

Surface-plasmon-polaritons - TE

TM - Continuity at the interface:

$$A_1 = A_2$$

$$\frac{k_2}{k_1} = -\frac{\epsilon_2}{\epsilon_1}$$

$$k_1^2 = \beta^2 - k_0^2 \epsilon_1$$

$$k_2^2 = \beta^2 - k_0^2 \epsilon_2$$

Dispersion relation:

$$\beta = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}.$$

For TE:

$$E_y(z) = A_2 e^{i\beta x} e^{-k_2 z}$$

$$H_x(z) = -i A_2 \frac{1}{\omega \mu_0} k_2 e^{i\beta x} e^{-k_2 z}$$

$$H_z(z) = A_2 \frac{\beta}{\omega \mu_0} e^{i\beta x} e^{-k_2 z}$$

TE - Continuity at the interface:

$$A_1 (k_1 + k_2) = 0$$

$$A_2 = A_1 = 0$$

condition is only fulfilled if $A_1 = 0$, so that also $A_2 = A_1 = 0$. Thus, no surface modes exist for TE polarization. *Surface plasmon polaritons only exist for TM polarization.*

$$E_y(z) = A_1 e^{i\beta x} e^{k_1 z}$$

$$H_x(z) = i A_1 \frac{1}{\omega \mu_0} k_1 e^{i\beta x} e^{k_1 z}$$

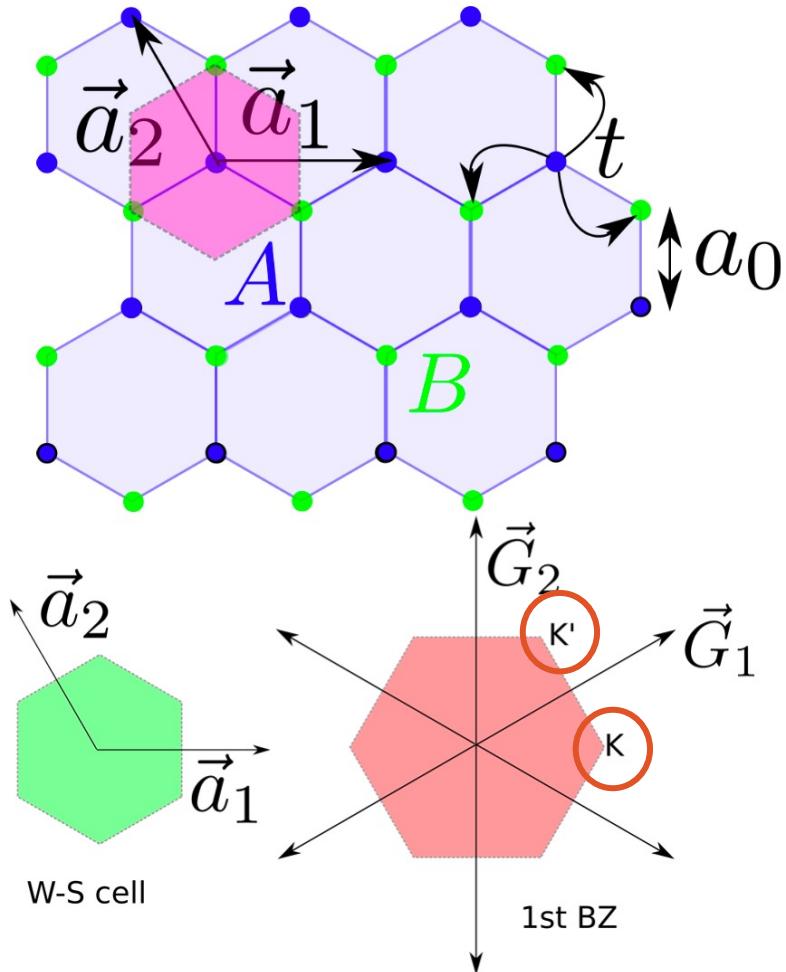
$$H_z(z) = A_1 \frac{\beta}{\omega \mu_0} e^{i\beta x} e^{k_1 z}$$

אופטיקה של חומרים דו-מימדיים - שיעור מס. 6

- Graphene •
- Graphene plasmons •
- Polaritons and the TMM •

Graphene

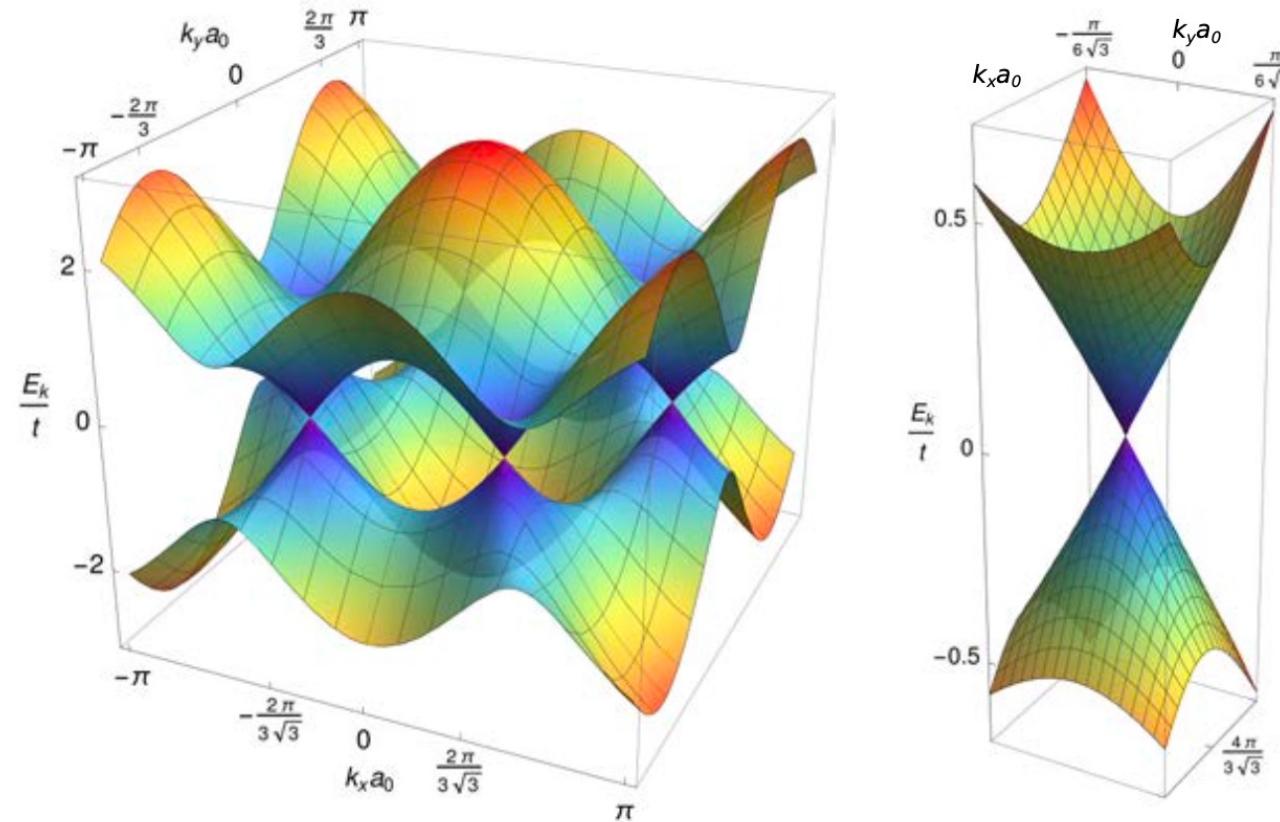
P.A.D Goncalves and N.M.R Peres - "An Introduction to Graphene Plasmonics"



- Hexagonal arrangement of carbon atoms
- Two sub-lattices

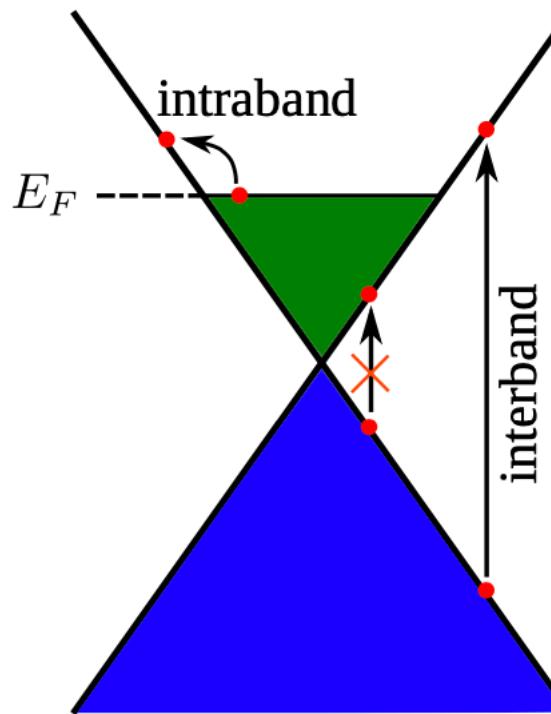
$t \simeq -2.7$ eV and $a_0 \simeq 1.42$ Å are the hopping integral and the carbon-carbon distance, respectively. The vectors $\vec{a}_1 = a(1, 0)$ and $\vec{a}_2 = a(-1/2, \sqrt{3}/2)$ are the primitive lattice-vectors, where $a = a_0\sqrt{3}$. Bottom: Wigner-Seitz cell (in green)

Graphene - band structure



- Linear dispersion relation $E_{\pm}(\mathbf{q}) = \pm \hbar v_F q$ $v_F \approx c/300$
- Electrons and holes behave the same.
- Half-filled – valence band full, conduction band empty
- Touching at the Dirac point - zero gap
- Absorbance of arbitrary low frequencies

Optical conductivity of graphene



$$\sigma_g(\omega) = \sigma_{\text{intra}}(\omega) + \sigma_{\text{inter}}(\omega)$$

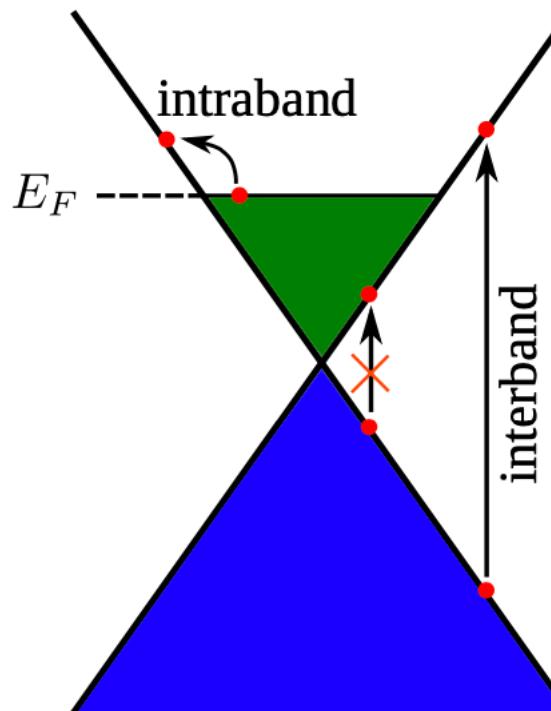
At zero temperature, or as long as the condition $E_F \gg k_B T$ above equations reduce to

$$\sigma_{\text{intra}}(\omega) = \frac{\sigma_0}{\pi} \frac{4E_F}{\hbar\gamma - i\hbar\omega} ,$$

$$\sigma_{\text{inter}}(\omega) = \sigma_0 \left[\Theta(\hbar\omega - 2E_F) + \frac{i}{\pi} \ln \left| \frac{\hbar\omega - 2E_F}{\hbar\omega + 2E_F} \right| \right] ,$$

tion to the conductivity due to Pauli blocking. Therefore, we note that for frequencies in the terahertz (THz) and mid-IR spectral region, at room temperature, and under typical doping levels, graphene's optical conductivity is mostly dominated by the term describing intraband electronic processes, following a Drude-like expression [Stauber *et al.* (2007); Peres *et al.* (2007)]

Optical conductivity of graphene



At non-zero temperatures:

$$\sigma_{\text{intra}}(\omega) = \frac{\sigma_0}{\pi} \frac{4}{\hbar\gamma - i\hbar\omega} \left[E_F + 2k_B T \ln \left(1 + e^{-E_F/k_B T} \right) \right]$$

$$\sigma_{\text{inter}}(\omega) = \sigma_0 \left[G(\hbar\omega/2) + i \frac{4\hbar\omega}{\pi} \int_0^\infty dE \frac{G(E) - G(\hbar\omega/2)}{(\hbar\omega)^2 - 4E^2} \right]$$

$$G(x) = \frac{\sinh \left(\frac{x}{k_B T} \right)}{\cosh \left(\frac{E_F}{k_B T} \right) + \cosh \left(\frac{x}{k_B T} \right)}$$

Optical conductivity of graphene

- Transmission and absorption of undoped graphene

$$\text{Absorbed intensity } W_a = \frac{e^2 E_0^2}{8\hbar}$$

$$\text{Input intensity } W_i = \frac{1}{2} c \epsilon_0 E_0^2$$

$$T = 1 - \frac{W_a}{W_i} = 1 - \pi\alpha \simeq 97.7\%$$

$\alpha = e^2/(4\pi\epsilon_0\hbar c)$ is the fine structure constant of atomic physics

absorption of 2.3%

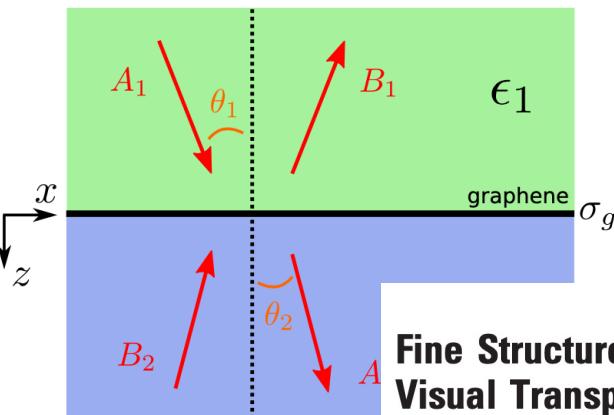
true only as long as the momentum expansion around the Dirac point remains valid, and only applies to neutral (undoped) graphene. In Fig. 2.3

Only for suspended graphene

$$T = 1/|1 + \sigma(\omega)/(2c\epsilon_0)|^2$$

$$\sigma(\omega) = \pi e^2 / 2h \equiv \sigma_0$$

$$\sigma_g(\omega) = \sigma_{\text{intra}}(\omega) + \sigma_{\text{inter}}(\omega)$$



Fine Structure Constant Defines Visual Transparency of Graphene

R. R. Nair,¹ P. Blake,¹ A. N. Grigorenko,¹ K. S. Novoselov,¹ T. J. Booth,¹ T. Stauber,² N. M. R. Peres,² A. K. Geim^{1*}

There are few phenomena in condensed matter physics that are defined only by the fundamental constants and do not depend on material parameters. Examples are the resistivity quantum, h/e^2 , that appears in a variety of transport experiments, including the quantum Hall effect and universal conductance fluctuations, and the magnetic flux quantum, $h/2e$, playing an important role in the physics of superconductivity (h is Planck's constant and e the electron charge). By and large, it requires sophisticated facilities and special measurement conditions to observe any of these phenomena. In contrast, we show that the opacity of suspended graphene (τ) is defined solely by the fine structure constant, $\alpha = e^2/\hbar c \approx 1/137$ (where c is the speed of light), the parameter that describes coupling between light and relativistic electrons and that is traditionally associated with quantum electrodynamics rather than materials science. Despite being only one atom thick, graphene is found to absorb a significant ($\pi\alpha = 2.3\%$) fraction of incident white light, a consequence of graphene's unique electronic

We have studied specially prepared graphene crystals (5) such that they covered submillimeter apertures in a metal scaffold (Fig. 1A inset). Such large one-atom-thick membranes suitable for

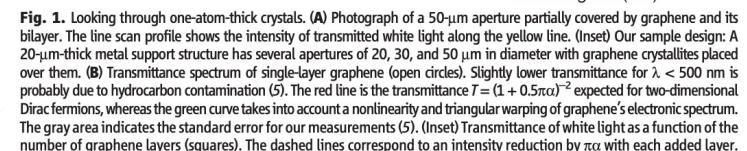
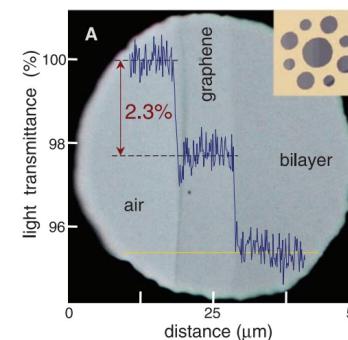
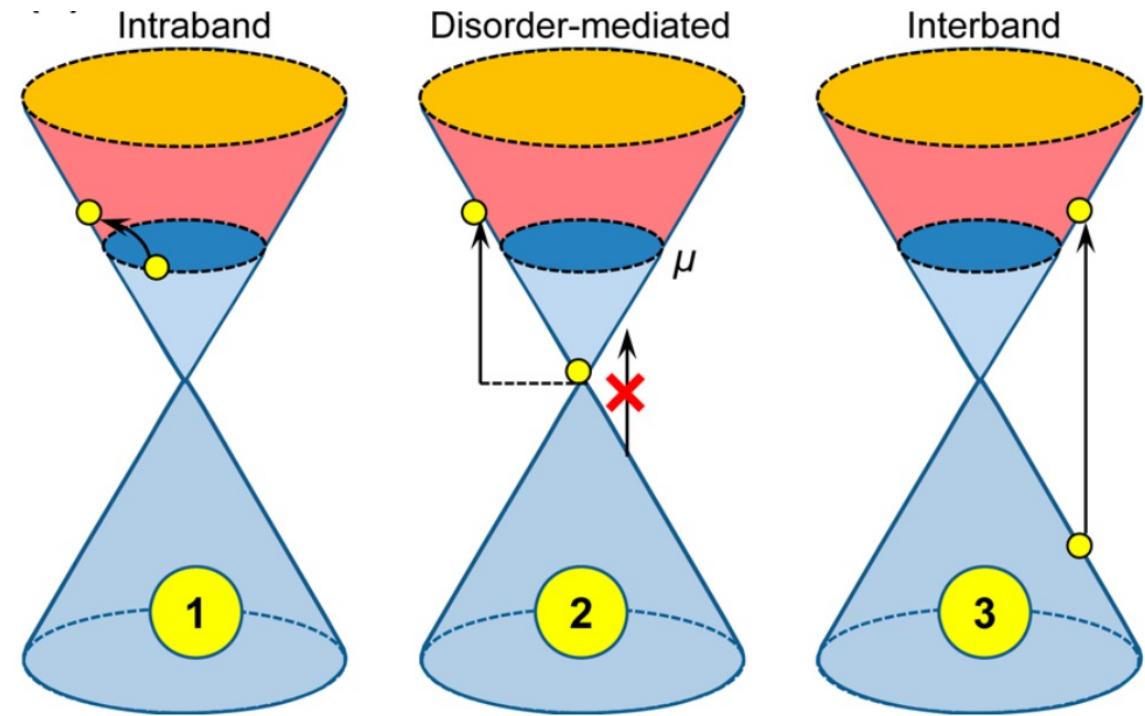
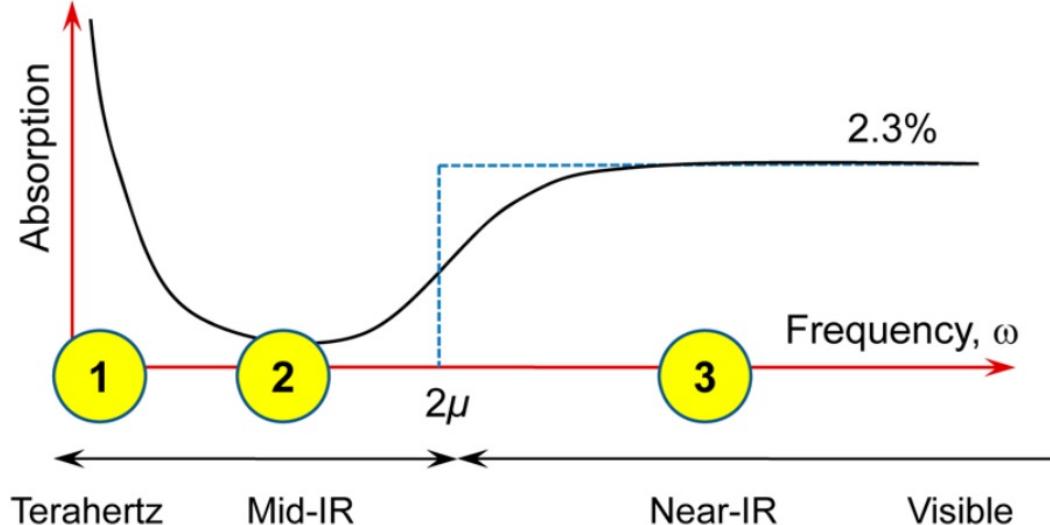


Fig. 1. Looking through one-atom-thick crystals. (A) Photograph of a 50-μm aperture partially covered by graphene and its bilayer. The line scan profile shows the intensity of transmitted white light along the yellow line. (Inset) Our sample design: A 20-μm-thick metal support structure has several apertures of 20, 30, and 50 μm in diameter with graphene crystallites placed over them. (B) Transmittance spectrum of single-layer graphene (open circles). Slightly lower transmittance for $\lambda < 500$ nm is probably due to hydrocarbon contamination (5). The red line is the transmittance $T = (1 + 0.5\pi\alpha)^{-2}$ expected for two-dimensional Dirac fermions, whereas the green curve takes into account a nonlinearity and triangular warping of graphene's electronic spectrum. The gray area indicates the standard error for our measurements (5). (Inset) Transmittance of white light as a function of the number of graphene layers (squares). The dashed lines correspond to an intensity reduction by $\pi\alpha$ with each added layer.

Optical conductivity of graphene



Optical conductivity of graphene

$$\sigma_g(\omega) = \sigma_{\text{intra}}(\omega) + \sigma_{\text{inter}}(\omega)$$

Kubo model (T=0):

$$\sigma_{\text{intra}}(\omega) = \frac{\sigma_0}{\pi} \frac{4E_F}{\hbar\gamma - i\hbar\omega} ,$$

$$\sigma_{\text{inter}}(\omega) = \sigma_0 \left[\Theta(\hbar\omega - 2E_F) + \frac{i}{\pi} \cancel{\ln} \left| \frac{\hbar\omega - 2E_F}{\hbar\omega + 2E_F} \right| \right]$$

Local model (T \neq 0):

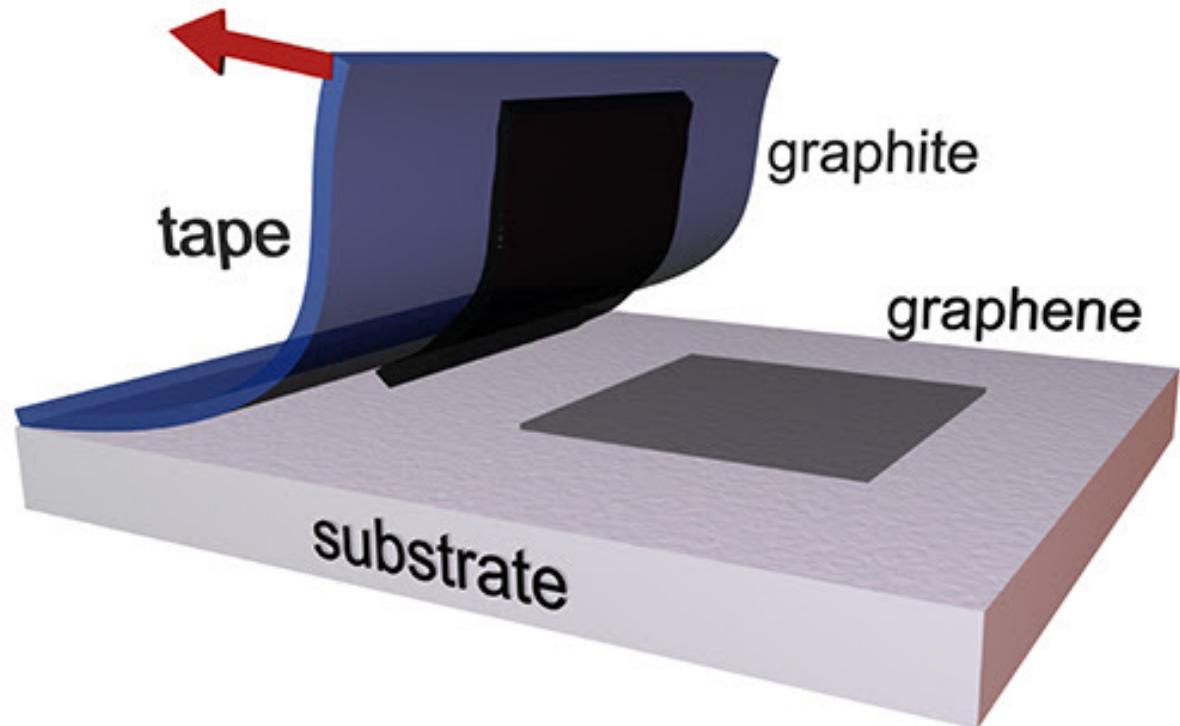
$$\sigma_{\text{intra}}(\omega) = \frac{\sigma_0}{\pi} \frac{4}{\hbar\gamma - i\hbar\omega} \left[E_F + 2k_B T \ln \left(1 + e^{-E_F/k_B T} \right) \right]$$

$$\sigma_{\text{inter}}(\omega) = \sigma_0 \left[G(\hbar\omega/2) + i \frac{4\hbar\omega}{\pi} \int_0^\infty dE \frac{G(E) - G(\hbar\omega/2)}{(\hbar\omega)^2 - 4E^2} \right]$$

$$G(x) = \frac{\sinh \left(\frac{x}{k_B T} \right)}{\cosh \left(\frac{E_F}{k_B T} \right) + \cosh \left(\frac{x}{k_B T} \right)}$$

Nonlocal model (T \neq 0): $\sigma(q, \omega)$

Graphene exfoliation



Graphene plasmons

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Milestone

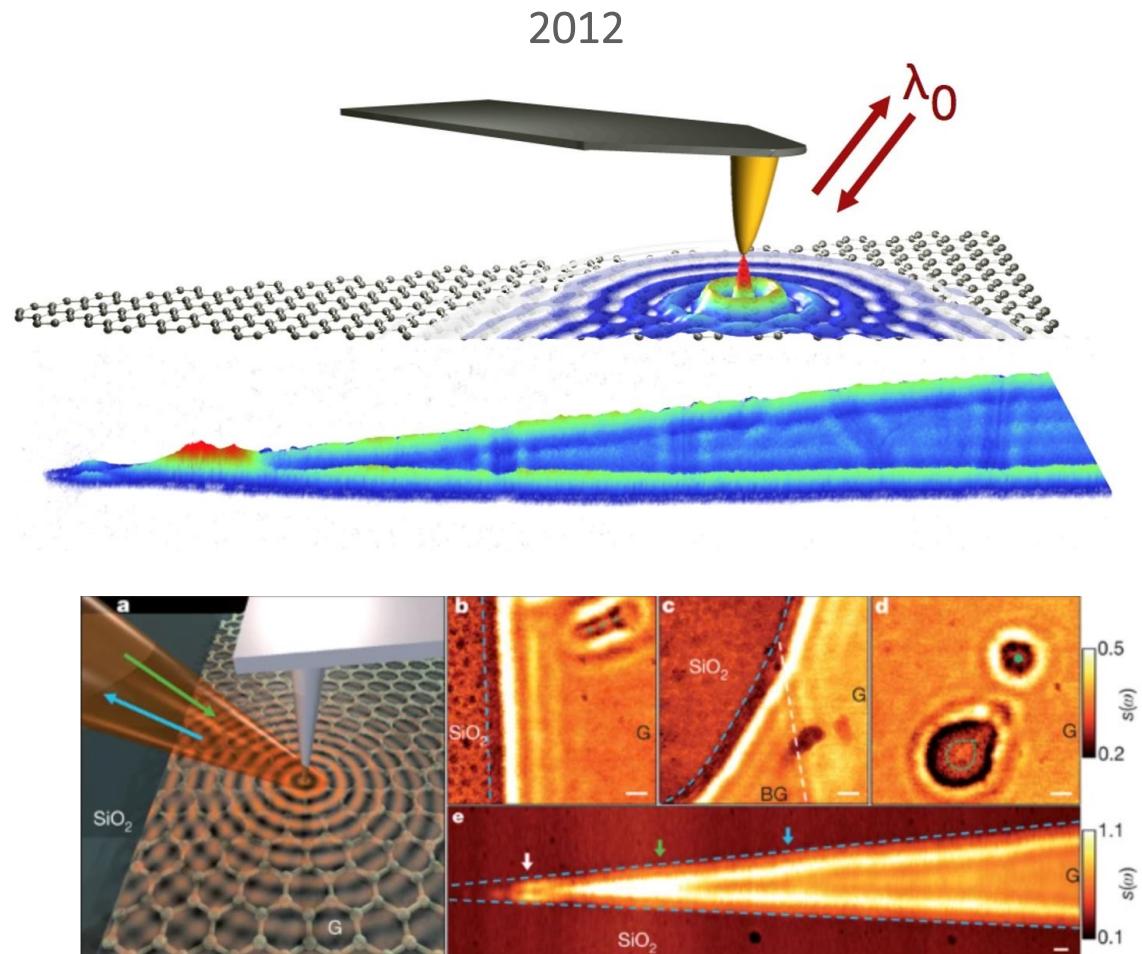
Plasmonics in graphene at infrared frequencies

Marinko Jablan, Hrvoje Buljan, and Marin Soljačić
Phys. Rev. B **80**, 245435 – Published 23 December 2009

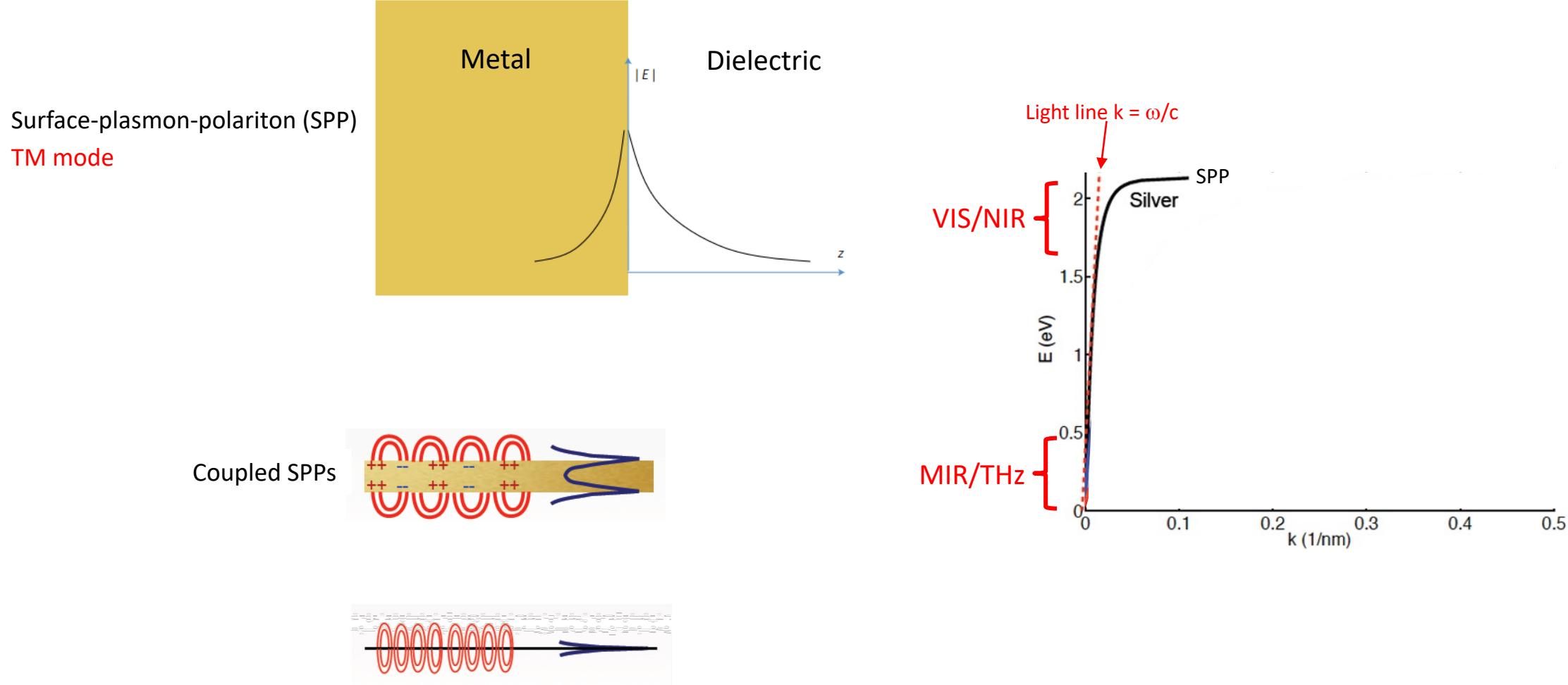
An article within the collection: *Physical Review B 50th Anniversary Milestones*

Article	References	Citing Articles (1,519)	PDF	HTML	Export Citation
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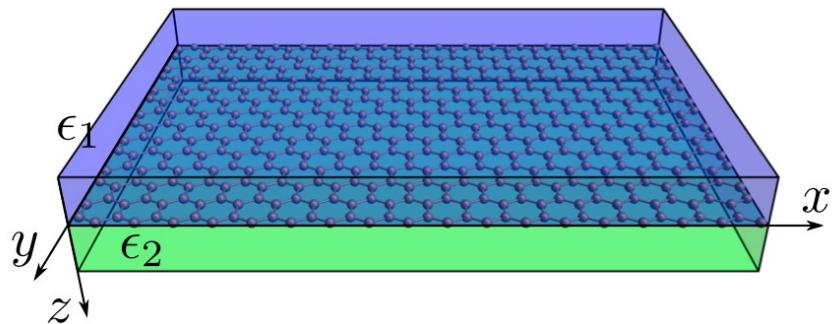
$$\varepsilon(\omega) = 1 + \frac{i\sigma(\omega)}{\varepsilon_0\omega}$$



Surface plasmons and graphene plasmons



Graphene plasmons - dispersion relation TM



The boundary conditions linking the electromagnetic fields at $z = 0$

$$E_{1,x}(x, z)|_{z=0} = E_{2,x}(x, z)|_{z=0} ,$$

$$B_{1,y}(x, z)|_{z=0} - B_{2,y}(x, z)|_{z=0} = \mu_0 J_x(x) = \mu_0 \sigma_{xx} E_{2,x}(x, z)|_{z=0} ,$$

$$\mathbf{E}_j = (E_{j,x}\hat{\mathbf{x}} + E_{j,z}\hat{\mathbf{z}}) e^{iqx} e^{-\kappa_j|z|} ,$$

$$\mathbf{B}_j = B_{j,y} e^{iqx} e^{-\kappa_j|z|} \hat{\mathbf{y}} ,$$

$$E_{j,x} = i \operatorname{sgn}(z) \frac{\kappa_j c^2}{\omega \epsilon_j} B_{j,y},$$

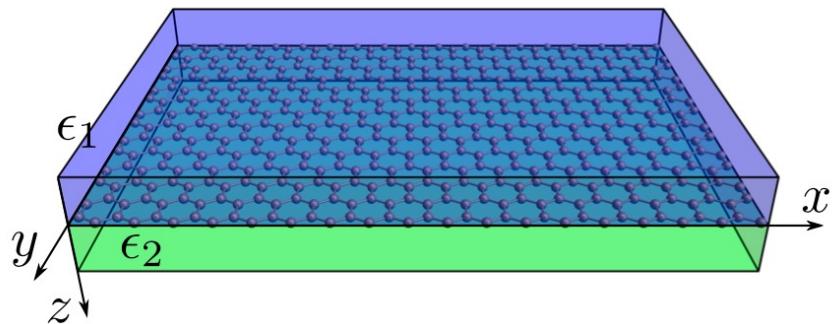
$$E_{j,z} = -\frac{q c^2}{\omega \epsilon_j} B_{j,y},$$

$$\kappa_j^2 = q^2 - \omega^2 \epsilon_j / c^2 .$$

$$\frac{\epsilon_1}{\kappa_1(q, \omega)} + \frac{\epsilon_2}{\kappa_2(q, \omega)} + i \frac{\sigma_g(\omega)}{\omega \epsilon_0} = 0$$

$$\frac{\epsilon_1}{\sqrt{q^2 - \epsilon_1 \omega^2 / c^2}} + \frac{\epsilon_2}{\sqrt{q^2 - \epsilon_2 \omega^2 / c^2}} + i \frac{\sigma_g(q, \omega)}{\omega \epsilon_0} = 0$$

Graphene plasmons – DR numerical



$$\frac{\epsilon_1}{\kappa_1(q, \omega)} + \frac{\epsilon_2}{\kappa_2(q, \omega)} + i \frac{\sigma(\omega)}{\omega \epsilon_0} = 0$$

$$\sigma_{\text{intra}}(\omega) = \frac{\sigma_0}{\pi} \frac{4E_F}{\hbar\gamma - i\hbar\omega}$$

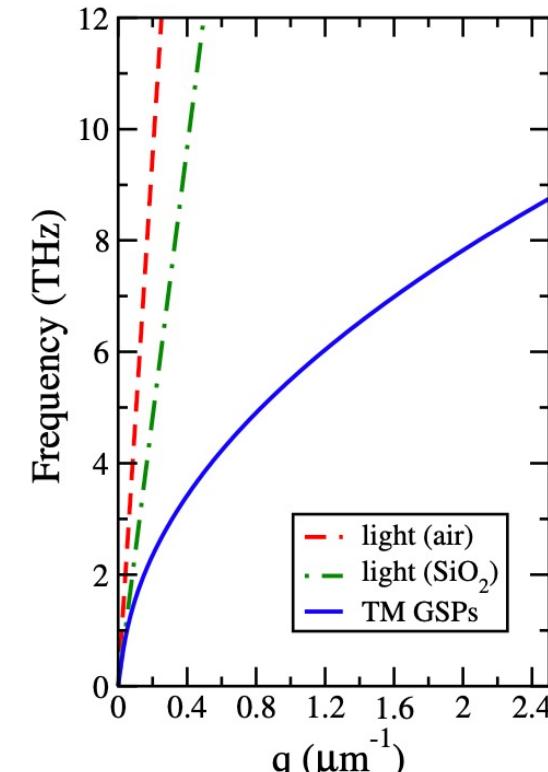
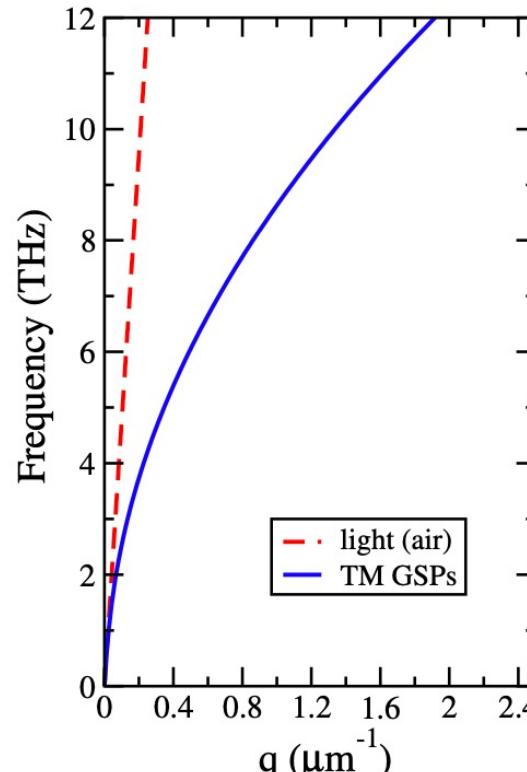
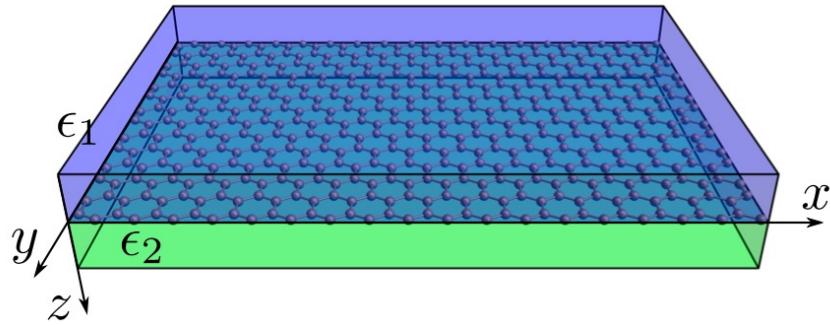


Figure 4.2: Dispersion relation of TM graphene surface plasmon-polaritons (GSPs) in a dielectric/graphene/dielectric heterostructure, as depicted in Fig. 4.1. Left: free-standing graphene in air; right: air/graphene/SiO₂.

Graphene plasmons – DR analytical



$$\frac{\epsilon_1}{\kappa_1(q, \omega)} + \frac{\epsilon_2}{\kappa_2(q, \omega)} + i \frac{\sigma(\omega)}{\omega \epsilon_0} = 0$$

$$\epsilon \equiv \epsilon_1 = \epsilon_2$$

$$1 + i \frac{\sigma(\omega)}{2\omega\epsilon\epsilon_0} \sqrt{q^2 - \epsilon\omega^2/c^2} = 0$$

$$q \gg \sqrt{\epsilon\omega}/c$$

$$q \approx i \frac{2\omega\epsilon\epsilon_0}{\sigma(\omega)}$$

$$\sigma_{\text{intra}}(\omega) = \frac{\sigma_0}{\pi} \frac{4E_F}{\hbar\omega - i\hbar\omega} \quad \sigma(\omega) = 4i\alpha\epsilon_0 c \frac{E_F}{\hbar\omega} \quad \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\hbar\omega_{GSP} \approx \sqrt{\frac{2\alpha}{\epsilon} E_F \hbar c q}$$

the plasmon dispersion $\omega_p \propto \sqrt{q}$ is inherent to all two-dimensional electronic systems [Stern (1967)]

With damping: $q' + iq'' \approx \frac{\epsilon\hbar}{2\alpha c} \frac{1}{E_F} (\omega^2 + i\gamma\omega)$

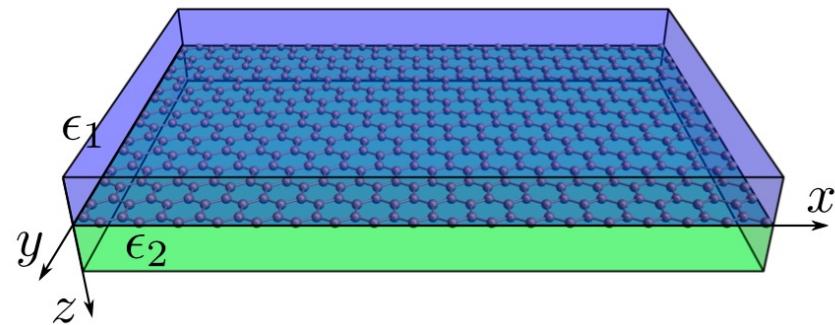
$$\lambda_{GSP} = 2\pi/\Re e\{q\} \quad L_{GSP} = \frac{1}{2q''} = \frac{\alpha}{\epsilon} \frac{\hbar c}{\Gamma} \frac{E_F}{\hbar\omega} \quad \Gamma = \hbar\gamma$$

$$\zeta_{GSP} = \frac{1}{\Re e\{\kappa\}} = \frac{2\alpha}{\epsilon} \hbar c \frac{E_F}{(\hbar\omega)^2} \quad \frac{\lambda_{GSP}}{\lambda_0} = \frac{2\alpha}{\epsilon} \frac{E_F}{\hbar\omega}$$

$\omega/(2\pi) = 10 \text{ THz}$, $E_F = 0.3 \text{ eV}$ and $\epsilon = 4$, we obtain $\lambda_{GSP}/\lambda_0 \approx 0.026$;

In the MIR: $\frac{\lambda_{GSP}}{\lambda_0} = \alpha \approx 1/137$

TE Graphene plasmons



$$\mathbf{E}_j(\mathbf{r}, t) = E_{j,y} \hat{\mathbf{y}} e^{-\kappa_j |z|} e^{i(qx - \omega t)},$$

$$\mathbf{B}_j(\mathbf{r}, t) = (B_{j,x} \hat{\mathbf{x}} + B_{j,z} \hat{\mathbf{z}}) e^{-\kappa_j |z|} e^{i(qx - \omega t)}$$

$$E_{1,y}(x, z)|_{z=0} = E_{2,y}(x, z)|_{z=0},$$

$$B_{2,x}(x, z)|_{z=0} - B_{1,x}(x, z)|_{z=0} = \mu_0 J_y(x, y) = \mu_0 \sigma_{yy} E_{2,y}(x, z)|_{z=0}$$

$$\kappa_1(q, \omega) + \kappa_2(q, \omega) = i\mu_0 \omega \sigma(\omega)$$

only has solutions if and only if
the imaginary part of $\sigma(\omega)$ is negative

$$\epsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

$$\epsilon(\omega) = 1 + \frac{i\sigma(\omega)}{\epsilon_0 \omega}$$

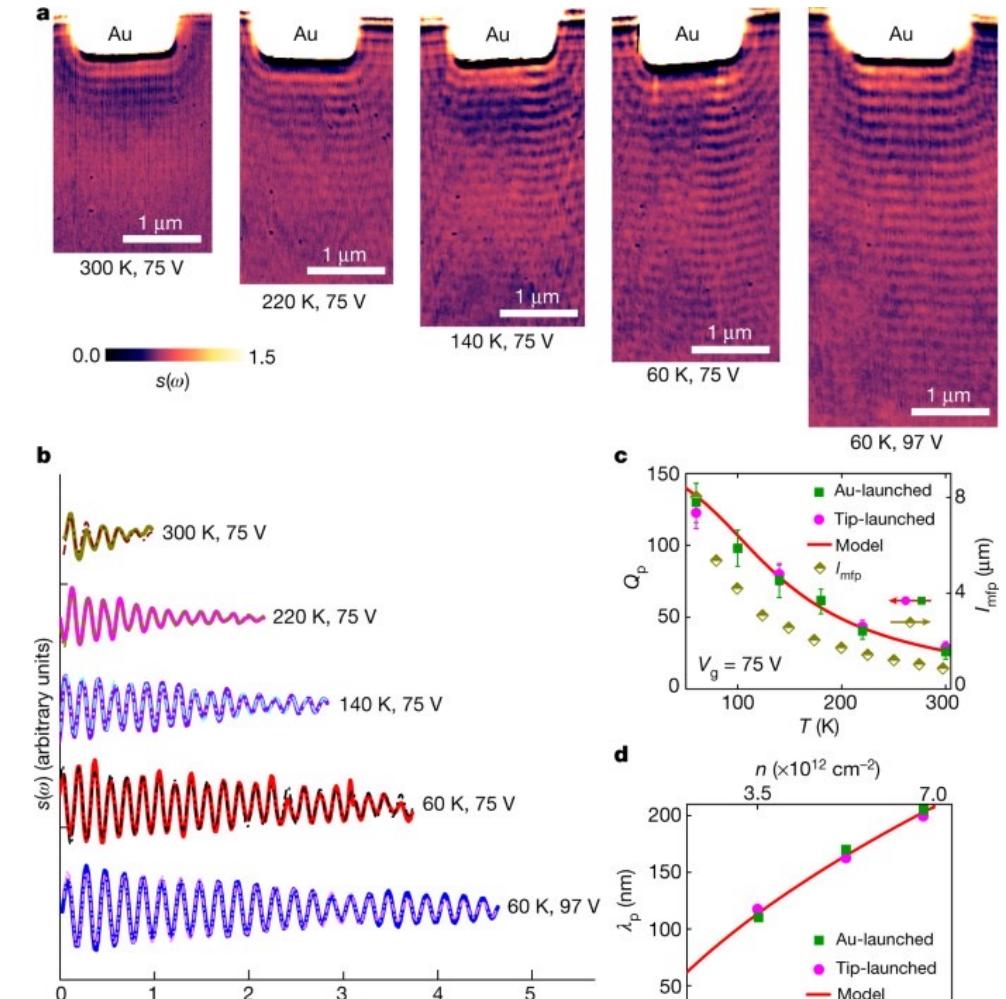
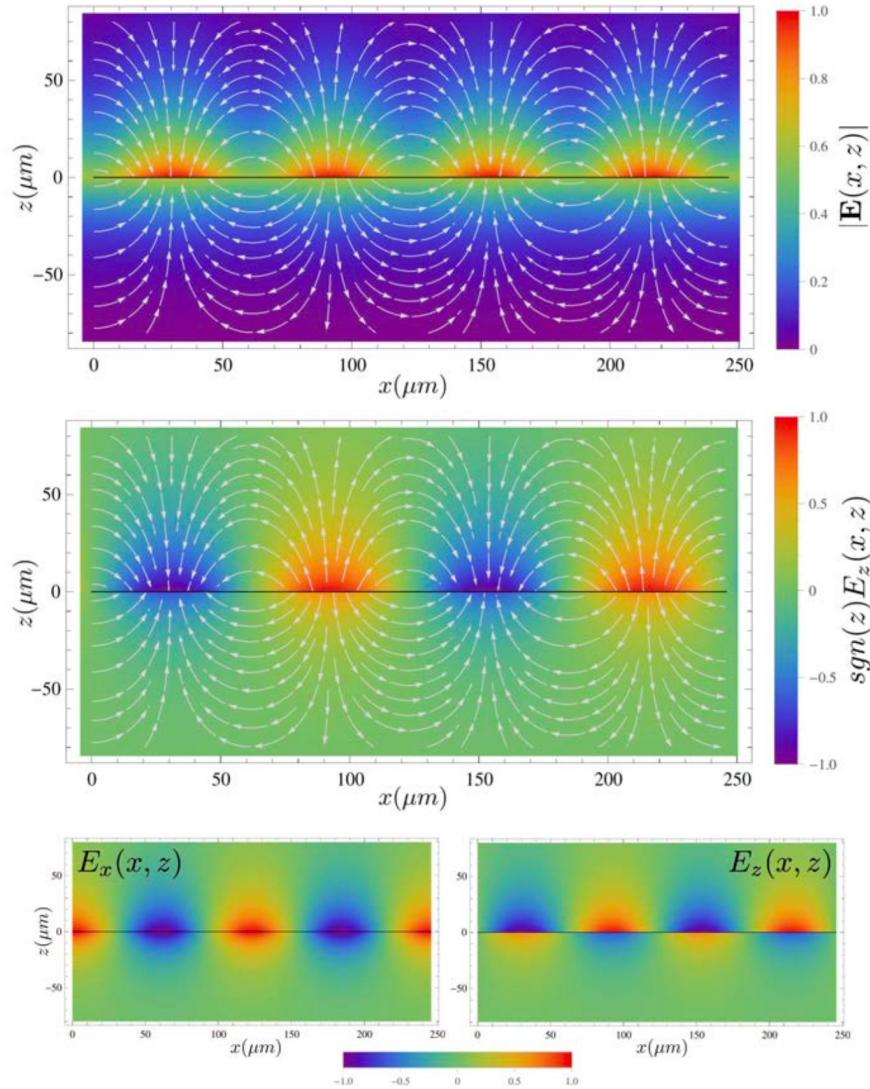
$$\gamma = 1/\tau \quad \sigma_0 = \frac{ne^2\tau}{m} = \omega_p^2 \tau \epsilon_0 \quad \sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \quad \text{Drude model}$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} = \frac{\sigma_0}{1 + \omega^2\tau^2} + i\omega\tau \frac{\sigma_0}{1 + \omega^2\tau^2}.$$

$$\sigma_g(\omega) = \sigma_{\text{intra}}(\omega) + \sigma_{\text{inter}}(\omega)$$

$$\hbar\omega \lesssim \hbar c q / \sqrt{\epsilon}$$

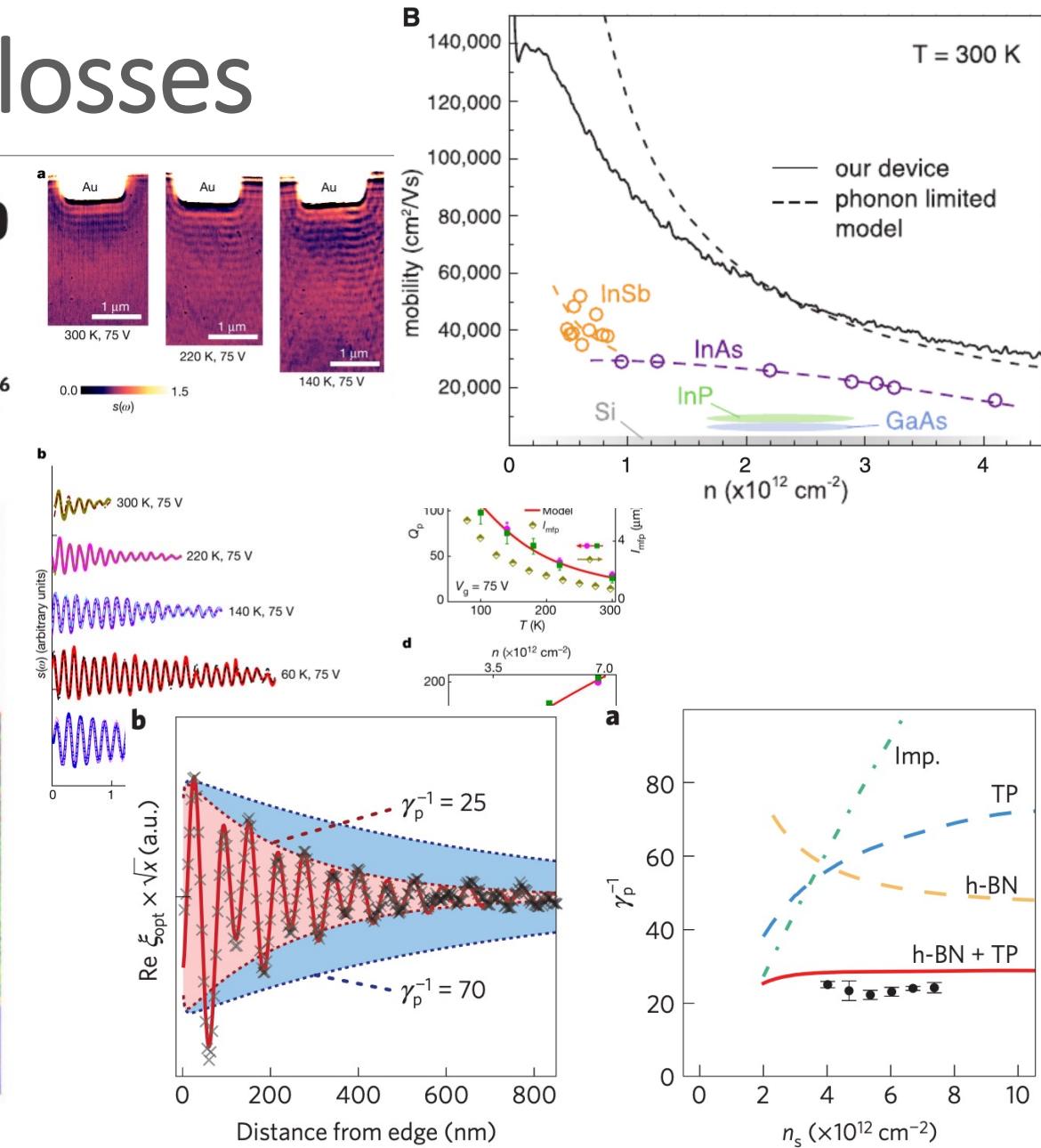
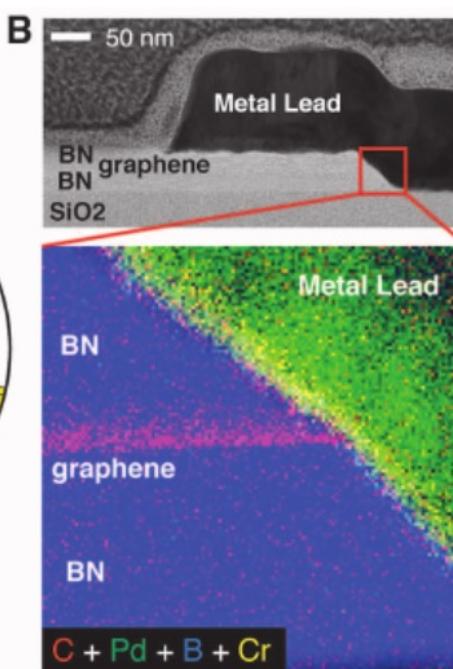
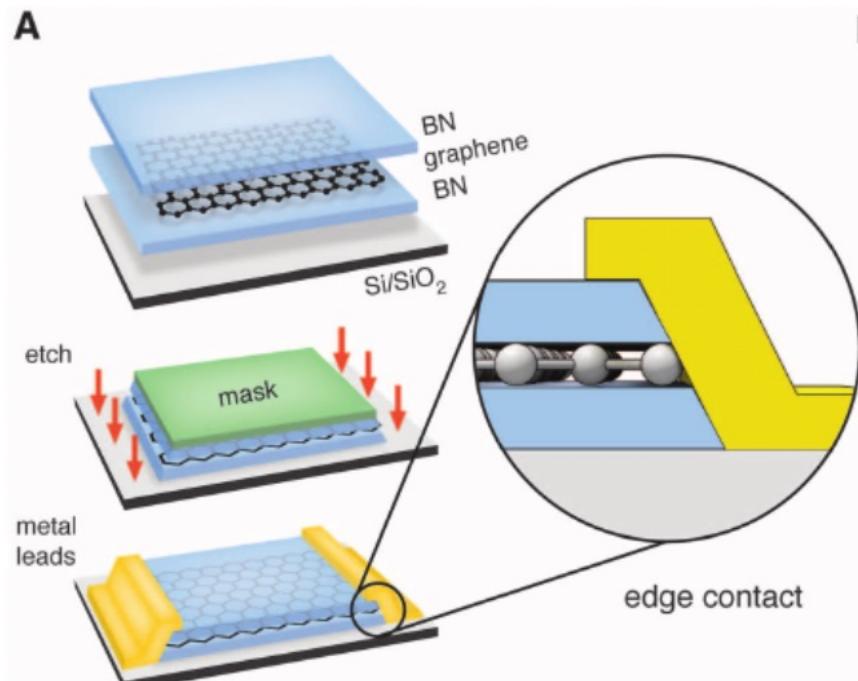
TM Graphene plasmons – fields



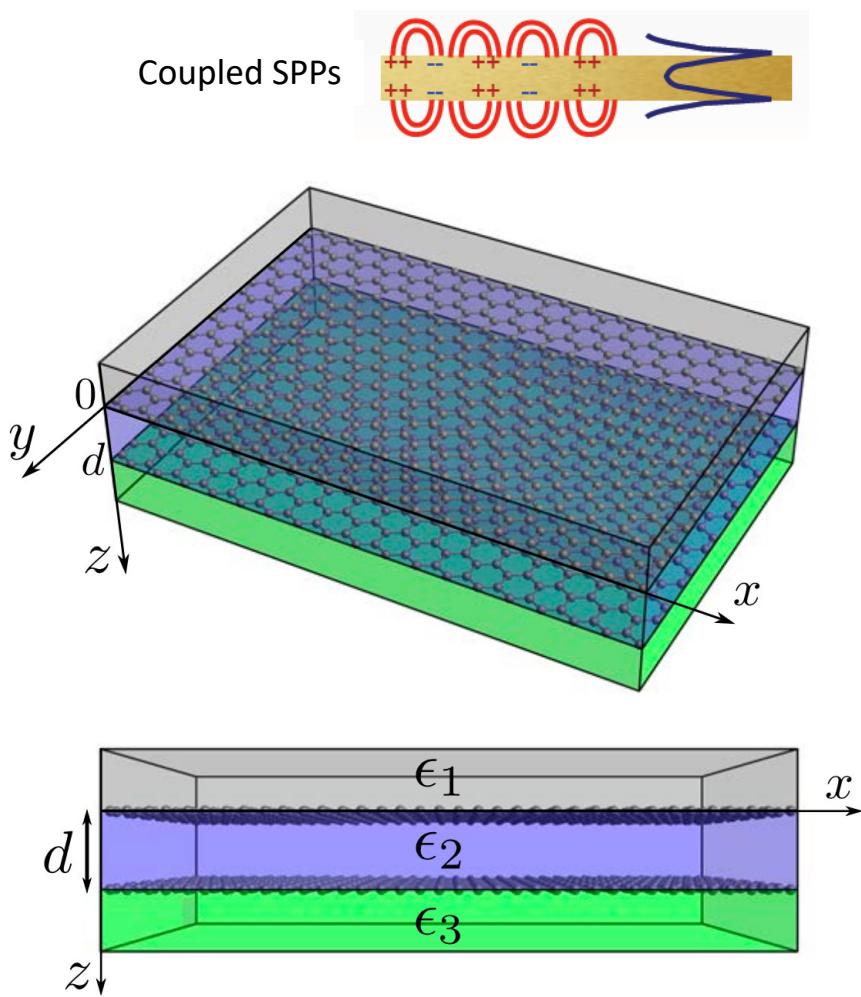
Graphene plasmons – losses

One-Dimensional Electrical Contact to a Two-Dimensional Material

L. Wang,^{1,2*} I. Meric,^{1*} P. Y. Huang,³ Q. Gao,⁴ Y. Gao,² H. Tran,⁵ T. Taniguchi,⁶ K. Watanabe,⁶ L. M. Campos,⁵ D. A. Muller,³ J. Guo,⁴ P. Kim,⁷ J. Hone,² K. L. Shepard,^{1,†} C. R. Dean^{1,2,8,†}

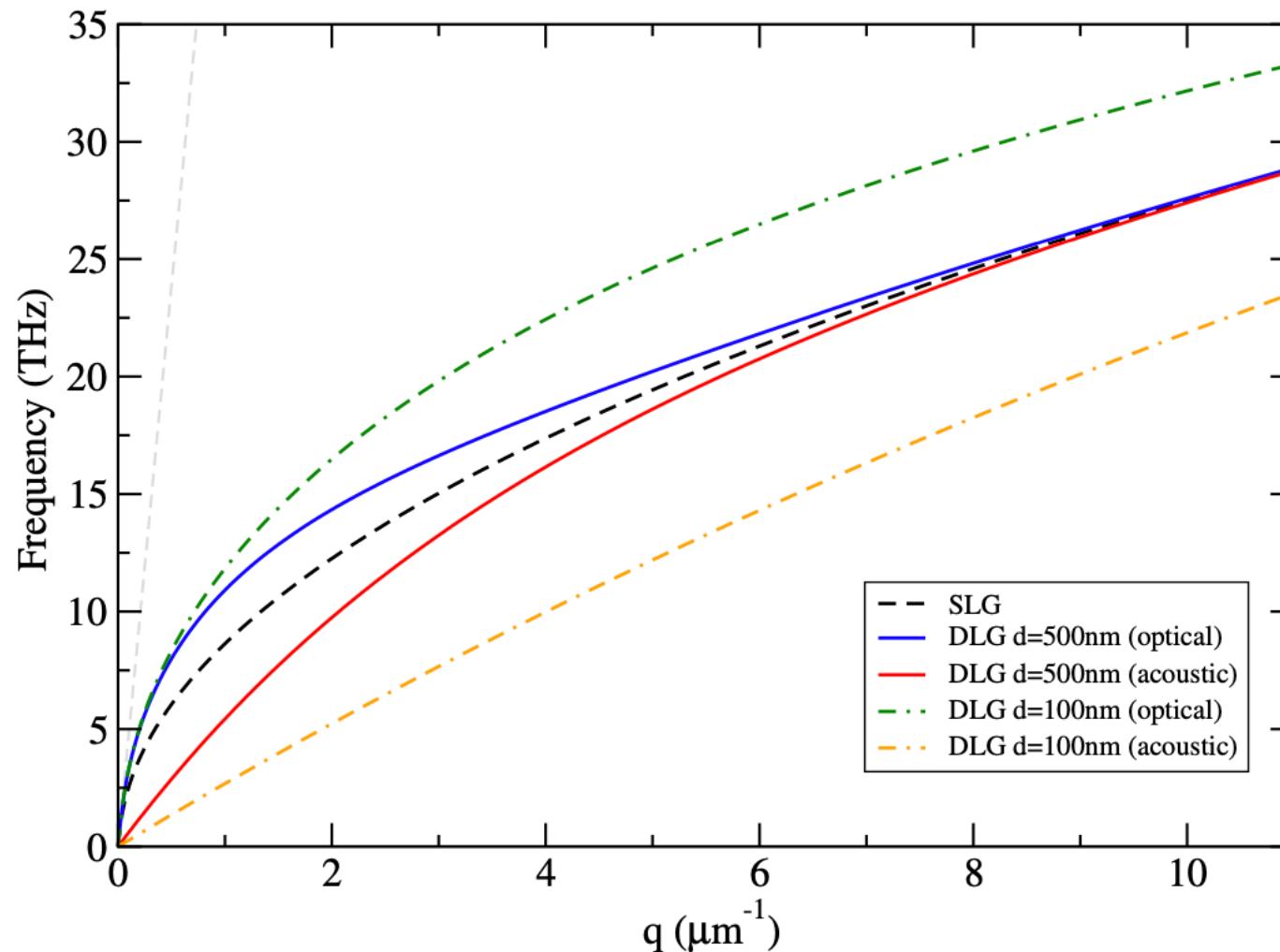


Acoustic graphene plasmons (AGP)

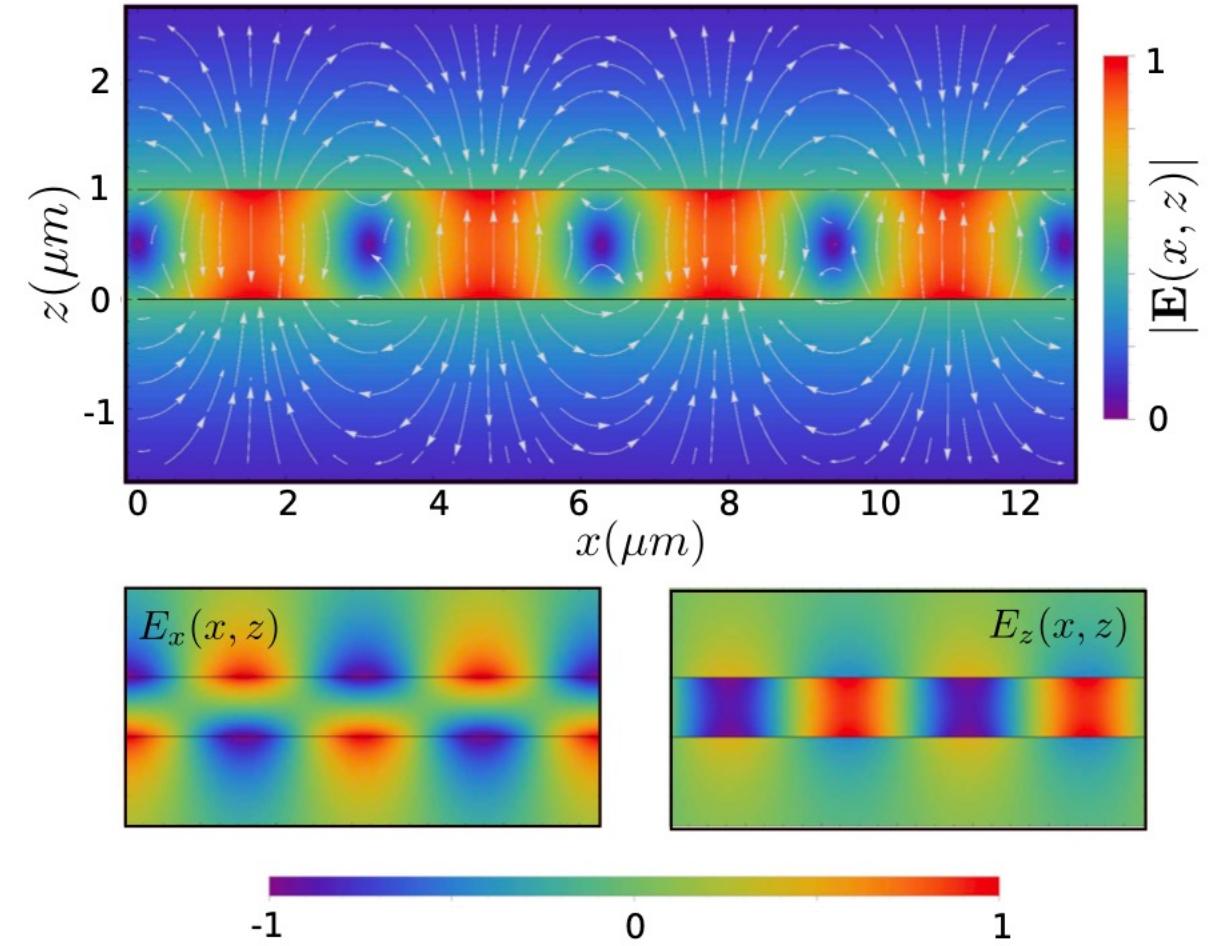
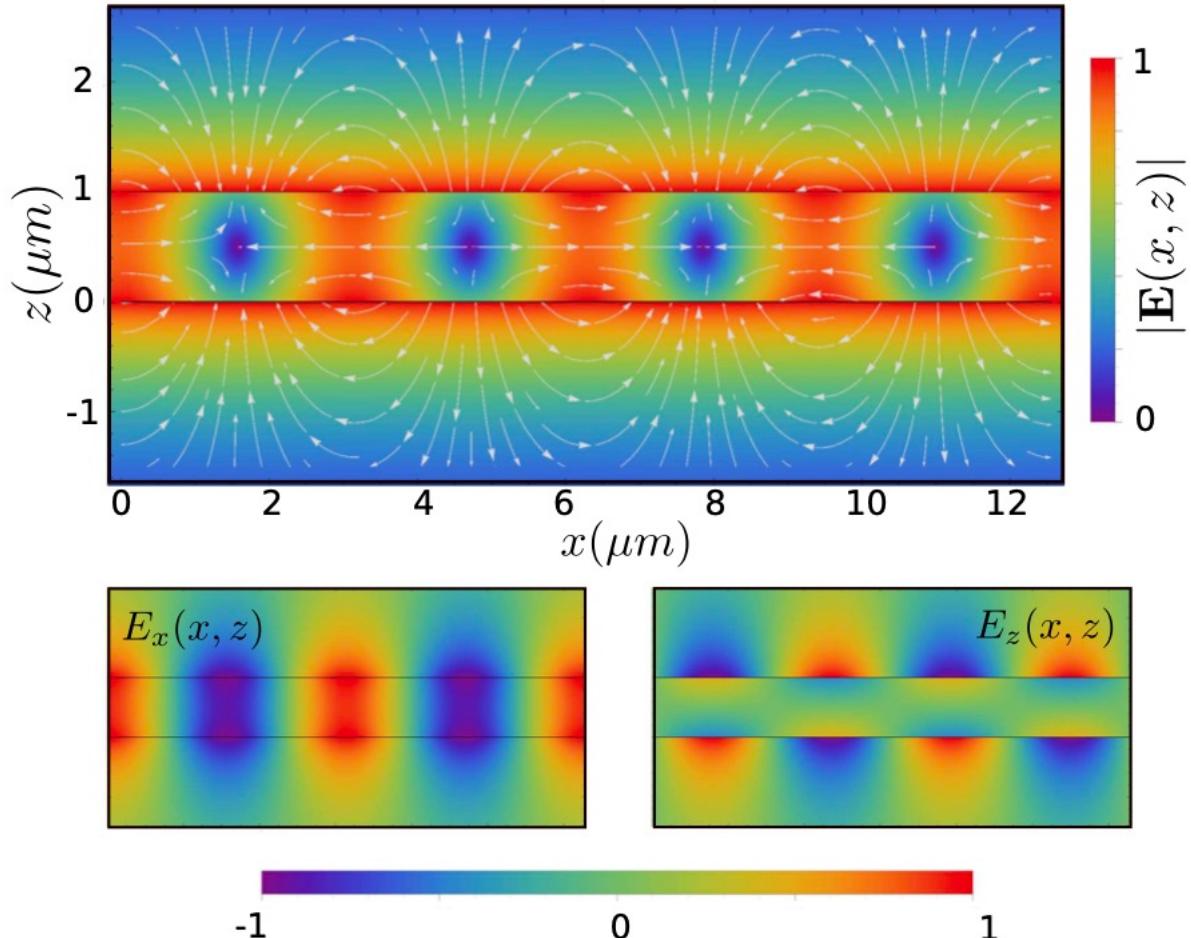


$$\begin{aligned}
 & e^{\kappa_2 d} \left(\frac{\epsilon_3}{\kappa_3} + i \frac{\sigma}{\omega \epsilon_0} + \frac{\epsilon_2}{\kappa_2} \right) \left(\frac{\epsilon_1}{\kappa_1} + i \frac{\sigma}{\omega \epsilon_0} + \frac{\epsilon_2}{\kappa_2} \right) = \\
 & e^{-\kappa_2 d} \left(\frac{\epsilon_3}{\kappa_3} + i \frac{\sigma}{\omega \epsilon_0} - \frac{\epsilon_2}{\kappa_2} \right) \left(\frac{\epsilon_1}{\kappa_1} + i \frac{\sigma}{\omega \epsilon_0} - \frac{\epsilon_2}{\kappa_2} \right) \\
 \kappa_2 d \rightarrow \infty : \quad & \left(\frac{\epsilon_3}{\kappa_3} + \frac{\epsilon_2}{\kappa_2} + i \frac{\sigma}{\omega \epsilon_0} \right) \left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} + i \frac{\sigma}{\omega \epsilon_0} \right) = 0 \\
 \epsilon_1 = \epsilon_3, \quad \kappa_1 = \kappa_3, \quad & \omega_{opt} : \quad \frac{\epsilon_2}{\kappa_2} \tanh(\kappa_2 d/2) + \frac{\epsilon_1}{\kappa_1} + i \frac{\sigma}{\omega \epsilon_0} = 0 \\
 & \omega_{ac} : \quad \frac{\epsilon_2}{\kappa_2} \coth(\kappa_2 d/2) + \frac{\epsilon_1}{\kappa_1} + i \frac{\sigma}{\omega \epsilon_0} = 0 \\
 \epsilon \equiv \epsilon_1 = \epsilon_2 = \epsilon_3, \quad & 2\epsilon + i \frac{q\sigma}{\omega \epsilon_0} (1 \pm e^{-qd}) = 0 \\
 (\hbar\omega_{GSP})^2 \approx \frac{2\alpha}{\epsilon} E_F \hbar c q (1 \pm e^{-qd}) & \\
 qd \rightarrow \infty : \quad & \hbar\omega_{GSP} \approx \sqrt{\frac{2\alpha}{\epsilon} E_F \hbar c q}
 \end{aligned}$$

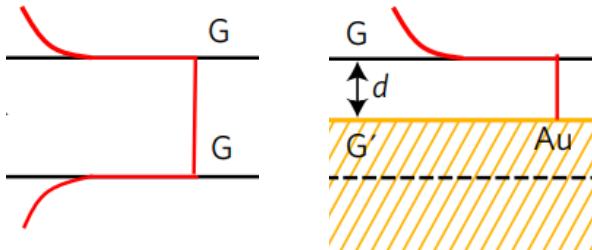
Acoustic graphene plasmons (AGP)



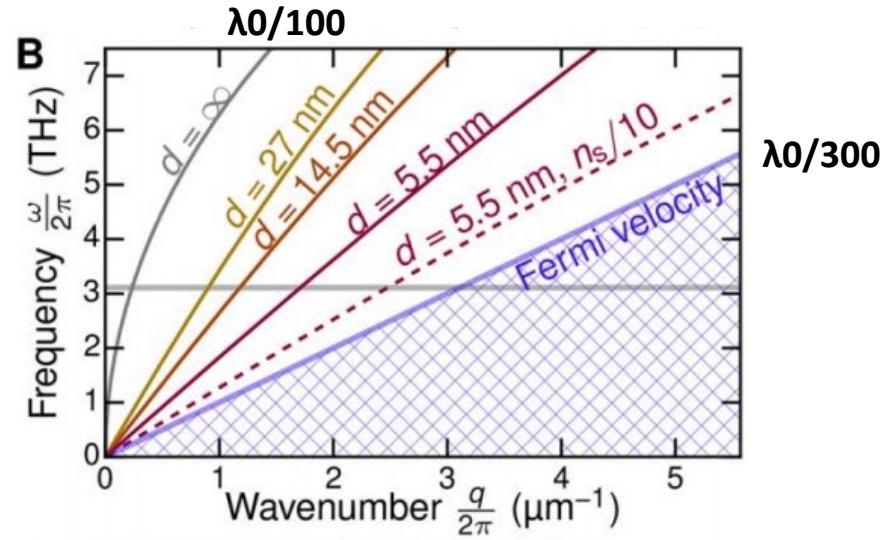
Acoustic graphene plasmons – fields



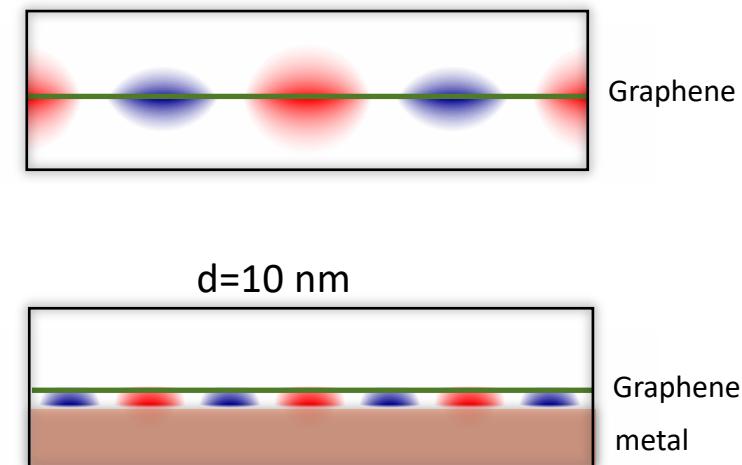
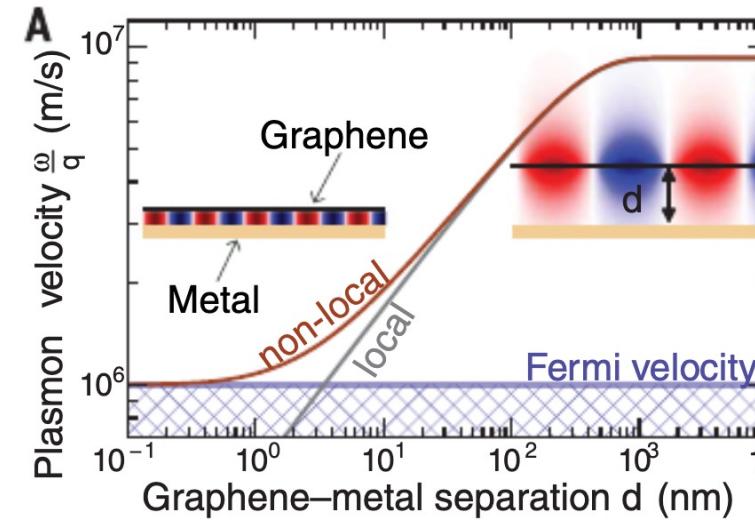
Acoustic graphene plasmons (AGP)



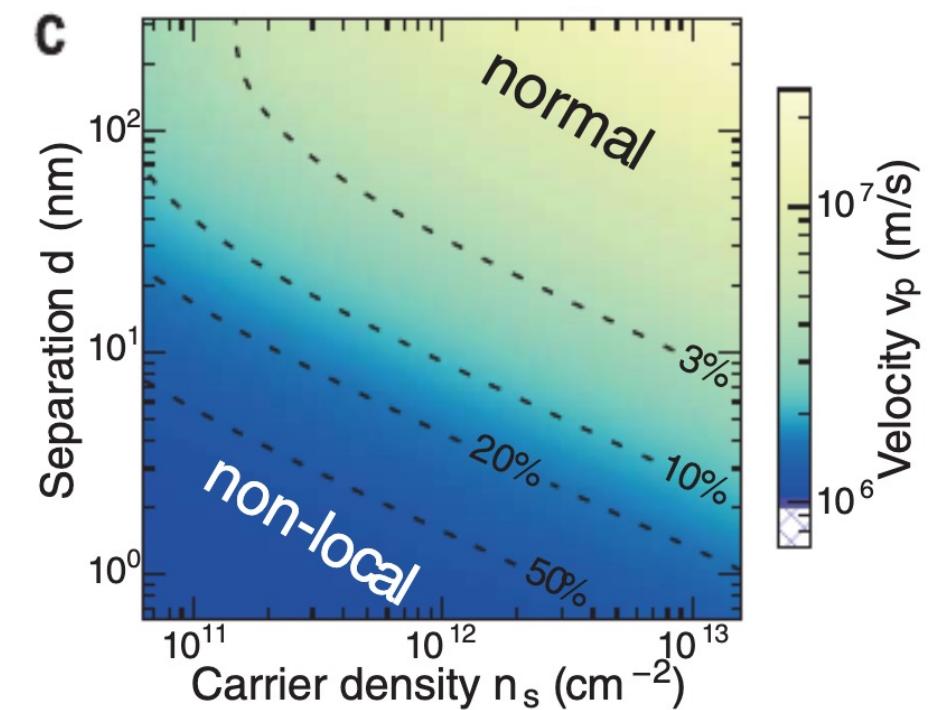
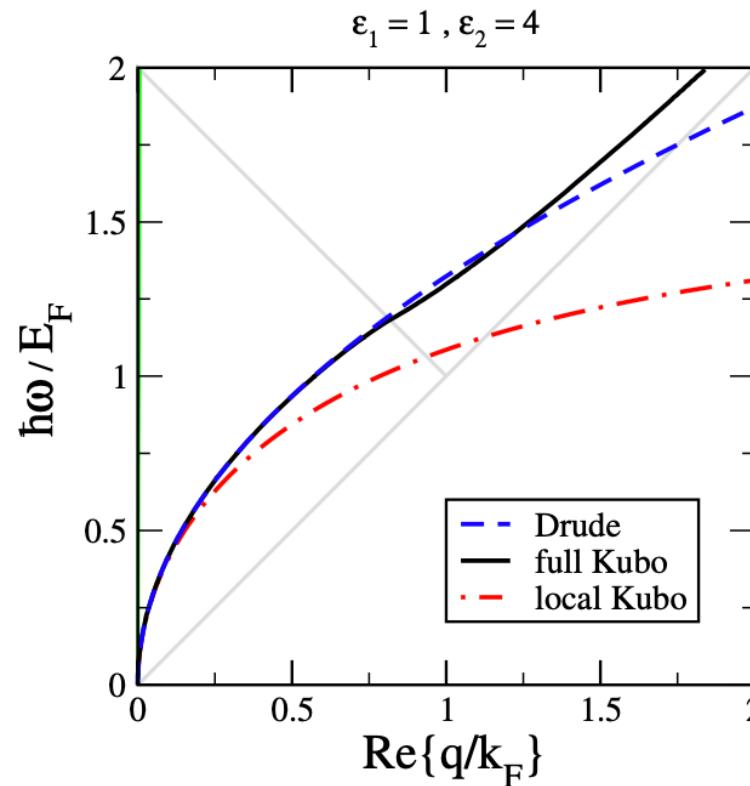
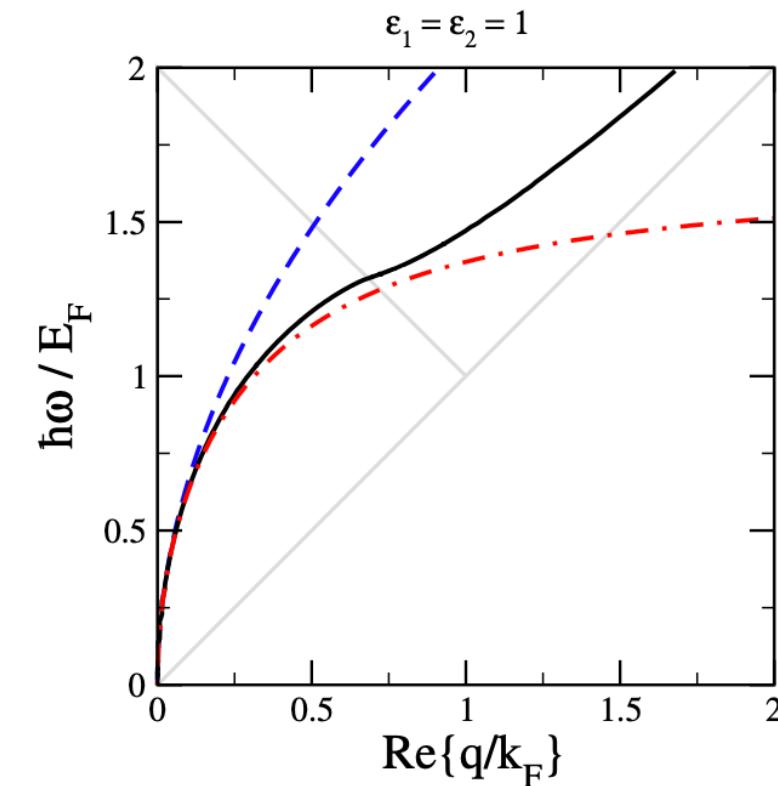
Alonso et al, Nat. Nanotech. (2017)



Lundeberg et al., Science (2017)

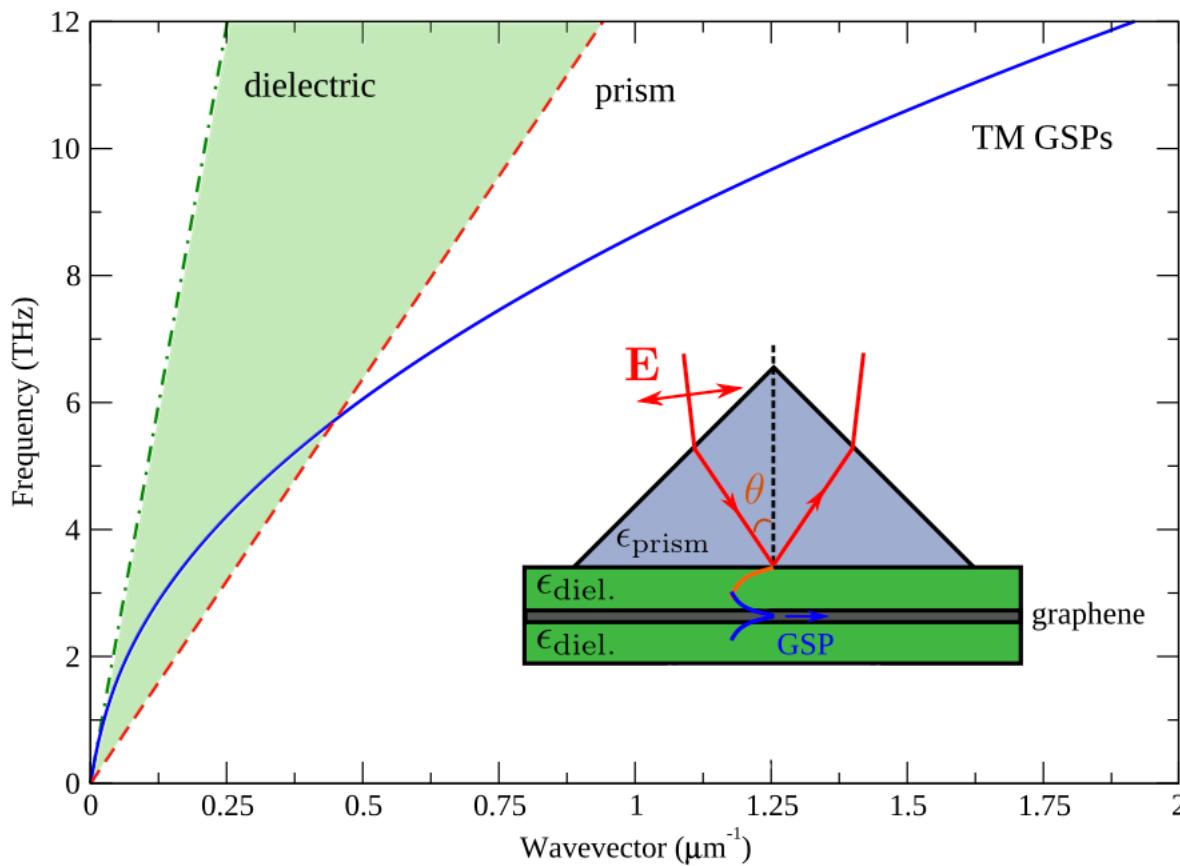


AGP and graphene conductivity

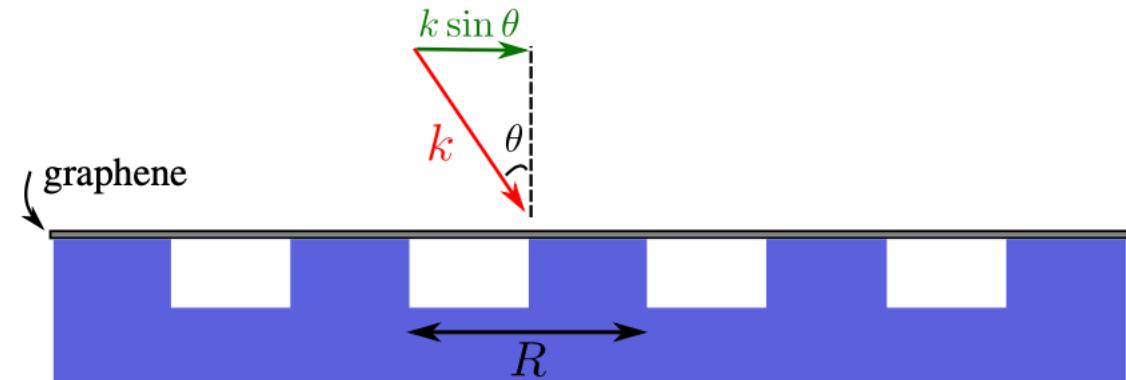


GPs excitation

Prism Coupling

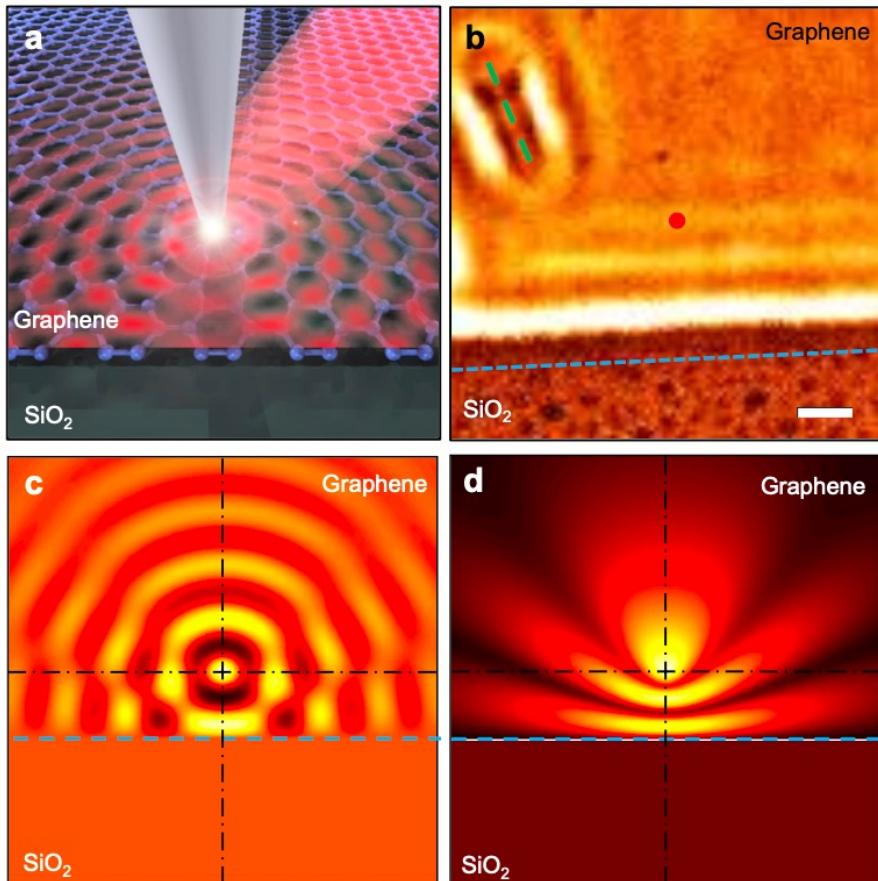


Grating Coupling

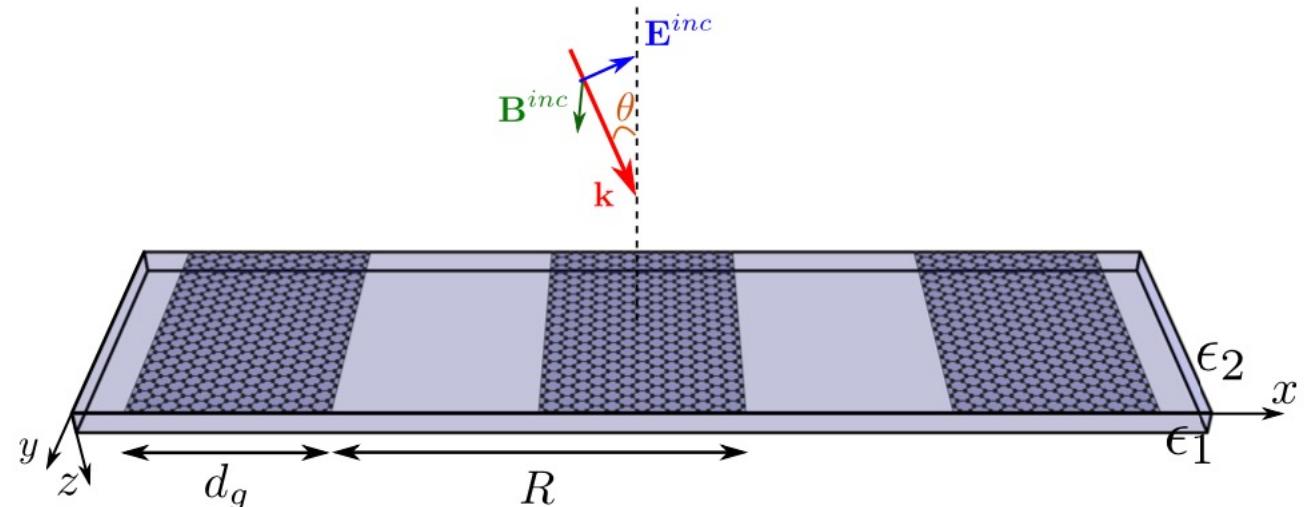


GPs excitation

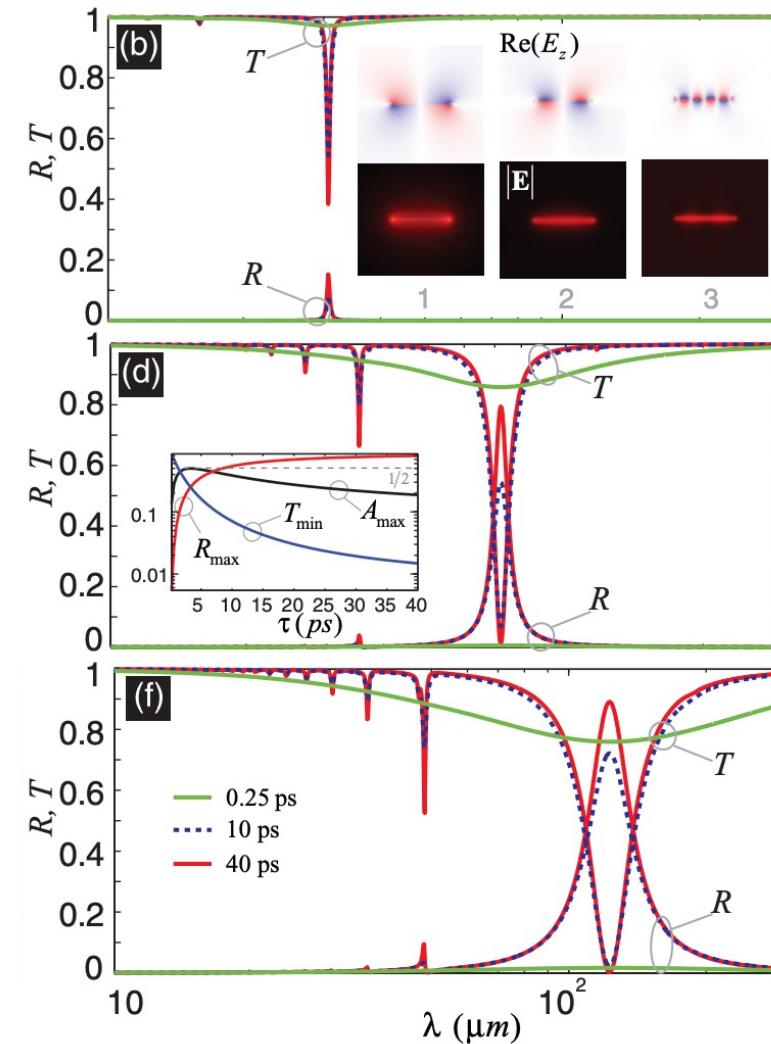
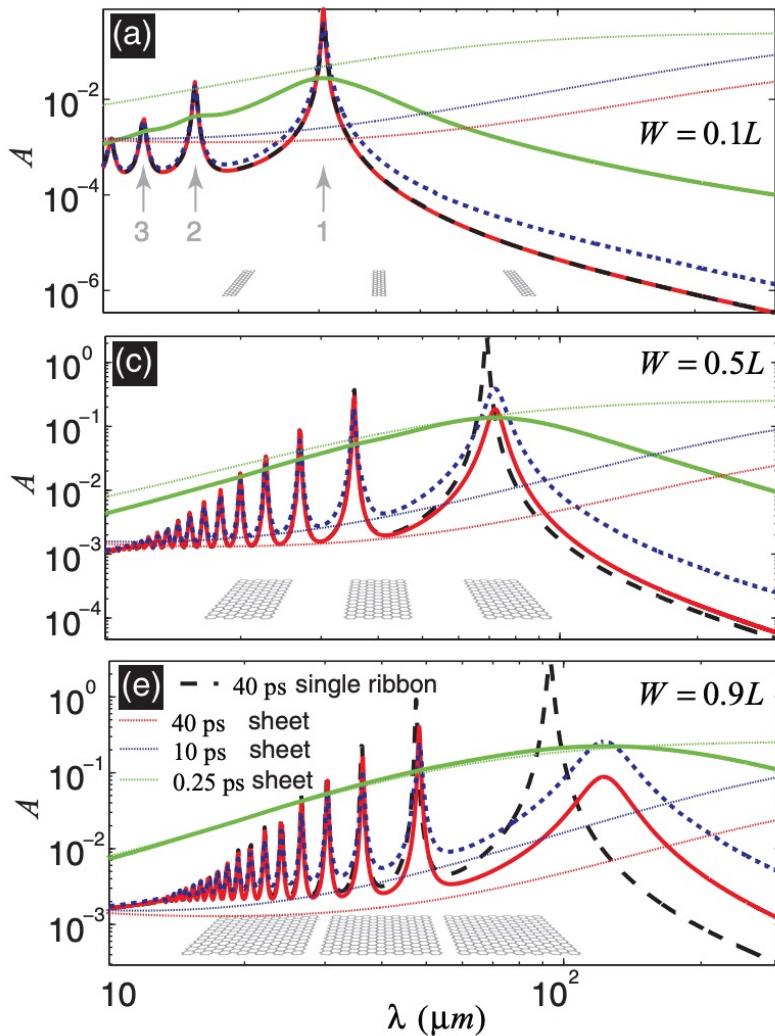
Near-field Excitation and Imaging of GSPs



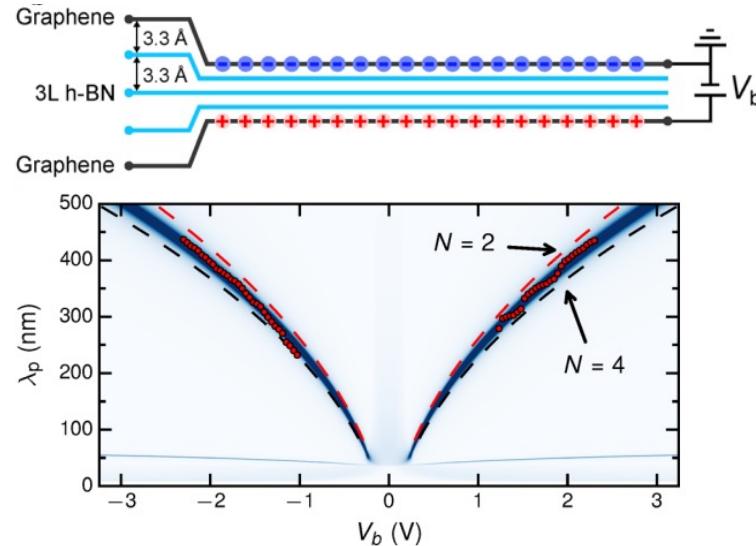
Periodic array of graphene ribbons



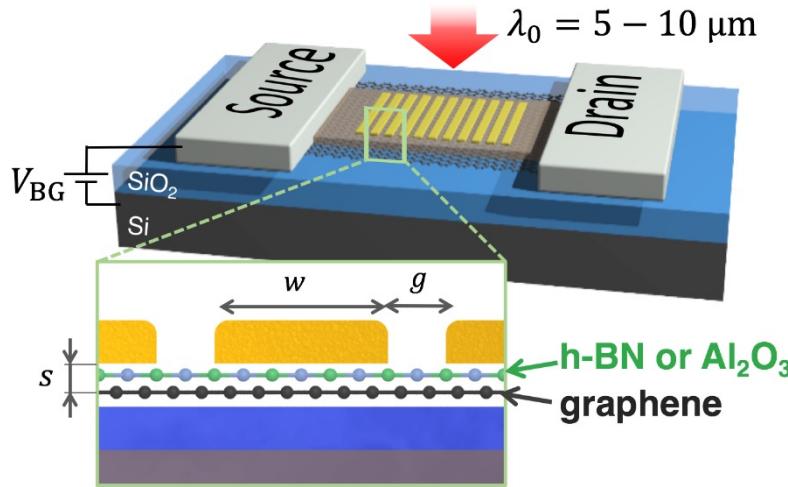
GPs in nanoribbons



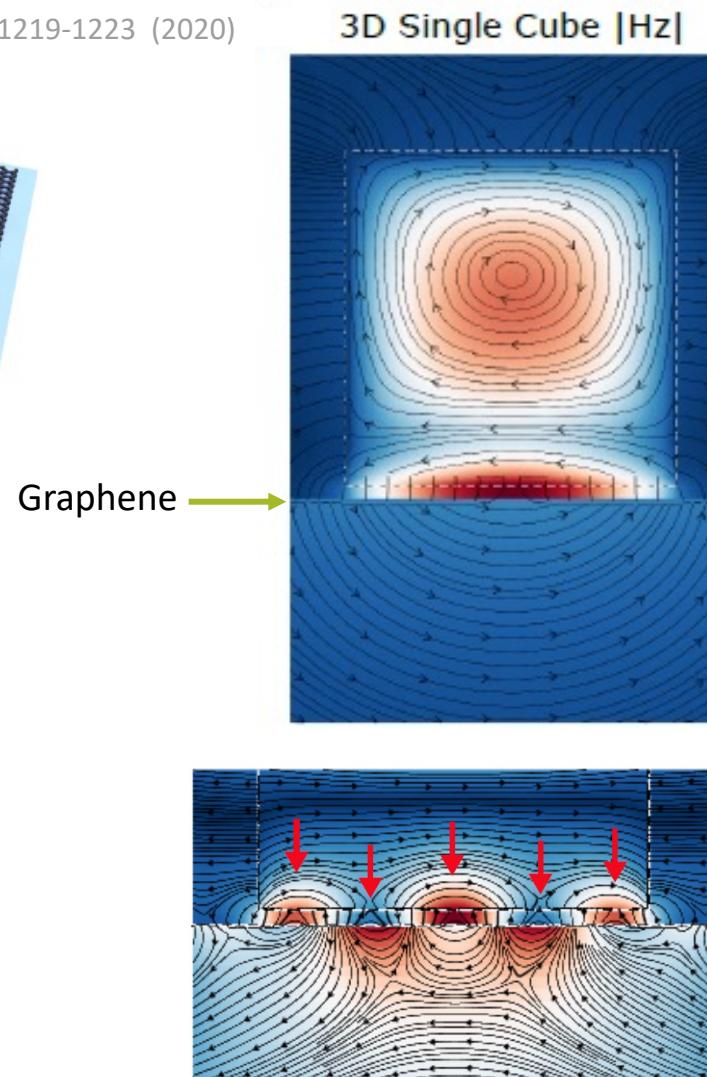
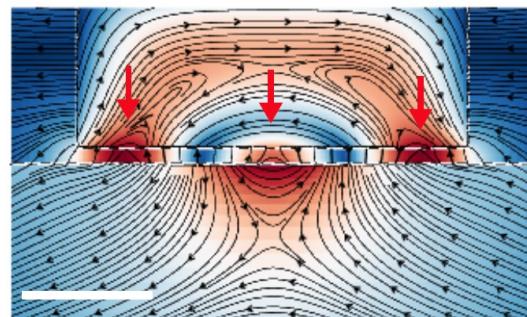
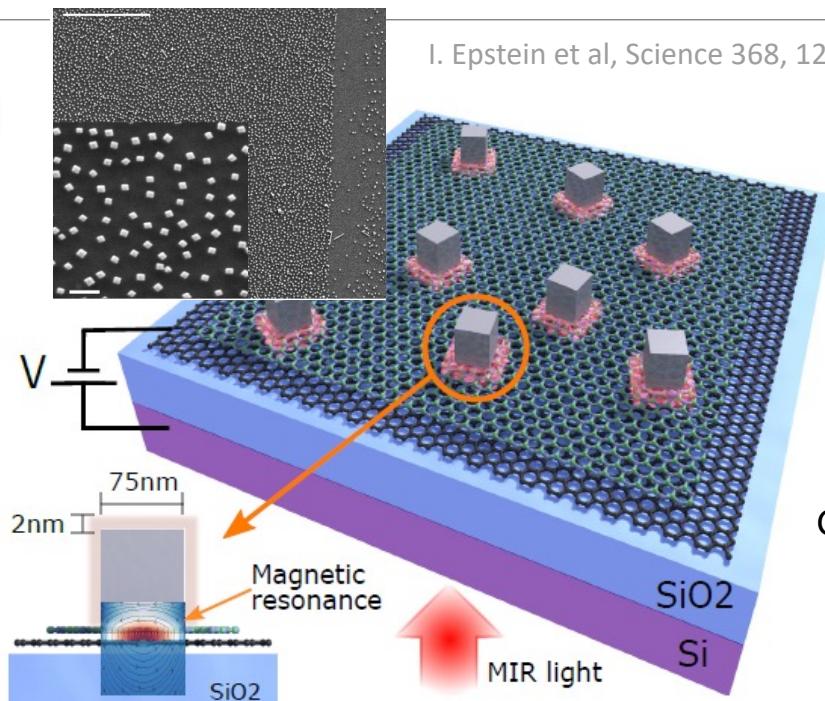
AGPs excitation



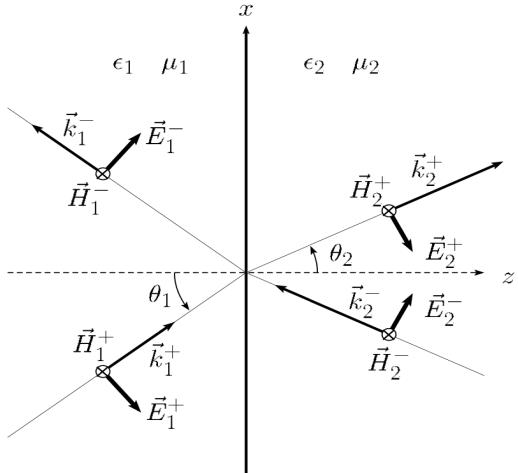
Far-field AGP excitation



Alcaraz, Nanot, Dias, Epstein... et al, Science 360, 291(2018)



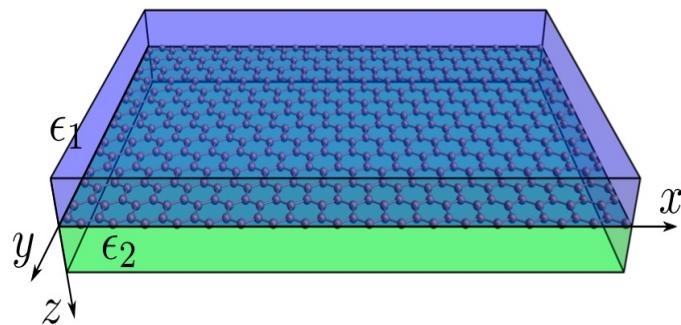
Polaritons, DR and the TMM



$$\tilde{\mathbf{M}}^{(p)} = \frac{1}{2} \begin{pmatrix} 1 + \frac{\epsilon_1}{\epsilon_2} \frac{k_{2z}}{k_{1z}} & 1 - \frac{\epsilon_1}{\epsilon_2} \frac{k_{2z}}{k_{1z}} \\ 1 - \frac{\epsilon_1}{\epsilon_2} \frac{k_{2z}}{k_{1z}} & 1 + \frac{\epsilon_1}{\epsilon_2} \frac{k_{2z}}{k_{1z}} \end{pmatrix} \quad r = \frac{M_{21}}{M_{11}} \quad \epsilon_2 k_{1z} + \epsilon_1 k_{2z} = 0 \quad \frac{k_2}{k_1} = -\frac{\epsilon_2}{\epsilon_1}$$

$$r = \frac{M_{21}}{M_{11}} = \frac{\epsilon_2 k_{1z} - \epsilon_1 k_{2z}}{\epsilon_2 k_{1z} + \epsilon_1 k_{2z}}$$

$$r = \frac{1 - \frac{\epsilon_1 k_{2,z}}{\epsilon_2 k_{1,z}} + \frac{\sigma_g k_{2,z}}{\omega \epsilon_0 \epsilon_2}}{1 + \frac{\epsilon_1 k_{2,z}}{\epsilon_2 k_{1,z}} + \frac{\sigma_g k_{2,z}}{\omega \epsilon_0 \epsilon_2}}$$



$$E_x^{(1)}(z=0) = E_x^{(2)}(z=0) ,$$

$$B_y^{(1)}(z=0) - B_y^{(2)}(z=0) = \mu_0 \sigma_g E_x^{(1)}(z=0),$$

$$A_1 - B_1 = \frac{\epsilon_1 k_{2,z}}{\epsilon_2 k_{1,z}} (A_2 - B_2) ,$$

$$A_1 + B_1 = \frac{\sigma_g k_{2,z}}{\omega \epsilon_0 \epsilon_2} (A_2 - B_2) + A_2 + B_2 .$$

$$1 + \frac{\epsilon_1 k_{2,z}}{\epsilon_2 k_{1,z}} + \frac{\sigma_g k_{2,z}}{\omega \epsilon_0 \epsilon_2} = 0$$

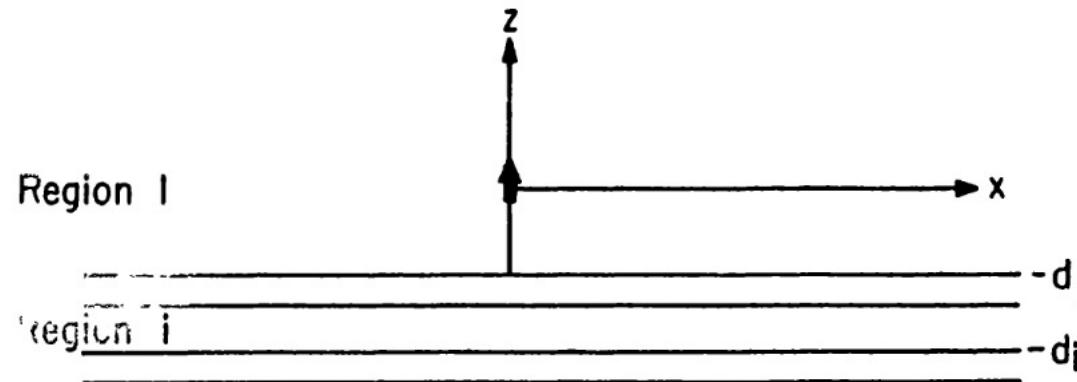
$$k_{j,z} = i \sqrt{q^2 - \epsilon_j \omega^2 / c^2} \equiv i \kappa_j$$

$$1 + \frac{\epsilon_1 \kappa_2}{\epsilon_2 \kappa_1} + \frac{i \sigma_g \kappa_2}{\omega \epsilon_0 \epsilon_2} = 0$$

$$\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} + \frac{i \sigma_g}{\omega \epsilon_0} = 0$$

The poles of r give the DR!!!

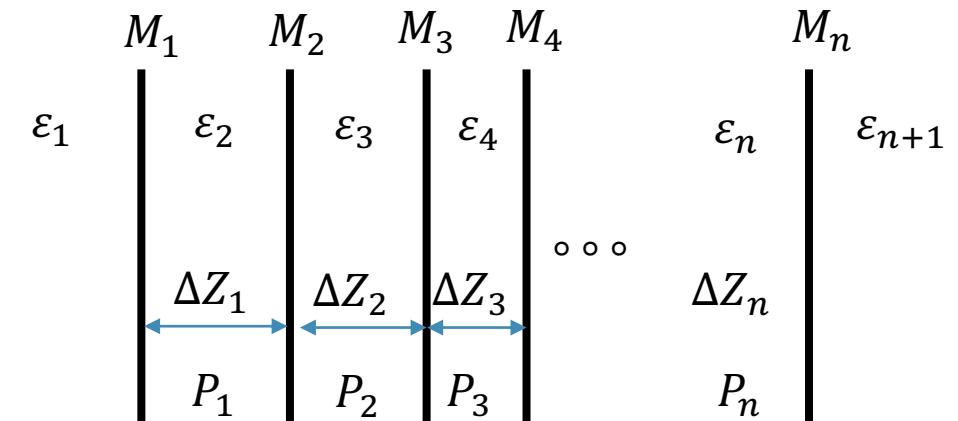
Polaritons, DR and the TMM



$$E_{1z} = \frac{-I\ell}{8\pi\omega\epsilon_1} \int_{-\infty}^{\infty} dk_{\rho} \frac{k_{\rho}^3}{k_{1z}} H_0^{(1)}(k_{\rho}\rho) \left[e^{ik_{1z}|z|} + \tilde{R}_{12}^{TM} e^{ik_{1z}z + 2ik_{1z}d_1} \right],$$

הקטבים של מקדם ההחזרה נתונים לנו
את יחס הדיספרסיה!

יחס הדיספרסיה נותן לנו את כל אופני
התנודה שנתמכים במערכת!



$$\begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix} = TM \begin{pmatrix} H_2^+ \\ H_2^- \end{pmatrix} \quad TM = M_1 P_1 M_2 P_2 \cdots M_{n+1}$$

$$r = \frac{M_{21}}{M_{11}}$$

The loss function $L_r(q, \omega) = \text{Im } \{r\}$

Needs to cover all the relevant
momentum/frequency space