Quantum Computing with Super-Conducting Qubits

A. Ner-Gaon and A. Ron
Raymond and Beverly Sackler School of Physics and Astronomy, Tel-Aviv University, Tel Aviv, 69978, Israel

Quantum computation promises to revolutionize various fields by solving complex problems that are difficult for classical computers. Originating from concepts proposed in the 1980s, quantum computing harnesses the principles of quantum mechanics for exponential speedups in certain computations. This paper focuses on superconducting qubits, a leading candidate for quantum computer implementation due to their macroscopic quantum coherence and compatibility with semiconductor technologies. Recent advancements have significantly improved their performance, making them a strong contender for scalable quantum computing. We provide a theoretical overview over the fundamentals of superconducting qubits and highlight key recent developments in the field.

INTRODUCTION

Rooted in the principles of quantum mechanics, the concept of a quantum computer was first proposed by Richard Feynman and others in the early 1980s [1]. They postulated that quantum systems could perform certain computations exponentially faster than classical systems. This idea was further refined by David Deutsch [2], who introduced the notion of a universal quantum computer, capable of simulating any physical system.

The motivation to build a quantum computer lies in its potential to solve complex problems that are currently intractable for classical computers. Problems in cryptography, optimization, materials science, and complex system simulations could be tackled more efficiently with quantum computing. For instance, Shor's algorithm for integer factorization [3] and Grover's algorithm for database search [4] offer exponential and quadratic speedups, respectively, compared to their best-known classical counterparts.

Among the various physical implementations of quantum bits (qubits), superconducting qubits have emerged as one of the most promising candidates. Superconducting qubits leverage the macroscopic quantum coherence of superconductors. In addition, their compatibility with existing semiconductor fabrication technologies also facilitates scalable production and integration into complex quantum circuits [5].

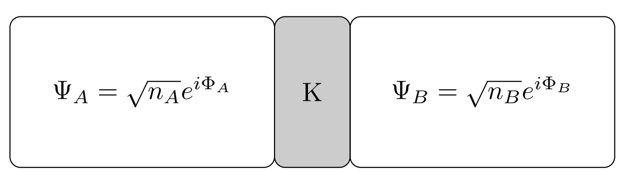
Recent advancements in superconducting qubits have significantly improved their coherence times, gate fidelities, and error rates [6]. This paper presents the theoretical background necessary to understand superconducting qubits and discusses recent breakthroughs that bring us closer to realizing practical quantum computers.

THEORETICAL BACKGROUND

Our first goal is to build a device that can hold quantum information. The most basic unit of information is the familiar classical bit which either holds the information 0 or 1 at any given time. On the other hand the most basic requirement of our fundamental quantum information unit (qubit) is that it can hold a superposition of the states 0 and 1.

$$|\psi_{\text{qubit}}\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{1}$$

To achieve this we can take the ground state and the first excited state of the quantum harmonic oscillator (QHO) to be our two states that our qubit can hold at a superposition. But there is a problem with this solution. We also require control over our qubit i.e. we need to be able to change its state from 0 to 1 and vice versa. If we recall the energy spectrum of the QHO, $E = \hbar\omega(n + \frac{1}{2})$, we'll see that in order to change the state of the qubit we'll need to give the qubit the energy quanta $\hbar\omega$. But the energy difference of the first transition $|0\rangle \leftrightarrow |1\rangle$ is the same for any other transition $|i\rangle \leftrightarrow |i+1\rangle$. This means that if our qubit was just the QHO nothing would stop it from going to any $|n\rangle$ from $|0\rangle$, in fact we would get a superposition of all $|n\rangle$ - this is not a qubit. We assume here that we do not want to work in single photon regime for its experimental drawbacks. We want our input to be classical and to get a quantum behaviour. To solve this issue we need to make our QHO have different energy difference for every transition, or in other words to introduce anharmonicity to our QHO. To do so we turn to the Josephson junction (and the Josephson effect).



We look at two super-conductors (SCs) A and B with a tunneling channel with a coupling constant K, this system is called the Josephson Junction (JJ). We assign each SC with its relative Ginzburg–Landau order parameter $\Psi_i = \sqrt{n_i}e^{i\Phi_i}$, were n_i is the density of the charge carrying cooper-pairs in each SC and Φ_i is their phase. The

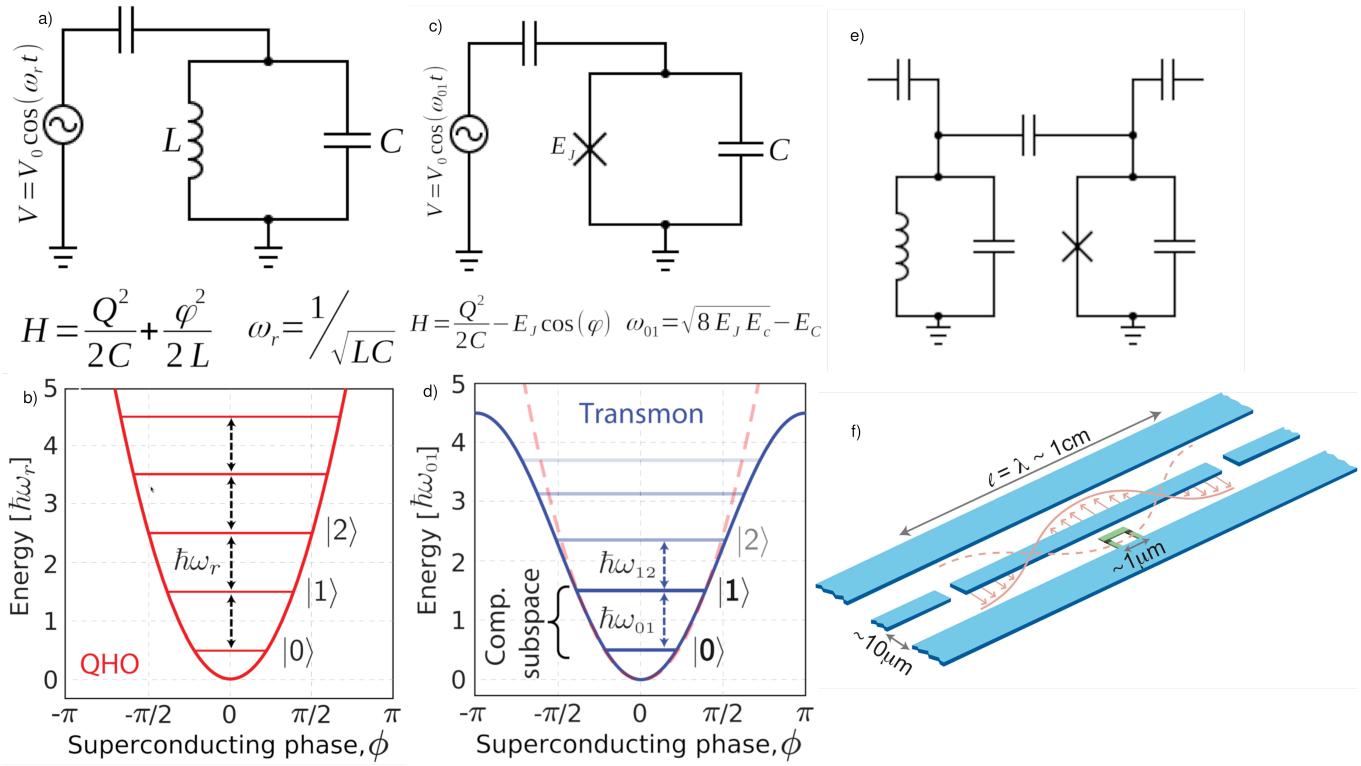


Figure 1: a) Circuit schematic and hemiltonain of the LC circuit, which is the CQED implementation of the QHO. b) Energy spectrum of the QHO where equal energy spacing is apparent. c) Circuit schematic and hemiltonain of the transmon, this is a CQED implementation of an atom (an artificial atom). d) Energy spectrum of the transmon (blue) overlaid by the energy spectrum of the QHO (dashed red). The difference between the two energy spectra (anharmonicity) gives rise to different energy transitions. This allows us to hone in on only the first two states. e) Circuit schematic of a transmon qubit coupled to an EM waveguide. The EM standing wave is represented by and harmonic oscillator of charge and phase and thus as an LC circuit. f) Diagram showing a transmon qubit in a waveguide. Conductors are in blue and white space is insulating, the transmon in green and the EM standing wave is drawn on the conducting island in the middle.

Ginzburg–Landau order parameter in general is not a wave-function but in this case it can be interpreted as such. By setting voltage V over the entire junction and setting the zero energy at the middle while considering that the charge of each cooper-pair is $2e$ - The coupled Schrödinger equations for said SCs are as follows:

$$i\hbar\partial_t\Psi_A = H\Psi_A = eV\Psi_A + K\Psi_B \tag{2}$$
$$i\hbar\partial_t\Psi_B = H\Psi_B = -eV\Psi_B + K\Psi_A \tag{3}$$

To solve these equations we take the time derivative of each Ψ_i explicitly, take the complex conjugate of both sides of each equation, and then add and subtract the complex conjugate for each equation at a time. By defining the Josephson phase $\varphi = \Phi_A - \Phi_B$ and by noting that $\dot{n}_i$ is proportional to the current through our SCs and that $n_A \approx n_B$ – we get the Josephson equations:

$$I(t) = I_c \sin(\varphi(t)) \tag{4}$$
$$\frac{\partial\varphi}{\partial t} = \frac{2e}{\hbar}V(t) \tag{5}$$

I_c is the critical current and it is dependent upon the physical characteristics of our junction, the temperature and an externally applied magnetic field. To get the energy of our junction we take the integral:

$$E = \int P\mathrm{d}t = \int IV\mathrm{d}t \propto \int \sin\varphi \mathrm{d}\varphi = -E_J\cos\varphi \tag{6}$$

Where E_J is the characteristic energy of the junction. Let us connect our JJ to a capacitor and write the total energy of the circuit.

$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\varphi} \tag{7}$$

Here we quantized the charge and phase of the circuit, these two variables are canonically conjugate variables and share the same commutation relation as x and p. It is common to relate to the charge as the number of cooper-pairs, $\hat{Q} = 2e\hat{n}$, and collect the constants to a single capacitive energy constant E_C.

$$\hat{H} = 4E_C\hat{n}^2 - E_J \cos\hat{\varphi} \tag{8}$$

We neglected the term allowing for charge fluctuation and for setting arbitrary charge, $(\hat{n} - \hat{n}_{\text{ground}})^2$, because from now on we will consider only the transmon regime $E_J \backslash E_C \gg 1$. To understand this let us explore our circuit. By setting a voltage over the capacitor and the junction we charge the capacitor until it is full and resist the flow of charge. But the junction constantly work to put charges on the other side of the capacitor via tunneling, in fact these two opposing forces at play cause the charge to oscillate as we will soon see more clearly. The transmon regime simply put means that the tunneling is stronger then the charging, this is why we can neglect any charge offset or fluctuations, because the junction will just keep putting those charges on the other side of the capacitor. This is the reason that the transmon qubit is the predominant SC qubit, it is less susceptible to charge fluctuations and thus has greater coherent times. Because the tunneling is in control the charge is changing rapidly and the uncertainty of the charge is large, hence the phase is well defined with small uncertainty/fluctuations. Therefore we can expand the cosine around small φ.

$$\hat{H}_{\text{Transmon}} \approx 4E_C\hat{n}^2 + \frac{1}{2}E_J\hat{\varphi}^2 - \frac{1}{4!}E_J\hat{\varphi}^4 \tag{9}$$

Now it is clear that the charge and phase oscillate but as a slightly anharmonic oscillator. We can define the corresponding creation and annihilation operators, $\hat{b}^\dagger$ and $\hat{b}$, and rewrite our hamiltonian while keeping only the terms with equal number of $\hat{b}^\dagger$ and $\hat{b}$. This is called the Rotating Wave Approximation (RWA) and it means that terms with unequal number of both latter operators rotate with much higher frequency than the ones that have an equal number and therefore average to zero over the time scale of the lowest frequency.

$$\hat{H}_{\text{T}} \approx (\sqrt{8E_J E_C} - E_C)\hat{b}^\dagger\hat{b} - \frac{E_C}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b} \tag{10}$$

$$\equiv \hbar\omega_q\hat{b}^\dagger\hat{b} - \frac{E_C}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b} \tag{11}$$

This is precisely what we wanted, a QHO with a small anharmonicity. Now we can tune our system such that it will be in resonance with only the first transition $E_{01} = \hbar\omega_q$. Our task now is to control and read the information of the qubit. In order to do so we capacitively couple our qubit to an EM waveguide (FIG.1f). The total hemiltonian of the system now reads:

$$\hat{H} = \left(\frac{\hat{Q}_r^2}{2C_r} + \frac{\hat{\Phi}_r^2}{2L_r}\right) + \left(4E_C\left(\hat{n} - \hat{n}_r\right)^2 - E_J\cos\hat{\varphi}\right) \tag{12}$$

Where the first term in parentheses is the quantized EM field that our qubit experiences via the coupling, and it translates to oscillations of charge and phase on the coupled capacitor. In general the EM field quantization is done by writing the sum over all possible frequencies $\sum_i \hbar\omega_i\hat{a}^\dagger\hat{a}$. Here we couple our transmon to a finite waveguide, therefore it experiences an EM standing wave of one frequency ω_r. In addition here we intentionally did not omit the charge fluctuation term on our qubit $\hat{n}_r$ because these are exactly the quantum fluctuations that our waveguide produces in our qubit and from this term we will get our coupling.

$$\hat{H} = \hbar\omega_r\hat{a}^\dagger\hat{a} + \left(\hbar\omega_q\hat{b}^\dagger\hat{b} - \frac{E_C}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}\right) - g\left(\hat{a}^\dagger - \hat{a}\right)\left(\hat{b}^\dagger - \hat{b}\right) \tag{13}$$

Now we can use the fact that we can be in resonance with only the first transition and throw the anharmonicity term of our transmon, as it does not affect the first transition. In other words, we approximate our transmon as a two level system (TLS): $\hat{b}^\dagger, \hat{b} \longrightarrow \hat{\sigma}_+, \hat{\sigma}_-$.

$$\hat{H} = \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q}{2}\hat{\sigma}_z - g\left(\hat{a}^\dagger - \hat{a}\right)\left(\hat{\sigma}_+ - \hat{\sigma}_-\right) \tag{14}$$

Finally we perform the RWA again to get the Jaynes–Cummings hemiltonian.

$$\hat{H}_{JC} = \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q}{2}\hat{\sigma}_z + g\left(\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+\right) \tag{15}$$

The Jaynes–Cummings hemiltonian is a general representation of a TLS coupled to a single mode of the EM field, it also applies to an atom inside an EM cavity, and it facilitates a general framework for quantum computation. We can see a glimpse of it in the hemiltonian itself where there are two coupling terms, $\hat{a}^\dagger\hat{\sigma}_-$ returns the qubit to the ground state while creating a photon and $\hat{a}\hat{\sigma}_+$ excites the qubit and destroys a photon. To control the qubit we use a resonant field with detuning $|\Delta| \equiv |\omega_q - \omega_r| \approx 0$. Then our qubit will perform (Rabi) oscillations between the ground and excited state, with the respective probabilities to find the qubit in each state:

$$P_g = \cos^2(\Omega t) \qquad P_e = \sin^2(\Omega t) \tag{16}$$

Therefore to perform a bit flip we need to drive our qubit for a duration of time $\Omega t_\pi = \pi$, this is called a π-pulse. In general we can get an arbitrarty superposition by driving the qubit for the appropriate time. To read the information off our qubit we work in the dispersive regime, commonly referred to as qubit readout mode, where $\Delta \gg g$. Then the reflected signal off the qubit will shift in frequency depending on the qubit state. If we send a signal with a very different frequency than that of the qubit we can treat the coupling term pertubetively, then the hemiltonian to first order will be:

$$\hat{H} \approx \hbar(\omega_r + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q}{2}\hat{\sigma}_z \tag{17}$$

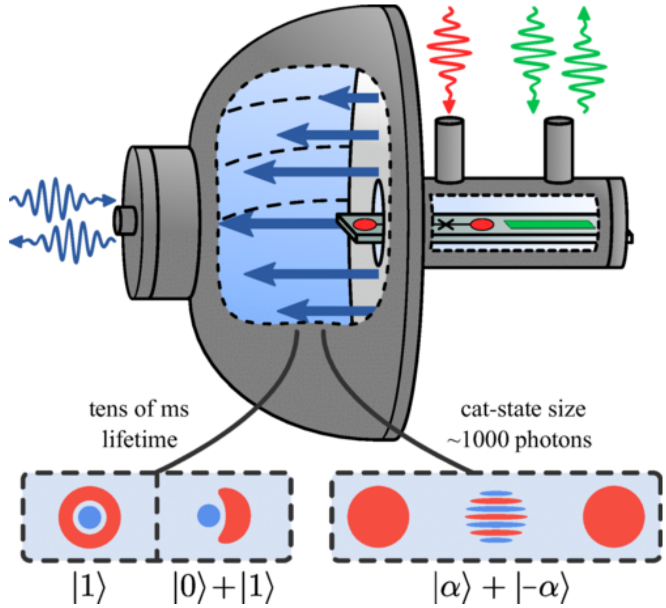
Where $\chi = g^2 \backslash \Delta$ is the shift in frequency. If the qubit is in the excited state the reflected signal will have a frequency $\omega_r + \chi$ and if it is in the ground state it will have $\omega_r - \chi$.

Just to summarise our results, we built from the ground up a physical system that can store quantum information that we can control and read. All that is left is to fabricate said device and test its capabilities.

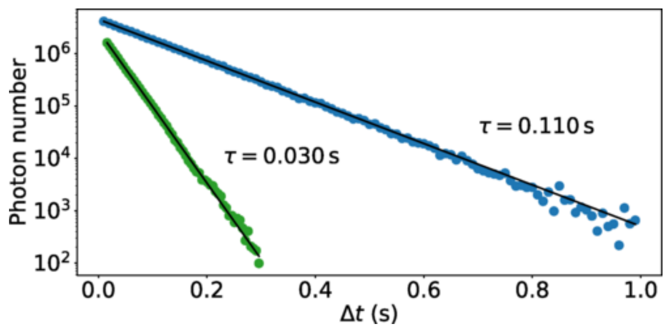
EXPERIMENTAL

Here we present a recent advancement in the field [7]. As a preface it is important to explain how the theoretical background relates to the experimental part of this work. Until now we described how to store, control and read quantum information off of a qubit using a coupled EM field. Theoretically we can reverse this idea on its head, store the quantum information in the EM field and control and read the information with a coupled transmon (artificial atom). This is called Bosonic Encoding and it holds few advantages over the traditional approach. Firstly the EM field lives inside a 3D cavity, which can hold many photons, this leads to long lived quantum states. Secondly, it allows the use of simpler and sometimes out right better error correcting codes [8, 9]. This does not retract the relevancy of the presented theoretical background as it is crucial for the understanding of how we interact with a TLS.

In this work [7] a superconducting Niobium cavity was manufactured and was chemically etched to reduce surface oxides which are thought to be a major contributor to photon loss in the cavity [10, 11]. Then it was cooled down to SC temperatures, 10mK, and a classical measurement of the electric field was conducted. In the first few cooldown the lifetime of the cavity was measured to be 110ms, but after a few cooldown and reheating cycles the lifetime



(a)



(b)

Figure 2: a) Illustration of the experimental system. The superconducting Niobium cavity is colored in grey and the predominant mode of the EM field inside is portrait in large blue arrows. A chip housing two transmons lies in a waveguide with an opening to the center of the cavity, protruding just enough for the transmon to couple to the EM field while keeping the field distortion by the chip to a minimum. The green pad is a resonator and is used to measure the cavity and the transmon. Classical field deployment and measurement (squiggly blue lines) are performed through the pin on the left. On the bottom left the Wigner distribution of a single-photon states is presented (will be discussed in this work). On the bottom right the Wigner distribution of a many-photon state is presented (will not be discussed in this work). b) In blue - classical measurements of photon number inside an empty cavity (without transmon chip and cooled to superconducting temperatures). The solid line is an exponential fit with a corresponding life time of 110ms. In green - same measurement but after cavity degradation, The solid line is an exponential fit with a corresponding life time of 30ms

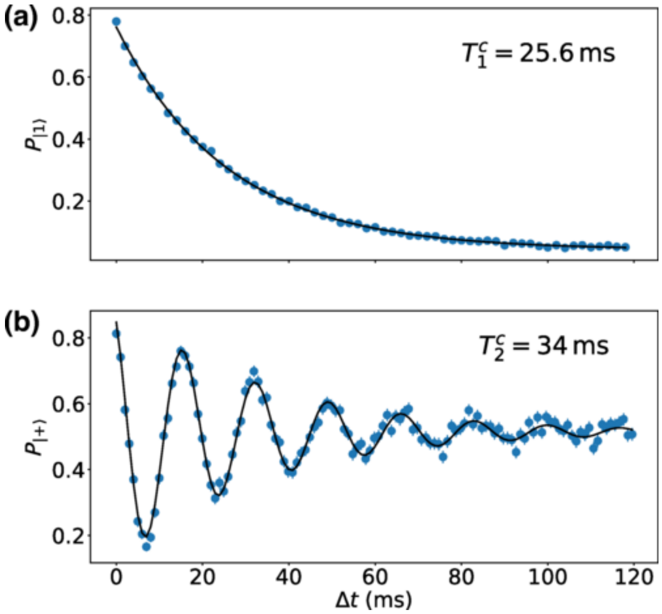


Figure 3: Lifetime measurements of the single photon states inside the cavity. The measurements were performed by preparing the cavity with the respective states and measuring the probability to get the state after Δt. a) Probability to find the photon in its prepared state $|1\rangle$ as a function of time. The solid line is an exponential fit with a characteristic lifetime of 25.6 ± 0.2 ms. b) Probability to find the photon in its prepared state $|+\rangle = (|0\rangle + |1\rangle)\backslash\sqrt{2}$ as a function of time. The solid line is an sinosodial exponential fit with a characteristic lifetime of 34 ± 1 ms.

degraded to 30ms (FIG. 2). This degradation persisted even after subsequent etching attempts. The rest of the measurements were performed in the latter state of the cavity. Next the transmon chip was inserted inside the cavity and the cavity was prepared with a single photon state. The experiment was conducted for both the Fock state $|1\rangle$ and for the superposition $|+\rangle = (|0\rangle + |1\rangle)\backslash\sqrt{2}$. The lifetimes of the single photon states were measured to be 25.6ms and 34ms respectively (FIG. 3).

CONCLUSION

In this work we presented the theoretical background necessary to get a grasp on a physical realisation of quantum computing and specifically with the use of SC qubits. In addition we presented a recent advancement in the field which showed record breaking time of qubit coherence, improvement of over an order of magnitude from previous attempts [7]. One major drawback of bosonic encoding in SC cavities is their scalability outlook. In one cavity one qubit worth of information can be stored. On the other hand if the information is stored in the transmons, a lot of them can be fabricated on one chip and inserted into one cavity. In other words, with bosonic encoding we have one qubit per unit of volume and with traditional transmon architecture we have orders of magnitude more per unit of volume. Therefore a quantum computer will probably never be built entirely out of SC cavities qubits. But these can be inserted in key positions in the computation pipeline to store important logical qubits for prolonged periods of time. In addition, as was previously mentioned before, bosonic encoding open the gate for strong error correcting codes, which is a crucial part of a universal quantum computer. Again, all the qubits in said quantum computer probably will not be of bosonic encoding, but these error correcting codes could be implemented on key logical qubits.

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