Q1

$$m^2\phi + \sum_{n=1}^{\infty}\lambda_n(2n+2)\phi^{2n+1} + \partial_\mu\partial^\mu\phi = 0$$

Q2

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = 0$$

$$\text{cross term} = -g(\phi_1^4 + \phi_2^4 + 2\phi_1^2\phi_2^2)$$

1) $\text{EOM1}: \ (\Box + m^2)\phi_1 = 4g(\phi_1^4 + \phi_2^2)$

$\text{EOM2}: \ (\Box + m^2)\phi_2 = 4g(\phi_2^4 + \phi_1^2)$

2) $$\mathcal{L} = \frac{1}{2}\underbrace{\partial_\mu\phi^*\partial^\mu\phi}_{(\partial_\mu\phi_1\ \partial_\mu\phi_2)\begin{pmatrix}\partial^\mu\phi_1\\ \partial^\mu\phi_2\end{pmatrix}} - \frac{1}{2}m^2\underbrace{\phi^+\phi}_{(\phi_1^2+\phi_2^2)} - g\underbrace{(\phi^*\phi)^2}$$

$$= (\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2)$$

וזה ייתן אותן משוואות.

3) רק צריך להסביר את שני הביטויים

$$\partial_\mu\phi^*\partial^\mu\phi, \ \phi^*\phi$$

כרס׳ :

נסמן

$$R = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

$$R^+R = \begin{pmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{pmatrix} = \mathbb{1}$$

$$\Rightarrow \ \phi^{*\prime}\phi' = \phi^+R^+R\phi = \phi^*\phi$$

כל $*$ מתחלפים ב- R^+

$$\partial_\mu\phi^{+\prime}\partial^\mu\phi' = \partial_\mu(\phi^*R^+)\partial^\mu(R\phi) = \partial(\phi^+)R^+R\partial^\mu(\phi)$$

פה שיניתי את הסדר
ו R לא תלוי בקואורדינטות

$$= \partial_\mu\phi^+\partial^\mu\phi$$

$$\Rightarrow \quad \mathcal{L}' = \mathcal{L}$$

Q3

1) Kinetic term: $\frac{1}{2}\partial_\mu \phi \partial^\mu \phi$

$$\partial_\mu = (\partial_t, \partial_i) \qquad \partial^\mu = \begin{pmatrix} \partial_t \\ -\partial_i \end{pmatrix}$$

$$\Rightarrow \frac{1}{2}\partial_\mu \phi \partial^\mu \phi = \frac{1}{2}(\partial_t \phi, \partial_i \phi)\begin{pmatrix} \partial_t \phi \\ -\partial_i \phi \end{pmatrix} = \frac{1}{2}\Big[(\underbrace{\partial_t \phi}_{\dot\phi})^2 - (\nabla \phi)^2\Big]$$

2)
$$\pi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \dot\phi$$

3)
$$\mathcal{H} = \sum \dot q \frac{\partial \mathcal{L}}{\partial \dot q} - \mathcal{L}$$

$$= \dot\phi^2 - \mathcal{L} = \frac{1}{2}\dot\phi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$$

4)
$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi = (\partial_t \phi, -\nabla \phi)$$

Q4

Cרנס' פאזה ב-θ שקול לסיבוב

ב-θ. הרי הכפלה במספר מרוכב זה סיבוב

במישור המרוכב.

Question 5

In the ABC model: Calculate the Matrix Element of $AA \to BB$ in the center-of-mass frame, assuming $m_B = m_C = 0$. Give the answer in terms of the energy of each particle (E), the mass of particle A (m_a) and the scattering angle θ. Use the results obtained in class for ease of calculations.

$$\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_c^2} + \frac{g^2}{(p_3 - p_2)^2 - m_c^2}$$

$$P_1 = (E, P, 0, 0) \qquad P_2 = (E, -P, 0, 0)$$

$$P_3 = (E, P\cos\theta, P\sin\theta, 0) \quad P_4 = (E, -P\cos\theta, -P\sin\theta, 0)$$

$$E^2 = P^2 + m^2$$

$$(P_4 - P_2)^2 = m^2 - 2E^2 - 2P^2\cos\theta$$

$$= m^2 - 2P^2(1+\cos\theta)$$

$$(P_3 - P_2)^2 = m^2 - 2P^2(1-\cos\theta)$$

$$\Rightarrow \mathcal{M} = g^2\left[\frac{1}{m^2 - 2P^2(1-\cos\theta)} + \frac{1}{m^2 - 2P^2(1+\cos\theta)}\right]$$

$$m^2 - 2P^2 + 2P^2\cos\theta \qquad m^2 - 2P^2 - 2P^2\cos\theta$$

$$P^2 = E^2 - m^2$$

$$= g^2\left[\frac{6m^2 - 4E^2}{(3m^2 - 2E^2) - 4(E^2 - m^2)^2\cos^2\theta}\right]$$

Question 6

In the ABC model:

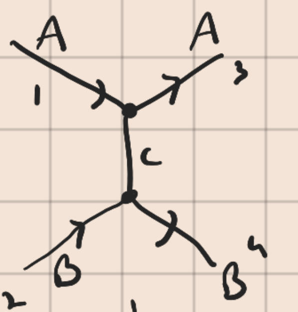
1. Draw the leading order diagrams and write an expression for the scattering amplitude $\mathcal{M}$ for the $AB \to AB$ process (there are 2 leading order diagrams).
2. Draw all the lowest order diagrams for the process $AA \to AA$ (there are six diagrams).

1) אין לנו דיאגרמת u channel כי A,B מובחנים, לכן

יש רק s,t:

s

$q = p_1 + p_2$



t

$q = p_1 - p_3$

$$\mathcal{M}_s = \int (-ig)(-ig)\frac{i}{q^2 - m_c^2}(2\pi)^4\delta(q - p_1 - p_2)(2\pi)^4\delta(q - p_3 - p_4)\frac{d^4q}{(2\pi)^4}$$

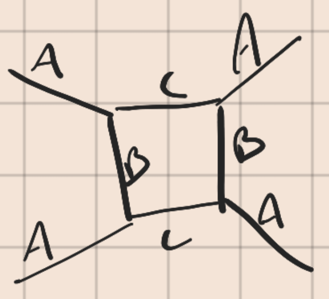
$$= \frac{g^2}{(p_1+p_2)^2 - m_c^2}$$

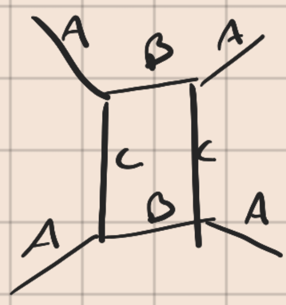
$$\mathcal{M}_t = \int (-ig)(-ig)\frac{i}{q^2 - m_c^2}(2\pi)^4\delta(q - p_1 + p_3)(2\pi)^4\delta(q - p_2 - p_4)\frac{d^4q}{(2\pi)^4}$$

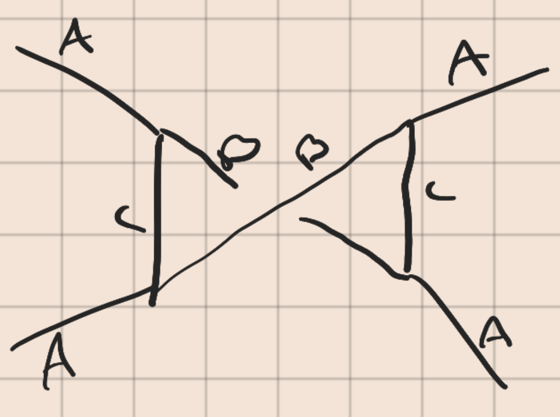
$$= \frac{g^2}{(p_1-p_3)^2 - m_c^2}$$

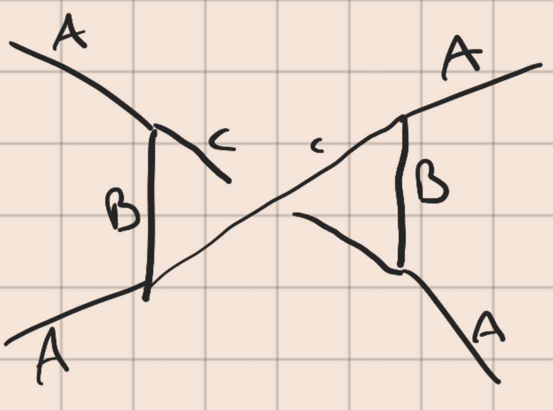
$$\mathcal{M} = \mathcal{M}_s + \mathcal{M}_t = \frac{g^2}{(p_1+p_2)^2 - m_c^2} + \frac{g^2}{(p_1-p_3)^2 - m_c^2}$$

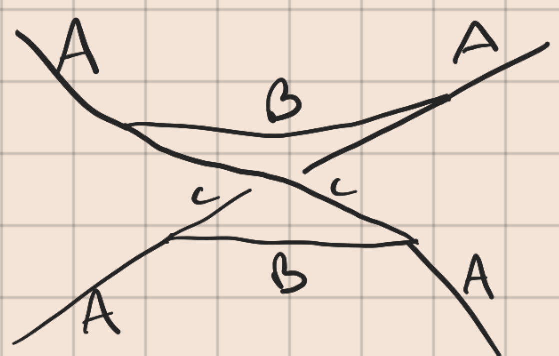
2)

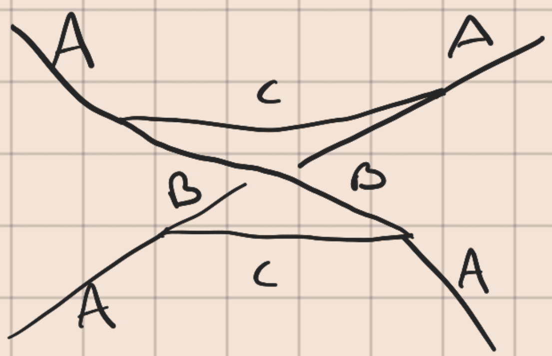












Q 7

בגבול הלא יחסותי $m \approx E$

$$p_i = (m_i, \bar{p}_i)$$

$$q^2 = 2m^2 - 2(\underbrace{m^2 + q^2}_{E^2})$$

$$V(r) = \int_{-\infty}^{\infty} \frac{g^2 e^{i\vec{q}\cdot\vec{r}}}{q^2 - m^2} \frac{d^3q}{(2\pi)^3}$$

$$= -\frac{g^2}{(2\pi)^3} \int \frac{e^{iqr\cos\theta}}{q^2+m^2} \, d\varphi \, \sin\theta \, d\theta \, q^2 dq \qquad \left.\frac{e^{iqrx}}{iqr}\right|_{x=1}^{x=-1}$$

$$= -\frac{g^2}{(2\pi)^2} \int \frac{e^{iqr\cos\theta}}{q^2+m^2} \, d(\cos\theta) \, q^2 dq$$

$$(q-im)(q+im)$$

$$= -\frac{g^2}{i(2\pi)^2} \frac{1}{r} \left[\int \frac{q e^{iqr}}{q^2+m^2} dq - \int \frac{q e^{-iqr}}{q^2+m^2} dq \right]$$

$$\cancel{2}\pi i \frac{\cancel{im} e^{-mr}}{\cancel{2im}} + 2\pi i \frac{im e^{-mr}}{2im} = 2\pi i e^{-mr}$$

$$= \boxed{-\frac{g^2 e^{-mr}}{2\pi r}}$$

עבור $m \to 0$ זה פוטנציאל קולון רק עם

מינוס. כלומר מושך לא דוחה.