⑥ $H = H_{matter} + H_{radiation} + V_{int}$.

① $H_m = \frac{\hat{L}^2}{2I}$; where I is the מומנט התמד : $I = 2\cdot m\left(\frac{L}{2}\right)^2 = \frac{1}{2}mL^2$ //

find e.s. & e.v. of H_m first: $H_m|\psi\rangle = \epsilon_m|\psi\rangle$

$\hat{L} = -i\hbar\frac{\partial}{\partial\varphi} \Rightarrow -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\varphi^2}\psi(\varphi) = \epsilon\psi(\varphi) \Rightarrow \boxed{\psi(\varphi) = Ae^{il\varphi}}$, $A \to \frac{1}{\sqrt{2\pi}}$

$\boxed{\epsilon_l = \frac{\hbar^2 l^2}{2I}}$

Quatization condition: $\psi(\varphi+2\pi) = \psi(\varphi) \Rightarrow e^{il(\varphi+2\pi)} = e^{il\varphi} \Rightarrow \boxed{l = 0, \pm1, \pm2 \ldots}$

② $\Gamma = \frac{2\pi}{\hbar}\left|\langle f|V_{int}|i\rangle\right|^2 \delta(\epsilon_f - \epsilon_i)$ (fermi Golden Rule)

absorption: $|i\rangle = |n, l\rangle$; $|f\rangle = |n-1^{k\alpha}, l+1\rangle$

need to calc.: $\langle n-1^{k\alpha}, l+1| -\sum_{k\alpha}\left(\frac{\hbar}{\omega\Omega}\right)^{1/2}\vec{J}_{-k}\cdot\vec{\lambda}_{k\alpha}(a_{k\alpha}+a^\dagger_{k\alpha})|n, l\rangle \equiv$ ☺

$= -\left(\frac{\hbar}{\omega\Omega}\right)^{1/2}\sqrt{n}\,\langle l+1|\vec{J}_{-k}|l\rangle\cdot\vec{\lambda}_{k\alpha}$

③ concentrate on: $\langle l+1|\vec{J}_{-k}|l\rangle = \int d^3r\,(1+i\vec{k}\cdot\vec{r}+\ldots)\langle l+1|\vec{J}(\vec{r})|l\rangle$

case I: $q_1 = -q_2 \longrightarrow$ dipole apx: $\langle l+1|\vec{J}_{-k}|l\rangle \simeq \int d^3r\,\langle l+1|\vec{J}(\vec{r})|l\rangle \equiv \langle l+1|\vec{J}_0(\vec{r})|l\rangle$

$\underset{\text{shiwos notes}}{=} -i\omega_{k\alpha}\langle l+1|\vec{d}|l\rangle$

$\underset{\text{in } \varphi \text{ basis}}{\langle l+1|\hat{d}|l\rangle} = \int d\varphi\,d\varphi'\langle l+1|\varphi\rangle\underbrace{\langle\varphi|\hat{d}|\varphi'\rangle}_{qL\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\delta(\varphi-\varphi')}\langle\varphi'|l\rangle = \int d\varphi\,\psi^*_{l+1}(\varphi)\,qL\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\psi_l(\varphi)$

classically: $\vec{d} = \sum_{i=1}^{2}\vec{r}_i q_i = q\frac{L}{2}\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix} + (-q)\frac{L}{2}\begin{pmatrix}-\cos\varphi\\ -\sin\varphi\\ 0\end{pmatrix} = qL\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}$

$= qL\int d\varphi\frac{1}{2\pi}e^{-i(\cancel{l}+1)\varphi}e^{\cancel{il\varphi}}\begin{pmatrix}\frac{1}{2}(e^{i\varphi}+e^{-i\varphi})\\ \frac{1}{2i}(e^{i\varphi}-e^{-i\varphi})\\ 0\end{pmatrix} = \frac{1}{2\pi}\int\frac{qL}{2}\begin{pmatrix}1+e^{-2i\varphi}\\ -i+ie^{-2i\varphi}\\ 0\end{pmatrix}d\varphi = \frac{qL}{2}\begin{pmatrix}1\\ -i\\ 0\end{pmatrix}\cancel{\frac{2\pi}{2\pi}}$

write $\vec{\lambda}$ in circular pol. basis: $\lambda_{k\alpha} = a\begin{pmatrix}1\\ i\\ 0\end{pmatrix} + b\begin{pmatrix}1\\ -i\\ 0\end{pmatrix} + c\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$ where a, b, c are real coef's

$\langle l+1|\vec{d}|l\rangle\cdot\vec{\lambda}_{k\alpha} = \frac{qL}{2}(2a+0\cdot b+0\cdot c) = qLa \equiv qL\lambda_+$ change name $a \to \lambda_+$

④ Put it all together:

$\Gamma = \frac{2\pi}{\hbar}\left|-\left(\frac{\hbar}{\omega\Omega}\right)^{1/2}\sqrt{n}\,(-i\omega_{k\alpha})\,qL\lambda_+\right|^2 = \frac{2\pi}{\hbar}\frac{\hbar}{\omega\Omega}n\,\omega_{k\alpha}^2 q^2L^2\lambda_+^2$

$\boxed{\Gamma_{k\alpha} = \frac{2\pi n}{\Omega}q^2L^2\omega_{k\alpha}\lambda_+^2\,\delta(\epsilon_f-\epsilon_i)}$

$\omega_{k\alpha} = \frac{\Delta\epsilon}{\hbar} = \frac{\epsilon(l+1)-\epsilon(l)}{\hbar} = \frac{\hbar^2}{2I\hbar}\left((l+1)^2-l^2\right) = \frac{\hbar}{2I}(2l+1)$ //

Needs to complete the $q_1 = q_2$ case (electric Quadrupole + Mag. dipole)

~~Put it all together~~

④ $\langle \ell+1 | \vec{x} | \ell \rangle = \frac{qL}{2}\begin{pmatrix}1\\-i\\0\end{pmatrix}$

$$\lambda_\mp = \frac{1}{\sqrt{2}}\left(\cos\theta\cos\varphi \mp i\sin\varphi,\ \cos\theta\sin\varphi \pm i\cos\varphi,\ -\sin\theta\right)$$

see tutorial

$$\Rightarrow \langle \ell+1 | \vec{x} | \ell \rangle \cdot \vec{\lambda}_\pm = \frac{qL}{2}\left(\cos\theta\cos\varphi \mp i\sin\varphi - i\cos\theta\sin\varphi \pm \cos\varphi\right)$$

$$\Big| \qquad \Big|^2 = \left(\frac{qL}{2}\right)^2\left((\cos\theta\cos\varphi \pm \cos\varphi)^2 + (\mp\sin\varphi - \sin\varphi\cos\theta)^2\right)$$

$$= \left(\frac{qL}{2}\right)^2\left[\cos^2\varphi(\cos\theta\pm1)^2 + \sin^2\varphi(\cos\theta\pm1)^2\right] = \left(\frac{qL}{2}\right)^2(\cos\theta\pm1)^2$$

⑤ Put it all together:

$$\Gamma = \frac{2\pi}{\hbar}\left|\langle f | V_{int} | i \rangle\right|^2 \delta(\varepsilon_f - \varepsilon_i) = \frac{2\pi}{\hbar}\left(\frac{\hbar}{2\omega\Omega}\right) n\,\omega^2\left(\frac{qL}{2}\right)^2(\cos\theta\pm1)^2\,\delta(\varepsilon_f-\varepsilon_i)$$

Now: $\omega_{ka} = \frac{\Delta\varepsilon}{\hbar} = \frac{\hbar^2\left((\ell+1)^2-\ell^2\right)}{2I\hbar} = \frac{\hbar}{2I}(2\ell+1) = \frac{\hbar}{mL^2}(2\ell+1)$

$I = \frac{1}{2}mL^2$

$$\boxed{\Gamma_{k\lambda_\pm} = \frac{2\pi}{\Omega} n\,\frac{\hbar}{mL^2}(2\ell+1)\left(\frac{qL}{2}\right)^2(\cos\theta\pm1)^2\cdot\delta\left(c|k| - \frac{\hbar}{mL^2}(2\ell+1)\right)}$$

direction of photon flight, i.e. of k