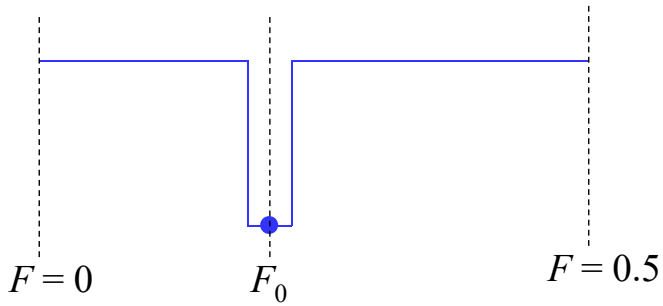
# 4. Some Popular Filters

## ◉ 4-A Popular Filters (1): Pass-Stop Band Filters

highpass bandpass lowpass allpass bandstop

notch filter: 想濾掉 $F = F_0$ 的 noise，但 stop band 越小越好




Question: Why the notch filter is hard to design?

## References

[1] K. Hirano, S. Nishimura, and S. K. Mitra, “Design of digital notch filters,” *IEEE Trans. Commun.*, vol. 22, no. 7, pp. 964-970, Jul. 1974.

[2] T. H. Yu, S. K. Mitra and H. Babic, “Design of linear phase FIR notch filters,” in *Sadhana*, Springer, vol. 15, issue 3, pp. 133-155, Nov. 1990.

[3] S. C. D. Roy, S. B. Jain, and B. Kumar, "Design of digital FIR notch filters," *Vision, Image and Signal Processing, IEE Proceedings*, vol.141, no. 5, pp.334-338, Oct. 1994.

[4] S. C. Pei and C. C. Tseng, “IIR multiple notch filter design based on allpass filter,” *IEEE Trans. Circuits Syst. II*, vol. 44, no.2, pp. 133-136, Feb. 1997.

[5] C. C. Tseng and S. C. Pei, “Stable IIR notch filter design with optimal pole placement,” *IEEE Trans. Signal Processing*, vol. 49, issue 11, pp. 2673-2681, Nov. 2001.

# ◉ 4-B Popular Filters (2): Smoother (Weighted Average)

近似 low-pass filter

最簡單的 smoother:

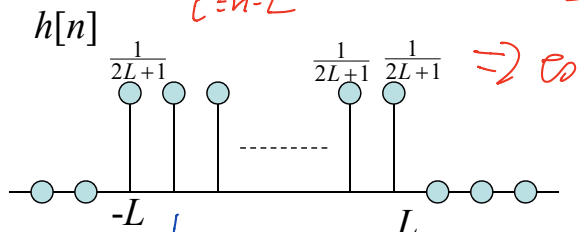
find the average $y[n]=\frac{1}{2L+1}\sum_{\tau=n-L}^{n+L}x[\tau]$

$\sum_{\tau=n-L}^{n+L}\frac{1}{2L+1} = n+L-(n-L) = 2L+1$

可改寫成

$y[n]=x[n]*h[n]$

$h[n]$ 如右圖




$\Rightarrow$ total : 1

$h[n] = \frac{1}{2L+1}(u[n+L]-u[n-L-1])$

$H(f) = \text{sinc}$ (periodic)

$$y[n]=\sum_{\tau}x[n-\tau]h[\tau]=\sum_{\tau=-L}^{L}x[n-\tau]\frac{1}{2L+1}=\frac{1}{2L+1}\sum_{\tau=-L}^{L}x[n+\tau]$$

if $x_1[n]=\frac{1}{2L+1}x[n]$, $y[n]=\sum_{\tau=n-L}^{n+L}x_1[n]$, $y[n]=y[n-1]+\cdots$

一般型態的 smoother

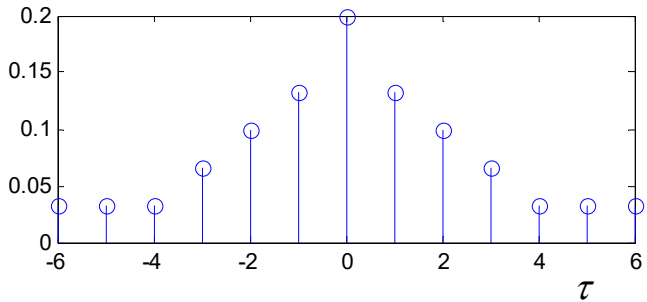
$$y[n] = x[n] * h[n] = \sum_{\tau} x[n-\tau]h[\tau]$$

Choose (1) $h[n] = h[-n]$

(2) $|h[n_1]| \le |h[n_2]|$ if $|n_1| > |n_2|$

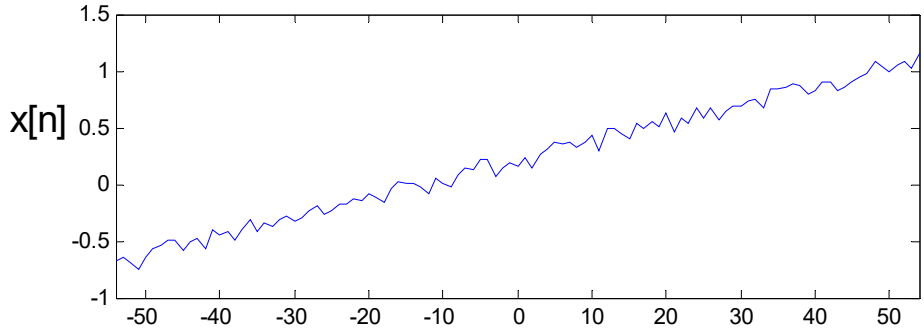
(3) $h[n] \ge 0$ for all $n$

(4) $\sum_{\tau} h[\tau] = 1$

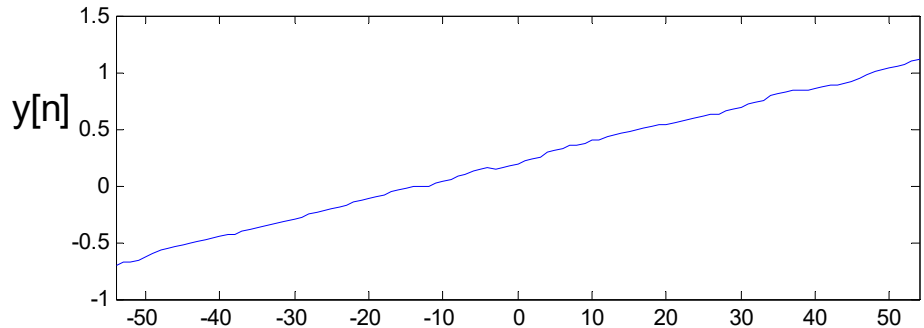



任何能量隨著 $|n|$ 遞減的 even function，都可以當成 smoother filter

Example




After applying the smoother filter



Smoother 是一種 lowpass filter (但不為 pass-stop band filter)

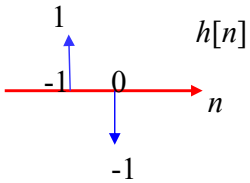
思考: smoother 在信號處理上有哪些功用？

# ◉ 4-C Popular Filters (3): Family of Odd Symmetric Filters

(a) Differentiation $H(f) = j2\pi f$ when $-f_s/2 < f < f_s/2$,

$$H(f) = H(f + f_s)$$

(b) Difference (一個簡單取代 differentiation 的方法)




$$x_1[n] = x[n] * h[n] = x[n+1] - x[n]$$

$h[n] = 1$ when $n = -1$, $h[n] = -1$ when $n = 0$,

$h[n] = 0$ otherwise

$$H(F) = j2e^{j\pi F}\sin(\pi F)$$

These two filters are equivalent only at low frequencies

(C) Discrete Hilbert Transform

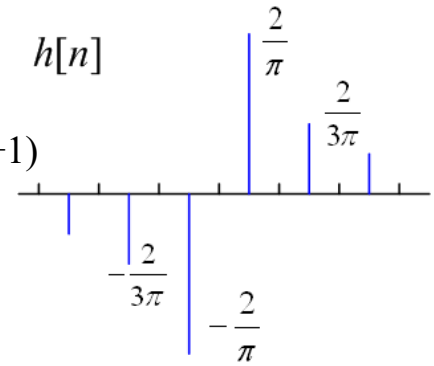
$H(F) = -j$ for $0 < F < 0.5$

$H(F) = j$ for $-0.5 < F < 0$

$H(0) = H(0.5) = 0$

$H(F) = H(F+1)$

$h[n] = \dfrac{2}{\pi n}$ when $n$ is odd, $h[n] = 0$ otherwise




Applications: (1) analytic function, (2) instantaneous frequency, (3) edge detection

Analytic function: $x_a[n] = x[n] + jx_H[n]$

where $x_H[n] = x[n] * h[n]$

(D) Edge Detection

(1) $h[n] = -h[-n]$
(2) $|h[n_1]| \le |h[n_2]|$ if $|n_1| > |n_2|$

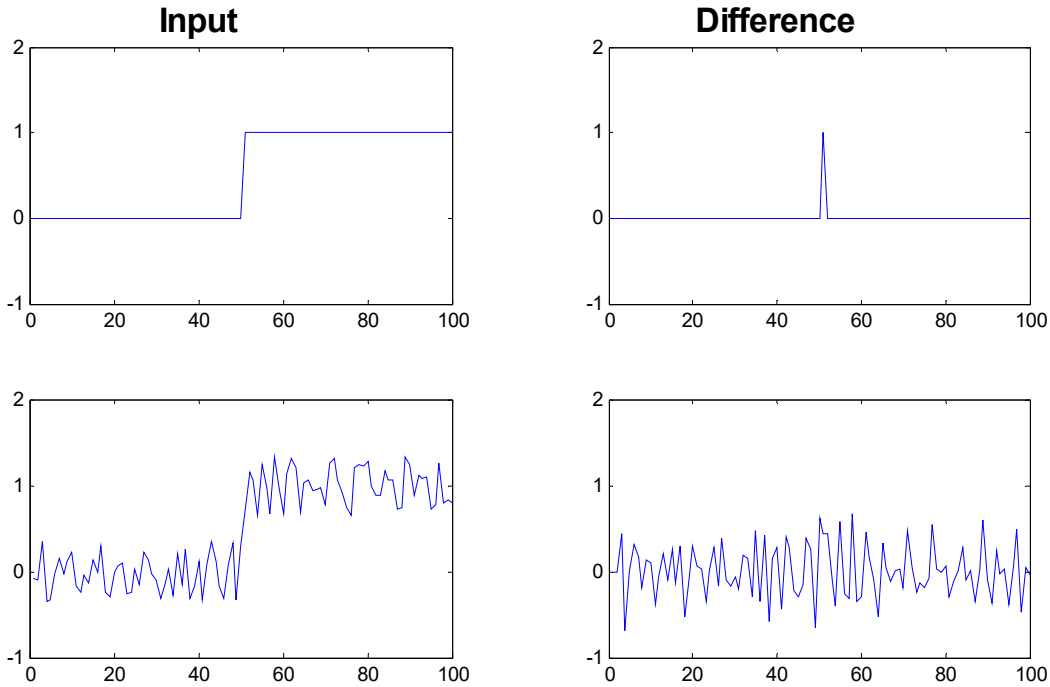
Difference 和 discrete Hilbert transform 都可用作 edge detection

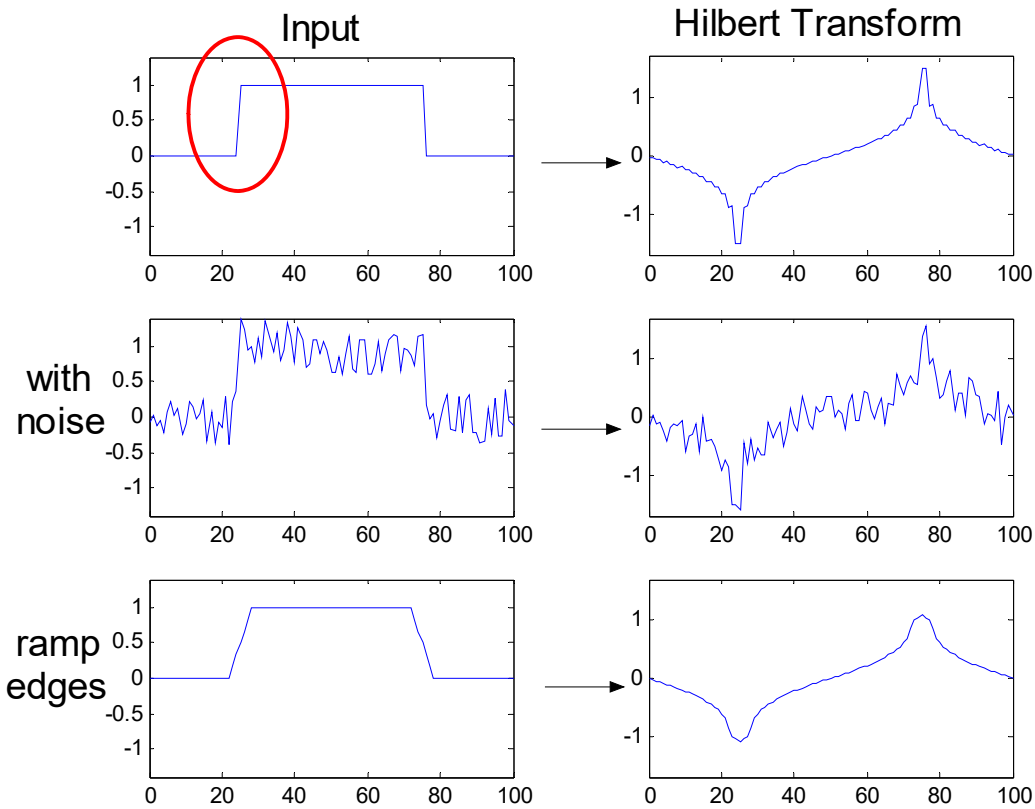
(1) 任何能量隨著 $|n|$ 遞減的 odd function，都可以當成 edge detection filter

(2) The edge detection filter is in fact a matched filter.

Reference

S. C. Pei and J. J. Ding, "Short response Hilbert transform for edge detection," *IEEE Asia Pacific Conference on Circuits and Systems*, Macao, China, pp. 340-343, Dec. 2008.


Input
Difference
2
1
0
-1
0
20
40
60
80
100


Input
Hilbert Transform
with noise
ramp edges
1
0.5
0
-0.5
-1
0
20
40
60
80
100

Other well-know edge detection filter:

Canny's Filter

L. Ding and A, Goshtasby. "On the Canny edge detector," *Pattern Recognition*, vol. 34, issue 3, pp. 721-725, 2001.

Sobel filter (A 2D Edge Detection Filter)

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

# Sobel Operator (horizontal)

$$\{2A[m, n+1] - 2A[m, n-1] + A[m+1, n+1] - A[m+1, n-1] + A[m-1, n+1] - A[m-1, n-1]\}/4$$

$$A * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} / 4$$

$n \longrightarrow$

$m \downarrow$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 11 | 10 | 10 | 10 | 12 | 11 | 10 | 9 | 10 |
| 10 | 10 | 11 | 10 | 10 | 10 | 10 | 11 | 9 |
| 10 | 10 | 9 | 150 | 150 | 150 | 10 | 10 | 10 |
| 10 | 10 | 160 | 160 | 155 | 160 | 158 | 10 | 11 |
| 10 | 10 | 158 | 160 | 161 | 161 | 160 | 150 | 10 |
| 10 | 155 | 160 | 163 | 164 | 165 | 160 | 151 | 10 |
| 10 | 148 | 160 | 160 | 162 | 160 | 155 | 10 | 12 |
| 8 | 10 | 140 | 150 | 152 | 150 | 10 | 11 | 10 |
| 9 | 12 | 10 | 10 | 10 | 10 | 9 | 10 | 10 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | 0 | 35 | 35.25 | 0.25 | -35.5 | -35 | -0.5 | |
| | 37.25 | 107.5 | 69 | 0 | -69.25 | -107.25 | -37 | |
| | 111.75 | 147.5 | 33.5 | 0.25 | -33.75 | -112.75 | -111 | |
| | 149 | 114.5 | 1.25 | 1 | -0.75 | -46.5 | -149.25 | |
| | 149.5 | 44.5 | 3.25 | 1.25 | -4 | -47.25 | -148.25 | |
| | 145.5 | 43 | 5 | 0.5 | -40 | -113.25 | -109 | |
| | 103.75 | 72.5 | 6.5 | 0 | -73 | -107 | -35.5 | |
| | | | | | | | | |

# Sobel Operator (vertical)

$$\{2A[m+1, n] - 2A[m-1, n] + A[m+1, n+1] - A[m-1, n+1] + A[m+1, n-1] - A[m-1, n-1]\}/4$$

$$A * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} / 4$$

$n \rightarrow$

$m \downarrow$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 11 | 10 | 10 | 10 | 12 | 11 | 10 | 9 | 10 |
| 10 | 10 | 11 | 10 | 10 | 10 | 10 | 11 | 9 |
| 10 | 10 | 9 | 150 | 150 | 150 | 10 | 10 | 10 |
| 10 | 10 | 160 | 160 | 155 | 160 | 158 | 10 | 11 |
| 10 | 10 | 158 | 160 | 161 | 161 | 160 | 150 | 10 |
| 10 | 155 | 160 | 163 | 164 | 165 | 160 | 151 | 10 |
| 10 | 148 | 160 | 160 | 162 | 160 | 155 | 10 | 12 |
| 8 | 10 | 140 | 150 | 152 | 150 | 10 | 11 | 10 |
| 9 | 12 | 10 | 10 | 10 | 10 | 9 | 10 | 10 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | -0.5 | 34.5 | 104.25 | 138.75 | 104 | 35 | 0.5 | |
| | 37.25 | 112 | 148.5 | 147.5 | 148.25 | 111.25 | 37 | |
| | 37.25 | 77 | 45 | 10.75 | 45.75 | 112.75 | 107.5 | |
| | 72.5 | 37 | 3.75 | 6.5 | 5.25 | 37.5 | 70.75 | |
| | 69.5 | 35.5 | 0.75 | 0.25 | -1.5 | -37.75 | -70.75 | |
| | -78 | -49.5 | -14.5 | -13 | -48 | -113.75 | -107.5 | |
| | -105.75 | -146.5 | -150.5 | -151 | -149.5 | -110.5 | -37 | |
| | | | | | | | | |

# Sobel Operator (45°)

$$\{2A[m\text{-}1, n\text{+}1] - 2A[m\text{+}1, n\text{-}1] + A[m\text{-}1, n] - A[m\text{+}1, n] + A[m, n\text{+}1] - A[m, n\text{-}1]\}/4$$

$$A * \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} / 4$$

$n \rightarrow$, $m \downarrow$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 11 | 10 | 10 | 10 | 12 | 11 | 10 | 9 | 10 |
| 10 | 10 | 11 | 10 | 10 | 10 | 10 | 11 | 9 |
| 10 | 10 | 9 | 150 | 150 | 150 | 10 | 10 | 10 |
| 10 | 10 | 160 | 160 | 155 | 160 | 158 | 10 | 11 |
| 10 | 10 | 158 | 160 | 161 | 161 | 160 | 150 | 10 |
| 10 | 155 | 160 | 163 | 164 | 165 | 160 | 151 | 10 |
| 10 | 148 | 160 | 160 | 162 | 160 | 155 | 10 | 12 |
| 8 | 10 | 140 | 150 | 152 | 150 | 10 | 11 | 10 |
| 9 | 12 | 10 | 10 | 10 | 10 | 9 | 10 | 10 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | 0.25 | 0.25 | -33.75 | -104 | -104.75 | -70.25 | -0.5 | |
| | 0.25 | -2.25 | -77.25 | -111.25 | -145 | -146.5 | -74.25 | |
| | 37 | 70.25 | -7.75 | -7.75 | -77.5 | -150.5 | -146.75 | |
| | 75.75 | 40 | -2.5 | -3.5 | -4.5 | -80.75 | -147.25 | |
| | 77 | 7.5 | 1.5 | 0.75 | -1.75 | -7.25 | -75 | |
| | 149.75 | 84.5 | 15.75 | 10.5 | 6 | 0.5 | -0.75 | |
| | 142.5 | 146.5 | 116.5 | 113 | 74.5 | 1.75 | 1.5 | |
| | | | | | | | | |

# ◉ 4-D Popular Filters (4): Matched Filter

Used for demodulation, similarity measurement, and pattern recognition

"Edge and corner detections" are special cases of pattern recognition.

To detect a pattern $h[n]$, we use its time-reverse and conjugation form as the filter

(correlation)

$$y[n] = x[n] * h^*[-n] = \sum_{\tau=-\tau_1}^{-\tau_2} x[n-\tau]h^*[-\tau] = \sum_{\tau=\tau_1}^{\tau_2} x[n+\tau]h^*[\tau]$$

if $h[n] \neq 0$ for $\tau_1 \leq n \leq \tau_2$

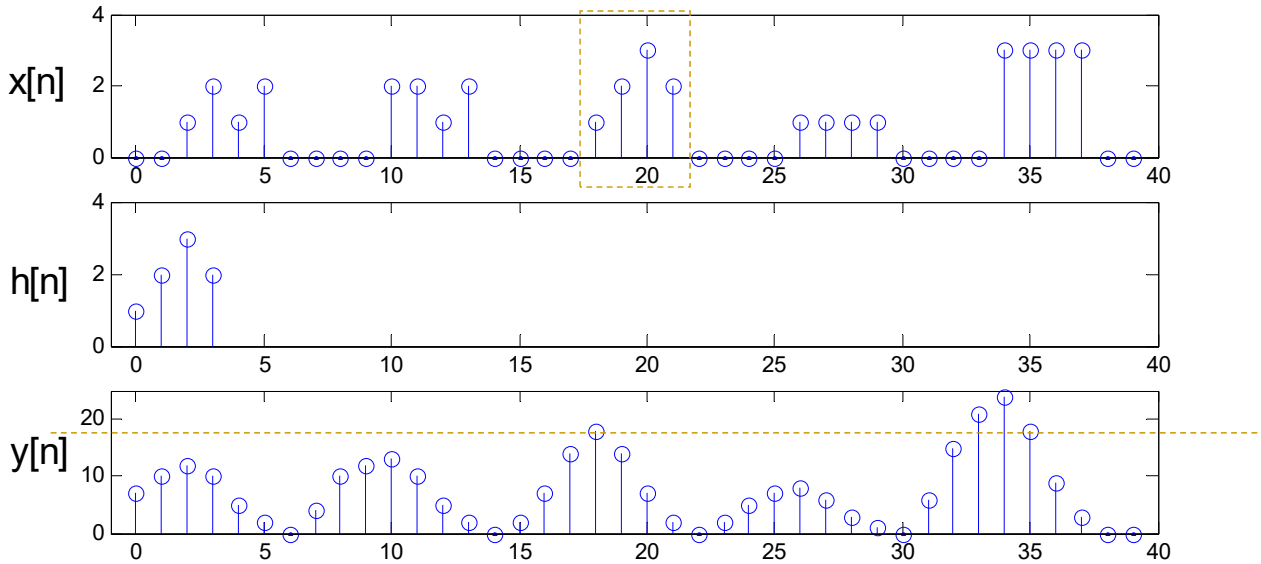
$x[n]$: input pattern, $h[n]$: the desired pattern

2-D form:

$$y[m,n] = x[m,n] * h^*[-m,-n] = \sum_{\tau=\tau_1}^{\tau_2} \sum_{\rho=\rho_1}^{\rho_2} x[m+\tau, n+\rho]h^*[\tau,\rho]$$

if $h[m, n] \neq 0$ for $\tau_1 \leq m \leq \tau_2$, $\rho_1 \leq n \leq \rho_2$,

# Example




$$y[n] = x[n] * \boxed{h^*[-n]}$$

The result of the convolution should be normalized!

- Normalization Form

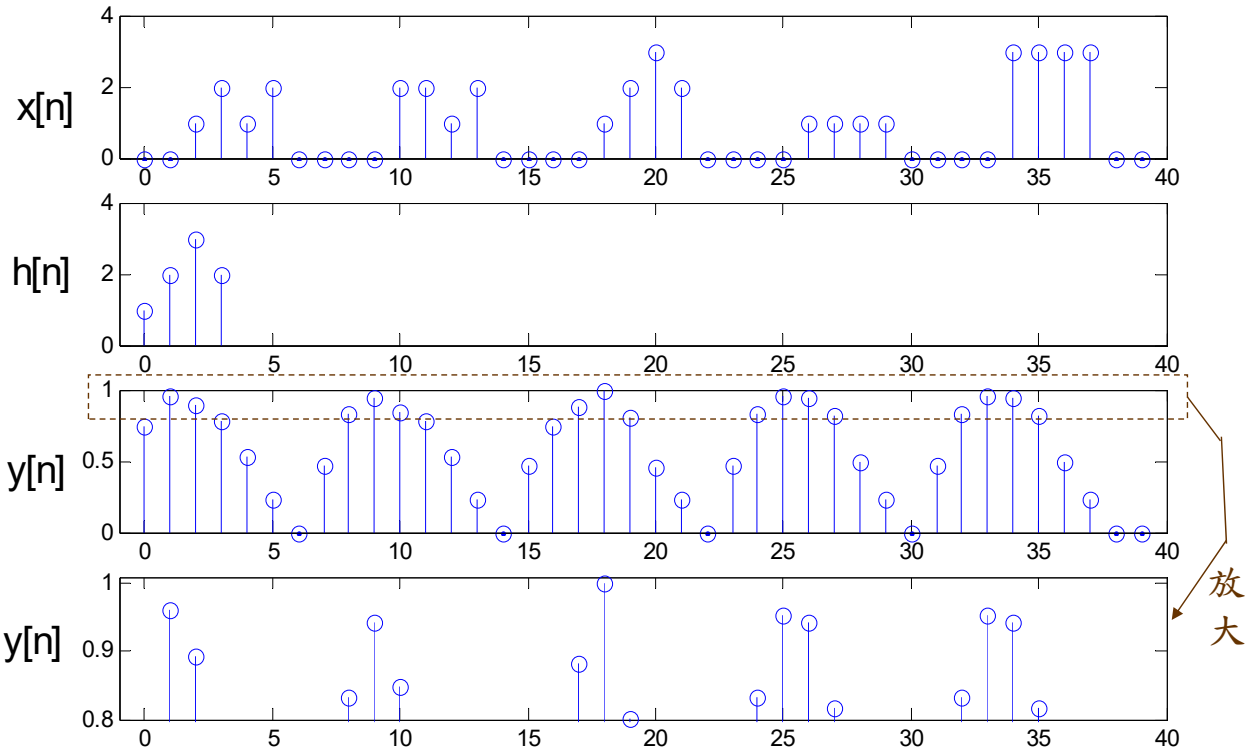
$$y[n]=\frac{\sum_{\tau=\tau_1}^{\tau_2} x[n+\tau]h^*[\tau]}{\sqrt{\sum_{s=n+\tau_1}^{n+\tau_2}|x[s]|^2\sum_{s=\tau_1}^{\tau_2}|h[s]|^2}} \qquad \text{when} \quad \sum_{s=n+\tau_1}^{n+\tau_2}|x[s]|^2\neq 0$$

$$y[n]=0 \qquad \text{when} \quad \sum_{s=n+\tau_1}^{n+\tau_2}|x[s]|^2=0$$

2-D Case

$$y[m,n]=\frac{\sum_{\tau=\tau_1}^{\tau_2}\sum_{\rho=\rho_1}^{\rho_2} x[m+\tau,n+\rho]h^*[\tau,\rho]}{\sqrt{\sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}|x[s,v]|^2\sum_{s=\tau_1}^{\tau_2}\sum_{v=\rho_1}^{\rho_2}|h[s,v]|^2}} \qquad \text{when} \quad \sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}|x[s,v]|^2\neq 0$$

$$y[m,n]=0 \qquad \text{when} \quad \sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}|x[s,v]|^2=0$$


x[n]
h[n]
y[n]
y[n]
放大

- Normalization and Offset Form

$$y[n] = \frac{\sum_{\tau=\tau_1}^{\tau_2} x[n+\tau] h_1^*[\tau]}{\sqrt{\sum_{s=n+\tau_1}^{n+\tau_2} |x[s]-x_0[s]|^2 \sum_{s=\tau_1}^{\tau_2} |h_1[s]|^2}} \qquad \text{when} \quad \sum_{s=n+\tau_1}^{n+\tau_2} |x[s]-x_0[s]|^2 \neq 0$$

$$y[n] = 0 \qquad \text{when} \quad \sum_{s=n+\tau_1}^{n+\tau_2} |x[s]-x_0[s]|^2 = 0$$

where $$h_1[s] = h[s] - \frac{1}{\tau_2 - \tau_1 + 1} \sum_{s=\tau_1}^{\tau_2} h[s] = h[s] - mean\big(h[s]\big)$$

$$x_0[s] = \frac{1}{\tau_2 - \tau_1 + 1} \sum_{s=n+\tau_1}^{n+\tau_2} x[s] \qquad \text{(local mean)}$$

- Normalization and Offset Form for the 2D Case

$$y[m,n]=\frac{\sum_{\tau=\tau_1}^{\tau_2}\sum_{\rho=\rho_1}^{\rho_2} x[m+\tau,n+\rho]h_1^*[\tau,\rho]}{\sqrt{\sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}\left|x[s,v]-x_0[s,v]\right|^2\sum_{s=\tau_1}^{\tau_2}\sum_{v=\rho_1}^{\rho_2}\left|h_1[s,v]\right|^2}}$$

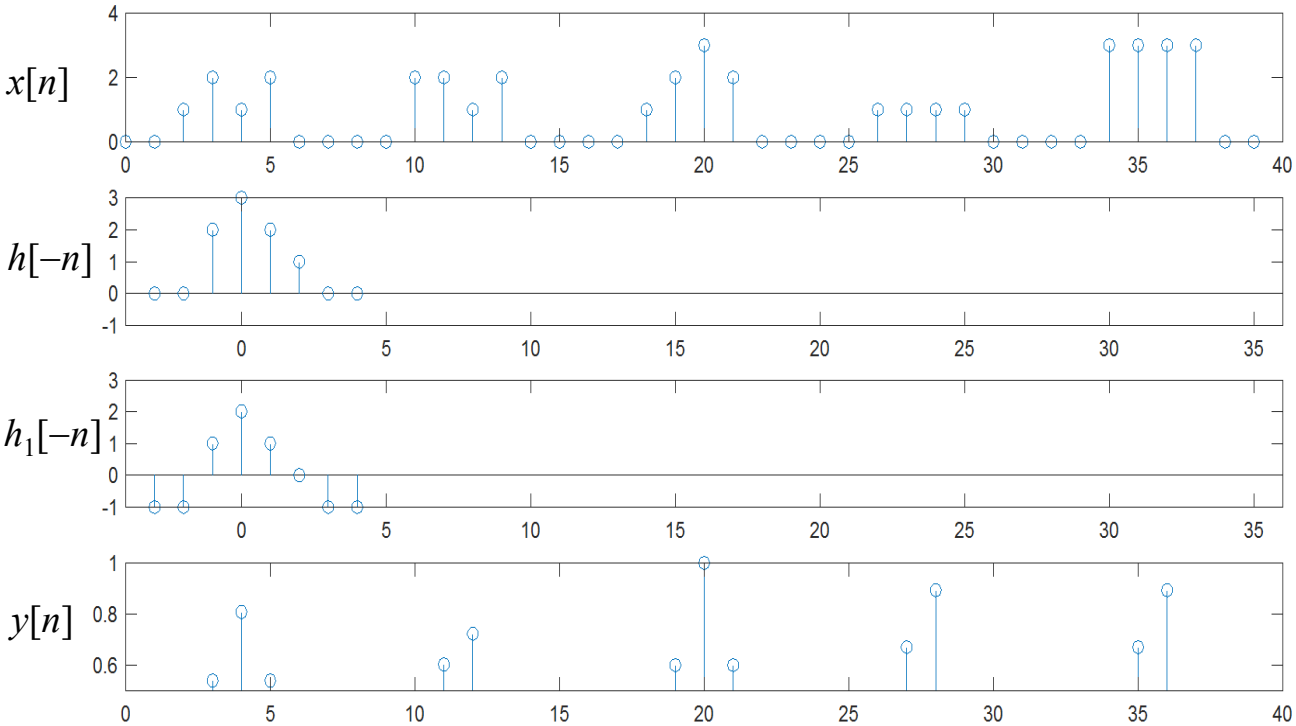
when $\sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}\left|x[s,v]-x_0[s,v]\right|^2\neq 0$

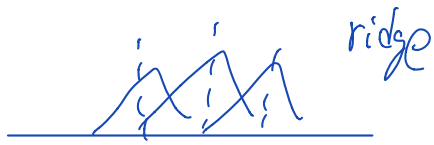
$$y[m,n]=0 \quad \text{when} \quad \sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}\left|x[s,v]-x_0[s,v]\right|^2=0$$

where $h_1[s,v]=h[s,v]-\frac{1}{\tau_2-\tau_1+1}\frac{1}{\rho_2-\rho_1+1}\sum_{s=\tau_1}^{\tau_2}\sum_{v=\rho_1}^{\rho_2}h[s,v]=h[s,v]-mean\left(h[s,v]\right)$

$$x_0[s]=\frac{1}{\tau_2-\tau_1+1}\frac{1}{\rho_2-\rho_1+1}\sum_{s=m+\tau_1}^{m+\tau_2}\sum_{v=n+\rho_1}^{n+\rho_2}x[s,v] \quad \text{(local mean)}$$

# Normalization and Offset Form



# ◉ 4-E Popular Filters (5): Particle Filter and Kalman Filter

Particle filter:

$$x[n+1] = f\left(x[n], m[n]\right)$$

$$x[n+1] = f\left(x[n], x[n-1], \cdots, x[n-K], m[n]\right)$$

$f(\ )$ is some mapping function and $m[n]$ is the noise

The goal of the particle filter is not to remove the noise.

It is used for system modeling or prediction.

When (i) $f(\ )$ is a linear function and (ii) $m[n]$ is a Gaussian noise, it becomes the Kalman filter.

Example: $$x[n+1] = \sum_{\tau=0}^{K} c_\tau x[n-\tau] + m[n]$$

# ◎ 4-F Popular Filters (5): Wiener Filter

(Nobert Wiener 維納, AD 1949)

- No specific passband and stop band

It is related to random process.

- The filter is designed based on the statistics of signal and noise

Suppose that 利用統計學去找 optimal filter

(a) the cross-correlation between the original signal $x_i[n]$ and the received signal $y_i[n]$ ($i$ = 1, 2, 3, ….) is $R_{xy}[n, \sigma]$,

(b) the auto-correlation of the received signal (denoted by $R_{yy}[n, \sigma]$),

then the transfer function of the optimal filter can be designed as

$E(x[n]) = E(y[n]) = 0$

$$H_{opt}(F) = R_{X,Y}(F,F) / R_{Y,Y}(F,F)$$

$$R_{x,y}(n,\sigma) = E\left[x[n]y^*[\sigma]\right] \qquad R_{yy}(n,\sigma) = E\left[y[n]y^*[\sigma]\right]$$

$$R_{X,Y}(F,F) = \sum_{\sigma}\sum_{n} e^{j2\pi(F\sigma - Fn)} R_{xy}[n,\sigma]$$

$$R_{Y,Y}(F,F) = \sum_{\sigma}\sum_{n} e^{j2\pi(F\sigma - Fn)} R_{yy}[n,\sigma]$$

$x[n] \overset{\checkmark}{=} IDTFT(Y(F)H_{opt}(F))$

## References

[1] N. Wiener, *Extrapolation, Interpolation, and Smoothing of Stationary Time Series*, M.I.T. Press, Cambridge, Mass. , 1964.

[2] S. S. Haykin, *Adaptive Filter Theory*, Prentice Hall, N.J., 2002.

[3] M. R. Banham and A. K. Katsaggelos, "Digital image restoration," *IEEE Signal Processing Magazine*, vol.14, no. 2, pp. 24-41, Mar. 1997

比較：noise removing $y[n] = x[n] + noise[n]$



# ◉ 4-G Popular Filters (6): Equalizer

Used for compensation (such as the multiple path problem)

$Y(F) = X(F)K(F)$ $\qquad$ $Y(F)\frac{1}{K(F)} = X(F)$

$$y[n] = x[n] * k[n]$$

$x[n]$: original signal, $y[n]$: received signal

$k[n]$: effect of the system

or $y[n] = x[n] * k[n] + noise[n]$

Equalizer:

$$x[n] = y[n] * h[n] \qquad H(F) = \frac{1}{K(F)}$$

If $K(F) \cong 0$, $H(F) \to \infty$

或者用 $Z$ transform 表示 $\quad H(z) = \dfrac{1}{K(z)}$

Gaussian:

$$e^{-\frac{\pi f^2}{\sigma^2}} \xrightarrow{FT} |\sigma| e^{-\pi\sigma^2 f^2}$$

$$y[n] = x[n] * k[n] \qquad \text{Equalizer:} \qquad H(F) = \frac{1}{K(F)}$$

Problem: If the system is interfered by noise $m[n]$

$$y[n] = x[n] * k[n] + m[n]$$

$$Y(F) = X(F)K(F) + M(F)$$

$$H(F)Y(F) = X(F)H(F)K(F) + H(F)M(F)$$

$$= X(F) + \frac{M(F)}{K(F)}$$

If $K(F)$ is near to 0, the effect of the noise is magnified.

Combined with the concept of the Wiener filter, the equalizer is modified as:

$$H(F) = \frac{1}{\frac{1}{K^*(F)} \frac{E\left(|M(F)|^2\right)}{E\left(|X(F)|^2\right)} + K(F)}$$

$E$: mean

$$= \frac{1}{\frac{1}{K^*(F)} \frac{1}{SNR(F)} + K(F)}$$

$$H(F) = \frac{1}{\frac{c}{K^*(F)} + K(F)}$$

$c$ is large when the SNR is small
$c$ is small when the SNR is large

If $K(F) = Ae^{j\phi}$

$K^*(F) = Ae^{-j\phi}$

$\frac{1}{K^*(F)} = \frac{1}{A} e^{j\phi}$

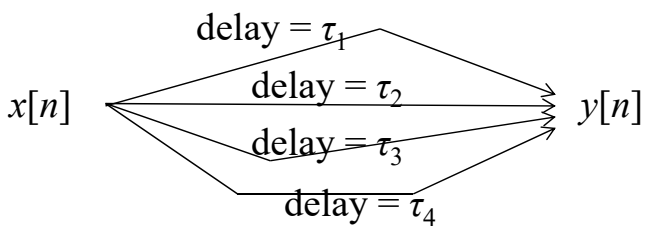
- Equalizer for the Multiple Path Problem

$$k[n] = \alpha_1 \delta[n-\tau_1] + \alpha_2 \delta[n-\tau_2] + \alpha_3 \delta[n-\tau_3] + \ldots\ldots$$
$$y[n] = x[n] * k[n] = \alpha_1 x[n-\tau_1] + \alpha_2 x[n-\tau_2] + \alpha_3 x[n-\tau_3] + \ldots\ldots$$
$$Y[z] = (\alpha_1 z^{-\tau_1} + \alpha_2 z^{-\tau_2} + \alpha_3 z^{-\tau_3} + \ldots\ldots) X[z]$$

Usually $\alpha_k$ is related to $\tau_k$, so it could be rewritten as $\alpha_k(\tau_k)$

Equalizer: $$H(z) = \frac{1}{\alpha_1 z^{-\tau_1} + \alpha_2 z^{-\tau_2} + \alpha_3 z^{-\tau_3} + \ldots\ldots}$$

$X(z) = Y(z) H(z)$




problems:

(1) $H(z)$ may be infinite

(2) $\tau_1, \tau_2$ .. are hard to known

(3) $\alpha_1, \alpha_2$ .. are hard to known

- 缺點: (1) $H(z)$可能unstable

  (2) $H(z)$ is usually a dynamic response

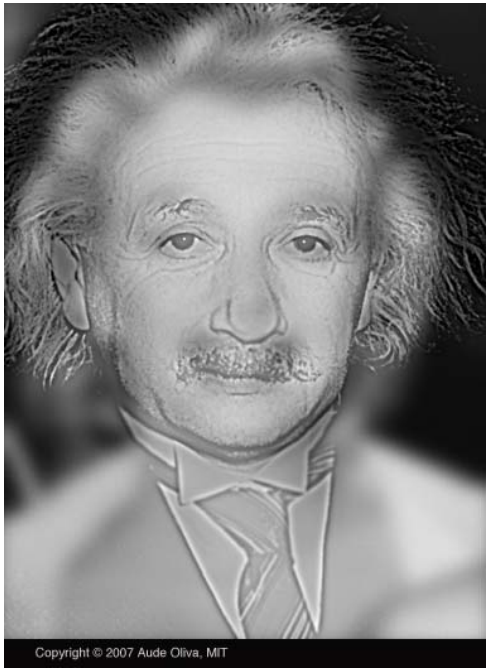
同態

- 可以用 homomorphic signal processing 來取代 equalizer 處理 multiple path problem.

References

S. S. Haykin, *Communication Systems*, John Wiley, N.J., 2010

W. D. Chang, J. J. Ding, Y. Chen, C. W. Chang, and C. C. Chang, "Edge-membership based blurred image reconstruction algorithm," *APSIPA Annual Summit and Conference*, Hollywood, USA, Dec. 2012




E : high frequency , edge

M : Low frequency , intensity

http://cvcl.mit.edu/hybrid_gallery/monroe_einstein.html

# 附錄五 讀論文的方法 (個人心得)

為了做研究和工作的需要，同學們將來都要經常閱讀論文，甚至於，有的時候可能要一週要閱讀三篇以上的論文，而且大部分的論文說得都沒有像大學課本那麼有條理。用大學以前的讀書習慣，恐怕將難以應付。

要如何在短時間之內讀懂那麼多的論文，甚至於發現論文所提的方法可以改良的地方，是上了研究所之後必需學會的能力。

以下是幾點原則 (根據我個人的經驗)：

## (A) 先判斷這篇論文是否應該被詳讀

(1) 越是核心，越是最早提出某個理論的論文，越是應該被詳讀

(2) 和自己目前研究密切相關的論文，當然有詳讀的必要

(3) Citation rate (引用次數) 較高的論文，可能也比較重要 (雖然不完全相關)。

至於比較支節的論文，大略讀過即可

## (B) 自己動手算

對於該「詳讀」的論文，可以自己動手來計算當中的幾個重要公式。

不是每篇論文都對論文中的理論和公式的來源有清楚的說明。在這個時候，還不如自己拿起筆來，親手證明論文當中的公式和理論。

自己動手算，不只能幫助自己了解論文當中的理論，而且，有時還可以「意外」的發現論文當中的理論可以進一步改良的地方，進而寫出新的論文出來。

## (C) 讀過論文之後，問自己一些問題

(1) 這篇論文所提的概念 (Concepts) 是什麼？

(2) 方法的優點何在 (Advantages)？

(3) 可能的應用 (Applications) 在何處？

若能回答這三個問題，表現你大致讀通了這篇論文

若回答不出來，可能要再把論文當中遺漏的地方，再好好看 一看

## (D) 進一步的分析

如果你不以讀懂一篇論文為滿足，想要進一步的發明創造之外，可以再問自己幾個問題

(1) Analysis for Advantages: 是什麼原因，造成這個方法有這樣的優點？

類似的概念，是否可以延伸、用在其他地方？

(2) Analysis for Disadvantages: 這方法有什麼問題？

是什麼原因，造成這些問題？

有什麼方法，可以改良這些問題？

(3) Innovations: 綜合以上的分析，再加上個人的靈感，想想這篇論文是否有可以再進一步發明創新的地方？

## (E) 註解

我經常看過一篇論文之後，會寫上幾行的文字，來描述這篇論文要點，以及在這個領域當中所扮演的角色。一方面有助於釐清概念，一方面也可以避免日後還要花時間來回憶這篇論文的內容是什麼

## (F) 做個整理

可以將多篇論文所提的許多種方法，做一個有系統的整理和比較。

總共有多少種方法被提出來處理這個問題？這些方法的優缺點和適用的地方是什麼？它們之間是否可以歸納成幾大類？這些方法的相似和相異之處是什麼？

有時，把各種不同的方法做個綜合，拮取各方法的優點，將有助出創造出效能更好的新方法