Lecture 7: Computational Geometry

CS 491 CAP

Uttam Thakore

Friday, October 7th, 2016

Credit for many of the slides on solving geometry problems goes to the Stanford CS 97SI course lecture on computational geometry (http://web.stanford.edu/class/cs97si/09-computational-geometry.pdf)

Table of Contents

Basic HS Geometry Review
- Trigonometry
- Vectors

Solving Geometry Problems
- Line-line intersection
- Sweep line algorithm
- Area of a polygon
- Binary search
- Ternary search
- Convex Hull

Other Geometry Concepts

More Resources

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sin, cos, & tan

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
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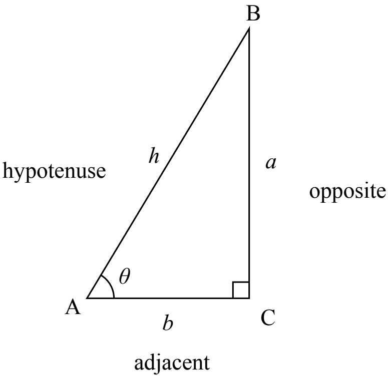

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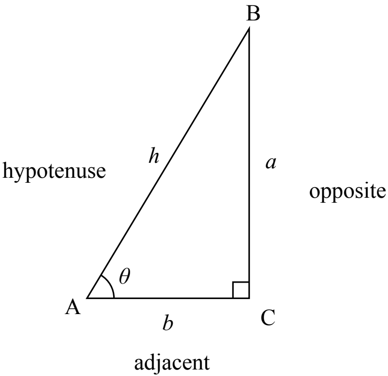

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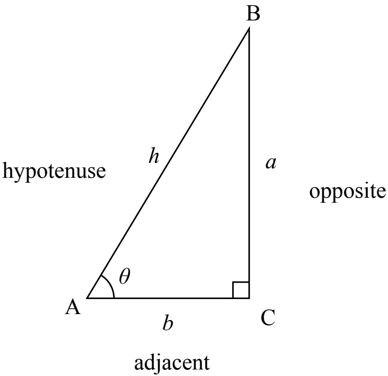

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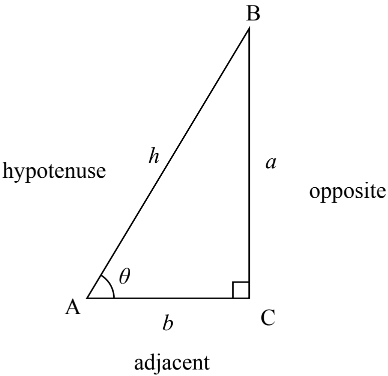
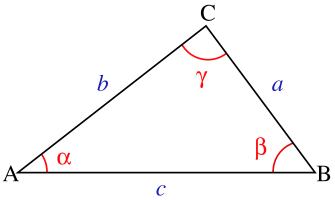


Law of sines & law of cosines



- Law of sines: $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$
- Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$
  - Put another way: $\cos(\alpha) = \frac{-a^2+b^2+c^2}{2bc}$
  - Similar for b and c

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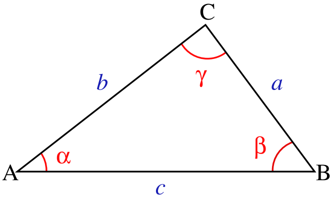

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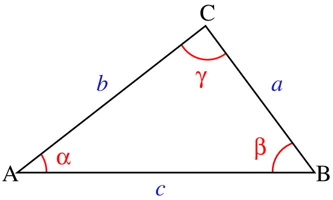

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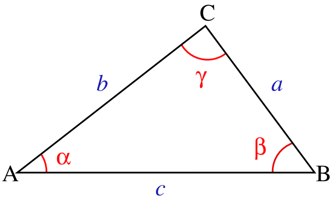

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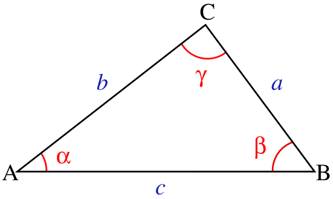

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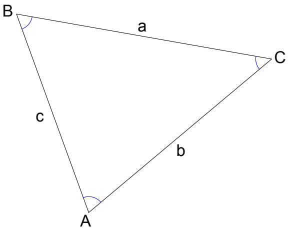

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- But there is an easier way to find the area of a triangle...

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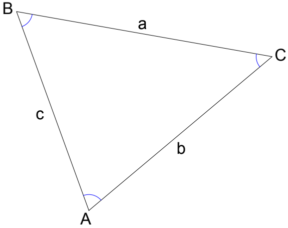

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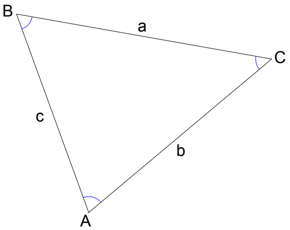

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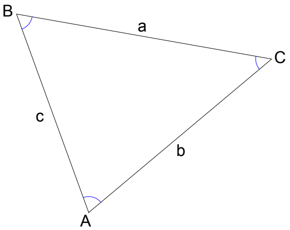

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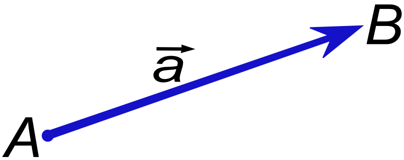
- Direction
- Magnitude

- Alternatively, magnitude & angle from positive x-axis



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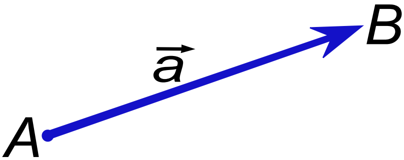

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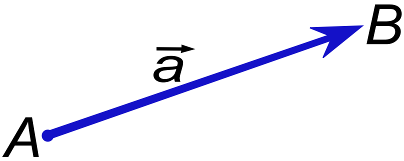

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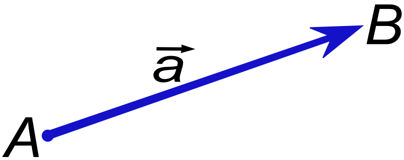

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- Basis for large number of geometry problems
- Basic operations on vectors provide very useful information

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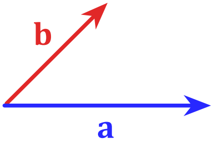
- Place beginning of one vector at end of other
- New vector from beginning of first vector to end of last



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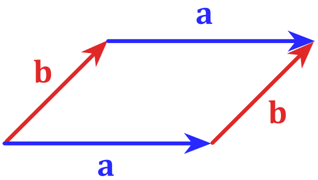

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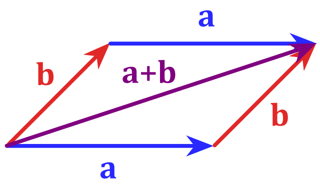
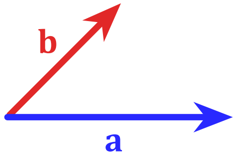


Vector Dot Product

- $\mathbf{A} \cdot \mathbf{B} = x_1 \times x_2 + y_1 \times y_2$
- Why is this important?
  - $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta$
  - Can use to find $\cos\theta$



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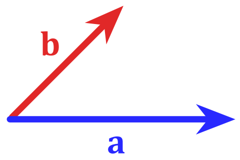

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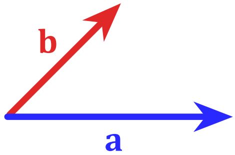

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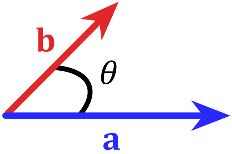

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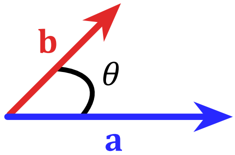

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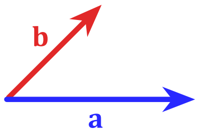

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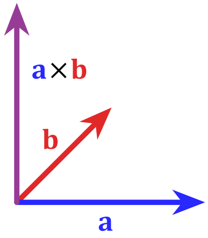

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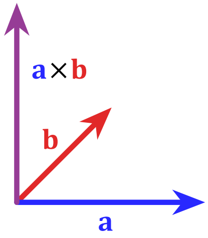

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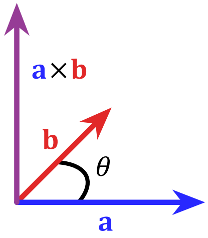

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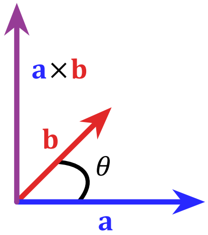

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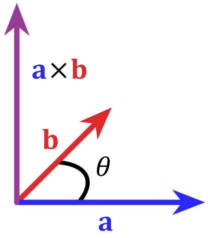

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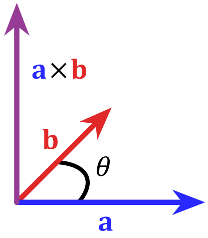

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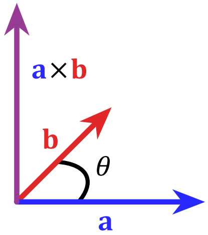

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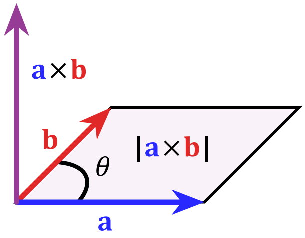

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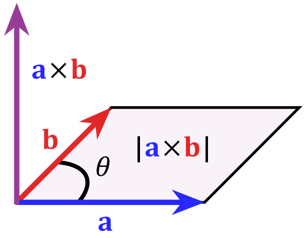

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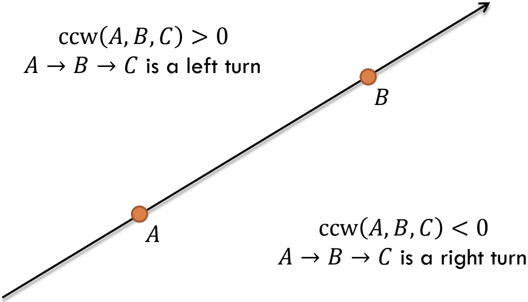
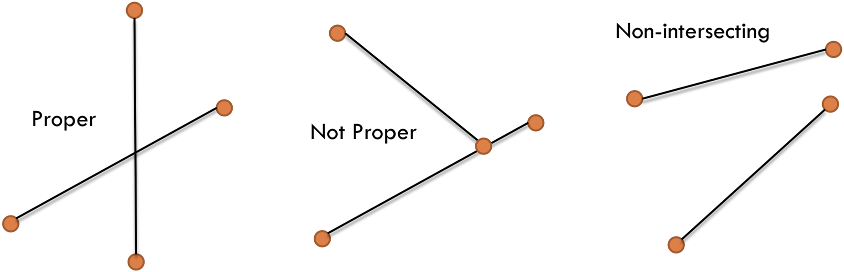
Cross Product

- Define $\text{ccw}(A, B, C) = (B - A) \times (C - A)$



Segment-Segment Intersection Test

- Given two segments AB and CD
- Want to determine if they intersect properly: two segments meet at a single point that are strictly inside both segments



Segment-Segment Intersection Test

- Assume that the segments intersect
  - From A's point of view, looking straight to B, C and D must lie on different sides
  - Holds true for the other segment as well
- The intersection exists and is proper if:
  - $\mathrm{ccw}(A, B, C) \times \mathrm{ccw}(A, B, D) < 0$
  - AND $\mathrm{ccw}(C, D, A) \times \mathrm{ccw}(C, D, B) < 0$

Segment-Segment Intersection Test

- Determining non-proper intersections
  - We need more special cases to consider!
  - e.g. If $\text{ccw}(A, B, C)$, $\text{ccw}(A, B, D)$, $\text{ccw}(C, D, A)$, $\text{ccw}(C, D, B)$ are all zeros, then two segments are collinear
  - Very careful implementation is required

Table of Contents

Basic HS Geometry Review
- Trigonometry
- Vectors

Solving Geometry Problems
- Line-line intersection
- Sweep line algorithm
- Area of a polygon
- Binary search
- Ternary search
- Convex Hull

Other Geometry Concepts

More Resources

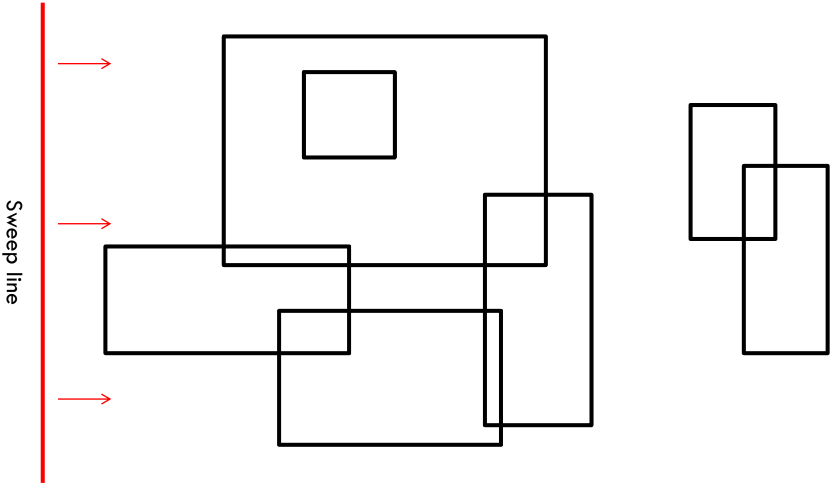
Sweep Line Algorithm

- A problem solving strategy for geometry problems
- The main idea is to maintain a line (with some auxiliary data structure) that sweeps through the entire plane and solve the problem locally
- We can't simulate a continuous process, (e.g. sweeping a line) so we define *events* that causes certain changes in our data structure
  - And process the events in the order of occurrence
- We'll cover one sweep line algorithm

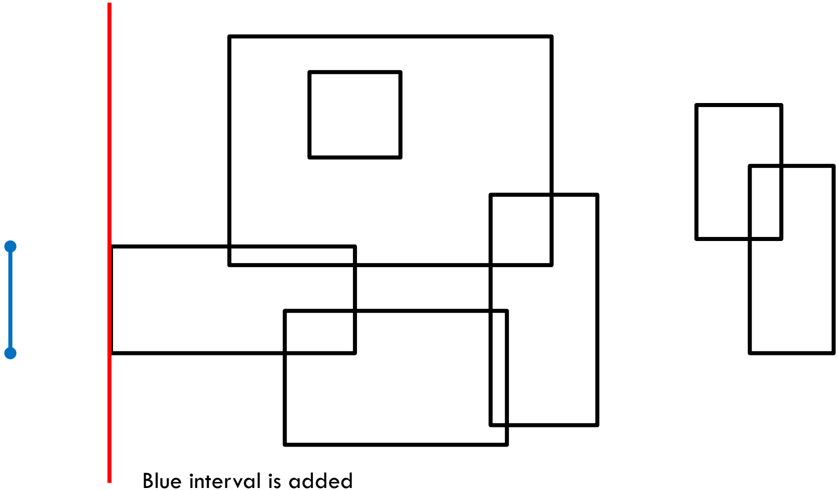
Sweep Line Algorithm

- Problem: Given n axis-aligned rectangles, find the area of the union of them
- We will sweep the plane from left to right
- Events: left and right edges of the rectangles
- The main idea is to maintain the set of "active" rectangles in order
  - It suffices to store the y-coordinates of the rectangles

Example

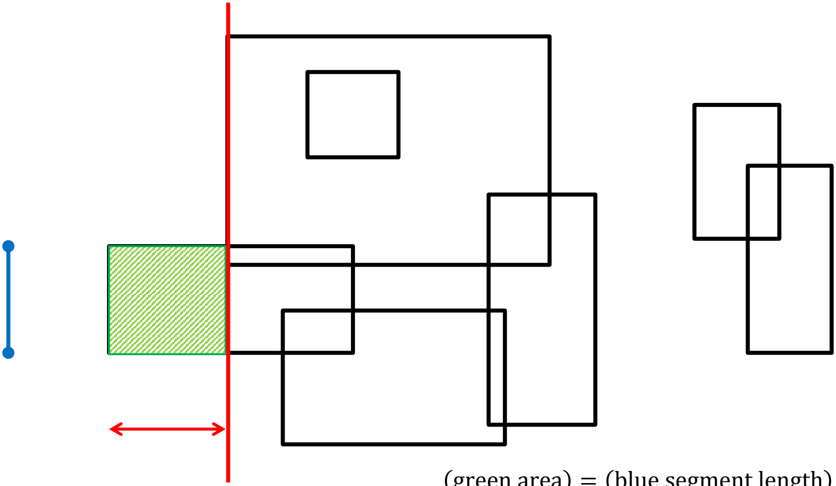


Example



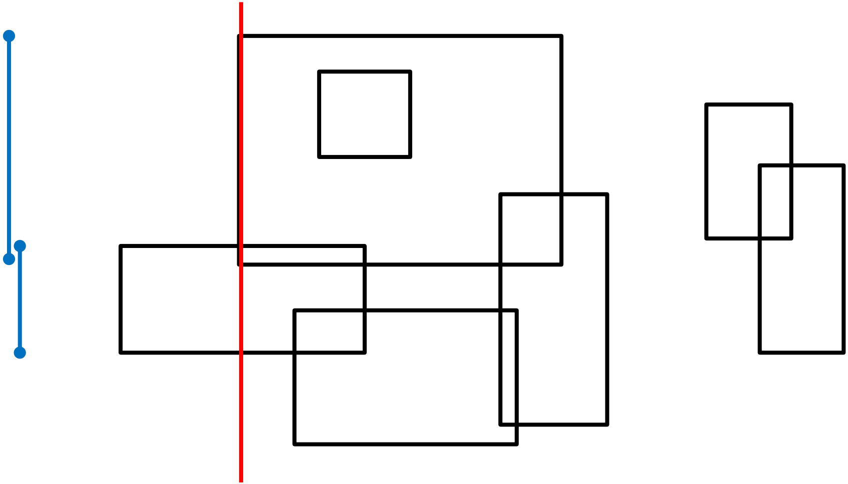
Blue interval is added
to the data structure

Example



(green area) = (blue segment length)
× (distance that the sweep line traveled)

Example



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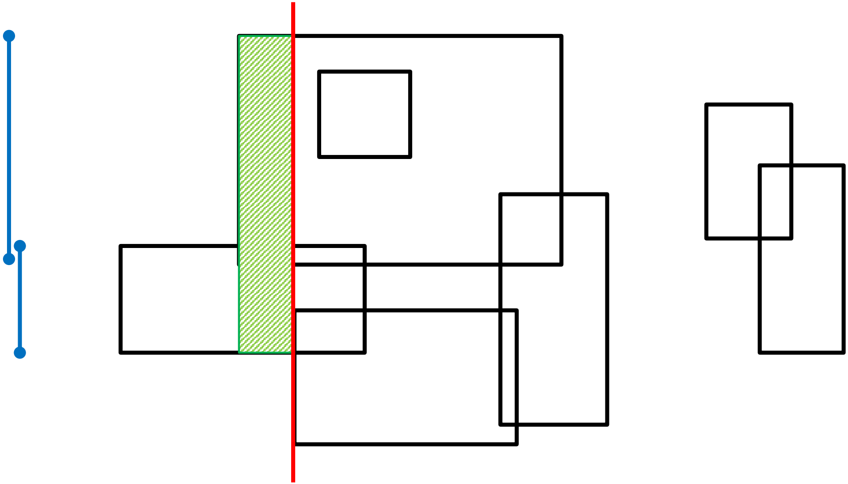

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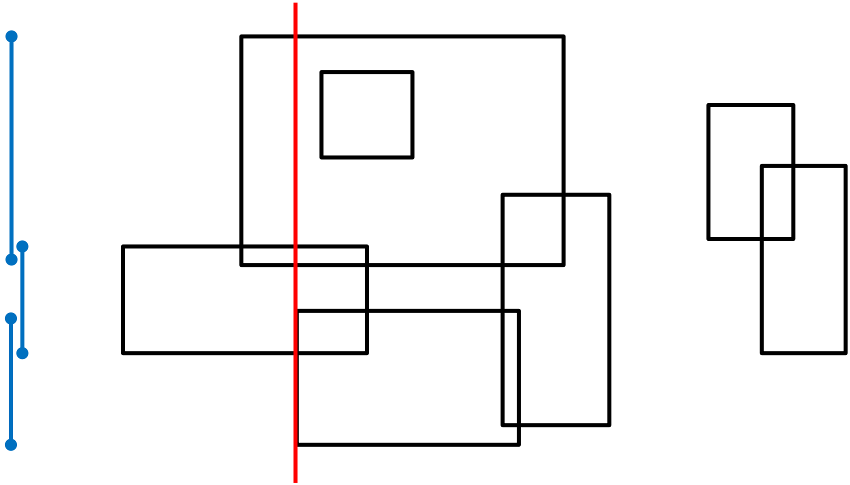
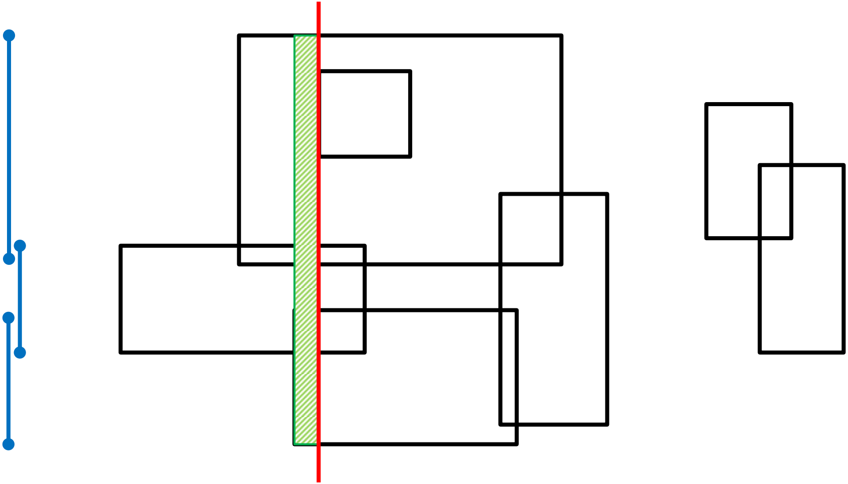


Example



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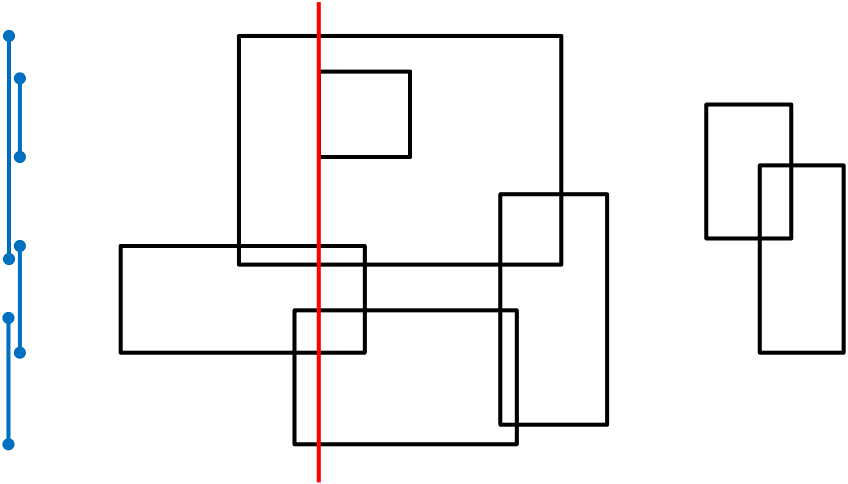

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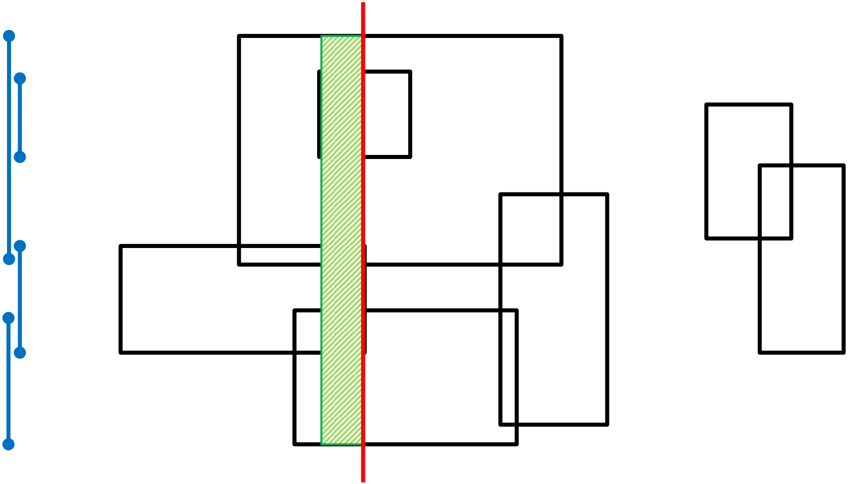
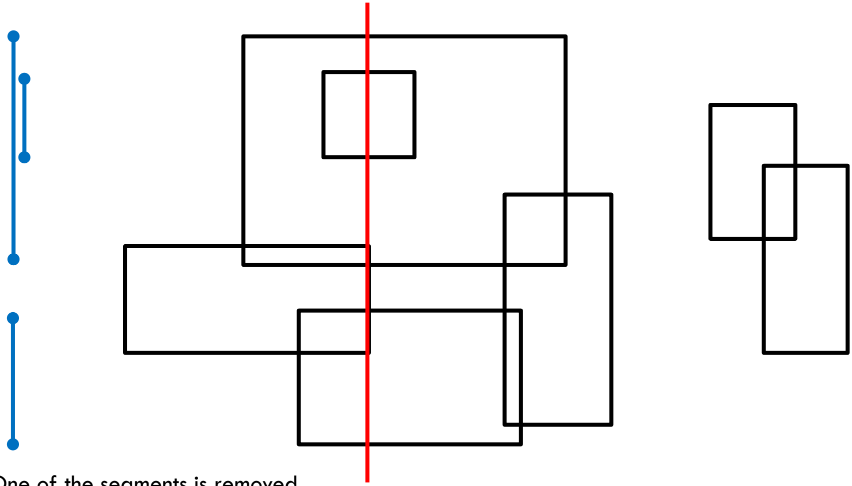
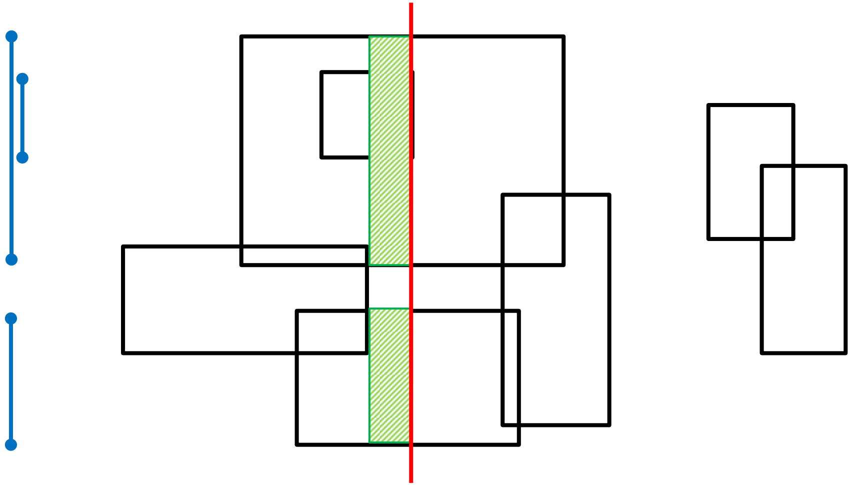


Example



One of the segments is removed

Example



Pseudopseudocode

- If the sweep line hits the left edge of a rectangle
  - Insert it to the data structure
- Right edge?
  - Remove it
- Move to the next event, and add the area(s) of the green rectangle(s)
  - Finding the length of the union of the blue segments is the hardest step
  - There is an easy $O(n)$ method for this step

Notes on Sweep Line Algorithms

- Sweep line algorithm is a generic concept
  - Come up with the right set of events and data structures for each problem
- Exercise problems
  - Finding the perimeter of the union of rectangles
  - Finding all k intersections of n line segments in $O\big((n+k)\log n\big)$ time

Table of Contents

Basic HS Geometry Review
- Trigonometry
- Vectors

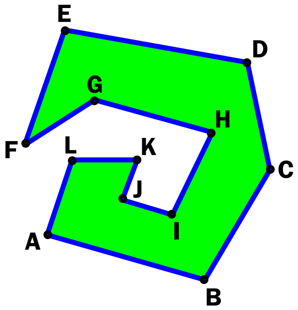
Solving Geometry Problems
- Line-line intersection
- Sweep line algorithm
- Area of a polygon
- Binary search
- Ternary search
- Convex Hull

Other Geometry Concepts

More Resources

Triangulation!

1. Start at any point (let's call it A)
2. Go through all other points, summing areas of triangles made by vectors from A (using cross product)



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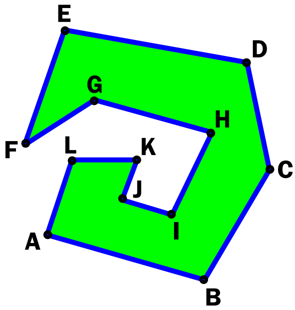

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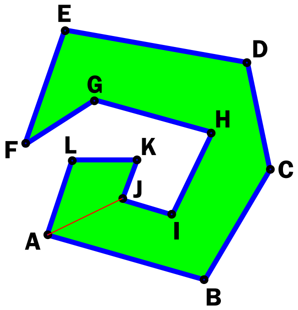

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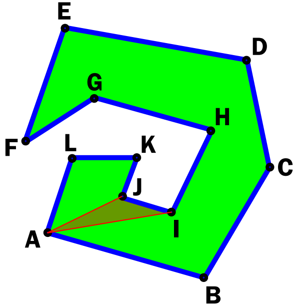

Why does this work?

- For convex polygons, obvious
- For concave polygons, when we turn inwards, area is negative
- Sum of the negative and positive triangles equals area of polygon

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Table of Contents

Basic HS Geometry Review
- Trigonometry
- Vectors

Solving Geometry Problems
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- Ternary search
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More Resources

Note on Binary Search

- Usually, binary search is used to find an item of interest in a sorted array
- There is a nice application of binary search, often used in geometry problems
  - Example: finding the largest circle that fits into a given polygon
    - Don't try to find a closed form solution or anything like that!
    - Instead, binary search on the answer

Table of Contents

Basic HS Geometry Review
- Trigonometry
- Vectors

Solving Geometry Problems
- Line-line intersection
- Sweep line algorithm
- Area of a polygon
- Binary search
- Ternary search
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Other Geometry Concepts

More Resources

Ternary Search

- Another useful method in many geometry problems
- Finds the minimum point of a "convex" function f
  - Not exactly convex, but let's use this word anyway
- Initialize the search interval $[s, e]$
- Until $e - s$ becomes small:
  - $m_1 = s + (e - s)/3, m_2 = e - (e - s)/3$
  - If $f(m_1) \leq f(m_2)$, then set e to m_2
  - Otherwise, set s to m_1

Table of Contents

Basic HS Geometry Review
- Trigonometry
- Vectors

Solving Geometry Problems
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- Area of a polygon
- Binary search
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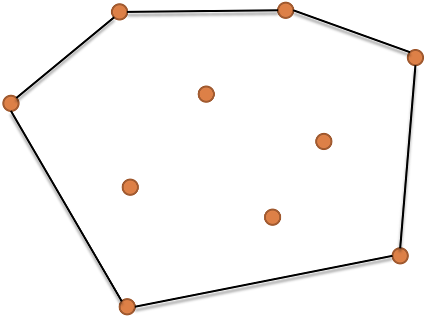
Other Geometry Concepts

More Resources

Convex Hull Problem

- Given n points on the plane, find the smallest convex polygon that contains all the given points
  - For simplicity, assume that no three points are collinear



Simple $O(n^3)$ algorithm

- AB is an edge of the convex hull iff $\text{ccw}(A, B, C)$ have the same sign for all other given points C
  - This gives us a simple algorithm
- For each A and B:
  - If $\text{ccw}(A, B, C) > 0$ for all $C \neq A, B$:
    - Record the edge $A \rightarrow B$
- Walk along the recorded edges to recover the convex hull

Faster Algorithm: Graham Scan

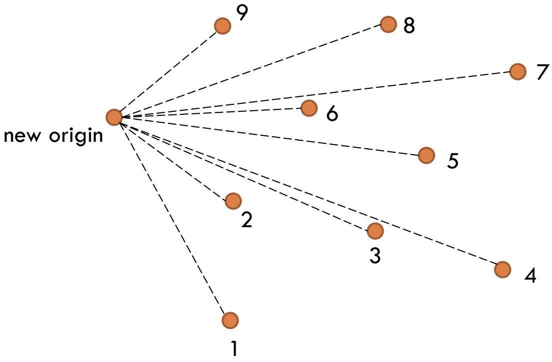
- We know that the leftmost given point has to be in the convex hull
  - We assume that there is a unique leftmost point
- Make the leftmost point the origin
  - So that all other points have positive x coordinates
- Sort the points in increasing order of y/x
  - Increasing order of angle, whatever you like to call it
- Incrementally construct the convex hull using a stack

Incremental Construction

- We maintain a *convex chain* of the given points
- For each i, we do the following:
  - Append point i to the current chain
  - If the new point causes a concave corner, remove the bad vertex from the chain that causes it
  - Repeat until the new chain becomes convex

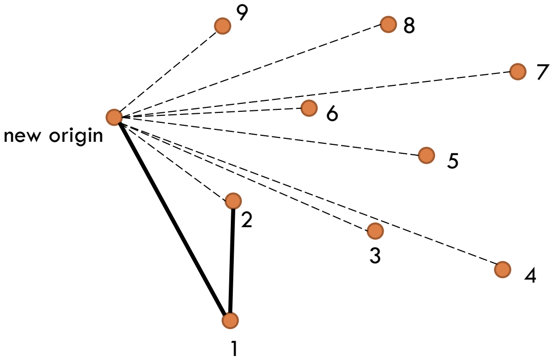
Example

- Points are numbered in increasing order of y/x



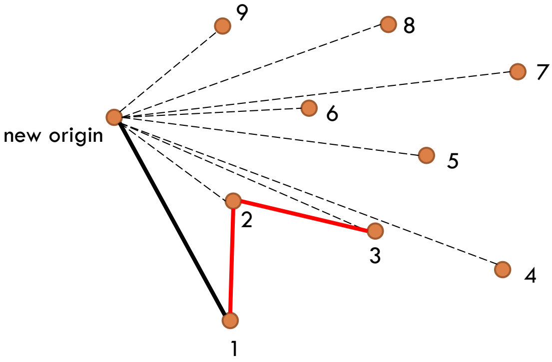
Example

- Add the first two points in the chain



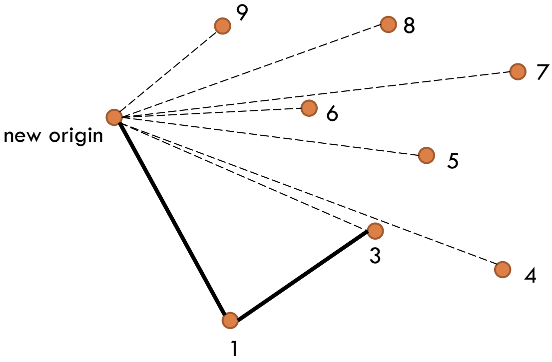
Example

- Adding point 3 causes a concave corner 1-2-3
  - Remove 2



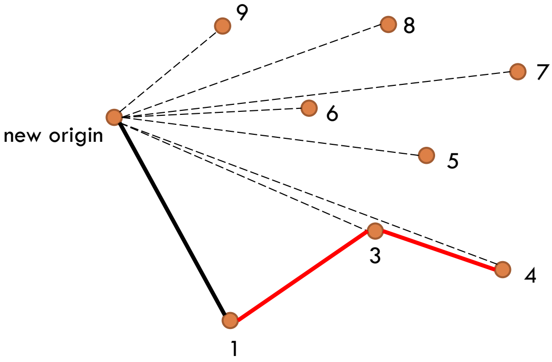
Example

- That's better…



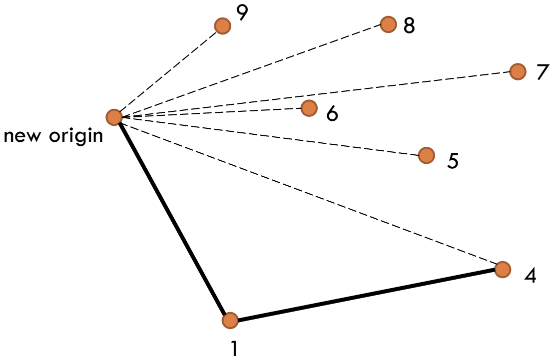
Example

- Adding 4 to the chain causes a problem
  - Remove 3



Example

- Continue adding points…



Example

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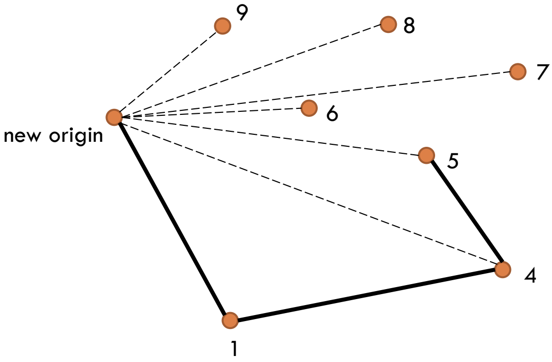
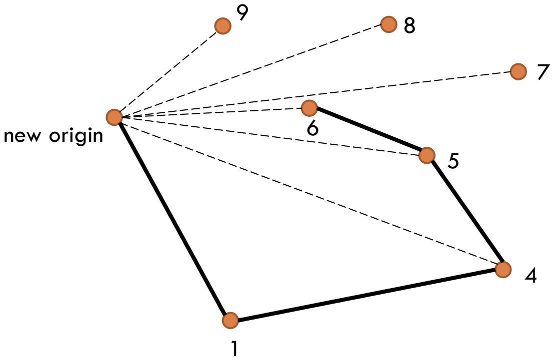


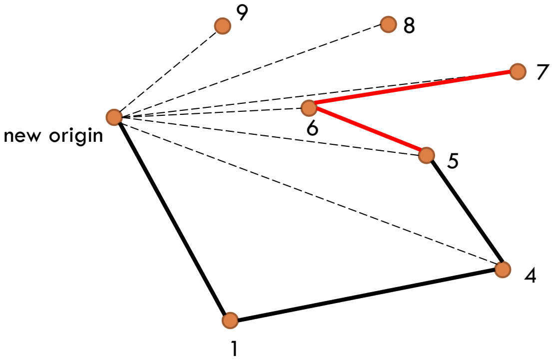
Example

- Continue adding points…



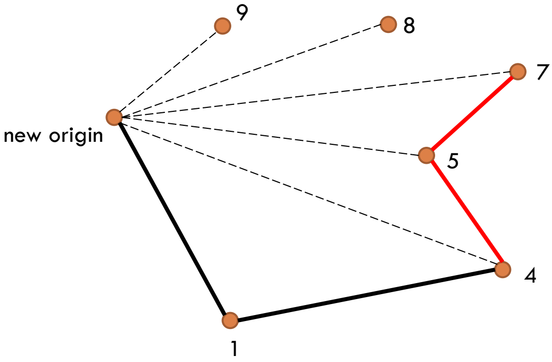
Example

- Bad corner!



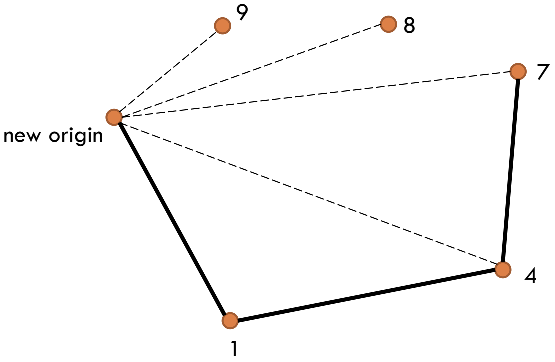
Example

- Bad corner again!



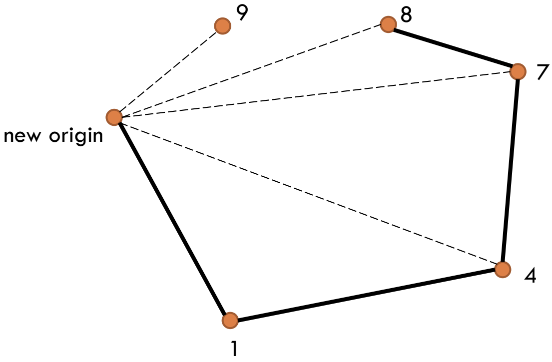
Example

- Continue adding points…



Example

- Continue adding points…



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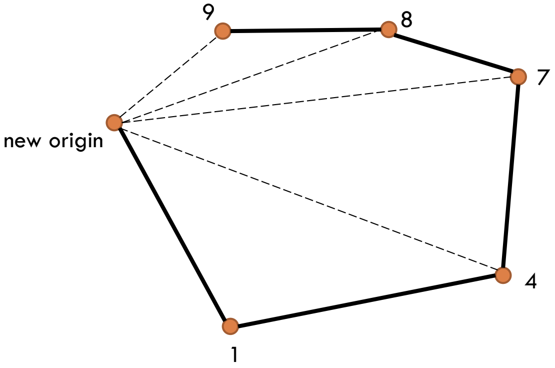
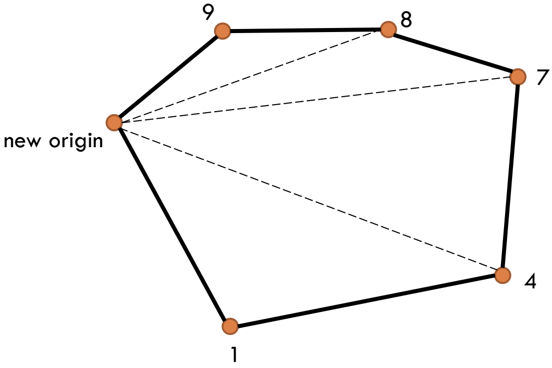


Example

- Done!



Pseudocode

- Set the leftmost point $(0,0)$, and sort the rest of the points in increasing order of y/x
- Initialize stack S
- For $i = 1 \dots n$:
  - Let A be the second topmost element of S, B be the topmost element of S, C be the ith point
  - If $\mathrm{ccw}(A, B, C) < 0$, pop S and go back
  - Push C to S
- Points in S form the convex hull

Graham Scan Complexity

- Sorting of points takes $O(n \log n)$ time
- Construction of the hull requires only traversal of all points once, with removal from the stack at most once, so $O(n)$ time
- Total runtime complexity: $O(n \log n)$

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Table of Contents

- Basic HS Geometry Review
  - Trigonometry
  - Vectors
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  - Line-line intersection
  - Sweep line algorithm
  - Area of a polygon
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There are more geometry concepts to know!

- Circle concepts – chord length, inscribed and circumscribed polygons, circles and intersecting tangent/secant lines
- Sphere concepts – great circle distance
- Polygons – checking if point is inside polygon (extension of concepts covered today)
- For formulas, algorithms, and implementations, see Chapter 7 of Competitive Programming by Steven Halim

Thankfully, most of these concepts do not appear in our regionals

- However, you should know these for World Finals!

Table of Contents

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  - Trigonometry
  - Vectors
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  - Line-line intersection
  - Sweep line algorithm
  - Area of a polygon
  - Binary search
  - Ternary search
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Some geometry resources

- TopCoder Algorithm Tutorials - Basic Geometry Concepts
- TopCoder Algorithm Tutorials - Line Intersection and its Applications
- TopCoder Algorithm Tutorials - Practice TopCoder geometry problems
- Ahmed-Aly list of geometry problems
- Stanford Introduction to ICPC Course
  - Geometry Lecture Slides
- Stanford ICPC Notebook (contains geometry algorithm implementations in C++)
- Chapter 7 of Competitive Programming by Steven Halim