Introduction to Particles and Nuclear Physics

Class Exercise 4

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20/06/2024

Today's Topics:

1. Feynman Diagrams

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$$A \sim M_{fi} = \langle\Psi_f|\mathcal{O}|\Psi_i\rangle \to P \propto A^2 \sim |M_{fi}|^2$$

Feynamn diagrams are graphs that give pictorial descriptions of a way for the state $|\Psi_i\rangle$ to propagate into state $|\Psi_f\rangle$. The full expression of $\mathcal{M}_{fi}$ is an expansion obtained using perturbation theory. Each diagram represents a contribution to $\mathcal{M}_{fi}$ and can be translated to a complex number.

$$M_{fi} = \sum_{i=0}^{\infty} M_{fi}^{(i)} \quad ; \quad M_{fi}^{(i)} \in \mathbb{C}$$

In this course we will not learn how to properly calculate the probabilities using Feynman diagrams. But even without extracting the quantitative expressions, Feynman diagrams are still of great importance, and they'll help us determine which process are allowed, and will give us ways to *qualitatively* compare between different processes and their probabilities.

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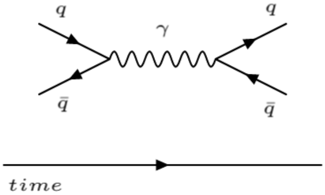

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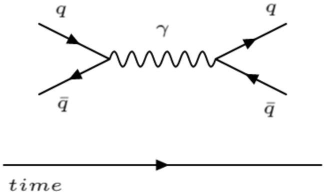
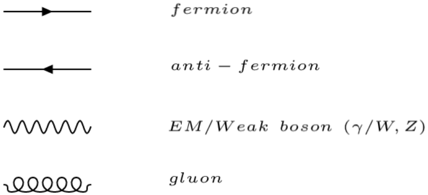


General Feynamn diagrams rules:



In our notation time goes from left to right

External lines: Left side = initial state particles = $|\Psi_i\rangle$, Right side = final state particles = $|\Psi_f\rangle$
Internal lines are called *propagators*

Feynman Diagrams

External lines:

- Represents the initial state (left) and final state particles (right)
- Each such line can be associated with "well-defined" (measurable) 4-momenta p_i^μ.
- These states are often referred to as "on-shell", "asymptotic" or "free" states and represent measureable particles.

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- Also called propagators, or "virtual" particles. They represents "intermediate states".
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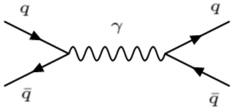
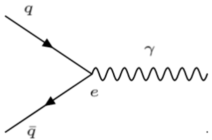
- This can explain how can we create a pair of $q\bar{q}$ from a "massless" photon



Feynman Diagrams

The Vertex:

- This is where particles are created or annihilated.
- Describes the type of interaction, so vertices are interaction-specific.
- It enforces all the conservation laws relevant for that interaction ("what goes in must come out").
  - ("Quantum numbers that enter the vertex = Quantum numbers that come out of the vertex")



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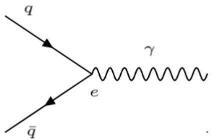

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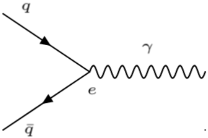

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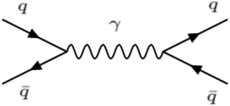
For example for the diagram we saw before, which has two "EM" vertices (that connect to a photon), we can write:

$$|\mathcal{M}| \propto e^2$$



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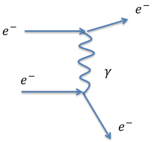
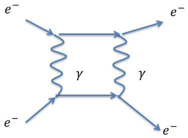
For the following diagrams, which describe an electromagnetic scattering between two electrons, what is the ratio between the contributions to the amplitude? What is the ratio between their "probabilities"?



First order / Leading order ("tree-level")



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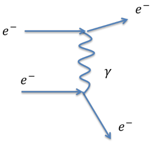

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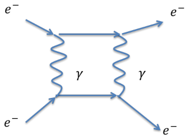

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Solution:

The leading order diagram (left) has two EM vertices, so we denote $|\mathcal{M}_\gamma| \propto e^2$. The loop diagram (right) has four EM vertices so we denote $|\mathcal{M}_{\gamma\gamma}| \propto e^4$. Hence the ratio of amplitudes is $\frac{|\mathcal{M}_{\gamma\gamma}|}{|\mathcal{M}_\gamma|} \approx e^2 \sim \alpha = \frac{1}{137}$

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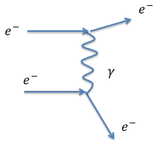

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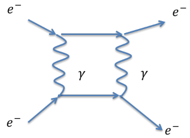

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The probability of a process is proportional to the square of the amplitude. This mean that the second order diagram is suppressed by a factor of $\sim \alpha^2 \approx 10^{-4}$ w.r.t leading order. In principle one should sum all of the diagrams to evaluate the probability of a processes. But this result shows it's sometimes enough to consider only the first order(s) in perturbation theory.

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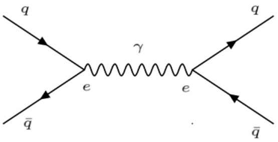
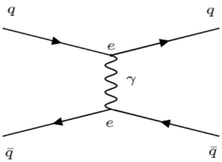


"s-channel"



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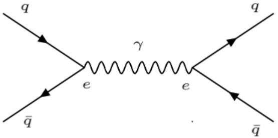

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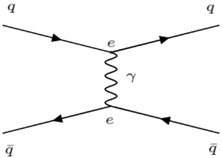

"t-channel"

Quantum Mechanics tells us that the two diagrams are coherent, and in order to evaluate the total amplitude we must sum them.
In general this is true for all orders of perturbation theory (diagrams of all orders), so actually:

$$\mathcal{M}_{fi} = \sum_{l \in Order}^{\infty} \sum_{m_l=0}^{N_l} M_{fi}^{(l,m)} = \sum_{i \in order\ 1} \mathcal{M}_i + \sum_{j \in order\ 2} \mathcal{M}_j + \sum_{k \in order\ 3} \mathcal{M}_k + \ ...$$

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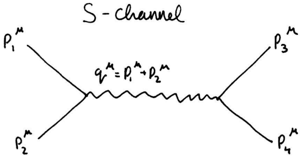
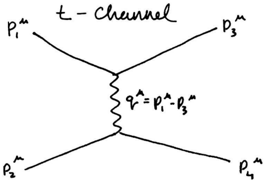
Mandelstam variables

The difference between the two types of processes can be more easily understood in terms of the *Mandelstam variables*.





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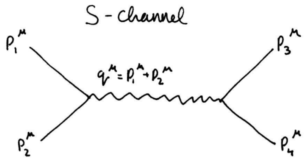

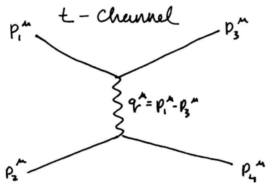

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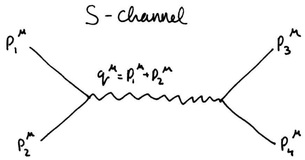

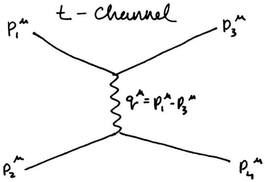

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Note: For s-channel process the total amount of energy available for creating new particles is higher than in t- or u- channels

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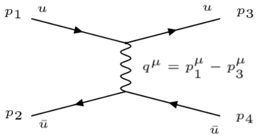

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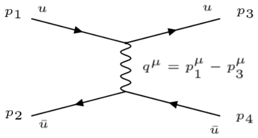
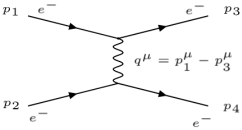


But if we look for example at $e^-e^- \to e^-e^-$ we need to consider both options:



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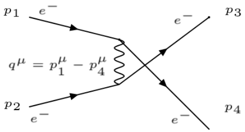

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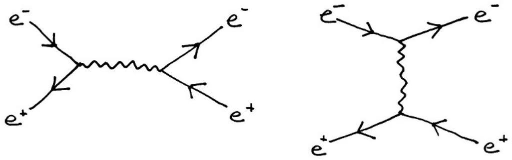

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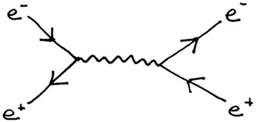

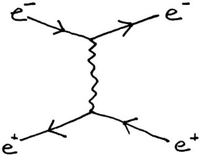

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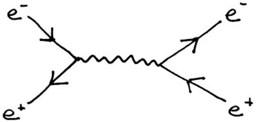

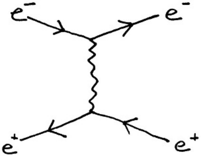

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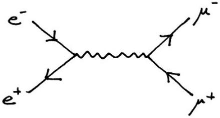
  Here we only have the two s-channel diagrams (via a γ or a Z). a t-channel diagram like in the previous example is not allowed as it would require a vertex connecting e and μ, and this would violate conservation of Lepton family (generation) number in each of the vertices.



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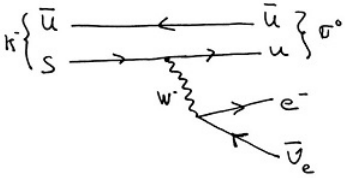
K^- is a meson comprised of $\bar{u}, s$. π^0 is a meson comprised of $\bar{u}, u$[1]. This means that for the process to take place a s quark must transform into an u quark, and this can only happen via the charged weak interaction (with a W^-). So the total flavor is not conserved (which is always the case for processes involving a W). In addition, since this decay involves quarks from different generations u, s , this means the Quark family number is also not conserved.



[1]Actually π^0 is a state of superposition of $\frac{\bar{u}u - \bar{d}d}{\sqrt{2}}$