Solid State
Exercise 3

Alon Ner Gaon

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Question 1

Show that:

$$R_H = \frac{1}{ce}\frac{n_h\mu_h^2 - n_e\mu_e^2}{(n_h\mu_h + n_e\mu_e)^2}.$$

Solution:
Firstly we'll note that there's no net flow in $\hat{y}$:

$$J_y = n_e q_e v_{ye} + n_h q_h v_{yh} = 0.$$

For every charge carier:

$$0 = -\frac{\vec{p}}{\tau} + q\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{H}\right].$$

The equation for $\hat{x}$:

$$mv_x = \tau q E_x + q\frac{\tau}{c}v_y H. \tag{1}$$

And the equation for $\hat{y}$:

$$mv_y = \tau q E_y - q\frac{\tau}{c}v_x H. \tag{2}$$

We'll take v_x from (1) and place in ()

$$mv_y = \tau q E_y - q\frac{\tau}{c}\left(\frac{\tau q}{m}E_x + q\frac{\tau}{cm}v_y H\right)H$$
$$mv_y\left(1 + \left(\frac{\tau q H}{cm}\right)^2\right) = \tau q E_y - \frac{\tau^2 q^2 H}{cm}E_x$$

And if $\omega_c\tau \equiv \frac{qH\tau}{cm} \ll 1$ we'll throw ω_c^2 and leave ω_c^1:

$$v_y = \underbrace{\frac{\tau q}{m}}_{\mu}\left(E_y \pm \omega_c\tau E_x\right).$$

We'll note that ω_c is positive or negative depending on the sign of q.

Now we'll put v_y for every charge carier in the expression for J_y:

$$J_y = n_e q_e \mu_e \left(E_y - \omega_c \tau_e E_x\right) + n_h q_h \mu_h \left(E_y + \omega_c \tau_h E_x\right) = 0$$
$$\Rightarrow E_y e \left(n_e \mu_e + n_h \mu_h\right) = \frac{He}{c} E_x \left(n_h \mu_h^2 - n_e \mu_e^2\right).$$

Again, if $\omega_c \tau \ll 1$:

$$v_x = \pm \mu E_x.$$

And:

$$J_x = n_e q_e v_{xe} + n_h q_h v_{xh}$$
$$\Rightarrow E_x = \frac{1}{e\left(n_h \mu_h + n_e \mu_e\right)} J_x$$

So finally:

$$E_y e \left(n_e \mu_e + n_h \mu_h\right) = \frac{He}{c} \frac{1}{e\left(n_h \mu_h + n_e \mu_e\right)} J_x \left(n_h \mu_h^2 - n_e \mu_e^2\right)$$
$$\Rightarrow R_H \equiv \frac{E_y}{HJ_x} = \frac{1}{ce} \frac{n_h \mu_h^2 - n_e \mu_e^2}{\left(n_h \mu_h + n_e \mu_e\right)^2}$$

Question 2

Solution:

$$n = \frac{1}{23} \frac{mol}{gr} \cdot 1 \frac{gr}{cm^3} = \frac{N_a}{23} \frac{particles}{cm^3} = \frac{N_a \cdot 10^6}{23} \frac{1}{m^3}.$$

$$[J] = \frac{qm}{m^3 s}$$
$$[I] = \frac{q}{s}$$
$$\Rightarrow I = AJ$$

$$E_y = R_H H J_x$$
$$\Rightarrow \underbrace{A}_{L^2} E_y = R_H H I$$
$$\underbrace{LE_y}_{V_y} = \frac{R_H H I}{L} = \frac{23}{5eN_a \cdot 10^3} \approx \boxed{-4.7675 \cdot 10^{-8} \quad V}.$$

Question 3

Solution:
The two surfaces genarate a field in the same diraction, so the total field is $E = 4\pi\sigma$.

$$\begin{aligned} F &= qE \\ m\ddot{d} &= -e4\pi\sigma = -e4\pi ned \\ \ddot{d} &= -\underbrace{\frac{4\pi ne^2}{m}}_{\omega_p^2} d \end{aligned}$$

Which means d oscillates with angular velocity ω_p.

Question 4.1

Solution:
We'll write the equation of motion for the electron:

$$0 = -\frac{\vec{p}}{\tau} - e\frac{1}{c}\vec{v} \times \vec{H}.$$

After a short time there's no magnetic field, so in $\hat{y}$:

$$v_y = \omega_c t v_x.$$

Question 4.2 + 4.3

Solution:
The flow of energy at x in the $\hat{y}$ direction due to the magnetic field:

$$\begin{aligned} J_{Q\ y}^{(magnetic)} &= \frac{1}{2} n v_y \left[\epsilon\left(x - v_x\tau\right) - \epsilon\left(x + v_x\tau\right)\right] \\ &= -\, n v_y v_x \tau \frac{\partial \epsilon}{\partial x} \\ &= -\, n \omega_c v_x^2 \tau^2 \frac{\partial \epsilon}{\partial x} \\ &= -\, \frac{1}{3} n \omega_c v^2 \tau^2 \frac{\partial \epsilon}{\partial T}\frac{\partial T}{\partial x} \\ &= -\, \frac{1}{3} \omega_c v^2 \tau^2 c_v \frac{\partial T}{\partial x} \end{aligned}$$

In the same fashion, the flow of energy due to the temperature gradient:

$$\begin{aligned} J_{Q\ y}^{(\nabla T)} &= \frac{1}{2} n v_y \left[\epsilon\left(y - v_y\tau\right) - \epsilon\left(y + v_y\tau\right)\right] \\ &= -\, n v_y^2 \tau \frac{\partial \epsilon}{\partial y} \\ &= -\, \frac{1}{3} n v^2 \tau \frac{\partial \epsilon}{\partial T}\frac{\partial T}{\partial y} \\ &= -\, \frac{1}{3} v^2 \tau c_v \frac{\partial T}{\partial y} \equiv -\kappa \frac{\partial T}{\partial y}. \end{aligned}$$

But the two flows must nullify each other for the conductor is finite in $\hat{y}$.

$$\begin{aligned}
J_{Q\ y}^{(\nabla T)} &= -J_{Q\ y}^{(magnetic)} \\
-\frac{\cancel{1}}{\cancel{3}}\cancel{v^2}\tau\cancel{c_v}\frac{\partial T}{\partial y} &= \frac{\cancel{1}}{\cancel{3}}\omega_c\cancel{v^2}\tau^{\cancel{2}}\cancel{c_v}\frac{\partial T}{\partial x} \\
\Rightarrow & \boxed{\frac{\partial T}{\partial y} = -\omega_c\tau\frac{\partial T}{\partial x}}.
\end{aligned}$$