Quantum Information 101 for physicists

Home exercise 8

May 21, 2023

1. Prove rigorously that $H(X) \geq 0$, with equality only when the probability of one of the $p(x) = \begin{cases} 1 & x = x_0 \\ 0 & \text{otherwise} \end{cases}$ for some x_0.

2. In this question you will improve the prefix code we saw in class:

$$p_a = \frac{1}{2}, p_b = p_c = p_d = \frac{1}{6}.$$

In class we were able to get to average character length $1\frac{5}{6}$, which was not equal to the best case, $H(X)/\log 2$. We will now get closer to the optimal case.

   (a) Instead of $\{a, b, c, d\}$, define your alphabet to be two-letter strings, and find the new probability distribution for this alphabet.
   $p_{aa} = \frac{1}{4}, p_{ai} = p_{ia} = \frac{1}{12}, p_{jk} = \frac{1}{36}$

   (b) Build a new prefix code tree for the new 16-letter alphabet. Find the average character length and compare to the minimal length expected by the entropy and to the one we found in class.

   (c) Can you think of a way to make the code even better, by defining different basic alphabet?