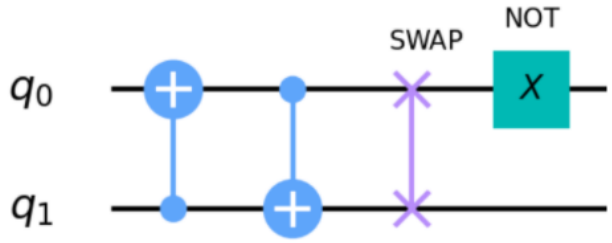
Quantum Computation 101 for Physicists

Home exercise 1

1. Translate the following reversible circuit into its equivalent permutation matrix:



2. Draw a reversible circuit that is equivalent to the following permutation matrix. You may use all the reversible gates we saw in class.

$$\begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array} \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

3. For this question we introduce the Toffoli gate, or the controlled-controlled not gate. The Toffoli gate gets as input two control bits and one target bit. The gate flips the target bit if and only if both control bits are equal to 1:

   (a) Show that the Toffoli gate alone is a universal set of gates, i.e., any boolean statement can be expressed using Toffoli gates and auxilary bits (with initial state of our choice) only. (Note: this will not be true when we move to quantum computation!)

   (b) Draw a reversible circuits that gets as input two two-bit numbers and two additional result bits initiated to 0. The circuit adds the two numbers modulo 4 and stores the result in the result bits, leaving the original numbers intact. You may use as many auxiliary bits (initiated to $|0\rangle$) as you need, CNOT and Toffoli gates.

4. Draw a reversible circuits that gets as input two two-bit numbers and two result bits initiated to $|0\rangle$. The circuit multiplies the two numbers modulo 4 and stores the result in the result bits, leaving the original numbers intact. You may use CNOT gates, NOT and Toffoli gates. It is useful to first understand the action of multiplication on binary numbers, and only then think of the circuit.