QM-1 **Selected Problems for the Exam** **Feb 2016**

The exam will consist of 2-3 short(!) problems which will be "motivated by" and "based upon" the ones of the following list.

You are advised to start with going over the theoretical material relevant for each problem, then understand how problems of this type should be set up in order to solve them. Then think of possible questions which could be asked in relation to various aspects of the solution paying attention to subtle points. PLEASE - always start with simple questions/assumptions making your way to more complicated issues.

1. Consider how the objects of the following type transform under local gauge transformation which was discussed in the class

$$\psi^*(\mathbf{r}_2)\psi(\mathbf{r}_1) \quad \text{and} \quad \int_{\mathbf{r}_1}^{\mathbf{r}_2} d\mathbf{r}\cdot\mathbf{A}(\mathbf{r})$$

2. Consider the problem of an electron placed in a uniform magnetic field and a constant electric field.

3. What will happen if a potential $U(x, y)$ is added to the problem of an electron in a uniform magnetic field. Consider simple forms of U and ask relevant questions.

4. Consider spinless charged particle in a magnetic field of an infinitely long cylinder of radius R with a uniform magnetic field inside and zero field outside.

5. Consider similarities and differences of the quantization of standing and running waves. Give examples and ask relevant questions.

6. Discuss a connection between the Hilbert space of photon states and the real space. Issues of the type - what is the electric or magnetic field $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ of a single photon? Etc....

7. Consider emission and absorption of radiation by the following systems

   a) A charged pendulum i.e. a point like massive charged particle hanging on a weightless string in the uniform gravitational field.

b) Two charged particles of equal masses m are attached to the ends of a massless rod of length L which can freely rotate in a plane around its midpoint.

8. Discuss solutions of the following Hamiltonian

$$H = \int \psi^+(x) \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x) \right) \psi(x) \, dx + \int \psi^+(x)\psi(x)V(x-y)\mu(y) \, dxdy \ ,$$

where $\psi(x)$ and $\psi^+(x)$ are boson operators,

$$\mu(y) = \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \xi^+_\alpha(y) \sigma^x_{\alpha\beta} \xi_\beta(y) \ ,$$

$\xi_\alpha(y)$ and $\xi^+_\beta(y)$ are fermion operators, $\alpha, \ \beta = 1, 2$, $\sigma^x_{\alpha\beta}$ – is the Pauli matrix and $U(x) = m\Omega^2 x^2/2$, $V(x-y) = m\omega^2(x-y)^2/2$, $\Omega > \omega$.

9. Consider possibilities of finding eigenfunctions and eigenvalues of the operators $\psi(\mathbf{r})$ and $\psi^+(\mathbf{r})$.

10. Problems similar to p. 4 of homework set 5.

11. Consider differences and similarities between second quantization of the Schrodinger equation for the harmonic oscillator and its description in terms of a and a^+ operators, i.e. $a = p/(\sqrt{2m\hbar\omega}) - i(\sqrt{m\omega/2\hbar})x$, etc.

12. Consider Bogoliubov spectrum as a function of the range of the boson-boson interaction

13. Consider the general many body Hamiltonian

$$H = \sum_\alpha \varepsilon_\alpha a^+_\alpha a_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a^+_\alpha a^+_\beta a_\delta a_\gamma$$

The interaction is such that only $V_{\alpha\beta\alpha\beta}$ and $V_{\alpha\beta\beta\alpha}$ are not zero. Discuss its solutions.

14. Consider a doubly degenerate energy level of a quantum system. Discuss various possibilities of a system to be in this level both in a pure and in a mixed state.

15. Consider various possible density matrices of a spin 1/2 state and their physical consequences.

16. Discuss the meaning of coherence in the language of the density matrix. Give examples, provide understanding.

17. An asymmetric double well changes in time in such a way that its lower minimum goes up while its higher minimum goes down so that their relative heights interchange (will be drawn on a blackboard in class at the beginning of the exam). Describe the time development of the system.

18. Consider a Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2M} + \frac{k(x^2+y^2)}{2} + \alpha xy \quad , \quad |\alpha| < k$$

and assume that $M \gg m$. Discuss the solution using the Born-Oppenheimer approximation.

19. This question draws heavily on Berry's original paper and tutorial. Consider problem described by the Hamiltonian

$$\hat{h} = \lambda(q\sigma_x + p\sigma_y) + \varepsilon\sigma_z$$

Discuss all aspects of the solution of it as a function of the parameters q and p.

20. Consider the problem of a scattering off a spherical potential well of depth V_0 and radius a.