Title of Thesis with all Formula, Symbols, or Greek Letters Written out in Words

by

Patrick Di Salvo

A Thesis

presented to

The University of Guelph

In partial fulfilment of requirements
for the degree of

Master of Pokemon, Poke master

in

Computer Science

Guelph, Ontario, Canada

ABSTRACT

Title of Thesis with all Formula,
Symbols, or Greek Letters Written out in Words

Patrick Di Salvo
University of Guelph, 2099

Advisor:
Dr. Sherlock Holmes

Present your abstract here. This template is just an example of an outline to consider for your thesis. It is a good idea to double-check with the current University's thesis requirements here:

`https://www.uoguelph.ca/graduatestudies/current-students/preparation-your-thesis`

Acknowledgments

Present your acknowledgements here.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Amidakuji is a custom in Japan which allows for a pseudo-random assignment of children to prizes [**?**]. Usually done in Japanese schools, a teacher will draw N vertical lines, hereby known as lines, where N is the number of students in class. At the bottom of each line will be a unique prize. And at the top of each line will be the name of one of the students. The teacher will then draw 0 or more horizontal lines, hereby known as *bars*, connecting two adjacent lines. The more bars there are the more complicated (and fun) the Amidakuji is. No two endpoints of two bars can be touching. Each student then traces their line, and whenever they encounter an end point of a bar along their line, they must cross the bar and continue going down the adjacent line. The student continues tracing down the lines and crossing bars until they get to the end of the ladder lottery. The prize at the bottom of the ladder lottery is their prize [**?**]. See Figure **??** for an example of a ladder lottery.

The exact date at which Amidakuji were invented is unknown. However, the word Amidakuji has an interesting etemology. In Japanese, Amida is the Japanese name for Amitabatha, the supreme Buddha of the Western Paradise. See image —image ref— for a picture af Amithaba. Amithaba is a Buddha from India and there is a cult based around him. The cult of Amida, otherwise known as Amidism, believes that by worshiping Amithaba, they shall enter into the his Western Paradie. Amidism began in India in the fourth century and made its way to China and Korea in the fifth centur, and finally came to Japan in ninth century. It was in Japan, where the game Amidakuji began. It is known as 'Ghost Legs' in China and Ladder Lotteries in English.

The game Amidakuji began in Japan in the Muromachi period, which spanned from 1336 to 1573. During the Muromachi period, the game was played by having players draw their names at the top of the lines, and at the bottom of the lines were pieces of paper that had the amount the players were willing to bet. The pieces of paper were folded in the shape of Amithaba's halo, which is why the game is called Amidakuji. Kuji is the Japanese word for lottery. Hence the name of the game being Amidakuji.

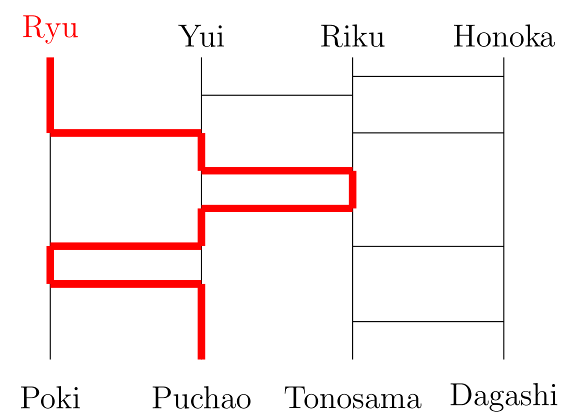

Figure 1.1: A ladder lottery where Ryu gets Puchao, Yui gets Dagashi, Riku gets Tonosama and Honoka gets Poki. You can see that Ryu's path is marked by red bars.

1.1 Thesis Statement

This thesis presents solutions to three problems surrounding ladder lotteries. The problems that this thesis presents solutions to are firstly the counting problem, secondly the minimial height problem and thirdly the problem of creating a Gray Code for generating canonical ladders within different sets of ladder lotteries.

1.2 Overview of Thesis

This thesis is broken down into several sections. Firstly, an introuduction to ladder lotteries, and how they pertain to computer science will be presented. This will be followed by a literature review of ladder lotteries in which discussions of solved problems will be provided, along with the commonalities between ladder lotteries and other mathematical objects. Following the literature review, the methodology used to conduct the research will be discussed. This section will focus primarily on the algorithms used to generate the data for this reasearch. Following the methodology section, the findings of the research will be discussed and analyzed. In this section there will be proofs and formulas for certain propositions made about ladder lotteries. This section will be the bulk of the findings for this thesis. Following the results section, a summary of future work will be provided. In this section, an of the failures and successes of this research will be analyzed. There will also be commentary on open (unsolved) problems related to ladder lotteries and a discussion of how research on ladder lotteries could be used in other fields. Finally a conlcusion that summarizes the thesis will be provided.

Chapter 2

Background

An interesting property about ladder lotteries is that they can be derived from a *permutation* which is a is a unique ordering of objects. For the purposes on this paper, the objects of a permutation will be integers ranging from $[1 \ldots N]$. *Optimal ladder lotteries* are a special case of ladder lotteries in which there is one bar in the ladder for each *inversion* in the permutation. An *inversion* is a relation between two elements in π, π_i and π_j, such that if $\pi_i > \pi_j$ and $i < j$ then π_i and π_j form an inversion. For example, given $\pi = (4, 3, 5, 1, 2)$, its iversion set is $Inv(\pi) = \{(4, 3), (4, 1), (4, 2), (3, 1), (3, 2), (5, 1), (5, 2)\}$. Every permutation has a unique, finite set of optimal ladder lotteries associated with it. Thus, the set of optimal ladder lotteries associated with π, hereby known as $OptL\{\pi\}$, is the set containing all ladder lotteries with a number of bars equal to the number if inversions in π. See figure **??** for an example of an optimal ladder in $OptL\{(4, 3, 2, 1)\}$. For each optimal ladder in $OptL\{\pi\}$, the N elements in π are listed at the top of a ladder and each element is given its own line. At the bottom of a ladder is the *sorted permutation*, hereby known as the *identity permutation* [**?**]. The identity permutation of size N is defined as follows - $I : (1, 2, 3, \ldots, N)$. Each ladder in $OptL\{\pi\}$ has the minimal number of horizontal bars to sort π into the identity permutation. Each bar in a ladder from $OptL\{\pi\}$ uninverts a single inversion in π exactly once. For the remainder of this paper, only optimal ladder lotteries will be discussed. Therefore when the term ladder lottery is used, assume optimal ladder lottery.

Figure 2.1: Two ladders for the permutation (4, 3, 2, 1). The left ladder is an optimal ladder and the right ladder is not. Therefore the left ladder belongs to $optL\{(4,3,2,1)\}$. The bold bars in the right ladder are redundant, thus the right ladder is not optimal

2.1 Literature Review

2.1.1 Literature Overview

The study of ladder lottieres as mathematical objects began in 2010, in the paper **Efficient Enumeration of Ladder Lotteries and its Application**. The paper was written by four authors, Yamanaka, Horiyama, Uno and Wasa. In this paper the authors present an algorithm for generating all the ladder lotteries of an arbitrary permutation, π. Since this paper emerged, there have been several other paper written directly about ladder lotteries. These papers include **The Ladder Lottery Realization Problem**, **Optimal Reconfiguration of Optimal Ladder Lotteries**, **Efficient Enumeration of all Ladder Lotteries with K Bars**, **Coding Ladder Lotteries** and **Enumeration, Counting, and Random Generation of Ladder Lotteries**.

2.1.2 Efficient Enumeration of Laddder Lotteries and its Application

In their paper, Efficient Enumeration of Ladder Lotteries and its Application, the authors provide an algorithm for generating $OptL\{\pi\}$ for any π, in $\mathcal{O}(1)$ per ladder.

This is the first and only published algroithm for generating $OptL\{\pi\}$. The paper also presents the number of ladder lotteries in $OptL\{(11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1)\}$ which is $5, 449, 192, 389, 984$. This is a very impressive accomplishment for reasons which will be discussed later in the literature review.

The authors' algorithm is based on several key concepts, the most important of which is the *local swap operation*. This is the minimal change operation that transitions from one ladder in $OptL\{\pi\}$ to the next ladder. The local swap operation is essentially a 180 degree rotation of three bars in the ladder, all at different levels in the ladder, such that the bottom bar is rotated to the top, the middle bar stays in the middle and the top bar is rotated to the bottom. If the bars undergo a 180 degree rotation to the right, then this is known as a *right swap operation* and if the bars udergo a 180 degree rotation to the left then this is known as a *left swap operation*. To go to the next ladder in the set, the current ladder, L_i udergoes a right swap operation to get toe ladder L_{i+1}. See figure –ref– for an exmaple of a local swap operation. The *route* of an element is the sequence of bars in the ladder, from top left to bottom right, that an element must cross in order to reach its correct position in the identity permutation. Note that each bar has two elements that cross it, therefore the bar belongs to the route of the greater of the two elements. The *clean level* refers to the smallest element in π such that none of its bars have a bar of a lesser element above its route. If there is no such element, then the clean level is the maximum element in $\pi + 1$.

Despite the fact that ladder lotteries have only been stuidied in and of themseleves for ten years, they are closely tied to other mathematical phenomena that have been studied for much longer. These mathematical phenomena include *Pseudo Lines* which are an arrangement of curves on a plane such that given two curves, they only intersect at most once and at each intersection, only two curves intersect.See figure –reference– for a wiring diagrams of the pseudo line arrangement for the permutation, $(5, 4, 3, 2, 1)$. The other mathematical phenomena is *adjacent transpositions* which is

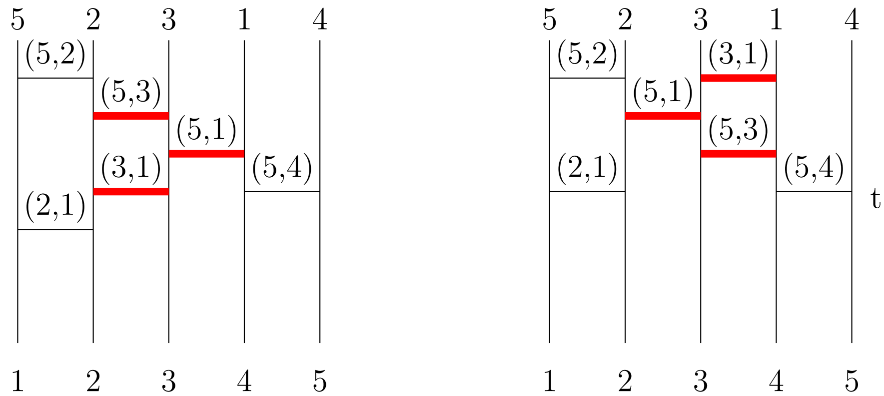

Figure 2.2: Example of a local swap operation. When a right swap operation is permformed on the left ladder, the result is the right ladder. When a left swap operation is permformed on the right ladder, the result is the left ladder.

a swap of two adjacent elements in a permutation.

2.1.3 Ladders and Adjacent Transpositions

A ladder lottery is a way of sorting a permutation, yet it can also be thought of as a decomposition of a permutation into *adjacent transpositions*. [**?**] An *adjacent transposition* is simply a swap of two adjacent elements in a permutation. For example, given the permutation (1, 3, 4, 2), an adjacent transposition could be done on the following pairs of elements: (1, 3), (3, 4) or (4, 2). Each would result in a unique permutation. Simply put, given any arbitrary starting permutation, π, keep swapping adjacent inversions until the identity permutation is reached. An optimal ladder lottery from $\pi's$ optimal ladder set is a minimal sequence of adjacent transpositions such that π is sorted into the identity permutation; each ladder in the set represents a sequence of adjacent transpositions for sorting π into the identity permutation. For example, given the permutation (4, 3, 2, 1) there exists eight ladders in this permutation's optimal ladder set. Two of these ladders are found in **??**:

From looking at the above ladders, going from top left to bottom right, the left ladder represents the sequence of adjacent transpositions (4,3), (4,2), (4,1),(3,2),(3,1),(2,1)

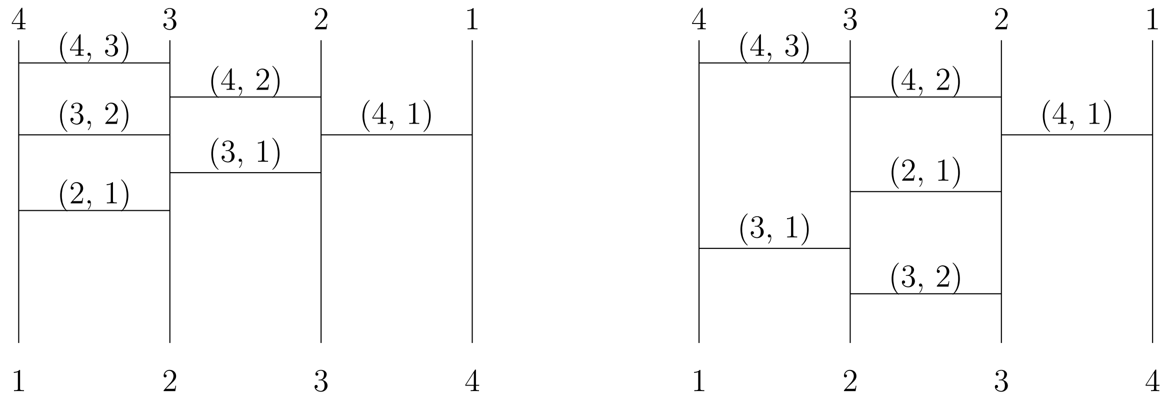

Figure 2.3: The left ladder is one of eight unique ladders from (4,3,2,1)'s optimal ladder set. The right ladder is another one of eight unique ladders form (4,3,2,1)'s optimal ladder set

whereas the right ladder represent the sequence of adjacent transpositions (4, 3),(4, 2),(4, 1),(2, 1),(3, 1),(3, 2). Notice how the length of the sequences are the same,because both lengths are equal to the minimal number of swaps to sort (4, 3, 2, 1) it is simply the order in which the adjacent transpositions occur in the sequence that makes the sequences different from each other.

Chapter 3

Methodology and Implementation

Chapter 4

Evaluation

Chapter 5

Summary and Future Work

Conclude your thesis with a re-cap of your major results and contributions. Then outline directions for further research and remaining open problems.