BASKIN SCHOOL OF ENGINEERING

Department of Applied Mathematics and Statistics

First Year Exam: September 2014

Problem AMS 203

Consider an electronic system comprised of three components denoted as C_1, C_2 and C_3. Let X_i be a random variable denoting the lifetime of component C_i, $i = 1, 2, 3$. Assume X_1, X_2 and X_3 are i.i.d. random variables each with p.d.f.

$$f(x|\beta) = \begin{cases} e^{-x}, & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that the system will operate as long as both component C_1 and <u>at least one</u> of the components C_2 and C_3 operate. Let Z be the random variable that denotes the lifetime of the system.

1. (60 points) Find the c.d.f. of Z.

2. (40 points) Find E(Z^2).

Useful information: A r.v. X follows an exponential distribution with parameter $\beta > 0$, $X \sim Exp(\beta)$, if its p.d.f. is

$$f(x) = \begin{cases} \beta e^{-\beta x}, & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

In this case $E(X) = 1/\beta$ and $V(X) = 1/\beta^2$.

1. Solution:

$$\begin{aligned} F(z) = Pr(Z \leq z) &= 1 - Pr(Z > z) = 1 - Pr((X_1 > z) \cap ((X_2 > z) \cup (X_3 > z))) \\ &= 1 - (Pr(X_1 > z)(1 - Pr(X_2 \leq z)Pr(X_3 \leq z))) \\ &= 1 - (e^{-z}(1 - (1 - e^{-z})^2)) \\ &= 1 - 2e^{-2z} - e^{-3z}. \end{aligned}$$

2. Solution: The p.d.f. of Z is $f(z) = 4e^{-2z} - 3e^{-3z}$ for $z > 0$. Therefore,

$$E(Z^2) = \int_0^{\infty} z^2(4e^{-2z} - 3e^{-3z})dz = \frac{7}{9}.$$

This can be computed using the following: if $X \sim Exp(\beta)$ $E(X^2) = (E(X))^2 + V(X)) = 2/\beta^2$.