(3) We expect very little difference from the problem $U = \frac{1}{2}gx^2$. In that problem we have small prob. to arrive at x_f, and high prob. to reflect back. Here we expect the opposit. The particle has a chance to bounce n times inside the barriar, then either cross to x_f with high prob. or reflect back with low prob.

First we nee to calculate $E(T)$:

$$(*) \quad T = \int_{x_i}^{x_f} dy \sqrt{\frac{m}{2(E + \frac{1}{2}gy^2)}} = \sqrt{\frac{m}{g}} \tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{2E(T)+gx^2}}\right)\Bigg|_{x_i}^{x_f}$$

(implicit)

Now we can plug this $E(T)$ into:

$$\frac{dx_f}{dp_i} = p(x_i)p(x_f)\int_{x_i}^{x_f} \frac{dx}{p^3(x)} = \sqrt{2m\left(E(T)+\frac{g}{2}x_i^2\right)}\sqrt{2m\left(E(T)+\frac{g}{2}x_f^2\right)}$$

$$\underbrace{\int \frac{dx}{\left(2m\left(E(T)+\frac{g}{2}x^2\right)\right)^{3/2}}}_{\frac{1}{2E(T)m}\left[\frac{x_f}{\sqrt{m(2E(T)+gx_f^2)}} - \frac{x_f}{\sqrt{m(2E(T)+gx_f^2)}}\right]}$$

$$\int_{x_i}^{x_f} p(x)\,dx = \int_{x_i}^{x_f} \sqrt{2m\left(E(T) + \frac{g}{2}x^2\right)}\,dx = \left[\frac{x}{2}\sqrt{m(2E(T)+gx^2)} + \frac{E(T)}{\sqrt{g}}\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{2E(T)+gx^2}}\right)\right]_{x_i}^{x_f}$$

now we plug everything back:

$$K(x_f,T,x_i,0) = \left\{2\pi\hbar\sqrt{2m\left(E(T)+\frac{g}{2}x_i^2\right)}\sqrt{2m\left(E(T)+\frac{2}{2}x_f^2\right)}\frac{1}{2E(T)m}\left[\frac{x_f}{\sqrt{m(2E(T)+gx_f^2)}} - \frac{x_f}{\sqrt{m(2E(T)+gx_f^2)}}\right]\right\}^{-1/2}$$

$$\times \exp\frac{i}{\hbar}\left\{\left[\frac{x}{2}\sqrt{m(2E(T)+gx^2)} + \frac{E(T)}{\sqrt{g}}\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{2E(T)+gx^2}}\right)\right]_{x_i}^{x_f} - E(T)\,T\right\}$$

where $E(T)$ is given by 8.

To get $G(x_f, x_i, E)$ we just need to do the Fourier Transform:

$$G(x_f, x_i, E) = \int_0^\infty \frac{dT}{2\pi\hbar}\, e^{-iET/\hbar} K(x_f, T, x_i, 0)$$

Or:

$$G(x_f, x_i, E) = \sum_{n=0}^{\infty} G_n$$

$$G_n = \frac{\frac{mt}{2T}\,(i/2)^n}{\sqrt{2m(E(T)+\frac{g}{2}x_i^2)}\sqrt{2m(E(T)+\frac{g}{2}x_f^2)}} \times$$

$$\times \exp \frac{1}{\hbar}\left[\left(i\int_{x_i}^{a} + (2n+1)\int_{a}^{b} + i\int_{b}^{x_f}\right)\sqrt{2m(E(T)+\frac{g}{2}x^2)}\,dx\right]$$

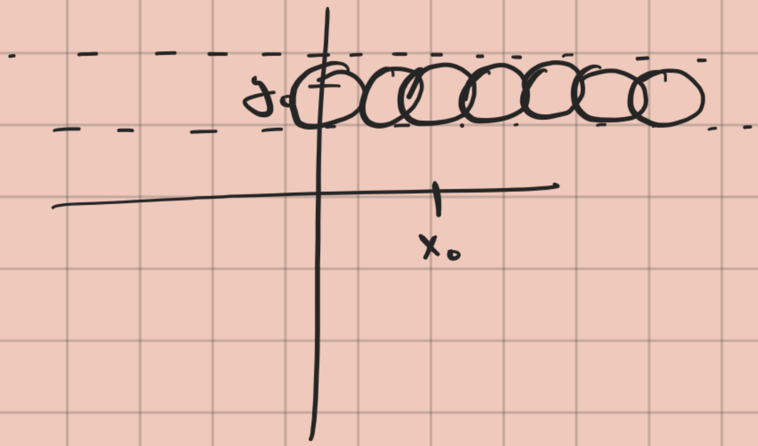
where again $E(T)$ is given by (*).

The Fourier transform is easier, cuz we need to calc $E(T)$ numerically anyway.

b) For the $n=0$ we use the classical path, i.e no bounces, which is the dominant contribution anyway. for the bounces we use complex time which are not classical paths.

② This is the same problem but now we have:

$$\ell = \sqrt{\frac{\hbar}{eB}} \qquad \omega_c = \frac{eB}{m}$$



$$\Rightarrow \quad \hat{X}_0 = x - \frac{\hat{v}_y}{\omega_c} = x - \frac{1}{\omega_c}\left(-i\hbar\partial_y + e\cancel{A_y}^{=0}\right) = x + \frac{i\hbar}{\omega_c}\partial_y$$

$$\hat{y}_0 = y + \frac{\hat{v}_x}{\omega_c} = y + \frac{1}{\omega_c}\left(-i\hbar\partial_x - eBy\right) = -i\frac{\hbar}{\omega_c}\partial_x = -i\ell^2\partial_x$$

$$\hat{v}_y = -i\hbar\partial_y \qquad \hat{v}_x = -i\hbar\partial_x - eBy$$

$$\hat{H} = \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2m}\underbrace{\left(-i\hbar\partial_x - eB\hat{y}\right)^2}_{(m v_x)^2 = (m(\hat{y}_0 - y)\omega_c)^2}$$

$$\Rightarrow \quad \psi(\bar{r}) = \frac{1}{\sqrt{L_z L_x}}\,\phi(y - y_0)\, e^{-i y_0 x/\ell^2} e^{-i k_z z}$$

$$\phi(y - y_0) \sim e^{-\frac{(y-y_0)^2}{\ell^2}} H\left(\frac{y - y_0}{\ell}\right)$$

$$\hat{H} = \frac{p_z^2}{2m} + \frac{m}{2}\left(\hat{v}_x^2 + \hat{v}_y^2\right)$$

$$[\hat{y}_0, v_y] = \left[y + \frac{v_x}{\omega_c}, \frac{v_y}{\omega_c}\right] = \frac{1}{\omega_c}[y, p_y] + \frac{1}{\omega_c^2}\underbrace{[v_x, v_y]}_{p_x - eBy,\ p_y} = [y, p_y] - [y, p_y] = 0$$

$$[y_0, V_x] = [y + V_x, V_x] = (y, \underset{P_x - eBy}{V_x}) = (y, y) = 0$$

Same with $[\hat{x}_0, \hat{V}_x) = 0 \{\hat{x}_0, \hat{V}_y\}$

$$\Rightarrow [\hat{x}_0, \hat{H}] = [\hat{y}_0, \hat{H}] = 0$$

$$[\hat{x}_0, \hat{y}_0] = \left[x - \frac{p_y}{\omega_c}, -\frac{p_x}{\omega_c}\right] = -\frac{1}{\omega_c}[x, p_x] = -\frac{i\hbar}{\omega_c} = -\frac{i\hbar m}{eB}$$

(there's an m factor missing in one of the deffinitions) $= -i\hbar l^2$

$$\Rightarrow \Delta x_0 \Delta y_0 \geq \frac{l^2}{2} \quad \Rightarrow A = 2\pi l^2$$

$$\Rightarrow \text{Flux} = BA = 2\pi B \frac{\hbar}{eB} = \frac{h}{e} = \Phi_0$$

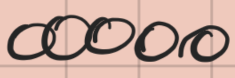
Var $\phi(y-y_0) = \langle(\hat{y} - \hat{y}_0)^2\rangle$

average potential energy $\overset{H.O.}{=} \frac{1}{2}$ total Energy

$$\bar{U} = \cancel{\frac{1}{2}} m\omega_c^{\cancel{2}} \langle(y-y_0)^2\rangle = \cancel{\frac{1}{2}} \hbar \cancel{\omega_c} (n + \tfrac{1}{2})$$

$$\Rightarrow \langle(y-y_0)^2\rangle = \frac{\hbar}{m\omega_c}(n + \tfrac{1}{2}) = l(n + \tfrac{1}{2})$$

small n



larg n

For $\vec{j}$ we obviously get $j_y = 0$

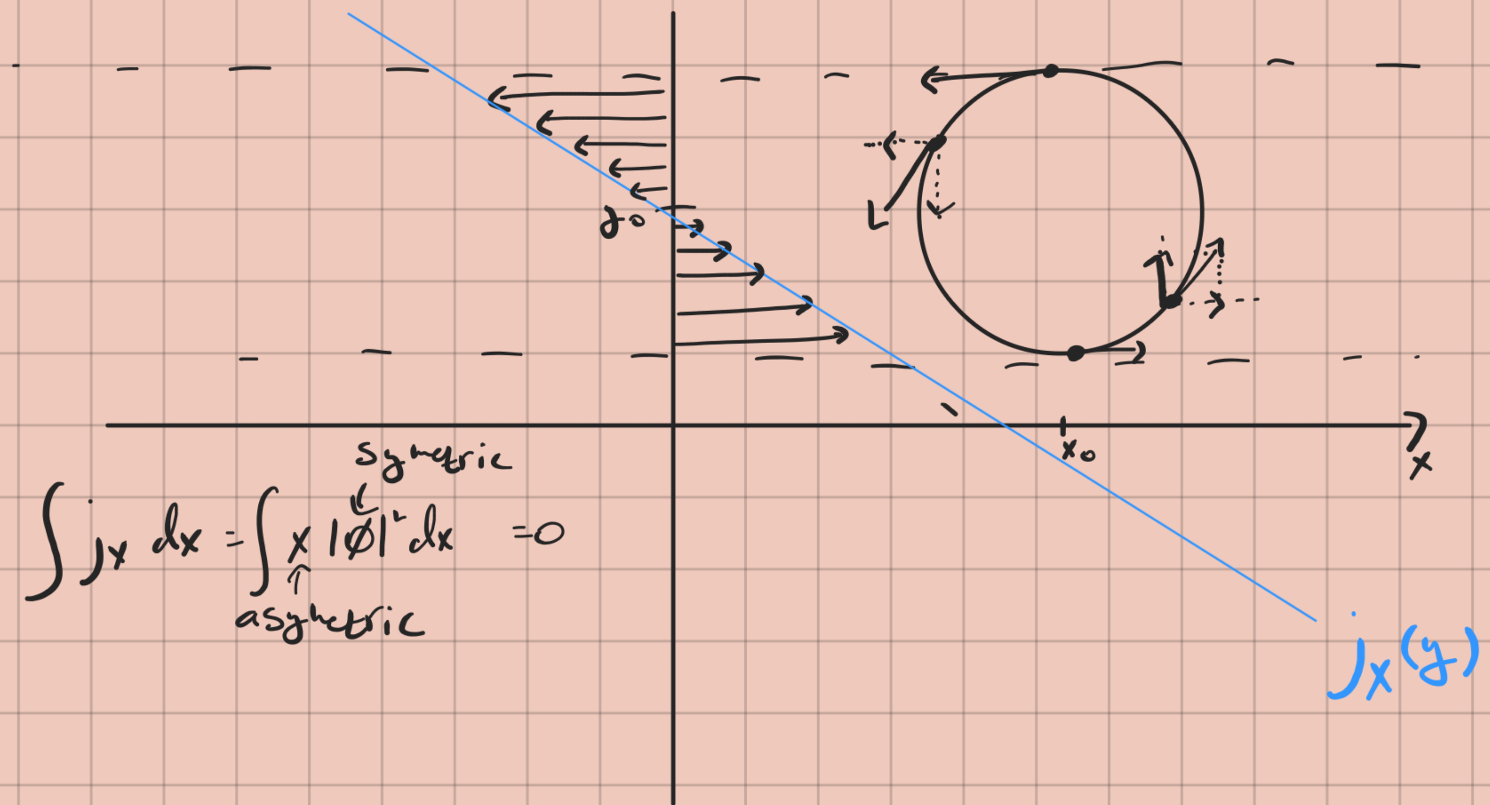
for j_x with $A_y = 0$:

$$j_x = \frac{ie\hbar}{2m}\left(\psi \partial_x \psi^* - \psi^* \partial_x \psi\right) = -\frac{ie\hbar}{2m}\frac{1}{L_z L_x}\left(\frac{iy_0}{\ell^2} + \frac{iy_0}{\ell^2}\right)\phi^2(y-y_0)$$

$$= \frac{e\hbar}{m}\frac{eB}{\hbar}\frac{1}{L_z L_x} y_0 \phi^2(y-y_0)$$

$$= e\omega_c y_0 \phi^2(y-y_0) \qquad \tilde{y} = y - y_0$$

$$j_x = e\omega_c(\tilde{y} - y_0)\phi^2(\tilde{y})$$



$$\int j_x \, dx = \int \underset{\substack{\uparrow \\ \text{asymetric}}}{x} \overset{\substack{\text{symetric} \\ \downarrow}}{|\phi|^2} dx = 0$$

b) when B grows ω_c grow, But ℓ decreases

$\Rightarrow$ smaller circles (But scales $B^{-1/2}$)

c) with $U(y)$ we would like to work in this gauge. That way the smearing in the x direction won't disturb us. we could solve for a particular X', and this solution would hold for every other degenerate state.