

電腦視覺與應用

Computer Vision and Applications

Lecture06-2-Two-views geometry-case study

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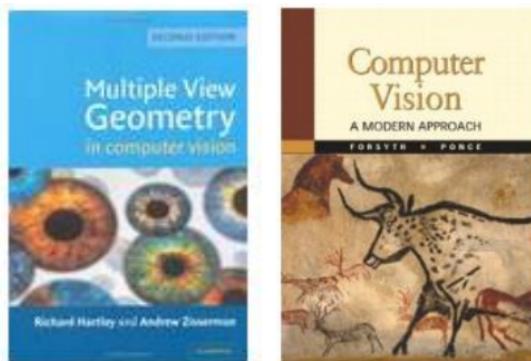
e-mail: thl@mail.ntust.edu.tw





Two-views geometry

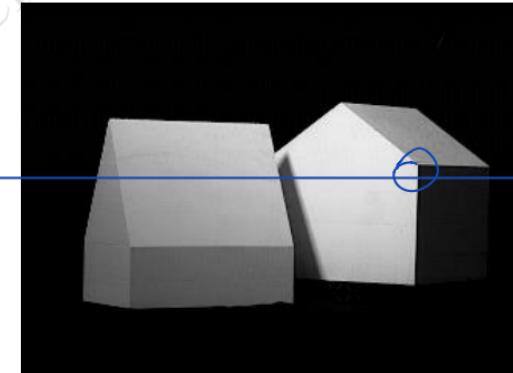
- Case study for stereo-vision & homography
- Lecture Reference at:
 - Multiple View Geometry in Computer Vision, Chapter 11
 - Computer Vision A Modern Approach, Chapter 11.



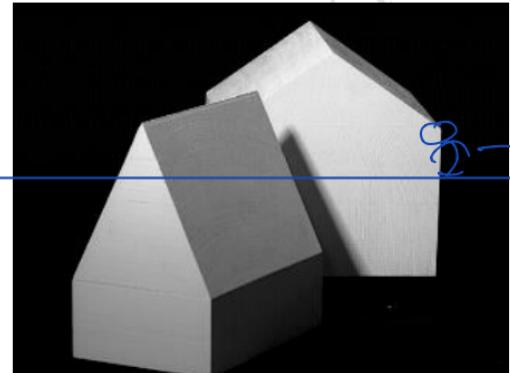


Stereo-image

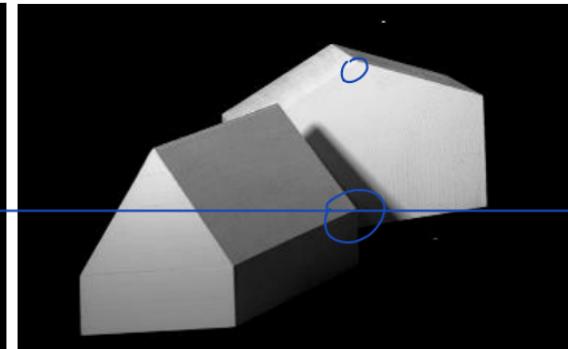
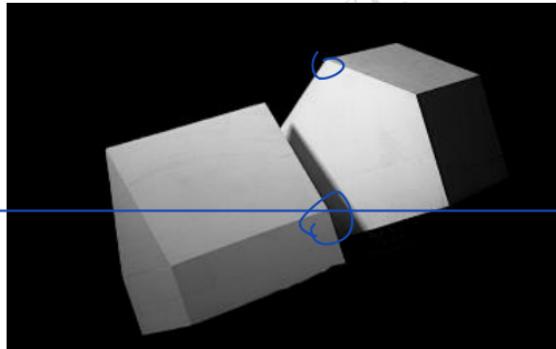
L



R

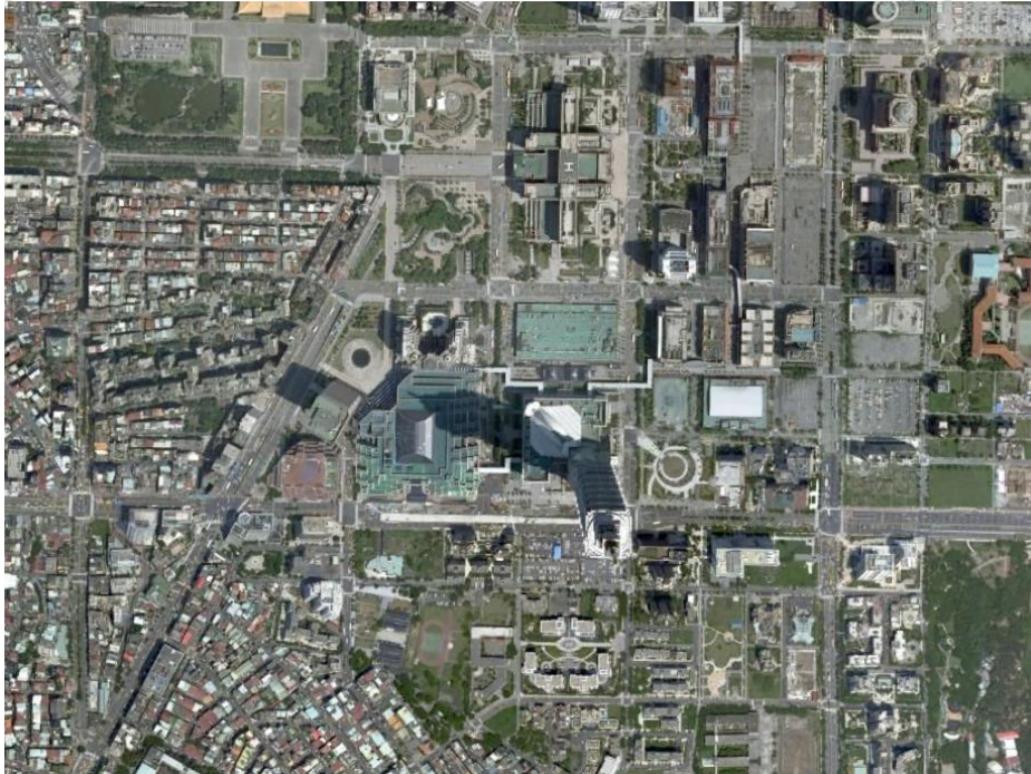


perfect





Rectified images





Rectification for stereo-image

$$[G.R.T] = H$$

- To projectively transfer images into a specific position

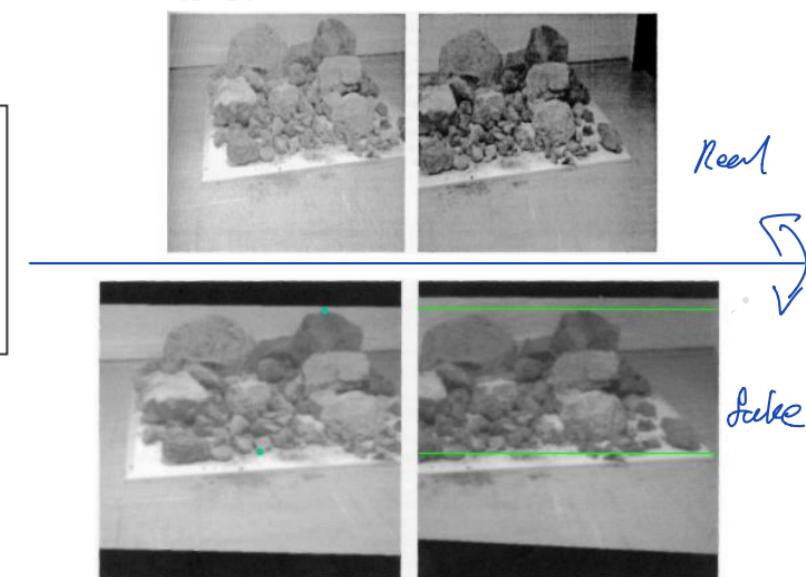
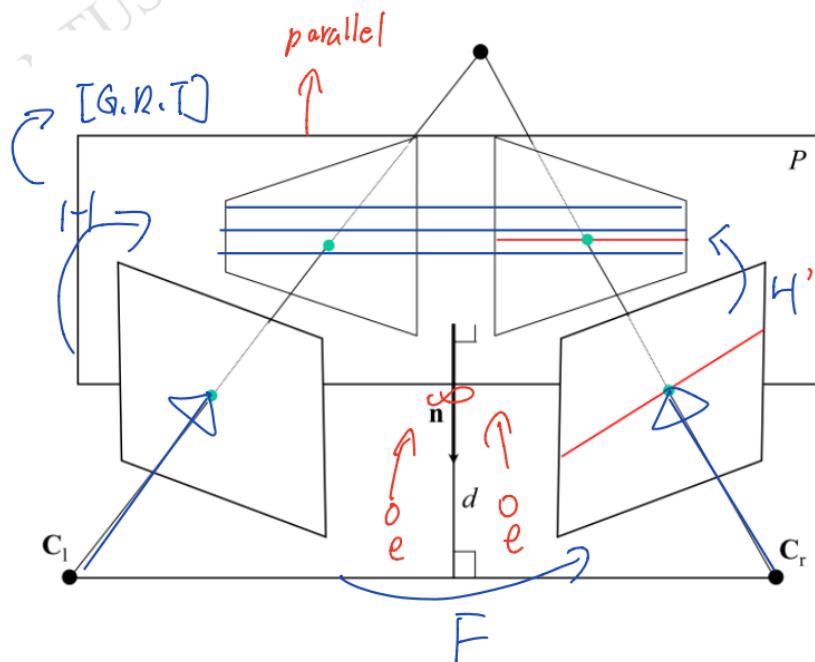
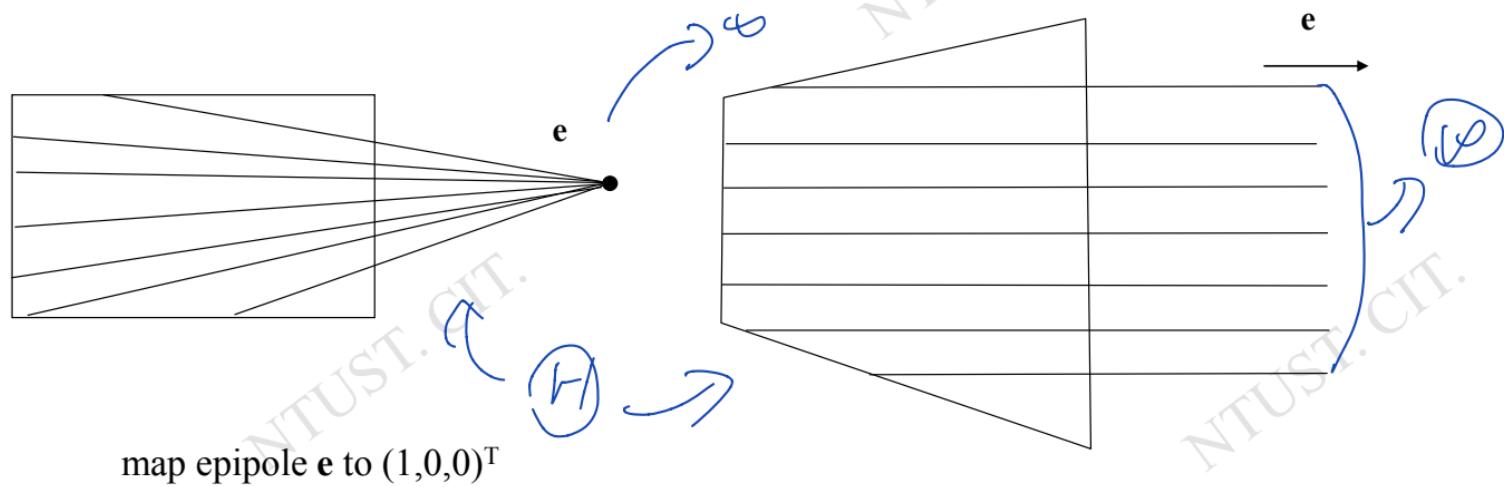




Image rectification

∞ : infinite

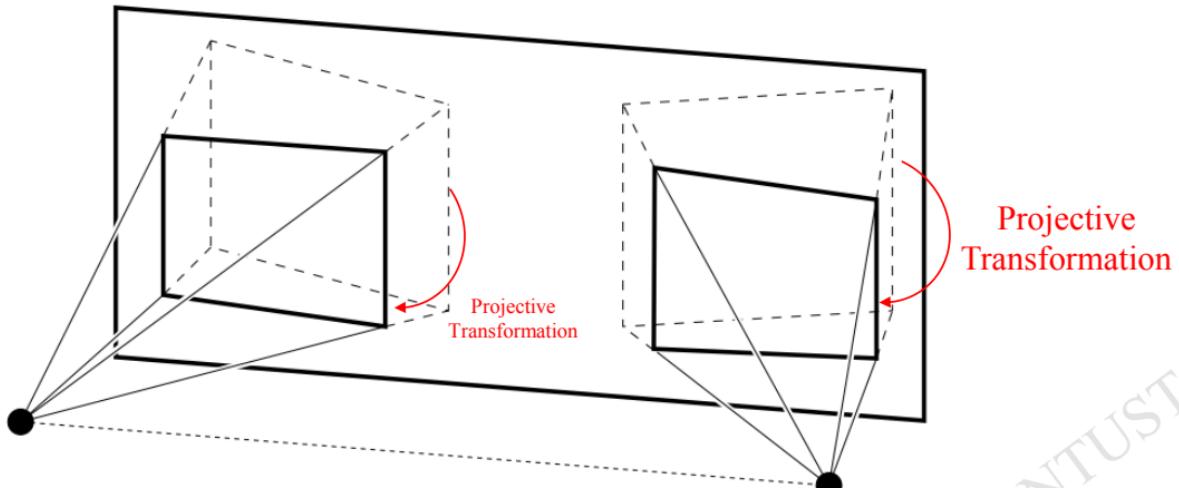
- Apply projective transformation so that epipolar lines correspond to horizontal line



try to minimize image distortion



Image rectification



NOTE! This projective transformation is so called **Homography**!



Image rectification—solution

- To transfer epipole to the point at infinity

Diagram illustrating the mapping of the epipole $e = (f, 0, 1)^T$ to the point at infinity $e_\infty = (f, 0, 0)^T$.

The epipole e is mapped to the point at infinity e_∞ via the homography G :

$$e_\infty = Ge$$

Given:

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{f} & 0 & 1 \end{bmatrix}$$
$$e = \begin{pmatrix} f \\ 0 \\ 1 \end{pmatrix}$$

Result:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{f} & 0 & 1 \end{bmatrix} \begin{pmatrix} f \\ 0 \\ 1 \end{pmatrix} \Rightarrow -1 + 1 = 0$$

Note! This is homography



Image rectification—solution, cont.

- However, in general, we have the configuration like the following figure.
- So, it needs a **translation** and a **rotation** to adjust the epipole on the special condition (on x -axis), and the homography will be derived as the format in the previous page.

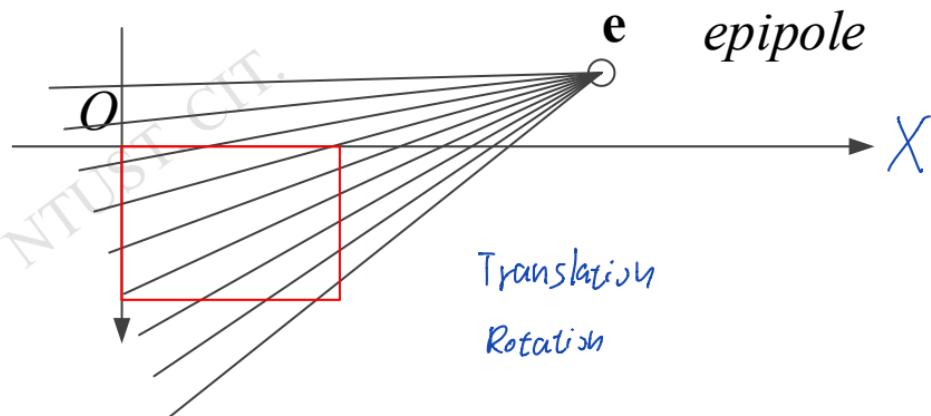




Image rectification—solution, cont.

- An appropriate choice of translation would be the center of image. For example, translate the image center to the origin.

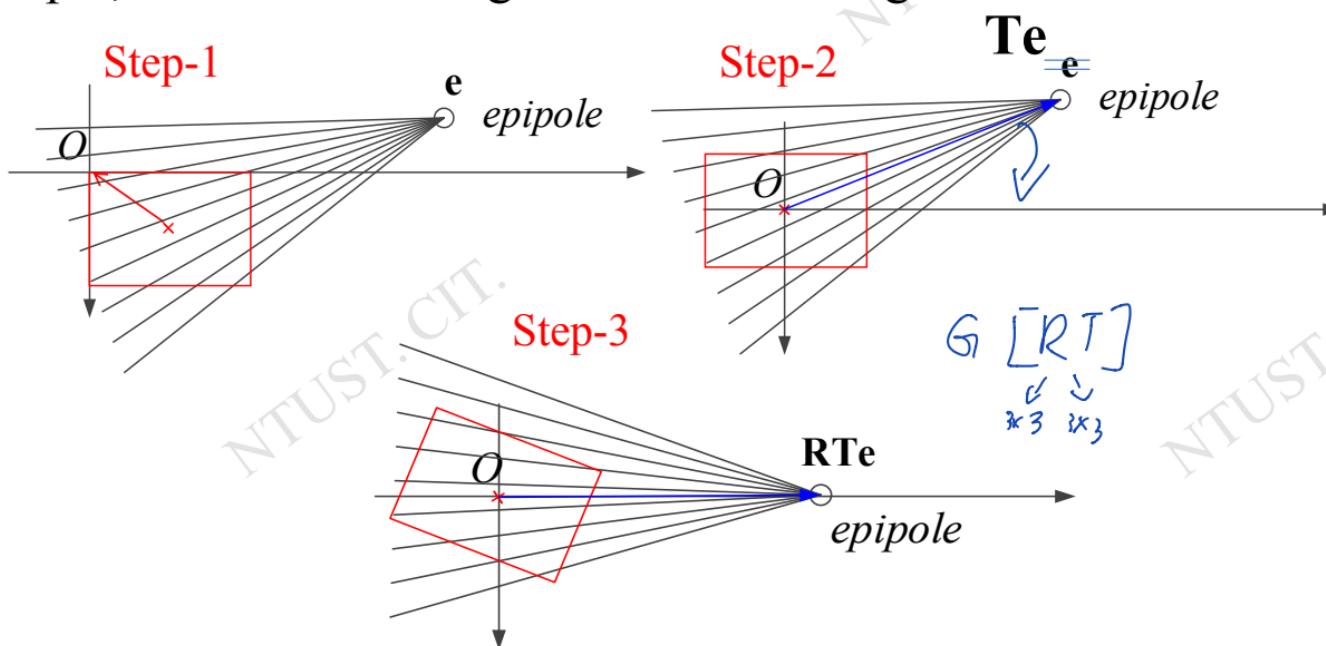
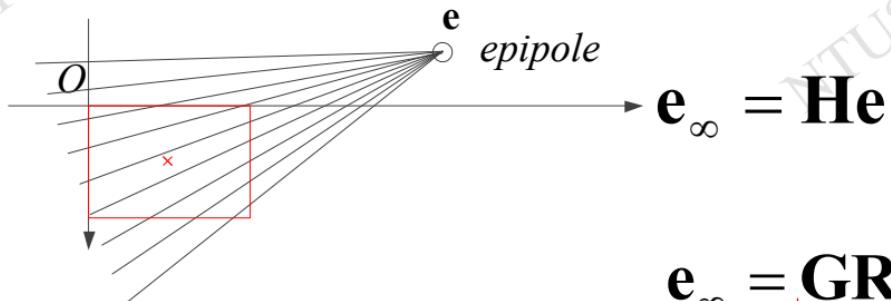




Image rectification—solution, cont.



$$\mathbf{e}_\infty = \mathbf{H}\mathbf{e}$$

$$\mathbf{e}_\infty = (f, 0, 0)^T$$

point at infinity

$$\mathbf{e}_\infty = \underbrace{\mathbf{GRTe}}_{\text{homography}}$$

$$\rightarrow \mathbf{e}_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/f & 0 & 1 \end{bmatrix} \mathbf{RTe}$$



Image rectification—solution, cont.

$$\rightarrow \mathbf{e}_\infty = \mathbf{He} = \mathbf{GRTe} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/f & 0 & 1 \end{bmatrix} \mathbf{RTe}$$

Review the procedure:

- Determine one \mathbf{F} from two image $\Rightarrow \mathbf{e}$
- Determine \mathbf{e} (use cross product of two epipolar lines, line eqs: $\mathbf{l} = \mathbf{F}^T \mathbf{x}'$)
- Determine \mathbf{T} (of course, you know the image resolution. use its center)
- Determine \mathbf{R} (you already have \mathbf{T}_e , rotation angle should be $-\tan^{-1} \frac{y}{x}$)

- Then, get f from \mathbf{RTe} .
- Finally, you have \mathbf{H} .

Note! \mathbf{H} is calculated from the projective mapping (homography) of point-point. If you need line mapping according this homography, use $\mathbf{l}_{rect} = \mathbf{H}^{-T} \mathbf{l}$

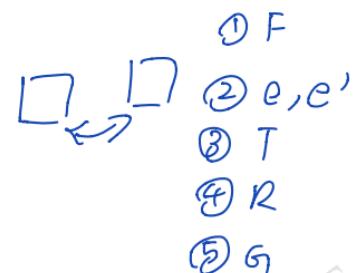




Image rectification—solution, cont.

- Call the process, again.

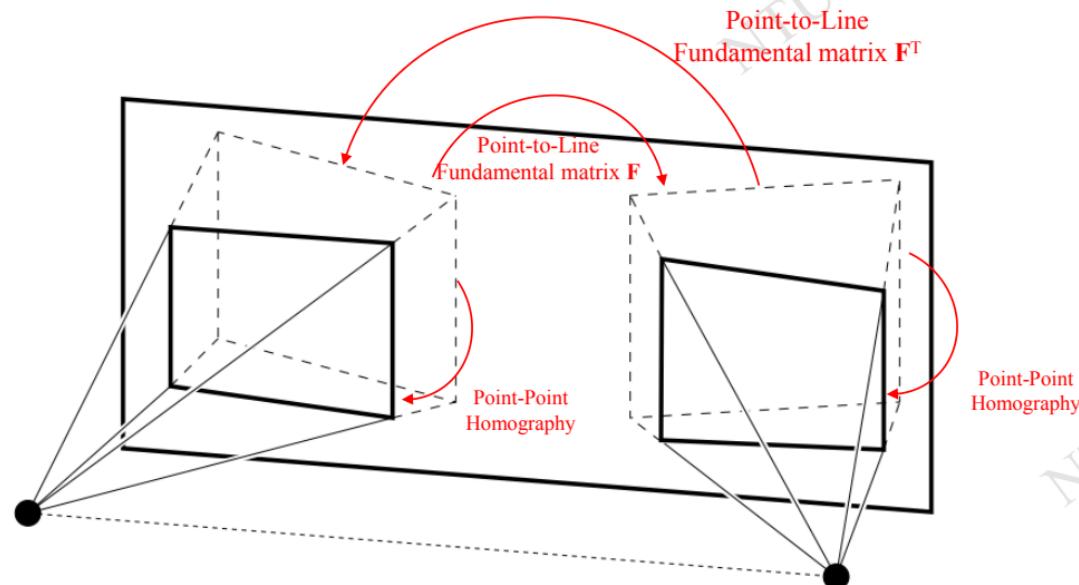
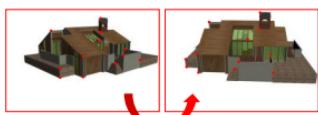




Image rectification—example

12 points $\rightarrow \hat{F}$

Recall the previous example.
Image resolution 720x480.



$$\mathbf{F} = \begin{bmatrix} -0.000219 & -0.000913 & 0.292220 \\ 0.000103 & -0.000245 & 0.737529 \\ -0.142952 & -0.450960 & 1.000000 \end{bmatrix}$$

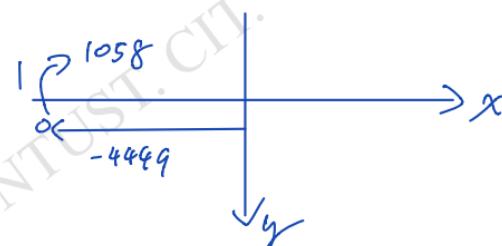
$$\mathbf{e} = \begin{bmatrix} -4089.085693 \\ 1298.432373 \\ 1.000000 \end{bmatrix}$$

$$\mathbf{e}' = \begin{bmatrix} -552.206970 \\ 217.436905 \\ 1.000000 \end{bmatrix}$$

For 1st image:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & -360 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}^* \mathbf{e} = \begin{bmatrix} -4449.085693 \\ 1058.432373 \\ 1.0 \end{bmatrix}$$



$$-\tan^{-1}\left(\frac{1058.432}{-4449.086}\right)$$

Rotation angle = 13.38°

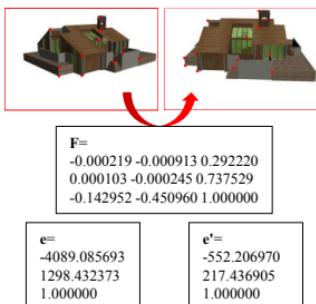
$$\mathbf{R} = \begin{bmatrix} \cos(13.38^\circ) & -\sin(13.38^\circ) & 0 \\ \sin(13.38^\circ) & \cos(13.38^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{RTe} = [-4573.3 \quad 0 \quad 1]^T$$

$$\therefore \mathbf{H} = \mathbf{GRT} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/4573.3 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(13.38^\circ) & -\sin(13.38^\circ) & 0 \\ \sin(13.38^\circ) & \cos(13.38^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -360 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{bmatrix}$$



Image rectification—example, cont.



For 2nd image:

$$\mathbf{T}' = \begin{bmatrix} 1 & 0 & -360 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}' * \mathbf{e}' = \begin{bmatrix} -912.2070 \\ -22.5631 \\ 1.0000 \end{bmatrix} \quad -\tan^{-1}\left(\frac{-22.56}{-912.2}\right)$$

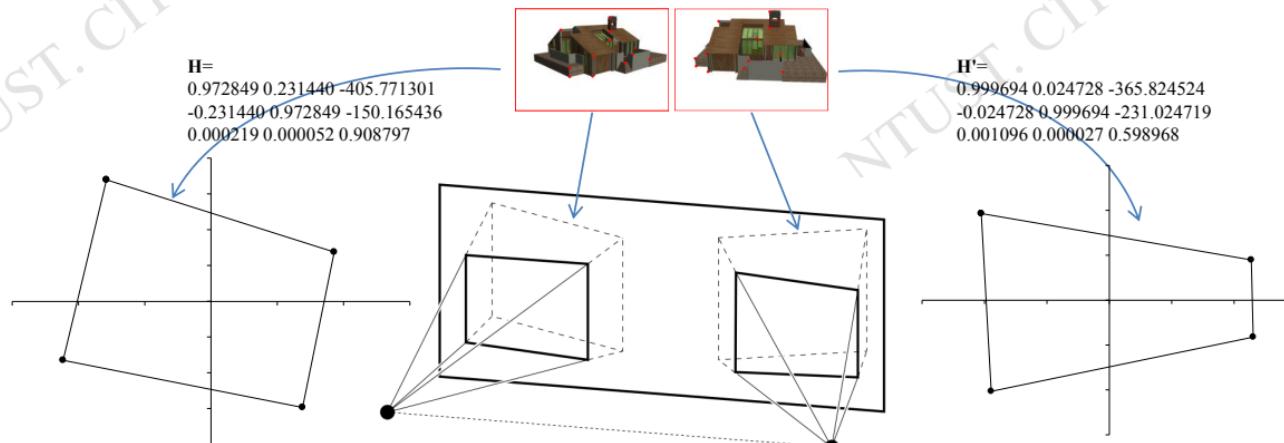
Rotation angle=-1.42

$$\mathbf{R}' = \begin{bmatrix} \cos(-1.42^\circ) & -\sin(-1.42^\circ) & 0 \\ \sin(-1.42^\circ) & \cos(-1.42^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \mathbf{H}' = \mathbf{G}' \mathbf{R}' \mathbf{T}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -912.486 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-1.42^\circ) & -\sin(-1.42^\circ) & 0 \\ \sin(-1.42^\circ) & \cos(-1.42^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -360 \\ 0 & 1 & -240 \\ 1 & 0 & 1 \end{bmatrix}$$



Image rectification—example, cont.



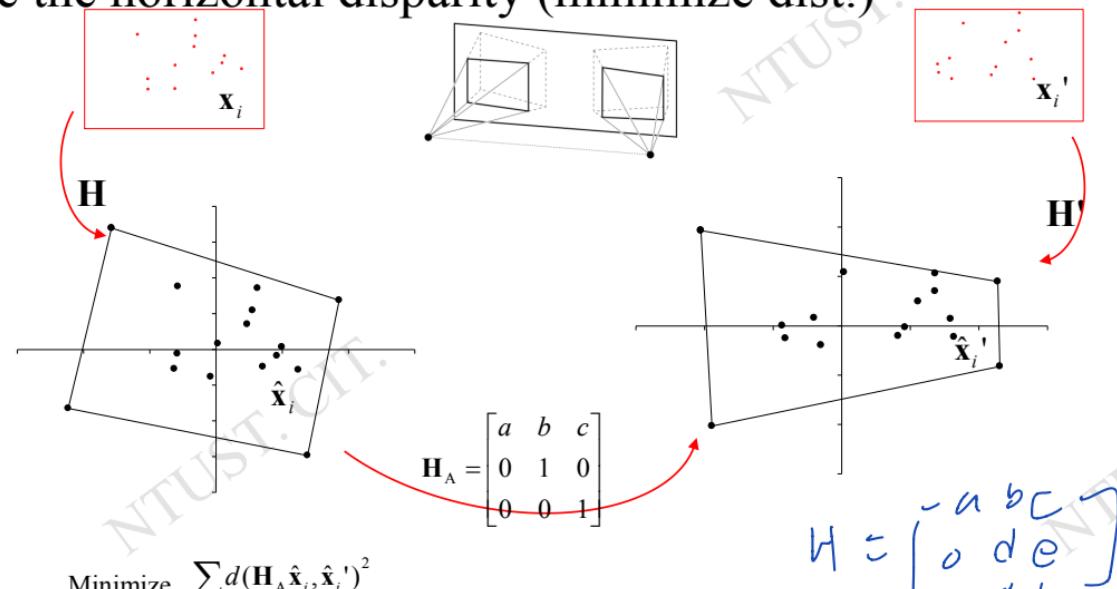
After rectification adjustment, two problems remain

1. Correspondences in two image may have disparity on y direction.
2. Pixel coordinates may not fall in positive region.



Image rectification—example, cont.

- Minimize the horizontal disparity (minimize dist.)



Note! The minimization is to minimize “distance” of correspondences (that are $\hat{\mathbf{x}}_i', \mathbf{H}_A \hat{\mathbf{x}}_i$)



Image rectification—example, cont.

- Minimize the horizontal disparity—cont.

$$\begin{aligned} \text{Minimize } & \sum_i d(\mathbf{H}_A \hat{\mathbf{x}}_i, \hat{\mathbf{x}}'_i)^2 \\ \Rightarrow & \sum_i [(a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i)^2 + (\hat{y}_i - \hat{y}'_i)^2] \end{aligned}$$

$(\hat{y}_i - \hat{y}'_i)^2$ is a constant, and will not affect the chosen of solution for (a, b, c)

So, minimization terms are reduced...

$$\text{Minimize } \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i)^2$$





Image rectification—example, cont.

- Minimize the horizontal disparity—cont.

$$\text{Minimize} \quad \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i)^2$$

For convenience

$$\text{let } g = \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i)^2$$

To determine extreme value of g , take partial difference for g . (偏微分)

$$\begin{cases} \frac{\partial g}{\partial a} = 0 \\ \frac{\partial g}{\partial b} = 0 \\ \frac{\partial g}{\partial c} = 0 \end{cases} \quad \left\{ \begin{array}{l} 2 \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i) \hat{x}_i = 0 \\ 2 \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i) \hat{y}_i = 0 \\ 2 \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i) = 0 \end{array} \right. \quad \left\{ \begin{array}{l} (\sum_i \hat{x}_i^2)a + (\sum_i \hat{x}_i \hat{y}_i)b + (\sum_i \hat{x}_i)c = \sum_i \hat{x}_i \hat{x}'_i \\ (\sum_i \hat{x}_i \hat{y}_i)a + (\sum_i \hat{y}_i^2)b + (\sum_i \hat{y}_i)c = \sum_i \hat{y}_i \hat{x}'_i \\ (\sum_i \hat{x}_i)a + (\sum_i \hat{y}_i)b + (\sum_i 1)c = \sum_i \hat{x}'_i \end{array} \right.$$



Image rectification—example, cont.

- Minimize the horizontal disparity—cont.

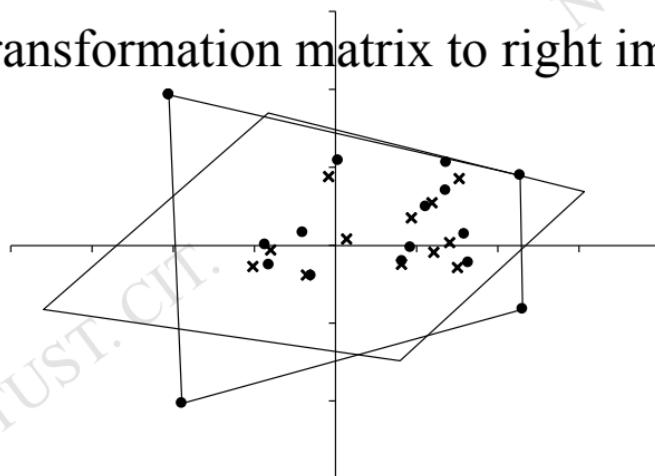
$$\begin{bmatrix} (\sum_i \hat{x}_i^2) & (\sum_i \hat{x}_i \hat{y}_i) & (\sum_i \hat{x}_i) \\ (\sum_i \hat{x}_i \hat{y}_i) & (\sum_i \hat{y}_i^2) & (\sum_i \hat{y}_i) \\ (\sum_i \hat{x}_i) & (\sum_i \hat{y}_i) & (\sum_i 1) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_i \hat{x}_i \hat{x}'_i \\ \sum_i \hat{y}_i \hat{x}'_i \\ \sum_i \hat{x}'_i \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} (\sum_i \hat{x}_i^2) & (\sum_i \hat{x}_i \hat{y}_i) & (\sum_i \hat{x}_i) \\ (\sum_i \hat{x}_i \hat{y}_i) & (\sum_i \hat{y}_i^2) & (\sum_i \hat{y}_i) \\ (\sum_i \hat{x}_i) & (\sum_i \hat{y}_i) & (\sum_i 1) \end{bmatrix}^{-1} \begin{bmatrix} \sum_i \hat{x}_i \hat{x}'_i \\ \sum_i \hat{y}_i \hat{x}'_i \\ \sum_i \hat{x}'_i \end{bmatrix}$$



Image rectification—example, cont.

- Minimize the horizontal disparity—cont.
- After applying transformation matrix to right image:





~~Image rectification—example, cont.~~

- Minimize the vertical disparity, as well as horizontal distance.

$$\begin{aligned} \text{Minimize} \quad & \sum_i d(\mathbf{H}_A \hat{\mathbf{x}}_i, \hat{\mathbf{x}}'_i)^2 \\ \rightarrow \quad & \sum_i [(a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i)^2 + (d\hat{y}_i + e - \hat{y}'_i)^2] \end{aligned}$$

$$\mathbf{H}_A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_{2D} = \begin{pmatrix} \text{New } x \\ \text{New } y \\ 1 \end{pmatrix}$$

↓

$$\left\{ \begin{array}{l} \frac{\partial}{\partial a} = 0 \\ \frac{\partial}{\partial b} = 0 \\ \frac{\partial}{\partial c} = 0 \\ \frac{\partial}{\partial d} = 0 \\ \frac{\partial}{\partial e} = 0 \end{array} \right. \quad \left\{ \begin{array}{l} 2 \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i) \hat{x}_i = 0 \\ 2 \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i) \hat{y}_i = 0 \\ 2 \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}'_i) = 0 \\ 2 \sum_i (d\hat{y}_i + e - \hat{y}'_i) \hat{y}_i = 0 \\ 2 \sum_i (d\hat{y}_i + e - \hat{y}'_i) = 0 \end{array} \right. \quad \left\{ \begin{array}{l} (\sum_i \hat{x}_i^2)a + (\sum_i \hat{x}_i \hat{y}_i)b + (\sum_i \hat{x}_i)c = \sum_i \hat{x}_i \hat{x}'_i \\ (\sum_i \hat{x}_i \hat{y}_i)a + (\sum_i \hat{y}_i^2)b + (\sum_i \hat{y}_i)c = \sum_i \hat{y}_i \hat{x}'_i \\ (\sum_i \hat{x}_i)a + (\sum_i \hat{y}_i)b + (\sum_i 1)c = \sum_i \hat{x}'_i \\ (\sum_i \hat{y}_i^2)d + (\sum_i \hat{y}_i)e = \sum_i \hat{y}_i \hat{y}'_i \\ (\sum_i \hat{y}_i)d + (\sum_i 1)e = \sum_i \hat{y}'_i \end{array} \right.$$

a, b, c, d, e



Image rectification—example, cont.

- Minimize the vertical disparity, as well as horizontal distance.—cont.

$$\begin{bmatrix}
 \sum_i \hat{x}_i^2 & \sum_i \hat{x}_i \hat{y}_i & \sum_i \hat{x}_i & 0 & 0 \\
 \sum_i \hat{x}_i \hat{y}_i & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i & 0 & 0 \\
 \sum_i \hat{x}_i & \sum_i \hat{y}_i & \sum_i 1 & 0 & 0 \\
 0 & 0 & 0 & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i \\
 0 & 0 & 0 & \sum_i \hat{y}_i & \sum_i 1
 \end{bmatrix} \cdot \begin{bmatrix}
 a \\
 b \\
 c \\
 d \\
 e
 \end{bmatrix} = \begin{bmatrix}
 \sum_i \hat{x}_i \hat{x}_i' \\
 \sum_i \hat{y}_i \hat{x}_i' \\
 \sum_i \hat{x}_i' \\
 \sum_i \hat{y}_i \hat{y}_i' \\
 \sum_i \hat{y}_i'
 \end{bmatrix}$$

↓ known ↓ 12

$$\begin{bmatrix}
 a \\
 b \\
 c \\
 d \\
 e
 \end{bmatrix} = \begin{bmatrix}
 \sum_i \hat{x}_i^2 & \sum_i \hat{x}_i \hat{y}_i & \sum_i \hat{x}_i & 0 & 0 \\
 \sum_i \hat{x}_i \hat{y}_i & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i & 0 & 0 \\
 \sum_i \hat{x}_i & \sum_i \hat{y}_i & \sum_i 1 & 0 & 0 \\
 0 & 0 & 0 & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i \\
 0 & 0 & 0 & \sum_i \hat{y}_i & \sum_i 1
 \end{bmatrix}^{-1} \begin{bmatrix}
 \sum_i \hat{x}_i \hat{x}_i' \\
 \sum_i \hat{y}_i \hat{x}_i' \\
 \sum_i \hat{x}_i' \\
 \sum_i \hat{y}_i \hat{y}_i' \\
 \sum_i \hat{y}_i'
 \end{bmatrix}$$

↑ unknown

Finally, recov

$$\mathbf{H}_A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix}$$



Image rectification—example, cont.

- Minimize the vertical disparity, as well as horizontal distance.—cont.

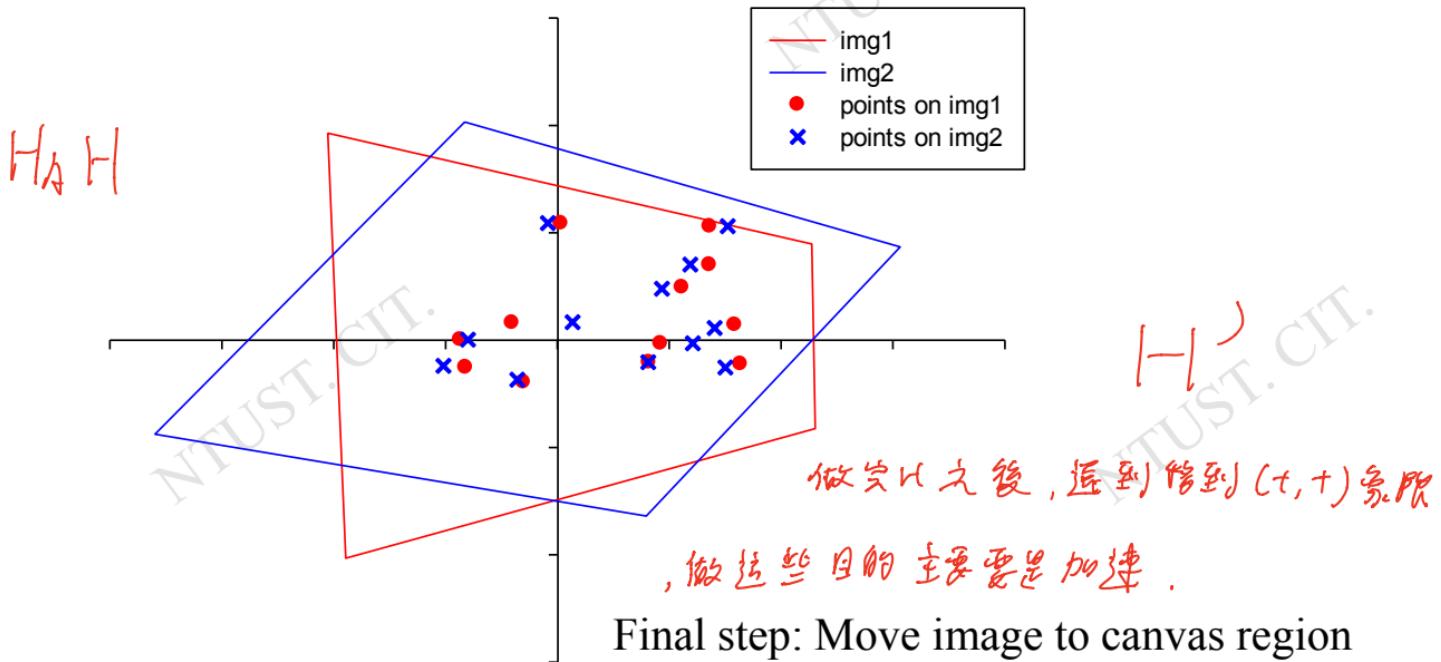




Image rectification—example, cont.

- Unique solution?

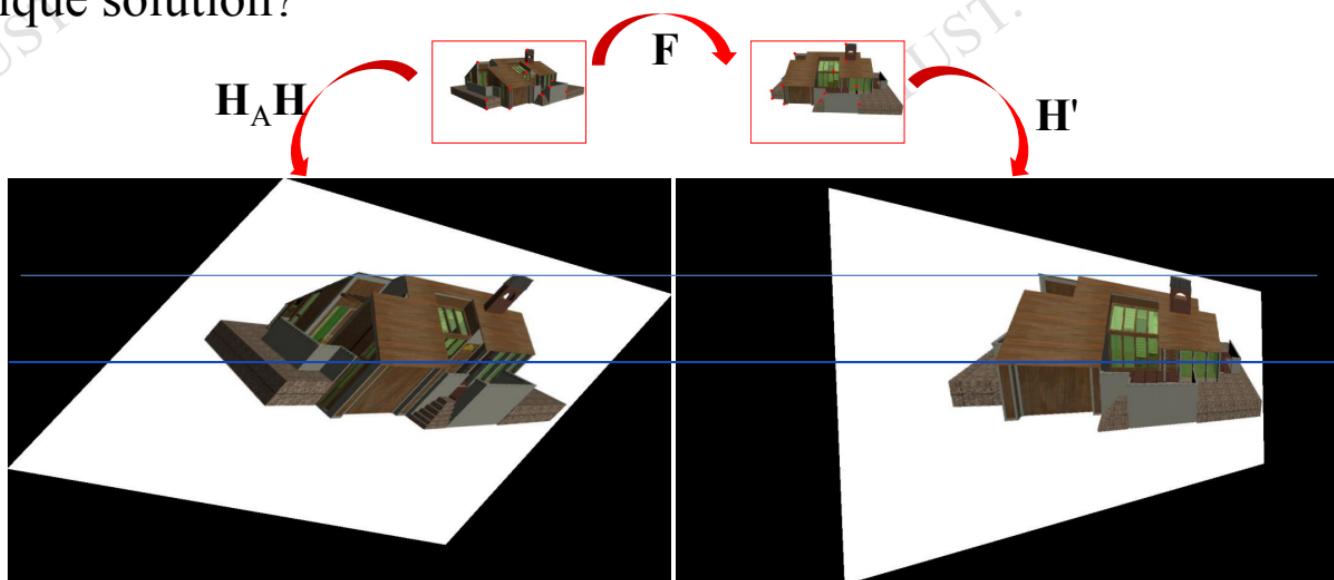




Image rectification—openCV

■ StereoRectifyUncalibrated

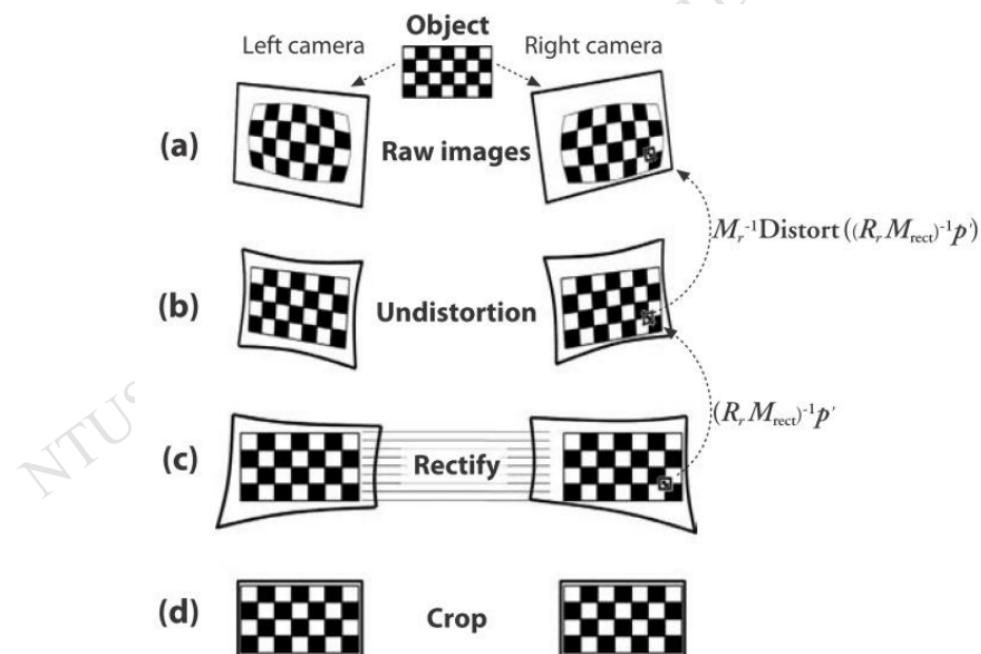




Image rectification—openCV

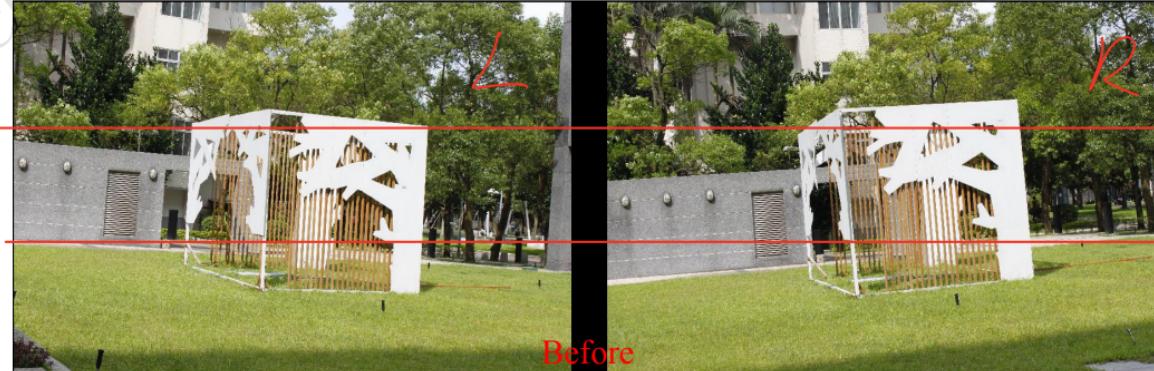
- The same operation in OpenCV

cvStereoRectifyUncalibrated

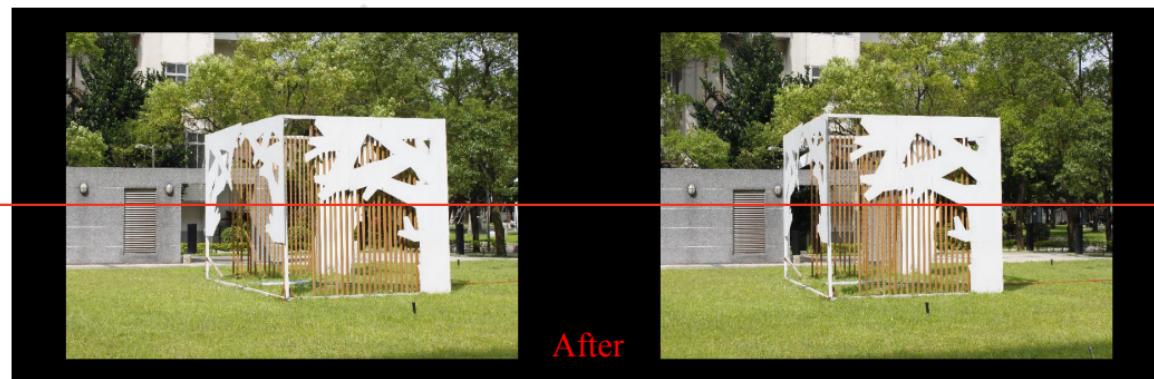
```
int cvStereoRectifyUncalibrated(  
    const CvMat* points1, _____ → The 2 arrays of corresponding 2D points. (input)  
    const CvMat* points2, _____ → Fundamental Matrix (input)  
    const CvMat* F, _____ → (input)  
    CvSize imageSize, _____ → (input)  
    CvMat* Hl, _____ → CvMat* Hr, _____ → Homography Matrix (output for Left, Right Images)  
    double threshold _____ → (input) for rejecting the outliers  
    _____ → for which  $|x^T Fx| >$  threshold  
);
```



Reference software (stereophoto maker)



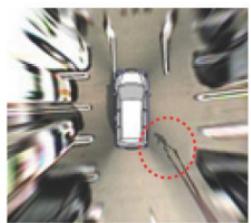
Real





Homography—Applications

■ Intelligent automobile



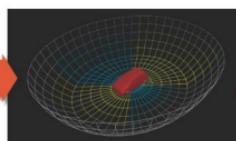
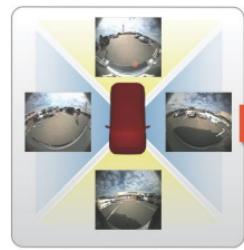
Conventional technology-based image,
vehicles and people are not visible.



Sample view using Fujitsu Laboratories' new
technology perspective from above-rear
(pedestrian is visible)



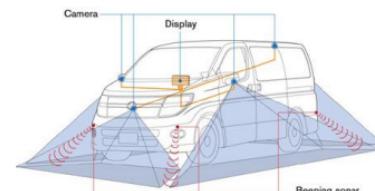
Sample view using Fujitsu's new
technology, perspective from front facing
vehicle (rearview pedestrian is visible)



Virtual 3D model for projection of video images is
synthesized (scene is projected virtually onto a 3D
curved plane). Image is changed to the
desired perspective



Desired view (perspective)
is displayed



NISSAN





Homography—Applications

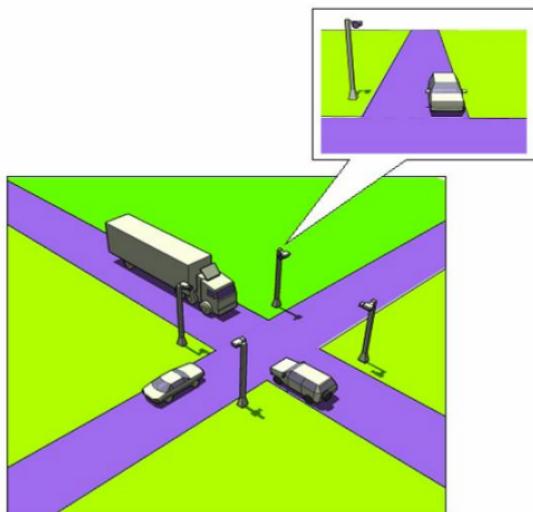
- Intelligent automobile—cont.



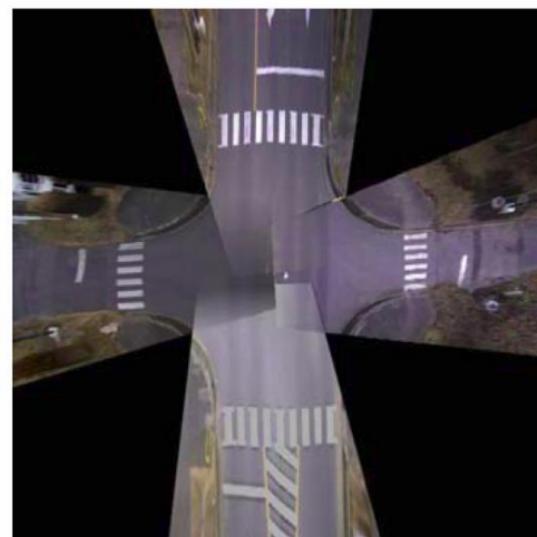


Homography—Applications

- Multi-camera surveillance
- 物聯網



筑波大學(JP)

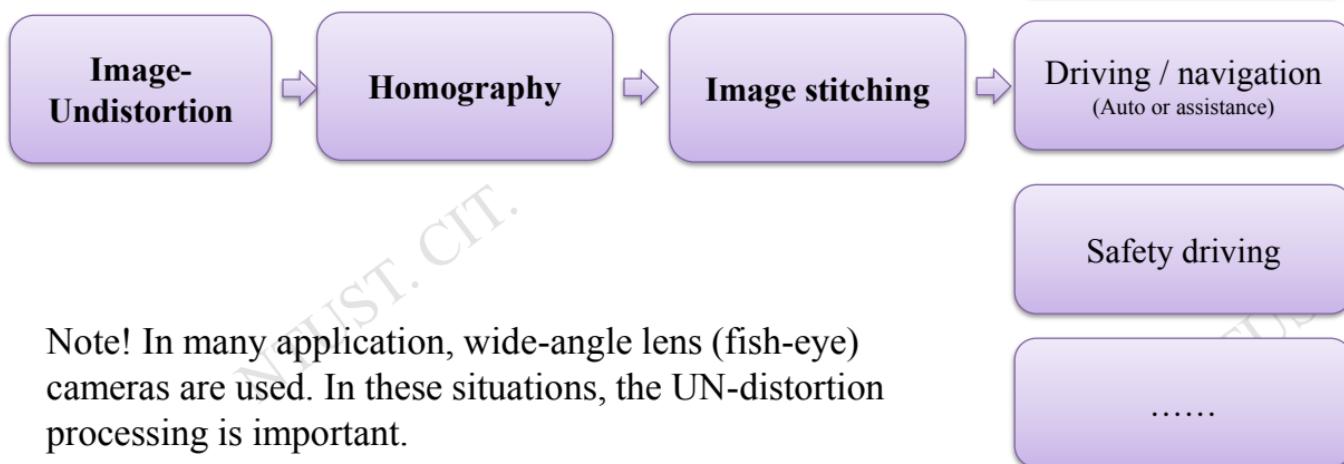


筑波大學(JP)



Homography—Applications

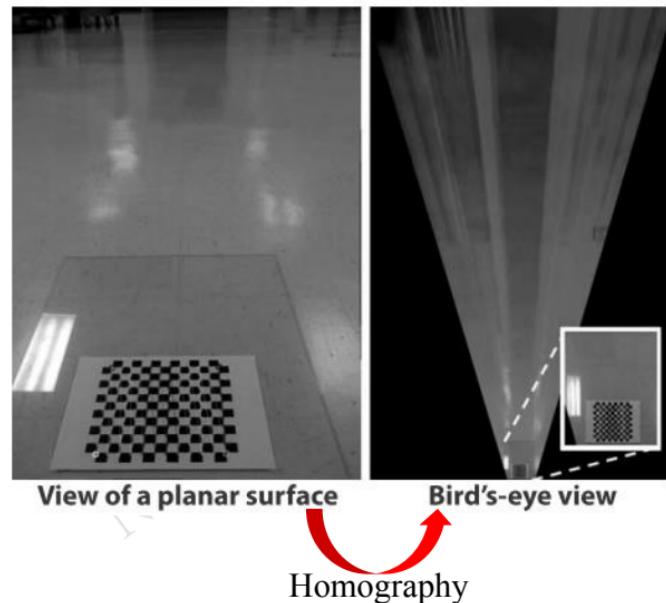
■ Application scenarios





Homography—Applications

- Once again, define your problem first!



- Pre-processing or post-processing for \mathbf{H}
- Constant \mathbf{H} or various \mathbf{H}

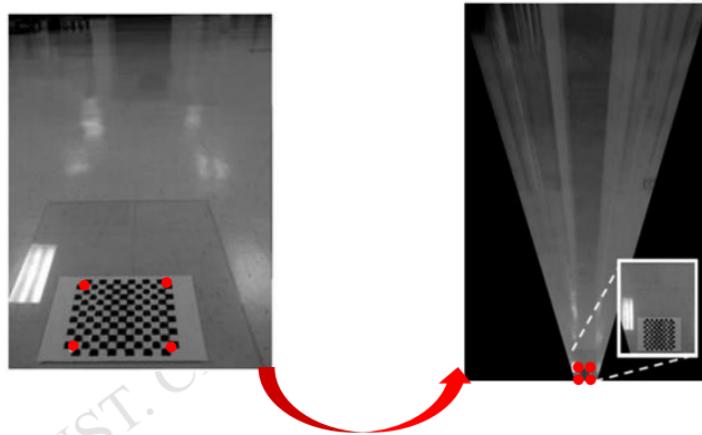
Solution: (homography)

- Line cue ? Point cue ?
- Correspondence
- Scale issue



Homography—Applications

- Solution for finding \mathbf{H}



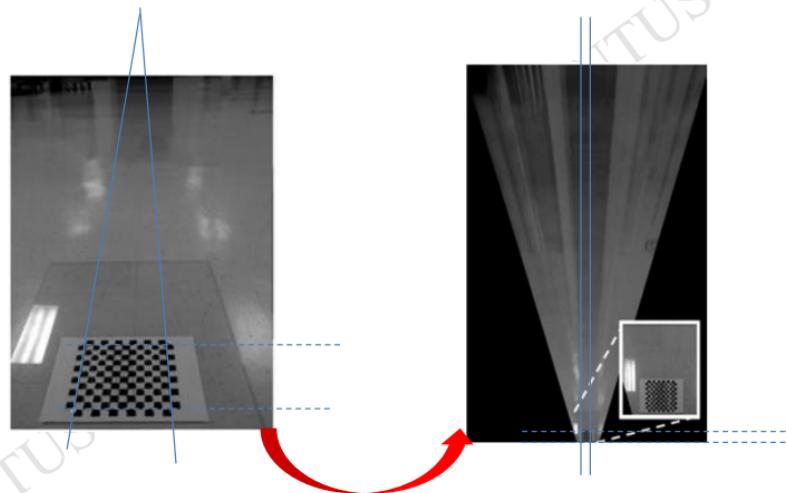
Possible method

Manually assign at least 4 correspondences (define your new image-resolution well)



Homography—Applications

- Solution for finding H —cont.



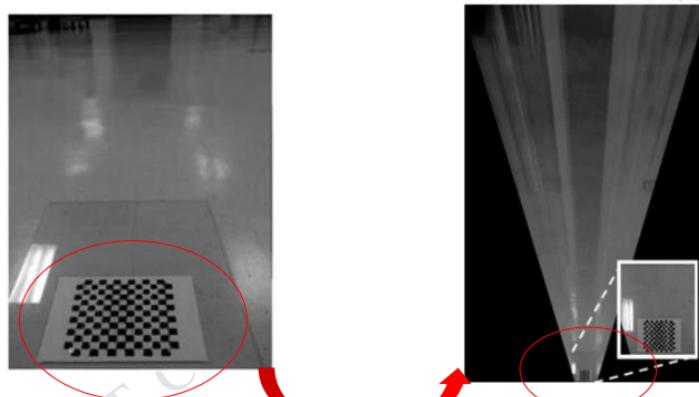
Possible method

Find vanishing points from parallel structure → can be an automatic method based on the line detection algorithm (line fitting, as well)



Homography—Applications

- Solution for finding \mathbf{H} —cont.



Pattern/object recognition

For a very simple example, the checkerboard has the fixed size and parallel grids. If you have a algorithm to determine the corners on the checkerboard, it will be easy to have correspondences for finding \mathbf{H} .



Homography—Applications

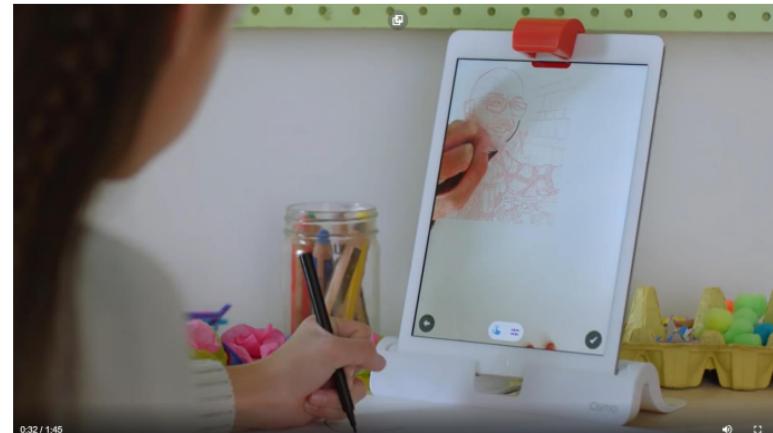
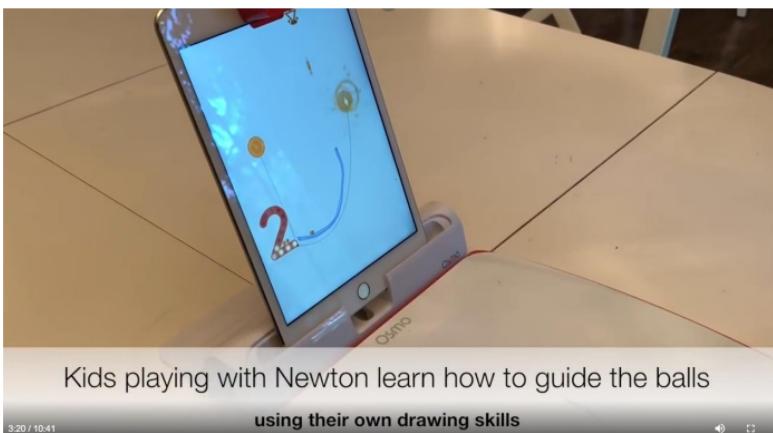
- Dynamic seethroughs (homography 應用)





Homography—Applications

- OSMO Kit: Interactive game





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