

1. Photon beam from a certain direction \mathbf{k} falls on a hydrogen atom in its ground state $1s$.
 - a) Calculate the absorption rate (probability of absorption per unit time) for a particular transition of the atom to the $2p$ excited state with $m = 0$. First assume that the photons are polarized and calculate the dependence of the rate on the polarization and the falling angles.
 - b) Then assume that the polarization was not specified. What is the angular dependence in this case? Help: not knowing the photon polarization means that there is equal probability of its different polarizations.
 - c) Indicate what will you have to change in the above treatment if instead of $2p$ excited state you'd need to find the absorption rate to the $3d$ state. Don't do the detailed calculations but indicate what will you need to calculate.
2. This problem draws heavily on Berry's original paper and lecture notes. Consider the two level Hamiltonian

$$H = \lambda(q\sigma_x + p\sigma_y) + \epsilon\sigma_z \quad ,$$

where $\sigma_{x,y,z}$ are Pauli matrices.

- By treating q and p as parameters calculate the adiabatic energies as functions of these two parameters. Plot them schematically for $\epsilon > 0$, < 0 and $= 0$.
- Consider the conditions of the above levels to cross. Does this happen on a surface, a line, a point in the parameter space of this model Hamiltonian? (For those curious) - how does this compare with what is discussed in the lecture notes about the levels crossings?
- Calculate the Berry connections (the "vector potential") $\bar{A}(p, q) \equiv (A_q, A_p)$ in the space of the two parameters p and q (Help: choose just one of the adiabatic states)

- (For extra credit - consult the lecture notes). Calculate the Berry curvature (the "magnetic field") $F_{pq}(p, q) = \partial_q A_p - \partial_p A_q$ where ∂_a is a short hand for $\partial/\partial\alpha$.

Note that the H has the form $\vec{d} \cdot \vec{\sigma}$ with the 3D vector $\vec{d} = (\lambda p, \lambda q, \epsilon)$. In this notation F_{pq} can be written as

$$F_{pq} = \frac{\vec{d} \cdot [\partial_p \vec{d} \times \partial_q \vec{d}]}{2|\vec{d}|^3},$$

- (not for credit - just for you) What is the "source" of this "magnetic field"?