

Q 1

$$\frac{d\theta}{d\xi} = \frac{\xi \cos \xi - \sin \xi}{\xi^2}$$

$$\theta_1 = \frac{\sin \xi}{\xi}$$

$$\frac{1}{\xi} \cdot \frac{d}{d\xi} \left(\xi \cdot \frac{d\theta}{d\xi} \right) = \left[\cancel{\cos \xi} - \xi \sin \xi - \cancel{\cos \xi} \right] \frac{1}{\xi}$$

$$= - \frac{\sin \xi}{\xi} = -\theta_1'$$

$$m_s = - \frac{4}{3} \pi R_1^3 \rho_c \frac{3}{\xi_1} \frac{\xi_1 \cos \xi_1 - \sin \xi_1}{\xi_1} \quad \xi_1 = \eta$$

$$= \frac{4}{3} \pi R_*^3 \rho_c$$

$$\frac{1}{\xi} \frac{d}{d\xi} \rightarrow - \frac{1}{3} \xi \left(1 + \frac{\xi}{3} \right)^{-3/2}$$

$$\theta_2 = \left(1 + \frac{\xi}{3} \right)^{-1/2}$$

$$- \left(1 + \frac{\xi}{3} \right)^{-3/2} + \frac{1}{3} \xi^{-1} \left(1 + \xi \right)^{-1/2}$$

$$\left(1 + \frac{\xi}{3} \right)^{-1/2} \left(- \left(1 + \frac{\xi}{3} \right) + \frac{1}{3} \xi^{-1} \right) = -\theta_2'$$

Q 2

$$L \propto T^n \frac{R^n}{M} = \frac{M^n}{R^n} \frac{R^n}{M} = M^n$$

$$\curvearrowleft T \propto \frac{M}{R} \text{ n. G. r. l. r. i. } \Rightarrow \text{ r. e. n.}$$

$$P \propto \frac{M^n}{R^n}$$

$$T^n \propto \frac{M^n}{R^n}$$

$$\text{r. r. r. f. P. r. r. r.}$$

$$\Rightarrow L \propto \frac{M^n}{R^n} \frac{R^n}{M} \propto M$$

Q 4

$$n + \frac{7}{2} = \frac{15}{2} = 7.5$$

$$n - 7.5 = 3.5 = -\frac{7}{2}$$

$$\Rightarrow L \propto \frac{R^3}{M} \left(\frac{M}{R}\right)^{\frac{n}{2}} \frac{R^n}{M} = R^{-\frac{1}{2}} M^{11/2}$$

$$L \propto P T^\alpha M = \frac{M^2}{R^3} \left(\frac{M}{R}\right)^\alpha = \frac{M^{2+\alpha}}{R^{3+\alpha}}$$

$$R^{-\frac{1}{2}} M^{11/2} \propto M^{2+\alpha} R^{-\alpha}$$

$$\frac{r+2\alpha}{2} = \frac{-\frac{1}{2} + 2 + \alpha}{2} \propto M^{2+\alpha - \frac{11}{2}} \frac{n+2\alpha-11}{2} = \frac{2\alpha-7}{2}$$

$$R \propto M^{\frac{2\alpha-7}{2\alpha+5}}$$

$$L \propto M^{-\frac{2\alpha-7}{2\alpha+10}} M^{\frac{11}{2}} = M^{\frac{10\alpha+31}{2\alpha+5}}$$

$$\frac{11}{2} - \frac{2\alpha-7}{2\alpha+10} = \frac{11(2\alpha+10) - 4\alpha + 14}{2\alpha+10}$$

$$= \frac{11(2\alpha+5) - 2\alpha + 7}{2\alpha+10} = \frac{20\alpha+62}{2\alpha+10}$$

$$= \frac{10\alpha+31}{2\alpha+5}$$

2) $\alpha = n \Rightarrow L \propto M^6 R^{-7}$

$$L \propto M^{71/13} \Rightarrow M \propto L^{13/71}$$

$$L \propto L^{78/71} R^{-7}$$

$$L^{-\frac{7}{71}} \propto R^{-7}$$

$$R \propto L^{\frac{1}{71}}$$

$$L \propto R^2 T_{\text{eff}}^n = L^{2/71} T_{\text{eff}}^n$$

$$L^{64/71} \propto T_{\text{eff}}^n$$

$$L \propto T_{\text{eff}}^{\frac{n-71}{64}} = L \cdot 116$$

Q5

$$1) L = \tilde{L} f_L(1) = \frac{\alpha c}{\kappa} \left(\frac{mnpG}{\kappa_0} \right)^n M_0^3$$

$$= \frac{7.6 \cdot 10^{15} \cdot 3 \cdot 10^6}{300} \left(\frac{0.7 \cdot 1.7 \cdot 10^{-24} \cdot 6.7 \cdot 10^{-8}}{1.1 \cdot 10^{-16}} \right)^n (2 \cdot 10^{33})^3$$

$$= \frac{7.6 \cdot 3.8}{300} \left(\frac{0.7 \cdot 1.7 \cdot 6.7}{1.1} \right)^n 10^{-5} \cdot 10^{-64} \cdot 10^{99} = 640 \cdot 10^{30}$$

$$= \boxed{6.4 \cdot 10^{32} \frac{\text{erg}}{\text{s}}}$$

$$2) \Sigma = -G \int \frac{mdm}{r} \quad U = \int \frac{3}{2} \kappa_0 T n \rightarrow \frac{dm}{dp}$$

$$r = R f_r(x) \quad x = \frac{r}{M} \quad m = xM \quad dm = dx \cdot M$$

$$3) \Sigma = -G \int \frac{M^2 \cdot x \cdot dx}{R f_r(x)} = -\frac{GM^2}{R} \int \underbrace{\frac{x \cdot dx}{f_r(x)}}_{\alpha_n}$$

$$U = \frac{3}{2} \kappa_0 \int f_r(x) TM x dx = \frac{3}{2} \frac{\kappa_0}{mp} TM \int \underbrace{f_r(x) x dx}_{\alpha_n}$$

$$4) U = -\frac{1}{2} \Sigma \Rightarrow \frac{3}{2} \frac{\kappa_0}{mp} TM \alpha_n = \frac{1}{2} G \frac{M}{R} \alpha_n$$

$$3 \frac{120}{mp} \underbrace{\frac{Mmp}{120} G \frac{M}{R} M \alpha_n}_T = G \frac{M^2}{R} \alpha_n$$

$$\Rightarrow 3M \alpha_n = \alpha \Sigma$$

$$5) E = U + \Sigma \quad U = 3rG \frac{M}{R} \alpha_n = \frac{1}{2} G \frac{M^2}{R}$$

$$= \frac{1}{2} G \frac{M^2}{R}$$

$$= \frac{1}{2} \frac{1}{K} \cdot 6.7 \cdot 10^{-8} \frac{4 \cdot 10^{66}}{100 \cdot 7 \cdot 10^6}$$

$$= 0.6 \cdot 10^{45} \text{ erg}$$

$$\Sigma = -\frac{1}{2} G \frac{M^2}{R} \alpha_n$$

$$= -\frac{1}{2} G \frac{M^2}{R}$$

$$6) L = - \frac{dE}{dt}$$

היכן ש $\dot{R} = \frac{dR}{dt}$

$$= - \frac{dE}{dR} \frac{dR}{dt} = \frac{1}{2} G \frac{M^2}{R^2} \frac{dR}{dt}$$

$$L_M = \frac{GM^2}{u} \frac{dR}{R^2}$$

$$L_t = \frac{GM^2}{c} \frac{1}{R} \Big|_{R(t=0)=100R_0}^{R(t)}$$

$$L_t = \frac{GM^2}{c} \left(\frac{1}{R(t)} - \frac{1}{100R_0} \right)$$

$$\frac{c}{Gm^2} L_t + \frac{1}{100R_0} = \frac{1}{R(t)}$$

$$\frac{c}{Gm^2} \frac{\alpha c}{12} \left(\frac{M \pi G}{120} \right)^4 M_t + \frac{1}{100R_0} = \frac{1}{R(t)}$$

Q6

$$\left(\frac{M}{\delta_n} \right)^{n-1} \left(\frac{R}{\delta_1} \right)^{3-n} = \frac{1}{n\pi} \left(\frac{(n+1)k_n}{G} \right)^n$$

1.3 1' C 1' 1.7

$$\left(\frac{M}{\delta_n} \right)^{n-1} \left(\frac{R}{\delta_1} \right)^{3-n} = \frac{1}{n\pi} \left(\frac{R_0}{\delta_1} \right)^{3-n} \left(\frac{\delta_n}{M_0} \right)^{1-n}$$

5 k_n $k = 1/2$

$$\Rightarrow \left(\frac{M}{M_0} \right)^{n-1} \left(\frac{R}{R_0} \right)^{3-n} = \frac{5}{n\pi}$$

$$\frac{R}{R_0} = \frac{5}{(n\pi)^{\frac{1}{3-n}}} \left(\frac{M}{M_0} \right)^{\frac{1-n}{3-n}}$$

$$n = \frac{3}{1.4} \cdot \left(\frac{M}{M_0} \right)^{\frac{1}{n}} = \frac{3}{1.1} \cdot X^2$$

$$y = \frac{5}{(n\pi)^{\frac{1}{3-n}}} \cdot X^{\frac{1-n}{3-n}}$$

```
| * ex10.py      *
| import numpy as np
| import matplotlib.pyplot as plt
|
| def func(x):
|     n = 3 / (1.4**2) * x**2
|     return (5 / ((4 * np.pi) ** (1 / (3 - n)))) * x ** ((1 - n) / (3 - n))
|
| x = np.linspace(0, 1.4, 500)
|
| plt.plot(x, func(x))
| plt.show()
```

