Introduction to Particles and Nuclear Physics - Home Exercise 5

Question 1

In class we found the cross section for two-body scattering in the lab frame, for the case of two massless particles in the final state.

a) Show that in the general case of the final state particles having masses, the cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{\mathbf{p_3}^2 \; \mathcal{S} \; |\mathcal{M}|^2}{m_2|\mathbf{p_1}| \; |(E_1 + m2)|\mathbf{p_3}| - |\mathbf{p_1}|E_3 \cos\theta|}$$

b) If the incident particle is massless ($m_1 = 0$) and the collision is "elastic" ($m_3 = m_1; m_2 = m_4$), show that the result of (a) simplifies to

$$\frac{d\sigma}{d\Omega} = \mathcal{S} \left(\frac{E_3}{8\pi m_2 E_1} \right)^2 |\mathcal{M}|^2$$

Question 2

In the cosmic background radiation, highly energetic protons reach earth's atmosphere. In the interactions between the protons and the atmosphere nuclei, many particles are created, the majority of which are *pions*. What is the initial momentum required for a π^+ created at height of 50 km above see level, in order for it to have a probability of $\frac{1}{e}$ to reach the surface?

Question 3

A collider provides a luminosity of $10^{34}\text{cm}^{-2}\text{s}^{-1}$ and runs for a year.

1. What is the integrated luminosity (in fb^{-1})?
2. How many events of a kind that has a cross section of 10 pb were produced during that year?

Question 4

Consider a beam of neutrinos moving in the x direction at energy of 200 GeV, and passing through a block of iron.

1. The density of iron is $\rho = 7.9\,g\,cm^{-3}$. In the rest frame of the neutrino, what is the flux of iron nuclei which are moving in its direction (give the answer in term of the neutrino velocity, the iron density, and the iron nucleus mass)?
2. The neutrino might scatter off a nucleus with cross section σ_ν. What is the probability for a neutrino to scatter in an infinitesimal length element dx? If $N(x)$ is the number of surviving neutrinos, - How many neutrino scatterings in total will occur in an infinitesimal length element dx?

3. Use the previous answer, and find the parameter λ in the equation

$$\frac{dN(x)}{dx} = -\lambda N(x)$$

Where N is the number of neutrino that did not scatter.
4. Assume that at high energies the neutrino-nucleon total cross section is given approximately by $\sigma_\nu \sim 10^{-38} \cdot E_\nu \, cm^2$, where E_ν is the neutrino energy given in GeV, and find the numerical value of λ.
5. Solve the equation from part (c). In your answer use the quantities N_0, the initial number of neutrinos in the beam, and λ.
6. Estimate the thickness of iron through which a beam of such neutrinos must travel if 1 in 10^9 of them is to interact.

Question 5

Starting from the general formula for the cross section of a $2 \to n$ scattering process:

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |M|^2 (2\pi)^4 \delta^4(p_1 + p_2 + \sum_{j=3}^{N} p_j) \prod_{k=3}^{N} 2\pi\delta(p_k^2 - m_k^2)\Theta(p_k^0) \frac{d^4 p_k}{(2\pi)^4}$$

Derive the expression for the differential cross section of a $2 \to 2$ process in the COM frame.

Instructions:

* Start with simplifying the general expression by performing the dp_k^0 integrals. Use the $\delta(p_k^2 - m_k^2)\Theta(p_k^0)$ term to do that.
* Show that in the COM frame, $\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2} = (E_1 + E_2)|\mathbf{p_1}|$, and plug this replacement into the denominator of the external fraction multiplying the integral.
* Use the spatial part of the $\delta^4(p_1 + p_2 + \sum_{j=3}^{N} p_j)$ term to perform the integral over $d^3\mathbf{p_4}$.
* Move to spherical coordinates ($d^3\mathbf{p_3} = |\mathbf{p_3}|^2 d|\mathbf{p_3}| d\Omega$) and perform the integral over $d|\mathbf{p_3}|$ using substitution of variables and the last remaining δ-function.
* Organize the expression to reach the final formula:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{S\ |M|^2}{(E_1 + E_2)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$

Question 6 (**OPTIONAL QUESTION**):

Consider the cross section ratio: $\frac{\sigma(e^+e^- \to e^+e^-)}{\sigma(e^+e^- \to \mu^+\mu^-)}$ at two different center-of-mass energies:

a) $E_{CM} = 500\ MeV$
b) $E_{CM} = M_{Z^0}$

At which COM energy will this ratio be closer to 1? Explain why.