Statistics for the Biological, Environmental and Health Sciences

STAT 007

Hypothesis Testing

Chapter 8

Testing a Claim about a Mean

Section 8-3

Hypothesis Testing

- We will introduce hypothesis tests for claims about the mean of a population when the standard deviation of the population, σ, is unknown.
- When σ is unknown we use the data to estimate it as the sample standard deviation, s.
- When we develop a hypothesis test for the mean when σ is unknown, then the sampling distribution of the test statistic will be a Student t distribution with $(n-1)$ degrees of freedom. Critical values and p-values will be computed using this distribution.

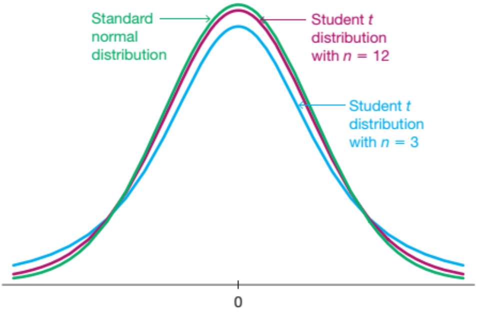
Hypothesis Testing for the population mean, μ

- **Step 1**: identify the claim of interest and its symbolic form.
- **Step 2**: identify the claim that is true when the one of interest is false.
- **Step 3**: write the null and alternative hypothesis.
- **Step 4**: select the significance level $\alpha = P(\textit{Type I error})$.
- **Step 5**: use the test statistic $t^{stat} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$, which under the assumption that H_0 is true follows a Student t distribution with $(n-1)$ degrees of freedom.
- **Step 6**:
  - find the critical region,
  - find the critical value(s) ($t_\alpha, -t_\alpha$, or $t_{\alpha/2}, -t_{\alpha/2}$),
  - find the p-value (area on the right of t^{stat}, area on the left of t^{stat}, or twice the area on the right of t^{stat}, when $t^{stat} > 0$ or twice the area on the left of t^{stat}, when $t^{stat} < 0$).
- **Step 7**: make a decision based on the critical value and/or the p-value.
- **Step 8**: restate your decision in simple nontechnical terms.

- **Requirement for step 5**: the population is normally distributed and/or $n > 30$. If $n \leq 30$ look at histograms, outliers, and quantile plots to check the normality assumption.

Properties of the Student t distribution

- It is different for different sample sizes.
- It has the same general bell shape as the standard normal distribution. The wider shape reflects the greater variability that is expected when σ is estimated by s.
- It has a mean equal to 0.
- It has a standard deviation that varies with the sample size and is larger than 1.
- As the sample size increases, it gets closer to the standard normal distribution.



Hypothesis Testing for μ

Example

A common recommendation is that adults should sleep between 7 hours and 9 hours each night.

From the National Health and Nutrition Examination Study the times of sleep (in hours) for 12 randomly selected adult subjects were obtained. Here are some of the statistics for this sample: $\overline{x} = 6.83$ hours and $s = 1.99$ hours.

a) Use 0.05 significance level to test the claim that the mean amount of sleep for adults is less than 7 hours. Base your conclusions on the critical value.

Types of errors

- The conclusions of an Hypothesis Test can be correct or incorrect.
- A **type I error** is the mistake of rejecting the null hypothesis when it is actually true (this is exactly the significance level).

$$\alpha = P(\textit{type I error}) = P(\textit{reject } H_0 \textit{ when } H_0 \textit{ is true}).$$

- A **type II error** is the mistake of failing to reject the null hypothesis when it is actually false.

$$\beta = P(\textit{type II error}) = P(\textit{failling to reject } H_0 \textit{ when } H_0 \textit{ is false}).$$

- α, β, and n are related. Common practices:
  - Choose the largest α that can be tolerated and the largest sample size n that is reasonable.
  - Choose α and β so that the required sample size is automatically determined.

Hypothesis Testing for μ

Example

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a) Use 0.05 significance level to test the claim that the mean amount of sleep for adults is less than 7 hours. Base your conclusions on the critical value.

b) Identify the type of error that you could be making.

Hypothesis Testing and Confidence Intervals

- Hypothesis Testing and Confidence Interval are both methods used in inferential statistics.
- Hypothesis testing is used when claims about the population parameters are made.
- Confidence Intervals are interval estimates for population parameters.
- When making inference regarding the population mean, μ, the confidence interval and the test of hypothesis will always agree:
  - Test statistic: $t^{stat} = \frac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}}$
  - Interval estimate: $\overline{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \overline{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$
- When making inference regarding the population proportion, p, the confidence interval and the test of hypothesis might disagree.
  - Test statistic: $z^{stat} = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$
  - Interval estimate: $\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Hypothesis Testing and Confidence Intervals

Example

Evaluations Data from student course evaluations were obtained from the University of Texas at Austin. The summary statistics are $n = 436$, $\overline{x} = 3.97$, $s = 0.55$.

a) Use a 0.05 significance level to test the claim that the population of student course evaluations has a mean equal to 4.00. Base your conclusions on the critical value.
b) Identify the type of error that you could be making.
c) Do the results apply to the population of all students?
d) Compute a 95% confidence interval for the mean value of the course evaluations.

Practice

Look at the exercises at the end of Section 8-3 in page 374.

Specially, look at exercises: 15-24. Base your conclusions on the critical value