Introduction to Particles and Nuclear Physics - Home Exercise 1

Question 1

1. Express a time interval of 1 ns in natural units.
2. In class we saw the wave function of a free particle in it's centre-of-mass frame.
   Write the wave function of the particle, as a function of it's energy and momentum, assuming it is moving in the x-direction.
3. What is the wavelength of the particle (in ordinary units) assuming it's momentum is 1 GeV?

Question 2

1. Using the relativistic energy and momentum definition, explicitly show that the norm of the 4-momentum is $p_\mu p^\mu = m^2$. Determine whether physical 4-momentum can have a negative norm.
2. Show that in the non-relativistic limit, the definitions of relativistic energy and momentum match those of Newtonian Mechanics

Question 3

Consider a particle of mass m that decays into two particles m_1, m_2. Using 4-momentum conservation, find the energy and momenta of the outgoing particles.

Question 4

1. Show that an elastic collision between 2 particles does not require conservation of mass (that is, that the initial and final masses do not need to be the same).
2. Show that the conservation of energy and momentum in an elastic collision between 2 particles holds in every inertial frame of reference: Write down the conservation law in one frame, apply Lorentz transformation on the 4-momentum of both particles, and check that the law holds in the new reference frame.

Question 5

The "***rapidity***" of a particle is denoted y and defined as: $y \equiv \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$

1. Explicitly show that the rapidity transforms under a boost in the $\hat{z}$ direction as: $y \to y' = y + \frac{1}{2} \ln \frac{1-\beta}{1+\beta}$.
2. Show that the difference in the rapidity of two particles Δy is invariant under a boost in the $\hat{z}$ direction.

3. Show that in the relativistic limit $E, p >> m$, the rapidity reduces to: $y \rightarrow \eta \equiv -\ln \tan \frac{\theta}{2}$, where θ is the polar angle in a spherical coordinate system.

   This η, known as "**pseudo-rapidity**", is the polar coordinate commonly used in analyzing particle collider kinematics.