

$$\textcircled{1} e^{i \frac{\varphi}{2} \hat{n} \cdot \vec{\sigma}} = 1 + i \frac{\varphi}{2} \hat{n} \cdot \vec{\sigma} - \frac{1}{2!} \left(\frac{\varphi}{2}\right)^2 (\hat{n} \cdot \vec{\sigma})^2 - i \frac{1}{3!} \left(\frac{\varphi}{2}\right)^3 (\hat{n} \cdot \vec{\sigma})^3 + \dots$$

$$\cdot \mathbb{1} \int \dots \approx \dots (\hat{n} \cdot \vec{\sigma})^2 \quad \text{...}$$

$$(\sigma_x + \sigma_y + \sigma_z)^2 = (\sigma_x + \sigma_y + \sigma_z)(\sigma_x + \sigma_y + \sigma_z) = (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) + 2(\sigma_x \sigma_y + \sigma_y \sigma_x + \sigma_x \sigma_z + \sigma_z \sigma_x + \sigma_y \sigma_z + \sigma_z \sigma_y)$$

$$\sigma_i \sigma_j + \sigma_j \sigma_i = 2 \delta_{ij} \mathbb{1} \rightarrow \text{Wikipedia}$$

$$\Rightarrow (\hat{n} \cdot \vec{\sigma})^2 = \sum_{i=1}^3 (\hat{n}_i \sigma_i)^2 = \sum_{i=1}^3 \hat{n}_i^2 \sigma_i^2 = \sum_{i=1}^3 \hat{n}_i^2 = 1$$

$$\Rightarrow \cos\left(\frac{\varphi}{2}\right) + i \sin\left(\frac{\varphi}{2}\right) (\hat{n} \cdot \vec{\sigma})$$

8/11

$$\textcircled{2} \frac{1}{2} \left(\mathbb{1} + i \frac{\sqrt{3}}{2} \sigma_z + i \frac{\sqrt{3}}{2} \sigma_x + i \frac{\sqrt{3}}{2} \sigma_y \right)$$

$$\cos \frac{1}{2} \Rightarrow \frac{\varphi}{2} = \frac{\pi}{3} \quad \underline{\underline{\varphi = \frac{2\pi}{3}}}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\frac{1}{2} \mathbb{1} + i \frac{\sqrt{3}}{2} \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \cdot \vec{\sigma}$$

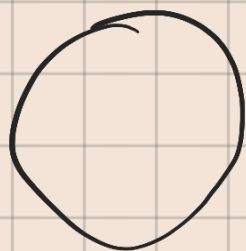
$$\textcircled{3} CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|000\rangle |001\rangle |010\rangle |011\rangle |100\rangle |101\rangle |110\rangle |111\rangle \text{ 8 bits } \approx 3$$

$$8 \times 8 \text{ matrix}$$

$$a) CNOT_{01} \otimes \mathbb{1}$$

$$\begin{array}{cccc}
 |000\rangle & |001\rangle & |010\rangle & |011\rangle \\
 = & \begin{array}{cccc}
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
 \end{array}
 \end{array}$$



$$\begin{array}{cccc}
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0
 \end{array}$$

b) $\underline{1} \times CNOT_{12}$

$$\begin{array}{cccc}
 |000\rangle & |001\rangle & |010\rangle & |011\rangle \\
 = & \begin{array}{cccc}
 \langle 000| & 1 & 0 & 0 \\
 \langle 001| & 0 & 1 & 0 \\
 \langle 010| & 0 & 0 & 1 \\
 \langle 011| & 0 & 0 & 1 \\
 \langle 100| & & & \\
 \langle 101| & & & \\
 \langle 110| & & & \\
 \langle 111| & & &
 \end{array}
 \end{array}$$



$$\begin{array}{cccc}
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & 1 & 0
 \end{array}$$

c) איינע אדער אים ארדע נאך אן
אפ'העט, און נאך אדער אן

$1000 \rangle \quad 1001 \rangle \quad 1010 \rangle \quad 1011 \rangle \quad 1100 \rangle \quad 1101 \rangle \quad 1110 \rangle \quad 1111 \rangle$

$= \langle 0000$

$\langle 0010$

$\langle 0101$

$\langle 0111$

$\langle 1001$

$\langle 1011$

$\langle 1101$

$\langle 1111$

