Homework 1 (Due: March 26th)

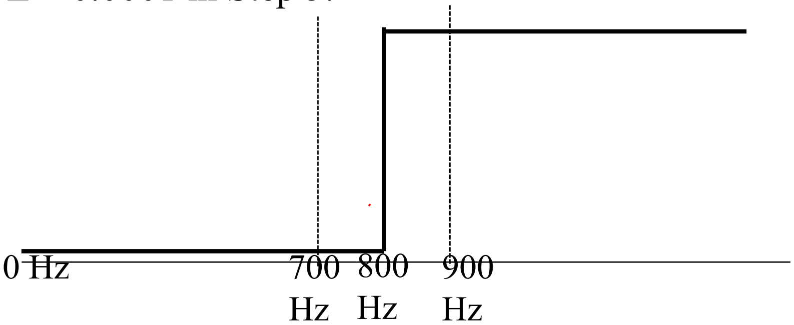
(1) Design a Mini-max **highpass** FIR filter such that (40 scores)

①Filter length = 19, ②Sampling frequency f_s = 4000Hz,

③Pass Band 800~2000Hz ④Transition band: 700~900 Hz,

⑤Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.5 for stop band .

⑥Set Δ = 0.0001 in Step 5.



※ Matlab or Python code should be handed out by ceiba
E-mail 主旨上註明學號

紙本上要有

(a) the Matlab program, (b) the frequency response,

(c) the impulse response $h[n]$, and (d) the maximal error for each iteration.

(a) The Matlab program M10907305

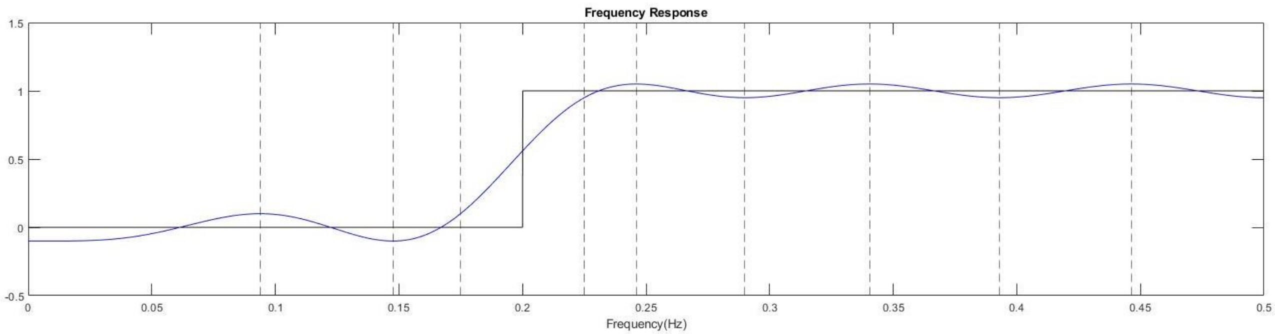
(19,0.0001,[0 0.05 0.09 0.13 0.175 0.25 0.26 0.33 0.38 0.43 0.5])

```
function ADSP_HW1_test(N,delta,F)
clc
K = (N-1) / 2;   %
transition_band_upper = 0.225;  %0.225
transition_band_lower = 0.175;  %0.175
transition_band_center = 0.2;   %0.2
W_lower =0.5;  %0.5
W_upper = 1;   %1
f_sampling = 0:delta:0.5;  %delta
H_sampling = f_sampling > transition_band_center ;        %setting


E1 = 9999;
E0 = 99;
aa = 0;
period = [0:K];
item_k = [0:K+1];
E0_register = [];


while( (0 > E1 - E0) || (E1 - E0 > delta ) )
    if( aa == 1)
        E1 = E0;
    end
    W = W_lower*(F<=transition_band_lower & F>= 0) + W_upper*(F>=transition_band_upper & F<= 0.5); %Setting range of F
    H = 1*(F>=transition_band_upper & F<= 0.5);          %Setting range of F
    %H = 1*(F<=transition_band_lower & F>= 0)          %Setting range of F
    s= inv(([cos(2*pi*F.'*period)  [(-1).^item_k./W].'])) *H.';
    RF_sampling = 0;
    RF = 0;
    for i = 1:K+1
        RF_sampling = RF_sampling + s(i)*cos(2*pi*(i-1)*f_sampling);
        RF = RF + s(i)*cos(2*pi*(i-1)*F);
    end
    %RF
    W_sampling = W_lower*(f_sampling<=transition_band_lower & f_sampling>= 0) + W_upper*(f_sampling>=transition_band_upper & f_sampling<= 0.5);
    err_sampling = [RF_sampling - H_sampling ].*W_sampling;
    err_sampling_zero = [0 err_sampling 0];
```

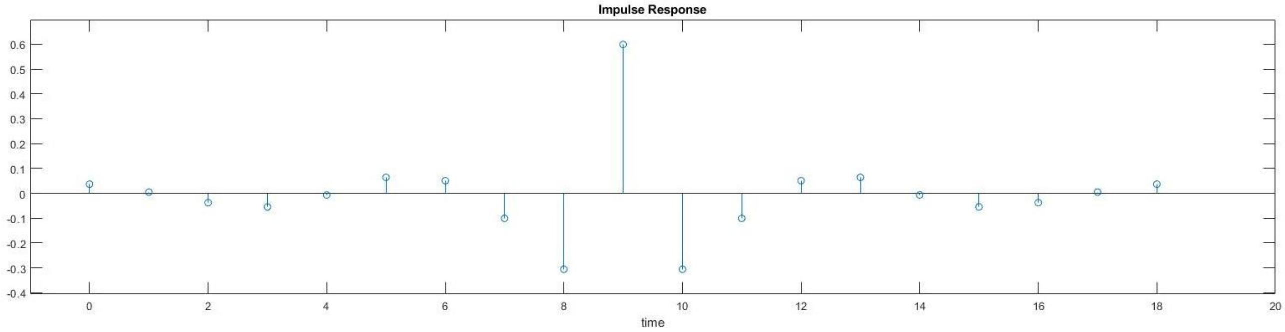
```
    err = [RF - H ].*W;
    P = [];
    for i = 2:length(err_sampling_zero)-1
        if( (err_sampling_zero(i) > err_sampling_zero(i+1)) & (err_sampling_zero(i) > err_sampling_zero(i-1)) )
            P = [P f_sampling(i-1)];
        elseif( (err_sampling_zero(i) < err_sampling_zero(i+1)) & (err_sampling_zero(i) < err_sampling_zero(i-1)) )
            P = [P f_sampling(i-1)];
        end
    end
    %P
    E0 = max(abs(err_sampling));
    E0_register = [E0_register E0];
    P1 = [];
    P2 = [];
    if((E1 - E0 > delta) || (E1 - E0 < 0))
        if(length(P) > K+2)
            for i = 1:length(P)
              if( (P(i) ~= 0) &  (P(i) ~= 0.5) & (P(i) ~= transition_band_lower) & (P(i) ~= transition_band_upper) )
                  P1 = [P1 P(i)];
              else
                  P2 = [P2 P(i)];
              end
            end
            %P1
            %P2
            if(length(P1) < K+2)
                point = (K+2)  -length(P1) ;
                P_abs_err = abs((err_sampling(P2/delta +1)));
                [b ,c ] = sort(P_abs_err,'descend');
                P  = sort([P1 P2(c(1:point))]);
            end
        end
        F = P;
        aa = 1;
    end
    %P
end
E0_register
subplot(2,1,1)
h_f = [(fliplr(s(2:end-1).'))/2 s(1) s(2:end-1).'/2];
x = 0:1:length(h_f)-1;
```

```
stem(x,h_f)
xlim([-1 length(h_f)+1])
ylim([min(h_f)-0.1 max(h_f)+0.1])
title('Impulse Response')
xlabel('time')


subplot(2,1,2)
plot(f_sampling,H_sampling,'k',f_sampling,RF_sampling,'b')
axis([0,0.5,-0.5,1.5])
title('Frequency Response')
xlabel('Frequency(Hz)')
for i = 2:length(F)-1
   xline(F(i),'--');
end
```

(b) the frequency response



(c) the impulse response $h[n]$



(d) the maximal error for each iteration

Iteration	1	2	3	4	5	6	7	8	9	10	11
Maximal Error	0.2421	0.0934	0.1444	0.2283	0.0948	0.1405	0.4724	0.0894	0.0508	0.05	0.05

(2) Suppose that $X(f)$ is the discrete-time Fourier transform of $x(n\Delta_t)$. Also suppose that we have known that $\Delta_t = 0.001$ sec and

$$X(f) = 1 \text{ for } |f| < 200 \text{ and } X(f) = 0 \text{ for } 200 < |f| < 500.$$

Determine (a) $X(900)$, (b) $X(-1900)$, (c) $X(6100)$. (10 scores)

(a)

$x(900) = 1$

$F = 1$

$900 - 1000$

$= -100$

(b)

$x(-1900) = 1$

$F = -2$

$-1900 - (-2000)$

$= 100$

(c)

$x(6100) = 1$

$F = 6$

$6100 - 6000$

$= 100$

(3) From the view point of implementation, what are the <u>disadvantages</u> of the discrete Fourier transform? (5 scores)

1. DFT 的 complexity 要 $O(n^2)$

2、如果直接做 sampling 失真、重疊、

(4) Suppose that $x[n] = y(0.0002n)$ and the length of $x[n]$ is 15000 and $X[m]$ is the FFT of $x[n]$. Find m_1 and m_2 such that $X[m_1]$ and $X[m_2]$ correspond to the 200Hz and -300Hz components of $y(t)$, respectively. (10 scores)

$x[n] = y(0.0002n) = y(\Delta_t n)$

$f_s = \frac{1}{\Delta_t} = 5000$, $N = 15000$

$-\frac{f_s}{2} = -2500 < f < 2500 = \frac{f_s}{2}$

$f_{m_1} = 200 = m_1 \cdot \frac{5000}{15000}$

$m_1 = 600$ #

$f_{m_2} = -300 = m_2 \cdot \frac{1}{3}$

$m_2 = -900$ #

(5) Which of the following filters are <u>odd</u>? (i) bandpass filter, (ii) edge detector, (iii) differentiation 2 times, (iv) integration 3 times, (v) particle filter, (vi) the Hilbert. (10 scores)

edge detector, integration 3 times, the Hilbert

(6) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta_t = 0.0002$, and the transition band is from 1600Hz to 1800Hz. (10 scores)

$\because \delta_1 = \delta_2 = 0.01$, $\Delta t = 0.0002$, $f_s = \frac{1}{\Delta t} = 5000$

$$\Delta F = \frac{1800-1600}{5000} = 0.04$$

$$\therefore N = \frac{2}{3} \frac{1}{\Delta F} \log_{10}\left(\frac{1}{10 \cdot \delta_1 \cdot \delta_2}\right) = \frac{2}{3} \cdot \frac{1}{0.04} \cdot \log_{10}\left(\frac{1}{10 \cdot 0.01^2}\right)$$

$$= \frac{2}{3} \cdot 25 \cdot \log_{10}\left(\frac{1}{10^{-3}}\right) = \frac{150}{3} = 50 \#$$

(7) Use the MSE method to design the 9-point FIR filter that approximates the lowpass filter of $H_d(F) = 1$ for $|F| < 0.2$ and $H_d(F) = 0$ for $0.2 < |F| < 0.5$. (15 scores)

$N=9,\ K = \frac{9-1}{2} = 4$

$$\frac{\partial MSE}{\partial S[0]} \Rightarrow S[0] = \int_{-0.2}^{0.2} H_d(F)\,dF = 0.4$$

$$S[n] = 2\int_{-0.2}^{0.2} \cos(2\pi nF)\cdot H_d(F)\,dF = 2\cdot\frac{1}{2\pi n}\cdot \sin(2\pi nF)\Big|_{-0.2}^{0.2} = \frac{2\sin(0.4n\pi)}{\pi n}$$

$$R(F) = \sum_{n=0}^{K} S[n]\cos(2\pi nF)$$

$$\Rightarrow \underbrace{0.4}_{S[0]} + \underbrace{0.6055}_{S[1]}\cdot\cos(2\pi F) + \underbrace{0.1871}_{S[2]}\cos(4\pi F) - \underbrace{0.1247}_{S[3]}\cos(6\pi F) - 0.1514\cdot\cos(8\pi F)$$

$h[4] = 0.4$

$h[0] = h[8] = 0.30245$

$h[1] = h[7] = 0.09355$

$h[2] = h[6] = -0.06235$

$h[3] = h[5] = -0.0457$

(8)Please describe under what special circumstances the IIR calculation will be less than the FIR.

(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 5, 6, 7)

等比級數 n項 $a^n u[n]$

在 IIR

$h[n] = a^n u[n]$, $\left\{ \begin{array}{l} u[n] = 1 \text{ for } n \geq 1 \\ \text{且 } u[n] = 0 \text{ for } n < 0 \end{array} \right\}$

$H(z) = \sum_n h[n] z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n$

$\Rightarrow \frac{1}{1 - a z^{-1}}$

$Y(z) = H(z) X(z) = \frac{X(z)}{1 - a z^{-1}}$

$Y(z) = X(z) + a z^{-1} Y(z)$

$z^{-1} \downarrow$

$y[n] = x[n] + a y[n-1]$ #

delay

在 FIR

$y[n] = x[n] * h[n]$

$= \sum_{\tau} x[n-\tau] h[\tau]$ #

Ans：在訊號是比等數列，在 IIR 的計算量小於 FIR