Electrolitic
Exercise 3

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$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}.$$

Question 1.1

Show that from the equation:

$$m\frac{\mathrm{d}U^{\mu}}{\mathrm{d}\tau} = \frac{q}{c}F^{\mu\nu}U_{\nu}.$$

The lorentz force and the work eq. can be derived.
Which of the fields $\vec{E}$, $\vec{B}$ execute work?

Solution:
For $\mu = 0$:

$$m\frac{\mathrm{d}U^{0}}{\mathrm{d}\tau} = \frac{q}{c}\left(\vec{E}\cdot\vec{v}\right) = \frac{1}{c}\left(\vec{f}\cdot\vec{v}\right) = \frac{1}{c}\frac{\mathrm{d}E}{\mathrm{d}t}.$$

For $\mu = i$:

$$m\frac{\mathrm{d}U^{i}}{\mathrm{d}\tau} = \frac{q}{c}\left(F^{i0}U_0 + F^{0i}U_i\right) = \frac{q}{c}\gamma\left(c\vec{E} + \vec{B}\times\vec{v}\right).$$

Only the electric field execute work because the term $\vec{B}\times\vec{v}$ is perpendicular to $\vec{v}$ and will be nullified in the dot product $\vec{f}\cdot\vec{v}$.

Question 1.2

Find $\vec{r}\left(t\right)$ of a relativistic particle in a constant magnetic field, and show the non-relativistic approximation.
What's the diffrence between the case of a constant electric field and the case of a magnetic one?

Solution:
We'll use our freedom of choise of a coordinate system and pick $\vec{B}$ such that:

$$\vec{B} = B\hat{z}$$
$$\vec{v}\left(t=0\right) = v_{0x}\hat{x} + v_{0y}\hat{y}.$$