(1.0) = 3.5$ sleepwalkers. Because 4 sleepwalkers is greater than or equal to 3.5 sleepwalkers, 4 sleepwalkers is a significantly high number.

21. Significantly high numbers of sleepwalkers are greater than or equal to $\mu + 2\sigma = 1.5 + 2(1.0) = 3.5$ sleepwalkers. Because 3 sleepwalkers is not greater than or equal to 3.5 sleepwalkers, 3 sleepwalkers is not a significantly high number.

22. a. $P(X = 4) = 0.027$

b. $P(X \geq 4) = 0.027 + 0.002 = 0.029$

c. The probability from part (b), since it is the probability of the given or more extreme result.

d. Yes, because the probability of four or more sleepwalkers is 0.029, which is very low (less than or equal to 0.05)

23. a. $P(X = 1) = 0.363$

b. $P(X \leq 1) = 0.172 + 0.363 = 0.535$

c. The probability from part (b), since it is the probability of the given or more extreme result.

d. No, because the probability of one or fewer sleepwalkers is 0.535, which is not low (less than or equal to 0.05)

Section 5-2: Binomial Probability Distributions

1. The given calculation assumes that the first two offspring peas have green pods and the last offspring peas have green pods, but there are other arrangements consisting of offspring peas have green pods and three that do not. The probabilities corresponding to those other arrangements should also be included in the result.

2. $n = 5, x = 2, p = 0.75, q = 0.25$

3. Because the 30 selections are made without replacement, they are dependent, not independent. Based on the 5% guideline for cumbersome calculations, the 30 selections can be treated as being independent. (The 30 selections constitute 3% of the population of 1020 responses, and 3% is not more than 5% of the population.) The probability can be found by using the binomial probability formula, but it would require application of that formula 21 times (or 10 times if we are clever), so it would be better to use technology.

4. The 0+ indicates that the probability is a very small positive value. (The actual value is 0.0000508.) The notation of 0+ does not indicate that the event is impossible; it indicates that the event is possible, but very unlikely.

5. Not binomial; each of the weights has more than two possible outcomes.

6. binomial

7. binomial

8. Not binomial; each of the responses has more than two possible outcomes.

9. Not binomial; there are more than two possible outcomes.

10. binomial

11. binomial

12. Not binomial; there are more than two possible outcomes.

13. a. $\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}=0.128$

 b. {WWC, WCW, CWW}; Each has a probability of 0.128.

 c. $0.128\cdot 3=0.384$

14. a. $P(\text{EEEN})=0.53\cdot 0.53\cdot 0.53\cdot 0.47=0.0700$

 b. {EEEN, EENE, ENEE, NEEE }; Each has a probability of 0.0700.

 c. $0.0700\cdot 4=0.28$

15. ${}_8C_7\cdot 0.2^7\cdot 0.8^1=0.0000819$ (Table: 0+)

16. ${}_8C_4\cdot 0.2^4\cdot 0.8^4+{}_8C_5\cdot 0.2^5\cdot 0.8^3+{}_8C_6\cdot 0.2^6\cdot 0.8^2+{}_8C_7\cdot 0.2^7\cdot 0.8^1+{}_8C_8\cdot 0.2^8\cdot 0.8^0=0.0563$ (Table: 0.056)

17. ${}_8C_0\cdot 0.2^0\cdot 0.8^8+{}_8C_1\cdot 0.2^1\cdot 0.8^7+{}_8C_2\cdot 0.2^2\cdot 0.8^6=0.797$ (Table: 0.798)

18. ${}_8C_0\cdot 0.2^0\cdot 0.8^8+{}_8C_1\cdot 0.2^1\cdot 0.8^7+{}_8C_2\cdot 0.2^2\cdot 0.8^6=0.797$ (Table: 0.798)

19. ${}_8C_0\cdot 0.2^0\cdot 0.8^8=0.168$

20. $1-{}_8C_0\cdot 0.2^0\cdot 0.8^8=0.832$ (Table: 0.833)

21. ${}_8C_6\cdot 0.21^2\cdot 0.79^6=0.300$

22. ${}_{20}C_5\cdot 0.21^5\cdot 0.79^{15}=0.184$

23. ${}_{10}C_3\cdot 0.21^3\cdot 0.79^7+{}_{10}C_4\cdot 0.21^4\cdot 0.79^6+\cdots+{}_{10}C_9\cdot 0.21^9\cdot 0.79^1+{}_{10}C_{10}\cdot 0.21^{10}\cdot 0.79^0=0.353$

24. ${}_{10}C_0\cdot 0.21^0\cdot 0.79^{10}+{}_{10}C_1\cdot 0.21^1\cdot 0.79^9+{}_{10}C_2\cdot 0.21^2\cdot 0.79^8=0.647$

25. a. $\mu=np=36\cdot 0.5=18.0$ girls, $\sigma=\sqrt{np(1-p)}=\sqrt{36\cdot 0.5\cdot 0.5}=3.0$ girls

 b. Values of $18.0-2(3.0)=12.0$ girls or fewer are significantly low, values of $18.0+2(3.0)=24.0$ girls or more are significantly high, and values between 12.0 girls and 24.0 girls are not significant.

 c. The result is significantly high because the result of 26 girls is greater than or equal to 24.0 girls. A result of 26 girls would suggest that the XSORT method is effective.

26. a. $\mu=np=16\cdot 0.5=8.0$ girls, $\sigma=\sqrt{np(1-p)}=\sqrt{16\cdot 0.5\cdot 0.5}=2.0$ girls

 b. Values of $8.0-2(2.0)=4.0$ girls or fewer are significantly low, values of $8.0+2(2.0)=12.0$ girls or more are significantly high, and values between 4.0 girls and 12.0 girls are not significant.

 c. The result is not significant because the result of 11 girls is not greater than or equal to 12.0 girls. A result of 11 girls would not suggest that the XSORT method is effective.

27. a. $\mu = np = 10 \cdot 0.75 = 7.5$ peas, $\sigma = \sqrt{np(1-p)} = \sqrt{10 \cdot 0.75 \cdot 0.25} = 1.4$ peas

b. Values of $7.5 - 2(1.4) = 4.7$ peas or fewer are significantly low, values of $7.5 + 2(1.4) = 10.3$ peas or more are significantly high, and values between 4.7 peas and 10.3 peas are not significant.

c. The result is not significant because the result of 9 peas is not greater than or equal to 10.3 peas.

28. a. $\mu = np = 16 \cdot 0.75 = 12.0$ peas, $\sigma = \sqrt{np(1-p)} = \sqrt{16 \cdot 0.75 \cdot 0.25} = 1.7$ peas

b. Values of $12.0 - 2(1.7) = 8.6$ peas or fewer are significantly low, values of $12.0 + 2(1.7) = 15.4$ peas or more are significantly high, and values between 4.7 peas and 10.3 peas are not significant.

c. The result is significant because the result of 7 peas is less than or equal to 8.6 peas.

29. $1 - {}_{36}C_0 \cdot 0.01^0 \cdot 0.99^{36} = 0.304$; It is not unlikely for such a combined sample to test positive.

30. $1 - {}_{16}C_0 \cdot 0.014^0 \cdot 0.986^{16} = 0.202$; It is somewhat likely for such a combined sample to test positive.

31. ${}_{40}C_1 \cdot 0.03^1 \cdot 0.97^{39} + {}_{40}C_0 \cdot 0.03^0 \cdot 0.97^{40} = 0.662$; The probability shows that about 2/3 of all shipments will be accepted. With about 1/3 of the shipments rejected, the supplier would be wise to improve quality.

32. About 92% of all shipments will be accepted. Almost all shipments will be accepted, and only 8% of the shipments will be rejected.

33. a. $\mu = np = 14 \cdot 0.5 = 7.0$ girls, $\sigma = \sqrt{np(1-p)} = \sqrt{14 \cdot 0.5 \cdot 0.5} = 1.8$ girls; Values between $14.0 - 2(1.8) = 3.4$ girls and $14.0 + 2(1.8) = 10.6$ girls are not significant (3.3 and 10.7 if using unrounded values). 13 girls lies outside these limits, so it is significant.

b. The probability of exactly 13 girls is ${}_{14}C_{13} \cdot 0.5^{13} \cdot 0.5^1 = 0.000854$.

c. The probability of 13 or more girls is ${}_{14}C_{13} \cdot 0.5^{13} \cdot 0.5^1 + {}_{14}C_{14} \cdot 0.5^{14} \cdot 0.5^0 = 0.000916$.

d. The probability from part (c) is relevant. The result of 13 girls is significantly high.

e. The results suggest that the XSORT method is effective in increasing the likelihood that a baby is a girl.

34. a. $\mu = np = 40 \cdot 0.6 = 24.0$ successes $\sigma = \sqrt{np(1-p)} = \sqrt{40 \cdot 0.6 \cdot 0.4} = 3.1$ successes; Values between $24.0 - 2(3.1) = 17.8$ successes and $24.0 + 2(3.1) = 30.2$ successes are not significant. The value of 29 successes lies between these boundaries, so it is neither significantly low or high.

b. The probability of exactly 29 successes is ${}_{40}C_{29} \cdot 0.6^{29} \cdot 0.4^{11} = 0.0357$.

c. The probability of 29 or more successes is ${}_{40}C_{29} \cdot 0.6^{29} \cdot 0.4^{11} + \cdots + {}_{40}C_{29} \cdot 0.6^{40} \cdot 0.4^0 = 0.0709$.

d. The probability from part (c) is relevant. The result of 29 successes is not significantly high.

e. The results do not provide strong evidence for the vaccine's effectiveness.

35. a. $\mu = np = 47 \cdot 0.75 = 35.3$ long stems, $\sigma = \sqrt{np(1-p)} = \sqrt{14 \cdot 0.75 \cdot 0.25} = 3.0$ long stems; Values between $35.3 - 2(3.0) = 29.3$ long stems and $35.3 + 2(3.0) = 14.3$ long stems are not significant (29.3 and 41.2 if using unrounded values). Because 34 falls between those limits, it is neither significantly low nor significantly high.

b. The probability of exactly 34 peas with long stems is ${}_{47}C_{34} \cdot 0.75^{47} \cdot 0.25^{13} = 0.118$.

c. The probability of 34 or more peas with long stems is ${}_{47}C_{34} \cdot 0.75^{34} \cdot 0.25^{13} + \cdots + {}_{47}C_{47} \cdot 0.75^{47} \cdot 0.25^0$ $= 0.390$.

d. The probability from part (c) is relevant. The value of 34 peas with long stems is not significantly low.

e. The results do not provide strong evidence against Mendel's claim of 75% for peas with long stems.

36. a. $\mu = np = 80 \cdot 0.05 = 4.0$ influenza cases, $\sigma = \sqrt{np(1-p)} = \sqrt{80 \cdot 0.05 \cdot 0.95} = 1.9$ influenza cases; Values between $4.0 - 2(1.9) = 0.2$ influenza cases and $4.0 + 2(1.9) = 7.8$ influenza cases are not significant (0.1 and 7.9 if using unrounded values). 1 influenza case lies between these limits, so it is not significantly low or high.

b. The probability of exactly 1 influenza case is ${}_{80}C_1 \cdot 0.05^1 \cdot 0.95^{79} = 0.0695$.

c. The probability of 1 or fewer influenza cases is ${}_{80}C_0 \cdot 0.05^0 \cdot 0.95^{70} + \cdots + {}_{80}C_1 \cdot 0.05^1 \cdot 0.95^{79} = 0.0861$.

d. The probability from part (c) is relevant. The value of influenza case is not significantly high.

e. The results do not provide strong evidence for the vaccine's effectiveness.

37. $P(5) = 0.06(1-0.06)^4 = 0.0468$

38. $\frac{10!}{5!2!3!} \cdot \left(\frac{78}{121}\right)^5 \cdot \left(\frac{22}{121}\right)^2 \cdot \left(\frac{21}{121}\right)^3 = 0.0485$; The expression could not be used without replacement, the outcomes would no longer be independent.

39. Without replacement: $P(3) = \frac{4!}{(4-3)!3!} \cdot \frac{16!}{(16-20+3)!(20-3)!} \div \frac{(4+16)!}{(4+16-20)!20!} = 0.139$

With replacement: ${}_8C_3 \cdot \left(\frac{4}{20}\right)^3 \cdot \left(\frac{16}{20}\right)^5 = 0.147$

Section 5-3: Poisson Probability Distributions

1. $\mu = 31{,}645/365 = 86.7$, which is the mean number of patient admissions per day. $x = 85$, because we want the probability that a randomly selected day has exactly 85 admissions, and $e \approx 2.71828$ which is a constant used in all applications of Formula 5-9.

2. The mean is $\mu = 31{,}645/365 = 86.7$, patients, the standard deviation is $\sigma = \sqrt{86.7} = 9.3$ patients, and the variance is $\sigma^2 = 86.7$ patients^2.

3. The possible values of *x* are 0, 1, 2, … (with no upper bound), so *x* is a discrete random variable. It is not possible to have $x = 90.3$ patient admissions in a day

4. $P(0)$ represents the probability of no occurrences of an event during the relevant interval. If $x = 0$, $P(0) = e^{-\mu}$.

5. $P(12) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{11.5863^{12} \cdot e^{-11.5863}}{12!} = 0.114$

6. $P(9) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{11.5863^{9} \cdot e^{-11.5863}}{9!} = 0.964$

7. $P(\text{At least one birth}) = 1 - P(0) = 1 - \frac{11.5863^0 \cdot e^{-11.5863}}{0!} = 0.999991$

8. $P(\text{At least two births}) = 1 - P(0) - P(1) = 1 - \frac{11.5863^0 \cdot e^{-11.5863}}{0!} - \frac{11.5863^1 \cdot e^{-11.5863}}{1!} = 0.99988$

9. $\mu = \frac{333}{365} = 0.9$ murders, $P(0) = \frac{0.9^0 \cdot e^{-0.9}}{0!} = 0.407$ (0.402 if using the unrounded mean.) There should be many days (roughly 40%) with no murders.

10. $\mu = \frac{196}{20} = 9.8$

a. $P(0) = \frac{9.8^0 \cdot e^{-9.8}}{0!} = 0.497$

b. $P(1) = \frac{9.8^1 \cdot e^{-9.8}}{1!} = 0.348$

c. $P(2) = \frac{9.8^2 \cdot e^{-9.8}}{2!} = 0.122$

d. $P(3) = \frac{9.8^3 \cdot e^{-9.8}}{3!} = 0.0284$

e. $P(4) = \frac{9.8^4 \cdot e^{-9.8}}{4!} = 0.00497$; The expected frequencies of 139, 97, 34, 8 , and 1.4 compare reasonably well to the actual frequencies, so the Poisson distribution does provide good results.

11. a. $P(2) = \frac{0.929^2 \cdot e^{-0.929}}{2!} = 0.170$

b. The expected number of regions with exactly 2 hits is between 97.9 and 98.2, depending on rounding.

c. The expected number of regions with 2 hits is close to 93, which is the actual number of regions with 2 hits.

12. a. $\mu = 12429 \cdot 0.000011 = 0.1367$

b. $P(0) = \frac{0.137^0 \cdot e^{-0.137}}{0!} = 0.872$ and $P(1) = \frac{0.137^1 \cdot e^{-0.137}}{1!} = 0.119$; So the probability of 0 or 1 is $0.872 + 0.119 = 0.991$.

c. $1 - 0.991 = 0.009$

d. No, the probability of more than one case is extremely small, so the probability of getting as many as four cases is even smaller.

13. $\mu = \frac{33,561}{2969} = 11.3$ fatalities, $1 - P(0) = 1 - \frac{11.3^0 \cdot e^{-11.3}}{0!} = 0.9999876$ or 0.9999877 if using the unrounded mean. There is a very high chance ("almost certain") that at least one fatality will occur.

14. a. $P(0) = \frac{7.0^0 \cdot e^{-7.0}}{0!} = 0.001$

b. $P(\text{At least one}) = 1 - P(0) = 1 - \frac{7.0^0 \cdot e^{-7.0}}{0!} = 0.999$

c. $P(\text{At most two}) = P(0) + P(1) + P(2) = \frac{7.0^0 \cdot e^{-7.0}}{0!} + \frac{7.0^1 \cdot e^{-7.0}}{1!} + \frac{7.0^2 \cdot e^{-7.0}}{2!} = 0.0296$

15. a. $\mu = \frac{138}{13} = 10.61538$ cases

b. $P(X = 0) = \frac{10.61538^0 \cdot e^{-10.61538}}{0!} = 0.0000245$

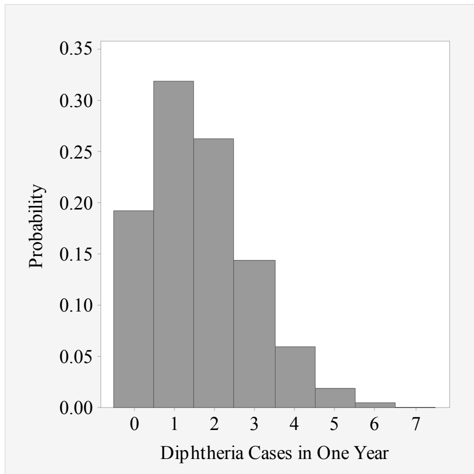
c. $P(X = 9) = \frac{10.61538^9 \cdot e^{-10.61538}}{9!} = 0.116$; With 2 of the 13 years having exactly 9 cases of rubella, the probability appears to be 2/13, or 0.154; the Poisson distribution yields a probability of 0.116, which is in the general ballpark, but it is off by a fairly large amount.

16. a. $\mu = \frac{56}{34} = 1.647$ cases per year

b. $P(X = 0) = \frac{1.647^0 \cdot e^{-1.647}}{0!} = 0.192$

c. $P(X \le 5) = \frac{1.647^0 \cdot e^{-1.647}}{0!} + \cdots + \frac{1.647^5 \cdot e^{-1.647}}{5!}$ =0.993; The probability of more than 5 cases is 0.007, which is significantly high.

17. The distribution is skewed to the right.



Chapter Quick Quiz

1. $\mu = 64 \cdot 0.206 = 13.2$ males

2. $\sigma = \sqrt{64 \cdot 0.206 \cdot 0.794} = 3.2$ males

3. parameters

4. Significantly low values are $13.2 - 2(3.2) = 6.8$ or fewer males. Significantly high values are $13.2 + 2(3.2) = 19.6$ or more males

5. ${}_8C_3 \cdot 0.206^3 \cdot 0.794^5 = 0.154$

6. This is probability distribution because the three requirements are satisfied. First, the variable x is a numerical random variable and its values are associated with probabilities. Second, $\Sigma P(x) = 0.762 + 0.213 + 0.024 + 0.001 + 0 + 0 = 1$. Third, each of the probabilities is between 0 and 1 inclusive, as required.

7. $\mu = 0 \cdot 0.762 + 1 \cdot 0.213 + 2 \cdot 0.024 + 3 \cdot 0.001 + 0 \cdot 0 + 0 \cdot 0 = 0.3$ male

8. $\sigma^2 = (0.5 \text{ male})^2 = 0.3 \text{ male}^2$; ($0.3 \text{ male}^2$ if calculated using values from table.)

9. 0+ indicates that the probability is a very small positive number. It does not indicate that it is impossible for all five adult males to be heavy drinkers.

10. $P(X < 3) = P(X \le 2) = 0.762 + 0.213 + 0.024 = 0.999$; Yes, 4 is significantly high.

Review Exercises

1. $P(X=3) = {}_5C_3 \cdot 0.28^3 \cdot 0.72^2 = 0.293$

2. $P(X \geq 1) = 1 - {}_5C_0 \cdot 0.28^0 \cdot 0.72^5 = 0.804$; No. The five subjects from the same family are not randomly selected from the population of adults. Because they are from the same family, they are likely to share similar diet and genetic factors, so they are not independent.

3. $\mu = 5 \cdot 0.28 = 1.4$ adults, $\sigma = \sqrt{5 \cdot 0.28 \cdot 0.72} = 1.0$ adult

4. Yes, the limit separating significantly high values is $1.4 + 2(1.0) = 3.4$ adults, and 5 is not greater than or equal to 3.4. Also, the probability that all five adults have high cholesterol is ${}_5C_5 \cdot 0.28^5 \cdot 0.72^0 = 0.00172$, which is very low (less than or equal to 0.05).

5. Yes, the limit separating significantly low values is $\mu - 2\sigma = 1.4 - 2(1.0) = -0.6$ adult, and 1 is not less than or equal to –0.6. Also, the probability of one or fewer adults having a credit card is ${}_5C_0 \cdot 0.28^0 \cdot 0.72^5$ $+ {}_5C_1 \cdot 0.28^1 \cdot 0.72^4 = 0.570$, which is not low (less than or equal to 0.05).

6. This is not a probability distribution because the responses are not values of a numerical random variable.

7. a. This is probability distribution because the three requirements are satisfied. First, the variable x is a numerical random variable and its values are associated with probabilities. Second, $\Sigma P(x) = 0.304 + 0.400 + 0.220 + 0.064 + 0.011 + 0.001 + 0 = 1$, Third, each of the probabilities is between 0 and 1 inclusive, as required.

   b. $\mu = 0 \cdot 0.304 + 1 \cdot 0.400 + 2 \cdot 0.220 + 3 \cdot 0.064 + 4 \cdot 0.011 + 5 \cdot 0.001 + 6 \cdot 0 = 1.1$ condoms

   c. $\sigma = \sqrt{(0-1.1)^2 \cdot 0.304 + (1-1.1)^2 \cdot 0.400 + (2-1.1)^2 \cdot 0.220 + \cdots + (5-1.1)^2 \cdot 0.001 + (6-1.1)^2 \cdot 0} = 0.9$ condom

   d. Yes, 5 failures is significantly high. A number is significantly high if it is equal to or greater than $\mu + 2\sigma = 1.1 + 2(0.9) = 2.9$, and 5 does exceed 2.9. Also, the probability of 5 or more failures is 0.001, which is a low probability.

   e. Here, the symbol 0+ represents a positive probability that is so small that it is 0.000 when rounded.

8. a. $\mu = 143/365$, or 0.392 death per day

   b. $P(X=0) = \dfrac{0.392^0 \cdot e^{-0.392}}{0!} = 0.676$

   c. $P(X > 2) = P(X \geq 2) = 1 - P(X \leq 1) = 1 - \dfrac{0.392^0 \cdot e^{-0.392}}{0!} - \dfrac{0.392^1 \cdot e^{-0.392}}{1!} = 0.00750$ (0.00749 if using unrounded value for the mean.)

   d. No, because the event is so rare. (But it is possible that more than one death occurs, so it might be wise to consider a contingency plan.)

Cumulative Review Exercises

1. a. The mean is $\bar{x} = \dfrac{69+72+73+79+81+83+88+90+92+97}{10} = 82.4$ manatees.

   b. The median is $\dfrac{81+83}{2} = 82.0$ manatees.

   c. The range is $97-69 = 28.0$ manatees.

   d. The standard deviation is $s = \sqrt{\dfrac{(69-82.4)^2+(79-82.4)^2+\cdots+(81-82.4)^2+(72-82.4)^2}{10-1}} = 9.3$ manatees.

   e. The variance is $s^2 = (9.3 \text{ manatees})^2 = 86.5 \text{ manatees}^2$. $\left(87.2 \text{ manatees}^2 \text{ using unrounded values.}\right)$

   f. The trend of the manatee deaths over time is not addressed by the preceding statistics.

   g. Significantly low numbers are $\mu-2\sigma = 82.4-2(9.3) = 63.8$ manatees or lower, and significantly high numbers are $\mu-2\sigma = 82.4+2(9.3) = 101$ manatees or higher. (If using unrounded statistics, the limits are 63.7 manatees and 101.1 manatees.)

   h. None of the listed numbers are significantly low or significantly high.

   i. ratio

   j. discrete

2. a. $\bar{x} = \dfrac{0(11)+1(11)+2(16)+3(14)+4(16)+5(24)+6(13)+7(14)+8(19)+9(9)}{11+11+16+14+16+24+13+18+19+9} = 4.6$ and

   $s = \sqrt{\dfrac{147\left(11\cdot 0^2+\cdots+9\cdot 9^2\right)-\left(11\cdot 0+\cdots+3\cdot 3\right)^2}{147(147-1)}} = 2.7$; They are statistics.

   b. The last digits appear to be random. None of the frequencies appears to be substantially different from the others.

   c. No. The values of x are numerical, but the frequencies are not probabilities, as required.

3. No vertical scale is shown, but a comparison of the numbers shows that 7,066,000 is roughly 1.2 times the number 6,000,000. However, the graph makes it appear that the goal of 7,066,000 people is roughly 3 times the number of people enrolled. The graph is misleading in the sense that it creates the false impression that actual enrollments are far below the goal, which is not the case. Fox News apologized for their graph and provided a corrected graph.

4. a. $1-0.115 = 0.885$

   b. $0.115\cdot 0.115 = 0.0132$

   b. $P(X \geq 1) = 1-P(X=0) = 1-{}_3C_0\cdot 0.115^0\cdot 0.885^3 = 0.307$ (Table: 0.256)

   c. $\mu = 40\cdot 0.115 = 4.6$ adults, $\sigma = \sqrt{40\cdot 0.115\cdot 0.885} = 2.0$ adults; There are parameters.

   d. Significantly low numbers are $\mu-2\sigma = 4.6-2(2.0) = 0.6$ or lower, and significantly high numbers are $\mu+2\sigma = 4.6+2(2.0) = 8.6$ and higher. Because 10 is greater than 8.6, it is a significantly high number of adults with diabetes (among 40).