XIII. Walsh Transform (Hadamard Transform)

◉ 13-A Ideas of Walsh Transforms

- 8-point Walsh transform

low frequency

$$W[m,n]=\begin{bmatrix}1&1&1&1&1&1&1&1\\1&1&1&1&-1&-1&-1&-1\\1&1&-1&-1&-1&-1&1&1\\1&1&-1&-1&1&1&-1&-1\\1&-1&-1&1&1&-1&-1&1\\1&-1&-1&1&-1&1&1&-1\\1&-1&1&-1&-1&1&-1&1\\1&-1&1&-1&1&-1&1&-1\end{bmatrix}\quad\begin{array}{cc}m & \text{zero crossing}\\0&0\\1&1\\2&2\\3&3\\4&4\\5&5\\6&6\\7&7\end{array}$$

DFT : $e^{-j\frac{2\pi}{N}mn}$

high frequency

- Advantages of the Walsh transform:

  (1) Real

  (2) No multiplication is required

  (3) Some properties are similar to those of the DFT

- Forward and inverse Walsh transforms are similar.

forward: $F[m]=\sum_{n=0}^{N-1} f[n]W[m,n]$,   inverse: $f[m]=\frac{1}{N}\sum_{n=0}^{N-1} W[m,n]F[n]$

$F = Wf$ $f = W^{-1}F = \frac{1}{N}WF$

- Alternative names of the Walsh transform:

Hadamard transform, Walsh-Hadamard transform

- Orthogonal Property $\sum_{n=0}^{N-1} W[m_0,n]W[m_1,n]=0$ if $m_0 \neq m_1$ $WW^T = N \cdot I$
- Zero-Crossing Property $W = W^T$ $W(\frac{1}{N} \cdot W^T) = I$
- Even / Odd Property $\frac{1}{N}W^T = W^{-1}$
- Fast Algorithm

Useful for spectrum analysis

Sometimes also useful for implementing the convolution

Walsh transform 和 DFT, DCT 有許多相似處

2nd row

Walsh

DCT

$$W[m,n]=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}, \; DFT[m,n]=\exp(-j2\pi\, m\, n/N),$$

$$\mathbf{DCT}=\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.3870 & 1.1759 & 0.7857 & 0.2759 & -0.2759 & -0.7857 & -1.1759 & -1.3870 \\ 1.3066 & 0.5412 & -0.5412 & -1.3066 & -1.3066 & -0.5412 & 0.5412 & 1.3066 \\ 1.1759 & -0.2759 & -1.3870 & -0.7857 & 0.7857 & 1.3870 & 0.2759 & -1.1759 \\ 1.0000 & -1.0000 & -1.0000 & 1.0000 & 1.0000 & -1.0000 & -1.0000 & 1.0000 \\ 0.7857 & -1.3870 & 0.2759 & 1.1759 & -1.1759 & -0.2759 & 1.3870 & -0.7857 \\ 0.5412 & -1.3066 & 1.3066 & -0.5412 & -0.5412 & 1.3066 & -1.3066 & 0.5412 \\ 0.2759 & -0.7857 & 1.1759 & -1.3870 & 1.3870 & -1.1759 & 0.7857 & -0.2759 \end{bmatrix}$$

References for Walsh Transforms

[1] K. G. Beanchamp, *Walsh Functions and Their Applications*, Academic Press, New York, 1975.

[2] B. I. Golubov, A. Efimov, and V. Skvortsov, *Walsh Series and Transforms: Theory and Applications*, Kluwer Academic Publishers, Boston, 1991.

[3] H. F. Harmuth, "Applications of Walsh functions in communications," *IEEE Spectrum*, vol. 6, no. 11, pp. 82-91, Nov. 1969.

[4] H. F. Harmuth, *Transmission of Information by Orthogonal Functions*, Springer-Verlag, New York, 1972.

◉ 13-B Generate the Walsh Transform

2-point Walsh transform

= 2-point DFT

$$\mathbf{W}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

4-point Walsh transform

$$\mathbf{W}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

DFT : $e^{-j\frac{2\pi}{N}mn}$, $N=4$, $(-j)^{mn}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

How do we obtain the 2^{k+1}-point Walsh transform from the 2^k-point Walsh transform ?

Step 1 $\quad \mathbf{V}_{2^{k+1}} = \begin{bmatrix} \mathbf{W}_{2^k} & \mathbf{W}_{2^k} \\ \mathbf{W}_{2^k} & \underline{-\mathbf{W}_{2^k}} \end{bmatrix}$

Step 2 根據 sign changes 將 rows 的順序重新排列

$$\mathbf{V}_{2^{k+1}} \xrightarrow{\textit{permutation}} \mathbf{W}_{2^{k+1}}$$

已知 $\mathbf{W}_{2^k}$ 每個 row 的 sign change 數，由上到下分別為

$0, 1, 2, 3, \ldots.., 2^k-1$

則 $\mathbf{V}_{2^{k+1}}$ 每個 row 的 sign change 數，由上到下分別為

$0, 3, 4, 7, \ldots.., 2^{k+1}-1, 1, 2, 5, 6, \ldots.., 2^{k+1}-2,$

若 row 的index 由0 開始

則 $\mathbf{V}_{2^{k+1}}$ 第 n 個 row (n is even and $n < N/2$) 的 sign change 為 $2n$

(n is odd and $n < N/2$) 的 sign change 為 $2n + 1$

(n is even and $n \geq N/2$) 的 sign change 為 $2n+1-N$

(n is odd and $n \geq N/2$) 的 sign change 為 $2n-N$

要根據 sign change 的數目將 $\mathbf{V}_{2^{k+1}}$ 的 row 重新排列

$$\mathbf{V}_{2^{k+1}} \xrightarrow{\textit{permutation}} \mathbf{W}_{2^{k+1}}$$

$$\mathbf{W}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{V}_4 = \begin{bmatrix} \mathbf{W}_2 & \mathbf{W}_2 \\ \mathbf{W}_2 & -\mathbf{W}_2 \end{bmatrix} = \begin{array}{c} \\ \left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ \hline 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right] \end{array} \begin{array}{c} \text{sign changes} \\ 0 \\ 3 \\ 1 \\ 2 \end{array}$$

6

$\frac{n}{2} \times (n-1) = 28$

$8 \times 15 = 120$

$$\mathbf{W}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\mathbf{V}_8 = \begin{array}{c} \begin{array}{cc} W4 & W4 \end{array} \\ \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ \hline 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \end{array}\right] \\ \begin{array}{cc} W4 & -W4 \end{array} \end{array} \begin{array}{c} \text{sign changes} \\ 0 \\ 3 \\ 4 \\ 7 \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

28

-28

1

4×3

$= 28$ 8 8

16

$$\begin{array}{r} 15 \\ \times\ 8 \\ \hline 120 \end{array}$$

8×8

Constraint for the number of points of the Walsh transform:

N must be a power of 2 (2, 4, 8, 16, 32, ……..) 2^k

Although in Matlab it is possible to define the $12 \cdot 2^k$ point or the $20 \cdot 2^k$ point Walsh transform, the inverse transform require the floating-point operation.

Matlab code: hadamard(2^k)

$\frac{1}{12}, \frac{1}{20}$

not trivial multiplications

◎ 13-C Alternative Forms of the Walsh Transform

標準定義　　from zero-crossing

- Sequency ordering (i.e., Walsh ordering) ……. using for signal processing
- Dyadic ordering (i.e., Paley ordering) ……….. using for control
- Natural ordering (i.e., Hadamard ordering) ……using for mathematics

matlab

Sequency ordering	Dyadic ordering	Natural ordering	$W[m, n]$
←→	(Gray Code) ←→	(Bit Reversal)	
000 row 0 =	000 row 0 =	000 row 0 =	[1, 1, 1, 1, 1, 1, 1, 1]
001 row 1 = 補0 0001 001 ⇒	001 row 1 = 001 ⇒	100 row 4 =	[1, 1, 1, 1, −1, −1, −1, −1]
010 row 2 =	011 row 3 =	110 row 6 =	[1, 1, −1, −1, −1, −1, 1, 1]
011 row 3 =	010 row 2 =	010 row 2 =	[1, 1, −1, −1, 1, 1, −1, −1]
100 row 4 =	110 row 6 = 110 ⇒	011 row 3 =	[1, −1, −1, 1, 1, −1, −1, 1]
101 row 5 = 0101 111 ⇒	111 row 7 =	111 row 7 =	[1, −1, −1, 1, −1, 1, 1, −1]
110 row 6 =	101 row 5 =	101 row 5 =	[1, −1, 1, −1, −1, 1, −1, 1]
111 row 7 =	100 row 4 =	001 row 1 =	[1, −1, 1, −1, 1, −1, 1, −1]

- Dyadic ordering Walsh transform

$$W[m,n]=\begin{bmatrix}1&1&1&1&1&1&1&1\\1&1&1&1&-1&-1&-1&-1\\1&1&-1&-1&1&1&-1&-1\\1&1&-1&-1&-1&-1&1&1\\1&-1&1&-1&1&-1&1&-1\\1&-1&1&-1&-1&1&-1&1\\1&-1&-1&1&1&-1&-1&1\\1&-1&-1&1&-1&1&1&-1\end{bmatrix}$$

- Natural ordering Walsh transform

$$W[m,n]=\begin{bmatrix}1&1&1&1&1&1&1&1\\1&-1&1&-1&1&-1&1&-1\\1&1&-1&-1&1&1&-1&-1\\1&-1&-1&1&1&-1&-1&1\\1&1&1&1&-1&-1&-1&-1\\1&-1&1&-1&-1&1&-1&1\\1&1&-1&-1&-1&-1&1&1\\1&-1&-1&1&-1&1&1&-1\end{bmatrix}$$

- binary code $n = \sum_{p=1}^{k} b_p 2^{p-1}$ to gray code

  When $N = 2^k$

  $g_k = b_k,\ \ g_q = \mathrm{XOR}(b_{q+1}, b_q)$ for $q = k-1, k-2, \ldots., 1$ $\qquad m = \sum_{q=1}^{k} g_q 2^{q-1}$

- gray code to binary code

  When $N = 2^k$

  $b_k = g_k,\ \ b_q = \mathrm{XOR}(b_{q+1}, g_q)$ for $q = k-1, k-2, \ldots., 1$

◉ 13-D Properties

(1) Orthogonal Property

(2) Zero-Crossing Property

(3) Even / Odd Property

$m = 0, 2, 4, 6$ $\quad$ $m = 1, 3, 5, 7$

(4) Linear Property

If $f[n] \Rightarrow F[m]$, $\quad g[n] \Rightarrow G[m]$, $\quad$ ($\Rightarrow$ means the Walsh transform)

then $a\, f[n] + b\, g[n] \Rightarrow a\, F[m] + b\, G[m]$

(5) Addition Property

$$W[m,n]\cdot W[l,n]=W[m\oplus l,n]$$

註： Addition modulo 2 (denoted by $\oplus$)

$0\oplus 0=1\oplus 1=0,\quad 0\oplus 1=1\oplus 0=1,$

$$(\sum_{p=0}^{k}a_k 2^p)\oplus(\sum_{p=0}^{k}b_k 2^p)=\sum_{p=0}^{k}(a_k\oplus b_k)2^p$$

Example:

3	0	1	1
7	1	1	1
4	1	0	0

, therefore $3\oplus 7=4$

$\oplus$: logic addition
(similar to XOR)

$0\oplus 1=1$
$1\oplus 0=1$
$0\oplus 0=0=1\oplus 1$

(6) Special functions

$\delta[n] = 1$ when $n = 0$, $\delta[n] = 0$ when $n \neq 0$

$\delta[n] \Rightarrow 1$, $1 \Rightarrow N \cdot \delta[n]$

(7) Shifting property

If $f[n] \Rightarrow F[m]$, then $f[n \oplus k] \Rightarrow W(k, m) \cdot F[m]$

(8) Modulation property

If $f[n] \Rightarrow F[m]$, then $W(k, n) \cdot f[n] \Rightarrow F[m \oplus k]$

(9) Parseval's Theorem

If $f[n] \Rightarrow F[m]$,

$$\sum_{n=0}^{N-1} |f[n]|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |F[m]|^2 ,$$

If $f[n] \Rightarrow F[m]$, $g[n] \Rightarrow G[m]$,

$$\sum_{n=0}^{N-1} f[n] g[n] = \frac{1}{N} \sum_{n=0}^{N-1} F[m] G[m]$$

(10) Convolution Property

If $f[n] \Rightarrow F[m]$, $\quad g[n] \Rightarrow G[m]$,

then $h[n] = f[n] \star g[n] \Rightarrow F[m]\,G[m]$

$\star$ means the "logical convolution" $\sum_{l=0}^{N-1} f[n \oplus l]\,g[l]$

linear $g[n-l]$

$$h[n] = f[n] \star g[n] = \sum_{l=0}^{N-1} f[l]\,g[n \oplus l] = \sum_{l=0}^{N-1} f[n \oplus l]\,g[l]$$

$$\begin{array}{r} 000 \\ \oplus\ 011 \\ \hline 011 \end{array}$$

$0 \oplus = 3 = 3$

$$\begin{array}{r} 001 \\ \oplus\ 011 \\ \hline 010 \end{array}$$

For example, when $N = 8$,

$$h[3] = f[0]g[3] + f[1]g[2] + f[2]g[1] + f[3]g[0] + f[4]g[7] + f[5]g[6] + f[6]g[5] + f[7]g[4]$$

$$h[2] = f[0]g[2] + f[1]g[3] + f[2]g[0] + f[3]g[1] + f[4]g[6] + f[5]g[7] + f[6]g[4] + f[7]g[5]$$

$$\begin{array}{r} 000 \\ \oplus\ 010 \\ \hline 010 \end{array} \qquad \begin{array}{r} 001 \\ \oplus\ 010 \\ \hline 011 \end{array} \qquad \begin{array}{r} 011 \\ \oplus\ 010 \\ \hline 001 \end{array}$$

Comparison：In digital signal processing, we often use

linear convolution

$$\sum_{l=0}^{N-1} f[l]g[n-l]$$

When will the circular convolution equal to the linear convolution? 3, 8

The condition where the circular convolution is equal to the linear convolution:

(i) $x[n]=0 \quad for\ n<0\ \ or\ \ n\geq N$

$\boxed{x_1[n]=x[n] \quad for\ 0\leq n<N, \quad x_1[n]=0 \quad for\ N\leq n<P-1}$

(ii) $h[n]=0 \quad for\ n<0\ \ or\ \ n\geq M$

$\boxed{h_1[n]=h[n] \quad for\ 0\leq n<M, \quad h_1[n]=0 \quad for\ M\leq n<P-1}$

(iii) $P\geq N+M-1$

circular convolution

$$\sum_{l=0}^{N-1} f[l]g[((n-l))_N]$$

$$IDFT_N\{DFT_N[f[n]]DFT_N[g[n]]\} = \sum_{l=0}^{N-1} f[l]g[((n-l))_N]$$

For example, when $N = 8$,

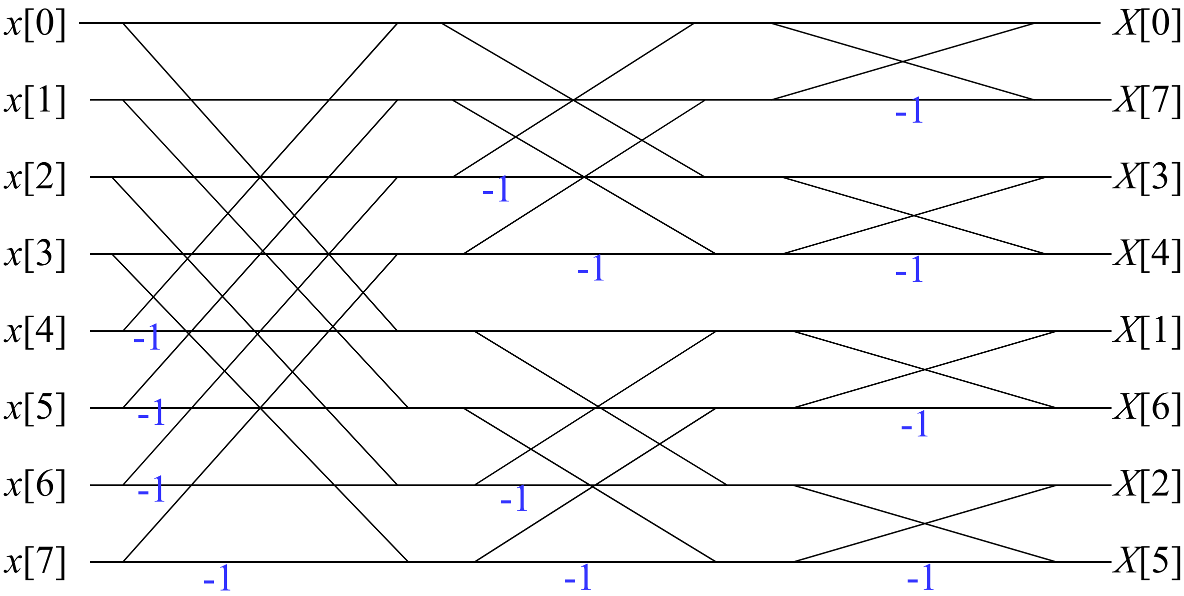
$$h[3] = f[0]g[3] + f[1]g[2] + f[2]g[1] + f[3]g[0] + f[4]g[7] + f[5]g[6] + f[6]g[5] + f[7]g[4]$$

$$h[2] = f[0]g[2] + f[1]g[1] + f[2]g[0] + f[3]g[7] + f[4]g[6] + f[5]g[5] + f[6]g[4] + f[7]g[\cancel{4}]$$

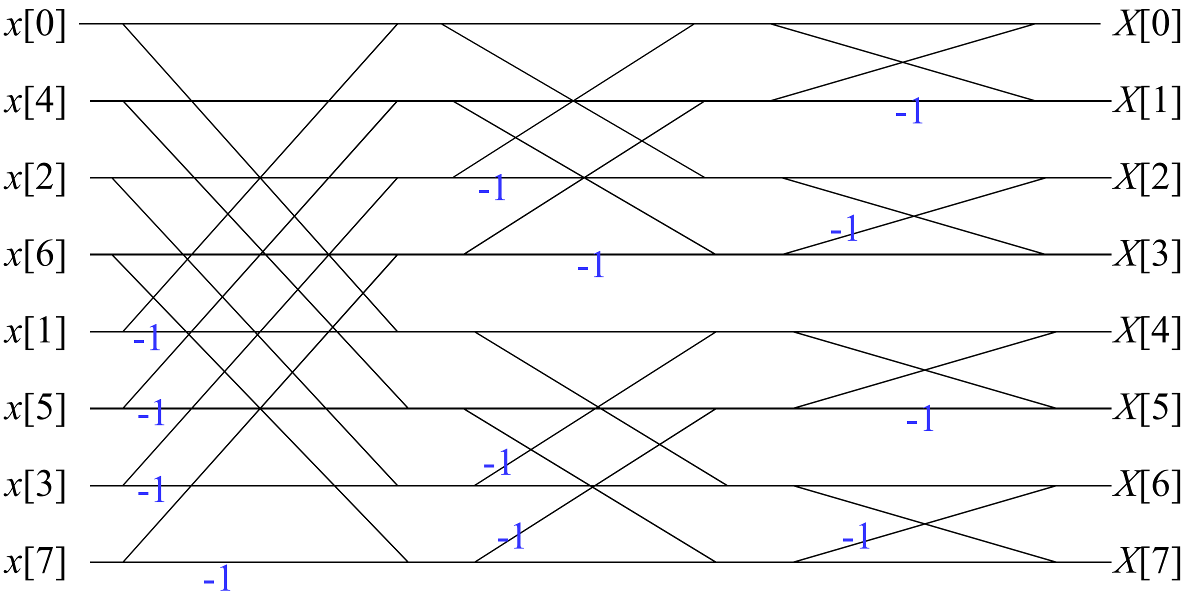
3

◉ 13-E Butterfly Fast Algorithm

(Method 1) John L. Shark's Algorithm



(Method 2) Manz's Sequence Algorithm



There are other fast implementation algorithm for the Walsh transform.

◉ 13-F Applications

Walsh transform 適合作 spectrum analysis，但未必適合作convolution

may not be better than DFT, DCT

Applications of the Walsh transform

Bandwidth reduction

High resolution

Modulation and Multiplexing

Information coding

Feature extraction

ECG signal (in medical signal processing) analysis

Hadamard spectrometer

Avoiding quantization error

- The Walsh transform is suitable for the function that is a combination of Step functions

New Applications: CDMA (code division multiple access)

modulation $\quad d_m\, x[n] \xrightarrow{\text{CDMA}} \underline{W[m,n]}\, x[n]\, d_m$

make a signal $\quad$ a row of the walsh transform

$d_0, d_1 \xrightarrow{\text{CDMA}} d_0 W[0,n] + d_1 W[1,n]$

demodulate for d_0

$$\frac{1}{N}\sum_{n=0}^{N-1} c[n]\, W[0,n] = d_0$$

$$= \frac{1}{N}\Big(d_0 \sum_{n=0}^{N-1} W[0,n]W[0,n] + \cancel{d_1 \sum_{n=0}^{N-1} \underbrace{\overbrace{W[1,n]\, W[0,n]}^{\text{不一樣位置}}}_{0}} \Big)$$

$$= \frac{1}{N} N \cdot d_0 = d_0$$

$e^{j2\pi f_0 t}\quad x(t)$

FT general modulation technology

$\cos(2\pi f_0 t)\, x(t)$

◉ 13-G Jacket Transform

把部分的 1 用 $\pm 2^k$ 取代

4-point Jacket transform

$$\mathbf{J}_4 = \begin{bmatrix} 1 & x & x & 1 \\ 1 & w & -w & -1 \\ 1 & -x & -x & 1 \\ 1 & -w & w & 1 \end{bmatrix} \qquad w = 2^k, \quad x = 2^h,$$

2^{k+1}-point Jacket

$$\mathbf{J}_{2^{k+1}} = \mathbf{P}\begin{bmatrix} \mathbf{J}_{2^k} & \mathbf{J}_{2^k} \\ \mathbf{J}_{2^k} & -\mathbf{J}_{2^k} \end{bmatrix}$$

P: row permutation

[Ref] M. H. Lee, "A new reverse Jacket transform and its fast algorithm," *IEEE Trans. Circuits Syst.-II* , vol. 47, pp. 39-46, 2000.

◉ 13-H Haar Transform

least computation loading

$$\mathbf{y} = \mathbf{H}\mathbf{x}$$

$$N = 2 \qquad \mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad\qquad N = 4 \qquad \mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$H_2 = W_2 = F_2$

$$N = 8 \qquad \mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

the same as W_8

4 zeros in each row
4 non-zero entries

6 zeros in each row
2 non-zero entries

$N = 32$

10^{th} row is ?

[Ref] H. F. Harmuth, *Transmission of Information by Orthogonal Functions*, Springer-Verlag, New York, 1972

8×8

4 2

$N = 16$

16

$2 \quad 2$

$-1 \quad 1$

$2 \quad 2^2$

$-1 \quad 1$

450

$$
\mathbf{H} = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}
$$

16

$8 \quad 2^1$

$4 \quad 2^2$

$16 + 8 + 8 = 32$

$2 \quad 2^3$

$H[m, n]$ 的值 ($m = 0, 1, \ldots, 2^k-1$, $n = 0, 1, \ldots, 2^k-1$)：

$H[0, n] = 1$ for all n

If $2^h \le m < 2^{h+1}$

$H[m, n] = 1$ for $(m - 2^h)2^{k-h} \le n < (m - 2^h + 1/2)2^{k-h}$

$H[m, n] = -1$ for $(m - 2^h + 1/2)2^{k-h} \le n < (m - 2^h + 1)2^{k-h}$

$H[m, n] = 0$ otherwise

analyze the features with different locations and scales

運算量比 Walsh transforms 更少

Applications: localized spectrum analysis, edge detection

256 points

Transforms	Running Time	terms required for NRMSE $< 10^{-5}$
DFT	9.5 sec $\Rightarrow 10^{-5}$	43
Walsh Transform	2.2 sec	65
Haar Transform	0.3 sec	128

Main Advantage of the Haar Transform $\xrightarrow{\text{extend}}$ wavelet transforms

(1) Fast (but this advantage is no longer important)

(2) Analysis of the local high frequency component
(The wavelet transform is a generalization of the Haar transform)

(3) Extracting local features → 数位 camera
(Example: Adaboost face detection)

附錄十三 SCI Papers 查詢方式

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Impact Factor (影響係數)

Key Indicators

Year	Total Cites Graph	Journal Impact Factor Graph	Impact Factor Without Journal Self Cites Graph	5 Year Impact Factor Graph	Immediacy Index Graph	Citable Items Graph	Cited Half-Life Graph	Citing Half-Life Graph	Eigenfacto Score Graph	Article Influence Score Graph	% Articles in Citable Items Graph	Normalizec Eigenfacto Graph	Average JIF Percentile Graph
2016	33,278	4.300	3.758	4.624	0.762	484	7.8	8.0	0.05...	1.550	99.79	6.73...	89.885
2015	22,917	2.624	2.133	3.157	0.462	496	7.8	7.6	0.06...	1.527	100.00	7.12...	86.965
2014	22,944	2.787	2.300	3.435	0.429	503	7.4	7.4	0.06...	1.473	100.00	6.90...	89.357
2013	22,913	3.198	2.658	3.592	0.439	508	7.2	7.2	0.07...	1.620	100.00	7.93...	90.927
2012	19,417	2.813	2.243	3.068	0.420	597	7.1	7.1	0.05...	1.291	100.00	Not ...	89.918
2011	16,727	2.628	2.111	2.829	0.421	527	6.9	7.2	0.05...	1.249	100.00	Not ...	88.776

(C) 關於 impact factor (影響係數)：

若一個 journal 裡面的文章，被別人引用的次數越多，則這個 journal 的 impact factor 越高

一般而言， impact factor 在 3.5 以上的 journals，已經算是高水準的期刊

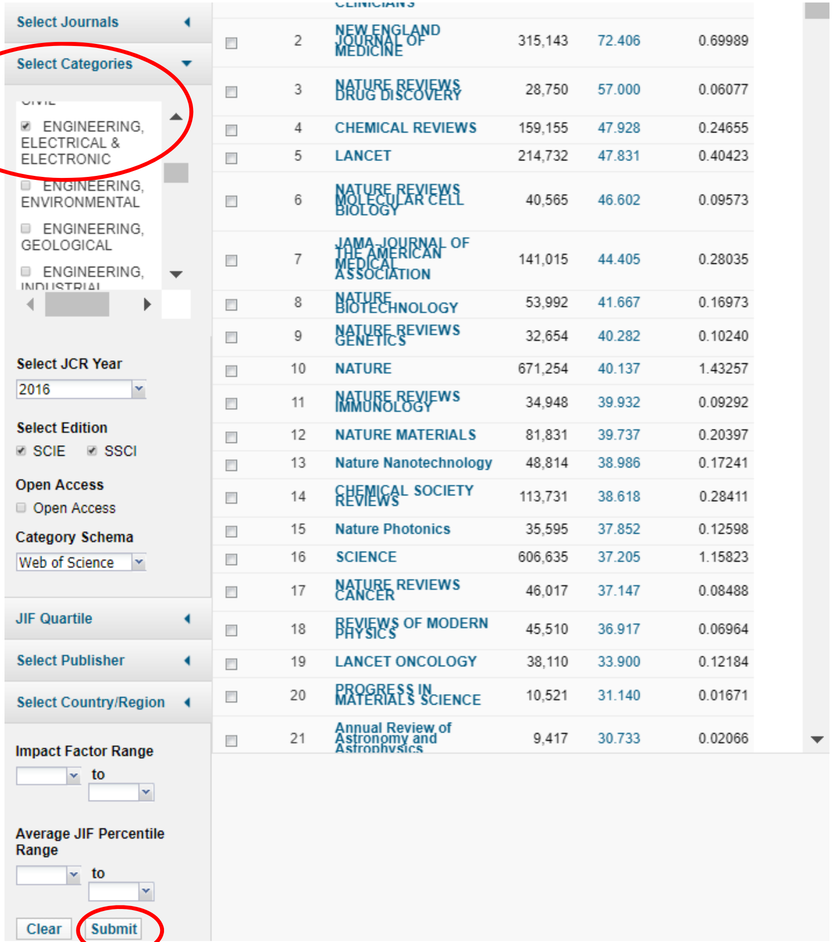
中等水準的期刊的 impact factors 在 1 到 3.5 之間

Nature 的 impact factor 為 42.778
Science 的 impact factor 為 41.845

IEEE 系列的期刊的 impact factors 通常在 2 到 8 之間
IEEE Trans. Image Processing的 impact factors 在 6 左右
IEEE Trans. Signal Processing的 impact factors 在 6 左右

(D) 要查詢一個領域有哪些 SCI journals

連結至 ISI Web of Knowledge 之後，點選「Select Category」



2	NEW ENGLAND JOURNAL OF MEDICINE	315,143	72.406	0.69989
3	NATURE REVIEWS DRUG DISCOVERY	28,750	57.000	0.06077
4	CHEMICAL REVIEWS	159,155	47.928	0.24655
5	LANCET	214,732	47.831	0.40423
6	NATURE REVIEWS MOLECULAR CELL BIOLOGY	40,565	46.602	0.09573
7	JAMA-JOURNAL OF THE AMERICAN MEDICAL ASSOCIATION	141,015	44.405	0.28035
8	NATURE BIOTECHNOLOGY	53,992	41.667	0.16973
9	NATURE REVIEWS GENETICS	32,654	40.282	0.10240
10	NATURE	671,254	40.137	1.43257
11	NATURE REVIEWS IMMUNOLOGY	34,948	39.932	0.09292
12	NATURE MATERIALS	81,831	39.737	0.20397
13	Nature Nanotechnology	48,814	38.986	0.17241
14	CHEMICAL SOCIETY REVIEWS	113,731	38.618	0.28411
15	Nature Photonics	35,595	37.852	0.12598
16	SCIENCE	606,635	37.205	1.15823
17	NATURE REVIEWS CANCER	46,017	37.147	0.08488
18	REVIEWS OF MODERN PHYSICS	45,510	36.917	0.06964
19	LANCET ONCOLOGY	38,110	33.900	0.12184
20	PROGRESS IN MATERIALS SCIENCE	10,521	31.140	0.01671
21	Annual Review of Astronomy and Astrophysics	9,417	30.733	0.02066

點選 category 之後再按 submit

(E) EI (Engineering Village)

官方網站： www.engineeringvillage.org

http://www.engineeringvillage.com/search/quick.url

查詢期刊或研討會是否為 EI

http://tul.blog.ntu.edu.tw/archives/4627

(F) SSCI (Social Science Citation Index)

比較偏向於社會科學

http://www.thomsonscientific.com/cgi-bin/jrnlst/jloptions.cgi?PC=J

(G) Conference 排名

Microsoft Academic Search 有列出各領域知名的 conferences 並加以排名(大致上也是被引用越多的排名越前面)

和通訊與信號處理相關的 conferences，大多排名於

http://academic.research.microsoft.com/RankList?entitytype=3&topDomainID=2&subDomainID=0&last=0&start=1&end=100

或

http://academic.research.microsoft.com/RankList?entitytype=3&topDomainID=8&subDomainID=0&last=0&start=1&end=100

(H) H Index

論文除了量以外，也要注意 citation 的次數

將發表的論文的 citation 次數從高到低做排序
如果排名第 N 名的論文 citation 數量大於等於 N
但是排名第 N+1名的論文 citation 數量小於等於 N+1

則 H index = N

Example: 假設有一個學者發表了10篇論文， citation 由多到少分別為
33, 24, 18, 13, 9, 7, 4, 3, 1, 1
則這個學者的 H-index 為 6

寫論文和投稿的經驗談

研究生經常會寫論文並且投稿。要如何讓論文投稿之後能夠順利的被接受，相信是同學們所期盼的，畢竟每篇論文都是大家花了不少時間的心血結晶，若論文能夠順利的被接受，也代表了自己的成果總算獲得了肯定。然而，影響論文是否被接受的因素很多，一個好的研究成果，還是配合好的編寫技巧，才可以被一流的期刊或研討會所接受。以下是個人關於論文編寫與投稿的經驗談：

(1) 你的論文的「賣點」(優點) 是什麼？人家有沒有辦法一眼看得出來你論文的「賣點」？

寫論文其實就是在推銷商品，而所謂的「商品」，就是你的「研究成果」。要說服人家接受你的商品，首先就是要強調你的商品的「賣點」。

(2) 和既有的方法的比較是否足夠？

要證明你所提出的方法是有效的，最好的方式，就是和既有的方法相比較，而且比較的對象越多越好，越新越好。

(3) 和前人的方法相比，你的方法創新的地方在何處？審稿者是否能看得出來你論文創新的地方？

(4) 就算你的文章和理論相關，最好也多提出實際應用的例子

(5) 參考資料越多越好，越新越好
(在研究一個領域時，論文 survey 的量要足夠)

(6) Previous work (前人已經提出的概念) 精簡介紹即可，多強調自己的貢獻。Introduction 加上 Previous work 最好不要超過一篇論文的四分之一

(7) 英文表達能力要有一定的水準

(8)可以多用數學式和圖來解釋概念，有時會比文字還清楚

通常東方人英文表達能力有限。審稿者經常會看你們的圖表和數學式(而非文字) 來判斷你們論文的品質

(9) 同樣的道理，可以用「條列式」的方式來取代一大段文字來描述方法的觀念、流程、或優點

(10) 可以用 Conference 的期限來要求自己多寫研討會論文，之後再陸續改成期刊論文投稿，如此一年的論文量將很可觀

(11)多注意格式，不同的期刊或研討會，對格式的要求也不同

(12) 最後，問自己一個問題：

如果你是審稿者，你會滿意你寫的這一篇論文嗎？

若答案是肯定的再投稿