Advanced Computer Graphics

Lecture-03 Math and Algebra

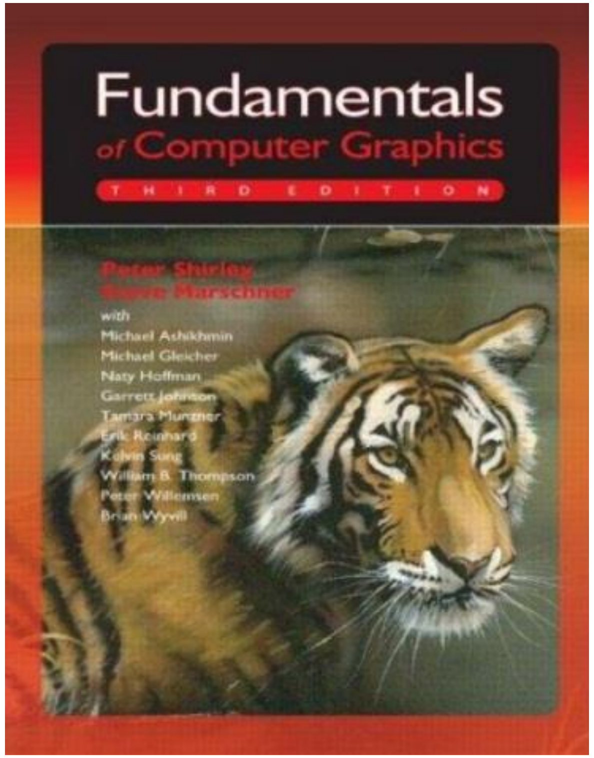
Tzung-Han Lin
National Taiwan University of Science and Technology
Graduate Institute of Color and Illumination Technology
e-mail: thl@mail.ntust.edu.tw

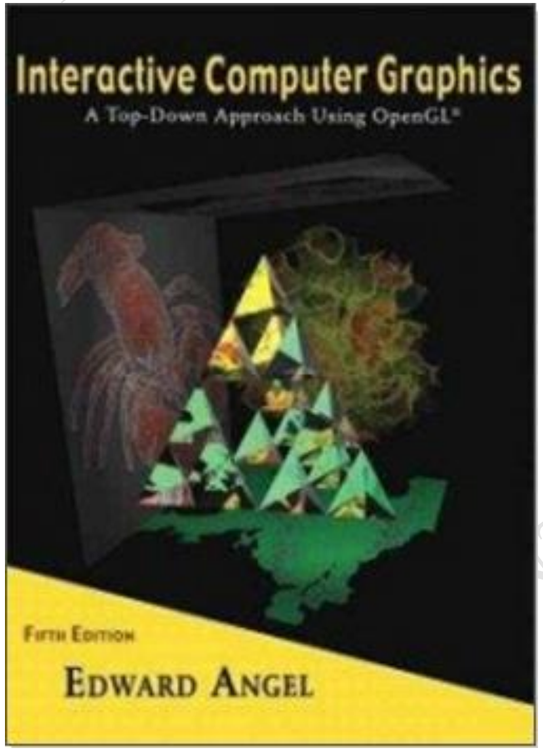
TAIWAN TECH National Taiwan University of Science and Technology

CIT 色彩與照明科技研究所 Graduate Institute of Color and Illumination Technology

Content in textbook

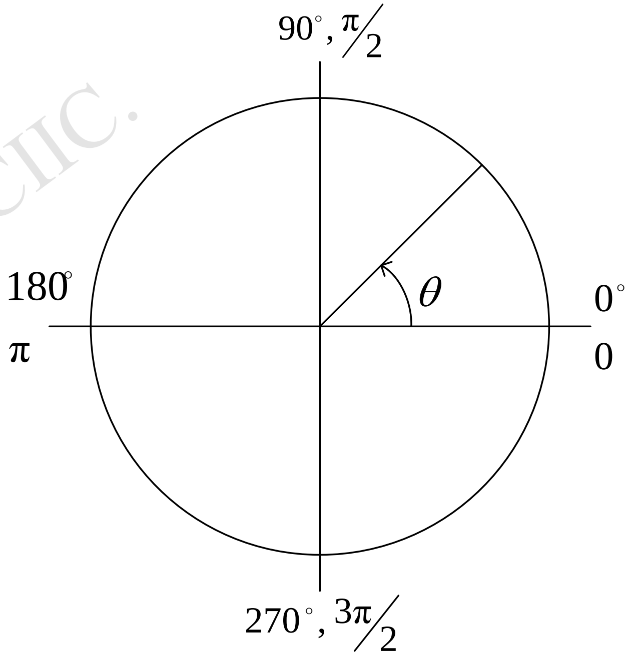
- Fundamentals of Computer Graphics: Chapter 2.

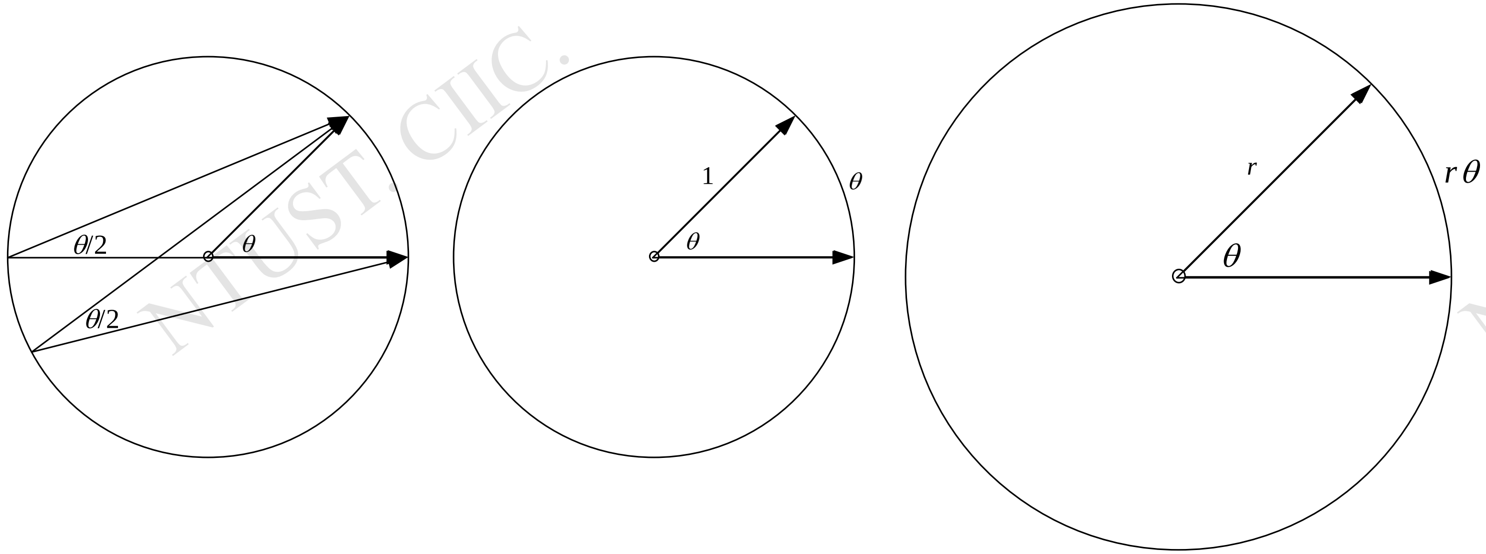




Trigonometry

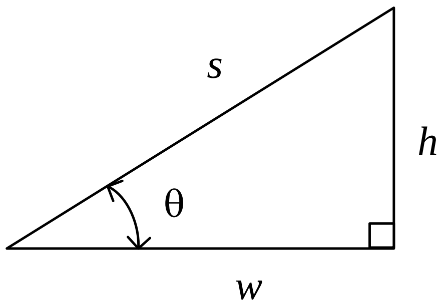
- Angles
- 1 radian =57.2957795 degrees
- 1 degree =0.0174532925 radian
- π=3.1415926574





Trigonometry

- Trigonometric functions

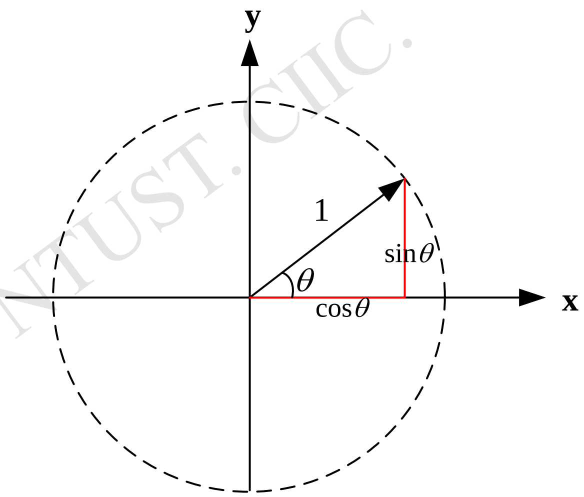


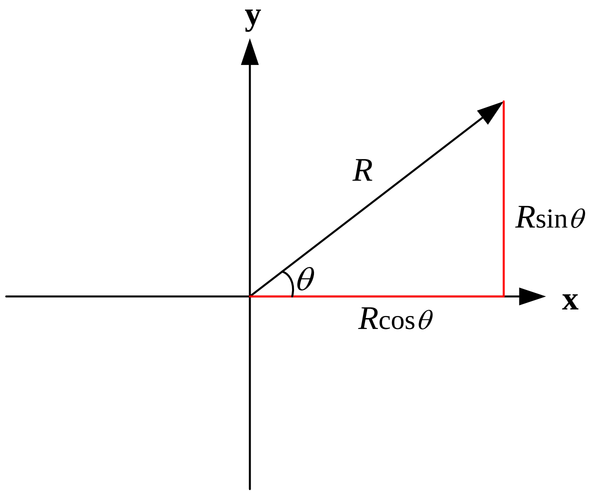
$$s^2 = h^2 + w^2$$

$$\sin\theta = \frac{h}{s} = \frac{h}{\sqrt{h^2 + w^2}}$$

$$\cos\theta = \frac{w}{s} = \frac{w}{\sqrt{h^2 + w^2}}$$

$$\tan\theta = \frac{h}{w}$$

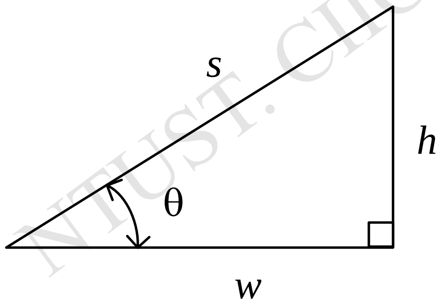




Trigonometry

Pythagorean identity

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A - \tan^2 A = 1$$
$$\csc^2 A - \cot^2 A = 1$$



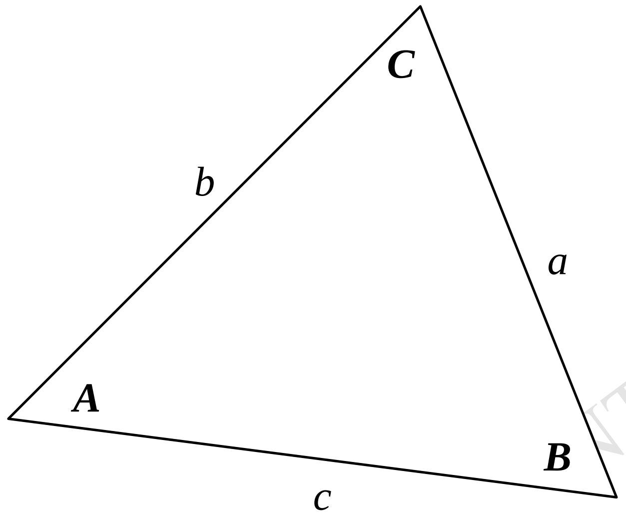
Law of sines

$$\frac{a}{\sin \mathbf{A}} = \frac{b}{\sin \mathbf{B}} = \frac{c}{\sin \mathbf{C}} = 2R$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



A, ***B***, ***C*** indicate angle
a, *b*, *c* represent edge-length

Trigonometry

Law of cosines: Proof

$\triangle ABC$中，$\overline{AB}=c$，$\overline{BC}=a$，$\overline{AC}=b$：

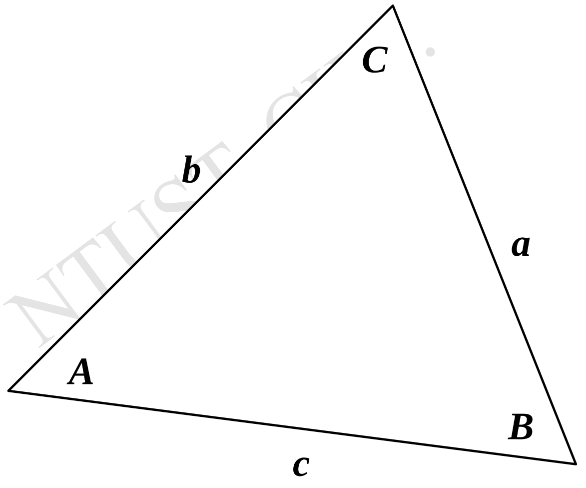
$$\left|\vec{BC}\right|^2=\vec{BC}\cdot\vec{BC}$$

$$\left|\vec{BC}\right|^2=(\vec{AC}-\vec{AB})(\vec{AC}-\vec{AB})$$

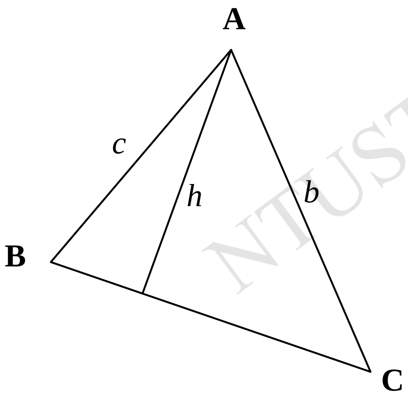
$$\left|\vec{BC}\right|^2=\left|\vec{AC}\right|^2+\left|\vec{AB}\right|^2-2\vec{AB}\vec{AC}$$

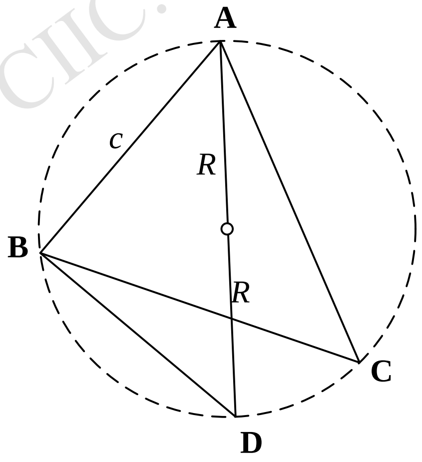
$$\left|\vec{BC}\right|^2=\left|\vec{AC}\right|^2+\left|\vec{AB}\right|^2-2\left|\vec{AB}\right|\left|\vec{AC}\right|\cos A$$

$$a^2=b^2+c^2-2bc\cos A$$



Law of sines: Proof





Since $c\sin\mathbf{B}=h=b\sin\mathbf{C}$

$$\frac{c}{\sin\mathbf{C}}=\frac{b}{\sin\mathbf{B}}$$

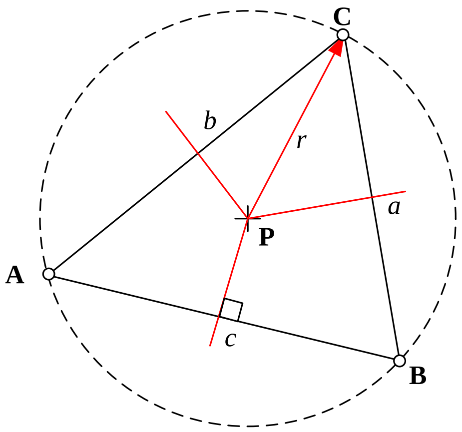
And, $\dfrac{c}{\sin\mathbf{C}}=\dfrac{b}{\sin\mathbf{B}}=\dfrac{a}{\sin\mathbf{A}}$

$$2R\sin\mathbf{D}=2R\sin\mathbf{C}=c$$

$$2R=\frac{c}{\sin\mathbf{C}}$$

http://zh.wikipedia.org/wiki/%E9%A4%98%E5%BC%A6%E5%AE%9A%E7%90%86

Trigonometry



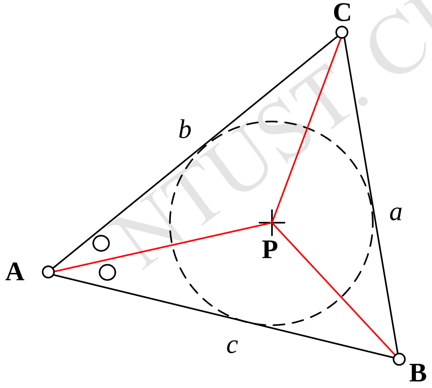
circumcentre (外心)

$$r = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}, \quad s = \frac{(a+b+c)}{2}$$

$$P_x = \frac{(x_A - x_B)(x_C^2 + y_C^2 - x_B^2 - y_B^2) - (x_C - x_B)(x_A^2 + y_A^2 - x_B^2 - y_B^2)}{2(x_A - x_B)(y_C - y_B) - 2(y_A - y_B)(x_C - x_B)}$$

$$P_y = \frac{(y_C - y_B)(x_A^2 + y_A^2 - x_B^2 - y_B^2) - (y_A - y_B)(x_C^2 + y_C^2 - x_B^2 - y_B^2)}{2(x_A - x_B)(y_C - y_B) - 2(y_A - y_B)(x_C - x_B)}$$

A, **B**, **C** indicate vertexes
a, *b*, *c* represent edge-length

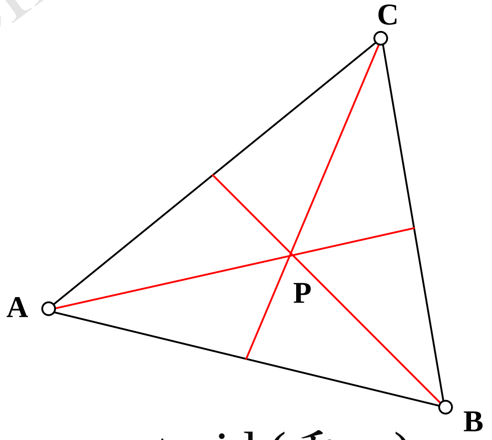


In centre (內心)

$$\mathbf{P} = \frac{a \cdot \mathbf{A} + b \cdot \mathbf{B} + c \cdot \mathbf{C}}{a+b+c}$$

A, **B**, **C** indicate vertexes
a, *b*, *c* represent edge-length

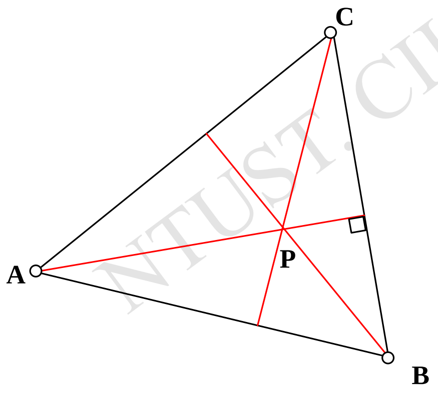
Trigonometry



centroid (重心)

$$\mathbf{P} = \frac{(\mathbf{A}+\mathbf{B}+\mathbf{C})}{3}$$

A, **B**, **C** indicate vertex coordinate



orthocentre垂心

$$\mathbf{P} = \frac{\alpha\mathbf{A}+\beta\mathbf{B}+\gamma\mathbf{C}}{\alpha+\beta+\gamma}$$

$$\alpha = (\overrightarrow{\mathbf{BC}}\cdot\overrightarrow{\mathbf{CA}})(\overrightarrow{\mathbf{BC}}\cdot\overrightarrow{\mathbf{AB}})$$

$$\beta = (\overrightarrow{\mathbf{CA}}\cdot\overrightarrow{\mathbf{AB}})(\overrightarrow{\mathbf{CA}}\cdot\overrightarrow{\mathbf{BC}})$$

$$\gamma = (\overrightarrow{\mathbf{AB}}\cdot\overrightarrow{\mathbf{BC}})(\overrightarrow{\mathbf{AB}}\cdot\overrightarrow{\mathbf{CA}})$$

A, **B**, **C** indicate vertex coordinate

Trigonometry

- Addition and subtraction identities
- Half-angle identities
- Product identities

$$\begin{aligned}
\sin(A+B) &= \sin A\cos B + \sin B\cos A \\
\sin(A-B) &= \sin A\cos B - \sin B\cos A \\
\sin(2A) &= 2\sin A\cos A \\
\cos(A+B) &= \cos A\cos B - \sin A\sin B \\
\cos(A-B) &= \cos A\cos B + \sin A\sin B \\
\cos(2A) &= \cos^2 A - \sin^2 A \\
\tan(A+B) &= \frac{\tan A + \tan B}{1-\tan A\tan B} \\
\tan(A-B) &= \frac{\tan A - \tan B}{1+\tan A\tan B} \\
\tan(2A) &= \frac{2\tan A}{1-\tan^2 A}
\end{aligned}$$

$$\begin{aligned}
\sin^2(A/2) &= (1-\cos A)/2 \\
\cos^2(A/2) &= (1+\cos A)/2
\end{aligned}$$

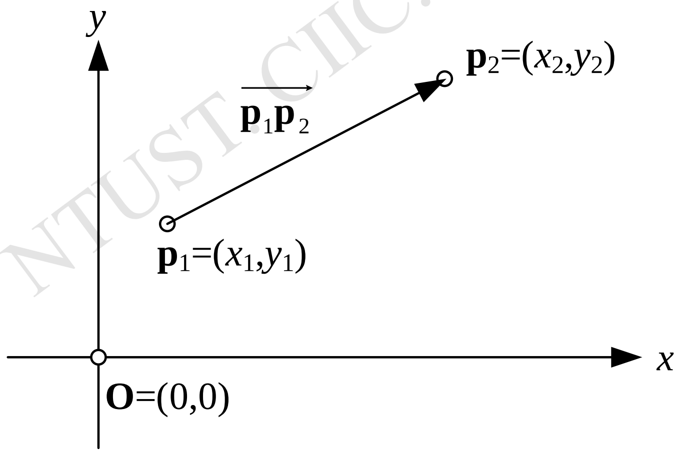
$$\begin{aligned}
\sin A\sin B &= -(\cos(A+B) - \cos(A-B))/2 \\
\sin A\cos B &= \quad (\sin(A+B) + \sin(A-B))/2 \\
\cos A\cos B &= \quad (\cos(A+B) + \cos(A-B))/2
\end{aligned}$$

$$\text{triangle area} = \frac{1}{2}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}$$

Vector operation

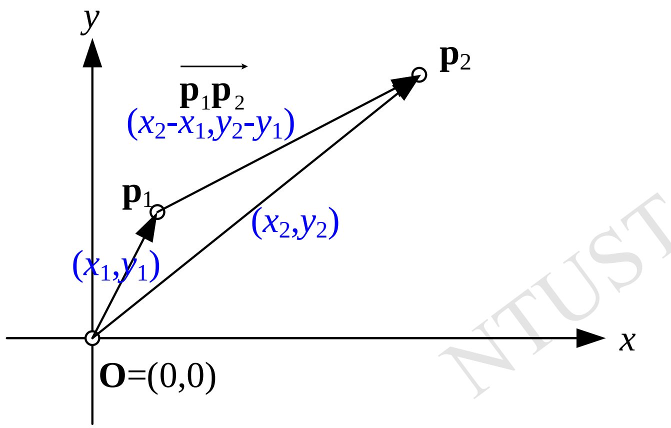
$\mathbf{p}_1=(x_1, y_1)$: a vector $\mathbf{p}_1$ relative to (0, 0) has an increment x_1 in x-axis, and y_1 in y-axis

In other words, $\mathbf{p}_1$ is equal to the vector $\overrightarrow{\mathbf{Op}_1}$, which starts from $\mathbf{O}$ and stops at $\mathbf{p}_1$

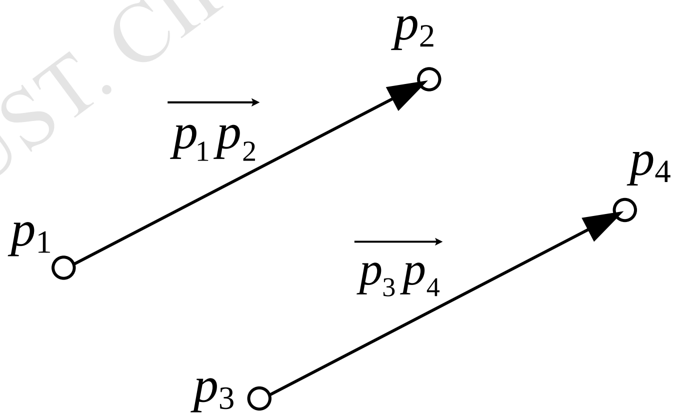


$\overrightarrow{\mathbf{p}_1\mathbf{p}_2}$

is the vector starts from $\mathbf{p}_1$ and stops at $\mathbf{p}_2$

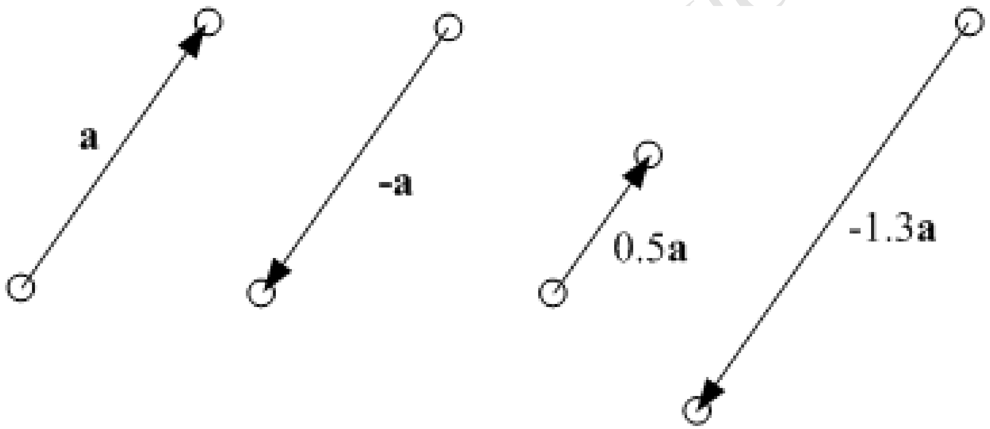


Vector operation



A vector includes the “direction” and its length

These two vectors have same value, however they have different beginning and different destination.



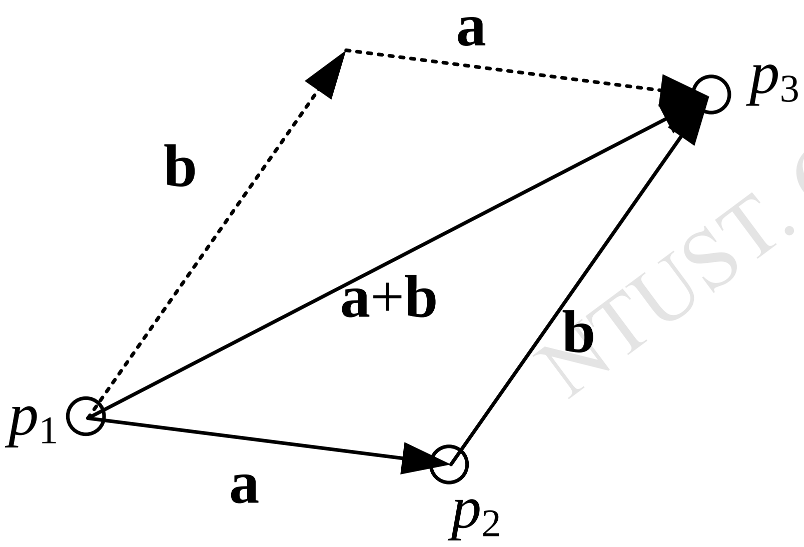
$$2D: \ |\mathbf{a}| = \sqrt{x^2 + y^2}$$

$$3D: \ |\mathbf{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$4D: \ |\mathbf{a}| = \sqrt{x^2 + y^2 + z^2 + w^2}$$

Vector operation

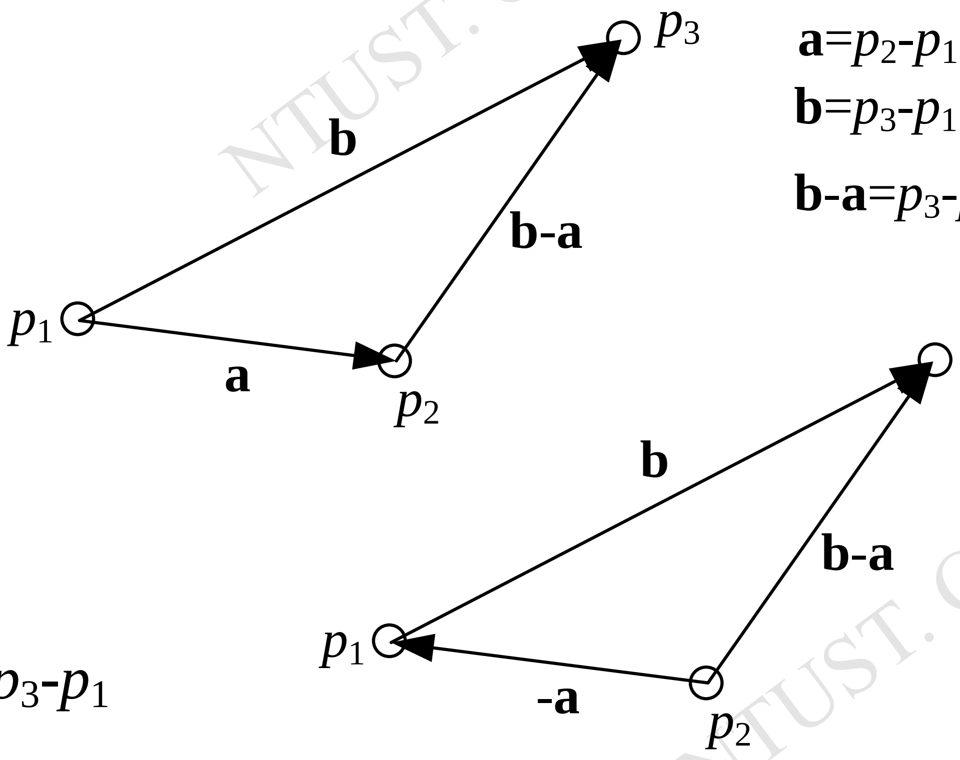
- Vector addition and subtraction
  - Parallelogram rule



$\mathbf{a}=p_2-p_1$

$\mathbf{b}=p_3-p_2$

$\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}=p_3-p_1$



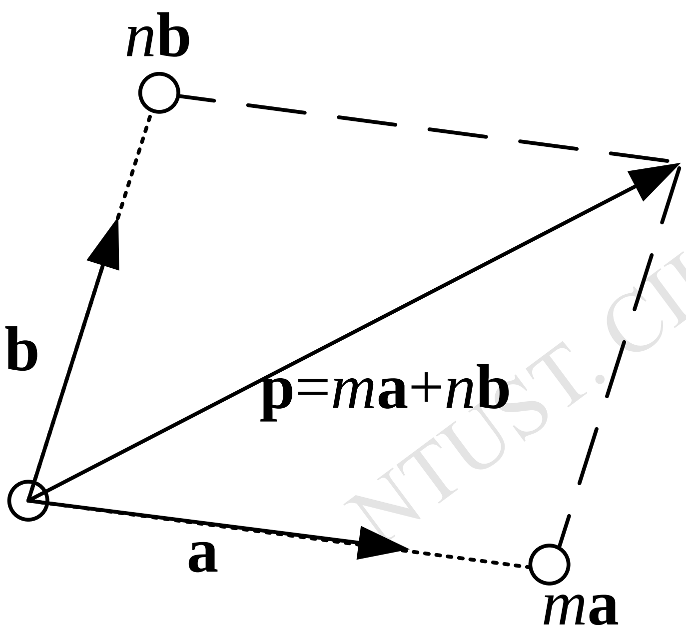
$\mathbf{a}=p_2-p_1$

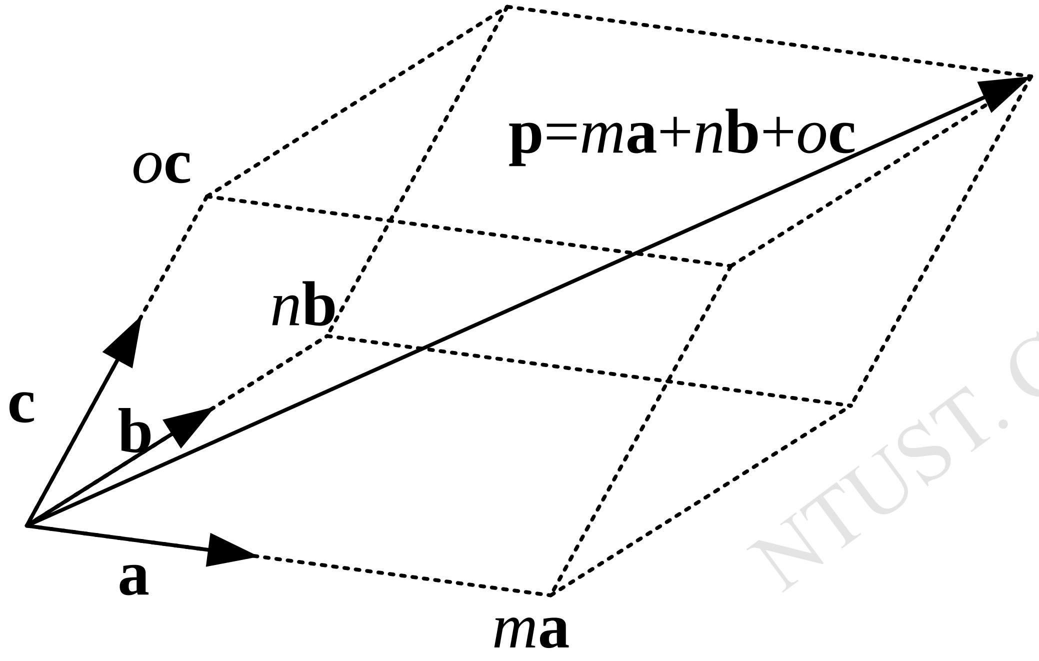
$\mathbf{b}=p_3-p_1$

$\mathbf{b}-\mathbf{a}=p_3-p_1=-\mathbf{a}+\mathbf{b}$

Vector operation

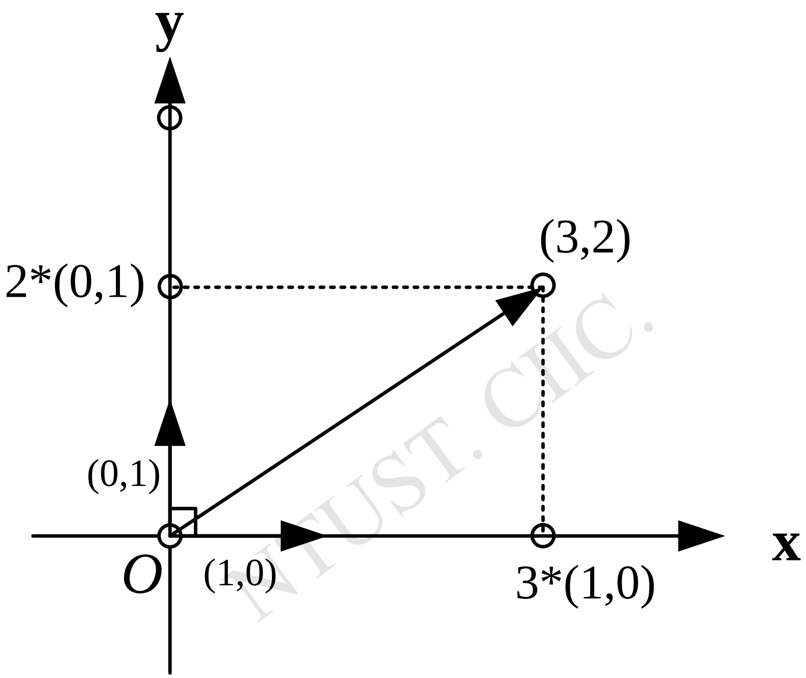
- Vector in general coordination
  - A 2D vector consists of two independent bases

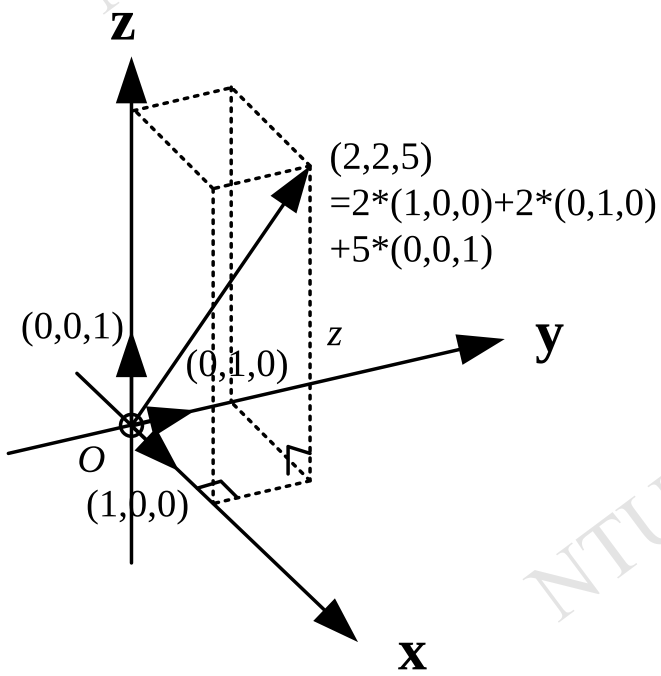




Vector operation

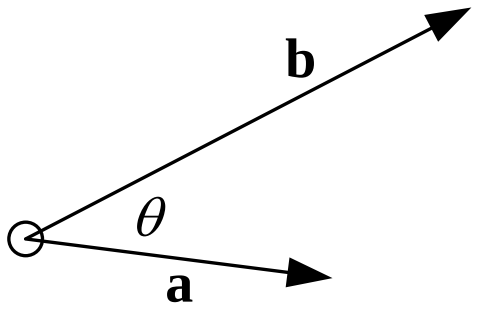
- Vector in Cartesian coordination
  - A 2D vector consists of two independent basis

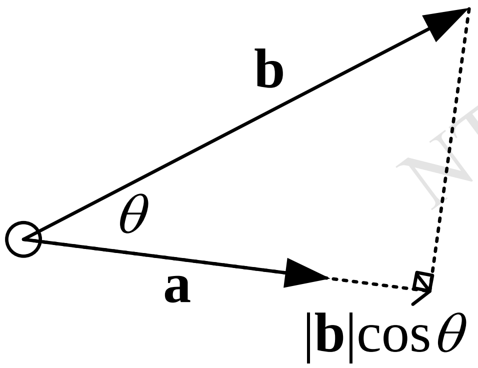


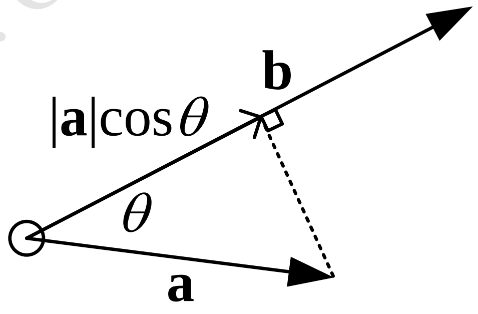


Vector operation

- Inner product (dot product)







$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

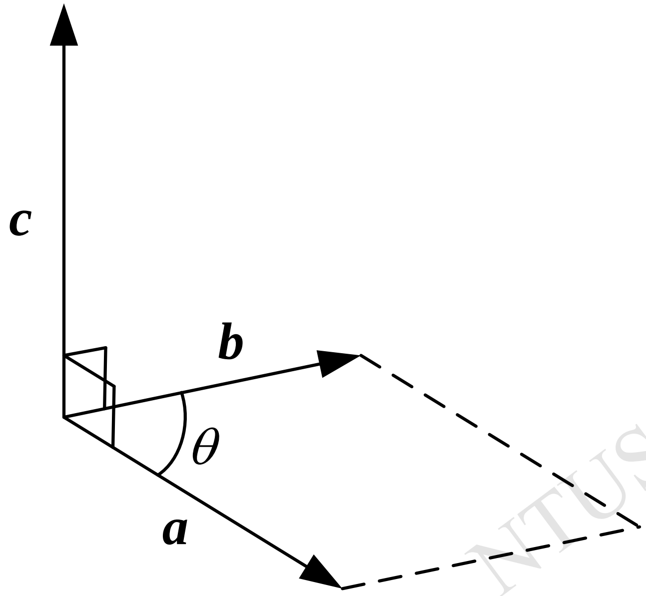
$$\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$$

$$\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$$

$$(k\mathbf{a})\cdot\mathbf{b} = \mathbf{a}\cdot(k\mathbf{b}) = k\mathbf{a}\cdot\mathbf{b}$$

Vector operation

- Cross product (outer product)



$$\mathbf{a} = [x_a, y_a, z_a]$$

$$\mathbf{b} = [x_b, y_b, z_b]$$

$$\mathbf{c} = \mathbf{a} \times \mathbf{b} = \begin{bmatrix} i & j & k \\ x_a & y_a & z_a \\ x_b & y_b & z_b \end{bmatrix} = (y_a z_b - y_b z_a)i + (x_b z_a - x_a z_b)j + (x_a y_b - x_b y_a)k$$

$$= [(y_a z_b - y_b z_a), (x_b z_a - x_a z_b), (x_a y_b - x_b y_a)]$$

Vector operation

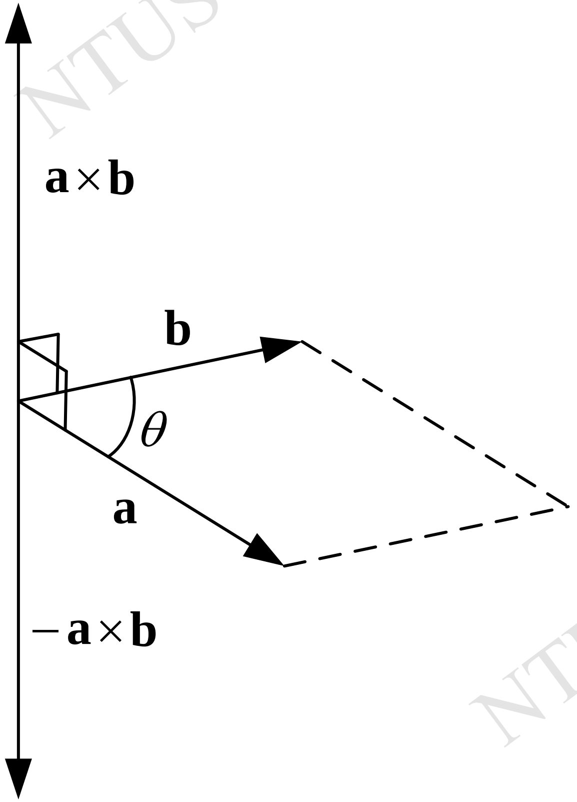
- Direction of cross production

$$|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}|\cdot|\mathbf{b}|\sin\theta$$

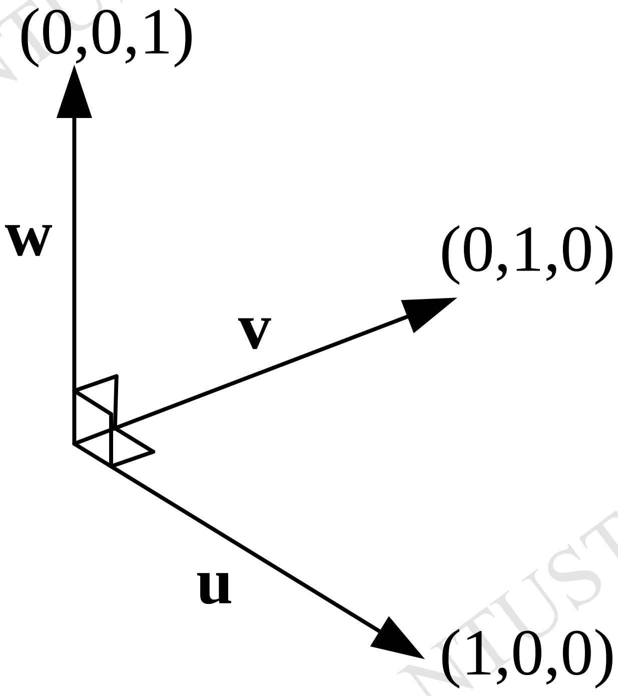
$$\mathbf{a}\times\mathbf{b}=-(\mathbf{b}\times\mathbf{a})$$

$$\mathbf{a}\times(\mathbf{b}+\mathbf{c})=\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$$

$$\mathbf{a}\times(k\mathbf{b})=(k\mathbf{a})\times\mathbf{b}=k(\mathbf{a}\times\mathbf{b})$$



Vector operation



Alternation (right-hand rule)

u → v → w

Orthogonal

$$\mathbf{u}\times\mathbf{v}=\mathbf{w} \qquad \mathbf{u}\cdot\mathbf{v}=0$$

$$\mathbf{v}\times\mathbf{w}=\mathbf{u} \qquad \mathbf{v}\cdot\mathbf{w}=0$$

$$\mathbf{w}\times\mathbf{u}=\mathbf{v} \qquad \mathbf{w}\cdot\mathbf{u}=0$$

$$\mathbf{v}\times\mathbf{u}=-\mathbf{w} \qquad \mathbf{v}\cdot\mathbf{u}=0$$

$$\mathbf{w}\times\mathbf{v}=-\mathbf{u} \qquad \mathbf{w}\cdot\mathbf{v}=0$$

$$\mathbf{u}\times\mathbf{w}=-\mathbf{v} \qquad \mathbf{u}\cdot\mathbf{w}=0$$

Vector operation

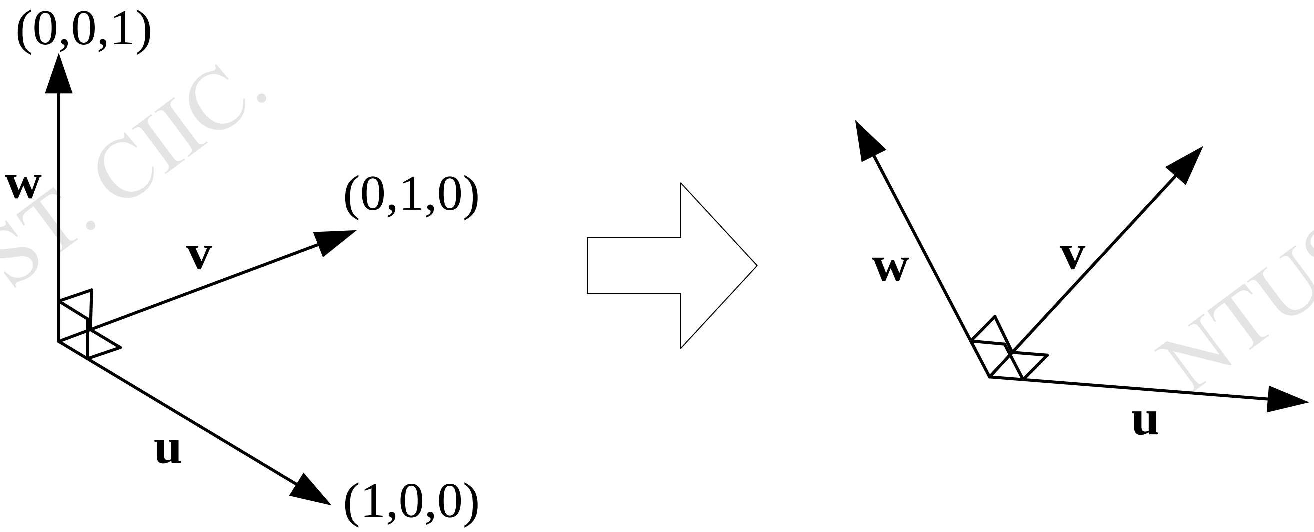
- Unit vector

$$\mathbf{w} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

- Dot product of two orthogonal vectors

$$\mathbf{u} \cdot \mathbf{v} = 0$$
$$\mathbf{v} \cdot \mathbf{w} = 0$$
$$\mathbf{w} \cdot \mathbf{u} = 0$$

$$\mathbf{v} \cdot \mathbf{u} = 0$$
$$\mathbf{w} \cdot \mathbf{v} = 0$$
$$\mathbf{u} \cdot \mathbf{w} = 0$$



Fundamental geometry & algebra

- Line equation
  - Implicit format

$$y = mx + b$$

  - If slope m and vertex (x0, y0) on line are known

$$y - y_0 = m(x - x_0)$$

$$y = \underbrace{mx}_{\text{known}} + (\underbrace{y_0 - mx_0}_{\text{known}})$$

Fundamental geometry & algebra

- Line equation by an independent parameter
  - The original format

$$ax+by+c=0 \quad \text{or} \quad (a,b,c)\cdot(x,y,1)=0$$

  - can be rewritten as

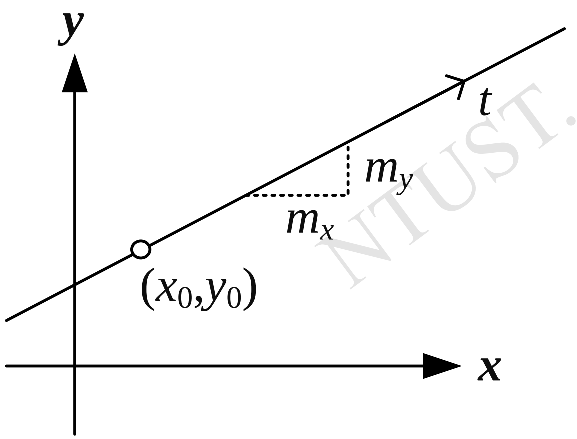
$$x = x_0 + m_x t$$

$$y = y_0 + m_y t$$

  - t is a parameter could be the distance

$$x = x_0 + \frac{m_x}{\sqrt{m_x^{\,2} + m_y^{\,2}}} t'$$

$$y = y_0 + \frac{m_y}{\sqrt{m_x^{\,2} + m_y^{\,2}}} t'$$



Fundamental geometry & algebra

- Line go through two points (x1,y1) and (x2,y2)

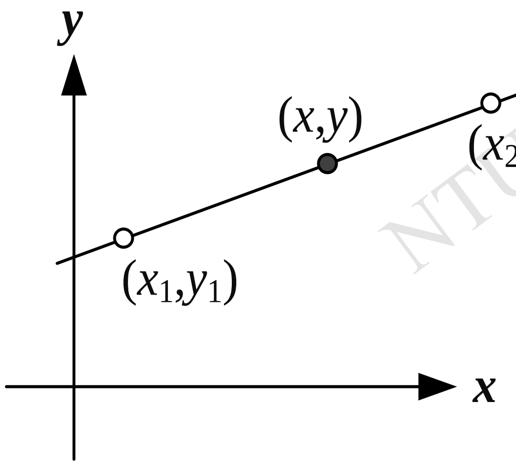
$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$(y_2-y_1)(x-x_1)-(x_2-x_1)(y-y_1)=0$$

$$(y_2-y_1)x-(x_2-x_1)y+[-(y_2-y_1)x_1+(x_2-x_1)y_1]=0$$

If $x_2-x_1\neq 0$

$$y=\frac{y_2-y_1}{x_2-x_1}x+y_1-\frac{y_2-y_1}{x_2-x_1}x_1$$



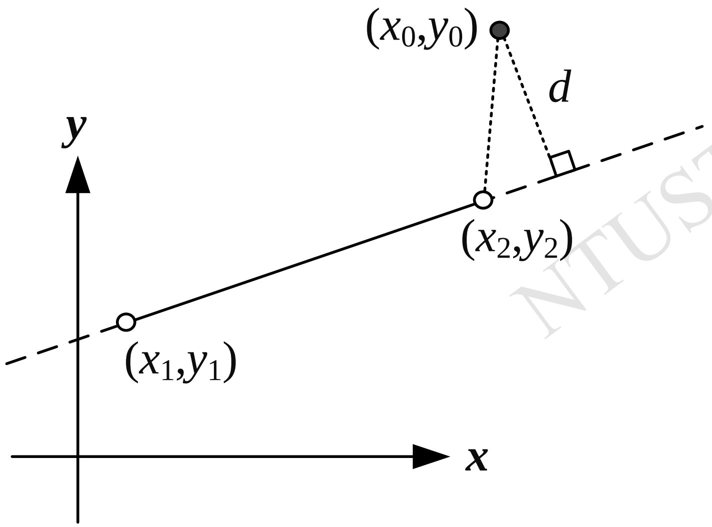
Fundamental geometry & algebra

- Distance from a point to one line

$$d = \frac{|(y_2 - y_1)x_0 - (x_2 - x_1)y_0 + [-(y_2 - y_1)x_1 + (x_2 - x_1)y_1]|}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}$$

Line equation: $ax+by+c=0$

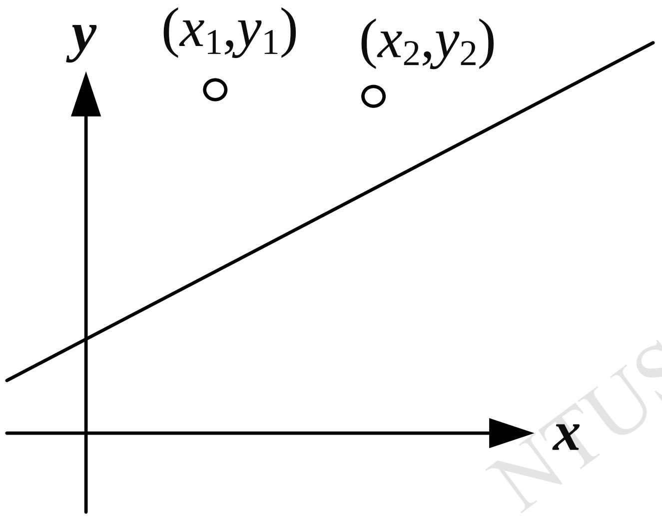
$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

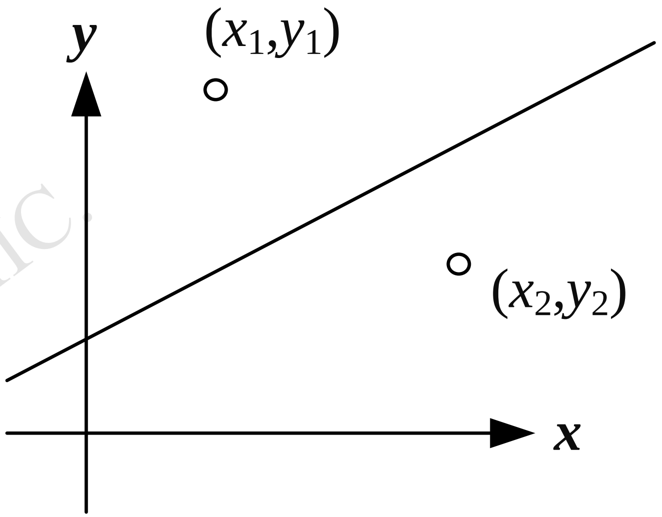


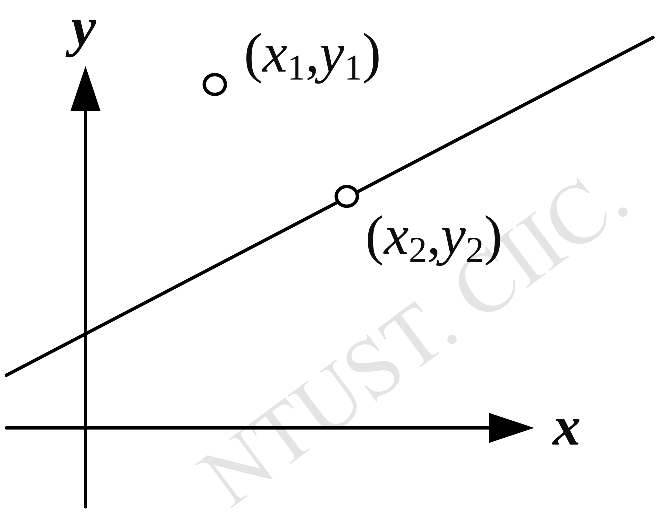
Fundamental geometry & algebra

- The relation of two points and one line

$$ax+by+c=0$$



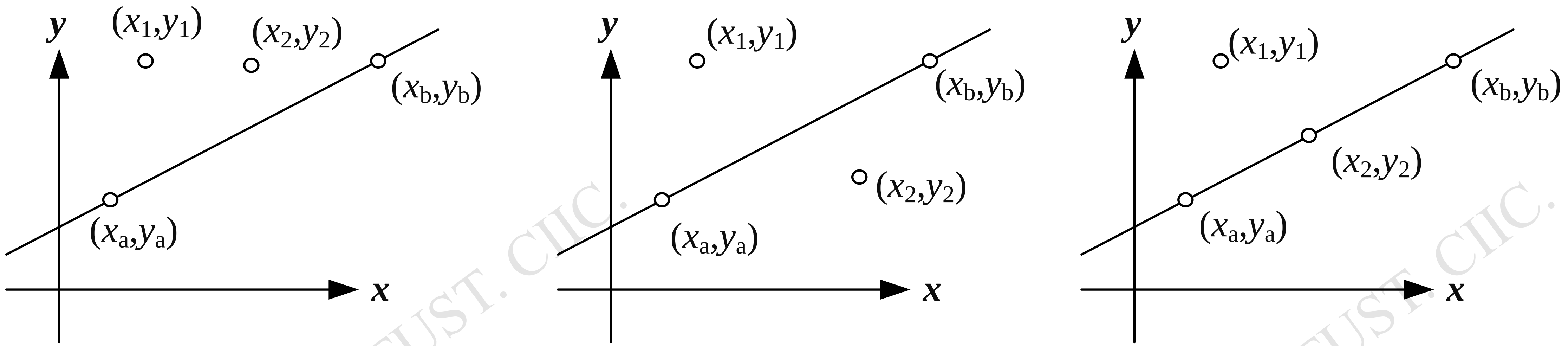




$$(ax_1+by_1+c)(ax_2+by_2+c)>0 \quad (ax_1+by_1+c)(ax_2+by_2+c)<0 \quad (ax_1+by_1+c)(ax_2+by_2+c)=0$$

Fundamental geometry & algebra

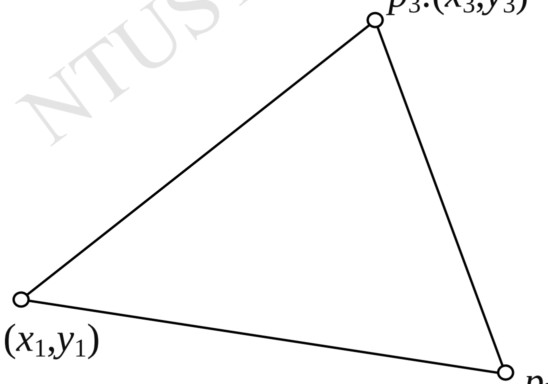
- Relation of two lines
  - Intersection of two line



$$\{(y_b - y_a)x_1 - (x_b - x_a)y_1 + [-(y_b - y_a)x_a + (x_b - x_a)y_a]\}\cdot$$
$$\{(y_b - y_a)x_2 - (x_b - x_a)y_2 + [-(y_b - y_a)x_a + (x_b - x_a)y_a]\}$$

Fundamental geometry & algebra

- Area of a triangle
  - Cross-production solution
  - Algebra solution

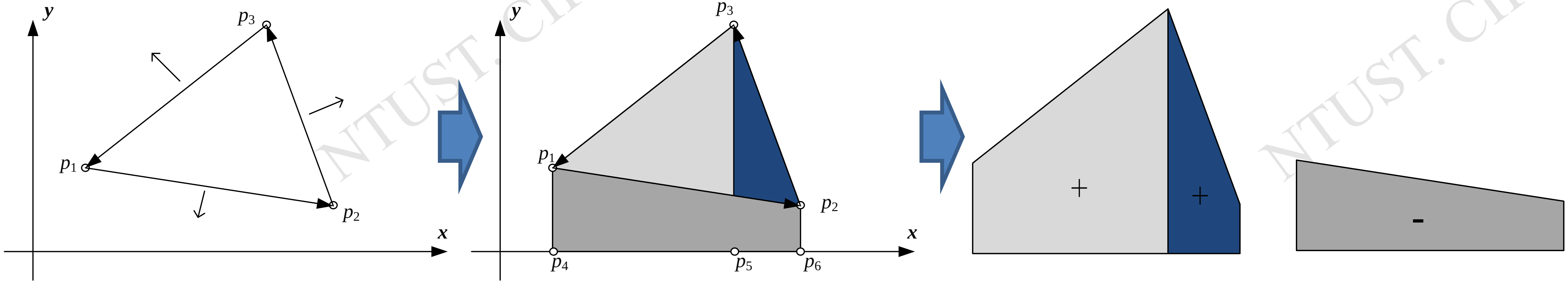


$$\Delta = \frac{1}{2}\left|\overrightarrow{p_1p_2} \times \overrightarrow{p_1p_3}\right|$$

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - y_2x_3 - y_1x_2 - x_1y_3)$$

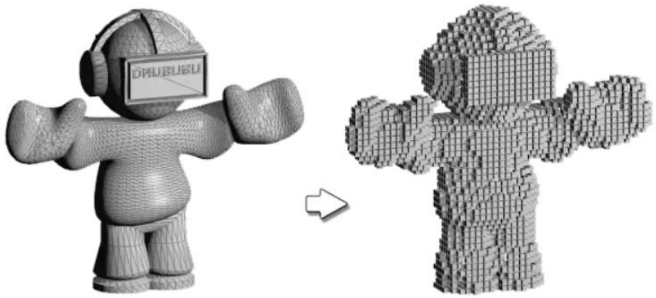
Fundamental geometry & algebra

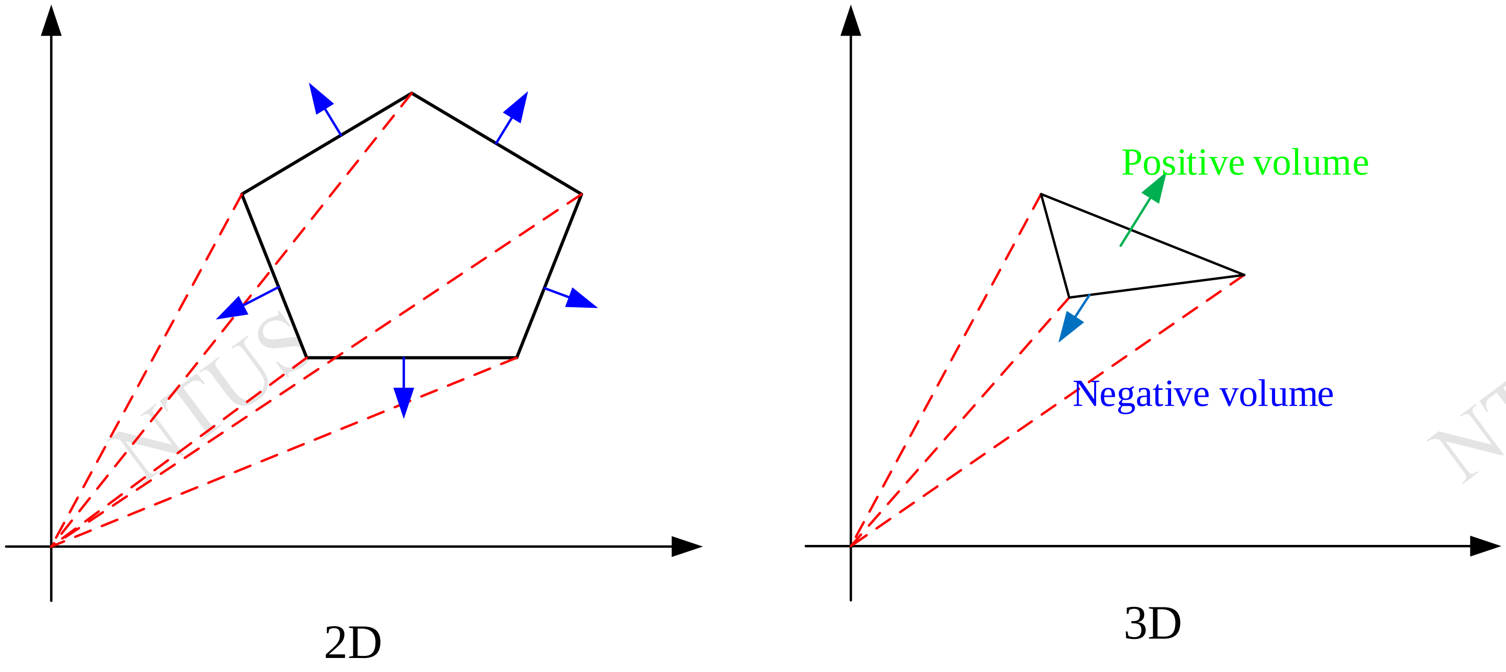
- Area of a triangle
  - Decompose the shape into several parts
  - The "up" part will give a positive area value, and the "down" part has negative area value. (similar to boolean operator)
  - Very useful for discrete and complex shape (must be close-volume)



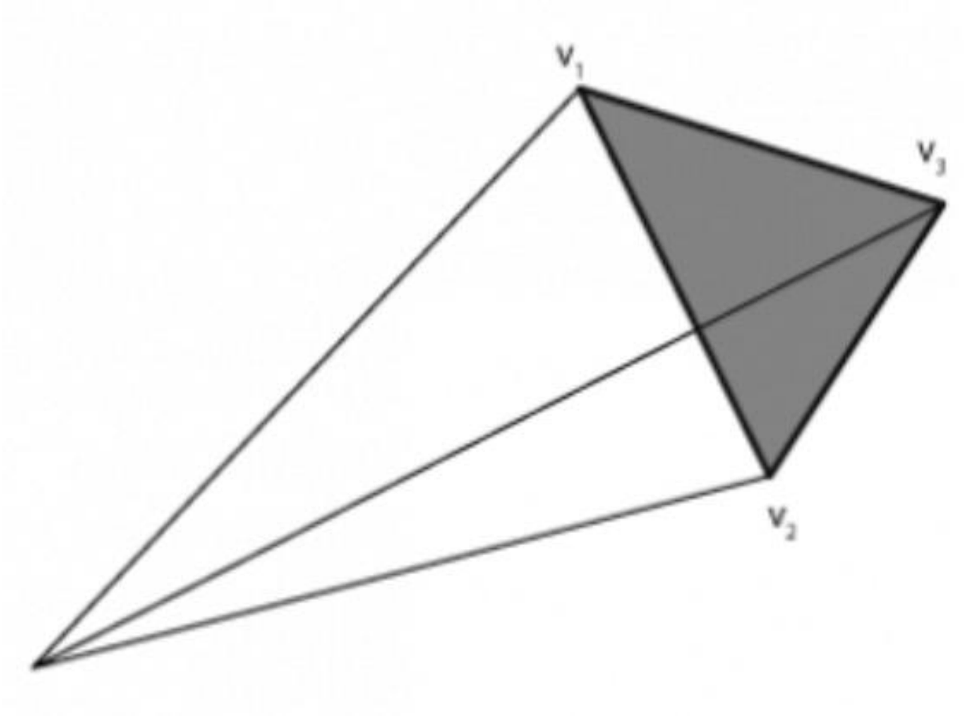
Fundamental geometry & algebra

- Volume of a close-volume mesh
  - Similar to above 2D method instead of triangular prism
  - By pyramid





Fundamental geometry & algebra



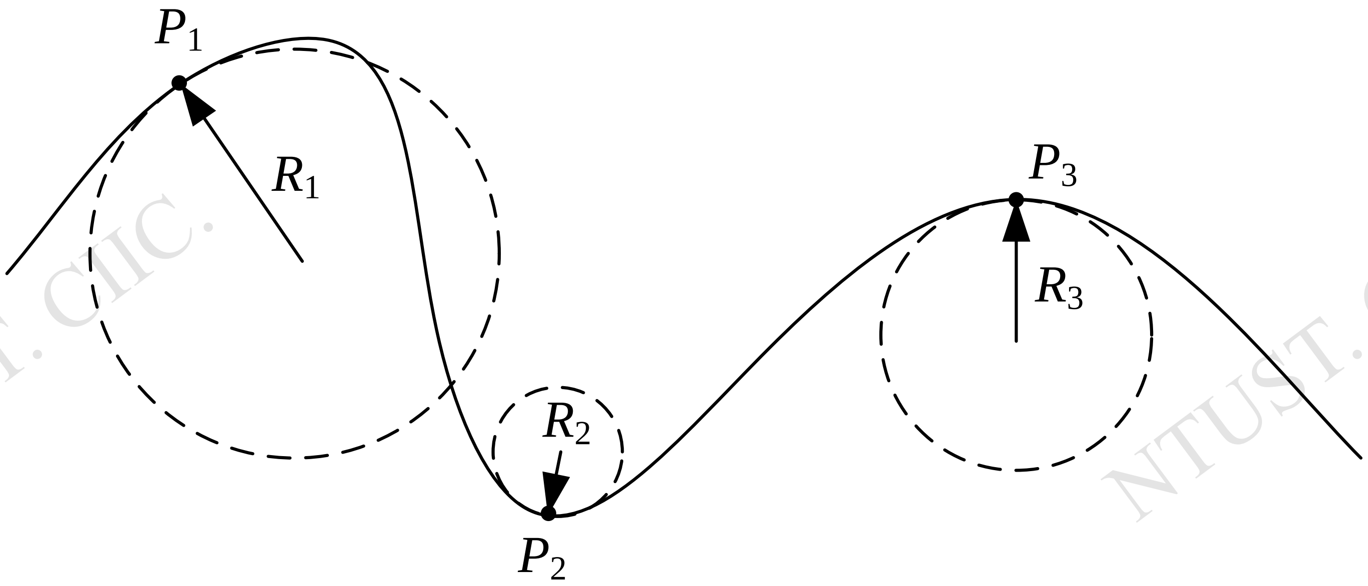
$$V = \frac{1}{6}(v_1 \times v_2) \cdot v_3$$

1D Curvature

- “curvature is the amount by which a geometric object deviates from being flat, or straight in the case of a line, but this is defined in different ways depending on the context” from Wikipedia.

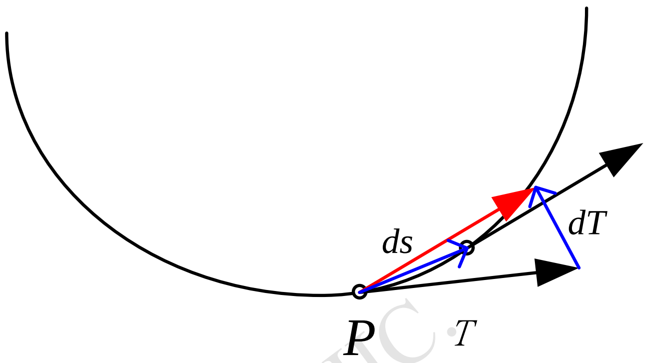
$$\kappa = \frac{1}{R}$$

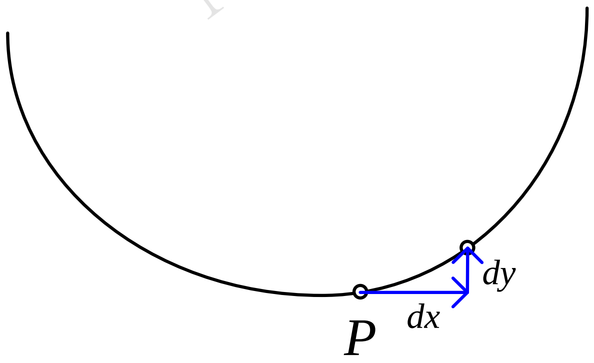
$$\kappa = \left| \frac{dT}{ds} \right|$$



1D Curvature

- Curvature on the plane (similar to slope)



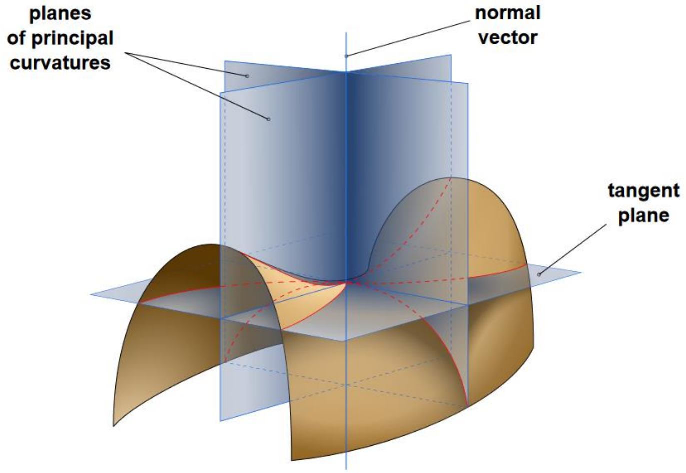


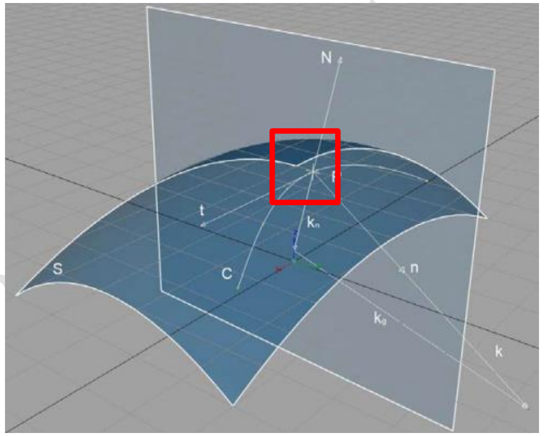
$$\kappa(s) = \left| \frac{\partial \mathbf{u}(s)}{\partial s} \right|$$

$$\mathbf{u}(s) = \frac{\partial \mathbf{r}(s)}{\partial s}$$

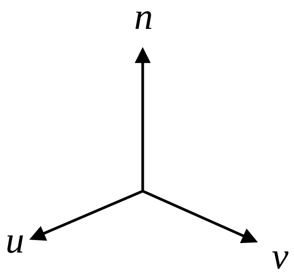
$$slope = \frac{dy}{dx}$$

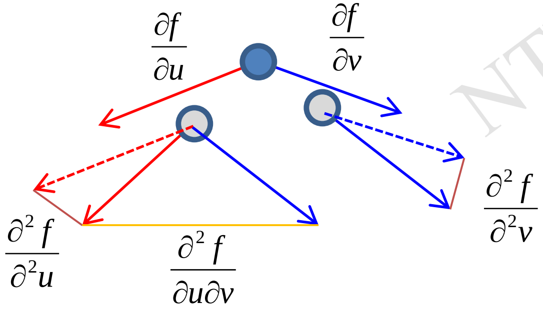
2D Curvature (curvature on surface)





$$\kappa = \begin{vmatrix} \dfrac{\partial^2 f}{\partial^2 u} & \dfrac{\partial^2 f}{\partial u \partial v} \\ \dfrac{\partial^2 f}{\partial v \partial u} & \dfrac{\partial^2 f}{\partial^2 v} \end{vmatrix}$$

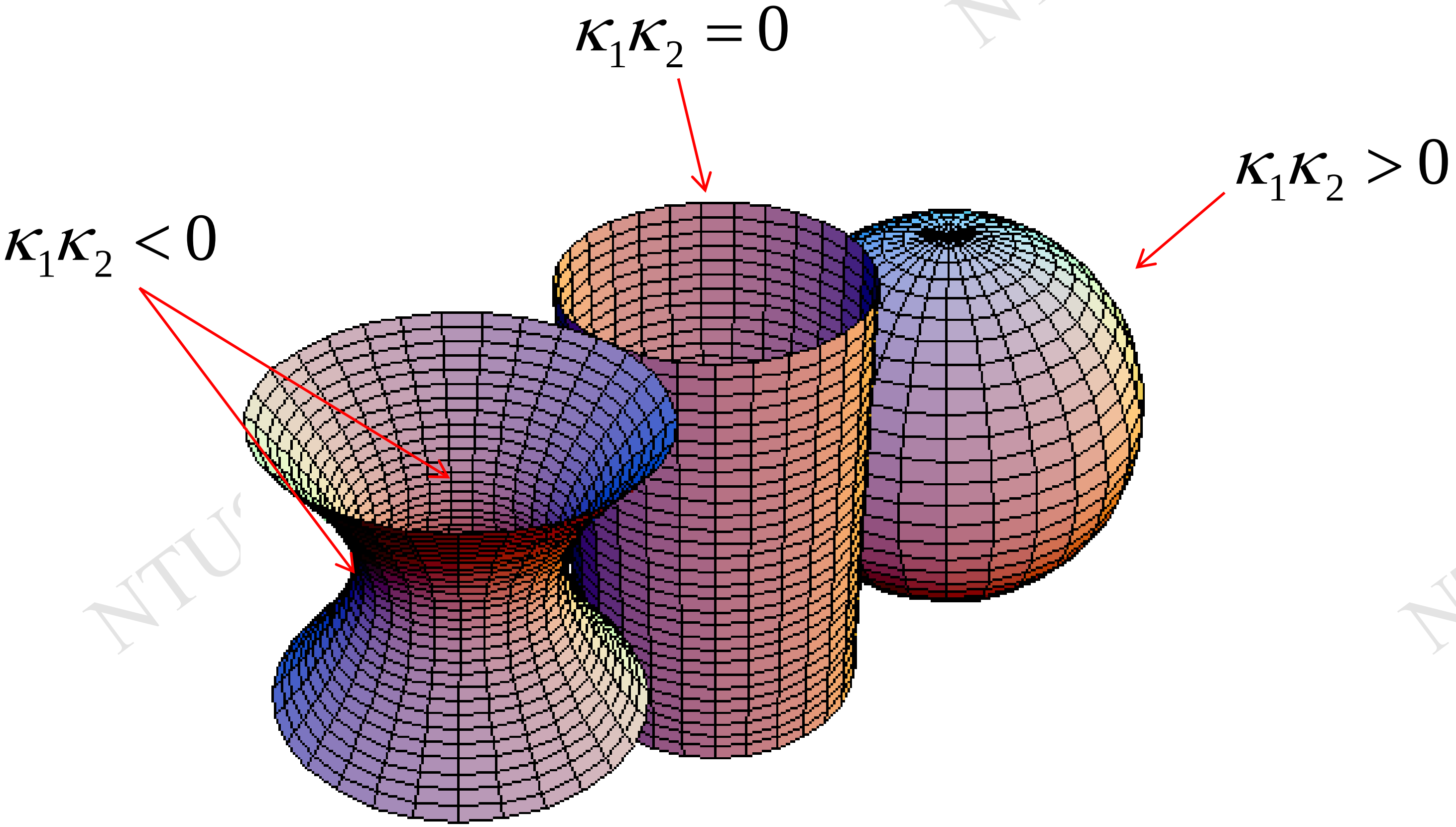




http://en.wikipedia.org/wiki/File:Minimal_surface_curvature_planes-en.svg

2D Curvature (curvature on surface)

- 2D curvature is defined by a 2x2 tensor (matrix)



http://en.wikipedia.org/wiki/File:Gaussian_curvature.PNG

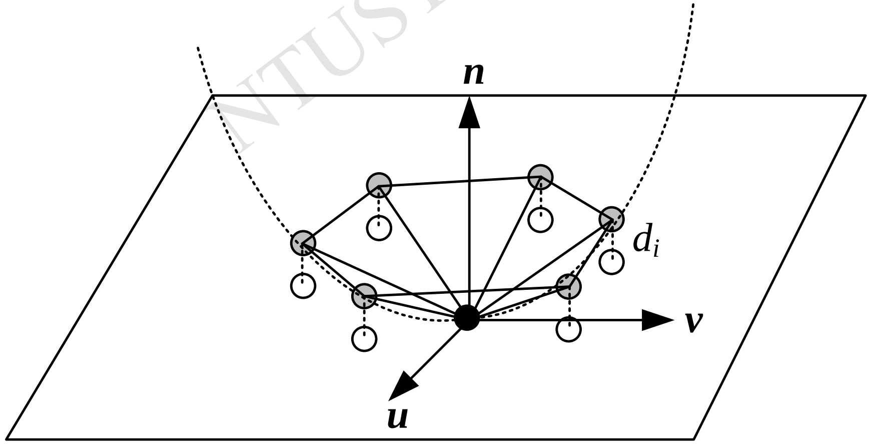
2D Curvature estimation (curvature on surface)

$$p = [x(u,v)\ y(u,v)\ z(u,v)]^T$$

$$p^* = [u\ v\ f(u,v)]^T$$

$$\kappa = \begin{vmatrix} \dfrac{\partial^2 f}{\partial^2 u} & \dfrac{\partial^2 f}{\partial u \partial v} \\ \dfrac{\partial^2 f}{\partial v \partial u} & \dfrac{\partial^2 f}{\partial^2 v} \end{vmatrix} = \begin{vmatrix} c_{2,0} & c_{1,1} \\ c_{1,1} & c_{0,2} \end{vmatrix}$$

$$f(u_i, v_i) = \frac{1}{2}(c_{2,0}u_i^2 + 2c_{1,1}u_i v_i + c_{0,2}v_i^2) = d_i$$



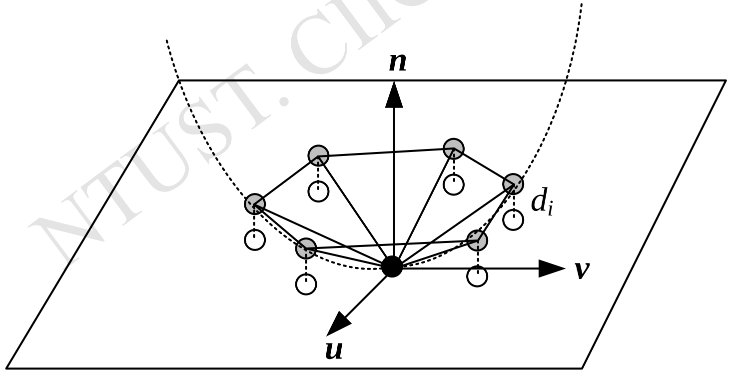
高斯曲率(Gaussian curvature): $\kappa_1\kappa_2$

平均曲率(mean curvature): $(\kappa_1+\kappa_2)/2$

B. Hamann, “Curvature approximation for triangulated surfaces,” *Computing*, vol. 8, pp. 139–153, 1993.

2D Curvature estimation (curvature on surface)

$$\begin{bmatrix} u_1^2 & 2u_1v_1 & v_1^2 \\ \vdots & \vdots & \vdots \\ u_n^2 & 2u_nv_n & v_n^2 \end{bmatrix}_{n\times 3} \begin{bmatrix} c_{2,0} \\ c_{1,1} \\ c_{0,2} \end{bmatrix}_{3\times 1} = \mathbf{U}_{n\times 3}\mathbf{c}_{3\times 1} = \mathbf{d}_{n\times 1} = \begin{bmatrix} 2d_1 \\ \vdots \\ 2d_n \end{bmatrix}_{n\times 1}$$



$$[\mathbf{U}^T]_{3\times n}[\mathbf{U}]_{n\times 3}\mathbf{c}_{3\times 1} = [\mathbf{U}^T]_{3\times n}\mathbf{d}_{n\times 1}$$

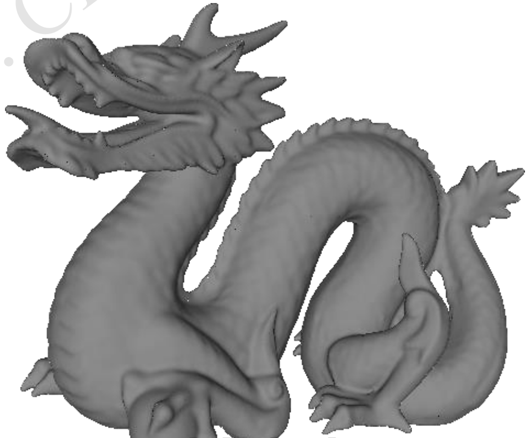
$$\mathbf{c}_{3\times 1} = \{[\mathbf{U}^T]_{3\times n}[\mathbf{U}]_{n\times 3}\}^{-1}[\mathbf{U}^T]_{3\times n}\mathbf{d}_{n\times 1}$$

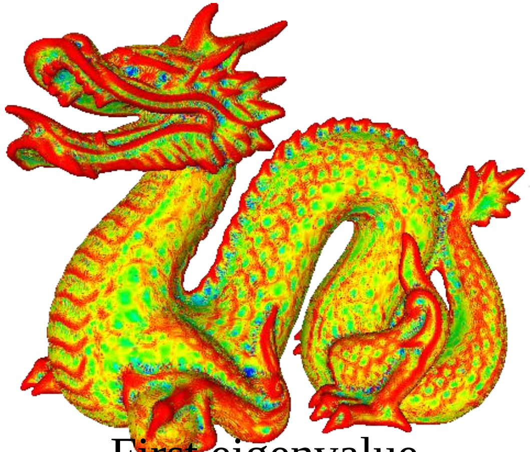
$$\kappa = \begin{vmatrix} c_{2,0} & c_{1,1} \\ c_{1,1} & c_{0,2} \end{vmatrix}$$

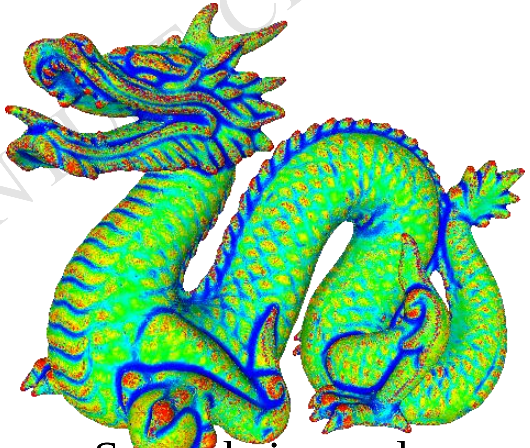
高斯曲率(Gaussian curvature): $\kappa_1\kappa_2$
平均曲率(mean curvature): $(\kappa_1+\kappa_2)/2$

B. Hamann, "Curvature approximation for triangulated surfaces," *Computing*, vol. 8, pp. 139–153, 1993.

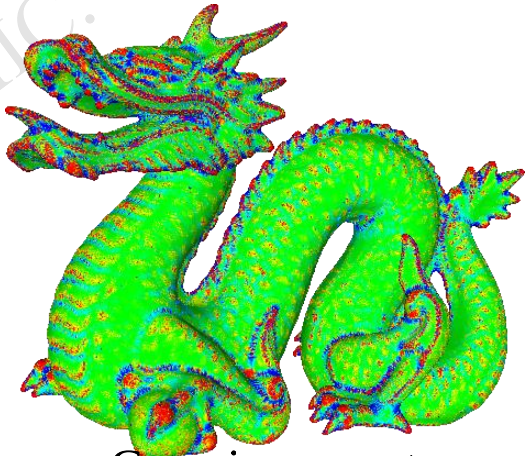
2D Curvature example:

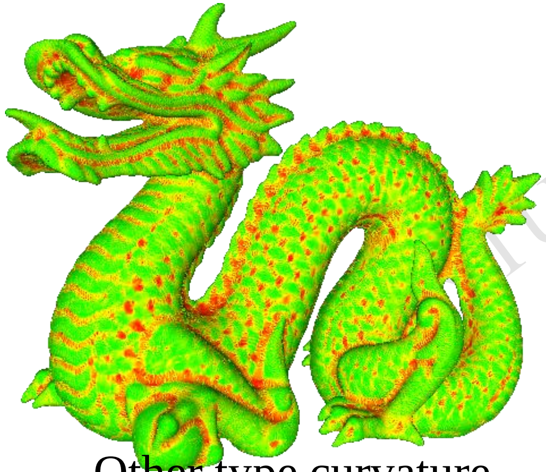

Model(shading)


First eigenvalue


Second eigenvalue

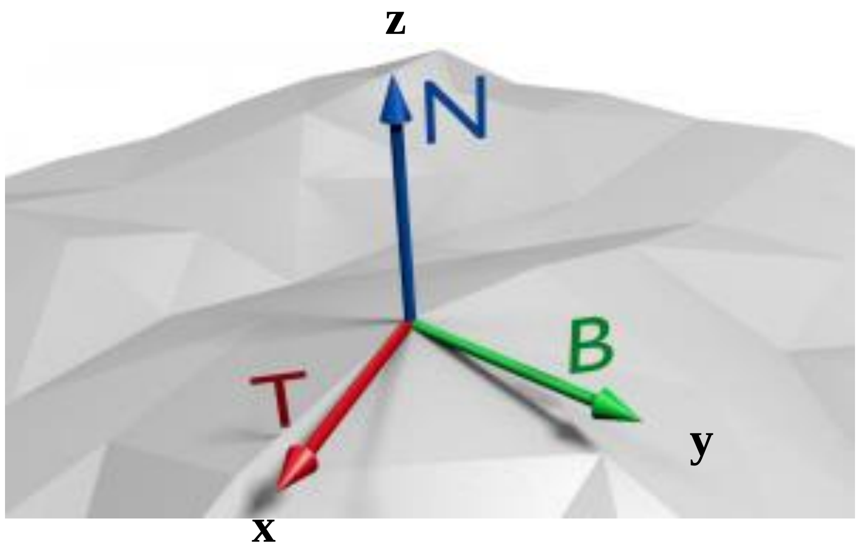

Mean curvature


Gaussian curvature


Other type curvature

TBN Matrix in computer graphics

- Tangent, Bitangent and Normal vectors
  - Represent the local x-y-z coordinate on the surface
  - Store as the normal-map



$$\begin{bmatrix} N_{objx} \\ N_{objy} \\ N_{objz} \end{bmatrix} = \begin{bmatrix} T_x & B_x & N_x \\ T_y & B_y & N_y \\ T_z & B_z & N_z \end{bmatrix} \begin{bmatrix} N_{tanx} \\ N_{tany} \\ N_{tanz} \end{bmatrix}$$

Pic from: http://www.txutxi.com/?p=316

TBN Matrix in computer graphics

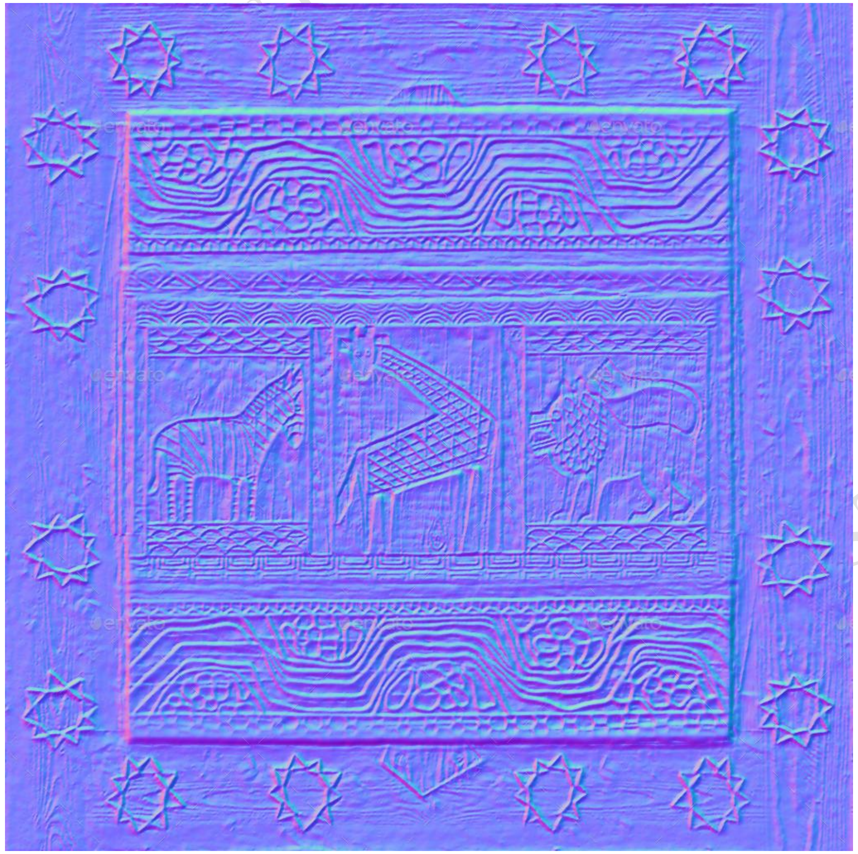
- Normal map
  - Either given by tools / devices
  - Or generated from high-polygon meshes
- Convertion between “Normal” and “RGB-image”

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \frac{1}{2}\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

TBN Matrix in computer graphics

- Normal map

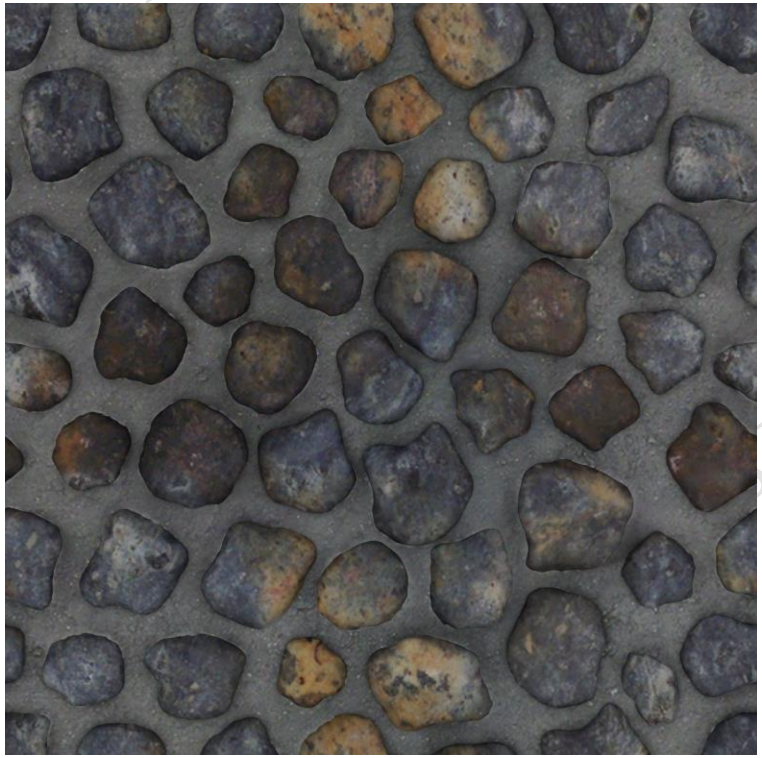
Diffuse



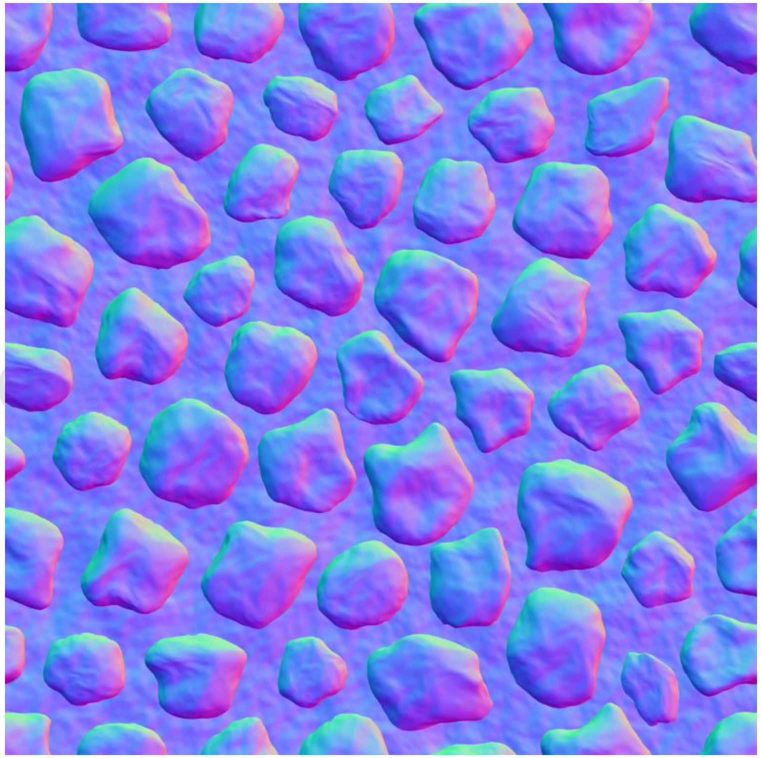
Normal

TBN Matrix in computer graphics

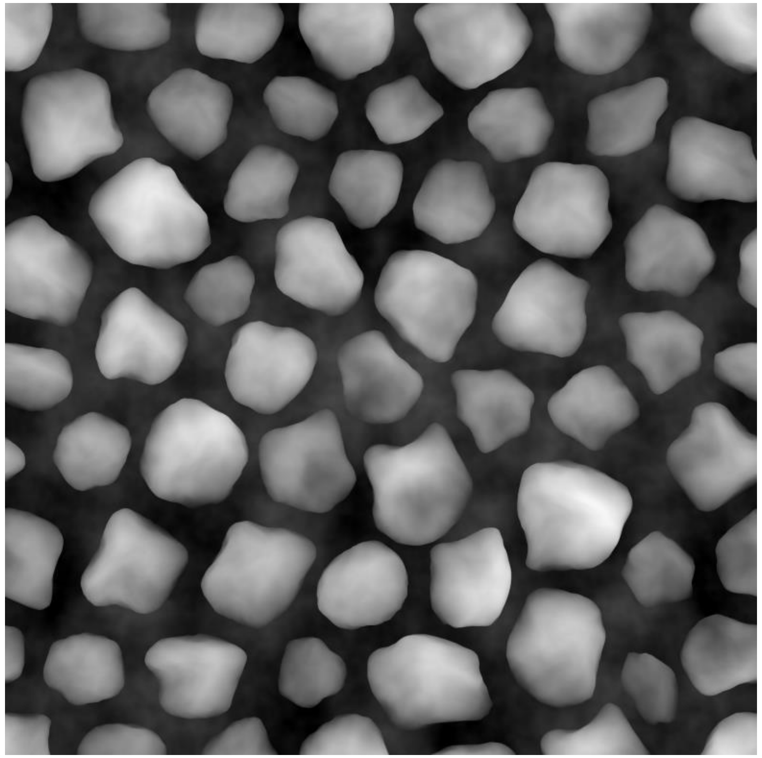
- Normal Map + High Map



Diffuse map



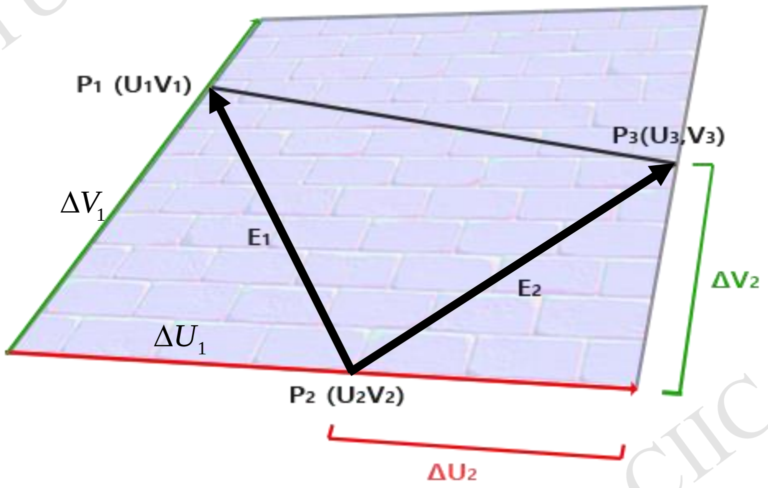
Normal map

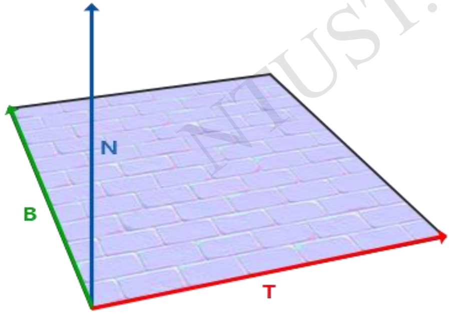


High map

TBN Matrix in computer graphics

- Determine T (tangent) and B (bitangent)





$$\begin{cases} \mathbf{E}_1 = -\Delta U_1 \mathbf{T} + \Delta V_1 \mathbf{B} \\ \mathbf{E}_2 = \Delta U_2 \mathbf{T} + \Delta V_2 \mathbf{B} \end{cases}$$

$$\begin{cases} [E_{1x} \quad E_{1y} \quad E_{1z}]^{\mathrm{T}} = -\Delta U_1 [T_x \quad T_y \quad T_z]^{\mathrm{T}} + \Delta V_1 [B_x \quad B_y \quad B_z]^{\mathrm{T}} \\ [E_{2x} \quad E_{2y} \quad E_{2z}]^{\mathrm{T}} = \Delta U_2 [T_x \quad T_y \quad T_z]^{\mathrm{T}} + \Delta V_2 [B_x \quad B_y \quad B_z]^{\mathrm{T}} \end{cases}$$

$$\begin{bmatrix} E_{1x} & E_{1y} & E_{1z} \\ E_{2x} & E_{2y} & E_{2z} \end{bmatrix} = \begin{bmatrix} -\Delta U_1 & \Delta V_1 \\ \Delta U_2 & \Delta V_2 \end{bmatrix} \begin{bmatrix} T_x & T_y & T_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$\begin{bmatrix} T_x & T_y & T_z \\ B_x & B_y & B_z \end{bmatrix} = \begin{bmatrix} -\Delta U_1 & \Delta V_1 \\ \Delta U_2 & \Delta V_2 \end{bmatrix}^{-1} \begin{bmatrix} E_{1x} & E_{1y} & E_{1z} \\ E_{2x} & E_{2y} & E_{2z} \end{bmatrix}$$

Pic from: http://www.txutxi.com/?p=316

色彩與照明科技研究所
Graduate Institute of
Color and Illumination Technology

Find us on: facebook