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1 vimrc

```
set nu
set cindent
set tabstop=4
set shiftwidth=4
set softtabstop=4
set expandtab
set autowrite
set mouse=a
autocmd filetype cpp nnoremap <F5> :w<CR> :!g++ % && ./
    a.out<CR>
```

2 defalt code

```cpp
#include <bits/stdc++.h>
using namespace std;
#define ld double
#define ll long long
#define pb push_back
#define endl '\n'
#define all(x) x.begin(), x.end()
#define IO ios::sync_with_stdio(0);cin.tie(0);cout.tie
    (0)
int main()
{
    IO;

    return 0;
}
```

3 Basic

3.1 stringstream

```cpp
//stringstream
string in, out, tmp;
cin >> in;
cin.ignore();
stringstream ss(in);
while(getline(ss, tmp, {char})) out += tmp;
//while(ss >> tmp) out += tmp;
```

3.2 check

```
for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done
```

3.3 python-related

```
parser:
int(eval(num.replace("/","//")))

from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

itwo = Decimal(0.5)
two = Decimal(2)

format(x, '0.10f') # set precision

N = 200
```

```
def angle(cosT):
  """given cos(theta) in decimal return theta"""
  for i in range(N):
    cosT = ((cosT + 1) / two) ** itwo
  sinT = (1 - cosT * cosT) ** itwo
  return sinT * (2 ** N)
pi = angle(Decimal(-1))
```

3.4 compare

```
struct Example{
  int x;
  friend bool operator>(const Example &a, const Example
       &b){
    return a.x > b.x;
  }
};
set<Example> s;// map<Example, int> mp;
struct cmp{
    bool operator()(const int &a,const int &b){return a
        >b;}
};
set<int, cmp> s;//map<int, int, cmp> mp;
```

4 flow

4.1 ISAP

```
struct Maxflow {
  static const int MAXV = 20010;
  static const int INF  = 1000000;
  struct Edge {
    int v, c, r;
    Edge(int _v, int _c, int _r):
      v(_v), c(_c), r(_r) {}
  };
  int s, t;
  vector<Edge> G[MAXV*2];
  int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
  void init(int x) {
    tot = x+2;
    s = x+1, t = x+2;
    for(int i = 0; i <= tot; i++) {
      G[i].clear();
      iter[i] = d[i] = gap[i] = 0;
  } }
  void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v]) ));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
  }
  int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
      Edge &e = G[p][i];
      if(e.c > 0 && d[p] == d[e.v]+1) {
        int f = dfs(e.v, min(flow, e.c));
        if(f) {
          e.c -= f;
          G[e.v][e.r].c += f;
          return f;
    } } }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
      d[p]++;
      iter[p] = 0;
      ++gap[d[p]];
    }
    return 0;
  }
  int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
  }
  void reset() {
    for(int i=0;i<=tot;i++) {
      iter[i]=d[i]=gap[i]=0;
} } }flow;
```

4.2 MinCostFlow

```
struct MinCostMaxFlow{
typedef int Tcost;
  static const int MAXV = 20010;
  static const int INFf = 1000000;
  static const Tcost INFc  = 1e9;
  struct Edge{
    int v, cap;
    Tcost w;
    int rev;
    Edge(){}
    Edge(int t2, int t3, Tcost t4, int t5)
    : v(t2), cap(t3), w(t4), rev(t5) {}
  };
  int V, s, t;
  vector<Edge> g[MAXV];
  void init(int n, int _s, int _t){
    V = n; s = _s; t = _t;
    for(int i = 0; i <= V; i++) g[i].clear();
  }
  void addEdge(int a, int b, int cap, Tcost w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
  }
  Tcost d[MAXV];
  int id[MAXV], mom[MAXV];
  bool inqu[MAXV];
  queue<int> q;
  pair<int,Tcost> solve(){
    int mxf = 0; Tcost mnc = 0;
    while(1){
      fill(d, d+1+V, INFc);
      fill(inqu, inqu+1+V, 0);
      fill(mom, mom+1+V, -1);
      mom[s] = s;
      d[s] = 0;
      q.push(s); inqu[s] = 1;
      while(q.size()){
        int u = q.front(); q.pop();
        inqu[u] = 0;
        for(int i = 0; i < (int) g[u].size(); i++){
          Edge &e = g[u][i];
          int v = e.v;
          if(e.cap > 0 && d[v] > d[u]+e.w){
            d[v] = d[u]+e.w;
            mom[v] = u;
            id[v] = i;
            if(!inqu[v]) q.push(v), inqu[v] = 1;
      } } }
      if(mom[t] == -1) break ;
      int df = INFf;
      for(int u = t; u != s; u = mom[u])
        df = min(df, g[mom[u]][id[u]].cap);
      for(int u = t; u != s; u = mom[u]){
        Edge &e = g[mom[u]][id[u]];
        e.cap              -= df;
        g[e.v][e.rev].cap += df;
      }
      mxf += df;
      mnc += df*d[t];
    }
    return {mxf,mnc};
} }flow;
```

4.3 Dinic

```
const int MXN = 10000;
struct Dinic{
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  }
  bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;
    queue<int> que;
    que.push(s);
```

```
    level[s] = 0;
    while (!que.empty()){
      int u = que.front(); que.pop();
      for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
          level[it.v] = level[u]+1;
          que.push(it.v);
    } } }
    return level[t] != -1;
  }
  int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
      if (it.f > 0 && level[it.v] == level[u]+1){
        int tf = DFS(it.v, min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
    } }
    if (!res) level[u] = -1;
    return res;
  }
  int flow(int res=0){
    while ( BFS() )
      res += DFS(s,2147483647);
    return res;
} }flow;
```

4.4 Kuhn Munkres 最大完美二分匹配

```
struct KM{ // max weight, for min negate the weights
  static const int MXN = 2001; // 1-based
  static const ll INF = 0x3f3f3f3f;
  int n, mx[MXN], my[MXN], pa[MXN];
  ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
  bool vx[MXN], vy[MXN];
  void init(int _n) {
    n = _n;
    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
  }
  void addEdge(int x, int y, ll w) {g[x][y] = w;}
  void augment(int y) {
    for(int x, z; y; y = z)
      x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
  }
  void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
      while(q.size()) {
        int x=q.front(); q.pop(); vx[x]=1;
        for(int y=1; y<=n; ++y) if(!vy[y]){
          ll t = lx[x]+ly[y]-g[x][y];
          if(t==0){
            pa[y]=x;
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
          }else if(sy[y]>t) pa[y]=x,sy[y]=t;
      } }
      ll cut = INF;
      for(int y=1; y<=n; ++y)
        if(!vy[y]&&cut>sy[y]) cut=sy[y];
      for(int j=1; j<=n; ++j){
        if(vx[j]) lx[j] -= cut;
        if(vy[j]) ly[j] += cut;
        else sy[j] -= cut;
      }
      for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
        if(!my[y]){augment(y);return;}
        vy[y]=1, q.push(my[y]);
  } } }
  ll solve(){
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
      lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
} }graph;
```

4.5 Directed MST

```
/* Edmond's algoirthm for Directed MST
 * runs in O(VE) */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF  = 2147483647;
struct Edge{
  int u, v, c;
  Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++ V; }
inline void addEdge(int u, int v, int c)
{ edges[++E] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
  fill(con, con+V+1, 0);
  int r1 = 0, r2 = 0;
  while(1){
    fill(mnInW, mnInW+V+1, INF);
    fill(prv, prv+V+1, -1);
    REP(i, 1, E){
      int u=edges[i].u, v=edges[i].v, c=edges[i].c;
      if(u != v && v != root && c < mnInW[v])
        mnInW[v] = c, prv[v] = u;
    }
    fill(vis, vis+V+1, -1);
    fill(cyc, cyc+V+1, -1);
    r1 = 0;
    bool jf = 0;
    REP(i, 1, V){
      if(con[i]) continue ;
      if(prv[i] == -1 && i != root) return -1;
      if(prv[i] > 0) r1 += mnInW[i];
      int s;
      for(s = i; s != -1 && vis[s] == -1; s = prv[s])
        vis[s] = i;
      if(s > 0 && vis[s] == i){
         // get a cycle
        jf = 1; int v = s;
        do{
          cyc[v] = s, con[v] = 1;
          r2 += mnInW[v]; v = prv[v];
        }while(v != s);
        con[s] = 0;
    } }
    if(!jf) break ;
    REP(i, 1, E){
      int &u = edges[i].u;
      int &v = edges[i].v;
      if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
      if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
      if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
      if(u == v) edges[i--] = edges[E--];
  } }
  return r1+r2;
}
```

4.6 SW min-cut (不限 S-T 的 min-cut)

```
// global min cut
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n; FZ(edge); FZ(del);
  }
  void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
  }
  void search(int &s, int &t){
    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
      int mx=-1, cur=0;
      for (int i=0; i<n; i++)
        if (!del[i] && !vst[i] && mx<wei[i])
          cur = i, mx = wei[i];
```

```
      if (mx == -1) break;
      vst[cur] = 1;
      s = t; t = cur;
      for (int i=0; i<n; i++)
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
  }
}graph;
```

4.7 Max flow with lower/upper bound

```
// flow use ISAP
// Max flow with lower/upper bound on edges
// source = 1 , sink = n
int in[ N ] , out[ N ];
int l[ M ] , r[ M ] , a[ M ] , b[ M ];//0-base,a下界,b
    上界
int solve(){
  flow.init( n );   //n為點的數量,m為邊的數量,點是1-
      base
  for( int i = 0 ; i < m ; i ++ ){
    in[ r[ i ] ] += a[ i ];
    out[ l[ i ] ] += a[ i ];
    flow.addEdge( l[ i ] , r[ i ] , b[ i ] - a[ i ] );
    // flow from l[i] to r[i] must in [a[ i ], b[ i ]]
  }
  int nd = 0;
  for( int i = 1 ; i <= n ; i ++ ){
    if( in[ i ] < out[ i ] ){
      flow.addEdge( i , flow.t , out[ i ] - in[ i ] );
      nd += out[ i ] - in[ i ];
    }
    if( out[ i ] < in[ i ] )
      flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
  }
  // original sink to source
  flow.addEdge( n , 1 , INF );
  if( flow.maxflow() != nd )
    // no solution
    return -1;
  int ans = flow.G[ 1 ].back().c; // source to sink
  flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
  // take out super source and super sink
  for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i
      ++ ){
    flow.G[ flow.s ][ i ].c = 0;
    Edge &e = flow.G[ flow.s ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
  }
  for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i
      ++ ){
    flow.G[ flow.t ][ i ].c = 0;
    Edge &e = flow.G[ flow.t ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
  }
  flow.addEdge( flow.s , 1 , INF );
  flow.addEdge( n , flow.t , INF );
  flow.reset();
  return ans + flow.maxflow();
}
```

4.8 Flow Conclusion

二分圖的最小點覆蓋數=該二分圖的最大匹配數
二分圖的最小邊覆蓋數=圖中的頂點數-(最小點覆蓋數)該二分圖的最大匹配數
DAG圖的最小路徑覆蓋可以轉化為二分圖的人後求解
二分圖的最大點獨立數=點的個數-最小點覆蓋數(最大匹配)
最大匹配:超級原點連一團,超級終點連另一團,流量即是最大匹配

5 Math

5.1 Fast power

```
int power(int a,int b){
    int ans=1,base=a;
    while(b!=0){
        if(b&1){//如果是奇数
            ans*=base;//向结果赋值
        }
        base*=base;//基础相乘
        b=b/2;
    }
    return ans;
}
```

5.2 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
  for(int i=0; i<=MAXN; i++)
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {
      cplx w = omega[inv ? MAXN-(i*theta%MAXN)
                         : i*theta%MAXN];
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
    } }
    theta = (theta * 2) % MAXN;
  }
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  }
  if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n,ll a[],int _m,ll b[],ll ans[])
{
  int n=1,sum=_n+_m-1;
  while(n<sum)
    n<<=1;
  for(int i=0;i<n;i++)
  {
    double x=(i<_n?a[i]:0),y=(i<_m?b[i]:0);
    arr[i]=complex<double>(x+y,x-y);
  }
  fft(n,arr);
  for(int i=0;i<n;i++)
    arr[i]=arr[i]*arr[i];
  fft(n,arr,true);
  for(int i=0;i<sum;i++)
    ans[i]=(long long int)(arr[i].real()/4+0.5);
}
```

5.3 Poly operator

```
struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
  NTT<P, root, MAXN> ntt;
  static int nxt2k(int x) {
    int i = 1; for (; i < x; i <<= 1); return i;
  }
```

```
// c[i]=sum{j=0~i}a[j]*b[i-j] -> c[i+j]+=a[i]*b[j](加
    分卷積)
// if c[i-j]+=a[i]*b[j]  (減法卷積)
// (轉換成加法捲積)  -> reverse(a); c=mul(a,b);
    reverse( c );
void Mul(int n, LL a[], int m, LL b[], LL c[]) {
  static LL aa[MAXN], bb[MAXN];
  int N = nxt2k(n+m);
  copy(a, a+n, aa); fill(aa+n, aa+N, 0);
  copy(b, b+m, bb); fill(bb+m, bb+N, 0);
  ntt.tran(N, aa); ntt.tran(N, bb);
  FOR(i, N) c[i] = aa[i] * bb[i] % P;
  ntt.tran(N, c, 1);
}
void Inv(int n, LL a[], LL b[]) {
  // ab = aa^-1 = 1 mod x^(n/2)
  // (b - a^-1)^2 = 0 mod x^n
  // bb - a^-2 + 2 ba^-1 = 0
  // bba - a^-1 + 2b = 0
  // bba + 2b = a^-1
  static LL tmp[MAXN];
  if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
  Inv((n+1)/2, a, b);
  int N = nxt2k(n*2);
  copy(a, a+n, tmp);
  fill(tmp+n, tmp+N, 0);
  fill(b+n, b+N, 0);
  ntt.tran(N, tmp); ntt.tran(N, b);
  FOR(i, N) {
    LL t1 = (2 - b[i] * tmp[i]) % P;
    if (t1 < 0) t1 += P;
    b[i] = b[i] * t1 % P;
  }
  ntt.tran(N, b, 1);
  fill(b+n, b+N, 0);
}
void Div(int n, LL a[], int m, LL b[], LL d[], LL r
    []) {
  // Ra = Rb * Rd mod x^(n-m+1)
  // Rd = Ra * Rb^-1 mod
  static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
  if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);
      return;}
  // d: n-1 - (m-1) = n-m (n-m+1 terms)
  copy(a, a+n, aa); copy(b, b+m, bb);
  reverse(aa, aa+n); reverse(bb, bb+m);
  Inv(n-m+1, bb, tb);
  Mul(n-m+1, ta, n-m+1, tb, d);
  fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
  // r: m-1 - 1 = m-2 (m-1 terms)
  Mul(m, b, n-m+1, d, ta);
  FOR(i, n) { r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
       += P; }
}
void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i
    -1] = i * a[i] % P; }
void Sx(int n, LL a[], LL b[]) {
  b[0] = 0;
  FOR(i, n) b[i+1] = a[i] * ntt.inv(i+1, P) % P;
}
void Ln(int n, LL a[], LL b[]) {
  // Integral a' a^-1 dx
  static LL a1[MAXN], a2[MAXN], b1[MAXN];
  int N = nxt2k(n*2);
  dx(n, a, a1); Inv(n, a, a2);
  Mul(n-1, a1, n, a2, b1);
  Sx(n+n-1-1, b1, b);
  fill(b+n, b+N, 0);
}
void Exp(int n, LL a[], LL b[]) {
  // Newton method to solve g(a(x)) = ln b(x) - a(x)
      = 0
  // b' = b - g(b(x)) / g'(b(x))
  // b' = b (1 - lnb + a)
  static LL lnb[MAXN], c[MAXN], tmp[MAXN];
  assert(a[0] == 0); // dont know exp(a[0]) mod P
  if (n == 1) {b[0] = 1; return;}
  Exp((n+1)/2, a, b);
  fill(b+(n+1)/2, b+n, 0);
  Ln(n, b, lnb);
  fill(c, c+n, 0); c[0] = 1;
    FOR(i, n) {
      c[i] += a[i] - lnb[i];
      if (c[i] < 0) c[i] += P;
      if (c[i] >= P) c[i] -= P;
    }
    Mul(n, b, n, c, tmp);
    copy(tmp, tmp+n, b);
  }
} polyop;
```

5.4 O(1)mul

```
LL mul(LL x,LL y,LL mod){
  LL ret=x*y-(LL)((long double)x/mod*y)*mod;
  // LL ret=x*y-(LL)((long double)x*y/mod+0.5)*mod;
  return ret<0?ret+mod:ret;
}
```

5.5 Miller Rabin

```
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383          6 :  pirmes <= 13
// n < 2^64                       7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[]={}
bool witness(LL a,LL n,LL u,int t){
  if(!a) return 0;
  LL x=mypow(a,u,n);
  for(int i=0;i<t;i++) {
    LL nx=mul(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  }
  return x!=1;
}
bool miller_rabin(LL n) {
  int s=(magic number size)
  // iterate s times of witness on n
  if(n<2) return 0;
  if(!(n&1)) return n == 2;
  ll u=n-1; int t=0;
  // n-1 = u*2^t
  while(!(u&1)) u>>=1, t++;
  while(s--){
    LL a=magic[s]%n;
    if(witness(a,n,u,t)) return 0;
  }
  return 1;
}
```

5.6 Faulhaber ($\sum_{i=1}^{n} i^p$)

```
/* faulhaber' s formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {
  int a=x,b=mod,a0=1,a1=0,b0=0,b1=1;
  while(b) {
    int q,t;
    q=a/b; t=b; b=a-b*q; a=t;
    t=b0; b0=a0-b0*q; a0=t;
    t=b1; b1=a1-b1*q; a1=t;
  }
  return a0<0?a0+mod:a0;
}
inline void pre() {
  /* combinational */
  for(int i=0;i<=MAXK;i++) {
    cm[i][0]=cm[i][i]=1;
    for(int j=1;j<i;j++)
      cm[i][j]=add(cm[i-1][j-1],cm[i-1][j]);
  }
  /* inverse */
```

```
  for(int i=1;i<=MAXK;i++) inv[i]=getinv(i);
  /* bernoulli */
  b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
  for(int i=2;i<MAXK;i++) {
    if(i&1) { b[i]=0; continue; }
    b[i]=1;
    for(int j=0;j<i;j++)
      b[i]=sub(b[i],
               mul(cm[i][j],mul(b[j], inv[i-j+1])));
  }
  /* faulhaber */
  // sigma_x=1~n {x^p} =
  //   1/(p+1) * sigma_j=0~p {C(p+1,j)*Bj*n^(p-j+1)}
  for(int i=1;i<MAXK;i++) {
    co[i][0]=0;
    for(int j=0;j<=i;j++)
      co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]))
          ;
  }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n,int p) {
  int sol=0,m=n;
  for(int i=1;i<=p+1;i++) {
    sol=add(sol,mul(co[p][i],m));
    m = mul(m, n);
  }
  return sol;
}
```

5.7 Chinese Remainder

```
LL x[N],m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
  LL g = __gcd(m1, m2);
  if((x2 - x1) % g) return -1;// no sol
  m1 /= g; m2 /= g;
  pair<LL,LL> p = gcd(m1, m2);
  LL lcm = m1 * m2 * g;
  LL res = p.first * (x2 - x1) * m1 + x1;
  return (res % lcm + lcm) % lcm;
}
LL solve(int n){ // n>=2,be careful with no solution
  LL res=CRT(x[0],m[0],x[1],m[1]),p=m[0]/__gcd(m[0],m
      [1])*m[1];
  for(int i=2;i<n;i++){
    res=CRT(res,p,x[i],m[i]);
    p=p/__gcd(p,m[i])*m[i];
  }
  return res;
}
```

5.8 Pollard Rho

```
// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
  if(!(n&1)) return 2;
  while(true){
    LL y=2, x=rand()%(n-1)+1, res=1;
    for(int sz=2; res==1; sz*=2) {
      for(int i=0; i<sz && res<=1; i++) {
        x = f(x, n);
        res = __gcd(abs(x-y), n);
      }
      y = x;
    }
    if (res!=0 && res!=n) return res;
} }
```

5.9 Josephus Problem

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

5.10 Gaussian Elimination

```
const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>> v;
    int ppow(int a , int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD ,
            k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD ,
            k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now , 0 , n){
            REP(i , now , n) if(v[now][now] == 0 && v[i
                ][now] != 0)
                swap(v[i] , v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now] , GAUSS_MOD - 2)
                ;
            REP(i , 0 , n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j , now , n + 1) (v[i][j] +=
                    GAUSS_MOD - tmp * v[now][j] %
                    GAUSS_MOD) %= GAUSS_MOD;
            }
        }
        REP(i , 0 , n) ans[i] = v[i][n + 1] * ppow(v[i
            ][i] , GAUSS_MOD - 2) % GAUSS_MOD;
        return ans;
    }
    // gs.v.clear() , gs.v.resize(n , vector<int>(n + 1
        , 0));
} gs;
```

5.11 ax+by=gcd

```
PII gcd(int a, int b){
  if(b == 0) return {1, 0};
  PII q = gcd(b, a % b);
  return {q.second, q.first - q.second * (a / b)};
}
```

5.12 Roots of Polynomial 找多項式的根

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ]; // a[0..n](coef) must be
    filled
int n; // degree of polynomial must be filled
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
  double tmp=1,sum=0;
  for(int i=0;i<=n;i++)
  { sum=sum+a[i]*tmp; tmp=tmp*x; }
  return sum;
}
double binary(double l,double r,double a[],int n){
  int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
  if(sl==0) return l; if(sr==0) return r;
  if(sl*sr>0) return inf;
  while(r-l>eps){
    double mid=(l+r)/2;
    int ss=sign(f(a,n,mid));
    if(ss==0) return mid;
    if(ss*sl>0) l=mid; else r=mid;
  }
  return l;
}
void solve(int n,double a[],double x[],int &nx){
  if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
  double da[10], dx[10]; int ndx;
  for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
  solve(n-1,da,dx,ndx);
  nx=0;
  if(ndx==0){
    double tmp=binary(-inf,inf,a,n);
    if (tmp<inf) x[++nx]=tmp;
    return;
  }
  double tmp;
  tmp=binary(-inf,dx[1],a,n);
  if(tmp<inf) x[++nx]=tmp;
```

```
  for(int i=1;i<=ndx-1;i++){
    tmp=binary(dx[i],dx[i+1],a,n);
    if(tmp<inf) x[++nx]=tmp;
  }
  tmp=binary(dx[ndx],inf,a,n);
  if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]
```

5.13 Primes

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 1000000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[ N ] , p_tbl[ N ];
vector<int> primes;
void sieve() {
  mu[ 1 ] = p_tbl[ 1 ] = 1;
  for( int i = 2 ; i < N ; i ++ ){
    if( !p_tbl[ i ] ){
      p_tbl[ i ] = i;
      primes.push_back( i );
      mu[ i ] = -1;
    }
    for( int p : primes ){
      int x = i * p;
      if( x >= M ) break;
      p_tbl[ x ] = p;
      mu[ x ] = -mu[ i ];
      if( i % p == 0 ){
        mu[ x ] = 0;
        break;
} } } }
vector<int> factor( int x ){
  vector<int> fac{ 1 };
  while( x > 1 ){
    int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
    while( x % p == 0 ){
      x /= p;
      for( int i = 0 ; i < fn ; i ++ )
        fac.PB( fac[ pos ++ ] * p );
  } }
  return fac;
}
```

5.14 Result

- Lucas' Theorem :
  For $n, m \in \mathbb{Z}^*$ and prime P, $C(m,n)$ mod $P = \Pi(C(m_i, n_i))$ where m_i is the i-th digit of m in base P.
- Stirling approximation :
  $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n e^{\frac{1}{12n}}$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
  $S(n,k) =$ coefficient of x^k in $\Pi_{i=0}^{n-1}(x+i)$
- Stirling Numbers(Partition n elements into k non-empty set):
  $S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$
- Pick' s Theorem : $A = i + b/2 - 1$
  其面積 A 和內部格點數目 i、邊上格點數目 b 的關係
- Catalan number : $C_n = \binom{2n}{n}/(n+1)$
  $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)!\frac{n-m+1}{n+1} \quad for \quad n \geq m$
  $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
  $C_0 = 1 \quad and \quad C_{n+1} = 2(\frac{2n+1}{n+2})C_n$
  $C_0 = 1 \quad and \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} \quad for \quad n \geq 0$
- Euler Characteristic:
  planar graph: $V - E + F - C = 1$
  convex polyhedron: $V - E + F = 2$
  V, E, F, C: number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :
  $A_{ii} = deg(i), A_{ij} = (i,j) \in E \ ? -1 : 0$, Deleting any one row, one column, and cal the det(A)
- Polya' theorem (c 為方法數，m 為總數):
  $(\sum_{i=1}^{m} c^{gcd(i,m)})/m$
- 錯排公式: (n 個人中，每個人皆不再原來位置的組合數):
  $dp[0] = 1; dp[1] = 0;$
  $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$
- Bell 數 (有 n 個人，把他們拆組的方法總數) :
  $B_0 = 1$
  $B_n = \sum_{k=0}^{n} s(n,k) \quad (second - stirling)$
  $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$
- Wilson's theorem :
  $(p-1)! \equiv -1 (mod\ p)$
- Fermat's little theorem :
  $a^p \equiv a (mod\ p)$
- Euler's totient function:
  $A^{B^C}\ mod\ p = pow(A, pow(B, C, p-1)) mod\ p$

6 Geometry

6.1 definition

```
typedef long double ld;
const ld eps = 1e-8;
int dcmp(ld x) {
  if(abs(x) < eps) return 0;
  else return x < 0 ? -1 : 1;
}
struct Pt {
  ld x, y;
  Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}

  Pt operator+(const Pt &a) const {
    return Pt(x+a.x, y+a.y);
  }
  Pt operator-(const Pt &a) const {
    return Pt(x-a.x, y-a.y);
  }
  Pt operator*(const ld &a) const {
    return Pt(x*a, y*a);
  }
  Pt operator/(const ld &a) const {
    return Pt(x/a, y/a);
  }
  ld operator*(const Pt &a) const {
    return x*a.x + y*a.y;
  }
  ld operator^(const Pt &a) const {
    return x*a.y - y*a.x;
  }
  bool operator<(const Pt &a) const {
    return x < a.x || (x == a.x && y < a.y);
    //return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
        dcmp(y-a.y) < 0);
  }
  bool operator==(const Pt &a) const {
    return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0;
  }
};
ld norm2(const Pt &a) {
  return a*a;
}
ld norm(const Pt &a) {
  return sqrt(norm2(a));
}
Pt perp(const Pt &a) {
  return Pt(-a.y, a.x);
}
Pt rotate(const Pt &a, ld ang) {
  return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y
      *cos(ang));
}
struct Line {
  Pt s, e, v; // start, end, end-start
  ld ang;
  Line(Pt _s=Pt(0, 0), Pt _e=Pt(0, 0)):s(_s), e(_e) { v
       = e-s; ang = atan2(v.y, v.x); }

  bool operator<(const Line &L) const {
    return ang < L.ang;
  }
};
struct Circle {
  Pt o; ld r;
```

```
  Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
};
```

6.2 Intersection of 2 lines

```
Pt LLIntersect(Line a, Line b) {
  Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
  ld f1 = (p2-p1)^(q1-p1),f2 = (p2-p1)^(p1-q2),f;
  if(dcmp(f=f1+f2) == 0)
    return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
  return q1*(f2/f) + q2*(f1/f);
}
```

6.3 halfPlaneIntersection

```
// for point or line solution, change > to >=
bool onleft(Line L, Pt p) {
  return dcmp(L.v^(p-L.s)) > 0;
}
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
  sort(L.begin(), L.end()); // sort by angle
  int n = L.size(), fir, las;
  Pt *p = new Pt[n];
  Line *q = new Line[n];
  q[fir=las=0] = L[0];
  for(int i = 1 ; i < n ; i++) {
    while(fir < las && !onleft(L[i], p[las-1])) las--;
    while(fir < las && !onleft(L[i], p[fir])) fir++;
    q[++las] = L[i];
    if(dcmp(q[las].v^q[las-1].v) == 0) {
      las--;
      if(onleft(q[las], L[i].s)) q[las] = L[i];
    }
    if(fir < las) p[las-1] = LLIntersect(q[las-1], q[
        las]);
  }
  while(fir < las && !onleft(q[fir], p[las-1])) las--;
  if(las-fir <= 1) return {};
  p[las] = LLIntersect(q[las], q[fir]);
  int m = 0;
  vector<Pt> ans(las-fir+1);
  for(int i = fir ; i <= las ; i++) ans[m++] = p[i];
  return ans;
}
```

6.4 Convex Hull

```
double cross(Pt o, Pt a, Pt b){
  return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<Pt> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){
    while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  stk.resize(top-1);
  return stk;
}
```

6.5 Intersection of 2 segments

```
int ori( const Pt& o , const Pt& a , const Pt& b ){
  LL ret = ( a - o ) ^ ( b - o );
  return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
             const Pt& q1 , const Pt& q2 ){
  if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
    if( ori( p1 , p2 , q1 ) ) return false;
    return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
           ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
           ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
           ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
  }
  return (ori( p1, p2, q1 ) * ori( p1, p2, q2 )<=0) &&
         (ori( q1, q2, p1 ) * ori( q1, q2, p2 )<=0);
}
```

6.6 Intersection of circle and segment

0

```
bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
  Pt dp = p2 - p1;
  double a = dp * dp;
  double b = 2 * ( dp * ( p1 - cc.O ) );
  double c = cc.O * cc.O + p1 * p1 - 2 * ( cc.O * p1 )
      - cc.R * cc.R;
  double bb4ac = b * b - 4 * a * c;
  return !( fabs( a ) < eps or bb4ac < 0 );
}
```

6.7 Circle cover

```
#define N 1021
#define D long double
struct CircleCover{
  int C; Circ c[ N ]; //填入C(圓數量),c(圓陣列)
  bool g[ N ][ N ], overlap[ N ][ N ];
  // Area[i] : area covered by at least i circles
  D Area[ N ];
  void init( int _C ){ C = _C; }
  bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
    Pt o1 = a.O , o2 = b.O;
    D r1 = a.R , r2 = b.R;
    if( norm( o1 - o2 ) > r1 + r2 ) return {};
    if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
        return {};
    D d2 = ( o1 - o2 ) * ( o1 - o2 );
    D d = sqrt(d2);
    if( d > r1 + r2 ) return false;
    Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
    Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
    p1 = u + v; p2 = u - v;
    return true;
  }
  struct Teve {
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[ N * 2 ];
  // strict: x = 0, otherwise x = -1
  bool disjuct( Circ& a, Circ &b, int x )
  {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
  bool contain( Circ& a, Circ &b, int x )
  {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
  bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
                contain(c[i], c[j], -1);
  }
  void solve(){
    for( int i = 0 ; i <= C + 1 ; i ++ )
      Area[ i ] = 0;
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        overlap[i][j] = contain(i, j);
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
    for( int i = 0 ; i < C ; i ++ ){
      int E = 0, cnt = 1;
      for( int j = 0 ; j < C ; j ++ )
        if( j != i && overlap[j][i] )
          cnt ++;
      for( int j = 0 ; j < C ; j ++ )
        if( i != j && g[i][j] ){
```

```
          Pt aa, bb;
          CCinter(c[i], c[j], aa, bb);
          D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E ++] = Teve(bb, B, 1);
          eve[E ++] = Teve(aa, A, -1);
          if(B > A) cnt ++;
        }
      if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
      else{
        sort( eve , eve + E );
        eve[E] = eve[0];
        for( int j = 0 ; j < E ; j ++ ){
          cnt += eve[j].add;
          Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
          D theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2.0 * pi;
          Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}}}}};
```

6.8 Convex Hull trick

```
/* Given a convexhull, answer querys in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
  int n;
  vector<Pt> a;
  vector<Pt> upper, lower;
  Conv(vector<Pt> _a) : a(_a){
    n = a.size();
    int ptr = 0;
    for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
    for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
    for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
    upper.push_back(a[0]);
  }
  int sign( LL x ){ // fixed when changed to double
    return x < 0 ? -1 : x > 0; }
  pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
    int l = 0, r = (int)conv.size() - 2;
    for( ; l + 1 < r; ){
      int mid = (l + r) / 2;
      if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
      else l = mid;
    }
    return max(make_pair(det(vec, conv[r]), r),
               make_pair(det(vec, conv[0]), 0));
  }
  void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
  }
  void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
      if (smid == sl) l = mid;
      else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
  }
  int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid = sign(det(v - u, a[mid % n] - u));
      if (smid == sl) l = mid;
      else r = mid;
    }
    return l % n;
  }
  // 1. whether a given point is inside the CH
  bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
      if (lower[id].Y > p.Y) return 0;
    }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
      if (upper[id].Y < p.Y) return 0;
    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
    return 1;
  }
  // 2. Find 2 tang pts on CH of a given outside point
  // return true with i0, i1 as index of tangent points
  // return false if inside CH
  bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
  }
  // 3. Find tangent points of a given vector
  // ret the idx of vertex has max cross value with vec
  int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
  }
  // 4. Find intersection point of a given line
  // return 1 and intersection is on edge (i, next(i))
  // return 0 if no strictly intersection
  bool get_intersection(Pt u, Pt v, int &i0, int &i1){
   int p0 = get_tang(u - v), p1 = get_tang(v - u);
   if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
     if (p0 > p1) swap(p0, p1);
     i0 = bi_search(u, v, p0, p1);
     i1 = bi_search(u, v, p1, p0 + n);
     return 1;
   }
   return 0;
}  };
```

6.9 Tangent line of two circles

```
vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = norm2( c1.O - c2.O );
  if( d_sq < eps ) return ret;
  double d = sqrt( d_sq );
  Pt v = ( c2.O - c1.O ) / d;
  double c = ( c1.R - sign1 * c2.R ) / d;
  if( c * c > 1 ) return ret;
  double h = sqrt( max( 0.0 , 1.0 - c * c ) );
  for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y ,
             v.Y * c + sign2 * h * v.X };
    Pt p1 = c1.O + n * c1.R;
    Pt p2 = c2.O + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
      p2 = p1 + perp( c2.O - c1.O );
    ret.push_back( { p1 , p2 } );
  }
  return ret;
}
```

6.10 KD Tree

```
const int MXN=100005;
const int MXK=10;
```

```
struct KDTree{
  struct Nd{
    LL x[MXK],mn[MXK],mx[MXK];
    int id,f;
    Nd *l,*r;
  }tree[MXN],*root;
  int n,k;
  LL dis(LL a,LL b){return (a-b)*(a-b);}
  LL dis(LL a[MXK],LL b[MXK]){
    LL ret=0;
    for(int i=0;i<k;i++) ret+=dis(a[i],b[i]);
    return ret;
  }
  void init(vector<vector<LL>> &ip,int _n,int _k){
    n=_n,k=_k;
    for(int i=0;i<n;i++){
      tree[i].id=i;
      copy(ip[i].begin(),ip[i].end(),tree[i].x);
    }
    root=build(0,n-1,0);
  }
  Nd* build(int l,int r,int d){
    if(l>r) return NULL;
    if(d==k) d=0;
    int m=(l+r)>>1;
    nth_element(tree+l,tree+m,tree+r+1,[&](const Nd &a,
        const Nd &b){return a.x[d]<b.x[d];});
    tree[m].f=d;
    copy(tree[m].x,tree[m].x+k,tree[m].mn);
    copy(tree[m].x,tree[m].x+k,tree[m].mx);
    tree[m].l=build(l,m-1,d+1);
    if(tree[m].l){
      for(int i=0;i<k;i++){
        tree[m].mn[i]=min(tree[m].mn[i],tree[m].l->mn[i
            ]);
        tree[m].mx[i]=max(tree[m].mx[i],tree[m].l->mx[i
            ]);
      }
    }
    tree[m].r=build(m+1,r,d+1);
    if(tree[m].r){
      for(int i=0;i<k;i++){
        tree[m].mn[i]=min(tree[m].mn[i],tree[m].r->mn[i
            ]);
        tree[m].mx[i]=max(tree[m].mx[i],tree[m].r->mx[i
            ]);
      }
    }
    return tree+m;
  }
  LL pt[MXK],md;
  int mID;
  bool touch(Nd *r){
    LL d=0;
    for(int i=0;i<k;i++){
      if(pt[i]<=r->mn[i]) d+=dis(pt[i],r->mn[i]);
        else if(pt[i]>=r->mx[i]) d+=dis(pt[i],r->mx[i])
            ;
    }
    return d<md;
  }
  void nearest(Nd *r){
    if(!r||!touch(r)) return;
    LL td=dis(r->x,pt);
    if(td<md) md=td,mID=r->id;
    nearest(pt[r->f]<r->x[r->f]?r->l:r->r);
    nearest(pt[r->f]<r->x[r->f]?r->r:r->l);
  }
  pair<LL,int> query(vector<LL> &_pt,LL _md=1LL<<57){
    mID=-1,md=_md;
    copy(_pt.begin(),_pt.end(),pt);
    nearest(root);
    return {md,mID};
  }
}tree;
```

6.11 Min Enclosing Circle

```
struct Mec{
  // return pair of center and r
  static const int N = 101010;
  int n;
  Pt p[ N ], cen;
  double r2;
  void init( int _n , Pt _p[] ){
    n = _n;
    memcpy( p , _p , sizeof(Pt) * n );
  }
  double sqr(double a){ return a*a; }
  Pt center(Pt p0, Pt p1, Pt p2) {
    Pt a = p1-p0;
    Pt b = p2-p0;
    double c1=norm2( a ) * 0.5;
    double c2=norm2( b ) * 0.5;
    double d = a ^ b;
    double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
    double y = p0.Y + (a.X * c2 - b.X * c1) / d;
    return Pt(x,y);
  }
  pair<Pt,double> solve(){
    random_shuffle(p,p+n);
    r2=0;
    for (int i=0; i<n; i++){
      if (norm2(cen-p[i]) <= r2) continue;
      cen = p[i];
      r2 = 0;
      for (int j=0; j<i; j++){
        if (norm2(cen-p[j]) <= r2) continue;
        cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
        r2 = norm2(cen-p[j]);
        for (int k=0; k<j; k++){
          if (norm2(cen-p[k]) <= r2) continue;
          cen = center(p[i],p[j],p[k]);
          r2 = norm2(cen-p[k]);
        }
      }
    }
    return {cen,sqrt(r2)};
  }
} mec;
```

6.12 Min Enclosing Ball

```
// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
  Pt q[3]; double m[3][3], sol[3], L[3], det;
  int i,j; res.x = res.y = res.z = radius = 0;
  switch ( nouter ) {
    case 1: res=outer[0]; break;
    case 2: res=(outer[0]+outer[1])/2; radius=norm2(res
        , outer[0]); break;
    case 3:
      for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
      for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q
          [i] * q[j])*2;
      for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
      if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps
          ) return;
      L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
      L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
      res=outer[0]+q[0]*L[0]+q[1]*L[1];
      radius=norm2(res, outer[0]);
      break;
    case 4:
      for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol
          [i]=(q[i] * q[i]);
      for (i=0;i<3;++i) for(j=0;j<3;++j) m[i][j]=(q[i]
          * q[j])*2;
      det= m[0][0]*m[1][1]*m[2][2]
        + m[0][1]*m[1][2]*m[2][0]
        + m[0][2]*m[2][1]*m[1][0]
        - m[0][2]*m[1][1]*m[2][0]
        - m[0][1]*m[1][0]*m[2][2]
        - m[0][0]*m[1][2]*m[2][1];
      if ( fabs(det)<eps ) return;
      for (j=0; j<3; ++j) {
        for (i=0; i<3; ++i) m[i][j]=sol[i];
        L[j]=( m[0][0]*m[1][1]*m[2][2]
               + m[0][1]*m[1][2]*m[2][0]
               + m[0][2]*m[2][1]*m[1][0]
               - m[0][2]*m[1][1]*m[2][0]
```

```
                - m[0][1]*m[1][0]*m[2][2]
                - m[0][0]*m[1][2]*m[2][1]
              ) / det;
        for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
      } res=outer[0];
      for (i=0; i<3; ++i ) res = res + q[i] * L[i];
      radius=norm2(res, outer[0]);
}}
void minball(int n){ ball();
  if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
    if( norm2(res, pt[i]) - radius > eps ){
      outer[ nouter ++ ] = pt[ i ]; minball(i); --
          nouter;
      if(i>0){ Pt Tt = pt[i];
        memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
            ;
}}}
double solve(){
  // n points in pt
  random_shuffle(pt, pt+n); radius=-1;
  for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
    nouter=1, outer[0]=pt[i], minball(i);
  return sqrt(radius);
}
```

6.13 Minkowski sum

```
vector<Pt> minkowski(vector<Pt> p, vector<Pt> q){
  int n = p.size() , m = q.size();
  Pt c = Pt(0, 0);
  for( int i = 0; i < m; i ++) c = c + q[i];
  c = c / m;
  for( int i = 0; i < m; i ++) q[i] = q[i] - c;
  int cur = -1;
  for( int i = 0; i < m; i ++)
    if( (q[i] ^ (p[0] - p[n-1])) > -eps)
      if( cur == -1 || (q[i] ^ (p[0] - p[n-1])) >
                       (q[cur] ^ (p[0] - p[n-1])) )
        cur = i;
  vector<Pt> h;
  p.push_back(p[0]);
  for( int i = 0; i < n; i ++)
    while( true ){
      h.push_back(p[i] + q[cur]);
      int nxt = (cur + 1 == m ? 0 : cur + 1);
      if((q[cur] ^ (p[i+1] - p[i])) < -eps) cur = nxt;
      else if( (q[nxt] ^ (p[i+1] - p[i])) >
               (q[cur] ^ (p[i+1] - p[i])) ) cur = nxt;
      else break;
    }
  for(auto &&i : h) i = i + c;
  return convex_hull(h);
}
```

6.14 Min dist on Cuboid

```
typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
  if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
  if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                        x0+L, y0, H, W, L);
  if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                        x0, y0+W, L, H, W);
  if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                        x0-H, y0, H, W, L);
  if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                        x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
  if( z1!=0 && z1!=H ){
    if( y1==0 || y1==W )
      swap(y1,z1), swap(y2,z2), swap(W,H);
  else swap(x1,z1), swap(x2,z2), swap(L,H);
  }
  if (z1==H) z1=0, z2=H-z2;
  r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
  return r;
}
```

6.15 Heart of Triangle

```
Pt inCenter( Pt &A,  Pt &B,  Pt &C) { // 內心
  double a = norm(B-C), b = norm(C-A), c = norm(A-B);
  return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a,  Pt &b,  Pt &c) { // 外心
  Pt bb = b - a, cc = c - a;
  double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
  return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a,  Pt &b,  Pt &c) { // 垂心
  Pt ba = b - a, ca = c - a, bc = b - c;
  double Y = ba.Y * ca.Y * bc.Y,
    A = ca.X * ba.Y - ba.X * ca.Y,
    x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
    y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
  return Pt(x0, y0);
}
```

7 Graph

7.1 MaximumClique 最大團

```
#define N 111
struct MaxClique{ // 0-base
  typedef bitset<N> Int;
  Int linkto[N] , v[N];
  int n;
  void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i ++){
      linkto[i].reset(); v[i].reset();
    }
  }
  void addEdge(int a , int b)
  { v[a][b] = v[b][a] = 1; }
  int popcount(const Int& val)
  { return val.count(); }
  int lowbit(const Int& val)
  { return val._Find_first(); }
  int ans , stk[N];
  int id[N] , di[N] , deg[N];
  Int cans;
  void maxclique(int elem_num, Int candi){
    if(elem_num > ans){
      ans = elem_num; cans.reset();
      for(int i = 0 ; i < elem_num ; i ++)
        cans[id[stk[i]]] = 1;
    }
    int potential = elem_num + popcount(candi);
    if(potential <= ans) return;
    int pivot = lowbit(candi);
    Int smaller_candi = candi & (~linkto[pivot]);
    while(smaller_candi.count() && potential > ans){
      int next = lowbit(smaller_candi);
      candi[next] = !candi[next];
      smaller_candi[next] = !smaller_candi[next];
      potential --;
      if(next == pivot || (smaller_candi & linkto[next
          ]).count()){
        stk[elem_num] = next;
        maxclique(elem_num + 1, candi & linkto[next]);
      }
    }
  }
  int solve(){
    for(int i = 0 ; i < n ; i ++){
      id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
          return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i ++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i ++)
      for(int j = 0 ; j < n ; j ++)
        if(v[i][j]) linkto[di[i]][di[j]] = 1;
    Int cand; cand.reset();
    for(int i = 0 ; i < n ; i ++) cand[i] = 1;
    ans = 1;
    cans.reset(); cans[0] = 1;
    maxclique(0, cand);
    return ans;
```

```
  }
} solver;
```

7.2 MaximalClique 極大團

```
#define N 80
struct MaxClique{ // 0-base
  typedef bitset<N> Int;
  Int lnk[N] , v[N];
  int n;
  void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i ++){
      lnk[i].reset(); v[i].reset();
    }
  }
  void addEdge(int a , int b)
  { v[a][b] = v[b][a] = 1; }
  int ans , stk[N], id[N] , di[N] , deg[N];
  Int cans;
  void dfs(int elem_num, Int candi, Int ex){
    if(candi.none()&&ex.none()){
      cans.reset();
      for(int i = 0 ; i < elem_num ; i ++)
        cans[id[stk[i]]] = 1;
      ans = elem_num; // cans is a maximal clique
      return;
    }
    int pivot = (candi|ex)._Find_first();
    Int smaller_candi = candi & (~lnk[pivot]);
    while(smaller_candi.count()){
      int nxt = smaller_candi._Find_first();
      candi[nxt] = smaller_candi[nxt] = 0;
      ex[nxt] = 1;
      stk[elem_num] = nxt;
      dfs(elem_num+1,candi&lnk[nxt],ex&lnk[nxt]);
    }
  }
  int solve(){
    for(int i = 0 ; i < n ; i ++){
      id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
          return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i ++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i ++)
      for(int j = 0 ; j < n ; j ++)
        if(v[i][j]) lnk[di[i]][di[j]] = 1;
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n,'1')), 0);
    return ans;
  }
} solver;
```

7.3 Strongly Connected Component

```
struct Scc{
  int n, nScc, vst[MXN], bln[MXN];
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
    for (int i=0; i<MXN; i++)
      E[i].clear(), rE[i].clear();
  }
  void addEdge(int u, int v){
    E[u].PB(v); rE[v].PB(u);
  }
  void DFS(int u){
    vst[u]=1;
    for (auto v : E[u]) if (!vst[v]) DFS(v);
    vec.PB(u);
  }
  void rDFS(int u){
    vst[u] = 1; bln[u] = nScc;
    for (auto v : rE[u]) if (!vst[v]) rDFS(v);
  }
  void solve(){
    nScc = 0;
    vec.clear();
    FZ(vst);
    for (int i=0; i<n; i++)
      if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    FZ(vst);
    for (auto v : vec)
      if (!vst[v]){
        rDFS(v); nScc++;
      }
  }
};
```

7.4 Dynamic MST

```
/* Dynamic MST O( Q lg^2 Q )
 (qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
 delete an edge: (i, \infty)
 add an edge: change from \infty to specific value   */
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
  int root=xx; while(a[root]) root=a[root];
  int next; while((next=a[xx])){a[xx]=root; xx=next; }
  return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
  if(Q==1){
    for(int i=1;i<=n;i++) a[i]=0;
    z[ qx[0] ]=qy[0]; tz = z;
    for(int i=0;i<m1;i++) id[i]=i;
    sort(id,id+m1,cmp); int ri,rj;
    for(int i=0;i<m1;i++){
      ri=find(x[id[i]]); rj=find(y[id[i]]);
      if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
    }
    printf("%lld\n",ans);
    return;
  }
  int ri,rj;
  //contract
  kt=0;
  for(int i=1;i<=n;i++) a[i]=0;
  for(int i=0;i<Q;i++){
    ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[
        ri]=rj;
  }
  int tm=0;
  for(int i=0;i<m1;i++) extra[i]=true;
  for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
  for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
  tz=z; sort(id,id+tm,cmp);
  for(int i=0;i<tm;i++){
    ri=find(x[id[i]]); rj=find(y[id[i]]);
    if(ri!=rj){
      a[ri]=rj; ans += z[id[i]];
      kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
  } }
  for(int i=1;i<=n;i++) a[i]=0;
  for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
  int n2=0;
  for(int i=1;i<=n;i++) if(a[i]==0)
  vd[i]=++n2;
  for(int i=1;i<=n;i++) if(a[i])
  vd[i]=vd[find(i)];
  int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
  for(int i=0;i<m1;i++) app[i]=-1;
  for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
    Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
    app[qx[i]]=m2; m2++;
  }
  for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[
      i]]; }
  for(int i=1;i<=n2;i++) a[i]=0;
  for(int i=0;i<tm;i++){
    ri=find(vd[ x[id[i]] ]);  rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
      a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
      Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
  } }
  int mid=Q/2;
```

```
  solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
  solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
  scanf("%d%d",&n,&m);
  for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
  scanf("%d",&Q);
  for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i
      ]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
```

7.5 Maximum General graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
  int to[E],bro[E],head[N],e;
  int lnk[N],vis[N],stp,n;
  void init( int _n ){
    stp = 0; e = 1; n = _n;
    for( int i = 1 ; i <= n ; i ++ )
      lnk[i] = vis[i] = 0;
  }
  void add_edge(int u,int v){
    to[e]=v,bro[e]=head[u],head[u]=e++;
    to[e]=u,bro[e]=head[v],head[v]=e++;
  }
  bool dfs(int x){
    vis[x]=stp;
    for(int i=head[x];i;i=bro[i]){
      int v=to[i];
      if(!lnk[v]){
        lnk[x]=v,lnk[v]=x;
        return true;
      }else if(vis[lnk[v]]<stp){
        int w=lnk[v];
        lnk[x]=v,lnk[v]=x,lnk[w]=0;
        if(dfs(w)){
          return true;
        }
        lnk[w]=v,lnk[v]=w,lnk[x]=0;
      }
    }
    return false;
  }
  int solve(){
    int ans = 0;
    for(int i=1;i<=n;i++)
      if(!lnk[i]){
        stp++; ans += dfs(i);
      }
    return ans;
  }
} graph;
```

7.6 Minimum General Weighted Matching

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match)
  static const int MXN = 105;
  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;
  void init(int _n) {
    n = _n;
    for( int i = 0 ; i < n ; i ++ )
      for( int j = 0 ; j < n ; j ++ )
        edge[ i ][ j ] = 0;
  }
  void add_edge(int u, int v, int w)
  { edge[u][v] = edge[v][u] = w; }
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.PB(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.PB(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
    } } }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  }
  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1;
      match[i+1] = i;
    }
    while (true){
      int found = 0;
      for( int i = 0 ; i < n ; i ++ )
        onstk[ i ] = dis[ i ] = 0;
      for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (SZ(stk)>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
      } } }
      if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

7.7 Minimum Steiner Tree

```
// Minimum Steiner Tree 重要點的mst
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
  int n , dst[V][V] , dp[1 << T][V] , tdst[V];
  void init( int _n ){
    n = _n;
    for( int i = 0 ; i < n ; i ++ ){
      for( int j = 0 ; j < n ; j ++ )
        dst[ i ][ j ] = INF;
      dst[ i ][ i ] = 0;
  } }
  void add_edge( int ui , int vi , int wi ){
    dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
    dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
  }
  void shortest_path(){
    for( int k = 0 ; k < n ; k ++ )
      for( int i = 0 ; i < n ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
          dst[ i ][ j ] = min( dst[ i ][ j ],
                dst[ i ][ k ] + dst[ k ][ j ] );
  }
  int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1 << t ) ; i ++ )
      for( int j = 0 ; j < n ; j ++ )
        dp[ i ][ j ] = INF;
    for( int i = 0 ; i < n ; i ++ )
      dp[ 0 ][ i ] = 0;
    for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
      if( msk == ( msk & (-msk) ) ){
        int who = __lg( msk );
        for( int i = 0 ; i < n ; i ++ )
          dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
        continue;
      }
      for( int i = 0 ; i < n ; i ++ )
        for( int submsk = ( msk - 1 ) & msk ; submsk ;
```

```
                submsk = ( submsk - 1 ) & msk )
            dp[ msk ][ i ] = min( dp[ msk ][ i ],
                                  dp[ submsk ][ i ] +
                                  dp[ msk ^ submsk ][ i ] );
      for( int i = 0 ; i < n ; i ++ ){
        tdst[ i ] = INF;
        for( int j = 0 ; j < n ; j ++ )
          tdst[ i ] = min( tdst[ i ],
                       dp[ msk ][ j ] + dst[ j ][ i ] );
      }
      for( int i = 0 ; i < n ; i ++ )
        dp[ msk ][ i ] = tdst[ i ];
    }
    int ans = INF;
    for( int i = 0 ; i < n ; i ++ )
      ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
    return ans;
} }solver;
```

7.8 BCC based on vertex

```
struct BccVertex {
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {
    n = _n; nScc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void addEdge(int u, int v)
  { E[u].PB(v); E[v].PB(u); }
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
          int z;
          sccv[nScc].clear();
          do {
            z = stk[--top];
            sccv[nScc].PB(z);
          } while (z != v);
          sccv[nScc++].PB(u);
        }
      }else
        low[u] = min(low[u],dfn[v]);
  } }
  vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++)
      dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
      if (dfn[i] == -1) {
        top = 0;
        DFS(i,i);
      }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
  }
}graph;
```

7.9 Min Mean Cycle

```
/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
  struct Edge { int v,u; double c; };
  int n, m, prv[V][V], prve[V][V], vst[V];
  Edge e[E];
  vector<int> edgeID, cycle, rho;
  double d[V][V];
  void init( int _n )
  { n = _n; m = 0; }
  // WARNING: TYPE matters
  void addEdge( int vi , int ui , double ci )
  { e[ m ++ ] = { vi , ui , ci }; }
  void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
      fill(d[i+1], d[i+1]+n, inf);
      for(int j=0; j<m; j++) {
        int v = e[j].v, u = e[j].u;
        if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
          d[i+1][u] = d[i][v]+e[j].c;
          prv[i+1][u] = v;
          prve[i+1][u] = j;
  } } } }
  double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
      double avg=-inf;
      for(int k=0; k<n; k++) {
        if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i
            ])/(n-k));
        else avg=max(avg,inf);
      }
      if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    fill(vst,0); edgeID.clear(); cycle.clear(); rho.
        clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
      vst[st]++;
      edgeID.PB(prve[i][st]);
      rho.PB(st);
    }
    while (vst[st] != 2) {
      if(rho.empty()) return inf;
      int v = rho.back(); rho.pop_back();
      cycle.PB(v);
      vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
} }mmc;
```

7.10 Directed Graph Min Cost Cycle

```
// works in O(N M)
#define INF 100000000000000LL
#define N 5010
#define M 200010
struct edge{
  int to; LL w;
  edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
  LL d; int u, next;
  node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
  vector<edge> g[N], grev[N];
  LL dp[N][N], p[N], d[N], mu;
  bool inq[N];
  int n, bn, bsz, hd[N];
  void b_insert(LL d, int u){
    int i = d/mu;
    if(i >= bn) return;
    b[++bsz] = node(d, u, hd[i]);
    hd[i] = bsz;
  }
  void init( int _n ){
    n = _n;
    for( int i = 1 ; i <= n ; i ++ )
      g[ i ].clear();
  }
  void addEdge( int ai , int bi , LL ci )
  { g[ai].push_back(edge(bi,ci)); }
  LL solve(){
    fill(dp[0], dp[0]+n+1, 0);
    for(int i=1; i<=n; i++){
      fill(dp[i]+1, dp[i]+n+1, INF);
      for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
        for(int k=0; k<(int)g[j].size(); k++)
```

```
        dp[i][g[j][k].to] =min(dp[i][g[j][k].to],
                               dp[i-1][j]+g[j][k].w);
  } }
  mu=INF; LL bunbo=1;
  for(int i=1; i<=n; i++) if(dp[n][i] < INF){
    LL a=-INF, b=1;
    for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
      if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
        a = dp[n][i]-dp[j][i];
        b = n-j;
    } }
    if(mu*b > bunbo*a)
      mu = a, bunbo = b;
  }
  if(mu < 0) return -1; // negative cycle
  if(mu == INF) return INF; // no cycle
  if(mu == 0) return 0;
  for(int i=1; i<=n; i++)
    for(int j=0; j<(int)g[i].size(); j++)
    g[i][j].w *= bunbo;
  memset(p, 0, sizeof(p));
  queue<int> q;
  for(int i=1; i<=n; i++){
    q.push(i);
    inq[i] = true;
  }
  while(!q.empty()){
    int i=q.front(); q.pop(); inq[i]=false;
    for(int j=0; j<(int)g[i].size(); j++){
      if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
        p[g[i][j].to] = p[i]+g[i][j].w-mu;
        if(!inq[g[i][j].to]){
          q.push(g[i][j].to);
          inq[g[i][j].to] = true;
  } } } }
  for(int i=1; i<=n; i++) grev[i].clear();
  for(int i=1; i<=n; i++)
    for(int j=0; j<(int)g[i].size(); j++){
      g[i][j].w += p[i]-p[g[i][j].to];
      grev[g[i][j].to].push_back(edge(i, g[i][j].w));
    }
  LL mldc = n*mu;
  for(int i=1; i<=n; i++){
    bn=mldc/mu, bsz=0;
    memset(hd, 0, sizeof(hd));
    fill(d+i+1, d+n+1, INF);
    b_insert(d[i]=0, i);
    for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
        b[k].next){
      int u = b[k].u;
      LL du = b[k].d;
      if(du > d[u]) continue;
      for(int l=0; l<(int)g[u].size(); l++) if(g[u][l
          ].to > i){
        if(d[g[u][l].to] > du + g[u][l].w){
          d[g[u][l].to] = du + g[u][l].w;
          b_insert(d[g[u][l].to], g[u][l].to);
    } } }
    for(int j=0; j<(int)grev[i].size(); j++) if(grev[
        i][j].to > i)
      mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
  }
  return mldc / bunbo;
} }graph;
```

7.11 K-th Shortest Path

```
// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
  struct nd{
    int u, v; ll d;
    nd(int ui = 0, int vi = 0, ll di = INF)
    { u = ui; v = vi; d = di; }
  };
  struct heap{
    nd* edge; int dep; heap* chd[4];
  };
  static int cmp(heap* a,heap* b)
  { return a->edge->d > b->edge->d; }
  struct node{
    int v; ll d; heap* H; nd* E;
    node(){}
    node(ll _d, int _v, nd* _E)
    { d =_d; v = _v; E = _E; }
    node(heap* _H, ll _d)
    { H = _H; d = _d; }
    friend bool operator<(node a, node b)
    { return a.d > b.d; }
  };
  int n, k, s, t;
  ll dst[ N ];
  nd *nxt[ N ];
  vector<nd*> g[ N ], rg[ N ];
  heap *nullNd, *head[ N ];
  void init( int _n , int _k , int _s , int _t ){
    n = _n; k = _k; s = _s; t = _t;
    for( int i = 1 ; i <= n ; i ++ ){
      g[ i ].clear(); rg[ i ].clear();
      nxt[ i ] = NULL; head[ i ] = NULL;
      dst[ i ] = -1;
  } }
  void addEdge( int ui , int vi , ll di ){
    nd* e = new nd(ui, vi, di);
    g[ ui ].push_back( e );
    rg[ vi ].push_back( e );
  }
  queue<int> dfsQ;
  void dijkstra(){
    while(dfsQ.size()) dfsQ.pop();
    priority_queue<node> Q;
    Q.push(node(0, t, NULL));
    while (!Q.empty()){
      node p = Q.top(); Q.pop();
      if(dst[p.v] != -1) continue;
      dst[ p.v ] = p.d;
      nxt[ p.v ] = p.E;
      dfsQ.push( p.v );
      for(auto e: rg[ p.v ])
        Q.push(node(p.d + e->d, e->u, e));
  } }
  heap* merge(heap* curNd, heap* newNd){
    if(curNd == nullNd) return newNd;
    heap* root = new heap;
    memcpy(root, curNd, sizeof(heap));
    if(newNd->edge->d < curNd->edge->d){
      root->edge = newNd->edge;
      root->chd[2] = newNd->chd[2];
      root->chd[3] = newNd->chd[3];
      newNd->edge = curNd->edge;
      newNd->chd[2] = curNd->chd[2];
      newNd->chd[3] = curNd->chd[3];
    }
    if(root->chd[0]->dep < root->chd[1]->dep)
      root->chd[0] = merge(root->chd[0],newNd);
    else
      root->chd[1] = merge(root->chd[1],newNd);
    root->dep = max(root->chd[0]->dep, root->chd[1]->
        dep) + 1;
    return root;
  }
  vector<heap*> V;
  void build(){
    nullNd = new heap;
    nullNd->dep = 0;
    nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd+4, nullNd);
    while(not dfsQ.empty()){
      int u = dfsQ.front(); dfsQ.pop();
      if(!nxt[ u ]) head[ u ] = nullNd;
      else head[ u ] = head[nxt[ u ]->v];
      V.clear();
      for( auto&& e : g[ u ] ){
        int v = e->v;
        if( dst[ v ] == -1 ) continue;
        e->d += dst[ v ] - dst[ u ];
        if( nxt[ u ] != e ){
          heap* p = new heap;
          fill(p->chd, p->chd+4, nullNd);
          p->dep = 1;
          p->edge = e;
          V.push_back(p);
        } }
      if(V.empty()) continue;
```

```
      make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
      for( size_t i = 0 ; i < V.size() ; i ++ ){
        if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
        else V[i]->chd[2]=nullNd;
        if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
        else V[i]->chd[3]=nullNd;
      }
      head[u] = merge(head[u], V.front());
  } }
  vector<ll> ans;
  void first_K(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;
    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )
      Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
      node p = Q.top(), q; Q.pop();
      ans.push_back( p.d );
      if(head[ p.H->edge->v ] != nullNd){
        q.H = head[ p.H->edge->v ];
        q.d = p.d + q.H->edge->d;
        Q.push(q);
      }
      for( int i = 0 ; i < 4 ; i ++ )
        if( p.H->chd[ i ] != nullNd ){
          q.H = p.H->chd[ i ];
          q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
              edge->d;
          Q.push( q );
  } }   }
  void solve(){ // ans[i] stores the i-th shortest path
    dijkstra();
    build();
    first_K(); // ans.size() might less than k
} }solver;
```

7.12 SPFA

```
bool spfa(){
    deque<int> dq;
    dis[0]=0;
    dq.push_back(0);
    inq[0]=1;
    while(!dq.empty()){
        int u=dq.front();
        dq.pop_front();
        inq[u]=0;
        for(auto i:edge[u]){
            if(dis[i.first]>i.second+dis[u]){
                dis[i.first]=i.second+dis[u];
                len[i.first]=len[u]+1;
                if(len[i.first]>n)  return 1;
                if(inq[i.first])  continue;
                if(!dq.empty()&&dis[dq.front()]>dis[i.
                    first])
                    dq.push_front(i.first);
                else
                    dq.push_back(i.first);
                inq[i.first]=1;
    } } }
    return 0;
}
```

7.13 kruskal

```
struct Edge{
    int u,v,w;
    friend bool operator<(const Edge& lhs,const Edge&
        rhs){
        return lhs.w<rhs.w;
    }
};
vector<Edge> graph;
void kruskal(){
    int sum=0;
    sort(graph.begin(),graph.end());
    for(auto i:graph){
        if(Find(i.u)!=Find(i.v)){
            Union(find(i.u),find(i.v));
            sum+=i.w;
        }
    }
    cout<<sum<<endl;
}
```

7.14 dijkstra

```
void dijkstra(int startPoint,int endPoint){
    priority_queue<pair<ll,int>,vector<pair<ll,int>>,
        greater<pair<ll,int>>> pq;
    v.clear();v.resize(n);
    dis.clear();dis.resize(n,INF);
    dis[startPoint]=0;
    pq.push({dis[startPoint],startPoint});
    //將起點加進去pq裡面
    while(!pq.empty()){
        //當pq還有東西時繼續做
        auto u=pq.top();pq.pop();
        //每次取出離起點最近的點
        if(v[u.second]) continue;
        //如果之前走過代表已經有更短的路經過不用在重新
            走
        v[u.second]=1;
        //設成走過
        for(auto i:edge[u.second]){
            if(dis[i.first]>u.first+i.second){
                //判斷是否可以鬆弛
                dis[i.first]=u.first+i.second;
                pq.push({dis[i.first],i.first});
                //將可鬆弛的路連出去
            }
        }
    }
    cout<<dis[endPoint]<<endl;
}
```

7.15 LCA

```
#define MXN 100005
#define LOG 20
vector<int> edge[MXN];
int anc[MXN][LOG];
int tin[MXN],tout[MXN],ti=0;
void build(int x,int f){
    for(int i=0;i<LOG;i++){
        anc[x][i] = f;
        f = anc[f][i];
    }
}
void dfs(int x,int f){
    tin[x] = ti++;
    build(x,f);
    for(auto i:edge[x]){
        if(i == f){
            continue;
        }
        dfs(i,x);
    }
    tout[x] = ti++;
}
bool isAnc(int x,int y){
    return tin[x] <= tin[y] && tout[x] >= tout[y];
}
int query(int x,int y){
    if(isAnc(x,y))  return x;
    if(isAnc(y,x))  return y;
    for(int i=LOG-1;i>=0;i--){
        if(! isAnc(anc[x][i],y)){
            x = anc[x][i];
        }
    }
    return anc[x][0];
}
```

7.16 Bellman

```
void bellman(int startPoint){
    dis.clear();dis.resize(n,INF);
    dis[startPoint]=0;
    for(int i=1;i<n;i++){    //iteration n-1 times
```

```
        for(auto x:edge){    //relaxation edge
            int a=x.u,b=u.v,c=u.w; //from a to b weight
                 c
            if(dis[a]+c<dis[b]){
                dis[b]=dis[a]+c;
            }
        }
    }
}
```

7.17 差分約束

約束條件 $V_j - V_i \leq W$ 建邊 $V_i -> V_j$ 權重為 W-> bellman-ford or spfa

7.18 eulerPath

```
#define FOR(i,a,b) for(int i=a;i<=b;i++)
int dfs_st[10000500],dfn=0;
int ans[10000500],cnt=0,num=0;
vector<int>G[1000050];
int cur[1000050];
int ind[1000050],out[1000050];
void dfs(int x){
    FOR(i,1,n)sort(G[i].begin(),G[i].end());
    dfs_st[++dfn]=x;
    memset(cur,-1,sizeof(cur));
    while(dfn>0){
        int u=dfs_st[dfn];
        int complete=1;
        for(int i=cur[u]+1;i<G[u].size();i++){
            int v=G[u][i];
            num++;
            dfs_st[++dfn]=v;
            cur[u]=i;
            complete=0;
            break;
        }
        if(complete)ans[++cnt]=u,dfn--;
    }
}
bool check(int &start){
    int l=0,r=0,mid=0;
    FOR(i,1,n){
        if(ind[i]==out[i]+1)l++;
        if(out[i]==ind[i]+1)r++,start=i;
        if(ind[i]==out[i])mid++;
    }
    if(l==1&&r==1&&mid==n-2)return true;
    l=1;
    FOR(i,1,n)if(ind[i]!=out[i])l=0;
    if(l){
        FOR(i,1,n)if(out[i]>0){
            start=i;
            break;
        }
        return true;
    }
    return false;
}
int main(){
    cin>>n>>m;
    FOR(i,1,m){
        int x,y;scanf("%d%d",&x,&y);
        G[x].push_back(y);
        ind[y]++,out[x]++;
    }
    int start=-1,ok=true;
    if(check(start)){
        dfs(start);
        if(num!=m){
            puts("What a shame!");
            return 0;
        }
        for(int i=cnt;i>=1;i--)
            printf("%d ",ans[i]);
        puts("");
    }
    else puts("What a shame!");
}
```

8 String

8.1 PalTree

```
// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴，aba的fail是a
const int MXN = 1000010;
struct PalT{
  int nxt[MXN][26],fail[MXN],len[MXN];
  int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
  int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
  char s[MXN]={-1};
  int newNode(int l,int f){
    len[tot]=l,fail[tot]=f,cnt[tot]=num[tot]=0;
    memset(nxt[tot],0,sizeof(nxt[tot]));
    diff[tot]=(l>0?l-len[f]:0);
    sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
    return tot++;
  }
  int getfail(int x){
    while(s[n-len[x]-1]!=s[n]) x=fail[x];
    return x;
  }
  int getmin(int v){
    dp[v]=fac[n-len[sfail[v]]-diff[v]];
    if(diff[v]==diff[fail[v]])
        dp[v]=min(dp[v],dp[fail[v]]);
    return dp[v]+1;
  }
  int push(){
    int c=s[n]-'a',np=getfail(lst);
    if(!(lst=nxt[np][c])){
      lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
      nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    }
    fac[n]=n;
    for(int v=lst;len[v]>0;v=sfail[v])
        fac[n]=min(fac[n],getmin(v));
    return ++cnt[lst],lst;
  }
  void init(const char *_s){
    tot=lst=n=0;
    newNode(0,1),newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
  }
}palt;
```

8.2 KMP

```
/*
len-failure[k]:
在k結尾的情況下，這個子字串可以由開頭
長度為(len-failure[k])的部分重複出現來表達

failure[k]:
failure[k]為次長相同前綴後綴
如果我們不只想求最多，而且以0-base做為考量
，那可能的長度由大到小會是
failuer[k]、failure[failuer[k]-1]
、failure[failure[failuer[k]-1]-1]..
直到有值為0為止
*/
int failure[MXN];
void KMP(string& t, string& p)
{
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i)
    {
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i)
    {
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1)
        {
            cout << i - p.size() + 1<<" ";
            j = failure[j];
}   }   }
```

8.3 SuffixAutomata

```
// any path start from root forms a substring of S
// occurrence of P : iff SAM can run on input word P
// number of different substring : ds[1]-1
// total length of all different substring : dsl[1]
// max/min length of state i : mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P : cnt[i]
// first occurrence position of P : fp[i]-|P|+1
// all position of P : fp of "dfs from i through rmom"
const int MXM = 1000010;
struct SAM{
  int tot, root, lst, mom[MXM], mx[MXM]; //ind[MXM]
  int nxt[MXM][33]; //cnt[MXM],ds[MXM],dsl[MXM],fp[MXM]
  // bool v[MXM]
  int newNode(){
    int res = ++tot;
    fill(nxt[res], nxt[res]+33, 0);
    mom[res] = mx[res] = 0; //cnt=ds=dsl=fp=v=0
    return res;
  }
  void init(){
    tot = 0;
    root = newNode();
    lst = root;
  }
  void push(int c){
    int p = lst;
    int np = newNode(); //cnt[np]=1
    mx[np] = mx[p]+1; //fp[np]=mx[np]-1
    for(; p && nxt[p][c] == 0; p = mom[p])
      nxt[p][c] = np;
    if(p == 0) mom[np] = root;
    else{
      int q = nxt[p][c];
      if(mx[p]+1 == mx[q]) mom[np] = q;
      else{
        int nq = newNode(); //fp[nq]=fp[q]
        mx[nq] = mx[p]+1;
        for(int i = 0; i < 33; i++)
          nxt[nq][i] = nxt[q][i];
        mom[nq] = mom[q];
        mom[q] = nq;
        mom[np] = nq;
        for(; p && nxt[p][c] == q; p = mom[p])
          nxt[p][c] = nq;
    } }
    lst = np;
  }
  void calc(){
    calc(root);
    iota(ind,ind+tot,1);
    sort(ind,ind+tot,[&](int i,int j){return mx[i]<mx[j
        ];});
    for(int i=tot-1;i>=0;i--)
    cnt[mom[ind[i]]]+=cnt[ind[i]];
  }
  void calc(int x){
    v[x]=ds[x]=1;dsl[x]=0; //rmom[mom[x]].push_back(x);
    for(int i=1;i<=26;i++){
      if(nxt[x][i]){
        if(!v[nxt[x][i]]) calc(nxt[x][i]);
        ds[x]+=ds[nxt[x][i]];
        dsl[x]+=ds[nxt[x][i]]+dsl[nxt[x][i]];
  } } }
  void push(const string& str){
    for(int i = 0; i < str.size() ; i++)
      push(str[i]-'a'+1);
  }
} sam;
```

8.4 Aho-Corasick

```
struct ACautomata{
  struct Node{
    int cnt,i;
    Node *go[26], *fail, *dic;
    Node (){
      cnt = 0; fail = 0; dic=0;
      memset(go,0,sizeof(go));
    }
  }pool[1048576],*root;
  int nMem,n_pattern;
  Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
  }
  void init() {nMem=0;root=new_Node();n_pattern=0;}
  void add(const string &str) { insert(root,str,0); }
  void insert(Node *cur, const string &str, int pos){
    for(int i=pos;i<str.size();i++){
      if(!cur->go[str[i]-'a'])
        cur->go[str[i]-'a'] = new_Node();
      cur=cur->go[str[i]-'a'];
    }
    cur->cnt++; cur->i=n_pattern++;
  }
  void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front(); que.pop();
      for (int i=0; i<26; i++){
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
          fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
          que.push(fr->go[i]);
  } } } }
  void query(string s){
      Node *cur=root;
      for(int i=0;i<(int)s.size();i++){
          while(cur&&!cur->go[s[i]-'a']) cur=cur->fail;
          cur=(cur?cur->go[s[i]-'a']:root);
          if(cur->i>=0) ans[cur->i]++;
          for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
              ans[tmp->i]++;
  } }// ans[i] : number of occurrence of pattern i
}AC;
```

8.5 Z Value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() { //z[i] = lcp(s[1...],s[i...])
  int i,j,left,right;
  left=right=0; z[0]=len;
  for(i=1;i<len;i++) {
    j=max(min(z[i-left],right-i),0);
    for(;i+j<len&&s[i+j]==s[j];j++);
    z[i]=j;
    if(i+z[i]>right) {
      right=i+z[i];
      left=i;
}   }   }
```

8.6 ZValue Palindrome

```
void z_value_pal(char *s,int len,int *z){
  len=(len<<1)+1;
  for(int i=len-1;i>=0;i--)
    s[i]=i&1?s[i>>1]:'@';
  z[0]=1;
  for(int i=1,l=0,r=0;i<len;i++){
    z[i]=i<r?min(z[l+l-i],r-i):1;
    while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
        ++z[i];
    if(i+z[i]>r) l=i,r=i+z[i];
} }
```

8.7 Smallest Rotation

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
  int a = 0, N = s.size(); s += s;
  rep(b,0,N) rep(k,0,N) {
    if(a+k == b || s[a+k] < s[b+k])
      {b += max(0, k-1); break;}
    if(s[a+k] > s[b+k]) {a = b; break;}
  } return a;
}
```

8.8 Cyclic LCS

```
#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
  int i=r+al,j=bl,l=0;
  while(i>r) {
    char dir=pred[i][j];
    if(dir==LU) l++;
    i+=mov[dir][0];
    j+=mov[dir][1];
  }
  return l;
}
inline void reroot(int r) { // r = new base row
  int i=r,j=1;
  while(j<=bl&&pred[i][j]!=LU) j++;
  if(j>bl) return;
  pred[i][j]=L;
  while(i<2*al&&j<=bl) {
    if(pred[i+1][j]==U) {
      i++;
      pred[i][j]=L;
    } else if(j<bl&&pred[i+1][j+1]==LU) {
      i++;
      j++;
      pred[i][j]=L;
    } else {
      j++;
} } }
int cyclic_lcs() {
  // a, b, al, bl should be properly filled
  // note: a WILL be altered in process
  //       -- concatenated after itself
  char tmp[MAXL];
  if(al>bl) {
    swap(al,bl);
    strcpy(tmp,a);
    strcpy(a,b);
    strcpy(b,tmp);
  }
  strcpy(tmp,a);
  strcat(a,tmp);
  // basic lcs
  for(int i=0;i<=2*al;i++) {
    dp[i][0]=0;
    pred[i][0]=U;
  }
  for(int j=0;j<=bl;j++) {
    dp[0][j]=0;
    pred[0][j]=L;
  }
  for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
      if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
      else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
      if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
      else if(a[i-1]==b[j-1]) pred[i][j]=LU;
      else pred[i][j]=U;
  } }
  // do cyclic lcs
  int clcs=0;
  for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
  }
  // recover a
  a[al]='\0';
  return clcs;
}
```

8.9 Rolling hash

```
const ll P1 = 75577;
const ll P2 = 12721;    // 多一個質數 p2
const ll MOD = 998244353;
pair<ll,ll> h1[100005];
void build(const string& s){
    pair<ll,ll> val = make_pair(0,0);
    h1[0] = {0, 0};
    for(int i=0; i<s.size(); i++){
        val.first = (val.first * P1 + s[i]) % MOD;
        val.second = (val.second * P2 + s[i]) % MOD;
        h1[i + 1] = val;
    }
}
```

9 Data Structure

9.1 Segment tree

```
struct seg_tree{
  ll a[MXN],val[MXN*4],tag[MXN*4],NO_TAG=0;
  void push(int i,int l,int r){
    if(tag[i]!=NO_TAG){
      val[i]+=tag[i]; // update by tag
      if(l!=r){
        tag[cl(i)]+=tag[i]; // push
        tag[cr(i)]+=tag[i]; // push
      }
      tag[i]=NO_TAG;
  } }
  void pull(int i,int l,int r){
    int mid=(l+r)>>1;
    push(cl(i),l,mid);push(cr(i),mid+1,r);
    val[i]=max(val[cl(i)],val[cr(i)]); // pull
  }
  void build(int i,int l,int r){
    if(l==r){
      val[i]=a[l]; // set value
      return;
    }
    int mid=(l+r)>>1;
    build(cl(i),l,mid);build(cr(i),mid+1,r);
    pull(i,l,r);
  }
  void update(int i,int l,int r,int ql,int qr,int v){
    push(i,l,r);
    if(ql<=l&&r<=qr){
      tag[i]+=v; // update tag
      return;
    }
    int mid=(l+r)>>1;
    if(ql<=mid) update(cl(i),l,mid,ql,qr,v);
    if(qr>mid) update(cr(i),mid+1,r,ql,qr,v);
    pull(i,l,r);
  }
  ll query(int i,int l,int r,int ql,int qr){
    push(i,l,r);
    if(ql<=l&&r<=qr)
      return val[i]; // update answer
      ll mid=(l+r)>>1,ret=0;
    if(ql<=mid) ret=max(ret,query(cl(i),l,mid,ql,qr));
    if(qr>mid) ret=max(ret,query(cr(i),mid+1,r,ql,qr));
    return ret;
} }tree;
```

9.2 Treap

```
struct Treap{
  int sz , val , pri , tag;
  Treap *l , *r;
  Treap( int _val ){
    val = _val; sz = 1;
    pri = rand(); l = r = NULL; tag = 0;
  }
};
void push( Treap * a ){
  if( a->tag ){
    Treap *swp = a->l; a->l = a->r; a->r = swp;
    int swp2;
    if( a->l ) a->l->tag ^= 1;
    if( a->r ) a->r->tag ^= 1;
    a->tag = 0;
} }
inline int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
  a->sz = Size( a->l ) + Size( a->r ) + 1;
```

```
}
Treap* merge( Treap *a , Treap *b ){
  if( !a || !b ) return a ? a : b;
  if( a->pri > b->pri ){
    push( a );
    a->r = merge( a->r , b );
    pull( a );
    return a;
  }else{
    push( b );
    b->l = merge( a , b->l );
    pull( b );
    return b;
} }
void split_kth( Treap *t , int k, Treap*&a, Treap*&b ){
  if( !t ){ a = b = NULL; return; }
  push( t );
  if( Size( t->l ) + 1 <= k ){
    a = t;
    split_kth( t->r , k - Size( t->l ) - 1 , a->r , b )
        ;
    pull( a );
  }else{
    b = t;
    split_kth( t->l , k , a , b->l );
    pull( b );
} }
void split_key(Treap *t, int k, Treap*&a, Treap*&b){
  if(!t){ a = b = NULL; return; }
  push(t);
  if(k<=t->val){
    b = t;
    split_key(t->l,k,a,b->l);
    pull(b);
  }
  else{
    a = t;
    split_key(t->r,k,a->r,b);
    pull(a);
} }
```

9.3 Link-Cut Tree

```
const int MXN = 100005;
const int MEM = 100005;
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay (int _val=-1) : val(_val), rev(0), size(1)
  { f = ch[0] = ch[1] = &nil; }
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d){
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push(){
    if( !rev ) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev=0;
  }
  void pull(){
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
  splayVec.clear();
  for (Splay *q=x;; q=q->f){
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir())
      rotate(x->f),rotate(x);
    else rotate(x),rotate(x);
  }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
  Splay *q = nil;
  for (;x!=nil;x=x->f){
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void chroot(Splay *x){
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y){
  access(x);
  splay(x);
  chroot(y);
  x->setCh(y, 1);
}
void cut_p(Splay *y) {
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
  chroot(x);
  cut_p(y);
}
Splay* get_root(Splay *x) {
  access(x);
  splay(x);
  for(; x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  x = get_root(x);
  y = get_root(y);
  return x == y;
}
Splay* lca(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  if (x->f == nil) return x;
  else return x->f;
}
```

9.4 Disjoint Set

```
struct DisjointSet {
  int fa[MXN], h[MXN], top;
  struct Node {
    int x, y, fa, h;
    Node(int _x = 0, int _y = 0, int _fa = 0, int _h =
        0)
        : x(_x), y(_y), fa(_fa), h(_h) {}
  } stk[MXN];
  void init(int n) {
```

```
    top = 0;
    for (int i = 1; i <= n; i++) fa[i] = i, h[i] = 0;
  }
  int find(int x) { return x == fa[x] ? x : find(fa[x])
      ; }
  void merge(int u, int v) {
    int x = find(u), y = find(v);
    if (h[x] > h[y]) swap(x, y);
    stk[top++] = Node(x, y, fa[x], h[y]);
    if (h[x] == h[y]) h[y]++;
    fa[x] = y;
  }
  void undo(int k=1) {  //undo k times
    for (int i = 0; i < k; i++) {
      Node &it = stk[--top];
      fa[it.x] = it.fa;
      h[it.y] = it.h;
} } }djs;
```

9.5 Trie

```
struct trie{
    trie *nxt[26];
    int cnt,sz;
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
trie *root = new trie();
void insert(string& s){
    trie *now = root;  // 每次從根結點出發
    for(auto i:s){
        now->sz++;
        if(now->nxt[i-'a'] == NULL){
            now->nxt[i-'a'] = new trie();
        }
        now = now->nxt[i-'a'];  //走到下一個字母
    }
    now->cnt++;
    now->sz++;
}
int query_prefix(string& s){  //查詢有多少前綴為 s
    trie *now = root;    // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL)return 0;
        now = now->nxt[i-'a'];
    }
    return now->sz;
}
int query_count(string& s){  //查詢字串 s 出現次數
    trie *now = root;    // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL)return 0;
        now = now->nxt[i-'a'];
    }
    return now->cnt;
}
```

10 DP

10.1 SOS dp

```
for(int i = 0; i<(1<<N); ++i)
  F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<
    N); ++mask){
  if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
}
```

10.2 subsum

```
bitset<MXN> dp;
int n, x;
cin >> n;
for(int i = 0; i < n; ++i){
  cin >> x;
  dp <<= dp | x;
}
```

10.3 knapsack

```
struct Data{
  int cost, w;
};
int dp[MXN] = {};
Data arr[MXN] = {};
int n, c, w, sum = 0;
cin >> n;
for(int i = 0; i < n; ++i){
  cin >> arr[i].cost >> arr[i].w;
  sum += arr[i].w;
}
dp[0] = 0;
for(int i = 0; i < n; ++i){
  for(int j = sum; j >= arr[i].w; --j){
    dp[j] = max(dp[j], dp[j - arr[i].w])
  }
}
```

10.4 knapsack unlimited

```
int cost[N], weight[N];
int c[W + 1];
void knapsack(int n, int w){
    for (int i=0; i<n; ++i)
        for (int j = weight[i]; j <= w; ++j)
            c[j] = max(c[j], c[j - weight[i]] + cost[i
                ]);
}
```

10.5 knapsack limited

```
dp[0] = 1;
for(int i = 1; i <= MXN; ++i){
  int tmp = i * w[i], now = i;
  while(tmp){
    tmp -= now;
    for(int j = now j <= sum; ++j){
      dp[j] |= dp[j - now];
    }
    now <<= 1;
    if(now > tmp) now = tmp;
  }
}
```

10.6 DP on dag

```
int dp[MXN];
bool v[MXN];

void dfs(int x){
    if(v[x])    return dp[x];
    v[x] = 1;
    dp[x] = 0;
    for(auto i:edge[x]){
        dp[x] = max(dp[x],dfs(i) + 1);
    }
    return dp[x];
}

int main(){

    for(int i=1;i<=n;i++){
        if(!v[i])   dfs(i);
    }
}
```

10.7 LIS

```
int LIS(vector<int>& s)
{
    if (s.size() == 0) return 0;
    vector<int> v;
    v.push_back(s[0]);
    for (int i=1; i<s.size(); ++i){
        int n = s[i];
        if (n > v.back())v.push_back(n);
        else *lower_bound(v.begin(), v.end(), n) = n;
    }
    return v.size();
}
```

10.8 Matrix fast power

```
#define MOD 1'000'000'007
#define ll long long
vector<vector<ll>> operator*(const vector<vector<ll>>&
    lhs,const vector<vector<ll>>& rhs){
    vector<vector<ll>> ret(lhs.size(),vector<ll>(rhs
        [0].size(),0));
    for(int i=0;i<lhs.size();i++){
        for(int j=0;j<rhs[0].size();j++){
            for(int k=0;k<rhs.size();k++){
                ret[i][j] += lhs[i][k] * rhs[k][j] %
                    MOD;
                ret[i][j] %= MOD;
    }   }   }
    return ret;
}
vector<vector<ll>> init_value={{1},{0}};  //第0,1項
vector<vector<ll>> base={{1,1},{1,0}};    //費式數列轉
    移式
vector<vector<ll>> matrix={{1,0},{0,1}};  //單位矩陣
while(y){// x^y
    if(y&1) matrix = matrix * base;
    base = base * base;
    y >>= 1;
}
matrix = matrix * init_value;
cout<< matrix[0][0] << endl;
```

11 Others

11.1 Closest Pair

```
// 最近點對
#include<bits/stdc++.h>
#define double long double
#define INF 1'000'000'000
#define MXN 3'000'005
using namespace std;
struct pt{
    double x,y;
}arr[MXN],sorted_y[MXN];
struct cmp_x{
    bool operator ()(const pt& lhs,const pt& rhs){
        if(lhs.x != rhs.x)  return lhs.x < rhs.x;
        return lhs.y < rhs.y;
    }
};
struct cmp_y{
    bool operator ()(const pt& lhs,const pt& rhs){
        if(lhs.y != rhs.y)  return lhs.y < rhs.y;
        return lhs.x < rhs.x;
    }
};
int n;
double ans=INF;
pt Left[MXN],Right[MXN],tmp[MXN];
inline double dist(int i,int j){
    return sqrt((arr[i].x-arr[j].x)*(arr[i].x-arr[j].x)
        +(arr[i].y-arr[j].y)*(arr[i].y-arr[j].y));
}
inline double dist(const pt& lhs,const pt& rhs){
    return sqrt((lhs.x-rhs.x)*(lhs.x-rhs.x)+(lhs.y-rhs.
        y)*(lhs.y-rhs.y));
}
void merge(int l,int r){
    int mid = (l+r)/2;
    int l_num=0,r_num=0;
    for(int i=l;i<=mid;i++){
        if(abs(sorted_y[i].x-arr[mid].x)<ans){
            Left[l_num] = sorted_y[i];
            l_num++;
        }
    }
    for(int i=mid+1;i<=r;i++){
        if(abs(sorted_y[i].x-arr[mid].x)<ans){
            Right[r_num] = sorted_y[i];
            r_num++;
        }
    }
    for(int i=0,j=0;i<l_num;i++){
        while(j+1 < r_num && Left[i].y > Right[j].y){
            j++;
        }
        for(int k=j;k<r_num && abs(Left[i].y-Right[k].y
            )<ans;k++){
            ans = min(ans, dist(Left[i],Right[k]));
        }
    }
    for(int i=0,j=0;i<r_num;i++){
        while(j+1 < l_num && Right[i].y > Left[j].y){
            j++;
        }
        for(int k=j;k<l_num && abs(Right[i].y-Left[k].y
            )<ans;k++){
            ans = min(ans, dist(Right[i],Left[k]));
        }
    }
    int l_id=l,r_id=mid+1,id=0; //merge sort
    while(l_id<=mid && r_id<=r){
        if(sorted_y[l_id].y < sorted_y[r_id].y){
            tmp[id++] = sorted_y[l_id++];
        }
        else{
            tmp[id++] = sorted_y[r_id++];
        }
    }
    while(l_id<=mid)    tmp[id++]=sorted_y[l_id++];
    while(r_id<=r)      tmp[id++]=sorted_y[r_id++];
    for(int i=l;i<=r;i++)   sorted_y[i] = tmp[i-l];
}
void closest_pair(int l,int r){
    if(r-l<=3){
        for(int i=l;i<=r;i++){
            for(int j=i+1;j<=r;j++){
                ans=min(ans,dist(i,j));
                sort(sorted_y+l,sorted_y+r+1,cmp_y());
            }
        }
        return;
    }
    int mid = (l+r)/2;
    closest_pair(l,mid);
    closest_pair(mid+1,r);
    merge(l,r);
}
int main(){
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n;
    for(int i=0;i<n;i++){
        cin>>arr[i].x>>arr[i].y;
        sorted_y[i] = arr[i];
    }
    sort(arr,arr+n,cmp_x());
    closest_pair(0,n-1);
    cout<<fixed<<setprecision(4)<<ans<<endl;
    return 0;
}
```

11.2 Reverse Pair

```
int ans;
vector<int>arr;
vector<int>temparr;
void msort(int s,int t) {
  if(s==t) return ;
  int mid=(s+t)>>1;
  msort(s,mid),msort(mid+1,t);
  int i=s,j=mid+1,k=s;
  while(i<=mid && j<=t) {
    if(arr[i]<=arr[j]) temparr[k]=arr[i],k++,i++;
    else temparr[k]=arr[j],k++,j++,ans+=mid-i+1;
  }
  while(i<=mid) temparr[k]=arr[i],k++,i++;
  while(j<=t) temparr[k]=arr[j],k++,j++;
  for(int i=s;i<=t;i++) arr[i]=temparr[i];
  return ;
}
```