

Team notebook

NCTU Daisengen

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Contents

1 Algorithms	2
1.1 Mo's algorithm on trees	2
1.2 Mo's algorithm	3
1.3 sliding window	4
2 Basics	4
2.1 default code	4
3 Data structures	5
3.1 binary index tree	5
3.2 dsu	5
3.3 hash table	5
3.4 heavy light decomposition	5
3.5 persistent array	6
3.6 persistent seg tree	7
3.7 persistent trie	8
3.8 segment tree	9
3.9 sparse table	11
3.10 splay tree	11
3.11 STL order statistics tree II	12

3.12 STL order statistics tree	13
3.13 STL Treap	14
3.14 trie	14
3.15 wavelet tree	15
4 DP Optimizations	15
4.1 convex hull trick	15
4.2 divide and conquer	17
4.3 dp on trees	17
5 Geometry	18
5.1 center 2 points + radius	18
5.2 closest pair problem	18
5.3 squares	19
5.4 triangles	20
6 Graphs	21
6.1 board	21
6.2 bridges	21
6.3 dijkstra	22
6.4 directed mst	22
6.5 eulerian path	23
6.6 karp min mean cycle	24
6.7 konig's theorem	24
6.8 minimum path cover in DAG	24
6.9 planar graph (euler)	25
6.10 query with lca	25
6.11 SCC kosaraju	26
6.12 tarjan scc	27
6.13 two sat (with kosaraju)	27

7 Math	29
7.1 counting	29
7.2 cumulative sum of divisors	29
7.3 fft	30
7.4 fibonacci properties	31
7.5 Lucas theorem	31
7.6 polynomials	32
7.7 sigma function	32
8 Matrix	32
8.1 matrix	32
9 Misc	33
9.1 dates	33
9.2 fraction	33
9.3 io	34
9.4 Template Java	34
10 Number theory	35
10.1 convolution	35
10.2 crt	36
10.3 diophantine equations	36
10.4 discrete logarithm	37
10.5 ext euclidean	37
10.6 highest exponent factorial	37
10.7 miller rabin	37
10.8 mod integer	38
10.9 mod inv	38
10.10mod mul	38
10.11mod pow	39
10.12number theoretic transform	39
10.13pollard rho factorize	39
10.14primes	40
10.15totient sieve	41
10.16totient	41
11 Strings	41
11.1 Incremental Aho Corasick	41
11.2 minimal string rotation	43
11.3 suffix array	43
11.4 suffix automaton	44

11.5 z algorithm	45
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1 Algorithms

1.1 Mo's algorithm on trees

```

/**
problems:
- https://codeforces.com/gym/101161 problem E
*/
void flat(vector<vector<edge>> &g, vector<int> &a,
          vector<int> &le, vector<int> &ri, vector<int> &cost,
          int node, int pi, int &ts, int w) {

    cost[node] = w;
    le[node] = ts;
    a[ts] = node;
    ts++;
    for (auto e : g[node]) {
        if (e.to == pi) continue;
        flat(g, a, le, ri, cost, e.to, node, ts, e.w);
    }
    ri[node] = ts;
    a[ts] = node;
    ts++;
}

/**
 * Case when the cost is in the edges.
 * */
void compute_queries(vector<vector<edge>> &g) {
    // g is undirected
    int n = g.size();

    lca_tree.init(g, 0);

    vector<int> a(2 * n), le(n), ri(n), cost(n);
    // a: nodes in the flatten array
    // le: left id of the given node
    // ri: right id of the given node
    // cost: cost of the edge from the node to the parent

```

```

int ts = 0; // timestamp
flat(g, a, le, ri, cost, 0, -1, ts, 0);

int q; cin >> q;
vector<query> queries(q);
for (int i = 0; i < q; i++) {
    int u, v;
    cin >> u >> v;
    u--; v--;
    int lca = lca_tree.query(u, v);
    if (le[u] > le[v])
        swap(u, v);
    queries[i].id = i;
    queries[i].lca = lca;
    queries[i].u = u;
    queries[i].v = v;
    if (lca == u) {
        queries[i].a = le[u] + 1;
        queries[i].b = le[v];
    } else {
        queries[i].a = ri[u];
        queries[i].b = le[v];
    }
}
solve_mo(queries, a, le, cost); // this is the usual algorithm
}

```

1.2 Mo's algorithm

```

const int MN = 5 * 100000 + 1;
const int SN = 708;

struct Query {
    int a, b, id;
    Query() {}
    Query(int x, int y, int i) : a(x), b(y), id(i) {}

    bool operator<(const Query &o) const {
        if (a / SN != o.a / SN) return a < o.a;
        return a / SN & 1 ? b < o.b : b > o.b;
    }
};

```

```

struct DS {
    DS() : {}

    void Insert(int x) {}

    void Erase(int x) {}

    long long Query() {}
};

Query s[MN];
int ans[MN];
DS active;

int main() {
    int n;
    cin >> n;
    vector<int> a(n);
    for (auto &i : a) cin >> i;

    int q;
    cin >> q;
    for (int i = 0; i < q; ++i) {
        int b, e;
        cin >> b >> e;
        b--;
        e--;
        s[i] = Query(b, e, i);
    }

    sort(s, s + q);

    int i = 0;
    int j = -1;
    for (int k = 0; k < (int)q; ++k) {
        int L = s[k].a;
        int R = s[k].b;

        while (j < R) active.Insert(a[++j]);
        while (j > R) active.Erase(a[j--]);
        while (i < L) active.Erase(a[i++]);
        while (i > L) active.Insert(a[--i]);
    }
}

```

```

    ans[s[k].id] = active.Query();
}

for (int i = 0; i < q; ++i) {
    cout << ans[i] << endl;
}

return 0;
};

```

1.3 sliding window

```

/*
 * Given an array ARR and an integer K, the problem boils down to computing
 * for each index i: min(ARR[i], ARR[i-1], ..., ARR[i-K+1]).
 * if mx == true, returns the maximum.
 * http://people.cs.uct.ac.za/~ksmith/articles/sliding_window_minimum.html
 * */

vector<int> sliding_window_minmax(vector<int> & ARR, int K, bool mx) {
    deque< pair<int, int> > window;
    vector<int> ans;
    for (int i = 0; i < ARR.size(); i++) {
        if (mx) {
            while (!window.empty() && window.back().first <= ARR[i])
                window.pop_back();
        } else {
            while (!window.empty() && window.back().first >= ARR[i])
                window.pop_back();
        }
        window.push_back(make_pair(ARR[i], i));

        while(window.front().second <= i - K)
            window.pop_front();

        ans.push_back(window.front().first);
    }
    return ans;
}

```

2 Basics

2.1 default code

```

#include<bits/stdc++.h>
using namespace std;

#define endl '\n'
#define pb emplace_back
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(x) x.begin(), x.end()
#define CLR(x, y) memset(x, y, sizeof(x))
#define IOS ios::sync_with_stdio(false); cin.tie(nullptr)
#define rep(i, begin, end) for (__typeof(end) i = (begin) - ((begin) > (end)); i != (end) - ((begin) > (end)); i += (begin > end ? -1 : 1))
#define debug(args...) { string _s = #args; replace(_s.begin(), _s.end(), ',', ' '); stringstream _ss(_s); istream_iterator<string> _it(_ss); err(_it, args); }
void err(istream_iterator<string> it) {}
template<typename T, typename... Args>
void err(istream_iterator<string> it, T a, Args... args) {
    cerr << *it << " = " << a << endl;
    err(++it, args...);
}

using ll = long long;
using vi = vector<int>;
using vii = vector<vi>;
using pii = pair<int, int>;
using pll = pair<ll, ll>;

const int MOD = 1000000007;
const int INF = INT_MAX;

signed main () {
    //    IOS;
    return 0;
}

```

3 Data structures

3.1 binary index tree

```

struct binary_index_tree {
    int n;
    int t[2 * N];

    void add(int where, long long what){
        for (where++; where <= n; where += where & -where){
            t[where] += what;
        }
    }

    void add(int from, int to, long long what) {
        add(from, what);
        add(to + 1, -what);
    }

    long long query(int where){
        long long sum = t[0];
        for (where++; where > 0; where -= where & -where){
            sum += t[where];
        }
        return sum;
    }
};

```

3.2 dsu

```

struct Dsu {
    vector<int> p;

    Dsu(int n) {
        p.resize(n);
        for (int i = 0; i < n; i++) {
            p[i] = i;
        }
    }

    int Find(int x) { return x == p[x] ? x : p[x] = Find(p[x]); }
}

```

```

int Join(int x, int y) {
    int px = Find(x), py = Find(y);
    if (px == py) return 0;
    p[px] = py;
    return 1;
}
};

```

3.3 hash table

```

/**
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 * */

const int MN = 1001;
struct ht {
    int _s[(MN + 10) >> 5];
    int len;
    void set(int id) {
        len++;
        _s[id >> 5] |= (1LL << (id & 31));
    }
    bool is_set(int id) {
        return _s[id >> 5] & (1LL << (id & 31));
    }
};

```

3.4 heavy light decomposition

```

// Heavy-Light Decomposition
struct TreeDecomposition {
    vector<int> g[MAXN], c[MAXN];
    int s[MAXN]; // subtree size
    int p[MAXN]; // parent id
    int r[MAXN]; // chain root id
    int t[MAXN]; // index used in segtree/bit/...
    int d[MAXN]; // depht
    int ts;
}

```

```

void dfs(int v, int f) {
    p[v] = f;
    s[v] = 1;
    if (f != -1) d[v] = d[f] + 1;
    else d[v] = 0;

    for (int i = 0; i < g[v].size(); ++i) {
        int w = g[v][i];
        if (w != f) {
            dfs(w, v);
            s[v] += s[w];
        }
    }
}

void hld(int v, int f, int k) {
    t[v] = ts++;
    c[k].push_back(v);
    r[v] = k;

    int x = 0, y = -1;
    for (int i = 0; i < g[v].size(); ++i) {
        int w = g[v][i];
        if (w != f) {
            if (s[w] > x) {
                x = s[w];
                y = w;
            }
        }
    }
    if (y != -1) {
        hld(y, v, k);
    }

    for (int i = 0; i < g[v].size(); ++i) {
        int w = g[v][i];
        if (w != f && w != y) {
            hld(w, v, w);
        }
    }
}

void init(int n) {

```

```

    for (int i = 0; i < n; ++i) {
        g[i].clear();
    }
}

void add(int a, int b) {
    g[a].push_back(b);
    g[b].push_back(a);
}

void build() {
    ts = 0;
    dfs(0, -1);
    hld(0, 0, 0);
}
};

```

3.5 persistent array

```

struct node {
    node *l, *r;
    int val;

    node (int x) : l(NULL), r(NULL), val(x) {}
    node () : l(NULL), r(NULL), val(-1) {}
};

typedef node* pnode;

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l == r) {
        ans->val = what;
        return ans;
    }
    int m = (l + r) >> 1;
    if (at <= m) ans->l = update(ans->l, l, m, at, what);
    else ans->r = update(ans->r, m + 1, r, at, what);
}

```

```

    return ans;
}

int get(pnode cur, int l, int r, int at) {
    if (cur == NULL) return 0;
    if (l == r) return cur->val;
    int m = (l + r) >> 1;
    if (at <= m) return get(cur->l, l, m, at);
    else return get(cur->r, m + 1, r, at);
}

```

3.6 persistent seg tree

```

/**
 * Problems:
 * http://codeforces.com/contest/813/problem/E
 *
 * Important:
 * When using lazy propagation remember to create new
 * versions for each push_down operation!!!
 */

struct node {
    node *l, *r;
    long long acc;
    int flip;

    node (int x) : l(NULL), r(NULL), acc(x), flip(0) {}
    node () : l(NULL), r(NULL), acc(0), flip(0) {}
};

typedef node* pnode;

pnode create(int l, int r) {
    if (l == r) return new node();
    pnode cur = new node();
    int m = (l + r) >> 1;
    cur->l = create(l, m);
    cur->r = create(m + 1, r);
    return cur;
}

```

```

pnode copy_node(pnode cur) {
    pnode ans = new node();
    *ans = *cur;
    return ans;
}

void push_down(pnode cur, int l, int r) {
    assert(cur);
    if (cur->flip) {
        int len = r - l + 1;
        cur->acc = len - cur->acc;
        if (cur->l) {
            cur->l = copy_node(cur->l);
            cur->l->flip ^= 1;
        }
        if (cur->r) {
            cur->r = copy_node(cur->r);
            cur->r->flip ^= 1;
        }
        cur->flip = 0;
    }
}

int get_val(pnode cur) {
    assert(cur);
    assert((cur->flip) == 0);
    if (cur) return cur->acc;
    return 0;
}

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = copy_node(cur);
    if (l == r) {
        assert(l == at);
        ans->acc = what;
        ans->flip = 0;
        return ans;
    }
    int m = (l + r) >> 1;
    push_down(ans, l, r);
    if (at <= m) ans->l = update(ans->l, l, m, at, what);
    else ans->r = update(ans->r, m + 1, r, at, what);

    push_down(ans->l, l, m);
}

```

```

push_down(ans-> r, m + 1, r);
ans-> acc = get_val(ans-> l) + get_val(ans-> r);
return ans;
}

pnode flip(pnode cur, int l, int r, int a, int b) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l > b || r < a)
        return ans;

    if (l >= a && r <= b) {
        ans-> flip ^= 1;
        push_down(ans, l, r);
        return ans;
    }

    int m = (l + r) >> 1;
    ans-> l = flip(ans-> l, l, m, a, b);
    ans-> r = flip(ans-> r, m + 1, r, a, b);
    push_down(ans-> l, l, m);
    push_down(ans-> r, m + 1, r);
    ans-> acc = get_val(ans-> l) + get_val(ans-> r);
    return ans;
}

long long get_all(pnode cur, int l, int r) {
    assert(cur);
    push_down(cur, l, r);
    return cur-> acc;
}

void traverse(pnode cur, int l, int r) {
    if (!cur) return;
    cout << l << " - " << r << " : " << (cur-> acc) << " " << (cur-> flip) <<
        endl;
    traverse(cur-> l, l, (l + r) >> 1);
    traverse(cur-> r, l + ((l + r) >> 1), r);
}

```

3.7 persistent trie

```

// both tries can be tested with the problem:
// http://codeforces.com/problemset/problem/916/D

// Persistent binary trie (BST for integers)
const int MD = 31;

struct node_bin {
    node_bin *child[2];
    int val;

    node_bin() : val(0) {
        child[0] = child[1] = NULL;
    }
};

typedef node_bin* pnode_bin;

pnode_bin copy_node(pnode_bin cur) {
    pnode_bin ans = new node_bin();
    if (cur) *ans = *cur;
    return ans;
}

pnode_bin modify(pnode_bin cur, int key, int inc, int id = MD) {
    pnode_bin ans = copy_node(cur);
    ans->val += inc;
    if (id >= 0) {
        int to = (key >> id) & 1;
        ans->child[to] = modify(ans->child[to], key, inc, id - 1);
    }
    return ans;
}

int sum_smaller(pnode_bin cur, int key, int id = MD) {
    if (cur == NULL) return 0;
    if (id < 0) return 0; // strictly smaller
    // if (id == - 1) return cur->val; // smaller or equal

    int ans = 0;

```



```

int to = (key >> id) & 1;
if (to) {
    if (cur->child[0]) ans += cur->child[0]->val;
    ans += sum_smaller(cur->child[1], key, id - 1);
} else {
    ans = sum_smaller(cur->child[0], key, id - 1);
}
return ans;
}

// Persistent trie for strings.
const int MAX_CHILD = 26;
struct node {
    node *child[MAX_CHILD];
    int val;
    node() : val(-1) {
        for (int i = 0; i < MAX_CHILD; i++) {
            child[i] = NULL;
        }
    }
};

typedef node* pnode;

pnode copy_node(pnode cur) {
    pnode ans = new node();
    if (cur) *ans = *cur;
    return ans;
}

pnode set_val(pnode cur, string &key, int val, int id = 0) {
    pnode ans = copy_node(cur);
    if (id >= int(key.size())) {
        ans->val = val;
    } else {
        int t = key[id] - 'a';
        ans->child[t] = set_val(ans->child[t], key, val, id + 1);
    }
    return ans;
}

pnode get(pnode cur, string &key, int id = 0) {
    if (id >= int(key.size()) || !cur)

```

```

return cur;
int t = key[id] - 'a';
return get(cur->child[t], key, id + 1);
}

```

3.8 segment tree

```

/**
 * Taken from: http://codeforces.com/blog/entry/18051
 */

const int MN = 1e5; // limit for array size

struct seg_tree {
    int n; // array size
    int t[2 * MN];

    seg_tree(int _n) : n(_n) {}

    void clear() {
        memset(t, 0, sizeof t);
    }

    void build() { // build the tree
        for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
    }

    // Single modification, range query.
    void modify(int p, int value) { // set value at position p
        for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
    }

    int query(int l, int r) { // sum on interval [l, r)
        int res = 0;
        for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
            if (l&1) res += t[l++];
            if (r&1) res += t[--r];
        }
        return res;
    }
};

```

```
// Range modification, single query.

void modify(int l, int r, int value) {
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) t[l++] += value;
        if (r&1) t[--r] += value;
    }
}

int query(int p) {
    int res = 0;
    for (p += n; p > 0; p >>= 1) res += t[p];
    return res;
}

/**
 * If at some point after modifications we need to inspect all the
 * elements in the array, we can push all the modifications to the
 * leaves using the following code. After that we can just traverse
 * elements starting with index n. This way we reduce the complexity
 * from O(n log(n)) to O(n) similarly to using build instead of n
 * modifications.
 */

void push() {
    for (int i = 1; i < n; ++i) {
        t[i<<1] += t[i];
        t[i<<1|1] += t[i];
        t[i] = 0;
    }
}

// Non commutative combiner functions.

void modify(int p, const S& value) {
    for (t[p += n] = value; p >>= 1; ) t[p] = combine(t[p<<1], t[p<<1|1]);
}

S query(int l, int r) {
    S resl, resr;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) resl = combine(resl, t[l++]);
        if (r&1) resr = combine(t[--r], resr);
    }
}
```

```
return combine(resl, resr);
}

/**
 * segment tree for intervals
 */

const int MN = 100000 + 100;
struct seg_tree {
    int val[MN * 4 + 4];
    int pending[MN * 4 + 4];

    seg_tree() {
        memset(val, -1, sizeof val);
        memset(pending, -1, sizeof pending);
    }

    void propagate(int node, int b, int e) {
        if (pending[node] != -1) {
            val[node] = pending[node];
            if (b < e) {
                pending[node << 1] = pending[node];
                pending[node << 1 | 1] = pending[node];
            }
            pending[node] = -1;
        }
    }

    void set(int node, int b, int e, int from, int to, int v) {
        if (b > to || e < from) return;

        if (b >= from && e <= to) {
            pending[node] = v;
            propagate(node, b, e);
            return;
        }

        int mid = (b + e) >> 1;

        set(node << 1, b, mid, from, to, v);
        set(node << 1 | 1, mid + 1, e, from, to, v);
    }
}
```

```

int query(int node, int b, int e, int pos) {
    propagate(node, b, e);

    if (b == e && b == pos) {
        return val[node];
    }

    int mid = (b + e) >> 1;
    if (pos <= mid)
        return query(node << 1, b, mid, pos);
    return query(node << 1 | 1, mid + 1, e, pos);
}

void set(int from, int to, int v) {
    return set(1, 0, MN - 1, from, to, v);
}

int query(int pos) {
    return query(1, 0, MN - 1, pos);
}
};

```

3.9 sparse table

```

// RMQ.
const int MN = 100000 + 10; // Max number of elements
const int ML = 18; // ceil(log2(MN));

struct st {
    int data[MN];
    int M[MN][ML];
    int n;

    void init(const vector<int> &d) {
        n = d.size();
        for (int i = 0; i < n; ++i)
            data[i] = d[i];

        build();
    }
};

```

```

}

void build() {
    for (int i = 0; i < n; ++i)
        M[i][0] = data[i];
    for (int j = 1, p = 2, q = 1; p <= n; ++j, p <= 1, q <= 1)
        for (int i = 0; i + p - 1 < n; ++i)
            M[i][j] = max(M[i][j - 1], M[i + q][j - 1]);
}

int query(int b, int e) {
    int k = log2(e - b + 1);
    return max(M[b][k], M[e + 1 - (1<<k)][k]);
}
};

```

3.10 splay tree

```

using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<x<<endl;

typedef int T;

struct node{
    node *left, *right, *parent;
    T key;
    node (T k) : key(k), left(0), right(0), parent(0) {}
};

struct splay_tree{

    node *root;

    void right_rot(node *x) {
        node *p = x->parent;
        if (x->parent == p->parent) {
            if (x->parent->left == p) x->parent->left = x;
            if (x->parent->right == p) x->parent->right = x;
        }
        if (p->left == x->right) p->left->parent = p;
        x->right = p;
    }
};

```

```

    p->parent = x;
}

void left_rot(node *x) {
    node *p = x->parent;
    if (x->parent = p->parent) {
        if (x->parent->left == p) x->parent->left = x;
        if (x->parent->right == p) x->parent->right = x;
    }
    if (p->right = x->left) p->right->parent = p;
    x->left = p;
    p->parent = x;
}

void splay(node *x, node *fa = 0) {

    while( x->parent != fa and x->parent != 0) {
        node *p = x->parent;
        if (p->parent == fa)
            if (p->right == x)
                left_rot(x);
            else
                right_rot(x);
        else {
            node *gp = p->parent; //grand parent
            if (gp->left == p)
                if (p->left == x)
                    right_rot(x),right_rot(x);
                else
                    left_rot(x),right_rot(x);
            else
                if (p->left == x)
                    right_rot(x), left_rot(x);
                else
                    left_rot(x), left_rot(x);
        }
    }
    if (fa == 0) root = x;
}

void insert(T key) {
    node *cur = root;
    node *pcur = 0;
    while (cur) {

```

```

        pcur = cur;
        if (key > cur->key) cur = cur->right;
        else cur = cur->left;
    }
    cur = new node(key);
    cur->parent = pcur;
    if (!pcur) root = cur;
    else if (key > pcur->key ) pcur->right = cur;
    else pcur->left = cur;
    splay(cur);
}

node *find(T key) {
    node *cur = root;
    while (cur) {
        if (key > cur->key) cur = cur->right;
        else if(key < cur->key) cur = cur->left;
        else return cur;
    }
    return 0;
}

splay_tree(){ root = 0;};
};

```

3.11 STL order statistics tree II

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef tree<int,null_type,less<int>,rb_tree_tag,
tree_order_statistics_node_update> order_set;

order_set X;

int get(int y) {
    int l=0,r=1e9+1;
    while(l<r) {

```

```

    int m=l+((r-l)>>1);
    if(m-X.order_of_key(m+1)<y)
        l=m+1;
    else
        r=m;
}
return l;
}

main(){
    ios::sync_with_stdio(0);
    cin.tie(0);
    int n,m;
    cin>>n>>m;

    for(int i=0;i<m;i++) {
        char a;
        int b;
        cin>>a>>b;
        if(a=='L')
            cout<<get(b)<<endl;
        else
            X.insert(get(b));
    }
}

/**
Input
20 7
L 5
D 5
L 4
L 5
D 5
L 4
L 5

Output
5
4
6
4
7
***/

```

3.12 STL order statistics tree

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <bits/stdc++.h>

using namespace __gnu_pbds;
using namespace std;

typedef
tree<
    pair<int,int>,
    null_type,
    less<pair<int,int>>,
    rb_tree_tag,
    tree_order_statistics_node_update>
ordered_set;

main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    int n;
    int sz=0;
    cin>>n;
    vector<int> ans(n,0);

    ordered_set t;
    int x,y;
    for(int i=0;i<n;i++)
    {
        cin>>x>>y;
        ans[t.order_of_key({x,++sz})]++;
        t.insert({x,sz});
    }

    for(int i=0;i<n;i++)
        cout<<ans[i]<<'\\n';
}

```

```

/**
Input
5
1 1
5 1
7 1
3 3
5 5

Output
1
2
1
1
0
***/

```

3.13 STL Treap

```

#include <ext/rope> //header with rope
using namespace std;
using namespace __gnu_cxx; //namespace with rope and some additional stuff
int main()
{
    ios_base::sync_with_stdio(false);
    rope <int> v; //use as usual STL container
    int n, m;
    cin >> n >> m;
    for(int i = 1; i <= n; ++i)
        v.push_back(i); //initialization
    int l, r;
    for(int i = 0; i < m; ++i)
    {
        cin >> l >> r;
        --l, --r;
        rope <int> cur = v.substr(l, r - l + 1);
        v.erase(l, r - l + 1);
        v.insert(v.mutable_begin(), cur);
    }
    for(rope <int>::iterator it = v.mutable_begin(); it != v.mutable_end();
        ++it)
        cout << *it << " ";

```

```

    return 0;
}

```

3.14 trie

```

const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

    void clear(){
        tree[nodes].c = 0;
        memset(tree[nodes].a, -1, sizeof tree[nodes].a);
        nodes++;
    }

    void init(){
        nodes = 0;
        clear();
    }

    int add(const string &s, bool query = 0){
        int cur_node = 0;
        for(int i = 0; i < s.size(); ++i){
            int id = gid(s[i]);
            if(tree[cur_node].a[id] == -1){
                if(query) return 0;
                tree[cur_node].a[id] = nodes;
                clear();
            }
            cur_node = tree[cur_node].a[id];
        }
        if(!query) tree[cur_node].c++;
        return tree[cur_node].c;
    }
}

```

```
};
```

3.15 wavelet tree

// this can be tested in the problem: <http://www.spoj.com/problems/ILKQUERY/>

```
struct wavelet {
    vector<int> values, ori;
    vector<int> map_left, map_right;
    int l, r, m;
    wavelet *left, *right;
    wavelet() : left(NULL), right(NULL) {}
    wavelet(int a, int b, int c) : l(a), r(b), m(c), left(NULL), right(NULL) {}
};

wavelet *init(vector<int> &data, vector<int> &ind, int lo, int hi) {
    if (lo > hi || (data.size() == 0)) return NULL;
    int mid = ((long long)(lo) + hi) / 2;
    if (lo + 1 == hi) mid = lo; // handle negative values

    wavelet *node = new wavelet(lo, hi, mid);

    vector<int> data_l, data_r, ind_l, ind_r;
    int ls = 0, rs = 0;
    for (int i = 0; i < data.size(); i++) {
        int value = data[i];
        if (value <= mid) {
            data_l.emplace_back(value);
            ind_l.emplace_back(ind[i]);
            ls++;
        } else {
            data_r.emplace_back(value);
            ind_r.emplace_back(ind[i]);
            rs++;
        }
        node->map_left.emplace_back(ls);
        node->map_right.emplace_back(rs);
        node->values.emplace_back(value);
        node->ori.emplace_back(ind[i]);
    }
}
```

```
if (lo < hi) {
    node->left = init(data_l, ind_l, lo, mid);
    node->right = init(data_r, ind_r, mid + 1, hi);
}
return node;
}
```

```
int kth(wavelet *node, int to, int k) {
    // returns the kth element in the sorted version of (a[0], ..., a[to])
    if (node->l == node->r) return node->m;
    int c = node->map_left[to];
    if (k < c)
        return kth(node->left, c - 1, k);
    return kth(node->right, node->map_right[to] - 1, k - c);
}
```

```
int pos_kth_occurrence(wavelet *node, int val, int k) {
    // returns the position on the original array of the kth occurrence of the
    // value "val"
    if (!node) return -1;

    if (node->l == node->r) {
        if ((int)(node->ori.size()) <= k)
            return -1;
        return node->ori[k];
    }

    if (val <= node->m)
        return pos_kth_occurrence(node->left, val, k);
    return pos_kth_occurrence(node->right, val, k);
}
```

4 DP Optimizations

4.1 convex hull trick

```
/**
 * Problems:
 * http://codeforces.com/problemset/problem/319/C
 * http://codeforces.com/contest/311/problem/B
 * https://csacademy.com/contest/archive/task/squared-ends
 */
```

```

*   http://codeforces.com/contest/932/problem/F
* */

struct line {
    long long m, b;
    line (long long a, long long c) : m(a), b(c) {}
    long long eval(long long x) {
        return m * x + b;
    }
};

long double inter(line a, line b) {
    long double den = a.m - b.m;
    long double num = b.b - a.b;
    return num / den;
}

/**
 * min m_i * x_j + b_i, for all i.
 *   x_j <= x_{j + 1}
 *   m_i >= m_{j + 1}
 */
struct ordered_cht {
    vector<line> ch;
    int idx; // id of last "best" in query
    ordered_cht() {
        idx = 0;
    }

    void insert_line(long long m, long long b) {
        line cur(m, b);
        // new line's slope is less than all the previous
        while (ch.size() > 1 &&
            (inter(cur, ch[ch.size() - 2]) >= inter(cur, ch[ch.size() - 1]))) {
            // f(x) is better in interval [inter(ch.back(), cur), inf)
            ch.pop_back();
        }

        ch.push_back(cur);
    }

    long long eval(long long x) { // minimum
        // current x is greater than all the previous x,
        // if that is not the case we can make binary search.

```

```

        idx = min<int>(idx, ch.size() - 1);
        while (idx + 1 < (int)ch.size() && ch[idx + 1].eval(x) <= ch[idx].eval(x))
            idx++;

        return ch[idx].eval(x);
    }
};

/**
 * Dynamic convex hull trick
 */

typedef long long int64;
typedef long double float128;

const int64 is_query = -(1LL<<62), inf = 1e18;

struct Line {
    int64 m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        int64 x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

struct HullDynamic : public multiset<Line> { // will maintain upper hull for
    maximum
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (float128)(x->b - y->b)*(z->m - y->m) >= (float128)(y->b -
            z->b)*(y->m - x->m);
    }

    void insert_line(int64 m, int64 b) {

```



```

auto y = insert({ m, b });
y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
if (bad(y)) { erase(y); return; }
while (next(y) != end() && bad(next(y))) erase(next(y));
while (y != begin() && bad(prev(y))) erase(prev(y));
}

int64 eval(int64 x) {
    auto l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
}
};

```

4.2 divide and conquer

```

/**
 * recurrence:
 *   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for all j > i;
 *
 * "comp" computes dp[k][i] for all i in O(n log n) (k is fixed)
 *
 * Problems:
 *   https://icpc.kattis.com/problems/branch
 *   http://codeforces.com/contest/321/problem/E
 */

void comp(int l, int r, int le, int re) {
    if (l > r) return;

    int mid = (l + r) >> 1;

    int best = max(mid + 1, le);
    dp[cur][mid] = dp[cur ^ 1][best] + cost(mid, best - 1);
    for (int i = best; i <= re; i++) {
        if (dp[cur][mid] > dp[cur ^ 1][i] + cost(mid, i - 1)) {
            best = i;
            dp[cur][mid] = dp[cur ^ 1][i] + cost(mid, i - 1);
        }
    }

    comp(l, mid - 1, le, best);
    comp(mid + 1, r, best, re);
}

```

```

}

```

4.3 dp on trees

```

/**
 * This trick is very useful when doing DP on trees, basically, you can save
 * the answer for each node as if it was the root of the tree. Partial results
 * are also stored in order to query subtrees (taking the root and exclude some
 * child).
 *
 * problems:
 *   - http://codeforces.com/gym/101161, problem I : Sky tax
 *   - http://codeforces.com/contest/791/problem/D
 */

struct edge {
    int to, p_id;
    edge(int a, int b) : to(a), p_id(b) {}
};

struct state {
    bool seen;
    long long missing;
    long long total;
    vector<long long> partial;

    state() { clear(); }

    void clear() {
        seen = false;
        missing = 0;
        total = 0;
        partial.clear();
    }
};

void add_edge(int u, int v) {
    int id_u_v = g[u].size();
    int id_v_u = g[v].size();
    g[u].emplace_back(v, id_v_u); // id of the parent in the child's list
    (g[v][id] -> u)
}

```

```

    g[v].emplace_back(u, id_u_v); // id of the parent in the child's list
    (g[u][id] -> v)
}

int go(int node, int id_parent) {
    state &s = dp[node];

    if (!s.seen) {
        int ans = 1;
        s.partial.assign(g[node].size(), 0); // create the list of partial results.
        for (int i = 0; i < int(g[node].size()); i++) {
            int to = g[node][i].to;
            int pid = g[node][i].p_id;
            if (i != id_parent) {
                int tmp = go(to, pid);
                ans += tmp;
                s.partial[i] = tmp;
            }
        }

        s.missing = id_parent;
        s.total = ans;
        s.seen = true;

        return ans;
    } else {

        if (s.missing == id_parent) { // the same id_parent than before, so we can
            not complete the results yet
            return s.total;
        }

        if (s.missing != -1) { // only one missing and is different of 'id_parent'
            int tmp = go(g[node][s.missing].to, g[node][s.missing].p_id);
            s.partial[s.missing] = tmp;
            s.total += tmp;
            s.missing = -1;
        }

        int extra = (id_parent == -1) ? 0 : s.partial[id_parent];
        return s.total - extra;
    }
}

```

```

}

```

5 Geometry

5.1 center 2 points + radius

```

vector<point> find_center(point a, point b, long double r) {
    point d = (a - b) * 0.5;
    if (d.dot(d) > r * r) {
        return vector<point> ();
    }
    point e = b + d;
    long double fac = sqrt(r * r - d.dot(d));
    vector<point> ans;
    point x = point(-d.y, d.x);
    long double l = sqrt(x.dot(x));
    x = x * (fac / l);
    ans.push_back(e + x);
    x = point(d.y, -d.x);
    x = x * (fac / l);
    ans.push_back(e + x);
    return ans;
}

```

5.2 closest pair problem

```

struct point {
    double x, y;
    int id;
    point() {}
    point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
    double a = p.x - o.x, b = p.y - o.y;
    return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x, vector<point> &y) {

```

```

if (p.size() < 4) {
    double best = 1e100;
    for (int i = 0; i < p.size(); ++i)
        for (int j = i + 1; j < p.size(); ++j)
            best = min(best, dist(p[i], p[j]));
    return best;
}

int ls = (p.size() + 1) >> 1;
double l = (p[ls - 1].x + p[ls].x) * 0.5;
vector<point> xl(ls), xr(p.size() - ls);
unordered_set<int> left;
for (int i = 0; i < ls; ++i) {
    xl[i] = x[i];
    left.insert(x[i].id);
}
for (int i = ls; i < p.size(); ++i) {
    xr[i - ls] = x[i];
}

vector<point> yl, yr;
vector<point> pl, pr;
yl.reserve(ls); yr.reserve(p.size() - ls);
pl.reserve(ls); pr.reserve(p.size() - ls);
for (int i = 0; i < p.size(); ++i) {
    if (left.count(y[i].id))
        yl.push_back(y[i]);
    else
        yr.push_back(y[i]);

    if (left.count(p[i].id))
        pl.push_back(p[i]);
    else
        pr.push_back(p[i]);
}

double dl = cp(pl, xl, yl);
double dr = cp(pr, xr, yr);
double d = min(dl, dr);
vector<point> yp; yp.reserve(p.size());
for (int i = 0; i < p.size(); ++i) {
    if (fabs(y[i].x - l) < d)
        yp.push_back(y[i]);
}

```

```

for (int i = 0; i < yp.size(); ++i) {
    for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
        d = min(d, dist(yp[i], yp[j]));
    }
}
return d;
}

double closest_pair(vector<point> &p) {
    vector<point> x(p.begin(), p.end());
    sort(x.begin(), x.end(), [](const point &a, const point &b) {
        return a.x < b.x;
    });
    vector<point> y(p.begin(), p.end());
    sort(y.begin(), y.end(), [](const point &a, const point &b) {
        return a.y < b.y;
    });
    return cp(p, x, y);
}

```

5.3 squares

```

typedef long double ld;

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

struct point{
    ld x, y;
    point(ld a, ld b) : x(a), y(b) {}
    point() {}
};

struct square{
    ld x1, x2, y1, y2,
    a, b, c;
    point edges[4];
    square(ld _a, ld _b, ld _c) {
        a = _a, b = _b, c = _c;
    }
}

```

```

    x1 = a - c * 0.5;
    x2 = a + c * 0.5;
    y1 = b - c * 0.5;
    y2 = b + c * 0.5;
    edges[0] = point(x1, y1);
    edges[1] = point(x2, y1);
    edges[2] = point(x2, y2);
    edges[3] = point(x1, y2);
}
};

ld min_dist(point &a, point &b) {
    ld x = a.x - b.x,
        y = a.y - b.y;
    return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
    if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) != -1 &&
        cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) != -1)
        return true;
    return false;
}

bool inside(square &s1, square &s2) {
    for (int i = 0; i < 4; ++i)
        if (point_in_box(s2, s1.edges[i]))
            return true;

    return false;
}

bool inside_vert(square &s1, square &s2) {
    if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1, s2.y2) != 1) ||
        (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2, s2.y2) != 1))
        return true;
    return false;
}

bool inside_hori(square &s1, square &s2) {
    if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1, s2.x2) != 1) ||
        (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2, s2.x2) != 1))
        return true;
    return false;
}

```

```

}

ld min_dist(square &s1, square &s2) {
    if (inside(s1, s2) || inside(s2, s1))
        return 0;

    ld ans = 1e100;
    for (int i = 0; i < 4; ++i)
        for (int j = 0; j < 4; ++j)
            ans = min(ans, min_dist(s1.edges[i], s2.edges[j]));

    if (inside_hori(s1, s2) || inside_hori(s2, s1)) {
        if (cmp(s1.y1, s2.y2) != -1)
            ans = min(ans, s1.y1 - s2.y2);
        else
            if (cmp(s2.y1, s1.y2) != -1)
                ans = min(ans, s2.y1 - s1.y2);
    }

    if (inside_vert(s1, s2) || inside_vert(s2, s1)) {
        if (cmp(s1.x1, s2.x2) != -1)
            ans = min(ans, s1.x1 - s2.x2);
        else
            if (cmp(s2.x1, s1.x2) != -1)
                ans = min(ans, s2.x1 - s1.x2);
    }

    return ans;
}

```

5.4 triangles

Let a, b, c be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

6 Graphs

6.1 board

```
struct board {
    int n, m, r;
    board(int a, int b, int c = 1) : n(a), m(b), r(c) {}

    long long frec(int x, int y) {
        // returns how many squares of r x r contain the cell (x, y)
        long long a = min(x, n - r) - max(x - r + 1, 0) + 1;
        long long b = min(y, m - r) - max(y - r + 1, 0) + 1;
        return a * b;
    }

    bool valid(int x, int y) {
        return x >= 0 && x < n && y >= 0 && y < m;
    }
};
```

6.2 bridges

```
struct Graph {
    vector<vector<Edge>> g;
    vector<int> vi, low, d, pi, is_b;
    int bridges_computed;

    int ticks, edges;

    Graph(int n, int m) {
        g.assign(n, vector<Edge>());
        is_b.assign(m, 0);
        vi.resize(n);
        low.resize(n);
    }
};
```

```
    d.resize(n);
    pi.resize(n);
    edges = 0;
    bridges_computed = 0;
}

void AddEdge(int u, int v) {
    g[u].push_back(Edge(v, edges));
    g[v].push_back(Edge(u, edges));
    edges++;
}

void Dfs(int u) {
    vi[u] = true;
    d[u] = low[u] = ticks++;
    for (int i = 0; i < (int)g[u].size(); ++i) {
        int v = g[u][i].to;
        if (v == pi[u]) continue;
        if (!vi[v]) {
            pi[v] = u;
            Dfs(v);
            if (d[u] < low[v]) is_b[g[u][i].id] = true;

            low[u] = min(low[u], low[v]);
        } else {
            low[u] = min(low[u], d[v]);
        }
    }
}

// Multiple edges from a to b are not allowed.
// (they could be detected as a bridge).
// If you need to handle this, just count
// how many edges there are from a to b.
void CompBridges() {
    fill(pi.begin(), pi.end(), -1);
    fill(vi.begin(), vi.end(), 0);
    fill(low.begin(), low.end(), 0);
    fill(d.begin(), d.end(), 0);
    ticks = 0;
    for (int i = 0; i < (int)g.size(); ++i)
        if (!vi[i]) Dfs(i);
    bridges_computed = true;
}
```

```

map<int, vector<Edge>> BridgesTree() {
    if (!bridges_computed) CompBridges();
    int n = g.size();
    Dsu dsu(g.size());
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (!is_b[e.id]) dsu.Join(i, e.to);

    map<int, vector<Edge>> tree;
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (is_b[e.id])
                tree[dsu.Find(i)].emplace_back(dsu.Find(e.to), e.id);

    return tree;
}
};

```

6.3 dijkstra

```

struct edge {
    int to;
    long long w;
    edge () {}
    edge (int a, long long b) : to(a), w(b) {}
    bool operator < (const edge &o) const {
        return w > o.w;
    }
};

typedef vector<vector<edge>> graph;

const long long inf = 1000000LL * 100000000LL;

pair<vector<int>, vector<long long>> dijkstra(graph &g, int start) {
    int n = g.size();
    vector<long long> d(n, inf);
    vector<int> p(n, -1);
    d[start] = 0;
    priority_queue<edge> q;
    q.push(edge(start, 0));

```

```

while (!q.empty()) {
    int node = q.top().to;
    long long dist = q.top().w;
    q.pop();

    if (dist > d[node]) continue;

    for (int i = 0; i < (int)g[node].size(); i++) {
        int to = g[node][i].to;
        long long w_extra = g[node][i].w;

        if (dist + w_extra < d[to]) {
            p[to] = node;
            d[to] = dist + w_extra;
            q.push(edge(to, d[to]));
        }
    }
}

return {p, d};
}

```

6.4 directed mst

```

const int inf = 1000000 + 10;

struct edge {
    int u, v, w;
    edge() {}
    edge(int a, int b, int c) : u(a), v(b), w(c) {}
};

/**
 * Computes the minimum spanning tree for a directed graph
 * - edges : Graph description in the form of list of edges.
 *   each edge is: From node u to node v with cost w
 * - root : Id of the node to start the DMST.
 * - n : Number of nodes in the graph.
 */

int dmst(vector<edge> &edges, int root, int n) {

```

```

int ans = 0;
int cur_nodes = n;
while (true) {
    vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
    for (int i = 0; i < edges.size(); ++i) {
        int u = edges[i].u, v = edges[i].v, w = edges[i].w;
        if (w < lo[v] and u != v) {
            lo[v] = w;
            pi[v] = u;
        }
    }

    lo[root] = 0;
    for (int i = 0; i < lo.size(); ++i) {
        if (i == root) continue;
        if (lo[i] == inf) return -1;
    }

    int cur_id = 0;
    vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
    for (int i = 0; i < cur_nodes; ++i) {
        ans += lo[i];
        int u = i;
        while (u != root and id[u] < 0 and mark[u] != i) {
            mark[u] = i;
            u = pi[u];
        }
        if (u != root and id[u] < 0) { // Cycle
            for (int v = pi[u]; v != u; v = pi[v])
                id[v] = cur_id;
            id[u] = cur_id++;
        }
    }

    if (cur_id == 0)
        break;

    for (int i = 0; i < cur_nodes; ++i)
        if (id[i] < 0) id[i] = cur_id++;

    for (int i = 0; i < edges.size(); ++i) {
        int u = edges[i].u, v = edges[i].v, w = edges[i].w;
        edges[i].u = id[u];
        edges[i].v = id[v];
        if (id[u] != id[v])

```

```

        edges[i].w -= lo[v];
    }
    cur_nodes = cur_id;
    root = id[root];
}

return ans;
}

```

6.5 eulerian path

// Taken from <https://github.com/lbv/pc-code/blob/master/code/graph.cpp>
// Eulerian Trail

```

struct Euler {
    ELV adj; IV t;
    Euler(ELV Adj) : adj(Adj) {}
    void build(int u) {
        while(! adj[u].empty()) {
            int v = adj[u].front().v;
            adj[u].erase(adj[u].begin());
            build(v);
        }
        t.push_back(u);
    }
};

bool eulerian_trail(IV &trail) {
    Euler e(adj);
    int odd = 0, s = 0;
    /*
        for (int v = 0; v < n; v++) {
            int diff = abs(in[v] - out[v]);
            if (diff > 1) return false;
            if (diff == 1) {
                if (++odd > 2) return false;
                if (out[v] > in[v]) start = v;
            }
        }
    */
    e.build(s);
    reverse(e.t.begin(), e.t.end());
    trail = e.t;
}

```

```

    return true;
}

```

6.6 karp min mean cycle

```

/**
 * Finds the min mean cycle, if you need the max mean cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
 * */

const int MN = 1000;
struct edge{
    int v;
    long long w;
    edge(){} edge(int v, int w) : v(v), w(w) {}
};

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge> > g) {
    int n = g.size();

    g.resize(n + 1); // this is important

    for (int i = 0; i < n; ++i)
        if (!g[i].empty())
            g[n].push_back(edge(i,0));
    ++n;

    for(int i = 0; i < n; ++i)
        fill(d[i], d[i] + (n+1), INT_MAX);

    d[n - 1][0] = 0;

    for (int k = 1; k <= n; ++k) for (int u = 0; u < n; ++u) {
        if (d[u][k - 1] == INT_MAX) continue;
        for (int i = g[u].size() - 1; i >= 0; --i)
            d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k - 1] + g[u][i].w);
    }
}

```

```

}

bool flag = true;

for (int i = 0; i < n && flag; ++i)
    if (d[i][n] != INT_MAX)
        flag = false;

if (flag) {
    return true; // return true if there is no a cycle.
}

double ans = 1e15;

for (int u = 0; u + 1 < n; ++u) {
    if (d[u][n] == INT_MAX) continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
        if (d[u][k] != INT_MAX)
            W = max(W, (double)(d[u][n] - d[u][k]) / (n - k));

    ans = min(ans, W);
}

// printf("%.2lf\n", ans);
cout << fixed << setprecision(2) << ans << endl;

return false;
}

```

6.7 konig's theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

6.8 minimum path cover in DAG

Given a directed acyclic graph $G = (V, E)$, we are to find the minimum number of vertex-disjoint paths to cover each vertex in V .

We can construct a bipartite graph $G' = (V_{out} \cup V_{in}, E')$ from G , where :

$$V_{out} = \{v \in V : v \text{ has positive out-degree}\}$$

$$V_{in} = \{v \in V : v \text{ has positive in-degree}\}$$

$$E' = \{(u, v) \in V_{out} \times V_{in} : (u, v) \in E\}$$

Then it can be shown, via König's theorem, that G' has a matching of size m if and only if there exists $n - m$ vertex-disjoint paths that cover each vertex in G , where n is the number of vertices in G and m is the maximum cardinality bipartite matching in G' .

Therefore, the problem can be solved by finding the maximum cardinality matching in G' instead.

NOTE: If the paths are not necessarily disjoint, find the transitive closure and solve the problem for disjoint paths.

6.9 planar graph (euler)

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and v is the number of vertices, e is the number of edges and f is the number of faces (regions bounded by edges, including the outer, infinitely large region), then:

$$f + v = e + 2$$

It can be extended to non connected planar graphs with c connected components:

$$f + v = e + c + 1$$

6.10 query with lca

```
struct lowest_ca {
    int T[MN], L[MN], W[MN];
    int P[MN][ML], MI[MN][ML], MA[MN][ML];

    void dfs(vector<vector<edge>> &g, int root, int pi = -1) {
        if (pi == -1) {
            L[root] = W[root] = 0;
            T[root] = -1;

```

```
        }
        for (int i = 0; i < (int)g[root].size(); ++i) {
            int to = g[root][i].v;
            if (to != pi) {
                T[to] = root;
                W[to] = g[root][i].w;
                L[to] = L[root] + 1;
                dfs(g, to, root);
            }
        }
    }
}

void init(vector<vector<edge>> &g, int root) {
    // g is undirected
    dfs(g, root);
    int N = g.size(), i, j;

    for (i = 0; i < N; i++) {
        for (j = 0; j < N; j++) {
            P[i][j] = -1;
            MI[i][j] = inf;
        }
    }

    for (i = 0; i < N; i++) {
        P[i][0] = T[i];
        MI[i][0] = W[i];
    }

    for (j = 1; j < N; j++)
        for (i = 0; i < N; i++)
            if (P[i][j-1] != -1) {
                P[i][j] = P[P[i][j-1]][j-1];
                MI[i][j] = min(MI[i][j-1], MI[P[i][j-1]][j-1]);
            }
    }

    int query(int p, int q) {
        int tmp, log, i;

        int mmin = inf;
        if (L[p] < L[q])
            tmp = p, p = q, q = tmp;

```

```

for (log = 1; 1 << log <= L[p]; log++);
log--;

for (i = log; i >= 0; i--)
    if (L[p] - (1 << i) >= L[q]) {
        mmin = min(mmin, MI[p][i]);
        p = P[p][i];
    }

if (p == q)
    // return p;
    return mmin;

for (i = log; i >= 0; i--)
    if (P[p][i] != -1 && P[p][i] != P[q][i]) {
        mmin = min(mmin, min(MI[p][i], MI[q][i]));
        p = P[p][i], q = P[q][i];
    }

// return T[p];
return min(mmin, min(MI[p][0], MI[q][0]));
}

int get_child(int p, int q) { // p is ancestor of q
    if (p == q) return -1;

    int i, log;
    for (log = 1; 1 << log <= L[q]; log++) {}
    log--;

    for (i = log; i >= 0; i--)
        if (L[q] - (1 << i) > L[p]) {
            q = P[q][i];
        }

    assert(P[q][0] == p);
    return q;
}

int is_ancestor(int p, int q) {
    if (L[p] >= L[q])
        return false;

    int dist = L[q] - L[p];

```

```

int cur = q;
int step = 0;
while (dist) {
    if (dist & 1)
        cur = P[cur][step];
    step++;
    dist >>= 1;
}

return cur == p;
}
};

```

6.11 SCC kosaraju

```

struct SCC {
    vector<vector<int> > g, gr;
    vector<bool> used;
    vector<int> order, component;
    int total_components;

    SCC(vector<vector<int> > &adj) {
        g = adj;
        int n = g.size();
        gr.resize(n);
        for (int i = 0; i < n; i++)
            for (auto to : g[i])
                gr[to].push_back(i);

        used.assign(n, false);
        for (int i = 0; i < n; i++)
            if (!used[i])
                GenTime(i);

        used.assign(n, false);
        component.assign(n, -1);
        total_components = 0;
        for (int i = n - 1; i >= 0; i--) {
            int v = order[i];
            if (!used[v]) {
                vector<int> cur_component;

```

```

        Dfs(cur_component, v);
        for (auto node : cur_component)
            component[node] = total_components;
        total_components++;
    }
}

void GenTime(int node) {
    used[node] = true;
    for (auto to : g[node])
        if (!used[to])
            GenTime(to);
    order.push_back(node);
}

void Dfs(vector<int> &cur, int node) {
    used[node] = true;
    cur.push_back(node);
    for (auto to : gr[node])
        if (!used[to])
            Dfs(cur, to);
}

vector<vector<int>> CondensedGraph() {
    vector<vector<int>> ans(total_components);
    for (int i = 0; i < int(g.size()); i++) {
        for (int to : g[i]) {
            int u = component[i], v = component[to];
            if (u != v)
                ans[u].push_back(v);
        }
    }
    return ans;
}
};

```

6.12 tarjan scc

```

const int MN = 20002;

struct tarjan_scc {

```

```

    int scc[MN], low[MN], d[MN], stacked[MN];
    int ticks, current_scc;
    deque<int> s; // used as stack.

    tarjan_scc() {}

    void init () {
        memset(scc, -1, sizeof scc);
        memset(d, -1, sizeof d);
        memset(stacked, 0, sizeof stacked);
        s.clear();
        ticks = current_scc = 0;
    }

    void compute(vector<vector<int> > &g, int u) {
        d[u] = low[u] = ticks++;
        s.push_back(u);
        stacked[u] = true;
        for (int i = 0; i < g[u].size(); ++i) {
            int v = g[u][i];
            if (d[v] == -1)
                compute(g, v);
            if (stacked[v]) {
                low[u] = min(low[u], low[v]);
            }
        }

        if (d[u] == low[u]) { // root
            int v;
            do {
                v = s.back(); s.pop_back();
                stacked[v] = false;
                scc[v] = current_scc;
            } while (u != v);
            current_scc++;
        }
    }
};

```

6.13 two sat (with kosaraju)

```

/**

```

```

* Given a set of clauses (a1 v a2)^(a2 v a3)....
* this algorithm find a solution to it set of clauses.
* test: http://lightoj.com/volume\_showproblem.php?problem=1251
**/

```

```

#include<bits/stdc++.h>
using namespace std;
#define MAX 100000
#define endl '\n'

vector<int> G[MAX];
vector<int> GT[MAX];
vector<int> Ftime;
vector<vector<int>> > SCC;
bool visited[MAX];
int n;

```

```

void dfs1(int n){
    visited[n] = 1;

    for (int i = 0; i < G[n].size(); ++i) {
        int curr = G[n][i];
        if (visited[curr]) continue;
        dfs1(curr);
    }

    Ftime.push_back(n);
}

```

```

void dfs2(int n, vector<int> &scc) {
    visited[n] = 1;
    scc.push_back(n);

    for (int i = 0; i < GT[n].size(); ++i) {
        int curr = GT[n][i];
        if (visited[curr]) continue;
        dfs2(curr, scc);
    }
}

```

```

void kosaraju() {
    memset(visited, 0, sizeof visited);

```

```

    for (int i = 0; i < 2 * n ; ++i) {
        if (!visited[i]) dfs1(i);
    }

    memset(visited, 0, sizeof visited);
    for (int i = Ftime.size() - 1; i >= 0; i--) {
        if (visited[Ftime[i]]) continue;
        vector<int> _scc;
        dfs2(Ftime[i], _scc);
        SCC.push_back(_scc);
    }
}

```

```

/**
 * After having the SCC, we must traverse each scc, if in one SCC are -b y b,
 * there is not a solution.
 * Otherwise we build a solution, making the first "node" that we find truth
 * and its complement false.
 */

```

```

bool two_sat(vector<int> &val) {
    kosaraju();
    for (int i = 0; i < SCC.size(); ++i) {
        vector<bool> tmpvisited(2 * n, false);
        for (int j = 0; j < SCC[i].size(); ++j) {
            if (tmpvisited[SCC[i][j] ^ 1]) return 0;
            if (val[SCC[i][j]] != -1) continue;
            else {
                val[SCC[i][j]] = 0;
                val[SCC[i][j] ^ 1] = 1;
            }
            tmpvisited[SCC[i][j]] = 1;
        }
    }
    return 1;
}

```

```

// Example of use

```

```

int main() {

    int m, u, v, nc = 0, t; cin >> t;

```

```

// n = "nodes" number, m = clauses number

while (t--) {
    cin >> m >> n;
    Ftime.clear();
    SCC.clear();
    for (int i = 0; i < 2 * n; ++i) {
        G[i].clear();
        GT[i].clear();
    }

    // (a1 v a2) = (a1 -> a2) = (a2 -> a1)
    for (int i = 0; i < m; ++i) {
        cin >> u >> v;
        int t1 = abs(u) - 1;
        int t2 = abs(v) - 1;
        int p = t1 * 2 + ((u < 0)? 1 : 0);
        int q = t2 * 2 + ((v < 0)? 1 : 0);
        G[p ^ 1].push_back(q);
        G[q ^ 1].push_back(p);
        GT[p].push_back(q ^ 1);
        GT[q].push_back(p ^ 1);
    }

    vector<int> val(2 * n, -1);
    cout << "Case " << ++nc << ": ";
    if (two_sat(val)) {
        cout << "Yes" << endl;
        vector<int> sol;
        for (int i = 0; i < 2 * n; ++i)
            if (i % 2 == 0 and val[i] == 1)
                sol.push_back(i / 2 + 1);
        cout << sol.size() ;

        for (int i = 0; i < sol.size(); ++i) {
            cout << " " << sol[i];
        }
        cout << endl;
    } else {
        cout << "No" << endl;
    }
}
return 0;
}

```

7 Math

7.1 counting

```

const int MN = 1e5 + 100;
long long fact[MN];

void fill_fact() {
    fact[0] = 1;
    for (int i = 1; i < MN; i++) {
        fact[i] = mult(fact[i - 1], i);
    }
}

long long perm_rep(vector<int> &frec) {
    int total = 0;
    long long den = 1;
    for (int i = 0; i < (int)frec.size(); i++) {
        den = mult(den, mod_inv(fact[frec[i]]));
        total += frec[i];
    }
    return mult(fact[total], den);
}

```

7.2 cumulative sum of divisors

/**
The function SOD(n) (sum of divisors) is defined
as the summation of all the actual divisors of
an integer number n. For example,

$$\text{SOD}(24) = 2+3+4+6+8+12 = 35.$$

The function CSOD(n) (cumulative SOD) of an integer n, is defined as below:

$$\text{csod}(n) = \sum_{i=1}^n \text{sod}(i)$$

It can be computed in $O(\sqrt{n})$:
*/

```
long long csod(long long n) {
    long long ans = 0;
    for (long long i = 2; i * i <= n; ++i) {
        long long j = n / i;
        ans += (i + j) * (j - i + 1) / 2;
        ans += i * (j - i);
    }
    return ans;
}
```

7.3 fft

```
/**
 * Fast Fourier Transform.
 * Useful to compute convolutions.
 * computes:
 *   C(f star g)[n] = sum_m(f[m] * g[n - m])
 * for all n.
 * test: icpc live archive, 6886 - Golf Bot
 */
```

```
using namespace std;
#include<bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'
```

```
const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];
```

```
const double PI = acos(-1.0);
```

```
struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
}
```

```
cpx(){}
};

cpx operator + (const cpx &c1, const cpx &c2) {
    return cpx(c1.real + c2.real, c1.image + c2.image);
}

cpx operator - (const cpx &c1, const cpx &c2) {
    return cpx(c1.real - c2.real, c1.image - c2.image);
}

cpx operator * (const cpx &c1, const cpx &c2) {
    return cpx(c1.real*c2.real - c1.image*c2.image, c1.real*c2.image +
        c1.image*c2.real);
}

int rev(int id, int len) {
    int ret = 0;
    for (int i = 0; (1 << i) < len; i++) {
        ret <<= 1;
        if (id & (1 << i)) ret |= 1;
    }
    return ret;
}

cpx A[1 << 20];

void FFT(cpx *a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT * 2 * PI / m));
        for(int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for(int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
}
```

```

    if (DFT == -1) for (int i = 0; i < len; i++) A[i].real /= len, A[i].image /=
        len;
    for (int i = 0; i < len; i++) a[i] = A[i];
    return;
}

cpx in[1 << 20];

void solve(int n) {
    memset(d, 0, sizeof d);
    int t;
    for (int i = 0; i < n; ++i) {
        cin >> t;
        d[t] = true;
    }
    int m;
    cin >> m;
    vector<int> q(m);
    for (int i = 0; i < m; ++i)
        cin >> q[i];

    for (int i = 0; i < MN; ++i) {
        if (d[i])
            in[i] = cpx(1, 0);
        else
            in[i] = cpx(0, 0);
    }

    FFT(in, MN, 1);
    for (int i = 0; i < MN; ++i) {
        in[i] = in[i] * in[i];
    }
    FFT(in, MN, -1);

    int ans = 0;
    for (int i = 0; i < q.size(); ++i) {
        if (in[q[i]].real > 0.5 || d[q[i]]) {
            ans++;
        }
    }
    cout << ans << endl;
}

int main() {

```

```

    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int n;
    while (cin >> n)
        solve(n);
    return 0;
}

```

7.4 fibonacci properties

Let A, B and n be integer numbers.

$$k = A - B \quad (1)$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \quad (2)$$

$$\sum_{i=0}^n F_i^2 = F_{n+1} F_n \quad (3)$$

$ev(n)$ = returns 1 if n is even.

$$\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - ev(n) \quad (4)$$

$$\sum_{i=0}^n F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \quad (5)$$

7.5 Lucas theorem

For non-negative integers m and n and a prime p , the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0$$

are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \leq n$.

7.6 polynomials

```
const double pi = acos(-1);
struct poly {
    deque<double> coef;
    double x_lo, x_hi;

    double evaluate(double x) {
        double ans = 0;
        for (auto it : coef)
            ans = (ans * x + it);
        return ans;
    }

    double volume(double x, double dx=1e-6) {
        dx = (x_hi - x_lo) / 1000000.0;
        double ans = 0;
        for (double ix = x_lo; ix <= x; ix += dx) {
            double rad = evaluate(ix);
            ans += pi * rad * rad * dx;
        }
        return ans;
    }
};
```

7.7 sigma function

the sigma function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when $x = 0$ is called the divisor function, that counts the number of positive divisors of n .

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

if n is written as prime factorization:

$$n = \prod_{i=1}^k P_i^{e_k}$$

we can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^k g(e_k + 1)$$

where $g(x)$ is the sum of the first x positive numbers:

$$g(x) = (x * (x + 1)) / 2$$

8 Matrix

8.1 matrix

```
const int MN = 111;
const int mod = 10000;

struct matrix {
    int r, c;
    int m[MN][MN];

    matrix(int _r, int _c) : r(_r), c(_c) {
        memset(m, 0, sizeof m);
    }

    void print() {
        for (int i = 0; i < r; ++i) {
            for (int j = 0; j < c; ++j)
                cout << m[i][j] << " ";
            cout << endl;
        }
    }
};
```



```

    }
}

int x[MN][MN];
matrix & operator *= (const matrix &o) {
    memset(x, 0, sizeof x);
    for (int i = 0; i < r; ++i)
        for (int k = 0; k < c; ++k)
            if (m[i][k] != 0)
                for (int j = 0; j < c; ++j) {
                    x[i][j] = (x[i][j] + ((m[i][k] * o.m[k][j]) % mod) ) % mod;
                }
    memcpy(m, x, sizeof(m));
    return *this;
}
};

void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
        if (e & 1) res *= b;
        if ((e >>= 1) == 0) break;
        b *= b;
    }
}

```

9 Misc

9.1 dates

```

//
// Time - Leap years
//

// A[i] has the accumulated number of days from months previous to i
const int A[13] = { 0, 0, 31, 59, 90, 120, 151, 181, 212, 243, 273, 304, 334 };
// same as A, but for a leap year

```

```

const int B[13] = { 0, 0, 31, 60, 91, 121, 152, 182, 213, 244, 274, 305, 335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y / 400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 == 0 && y % 100 != 0); }
// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
    return (y - 1) * 365 + leap_years(y - 1) + (is_leap(y) ? B[m] : A[m]) + d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400 block?
    bool top4;   // are we in the top 4 years of a 100 block?
    bool top1;   // are we in the top year of a 4 block?

    y = 1;
    top100 = top4 = top1 = false;

    y += ((days-1) / p400) * 400;
    d = (days-1) % p400 + 1;

    if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
    else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

    if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
    else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

    if (d > p1*3) top1 = true, d -= p1*3, y += 3;
    else y += (d-1) / p1, d = (d-1) % p1 + 1;

    const int *ac = top1 && (!top4 || top100) ? B : A;
    for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
    d -= ac[m];
}

```

9.2 fraction

```

struct frac{

```

```

long long x, y;
frac(long long a, long long b) {
    long long g = __gcd(a, b);
    x = a / g;
    y = b / g;
}
bool operator < (const frac &o) const {
    return (x * o.y < y * o.x);
}
};

```

9.3 io

// taken from :
<https://github.com/lbv/pc-code/blob/master/solved/c-e/diablo/diablo.cpp>
 // this is very fast as well :
<https://github.com/lbv/pc-code/blob/master/code/input.cpp>

```

typedef unsigned int u32;
#define BUF 524288
struct Reader {
    char buf[BUF]; char b; int bi, bz;
    Reader() { bi=bz=0; read(); }
    void read() {
        if (bi==bz) { bi=0; bz = fread(buf, 1, BUF, stdin); }
        b = bz ? buf[bi++] : 0; }
    void skip() { while (b > 0 && b <= 32) read(); }
    u32 next_u32() {
        u32 v = 0; for (skip(); b > 32; read()) v = v*10 + b-48; return v; }
    int next_int() {
        int v = 0; bool s = false;
        skip(); if (b == '-') { s = true; read(); }
        for (; 48<=b&&b<=57; read()) v = v*10 + b-48; return s ? -v : v; }
    char next_char() { skip(); char c = b; read(); return c; }
};

```

9.4 Template Java

```

import java.io.*;
import java.util.StringTokenizer;

```

```

public class Template {

    public static void main(String []args) throws IOException {
        Scanner in = new Scanner(System.in);
        OutputWriter out = new OutputWriter(System.out);
        Task solver = new Task();
        solver.solve(in, out);
        out.close();
    }
}

class Task{
    public void solve(Scanner in, OutputWriter out){

    }
}

class Scanner{
    public BufferedReader reader;
    public StringTokenizer st;

    public Scanner(InputStream stream){
        reader = new BufferedReader(new InputStreamReader(stream));
        st = null;
    }

    public String next(){
        while(st == null || !st.hasMoreTokens()){
            try{
                String line = reader.readLine();
                if(line == null) return null;
                st = new StringTokenizer(line);
            }catch (Exception e){
                throw (new RuntimeException());
            }
        }
        return st.nextToken();
    }

    public int nextInt(){
        return Integer.parseInt(next());
    }
}

```

```

public long nextLong(){
    return Long.parseLong(next());
}
public double nextDouble(){
    return Double.parseDouble(next());
}
}

class OutputWriter{
    BufferedWriter writer;

    public OutputWriter(OutputStream stream){
        writer = new BufferedWriter(new OutputStreamWriter(stream));
    }

    public void print(int i) throws IOException {
        writer.write(i);
    }

    public void print(String s) throws IOException {
        writer.write(s);
    }

    public void print(char []c) throws IOException {
        writer.write(c);
    }

    public void close() throws IOException {
        writer.close();
    }
}

```

10 Number theory

10.1 convolution

```

typedef long long int LL;
typedef pair<LL, LL> PLL;

inline bool is_pow2(LL x) {
    return (x & (x-1)) == 0;
}

```

```

}

inline int ceil_log2(LL x) {
    int ans = 0;
    --x;
    while (x != 0) {
        x >>= 1;
        ans++;
    }
    return ans;
}

/* Returns the convolution of the two given vectors in time proportional to
n*log(n).
* The number of roots of unity to use nroots_unity must be set so that the
product of the first
* nroots_unity primes of the vector nth_roots_unity is greater than the
maximum value of the
* convolution. Never use sizes of vectors bigger than 2^24, if you need to
change the values of
* the nth roots of unity to appropriate primes for those sizes.
*/
vector<LL> convolve(const vector<LL> &a, const vector<LL> &b, int nroots_unity
= 2) {
    int N = 1 << ceil_log2(a.size() + b.size());
    vector<LL> ans(N,0), fA(N), fB(N), fC(N);
    LL modulo = 1;
    for (int times = 0; times < nroots_unity; times++) {
        fill(fA.begin(), fA.end(), 0);
        fill(fB.begin(), fB.end(), 0);
        for (int i = 0; i < a.size(); i++) fA[i] = a[i];
        for (int i = 0; i < b.size(); i++) fB[i] = b[i];
        LL prime = nth_roots_unity[times].first;
        LL inv_modulo = mod_inv(modulo % prime, prime);
        LL normalize = mod_inv(N, prime);
        ntfft(fA, 1, nth_roots_unity[times]);
        ntfft(fB, 1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i]) % prime;
        ntfft(fC, -1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) {
            LL curr = (fC[i] * normalize) % prime;
            LL k = (curr - (ans[i] % prime) + prime) % prime;
            k = (k * inv_modulo) % prime;
            ans[i] += modulo * k;
        }
    }
}

```

```

    }
    modulo *= prime;
}
return ans;
}

```

10.2 crt

```

/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 */
long long crt(vector<long long> &a, vector<long long> &x) {
    long long z = 0;
    long long n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        long long tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}

```

10.3 diophantine equations

```

long long gcd(long long a, long long b, long long &x, long long &y) {
    if (a == 0) {
        x = 0;
        y = 1;
        return b;
    }
    long long x1, y1;
    long long d = gcd(b % a, a, x1, y1);
    x = y1 - (b / a) * x1;
    y = x1;
    return d;
}

```

```

}

bool find_any_solution(long long a, long long b, long long c, long long &x0,
    long long &y0, long long &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }

    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}

void shift_solution(long long &x, long long &y, long long a, long long b,
    long long cnt) {
    x += cnt * b;
    y -= cnt * a;
}

long long find_all_solutions(long long a, long long b, long long c,
    long long minx, long long maxx, long long miny,
    long long maxy) {
    long long x, y, g;
    if (!find_any_solution(a, b, c, x, y, g)) return 0;
    a /= g;
    b /= g;

    long long sign_a = a > 0 ? +1 : -1;
    long long sign_b = b > 0 ? +1 : -1;

    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx) shift_solution(x, y, a, b, sign_b);
    if (x > maxx) return 0;
    long long lx1 = x;

    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx) shift_solution(x, y, a, b, -sign_b);
    long long rx1 = x;

    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny) shift_solution(x, y, a, b, -sign_a);
}

```

```

if (y > maxy) return 0;
long long lx2 = x;

shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy) shift_solution(x, y, a, b, sign_a);
long long rx2 = x;

if (lx2 > rx2) swap(lx2, rx2);
long long lx = max(lx1, lx2);
long long rx = min(rx1, rx2);

if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}

```

10.4 discrete logarithm

// Computes x which $a^x = b \pmod n$.

```

long long d_log(long long a, long long b, long long n) {
    long long m = ceil(sqrt(n));
    long long aj = 1;
    map<long long, long long> M;
    for (int i = 0; i < m; ++i) {
        if (!M.count(aj))
            M[aj] = i;
        aj = (aj * a) % n;
    }

    long long coef = mod_pow(a, n - 2, n);
    coef = mod_pow(coef, m, n);
    // coef = a ^ (-m)
    long long gamma = b;
    for (int i = 0; i < m; ++i) {
        if (M.count(gamma)) {
            return i * m + M[gamma];
        } else {
            gamma = (gamma * coef) % n;
        }
    }
    return -1;
}

```

```

}

```

10.5 ext euclidean

```

void ext_euclid(long long a, long long b, long long &x, long long &y, long
    long &g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}

```

10.6 highest exponent factorial

```

int highest_exponent(int p, const int &n){
    int ans = 0;
    int t = p;
    while(t <= n){
        ans += n/t;
        t*=p;
    }
    return ans;
}

```

10.7 miller rabin

```

const int rounds = 20;

// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
    // check as in Miller Rabin Primality Test described
    long long u = n - 1;
    int t = 0;
    while (u % 2 == 0) {

```

```

    t++;
    u >>= 1;
}
long long next = mod_pow(a, u, n);
if (next == 1) return false;
long long last;
for (int i = 0; i < t; ++i) {
    last = next;
    next = mod_mul(last, last, n);
    if (next == 1) {
        return last != n - 1;
    }
}
return next != 1;
}

// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(99999999999999997LL) == 1);
// D(miller_rabin(999999999999999971LL) == 1);
// D(miller_rabin(7907) == 1);
bool miller_rabin(long long n, int it = rounds) {
    if (n <= 1) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (int i = 0; i < it; ++i) {
        long long a = rand() % (n - 1) + 1;
        if (witness(a, n)) {
            return false;
        }
    }
    return true;
}

```

10.8 mod integer

```

template<class T, T mod>
struct mint_t {
    T val;
    mint_t() : val(0) {}
    mint_t(T v) : val(v % mod) {}

```

```

    mint_t operator + (const mint_t& o) const {
        return (val + o.val) % mod;
    }
    mint_t operator - (const mint_t& o) const {
        return (val - o.val) % mod;
    }
    mint_t operator * (const mint_t& o) const {
        return (val * o.val) % mod;
    }
};

typedef mint_t<long long, 998244353> mint;

```

10.9 mod inv

```

long long mod_inv(long long n, long long m) {
    long long x, y, gcd;
    ext_euclid(n, m, x, y, gcd);
    if (gcd != 1)
        return 0;
    return (x + m) % m;
}

```

10.10 mod mul

```

// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long mod) {
    long long x = 0, y = a % mod;
    while (b > 0) {
        if (b & 1)
            x = (x + y) % mod;
        y = (y * 2) % mod;
        b /= 2;
    }
    return x % mod;
}

```

10.11 mod pow

```
// Computes ( a ^ exp ) % mod.
long long mod_pow(long long a, long long exp, long long mod) {
    long long ans = 1;
    while (exp > 0) {
        if (exp & 1)
            ans = mod_mul(ans, a, mod);
        a = mod_mul(a, a, mod);
        exp >>= 1;
    }
    return ans;
}
```

10.12 number theoretic transform

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime, generator)
 * where the prime has an Nth root of unity for N being a power of two.
 * The generator is a number g s.t g^(p-1)=1 (mod p)
 * but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
    {1224736769,330732430},{1711276033,927759239},{167772161,167489322},
    {469762049,343261969},{754974721,643797295},{1107296257,883865065}};

PLL ext_euclid(LL a, LL b) {
    if (b == 0)
        return make_pair(1,0);
    pair<LL,LL> rc = ext_euclid(b, a % b);
    return make_pair(rc.second, rc.first - (a / b) * rc.second);
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
    PLL p = ext_euclid(x, modulo);
    if ( (p.first * x + p.second * modulo) != 1 )
        return -1;
    return (p.first+modulo) % modulo;
}
```

```
//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
    int n = a.size();
    LL prime = root_unity.first;
    LL basew = mod_pow(root_unity.second, (prime-1) / n, prime);
    if (dir < 0) basew = mod_inv(basew, prime);
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        LL w = 1;
        for (int i = 0; i < mh; i++) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = (a[j] - a[k] + prime) % prime;
                a[j] = (a[j] + a[k]) % prime;
                a[k] = (w * x) % prime;
            }
            w = (w * basew) % prime;
        }
        basew = (basew * basew) % prime;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
}
```

10.13 pollard rho factorize

```
long long pollard_rho(long long n) {
    long long x, y, i = 1, k = 2, d;
    x = y = rand() % n;
    while (1) {
        ++i;
        x = mod_mul(x, x, n);
        x += 2;
        if (x >= n) x -= n;
        if (x == y) return 1;
        d = __gcd(abs(x - y), n);
        if (d != 1) return d;
    }
}
```

```

    if (i == k) {
        y = x;
        k *= 2;
    }
}
return 1;
}

// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
    vector<long long> ans;
    if (n == 1)
        return ans;
    if (miller_rabin(n)) {
        ans.push_back(n);
    } else {
        long long d = 1;
        while (d == 1)
            d = pollard_rho(n);
        vector<long long> dd = factorize(d);
        ans = factorize(n / d);
        for (int i = 0; i < dd.size(); ++i)
            ans.push_back(dd[i]);
    }
    return ans;
}

```

10.14 primes

```

namespace primes {
    const int MP = 100001;
    bool sieve[MP];
    long long primes[MP];
    int num_p;
    void fill_sieve() {
        num_p = 0;
        sieve[0] = sieve[1] = true;
        for (long long i = 2; i < MP; ++i) {
            if (!sieve[i]) {
                primes[num_p++] = i;
                for (long long j = i * i; j < MP; j += i)

```

```

                sieve[j] = true;
            }
        }
    }

    // Finds prime numbers between a and b, using basic primes up to sqrt(b)
    // a must be greater than 1.
    vector<long long> seg_sieve(long long a, long long b) {
        long long ant = a;
        a = max(a, 3LL);
        vector<bool> pmap(b - a + 1);
        long long sqrt_b = sqrt(b);
        for (int i = 0; i < num_p; ++i) {
            long long p = primes[i];
            if (p > sqrt_b) break;
            long long j = (a + p - 1) / p;
            for (long long v = (j == 1) ? p + p : j * p; v <= b; v += p) {
                pmap[v - a] = true;
            }
        }
        vector<long long> ans;
        if (ant == 2) ans.push_back(2);
        int start = a % 2 ? 0 : 1;
        for (int i = start, I = b - a + 1; i < I; i += 2)
            if (pmap[i] == false)
                ans.push_back(a + i);
        return ans;
    }

    vector<pair<int, int>> factor(int n) {
        vector<pair<int, int>> ans;
        if (n == 0) return ans;
        for (int i = 0; primes[i] * primes[i] <= n; ++i) {
            if ((n % primes[i]) == 0) {
                int expo = 0;
                while ((n % primes[i]) == 0) {
                    expo++;
                    n /= primes[i];
                }
                ans.emplace_back(primes[i], expo);
            }
        }

        if (n > 1) {

```



```

    ans.emplace_back(n, 1);
}
return ans;
}
}

```

10.15 totient sieve

```

for (int i = 1; i < MN; i++)
    phi[i] = i;

for (int i = 1; i < MN; i++)
    if (!sieve[i]) // is prime
        for (int j = i; j < MN; j += i)
            phi[j] -= phi[j] / i;

```

10.16 totient

```

long long totient(long long n) {
    if (n == 1) return 0;
    long long ans = n;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            while ((n % primes[i]) == 0) n /= primes[i];
            ans -= ans / primes[i];
        }
    }
    if (n > 1) {
        ans -= ans / n;
    }
    return ans;
}

```

11 Strings

11.1 Incremental Aho Corasick

```

class IncrementalAhoCorasick {
    static const int Alphabets = 26;
    static const int AlphabetBase = 'a';
    struct Node {
        Node *fail;
        Node *next[Alphabets];
        int sum;
        Node() : fail(NULL), next{}, sum(0) { }
    };

    struct String {
        string str;
        int sign;
    };

public:
    //totalLen = sum of (len + 1)
    void init(int totalLen) {
        nodes.resize(totalLen);
        nNodes = 0;
        strings.clear();
        roots.clear();
        sizes.clear();
        que.resize(totalLen);
    }

    void insert(const string &str, int sign) {
        strings.push_back(String{ str, sign });
        roots.push_back(nodes.data() + nNodes);
        sizes.push_back(1);
        nNodes += (int)str.size() + 1;
        auto check = [&]() { return sizes.size() > 1 && sizes.end()[-1] ==
            sizes.end()[-2]; };
        if(!check())
            makePMA(strings.end() - 1, strings.end(), roots.back(), que);
        while(check()) {
            int m = sizes.back();
            roots.pop_back();
            sizes.pop_back();
            sizes.back() += m;
            if(!check())
                makePMA(strings.end() - m * 2, strings.end(), roots.back(), que);
        }
    }
}

```

```

}

int match(const string &str) const {
    int res = 0;
    for(const Node *t : roots)
        res += matchPMA(t, str);
    return res;
}

private:
static void makePMA(vector<String>::const_iterator begin,
    vector<String>::const_iterator end, Node *nodes, vector<Node*> &que) {
    int nNodes = 0;
    Node *root = new(&nodes[nNodes++]) Node();
    for(auto it = begin; it != end; ++ it) {
        Node *t = root;
        for(char c : it->str) {
            Node *&n = t->next[c - AlphabetBase];
            if(n == nullptr)
                n = new(&nodes[nNodes++]) Node();
            t = n;
        }
        t->sum += it->sign;
    }
    int qt = 0;
    for(Node *&n : root->next) {
        if(n != nullptr) {
            n->fail = root;
            que[qt++] = n;
        } else {
            n = root;
        }
    }
    for(int qh = 0; qh != qt; ++ qh) {
        Node *t = que[qh];
        int a = 0;
        for(Node *n : t->next) {
            if(n != nullptr) {
                que[qt++] = n;
                Node *r = t->fail;
                while(r->next[a] == nullptr)
                    r = r->fail;
                n->fail = r->next[a];
                n->sum += r->next[a]->sum;
            }
        }
    }
}

```

```

    }
    ++ a;
}
}

static int matchPMA(const Node *t, const string &str) {
    int res = 0;
    for(char c : str) {
        int a = c - AlphabetBase;
        while(t->next[a] == nullptr)
            t = t->fail;
        t = t->next[a];
        res += t->sum;
    }
    return res;
}

vector<Node> nodes;
int nNodes;
vector<String> strings;
vector<Node*> roots;
vector<int> sizes;
vector<Node*> que;
};

int main() {
    int m;
    while(~scanf("%d", &m)) {
        IncrementalAhoCorasic iac;
        iac.init(600000);
        rep(i, m) {
            int ty;
            char s[300001];
            scanf("%d%s", &ty, s);
            if(ty == 1) {
                iac.insert(s, +1);
            } else if(ty == 2) {
                iac.insert(s, -1);
            } else if(ty == 3) {
                int ans = iac.match(s);
                printf("%d\n", ans);
                fflush(stdout);
            }
        }
    }
}

```

```

    } else {
        abort();
    }
}
return 0;
}

```

11.2 minimal string rotation

```
// Lexicographically minimal string rotation
```

```

int lmsr() {
    string s;
    cin >> s;
    int n = s.size();
    s += s;
    vector<int> f(s.size(), -1);
    int k = 0;
    for (int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while (i != -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) {
                k = j - i - 1;
                i = f[i];
            }
            if (i == -1 && s[j] != s[k + i + 1]) {
                if (s[j] < s[k + i + 1]) {
                    k = j;
                }
                f[j - k] = -1;
            } else {
                f[j - k] = i + 1;
            }
        }
    }
    return k;
}

```

11.3 suffix array

```

/**
 * O (n log^2 (n))
 * See http://web.stanford.edu/class/cs97si/suffix-array.pdf for reference
 */

struct entry{
    int a, b, p;
    entry(){}
    entry(int x, int y, int z): a(x), b(y), p(z){}
    bool operator < (const entry &o) const {
        return (a == o.a) ? (b == o.b) ? (p < o.p) : (b < o.b) : (a < o.a);
    }
};

struct SuffixArray{
    const int N;
    string s;
    vector<vector<int> > P;
    vector<entry> M;

    SuffixArray(const string &s) : N(s.length()) , s(s), P(1, vector<int> (N,
        0)), M(N) {
        for (int i = 0; i < N; ++i)
            P[0][i] = (int) s[i];

        for (int skip = 1, level = 1; skip < N; skip *= 2, level++) {
            P.push_back(vector<int>(N, 0));
            for (int i = 0 ; i < N; ++i) {
                int next = ((i + skip) < N) ? P[level - 1][i + skip] : -10000;
                M[i] = entry(P[level - 1][i], next, i);
            }
            sort(M.begin(), M.end());
            for (int i = 0; i < N; ++i)
                P[level][M[i].p] = (i > 0 and M[i].a == M[i - 1].a and M[i].b == M[i -
                    1].b) ? P[level][M[i - 1].p] : i;
        }
    }

    vector<int> getSuffixArray(){
        vector<int> &rank = P.back();
        vector<pair<int, int> > inv(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            inv[i] = make_pair(rank[i], i);
        sort(inv.begin(), inv.end());
    }
}

```

```

vector<int> sa(rank.size());
for (int i = 0; i < rank.size(); ++i)
    sa[i] = inv[i].second;
return sa;
}

// returns the length of the longest common prefix of s[i...L-1] and
// s[j...L-1]
int lcp(int i, int j) {
    int len = 0;
    if (i == j) return N - i;
    for (int k = P.size() - 1; k >= 0 && i < N && j < N; --k) {
        if (P[k][i] == P[k][j]) {
            i += 1 << k;
            j += 1 << k;
            len += 1 << k;
        }
    }
    return len;
}
};

```

11.4 suffix automaton

```

/*
 * Suffix automaton:
 * This implementation was extended to maintain (online) the
 * number of different substrings. This is equivalent to compute
 * the number of paths from the initial state to all the other
 * states.
 *
 * The overall complexity is O(n)
 * can be tested here:
 *   https://www.urionlinejudge.com.br/judge/en/problems/view/1530
 */

struct state {
    int len, link;
    long long num_paths;
    map<int, int> next;
};

```

```

const int MN = 200011;
state sa[MN << 1];
int sz, last;
long long tot_paths;

void sa_init() {
    sz = 1;
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    sa[0].next.clear();
    sa[0].num_paths = 1;
    tot_paths = 0;
}

void sa_extend(int c) {
    int cur = sz++;
    sa[cur].len = sa[last].len + 1;
    sa[cur].next.clear();
    sa[cur].num_paths = 0;
    int p;
    for (p = last; p != -1 && !sa[p].next.count(c); p = sa[p].link) {
        sa[p].next[c] = cur;
        sa[cur].num_paths += sa[p].num_paths;
        tot_paths += sa[p].num_paths;
    }

    if (p == -1) {
        sa[cur].link = 0;
    } else {
        int q = sa[p].next[c];
        if (sa[p].len + 1 == sa[q].len) {
            sa[cur].link = q;
        } else {
            int clone = sz++;
            sa[clone].len = sa[p].len + 1;
            sa[clone].next = sa[q].next;
            sa[clone].num_paths = 0;
            sa[clone].link = sa[q].link;
            for (; p != -1 && sa[p].next[c] == q; p = sa[p].link) {
                sa[p].next[c] = clone;
                sa[q].num_paths -= sa[p].num_paths;
                sa[clone].num_paths += sa[p].num_paths;
            }
        }
    }
}

```

```

    sa[q].link = sa[cur].link = clone;
}
}
last = cur;
}

```

11.5 z algorithm

```

using namespace std;
#include<bits/stdc++.h>

vector<int> compute_z(const string &s){
    int n = s.size();
    vector<int> z(n,0);
    int l,r;
    r = l = 0;
    for(int i = 1; i < n; ++i){
        if(i > r) {
            l = r = i;
            while(r < n and s[r - 1] == s[r])r++;
            z[i] = r - l;r--;
        }else{
            int k = i-l;
            if(z[k] < r - i +1) z[i] = z[k];
            else {

```

```

                l = i;
                while(r < n and s[r - 1] == s[r])r++;
                z[i] = r - l;r--;
            }
        }
        return z;
    }

int main(){

    //string line;cin>>line;
    string line = "alfalfa";
    vector<int> z = compute_z(line);

    for(int i = 0; i < z.size(); ++i ){
        if(i)cout<<" ";
        cout<<z[i];
    }
    cout<<endl;

    // must print "0 0 0 4 0 0 1"

    return 0;
}

```
