## Homework 4 (Due: 6/18)

(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two gray images A and B. The sizes of A and B are equivalent.

SSIM(A, B, c1, c2)

where c1 and c2 are some adjust constants.

The code should be handed out by ceiba. (20 scores)

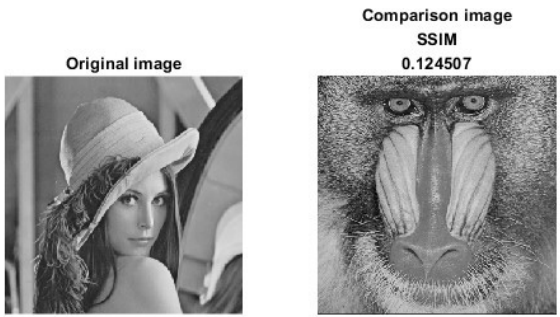


Fig 1. Result 1

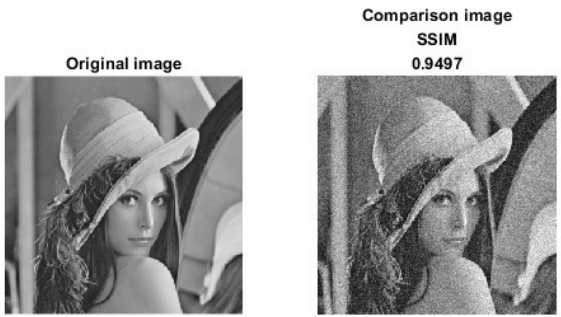


Fig 2. Result 2

(2) State at least three examples that using the PSNR cannot reflect the similarity of two vocal signals. (10 scores)

① 延遲或相位的變化

```
>> A = cos(2*pi*1000*(t-0.0001)) ;B = sin(2*pi*1000*t);
>> psnr(A,B)

ans =

    3.8488
```

② 頻率(Frequency)的調整

```
>> A = 5*cos(2*pi*1001*(t)); B = 5*cos(2*pi*1000*(t));
>> psnr(A,B)

ans =

  -13.9794
```

③ 振幅(amplitude)的變化

```
>> A = 5*cos(2*pi*1000*(t)); B = 4.99999999*cos(2*pi*1000*(t));
>> psnr(A,B)

ans =

  163.0103
```

(3) How do we implement the following matrix operation with the least number of multiplications? (15 scores)

$a = 0.7010,\ b = 0.9239,\ c = 0.3827$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.7010 & 0.7010 & 0.7010 & 0.7010 \\ 0.9239 & 0.3827 & -0.3827 & -0.9239 \\ 0.7010 & -0.7010 & -0.7010 & 0.7010 \\ 0.3827 & -0.9239 & 0.9239 & -0.3827 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

①

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ b & c & -c & -b \\ a & -a & -a & a \\ c & -b & b & -c \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

②

$$\begin{array}{c|cc|c|c} y_0 & a & a & a & a & x_0 \\ y_2 & a & -a & -a & a & x_2 \\ \hline y_1 = & b & -c & c & -b & x_1 \\ y_3 & c & b & -b & -c & x_3 \end{array}$$

$$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \begin{bmatrix} b & -c \\ c & b \end{bmatrix} \begin{bmatrix} x_0 - x_3 \\ x_2 - x_1 \end{bmatrix}$$

$bx_0 - bx_3 - cx_2 + cx_1$

$cx_0 - cx_3 + bx_2 - bx_1$

$\Rightarrow$ 2 MUL

③

$$\begin{bmatrix} z_0 \\ z_1 \end{bmatrix} = \begin{bmatrix} a & a \\ a & a \end{bmatrix} \begin{bmatrix} x_0 \\ x_3 \end{bmatrix} \Rightarrow 1\,\text{MUL}$$

$z_0 = a(x_0 + x_3) = z_1$

$$\begin{bmatrix} z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} a & a \\ -a & -a \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} \Rightarrow 1\,\text{MUL}$$

$z_2 = a(x_2 + x_1) = -z_3$

$$\Rightarrow \begin{bmatrix} y_0 \\ y_2 \end{bmatrix} = \begin{bmatrix} z_0 + z_2 \\ z_1 + z_3 \end{bmatrix}$$

Ans: 4 MUL ※

(4) Suppose that $x$ is a complex number. What are the constraints of $\theta$ such that the multiplication of $x$ and $\exp(j\theta)$ required only 2 real multiplications?

(10 scores)

$\exp(j\theta) = \underset{c}{\cos\theta} + j\underset{d}{\sin\theta}$, $x = a + jb \Rightarrow$

if $\cos\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{4} \Rightarrow c = d$

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & -c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -2c \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

1MUL + 1MUL = 2MUL ※

if $\theta = \frac{\pi}{6}$, $\cos\theta$ or $\sin\theta \Rightarrow$ 其中一個等0

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -c \\ -c & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

1MUL + 1MUL = 2MUL ※

(5) Determining the numbers of real multiplications for the (a) 165-point DFT, (b) 242-point DFT. (10 scores)

(a) $165 = 15 \times 11$

$\Rightarrow 11\,MVL_{15} + 15\,MVL_{11} = 11 \times 40 + 15 \times 40$

$\Rightarrow 1040$ ※

(b) $242 = 2 \times 11^2$

$121 \times 0 + 2 \times MVL_{121}$

$= 2\,(11\,MVL_{11} + 11\,MVL_{11} + 3 \times (11-1)(11-1))$

$= 2\,(11 \times 40 + 11 \times 40 + 3 \times 100) = 2360$ ※

(6) Suppose that we want to implement the convolution of two complex sequences $x[n]$ and $h[n]$ where length($x[n]$) = 300 and length($h[n]$) = 200 .

(a) What is the number of the points of the DFT that should be used for implementation?

(b) How many real multiplications are required? (10 scores)

(a) $P \geq 300 + 200 - 1 = 499$

透過查表可用 $P = 504$ ※

(b) $MUL_{504} = 2 \times 2300 + 3 \times 504$

$= 6112$ ※

(7) Suppose that a 1-D edge detector is:

$x_s[n] = x[n] * h[n]$ $\quad h[1] = -h[-1] = 0.8$ $\quad h[2] = -h[-2] = 0.15$

$h[3] = -h[-3] = 0.075$ $\quad h[0] = 0$ $\quad h[n] = 0$ otherwise

Design an efficient way with least number of real multiplications to implement the above filter operation. (10 scores)

$h[n] = [-0.075 \quad -0.15 \quad -0.8 \quad 0 \quad 0.8 \quad 0.15 \quad 0.075]$ for $n = -3 \sim 3$

(-3 -2 -1 0 1 2 3)

↳ real number

Using the directly computing method.

$x_s[n] = x[n] \times h[n]$

$\Rightarrow -0.075\,x[n+3] - 0.15\,x[n+2] - 0.8\,x[n+1] + 0 \cdot x[n] + 0.8\,x[n-1] + 0.15\,x[n-2] + 0.075\,x[n-3]$

$\Rightarrow 0.075\,(x[n-3] - x[n+3] + 2[x[n-2] - x[n+2]]) + 0.8\,(x[n-1] - x[n+1])$

↳ 2 "係"

$\Rightarrow$ nearly 2 MUL for each output

$N = 1600$

(8) Suppose that length($x[n]$) = 1600. What is the best way to implement the convolution of two complex sequences $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 450, (b) length($y[n]$) = 30, and

(c) length($y[n]$) = 2, (15 scores)

$M$

section convolution

要比較

(a)

Directly computing

$3NxM = 2160000$

Using the 2304-point DFT

$P \geq N+M-1 = 2049$

$P = 2304$ , 15868

$2 \cdot 15868 + 3 \cdot 2304$

$= 38648$ ※(最大)

(b)

section convolution

$M << N$

$L = 174$ (by estimation)

$P \geq 174 + 30 - 1 = 203$

if $P = 240$, $MUL = 940$

$L = 211$, $S = 8$

$2 \cdot 8 \cdot 940 + 3 \cdot 8 \cdot 240$

$= 20800$ ※(較大)

If $P = 204$, $MUL = 936$

$L = 175$, $S = 10$

$2 \cdot 10 \cdot 936 + 3 \cdot 10 \cdot 204$

$= 25640$

(c)

Directly computing

$3 \times N \times M = 8000$

Using section convolution

$N = 1600$, $M = 2$

$L = 2$

$P \geq 2 + 2 - 1 = 3$

If $P = 3$, $MUL = 2$

$L = 2$, $S = 800$

$2 \cdot 800 \cdot 2 + 3 \cdot 800 \cdot 3$

$= 10400$

if $P = 4$, $MUL = 0$

$L = 3$, $S = 534$

$0 + 3 \cdot 534 \cdot 4$

$= 6408$ ※(最大)

(Extra): Answer the questions according to your student ID number.
(ended with 0, 1, 4, 5, 6, 9)

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$MUL_{44} = ?$

$MUL_{44} = 4\,MUL_{11} + 11\,MUL_4 = 4 \times 40 + 11 \times 16 = 456$