

Contents

1 Basic

1.1 Increase Stack Size

```
//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        }
    }
}
```

1.2 Misc

```
□ □ □ □ -std=c++14 -Wall -Wshadow (-fsanitize=
    undefined)
//check special cases for example (n==1)
//check size arrays

#include <random>
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

#define SECS ((double)clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;

__builtin_popcountll //□ □ □ □ □ □ □ □ 1
__builtin_clzll //□ □ □ □ □ □ □ □ 1 □ □ 0 □ □ □
__builtin_parityll //□ □ 1 □ □ □ □ □ □ □
__builtin_mul_overflow(a,b,&h) //□ □ a*b □ □ □ □
```

1.3 check

```
for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done
```

1.4 python-related

```
parser:
int(eval(num.replace("/", "/")))

from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

itwo = Decimal(0.5)
two = Decimal(2)

format(x, '0.10f') # set precision

N = 200
def angle(cosT):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cosT = ((cosT + 1) / two) ** itwo
        sinT = (1 - cosT * cosT) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))
```

2 flow

2.1 MinCostFlow

```
struct MinCostMaxFlow{
typedef int Tcost;
static const int MAXV = 20010;
static const int INFF = 1000000;
static const Tcost INFC = 1e9;
struct Edge{
    int v, cap;
    Tcost w;
    int rev;
    Edge(){
    }
    Edge(int t2, int t3, Tcost t4, int t5)
        : v(t2), cap(t3), w(t4), rev(t5) {}
};
int V, s, t;
vector<Edge> g[MAXV];
void init(int n, int _s, int _t){
    V = n; s = _s; t = _t;
    for(int i = 0; i <= V; i++) g[i].clear();
}
void addEdge(int a, int b, int cap, Tcost w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
}
Tcost d[MAXV];
int id[MAXV], mom[MAXV];
bool inqu[MAXV];
queue<int> q;
pair<int,Tcost> solve(){
    int mxf = 0; Tcost mnc = 0;
    while(1){
        fill(d, d+1+V, INFC);
        fill(inqu, inqu+1+V, 0);
        fill(mom, mom+1+V, -1);
        mom[s] = s;
        d[s] = 0;
        q.push(s); inqu[s] = 1;
        while(q.size()){
            int u = q.front(); q.pop();
            inqu[u] = 0;
            for(int i = 0; i < (int) g[u].size(); i++){
                Edge &e = g[u][i];
                int v = e.v;
                if(e.cap > 0 && d[v] > d[u]+e.w){
                    d[v] = d[u]+e.w;
                    mom[v] = u;
                    id[v] = i;
                    if(!inqu[v]) q.push(v), inqu[v] = 1;
                }
            }
            if(mom[t] == -1) break;
            int df = INFF;
            for(int u = t; u != s; u = mom[u])
                df = min(df, g[mom[u]][id[u]].cap);
            for(int u = t; u != s; u = mom[u]){
                Edge &e = g[mom[u]][id[u]];
                e.cap -= df;
                g[e.v][e.rev].cap += df;
            }
            mxf += df;
            mnc += df*d[t];
        }
        return {mxf,mnc};
    }
}
} }flow;
```

2.2 Dinic

```
const int MXN = 10000;
struct Dinic{
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
```

```

level[s] = 0;
while (!que.empty()){
    int u = que.front(); que.pop();
    for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
            level[it.v] = level[u]+1;
            que.push(it.v);
        }
    }
    return level[t] != -1;
}
int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
        if (it.f > 0 && level[it.v] == level[u]+1){
            int tf = DFS(it.v, min(nf,it.f));
            res += tf; nf -= tf; it.f -= tf;
            E[it.v][it.re].f += tf;
            if (nf == 0) return res;
        }
    }
    if (!res) level[u] = -1;
    return res;
}
int flow(int res=0){
    while ( BFS() )
        res += DFS(s,2147483647);
    return res;
} }flow;

```

2.3 Kuhn Munkres

```

struct KM{ // max weight, for min negate the weights
    static const int MXN = 2001; // 1-based
    static const ll INF = 0x3f3f3f3f;
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) {
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;}
    void augment(int y) {
        for(int x, z; y; y = z)
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]){
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0){
                        pa[y]=x;
                        if(!my[y]){augment(y);return;}
                        vy[y]=1, q.push(my[y]);
                    }else if(sy[y]>t) pa[y]=x,sy[y]=t;
                }
            }
            ll cut = INF;
            for(int y=1; y<=n; ++y)
                if(!vy[y]&&cut>sy[y]) cut=sy[y];
            for(int j=1; j<=n; ++j){
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
                if(!my[y]){augment(y);return;}
                vy[y]=1, q.push(my[y]);
            }
        }
    }
    ll solve(){
        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    } }graph;

```

2.4 Max flow with lower/upper bound

```

// flow use ISAP
// Max flow with lower/upper bound on edges
// source = 1, sink = n
int in[ N ], out[ N ];
int l[ M ], r[ M ], a[ M ], b[ M ]; //0-base,a[],b[]
int solve(){
    flow.init( n ); //n,m,base
    for( int i = 0 ; i < m ; i ++ ){
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ], r[ i ], b[ i ] - a[ i ] );
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd = 0;
    for( int i = 1 ; i <= n ; i ++ ){
        if( in[ i ] < out[ i ] ){
            flow.addEdge( i, flow.t, out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
        if( out[ i ] < in[ i ] )
            flow.addEdge( flow.s, i, in[ i ] - out[ i ] );
    }
    // original sink to source
    flow.addEdge( n, 1, INF );
    if( flow.maxflow() != nd )
        // no solution
        return -1;
    int ans = flow.G[ 1 ].back().c; // source to sink
    flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
    // take out super source and super sink
    for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i ++ ){
        flow.G[ flow.s ][ i ].c = 0;
        Edge &e = flow.G[ flow.s ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i ++ ){
        flow.G[ flow.t ][ i ].c = 0;
        Edge &e = flow.G[ flow.t ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    flow.addEdge( flow.s, 1, INF );
    flow.addEdge( n, flow.t, INF );
    flow.reset();
    return ans + flow.maxflow();
}

```

2.5 Flow Method

Maximize $c^T x$ subject to $Ax \leq b, x \geq 0$;
with the corresponding symmetric dual problem,
Minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$.

Maximize $c^T x$ subject to $Ax \leq b$;
with the corresponding asymmetric dual problem,
Minimize $b^T y$ subject to $A^T y = c, y \geq 0$.

Minimum vertex cover on bipartite graph =
Maximum matching on bipartite graph

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Maximum density subgraph $(\sum W_e + \sum W_v) / |V|$

Binary search on answer:

For a fixed D, construct a Max flow model as follow:
Let S be Sum of all weight (or inf)

1. from source to each node with cap = S
2. For each (u,v,w) in E, $(u \rightarrow v, \text{cap}=w)$, $(v \rightarrow u, \text{cap}=w)$
3. For each node v, from v to sink with cap = $S + 2 * D - \text{deg}[v] - 2 * (W \text{ of } v)$

where $\text{deg}[v] = \sum \text{weight of edge associated with } v$
If $\text{maxflow} < S * |V|$, D is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.

3 Math

3.1 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++){
        omega[i] = exp(i * 2 * PI / MAXN * I);
    }
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n, ll a[], int _m, ll b[], ll ans[])
{
    int n=1, sum=_n+_m-1;
    while(n<sum)
        n<<=1;
    for(int i=0; i<n; i++){
        double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y, x-y);
    }
    fft(n, arr);
    for(int i=0; i<n; i++){
        arr[i]=arr[i]*arr[i];
    }
    fft(n, arr, true);
    for(int i=0; i<sum; i++){
        ans[i]=(long long int)(arr[i].real()/4+0.5);
    }
}
```

3.2 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
n    2^n    p    a    root
16   65536   65537   1    3
20   1048576 7340033   7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1)return 1;
        return ((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
};
```

```
}
LL omega[MAXN+1];
NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)
        omega[i] = (omega[i-1]*r)%P;
}
// n must be 2^k
void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n, theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            LL w = omega[i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = a[j] - a[k];
                if (x < 0) x += P;
                a[j] += a[k];
                if (a[j] > P) a[j] -= P;
                a[k] = (w * x) % P;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv_ntt) {
        LL ni = inv(n, P);
        reverse(a+1, a+n);
        for (i = 0; i < n; i++)
            a[i] = (a[i] * ni) % P;
    }
}
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

3.3 Fast Walsh Transform

```
/* xor convolution:
* x = (x0,x1) , y = (y0,y1)
* z = ( x0y0 + x1y1 , x0y1 + x1y0 )
* =>
* x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
* z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
* z = (1/2) * z'
* or convolution:
* x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
* and convolution:
* x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL inv( LL x ) {
    return mypow( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
        if( inv )
            for( int i = 0 ; i < N ; i++ ) {
                x[ i ] *= inv( N );
                x[ i ] %= MOD;
            }
    }
}
```

3.4 Poly operator

```
struct PolyOp {
```

```

#define FOR(i, c) for (int i = 0; i < (c); ++i)
NTT<P, root, MAXN> ntt;
static int nxt2k(int x) {
    int i = 1; for (; i < x; i <= 1); return i;
}
// c[i]=sum[j=0~i]a[j]*b[i-j] -> c[i+j]=a[i]*b[j](□
// if c[i-j]=a[i]*b[j] (□□□□)
// (□□□□□□□) -> reverse(a); c=mul(a,b);
reverse(c);
void Mul(int n, LL a[], int m, LL b[], LL c[]) {
    static LL aa[MAXN], bb[MAXN];
    int N = nxt2k(n+m);
    copy(a, a+n, aa); fill(aa+n, aa+N, 0);
    copy(b, b+m, bb); fill(bb+m, bb+N, 0);
    ntt.tran(N, aa); ntt.tran(N, bb);
    FOR(i, N) c[i] = aa[i] * bb[i] % P;
    ntt.tran(N, c, 1);
}
void Inv(int n, LL a[], LL b[]) {
    // ab = aa^-1 = 1 mod x^(n/2)
    // (b - a^-1)^2 = 0 mod x^n
    // bb - a^-2 + 2ba^-1 = 0
    // bba - a^-1 + 2b = 0
    // bba + 2b = a^-1
    static LL tmp[MAXN];
    if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
    Inv((n+1)/2, a, b);
    int N = nxt2k(n*2);
    copy(a, a+n, tmp);
    fill(tmp+n, tmp+N, 0);
    fill(b+n, b+N, 0);
    ntt.tran(N, tmp); ntt.tran(N, b);
    FOR(i, N) {
        LL t1 = (2 - b[i] * tmp[i]) % P;
        if (t1 < 0) t1 += P;
        b[i] = b[i] * t1 % P;
    }
    ntt.tran(N, b, 1);
    fill(b+n, b+N, 0);
}
void Div(int n, LL a[], int m, LL b[], LL d[], LL r
    []) {
    // Ra = Rb * Rd mod x^(n-m+1)
    // Rd = Ra * Rb^-1 mod
    static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
    if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);
        return;}
    // d: n-1 - (m-1) = n-m (n-m+1 terms)
    copy(a, a+n, aa); copy(b, b+m, bb);
    reverse(aa, aa+n); reverse(bb, bb+m);
    Inv(n-m+1, bb, tb);
    Mul(n-m+1, ta, n-m+1, tb, d);
    fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
    // r: m-1 - 1 = m-2 (m-1 terms)
    Mul(m, b, n-m+1, d, ta);
    FOR(i, n) {r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
        += P;}
}
void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i]
    -1] = i * a[i] % P;}
void Sx(int n, LL a[], LL b[]) {
    b[0] = 0;
    FOR(i, n) b[i+1] = a[i] * ntt.inv(i+1, P) % P;
}
void Ln(int n, LL a[], LL b[]) {
    // Integral a' a^-1 dx
    static LL a1[MAXN], a2[MAXN], b1[MAXN];
    int N = nxt2k(n*2);
    dx(n, a, a1); Inv(n, a, a2);
    Mul(n-1, a1, n, a2, b1);
    Sx(n+n-1-1, b1, b);
    fill(b+n, b+N, 0);
}
void Exp(int n, LL a[], LL b[]) {
    // Newton method to solve g(a(x)) = ln b(x) - a(x)
    // = 0
    // b' = b - g(b(x)) / g'(b(x))
    // b' = b (1 - ln b + a)
    static LL lnb[MAXN], c[MAXN], tmp[MAXN];
    assert(a[0] == 0); // dont know exp(a[0]) mod P
    if (n == 1){b[0] = 1; return;}

```

```

    Exp((n+1)/2, a, b);
    fill(b+(n+1)/2, b+n, 0);
    Ln(n, b, lnb);
    fill(c, c+n, 0); c[0] = 1;
    FOR(i, n) {
        c[i] += a[i] - lnb[i];
        if (c[i] < 0) c[i] += P;
        if (c[i] >= P) c[i] -= P;
    }
    Mul(n, b, n, c, tmp);
    copy(tmp, tmp+n, b);
}
} polyop;

```

3.5 O(1)mul

```

LL mul(LL x, LL y, LL mod){
    LL ret=x*y-(LL)((long double)x/mod*y)*mod;
    // LL ret=x*y-(LL)((long double)x*y/mod+0.5)*mod;
    return ret<0?ret+mod:ret;
}

```

3.6 Linear Recurrence

```

// Usage: linearRec({0, 1}, {1, 1}, k) //k'th fib
typedef vector<ll> Poly;
//S: □ i □ □ □, tr: □ □ □ □, k: □ □ □ □
ll linearRec(Poly& S, Poly& tr, ll k) {
    int n = tr.size();
    auto combine = [&](Poly& a, Poly& b) {
        Poly res(n * 2 + 1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            res[i+j] = (res[i+j] + a[i]*b[j])%mod;
        for(int i = 2*n; i > n; --i) rep(j, 0, n)
            res[i-1-j] = (res[i-1-j] + res[i]*tr[j])%mod;
        res.resize(n + 1);
        return res;
    };
    Poly pol(n + 1, e(pol));
    pol[0] = e[1] = 1;
    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }
    ll res = 0;
    rep(i, 0, n) res = (res + pol[i+1]*S[i])%mod;
    return res;
}

```

3.7 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[]={
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x=mpow(a,u,n);
    for(int i=0;i<t;i++){
        LL nx=mul(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(LL n) {
    int s=(magic number size)
    // iterate s times of witness on n
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=magic[s]%n;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

3.8 Faulhaber ($\sum_{i=1}^n i^p$)

```

/* faulhaber 's formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for(int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2; i<MAXK; i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0; j<i; j++)
            b[i]=sub(b[i],
                mul(cm[i][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1, j)*Bj*n^(p-j+1)}
    for(int i=1; i<MAXK; i++) {
        co[i][0]=0;
        for(int j=0; j<=i; j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
    int sol=0, m=n;
    for(int i=1; i<=p+1; i++) {
        sol=add(sol, mul(co[p][i], m));
        m = mul(m, n);
    }
    return sol;
}

```

3.9 Chinese Remainder

```

LL x[N], m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
LL solve(int n) { // n>=2, be careful with no solution
    LL res=CRT(x[0], m[0], x[1], m[1]), p=m[0]/__gcd(m[0], m[1])*m[1];
    for(int i=2; i<n; i++) {
        res=CRT(res, p, x[i], m[i]);
        p=p/__gcd(p, m[i])*m[i];
    }
    return res;
}

```

3.10 Pollard Rho

```

// does not work when n is prime
LL f(LL x, LL mod) { return add(mul(x, x, mod), 1, mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true) {
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

3.11 Josephus Problem

```

int josephus(int n, int m) { // n □ m □
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}

```

3.12 Gaussian Elimination

```

const int GAUSS_MOD = 1000000007LL;
struct GAUSS {
    int n;
    vector<vector<int>> v;
    int ppow(int a, int k) {
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD,
            k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD,
            k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve() {
        vector<int> ans(n);
        REP(now, 0, n) {
            REP(i, now, n) if(v[now][now] == 0 && v[i][now] != 0)
                swap(v[i], v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            REP(i, 0, n) if(i != now) {
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j, now, n + 1) (v[i][j] +=
                    GAUSS_MOD - tmp * v[now][j] %
                    GAUSS_MOD) %= GAUSS_MOD;
            }
            REP(i, 0, n) ans[i] = v[i][n + 1] * ppow(v[i][n + 1],
                GAUSS_MOD - 2) % GAUSS_MOD;
            return ans;
        }
    }
    // gs.v.clear(), gs.v.resize(n, vector<int>(n + 1, 0));
} gs;

```

3.13 ax+by=gcd

```

PII gcd(int a, int b) {
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

3.14 Discrete sqrt

```

void calch(LL &t, LL &h, const LL p) {
    LL tmp=p-1; for(t=0; (tmp&1)==0; tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(LL a, LL p, LL &x, LL &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
}

```



```

if ((p + 1) % 4 == 0) {
    x=mypow(a,(p+1)/4,p); y=p-x; return true;
} else {
    LL t, h, b, pb; calcH(t, h, p);
    if (t >= 2) {
        do {b = rand() % (p - 2) + 2;
            } while (mypow(b, p / 2, p) != p - 1);
        pb = mypow(b, h, p);
    } int s = mypow(a, h / 2, p);
    for (int step = 2; step <= t; step++) {
        int ss = (((LL)(s * s) % p) * a) % p;
        for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
        if (ss + 1 == p) s = (s * pb) % p;
        pb = ((LL)pb * pb) % p;
    } x = ((LL)s * a) % p; y = p - x;
} return true;
}

```

3.15 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```

3.16 Roots of Polynomial ??????

```

const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ]; // a[0..n](coef) must be
    filled
int n; // degree of polynomial must be filled
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++){
        sum=sum+a[i]*tmp; tmp=tmp*x; }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]

```

3.17 Primes

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 98777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847

```

```

* 9223372036854775783, 18446744073709551557 */
int mu[ N ] , p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}

```

3.18 Result

- Lucas' Theorem :
For $n, m \in \mathbb{Z}^+$ and prime P , $C(m, n) \bmod P = \prod (C(m_i, n_i))$ where m_i is the i -th digit of m in base P .
- Stirling approximation :
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$
- Stirling Numbers(Partition n elements into k non-empty set):
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick's Theorem : $A = i + b/2 - 1$
??? A ?????? i ?????? b ???
- Catalan number : $C_n = \binom{2n}{n} / (n+1)$
$$C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)n!}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$
- Euler Characteristic:
planar graph: $V - E + F - C = 1$
convex polyhedron: $V - E + F = 2$
 V, E, F, C : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$, Deleting any one row, one column, and cal the $\det(A)$
- Polya' theorem (c ?????m ???):
$$\left(\sum_{i=1}^m e^{gcd(i,m)} \right) / m$$
- ????: (n ?????????????????):
 $dp[0] = 1; dp[1] = 0;$
 $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$
- Bell B (B n ??, ?????????):
 $B_0 = 1$
 $B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$
 $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$
- Wilson's theorem :
 $(p-1)! \equiv -1 \pmod{p}$
- Fermat's little theorem :
 $a^p \equiv a \pmod{p}$
- Euler's totient function:
 $A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$

4 Geometry

4.1 definition

```
typedef long double ld;
const ld eps = 1e-8;
int dcmp(ld x) {
    if(abs(x) < eps) return 0;
    else return x < 0 ? -1 : 1;
}
struct Pt {
    ld x, y;
    Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}

    Pt operator+(const Pt &a) const {
        return Pt(x+a.x, y+a.y);
    }
    Pt operator-(const Pt &a) const {
        return Pt(x-a.x, y-a.y);
    }
    Pt operator*(const ld &a) const {
        return Pt(x*a, y*a);
    }
    Pt operator/(const ld &a) const {
        return Pt(x/a, y/a);
    }
    ld operator*(const Pt &a) const {
        return x*a.x + y*a.y;
    }
    ld operator^(const Pt &a) const {
        return x*a.y - y*a.x;
    }
    bool operator<(const Pt &a) const {
        return x < a.x || (x == a.x && y < a.y);
        //return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
        //    dcmp(y-a.y) < 0);
    }
    bool operator==(const Pt &a) const {
        return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0;
    }
};
ld norm2(const Pt &a) {
    return a*a;
}
ld norm(const Pt &a) {
    return sqrt(norm2(a));
}
Pt perp(const Pt &a) {
    return Pt(-a.y, a.x);
}
Pt rotate(const Pt &a, ld ang) {
    return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y*cos(ang));
}
struct Line {
    Pt s, e, v; // start, end, end-start
    ld ang;
    Line(Pt _s=Pt(0, 0), Pt _e=Pt(0, 0)):s(_s), e(_e) { v = e-s; ang = atan2(v.y, v.x); }

    bool operator<(const Line &l) const {
        return ang < l.ang;
    }
};
struct Circle {
    Pt o; ld r;
    Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
};
```

4.2 Intersection of 2 lines

```
Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    ld f1 = (p2-p1)^(q1-p1), f2 = (p2-p1)^(p1-q2), f;
    if(dcmp(f=f1+f2) == 0)
        return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
    return q1*(f2/f) + q2*(f1/f);
}
```

4.3 halfPlaneIntersection

// for point online solution, change > to >=

```
bool onleft(Line L, Pt p) {
    return dcmp(L.v^(p-L.s)) > 0;
}
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
    sort(L.begin(), L.end()); // sort by angle
    int n = L.size(), fir, las;
    Pt *p = new Pt[n];
    Line *q = new Line[n];
    q[fir=las=0] = L[0];
    for(int i = 1; i < n; i++) {
        while(fir < las && !onleft(L[i], p[las-1])) las--;
        while(fir < las && !onleft(L[i], p[fir])) fir++;
        q[++las] = L[i];
        if(dcmp(q[las].v^q[las-1].v) == 0) {
            las--;
            if(onleft(q[las], L[i].s)) q[las] = L[i];
        }
        if(fir < las) p[las-1] = LLIntersect(q[las-1], q[las]);
    }
    while(fir < las && !onleft(q[fir], p[las-1])) las--;
    if(las-fir <= 1) return {};
    p[las] = LLIntersect(q[las], q[fir]);
    int m = 0;
    vector<Pt> ans(las-fir+1);
    for(int i = fir; i <= las; i++) ans[m++] = p[i];
    return ans;
}
```

4.4 Convex Hull

```
double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2], stk[top-1], pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2], stk[top-1], pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}
```

4.5 Intersection of 2 segments

```
int ori(const Pt& o, const Pt& a, const Pt& b){
    LL ret = (a-o) ^ (b-o);
    return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana(const Pt& p1, const Pt& p2, const Pt& q1, const Pt& q2){
    if(( (p2-p1) ^ (q2-q1) ) == 0){ // parallel
        if(ori(p1, p2, q1)) return false;
        return ( (p1-q1) * (p2-q1) ) <= 0 ||
            ( (p1-q2) * (p2-q2) ) <= 0 ||
            ( (q1-p1) * (q2-p1) ) <= 0 ||
            ( (q1-p2) * (q2-p2) ) <= 0;
    }
    return (ori(p1, p2, q1) * ori(p1, p2, q2) <= 0) &&
        (ori(q1, q2, p1) * ori(q1, q2, p2) <= 0);
}
```

4.6 Intersection of circle and segment

```
bool Inter(const Pt& p1, const Pt& p2, Circle& cc){
    Pt dp = p2 - p1;
    double a = dp * dp;
    double b = 2 * (dp * (p1 - cc.o));
```

```
double c = cc.0 * cc.0 + p1 * p1 - 2 * ( cc.0 * p1 )
    - cc.R * cc.R;
double bb4ac = b * b - 4 * a * c;
return !( fabs( a ) < eps or bb4ac < 0 );
}
```

4.7 Intersection of 2 circles

4.8 Circle cover

```
#define N 1021
#define D long double
struct CircleCover{
    int C; Circ c[ N ]; //□□C(□□□□),c(□□□□)
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool Cinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.0 , o2 = b.0;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return false;
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return true;
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
    struct Teve {
        Pt p; D ang; int add;
        Teve() {}
        Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang;
        }
    }eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
    bool disjunct( Circ& a, Circ &b, int x )
    {return sign( norm( a.0 - b.0 ) - a.R - b.R ) > x;}
    bool contain( Circ& a, Circ &b, int x )
    {return sign( a.R - b.R - norm( a.0 - b.0 ) ) > x;}
    bool contain(int i, int j){
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
    }
    void solve(){
        for( int i = 0 ; i <= C + 1 ; i ++ )
            Area[ i ] = 0;
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                overlap[i][j] = contain(i, j);
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjunct(c[i], c[j], -1));
        for( int i = 0 ; i < C ; i ++ ){
            int E = 0, cnt = 1;
            for( int j = 0 ; j < C ; j ++ )
                if( j != i && overlap[j][i] )
                    cnt ++;
            for( int j = 0 ; j < C ; j ++ )
                if( i != j && g[i][j] ){
                    Pt aa, bb;
                    Cinter(c[i], c[j], aa, bb);
                    D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                    D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                    eve[E ++] = Teve(bb, B, 1);
                    eve[E ++] = Teve(aa, A, -1);
                    if(B > A) cnt ++;
                }
            if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
            else{
                sort( eve , eve + E );
                eve[E] = eve[0];
                for( int j = 0 ; j < E ; j ++ ){
                    cnt += eve[j].add;

```

```
Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
D theta = eve[j + 1].ang - eve[j].ang;
if (theta < 0) theta += 2.0 * pi;
Area[cnt] +=
    (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}}}};
```

4.9 Convex Hull trick

```
/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt(
            p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
        return 1;
    }
    // 2. Find 2 tang pts on CH of a given outside point
    // return true with i0, i1 as index of tangent points

```



```

// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
}
// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second + (int)lower.size() - 1) % n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){
    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if(sign(det(v-u, a[p0]-u))*sign(det(v-u, a[p1]-u)) < 0){
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);
        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
    return 0;
}
}
}

```

4.10 Tangent line of two circles

```

vector<Line> go(const Cir& c1, const Cir& c2, int
    sign1){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2(c1.0 - c2.0);
    if(d_sq < eps) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.0 - c1.0) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if(c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for(int sign2 = 1; sign2 >= -1; sign2 -= 2){
        Pt n = {v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X};
        Pt p1 = c1.0 + n * c1.R;
        Pt p2 = c2.0 + n * (c2.R * sign1);
        if(fabs(p1.X - p2.X) < eps and
            fabs(p1.Y - p2.Y) < eps)
            p2 = p1 + perp(c2.0 - c1.0);
        ret.push_back({p1, p2});
    }
    return ret;
}

```

4.11 KD Tree

```

const int MXN=100005;
const int MXK=10;
struct KDTree{
    struct Nd{
        LL x[MXK], mn[MXK], mx[MXK];
        int id, f;
        Nd *l, *r;
    } tree[MXN], *root;
    int n, k;
    LL dis(LL a, LL b){return (a-b)*(a-b);}
    LL dis(LL a[MXK], LL b[MXK]){
        LL ret=0;
        for(int i=0; i<k; i++) ret+=dis(a[i], b[i]);
        return ret;
    }
    void init(vector<vector<LL>> &ip, int _n, int _k){

```

```

        n=_n, k=_k;
        for(int i=0; i<n; i++){
            tree[i].id=i;
            copy(ip[i].begin(), ip[i].end(), tree[i].x);
        }
        root=build(0, n-1, 0);
    }
    Nd* build(int l, int r, int d){
        if(l>r) return NULL;
        if(d==k) d=0;
        int m=(l+r)>>1;
        nth_element(tree+l, tree+m, tree+r+1, [&](const Nd &a,
            const Nd &b){return a.x[d]<b.x[d];});
        tree[m].f=d;
        copy(tree[m].x, tree[m].x+k, tree[m].mn);
        copy(tree[m].x, tree[m].x+k, tree[m].mx);
        tree[m].l=build(l, m-1, d+1);
        if(tree[m].l){
            for(int i=0; i<k; i++){
                tree[m].mn[i]=min(tree[m].mn[i], tree[m].l->mn[i]);
                tree[m].mx[i]=max(tree[m].mx[i], tree[m].l->mx[i]);
            }
        }
        tree[m].r=build(m+1, r, d+1);
        if(tree[m].r){
            for(int i=0; i<k; i++){
                tree[m].mn[i]=min(tree[m].mn[i], tree[m].r->mn[i]);
                tree[m].mx[i]=max(tree[m].mx[i], tree[m].r->mx[i]);
            }
        }
        return tree+m;
    }
    LL pt[MXK], md;
    int mID;
    bool touch(Nd *r){
        LL d=0;
        for(int i=0; i<k; i++){
            if(pt[i]<=r->mn[i]) d+=dis(pt[i], r->mn[i]);
            else if(pt[i]>=r->mx[i]) d+=dis(pt[i], r->mx[i]);
        }
        return d<md;
    }
    void nearest(Nd *r){
        if(!r||!touch(r)) return;
        LL td=dis(r->x, pt);
        if(td<md) md=td, mID=r->id;
        nearest(pt[r->f]<r->x[r->f]?r->l:r->r);
        nearest(pt[r->f]>r->x[r->f]?r->r:r->l);
    }
    pair<LL, int> query(vector<LL> &_pt, LL _md=1LL<<57){
        mID=-1, md=_md;
        copy(_pt.begin(), _pt.end(), pt);
        nearest(root);
        return {md, mID};
    }
}tree;

```

4.12 Lower Concave Hull

```

const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        return s ? b - s->b < (s->m - m) * rhs.m : 0;
    }
}; // maintain upper hull for maximum
struct HullDynamic : public multiset<Line> {
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);

```

```

    if(z==end())return y->m==x->m&&y->b<=x->b;
    return (x->b-y->b)*(z->m-y->m)>=
        (y->b-z->b)*(y->m-x->m);
}
void insert_line(ll m, ll b) {
    auto y = insert({m, b});
    y->succ = [=]{return next(y)==end()?0:&*next(y);};
    if(bad(y)) {erase(y); return;}
    while(next(y)!=end()&&bad(next(y)))erase(next(y));
    while(y!=begin()&&bad(prev(y)))erase(prev(y));
}
ll eval(ll x) {
    auto l = *lower_bound((Line) {x, is_query});
    return l.m * x + l.b;
}
};

```

4.13 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[ N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (norm2(cen-p[j]) <= r2) continue;
                cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
                r2 = norm2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (norm2(cen-p[k]) <= r2) continue;
                    cen = center(p[i],p[j],p[k]);
                    r2 = norm2(cen-p[k]);
                }
            }
        }
        return {cen,sqrt(r2)};
    }
} mec;

```

4.14 Min Enclosing Ball

```

// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
    Pt q[3]; double m[3][3], sol[3], L[3], det;
    int i,j; res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2; radius=norm2(res, outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q[i] * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=q[i] * q[i];
            if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
                return;

```

```

        L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
        L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
        res=outer[0]+q[0]*L[0]+q[1]*L[1];
        radius=norm2(res, outer[0]);
        break;
    case 4:
        for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=(q[i] * q[i]);
        for (i=0; i<3; ++i) for(j=0; j<3; ++j) m[i][j]=(q[i] * q[j])*2;
        det= m[0][0]*m[1][1]*m[2][2]
            + m[0][1]*m[1][2]*m[2][0]
            + m[0][2]*m[1][1]*m[1][0]
            - m[0][2]*m[1][1]*m[2][0]
            - m[0][1]*m[1][0]*m[2][2]
            - m[0][0]*m[1][2]*m[2][1];
        if ( fabs(det)<eps ) return;
        for (j=0; j<3; ++j) {
            for (i=0; i<3; ++i) m[i][j]=sol[i];
            L[j]=( m[0][0]*m[1][1]*m[2][2]
                + m[0][1]*m[1][2]*m[2][0]
                + m[0][2]*m[1][1]*m[1][0]
                - m[0][2]*m[1][1]*m[2][0]
                - m[0][1]*m[1][0]*m[2][2]
                - m[0][0]*m[1][2]*m[2][1]
            ) / det;
            for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
        }
        res=outer[0];
        for (i=0; i<3; ++i) res = res + q[i] * L[i];
        radius=norm2(res, outer[0]);
    }
}
void minball(int n){ ball();
    if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
        if( norm2(res, pt[i]) - radius > eps ){
            outer[ nouter ++ ] = pt[ i ]; minball(i); --
            nouter;
            if(i>0){ Pt Tt = pt[i];
                memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt;
            }
        }
}
double solve(){
    // n points in pt
    random_shuffle(pt, pt+n); radius=-1;
    for(int i=0; i<n; i++) if(norm2(res,pt[i])-radius>eps)
        nouter=1, outer[0]=pt[i], minball(i);
    return sqrt(radius);
}

```

4.15 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
    if(i>=0 && i<2) turn(i+1, j, x0+L+z, y, x0+L-x,
                          x0+L, y0, H, W, L);
    if(j>=0 && j<2) turn(i, j+1, x, y0+W+z, y0+W-y,
                          x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                          x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-W-z, y-y0,
                          x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
        else swap(x1,z1), swap(x2,z2), swap(L,H);
    }
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}

```

4.16 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // □ □
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}

```

```

Pt circumCenter( Pt &a, Pt &b, Pt &c) { // □ □
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a, Pt &b, Pt &c) { // □ □
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
        A = ca.X * ba.Y - ba.X * ca.Y,
        x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
        y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}

```

5 Graph

5.1 DominatorTree

```

const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n, m, s;
    vector< int > g[ MAXN ], pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ], nfd[ MAXN ], ts;
    int par[ MAXN ]; //idom[u] s □ u □ □ □ □ □ □ □
    int sdom[ MAXN ], idom[ MAXN ];
    int mom[ MAXN ], mn[ MAXN ];
    inline bool cmp( int u, int v )
    { return dfn[ u ] < dfn[ v ]; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if(cmp( sdom[ mn[ mom[ u ] ] ], sdom[ mn[ u ] ] ))
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n, int _m, int _s ){
        ts = 0; n = _n; m = _m; s = _s;
        REP( i, 1, n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u, int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i, 1, n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i, n, 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ], sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ], par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i, 2, n ){
            int u = nfd[ i ];

```

```

        if( u == 0 ) continue;
        if( idom[ u ] != sdom[ u ] )
            idom[ u ] = idom[ idom[ u ] ];
    }
} domT;

```

5.2 MaximumClique

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int linkto[N], v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; i++){
            linkto[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a, int b)
    { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans, stk[N];
    int id[N], di[N], deg[N];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num; cans.reset();
            for(int i = 0; i < elem_num; i++)
                cans[id[stk[i]]] = 1;
        }
        int potential = elem_num + popcount(candi);
        if(potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while(smaller_candi.count() && potential > ans){
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential--;
            if(next == pivot || (smaller_candi & linkto[next]
                ).count()){
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    int solve(){
        for(int i = 0; i < n; i++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id, id + n, [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0; i < n; i++) di[id[i]] = i;
        for(int i = 0; i < n; i++)
            for(int j = 0; j < n; j++)
                if(v[i][j]) linkto[di[i]][di[j]] = 1;
        Int cand; cand.reset();
        for(int i = 0; i < n; i++) cand[i] = 1;
        ans = 1;
        cans.reset(); cans[0] = 1;
        maxclique(0, cand);
        return ans;
    }
} solver;

```

5.3 MaximalClique

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N], v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; i++){
            lnk[i].reset(); v[i].reset();
        }
    }

```

```

}
void addEdge(int a , int b)
{ v[a][b] = v[b][a] = 1; }
int ans , stk[N], id[N] , di[N] , deg[N];
int cans;
void dfs(int elem_num, Int candi, Int ex){
    if(candi.none() && ex.none()){
        cans.reset();
        for(int i = 0 ; i < elem_num ; i ++){
            cans[id[stk[i]]] = 1;
            ans = elem_num; // cans is a maximal clique
            return;
        }
        int pivot = (candilex)._Find_first();
        Int smaller_candi = candi & (~lnk[pivot]);
        while(smaller_candi.count()){
            int nxt = smaller_candi._Find_first();
            candi[nxt] = smaller_candi[nxt] = 0;
            ex[nxt] = 1;
            stk[elem_num] = nxt;
            dfs(elem_num+1, candi&lnk[nxt], ex&lnk[nxt]);
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i ++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i ++){ di[id[i]] = i; }
        for(int i = 0 ; i < n ; i ++){
            for(int j = 0 ; j < n ; j ++){
                if(v[i][j]) lnk[di[i]][di[j]] = 1;
            }
        }
        ans = 1; cans.reset(); cans[0] = 1;
        dfs(0, Int(string(n, '1')), 0);
        return ans;
    }
}
} solver;

```

5.4 Strongly Connected Component

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear(), rE[i].clear();
        }
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++){
            if (!vst[i]) DFS(i);
            reverse(vec.begin(), vec.end());
            FZ(vst);
            for (auto v : vec)
                if (!vst[v]){
                    rDFS(v); nScc++;
                }
        }
    }
};

```

5.5 Dynamic MST

```

/* Dynamic MST O( Q lg^2 Q )
(qx[i], qy[i]) -> chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infy)
add an edge: change from \infy to specific value */

```

```

const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
            int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)
        vd[i]=++n2;
    for(int i=1;i<=n;i++) if(a[i])
        vd[i]=vd[find(i)];
    int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
        Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
        app[qx[i]]=m2; m2++;
    }
    for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
        if(ri!=rj){
            a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
            Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
        }
    }
    int mid=Q/2;
    solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
    solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }

```

5.6 Maximum General graph Matching

```

const int N = 514, E = (2e5) * 2;
struct Graph{
    int to[E],bro[E],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init( int _n ){
        stp = 0; e = 1; n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            lnk[i] = vis[i] = 0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v,lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)){
                    return true;
                }
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;
        for(int i=1;i<=n;i++){
            if(!lnk[i]){
                stp++; ans += dfs(i);
            }
        }
        return ans;
    }
} graph;

```

5.7 Minimum General Weighted Matching

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
    }
}

```

```

while (true){
    int found = 0;
    for( int i = 0 ; i < n ; i ++ )
        onstk[ i ] = dis[ i ] = 0;
    for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
            found = 1;
            while (SZ(stk)>=2){
                int u = stk.back(); stk.pop_back();
                int v = stk.back(); stk.pop_back();
                match[u] = v;
                match[v] = u;
            }
        }
        if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++){
        ret += edge[i][match[i]];
    }
    ret /= 2;
    return ret;
}
}graph;

```

5.8 Minimum Steiner Tree

```

// Minimum Steiner Tree □ □ □ □ mst
//  $O(V \cdot 3^T + V^2 \cdot 2^T)$ 
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1 << T][V], tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui, int vi, int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ], wi );
        dst[ vi ][ ui ] = min( dst[ vi ][ ui ], wi );
    }
    void shortest_path(){
        for( int k = 0 ; k < n ; k ++ )
            for( int i = 0 ; i < n ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dst[ i ][ j ] = min( dst[ i ][ j ],
                        dst[ i ][ k ] + dst[ k ][ j ] );
    }
    int solve( const vector<int>& ter ){
        int t = (int)ter.size();
        for( int i = 0 ; i < ( 1 << t ) ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                dp[ i ][ j ] = INF;
        for( int i = 0 ; i < n ; i ++ )
            dp[ 0 ][ i ] = 0;
        for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
            if( msk == ( msk & (-msk) ) ){
                int who = __lg( msk );
                for( int i = 0 ; i < n ; i ++ )
                    dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                continue;
            }
            for( int i = 0 ; i < n ; i ++ )
                for( int submsk = ( msk - 1 ) & msk ; submsk ;
                    submsk = ( submsk - 1 ) & msk )
                    dp[ msk ][ i ] = min( dp[ msk ][ i ],
                        dp[ submsk ][ i ] +
                        dp[ msk ^ submsk ][ i ] );
            for( int i = 0 ; i < n ; i ++ ){
                tdst[ i ] = INF;
                for( int j = 0 ; j < n ; j ++ )
                    tdst[ i ] = min( tdst[ i ],
                        dp[ msk ][ j ] + dst[ j ][ i ] );
            }
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = tdst[ i ];
        }
        int ans = INF;
        for( int i = 0 ; i < n ; i ++ )

```



```

    ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
    return ans;
} }solver;

```

5.9 BCC based on vertex

```

struct BccVertex {
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u], dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i,i);
            }
        REP(i,nScc) res.PB(sccv[i]);
        return res;
    }
}graph;

```

5.10 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;

```

```

        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        fill(vst,0); edgeID.clear(); cycle.clear(); rho.clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.PB(prve[i][st]);
            rho.PB(st);
        }
        while (vst[st] != 2) {
            if(rho.empty()) return inf;
            int v = rho.back(); rho.pop_back();
            cycle.PB(v);
            vst[v]++;
        }
        reverse(ALL(edgeID));
        edgeID.resize(SZ(cycle));
        return mmc;
    } }mmc;

```

5.11 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[++bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i+1], dp[i+1]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] = min(dp[i][g[j][k].to],
                                            dp[i-1][j]+g[j][k].w);
                }
            }
        }
        mu=INF; LL bunbo=1;
        for(int i=1; i<=n; i++) if(dp[n][i] < INF){
            LL a=-INF, b=1;
            for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                    a = dp[n][i]-dp[j][i];
                    b = n-j;
                }
            }
            if(mu*b > bunbo*a)
                mu = a, bunbo = b;
        }
        if(mu < 0) return -1; // negative cycle

```

```

if(mu == INF) return INF; // no cycle
if(mu == 0) return 0;
for(int i=1; i<=n; i++){
    for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w *= bunbo;
        memset(p, 0, sizeof(p));
        queue<int> q;
        for(int i=1; i<=n; i++){
            q.push(i);
            inq[i] = true;
        }
        while(!q.empty()){
            int i=q.front(); q.pop(); inq[i]=false;
            for(int j=0; j<(int)g[i].size(); j++){
                if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                    p[g[i][j].to] = p[i]+g[i][j].w-mu;
                    if(!inq[g[i][j].to]){
                        q.push(g[i][j].to);
                        inq[g[i][j].to] = true;
                    }
                }
            }
        }
        for(int i=1; i<=n; i++) grev[i].clear();
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w += p[i]-p[g[i][j].to];
                grev[g[i][j].to].push_back(edge(i, g[i][j].w));
            }
        }
        LL mldc = n*mu;
        for(int i=1; i<=n; i++){
            bn=mldc/mu, bs=0;
            memset(hd, 0, sizeof(hd));
            fill(d+i+1, d+n+1, INF);
            b_insert(d[i]=0, i);
            for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
                b[k].next){
                int u = b[k].u;
                LL du = b[k].d;
                if(du > d[u]) continue;
                for(int l=0; l<(int)g[u].size(); l++) if(g[u][l]
                    .to > i){
                    if(d[g[u][l].to] > du + g[u][l].w){
                        d[g[u][l].to] = du + g[u][l].w;
                        b_insert(d[g[u][l].to], g[u][l].to);
                    }
                }
            }
            for(int j=0; j<(int)grev[i].size(); j++) if(grev[i]
                [j].to > i)
                mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
        }
        return mldc / bunbo;
    }
} }graph;

```

5.12 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v; ll d;
        nd(int ui = 0, int vi = 0, ll di = INF)
        { u = ui; v = vi; d = di; }
    };
    struct heap{
        nd* edge; int dep; heap* chd[4];
    };
    static int cmp(heap* a, heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; ll d; heap* H; nd* E;
        node(){
            node(ll _d, int _v, nd* _E)
            { d = _d; v = _v; E = _E; }
            node(heap* _H, ll _d)
            { H = _H; d = _d; }
            friend bool operator<(node a, node b)
            { return a.d > b.d; }
        };
        int n, k, s, t;
        ll dst[ N ];
        nd *nxt[ N ];
        vector<nd*> g[ N ], rg[ N ];
        heap *nullNd, *head[ N ];
        void init( int _n , int _k , int _s , int _t ){
            n = _n; k = _k; s = _s; t = _t;

```

```

        for( int i = 1 ; i <= n ; i ++ ){
            g[ i ].clear(); rg[ i ].clear();
            nxt[ i ] = NULL; head[ i ] = NULL;
            dst[ i ] = -1;
        }
        void addEdge( int ui , int vi , ll di ){
            nd* e = new nd(ui, vi, di);
            g[ ui ].push_back( e );
            rg[ vi ].push_back( e );
        }
        queue<int> dfsQ;
        void dijkstra(){
            while(dfsQ.size()) dfsQ.pop();
            priority_queue<node> Q;
            Q.push(node(0, t, NULL));
            while (!Q.empty()){
                node p = Q.top(); Q.pop();
                if(dst[p.v] != -1) continue;
                dst[ p.v ] = p.d;
                nxt[ p.v ] = p.E;
                dfsQ.push( p.v );
                for(auto e: rg[ p.v ])
                    Q.push(node(p.d + e->d, e->u, e));
            }
            heap* merge(heap* curNd, heap* newNd){
                if(curNd == nullNd) return newNd;
                heap* root = new heap;
                memcpy(root, curNd, sizeof(heap));
                if(newNd->edge->d < curNd->edge->d){
                    root->edge = newNd->edge;
                    root->chd[2] = newNd->chd[2];
                    root->chd[3] = newNd->chd[3];
                    newNd->edge = curNd->edge;
                    newNd->chd[2] = curNd->chd[2];
                    newNd->chd[3] = curNd->chd[3];
                }
                if(root->chd[0]->dep < root->chd[1]->dep)
                    root->chd[0] = merge(root->chd[0], newNd);
                else
                    root->chd[1] = merge(root->chd[1], newNd);
                root->dep = max(root->chd[0]->dep, root->chd[1]->
                    dep) + 1;
                return root;
            }
            vector<heap*> V;
            void build(){
                nullNd = new heap;
                nullNd->dep = 0;
                nullNd->edge = new nd;
                fill(nullNd->chd, nullNd->chd+4, nullNd);
                while(not dfsQ.empty()){
                    int u = dfsQ.front(); dfsQ.pop();
                    if(!nxt[ u ]) head[ u ] = nullNd;
                    else head[ u ] = head[nxt[ u ]->v];
                    V.clear();
                    for( auto&& e : g[ u ] ){
                        int v = e->v;
                        if( dst[ v ] == -1 ) continue;
                        e->d += dst[ v ] - dst[ u ];
                        if( nxt[ u ] != e ){
                            heap* p = new heap;
                            fill(p->chd, p->chd+4, nullNd);
                            p->dep = 1;
                            p->edge = e;
                            V.push_back(p);
                        }
                    }
                    if(V.empty()) continue;
                    make_heap(V.begin(), V.end(), cmp);
                }
                #define L(X) ((X<<1)+1)
                #define R(X) ((X<<1)+2)
                for( size_t i = 0 ; i < V.size() ; i ++ ){
                    if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
                    else V[i]->chd[2]=nullNd;
                    if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
                    else V[i]->chd[3]=nullNd;
                }
                head[u] = merge(head[u], V.front());
            }
            vector<ll> ans;
            void first_K(){
                ans.clear();
                priority_queue<node> Q;

```

```

if( dst[ s ] == -1 ) return;
ans.push_back( dst[ s ] );
if( head[s] != nullNd )
    Q.push(node(head[s], dst[s]+head[s]->edge->d));
for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
    node p = Q.top(), q; Q.pop();
    ans.push_back( p.d );
    if(head[ p.H->edge->v ] != nullNd){
        q.H = head[ p.H->edge->v ];
        q.d = p.d + q.H->edge->d;
        Q.push(q);
    }
}
for( int i = 0 ; i < 4 ; i ++ )
    if( p.H->chd[ i ] != nullNd ){
        q.H = p.H->chd[ i ];
        q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
            edge->d;
        Q.push( q );
    }
}
void solve(){ // ans[i] stores the i-th shortest path
    dijkstra();
    build();
    first_K(); // ans.size() might less than k
}
} solver;

```

5.13 SPFA

```

bool spfa(){
    deque<int> dq;
    dis[0]=0;
    dq.push_back(0);
    inq[0]=1;
    while(!dq.empty()){
        int u=dq.front();
        dq.pop_front();
        inq[u]=0;
        for(auto i:edge[u]){
            if(dis[i.first]>i.second+dis[u]){
                dis[i.first]=i.second+dis[u];
                len[i.first]=len[u]+1;
                if(len[i.first]>n) return 1;
                if(inq[i.first]) continue;
                if(!dq.empty()&&dis[dq.front()]>dis[i.first])
                    dq.push_front(i.first);
            }
            else
                dq.push_back(i.first);
            inq[i.first]=1;
        }
    }
    return 0;
}

```

5.14 0000

0000 $V_j - V_i \leq W$ 00 $V_i - > V_j$ 000 $W \rightarrow$ bellman-ford or spfa

6 String

6.1 PalTree

```

// len[s] 0 0 0 0 0 0 0
// num[s] 0 0 0 0 0 0 0
// cnt[s] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
// fail[s] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN] = {"-1"};
    int newNode(int l, int f){
        len[tot] = l, fail[tot] = f, cnt[tot] = num[tot] = 0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot] = (l > 0 ? l - len[f] : 0);
        sfail[tot] = (l > 0 && diff[tot] == diff[f] ? sfail[f] : f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1] != s[n]) x = fail[x];
        return x;
    }
    int getmin(int v){
        dp[v] = fac[n-len[sfail[v]]-diff[v]];
    }
}

```

```

if(diff[v]==diff[fail[v]])
    dp[v]=min(dp[v],dp[fail[v]]);
return dp[v]+1;
}
int push(){
    int c=s[n]-'a', np=getfail(lst);
    if(!(lst=nxt[np][c])){
        lst=newNode(len[np]+2, nxt[getfail(fail[np])][c]);
        nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    }
    fac[n]=n;
    for(int v=lst; len[v]>0; v=sfail[v])
        fac[n]=min(fac[n], getmin(v));
    return ++cnt[lst], lst;
}
void init(const char *_s){
    tot=lst=n=0;
    newNode(0, 1), newNode(-1, 1);
    for(; s[n];) s[n+1]=s[n], ++n, state[n-1]=push();
    for(int i=tot-1; i>1; i--) cnt[fail[i]]+=cnt[i];
}
} palt;

```

6.2 KMP

```

/*
len-failure[k]:
0 k 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 (len-failure[k]) 0 0 0 0 0 0 0 0 0 0 0 0 0 0

failure[k]:
failure[k] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-base 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
failuer[k] 0 failure[failuer[k]-1]
0 failure[failure[failuer[k]-1]-1]..
0 0 0 0 0 0 0 0
*/
int failure[MXN];
void KMP(string& t, string& p)
{
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i)
    {
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i)
    {
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1)
        {
            cout << i - p.size() + 1 << " ";
            j = failure[j];
        }
    }
}

```

6.3 SAIS

```

const int N = 300010;
struct SA{
    #define REP(i,n) for ( int i=0; i<int(n); i++ )
    #define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i > 0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
}

```

```

    }
}
void sais(int *s, int *sa, int *p, int *q, bool *t,
          int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        ]+1) ? t[i+1] : s[i]<s[i+1];
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]
        ]]=p[q[i]-nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]] = p[nsa[i]]);
}
}
}
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

6.4 SuffixAutomata

```

// any path start from root forms a substring of S
// occurrence of P : iff SAM can run on input word P
// number of different substring : ds[1]-1
// total length of all different substring : dsl[1]
// max/min length of state i : mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P : cnt[i]
// first occurrence position of P : fp[i]-lpl+1
// all position of P : fp of "dfs from i through rmom"
const int MXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MXM], mx[MXM]; //ind[MXM]
    int nxt[MXM][33]; //cnt[MXM],ds[MXM],dsl[MXM],fp[MXM]
    // bool v[MXM]
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = 0; //cnt=ds=dsl=fp=v=0
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode(); //cnt[np]=1
        mx[np] = mx[p]+1; //fp[np]=mx[np]-1
        for(; p && nxt[p][c] == 0; p = mom[p])

```

```

        nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode(); //fp[nq]=fp[q]
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for(; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void calc(){
        calc(root);
        iota(ind,ind+tot,1);
        sort(ind,ind+tot,&)(int i,int j){return mx[i]<mx[j]
            ;};
        for(int i=tot-1;i>=0;i--){
            cnt[mom[ind[i]]]+=cnt[ind[i]];
        }
    }
    void calc(int x){
        v[x]=ds[x]=1;dsl[x]=0; //rmom[mom[x]].push_back(x);
        for(int i=1;i<=26;i++){
            if(nxt[x][i]){
                if(!v[nxt[x][i]]) calc(nxt[x][i]);
                ds[x]+=ds[nxt[x][i]];
                dsl[x]+=dsl[nxt[x][i]]+dsl[nxt[x][i]];
            }
        }
    }
    void push(const string& str){
        for(int i = 0; i < str.size(); i++){
            push(str[i]-'a'+1);
        }
    }
} sam;

```

6.5 Aho-Corasick

```

struct AAutomata{
    struct Node{
        int cnt,i;
        Node *go[26], *fail, *dic;
        Node(){
            cnt = 0; fail = 0; dic=0;
            memset(go,0,sizeof(go));
        }
    }pool[1048576],*root;
    int nMem,n_pattern;
    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init() {nMem=0;root=new_Node();n_pattern=0;}
    void add(const string &str) {insert(root,str,0);}
    void insert(Node *cur, const string &str, int pos){
        for(int i=pos;i<str.size();i++){
            if(!cur->go[str[i]-'a'])
                cur->go[str[i]-'a'] = new_Node();
            cur=cur->go[str[i]-'a'];
        }
        cur->cnt++; cur->i=n_pattern++;
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front(); que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
                    fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
                    que.push(fr->go[i]);
                }
            }
        }
    }
    void query(string s){
        Node *cur=root;
        for(int i=0;i<(int)s.size();i++){

```

```

    while(cur&&!cur->go[s[i]-'a']) cur=cur->fail;
    cur=(cur?cur->go[s[i]-'a']:root);
    if(cur->i>=0) ans[cur->i]++;
    for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
        ans[tmp->i]++;
} } // ans[i] : number of occurrence of pattern i
}AC;

```

6.6 Z Value

```

char s[MAXN];
int len,z[MAXN];
void Z_value() { //z[i] = lcp(s[1...],s[i...])
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

6.7 ZValue Palindrome

```

void z_value_pal(char *s,int len,int *z){
    len=(len<1)+1;
    for(int i=len-1;i>=0;i--)
        s[i]=i&1?s[i>>1]:'@';
    z[0]=1;
    for(int i=1,l=0,r=0;i<len;i++){
        z[i]=i<r?min(z[l+l-i],r-i):1;
        while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
            ++z[i];
        if(i+z[i]>r) l=i,r=i+z[i];
    }
}

```

6.8 Smallest Rotation

```

//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}

```

6.9 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
        }
    }
}

```

```

    pred[i][j]=L;
} else {
    j++;
} } }
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<=2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<=2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

7 Data Structure

7.1 Segment tree

```

struct seg_tree{
    ll a[MXN],val[MXN*4],tag[MXN*4],NO_TAG=0;
    void push(int i,int l,int r){
        if(tag[i]!=NO_TAG){
            val[i]+=tag[i]; // update by tag
            if(l!=r){
                tag[cl(i)]+=tag[i]; // push
                tag[cr(i)]+=tag[i]; // push
            }
            tag[i]=NO_TAG;
        }
    }
    void pull(int i,int l,int r){
        int mid=(l+r)>>1;
        push(cl(i),l,mid);push(cr(i),mid+1,r);
        val[i]=max(val[cl(i)],val[cr(i)]); // pull
    }
    void build(int i,int l,int r){
        if(l==r){
            val[i]=a[l]; // set value
            return;
        }
        int mid=(l+r)>>1;
        build(cl(i),l,mid);build(cr(i),mid+1,r);
        pull(i,l,r);
    }
    void update(int i,int l,int r,int ql,int qr,int v){
        push(i,l,r);
        if(ql<=l&&r<=qr){
            tag[i]+=v; // update tag
            return;
        }
        int mid=(l+r)>>1;
        if(ql<=mid) update(cl(i),l,mid,ql,qr,v);
    }
}

```



```

    if(qr>mid) update(cr(i),mid+1,r,ql,qr,v);
    pull(i,l,r);
}
ll query(int i,int l,int r,int ql,int qr){
    push(i,l,r);
    if(ql<=l&&r<=qr)
        return val[i]; // update answer
    ll mid=(l+r)>>1,ret=0;
    if(ql<=mid) ret=max(ret,query(cl(i),l,mid,ql,qr));
    if(qr>mid) ret=max(ret,query(cr(i),mid+1,r,ql,qr));
    return ret;
} }tree;

```

7.2 Treap

```

struct Treap{
    int sz , val , pri , tag;
    Treap *l , *r;
    Treap( int _val ){
        val = _val; sz = 1;
        pri = rand(); l = r = NULL; tag = 0;
    }
};
void push( Treap * a ){
    if( a->tag ){
        Treap *swp = a->l; a->l = a->r; a->r = swp;
        int swp2;
        if( a->l ) a->l->tag ^= 1;
        if( a->r ) a->r->tag ^= 1;
        a->tag = 0;
    }
}
inline int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
    a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a , Treap *b ){
    if( !a || !b ) return a ? a : b;
    if( a->pri > b->pri ){
        push( a );
        a->r = merge( a->r , b );
        pull( a );
        return a;
    }else{
        push( b );
        b->l = merge( a , b->l );
        pull( b );
        return b;
    }
}
void split_kth( Treap *t , int k , Treap*&a , Treap*&b ){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( Size( t->l ) + 1 <= k ){
        a = t;
        split_kth( t->r , k - Size( t->l ) - 1 , a->r , b );
        pull( a );
    }else{
        b = t;
        split_kth( t->l , k , a , b->l );
        pull( b );
    }
}
void split_key( Treap *t , int k , Treap*&a , Treap*&b ){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( k <= t->val ){
        b = t;
        split_key( t->l , k , a , b->l );
        pull( b );
    }else{
        a = t;
        split_key( t->r , k , a->r , b );
        pull( a );
    }
}
}

```

7.3 Disjoint Set

```

struct DisjointSet{
    // save() is like recursive
    // undo() is like return
    int n , fa[ N ] , sz[ N ];
    vector< pair<int*,int> > h;
}

```

```

vector<int> sp;
void init( int tn ){
    n=tn;
    for( int i = 0 ; i < n ; i ++ ){
        fa[ i ]=i;
        sz[ i ]=1;
    }
    sp.clear(); h.clear();
}
void assign( int *k , int v ){
    h.PB( {k , *k} );
    *k = v;
}
void save(){ sp.PB(SZ(h)); }
void undo(){
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while( SZ(h)!=last ){
        auto x=h.back(); h.pop_back();
        *x.first = x.second;
    }
}
void uni( int x , int y ){
    x = f( x ); y = f( y );
    if( x == y ) return;
    if( sz[ x ] < sz[ y ] ) swap( x , y );
    assign( &sz[ x ] , sz[ x ] + sz[ y ] );
    assign( &fa[ y ] , x );
} }djs;

```

7.4 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
    r[ 1 ].erase( 1 , 1 );
    cout << r[ 1 ].substr( 0 , 2 );
}

```

8 Others

8.1 Find max tangent(x,y is increasing)

```

const int MAXN = 100010;
Pt sum[MAXN], pnt[MAXN], ans, calc;
inline bool cross(Pt a, Pt b, Pt c){
    return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
} //pt[0]=(0,0);pt[i]=(i,pt[i-1].y+dy[i-1]),i=1~n;dx>=1
double find_max_tan(int n,int l,LL dy[]){
    int np, st, ed, now;
    sum[0].x = sum[0].y = np = st = ed = 0;
    for( int i = 1 , v; i <= n; i++){
        sum[i].x=i,sum[i].y=sum[i-1].y+dy[i-1];
        ans.x = now = 1,ans.y = -1;
        for( int i = 0; i <= n - l; i++){

```

```

while(np>1&&cross(pnt[np-2],pnt[np-1],sum[i]))
    np--;
if (np < now && np != 0) now = np;
pnt[np++] = sum[i];
while(now<np&&!cross(pnt[now-1],pnt[now],sum[i+1]))
    now++;
calc = sum[i + 1] - pnt[now - 1];
if (ans.y * calc.x < ans.x * calc.y)
    ans = calc,st = pnt[now - 1].x,ed = i + 1;
}
return (double)(sum[ed].y-sum[st].y)/(sum[ed].x-sum[
st].x);
}

```

8.2 Exact Cover Set

```

// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
    L[R[c]]=L[c]; R[L[c]]=R[c];
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=R[i]; j!=i; j=R[j] ){
            U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
        }
}
void resume(int c){
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=L[i]; j!=i; j=L[j] ){
            U[D[j]]=D[U[j]]=j; S[C[j]]++;
        }
    L[R[c]]=R[L[c]]=c;
}
int dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for( int i=R[0]; i!=0; i=R[i] )
        if(S[i]<md){ md=S[i]; c=i; }
    if(md==0) return 0;
    remove(c);
    for( int i=D[c]; i!=c; i=D[i] ){
        used[ROW[i]]=1;
        for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
        if(dfs()) return 1;
        for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
        used[ROW[i]]=0;
    }
    resume(c);
    return 0;
}
int exact_cover(int n,int m){
    for( int i=0; i<=m; i++ ){
        R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
        S[i]=0; C[i]=i;
    }
    R[m]=0; L[0]=m;
    int t=m+1;
    for( int i=0; i<n; i++ ){
        int k=-1;
        for( int j=0; j<m; j++ ){
            if(!A[i][j]) continue;
            if(k==-1) L[t]=R[t]=t;
            else{ L[t]=k; R[t]=R[k]; }
            k=t; D[t]=j+1; U[t]=U[j+1];
            L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
            C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
        }
    }
    for( int i=0; i<n; i++ ) used[i]=0;
    return dfs();
}

```