E20 - CALORIMETRIA E DILATAZIONE DEI SOLIDI

1) $m = 2g$ (Hg)
$h_1 = 5\,cm$
$T_1 = 0\,°C = 273{,}15\,K$
$p_1 = p_{atm} = 1{,}013 \cdot 10^5\,Pa$
$h_2 = 10\,cm$
$T_2 = 100\,°C = 373{,}15\,K$

? costanti termometriche
? $T_3 \mid h_3 = 8\,cm$
? $Q_{14} \mid h_4 = 6{,}9\,cm$

$c_{Hg} = 3{,}3 \cdot 10^{-2}\,cal/g\,°C$

$$T(h) = a \cdot h + b$$

$$\begin{cases} T_1 = a h_1 + b \\ T_2 = a h_2 + b \end{cases} \Rightarrow \begin{cases} 0 = a \cdot 5 + b \\ 100 = a \cdot 10 + b \end{cases}$$

$$\Rightarrow \begin{cases} b = -5a \\ 100 = 10a - 5a \end{cases} \Rightarrow \begin{cases} b = -5a = -100 \\ a = 20 \end{cases}$$

$$\begin{cases} a = 20\,°C/\cancel{m}\ \text{cm!!} \\ b = -100\,°C \end{cases} \rightarrow T(h) = 20h - 100$$

$$\Rightarrow T_3 = 20 \cdot 8 - 100 = 60\,°C = 333{,}15\,K$$

$$\Rightarrow Q = m c_{Hg} \Delta T = m c_{Hg} (T_4 - T_1) = 2 \cdot 3{,}3 \cdot 10^{-2} (20 \cdot 6{,}9 - 100)$$
$$= 2{,}508\,cal = 10{,}49\,J$$

2) $V_{H_2O} = 500\,cm^3 = 0{,}5\,dm^3$
$T_i = 20\,°C$
$T_G = -5\,°C$
? $m_{G,min} \mid T_f = 0\,°C$

$\lambda_F = 80\,cal/g$
$c_G = 0{,}49\,cal/g\,°C$

$\rho_{H_2O} = 10^3\,kg/m^3$
$= 10^3\,g/dm^3$
$= 1\,g/cm^3$

SISTEMA ISOLATO : $\Delta Q_{SIST} = 0$

$$\Rightarrow Q_{ASS,G} + Q_{CED,H_2O} + Q_{cambio\,fase} = 0$$

$$\Rightarrow m_G c_G (T_f - T_G) + m_{H_2O} c_{H_2O} (T_f - T_i) + m_G \lambda_G = 0$$

$$\Rightarrow m_G [c_G (T_f - T_G) + \lambda_G] = m_{H_2O} c_{H_2O} (T_i - T_f)$$

$$\Rightarrow m_G = \frac{500 \cdot 1 \cdot 20}{0{,}49 \cdot 5 + 80} = 121{,}3\,g$$

! in Kelvin

3) $m_1 = 1\,kg$
$m_2 = 2\,kg$
$T_1 = 10\,°C = 283{,}15\,K$
$T_2 = 50\,°C = 323{,}15\,K$
$c(T) = aT^2 \rightarrow$ non è cost

? T_{eq}

$$Q = \int_{T_i}^{T_f} c(T)\, m\, dT = \int_{T_i}^{T_f} aT^2 m\, dT = am \left[\frac{T^3}{3}\right]_{T_i}^{T_f} = am \frac{T_f^3 - T_i^3}{3}$$

SISTEMA ISOLATO : $Q_1 + Q_2 = 0 \Rightarrow Q_1 = -Q_2$

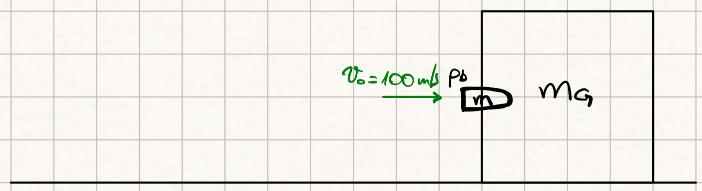
$$\Rightarrow \cancel{a} m_1 \frac{T_{eq}^3 - T_1^3}{\cancel{3}} = \cancel{a} m_2 \frac{T_2^3 - T_{eq}^3}{\cancel{3}}$$

$$\Rightarrow T_{eq} = \sqrt[3]{\frac{m_1 T_1^3 + m_2 T_2^3}{m_1 + m_2}} = \sqrt[3]{\frac{1 \cdot \cancel{10}^3 + 2 \cdot \cancel{50}^3}{1 + 2}} = 310{,}9\,K$$

$283{,}15^3$ $323{,}15^3$

$= 37{,}8\,°C$

4)



$m = 50\,g = 0{,}05\ kg$

$T_{pb} = 20\ °C = 293{,}15\ K$

$v_0 = 100\ m/s$

$m_G = 500\,g = 0{,}5\ kg$

$\lambda_G = 3{,}3 \cdot 10^5\ J/kg$

$c_{pb} = 130\ J/kg\,°C$

? m_g fusa

URTO ANAELASTICO: $m v_0 = (m+m_G) v$

$\rightarrow v = \dfrac{m v_0}{m+m_G}$

dissipazione di energia: $E_{diss} = \Delta E_k = -\frac{1}{2} m v_0^2 + \frac{1}{2}(m+m_G) v^2 = -\frac{1}{2} m v_0^2 + \frac{1}{2}\cancel{(m+m_G)} \dfrac{m^2 v_0^2}{(m+m_G)^{\cancel{2}}}$

$$= \frac{1}{2} m v_0^2 \left(-1 + \frac{m}{m+m_G}\right) = \frac{1}{2} m v_0^2 \left(\frac{-m_G}{m+m_G}\right) = -\frac{1}{2} \frac{m m_G}{m+m_G} v_0^2$$

SISTEMA ISOLATO: $Q_{SIST} = 0$

$\rightarrow \underset{E_{diss}}{\underset{\shortparallel}{Q_{URTO}}} + Q_{pb} + Q_{ghiaccio\ fuso} = 0 \quad \Rightarrow -\frac{1}{2} \frac{m m_G}{m+m_G} v_0^2 + m c_{pb}(T_f - T_i) + \Delta m_G \lambda = 0$

$$\rightarrow \Delta m_G = \frac{1}{\lambda}\left[\frac{1}{2}\frac{m m_G}{m+m_G} v_0^2 + m c_{pb}(T_i - T_f)\right] = \frac{1}{3{,}3 \cdot 10^5}\left[\frac{1}{2}\,\frac{0{,}05 \cdot 0{,}5}{0{,}55}\,10^4 + 0{,}05 \cdot 130 \cdot 20\right] =$$

$$= 1{,}08 \cdot 10^{-3}\ kg = 1{,}08\ g$$

5) $\beta = 182 \cdot 10^{-6}\ °C^{-1}$

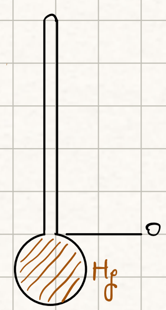
$d_1 = 7 \cdot 10^{-3}\, m = 7\, mm$

$d_2 = 0{,}5 \cdot 10^{-3}\, m = 0{,}5\, mm$

$\alpha = 8 \cdot 10^{-6}\ °C^{-1}$

a) ? h | $\Delta T = 25\,°C$ (no α)

b) ? h' (si α)



a) $\beta = \dfrac{1}{V}\dfrac{\Delta V}{\Delta T} = \dfrac{1}{\frac{4}{3}\left(\frac{d_1}{2}\right)^3 \pi} \, \dfrac{\left(\frac{d_2}{2}\right)^2 \pi \cdot h}{\Delta T}$

$\rightarrow h = \dfrac{\beta \Delta T \frac{d_1^3}{6} \cancel{\pi} \cdot 4^2}{d_2^2 \cancel{\pi}} = 4{,}16\ mm$

b) $\alpha = \dfrac{1}{d_1}\dfrac{\Delta d}{\Delta T} = \dfrac{1}{d_1}\dfrac{(d_1' - d_1)}{\Delta T} \rightarrow d_1' = \alpha d_1 \Delta T + d_1 =$

$= 7{,}0016\ mm$

$d_2' = \alpha d_2 \Delta T + d_2 = 0{,}5001\ mm$

$V_{Hg} = \dfrac{4}{3}\dfrac{d_1^3}{8}\pi + \dfrac{d_2^2}{4}\pi \cdot h \quad ; \quad V'_{Hg} = \dfrac{4}{3}\dfrac{d_1'^3}{8}\pi + \dfrac{d_2'^2}{4}\pi \cdot h'$

$\Rightarrow V_{Hg} = V'_{Hg}$: $h' = \left(\dfrac{\cancel{4}^2}{3}\dfrac{d_1^3}{\cancel{8}_2}\cancel{\pi} + \dfrac{d_2^2}{\cancel{4}}\cancel{\pi} \cdot h - \dfrac{\cancel{4}^2}{3}\dfrac{d_1'^3}{\cancel{8}_2}\cancel{\pi}\right)\dfrac{\cancel{4}}{d_2'^2 \cancel{\pi}} = \left(\dfrac{2}{3} d_1^3 + d_2^2 h - \dfrac{2}{3} d_1'^3\right)\dfrac{1}{d_2'^2} = 3{,}53\ mm$

6) $\alpha_{acc} = 11 \cdot 10^{-6}\ °C^{-1}$
$\tau = 1 s$
$T = 20\ °C$
$\Delta T = 5\ °C$

? $\Delta\tau$
? $\Delta t_{sett.}$

$$\alpha_{acc} = \frac{1}{\ell}\frac{\Delta\ell}{\Delta T} \Rightarrow \Delta\ell = \alpha\Delta T\ell \quad ; \quad \tau = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Delta\tau = \frac{2\pi}{\sqrt{g}}\left(\sqrt{\ell+\alpha\Delta T\ell} - \sqrt{\ell}\right) = \frac{2\pi}{\sqrt{g}}\left(\sqrt{(1+\alpha\Delta T)\ell} - \sqrt{\ell}\right)$$

$$= \frac{2\pi}{\sqrt{g}}\sqrt{\ell}\left(\sqrt{1+\alpha\Delta T} - 1\right) = \tau\left(\sqrt{1+\alpha\Delta T} - 1\right) = 2,75 \cdot 10^{-5} s$$

$$\Delta t_{sett} = 16,632 s$$

E21 — GAS PERFETTI

1) $T = 30 K$
$\eta = 10^{25}$ molecole $/m^3$

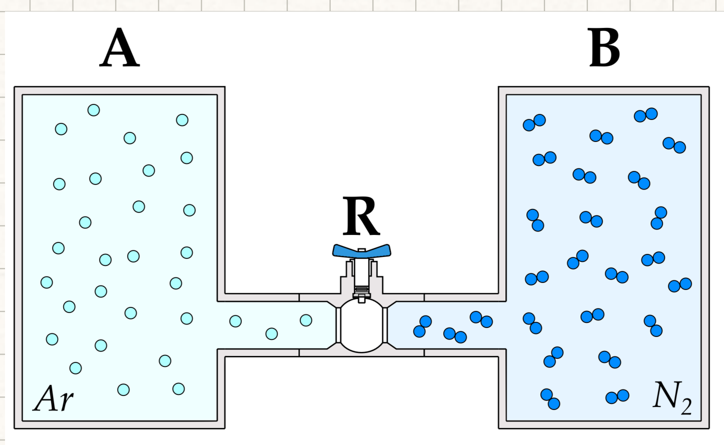
? p

EQUAZIONE DI STATO DEL GAS IDEALE:

$$pV = nRT \Rightarrow p = \frac{n}{V}RT \quad ; \quad \frac{n}{V} = \frac{\eta}{N_A}$$

$$\Rightarrow p = \frac{\eta}{N_A}RT = \frac{10^{\cancel{25}\,2}}{6,022 \cdot \cancel{10^{23}}}\, 8,31 \cdot 30 = 4138,8\ Pa = 4,1388\ kPa$$

2)



$V_A = 400\ cm^3 = 0,4 \cdot 10^{-3} m^3$
$n_{Ar} = 0,06$ mol (Ar)
$V_B = 550\ cm^3$ (N_2) $= 0,55 \cdot 10^{-3} m^3$
$p_{N_2} = 450$ torr $= 59980$ Pa $= 59,98$ kPa
$T_B = 23\ °C = 296,15 K$

? p'_{Ar} ? p'_{N_2} ($T =$ cost)

$$1 \text{ torr} = 1 \text{ mmHg} = \frac{1}{760} \text{ atm} = \frac{1}{760} 1,013 \cdot 10^5 \text{ Pa}$$

ISOTERMA $\Rightarrow pV =$ cost

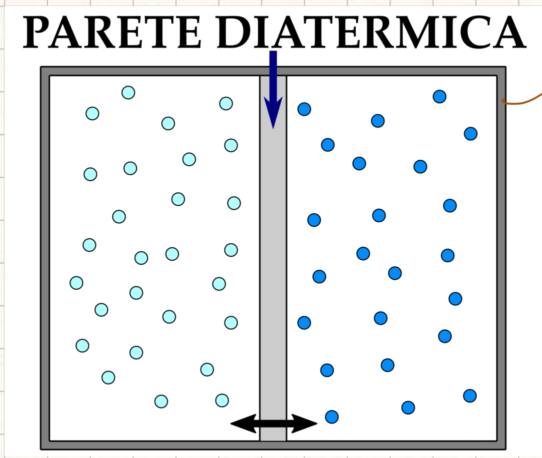
$$\Rightarrow p_{Ar}V_A = p'_{Ar}(V_A+V_B) \Rightarrow p'_{Ar} = p_{Ar}\frac{V_A}{V_A+V_B} = \frac{n_{Ar}RT}{\cancel{V_A}}\frac{\cancel{V_A}}{V_A+V_B} = \frac{0,06 \cdot 8,31 \cdot 296,15}{(0,4+0,55)10^{-3}} = 1,53 \text{ atm}$$

$$\Rightarrow p_{N_2}V_B = p'_{N_2}(V_A+V_B) \Rightarrow p'_{N_2} = p_{N_2}\frac{V_B}{V_A+V_B} = 0,34 \text{ atm}$$

oppure LEGGE DI DALTON: $p_{tot} = \sum_{i=1}^{N} p_i$ $(= p_{Ar} + p_{N_2})$

$$\Rightarrow \begin{cases} p_i V = n_i RT \\ p_{tot}V = n_{tot}RT \end{cases} \Rightarrow \frac{p_i}{p_{tot}} = \frac{n_i}{n_{tot}} \Rightarrow p_i = \frac{n_i}{n_{tot}} p_{tot}$$

3)



→ adiabatica

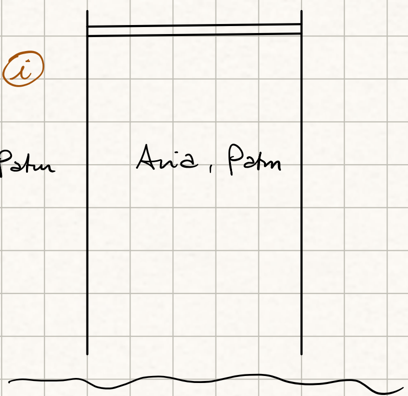
? relaz. tra ρ_1, ρ_2, M_1, M_2

$$\begin{cases} p_1 V_1 = n_1 R T_{eq} \\ p_2 V_2 = n_2 R T_{eq} \end{cases} \Rightarrow \begin{cases} p_1 \dfrac{\cancel{n_1} M_1}{\rho_1} = \cancel{n_1} R T_{eq} \\ p_2 \dfrac{\cancel{n_2} M_2}{\rho_2} = \cancel{n_2} R T_{eq} \end{cases}$$

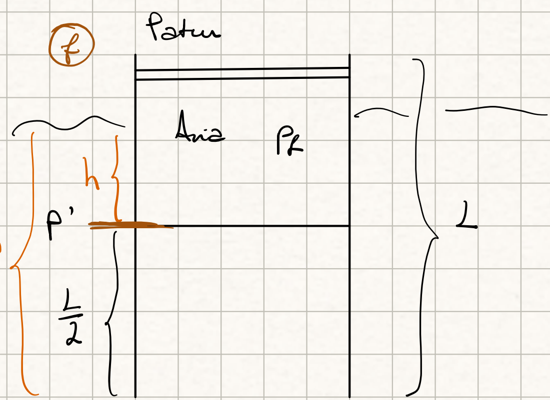
EQUILIBRIO : $p_1 = p_2 = p_{est}$; $T_1 = T_2 = T_{eq}$

$$\Rightarrow \frac{M_1}{\rho_1} = \frac{M_2}{\rho_2}$$

4) (i)



(f)



$L = 4m$
T uniforme

? h'

(i) $pV = nRT \Rightarrow n = \dfrac{p_{atm}(r^2\pi)L}{RT}$

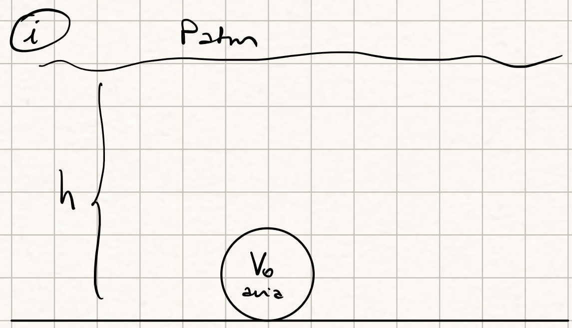
(f) $p_f V_f = nRT \Rightarrow p_f = \dfrac{nRT}{V_f} = \dfrac{p_{atm}\cancel{(r^2\pi)}\cancel{L}\,\cancel{RT}}{\cancel{RT}\cancel{(r^2\pi)}\frac{\cancel{L}}{2}} = 2p_{atm}$

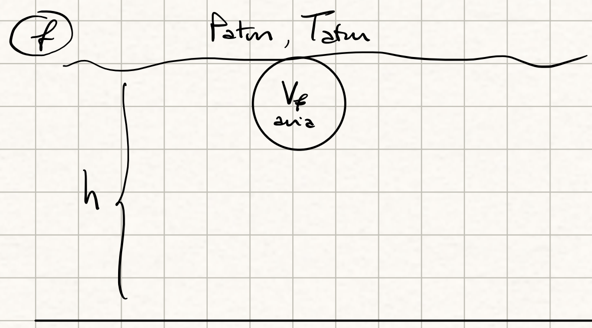
STEVINO : $p' = p_{atm} + \rho_{H_2O}\, g\, h$

$\Rightarrow p_f = p'$: $\quad 2p_{atm} = p_{atm} + \rho_{H_2O}\, g\, h \Rightarrow 2p_{atm} = p_{atm} + \rho_{H_2O}\, g \left(h' - \frac{L}{2}\right)$

$$\Rightarrow h' = \frac{L}{2} + \frac{p_{atm}}{\rho_{H_2O}\, g} = 2 + \frac{1{,}013 \cdot 10^{\cancel{5}\,2}}{\cancel{10^3} \cdot 9{,}8} = 12{,}33\ m$$

5)





$V_0 = 1\,dm^3$; $h = 10\,m$; $T_i = 6{,}8\,°C = cost$; $T_{atm} = 21{,}8\,°C$

? n ? V_f

i $\quad pV_0 = nRT_i \quad ; \quad p = \rho_{H_2O}\,g\,h + p_{atm}$

$$\rightarrow n = \frac{(\rho_{H_2O}gh + p_{atm})V_0}{RT_i} = \frac{(10^3 \cdot 9{,}8 \cdot 10 + 1{,}013 \cdot 10^5)10^{-3}}{8{,}31 \cdot (273{,}15 + 6{,}8)} = 0{,}0856\ mol = 85{,}6 \cdot 10^{-3}\ mol$$

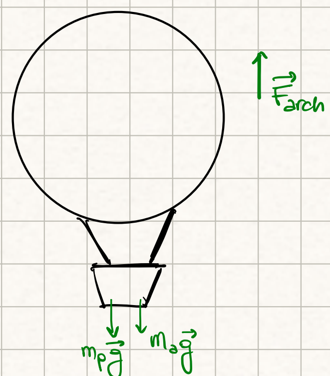
f $\quad$ $$p_{atm}V_f = nRT_f \Rightarrow V_f = \frac{nRT_f}{p_{atm}} = \frac{0{,}0856 \cdot 8{,}31 \cdot (273{,}15 + 21{,}8)}{1{,}013 \cdot 10^5} = 2{,}07 \cdot 10^{-3}\ m^3 = 2{,}07\ dm^3$$

6) $m_p = 200\,kg$
$V_p = 400\,m^3$
$T_{est} = 10\,°C$
$\rho_{aria} = 1{,}25\,kg/m^3$

20% O_2 ; 80% N_2

? T_{int} | decollo

p_{atm}, T_{est}



$p_{int} \geq p_{atm}$

DECOLLO: $\quad F_A \geq m_p g + m_a g = (m_p + m_a)g$

$F_A = \rho_{aria} V_p \cancel{g} \geq (m_p + m_a)\cancel{g} \quad \rightarrow$ LIMITE: $\rho_{aria} V_p = m_p + m_a$

$\Rightarrow m_a = \rho_{aria} V_p - m_p = 1{,}25 \cdot 400 - 200 = 300\ kg$

$$n = \frac{m_{a}}{0{,}2 \cdot M_{O_2} + 0{,}8 \cdot M_{N_2}} = \frac{3 \cdot 10^5}{0{,}2 \cdot 32 + 0{,}8 \cdot 28} = 1{,}04 \cdot 10^4\ mol$$

$$p_{atm}V_p = nRT_f \Rightarrow T_f = \frac{p_{atm}V_p}{nR} = \frac{1{,}013 \cdot 10^{\cancel{5}} \cdot 4 \cdot 10^2}{1{,}04 \cdot \cancel{10^4} \cdot 8{,}31} = 468\ K$$

E22 - Trasformazioni termodinamiche

1) $T_1 = 300K$
$n = 1$ mol (MONOATOMICO)
ESPANSIONE ADIAB. REV.
$V_1 = 1m^3$
$V_2 = 2m^3$

$?T_2 \quad ?L_{12}$

ADIAB $\Rightarrow Q = 0 \Rightarrow \Delta U = -L$

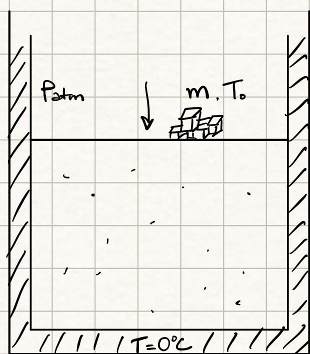
$\rightarrow TV^{\gamma-1} = cost$; monoatomico: $\gamma = \frac{5}{3}$

$$T_1 V_1^{\frac{5}{3}-1} = T_2 V_2^{\frac{5}{3}-1} \Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\frac{2}{3}} = 300\left(\frac{1}{2}\right)^{\frac{2}{3}} = 189K$$

$$L_{12} = -\Delta U_{12} = -nc_v(T_2 - T_1) = nc_v(T_1 - T_2) = 1 \cdot \frac{3}{2}R(300-189)$$

$$= \frac{3}{2} 8{,}31 \cdot 111 = 1383{,}6 \text{ J}$$

2) $n = 10$ mol
COMPRESSIONE ISOTERMICA
REVERSIBILE
$V_i = 1m^3$
recipiente adiabatico
$m = 0{,}1$ kg
$T_0 = 0°C = 273{,}15K$
$\lambda_f = 80$ kcal/kg $= 334880$ J/kg



$?V_f$ | completa fusione

- $T = cost \rightarrow pV = cost$; $\Delta U = 0$
- $Q = 0$
  $\hookrightarrow Q_{Gas} + Q_{Sorb} = 0$
  Q_{Gas}: cede <0 ; Q_{Sorb}: assorbe >0

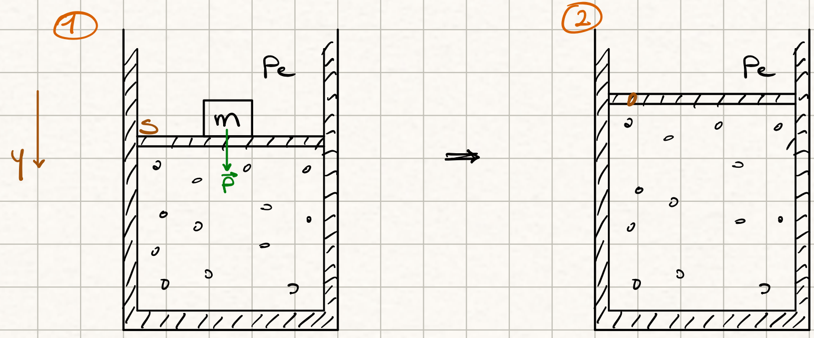
$$Q_G = \underbrace{\Delta U}_{0\ (\Delta T = 0)} + L^{<0\ (\text{compiuto sul gas})} \rightarrow L = Q_G = -Q_s = -m\lambda_f = -0{,}1 \cdot 334880 = -33488 \text{ J}$$

$$L = \int p\,dV \quad ; \quad p_i V_i = p_f V_f$$

$$= \int \frac{nRT}{V} dV = nRT\int\frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right) = -m\lambda_f$$

$$\rightarrow V_f = V_i e^{-\frac{m\lambda_F}{nRT}} = 1 e^{-\frac{Q_s}{10 \cdot 8{,}31 \cdot 273{,}15}} = 0{,}23 \text{ m}^3$$

3) $n = 2$ mol (mono)
adiabatico
$S = 10\ cm^2 = 10^{-3}\ m^2$
$m = 20$ kg
equilibrio
$T_1 = 100$ K
$p_e = 10^5$ Pa
? V_1
? T_2



① $Q = 0$

equilibrio: - meccanico: $F_{tot} = 0 \Rightarrow mg - p_1 S + p_e S = 0$

$$\Rightarrow p_1 = \frac{mg + p_e S}{S} = \frac{mg}{S} + p_e = \frac{20 \cdot 9{,}8}{10^{-3}} + 10^5 = 2{,}96 \cdot 10^5\ Pa$$

$$p_1 V_1 = nRT_1 \Rightarrow V_1 = \frac{nRT_1}{p_1} = \frac{2 \cdot 8{,}31 \cdot \cancel{10^2}}{2{,}96 \cdot \cancel{10^5}\,^{3}} = 5{,}6 \cdot 10^{-3}\ m^3$$

② $Q = 0$; $\Delta U = -L$

$$\left.\begin{aligned} \Delta U &= nc_v (T_2 - T_1) \\ \delta L &= p_e\, dV \Rightarrow L = p_e (V_2 - V_1) \end{aligned}\right\} \quad nc_v (T_1 - T_2) = p_e (V_2 - V_1)$$

$$\Rightarrow nc_v (T_1 - T_2) = p_e \left(\frac{nRT_2}{p_e} - V_1 \right) \Rightarrow nc_v T_1 - nc_v T_2 = nRT_2 - p_e V_1$$

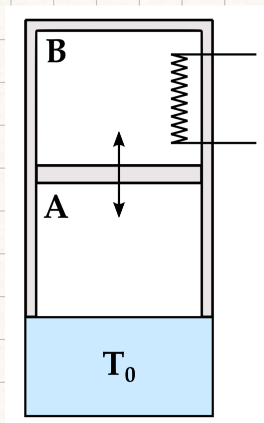
$$\Rightarrow (c_v + R) n T_2 = nc_v T_1 + p_e V_1 \Rightarrow T_2 = \frac{nc_v T_1 + p_e V_1}{(c_v + R) n} = 73{,}47\ K$$

$$T p^{\frac{1-\gamma}{\gamma}} = cost \Rightarrow T_1 p_1^{\frac{1-\gamma}{\gamma}} = T_2 p_e^{\frac{1-\gamma}{\gamma}} \quad ; \quad \gamma = \frac{5}{3}$$

$$\Rightarrow T_2 = T_1 \left(\frac{p_1}{p_e} \right)^{\frac{1-\frac{5}{3}}{\frac{5}{3}}} = 100 \left(\frac{2{,}96 \cdot \cancel{10^5}}{\cancel{10^5}} \right)^{-\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{5}} =$$

perchè viene sbagliato ?

4)



$V_A = V_B$
$n_A = n_B = 1{,}2 \text{ mol (gas ideale)}$
$p_0 = 1 \text{atm} = 1{,}013 \cdot 10^5 \text{ Pa}$
$T_0 = 300 \text{K}$
$p_f = 1{,}3 \text{ atm}$

a) $?V_0$ $(=V_A=V_B)$
b) $?V_{A_f}$, $?V_{B_f}$
c) $?Q_A$
d) $?Q_B$

a) $p_0 V_0 = nRT_0 \Rightarrow V_0 = \dfrac{nRT_0}{p_0} = 2{,}95 \cdot 10^{-2} \text{ m}^3$

b) $p_f = 131690 \text{ Pa}$

$T_A = \text{cost} \Rightarrow p_f V_{f,A} = p_0 V_0 \rightarrow V_{f,A} = \dfrac{p_0}{p_f} V_0 = 2{,}27 \cdot 10^{-2} \text{ m}^3$

$V_{f,B} = 2 \cdot V_0 - V_{f,A} = 3{,}63 \cdot 10^{-2} \text{ m}^3$

c) ISOTERMA : $\Delta U = 0$;

$$Q = L = \int_0^f p\, dV = \int_{V_0}^{V_{f,A}} \frac{nRT}{V}\, dV = nRT_0 \ln\left(\frac{V_{f,A}}{V_0}\right) = -783{,}87 \text{ J}$$

$< 0 \rightarrow$ ceduto

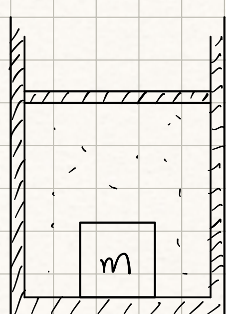
d) $$Q = \Delta U + \underset{\substack{\| \\ -L_A}}{L} = nc_v \Delta T + \int_0^f p\, dV = nc_v\left(\frac{p_{fB} V_{fB}}{nR} - T_0\right) + nRT_0 \ln\left(\frac{V_{f,B}}{V_0}\right)$$

$$= \frac{5}{2} p_{f,B} V_{fB} - nc_v T_0 + nRT_0 \ln\left(\frac{V_{fB}}{V_0}\right) = 5032 \text{ J}$$

$> 0 \Rightarrow$ assorbito

5) $C_m = 1 \text{ J/°C}$
$T_m = 100 \text{ °C}$
Adiabatico
$V_m \sim 0$
$n = 1 \text{ mol (MONO)}$
$T_a = 25 \text{ °C} = 298{,}15 \text{ K}$

a) $?T_f$
b) $?Q_m$ (ceduto)
c) $?T_f'$ (pistone libero)
d) $?L_G$ $\quad ?L_G'$



a) SISTEMA ISOLATO : $Q_{SIST} = 0$
$Q_m + Q_G = 0 \Rightarrow Q_G = -Q_m$

$$\begin{cases} Q_m = C_m \Delta T = C_m (T_f - T_m) \\ Q_G = n c_v \Delta T = n c_v (T_f - T_a) \end{cases}$$

$\Rightarrow n \frac{3}{2} R (T_f - T_a) = C_m (T_m - T_f)$

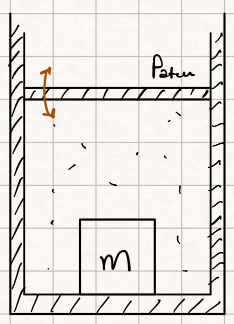
$\Rightarrow \left(\frac{3}{2} nR + C_m\right) T_f = \frac{3}{2} nRT_a + C_m T_m$

$$\Rightarrow T_f = \frac{\frac{3}{2} nRT_a + C_m T_m}{\frac{3}{2} nR + C_m} = \frac{\frac{3}{2} 8{,}31 \cdot 25 + 1 \cdot 100}{\frac{3}{2} 8{,}31 + 1}$$

$= 30{,}57 \text{ °C} = 303{,}7 \text{ K}$

b) $Q_m = C_m \Delta T = 1 \left[\frac{J}{°C}\right] (30{,}57 - 100)\,[°C] = \boxed{-69{,}43\ J}$ calore ceduto

c)



pistone libero → equilibrio meccanico : $p = p_e = p_{atm}$

$Q_G = -Q_m = C_m (T_m - T_f') = n c_p (T_f' - T_a)$

$\Rightarrow (n c_p + C_m) T_f' = C_m T_m + n c_p T_a \Rightarrow$

$\Rightarrow T_f' = \frac{C_m T_m + n c_p T_a}{n c_p + C_m} = \frac{1 \cdot 100 + 1 \cdot \frac{5}{2} 8{,}31 \cdot 25}{1 \cdot \frac{5}{2} 8{,}31 + 1} = 28{,}4\ °C$
$= \boxed{301{,}6\ K}$

d) $L = 0\ J$

$L' = \int p\, dV = p_e \Delta V = \cancel{p_e} \frac{nRT_f' - nRT_a}{\cancel{p_e}} = nR(T_f' - T_a) = \boxed{28{,}67\ J}$

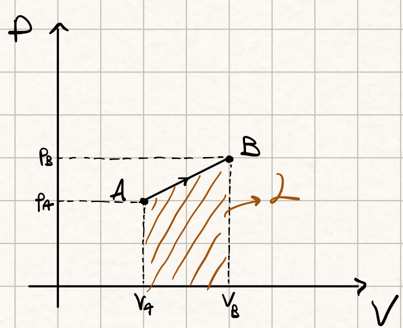
$p_e V_f' = nRT_f' \rightarrow V_f' = \frac{nRT_f'}{p_e}$

$p_e V = nRT_a \rightarrow V = \frac{nRT_a}{p_e}$

6) MONOATOMICO
REVERSIBILE
$A(p_A, V_A)$
$B(p_B, V_B)$
$V_A = \frac{2}{3} V_B$

$p_A = \frac{1}{2} p_B$

a) ? L ? Q
b) ? $c_S(V)$



a) $L = (p_B + p_A) \frac{(V_B - V_A)}{2}$
$= \frac{1}{2}\left(p_B + \frac{1}{2} p_B\right)\left(V_B - \frac{2}{3} V_B\right)$
$= \frac{1}{2} \frac{\cancel{3}}{2} \frac{1}{\cancel{3}} p_B V_B = \boxed{\frac{1}{4} p_B V_B}$

$Q = \Delta U + L$

$\Delta U = n c_v \Delta T$

$\begin{cases} p_A V_A = nRT_A \\ p_B V_B = nRT_B \end{cases} \rightarrow \frac{1}{\cancel{2}} \frac{\cancel{2}}{3} = \frac{T_A}{T_B} \rightarrow T_A = \frac{1}{3} T_B$

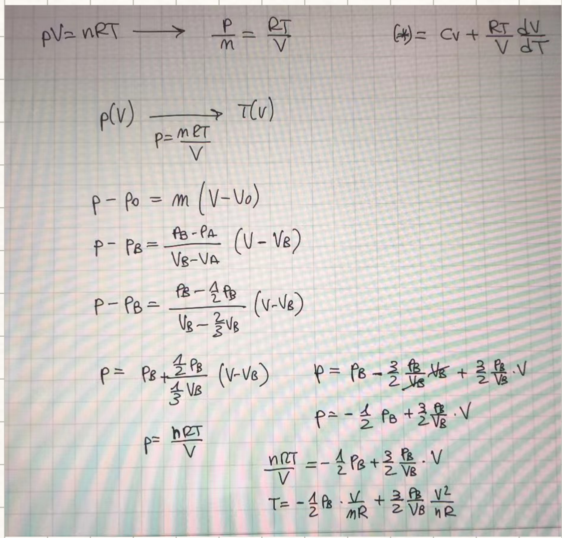
$\Rightarrow n = \frac{p_B V_B}{R T_B}$

$\Delta U = \frac{p_B V_B}{\cancel{R T_B}} \frac{3}{2} \cancel{R} \left(1 - \frac{1}{3}\right) \cancel{T_B} = p_B V_B$

$\Rightarrow Q = p_B V_B + \frac{1}{4} p_B V_B = \frac{5}{4} p_B V_B$

b) $c_{AB} = \frac{1}{n} \frac{\delta Q}{dT} = \frac{1}{n} \frac{dU + \delta L}{dT} = \frac{1}{n} \frac{n c_v dT + p\, dV}{dT} = \frac{1}{\cancel{n}} \cancel{n} c_v + \frac{1}{n} p \frac{dV}{dT} =$

$= c_v + \frac{1}{\cancel{n}} \cancel{p} \frac{\cancel{n} R}{\cancel{p} V} \frac{\cancel{dV}}{\cancel{dV}} = c_v + \frac{R}{V}$??? soluzione: $c_v + R \frac{3V - V_B}{6V - V_B}$



$$pV = nRT \longrightarrow \frac{p}{n} = \frac{RT}{V} \qquad (*) = c_V + \frac{RT}{V}\frac{dV}{dT}$$

$$p(V) \xrightarrow[p = \frac{nRT}{V}]{} T(V)$$

$$p - p_0 = m(V - V_0)$$

$$p - p_B = \frac{p_B - p_A}{V_B - V_A}(V - V_B)$$

$$p - p_B = \frac{p_B - \frac{1}{2}p_B}{V_B - \frac{2}{3}V_B}(V - V_B)$$

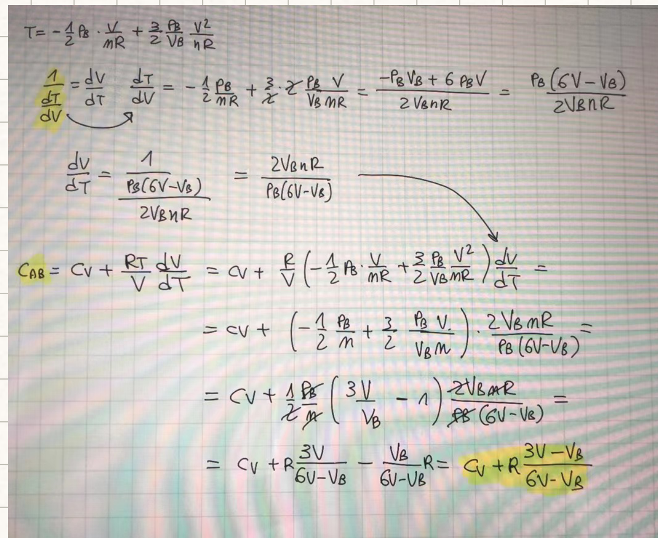
$$p = p_B + \frac{\frac{1}{2}p_B}{\frac{1}{3}V_B}(V - V_B) \qquad p = p_B - \frac{3}{2}\frac{p_B}{\cancel{V_B}}\cancel{V_B} + \frac{3}{2}\frac{p_B}{V_B}\cdot V$$

$$p = -\frac{1}{2}p_B + \frac{3}{2}\frac{p_B}{V_B}\cdot V$$

$$p = \frac{nRT}{V}$$

$$\frac{nRT}{V} = -\frac{1}{2}p_B + \frac{3}{2}\frac{p_B}{V_B}\cdot V$$

$$T = -\frac{1}{2}p_B\cdot\frac{V}{nR} + \frac{3}{2}\frac{p_B}{V_B}\frac{V^2}{nR}$$



$$T = -\frac{1}{2}p_B\cdot\frac{V}{nR} + \frac{3}{2}\frac{p_B}{V_B}\frac{V^2}{nR}$$

$$\frac{1}{\frac{dT}{dV}} = \frac{dV}{dT} \qquad \frac{dT}{dV} = -\frac{1}{2}\frac{p_B}{nR} + \frac{3}{\cancel{2}}\cdot\cancel{2}\frac{p_B}{V_B}\frac{V}{nR} = \frac{-p_BV_B + 6p_BV}{2V_BnR} = \frac{p_B(6V - V_B)}{2V_BnR}$$

$$\frac{dV}{dT} = \frac{1}{\frac{p_B(6V - V_B)}{2V_BnR}} = \frac{2V_BnR}{p_B(6V - V_B)}$$

$$C_{AB} = c_V + \frac{RT}{V}\frac{dV}{dT} = c_V + \frac{R}{V}\left(-\frac{1}{2}p_B\cdot\frac{V}{nR} + \frac{3}{2}\frac{p_B}{V_B}\frac{V^2}{nR}\right)\frac{dV}{dT} =$$

$$= c_V + \left(-\frac{1}{2}\frac{p_B}{n} + \frac{3}{2}\frac{p_B V}{V_B n}\right)\cdot\frac{2V_BnR}{p_B(6V - V_B)} =$$

$$= c_V + \frac{1}{\cancel{2}}\frac{\cancel{p_B}}{\cancel{n}}\left(3\frac{V}{V_B} - 1\right)\frac{\cancel{2}V_B\cancel{n}R}{\cancel{p_B}(6V - V_B)} =$$

$$= c_V + R\frac{3V}{6V - V_B} - \frac{V_B}{6V - V_B}R = c_V + R\frac{3V - V_B}{6V - V_B}$$

E23 - CICLI TERMODINAMICI

1) $n = 10\ \text{mol}$ (MONO)

CICLO REVERSIBILE

A : $p_A = 10^5\ N/m^2$

$V_A = 1\ m^3 \quad = V_C$

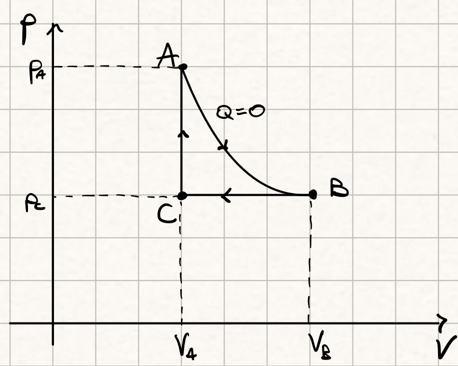
AB : esp. adiab

BC : compr. isobara

CA : tr. isocora

$p_C = \frac{1}{2}p_A = \frac{1}{2}10^5\ Pa \quad = p_B$

a)



b) A : $T_A = \frac{p_AV_A}{nR}$

B : $p_AV_A^\gamma = p_BV_B^\gamma \rightarrow V_B = \left(\frac{p_A}{p_B}\right)^{\frac{1}{\gamma}}V_A = \left(\frac{\cancel{p_A}}{\frac{1}{2}\cancel{p_A}}\right)^{\frac{3}{5}}V_A = 2^{\frac{3}{5}}V_A \quad ; \quad T_B = \frac{\frac{1}{2}p_A\cdot 2^{3/5}V_A}{nR}$

C : $T_C = \frac{\frac{1}{2}p_AV_A}{nR}$

AB : ADIAB. $\Rightarrow$ $Q_{AB} = 0$;

$$L_{AB} = -\Delta U = -nc_V\Delta T = -\cancel{n}\frac{3}{2}\cancel{R}\frac{p_AV_A}{\cancel{nR}}\left(\frac{1}{2}\cdot 2^{3/5} - 1\right) = 3{,}63\cdot 10^4\ J$$

BC : ISOBARA $\rightarrow$ $Q_{BC} = nc_p\Delta T = nc_p(T_C - T_B) = \cancel{n}c_p\left(\frac{\frac{1}{2}p_AV_A}{\cancel{n}R} - \frac{2^{\frac{3}{5}-1}p_AV_A}{\cancel{n}R}\right)$

$$= \frac{5}{2}p_AV_A\left(\frac{1}{2} - 2^{-\frac{3}{5}}\right) = -6{,}45\cdot 10^4\ J$$

$$L = \int_B^C p\,dV = \frac{1}{2}p_A(V_C - V_B) = -2{,}58\cdot 10^4\ J$$

CA : ISOCORA $\rightarrow$ $L_{CA} = 0$

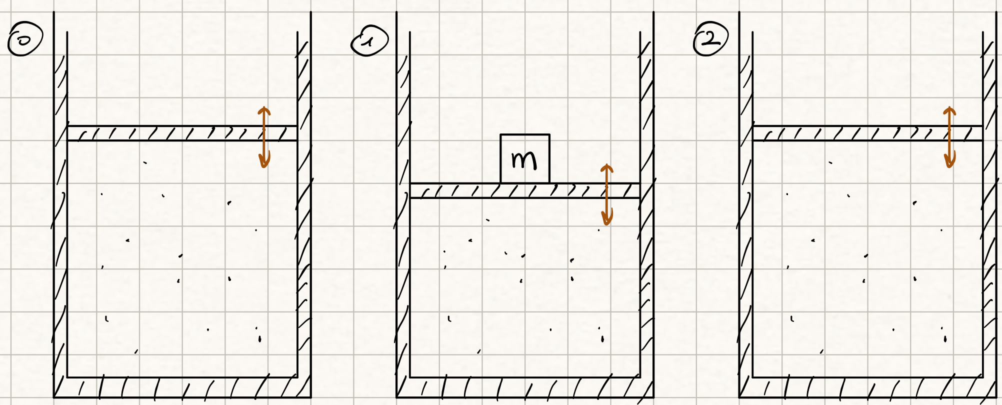
$$Q_{CA} = \Delta U = n c_v (T_A - T_C) = \cancel{n} \frac{3}{2} \cancel{R} \left(\frac{P_A V_A}{\cancel{nR}} - \frac{\frac{1}{2} P_A V_A}{\cancel{nR}} \right) = \frac{3}{2} P_A V_A \left(1 - \frac{1}{2}\right)$$

$$= 7{,}5 \cdot 10^4 \, J$$

c) $\eta = \frac{L}{Q_{ass}} = 1 + \frac{Q_{ced}}{Q_{ass}} = 1 + \frac{Q_{BC}}{Q_{CA}} = 0{,}14$

d) $\eta_c = 1 - \frac{T_{MIN}}{T_{MAX}} = 1 - \frac{T_C}{T_A} = 1 - \frac{1}{2} = 0{,}5$

2)



n mol

MONO

0 EQ. TERMOD. a T_0

1 $p_1 = 6 p_0$

2 IRR

a) ? T_1

b) ? ΔU

a) 0-1 : COMPRESSIONE ADIABATICA : $Q = 0$ $\rightarrow$ $\Delta U = -L$

$$\Delta U = n c_v (T_1 - T_0) \; ; \; L = 6 p_0 (V_1 - V_0) = 6 p_0 \left(\frac{nRT_1}{6 p_0} - \frac{nRT_0}{p_0} \right) = nR (T_1 - 6 T_0)$$

$$\Rightarrow \cancel{n} \frac{3}{2} \cancel{R} (T_1 - T_0) = -\cancel{n} \cancel{R} (T_1 - 6 T_0) \quad \Rightarrow \left(\frac{3}{2} + 1 \right) T_1 = \left(\frac{3}{2} + 6 \right) T_0$$

$$\Rightarrow T_1 = \frac{\overset{3}{\cancel{15}}}{\cancel{2}} T_0 \, \frac{\cancel{2}}{\cancel{5}} = 3 T_0$$

b) 0-1 : $\Delta U = n c_v (3T_0 - T_0) = \frac{3}{2} nR \, 2T_0$

1-2 : $\Delta U = -L_{12}$: $\frac{3}{2} \cancel{nR} (T_2 - 3T_0) = \cancel{p_0} \left(\frac{\cancel{nR} 3 T_0}{\cancel{2}6 \cancel{p_0}} - \frac{\cancel{nR} T_2}{\cancel{p_0}} \right)$

$$\Rightarrow \left(\frac{3}{2} + 1 \right) T_2 = \left(\frac{8}{2} + \frac{1}{2} \right) T_0 \quad \Rightarrow \quad T_2 = T_0 \, \cancel{5} \cdot \frac{2}{\cancel{5}} = 2 T_0$$

$$\rightarrow \Delta U = n \frac{3}{2} R (2T_0 - 3T_0) = -\frac{3}{2} nRT_0$$

$$\Rightarrow \Delta U_{tot} = \frac{3}{2} nR \, 2T_0 - \frac{3}{2} nRT_0 = \frac{3}{2} nRT_0$$

3) REVERSIBILE

A : $p_A = 10^6\ Pa$
$V_A = 2\ m^3$
$T_A = 200 K$

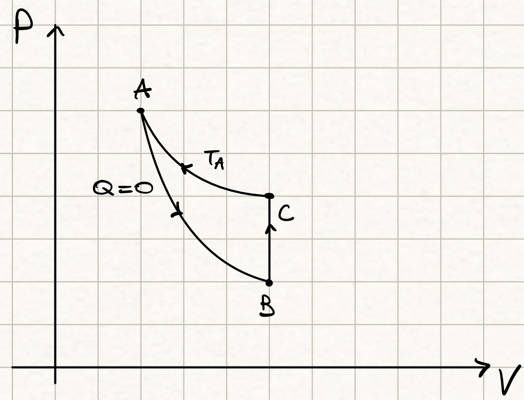
B : $p_B = 68{,}4 \cdot 10^3\ Pa$
$V_B = 10\ m^3$
$T_B = 68{,}4\ K$

BC : ISOCORA

C : $V_C = V_B$
$Q_{BC} \neq 0$
$p_C > p_B$
$T_C = T_A$

CA : ISOTERMA

a)



b) ? n ? c_V

$$p_A V_A = nRT_A \Rightarrow n = \frac{p_A V_A}{RT_A} = \frac{\cancel{10^6}^{\,4} \cdot \cancel{2}}{8{,}31 \cdot \cancel{200}} = 1{,}2 \cdot 10^3\ mol$$

AB : COMPRESS. ADIAB. REV

$$p_A V_A^{\gamma} = p_B V_B^{\gamma} \quad ; \quad \gamma = \frac{c_p}{c_V} = \frac{c_V + R}{c_V} = 1 + \frac{R}{c_V} \rightarrow c_V = \frac{R}{\gamma - 1}$$

$$\rightarrow \frac{p_A}{p_B} = \left(\frac{V_B}{V_A}\right)^{\gamma} \rightarrow \gamma = \log_{\frac{V_B}{V_A}} \frac{p_A}{p_B} = \frac{\log \frac{p_A}{p_B}}{\log \frac{V_B}{V_A}} = 1{,}666$$

$$\rightarrow c_V = \frac{1}{\gamma - 1} R = 1{,}5 R = \frac{3}{2} R \quad J/mol \cdot K$$

c) $\varepsilon = \dfrac{Q_{ASS}}{|L|}$

$$Q_{BC} = nc_V \Delta T = 1{,}2 \cdot 10^3 \ \frac{3}{2} R \,(200 - 68{,}4) = 1{,}57 \cdot 10^6 J$$

$$Q_{CA} = L_{CA} = \int p\, dV = \int_C^A \frac{nRT}{V}\, dV = nRT_A \ln \frac{V_A}{V_B} = -3{,}21 \cdot 10^6 J$$

$$\varepsilon = \frac{Q_{ASS}}{|Q_{ASS} + Q_{CED}|} = 1{,}6$$

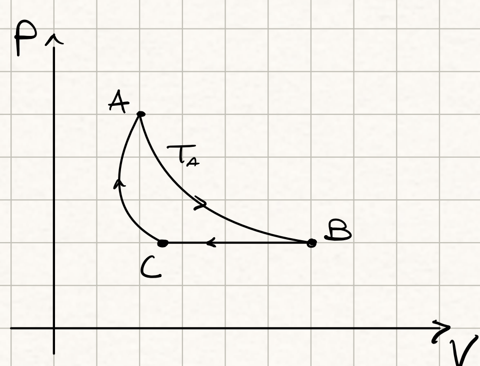
4) ESP. ISOTERMA AB

A : $p_A = 1 atm$
$V_A = 20\ \ell = 20\ dm^3 = 2 \cdot 10^{-2} m^3$

B : $V_B = x^2 V_A$; $x = 1{,}1$

COMPR. ISOBARA REV. BC : $p_B = p_C$

CA : rev. $pV^2 = cost$



? V_C
? L_{tot}

$$\begin{cases} p_A V_A = p_B V_B \\ p_B V_C^2 = p_A V_A^2 \end{cases} \Rightarrow p_B = p_A \frac{V_A}{V_B}$$

$$\hookrightarrow \cancel{p_A} \frac{\cancel{V_A}}{V_B} V_C^2 = \cancel{p_A} V_A^{\cancel{2}} \Rightarrow V_C = \sqrt{V_A V_B}$$

$$\Rightarrow V_C = \sqrt{2 \cdot 10^{-2} \cdot 1{,}1^2 \cdot 2 \cdot 10^{-2}} = 0{,}022\ m^3 = 22\ \ell$$

$$L_{tot} = L_{AB} + L_{BC} + L_{CA} =$$

AB: $\Delta U_{AB} = 0 \Rightarrow Q_{AB} = L_{AB} = nRT_A \ln \frac{V_B}{V_A} = \cancel{nR} \frac{p_A V_A}{\cancel{nR}} \ln \frac{x^2 \cancel{V_A}}{\cancel{V_A}}$

$$= p_A V_A \ln x^2 = 1{,}013 \cdot 10^5 \cdot 2 \cdot 10^{-2} \ln 1{,}1^2 = 386{,}2\ J$$

BC: $L_{BC} = p_B \Delta V = p_A \frac{V_A}{V_B}(V_C - V_B) = p_A \frac{\cancel{V_A}}{x^2 \cancel{V_A}}(V_C - x^2 V_A) = -184{,}2\ J$

CA: $L_{CA} = \int_C^A p\, dV = \int_{V_C}^{V_A} \frac{p_A V_A^2}{V^2} dV = p_A V_A^2 \left[-\frac{1}{V}\right]_{V_A}^{V_C} = p_A V_A^2 \left[\frac{1}{V_C} - \frac{1}{V_A}\right] = -184{,}2\ J$

$\Rightarrow L_{tot} = 17{,}8\ J$

5) MACCH. FRIG. REV
$n = 2$ mol (BIAT)
AB: ESP. ISOTERMA
$V_B = 0{,}074\ m^3$
$m_G = 12{,}3 \cdot 10^{-3}$ kg
BC: COMPR. ADIAB
CA: COMPRESS. ISOBARA
$\lambda_F = 3{,}33 \cdot 10^5$ J/kg

a) ? V_A
b) ? V_C

AB: $Q_{AB} = -Q_G$; $Q_{AB} = L_{AB}$ $(\Delta U_{AB} = 0)$
$pV = cost$; $Q_G = m_G \lambda$ (ceduto)

$\Rightarrow m_G \lambda = \int_A^B p\, dV = p_e (V_B - V_A)$

$\Rightarrow V_A = V_B - \frac{m_G \lambda}{p_e} = 0{,}03\ m^3$

$\Rightarrow L_{AB} = nRT \ln \frac{V_B}{V_A} = m_G \lambda$

$\Rightarrow T = \ln \frac{V_A}{V_B} \frac{m_G \lambda}{nR} = 222{,}5\ K$

BC: $Q_{BC} = 0 \Rightarrow \Delta U_{BC} = -L_{BC}$
$\Delta U_{BC} = n c_V (T_C - T)$

$p_B V_B^\gamma = p_A V_C^\gamma \Rightarrow \frac{\cancel{nRT}}{V_B} V_B^\gamma = \frac{\cancel{nRT}}{V_A} V_C^\gamma \Rightarrow V_C = \left(\frac{V_A}{V_B}\right)^{\frac{1}{\gamma}} V_B = 0{,}035\ m^3$

E24 - ENTROPIA

1) $m = 10\,Kg$
$T_1 = 12\,°C$
$T_2 = 27\,°C$
? ΔS_U
$c = 2{,}38 \cdot 10^3\, J/kg\,K$

$$\Delta S_U = \Delta S_{SIST} + \Delta S_{AMB}$$

$$\Delta S_{SIST} = \int_1^2 \frac{(\delta Q)_{rev}}{T} = \int_1^2 \frac{mc\,dT}{T} = mc \ln \frac{T_2}{T_1}$$

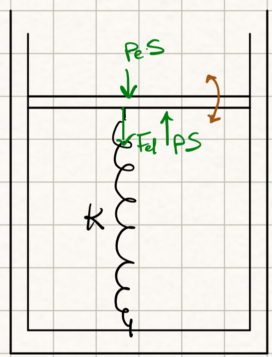
$$\Delta S_{AMB} = \int_1^2 \frac{(\delta Q)_{rev}}{T} = \frac{Q_{AMB}}{T_2}$$

$$Q_{SIST} + Q_{AMB} = 0 \rightarrow Q_{AMB} = -Q_{SIST} = mc(T_1 - T_2)$$

$$\Delta S_U = mc \ln\frac{T_2}{T_1} + \frac{mc(T_1-T_2)}{T_2} = 30{,}75\, J/K$$

2) $V_1 = 20\,\ell = 20 \cdot 10^{-3}\, m^3$
$n = 1\,mol\ (He)$
$\Delta T = 100\,°C$
$T_1 = 25\,°C$
$Q = 2 \cdot 10^3\, J$

?ΔV
?ΔS_g



Equilibrio meccanico : $F_{tot} = 0$

① $p_1 S = p_e S + F_{el}$

$$\rightarrow p_1 = p_e + \frac{F_{el}}{S} = p_e + \frac{kh_1}{V_1/h_1} = p_e + \frac{kh_1^2}{V_1}$$

② $p_2 = p_e + \dfrac{kh_2^2}{V_2}$

$$\begin{cases} p_1 V_1 = nRT_1 \\ p_2 V_2 = nRT_2 \end{cases} \rightarrow \begin{cases} \left(p_e + \frac{kh_1^2}{V_1}\right)V_1 = nRT_1 \; * \\ \left(p_e + \frac{kh_2^2}{V_2}\right)V_2 = nRT_2 \end{cases}$$

$$nR\Delta T = \left(p_e + \frac{kh_2^2}{V_2}\right)V_2 - \left(p_e + \frac{kh_1^2}{V_1}\right)V_1 = p_e \Delta V + k(h_2^2 - h_1^2)$$

$$\Delta U = Q - L = nc_V\Delta T \;;$$

$$L = \int_1^2 p\,dV = \int_1^2 \left(p_e + \frac{kh^2}{V}\right)dV = p_e\Delta V + k\int_{h_1}^{h_2} \frac{h^2}{\cancel{S}h}\cancel{S}dh = p_e\Delta V + k\int_{h_1}^{h_2} h\,dh = p_e\Delta V + \frac{1}{2}k(h_2^2 - h_1^2)$$

$$Q = 2000 = nc_V\Delta T + p_e\Delta V + \frac{1}{2}k(h_2^2 - h_1^2)$$

$$k(h_2^2 - h_1^2) = nR\Delta T - p_e\Delta V$$

$$\Rightarrow Q = nc_V\Delta T + p_e\Delta V + \frac{1}{2}(nR\Delta T - p_e\Delta V) = \left(c_V + \frac{1}{2}R\right)n\Delta T + \frac{1}{2}p_e\Delta V = 2nR\Delta T + \frac{1}{2}p_e\Delta V$$

$$\Rightarrow \Delta V = 2\frac{Q - 2nR\Delta T}{p_e} = 6{,}67 \cdot 10^{-3}\, m^3 = 6{,}67\,\ell$$

Per trovare ΔS_g considero trasformazioni equivalenti REVERSIBILI : ISOTERMA ① + ISOCORA ②

$$\Delta S_g = \Delta S_1 + \Delta S_2$$

① ISOTERMA : $T = 25°C = \text{cost}$, $V \to \Delta V + V_1$ $\Rightarrow \Delta S_1 = \int \frac{(\delta Q)_{rev}}{T} = \int \frac{\delta L}{T} = \frac{n R \cancel{T} \ln\left(\frac{V_f}{V_i}\right)}{\cancel{T}}$

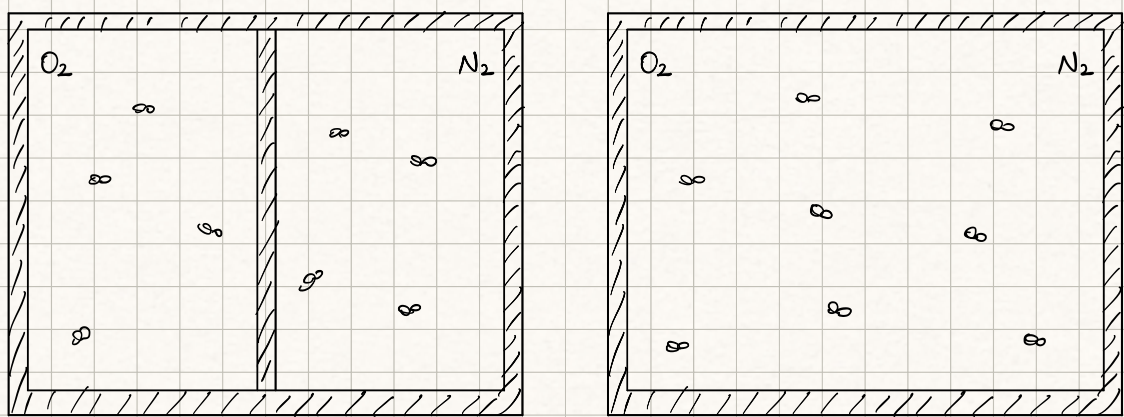
$$\Rightarrow \Delta S_1 = nR \ln\left(\frac{V_f}{V_i}\right) = nR \ln\left(\frac{V_i + \Delta V}{V_i}\right) = nR \ln\left(1 + \frac{\Delta V}{V_i}\right)$$

② ISOCORA : $V_1 + \Delta V = \text{cost}$, $T \to T_1 + \Delta T$

$$\Delta S_2 = \int \frac{(\delta Q)_{rev}}{T} = \int \frac{dU}{T} = \int \frac{n c_v \, dT}{T} = n c_v \ln\left(\frac{T_f}{T_i}\right) = n c_v \ln\left(1 + \frac{\Delta T}{T_i}\right)$$

$$\Rightarrow \Delta S = \Delta S_1 + \Delta S_2 = nR \ln\left(1 + \frac{\Delta V}{V_i}\right) + n c_v \ln\left(1 + \frac{\Delta T}{T_i}\right) = 6\ J/K$$

3)



EQUILIBRIO

$V_1 = V_2$; $V = 4\ \ell$

ADIABATICO

O_2 : $T_1 = 300K$, $p_1 = 10^5\ Pa$

N_2 : $T_2 = 500K$, $p_2 = 3 \cdot 10^5\ Pa$

? T_f ? p_f

? ΔS

$$V_1 = V_2 = \frac{V}{2}$$

$$p_1 \frac{V}{2} = n_1 R T_1 \Rightarrow n_1 = \frac{p_1 V}{2RT_1} = 0{,}08\ mol \quad ; \quad n_2 = \frac{p_2 V}{2RT_2} = 0{,}14\ mol$$

SISTEMA ISOLATO : $Q_1 + Q_2 = 0$

$$\Delta U = \underbrace{Q}_{\to 0} - \underbrace{L}_{\to 0 \text{ (per il sistema complessivo } \Delta V = 0)} = 0 \Rightarrow \Delta U_1 + \Delta U_2 = 0 \Rightarrow n_1 \cancel{c_v}(T_f - T_1) + n_2 \cancel{c_v}(T_f - T_2) = 0$$

$$\Rightarrow T_f = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)} = 427\ K$$

Legge di Dalton: $\sum_i^2 p_i = p_{tot} = p_f \Rightarrow p_f = p_{1f} + p_{2f}$

$$\begin{cases} p_{1f} V = n_1 R T_f \\ p_{2f} V = n_2 R T_f \end{cases} \Rightarrow (p_{1f} + p_{2f}) V = (n_1 + n_2) R T_f \Rightarrow p_f = \frac{(n_1 + n_2) R T_f}{V}$$

$$\Rightarrow p_f = \left(\frac{p_1 \cancel{V}}{2\cancel{R}T_1} + \frac{p_2 \cancel{V}}{2\cancel{R}T_2}\right) \frac{\cancel{R} T_f}{\cancel{V}} = \frac{1}{2} T_f \left(\frac{p_1}{T_1} + \frac{p_2}{T_2}\right) = 2 \cdot 10^5\ Pa$$

$$\Delta S = \Delta S_1 + \Delta S_2 = \int_1 \frac{(\delta Q)_{rev}}{T} + \int_2 \frac{(\delta Q)_{rev}}{T}$$

① : ISOBARA + ISOTERMA : $\Delta S_1 = n_1 c_p \ln\left(\frac{T_f}{T_1}\right) + n_1 R \ln\left(\frac{V_f}{V_1}\right)$

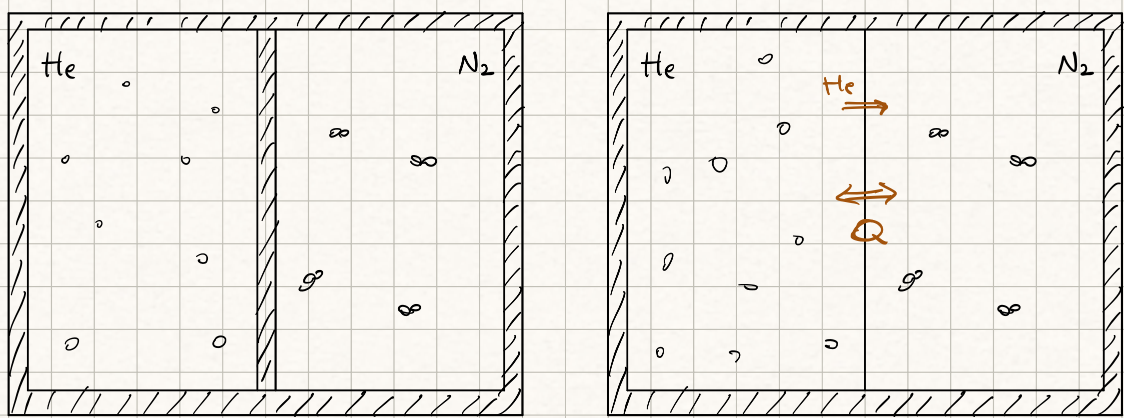
$T_1 \to T_f$ $\quad V_1 \to V$

② $\Delta S_2 = n_2 c_p \ln\left(\frac{T_f}{T_2}\right) + n_2 R \ln\left(\frac{V_f}{V_2}\right)$

$$\Rightarrow \Delta S = \Delta S_1 + \Delta S_2 = n_1 c_p \ln\left(\frac{T_f}{T_1}\right) + n_1 R \ln\left(\frac{V_f}{V_1}\right) + n_2 c_p \ln\left(\frac{T_f}{T_2}\right) + n_2 R \ln\left(\frac{V_f}{V_2}\right)$$

$$= n_1 c_p \ln\left(\frac{T_f}{T_1}\right) + n_2 c_p \ln\left(\frac{T_f}{T_2}\right) + (n_1+n_2) R \ln 2 = 1,46\ J/K$$

4)



$V_1 = V_2 = \frac{V}{2}$

$n_1 = 1$ mol (He)

$T_1 = 445$ K

$n_2 = 1$ mol (N_2)

$T_2 = 280$ K

? T_f ? ΔS

$R = 2$ cal/mol K $= 8,372$ J/mol·K

SISTEMA ISOLATO : $Q_1 + Q_2 = 0 \Rightarrow \Delta U = 0 = \Delta U_1 + \Delta U_2$

$p_1 V_1 = nRT_1$; $p_2 V_2 = nRT_2$

$$\Rightarrow \Delta U_1 + \Delta U_2 = 0 \Rightarrow n_1 c_v (T_f - T_1) + n_2 c_v (T_f - T_2) = 0$$

$$\Rightarrow 1 \cdot \frac{3}{2}\cancel{R}(T_f - T_1) + 1 \frac{5}{2}\cancel{R}(T_f - T_2) = 0$$

$$\Rightarrow \left(\frac{3}{2} + \frac{5}{2}\right) T_f = \frac{3}{2} T_1 + \frac{5}{2} T_2 \Rightarrow T_f = \frac{3T_1 + 5T_2}{8} = 341,9\ K$$

$$\Delta S = \Delta S_1 + \Delta S_2$$

① : ISOCORA + ISOTERMA : $\Delta S_1 = n c_v \ln\left(\frac{T_f}{T_1}\right) + nR \ln\left(\frac{V}{V_1}\right) = n c_v \ln\left(\frac{T_f}{T_1}\right) + nR \ln 2$

$T_1 \to T_f$ $\quad V_1 \to V$

② : ISOCORA : $\Delta S_2 = n c_v \ln\left(\frac{T_f}{T_2}\right)$

$T_2 \to T_f$

$$\Rightarrow \Delta S = n c_v \ln\left(\frac{T_f}{T_1}\right) + nR \ln 2 + n c_v \ln\left(\frac{T_f}{T_2}\right) = \frac{3}{2} R \ln\left(\frac{T_f}{T_1}\right) + R \ln 2 + \frac{5}{2} R \ln\left(\frac{T_f}{T_2}\right) = 6,68\ J/K$$

5) $S = aT^\alpha$, $a > 0$, $\alpha > 0$

n moli

TRASF. REV.

?c

$$c = \frac{1}{n}\frac{\delta Q}{dT} \Rightarrow \delta Q = nc\,dT$$

$$S = \int \frac{(\delta Q)_{rev}}{T} = aT^\alpha$$

$$\Rightarrow \int \frac{nc\,dT}{T} = aT^\alpha \Rightarrow n\int \frac{c(T)}{T}dT = aT^\alpha$$

$$\Rightarrow n\frac{c}{T} = a\frac{dT^\alpha}{dT} \Rightarrow c = \frac{T}{n}a\,\alpha T^{\alpha-1}$$

$$\Rightarrow c = \frac{\alpha}{n}aT^\alpha$$

6) $n = 2$ mol (MONO)

$p_1 = 10^6$ Pa

$V = 5\,\ell$

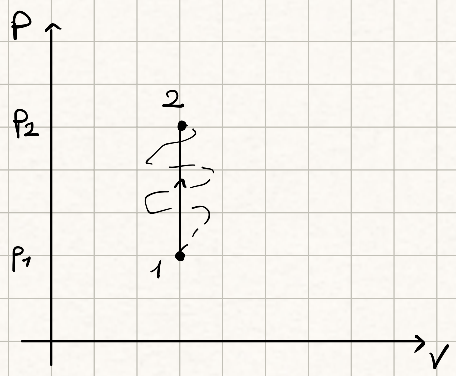
$p_2 = 3p_1$

a) ?$\mathcal{L}_f$

b) ?ΔU

c) ?Q

d) ?ΔS_f



$V_1 = V_2$

! ma non è detto che rimanga costante durante tutta la trasformazione

$\Rightarrow$ "Una" trasformazione — Rev. / Irrev.

Sono calcolabili solo le funzioni di stato : ΔU ΔS

perché si conoscono solo le cond. iniz./finali

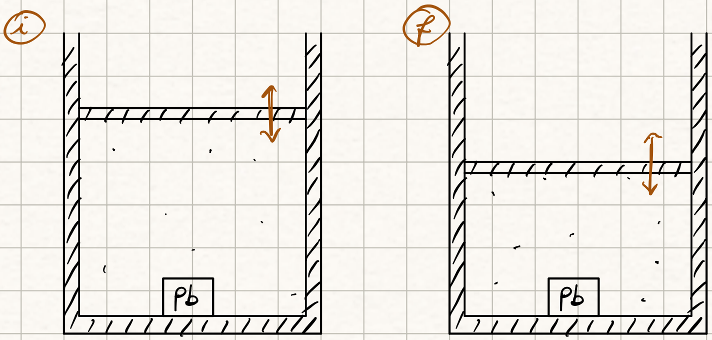
b) $\Delta U = n c_v (T_2 - T_1) = \cancel{n}\frac{3}{2}\cancel{R}\left(\frac{p_2V_2}{\cancel{n}\cancel{R}} - \frac{p_1V_1}{\cancel{n}\cancel{R}}\right) = \frac{3}{2}V(3p_1 - p_1) = \frac{3}{\cancel{2}}\cancel{2}p_1V = 3p_1V$

$= 15 \cdot 10^3$ J

d) $\Delta S = \int \frac{(\delta Q)_{rev}}{T} \overset{\text{ISOCORA}}{=} nc_v \ln\left(\frac{T_2}{T_1}\right) = nc_v \ln\left(\frac{p_2\cancel{V_2}}{p_1\cancel{V_1}}\right) = nc_v \ln 3 = 27,4$ J/K

E25 - ENTROPIA

1)



$n = 1\,mol$ (reono)
$m_{Pb} = 0{,}5\,kg$
$Q = 0$
$T_2 = T_1 + 0{,}1\,T_1 = 1{,}1\,T_1$
COMPRESSIONE
$\Delta S = 0$
$C_{Pb} = 130\,J/kg\,K$
? $\dfrac{p_f}{p_i}$

ISOENTROPICO : COMPRESSIONE ADIABATICA REVERSIBILE

$$Q = 0 \Rightarrow Q_g = -Q_{Pb} = -m\,c_{Pb}\,\Delta T$$

$$\Delta U = -L \Rightarrow n c_v \Delta T = -\int_i^f p_e\, dV$$

$$p_i V_i = nRT_i \quad ; \quad p_f V_f = nRT_f$$

$$\Rightarrow \frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

$T_i \to T_f$ ISOBARA ; $p_i \to p_f$ ISOTERMA

$$\Delta S = \int \frac{(dQ)_{rev}}{T} = n\cancel{c_v}^{\,c_p} \ln\left(\frac{T_f}{T_i}\right) - nR\ln\left(\frac{p_f}{p_i}\right) + m c_{Pb} \ln\frac{T_f}{T_i} = 0$$

$$\Rightarrow (n c_p + m c_{Pb}) \ln 1{,}1 = nR \ln\left(\frac{p_f}{p_i}\right) \Rightarrow \ln\frac{p_f}{p_i} = \frac{n c_p + m c_{Pb}}{nR} \ln 1{,}1 \Rightarrow$$

$$\frac{p_f}{p_i} = 1{,}1^{\frac{5}{2}\cancel{\frac{3}{2}} + \frac{m c_{Pb}}{nR}} = \cancel{2{,}43} \quad 2{,}67$$

2) $m_G = 20 \cdot 10^{-3}\,kg = 2 \cdot 10^{-2}\,kg$
$T_G = -5\,°C = 268{,}15\,K$
$V_A = 0{,}2\,\ell = 0{,}2 \cdot 10^{-3}\,m^3$
$T_A = 25\,°C = 298{,}15\,K$
→ EQUILIBRIO TERMODINAMICO
? ΔS_U

$C_G = 2051\,J/kg\,K$
$C_A = 4186{,}8\,J/kg\,K$
$\lambda = 3{,}33 \cdot 10^5\,J/kg\,K$

Acqua : $T_A \to T_{eq}$; Ghiaccio : $T_G \to T_0 \to T_{eq}$

$$\Delta S_U = \Delta S_{SIST} + \cancel{\Delta S_{AMB}}^{\,\text{ISOLATO}}$$

$$Q_{TOT} = 0 : \quad \underset{1}{\rho_A V_A C_A (T_{eq} - T_A)} + \underset{2}{m_G C_G (T_0 - T_G)} + \underset{3}{m_G \lambda_G} + \underset{4}{m_G C_A (T_{eq} - T_0)} = 0$$

$$T_{eq} = \frac{\rho_A V_A C_A T_A - m_G C_G (T_0 - T_G) - m_G \lambda_G + m_G C_A T_0}{(\rho_A V_A C_A + m_G C_A)} = \frac{0{,}2\, C_A T_A - m_G (C_G T_0 - C_G T_G - \lambda_G + C_A T_0)}{(\rho_A V_A + m_G) C_A} = 253{,}2\,K$$

$m_A = V_A \rho_A = 0{,}2\ kg$

$\Delta S_U = \Delta S_{SIST} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4$

$\Delta S_1 = \rho_A V_A c_A \ln\frac{T_{eq}}{T_A}$; $\Delta S_2 = m_G c_G \ln\frac{T_0}{T_G}$; $\Delta S_3 = \frac{m_G \lambda_G}{T_0}$; $\Delta S_4 = m_G c_A \ln\frac{T_{eq}}{T_0}$

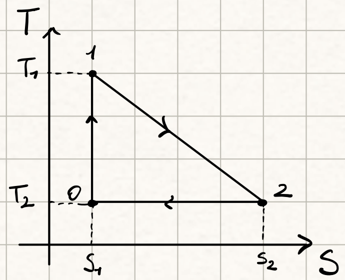
$\Rightarrow \Delta S_U = 1{,}57\ J/K$

3) $T_1 = 500\ K$

$T_2 = 300\ K$

$?\ \eta$

$?\ P_m(\tau)$



0-1 : $\Delta S = 0$, $Q = 0$

1-2 : $Q = (T_1 + T_2)\cdot\frac{(S_2 - S_1)}{2} = 400\Delta S$

2-0 : ISOTERMA : $Q = 300\Delta S \quad < 0$

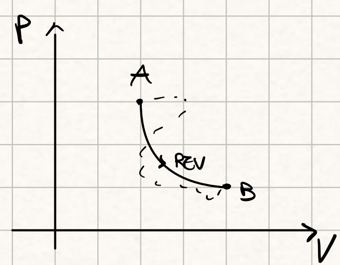
ciclo : $\Delta U = 0 \Rightarrow Q = L$

$\Rightarrow \eta = 1 - \frac{3\cancel{00\Delta S}}{4\cancel{00\Delta S}} = \frac{4-3}{4} = 0{,}25$

$P_u = \frac{|L|}{\tau} = \frac{|Q_{ASS} + Q_{CED}|}{\tau} = \frac{400\Delta S - 300\Delta S}{\tau} = 100\frac{\Delta S}{\tau} = 100\frac{(S_2 - S_1)}{\tau}$

4) AB : ESP. ADIAB — REV / IRR

$T_{B\,REV} \overset{?}{>} T_{B\,IRR} \Rightarrow \frac{T_{B\,IRR}}{T_{B\,REV}}$?



$\Delta S_{REV} = 0$

$\Delta S_{IRR} > 0$

$\frac{c_p}{c_v} - 1 = \frac{R + c_v}{c_v} - 1 = \frac{R + c_v - c_v}{c_v} = \frac{R}{c_v}$

ADIAB : REV $\Rightarrow T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \Rightarrow T_B^{REV} = T_A\left(\frac{V_A}{V_B}\right)^{\gamma-1}$

IRR $\Rightarrow$ ISOCORA $(T_A \to T_B)$ + ISOTERMA $(V_A \to V_B)$

$$\Delta S = n c_v \ln\left(\frac{T_B}{T_A}\right) + nR\ln\left(\frac{V_B}{V_A}\right)$$

$$\Rightarrow \ln\frac{T_B}{T_A} = \frac{1}{n c_v}\left(\Delta S - nR\ln\frac{V_B}{V_A}\right)$$

$$\Rightarrow T_B = T_A e^{\frac{\Delta S}{n c_v} - \frac{R}{c_v}\ln\frac{V_B}{V_A}} = \frac{T_A e^{\frac{\Delta S}{n c_v}}}{e^{\ln\left(\frac{V_B}{V_A}\right)^{\frac{R}{c_v}}}} = T_A e^{\frac{\Delta S}{n c_v}}\left(\frac{V_B}{V_A}\right)^{-\frac{R}{c_v}}$$

$$\Rightarrow \frac{T_B^{REV}}{T_B^{IRR}} = \cancel{T_A}\cancel{\left(\frac{V_A}{V_B}\right)^{\frac{R}{c_v}}}\left(\cancel{T_A} e^{\frac{\Delta S}{n c_v}}\cancel{\left(\frac{V_B}{V_A}\right)^{-\frac{R}{c_v}}}\right)^{-1} = e^{-\overset{>0}{\frac{\Delta S}{n c_v}}} = \frac{1}{e^{\frac{\Delta S}{n c_v}}} \Rightarrow T_B^{REV} < T_B^{IRR}$$

5) $T_2 = 800\,°C = 1073,15\ K$

$T_1 = 300\,°C = 573,15\ K$

$P = 10^4\ W$

$\eta = 60\%\ \eta_c$

$?Q \quad ?\Delta S_U \quad \Delta t = 1h$

ciclo $\Delta S_{SIST} = 0$

$$\eta_c = 1 - \frac{T_1}{T_2} = 0,47$$

$$P = \frac{L}{\Delta t} \Rightarrow L = P\Delta t$$

$$\eta = 0,6\,\eta_c = \frac{|L|}{Q_{ASS}} = \frac{P\Delta t}{Q_{ASS}}$$

$$\Rightarrow Q_{ASS} = \frac{P\Delta t}{0,6\,\eta_c} = 1,27 \cdot 10^8\ J$$

$$Q_{CED} = L - Q_{ASS} = -9,2 \cdot 10^7\ J$$

per la sorgente a T_2 il calore è ceduto

$$\Delta S_U = \Delta S_{AMB} = \int_1^2 \frac{(\delta Q)_{rev}}{T} = -\frac{Q_{ASS}}{T_2} + \frac{|Q_{CED}|}{T_1} = 4,35 \cdot 10^4\ J/K$$

per la sorgente a T_1 il calore è assorbito

6) CICLO FRIGORIFERO

$n = 2\,mol$ (BIAT.)

AB: COMPR. ADIAB. ($Q_{AB} = 0$)

$T_A \rightarrow T_B$

BC: RAFFREDD. ISOCORO ($V_C = V_B$)

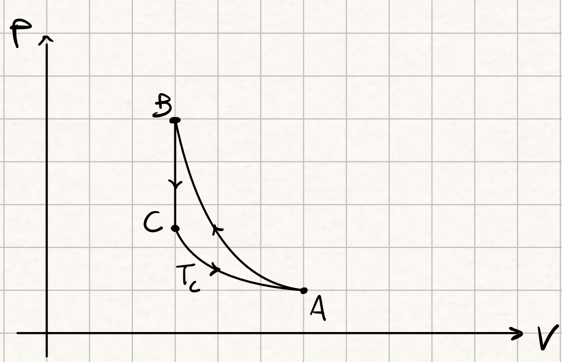
$T_B \rightarrow T_C$ ($< T_B$) $\quad \hookrightarrow L_{BC} = 0$; $\Delta U_{BC} = Q_{BC}$

CA: ESP. ISOTERMA ($T_C = T_A$)

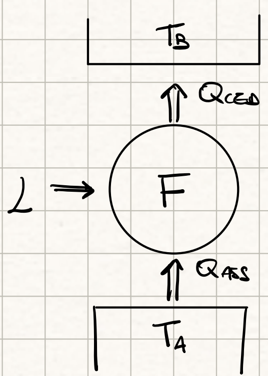
$V_C \rightarrow 2V_C = V_A \quad \hookrightarrow \Delta U_{CA} = 0$; $Q_{CA} = L_{CA}$

$T_A = 266\ K$

$T_B = 351\ K$



$?Q_{ASS} \quad ?\varepsilon \quad ?\Delta S_{AB} \quad ?\Delta S_{BC} \quad ?\Delta S_{CA}$



$Q_{AB} = 0$

$$Q_{BC} = \Delta U_{BC} = n c_v \Delta T = n \tfrac{5}{2} R (T_C - T_B) = 5R(T_A - T_B)$$

$$= -3,5 \cdot 10^3\ J = Q_{CED}$$

$$Q_{CA} = L_{CA} = nRT_A \ln\left(\frac{V_A}{V_C}\right) = 2RT_A \ln 2 = 3,06 \cdot 10^3\ J = Q_{ASS}$$

$$\varepsilon = \frac{Q_{ASS}}{|L|} = \frac{Q_{ASS}}{|Q_{CED} + Q_{ASS}|} = 6,5$$

$\Delta S_{AB} = 0\ J/K$; $\quad \Delta S_{BC} = n c_v \ln\left(\frac{T_C}{T_B}\right) = -11,5\ J/K$

$\Delta S_{CA} = nR \ln\frac{V_A}{V_C} = 11,5\ J/K = -\Delta S_{BC} \quad \Rightarrow \quad \Delta S_{SIST} = 0$