電腦視覺與應用
Computer Vision and Applications

Lecture06-2-Two-views geometry-case study

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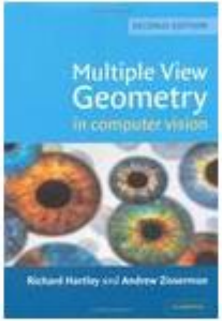
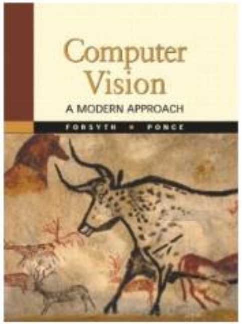
Two-views geometry

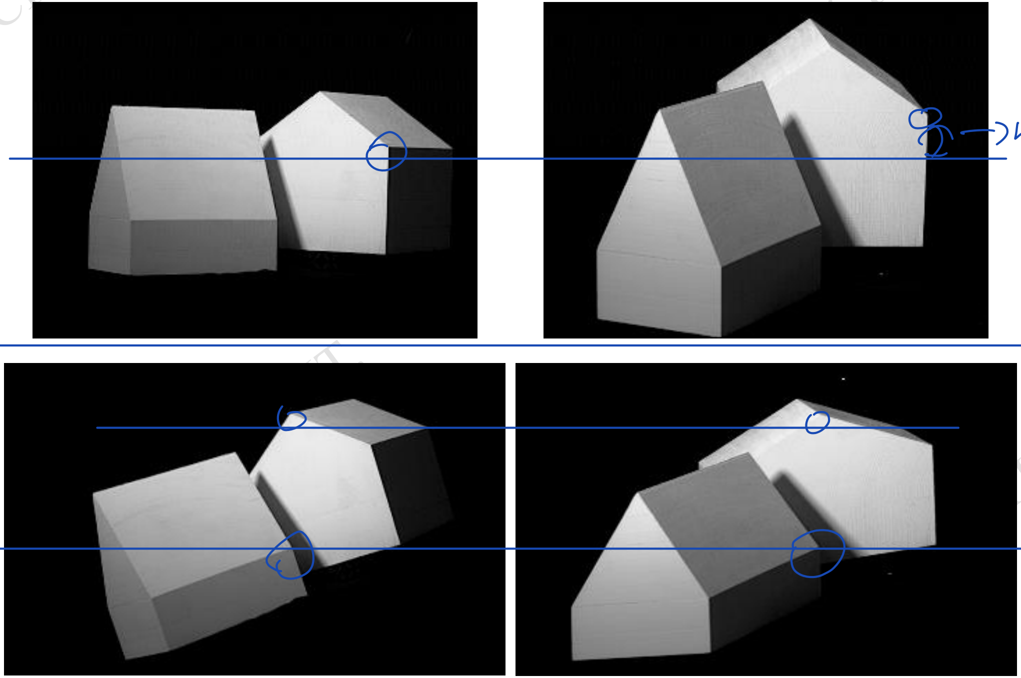
- Case study for stereo-vision & homography
- Lecture Reference at:
  - Multiple View Geometry in Computer Vision, Chapter 11
  - Computer Vision A Modern Approach, Chapter 11.





Stereo-image

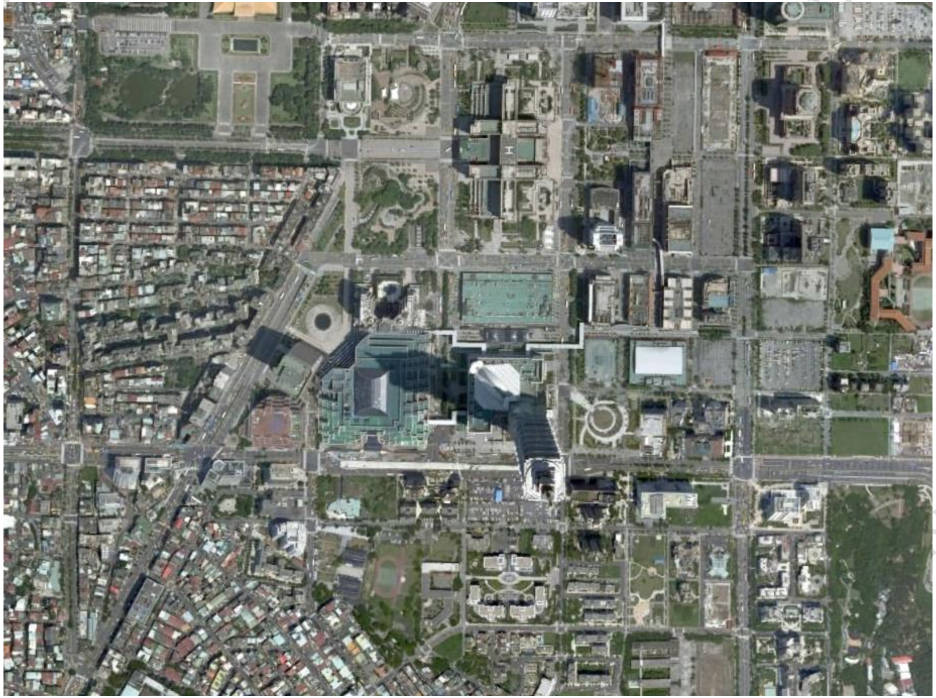
L R



→ not good

perfect

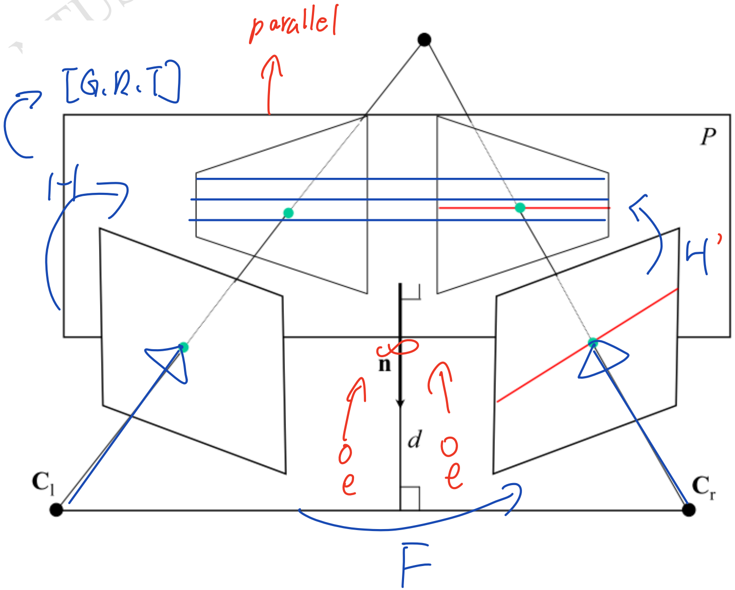
Rectified images



Rectification for stereo-image

[G R T] = H

- To projectively transfer images into a specific position



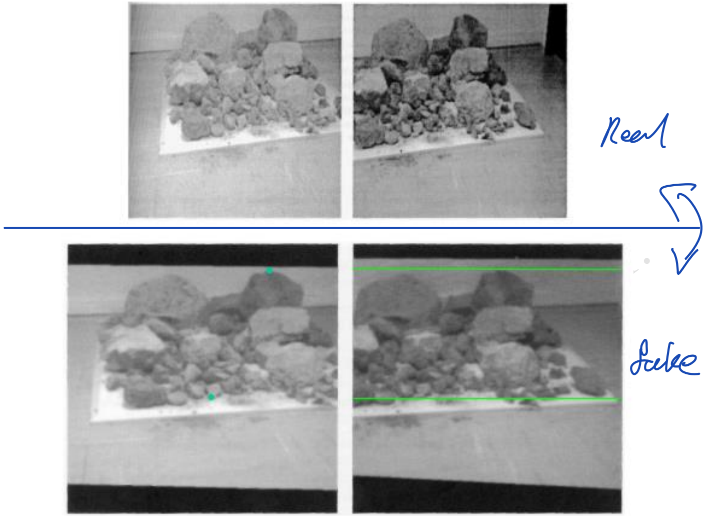
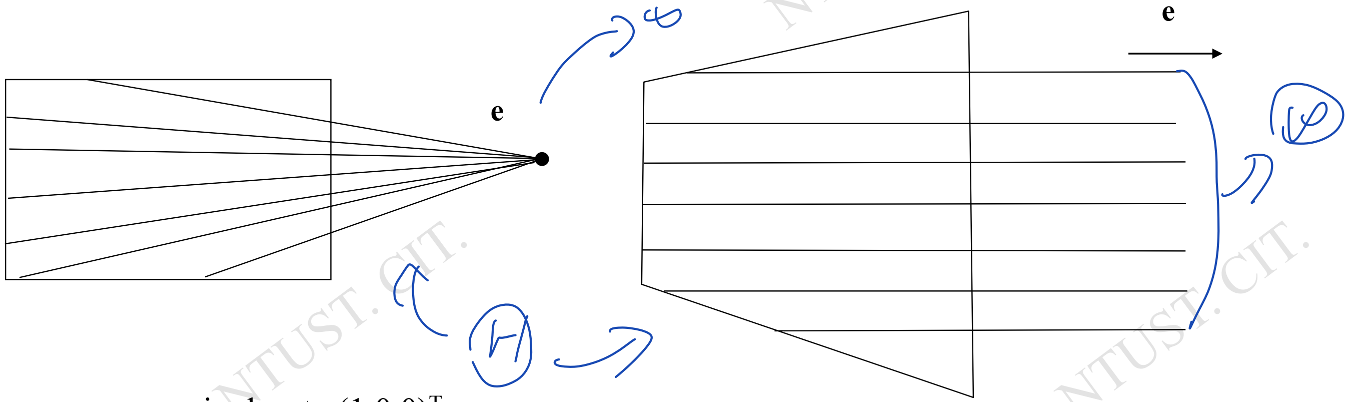


Image rectification

∞ : infinite

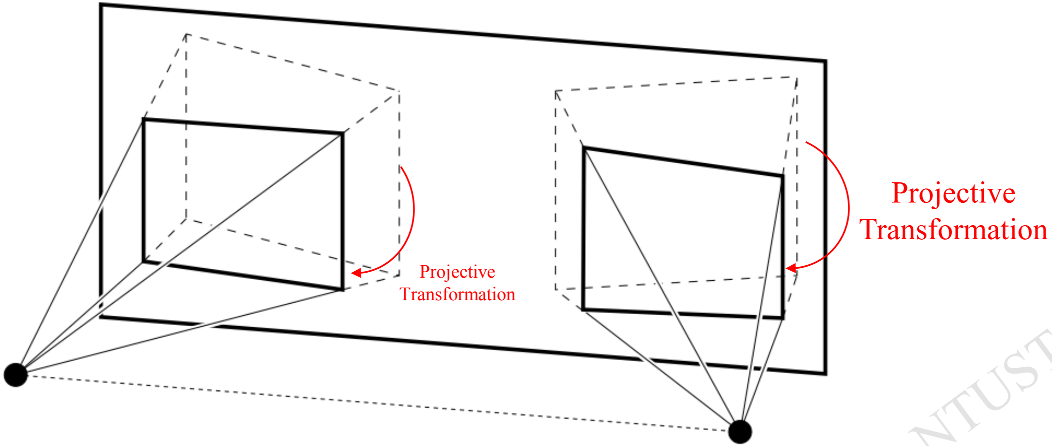
- Apply projective transformation so that epipolar lines correspond to horizontal line



map epipole **e** to $(1,0,0)^T$

try to minimize image distortion

Image rectification



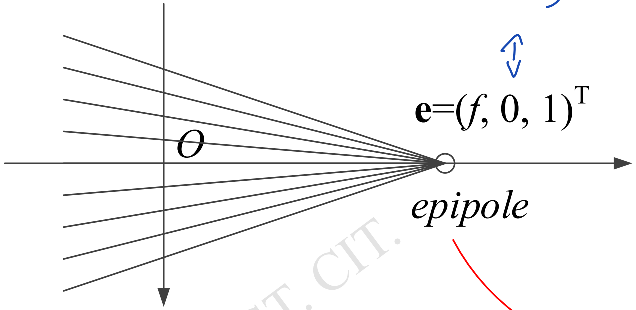
NOTE! This projective transformation is so called Homography!

Image rectification—solution

- To transfer epipole to the point at infinity

$(1, 0, 0)$ $(?, 0, 0)$

non. 0



$\mathbf{e}=(f, 0, 1)^T$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/f & 0 & 1 \end{bmatrix}$$

$\mathbf{e}_\infty = (f, 0, 0)^T$

point at infinity

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{f} & 0 & 1 \end{bmatrix}}_{G} \underbrace{\begin{pmatrix} f \\ 0 \\ 1 \end{pmatrix}}_{e} \Rightarrow -1+1=0$$

$$\mathbf{e}_\infty = \mathbf{G}\mathbf{e}$$

Note! This is homography

Image rectification—solution, cont.

- However, in general, we have the configuration like the following figure.
- So, it needs a translation and a rotation to adjust the epipole on the special condition (on *x*-axis), and the homography will be derived as the format in the previous page.

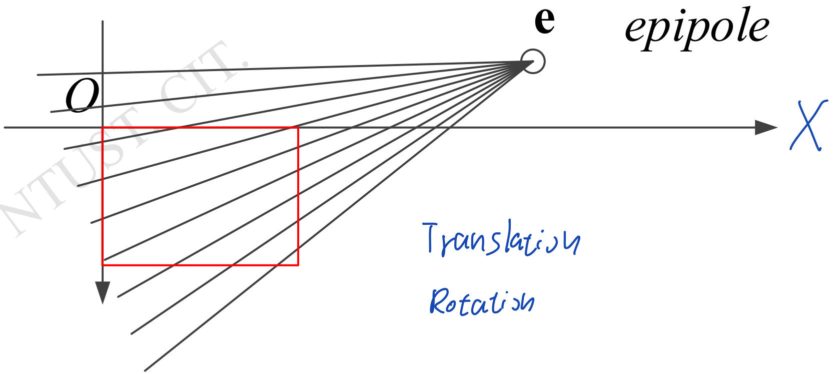


Image rectification—solution, cont.

- An appropriate choice of translation would be the center of image. For example, translate the image center to the origin.

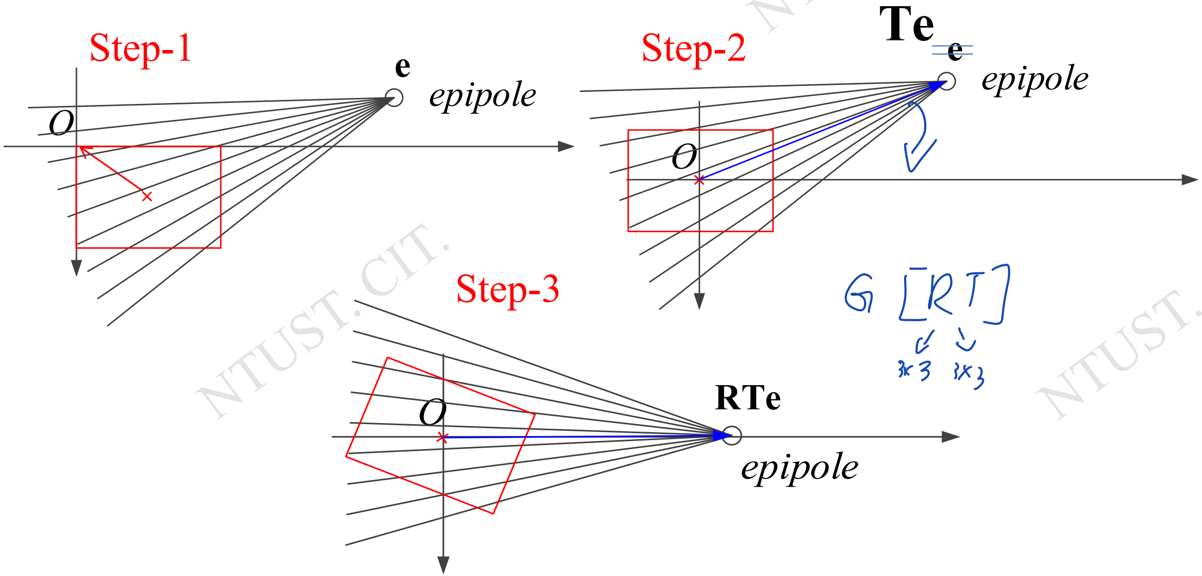
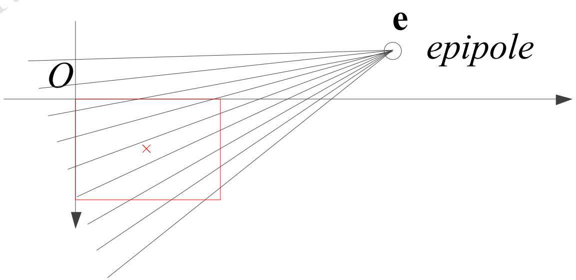


Image rectification—solution, cont.



$$\mathbf{e}_\infty = \mathbf{He}$$

$\mathbf{e}_\infty = (f, 0, 0)^T$

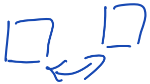
point at infinity

$$\mathbf{e}_\infty = \underbrace{\mathbf{GRT}}_{\text{homography}}\mathbf{e}$$

$$\rightarrow \quad \mathbf{e}_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/f & 0 & 1 \end{bmatrix} \mathbf{RTe}$$

Image rectification—solution, cont.

$$\rightarrow \mathbf{e}_\infty = \mathbf{He} = \mathbf{GRTe} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/f & 0 & 1 \end{bmatrix} \underbrace{\mathbf{RTe}}_{φφ}$$



① F
② e, e'
③ T
④ R
⑤ G

Review the procedure:

- Determine one **F** from two image ⇒ e
- Determine e (use cross product of two epipolar lines,line eqs: $\mathbf{l}=\mathbf{F}^T\mathbf{x}'$)
- Determine **T** (of course, you know the image resolution. use its center)
- Determine **R** (you already have $\mathbf{T}_e$, rotation angle should be $-\tan^{-1}\frac{y}{x}$

- Then, get f from RTe.
- Finally, you have H.

Note! **H** is calculated from the projective mapping (homography) of point-point. If you need line mapping according this homography, use $\mathbf{l}_{rect} = \mathbf{H}^{-T}\mathbf{l}$

Image rectification—solution, cont.

- Call the process, again.

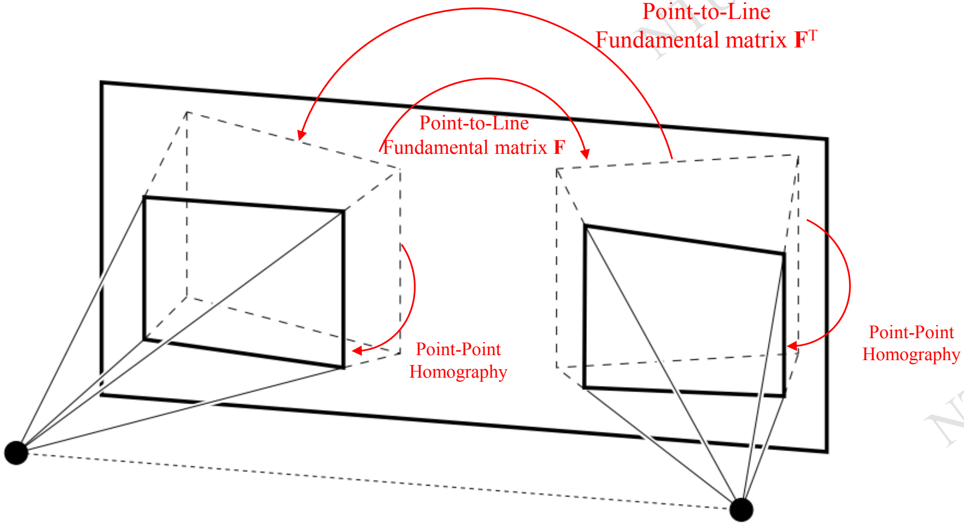
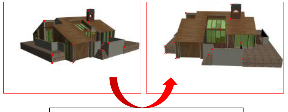


Image rectification—example

12 point → F

Recall the previous example.
Image resolution 720x480.

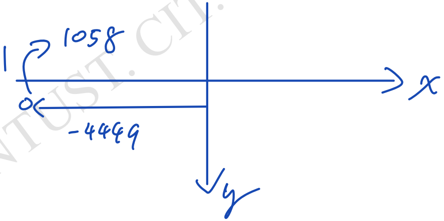


F=
-0.000219 -0.000913 0.292220
0.000103 -0.000245 0.737529
-0.142952 -0.450960 1.000000

e=	e'=
-4089.085693	-552.206970
1298.432373	217.436905
1.000000	1.000000

For 1st image:

$$\mathbf{T}=\begin{bmatrix}1 & 0 & -360\\ 0 & 1 & -240\\ 0 & 0 & 1\end{bmatrix}$$



$$\mathbf{T*e}=\boxed{\begin{matrix}-4449.085693\\ 1058.432373\\ 1.0\end{matrix}}$$

$$-\tan^{-1}\left(\frac{1058.432}{-4449.086}\right)$$

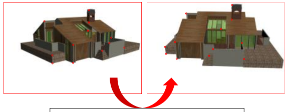
Rotation angle= 13.38°

$$\mathbf{R}=\begin{bmatrix}\cos(13.38^\circ) & -\sin(13.38^\circ) & 0\\ \sin(13.38^\circ) & \cos(13.38^\circ) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\mathbf{RTe}=[\boxed{-4573.3} \quad 0 \quad 1]^{\mathrm{T}}$$

$$\therefore \mathbf{H}=\mathbf{GRT}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -1/-4573.3 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(13.38^\circ) & -\sin(13.38^\circ) & 0\\ \sin(13.38^\circ) & \cos(13.38^\circ) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -360\\ 0 & 1 & -240\\ 0 & 0 & 1\end{bmatrix}$$

Image rectification—example, cont.



F=
-0.000219 -0.000913 0.292220
0.000103 -0.000245 0.737529
-0.142952 -0.450960 1.000000

e=
-4089.085693
1298.432373
1.000000

e'=
-552.206970
217.436905
1.000000

For 2nd image:

$$\mathbf{T'} = \begin{bmatrix} 1 & 0 & -360 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T'*e'} = \boxed{\begin{matrix} -912.2070 \\ -22.5631 \\ 1.0000 \end{matrix}}$$

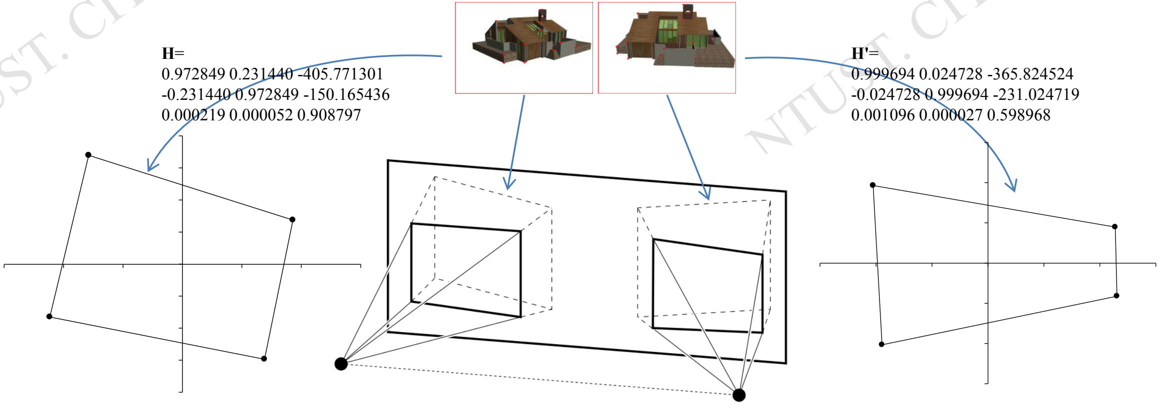
$$-\tan^{-1}\left(\frac{-22.56}{-912.2}\right)$$

Rotation angle=-1.42

$$\mathbf{R'} = \begin{bmatrix} \cos(-1.42^\circ) & -\sin(-1.42^\circ) & 0 \\ \sin(-1.42^\circ) & \cos(-1.42^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \mathbf{H'} = \mathbf{G'R'T'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{-912.486} & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-1.42^\circ) & -\sin(-1.42^\circ) & 0 \\ \sin(-1.42^\circ) & \cos(-1.42^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -360 \\ 0 & 1 & -240 \\ 0 & 0 & 1 \end{bmatrix}$$

Image rectification—example, cont.



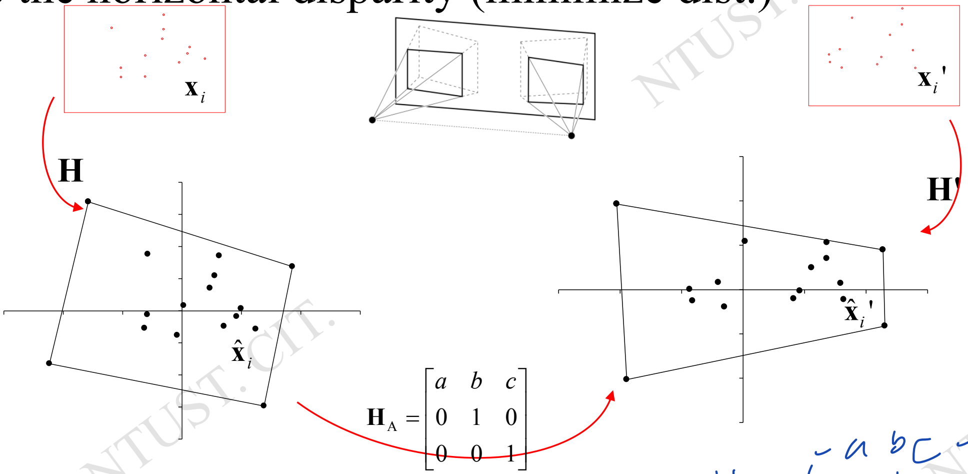
After rectification adjustment, two problems remain

1.Correspondences in two image may have disparity on *y* direction.

2.Pixel coordinates may not fall in positive region.

Image rectification—example, cont.

- Minimize the horizontal disparity (minimize dist.)



$$\mathbf{H}_A = \begin{bmatrix} a & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

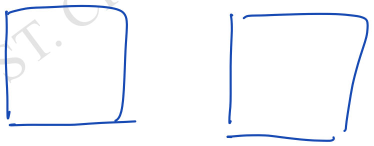
$$H = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix}$$

Minimize $\sum_i d(\mathbf{H}_A \hat{\mathbf{x}}_i, \hat{\mathbf{x}}_i')^2$

Note! The minimization is to minimize "distance" of correspondences (that are $\hat{\mathbf{x}}_i', \mathbf{H}_A \hat{\mathbf{x}}_i$)

Image rectification—example, cont.

- Minimize the horizontal disparity—cont.



$$\text{Minimize} \quad \sum_i d(\mathbf{H}_A \hat{\mathbf{x}}_i, \hat{\mathbf{x}}_i')^2$$

$$\rightarrow \quad \sum_i [(a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')^2 + (\hat{y}_i - \hat{y}_i')^2]$$

$(\hat{y}_i - \hat{y}_i')^2$ is a constant, and will not affect the chosen of solution for (a, b, c)

So, minimization terms are reduced…

$$\text{Minimize} \quad \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')^2$$

Image rectification—example, cont.

- Minimize the horizontal disparity—cont.

$$\text{Minimize} \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')^2$$

For convenience

$$\text{let} \quad g = \sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')^2$$

To determine extreme value of g, take partial difference for g. (偏微分)

$$\begin{cases} \dfrac{\partial g}{\partial a} = 0 \\ \dfrac{\partial g}{\partial b} = 0 \\ \dfrac{\partial g}{\partial c} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')\hat{x}_i = 0 \\ 2\sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')\hat{y}_i = 0 \\ 2\sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i') = 0 \end{cases} \Rightarrow \begin{cases} (\sum_i \hat{x}_i^2)a + (\sum_i \hat{x}_i\hat{y}_i)b + (\sum_i \hat{x}_i)c = \sum_i \hat{x}_i\hat{x}_i' \\ (\sum_i \hat{x}_i\hat{y}_i)a + (\sum_i \hat{y}_i^2)b + (\sum_i \hat{y}_i)c = \sum_i \hat{y}_i\hat{x}_i' \\ (\sum_i \hat{x}_i)a + (\sum_i \hat{y}_i)b + (\sum_i 1)c = \sum_i \hat{x}_i' \end{cases}$$

Image rectification—example, cont.

- Minimize the horizontal disparity—cont.

$$\begin{bmatrix} (\sum_i \hat{x}_i^2) & (\sum_i \hat{x}_i\hat{y}_i) & (\sum_i \hat{x}_i) \\ (\sum_i \hat{x}_i\hat{y}_i) & (\sum_i \hat{y}_i^2) & (\sum_i \hat{y}_i) \\ (\sum_i \hat{x}_i) & (\sum_i \hat{y}_i) & (\sum_i 1) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_i \hat{x}_i\hat{x}_i' \\ \sum_i \hat{y}_i\hat{x}_i' \\ \sum_i \hat{x}_i' \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} (\sum_i \hat{x}_i^2) & (\sum_i \hat{x}_i\hat{y}_i) & (\sum_i \hat{x}_i) \\ (\sum_i \hat{x}_i\hat{y}_i) & (\sum_i \hat{y}_i^2) & (\sum_i \hat{y}_i) \\ (\sum_i \hat{x}_i) & (\sum_i \hat{y}_i) & (\sum_i 1) \end{bmatrix}^{-1} \begin{bmatrix} \sum_i \hat{x}_i\hat{x}_i' \\ \sum_i \hat{y}_i\hat{x}_i' \\ \sum_i \hat{x}_i' \end{bmatrix}$$

Image rectification—example, cont.

- Minimize the horizontal disparity—cont.

- After applying transformation matrix to right image:

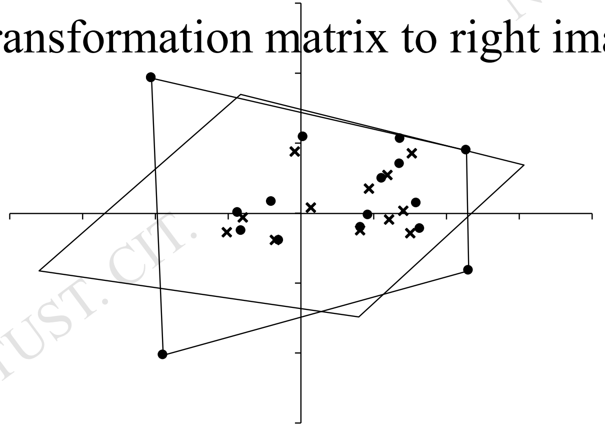


Image rectification—example, cont.

- Minimize the vertical disparity, as well as horizontal distance.

$$\text{Minimize} \quad \sum_i d(\mathbf{H}_A \hat{\mathbf{x}}_i, \hat{\mathbf{x}}_i')^2 \quad \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = H_A \cdot \hat{x}_i$$

$$\rightarrow \sum_i [(\underbrace{a\hat{x}_i + b\hat{y}_i + c}_{H_A \cdot \hat{x}_i} - \hat{x}_i')^2 + (d\hat{y}_i + e - \hat{y}_i')^2]$$

$$\mathbf{H}_A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_{2d} = \begin{pmatrix} \text{New } x \\ \text{New } y \\ 1 \end{pmatrix}$$

5 unknow

$x \frac{\partial}{\partial x} = 0$

a, b, c, d, e

$\frac{\partial}{\partial x}$

$$\begin{cases} \frac{\partial}{\partial a} = 0 \\ \frac{\partial}{\partial b} = 0 \\ \frac{\partial}{\partial c} = 0 \\ \frac{\partial}{\partial d} = 0 \\ \frac{\partial}{\partial e} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')\hat{x}_i = 0 \\ 2\sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i')\hat{y}_i = 0 \\ 2\sum_i (a\hat{x}_i + b\hat{y}_i + c - \hat{x}_i') = 0 \\ 2\sum_i (d\hat{y}_i + e - \hat{y}_i')\hat{y}_i = 0 \\ 2\sum_i (d\hat{y}_i + e - \hat{y}_i') = 0 \end{cases} \Rightarrow \begin{cases} (\sum_i \hat{x}_i^2)a + (\sum_i \hat{x}_i\hat{y}_i)b + (\sum_i \hat{x}_i)c = \sum_i \hat{x}_i\hat{x}_i' \\ (\sum_i \hat{x}_i\hat{y}_i)a + (\sum_i \hat{y}_i^2)b + (\sum_i \hat{y}_i)c = \sum_i \hat{y}_i\hat{x}_i' \\ (\sum_i \hat{x}_i)a + (\sum_i \hat{y}_i)b + (\sum_i 1)c = \sum_i \hat{x}_i' \\ (\sum_i \hat{y}_i^2)d + (\sum_i \hat{y}_i)e = \sum_i \hat{y}_i\hat{y}_i' \\ (\sum_i \hat{y}_i)d + (\sum_i 1)e = \sum_i \hat{y}_i' \end{cases}$$

a, b, c, d, e

Image rectification—example, cont.

- Minimize the vertical disparity, as well as horizontal distance.—cont.

known

12

$$\begin{bmatrix} \sum_i \hat{x}_i^2 & \sum_i \hat{x}_i\hat{y}_i & \sum_i \hat{x}_i & 0 & 0 \\ \sum_i \hat{x}_i\hat{y}_i & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i & 0 & 0 \\ \sum_i \hat{x}_i & \sum_i \hat{y}_i & \sum_i 1 & 0 & 0 \\ 0 & 0 & 0 & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i \\ 0 & 0 & 0 & \sum_i \hat{y}_i & \sum_i 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} \sum_i \hat{x}_i\hat{x}_i' \\ \sum_i \hat{y}_i\hat{x}_i' \\ \sum_i \hat{x}_i' \\ \sum_i \hat{y}_i\hat{y}_i' \\ \sum_i \hat{y}_i' \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} \sum_i \hat{x}_i^2 & \sum_i \hat{x}_i\hat{y}_i & \sum_i \hat{x}_i & 0 & 0 \\ \sum_i \hat{x}_i\hat{y}_i & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i & 0 & 0 \\ \sum_i \hat{x}_i & \sum_i \hat{y}_i & \sum_i 1 & 0 & 0 \\ 0 & 0 & 0 & \sum_i \hat{y}_i^2 & \sum_i \hat{y}_i \\ 0 & 0 & 0 & \sum_i \hat{y}_i & \sum_i 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum_i \hat{x}_i\hat{x}_i' \\ \sum_i \hat{y}_i\hat{x}_i' \\ \sum_i \hat{x}_i' \\ \sum_i \hat{y}_i\hat{y}_i' \\ \sum_i \hat{y}_i' \end{bmatrix}$$

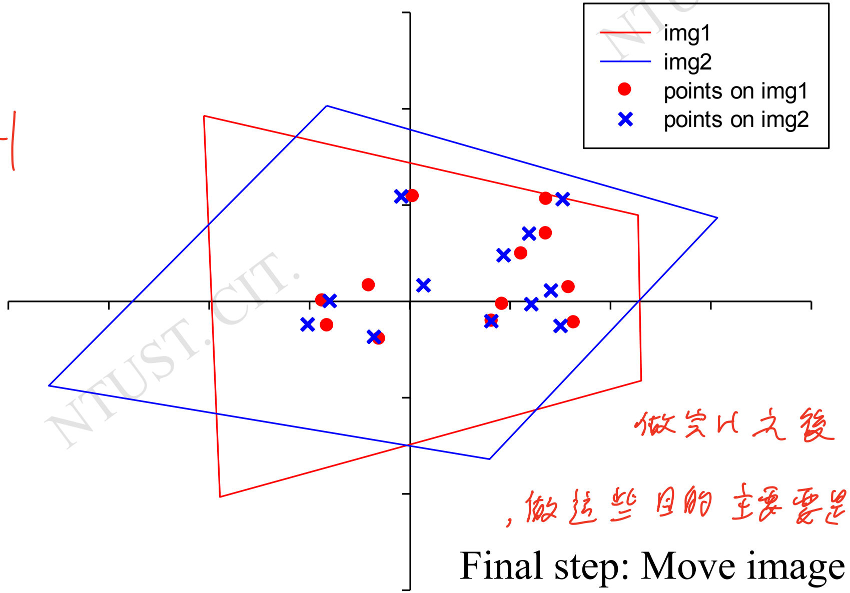
unknown

Finally, recover $\boxed{\mathbf{H}_A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix}}$

Image rectification—example, cont.

- Minimize the vertical disparity, as well as horizontal distance.—cont.

$H_A H$



H'

做完H之後，還要移到(+,+)象限
，做這些目的主要是加速.

Final step: Move image to canvas region

Image rectification—example, cont.

- Unique solution?

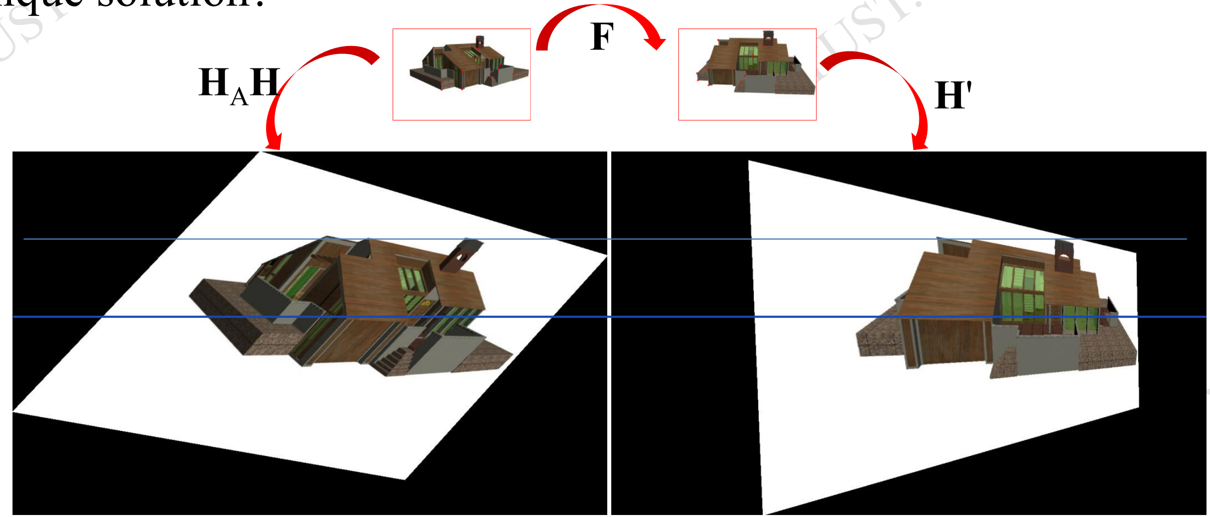


Image rectification—openCV

- StereoRectifyUncalibrated

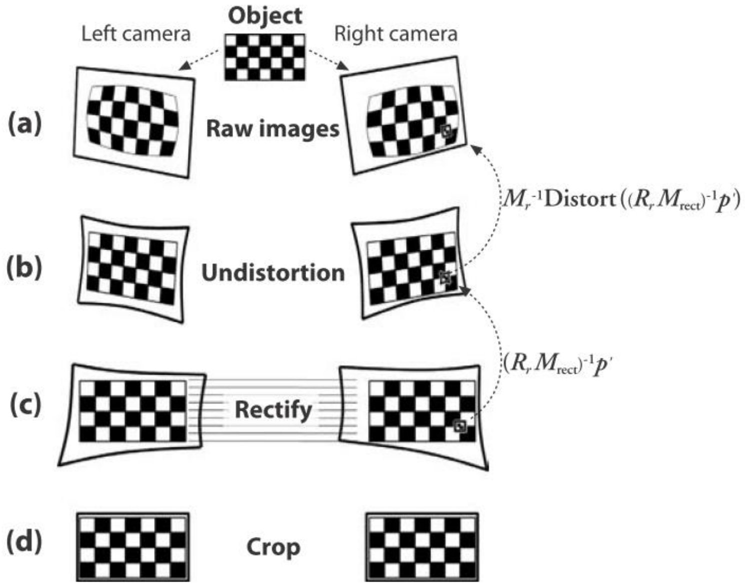


Image rectification—openCV

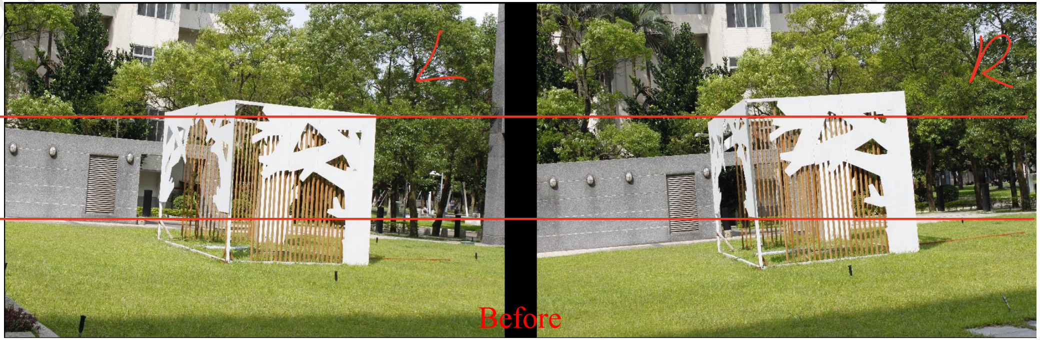
- The same operation in OpenCV

cvStereoRectifyUncalibrated

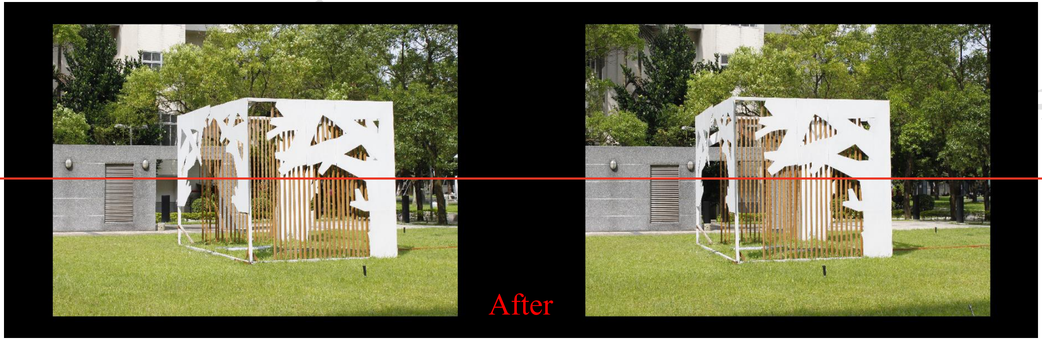
```
int cvStereoRectifyUncalibrated(
    const CvMat* points1,
    const CvMat* points2,
    const CvMat* F,
        CvSize imageSize,
        CvMat* Hl,
        CvMat* Hr,
        double threshold
);
```

- points1, points2 → The 2 arrays of corresponding 2D points. (input)
- F → Fundamental Matrix (input)
- imageSize → (input)
- Hl, Hr → Homography Matrix (output for Left, Right Images)
- threshold → (input) for rejecting the outliers for which $|\mathbf{x}'^T\mathbf{F}\mathbf{x}| >$ threshold

Reference software (stereophoto maker)



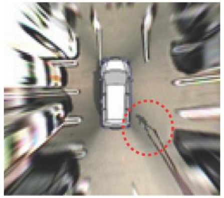
Real



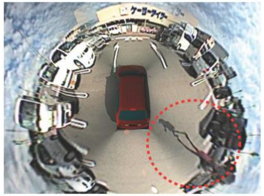
http://stereo.jpn.org/eng/stphmkr/

Homography—Applications

- Intelligent automobile

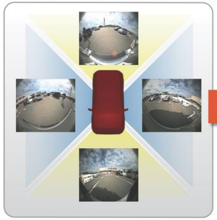


Conventional technology-based image, vehicles and people are not visible.

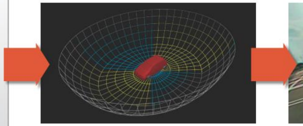


Sample view using Fujitsu Laboratories' new technology perspective from above-rear (pedestrian is visible)

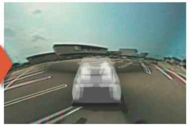
Sample view using Fujitsu's new technology, perspective from front facing vehicle (rearview pedestrian is visible)



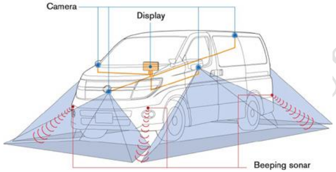
4 cameras capture video images of 4 different views (perspectives)



Virtual 3D model for projection of video images is synthesized (scene is projected virtually onto a 3D curved plane). Image is changed to the desired perspective



Desired view (perspective) is displayed



NISSAN

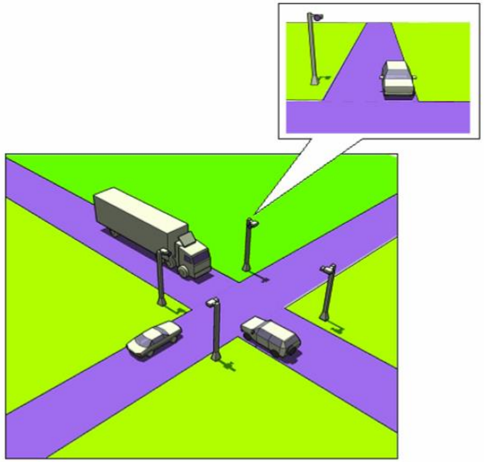
FUJITSU
http://www.fujitsu.com/downloads/MICRO/fma/pdf/360_OmniView_AppNote.pdf

Homography—Applications

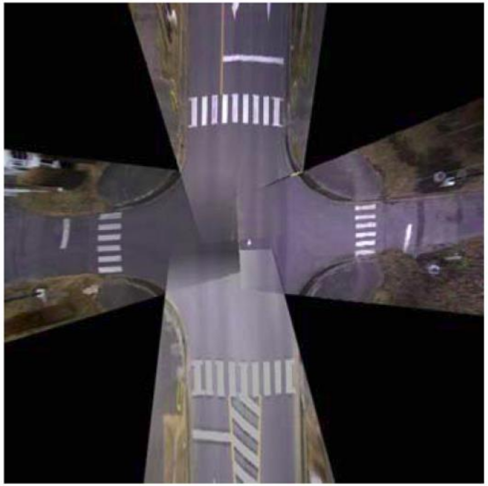
- Intelligent automobile—cont.

Homography—Applications

- Multi-camera surveillance
- 物聯網



筑波大學(JP)



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Homography—Applications

- Application scenarios

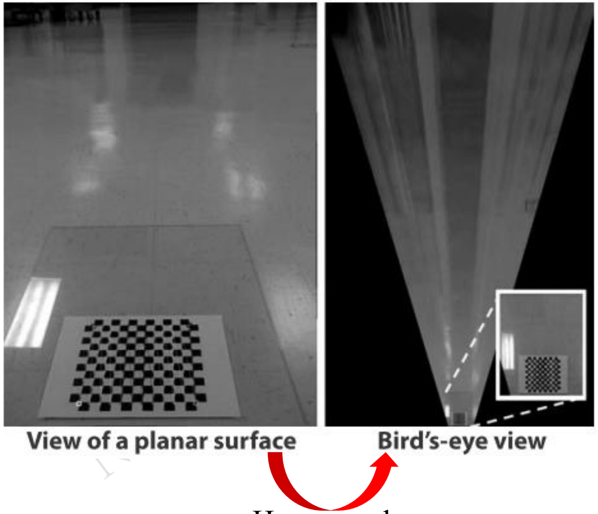
Image-Undistortion → **Homography** → **Image stitching** →

- Auto parking / Parking assistant system
- Obstacle detection (distance measurement)
- Driving / navigation (Auto or assistance)
- Safety driving
- ……

Note! In many application, wide-angle lens (fish-eye) cameras are used. In these situations, the UN-distortion processing is important.

Homography—Applications

- Once again, define your problem first!



1. Pre-processing or post-processing for **H**
2. Constant **H** or various **H**

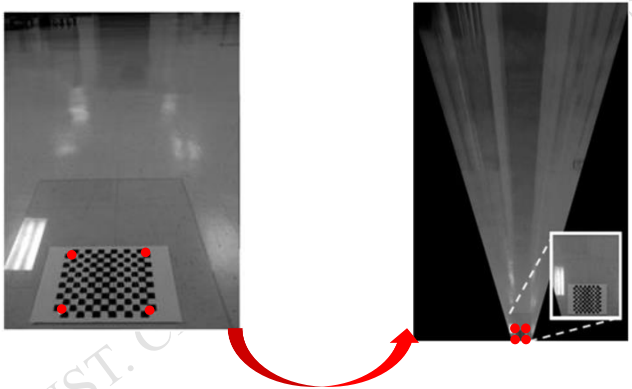
Solution: (homography)

1. Line cue ? Point cue ?
2. Correspondence
3. Scale issue

Picture from OReilly Learning OpenCV

Homography—Applications

- Solution for finding **H**

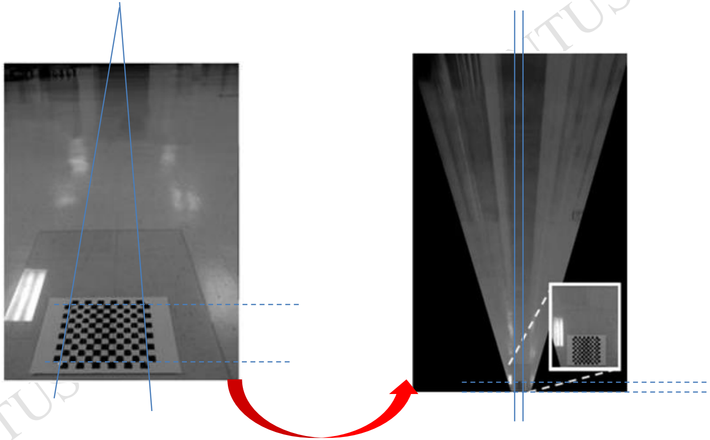


Possible method
Manually assign at least 4 correspondences (define your new image-resolution well)

Picture from OReilly Learning OpenCV

Homography—Applications

- Solution for finding **H**—cont.

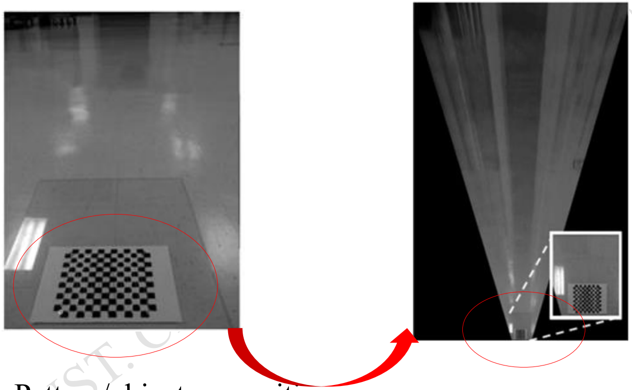


Possible method
Find vanishing points from parallel structure →can be an automatic method based on the line detection algorithm (line fitting, as well)

Picture from OReilly Learning OpenCV

Homography—Applications

- Solution for finding **H**—cont.



Pattern/object recognition
For a very simple example, the checkerboard has the fixed size and parallel grids. If you have a algorithm to determine the corners on the checkerboard, it will be easy to have correspondences for finding **H**.

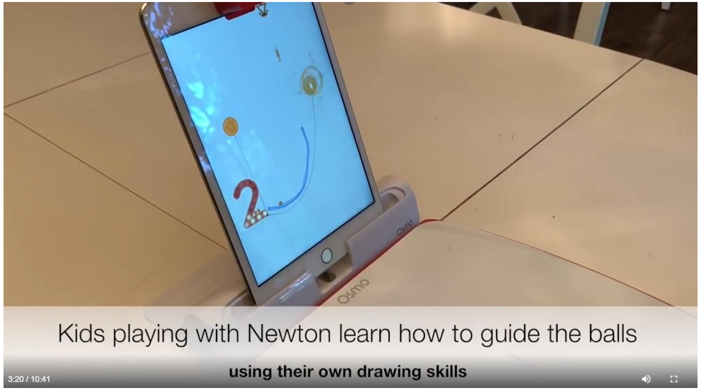
Homograhy—Applications

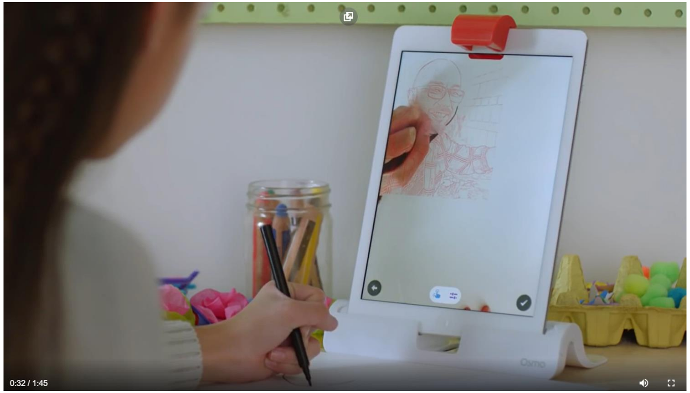
- Dynamic seethroughs (homography應用)

P. Barnum, Y. Sheikh, A. Datta, and T. Kanade, "Dynamic seethroughs : synthesizing hidden views of moving objects," IEEE Int. Symp. Mix. Augment. Real., pp. 111–114, Oct. 2009.

Homography—Applications

- OSMO Kit: Interactive game





https://www.youtube.com/watch?v=0upQlA6K5YI

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