QM-1 Problems Themes for the Exam February 2025

The exam will consist of 1-3 short problems from the list of subjects in the following list. Please note that the idea of the list is to focus the preparations for the exams to relevant physics issues.

In accordance with this you are advised to follow the following steps.

Start with going over the theoretical material and homework problems relevant for each topic on the list. Then understand how problems of this type should be SET UP in order to solve them.

Then "train" yourself considering the homework problems as well as the problems given below as examples. Note that the problems below just provide a general direction, motivate what to "train", etc. Also note that the problems in the exams will be of a similar type but with many questions, details, extensions, etc.

PLEASE - always start with simple questions/assumptions making your way to more complicated issues.

1. Density matrix

   a. Basics of density matrices. Probabilities to measure physical observables, spectral representation...

   b. Pure vs mixed states. Coherence and its loss.

   Example:

   1) How quantum interference of two waves is modified if the system is in a mixed state?

   2) An atom in an excited state emits a photon. The excited state contains a multiplet of degenerate states (like electronic ψ_{nlmm_s} in a hydrogen atom) but it is not known (measured) which of it, only its energy. What is the most reasonable assumption using the density matrix understanding to such an undetermined state.

   c. Dynamics of density matrix. Simple (relaxation time) approximations.

   What is the physics of the different relaxation times T_1 and T_2? Which one is usually longer? Why?

   d. All of the above for examples of density matrices of a finite small size - spin 1/2, spin 1.

Example :

Consider a 3 level system - angular momentum 1 is a good example. Work in the basis of J_z. Given a density matrix

$$\rho = \frac{1}{4}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Is this a permissible density matrix? Give your reasoning. Does it describe a pure or mixed state? What is the probability to measure J_x? Assuming a magnetic field in the z direction write the dynamical equation for ρ in the relaxation time approximation and discuss what happens in the $t \to \infty$ limit.

2. Path Integrals.

   a) Time ordered exponentials.

   Example:

   Use the example of time ordered perturbation expansion of $U(T) = e^{-i(H_0+V)T}$ in powers of V to calculate derivative w.r.t. x of the exponential e^{A+xB} with A and B non commuting operators (matrices).

   b) Path integrals for quadratic Lagrangian.

   Example :

   Calculate the path integral for a propagator of a particle in 3 dim in a harmonic oscillator potential and a uniform electric field.

   c) Semiclassical approximation. Van Vleck determinant.

   Example : Semiclassical approximation for a propagator in $1/x^2$ potential (see an update of this chapter lecture notes planned to be posted in a few days).

3. External EM field

   a. Start with a uniform magnetic field **B**. Add a uniform electric field **E** to the uniform **B**. Assume perpendicular fields.

   b. Consider different gauges.

   c. Add external potential.

Example:

What will happen if a potential $U(x)$ is added to the problem of an electron in a uniform magnetic field? Discuss the currents flowing in this system.

d. Add AB flux.

Example:

Consider the energy levels of a (spinless) charged particle moving in the (x, y) plane in a magnetic field of an infinitely long cylinder of radius R placed parallel to the z axis with a uniform magnetic field inside and zero field outside.

e. In all the above discuss currents.

4. QM of fields

a. Standing vs running. Wave functionals.

Example:

Consider similarities and differences of the quantization of standing and running waves.

b. Linear vs non linear field equations.

Example:

Consider a complex classical field $\phi(x)$ in one space dim. which is described by the Hamiltonian

$$H = \int_{-\infty}^{\infty} dx \left[\frac{1}{2}|\pi(x)|^2 + \frac{1}{2}|\partial_x \phi(x)|^2 + U(\phi) \right]$$

with $\pi(x)$ - canonical momenta and various forms of the function $U(\phi)$. Discuss quantization of this field and its "particle" content.

c. Relation between the field wave functions/functional and real space measurements.

What does *classical* electric or magnetic fields $\mathbf{E}(\mathbf{r})$, $\mathbf{B}(\mathbf{r})$ mean in terms of photons and their wave functions?

d. Different (common) states of light.

Example:

What quantum states describe radiation emitted by common radiation sources?

5. Emission and absorption of photons

   a. Photon emission and/or absorption rates for transitions between levels of simple QM systems - atoms, oscillator, potential well, spherical rotor, ...

   b. Selection rules in the above systems and consequently what type of the radiation dominates - dipole, quadrupole, ...

   c. Angular dependence of the emission/absorption.

   d. The role of the polarization direction in all of the above.

   Example:

   A charged particle with spin 1/2 (electron) is in a tightly bound level of an atom. It is subjected to a uniform constant magnetic field $\mathbf{B}_0$. Discuss the interaction of such a particle with (quantized) EM field and discuss all the above issues.

6. Second quantization

   a. Calculations in the 2nd quantization of normalization of states, action of operators on various states, matrix elements of operators.

   Example:

   Calculate the norm of the following states for both bosons and fermions

$$a)\ \sum_{ij} C_{ij} a_i^+ a_j^+ |0> \quad , \quad b)\ a_i^+ a_j^+ a_k^+ |0> \ .$$

   b. Solving eigenvalue problems for various second quantized operators.

   c. Heisenberg equations for various 2nd quantized operators.

   d. Solving simple problems in second quantization.

   Examples:

   1) Describe differences and similarities between second quantization of the harmonic oscillator and its description in terms of a and a^+ operators, i.e. $a = p/(\sqrt{2m\hbar\omega}) - i(\sqrt{m\omega/2\hbar})x$, etc. Provide explanations and give examples.

2) Consider the following simplified model of spinless fermions placed in a potential well and interacting via a very long range interaction

$$H = \sum_{i,j=1}^{\infty} h_{ij} a_i^+ a_j + \frac{V_0}{2} (\hat{N} - N_0)^2 \ .$$

Here h_{ij} is a single particle hamiltonian of the well. It is a hermitian matrix with known eigenvalues ε_n and eigenfunctions $\psi_n(i)$. The operator $\hat{N}$ is $\hat{N} = \sum_{j=1}^{\infty} a_j^+ a_j$ and N_0 is a fixed integer.

7. Adiabatic theory

Using adiabatic approximation for (relatively) simple Hamiltonians slowly changing in time.

Example: An asymmetric double well changes in time in such a way that its lower minimum goes up while its higher minimum goes down so that their relative heights interchange. Assume that the change is very slow and consider what will happen to a particle which was originally at the bottom of the lowest well.