$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar\lambda(\sigma_+ + \sigma_-)(a + a^\dagger) \quad (k \quad 1$$

$$U = e^{i\omega t(a^\dagger a + \frac{1}{2}\sigma_z)} \qquad U^\dagger = e^{-i\omega t(a a^\dagger + \frac{1}{2}\sigma_z)}$$

$$H_{eff} = U(H-K)U^\dagger$$

$$= UHU^\dagger - UKU^\dagger$$

$$e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \dots$$

$$UKU^\dagger = e^{itK/\hbar} K e^{-itK/\hbar} = K + i\frac{t}{\hbar}[K,K] = K$$

$$UHU^\dagger = e^{itK/\hbar} H e^{-itK/\hbar} = H + i\frac{t}{\hbar}[K,H] + \left(i\frac{t}{\hbar}\right)^2[K,[K,H]]$$

$$[K,H] = \frac{\hbar\omega\omega_0}{2}\overset{=0}{\cancel{\left[a^\dagger a + \frac{1}{2}\sigma_z, \sigma_z\right]}} + \hbar^2\omega^2\overset{=0}{\cancel{\left[a^\dagger a + \frac{1}{2}\sigma_z, a^\dagger a\right]}}$$

$$+ \hbar^2\omega\lambda\Big[a^\dagger a + \frac{1}{2}\sigma_z, \underbrace{(\sigma_+ + \sigma_-)(a + a^\dagger)}_{\sigma_+ a + \sigma_+ a^\dagger + \sigma_- a + \sigma_- a^\dagger}\Big]$$

$$= \hbar^2\omega\lambda\Big\{\overset{\cancel{-\sigma_+ a}}{[a^\dagger a, \sigma_+ a]} + \overset{+\sigma_+ a^\dagger}{[a^\dagger a, \sigma_+ a^\dagger]} + \overset{-\sigma_- a}{[a^\dagger a, \sigma_- a]} + \overset{\cancel{+\sigma_- a^\dagger}}{[a^\dagger a, \sigma_- a^\dagger]}$$
$$+ \overset{\cancel{+\sigma_+ a}}{\left[\frac{1}{2}\sigma_z, \sigma_+ a\right]} + \overset{+\sigma_+ a^\dagger}{\left[\frac{1}{2}\sigma_z, \sigma_+ a^\dagger\right]} + \overset{-\sigma_- a}{\left[\frac{1}{2}\sigma_z, \sigma_- a\right]} + \overset{\cancel{-\sigma_- a^\dagger}}{\left[\frac{1}{2}\sigma_z, \sigma_- a^\dagger\right]}\Big\}$$

$$[a^\dagger a, a] = a^\dagger[a,a] + [a^\dagger,a]a = [a^\dagger,a]a = -a$$

$$[a^\dagger a, a^\dagger] = [a, a^\dagger]a^\dagger = a^\dagger$$

$$\left[\frac{1}{2}\sigma_z, \sigma_+ a\right] = \frac{1}{2}[\sigma_z, \sigma_+]a$$

$$\Rightarrow 2\hbar^2\omega\lambda\left[\sigma_+ a^\dagger - \sigma_- a\right]$$

$$[K,[K,H]] = 2\hbar^3\omega^2\lambda\left[a^\dagger a + \frac{1}{2}\sigma_z, \sigma_+ a^\dagger - \sigma_- a\right]$$

$$= 2\hbar^3\omega^2\lambda\Big\{\overset{\sigma_+ a^\dagger}{[a^\dagger a, \sigma_+ a^\dagger]} - \overset{+\sigma_- a}{[a^\dagger a, \sigma_- a]} + \overset{+\sigma_+ a^\dagger}{\left[\frac{1}{2}\sigma_z, \sigma_+ a^\dagger\right]} - \overset{+\sigma_- a}{\left[\frac{1}{2}\sigma_z, \sigma_- a\right]}\Big\}$$

$$= 4\hbar^3\omega^2\lambda\left\{\sigma_+ a^\dagger + \sigma_- a\right\}$$

$$[K,[K,[K,H]]] = 4\hbar^4\omega^3\lambda\left[a^\dagger a + \frac{1}{2}\sigma_z, \sigma_+ a^\dagger + \sigma_- a\right]$$

$$= 4\hbar^4\omega^3\lambda\Big\{[a^\dagger a, \sigma_+ a^\dagger] + [a^\dagger a, \sigma_- a] + \left[\frac{1}{2}\sigma_z, \sigma_+ a^\dagger\right] + \left[\frac{1}{2}\sigma_z, \sigma_- a\right]\Big\}$$

$$\frac{\hbar\omega}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar\lambda\left(\sigma_+ a + \sigma_+ a^\dagger + \sigma_- a + \sigma_- a^\dagger\right) + \frac{t}{i\hbar}\,\hbar^2\omega\lambda\left(\sigma_+ a^\dagger - \sigma_- a\right) + \left(\frac{t}{i\hbar}\right)^2 \hbar^2\omega^2\lambda\left\{\sigma_+ a^\dagger + \sigma_- a\right\}$$

$$\sigma_+ a + a^\dagger \sigma_-$$

$$H_{eff} = \frac{\hbar\Delta}{2}\sigma_z + \hbar\lambda(\sigma_+ a + a^\dagger \sigma_-)$$ (P

$$= \begin{pmatrix} \frac{\hbar\Delta}{2} & 0 \\ 0 & -\frac{\hbar\Delta}{2} \end{pmatrix} + \hbar\lambda \begin{pmatrix} & a \\ a^\dagger & \end{pmatrix} = \hbar \begin{pmatrix} \frac{\Delta}{2} & \lambda a \\ \lambda a^\dagger & -\frac{\Delta}{2} \end{pmatrix}$$

$$\begin{vmatrix} \frac{\Delta}{2} - \varepsilon & \lambda a \\ \lambda a^\dagger & -\left(\frac{\Delta}{2} + \varepsilon\right) \end{vmatrix} = -\left(\frac{\Delta^2}{4} - \varepsilon^2\right) - \lambda^2 \underbrace{a a^\dagger}_{\to n+1}$$

$$= \varepsilon^2 - \frac{\Delta^2}{4} - \lambda^2(n+1)$$

$$\frac{\varepsilon_\pm}{\hbar} = \pm\sqrt{\frac{\Delta^2}{4} + \lambda^2 n} \Rightarrow E_{n\pm} = \pm\frac{\hbar}{2}\sqrt{\Delta^2 + 4\lambda^2(n+1)}$$

$$\Omega = \Delta\sqrt{1 + \frac{4\lambda^2(n+1)}{\Delta^2}}$$

$$x \equiv \frac{2\lambda\sqrt{(n+1)}}{\Delta}$$ (c

$$x = \tan\varphi$$

$$\sqrt{1+x^2} = \sqrt{1 + \frac{\sin^2\varphi}{\cos^2\varphi}}$$

$$= \sqrt{\frac{1}{\cos^2\varphi}} = \frac{1}{\cos\varphi} \Rightarrow \Omega(\Delta) = \frac{\Delta}{\cos\varphi}$$

$$E_{n\pm} = \pm\frac{\hbar}{2}\frac{\Delta}{\cos\varphi}$$

$$U^{\dagger}|n+\rangle = e^{-itH/\hbar}\left[\cos\frac{\varphi_n}{2}|e,n\rangle + \sin\frac{\varphi_n}{2}|g,n+1\rangle\right] \quad (2$$

$$\sigma_z|e\rangle = |e\rangle \qquad\qquad H = \hbar\omega\left(n + \frac{1}{2}\sigma_z\right)$$

$$\sigma_z|g\rangle = -|g\rangle$$

$$= \cos\frac{\varphi_n}{2}\, e^{-i\omega t(n+\frac{1}{2})}|e,n\rangle + \sin\frac{\varphi_n}{2}\, e^{-i\omega t(n-\frac{1}{2})}|g,n+1\rangle$$

$$= e^{-i\omega t n}\frac{1}{2}\left[\left(e^{i\frac{\varphi_n}{2}} + e^{-i\frac{\varphi_n}{2}}\right)e^{-i\frac{\omega t}{2}}\right.$$

$$e^{i\frac{1}{2}(\varphi_n - \omega t)} + e^{-i\frac{1}{2}(\varphi_n + \omega t)}$$

2 א) עבור $|\Delta| \gg \lambda\sqrt{n}$ נפתח לסדר ראשון

$$\Omega(\Delta) = \Delta\sqrt{1+x^2} \approx \Delta\left[1+\frac{x^2}{2}\right] \qquad x \equiv \frac{2\lambda\sqrt{n+1}}{\Delta}$$

$$= \Delta + \frac{\cancel{\Delta}}{2}\frac{4\lambda^2(n+1)}{\Delta^2} = \Delta + \frac{2\lambda^2(n+1)}{\Delta}$$

$$= \Delta + 2\frac{\lambda^2}{\Delta}n + 2\frac{\lambda^2}{\Delta}$$

$$E_{n+} = \hbar\omega n + \frac{\hbar\omega}{2} + \frac{\hbar}{2}\Delta + \hbar\frac{\lambda^2}{\Delta}n + \hbar\frac{\lambda^2}{\Delta}$$

$$= \hbar\left(\omega + \frac{\lambda^2}{\Delta}\right)n + \underbrace{\frac{\hbar}{2}\left(\omega + \overset{\omega_0-\omega}{\Delta} + \frac{2\lambda^2}{\Delta}\right)}_{\frac{\hbar}{2}\omega_0 + \hbar\frac{\lambda^2}{\Delta}}$$

$$E_{n-} = \hbar\omega n + \frac{\hbar\omega}{2} - \frac{\hbar}{2}\Delta - \hbar\frac{\lambda^2}{\Delta}n - \hbar\frac{\lambda^2}{\Delta}$$

$$= \hbar\left(\omega - \frac{\lambda^2}{\Delta}\right)n + \frac{\hbar\omega}{2} - \frac{\hbar}{2}\omega_0 + \frac{\hbar}{2}\omega - \hbar\frac{\lambda^2}{\Delta}$$

ב)

$$e^{-S} = e^{\frac{\lambda}{\Delta}(a^\dagger\sigma_- - \sigma_+ a)}$$

$$e^{-S}|e n\rangle = e^{\frac{\lambda}{\Delta}(\sqrt{n+1} - \sqrt{n})},$$