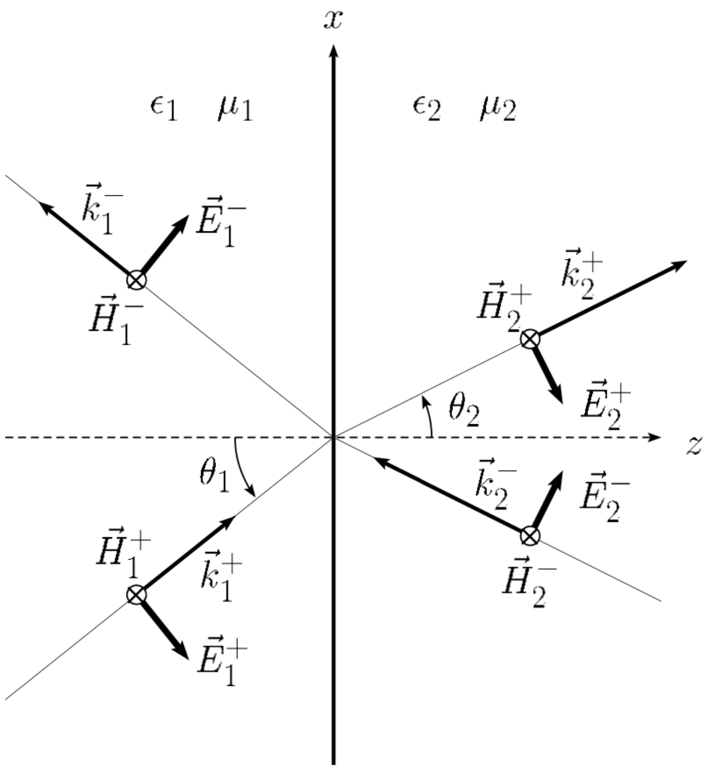
# אופטיקה של חומרים דו-מימדיים - שיעור מס. 3

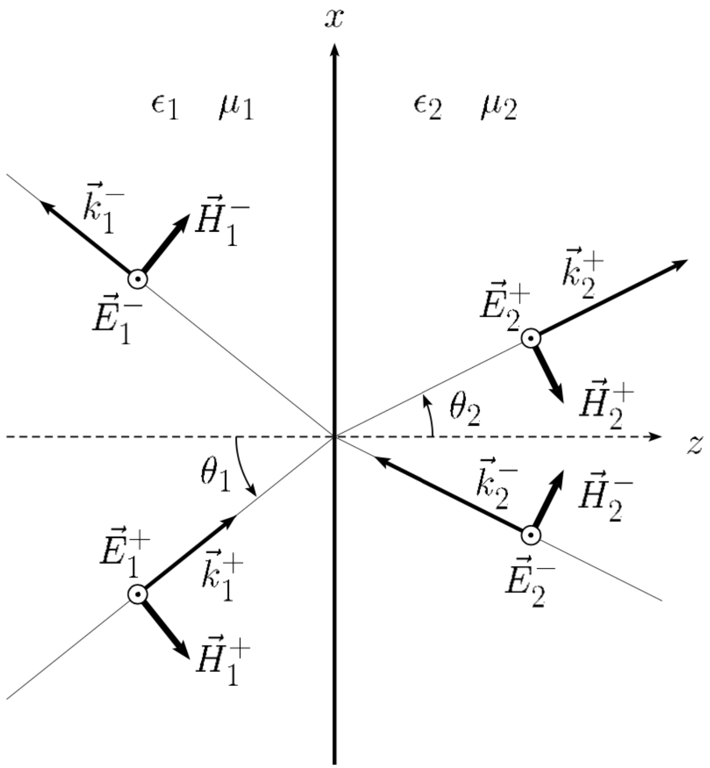
- The Transfer-Matrix-Method
- מבוא לפולריטונים
- Surface-plasmon-polaritons

# מעבר גלים בין שתי שכבות

TM




TE




$$\hat{n}\cdot\left(\vec{B}_2-\vec{B}_1\right)=0 \Rightarrow B_{1\perp}=B_{2\perp}$$

$$\hat{n}\times\left(\vec{E}_2-\vec{E}_1\right)=0 \Rightarrow \vec{E}_{1\|}=\vec{E}_{2\|}$$

$$\hat{n}\cdot\left(\vec{D}_2-\vec{D}_1\right)=\sigma \;\Rightarrow D_{2\perp}-D_{1\perp}=\sigma$$

$$(\sigma = \text{surface charge density})$$

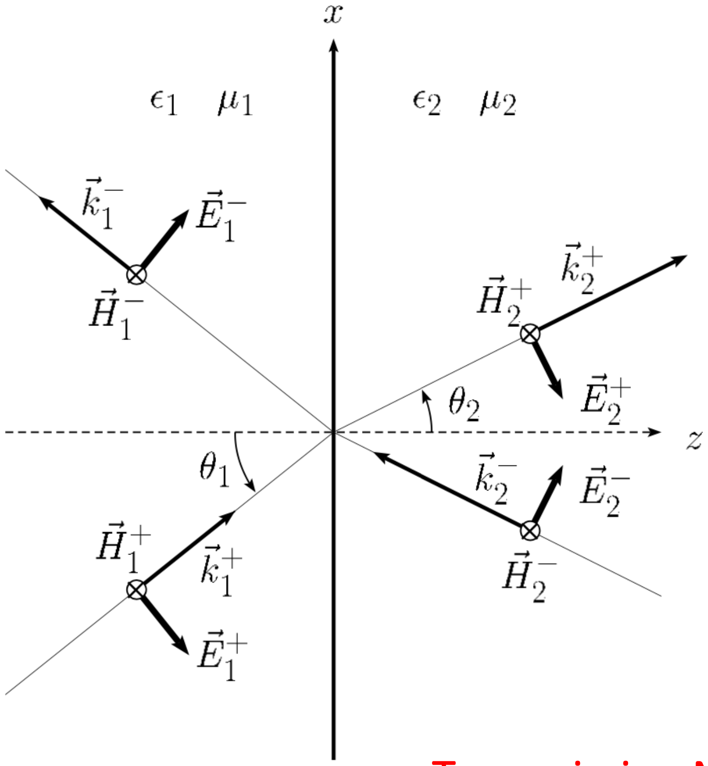
$$\hat{n}\times\left(\vec{H}_2-\vec{H}_1\right)=\vec{K} \Rightarrow \vec{H}_{2\|}-\vec{H}_{1\|}=\vec{K}$$

$$(\vec{K} = \text{surface current density})$$

# מעבר גלים בין שתי שכבות -TM

$$\vec{H} = (0,\ H,\ 0)$$

$$\vec{E} = (E_x,\ 0,\ E_z)$$




מתנאי השפה נדרוש:

$$H_1^+ + H_1^- = H_2^+ + H_2^-$$

$$E_{1x}^+ + E_{1x}^- = E_{2x}^+ + E_{2x}^-$$

$$\vec{k} \times \vec{H} = -\frac{\varepsilon\omega}{c}\ \vec{E},$$

$$\frac{\varepsilon_1\omega}{c} E_{1x}^+ = k_{1z} H_1^+ \quad \text{and} \quad \frac{\varepsilon_1\omega}{c} E_{1x}^- = -k_{1z} H_1^-$$

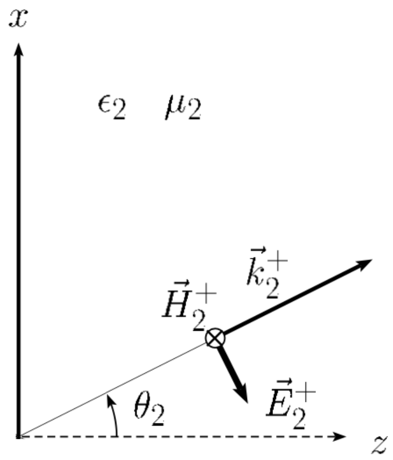
$$\begin{pmatrix} 1 & 1 \\ \frac{k_{1z}}{\varepsilon_1} & -\frac{k_{1z}}{\varepsilon_1} \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -\frac{k_{2z}}{\varepsilon_2} & +\frac{k_{2z}}{\varepsilon_2} \end{pmatrix} \begin{pmatrix} H_2^+ \\ H_2^- \end{pmatrix}$$

$$\begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix} = \tilde{\mathbf{M}}^{(p)} \begin{pmatrix} H_2^+ \\ H_2^- \end{pmatrix} = \begin{matrix} M_{11} H_2^+ + M_{12} H_2^- \\ M_{21} H_2^+ + M_{22} H_2^- \end{matrix}$$

$$\tilde{\mathbf{M}}^{(p)} = \frac{1}{2} \begin{pmatrix} 1 + \frac{\varepsilon_1}{\varepsilon_2}\frac{k_{2z}}{k_{1z}} & 1 - \frac{\varepsilon_1}{\varepsilon_2}\frac{k_{2z}}{k_{1z}} \\ 1 - \frac{\varepsilon_1}{\varepsilon_2}\frac{k_{2z}}{k_{1z}} & 1 + \frac{\varepsilon_1}{\varepsilon_2}\frac{k_{2z}}{k_{1z}} \end{pmatrix} \qquad r = \frac{M_{21}}{M_{11}} \qquad t = \frac{1}{M_{11}}$$

Transmission Matrix

התקדמות גלים בתוך שכבה:

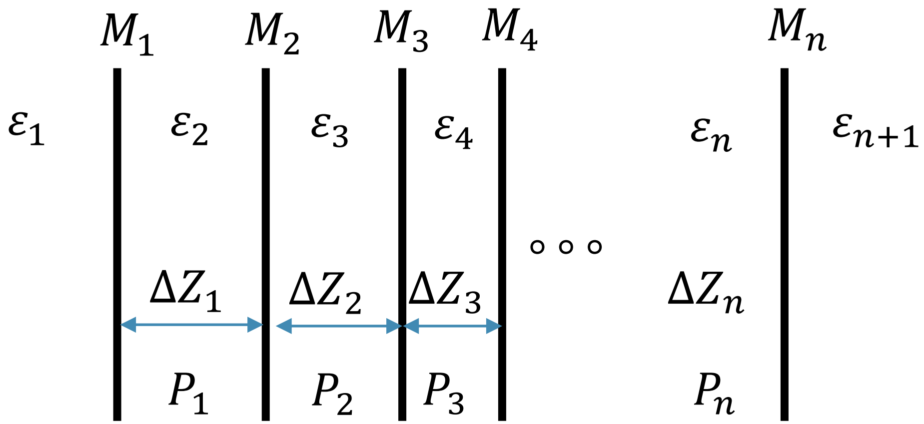



$$\begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix} = P(\Delta z) \begin{pmatrix} H_2^+ \\ H_2^- \end{pmatrix}$$

$$P(\Delta z) = \begin{bmatrix} e^{-ik_z\Delta z} & 0 \\ 0 & e^{ik_z\Delta z} \end{bmatrix}$$

Propagation Matrix

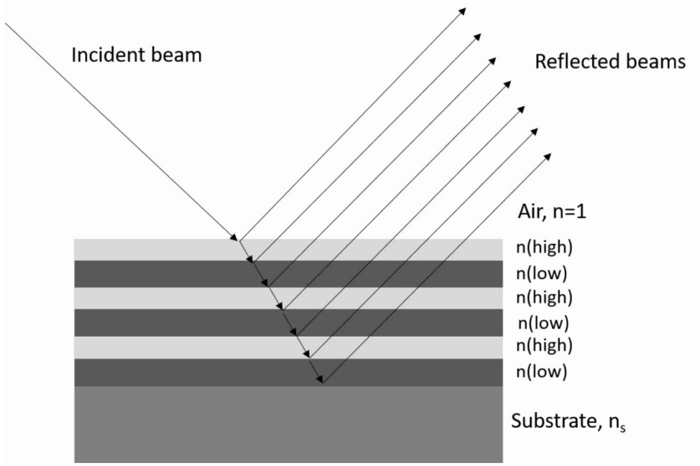
# Transfer-Matrix-Method

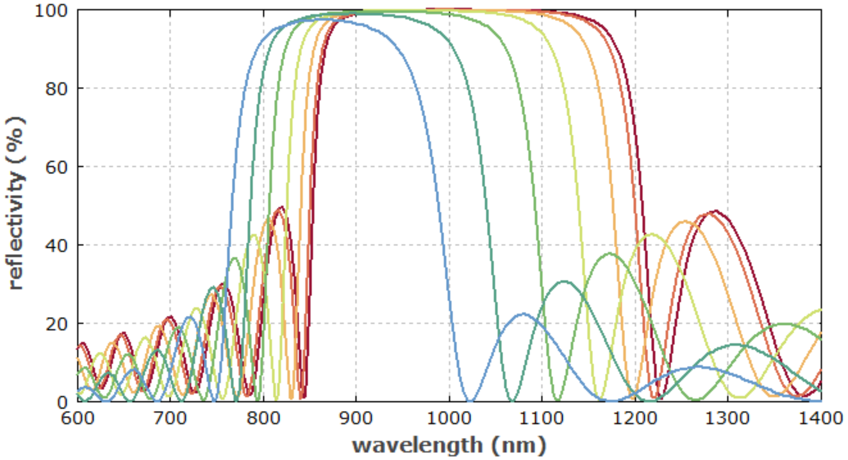



$$\begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix} = TM \begin{pmatrix} H_2^+ \\ H_2^- \end{pmatrix}$$

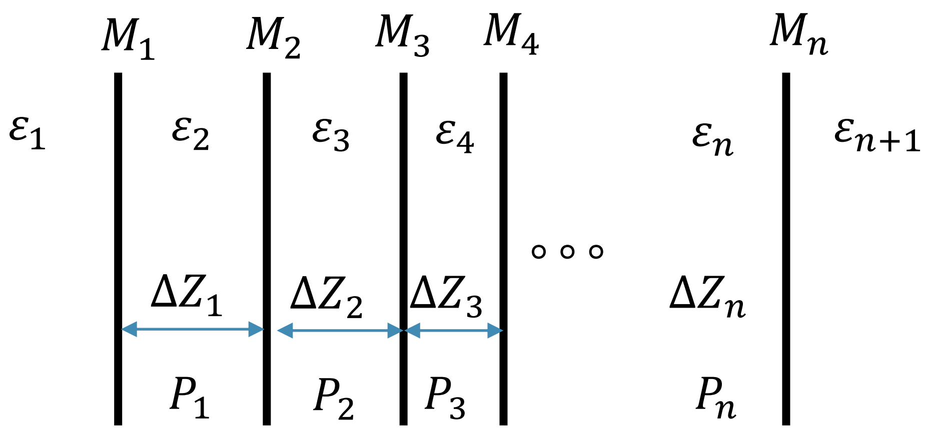
$$TM = M_1 P_1 M_2 P_2 \cdots M_{n+1}$$

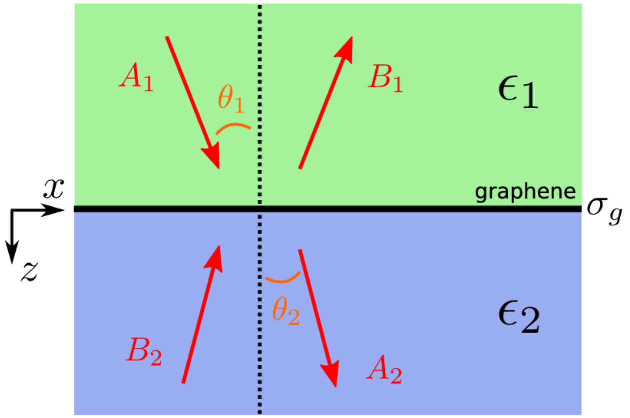
Bragg mirrors



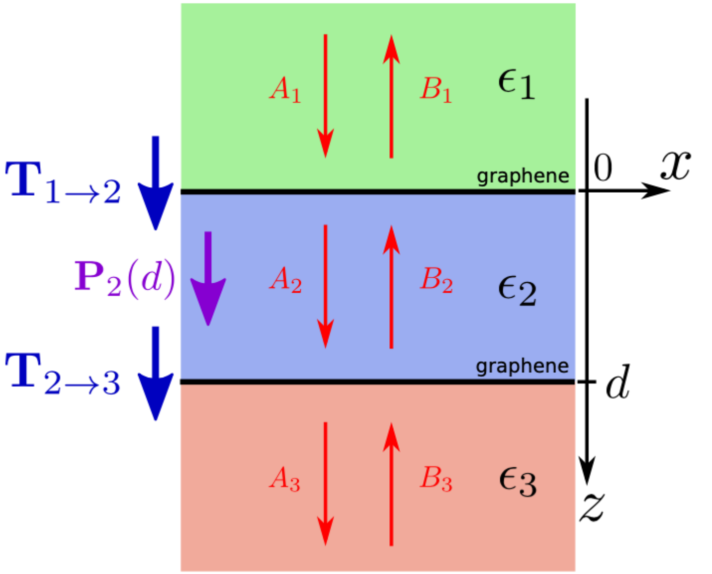



# חומרים דו-מימדיים



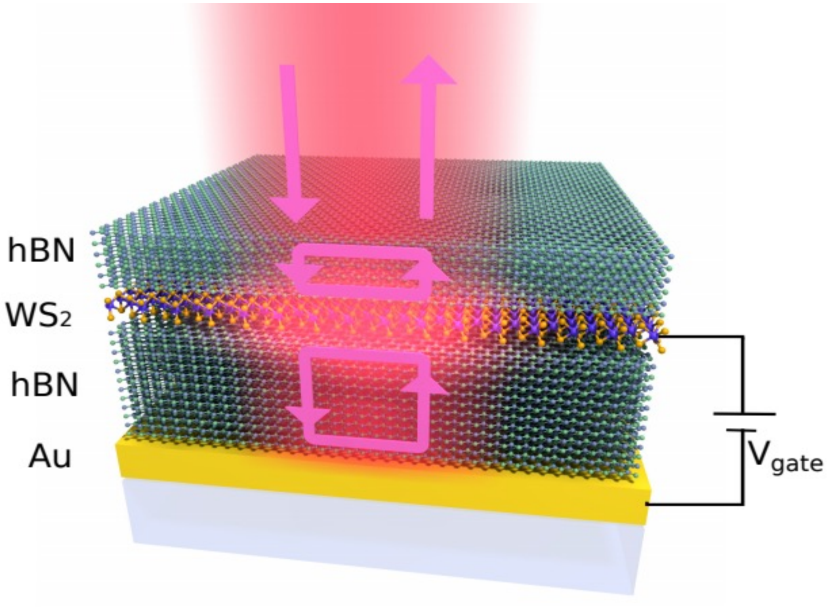

$$E_{1,x}(x,z)|_{z=0} = E_{2,x}(x,z)|_{z=0} \ ,$$
$$B_{1,y}(x,z)|_{z=0} - B_{2,y}(x,z)|_{z=0} = 0$$

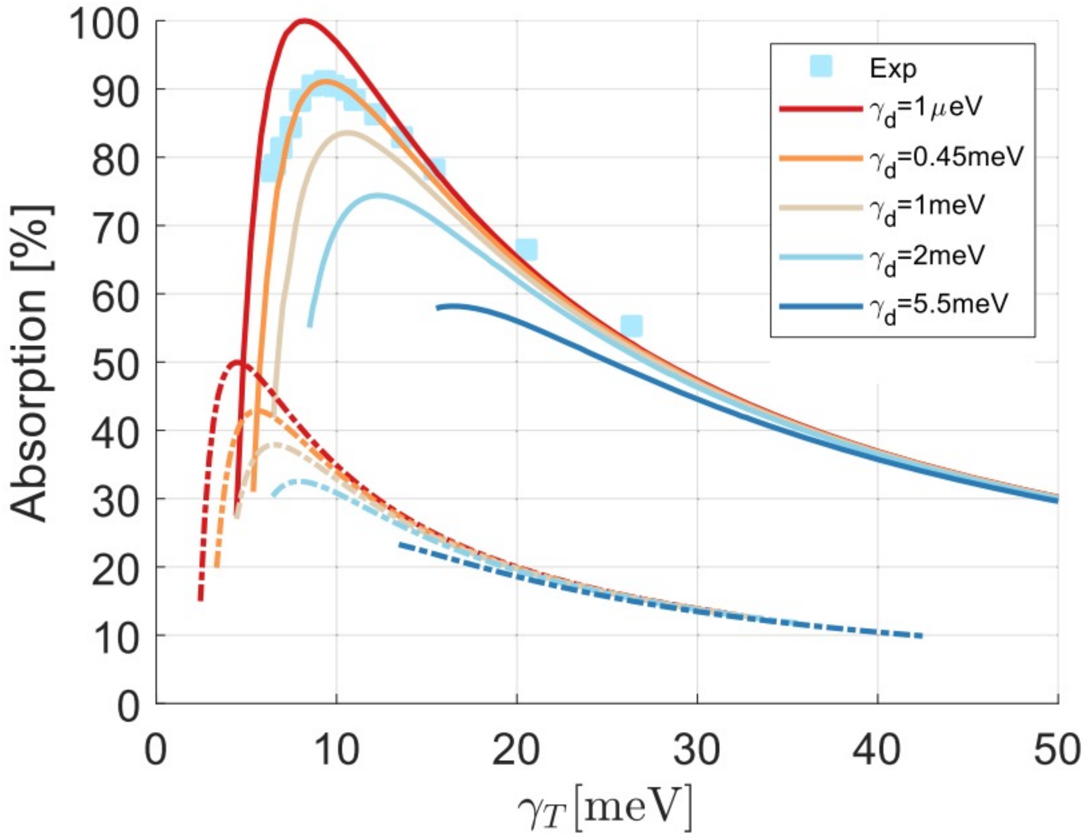



$$E_{1,x}(x,z)|_{z=0} = E_{2,x}(x,z)|_{z=0} \ ,$$
$$B_{1,y}(x,z)|_{z=0} - B_{2,y}(x,z)|_{z=0} = \mu_0 J_x(x) = \mu_0 \sigma_{xx} E_{2,x}(x,z)|_{z=0}$$

# חומרים דו-מימדיים

<u>דוגמה: 100% בליעה בשכבה אטומית של מוליך למחצה</u>



# Introduction to Polaritons

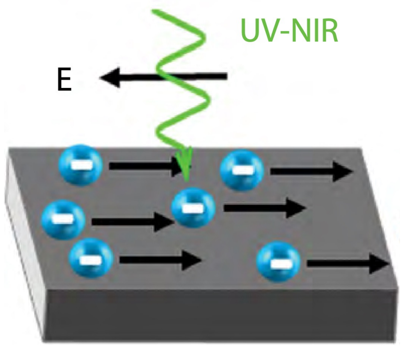
Polaritons = coupled to a photon

Matter excitation ⟷ Light

Electric dipole oscillations ∿ Electromagnetic field oscillations

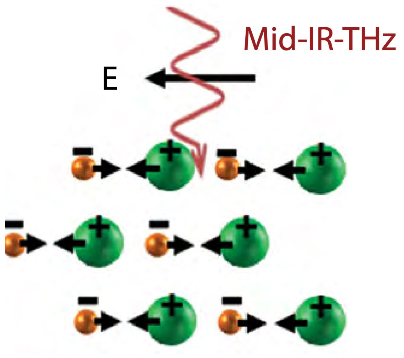
Caldwell et al, Nanophotonics 4, 44 (2015)

Free carriers



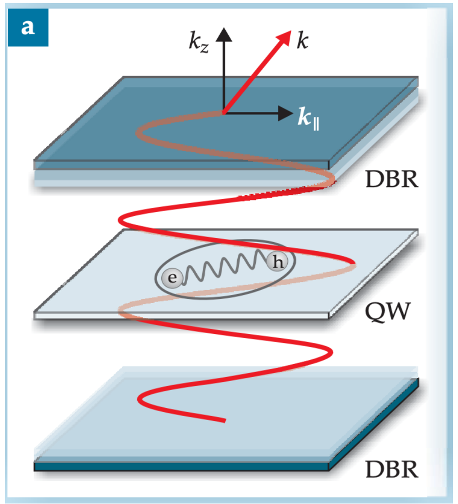

Surface-plasmon-polaritons

Optical phonons



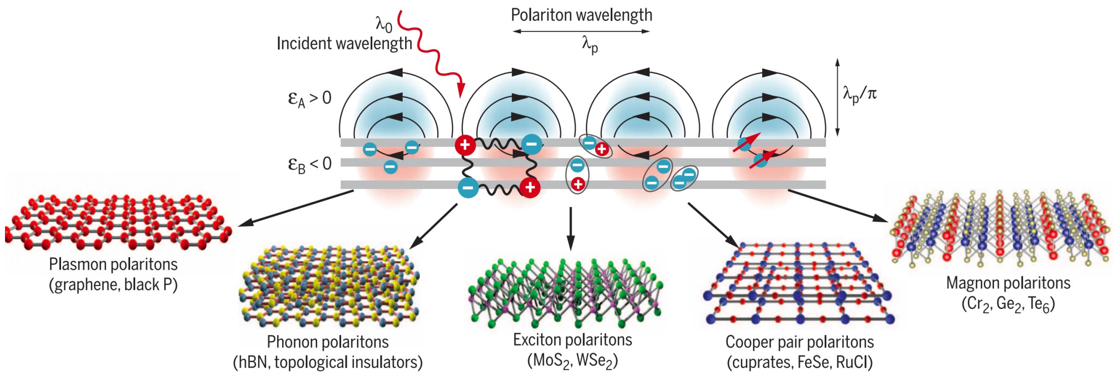

Surface-phonon-polaritons

Exciton-polaritons

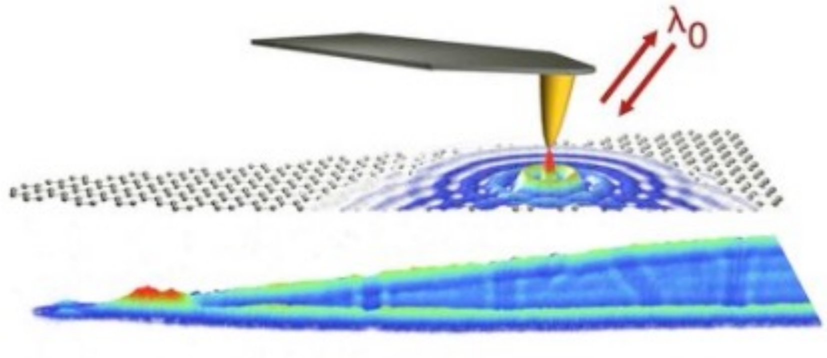



D. Snoke, and Peter Littlewood, Physics Today 63, 42(2010)

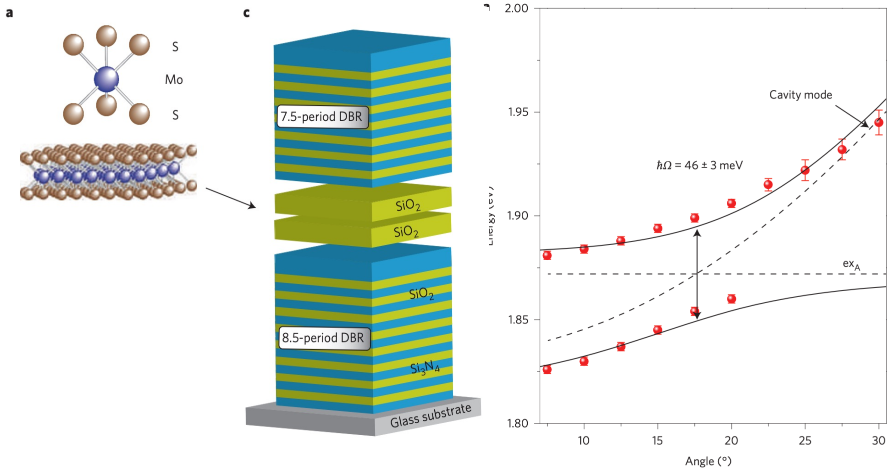
# Polaritons in 2D materials



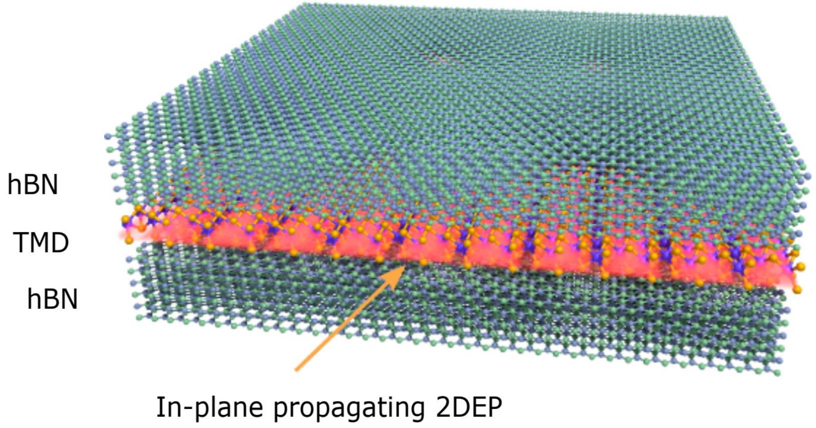

D. N. Basov, M. M. Fogler, F. J. García de Abajo, Science 354, 6309(2016)
Low, T. et al, Nat. Mater. 16, 182–194 (2017).
D. N. Basov et al, Nanophotonics 10, 549-577(2021).




Fei, Z. et al, Nature 487, 82–85 (2012).
Chen, J. et al, Nature 487, 77–81 (2012).



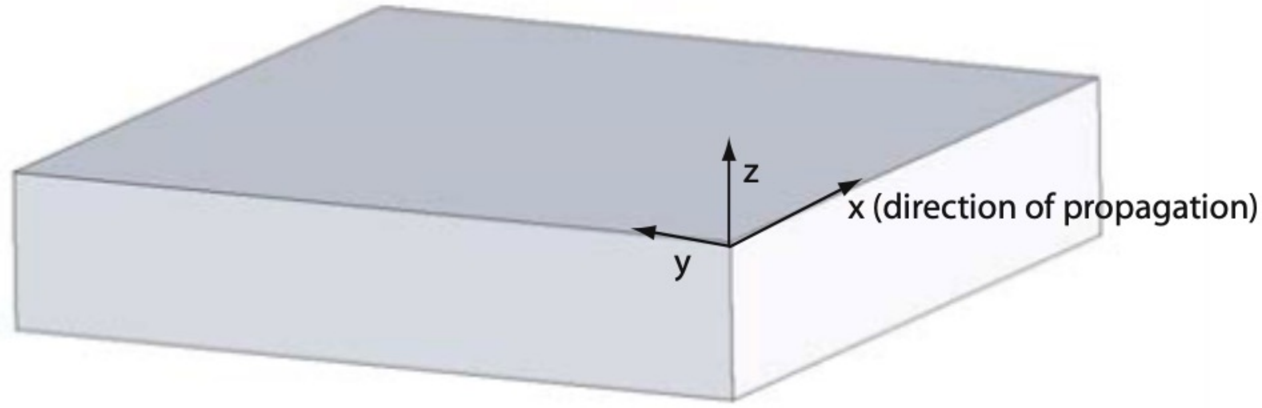

Liu, X., Galfsky, T., Sun, Z. et al. Nat. Phot. 9, 30–34 (2015).




I. Epstein et al, 2D Materials 7, 035031 (2020)

# Surface-plasmon-polaritons (SPPs)

- S.A. Maier- “plasmonics: fundamentals and applications”




$$\nabla^2 \mathbf{E} + k_0^2 \varepsilon \mathbf{E} = 0$$

$$\mathbf{E}(x, y, z) = \mathbf{E}(z) e^{i\beta x}$$

$$\frac{\partial^2 \mathbf{E}(z)}{\partial z^2} + \left(k_0^2 \varepsilon - \beta^2\right) \mathbf{E} = 0$$

$$\cancel{\frac{\partial E_z}{\partial y}} - \frac{\partial E_y}{\partial z} = i\omega\mu_0 H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}} = i\omega\mu_0 H_z$$

$$\cancel{\frac{\partial H_z}{\partial y}} - \frac{\partial H_y}{\partial z} = -i\omega\varepsilon_0\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \cancel{\frac{\partial H_x}{\partial y}} = -i\omega\varepsilon_0\varepsilon E_z$$

# Surface-plasmon-polaritons

$$\frac{\partial E_y}{\partial z} = -i\omega\mu_0 H_x$$

$$\frac{\partial E_x}{\partial z} - i\beta E_z = i\omega\mu_0 H_y$$

$$i\beta E_y = i\omega\mu_0 H_z$$

$$\frac{\partial H_y}{\partial z} = i\omega\varepsilon_0\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - i\beta H_z = -i\omega\varepsilon_0\varepsilon E_y$$

$$i\beta H_y = -i\omega\varepsilon_0\varepsilon E_z.$$

For TM:

$E_x$, $E_z$ and $H_y$ are nonzero

$$E_x = -i\frac{1}{\omega\varepsilon_0\varepsilon}\frac{\partial H_y}{\partial z}$$

$$E_z = -\frac{\beta}{\omega\varepsilon_0\varepsilon}H_y,$$

wave equation for TM modes is

$$\frac{\partial^2 H_y}{\partial z^2} + \left(k_0^2\varepsilon - \beta^2\right) H_y = 0.$$

For a bound mode at the interface:

for $z > 0$ $\quad k_i \equiv k_{z,i} (i = 1, 2)$

$$H_y(z) = A_2 e^{i\beta x} e^{-k_2 z}$$

Re(K)>0

$$E_x(z) = iA_2\frac{1}{\omega\varepsilon_0\varepsilon_2}k_2 e^{i\beta x} e^{-k_2 z}$$

$$E_z(z) = -A_1\frac{\beta}{\omega\varepsilon_0\varepsilon_2}e^{i\beta x} e^{-k_2 z}$$

for $z < 0$

$$H_y(z) = A_1 e^{i\beta x} e^{k_1 z}$$

$$E_x(z) = -iA_1\frac{1}{\omega\varepsilon_0\varepsilon_1}k_1 e^{i\beta x} e^{k_1 z}$$

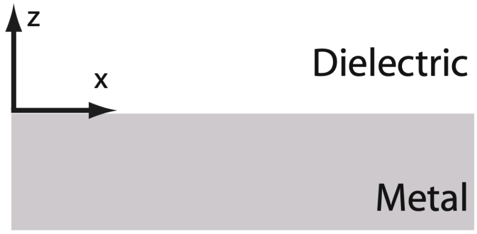
$$E_z(z) = -A_1\frac{\beta}{\omega\varepsilon_0\varepsilon_1}e^{i\beta x} e^{k_1 z}$$

# Surface-plasmon-polaritons

Continuity at the interface:

$$A_1 = A_2$$

$$\frac{k_2}{k_1} = -\frac{\varepsilon_2}{\varepsilon_1}$$

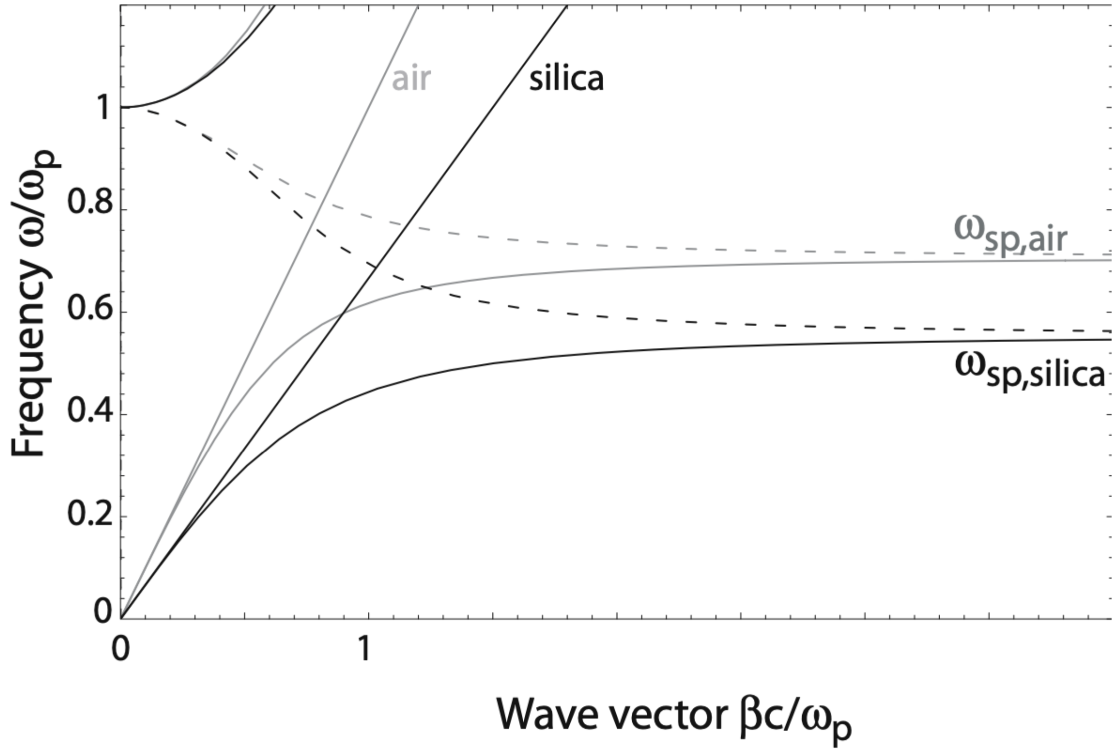



From the wave equation:

$$k_1^2 = \beta^2 - k_0^2\varepsilon_1$$
$$k_2^2 = \beta^2 - k_0^2\varepsilon_2$$

Dispersion relation:

$$\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}.$$



# Surface-plasmon-polaritons

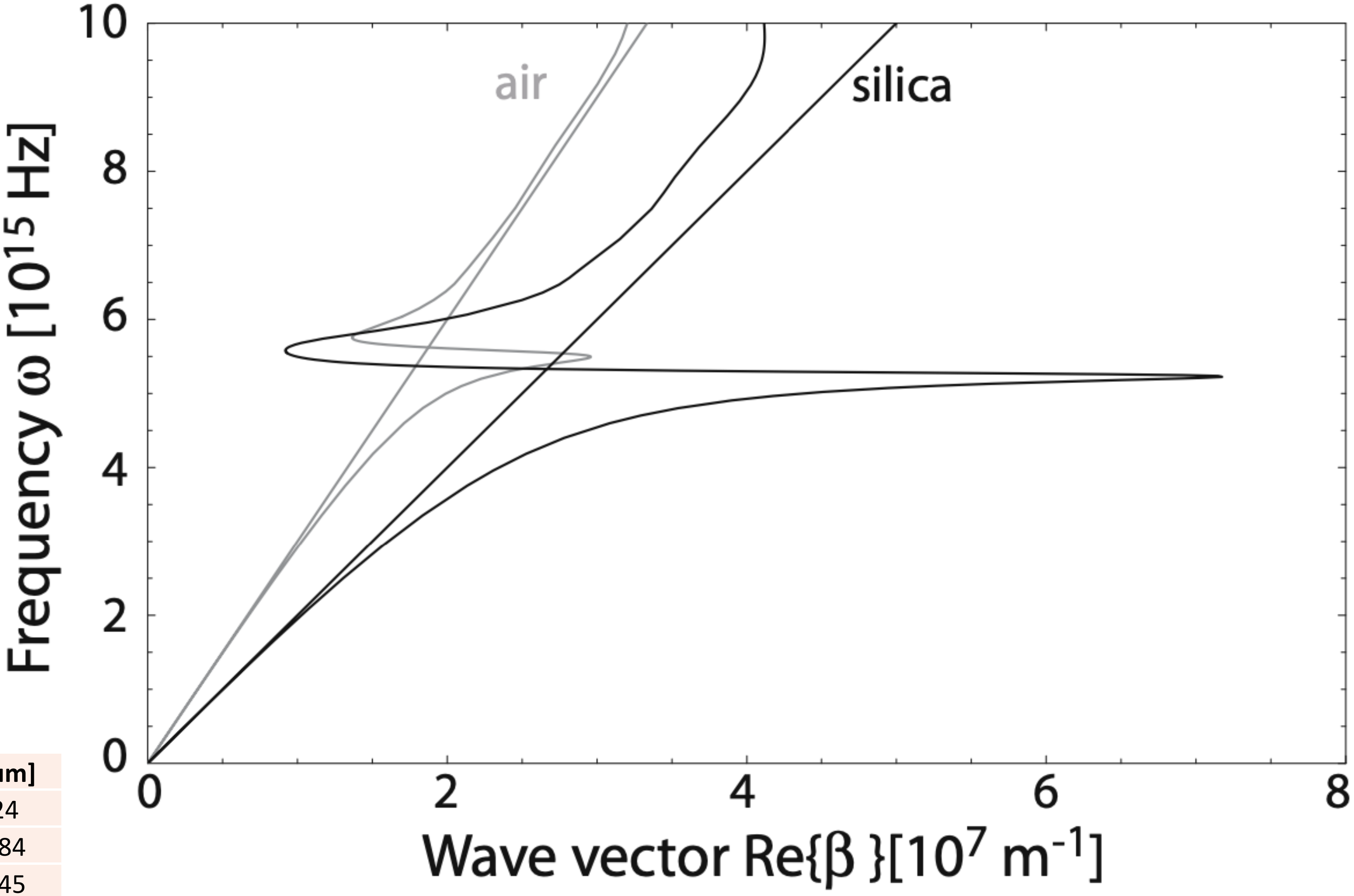
Dispersion relation with real metals:

$$\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}}.$$

$$\lambda_{sp} = 2\pi/\mathrm{Re}\left[\beta\right]$$

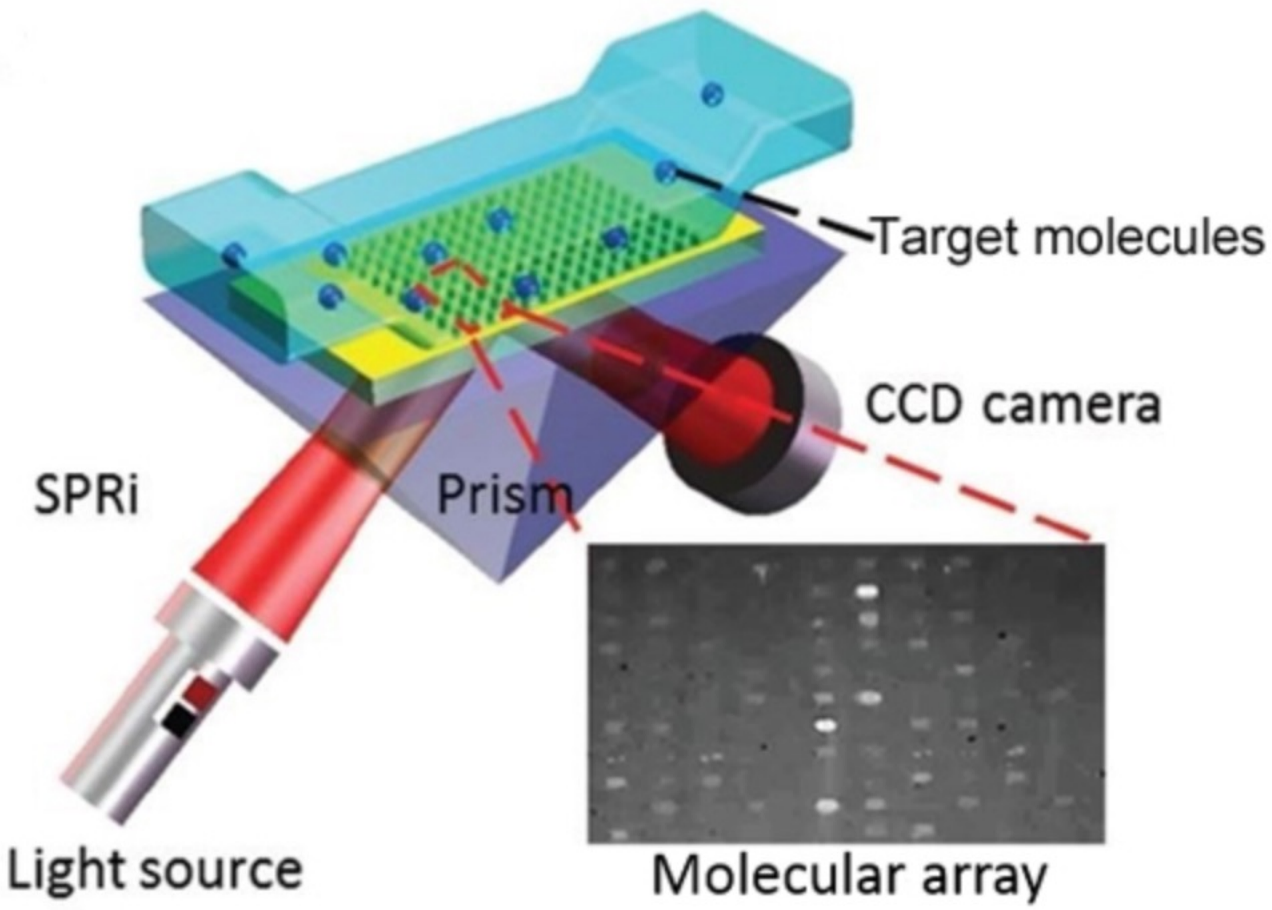
$$L = (2\mathrm{Im}\left[\beta\right])^{-1}$$

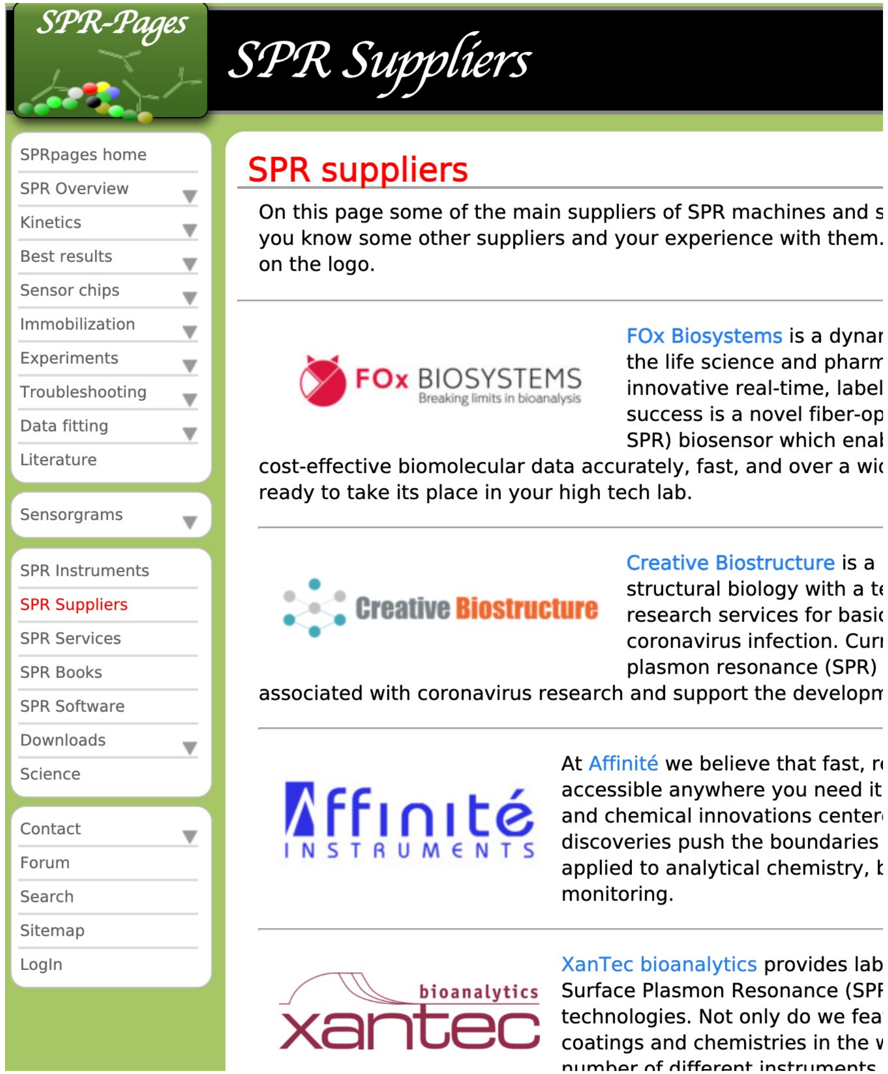
$$\hat{z} = 1/\left|k_z\right|$$



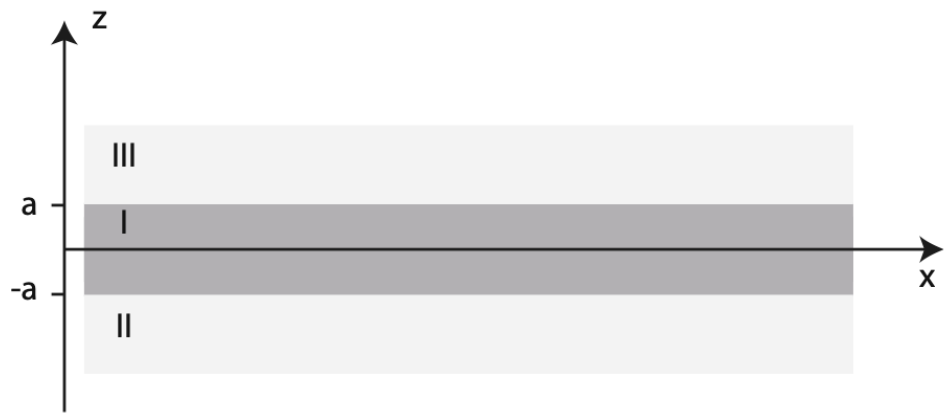

| lam0 [um] | lamSP [um] | L [um] | Zd [um] | Zm [um] |
|---|---|---|---|---|
| 1.064 | 1.053 | 108 | 1.2 | 0.024 |
| 0.623 | 0.595 | 8.11 | 0.32 | 0.0284 |
| 0.45 | 0.442 | 0.4 | 0.22 | 0.0345 |

# Surface Plasmon Resonance (SPR)



# Surface-plasmon-polaritons - slab geometry




For $z > a$

$$H_y = A e^{i\beta x} e^{-k_3 z}$$

$$E_x = iA \frac{1}{\omega \varepsilon_0 \varepsilon_3} k_3 e^{i\beta x} e^{-k_3 z}$$

$$E_z = -A \frac{\beta}{\omega \varepsilon_0 \varepsilon_3} e^{i\beta x} e^{-k_3 z},$$

for $z < -a$

$$H_y = B e^{i\beta x} e^{k_2 z}$$

$$E_x = -iB \frac{1}{\omega \varepsilon_0 \varepsilon_2} k_2 e^{i\beta x} e^{k_2 z}$$

$$E_z = -B \frac{\beta}{\omega \varepsilon_0 \varepsilon_2} e^{i\beta x} e^{k_2 z}.$$

# Surface-plasmon-polaritons - slab geometry

$$-a < z < a$$

$$H_y = C\mathrm{e}^{i\beta x}\mathrm{e}^{k_1 z} + D\mathrm{e}^{i\beta x}\mathrm{e}^{-k_1 z}$$

$$E_x = -iC\frac{1}{\omega\varepsilon_0\varepsilon_1}k_1\mathrm{e}^{i\beta x}\mathrm{e}^{k_1 z} + iD\frac{1}{\omega\varepsilon_0\varepsilon_1}k_1\mathrm{e}^{i\beta x}\mathrm{e}^{-k_1 z}$$

$$E_z = C\frac{\beta}{\omega\varepsilon_0\varepsilon_1}\mathrm{e}^{i\beta x}\mathrm{e}^{k_1 z} + D\frac{\beta}{\omega\varepsilon_0\varepsilon_1}\mathrm{e}^{i\beta x}\mathrm{e}^{-k_1 z}.$$

The requirement of continutity of $H_y$ and $E_x$ leads to

$$A\mathrm{e}^{-k_3 a} = C\mathrm{e}^{k_1 a} + D\mathrm{e}^{-k_1 a}$$

$$\frac{A}{\varepsilon_3}k_3\mathrm{e}^{-k_3 a} = -\frac{C}{\varepsilon_1}k_1\mathrm{e}^{k_1 a} + \frac{D}{\varepsilon_1}k_1\mathrm{e}^{-k_1 a}$$

$$k_i^2 = \beta^2 - k_0^2\varepsilon_i$$

at $z = a$ and at $z = -a$

$$\mathrm{e}^{-4k_1 a} = \frac{k_1/\varepsilon_1 + k_2/\varepsilon_2}{k_1/\varepsilon_1 - k_2/\varepsilon_2}\frac{k_1/\varepsilon_1 + k_3/\varepsilon_3}{k_1/\varepsilon_1 - k_3/\varepsilon_3}$$

$$B\mathrm{e}^{-k_2 a} = C\mathrm{e}^{-k_1 a} + D\mathrm{e}^{k_1 a}$$

$$-\frac{B}{\varepsilon_2}k_2\mathrm{e}^{-k_2 a} = -\frac{C}{\varepsilon_1}k_1\mathrm{e}^{-k_1 a} + \frac{D}{\varepsilon_1}k_1\mathrm{e}^{k_1 a}$$

# Surface-plasmon-polaritons - slab geometry

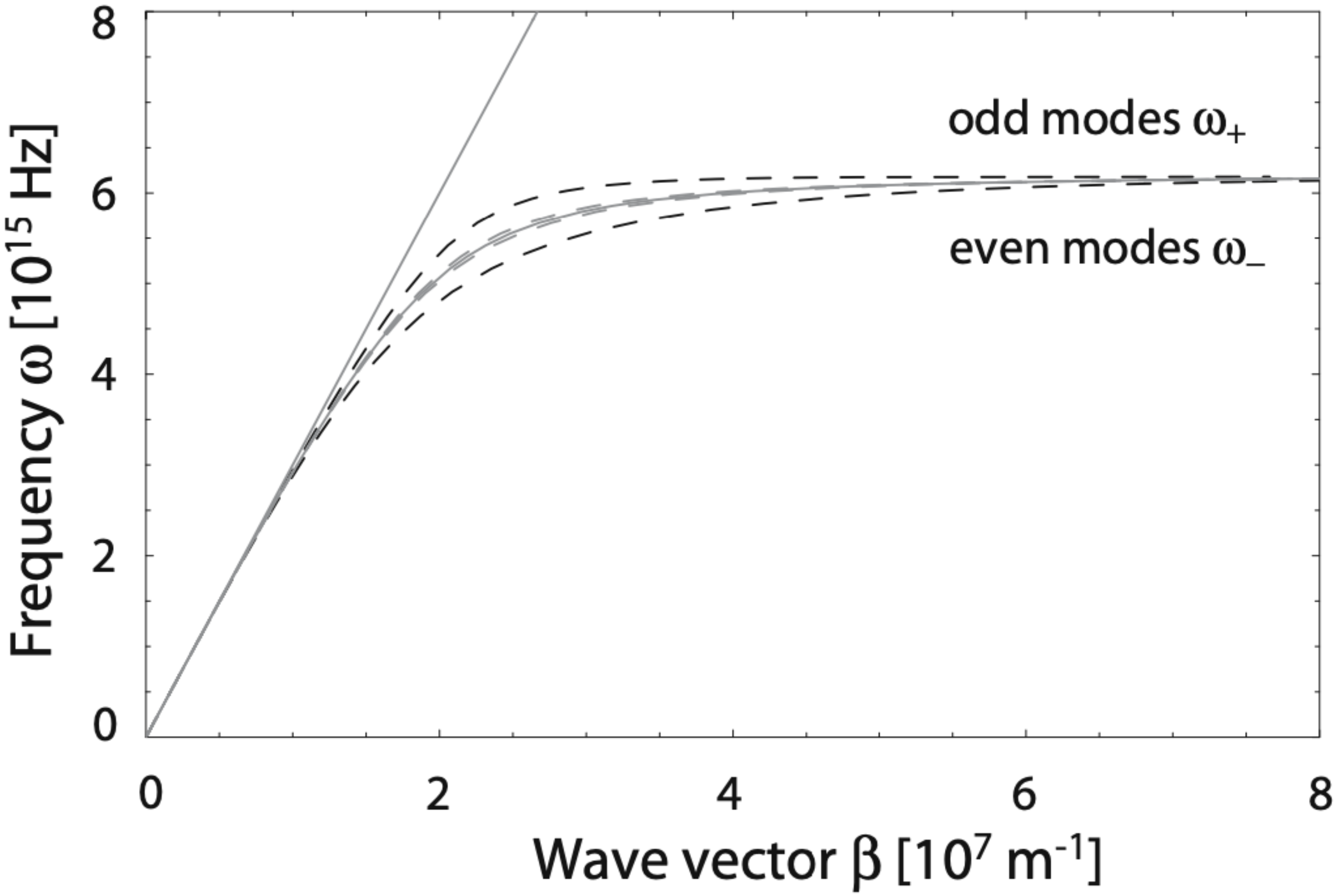
$$e^{-4k_1 a} = \frac{k_1/\varepsilon_1 + k_2/\varepsilon_2}{k_1/\varepsilon_1 - k_2/\varepsilon_2} \frac{k_1/\varepsilon_1 + k_3/\varepsilon_3}{k_1/\varepsilon_1 - k_3/\varepsilon_3}$$

$$\varepsilon_2 = \varepsilon_3 \text{ and thus } k_2 = k_3$$

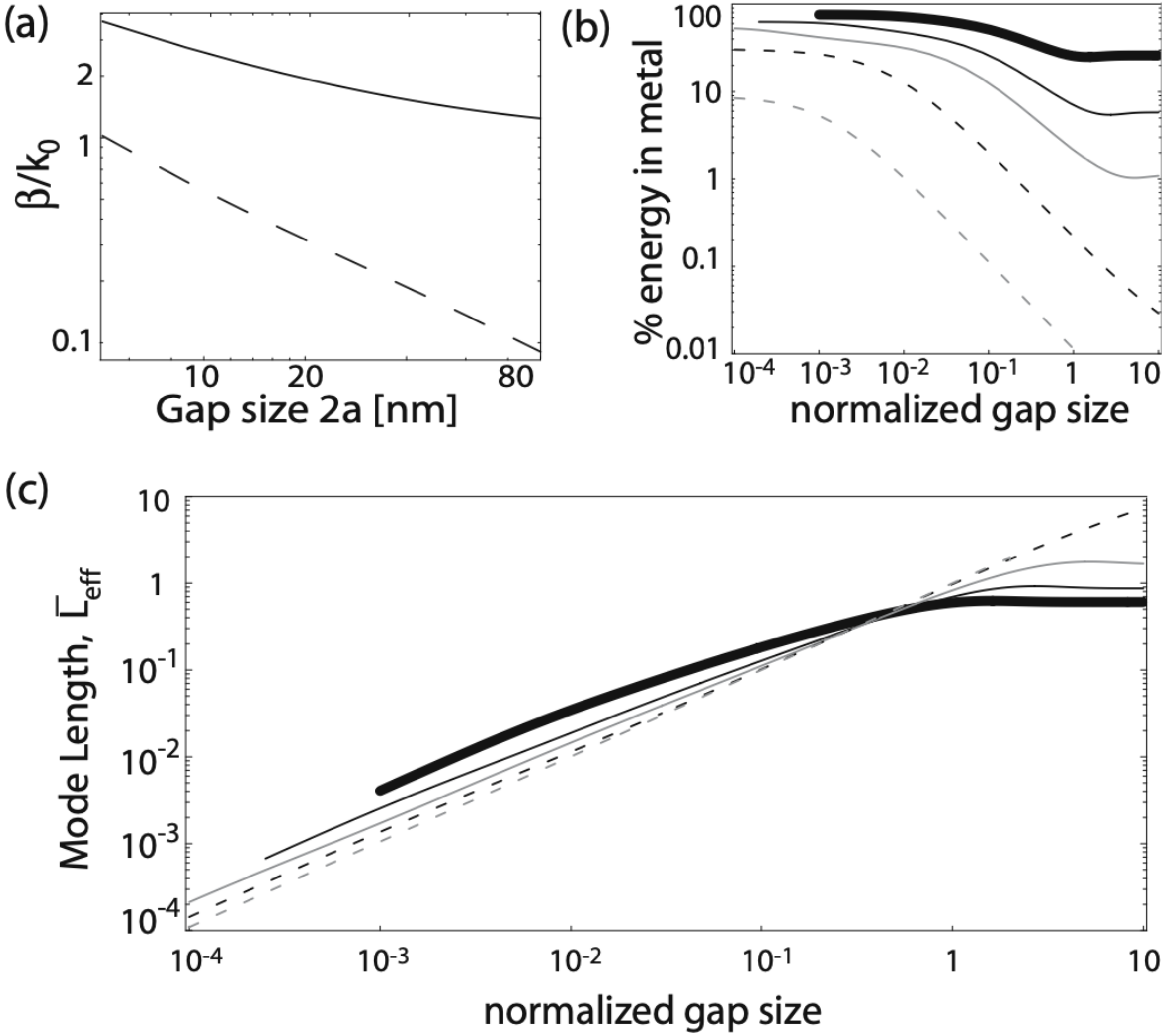
$$\tanh k_1 a = -\frac{k_2 \varepsilon_1}{k_1 \varepsilon_2}$$

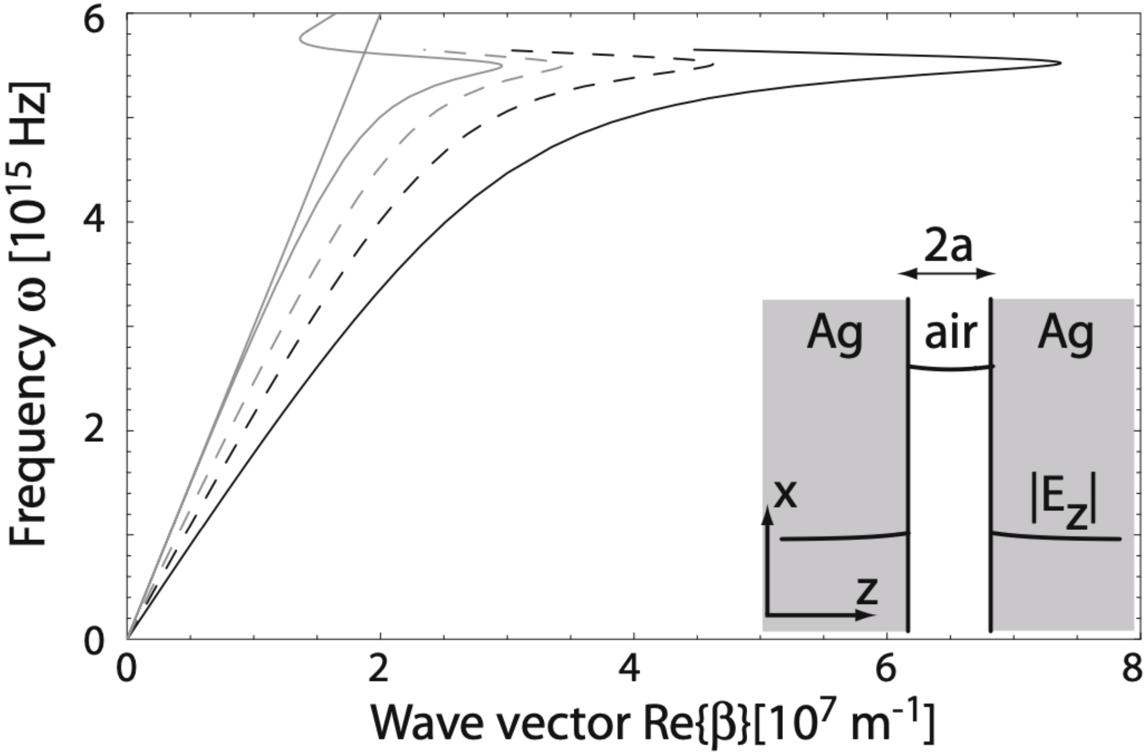
$$\tanh k_1 a = -\frac{k_1 \varepsilon_2}{k_2 \varepsilon_1}$$

Dispersion relation:

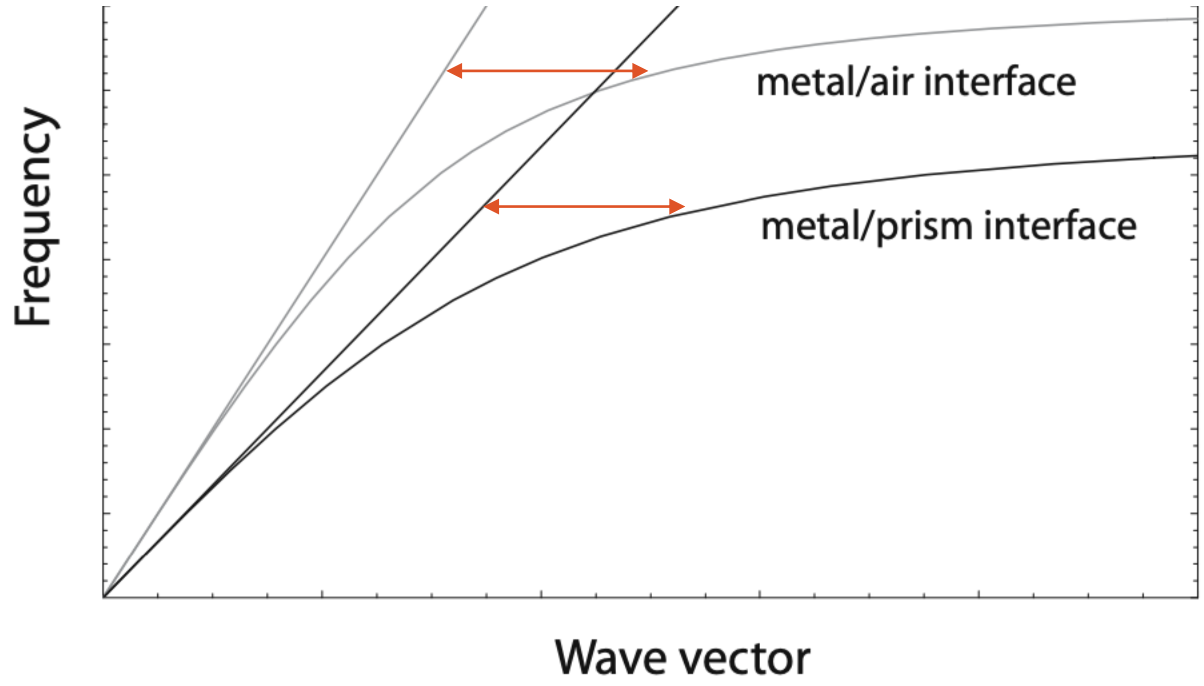


# SPPs – confinement and losses



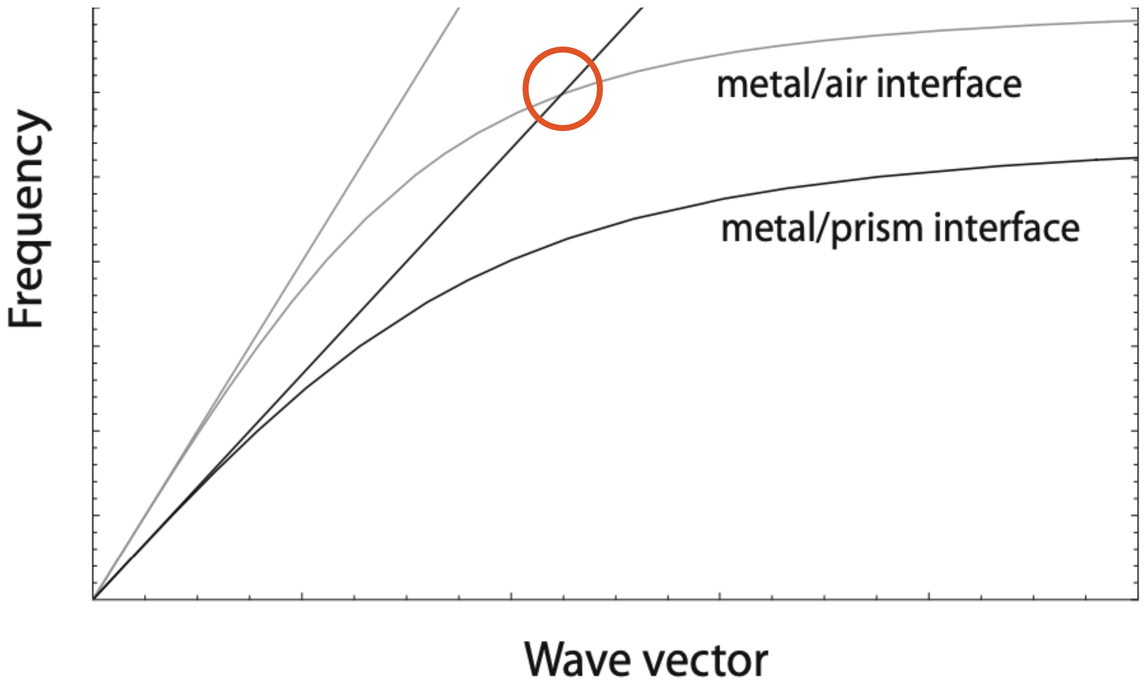



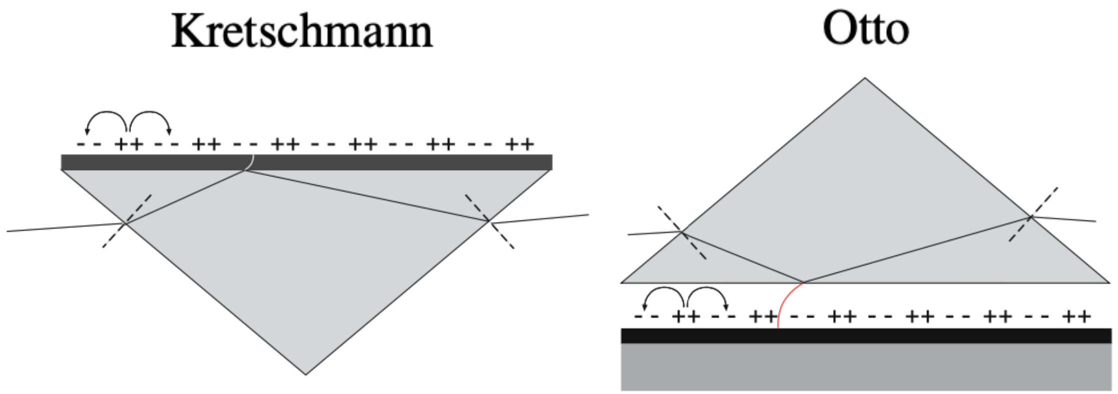
# SPPs – excitation



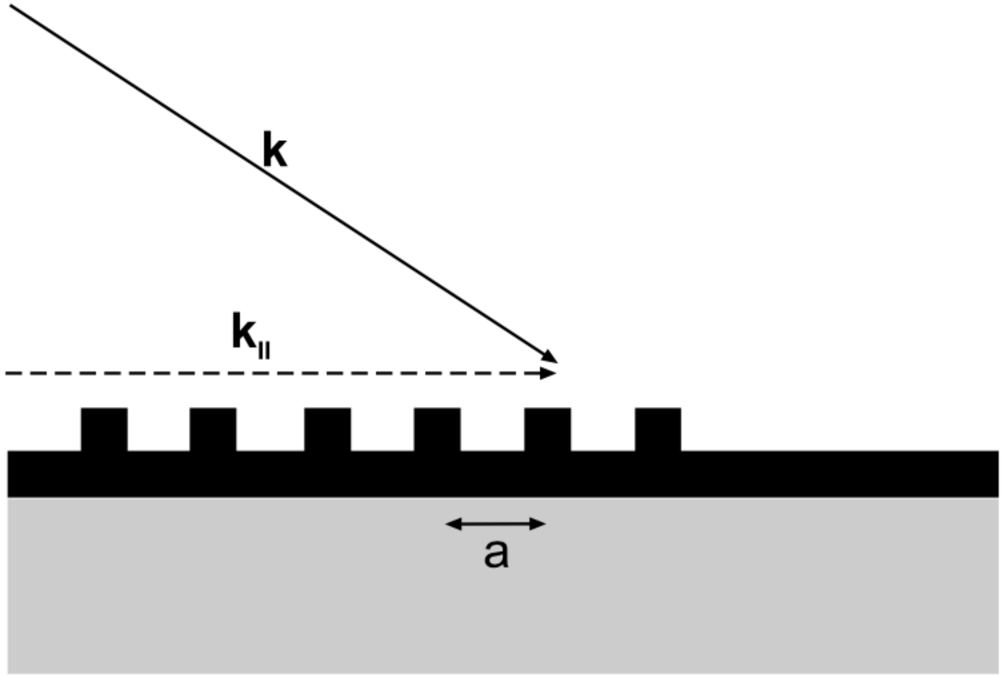

Momentum mismatch – Phase mismatch

# SPPs – prism coupling

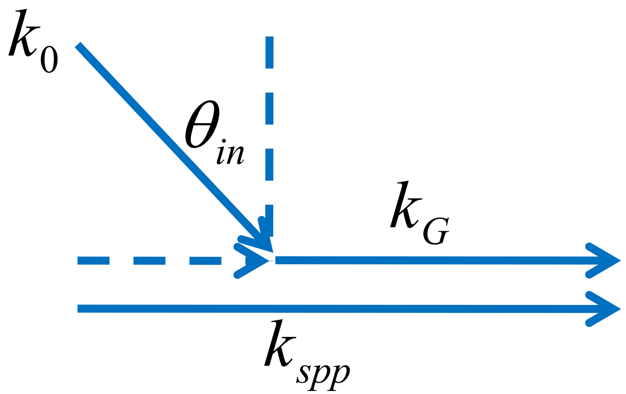


# SPPs – grating coupling




$$k_G = \frac{2\pi m}{\Lambda}$$




$$k_0 \sin\theta_{in} + mk_G = k_{spp}$$