STATICA C.R.

$$\begin{cases} \vec{F}_{ext} = \vec{0} \\ \vec{\tau}_{ext} = \vec{0} \end{cases}$$

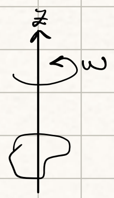
NO TRASLAZIONE
NO ROTAZIONE

⇒ DINAMICA C.R.

$$\begin{cases} \vec{F}_{ext} = \dfrac{d\vec{Q}}{dt} = M\vec{a}_{cm} \\ \vec{\tau}_{ext} = \dfrac{d\vec{L}_\Omega}{dt} \end{cases}$$

con $\vec{v}_\Omega = 0$ (polo fermo, o solidale al moto di CM)

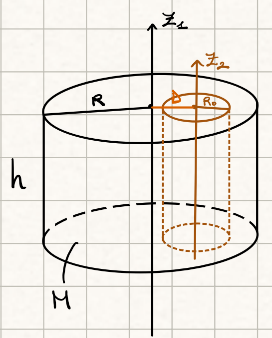
- ASSE FERMO



$\vec{L}_z = I_z \vec{\omega}$ se asse z = asse di simmetria

$$\vec{\tau}_e = I_z \frac{d\vec{\omega}}{dt} = I_z \vec{\alpha}$$

1)



$I_{DISCO} = \frac{1}{2} m r^2$

! I = proprietà additiva se rispetto stessa asse

$\rho = \frac{M}{V}$ (omogeneo) → disco forato

disco non forato
$$I_{\left(\substack{R\\ z_1}\right)} = \frac{1}{2} M R^2 \quad ; \quad I_{\left(\substack{R_0\\ z_2}\right)} = \frac{1}{2} M_{R_0} R_0^2$$

$$M = M_R - M_{R_0} \quad ; \quad V = V_R - V_{R_0}$$

formula di H-S :

$$I_{\left(\substack{R_0\\ z_1}\right)} = I_{\left(\substack{R_0\\ z_2}\right)} + M_{R_0} \underbrace{D^2}_{\text{distanza } z_1 - z_2} = \frac{1}{2} M_{R_0} R_0^2 + M_{R_0} D^2 \quad *$$

$$M_R = \rho V_R = \rho \pi R^2 h \quad ; \quad M_{R_0} = \rho V_{R_0} = \rho \pi R_0^2 h \quad ; \quad M = \rho(V_R - V_{R_0}) = \rho \pi h (R^2 - R_0^2)$$

$$\Rightarrow \frac{M_{R_0}}{M} = \frac{R_0^2}{R^2 - R_0^2} \quad \Rightarrow \quad M_{R_0} = \frac{R_0^2}{R^2 - R_0^2} M$$

$$M_R = \frac{R^2}{R^2 - R_0^2} M$$

$$* \Rightarrow I_{\left(\substack{R_0\\ z_1}\right)} = \frac{1}{2} \frac{R_0^4}{R^2 - R_0^2} M + \frac{R_0^2 D^2}{R^2 - R} M$$

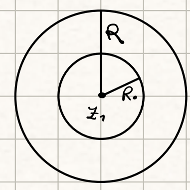
⇒ PROPR. ADDITIVA :

$$I = I_{\left(\substack{R\\ z_1}\right)} - I_{\left(\substack{R_0\\ z_1}\right)} = \frac{1}{2} \frac{R^4}{R^2 - R_0^2} M - \frac{1}{2} \frac{R_0^4}{R^2 - R_0^2} M - \frac{R_0^2 D^2}{R^2 - R_0^2} M =$$

$$= \frac{1}{2} \frac{M}{R^2 - R_0^2} (R^4 - R_0^4) - \frac{R_0^2 D^2}{R^2 - R_0^2} M = \frac{1}{2} \frac{M}{\cancel{R^2 - R_0^2}} (R^2 + R_0^2)\cancel{(R^2 - R_0^2)} - \frac{R_0^2 D^2}{R^2 - R_0^2} M$$

$$= \frac{1}{2} M \left(R^2 + R_0^2 - \frac{2R_0^2 D^2}{R^2 - R_0} \right)$$

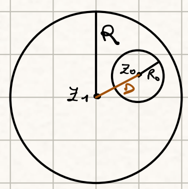
- se $D = 0 \Rightarrow I$ è max



I_{MAX}

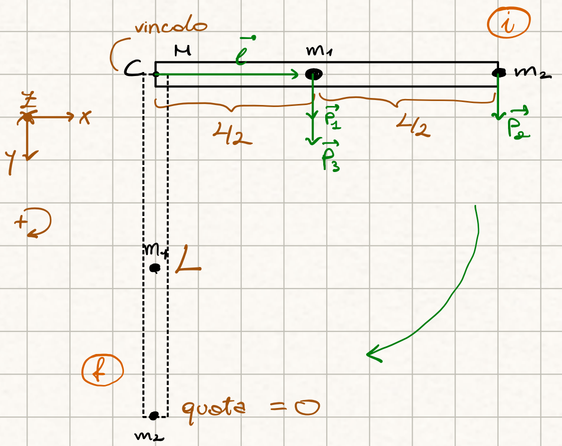
- se $D \neq 0 \Rightarrow I < I_{max}$

$\hookrightarrow \vec{\tau} = I \vec{\alpha} \Rightarrow$ è più facile da mettere in rotazione

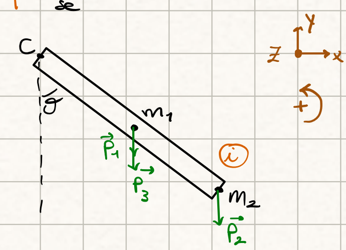


$$I = \int d^2 \, dm$$

2) $m_1 = 2$ kg
$m_2 = 1$ kg
$M = 3$ kg
$L = 1$ m



1. ? I_C
2. ? $\vec{\alpha}$ (orizzontale) ? $\vec{\omega}$ (verticale)
3. ? T se



1. $I_1 = m_1 \frac{L^2}{4}$; $I_2 = m_2 L^2$; I_{ASTA} (rispetto C) $= \frac{1}{12} ML^2 + M \frac{L^2}{4} = \frac{1}{3} ML^3$

$$\Rightarrow I_C = I_1 + I_2 + I_{ASTA} = \frac{L^2}{12}(3m_1 + 12 m_2 + 4M) = 2{,}5 \ \text{kg} \cdot \text{m}^2$$

2. ! ci sono vincoli applicati a C, ma il momento rispetto a C di forze applicate in C = 0

$$\vec{\tau}_{est} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3$$

$$\vec{\tau}_1 = \vec{l} \times \vec{P}_1 = \frac{L}{2} \vec{u}_x \times m_1 g \vec{u}_y = m_1 g \frac{L}{2} \vec{u}_z$$ entrante

$$\vec{\tau}_2 = m_2 g L \vec{u}_z$$

$$\vec{\tau}_3 = M g \frac{L}{2} \vec{u}_z$$

verso entrante lungo $\vec{u}_z$

$$\Rightarrow \tau = g \frac{L}{2}(m_1 + 2m_2 + M) = I_C \alpha \Rightarrow \alpha = \frac{6g}{L} \frac{m_1 + 2m_2 + M}{3m_1 + 2m_2 + 4M} = 13{,}7 \ \text{rad/s}^2$$

CONSERVAZIONE E_m : no forze dissipative $\Delta E_m = 0$

$(E_{pf}$ di $m_2 = 0)$

$$\Rightarrow \frac{1}{2} I_C \omega^2 - \underbrace{0}_{E_{Ki}} + (m_1 + M) g \frac{L}{2} - (M + m_1 + m_2) g L = 0 \Rightarrow \omega = \sqrt{\frac{12}{L} g \frac{m_1 + 2m_2 + M}{3m_1 + 2m_2 + 4M}}$$

$\vec{u}_z$, positivo

$$= 5{,}24 \ \text{rad/s}$$

3. $\vec{\tau}_1 = (l_x \vec{u}_x + l_y \vec{u}_z) \times (-m_1 g\, \vec{u}_y) = -l_x m_1 g\, \vec{u}_z = -\frac{L}{2} \sin\theta\, m_1 g\, \vec{u}_z$ $\quad \vec{u}_z \times \vec{u}_z = 0$

$\vec{\tau}_2 = -m_2 g L \sin\theta\, \vec{u}_z$

$\vec{\tau}_3 = -M g \frac{L}{2} \sin\theta\, \vec{u}_z$

$\Rightarrow \tau = I_c \alpha = I_c \frac{d^2\theta}{dt^2}$

$-\frac{L}{2} g \sin\theta\, (m_1 + 2m_2 + M) = I_c \frac{d^2\theta}{dt^2}$

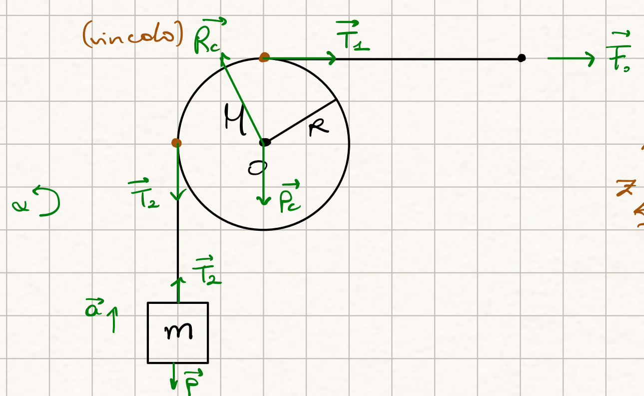
$\frac{d^2\theta}{dt^2} + \underbrace{\frac{gL}{2I_c}(m_1 + 2m_2 + M)}_{\omega^2\ [rad/s^2]} \sin\theta = 0 \quad \Rightarrow \frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0$

considerando piccole oscillazioni: $\sin\theta \sim \theta \quad \Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ MOTO ARMONICO

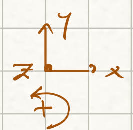
$\hookrightarrow T = \frac{2\pi}{\omega}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2I_c}{gL(m_1 + 2m_2 + M)}} = 1{,}7\ s$

4) CARRUCOLA __NON__ IDEALE



$m = 1\,kg$
$F_0 = 15\,N$
$R = 10\,cm$
$M = 2\,kg$



1. $T_1 \overset{?}{=} T_2$
2.

1.
forze agenti sulla carrucola

$$\begin{cases} \vec{F}_{est} = M\vec{a}_{CM} = 0 \quad \text{(fermo)} \\ \vec{T}_1 + \vec{T}_2 + \vec{P}_c + \vec{R}_c = 0 \\ \vec{\tau} = I\vec{\alpha} \end{cases}$$

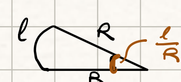
la fa girare

$\vec{\tau}_{T_1} = -RT_1 \vec{u}_z$
$\vec{\tau}_{T_2} = RT_2 \vec{u}_z$
$\Rightarrow \vec{\tau} = R(T_2 - T_1)\vec{u}_z$

se $T_1 = T_2 \rightarrow \vec{\tau} = I\vec{\alpha} = \vec{0} \Rightarrow$ carrucola non ruota

$\hookrightarrow T_1 \neq T_2$ il testo dice che la carrucola ruota

2.
$$\begin{cases} T_2 - mg = ma \\ T_1 = F_0 \\ R(T_2 - F_0) = -I\alpha \end{cases} \qquad I = \frac{1}{2}MR^2$$



! la fune non scivola sulla carrucola $\Rightarrow$ per uno spostamento l si ha una corrispondente rotaz. della carrucola di angolo θ $\Rightarrow \alpha = \frac{a}{R}$

$$\ldots \rightarrow \quad a = \frac{F_0 - mg}{I + mR^2} = 2{,}6\ m/s^2$$

3. $\mathcal{L} = \vec{F} \cdot \Delta\vec{s} = F_0 \ell = F_0 \left(\frac{1}{2} a \Delta t^2\right) = 4{,}88\ J$