Introduction to Particles and Nuclear Physics - Home Exercise 7

Question 1

Express the ratio of the cross-sections for the reactions $K^-p \to \pi^-\Sigma^+$ and $K^-p \to \pi^+\Sigma^-$ in terms of the two possibles isospin amplitudes.
The isospin wave-functions of the relevant particles are:

$$
\begin{aligned}
|p\ \rangle =& |\frac{1}{2}, \frac{1}{2}\rangle \\
|K^{\pm}\rangle =& |\frac{1}{2}, \pm\frac{1}{2}\rangle \\
|\pi^{\pm}\rangle =& |1, \pm 1\rangle \\
|\Sigma^{\pm}\rangle =& |1, \pm 1\rangle
\end{aligned}
$$

Question 2

$\pi N \to \pi N$ scattering:
One of the simplest calculations is nucleon-pion scattering. The possible channels through which the interaction may occur are six *elastic* processes (same particles come out as the ones that went in):

$$
\begin{aligned}
&(El.a)\ \pi^+p \to \pi^+p \\
&(El.b)\ \pi^-p \to \pi^-p \\
&(El.c)\ \pi^0n \to \pi^0n \\
&(El.d)\ \pi^0p \to \pi^0p \\
&(El.e)\ \pi^+n \to \pi^+n \\
&(El.f)\ \pi^-n \to \pi^-n
\end{aligned}
$$

and four *charge-exchange* processes:

$$
\begin{aligned}
&(CE.a)\ \pi^+n \to \pi^0p \\
&(CE.b)\ \pi^0n \to \pi^-p \\
&(CE.c)\ \pi^0p \to \pi^+n \\
&(CE.d)\ \pi^-p \to \pi^0n
\end{aligned}
$$

that take place via an exchange of charged particles. The pion has $I = 1$ and the nucleon has $I = \frac{1}{2}$ so the total isospin of the composite system must be $\frac{3}{2}$ or $\frac{1}{2}$.

1. Use the Clebsch-Gordan coefficients to compute the isospin decompositions for $|\pi^+p\rangle$, $|\pi^0p\rangle$, $|\pi^+n\rangle$, $|\pi^0n\rangle$, $|\pi^-p\rangle$, $|\pi^-n\rangle$
2. The cross section is proportional to the square of the scattering amplitude, i.e. $\sigma \propto |\mathcal{M}|^2$. Note that there are only one or two amplitudes that contribute to each of the processes, corresponding to the isospin of the states. Denote these as $\mathcal{M}_{\frac{3}{2}}$ and $\mathcal{M}_{\frac{1}{2}}$ and write down the squared amplitudes of each of the processes.

3. Write down the ratio between the cross sections of the ten processes. Note that among the ten **total** amplitudes only four are independent. To clarify, you have to write this ratio $\sigma_{\mathrm{i}} : \sigma_{\mathrm{ii}} : \sigma_{\mathrm{iii}} : \sigma_{\mathrm{iv}}$ where the four subscript indexes correspond to the four independent amplitudes.
4. In certain experimental settings, a single isospin channel may dominate and a "resonance particle" may be produced. This means that a peak in the scattering cross section as a function of the center of mass (CM) energy will be observed. The peak will correspond exactly to a short-lived particle of certain mass (remember the Breit-Wigner derivation from week 04 ?). Concretely, the first resonance in πN scattering is the Δ baryon at $m = 1232$ MeV with $I = \frac{3}{2}$, as can be seen in figure 1. When the CM energy reaches 1232 MeV, the probability of the π and the N converting into a Δ is much larger than that of any other interaction. So we say that the isospin $\frac{3}{2}$ channel dominates and thus the scattering amplitude for $I = \frac{3}{2}$ is much larger than for $I = \frac{1}{2}$. In this limit, simplify the previous result to a simple numeric ratio.

Question 3

Consider the Scalar QED Lagrangian we've seen in class (without a scalar potential):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\phi(D^\mu\phi)^* - m^2\phi\phi^*, D_\mu = \partial_\mu - ieA_\mu, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

1. Show this Lagrangian under the gauge transformation: $\phi \to \phi' = e^{ie\theta(x)}\phi, A_\mu \to A'_\mu = A_\mu + \partial_\mu\theta(x)$
2. Write out the interaction terms explicitly and draw the Feynman rules (draw out the vertices and their value, as well as the values for the propagators).
3. Draw out the relevant diagrams for a $e^-e^- \to e^-e^-$ and evaluate the matrix elements for each of the channels you find. (Recall, the total matrix element is the sum of all the channels $\mathcal{M} = \sum_i \mathcal{M}_i$), write your result with Mandelstam variables.
4. Using the results you've got to calculate the cross section:

$$\frac{d\sigma(e^-e^- \to e^-e^-)}{d\Omega} = \frac{1}{(8\pi)^2 s}|\mathcal{M}|^2$$

5. Express your solution in the COM frame, and find the results both at the ultra-relativistic regime, $|\vec{p}| >> m$, and the non-relativistic regime, $m >> |\vec{p}|$.

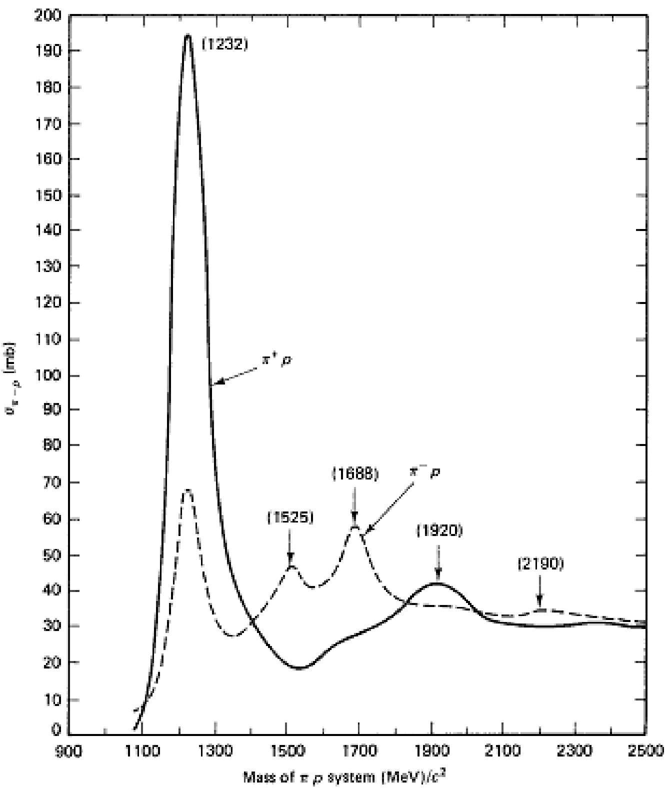

Figure 1: Total cross sections for π^+p and π^-p, shown with solid and dashed lines, respectively.