מרצה: רון ליפשיץ   מתרגל: נועם רימוק

דף נוסחאות למבחן

משוואות מקסוול

$A^\mu = (\varphi, \mathbf{A})$; $A'^\mu = A^\mu + \partial^\mu \chi$;

$\mathbf{E} = -\boldsymbol{\nabla}\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$; $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$.

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix};$$

$$(F^*)^{\mu\nu} = \frac{1}{2}\mathcal{E}^{\mu\nu\rho\sigma}F_{\rho\sigma} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}.$$

$F^{\mu\nu}F_{\mu\nu} = 2(|\mathbf{B}|^2 - |\mathbf{E}|^2)$;

$(F^*)^{\mu\nu}F_{\mu\nu} = 2\partial_\mu\left(\epsilon^{\mu\nu\rho\sigma}A_\nu \partial_\rho A_\sigma\right) = -4\mathbf{E}\cdot\mathbf{B}$.

$\mathcal{L} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}A_\mu J^\mu$; $J^\mu = (c\rho, \mathbf{J})$;

$\frac{\partial \mathcal{L}}{\partial A_\mu} = -\frac{1}{c}J^\mu$; $\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\frac{1}{4\pi}F^{\mu\nu}$.

$\partial_\mu (F^*)^{\mu\nu} = 0 \iff \boldsymbol{\nabla}\cdot\mathbf{B} = 0$; $\boldsymbol{\nabla}\times\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$;

$\partial_\nu F^{\nu\mu} = \frac{4\pi}{c}J^\mu \iff \boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi\rho$; $\boldsymbol{\nabla}\times\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}$.

$\int_S \mathbf{B}\cdot \mathrm{d}\mathbf{a} = 0$; $\oint_C \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} = -\frac{1}{c}\frac{\partial \phi_B}{\partial t}$, $\phi_B = \int_S \mathbf{B}\cdot\mathrm{d}\mathbf{a}$;

$\int_S \mathbf{E}\cdot\mathrm{d}\mathbf{a} = 4\pi Q_{in}$; $\oint_C \mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell} = \frac{4\pi}{c}\int_S \left(\mathbf{J} + \frac{1}{4\pi}\frac{\partial \mathbf{E}}{\partial t}\right)\cdot \mathrm{d}\mathbf{a}$.

יחסות פרטית

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix};\ \beta = \frac{v}{c};\ \gamma = \frac{1}{\sqrt{1-\beta^2}};$$

$x^\mu = (ct, x, y, z)$; $d\tau = \frac{ds}{c} = \frac{1}{c}\sqrt{dx_\mu dx^\mu} = \frac{1}{\gamma}dt$.

$U^\mu = \frac{dx^\mu}{d\tau} = \gamma(c, \mathbf{v})$; $U^\mu U_\mu = c^2$; $a^\mu = \frac{dU^\mu}{d\tau}$; $a^\mu U_\mu = 0$.

$p^\mu = mU^\mu$; $E_k = \gamma mc^2$; $\mathbf{p} = \gamma m\mathbf{v}$; $E_k^2 = m^2c^4 + p^2c^2$.

$f^\mu = \frac{dp^\mu}{d\tau} = \gamma(\mathbf{f}\cdot\mathbf{v}, \mathbf{f})$; $\mathbf{f} = \frac{d\mathbf{p}}{dt}$; $\mathbf{f}\cdot\mathbf{v} = \frac{1}{c}\frac{dE_k}{dt}$.

$x'^\mu = \Lambda^\mu{}_\nu x^\nu$; $g_{\mu\nu} = \Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu g_{\rho\sigma} \iff g = \Lambda^T g \Lambda$.

boost בכיוון $\hat{z}$:

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

חלקיק בשדה א״מ

$m\frac{dU^\mu}{d\tau} = \frac{q}{c}F^{\mu\nu}U_\nu$; $\frac{dE_k}{dt} = q\mathbf{E}\cdot\mathbf{v}$; $\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}\mathbf{v}\times\mathbf{B}$;

$m\mathbf{a} = q\sqrt{1-\beta^2}\,[\mathbf{E} + \boldsymbol{\beta}\times\mathbf{B} - (\mathbf{E}\cdot\boldsymbol{\beta})\boldsymbol{\beta}]$.

מכניקה אנליטית של שדות

$S = \int L dt = \int \mathcal{L}(\phi_k, \partial_\nu\phi_k, x^\mu) d^3r dt$; $d\Omega = c dt d^3 r$;

$\partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi_k)} = \frac{\partial \mathcal{L}}{\partial \phi_k}$; $\pi_k = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi_k)}$; $\mathcal{H} = \sum_k \pi_k \frac{\partial \phi_k}{\partial t} - \mathcal{L}$.

$S = -\sum m_i c\, ds_i - \frac{1}{c^2}\int A_\mu J^\mu d\Omega - \frac{1}{16\pi c}\int F_{\mu\nu}F^{\mu\nu} d\Omega$.

משפט נתר וחוקי שימור

$\varphi_k(x^\alpha) \xrightarrow{s} \phi_k(x^\alpha, s)$; $\left.\frac{\partial \mathcal{L}}{\partial s}\right|_{s=0} = \partial_\alpha \Lambda^\alpha$;

$I^\alpha = \sum_k \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \varphi_k)} \left.\frac{\partial \phi_k}{\partial s}\right|_{s=0} - \Lambda^\alpha$; $\partial_\alpha I^\alpha = 0$.

$T^{\alpha\beta} = \frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\lambda)}\partial^\beta A_\lambda - g^{\alpha\beta}\mathcal{L}$; $\partial_\alpha T^{\alpha\beta} = 0$.

$\partial_\mu J^\mu = \frac{\partial \rho}{\partial t} + \nabla\cdot\mathbf{J} = 0$; $\frac{\partial u}{\partial t} + \frac{\partial \mathcal{E}_k}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{S} = 0$;

$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$; $\frac{\partial \mathcal{E}_k}{\partial t} = \mathbf{J}\cdot\mathbf{E}$; $\mathbf{S} = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{H})$;

$\int_S \mathbf{J}\cdot\mathrm{d}\mathbf{a} = -\frac{dQ}{dt}$; $\int_S \mathbf{S}\cdot\mathrm{d}\mathbf{a} = -\frac{d}{dt}[U + E_k]$.

עבור הלגראנג׳יאן $\mathcal{L} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$:

$T^{\mu\nu} = -\frac{1}{4\pi}\left(g^{\mu\rho}F_{\rho\sigma}\partial^\nu A^\sigma - \frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right) = \Theta^{\mu\nu} + \tau^{\mu\nu}$

$\Theta^{\mu\nu} = \frac{1}{4\pi}\left(F^\mu{}_\lambda F^{\lambda\nu} + \frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)$; $\tau^{\mu\nu} = \frac{1}{4\pi}F^{\lambda\mu}\partial_\lambda A^\nu$.

$\Theta^{\mu\nu} = \Theta^{\nu\mu}$; $\Theta^\mu{}_\mu = 0$;

$\Theta^{00} = u = \frac{1}{8\pi}(E^2 + B^2)$; $\Theta^{0i} = \frac{S_i}{c} = \frac{1}{4\pi}(\mathbf{E}\times\mathbf{B})_i$;

$\Theta^{ij} = -\frac{1}{4\pi}\left[E_iE_j + B_iB_j - \frac{1}{2}\delta_{ij}(E^2 + B^2)\right]$.

$M^{\mu\nu\rho} = \Theta^{\mu\nu}x^\rho - \Theta^{\mu\rho}x^\nu$; $M^{\mu\rho\nu} = -M^{\mu\nu\rho}$;

$M^{00i} = (u\mathbf{r} - t\mathbf{S})_i$; $M^{0ij} = \frac{1}{c}\left(S_i r_j - S_j r_i\right)$.

$\partial_\mu \Theta^{\mu\nu} + f^\nu = 0$; $\partial_\mu M^{\mu\nu\rho} + f^\nu x^\rho - f^\rho x^\nu = 0$;

$f^\mu = \frac{1}{c}F^{\mu\nu}J_\nu$.

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סטטיקה

$$\nabla^2\varphi = -4\pi\rho; \quad \nabla^2\mathbf{A} = -\frac{4\pi}{c}\mathbf{J}; \quad \nabla\cdot\mathbf{A} = 0;$$

$$\nabla^2 G = -4\pi\delta(\mathbf{r}-\mathbf{r}') \longrightarrow G(\mathbf{r},\mathbf{r}') = \frac{1}{|\mathbf{r}-\mathbf{r}'|}.$$

$$\varphi(\mathbf{r}) = \int G(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')d^3r' = \int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r';$$

$$\mathbf{A}(\mathbf{r}) = \int G(\mathbf{r},\mathbf{r}')\frac{\mathbf{J}(\mathbf{r}')}{c}d^3r' = \frac{1}{c}\int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r';$$

$$\mathbf{E}(\mathbf{r}) = \int \rho(\mathbf{r}')\frac{\hat{\mathbf{e}}_{\mathbf{r}-\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^2}d^3r'; \ \mathbf{B}(\mathbf{r}) = \frac{1}{c}\int \frac{\mathbf{J}(\mathbf{r}')\times\hat{\mathbf{e}}_{\mathbf{r}-\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^2}d^3r'.$$

$$\mathbf{A}(\mathbf{r}) = \frac{1}{c}\int \frac{I d\boldsymbol{\ell}'}{|\mathbf{r}-\mathbf{r}'|}; \quad \mathbf{B}(\mathbf{r}) = \frac{1}{c}\int \frac{I d\boldsymbol{\ell}'\times\hat{\mathbf{e}}_{\mathbf{r}-\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^2};$$

$$\mathbf{F}_{\text{mag}} = \frac{I}{c}\int d\boldsymbol{\ell}'\times\mathbf{B} = \frac{1}{c}\int \mathbf{J}(\mathbf{r})\times\mathbf{B}(\mathbf{r})\,d^3r;$$

$$\mathbf{F}_{1\to2} = -\frac{I_1 I_2}{c^2}\oint\oint \frac{d\boldsymbol{\ell}_1\cdot d\boldsymbol{\ell}_2\hat{\mathbf{e}}_{\mathbf{r}_2-\mathbf{r}_1}}{|\mathbf{r}_2-\mathbf{r}_1|^2} = -\mathbf{F}_{2\to1}.$$

$$\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int \frac{\mathbf{B}(\mathbf{r}')\times\hat{\mathbf{e}}_{\mathbf{r}-\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^2}d^3r'.$$

$$U = \frac{1}{8\pi}\int |E|^2 \mathrm{d}V = \frac{1}{2}\int \rho\varphi\,\mathrm{d}V = \frac{1}{2}\int\int \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r\,d^3r';$$

$$U = \frac{1}{2}\sum_{i\neq j}\frac{q_i q_j}{|\mathbf{r}_i-\mathbf{r}_j|}.$$

מולטיפולים חשמליים

$$\varphi(\mathbf{r}) = \frac{1}{r}\sum_{n=0}^{\infty}\int\left(\frac{r'}{r}\right)^n \rho(\mathbf{r}')\,P_n(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')\,d^3r'.$$

$$Q = \int \rho(\mathbf{r}')\mathrm{d}^3r'; \ \varphi_{\text{mono}}(\mathbf{r}) = \frac{Q}{r}; \ \mathbf{E}_{\text{mono}}(\mathbf{r}) = \frac{Q}{r^2}\hat{\mathbf{r}}.$$

$$\mathbf{p} = \int \rho(\mathbf{r}')\mathbf{r}'\mathrm{d}^3r'; \ \varphi_{\text{dip}}(\mathbf{r}) = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}; \mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{p}}{r^3};$$

$$U = \frac{1}{R^3}\left(\mathbf{p}_1\cdot\mathbf{p}_2 - 3(\mathbf{p}_1\cdot\hat{\mathbf{R}})(\mathbf{p}_2\cdot\hat{\mathbf{R}})\right).$$

$\underline{\mathbf{p} = p\hat{\mathbf{z}}}$:

$$\varphi_{\text{dip}}(\mathbf{r}) = \frac{p\cos\theta}{r^2}; \ \mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{p}{r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right).$$

$$Q_{ij} = \int \rho(\mathbf{r}')\left(3r_i'r_j' - \delta_{ij}r'^2\right)\mathrm{d}^3r'; \ \varphi_{\text{quad}}(\mathbf{r}) = \sum\frac{Q_{ij}\hat{r}_i\hat{r}_j}{2r^3}$$

$$U = Q\varphi(0) - \mathbf{p}\cdot\mathbf{E}(0) + \frac{1}{6}\sum_{i,j}Q_{ij}\left.\frac{\partial^2\varphi}{\partial r_i\partial r_j}\right|_{\mathbf{r}=\mathbf{0}} + \cdots$$

מולטיפולים מגנטיים

$$\mathbf{A}(\mathbf{r}) = \frac{1}{cr}\sum_{n=0}^{\infty}\int\left(\frac{r'}{r}\right)^n \mathbf{J}(\mathbf{r}')\,P_n(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')\,d^3r';$$

$$\mathbf{m} = \frac{1}{2c}\int \mathbf{r}'\times\mathbf{J}(\mathbf{r}')d^3r';$$

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}; \ \mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{m}}{r^3}.$$

$\underline{\mathbf{m} = m\hat{\mathbf{z}}}$:

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{m\sin\theta}{r^2}\hat{\boldsymbol{\phi}}; \ \mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{m}{r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right).$$

עבור לולאת זרם בשטח a:

$$\mathbf{m} = \frac{Ia}{c}\hat{\mathbf{n}}$$

$$U = -\mathbf{m}\cdot\mathbf{B} + \cdots$$

מולטיפולים חשמליים כדוריים

$$\varphi(\mathbf{r}) = \sum_{l,m}\frac{4\pi}{2l+1}\left(\frac{M_{lm}}{r^{l+1}}Y_{lm}(\theta,\phi) + \overline{M}_{lm}r^l Y_{lm}^*(\theta,\phi)\right);$$

$$M_{lm} = \int_{r'\leq r} Y_{lm}^*(\theta',\phi')(r')^l\rho(\mathbf{r}')d^3r';$$

$$\overline{M}_{lm} = \int_{r'\geq r}\frac{Y_{lm}(\theta',\phi')}{(r')^{l+1}}\rho(\mathbf{r}')d^3r'.$$

$$M_{0,0} = \frac{1}{\sqrt{4\pi}}Q.$$

$$M_{1,0} = \sqrt{\frac{3}{4\pi}}p_z; \quad M_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\left(p_x \mp ip_y\right).$$

$$M_{2,0} = \sqrt{\frac{5}{16\pi}}Q_{zz}; \ M_{2,\pm1} = \mp\sqrt{\frac{5}{24\pi}}\left(Q_{xz} \mp iQ_{yz}\right);$$

$$M_{2,\pm2} = \sqrt{\frac{5}{96\pi}}\left(Q_{xx} - Q_{yy} \mp 2iQ_{xy}\right).$$

<u>בסימטריה כדורית</u>:

$$\varphi(\mathbf{r}) = \frac{4\pi}{r}\int_0^r (r')^2\rho(r')dr' + 4\pi\int_r^{\infty} r'\rho(r')dr'$$

<u>בסימטריה אזימותית</u>:

$$\varphi(\mathbf{r}) = \sum_l\left(\frac{M_l}{r^{l+1}} + \overline{M}_l r^l\right)P_l(\cos\theta)$$

$$M_l = \int_{r'\leq r} P_l(\cos\theta')(r')^l\rho(\mathbf{r}')d^3r';$$

$$\overline{M}_l = \int_{r'\geq r}\frac{P_l(\cos\theta')}{(r')^{l+1}}\rho(\mathbf{r}')d^3r'.$$

בעיות תנאי שפה

$$\varphi(\mathbf{r}) = \int_V \rho(\mathbf{r}')G_D(\mathbf{r},\mathbf{r}')d^3r' - \frac{1}{4\pi}\int_S \varphi(\mathbf{r}')\frac{\partial G_D(\mathbf{r},\mathbf{r}')}{\partial n'}da';$$

<u>מחוץ לכדור ברדיוס a</u>:

$$G_D(\mathbf{r},\mathbf{r}') = \frac{1}{\sqrt{r^2+r'^2-2rr'\cos\gamma}} - \frac{1}{\sqrt{\frac{r^2r'^2}{a^2}+a^2-2rr'\cos\gamma}};$$

$$\left.\frac{\partial G_D}{\partial n'}\right|_S = -\frac{r^2-a^2}{a(r^2+a^2-2ar\cos\gamma)^{\frac{3}{2}}};$$

$$\cos\gamma = \hat{\mathbf{r}}\cdot\hat{\mathbf{r}}' = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi').$$

מטעני דמות

מישור $x-y$ מוארק: $q' = -q; \quad z' = -z;$

כדור מוארק ברדיוס a: $q' = -\frac{a}{r}q; \quad r' = \frac{a^2}{r};$

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שדות בחומר

$$\nabla\cdot\mathbf{D}=4\pi\rho;\quad \nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t};$$
$$\nabla\cdot\mathbf{B}=0;\quad \nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}+\frac{4\pi}{c}\mathbf{J}.$$

$$\mathbf{E}=\mathbf{D}-4\pi\mathbf{P};\quad \nabla\cdot\mathbf{P}=-\rho_b;\quad \hat{\mathbf{n}}\cdot(\mathbf{P}_2-\mathbf{P}_1)=-\sigma_b;$$

$$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M};\quad \nabla\times\mathbf{M}=\frac{1}{c}\mathbf{J}_b;\quad \hat{\mathbf{n}}\times(\mathbf{M}_2-\mathbf{M}_1)=\frac{1}{c}\mathbf{K}_b.$$

$$\hat{\mathbf{n}}\cdot(\mathbf{D}_2-\mathbf{D}_1)=4\pi\sigma;\quad \hat{\mathbf{n}}\times(\mathbf{E}_2-\mathbf{E}_1)=0;$$

$$\hat{\mathbf{n}}\cdot(\mathbf{B}_2-\mathbf{B}_1)=0;\quad \hat{\mathbf{n}}\times(\mathbf{H}_2-\mathbf{H}_1)=\frac{4\pi}{c}\mathbf{K}.$$

<u>בחומרים לינאריים איזוטרופיים</u>

$$\mathbf{P}=\chi_e\mathbf{E};\quad \mathbf{D}=\epsilon\mathbf{E};\quad \epsilon=1+4\pi\chi_e;$$

$$\mathbf{M}=\chi_m\mathbf{H};\quad \mathbf{B}=\mu\mathbf{H};\quad \mu=1+4\pi\chi_m.$$

<u>בחומרים לינאריים לא-איזוטרופיים</u>

$$P_i=\sum_j\chi_{ij}E_j;\quad D_i=\sum_j\epsilon_{ij}E_j;\quad \epsilon_{ij}=\delta_{ij}+4\pi\chi_{ij};$$

$$M_i=\sum_j\chi^m_{ij}H_j;\quad B_i=\sum_j\mu_{ij}H_j;\quad \mu_{ij}=\delta_{ij}+4\pi\chi^m_{ij}.$$

<u>פוטנציאל מגנטי סקלרי אפקטיבי</u>

$$\mathbf{H}=-\nabla\varphi_m;\quad \nabla^2\varphi_m=-4\pi\rho_m;\quad \rho_m=-\nabla\cdot\mathbf{M};$$
$$\sigma_m=-\hat{\mathbf{n}}\cdot(\mathbf{M}_{out}-\mathbf{M}_{in}).$$

דינמיקה

$$\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-4\pi\rho;\ \nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=-\frac{4\pi}{c}\mathbf{J};\ \partial_\mu A^\mu=0;$$

$$\nabla^2G^\pm-\frac{1}{c^2}\frac{\partial^2G^\pm}{\partial t^2}=-4\pi\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\longrightarrow$$

$$G^\pm(\mathbf{r},t,\mathbf{r}',t')=\frac{\delta(t'-[t\mp|\mathbf{r}-\mathbf{r}'|/c])}{|\mathbf{r}-\mathbf{r}'|}.$$

$$\varphi_{\rm ret}(\mathbf{r},t)=\int d^3r'dt'G^+(\mathbf{r},t,\mathbf{r}',t')\rho(\mathbf{r}',t')$$
$$=\int\frac{\rho(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}d^3r';$$

$$\mathbf{A}_{\rm ret}(\mathbf{r},t)=\int d^3r'dt'G^+(\mathbf{r},t,\mathbf{r}',t')\frac{\mathbf{J}(\mathbf{r}',t')}{c}$$
$$=\frac{1}{c}\int\frac{\mathbf{J}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}d^3r'.$$

הספק קרינה

$$P=\int(\mathbf{S}\cdot\hat{\mathbf{r}})r^2d\Omega;\ \frac{dP}{d\Omega}=(\mathbf{S}\cdot\hat{\mathbf{r}})r^2.$$

פוטנציאל לינרד-ויכרט

$$t-t_{\rm ret}=\frac{|\mathbf{r}-\mathbf{r}_q(t_{\rm ret})|}{c};\ \mathbf{R}=\mathbf{r}-\mathbf{r}_q;\ \hat{\mathbf{n}}=\frac{\mathbf{R}}{R};\ K=R-\mathbf{R}\cdot\boldsymbol{\beta};$$

$$\varphi_{\rm ret}(\mathbf{r},t)=\frac{q}{[K]};\quad \mathbf{A}_{\rm ret}(\mathbf{r},t)=\frac{q[\boldsymbol{\beta}]}{[K]};$$

$$\mathbf{E}_v=q\left[\frac{(\mathbf{R}-R\boldsymbol{\beta})(1-\beta^2)}{K^3}\right];\quad \mathbf{E}_a=\frac{q}{c^2}\left[\frac{\mathbf{R}\times\{(\mathbf{R}-R\boldsymbol{\beta})\times\mathbf{a}_q\}}{K^3}\right];$$

$$\mathbf{B}_v=[\hat{\mathbf{n}}]\times\mathbf{E}_v;\quad \mathbf{B}_a=[\hat{\mathbf{n}}]\times\mathbf{E}_a;\quad \mathbf{S}=\frac{c}{4\pi}E_a^2[\hat{\mathbf{n}}].$$

במערכת החלקיק $\frac{dP}{d\Omega}=\frac{c}{4\pi}E_a^2R^2$

נוסחת לרמור $\frac{dP}{d\Omega}=\frac{q^2}{4\pi c^3}a^2\sin^2\theta\ ;\quad P=\frac{2q^2}{3c^3}a^2\ .$

נוסחת לינרד $\frac{dP}{d\Omega}=\frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{n}}\times\{(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right|^2}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^5}$

$$=\frac{q^2}{4\pi c}\left(\frac{\dot\beta^2}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3}+\frac{2(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}})(\hat{\mathbf{n}}\cdot\dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^4}-\frac{(1-\beta^2)(\hat{\mathbf{n}}\cdot\dot{\boldsymbol{\beta}})^2}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^5}\right);$$

$$P=\frac{2q^2}{3c}\gamma^6\left(\dot\beta^2-\left(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right)^2\right).$$

קרינה מהתפלגות רציפה

$$\mathbf{J}(\mathbf{r},t)=\mathbf{J}(\mathbf{r})e^{-i\omega t};\ \rho(\mathbf{r},t)=\rho(\mathbf{r})e^{-i\omega t};\ k=\frac{\omega}{c};$$

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}(\mathbf{r})e^{-i\omega t};$$
$$\mathbf{A}(\mathbf{r})=\frac{e^{ikr}}{cr}\int\mathbf{J}(\mathbf{r}')e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}d^3r';\quad (kr\gg1, r\gg r')$$

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})e^{-i\omega t};\mathbf{B}(\mathbf{r},t)=\mathbf{B}(\mathbf{r})e^{-i\omega t};$$

$$\langle\mathbf{S}\rangle=\frac{c}{8\pi}\mathrm{Re}\big(\mathbf{E}(\mathbf{r})\times\mathbf{B}^*(\mathbf{r})\big).$$

קרינה מולטיפולית

$$\mathbf{A}(\mathbf{r})=\frac{e^{ikr}}{cr}\sum_{n=0}^{\infty}\frac{(-ik)^n}{n!}\int\mathbf{J}(\mathbf{r}')(\hat{\mathbf{r}}\cdot\mathbf{r}')^nd^3r'\quad (kr'\ll1)$$

$$\mathbf{A}_{\rm dip}(\mathbf{r})=-ik\mathbf{p}\frac{e^{ikr}}{r};\ \mathbf{A}_{\rm dip}(\mathbf{r},\mathrm{t})=\frac{\dot{\mathbf{p}}(t-r/c)}{cr};$$

$$\frac{d\langle P\rangle}{d\Omega}=\frac{c}{8\pi}k^4|(\hat{\mathbf{r}}\times\mathbf{p})\times\hat{\mathbf{r}}|^2=\frac{c}{8\pi}k^4(|\mathbf{p}|^2-|\hat{\mathbf{r}}\cdot\mathbf{p}|^2).$$

אם לכל הרכיבים של $\mathbf{p}$ יש אותה הפאזה

$$\frac{d\langle P\rangle}{d\Omega}=\frac{c}{8\pi}k^4|\mathbf{p}|^2\sin^2\theta;\ \langle P\rangle=\frac{ck^4}{3}|\mathbf{p}|^2.$$

$$\mathbf{A}_{\rm dip}(\mathbf{r})=ik(\hat{\mathbf{r}}\times\mathbf{m})\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right);$$

$$\frac{d\langle P\rangle}{d\Omega}=\frac{c}{8\pi}k^4|(\hat{\mathbf{r}}\times\mathbf{m})\times\hat{\mathbf{r}}|^2=\frac{c}{8\pi}k^4(|\mathbf{m}|^2-|\hat{\mathbf{r}}\cdot\mathbf{m}|^2).$$

אם לכל הרכיבים של $\mathbf{m}$ יש אותה הפאזה

$$\frac{d\langle P\rangle}{d\Omega}=\frac{c}{8\pi}k^4|\mathbf{m}|^2\sin^2\theta;\ \langle P\rangle=\frac{ck^4}{3}|\mathbf{m}|^2.$$

$$Q_j(\mathbf{r})=\sum\hat{r}_iQ_{ij};\quad \mathbf{A}_{\rm quad}(\mathbf{r})=-\frac{k^2e^{ikr}}{6r}\mathbf{Q}(\mathbf{r});$$

$$\frac{d\langle P\rangle}{d\Omega}=\frac{c}{288\pi}k^6(|\mathbf{Q}|^2-|\hat{\mathbf{r}}\cdot\mathbf{Q}|^2);\quad \langle P\rangle=\frac{ck^6}{360}\sum_{i,j}\left|Q_{ij}\right|^2;$$

$$\langle P\rangle=\frac{1}{2}{I_0}^2R_{\rm rad}.$$

מרצה: רון ליפשיץ   מתרגל: נועם רימוק

גלים בחומר

$n(\omega) = \sqrt{\mu(\omega)\epsilon(\omega)}; \quad ck = \omega n(\omega);$

$\hat{\mathbf{k}} \times \mathbf{E} = Z(\omega)\mathbf{H}; \; Z(\omega) = \sqrt{\frac{\mu(\omega)}{\epsilon(\omega)}} = \frac{\mu\omega}{ck}.$

יחסי קרמרס-קרוניג

$\mathbf{E}(-\omega) = \mathbf{E}^*(\omega); \; \chi(-\omega) = \chi^*(\omega);$

$$\mathrm{Re}\big(\chi(\omega)\big) = \frac{2}{\pi} P \int_0^\infty \frac{\omega' \mathrm{Im}\big(\chi(\omega')\big)}{\omega'^2 - \omega^2} d\omega';$$

$$\mathrm{Im}\big(\chi(\omega)\big) = -\frac{2\omega}{\pi} P \int_0^\infty \frac{\mathrm{Re}\big(\chi(\omega')\big)}{\omega'^2 - \omega^2} d\omega'.$$

נספח מתמטי

$\sin 2\alpha = 2 \sin\alpha \cos\alpha; \quad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha.$

$\delta[f(x)] = \sum \frac{1}{|f'(x_n)|} \delta(x - x_n); \; f(x_n) = 0, f'(x_n) \neq 0.$

$x\frac{d\delta(x)}{dx} = -\delta(x); \; \delta(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} e^{ikx};$

$\theta(x) = \int_{-\infty}^{x} \delta(x')dx' = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}; \quad \delta(x) = \frac{d\theta(x)}{dx}.$

$\mathbf{a} \cdot \mathbf{b} = \delta^{ij} a_i b_j; \; (\mathbf{a} \times \mathbf{b})_k = \epsilon^{ijk} A_i B_j;$

$$\sum_k \epsilon^{ijk} \epsilon^{mnk} = \delta^{im}\delta^{jn} - \delta^{in}\delta^{jm}.$$

$\boldsymbol{\nabla} f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}; \;\; \boldsymbol{\nabla} \cdot \mathbf{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z};$

$\nabla^2 f = \boldsymbol{\nabla} \cdot \boldsymbol{\nabla} f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2};$

$\boldsymbol{\nabla} \times \mathbf{f} = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\hat{\mathbf{z}}.$

$\int_V \boldsymbol{\nabla} \cdot \mathbf{F} dV = \int_S \mathbf{F} \cdot \mathrm{d}\mathbf{a}; \; \int_S (\boldsymbol{\nabla} \times \boldsymbol{F}) \cdot \mathrm{d}\mathbf{a} = \oint_C \mathbf{F} \cdot \mathrm{d}\boldsymbol{\ell}.$

$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b});$

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b});$

$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c});$

$\boldsymbol{\nabla} \times \boldsymbol{\nabla} f = \mathbf{0}; \quad \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{f}) = 0;$

$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{f}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{f}) - \nabla^2 \mathbf{f};$

$\boldsymbol{\nabla} \cdot (f\mathbf{g}) = \boldsymbol{\nabla} f \cdot \mathbf{g} + f \boldsymbol{\nabla} \cdot \mathbf{g};$

$\boldsymbol{\nabla} \times (f\mathbf{g}) = \boldsymbol{\nabla} f \times \mathbf{g} + f \boldsymbol{\nabla} \times \mathbf{g};$

$\boldsymbol{\nabla}(\mathbf{f} \cdot \mathbf{g}) = (\mathbf{f} \cdot \boldsymbol{\nabla})\mathbf{g} + (\mathbf{g} \cdot \boldsymbol{\nabla})\mathbf{f} + \mathbf{f} \times (\boldsymbol{\nabla} \times \mathbf{g}) + \mathbf{g} \times (\boldsymbol{\nabla} \times \mathbf{f});$

$\boldsymbol{\nabla} \cdot (\mathbf{f} \times \mathbf{g}) = \mathbf{g} \cdot (\boldsymbol{\nabla} \times \mathbf{f}) - \mathbf{f} \cdot (\boldsymbol{\nabla} \times \mathbf{g});$

$\boldsymbol{\nabla} \times (\mathbf{f} \times \mathbf{g}) = \mathbf{f}(\boldsymbol{\nabla} \cdot \mathbf{g}) - \mathbf{g}(\boldsymbol{\nabla} \cdot \mathbf{f}) + (\mathbf{g} \cdot \boldsymbol{\nabla})\mathbf{f} - (\mathbf{f} \cdot \boldsymbol{\nabla})\mathbf{g}.$

קואורדינטות כדוריות

$x = r\sin\theta\cos\phi; \; y = r\sin\theta\sin\phi; \; z = r\cos\theta;$

$\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}; \;\; \hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{r}}; \;\; \hat{\boldsymbol{\phi}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\theta}};$

$$\begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix};$$

$$\begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix};$$

$\boldsymbol{\nabla} f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}};$

$\boldsymbol{\nabla} \cdot \mathbf{f} = \frac{1}{r^2}\frac{\partial(r^2 f_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta f_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(f_\phi)}{\partial\phi};$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2};$

$$\boldsymbol{\nabla} \times \mathbf{f} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\, f_\phi) - \frac{\partial f_\theta}{\partial\phi}\right)\hat{\mathbf{r}} + \left(\frac{1}{r\sin\theta}\frac{\partial f_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rf_\phi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rf_\theta) - \frac{\partial f_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}};$$

קואורדינטות גליליות

$x = \rho\cos\phi; \;\; y = \rho\sin\phi; \;\; z = z;$

$\hat{\boldsymbol{\rho}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}; \;\; \hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\boldsymbol{\rho}}; \;\; \hat{\mathbf{z}} \times \hat{\boldsymbol{\rho}} = \hat{\boldsymbol{\phi}};$

$$\begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} \end{pmatrix};$$

$$\begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix};$$

$\boldsymbol{\nabla} f = \frac{\partial f}{\partial\rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}};$

$\boldsymbol{\nabla} \cdot \mathbf{f} = \frac{1}{\rho}\frac{\partial(\rho f_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial f_\phi}{\partial\phi} + \frac{\partial f_z}{\partial z};$

$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2};$

$$\boldsymbol{\nabla} \times \mathbf{f} = \left(\frac{1}{\rho}\frac{\partial f_z}{\partial\phi} - \frac{\partial f_\phi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial f_\rho}{\partial z} - \frac{\partial f_z}{\partial\rho}\right)\hat{\boldsymbol{\phi}} + \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho f_\phi) - \frac{\partial f_\rho}{\partial\phi}\right)\hat{\mathbf{z}};$$