1

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{1 + i\frac{\gamma}{\omega}}$$

$$\frac{\gamma}{\omega} \ll 1 \Rightarrow \frac{1}{1+i\frac{\gamma}{\omega}} \simeq 1 - i\frac{\gamma}{\omega}$$

$$\varepsilon(\omega) = 1 - \underbrace{\frac{\omega_p^2}{\omega^2}}_{<1}\left(1 - i\frac{\gamma}{\omega}\right)$$

$\omega > \omega_p$

(א

$$k^2 = \mu\,\varepsilon(\omega)\frac{\omega^2}{c^2}$$

$$= \mu\frac{1}{c^2}\underbrace{\left[\omega^2 - \omega_p^2\left(1 - i\frac{\gamma}{\omega}\right)\right]}_{\omega^2 - \omega_p^2 + i\omega_p^2\frac{\gamma}{\omega}}$$

$$k \approx \underbrace{\sqrt{\frac{\mu}{c^2}(\omega^2 - \omega_p^2)}}_{\equiv k_r}\left(1 + \frac{1}{2(\omega^2-\omega_p^2)}\,i\omega_p^2\frac{\gamma}{\omega}\right)$$

$$k_i \equiv \sqrt{\frac{\mu}{c^2}}\frac{\omega_p^2}{2\underbrace{\sqrt{\omega^2-\omega_p^2}}_{\omega>\omega_p}}\frac{\gamma}{\omega} \in \mathbb{R} > 0$$

(ב

$$\bar{E} = E_0\hat{x}\,e^{i(k_r z - \omega t)}e^{-k_i z}$$

$$\bar{H} = \frac{\mu\omega}{ck}\hat{z}\times\bar{E} = \frac{\mu\omega}{c\,k_r e^{i\frac{k_i}{k_r}}}\hat{y}E_0 e^{i(k_r z - \omega t)}e^{-k_i z}$$

$$= \hat{y}\underbrace{\frac{\mu\omega}{ck_r}}_{E_0\text{ פקטור ל}}E_0\,e^{\overbrace{i\left(k_r z - \omega t - \frac{k_i}{k_r}\right)}^{\text{פאזה}}}e^{-k_i z}$$

וכדי לקחת את הממשי רק צריך לקחת $e^{i\cdots} \to \cos(\cdots)$

$$u(t)=\frac{1}{8\pi}e^{-2k_iz}E_0^2\left[\varepsilon(\omega)\cos^2(k_iz-\omega t)+\frac{\mu^2\omega}{ck_r}\cos^2\left(k_iz-\omega t-\frac{k_i}{k_r}\right)\right]$$ (ב

ממוצע על זמן מחזור "יתן $\frac{1}{2}$ לכל משהו:

$$\Rightarrow\quad \langle u\rangle=\frac{1}{16\pi}e^{-2k_iz}E_0^2\left[\varepsilon(\omega)+\frac{\mu^2\omega}{ck_r}\right]$$

$$\bar S=\bar D\times\bar H$$ (ג

$$=\hat z\,e^{-2k_iz}E_0^2\,\varepsilon(\omega)\frac{\mu\omega}{ck_r}\,e^{i\left(2k_rz-2\omega t-\frac{k_i}{k_r}\right)}$$

n^2

$$\langle\bar S\rangle=\hat z\,e^{-2k_iz}E_0^2\,n^2\frac{\omega}{2ck_r}$$

(א 2

$$m\bar a=-e\bar E(t)-e\frac{\bar v}{c}\times(\bar B_0+\cancel{\bar B})-m\gamma\bar v$$

זניח
כי קטן

$$\bar r(t)=\mathrm{Re}\left(\bar r_0e^{-i\omega t}\right)$$

נגזר

$$(-i\omega)^2=-\omega^2$$

$$-\omega^2m\bar r_0=-e\bar E+i\frac{e}{c}\omega\,\bar r_0\times B_0\hat z+im\omega\gamma\bar r_0$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\times\begin{pmatrix}0\\0\\B_0\end{pmatrix}=B_0\begin{pmatrix}y\\-x\\0\end{pmatrix}$$

$$e\bar E=\omega^2m\bar r_0+i\frac{e}{c}\omega\,\bar r_0\times B_0\hat z+im\omega\gamma\bar r_0$$

$$\frac{e}{m}\bar E=\omega^2\begin{pmatrix}r_x\\r_y\\r_z\end{pmatrix}+i\omega_B\omega\begin{pmatrix}r_y\\-r_x\\0\end{pmatrix}+i\gamma\omega\begin{pmatrix}r_x\\r_y\\r_z\end{pmatrix}$$

$$\frac{e}{m}E_z=(\omega^2+i\gamma\omega)z(t)$$

$$\frac{e}{m}\begin{pmatrix}E_x\\E_y\end{pmatrix}=\begin{pmatrix}\omega^2+i\gamma\omega & i\omega\omega_B\\-i\omega\omega_B & \omega^2+i\gamma\omega\end{pmatrix}\begin{pmatrix}r_x\\r_y\end{pmatrix}\qquad \omega_B=\frac{eB_0}{mc}$$

$$\begin{pmatrix}a&b\\c&d\end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}\qquad\frac{1}{(\omega^2+i\gamma\omega)^2-(i\omega\omega_B)(-i\omega\omega_B)}$$

$$-\omega^2\omega_B^2$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{e}{m} \frac{1}{(\omega^2+i\gamma\omega)^2-\omega^2\omega_B^2} \begin{pmatrix} \omega^2+i\gamma\omega & -i\omega\omega_B \\ i\omega\omega_B & \omega^2+i\gamma\omega \end{pmatrix} \begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}$$

$$\frac{e}{m} E_z = \omega^2 + i\gamma\omega \; z(t)$$

(ד

$$\bar{P} = n(-e)\bar{r}$$

$$= \underbrace{\frac{ne^2}{m} \frac{1}{\omega^2\omega_B^2-(\omega^2+i\gamma\omega)^2} \begin{pmatrix} \omega^2+i\gamma\omega & -i\omega\omega_B \\ i\omega\omega_B & \omega^2+i\gamma\omega \end{pmatrix}}_{\chi_{ij}} \bar{E}$$

$$\varepsilon_{ij} = \mathbb{1} + 4\pi \chi_{ij}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\omega_p^2}{\omega^2\omega_B^2-(\omega^2+i\gamma\omega)^2} \begin{pmatrix} \omega^2+i\gamma\omega & -i\omega\omega_B \\ i\omega\omega_B & \omega^2+i\gamma\omega \end{pmatrix}$$

(c

$$\hat{e}_\pm = \frac{\hat{x} \pm i\hat{y}}{\sqrt{2}}$$

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$= \begin{pmatrix} a \mp ib \\ b \pm ia \end{pmatrix} = a \begin{pmatrix} 1 \\ \pm i \end{pmatrix} + ib \begin{pmatrix} \mp 1 \\ -i \end{pmatrix}$$

$$= a \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \mp b \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$= (a \mp b) \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$\varepsilon_{ij} \hat{e}_\pm = \underbrace{\left[1 + \frac{\omega_p^2}{\omega^2\omega_B^2-(\omega^2+i\gamma\omega)^2} \left(\omega^2 + \overbrace{i\gamma\omega \mp i\omega\omega_B}^{i\omega(\gamma\mp\omega_B)} \right) \right]}_{\varepsilon_\pm} \hat{e}_\pm,$$

(2

$$k_\pm = \frac{\omega}{c} \sqrt{\varepsilon_\pm}$$

יחס הנפיצה

$$\bar{E}_+ = \hat{e}_+ E_0 e^{i(k_+ z - \omega t)} e^{-k_i z}$$

$$\bar{E}_- = \hat{e}_- E_0 e^{i(k_- z - \omega t)} e^{-k_i z}$$

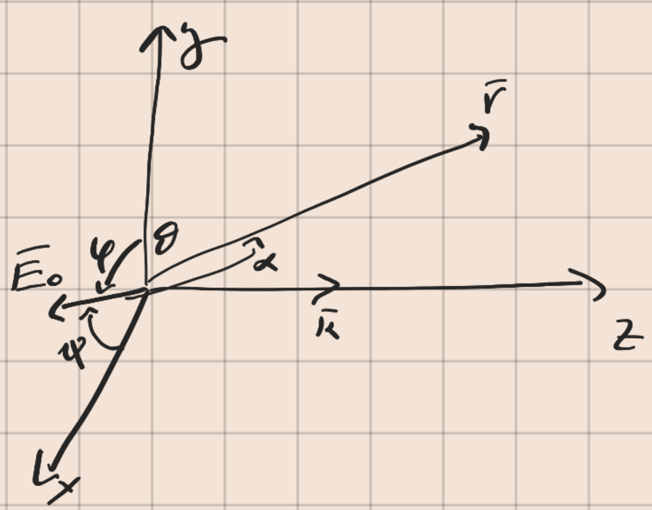
ה) אותו פתרון כמו בתרגול רק עם דעיכה

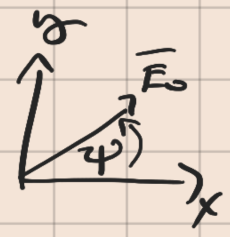
$$= E_0 e^{i(kz-\omega t)} \begin{pmatrix} \cos(\Delta k z) \\ -\sin(\Delta k z) \end{pmatrix} e^{-k_i z}$$

ב) א) בהרצאה ראינו:

$$\frac{dP}{d\Omega} = \frac{E_0^2 q^2}{8\pi m^2 c^3} \left| \frac{-\omega^2}{(\omega_0^2-\omega^2)-i\gamma\omega} \right|^2 \sin^2\alpha$$

$$= \frac{E_0^2 e^2}{8\pi m^2 c^3} \frac{\omega^4}{(\omega_0^2-\omega^2)^2} \left[1 - \sin^2\varphi\,(1-\cos^2\theta)\right]$$





מותר להשתמש בקירוב הדיפול כי בבעיות פיזור בכל מקרה הגלאי נמצא רחוק מהמפזר וגם הפאזות השונות לא מתואמות, הם מתמצעות כי מחפשים הספק.

ב) רק צריך לחלק ב $\langle S\rangle = \frac{c}{8\pi} E_0^2$

$$\frac{d\sigma}{d\Omega} = \frac{e^2}{m^2c^4} \frac{\omega^4}{(\omega_0^2-\omega^2)^2} \left[1 - \sin^2\varphi\,(1-\cos^2\theta)\right]$$ מקוטב:

$$1 - \tfrac{1}{2} + \tfrac{1}{2}\cos^2\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{e^2}{m^2c^4} \frac{\omega^4}{(\omega_0^2-\omega^2)^2} \frac{1+\cos^2\theta}{2}$$ לא מקוטב:

ג) $$\sigma = \frac{8\pi}{3} \frac{e^2}{m^2c^4} \frac{\omega^4}{(\omega_0^2-\omega^2)^2}$$

4

$$\chi_1(\omega) = \frac{2}{\pi} \int_0^\infty \frac{\omega' \chi_2(\omega')}{\omega'^2 - \omega^2}\, d\omega'$$

$$\chi_2(\omega) = -\frac{2\omega}{\pi} \int_0^\infty \frac{\chi_1(\omega')}{\omega'^2 - \omega^2}\, d\omega'$$

$$\varepsilon_r = 1 + 4\pi \chi_1$$

$$\varepsilon_i = 4\pi \chi_2 = \varepsilon_0 \left[\Theta(\omega - \omega_1) - \Theta(\omega - \omega_2)\right]$$

$$\Rightarrow \chi_1 = \frac{2}{\pi} \frac{\varepsilon_0}{4\pi} \int_{\omega_1}^{\omega_2} \frac{\omega'}{\omega'^2 - \omega^2}\, d\omega'$$

$$\frac{\omega'}{\omega'^2 - \omega^2} = \frac{\omega'}{(\omega' - \omega)(\omega' + \omega)} = \frac{1}{2}\left[\frac{1}{\omega' - \omega} + \frac{1}{\omega' + \omega}\right]$$

$$\Rightarrow \chi_1 = \frac{\varepsilon_0}{4\pi^2} \left\{ \int_{\omega_1}^{\omega_2} \frac{1}{\omega' - \omega}\, d\omega' + \int_{\omega_1}^{\omega_2} \frac{1}{\omega' + \omega}\, d\omega' \right\}$$

$$= \frac{\varepsilon_0}{4\pi^2} \left[\ln|\omega' - \omega| \Big|_{\omega_1}^{\omega_2} + \ln|\omega' + \omega| \Big|_{\omega_1}^{\omega_2} \right]$$

$$= \frac{\varepsilon_0}{4\pi^2} \left[\ln|\omega_2 - \omega| - \ln|\omega_1 - \omega| + \ln|\omega_2 + \omega| - \ln|\omega_1 + \omega| \right]$$

$$= \frac{\varepsilon_0}{4\pi^2} \ln\left[\frac{\omega^2 - \omega_2^2}{\omega^2 - \omega_1^2}\right]$$