1 א) קיומם של מ"ג רצויים ש- $\langle n'|n\rangle=\delta_{nn'}$

ולכן מדובר על סכום אורתו' $\sum_n |n\rangle\langle n| = \mathbb{1}$

והאופרטורים מטלים בה את בה דהבמצב.

לכן שקולים רק אם $\frac{\mathbb{1}}{2\pi}\int_0^{2\pi} d\phi = \mathbb{1}$

ב) $P(\varphi) = \frac{1}{2\pi}\langle\varphi|\rho|\varphi\rangle$

$$\rho = \sum \rho_n |n\rangle\langle n|$$

$$\langle\varphi|\rho|\varphi\rangle = \sum_{nn'n''} e^{-in\varphi}\langle n|\rho_n|n'\rangle\langle n'|e^{in''\varphi}|n''\rangle$$

$$= \sum_n \rho_n = 1$$

$$\Rightarrow \quad \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, P(\varphi) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi = 1$$

3 ד) $\frac{1}{2\pi}\langle\varphi|\rho|\varphi\rangle = \frac{1}{2\pi}\frac{1}{2}\sum_n \langle n|(|0\rangle\langle 0| + |1\rangle\langle 1|)|n\rangle$

$= \frac{1}{4\pi}[1+1] = \frac{1}{2\pi} \rightarrow$ התפלגות אחידה

$$\sum_n \langle n|0\rangle\langle 0|n\rangle = 1$$

א) $\frac{1}{2\pi}\langle\varphi|\rho|\varphi\rangle = \frac{1}{2\pi}\sum_{nn'} e^{-in\varphi}\langle n|\psi\rangle\langle\psi|e^{in'\varphi}|n'\rangle$

$$= \frac{1}{4\pi}\sum_{nn'} e^{-in\varphi}\langle n|(|0\rangle + e^{i\theta}|1\rangle)(\langle 0| + \langle 1|e^{-i\theta})e^{in'\varphi}|n'\rangle$$

$$(e^{-in\varphi}\delta_{n0} + e^{i\theta}\delta_{n1}e^{-in\varphi})(e^{in'\varphi}\delta_{0n'} + \delta_{n'1}e^{-i\theta}e^{in'\varphi})$$

$$\Big(\underbrace{\delta_{n0}\delta_{0n'}e^{-in\varphi}e^{in'\varphi}}_{1} + \cancel{\delta_{n0}\delta_{n1}}\underbrace{e^{i\theta}\cancel{e^{-in\varphi}}\cancel{e^{in'\varphi}}}_{e^{i(\theta-\varphi)}}$$

$$\cancel{\delta_{n0}\delta_{n'1}}\cancel{e^{-in\varphi}}\,e^{-i\theta}e^{in'\varphi} = e^{-i(\theta-\varphi)} \qquad \delta_{n1}\delta_{n'1}e^{-in\varphi}e^{in'\varphi}\Big)$$

$$\Rightarrow \frac{1}{4\pi}\left(2 + e^{+i(\theta-\varphi)} + e^{-i(\theta-\varphi)}\right) = \frac{1}{2\pi}\left(1 + \cos(\theta-\varphi)\right)$$

$$= \frac{1}{2\pi}\left(1 + 2\cos^2\frac{\theta-\varphi}{2} - 1\right)$$

$$= \frac{1}{\pi}\cos^2\left(\frac{\theta-\varphi}{2}\right)$$

(ג 3

$$\langle n^2\rangle = \mathrm{Tr}(n^2 \rho_{Th})$$

$$= \sum_n \langle n|\hat n^2 \rho_{Th}|n\rangle = \sum_{nn'} \langle n|\hat n^2 \frac{1}{1+\bar n}\left(\frac{\bar n}{1+\bar n}\right)^{n'} |n'\rangle\langle n'|n\rangle$$

$$= \frac{1}{1+\bar n} \sum_{nn'} \left(\frac{\bar n}{1+\bar n}\right)^{n'} \langle n|\hat n \hat n|n'\rangle\langle n'|n\rangle$$

$$= \frac{1}{1+\bar n} \sum_n \left(\frac{\bar n}{1+\bar n}\right)^{n} n^2 = \underline{\underline{\bar n(2\bar n+1)}} \qquad q \equiv \frac{\bar n}{1+\bar n} < 1$$

$$\sum_{n=0}^{\infty} q^n n^2 = -\frac{q(q+1)}{(q-1)^3} = \bar n(\bar n+1)(2\bar n+1)$$

$$\langle n^2\rangle - \langle n\rangle^2 = \bar n(2\bar n+1) - \bar n^2 = 2\bar n^2 + \bar n - \bar n^2 = \boxed{\bar n^2 + \bar n}$$

$$\sum_{n=0}^{\infty} q^n n^2 = \sum$$

$$q\partial_q q^n = n q^n$$

$$q^2\partial_q^2 q^n = q^2 \partial_q n q^{n-1} = \overset{(n^2-n)}{n(n-1)} q^n = q^n n^2 - n q^n$$

$$q^2 \partial_q^2 q^n + \underset{q\partial_q q^n}{n q^n} = q^n n^2$$

$$\Rightarrow \sum_{n=0}^{\infty} q^n n^2 = q^2 \partial_q^2 \sum q^n + q\,\partial_q \sum q^n$$

$$= q^2 \partial_q^2 \frac{1}{1-q} + q\partial_q \frac{1}{1-q} \qquad \frac{1}{(1-q)^2} \to \frac{-2(q-1)}{(q-1)^3}$$

להוכיח
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$$\sum_{n=0}^{\infty} q^n n^i = \sum_i q^i \partial_q^i \left[\frac{1}{1-q}\right] = \sum_{n=0}^{i} \frac{(-1)^n n!\, q^n}{(q-1)^{n+1}}$$

$$\Rightarrow \frac{-2q^2}{(q-1)^3} + \frac{\overset{q^2-q}{q(q-1)}}{(q-1)^3} = \frac{-q^2-q}{(q-1)^3} = \underline{\underline{\frac{-q(q+1)}{(q-1)^3}}}$$

ב) כה נובע מ"ב כי כרגיל $\langle\varphi|\rho_{Th}|\varphi\rangle = \frac{1}{Z}\sum e^{-\beta E_n} = 1$

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$$|\psi(0)\rangle = |\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

$$\Rightarrow |\psi(t)\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}e^{-i\omega t(n+\frac{1}{2})}|n\rangle$$

$$\to e^{-i\omega t n}e^{-i\omega t\frac{1}{2}}$$

$$= e^{-i\omega t/2}|\alpha e^{-i\omega t}\rangle$$