Quantum Computation 101 for Physicists

Home exercise 7

1. Show that for the 7-qubit code, the logical CNOT is implemented by a product of 7 CNOTs: $\overline{\mathrm{CNOT}}(x, y) = \mathrm{CNOT}(x_0, y_0) \otimes \mathrm{CNOT}(x_1, y_1) \otimes \cdots \otimes \mathrm{CNOT}(x_6, y_6)$.

   (a) Show that if the control qubit is in the state $M_0^r M_1^s M_2^t |0\rangle^{\otimes 7}$ where $r, s, t \in \{0, 1\}$, then the operator acting on the target qubit is $M_0^r M_1^s M_2^t$.

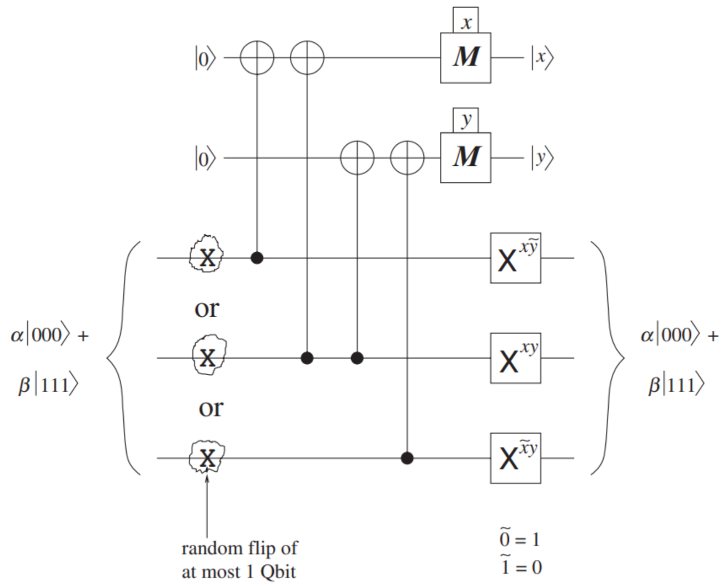
   (b) Show that $2^{-3/2}(1 + M_0)(1 + M_1)(1 + M_2)$ acts as the identity on $|\bar{0}\rangle$ and $|\bar{1}\rangle$, up to a normalization constant.

   (The operator here is defined up to a normalization constant since it is not a unitary, it is a projector operator. Therefore, we cannot apply it, but we can use it in our analysis.)

   (c) Use (a) and (b) to show that the definition above implements the logical CNOT gate. Hint: Do not expand all terms, but instead understand which operator is being applied to th target qubit when the control qubit is in each of the states $|\bar{0}\rangle, |\bar{1}\rangle$.

2. In class we saw that the 5-qubit code is a $[\![5, 1, 3]\!]$ code, i.e., its distance is 3. Give one example to a 3-qubit error that cannot be identified in the code, and one example for a 3 qubit error that can be detected.

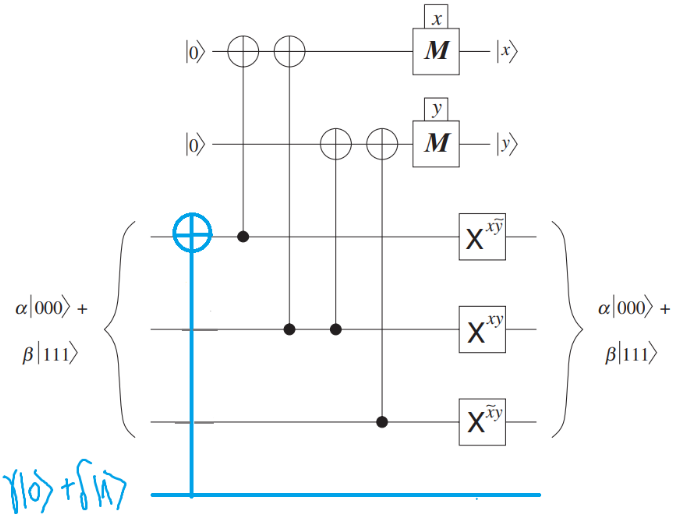
3. Think of the error correcting circuit for the 3-qubit code we saw in class, (Fig. 5.3 in Mermin's book):



   (a) Assume some continuous error occurred to one of the logical qubits in the form of an unwanted operator $U_e = e^{i\phi X} = \cos\phi\mathbb{I} + i\sin\phi X$. For example assume that is is the second logical qubit is the one on which the error is applied. Show explicitly that by applying controlled-fixes, the ancilla and physical qubits become disentangled, and the state of the physical qubits is fixed.

   By controlled-fixes, we mean some sort of controlled gates such that the syndrome qubits are the control qubits, and one of the physical qubits is the target qubit. Which gate should we apply in order to fix an error in the second qubit?

(b) Now add to the circuit an additional qubit e, which is the environment of the quantum computer. Assume that e is in th state $\gamma|0\rangle + \delta|1\rangle$, and that the error applied to the circuit is $U_e = CNOT_{e,q_0}$:



Show that the error correcting circuit (with measurements of the syndrome qubits, and not controlled-fixes) disentangles the qubit from the environment and allows us to fix the error.