Introduction to Particles and Nuclear Physics

Class Exercise 1

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Opening Remarks

- Welcome to the first class exercise in "Introduction to Particle and Nuclear Physics"
- Teaching Assistant: Matan Parnas, Kaplun 417, matanparnas@mail.tau.ac.il
- Class exercises will be uploaded to Moodle shortly after the class / video session.
- Home exercises:
  - Will also be uploaded and submitted via Moodle.
  - Each exercise will be graded in the following way:
    - $1 =$ If it's clear that the student put in effort to solve the questions in the exercise, even if the solution is not necessarily correct.
    - $\frac{1}{2} =$ The student put in "partial" effort, or did not attempt to solve the all of the questions in the exercise
    - $0 =$ It's clear that the student put in little to no effort when solving the exercise
  - The final grade in the course is comprised of 80% - final exam grade and 20% - combined score of student's top 9 home exercises out of 11 assignments handed out.

Today's Topics:

1. Natural Units

2. Relativistic Kinematics

Natural Units

In high-energy physics we often use two fundamental physical constants:

- $\hbar = \frac{h}{2\pi} = 1.055 \cdot 10^{-34} J \cdot sec \quad ; \qquad [M]\,[L]^2\,[T]^{-1}$
- $c = 2.998 \cdot 10^{8} \frac{m}{sec} \quad ; \qquad [L]\,[T]^{-1}$

We will chose to work with $\hbar = c = \epsilon_0 = \mu_0 = 1$

When required to, we calculate a numerical result by performing a "dimensional analysis" and multiplying the expression we want to evaluate by the proper combination of $\hbar$, c.

Natural Units

We will often use $[eV]$ ("electron-Volt") units for energy.
eV is defined as the kinetic energy obtain by a particle with the electric charge and mass of an electron under an electric potential of 1 *Volt*.

In Joules: $1eV = 1.6 \cdot 10^{-19} J$

Under this choice of units we have:

- energy
- momentum
- mass

Given in units of eV.

- length
- time

Given in units of eV^{-1}

Question 1:

1. What is $\hbar \cdot c$ in "ordinary" units ?
2. What is $1fm$ in natural units ?
3. Find the period time of the wave function of a free particle with a mass of 1 GeV in "ordinary" units, in the particle's frame.

Solution:

1. $\hbar c = \frac{1.055 \cdot 10^{-34} J \cdot sec}{1.6 \cdot 10^{-19} \ J/eV} \times 2.998 \cdot 10^{8} \frac{m}{sec} \approx 0.2 \ GeV \cdot fm$
   Note: the relation $\hbar c = 0.2 \ GeV \cdot fm$ will be very useful when we'll want to switch from natural to ordinary units (and the other way around) as we'll soon see...
2. We need to express the length of 1 fm in units of energy. In part (1) we saw that $\hbar c = 0.2 \ GeV \cdot fm$. Hence we can write:
   $1 \ fm = \frac{\hbar c}{0.2 \ GeV} = \frac{1}{0.2 \ GeV} = 5 \ GeV^{-1}$
3. The wave-function of a free particle is $\psi \sim e^{-iEt}$ and in "ordinary" units $\psi \sim e^{-i\frac{E}{\hbar}t}$. To find the period time T we will look for when: $\frac{ET}{\hbar} = 2\pi$

$$T = \frac{2\pi \cdot \hbar}{E} = \frac{2\pi \cdot \hbar}{mc^2} = \frac{2\pi \cdot 1.055 \cdot 10^{-34} J \cdot s}{1 GeV}$$
$$= \frac{2\pi \cdot 1.055 \cdot 10^{-34} eV \cdot s}{1.6 \cdot 10^{-19} GeV} = \frac{2\pi \cdot 6.6 \cdot 10^{-25} GeV \cdot s}{GeV} \approx 4 \cdot 10^{-24} sec$$

Relativistic Kinematics

In particle physics we work in a quantum, relativistic regime (quantum field theory, *QFT*), and with inertial frames of references (where Newton's first law holds).
We will now review some basic concepts in special relativity

Relativistic Kinematics

4-vector:
An event is defined by it's 4-coordinates in a given frame of reference S:

$$x^{\mu} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ \vec{r} \end{pmatrix}$$

Note: Sometimes in the literature 4-vectors are labeled simply as x (without the index), whereas "regular" (3-vectors) are labeled: $\vec{x}$ or x_i.

Lorentz Transformation:
To describe the same vector in another frame, S', which is moving with velocity β with respect to S, we use Lorentz Transformation:

$$x^{\mu'} = \Lambda^{\mu}_{\nu}(\beta) x^{\nu}$$

With:

$$\Lambda^{\mu}_{\nu}(\beta) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ; \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Relativistic Kinematics

Minkowski metric: In special relativity we are working in Minkowski space, where the metric $g_{\mu\nu}$ is given by:

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

The scalar product between two 4-vectors a^μ, b^μ in Minkwoski space is given by:

$$a \cdot b = a^\mu g_{\mu\nu} b^\nu = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Note: the scalar (or dot) product is invariant under Lorentz transformation - it's value is the same in all inertial frames.

Relativistic Kinematics

Energy and Momentum:
Let us define the relativistic energy and momentum:

$$E = \gamma m$$
$$\vec{p} = \gamma m \vec{\beta}$$

We can now define the relativistic 4-momentum:

$$p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix} = \gamma m \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix}$$

The 4-momentum is the fundamental kinematic property that's conserved in nature.
Let us look at the norm of the 4-momentum:

$$p^2 = p^\mu p_\mu = E^2 - \vec{p}^2 \equiv m^2$$

Since the scalar product is Lorentz invariant (also called "Lorentz scalar") we can find the mass of any particle by calculating the norm of it's 4-momentum in any given frame of reference.

Question 2:

1. A proton is traveling at $\beta = 0.758$ in the lab frame. Find the proton's energy and momentum in the lab frame.
2. Find the proton's energy and momentum in frame S', which is moving at $u = 0.8c$ in a direction perpendicular to the protons velocity in the lab frame.
3. Calculate the proton's mass in frame S' to explicitly show that it's invariant under Lorentz transformation.

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Solution:

1. We'll use in this part the known mass of the proton $m_0 = 938MeV$. Without loss of generality, let the proton motion in the lab frame be in the $\hat{y}$ direction, and let S' motion with respect to the lab frame be in $\hat{x}$ direction (as we're told that it's movement is perpendicular to that of the proton's).
   In the lab frame the proton's 4-momentum is:

$$E = \gamma m_0 = \frac{938MeV}{\sqrt{1 - 0.758^2}} = 1438.09MeV$$
$$\vec{p} = (0, \gamma m_0 \beta, 0) = 1090.07MeV\hat{y}$$

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2. We'll use Lorentz transformation to find the energy and momentum in S':

$$E' = \gamma (E - up_x) = \frac{1}{\sqrt{1 - 0.8^2}} (1438.09MeV - 0) = 2396.8MeV$$
$$p'_x = \gamma (p_x - uE) = \frac{1}{\sqrt{1 - 0.8^2}} (0 - 0.8 \cdot 1438.09MeV) = -1917.45MeV$$
$$p'_y = p_y = 1090.07MeV$$
$$p'_z = p_z = 0$$

Solution:

1.
2. We'll use Lorentz transformation to find the energy and momentum in S':

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$$p'_y = p_y = 1090.07MeV$$
$$p'_z = p_z = 0$$

3. We'll use the values we found for E', $\vec{p'}$ in the relation that holds in every inertial frame:

$$m_0^2 = E'^2 - \left(p'\right)^2 = 2396.8^2 - 1917.45^2 - 1090.07^2 = 879,783.1MeV^2$$
$$m_0 = 937.96MeV \approx 938MeV$$