電腦視覺與應用
Computer Vision and Applications

Lecture08-3D reconstruction

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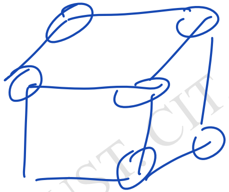
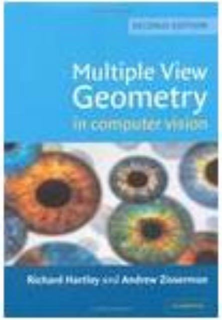
3D reconstruction

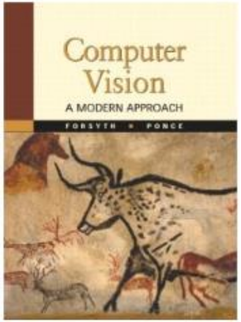
- Either from calibrated images ⇒ Real distance
- Or from uncalibrated images ⇒ 3D without size



3D reconstruction

- Lecture Reference at:
  - Multiple View Geometry in Computer Vision, (*Chapter10 and 12)
  - Computer Vision A Modern Approach, Chapter 13



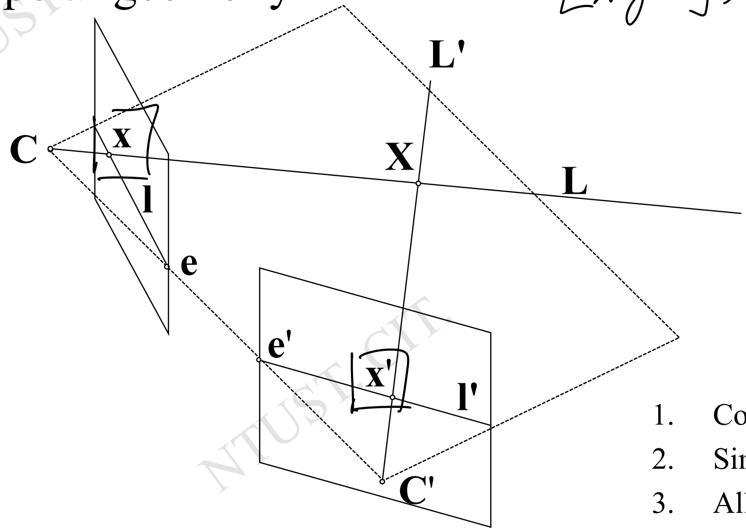


3D reconstruction

- The questions?
  - Correspondence geometry: Given an image point x in the first image, how does this constrain the position of the corresponding point x' in the second image?
  - Camera geometry (motion): Given a set of corresponding image points $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, i=1,…,n, what are the cameras **P** and **P'** for the two views?
  - Scene geometry (structure): Given corresponding image points $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ and cameras **P**, **P'**, what is the position of **X** in space?

3D reconstruction

- Epipolar geometry



$$\begin{array}{ccc} 1\times 3 & 3\times 3 & 3\times 1 \\ [x\ y\ 1] & \times \begin{bmatrix} 3\times 3 \end{bmatrix} & \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \end{array}$$

$\mathbf{l'}$

$Fx = l'$

$$\underbrace{\mathbf{x}'^{\mathrm{T}}\mathbf{F}}_{\mathbf{l}^{\mathrm{T}}}\overbrace{\mathbf{F}\mathbf{x}}^{\mathbf{l'}} = 0$$

$x'^T F = l^T$

l'

1. Computable from corresponding points
2. Simplifies matching
3. Allows to detect outliers
4. Related to calibration

3D reconstruction of cameras and structure

- Reconstruction problem:

$$\text{Given } \underline{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i}\text{, compute } \mathbf{P}, \boxed{\mathbf{P}'} \text{ and } \mathbf{X}_i$$

$$\underline{\mathbf{x}_i = \mathbf{P}\mathbf{X}_i} \text{ and } \mathbf{x}_i' = \mathbf{P}'\mathbf{X}_i \quad \text{for all } i$$

without additional constraints possible up to projective ambiguity

$x_i \rightarrow x_i^T$ (2D)

$\Downarrow$

P

unknown 3D

x_{3D}

P P'

L R

3D reconstruction of cameras and structure

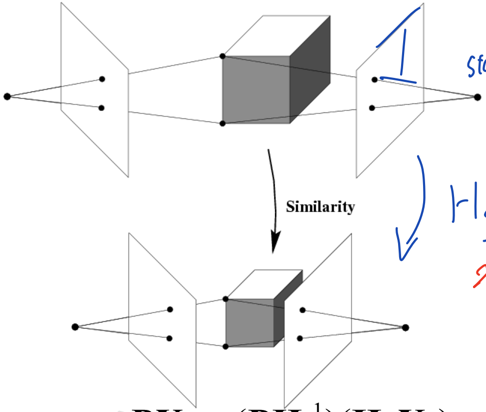
- Reconstruction ambiguity (projective ambi.)

$x_{2D} = PX$ $\quad x_{2D} = P H^{-1} H X_{3D}$

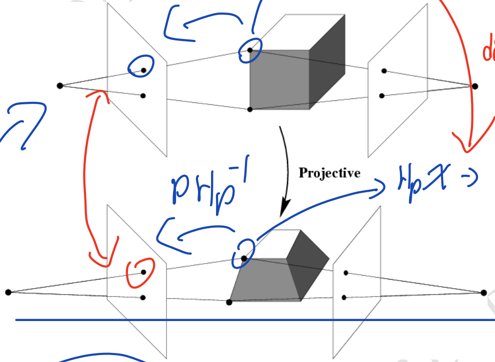
P_2 $\quad X_{3D}$

0.5

$$\begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$x = PX$



$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = (\mathbf{P}\mathbf{H}_S^{-1})\underbrace{(\mathbf{H}_S\mathbf{X}_i)}$$

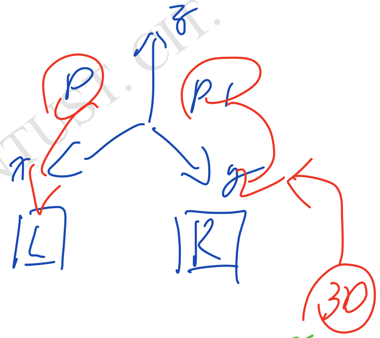
$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = (\mathbf{P}\mathbf{H}_P^{-1})\underbrace{(\mathbf{H}_P\mathbf{X}_i)}$$

PH_P^{-1}

One solution which is projected to $\mathbf{x}_i$, as well.

Outline of 3D reconstruction (from uncalibrated images)

- Compute F from correspondences
- Compute camera matrices from F
- Compute 3D point for each pair of corresponding points

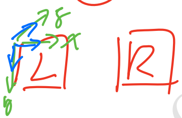


computation of **F**

use $\mathbf{x}'_i\mathbf{F}\mathbf{x}_i=0$ equations, linear in coeff. F

computation of camera matrices

use $$\mathbf{P}=[\mathbf{I}\,|\,0] \quad \mathbf{P}'=[[\mathbf{e}']_\times\mathbf{F}+\mathbf{e}'\mathbf{v}^T\,|\,\lambda\mathbf{e}']$$ → General formula



or $$\mathbf{P}=[\mathbf{I}\,|\,0] \quad \mathbf{P}'=[[\mathbf{e}']_\times\mathbf{F}\,|\,\mathbf{e}']$$

detail at

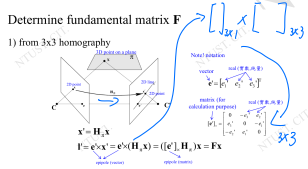
Q. Luong and T. Vieville, "Canonical representations for the geometries of multiple projective views," *Computer vision and image understanding*, vol. 64, no. 2, pp. 193-229, 1996.

F, e, e'

triangulation

compute intersection of two back-projected rays

x, x' (2D) $\underset{6}{\Rightarrow}$ F, e, e' ⇒ P, P'

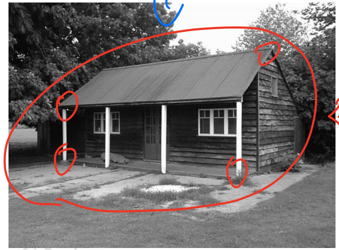


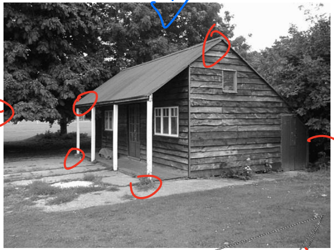
Projective reconstruction

Input (2D Image)
Output 3D (def)

- The reconstruction required no information about camera matrices, or information about the scene geometry. The fundamental matrix F is computed from point correspondences between the iamges, camera matrices are retrived from F, and then 3D points are computed by triangulation from triangulation from the correspondences.

6 → F → e, e'
⇓
P, P'

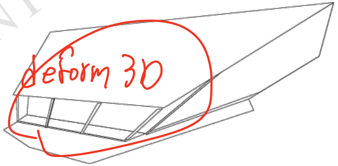




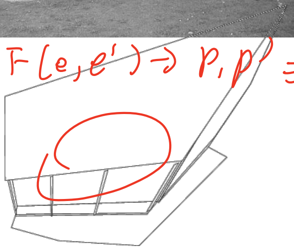
P'

P [I|0]

2D ↔ F (e, e') → P, P' ⇒ | P P' x, x' | → X (deform)



deform 3D



Theorem for the projective reconstruction

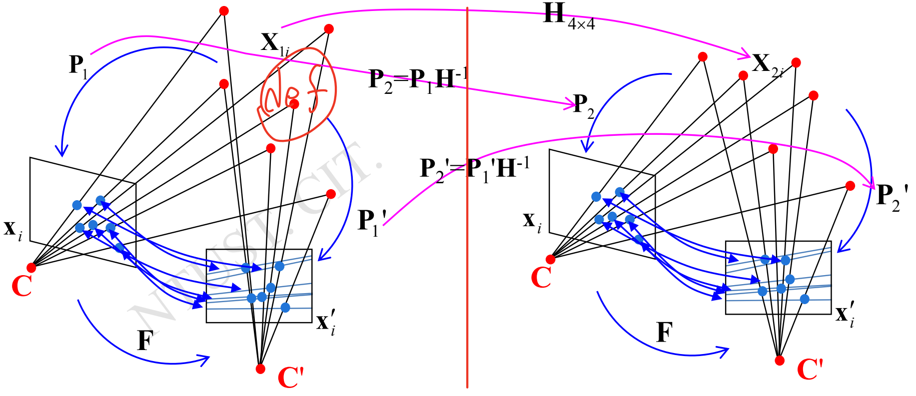
correct deform
$x_{2i} = H x_{1i}$
New

> Suppose that $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ is a set of correspondences between points in two images and that the fundamental matrix $\mathbf{F}$ is uniquely determined by the condition $\mathbf{x}'^{\mathsf{T}}_i \mathbf{F}\mathbf{x}_i = 0$ for all i. Let $(\mathbf{P}_1, \mathbf{P}_1', \{\mathbf{X}_{1i}\})$ and $(\mathbf{P}_2, \mathbf{P}_2', \{\mathbf{X}_{2i}\})$ be two reconstructions of the correspondences $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$. Then there exists a non-singular matrix $\mathbf{H}$ such that $\mathbf{P}_2 = \mathbf{P}_1\mathbf{H}^{-1}$, $\mathbf{P}_2' = \mathbf{P}_1'\mathbf{H}^{-1}$ and $\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$ for all i, except for those i such $\mathbf{F}\mathbf{x}_i = \mathbf{x}_i'\mathbf{F} = 0$

Input
(2D image)

P, P'

$x = P X$



$\mathbf{P}_2 = \mathbf{P}_1\mathbf{H}^{-1}$

$\mathbf{P}_2' = \mathbf{P}_1'\mathbf{H}^{-1}$

$\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$

$x_i = P H^{-1} H X_i$

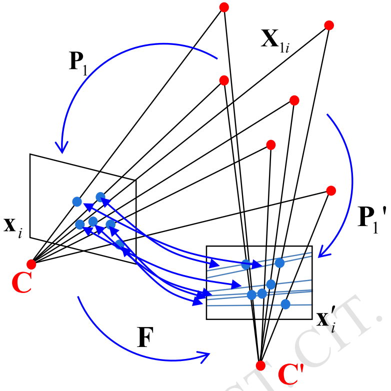
$H^{-1} H \Rightarrow I$

New Corrected P

key result:
allows reconstruction from pair of uncalibrated images

Theorem for the projective reconstruction

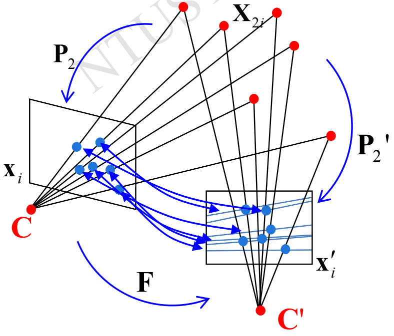
- In practice



[Projective reconstruction]

$\mathbf{H}_{4x4}$

How to get **H?**



Affine reconstruction
or Metric reconstruction

$\mathbf{P}_2 = \mathbf{P}_1\mathbf{H}^{-1}$

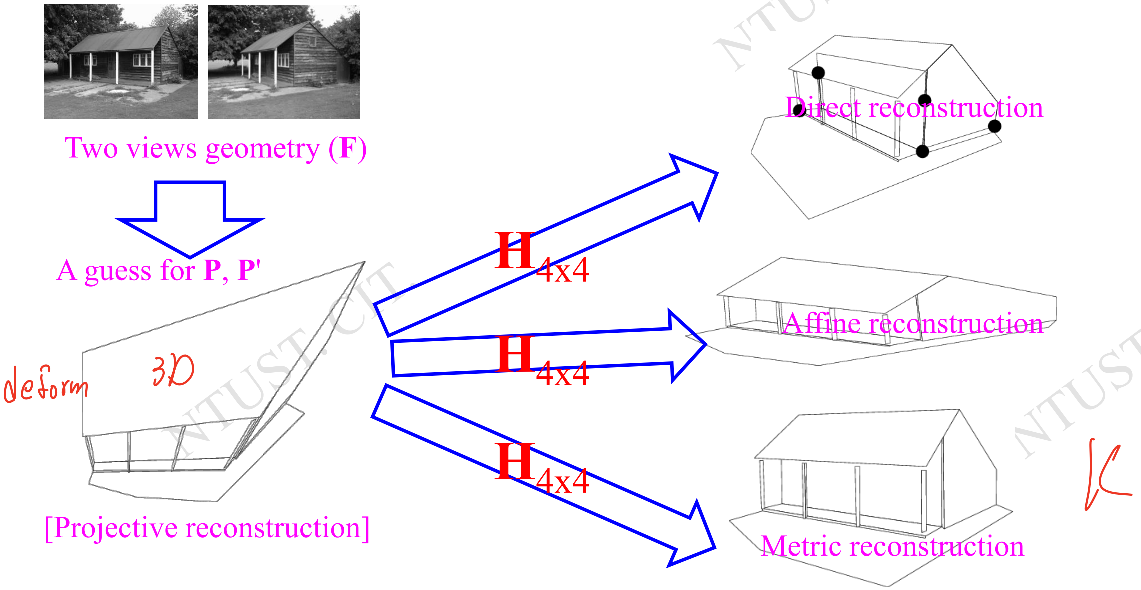
$\mathbf{P}_2' = \mathbf{P}_1'\mathbf{H}^{-1}$

$\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$

Step-1: solve epipolar geometry (**F**)
Step-2: guess **P** & **P'** for intitalization, then determine **X**.
Step-3: determine **H**, then, finding new **P** & **P'** & **X**.

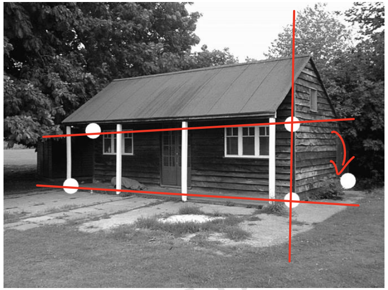
3D reconstruction for projective geometry

- (in practice)



Direct reconstruction using ground truth

- use control points $\mathbf{X}_{Ei}$ with know coordinates, to go from projective to metric



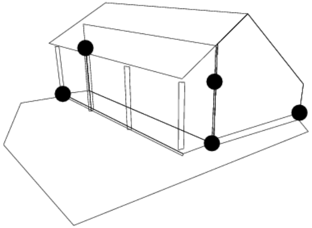
$$\boxed{\mathbf{X}_{Ei}} = \mathbf{H}\boxed{\mathbf{X}_{\mathrm{i}}}$$

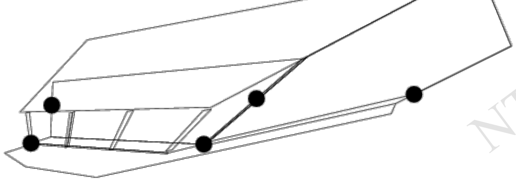
Ground truth (measurement in real environment)

Homography (4x4)

3D points for an estimated projective reconstruction.

$$\mathbf{x}_i = \mathbf{P}\mathbf{H}^{-1}\mathbf{X}_{Ei}$$

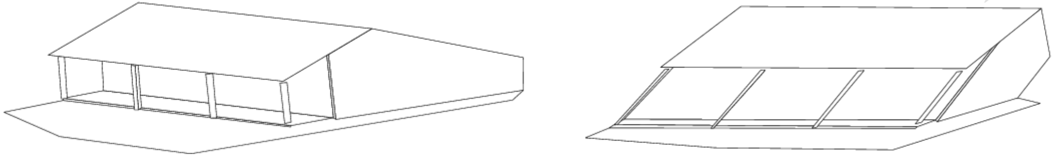




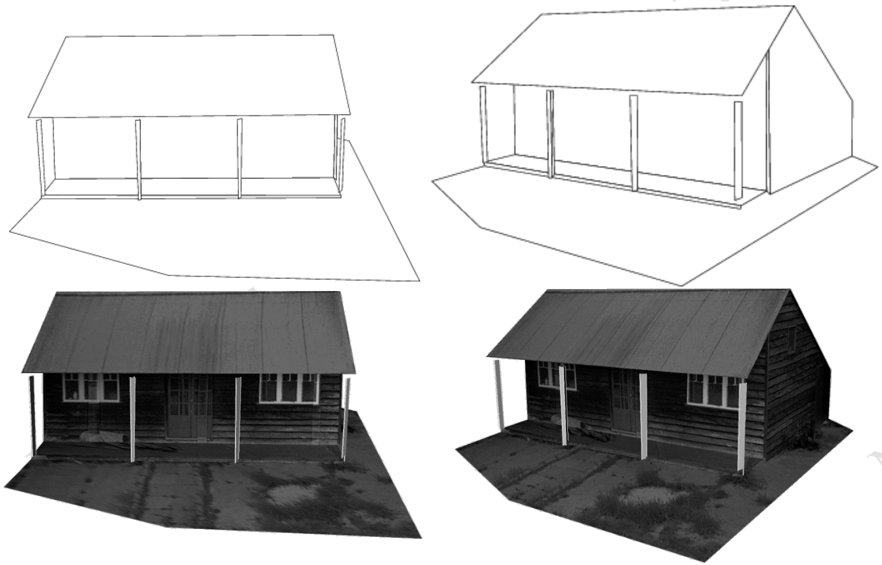
Note! do NOT select degenerated cases,
Ex. 4 points on a plane, 3 points on a line.

Affine reconstruction

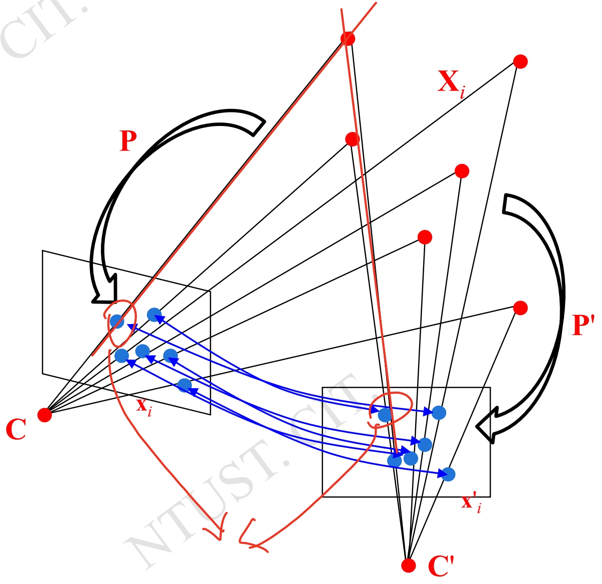
- There are 3 sets of parallel lines in the scene, each set with a different direction. These 3 sets enable the position of the plnae at infinity to be computeed in the projective reconstruction. Note that parallel scene lines are parallel in reconstruction, but lines are NOT perpendicular in the reconstruction.



Metric reconstruction



3D estimation Direct Triangulation method



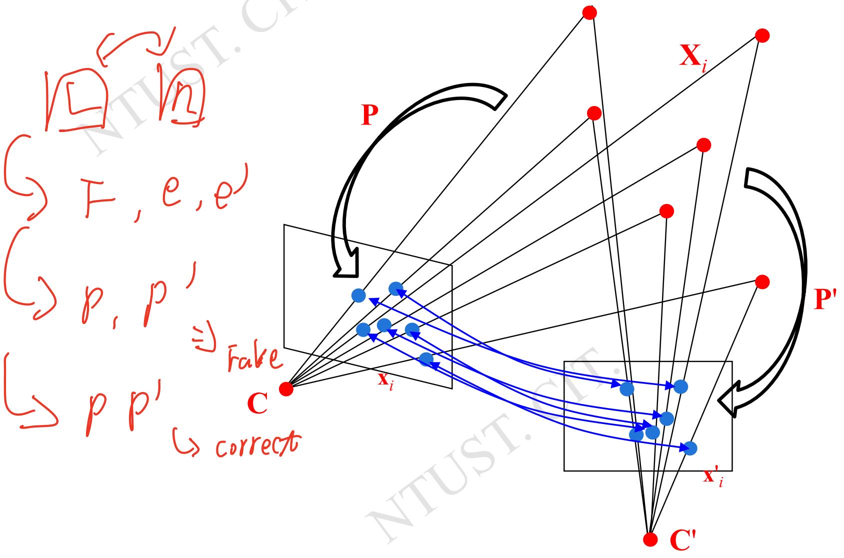
- Condtion-1

P, P', x_i, x'_i : known
X : unknown (to be solved)

- Condtion-2

x_i, x'_i, X: known
P, P': unknown (to be solved)
→become a calibration problem

3D estimation Direct Triangulation method



- Condtion-3 **(direct reconstruction)**

$\mathbf{x}_i$, $\mathbf{x}'_i$: known
5 points **X**: known
P, P': unknown (to be solved)

- Condtion-4 **(affine reconstruction)**

$\mathbf{x}_i$, $\mathbf{x}'_i$, other cues: known
P, P': unknown (to be solved)

- Condtion-5 **(metric reconstruction)**

$\mathbf{x}_i$, $\mathbf{x}'_i$, **K**: known
X, P, P': unknown (to be solved)

Objective

Given two uncalibrated images compute ($\mathbf{P}_M$,$\mathbf{P}'_M$,{$\mathbf{X}_{Mi}$})
(i.e. within similarity of original scene and cameras)

Algorithm [Textbook: Hartley04, Algorithm 10.1]

(i) Compute projective reconstruction ($\mathbf{P}$, $\mathbf{P}'$,{$\mathbf{X}_i$})
- (a) Compute $\mathbf{F}$ from $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$
- (b) Compute $\mathbf{P}$, $\mathbf{P}'$ from $\mathbf{F}$
- (c) Triangulate $\mathbf{X}_i$ from $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$

(ii) Rectify reconstruction from projective to metric
- (a) **Direct method**: compute $\mathbf{H}$ from control points →(condition-3 in the previous slide)

$$\mathbf{P}_M = \mathbf{P}\mathbf{H}^{-1}, \quad \mathbf{P}_M' = \mathbf{P}'\mathbf{H}^{-1}, \quad \mathbf{X}_{Mi} = \mathbf{H}\mathbf{X}_i$$

Stratified method:

- (a) **Affine reconstruction**: →(condition-4 in the previous slide)

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid 0 \\ \boldsymbol{\pi}_\infty^T \end{bmatrix}$$

- (b) **Metric reconstruction**: compute IAC $\boldsymbol{\omega}$ →(condition-5 in the previous slide)

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{A}\mathbf{A}^T = (\mathbf{M}^T\boldsymbol{\omega}\mathbf{M})^{-1}$$

3D reconstruction

Image information provided	View relations and projective objects	3-space objects	reconstruction ambiguity
point correspondences	$\mathbf{F}$		projective
point correspondences including vanishing points	$\mathbf{F}, \mathbf{H}_\infty$	$\mathbf{p}_\infty$	affine
Points correspondences and internal camera calibration	$\mathbf{F}, \mathbf{H}_\infty$ ω, ω'	$\mathbf{p}_\infty$ Ω_∞	metric

The two-view relations, image entities, and their 3-space conterpart for various classes of reconstruction ambiguity.

3D estimation Direct Triangulation method

- If the projected point **PX** is very close to x, then

$$\mathbf{x} \times \mathbf{x} = \mathbf{0} \quad \rightarrow \text{zero vector}$$

So, one constraint for **X** is

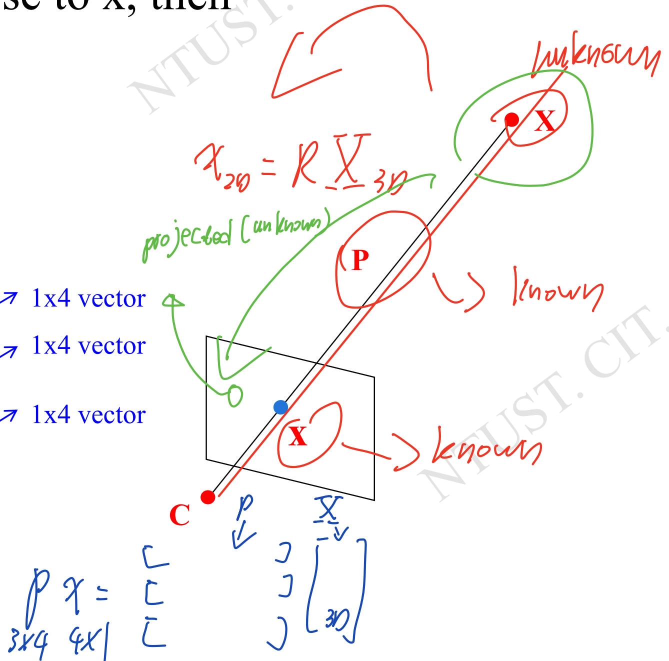
$$\mathbf{x} \times (\mathbf{PX}) = \mathbf{0}$$

For convenience, rewrite **P** as

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{31} & p_{32} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^{\mathrm{T}} \\ \mathbf{p}_2^{\mathrm{T}} \\ \mathbf{p}_3^{\mathrm{T}} \end{bmatrix}_{3\times 4}$$

1x4 vector
1x4 vector
1x4 vector

$$\mathbf{p}_1^{\mathrm{T}} = [p_{11} \quad p_{12} \quad p_{13} \quad p_{14}]$$
$$\mathbf{p}_2^{\mathrm{T}} = [p_{21} \quad p_{22} \quad p_{23} \quad p_{24}]$$
$$\mathbf{p}_3^{\mathrm{T}} = [p_{31} \quad p_{32} \quad p_{33} \quad p_{34}]$$



3D estimation Direct Triangulation method-co

- To solve the equation: $\Rightarrow PX$ $\qquad \mathbf{x} \times (\mathbf{P}\mathbf{X}) = \mathbf{0}$

$$\rightarrow \begin{bmatrix} x \\ y \\ w \end{bmatrix} \times \begin{bmatrix} \mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} \\ \mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} \\ \mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{cases} y\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} - w\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} = 0 \\ -x\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} + w\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} = 0 \\ x\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} - y\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} = 0 \end{cases} \Rightarrow x \times PX = 0$$

x

$$\rightarrow \begin{cases} -\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} + \dfrac{y}{w}\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} = 0 \\ -\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} + \dfrac{x}{w}\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} = 0 \\ y\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} - \cancel{x}\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} = 0 \end{cases}$$

Two independent eqs. only. (neglect 3rd eq.)

Let $(u,v) = (x/w, y/w)$ as the image point

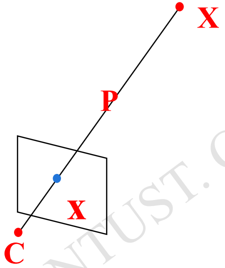
$$\rightarrow \begin{cases} (v\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases}$$

4x3

$V = y/w$

3Eq ⇒ 2DOF

$$\begin{bmatrix} \quad \\ \quad \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = 0$$



The unknown **X** must statisfy the above eqs. for this image (says **C**)

3D estimation Direct Triangulation method-co

- Of course, in the second image (says C'), the point X will have the same property as:

$$\begin{cases} (v'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases}$$

⇒ right

(1 point 3D)

2 Eq

- Finally, from one correspondence (u, v) and (u', v'), we will have

3 Eq

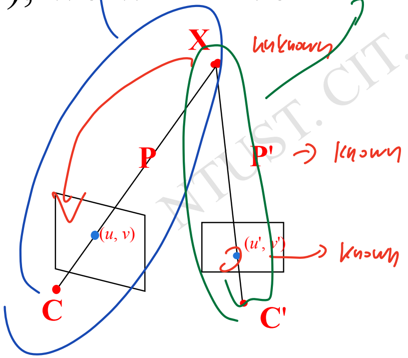
L

$$\begin{cases} (v\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases}$$

R

$$\begin{cases} (v'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases} \rightarrow \boxed{\begin{bmatrix} u\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_1{}^{\mathrm{T}} \\ v\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_2{}^{\mathrm{T}} \\ u'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_1{}^{\mathrm{T}} \\ v'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_2{}^{\mathrm{T}} \end{bmatrix} \mathbf{X}_{4\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

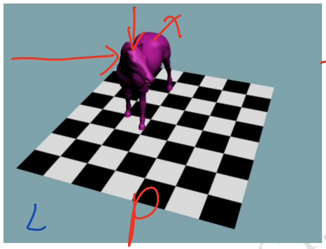
Solve it by SVD

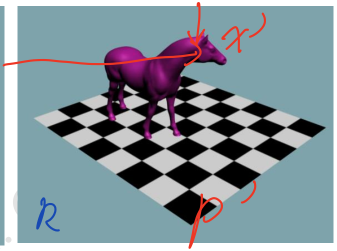


remeber: $\mathbf{p}_1{}^{\mathrm{T}} = [p_{11} \quad p_{12} \quad p_{13} \quad p_{14}]$ ...

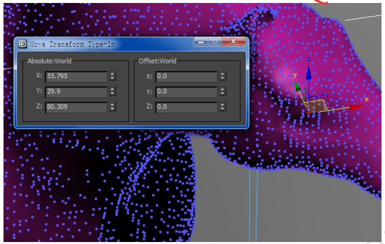
3D estimation Direct Triangulation method

- Example1 for Condition-1
  - **P**, **P'**, **x**$_i$, **x'**$_i$: known
  - **X** : unknown (to be solved)





3DMax



P =

$$\begin{matrix} 2.0179 & 1.5967 & -0.5695 & 113.8802 \\ 0.2820 & -0.7636 & -2.4258 & 305.7125 \\ -0.0009 & 0.0023 & -0.0018 & 1.0000 \end{matrix}$$

Pp=

$$\begin{matrix} 2.8143 & -1.3450 & -0.5673 & 347.4957 \\ -0.4439 & -0.4444 & -3.0134 & 371.1864 \\ 0.0023 & 0.0023 & -0.0018 & 1.0000 \end{matrix}$$

Ground truth :
$[55.795, 29.9, 80.309, 1]^T$

To determine the 3D coordinate of the horse's right eye, we pick up this feature on these two images, then we have
$x=[259, 120, 1]^T$
$x'=[395, 89, 1]^T$

→**X**?

3D estimation Direct Triangulation method

- Example1 for Condition-1—cont.

Final project

P, P', $\mathbf{x}_i$, $\mathbf{x}'_i$: known

X : unknown (to be solved)

$$\begin{bmatrix} u\mathbf{p}_3^{\mathrm{T}} - \mathbf{p}_1^{\mathrm{T}} \\ v\mathbf{p}_3^{\mathrm{T}} - \mathbf{p}_2^{\mathrm{T}} \\ u'\mathbf{p}'^{\mathrm{T}}_3 - \mathbf{p}'^{\mathrm{T}}_1 \\ v'\mathbf{p}'^{\mathrm{T}}_3 - \mathbf{p}'^{\mathrm{T}}_2 \end{bmatrix} \mathbf{X}_{4\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4x4

SVD

$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$
p1 =	p2 =	p3 =
2.0179	0.2820	-0.0009
1.5967	-0.7636	0.0023
-0.5695	-2.4258	-0.0018
113.8802	305.7125	1.0000

$[u \quad v]$

u = 259
v = 120

$\mathbf{p}'_1$	$\mathbf{p}'_2$	$\mathbf{p}'_3$
pp1 =	pp2 =	pp3 =
2.8143	-0.4439	0.0023
-1.3450	-0.4444	0.0023
-0.5673	-3.0134	-0.0018
347.4957	371.1864	1.0000

$[u' \quad v']$

up = 395
vp = 89

In Matlab (note the notation ' in Matlab means transpose operation)

```
A=[u*p3'-p1';
v*p3'-p2';
up*pp3'-pp1';
vp*pp3'-pp2'];
[U,S,V]=svd(A)
…
V=
```

0.0038	0.7662	-0.3417	-0.5441
0.0030	-0.5582	-0.7735	-0.3002
0.0088	-0.3183	0.5337	-0.7834
-0.9999	-0.0015	0.0010	-0.0099

normalize

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{31} & p_{32} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^{\mathrm{T}} \\ \mathbf{p}_2^{\mathrm{T}} \\ \mathbf{p}_3^{\mathrm{T}} \end{bmatrix}_{3\times4}$$

1x4 vector
1x4 vector
1x4 vector

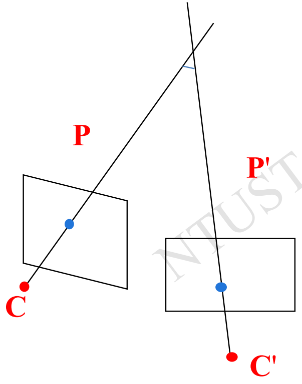
X =
55.1137
30.4065
79.3529
1.0000

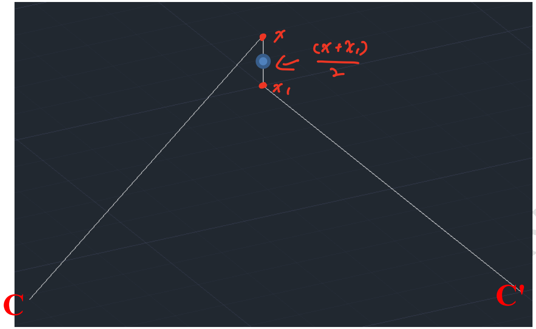
Ground truth :
$[55.795, 29.9, 80.309, 1]^{\mathrm{T}}$

3D estimation Direct Triangulation method

- Example1 for Condition-1—cont.

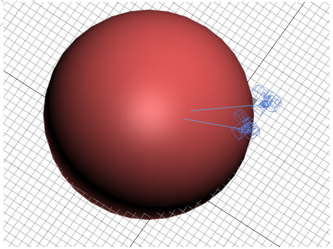
Note!
These two spatial lines are not necessary to intersect. The solution is the point whose residual error is smallest.





3D estimation Direct Triangulation method

- Example2 for Condtion-1



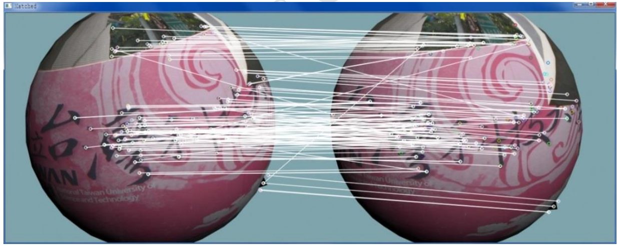
P=

$$\begin{matrix} 2.5873 & -0.3302 & -0.9174 & 246.8139 \\ -0.5844 & -1.8389 & -1.8654 & 401.7092 \\ 0.0004 & 0.0014 & -0.0029 & 1.0000 \end{matrix}$$

Pp =

$$\begin{matrix} 2.5737 & 0.2628 & -0.9172 & 207.1313 \\ -0.1437 & -1.9363 & -1.8245 & 382.3570 \\ 0.0001 & 0.0014 & -0.0029 & 1.0000 \end{matrix}$$

160 point



The question is:

P, P' : known

$\mathbf{x}_i$, $\mathbf{x}'_i$: Generated by Feature Matching algorithm (including outlier)

X?

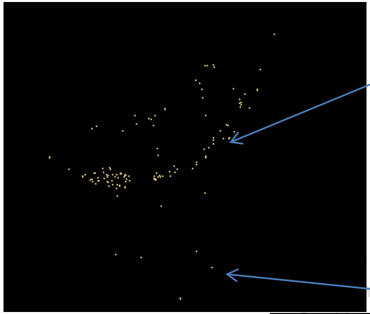
3D estimation Direct Triangulation method

- Example2 for Condtion-1—cont.

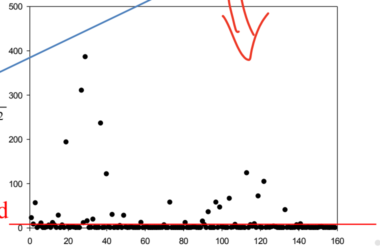
$$\begin{bmatrix} u\mathbf{p}_3^{\mathrm{T}}-\mathbf{p}_1^{\mathrm{T}} \\ v\mathbf{p}_3^{\mathrm{T}}-\mathbf{p}_2^{\mathrm{T}} \\ u'\mathbf{p}'^{\mathrm{T}}_3-\mathbf{p}'^{\mathrm{T}}_1 \\ v'\mathbf{p}'^{\mathrm{T}}_3-\mathbf{p}'^{\mathrm{T}}_2 \end{bmatrix} \mathbf{X}_{4\times 1} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$$

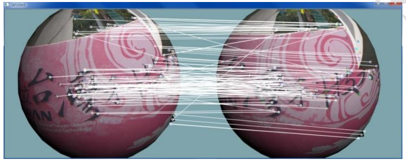
3D

0
0
0
1



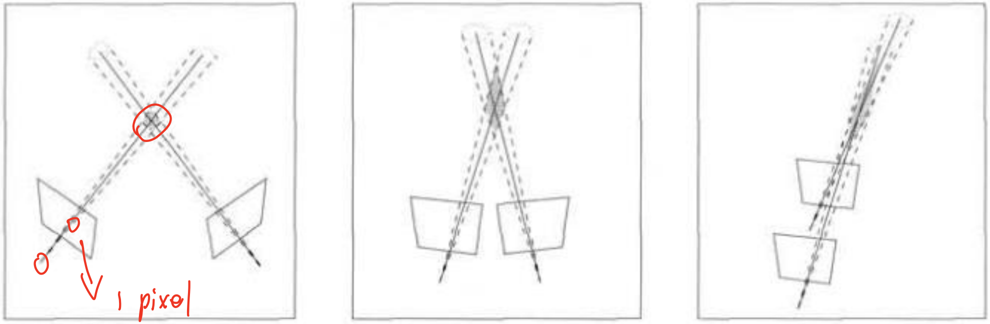
$$\sqrt{\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2+\varepsilon_4^2}$$

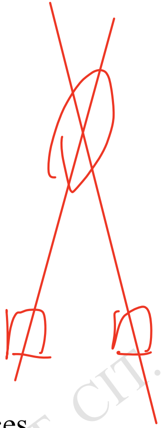




In practice, the outliers are usually removed by Fundamental Matrix constraints.

3D estimation Direct Triangulation method





More Difficult to find correspondences (feature matching)

Smaller uncertainty (Better 3D reconstruction)

Easier to find correspondences (feature matching)

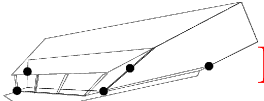
Larger uncertainty (Poor 3D reconstruction)

3D reconstruction for projective geometry

- Direction reconstruction

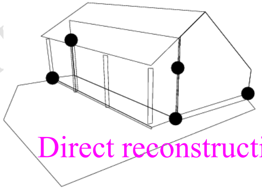
deformed

Two views geometry (**F**)



A guess for **P**, **P'**
[Projective reconstruction]

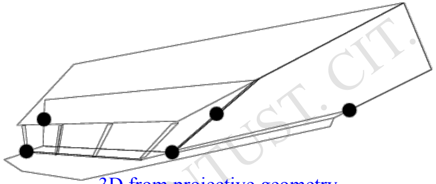
$\mathbf{H}_{4x4}$



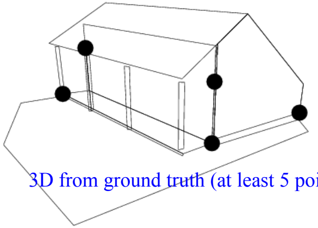
Direct reconstruction

5 moa

3D



3D from projective geometry
(at least 5 points)



3D from ground truth (at least 5 points)

$$\begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ X_4' \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

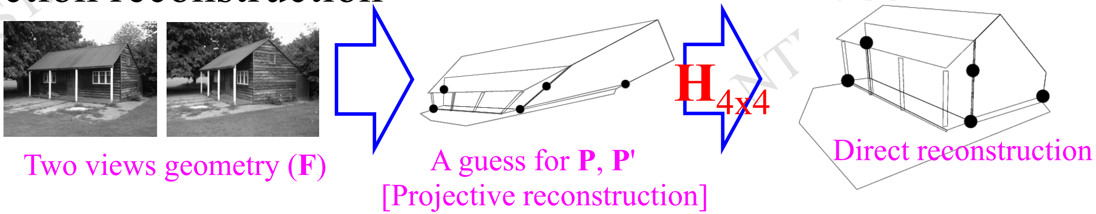
$$\boxed{\begin{aligned} \tilde{\mathrm{H}}_1^{\mathrm{T}} &= [H_{11} \quad H_{12} \quad H_{13} \quad H_{14}] \\ \tilde{\mathrm{H}}_2^{\mathrm{T}} &= [H_{21} \quad H_{22} \quad H_{23} \quad H_{24}] \\ \tilde{\mathrm{H}}_3^{\mathrm{T}} &= [H_{31} \quad H_{32} \quad H_{33} \quad H_{34}] \\ \tilde{\mathrm{H}}_4^{\mathrm{T}} &= [H_{41} \quad H_{42} \quad H_{43} \quad H_{44}] \\ \mathbf{X}^{\mathrm{T}} &= [X \quad Y \quad Z \quad 1] \end{aligned}}$$

$$\begin{bmatrix} \mathbf{X}^{\mathrm{T}} & 0^{\mathrm{T}} & 0^{\mathrm{T}} & -X'\mathbf{X}^{\mathrm{T}} \\ 0^{\mathrm{T}} & \mathbf{X}^{\mathrm{T}} & 0^{\mathrm{T}} & -Y'\mathbf{X}^{\mathrm{T}} \\ 0^{\mathrm{T}} & 0^{\mathrm{T}} & \mathbf{X}^{\mathrm{T}} & -Z'\mathbf{X}^{\mathrm{T}} \end{bmatrix}_{3\times16} \begin{bmatrix} \tilde{\mathrm{H}}_1 \\ \tilde{\mathrm{H}}_2 \\ \tilde{\mathrm{H}}_3 \\ \tilde{\mathrm{H}}_4 \end{bmatrix}_{16\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

Select at least 5 correspondences→computer **H**
(please review chapter "Projective 3D geometry" for detail)

3D reconstruction for projective geometry

- Direction reconstruction



Once having **H**:

$\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$

$\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$

$\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$

Either

Apply homography **H** to all old 3D points ($\mathbf{X}_{1i}$), then gel all new 3D points ($\mathbf{X}_{2i}$)

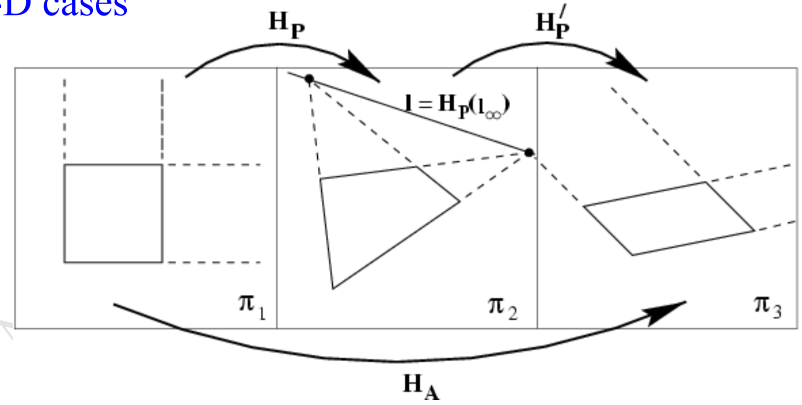
Or

Compute new **P** & **P'**, and use triangulation (back-projection) from all 2D correspondences, then generate new 3D points ($\mathbf{X}_{2i}$)

3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

2-D cases



3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

$$(\mathbf{P}, \mathbf{P}', \{\mathbf{X}_i\})$$

$$\boldsymbol{\pi}_\infty = (A, B, C, D)^\mathrm{T} \rightarrow (0,0,0,1)^\mathrm{T}$$

$\mathbf{H}^{-\mathrm{T}}\boldsymbol{\pi}_\infty = (0,0,0,1)^\mathrm{T}$ →to find a $\boldsymbol{\pi}_\infty$ under this constraint, but how?

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid 0 \\ \boldsymbol{\pi}_\infty^\mathrm{T} \end{bmatrix}$$

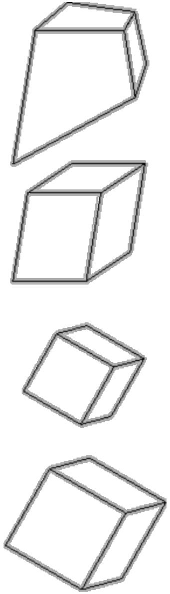
(if determinent ≠0)

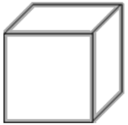
A desired solution if $\boldsymbol{\pi}_\infty$ can be determined, the mapped 3D points will be an affine-mapping

3D reconstruction for projective geometry

- Hierarchy of transformations

Projective 15dof	$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$	Intersection and tangency
Affine 12dof	$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$	Parallellism of planes, Volume ratios, centroids, **The plane at infinity π_∞**
Similarity 7dof	$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$	**The absolute conic Ω_∞**
Euclidean 6dof	$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$	Volume





3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

How to determine $\boldsymbol{\pi}_\infty$?

Here are various cases:

$$\mathbf{H}^{-T}\boldsymbol{\pi}_\infty = (0,0,0,1)^T$$

Translation motion

Parallel lines

Distance ratios on a lines (angle)

The infinite homography

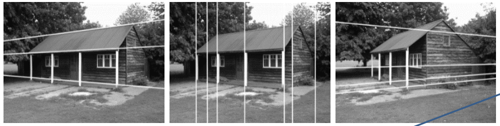
One of the cameras is affine

3D plane at infinity

3D plane: the scene in the projective geometry
(need to be determined, ex. use 3 points to find out)

3D reconstruction for projective geometry

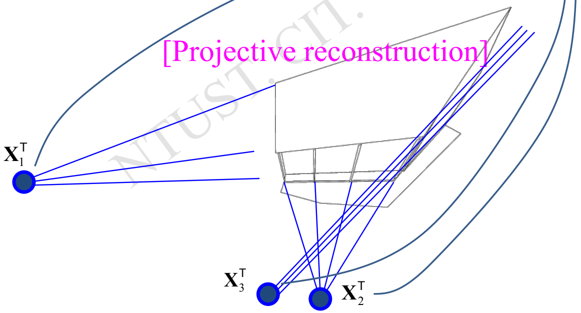
- Affine reconstruction
  - Projective transformation, then affine trans..



$\mathbf{X}_1^{\mathrm{T}}$ $\mathbf{X}_2^{\mathrm{T}}$ $\mathbf{X}_3^{\mathrm{T}}$

Recall slide: "Projective 3D geometry“
Determine a 3D plane

$$\begin{bmatrix} \mathbf{X}_1^{\mathrm{T}} \\ \mathbf{X}_2^{\mathrm{T}} \\ \mathbf{X}_3^{\mathrm{T}} \end{bmatrix} \pi = 0 \qquad \begin{bmatrix} (X_1)_1 & (X_1)_2 & (X_1)_3 & (X_1)_4 \\ (X_2)_1 & (X_2)_2 & (X_2)_3 & (X_2)_4 \\ (X_3)_1 & (X_3)_2 & (X_3)_3 & (X_3)_4 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = 0$$

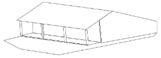


Find $\boldsymbol{\pi}_\infty$

Then, $\mathbf{H} = \begin{bmatrix} \mathbf{I} \,|\, 0 \\ \boldsymbol{\pi}_\infty^{\mathrm{T}} \end{bmatrix}$

Again, calculate final **X**
(here is affine reconstruction)

$\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$
$\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$
$\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$

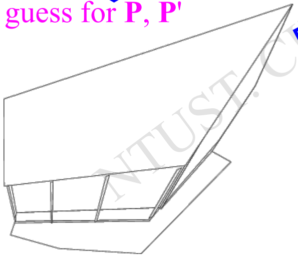


3D reconstruction for projective geometry

- Metric reconstruction

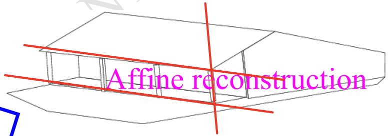
Two views geometry (**F**)

A guess for **P**, **P'**



[Projective reconstruction]

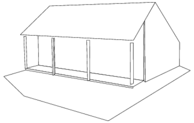
$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \,|\, 0 \\ \boldsymbol{\pi}_\infty^{\mathrm{T}} \end{bmatrix}$$



Affine reconstruction

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

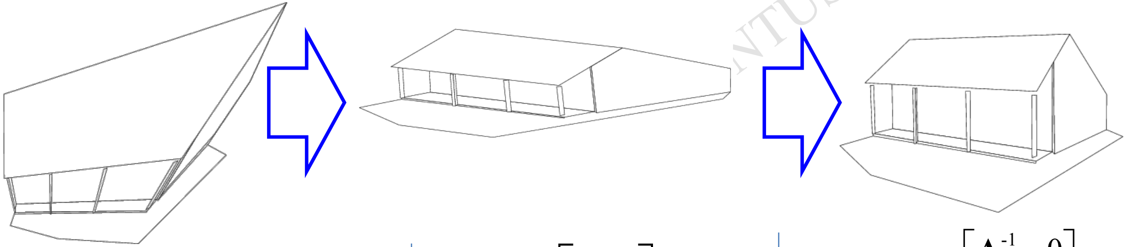
$$\mathbf{A}\mathbf{A}^{\mathrm{T}} = (\mathbf{M}^{\mathrm{T}}\boldsymbol{\omega}\mathbf{M})^{-1}$$



Metric reconstruction

3D reconstruction for projective geometry

- Metric reconstruction—cont.



Either

$\mathbf{P}_1 = [\mathbf{I} \mid 0] \quad \mathbf{P}_1' = [[\mathbf{e}']_\times \mathbf{F} + \mathbf{e}'\mathbf{v}^T \mid \lambda \mathbf{e}']$

Or

$\mathbf{P}_1 = [\mathbf{I} \mid 0] \quad \mathbf{P}_1' = [[\mathbf{e}']_\times \mathbf{F} \mid \mathbf{e}']$

Triangulation for all 3D points ($\mathbf{X}_{1i}$), ex

$$\begin{bmatrix} u\mathbf{p}_3^T - \mathbf{p}_1^T \\ v\mathbf{p}_3^T - \mathbf{p}_2^T \\ u'\mathbf{p}'^T_3 - \mathbf{p}'^T_1 \\ v'\mathbf{p}'^T_3 - \mathbf{p}'^T_2 \end{bmatrix} \mathbf{X}_{4\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid 0 \\ \boldsymbol{\pi}_\infty^T \end{bmatrix}$$

$\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$

$\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$

$\mathbf{X}_{2i} = \mathbf{H}\mathbf{X}_{1i}$

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A}\mathbf{A}^T = (\mathbf{M}^T \boldsymbol{\omega} \mathbf{M})^{-1}$$

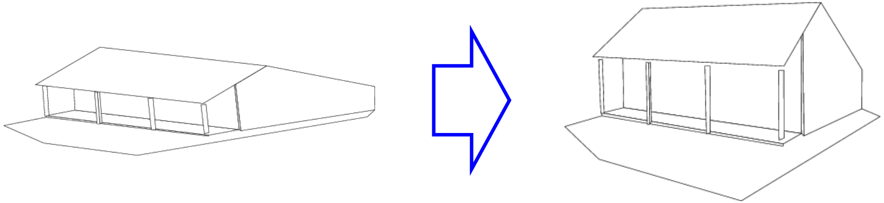
$\mathbf{P}_3 = \mathbf{P}_2 \mathbf{H}^{-1}$

$\mathbf{P}_3' = \mathbf{P}_2' \mathbf{H}^{-1}$

$\mathbf{X}_{3i} = \mathbf{H}\mathbf{X}_{2i}$

3D reconstruction for projective geometry

- Metric reconstruction—cont.



$$\mathbf{H}=\begin{bmatrix}\mathbf{A}^{-1} & 0\\ 0 & 1\end{bmatrix}$$

$$\mathbf{A}\mathbf{A}^{T}=(\mathbf{M}^{T}\boldsymbol{\omega}\mathbf{M})^{-1}$$

Cholesky factorization

$$\boxed{\begin{aligned}&\mathbf{P}_3=\mathbf{P}_2\mathbf{H}^{-1}\\ &\mathbf{P}_3'=\mathbf{P}_2'\mathbf{H}^{-1}\\ &\mathbf{X}_{3i}=\mathbf{H}\mathbf{X}_{2i}\end{aligned}}$$

Here

$$\boldsymbol{\omega}=(\mathbf{K}\mathbf{K}^{T})^{-1}$$

→you may assume intrinsic parameter **K** is known, or under some constraints.

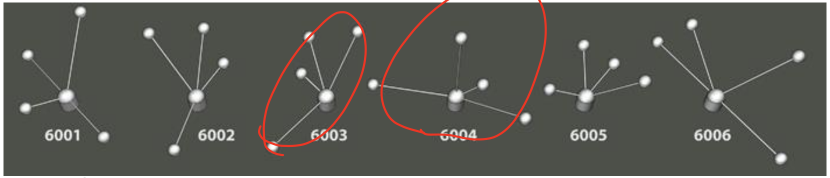
$$\mathbf{P}_2=[\mathbf{M}\,|\,\mathbf{m}]$$

The projection matrix of affine reconstruction

3D reconstruction–Applications

- Motion Tracking (other tracking issue)
- How to? 3D→2D (handling features)

5



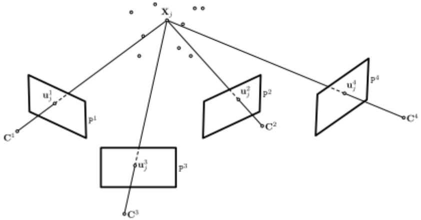
[R|T]

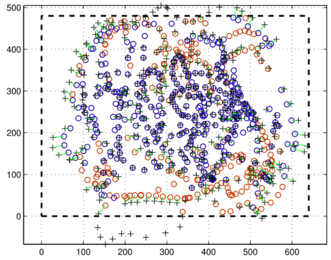
K

[R|T]

3D reconstruction–Applications

- Multi-camera calibration





T. Svoboda, D. Martinec, and T. Pajdla, “A convenient multicamera self-calibration for virtual environments,” *Teleoperators Virtual Environments*, vol. 14, no. 4, pp. 407-422, 2005.

3D reconstruction–Applications

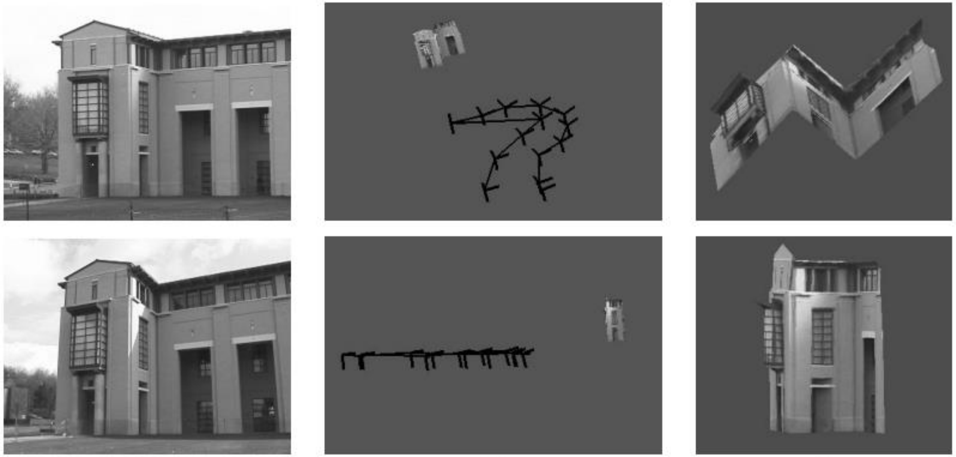
- Other issues
  - 1. Bundle adjustment?
  - 2. Structure From motion?
  - 3. Factorization?

- Select paper:

M. Han and T. Kanade, “Creating 3D models with uncalibrated cameras,” in IEEE Workshop on Applications of Computer Vision, 2000, pp. 178-185.

3D reconstruction–Applications

- Factorization

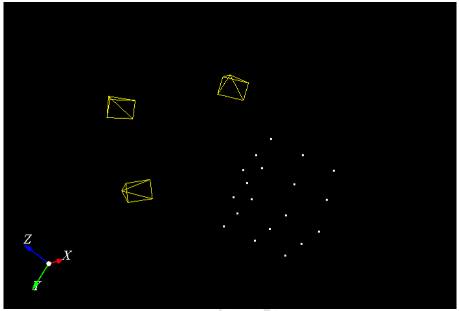


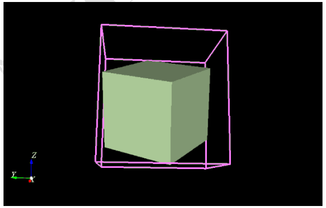
(a) (b) (c)

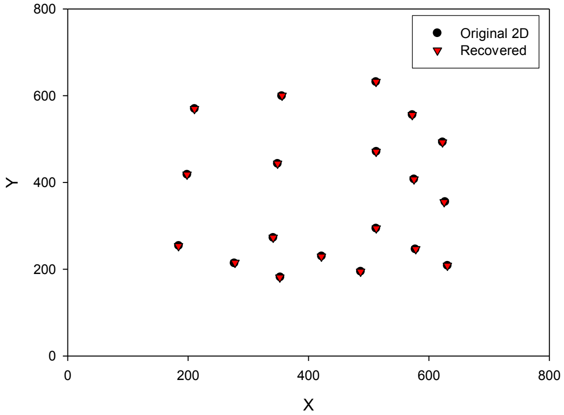
M. Han and T. Kanade, "Creating 3D models with uncalibrated cameras," in *IEEE Workshop on Applications of Computer Vision*, 2000, pp. 178-185.

3D reconstruction–Applications

- Factorization—Implementation result

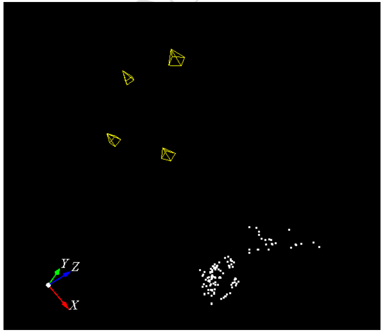






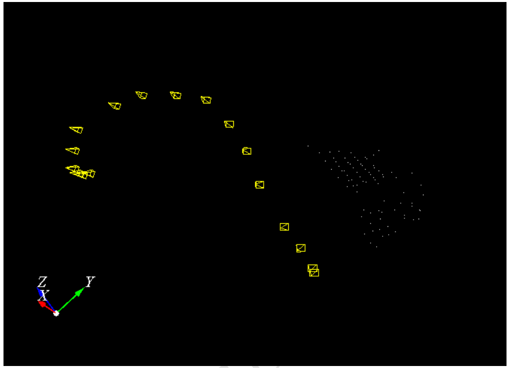
3D reconstruction–Applications

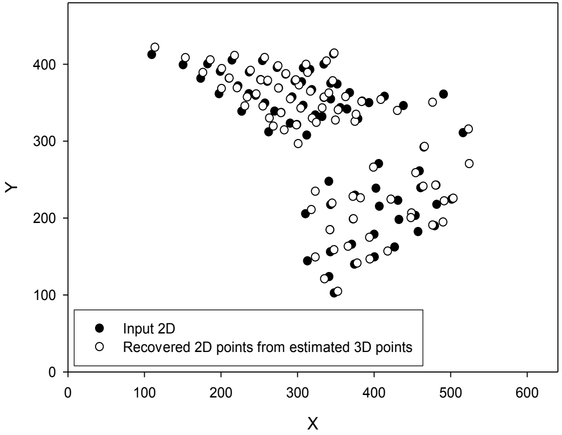
- Factorization—Implementation result—cont.



3D reconstruction–Applications

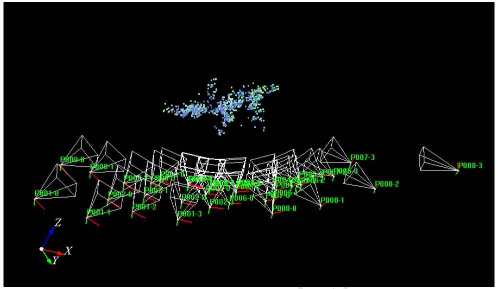
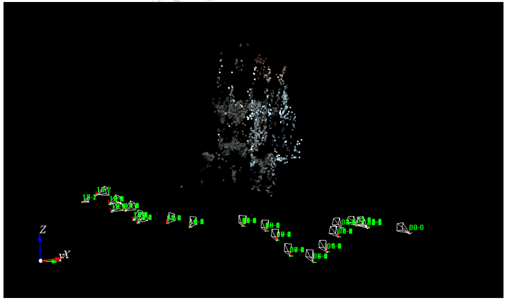
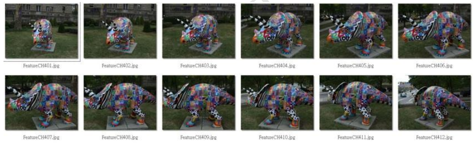
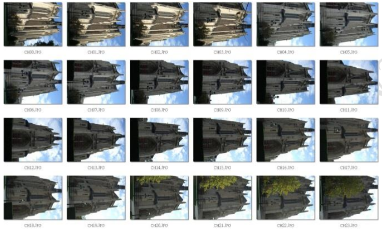
- Factorization—Implementation result—cont.





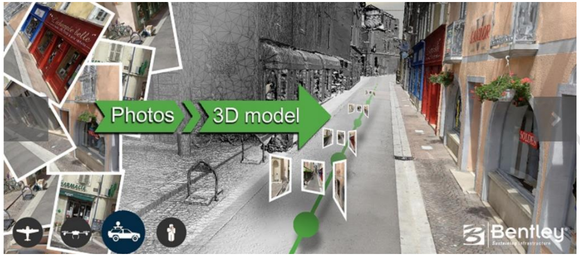
3D reconstruction–Applications

- Factorization—Implementation result—cont.

Commercial / Noncommercial Tools

- Agisoft PhotoScan
- RealityCapture
- PhotoModeler
- Autodesk Remake
- Strata Foto 3D CX2



http://www.agisoft.com/
https://www.capturingreality.com/
http://www.photomodeler.com/index.html
https://remake.autodesk.com/about
https://www.strata.com/foto-3d-cx-create-textured-3d-models-from-your-digital-camera/

Commercial / Noncommercial Tools

- 3DF Zephyr Pro
- PIX4D
- DroneDeploy
- senseFly

…

http://www.3dflow.net/3df-zephyr-pro-3d-models-from-photos/
https://pix4d.com/
https://www.dronedeploy.com/
https://www.sensefly.com/drones/ebee.html

Structure from motion

- CMPMVS
- Image-based 3D reconstruction (freeware)

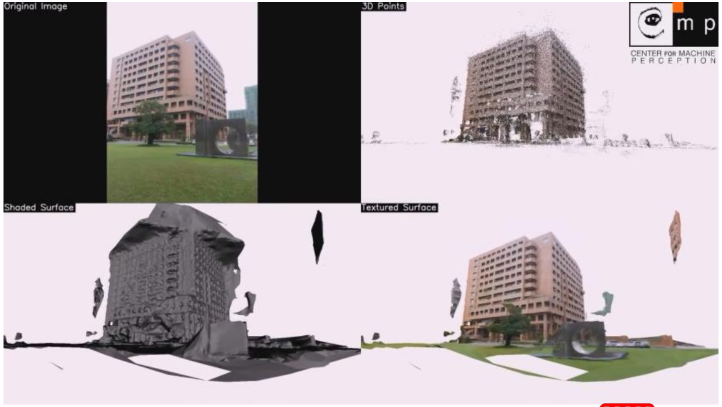

CMPMVS - Multi-View Reconstruction Software

Authors: Michal Jancosek & Tomas Pajdla
Software written by: Michal Jancosek
Latest version: 0.6.0
Release date: September 28, 2012
Reference to cite:
[1] *M. Jancosek, T. Pajdla. Multi-View Reconstruction Preserving Weakly-Supported Surfaces*, CVPR 2011 - IEEE Conference on Computer Vision and Pattern Recognition 2011 (pdf).

Introduction
CMPMVS is a multi-view reconstruction software. The input to our software is a set of perspective images and camera parameters (internal and external camera calibrations). The output is a textured mesh of the rigid scene visible in the images. Non-rigid objects are implicitly ignored.

For discussion on the software please visit our Google group at

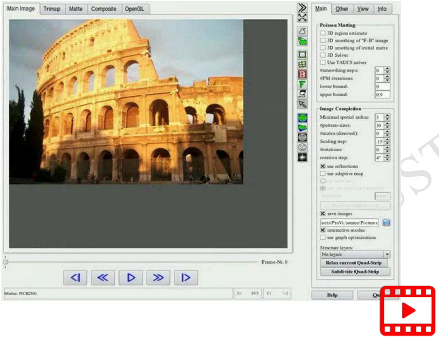
http://groups.google.com/group/cmpmvs

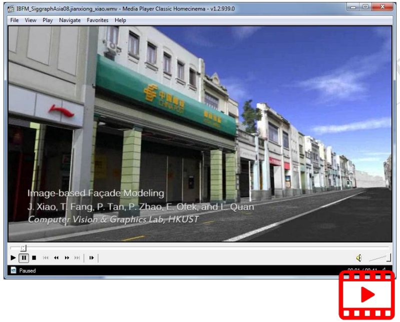


http://ptak.felk.cvut.cz/sfmservice/websfm.pl?menu=cmpmvs

Structure from motion (software pre-product)

- Applications
- Video trace: image synthesis
- Authoring tools to 3D (static object)
- Reconstruct urban 3D:Deal with frontal textures



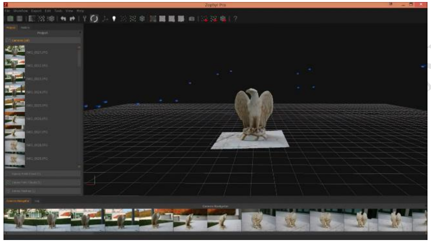


2006 - Pavić, Schönefeld, Kobbelt - Interactive image completion with perspective correction
Siggraph 2007 Video trace
Image based façade modeling, SIGGRAPH ASIA 2008

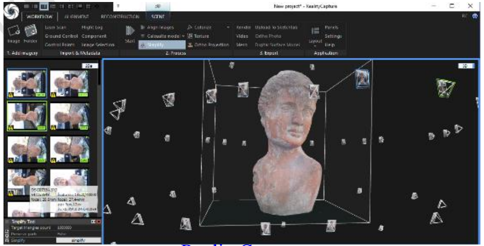
Multi-view in 3D reconstruction (photometric)

- 3D software or services

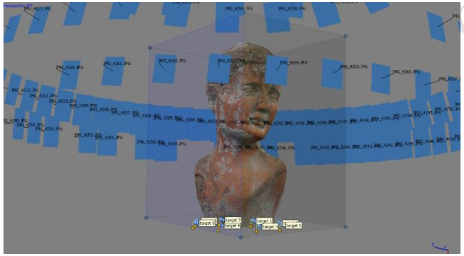
Autodesk Remake



3DF Zephyr Pro

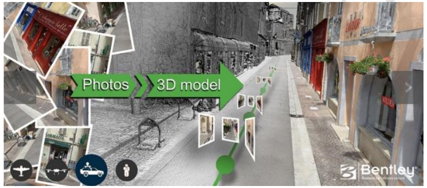


RealityCapture



Agisoft PhotoScan

Photo Modeler



http://www.acute3d.com/

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