Homework 2 (Due: 4/23)

(1) Write a Matlab or Python program that uses the frequency sampling method to design a $(2k+1)$-point discrete differentiation filter $H(F) = j2\pi F$ (k is an input parameter and can be any integer). (25 scores)

The transition band can be assigned to reduce the error (unnecessary to optimize). The impulse response of the designed filter should be shown. The code should be handed out by ceiba.

(1) Matlab code

```
function ADSP_HW2_M10907305(k,f_sampling,transition_band)
%example:ADSP_HW2_M10907305(10,1000,0.1)
%transition_band is Symmetrical,so you cann't set two
% How to use a k point to design.
% f_sampling is sampling of frequency.
N = 2*k+1;
F = [ (0:k)*(1-transition_band)/N (k+1:N-1)*(1-transition_band)/N
+transition_band];

Hd_f = j*2*pi*(F-(F>=0.5));
r1 = ifft(Hd_f);
r  = circshift(r1,k);

Hd_sampling1 = (0:f_sampling*(1-transition_band)/2)/f_sampling;
Hd_sampling2 = (f_sampling*(transition_band+(1-
transition_band)/2):f_sampling)/f_sampling;
Hd_f_sampling1 = j*2*pi*(Hd_sampling1-(Hd_sampling1>=0.5));
Hd_f_sampling2 = j*2*pi*(Hd_sampling2-(Hd_sampling2>=0.5));
transition =
 linspace(Hd_f_sampling1(end),Hd_f_sampling2(1),transition_band*f_sampling
+1);
Hd_f_sampling = [ Hd_f_sampling1(1:end-1) transition
 Hd_f_sampling2(1:end-1)];
ff = [];
for i = 1:N
    ff = [ff Hd_f_sampling(round((i-1)/N*1000)+1) ]   ;
end
fff = circshift(ifft(ff),k);
fftt = fft(circshift([fff zeros(1, f_sampling-2*k-1)], [0 -k]));
fs_sampling = (0:f_sampling-1)/f_sampling;

figure;
% plot([F 1],imag([Hd_f 0]),'blue');
plot((0:f_sampling)/
f_sampling,imag(Hd_f_sampling),'blue',fs_sampling,imag(fftt),'red');
title('Frequency Response')
xlabel('F')
figure;
subplot(3,1,1);
stem(0:N-1,real(r1));
xlim([-N-5,N+5]);
title('r1[n]');
subplot(3,1,2);
stem(-k:k,real(r));
xlim([-N-5,N+5]);
title('r[n]');
subplot(3,1,3);
stem(0:N-1,real(r));
xlim([-N-5,N+5]);
title('h[n]');
```

Fig1. Frequency sampling method code.

(1) Impulse response

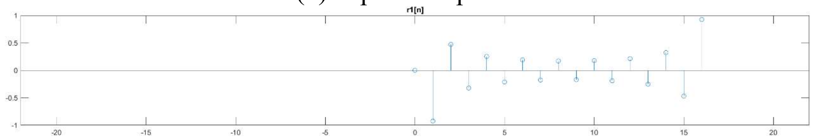


Fig2. The impulse response of r1[n]

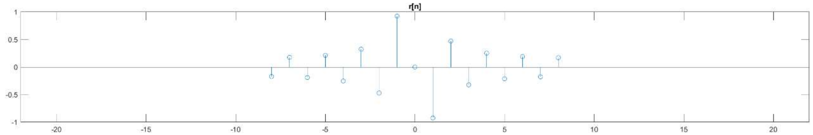


Fig3. The impulse response of r[n]

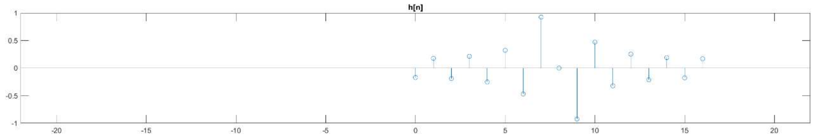


Fig4. The impulse response of h[n].

(2) Frequency response

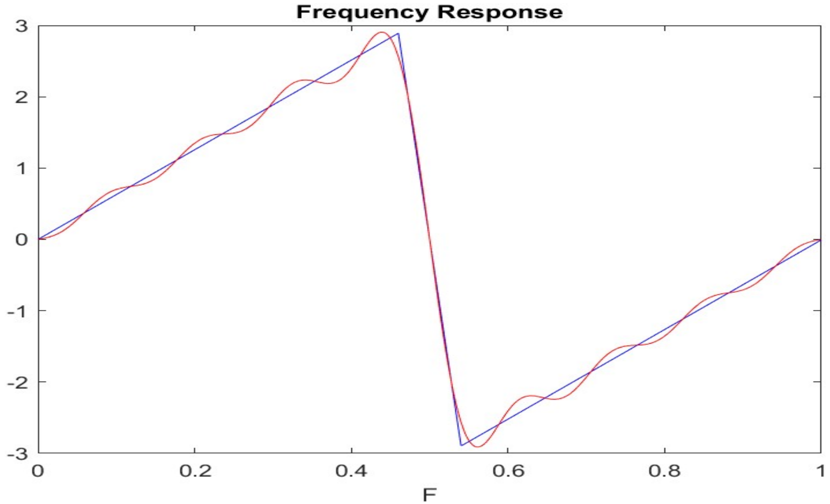


Fig5. k = 8 and transition band = 0.08

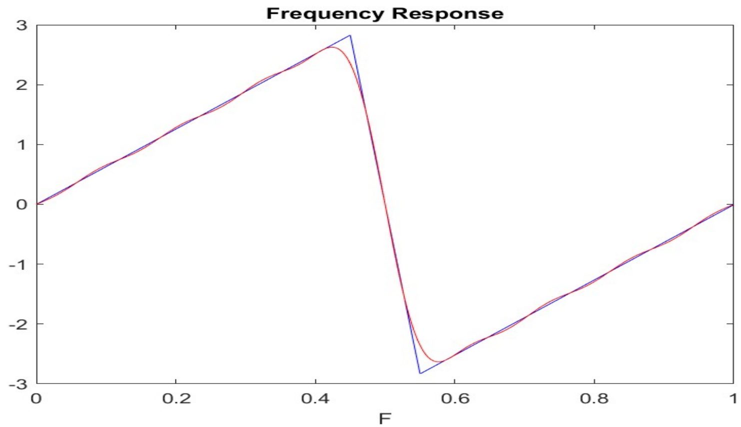


Fig6. k = 8 and transition band = 0.1

(2) What are the advantages of using the Wiener filter for <u>equalizer</u>? (10 scores)

① 當 equalizer 加入 Wiener filter，如果當 $K(F) \cong 0$，不會導致無限大，造成不穩定的情況發生。

② 當 SNR 較大時，可以調大 Wiener filter 之 C，反之 SNR 較小時 C 值調小，有效的使 equalizer 在不同 SNR 情況下保持 stable。

ex.

$$H(F) = \frac{1}{\underbrace{\frac{1}{K^*(F)}\ \frac{1}{SNR(F)}}_{\text{Wiener filter}} + K(F)}$$

$K(F) = Ae^{j\phi}$

$K^*(F) = Ae^{-j\phi} \Rightarrow$ 保證不會負值

C 可調大調小，但不可調到 0 以下

(3) Derive the way to design the FIR filter of Type IV using the method of the FIR filter of Type I. (10 scores)

$\because$ Type 4 : $R(F) = \sum_{n=1}^{k+1/2} S[n]\sin(2\pi(n-\frac{1}{2})F)$, $h[n] = -h[N-1-n]$

利用積化和差： $\sin(2\pi(n+\frac{1}{2})F) - \sin(2\pi(n-\frac{1}{2})F) = \underline{2\cos(2\pi nF)}\sin(\pi F)$ Type 1

求 $S_1[n]$ & $S[n]$ 關係： $R(F) = \sum_{n=0}^{k_1} S_1[n]\sin(\pi F)\cos(2\pi nF) = \sum_{n=0}^{k_1}[\frac{1}{2}S_1[n]\sin(2\pi(n+\frac{1}{2})F)] - \sum_{n=0}^{k_1}[\frac{1}{2}S_1[n]\sin(2\pi(n-\frac{1}{2})F)]$

$\Rightarrow \sum_{n=1}^{k_1+1}[\frac{1}{2}S_1[n-1]\sin(2\pi(n-\frac{1}{2})F)] - \sum_{n=0}^{k_1}[\frac{1}{2}S_1[n]\sin(2\pi(n-\frac{1}{2})F)] = -\frac{1}{2}(S_1[0]\sin(-\pi F)) + \sum_{n=1}^{k_1}[\frac{1}{2}(S_1[n-1]-S_1[n])\sin(2\pi(n-\frac{1}{2})F)] + \frac{1}{2}S_1[k_1]\sin(2\pi(k_1+\frac{1}{2})F)$

$\Rightarrow (S[0] - \frac{1}{2}S_1[1])\sin(\pi F) + \sum_{n=2}^{k_1 \to k-\frac{1}{2}}[\frac{1}{2}(S_1[n-1]-S_1[n])\sin(2\pi(n-\frac{1}{2})F)] + \frac{1}{2}S_1[\overset{k-\frac{1}{2}}{k_1}]\sin(2\pi(\underbrace{\overset{k-\frac{1}{2}}{k_1}+\frac{1}{2}}_{k})F)$

令 $k_1 + \frac{1}{2} = k$, $k_1 = k - \frac{1}{2}$

$S[n]$ 和 $S_1[n]$ 關係

$\because S[1] = S_1[0] - \frac{1}{2}S_1[1]$

$S[n] = \frac{1}{2}(S_1[n-1] - S_1[n])$, for $n = 2, 3, \dots, k-\frac{1}{2}$

$S[k-\frac{1}{2}] = \frac{1}{2}S_1[k-\frac{1}{2}]$ #

$err(F) = [H_d(F) - R(F)]W(F)$

$R(F) = \sum_{n=0}^{k} S[n]\cos(2\pi nF) \Rightarrow \sum_{n=0}^{k_1} S_1[n]\sin(\pi F)\cos(2\pi nF)$

$\Rightarrow [H_d(F) - \sin(\pi F)\sum_{n=0}^{k-\frac{1}{2}} S_1[n]\cos(2\pi nF)]W(F)$ 提出 $\sin(\pi F)$

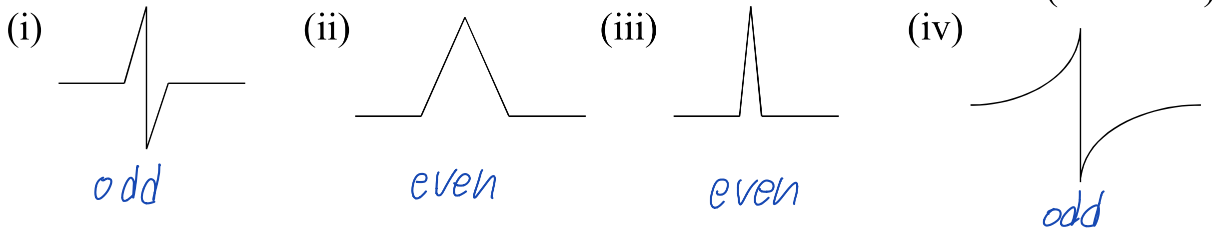
$\Rightarrow [\csc(\pi F)H_d(F) - \sum_{n=0}^{k-\frac{1}{2}} S_1[n]\cos(2\pi nF)]\sin(\pi F)W(F)$

$H_d(F) \Rightarrow \csc(\pi F)H_d(F)$

$W(F) \Rightarrow \sin(\pi F)W(F)$

$k \Rightarrow k-\frac{1}{2} = \frac{N}{2} - 1$ #

(4) The following figures are the impulse responses of some filters. Which one is suitable for ridge detection when the SNR is low? Also illustrate the reasons. (10 scores)



ridge detection 為 even-symmetic，因此選擇 (ii)、(iii)

，但 SNR is low，而 $SNR = \frac{P_{signal}}{P_{noise}}$，在 noise 較大的情況下，

要選擇 (ii) 效果比較好

(5) Suppose that the smooth filter is $h[n] = 0.05$ for $|n| \leqq 5$, $h[n] = a$ for $6 \leqq |n| \leqq 15$, and $h[n] = 0$ otherwise. (a) What is the value of a? (b) What is the efficient way to implement the convolution $y[n] = x[n] * h[n]$? (10 scores)

(a) 在 smooth filter 特性中:

$$\sum_{\tau} h[\tau] = 1$$

$$h[n] = 0.05 \;,\; |n| \leq 5$$

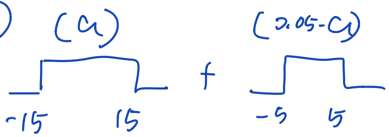
$$\Rightarrow 0.05\,(5-(-5)+1) = 0.55$$

$$2(15-6+1)\,a = 1-0.55$$

$$20a = 0.45$$

$$a = 0.0225 \;\#$$

(b)



等比級數方法, $L_1 = 15$, $L_2 = 5$

$$\Rightarrow h[n] = \frac{1}{2L+1}\left(\underbrace{u[n+L]} - u[n-L-1]\right)$$

$$H(z) = \sum_{n} h[n] z^{-n} = \sum_{n=0}^{\infty} 1^n \cdot z^{-n} \Rightarrow \sum_{n=0}^{\infty} (1 \cdot z^{-1})^n$$

$$\Rightarrow \begin{array}{l} q = n+L \\ n = q-L \end{array} \Rightarrow \sum_{q=0}^{+\infty} z^{-(q-L)} = \frac{z^{-(-L)}}{1-z^{-1}}$$

$L_1, L_2 = 15, 5$

$$\Rightarrow y(z) = 0.0225 \cdot x(z)\left[\left(\frac{z^{-(-L_1)}}{1-z^{-1}} - \frac{z^{-(L_1+1)}}{1-z^{-1}}\right) - \left(\frac{z^{-(-L_2)}}{1-z^{-1}} - \frac{z^{-(L_2+1)}}{1-z^{-1}}\right)\right]$$

$$\Rightarrow y(z) - z^{-1}y(z) = 0.0225\, x(z)\left[(z^{15} - z^{-16}) - (z^{10} - z^{-11})\right]$$

$$y[n] = 0.0225\left[(x(n+15) - x(n-16)) - (x(n+10) - x(n-11))\right] + y[n-1]$$

(6) Suppose that an IIR filter is $H(z) = \dfrac{3z^3 - 4z^2 - 3z - 2}{2z^2 - 1}$

(a) Find its <u>cepstrum</u>.

(b) Convert it into the <u>minimum phase filter</u>.

(c) Compared to the original IIR filter, what <u>are two advantages of the minimum phase filter</u>? (20 scores)

(a)

$$\because H(z) = \frac{3z^3 - 4z^2 - 3z - 2}{2z^2 - 1}$$

$$\Rightarrow \frac{3 \cdot -2(1-0.5z)\, z^2 \left(1-\left(-\frac{1}{3}+\frac{\sqrt{2}}{3}i\right)z^{-1}\right)\left(1-\left(-\frac{1}{3}-\frac{\sqrt{2}}{3}i\right)z^{-1}\right)}{2z^2\left(1-\left(-\sqrt{1/2}\right)z^{-1}\right)\left(1-\sqrt{1/2}\, z^{-1}\right)}$$

$$\Rightarrow \frac{-3(1-0.5z)\left(1-\left(-\frac{1}{3}+\frac{\sqrt{2}}{3}i\right)z^{-1}\right)\left(1-\left(-\frac{1}{3}-\frac{\sqrt{2}}{3}i\right)z^{-1}\right)}{\left(1-\left(-\sqrt{1/2}\right)z^{-1}\right)\left(1-\sqrt{1/2}\, z^{-1}\right)}$$

$$\hat{x}[n] = \begin{cases} \log(-3) = 1.0986 + 3.1416i, & n = 0 \\ -\dfrac{\left(-\frac{1}{3}+\frac{\sqrt{2}}{3}i\right)^n}{n} - \dfrac{\left(-\frac{1}{3}-\frac{\sqrt{2}}{3}i\right)^n}{n} + \dfrac{\left(-\frac{\sqrt{2}}{2}\right)^n}{n} + \dfrac{\left(\frac{\sqrt{2}}{2}\right)^n}{n}, & n > 0 \\ \dfrac{(0.5)^{-n}}{n}, & n < 0 \end{cases}$$

(b)

$$H(z) = \frac{3(z-2)\left(z-\left(-\frac{1}{3}+\frac{\sqrt{2}}{3}i\right)\right)\left(z-\left(-\frac{1}{3}-\frac{\sqrt{2}}{3}i\right)\right)}{2(z-1)(z+1)}$$

$z=2$ 超出 range

$\because \dfrac{z-0.5}{z-2}$ is an all-pass filter

$$\Rightarrow \because H(z)\, H_{ap}(z) = H_{mp}(z)$$

$$H_{mp}(z) = \frac{3(z-0.5)\left(z-\left(-\frac{1}{3}+\frac{\sqrt{2}}{3}i\right)\right)\left(z-\left(-\frac{1}{3}-\frac{\sqrt{2}}{3}i\right)\right)}{(z-\sqrt{0.5})(z+\sqrt{0.5})}$$

(c)

① try to make the energy concentrating on the region near to $n=0$.

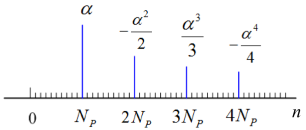
② try to make both the forward and the inverse transforms stable.

上課解

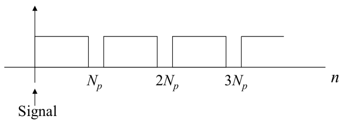
(7) (a) Why the <u>cepstrum</u> is more suitable for dealing with the <u>multipath problem</u> than the equalizer $1/H(z)$ where $H(z)$ is the z transform of the impulse response? (Write at least 2 reasons) (b) Why the <u>Mel-cepstrum</u> is more suitable for dealing with the acoustic signal than the original cepstrum? (Write at least 3 reasons) (15 scores)

(a)

① 與 α 沒有關係



Equalization for echo



lifter

② 可以用 cepstrum 將 mulitpath 的影響去除

③ 能用方便提取在分析原頻譜圖上難以辨別的週期訊號。

(b)

① $B_m[k]$ 為等比級數，符合人類聽覺特性。

② The probability that $\sum |x[k]|^2 B_m[k] = 0$ is much less

③ No phase ambiguity since $\sum |x[k]|^2 B_m[k]$ is real

④ For the inverse, the DCT is applied to reduce the computation

(Extra): Answer the questions according to your student ID number.

(ended with 0, 3, 4, 5, 8, 9)

計算等比級數的cepstrum: student ID : M10909305

$$x[n] = \begin{cases} 0.6^n, & \text{for } n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 0.6^n u[n] z^{-n} = \sum_{n=0}^{\infty} (0.6 z^{-1})^n$$

$$\Rightarrow X(z) = \frac{1}{1 - 0.6 z^{-1}} = \frac{z}{z - 0.6}, \quad |z| > |0.6|$$

cepstrum :

$$X(z) = \frac{1}{(1 - 0.6 z^{-1})} \Rightarrow \begin{cases} \sum_{n=1}^{\infty} \frac{(0.6)^n}{n}, & n > 0 \\ 0, & \text{otherwise} \end{cases}$$