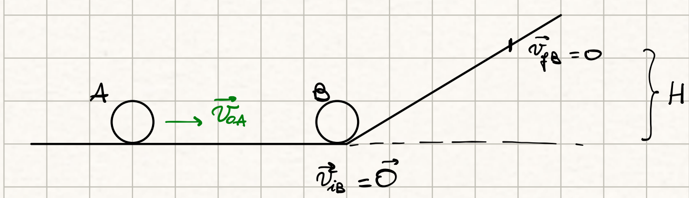
6.1



a) ?H se urto elastico

b) ?H se urto completamente anelastico

a) URTO ELASTICO : - conservazione $\vec{Q}$
- conservazione E_k , E_{tot}
- conservazione $\vec{L}$

$$\begin{cases} \Delta\vec{Q} = 0 \\ \Delta E_k = 0 \end{cases} \Rightarrow \begin{matrix} \vec{Q}_i = \vec{Q}_f \\ E_{ki} = E_{kf} \end{matrix} \Rightarrow \begin{cases} \cancel{m}v_{0A} + 0 = \cancel{m}v_A + \cancel{m}v_B \\ \cancel{\frac{1}{2}m}v_{0A}^2 + 0 = \cancel{\frac{1}{2}m}v_A^2 + \cancel{\frac{1}{2}m}v_B^2 \end{cases} \Rightarrow$$

$$\begin{cases} v_{0A} = v_A + v_B \\ v_{0A}^2 = v_A^2 + v_B^2 \end{cases} \rightarrow \begin{cases} v_A = v_B - v_{0A} \\ \cancel{v_{0A}^2} = v_B^2 + \cancel{v_{0A}^2} - 2v_Bv_{0A} + v_B^2 \end{cases} \rightarrow \begin{cases} v_A = 0 \\ v_B = v_{0A} \end{cases}$$

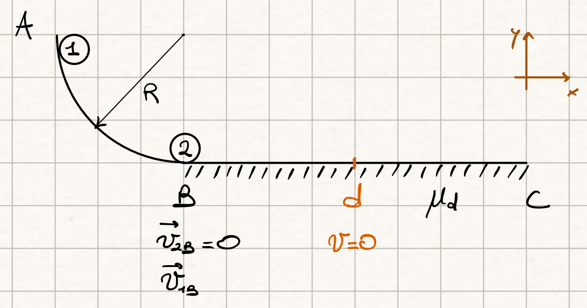
$$\cdot\ \Delta E_{mB} = 0 \Rightarrow \frac{1}{2}\cancel{m}v_B^2 = \cancel{m}gH \Rightarrow H = \frac{v_B^2}{2g} = \frac{v_{0A}^2}{2g}$$

b) URTO COMPLETAMENTE ANAELASTICO : - conservazione $\vec{Q}$
- conservazione E_{tot}
- conservazione $\vec{L}$

$$\Delta Q = 0 \rightarrow \cancel{m}v_{0A} = 2\cancel{m}v \Rightarrow v = \frac{v_{0A}}{2}$$

$$\Delta E_m = 0 \Rightarrow \frac{1}{2}\cancel{2m}v^2 = \cancel{2m}gH \Rightarrow H = \frac{v^2}{2g} = \frac{v_{0A}^2}{4\cdot 2g} = \frac{v_{0A}^2}{8g}$$

6.2



$R = 3m$
$\mu_d = 0,3$
$m_1 = 2kg$
$m_2 = 3kg$

? $\vec{v}$ dopo l'urto
? d

URTO COMPLETAMENTE ANAELASTICO $\rightarrow$ $\Delta E_{tot} = 0$
$\Delta\vec{Q} = 0$
$\Delta\vec{L} = 0$

$$\cdot\ \Delta E_m = 0 \text{ in } AB : \quad \cancel{m_1}gR = \frac{1}{2}\cancel{m_1}v_{1B}^2 \Rightarrow v_{1B} = \sqrt{2gR}$$

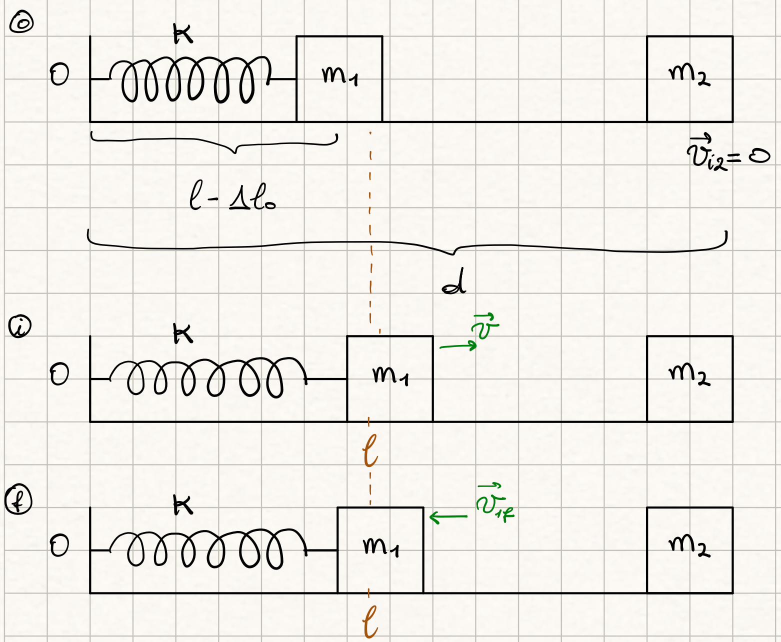
$$\cdot\ \Delta\vec{Q} = 0 : \quad \vec{Q}_i = \vec{Q}_f \Rightarrow m_1v_{1B} + 0 = (m_1+m_2)v$$

$$\Rightarrow v = \frac{m_1v_{1B}}{m_1+m_2} = \frac{m_1\sqrt{2gR}}{m_1+m_2} = \frac{2}{2+3}\sqrt{2\cdot 9{,}8\cdot 3} = 3{,}07\ m/s$$

$$\cdot\ \Delta E_{mBC} = W_{attr} \Rightarrow \Delta E_{kBC} + \underbrace{\cancel{\Delta E_{pBC}}}_{=0} = W_{attr} \Rightarrow \Delta E_{kBC} = W_{attr} \rightarrow 0 \cancel{-} \frac{1}{2}(m_1+m_2)v^2 = \cancel{-}\mu_d R_N d$$

$$\frac{1}{2}\cancel{(m_1+m_2)}v^2 = \mu_d \cancel{(m_1+m_2)} g\, d \quad\Rightarrow\quad d = \frac{v^2}{2g\mu_d} = \frac{3{,}07^2}{2\cdot 9{,}8\cdot 0{,}3} = 1{,}6\ m$$

6.3



$m_1 = 1\ kg = m_2$
$\ell = 50\ cm = 0{,}5\ m$ (a riposo)
$K = 500\ N/m$
$\Delta\ell_0 = 5\ cm = 0{,}05\ m$ (compressione)

URTO ELASTICO
$d = 53\ cm = 0{,}53\ m$

? $\vec{v}_{f2}$
? A (ampiezza di oscillazione) di m_1

ⓞ - ⓘ : $\Delta E_m = 0 \Rightarrow \cancel{\frac{1}{2}}k\,\Delta\ell_0^2 = \cancel{\frac{1}{2}}m_1 v^2 + \cancel{\frac{1}{2}}k(d-\ell)^2 \Rightarrow$

$$\Rightarrow v = \sqrt{\frac{k[\Delta\ell_0^2-(d-\ell)^2]}{m_1}} = \sqrt{\frac{500(0{,}05^2-0{,}03^2)}{1}} = 0{,}89\ m/s$$

URTO : $\Delta\vec{Q} = 0 \Rightarrow m_1 v = -m_1 v_{1f} + m_2 v_{2f} \Rightarrow \cancel{m}v = \cancel{m}(-v_{1f}+v_{2f}) \Rightarrow v_{1f} = v_{2f} - v$

$\Delta E_K = 0 \Rightarrow \cancel{\frac{1}{2}}m_1 v^2 = \cancel{\frac{1}{2}}m_1 v_{1f}^2 + \cancel{\frac{1}{2}}m_1 v_{2f}^2 \Rightarrow \cancel{m}v^2 = \cancel{m}(v_{1f}^2 + v_{2f}^2)$

$\Rightarrow \cancel{v^2} = \cancel{v^2} + v_{2f}^2 - 2v v_{2f} + v_{2f}^2 \Rightarrow v_{2f}^2 - v v_{2f} = 0 \Rightarrow v_{2f}(v_{2f} - v) = 0$

$$\Rightarrow \begin{cases} v_{1f} = 0 \\ v_{2f} = v = 0{,}89\ m/s \end{cases}$$

Dopo l'urto m_1 ha velocità nulla, ma è soggetta alla forza elastica, perciò riprende il suo moto armonico con ampiezza come nel caso ⓞ $\Rightarrow A = d - \ell = 0{,}03\ m$

6.5

$m_B = 2m_A$

$|\vec{v}_A| = |\vec{v}_B|$

$\vec{v}_A = -\vec{v}_B$

Ek dopo l'urto : $E_{kf} = \frac{1}{6} m_A (\vec{v}_A - 2\vec{v}_B)^2 = \frac{1}{6} m_A (\vec{v}_A - 2\vec{v}_A)^2 = \frac{1}{6} m_A v_A^2$

? tipo di urto

Ek prima dell'urto : $E_{Ki} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{\cancel{2}} \cancel{2} m_A v_A^2 = \left(\frac{1}{2} + 1\right) m_A v_A^2 = \frac{3}{2} m_A v_A^2$

⇒ Ek non si conserva : NO URTO ELASTICO $\neq E_{kf}$

→ $\Delta E_K < 0$: URTO ANAELASTICO ENDOTERMICO

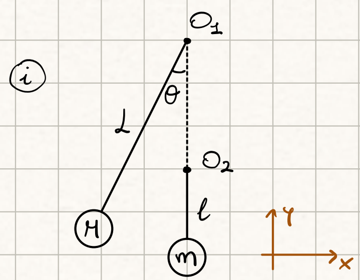
⇒ $\Delta \vec{Q} = 0$ se fosse completamente anaelastico →

$$m_A v_A - m_B v_B = (m_A + m_B) v$$
$$m_A v_A - 2m_A v_A = 3m_A v$$
$$-m_A v_A = 3 m_A v \Rightarrow v = -\frac{1}{3} v_A$$

⇒ $E_{kf} = \frac{1}{2} 3m_A \left(-\frac{1}{3} v_A\right)^2 = \frac{3}{2} m_A \frac{1}{9} v_A^2 = \frac{1}{6} m v_A^2$

↳ È URTO COMPLETAMENTE ANAELASTICO

6.7



$M = 2\,kg$

$L = 50\,cm = 0{,}5\,m$

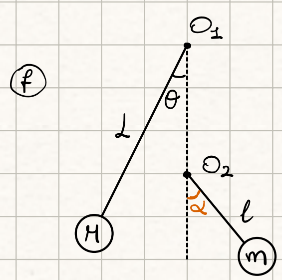
$\theta = 30°$

$m = 1\,kg$

$l = 20\,cm = 0{,}2\,m$

? α

URTO ELASTICO



Conservazione Em per M : $\cancel{M} g L (1 - \cos\theta) = \frac{1}{2} \cancel{M} v_1^2$

⇒ $v_1 = \sqrt{2gL(1-\cos\theta)}$

URTO :
$$\begin{cases} \Delta \vec{Q} = 0 : & M\vec{v}_1 = M\vec{v}_1' + m\vec{v}_2' \\ \Delta E_K = 0 : & \frac{1}{\cancel{2}} M v_1^2 = \frac{1}{\cancel{2}} M v_1'^2 + \frac{1}{\cancel{2}} m v_2'^2 \end{cases}$$

direzione lungo asse x

$$\Rightarrow \begin{cases} v_2' = \frac{M(v_1 - v_1')}{m} \\ Mv_1^2 = Mv_1'^2 + \cancel{m} \frac{M^2}{m^{\cancel{2}}} (v_1 - v_1')^2 \end{cases}$$

→ $\cancel{M}(v_1^2 - v_1'^2) = \frac{M^{\cancel{2}}}{m}(v_1 - v_1')^2$ → $(v_1 + v_1')\cancel{(v_1 - v_1')} = \frac{M}{m}(v_1 - v_1')^{\cancel{2}}$ se $v_1 \neq v_1'$ (se fossero = non ci sarebbe urto)

→ $\left(\frac{M}{m} - 1\right) v_1 - \left(\frac{M}{m} + 1\right) v_1' = 0$ ⇒ $v_1' = \frac{\left(\frac{M}{m} - 1\right) v_1}{\frac{M}{m} + 1} = \frac{M-m}{\cancel{m}} \frac{\cancel{m}}{M+m} v_1 = \frac{M-m}{M+m} v_1$

$$\Rightarrow v_2' = \frac{M}{m}\left[v_1 - \left(\frac{M-m}{M+m}\right)v_1\right] = \frac{M}{m}\left(1-\frac{M-m}{M+m}\right)v_1 = \frac{M}{m}\left(\frac{\cancel{M}+m\cancel{-M}+m}{M+m}\right)v_1 = \frac{M\cdot 2\cancel{m}}{\cancel{m}(M+m)}v_1$$

$$= \frac{2M}{M+m}v_1 = \frac{2M}{M+m}\sqrt{2gL(1-\cos\theta)}$$

POST-URTO: $\Delta E_{m_2} = 0 \Rightarrow \frac{1}{2}m v_2'^2 = mg\ell(1-\cos\alpha)$

$$\Rightarrow \frac{1}{\cancel{2}}\cancel{m}\frac{4M^2}{(M+m)^2}\cancel{2g}L(1-\cos\theta) = \cancel{m}\cancel{g}\ell(1-\cos\alpha)$$

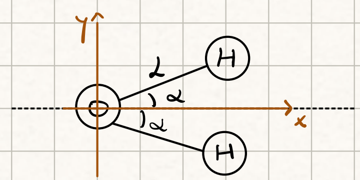
$$\Rightarrow \cos\alpha = \frac{1}{\ell}\left[\ell - \frac{4M^2L(1-\cos\theta)}{(M+m)^2}\right] = 1-\frac{4M^2L(1-\cos\theta)}{\ell(M+m)^2} = 1-\frac{4\cdot\cancel{4}^{\,2}\cdot\cancel{\frac{1}{2}}\left(1-\frac{\sqrt{3}}{2}\right)}{\frac{1}{5}\;9} =$$

$$= 1-\frac{\cancel{40}^{\,20}}{9}\,\frac{2-\sqrt{3}}{\cancel{2}} = \frac{9-40+20\sqrt{3}}{9}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{9-40-20\sqrt{3}}{9}\right) = 66{,}13°$$

ESERCITAZIONE I:

1)



$m_O = 16\, m_H$
$\alpha = 53°$
$L = 0{,}1\cdot 10^{-9}\, m$

?CM

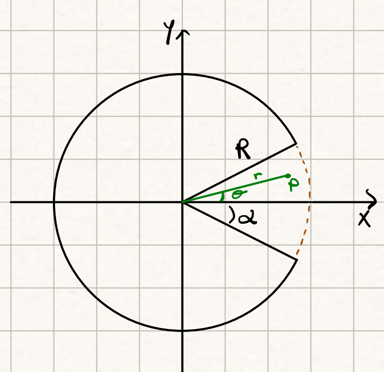
$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$; $x_{CM} = \frac{\sum m_i x_i}{\sum m_i}$; $y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$

$$\Rightarrow x_{CM} = \frac{\sum_{i=1}^{3} m_i x_i}{\sum_{i=1}^{3} m_i} = \frac{m_H(L\cos\alpha)+m_H(L\cos\alpha)+0}{2m_H+16m_H} = \frac{\cancel{2m_H}L\cos\alpha}{\cancel{18}^{\,9}\,\cancel{m_H}} = \frac{10^{-10}\cos 53°}{9} = 6{,}69\cdot 10^{-12}\, m = 6{,}69\ pm$$

$$\rightarrow y_{CM} = \frac{\sum_{i=1}^{3} m_i y_i}{\sum_{i=1}^{3} m_i} = \frac{m_H(L\sin\alpha)-m_H(L\sin\alpha)+0}{2m_H+16m_H} = 0\, m$$

CM $(6{,}7\ ;\ 0)$

2)

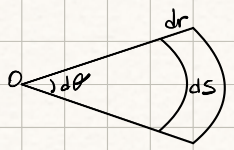


?CM (asse x è l'asse di simmetria : $y_{CM} = 0$)

$\sigma = \text{cost}$; $\sigma = \frac{M_\alpha}{S_\alpha} \Rightarrow M_\alpha = \sigma\cdot S_\alpha = \sigma(\alpha\cdot R^2)$

$\begin{cases} x_P = r\cos\theta \\ y_P = r\sin\theta \end{cases}$; $0 \leq r \leq R$; $-\alpha \leq \theta \leq +\alpha$

$$CM_\alpha = \begin{cases} x_\alpha = \dfrac{1}{M_\alpha}\displaystyle\int_S x_p \, dm \\ y_\alpha = \dfrac{1}{M_\alpha}\displaystyle\int_S y_p \, dm = 0 \end{cases}$$ perchè asse di simmetria



$$dS = r\, d\theta \cdot dr$$
$$dm = \sigma dS = \sigma r d\theta dr$$

$$x_\alpha = \frac{1}{\sigma \alpha R^2}\int_S x_p \, \sigma r d\theta dr = \frac{1}{\cancel{\sigma}\alpha R^2}\cancel{\sigma}\int_S r\cos\theta \, r d\theta dr = \frac{1}{\alpha R^2}\int_{-\alpha}^{+\alpha}\int_0^R r^2\cos\theta \, d\theta dr =$$

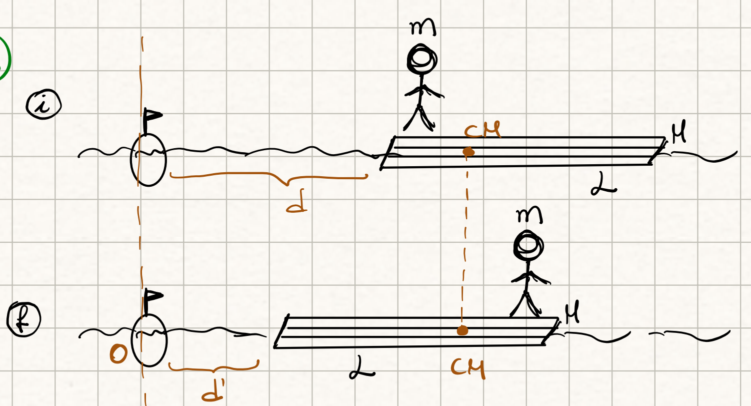
$$\frac{1}{\alpha R^2}\int_{-\alpha}^{+\alpha}\cos\theta \, d\theta \int_0^R r^2 dr = \frac{1}{\alpha R^2}\left[\sin\theta\right]_{-\alpha}^{+\alpha}\left[\frac{r^3}{3}\right]_0^R = \frac{1}{\alpha \cancel{R^2}}\left[\sin\alpha - \sin(-\alpha)\right]\left[\frac{R^{\cancel{3}}}{3}\right] = \frac{2\sin\alpha}{3\alpha}R$$

$$\Rightarrow CM_\alpha\left(\frac{2\sin\alpha}{3\alpha}R \, ; \, 0\right) \quad ; \quad CM_0(0 \, ; \, 0)$$

$$CM = \frac{M_0 CM_0 - M_\alpha CM_\alpha}{M_0 - M_\alpha} \rightarrow \begin{cases} x_{CM} = \left(0 - \cancel{\sigma R^2 \alpha}\,\dfrac{2\sin\alpha}{3\cancel{\alpha}}R\right)\dfrac{1}{\cancel{\sigma R^2}(\pi-\alpha)} = -\dfrac{2\sin\alpha}{3(\pi-\alpha)}R \\ y_{CM} = 0 \end{cases}$$

$$\rightarrow CM\left(-\frac{2}{3}\frac{\sin\alpha}{\pi-\alpha}R \, ; \, 0\right)$$

3)



$m = 50\,kg$
$M = 100\,kg$
$L = 3\,m$
$d = 2\,m$

? d'

NO FORZE ESTERNE $\rightarrow$ sistema isolato $\rightarrow \Delta\vec{Q} = 0 \Rightarrow \Delta\vec{v}_{CM} = \dfrac{\Delta\vec{Q}}{M_{tot}} = 0 \Rightarrow \vec{v}_{CM_i} = \vec{v}_{CM_f} = 0$

$\Rightarrow$ CONSERVAZIONE DEL CENTRO DI MASSA : $CM_i = CM_f \Rightarrow x_{CM} = x'_{CM}$

$$\begin{cases} x_{CM} = \dfrac{1}{M+m}\displaystyle\sum_{i=1}^{2} m_i x_i = \dfrac{1}{M+m}\left[md + M\left(d + \dfrac{L}{2}\right)\right] \\ x'_{CM} = \dfrac{1}{M+m}\left[m(d'+L) + M\left(d' + \dfrac{L}{2}\right)\right] \end{cases} \Rightarrow x_{CM} = x'_{CM}$$

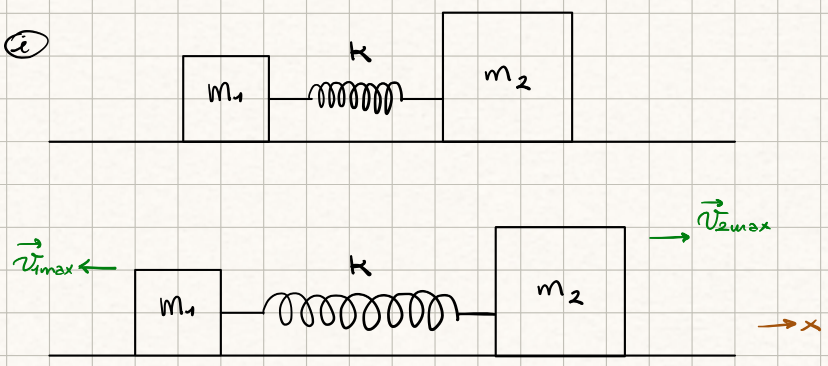
$$\Rightarrow \cancel{\frac{1}{M+m}}\left[md+M\left(d+\frac{L}{2}\right)\right] = \cancel{\frac{1}{M+m}}\left[m(d'+L)+M\left(d'+\frac{L}{2}\right)\right]$$

$$\Rightarrow md+Md+\cancel{M\frac{L}{2}} = md'+mL+Md'+\cancel{M\frac{L}{2}}$$

$$\Rightarrow d(m+M) = d'(m+M)+mL$$

$$\Rightarrow d' = d-\frac{mL}{m+M} = 2-\frac{\cancel{50}\cdot 3}{\cancel{150}} = 1\,m$$

4)



$m_2 = 2m_1$

L lunghezza a riposo

l_0 compressione

? $\vec{v}_{1max}$, $\vec{v}_{2max}$

le due masse raggiungono la propria velocità massima quando la molla raggiunge la sua lunghezza a riposo, infatti a L le due masse sono all'a_{max}, oltre L iniziano a rallentare

→ solo forze conservative, no forze esterne :

- $\Delta E_m = 0$
- $\Delta \vec{Q} = 0$

$$\Rightarrow \begin{cases} \cancel{\frac{1}{2}}K l_0^2 = \cancel{\frac{1}{2}} m_1 v_{1max}^2 + \cancel{\frac{1}{2}} m_2 v_{2max}^2 \\ m_1 \vec{v}_{1max} + m_2 \vec{v}_{2max} = 0 \end{cases}$$ perché in (i) la $\vec{Q}_i = 0$

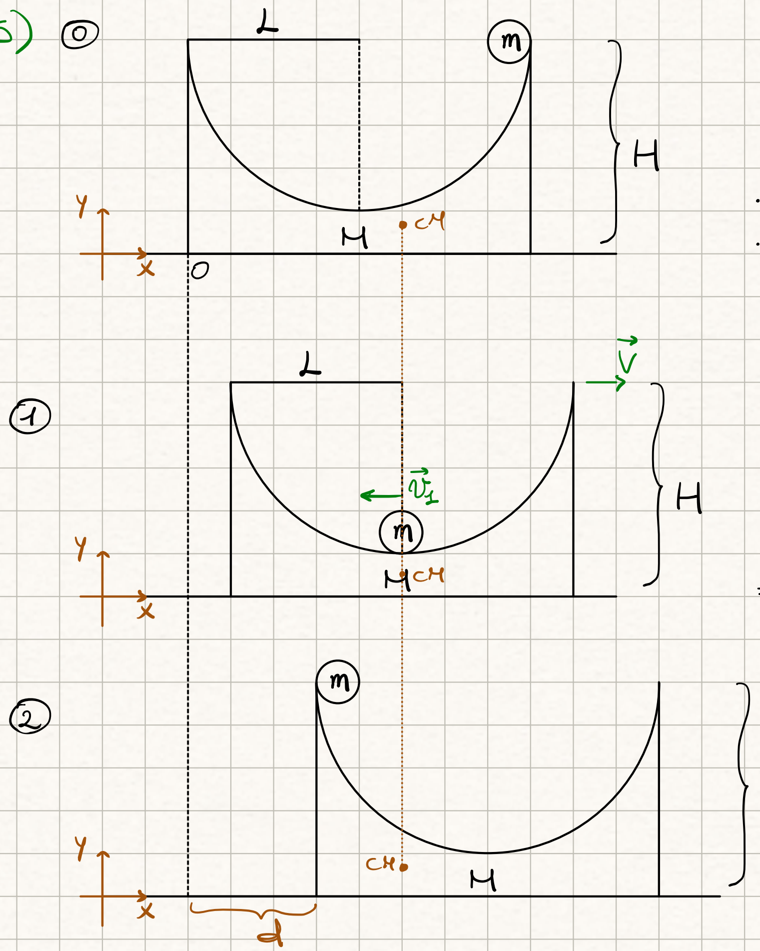
$\Rightarrow m_1 v_{1max} + m_2 v_{2max} = 0$ moto lungo asse x

$\Rightarrow v_{1max} = -\frac{m_2}{m_1} v_{2max} = -\frac{2\cancel{m_1}}{\cancel{m_1}} v_{2max} = -2 v_{2max}$

$$\Rightarrow \begin{cases} Kl_0^2 = m_1 4 v_{2max}^2 + m_2 v_{2max}^2 \\ v_{1max} = -2v_{2max} \end{cases} \Rightarrow Kl_0^2 = m_1 v_{2max}^2(4+2) \Rightarrow v_{2max} = \sqrt{\frac{Kl_0^2}{6m_1}} = l_0\sqrt{\frac{K}{6m_1}}$$

$$\Rightarrow v_{1max} = -2 l_0\sqrt{\frac{K}{6m_1}} = -l_0\sqrt{\frac{\cancel{4}^{2}K}{\cancel{6}_{3}m_1}} = -l_0\sqrt{\frac{2K}{3m_1}}$$

5)



? $\vec{v}_1$
? d

- solo forze conservative : $\Delta E_m = 0$
- NO forze esterne : $\Delta \vec{Q} = 0$; CM fermo

$$\vec{Q}_i = \vec{Q}_f \; ; \; \vec{Q}_i = 0 \Rightarrow \vec{Q}_f = 0$$

$$\begin{cases} m\vec{v}_1 + M\vec{V} = 0 \\ mgH = \frac{1}{2}m v_1^2 + \frac{1}{2}MV^2 \end{cases} \xRightarrow[\text{asse } x]{\text{moto lungo}} m v_1 + MV = 0$$

$$\Rightarrow \begin{cases} V = -\frac{m v_1}{M} \\ \cancel{m}gH = \frac{1}{2}\cancel{m} v_1^2 + \frac{1}{2}\cancel{M}\frac{m^{\cancel{2}}}{M^{\cancel{2}}} v_1^2 \end{cases}$$

$$\Rightarrow \frac{1}{2}v_1^2\left(1+\frac{m}{M}\right) - gH = 0 \Rightarrow \boxed{v_1 = -\sqrt{\frac{2gH}{1+\frac{m}{M}}}}$$

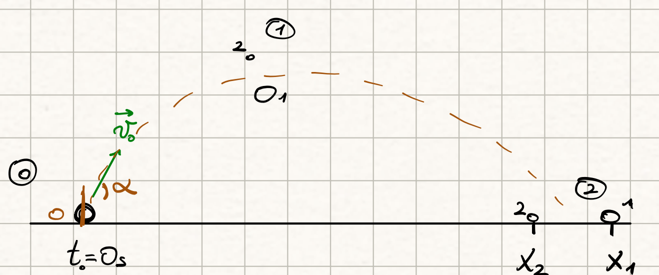
Calcolo il CM nel caso ⓪ : $x_{CM} = \frac{m \cdot 2L}{m+M}$

mentre il CM nel caso ② : $x'_{CM} = \frac{md + Md}{m+M} = d$

$\Rightarrow$ per la conservazione del centro di massa si ha : $x_{CM} = x'_{CM}$

$$\Rightarrow \frac{m \cdot 2L}{m+M} = d \qquad \Rightarrow \qquad \boxed{d = \frac{2mL}{m+M}}$$

6)



$v_0 = 10\,m/s$
$\alpha = 60°$
$m_1 = \frac{2}{3}m$; $m_2 = \frac{1}{3}m$
$x_2 = 11\,m$
? x_1

No forze est., √ forze conservative $\rightarrow$ $\Delta E_m = 0$; $\Delta \vec{Q} = 0$; CM fermo

② : $x_{CM} = \frac{\frac{2}{3}m x_1 + \frac{1}{3}m x_2}{m} = \frac{2}{3}x_1 + \frac{1}{3}x_2 \Rightarrow x_1 = \left(x_{CM} - \frac{1}{3}x_2\right)\frac{3}{2}$

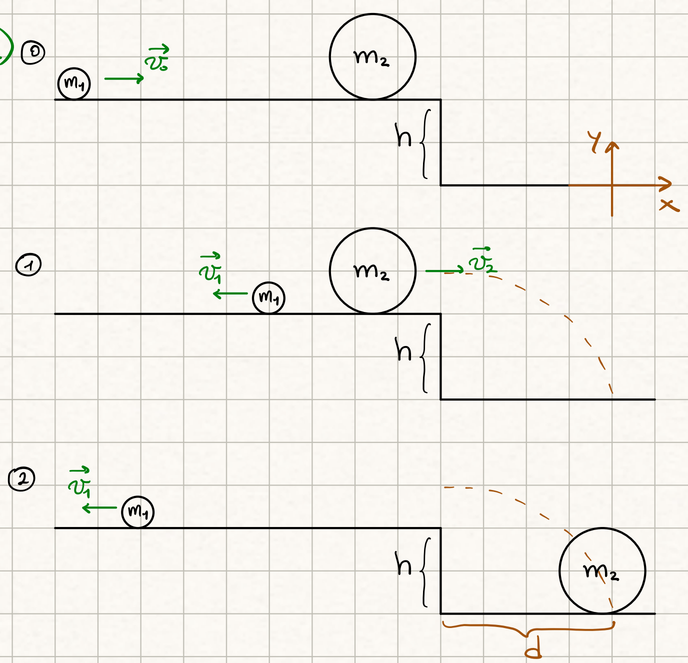
$x_{CM} = \text{gittata} = \frac{v_0^2 \sin 2\alpha}{g}$

$$\begin{cases} x = v_0 \cos\alpha \cdot t \\ y = v_0 \sin\alpha t - \frac{1}{2} g t^2 = 0 \end{cases} \rightarrow t = 0 \wedge t = \frac{2 v_0 \sin\alpha}{g} \rightarrow x_2 = \frac{2 v_0^2 \sin\alpha\cos\alpha}{g} = \frac{v_0^2 \sin 2\alpha}{g}$$

$$\rightarrow x_1 = \left(\frac{v_0^2 \sin 2\alpha}{g} - \frac{1}{3} x_2\right)\frac{3}{2} = \left(\frac{100 \sin 120^\circ}{9{,}8} - \frac{1}{3} 11\right)\frac{3}{2} = \left(\frac{\cancel{100}^{50}}{9{,}8}\frac{\sqrt{3}}{\cancel{2}} - \frac{1}{3} 11\right)\frac{3}{2} = 7{,}75 \text{ m}$$

ESERCITAZIONE 2

1)



$m_2 = 3 m_1$

$h = 0{,}5$ m

$? v_0 \mid d = 5$ m

→ URTO ELASTICO

$\Delta \vec{Q} = 0$

$\Delta E_m = 0$ (solo f. cons.)

$\Delta E_K = 0$

$$\begin{cases} m_1 \vec{v}_0 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \\ \cancel{\frac{1}{2}} m_1 v_0^2 = \cancel{\frac{1}{2}} m_1 v_1^2 + \cancel{\frac{1}{2}} m_2 v_2^2 \end{cases}$$ direzione lungo asse x

$$\rightarrow \begin{cases} \cancel{m_1} v_0 = \cancel{m_1} v_1 + 3\cancel{m_1} v_2 \\ \cancel{m_1} v_0^2 = \cancel{m_1} v_1^2 + 3\cancel{m_1} v_2^2 \end{cases} \Rightarrow \begin{cases} v_0 = v_1 + 3 v_2 \\ v_0^2 = v_1^2 + 3 v_2^2 \end{cases}$$

moto parabolico di m_2 :

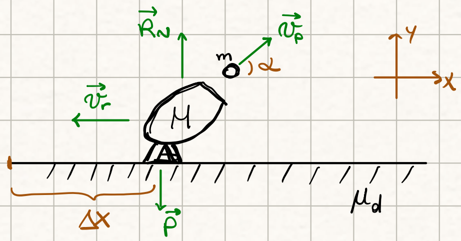
$$\begin{cases} x = v_2 t \\ y = h - \frac{1}{2} g t^2 = 0 \Rightarrow t = \sqrt{\frac{2h}{g}} \end{cases} \rightarrow d = v_2 \sqrt{\frac{2h}{g}} \rightarrow v_2 = d\sqrt{\frac{g}{2h}}$$

$$\Rightarrow \begin{cases} v_1 = v_0 - 3 v_2 \\ v_0^2 = (v_0 - 3 v_2)^2 + 3 v_2^2 \\ v_2 = d\sqrt{\frac{g}{2h}} \end{cases} \Rightarrow \cancel{v_0^2} = \cancel{v_0^2} + 9 v_2^2 - 6 v_0 v_2 + 3 v_2^2 \Rightarrow 4 v_2^2 - 2 v_0 v_2 = 0 \Rightarrow 2 v_2 (2 v_2 - v_0) = 0$$

$$\Rightarrow v_2 = 0 \wedge v_2 = \frac{v_0}{2}$$

$$\Rightarrow \frac{v_0}{2} = d\sqrt{\frac{g}{2h}} \Rightarrow v_0 = d\sqrt{\frac{2g}{h}} = 5\sqrt{\frac{2 \cdot 9{,}8}{1/2}} = 31{,}3 \text{ m/s}$$

2)



? Δx

? $\vec{I} = \int \vec{R}_N \, dt \; = \Delta \vec{Q}$

$$\vec{I} = \int_{t_0}^{t} \vec{R}_N \, dt = \int_{t_0}^{t} Mg \, dt = \int_{\vec{Q}_0}^{\vec{Q}} d\vec{Q} = \vec{Q} - \vec{Q}_0$$

$\Delta \vec{Q} = 0 \; ; \; \vec{Q}_0 = 0 \; : \qquad M\vec{v}_r + m\vec{v}_p = 0$

$$\Rightarrow M v_r \vec{u}_x + m\left(v_p \cos\alpha \, \vec{u}_x + v_p \sin\alpha \, \vec{u}_y\right) = 0$$

$$\Rightarrow M v_r + m v_p \cos\alpha = 0 \Rightarrow v_r = -\frac{m}{M} v_p \cos\alpha$$

$$\begin{cases} W_{nc} = \int_{\Delta x}^{0} -F_{attr} \cdot dx = \int_{\Delta x}^{0} -\mu_d Mg \, dx = +\mu_d Mg \Delta x \\ = \Delta E_m = \Delta E_K + \cancel{\Delta E_p} = \Delta E_K = \frac{1}{2} M v_r^2 = \frac{1}{2}\frac{m^2}{M} v_p^2 \cos^2\alpha \end{cases}$$

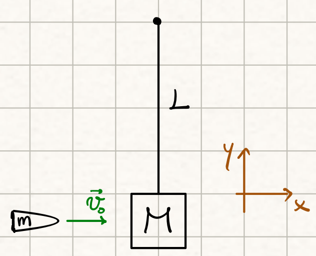
$$\rightarrow \mu_d Mg \Delta x = \frac{1}{2}\frac{m^2}{M} v_p^2 \cos^2\alpha$$

$$\Rightarrow \boxed{\Delta x = \left(\frac{m}{M}\right)^2 \frac{v_p^2 \cos^2\alpha}{2\mu_d g}}$$

$\vec{I} = \vec{Q} - \vec{Q}_0 \; ; \; \vec{Q}_0 = 0$ l'impulso della $\vec{R}_N$ ha direzione lungo y, dunque $\Rightarrow$

$$\vec{I} = Q_y \vec{u}_y = \boxed{m v_p \sin\alpha \, \vec{u}_y}$$

3)



a) ? v_{0min} | il blocco compie un giro
? E_{diss}

b) ? se asta rigida

a) URTO ANAELASTICO :
- $\Delta E_m \neq 0$
- $\Delta \vec{Q} = 0$
- $\Delta \vec{L} = 0$

$\vec{Q}_f = \vec{Q}_i \; : \quad (m+M)\vec{v} = m\vec{v}_0 \quad \Longrightarrow$ direzione $\vec{u}_x \qquad v = \frac{m}{m+M} v_0$

Affinché il blocco compia un giro intero : $\vec{T} + (m+M)\vec{g} = (m+M)\vec{a}$

$\Rightarrow \vec{T}$ e $\vec{g}$ hanno direzione $\vec{u}_y$: $\quad T + (m+M)g = (m+M)\frac{v_1^2}{L} \rightarrow T = (m+M)\left[\frac{v_1^2}{L} - g\right] \geq 0$

caso limite : $\quad (m+M)\left[\frac{v_1^2}{L} - g\right] = 0 \rightarrow v_1 = \sqrt{gL}$

$$\Delta E_m = 0 : \quad \frac{1}{2}(\cancel{m+M})v^2 = (\cancel{m+M})g2L + \frac{1}{2}(\cancel{m+M})v_1^2$$

$$\frac{1}{2}\left(\frac{m}{M+m}\right)^2 v_0^2 = 2gL + \frac{1}{2}gL \Rightarrow \left(\frac{m}{M+m}\right)^2 v_0^2 = 5gL \Rightarrow v_{0\,min} = \frac{M+m}{m}\sqrt{5gL}$$

$$E_{diss} = -\Delta E_K = -\frac{1}{2}(M+m)v^2 + \frac{1}{2}mv_0^2 = -\frac{1}{2}(\cancel{M+m})\frac{m^2}{(M+m)^{\cancel{2}}}v_0^2 + \frac{1}{2}mv_0^2 =$$

$$= \frac{1}{2}\left(-\frac{m^{\cancel{2}}}{(\cancel{M+m})}\frac{(M+m)^{\cancel{2}}}{\cancel{m^2}}5gL + \cancel{m}\frac{(M+m)^2}{m^{\cancel{2}}}5gL\right) = \frac{5}{2}gL\left(-(M+m) + \frac{(M+m)^2}{m}\right) =$$

$$= \frac{5}{2}gL(M+m)\left[-1 + \frac{M+m}{m}\right] = \frac{5}{2}gL(M+m)\frac{M+\cancel{m}-\cancel{m}}{m} = \frac{5}{2}gL\frac{M}{m}(M+m)$$

b) se asta rigida: $\vec{v}_1 \geq 0 \Rightarrow v_{1min} = 0$

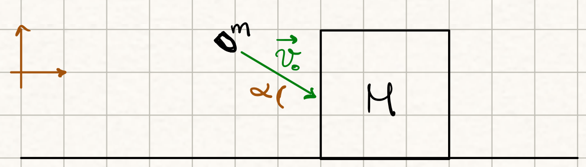
$$\to \Delta E_m = 0 : \quad \frac{1}{2}(\cancel{m+M})v^2 = (\cancel{m+M})g2L \Rightarrow v_{min} = \sqrt{4gL} = 2\sqrt{gL}$$

$$\to v_{0min} = \frac{m+M}{m}v_{min} = \frac{m+M}{m}2\sqrt{gL}$$

$$E_{diss} = -\frac{1}{2}(M+m)v^2 + \frac{1}{2}mv_0^2 = -\frac{1}{\cancel{2}}(M+m)\cancel{4}^{2}gL + \frac{1}{\cancel{2}}\cancel{m}\frac{m+M}{m^{\cancel{2}}}\cancel{4}^{2}gL = \frac{M+m}{m}2gL - (M+m)2gL$$

$$= 2gL(M+m)\left(\frac{1}{m} - 1\right) = 2gL(M+m)\left(\frac{1-m}{m}\right)$$

4)



URTO COMPLETAMENTE ANAELASTICO

? $\vec{v}$ dopo l'urto

? $\vec{I} = \int_{t_0}^{t} \vec{R}_N \, dt$

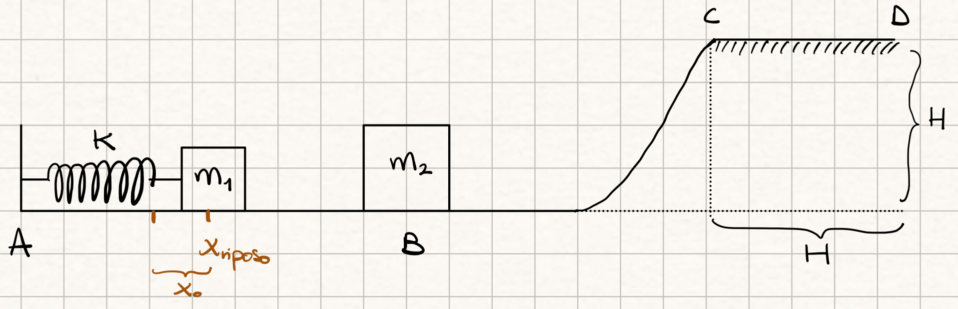
$$mv_0\cos\alpha = (m+M)v \Rightarrow v = \frac{m}{m+M}v_0\cos\alpha$$

$\vec{I} = \Delta\vec{Q} = \vec{Q} - \vec{Q}_0$, l'impulso della reazione vincolare agisce in direzione $\vec{u}_y$

$$\to \vec{Q}_0 = 0 \to \vec{I} = \vec{Q} \Rightarrow I\vec{u}_y = (\cancel{m+M})\frac{m}{\cancel{m+M}}v_{0y}\vec{u}_y = mv_0\sin\alpha\,\vec{u}_y$$

5)

0



$K = 400\,N/m$

$m_1 = 200\,g = 0{,}2\,kg$

compressione x_0 (cm)

URTO ELASTICO

$m_2 = 2m_1 = 0{,}4\,kg$

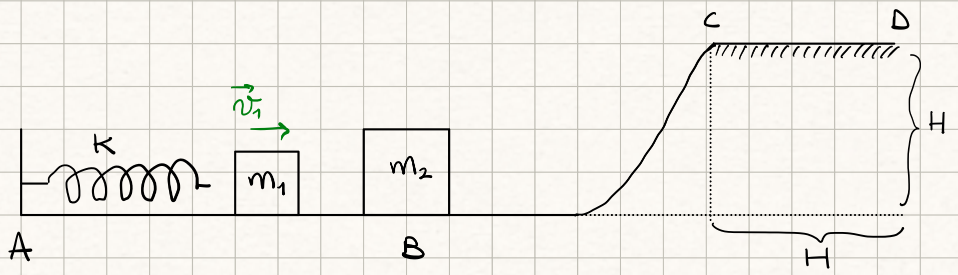
$x_1 = 3{,}5\,cm = 3{,}5 \cdot 10^{-2}\,m$

$\mu_d = 0{,}5$

? x_0

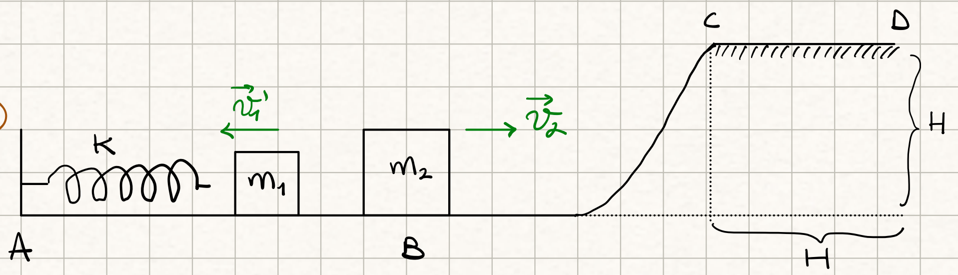
? H

1



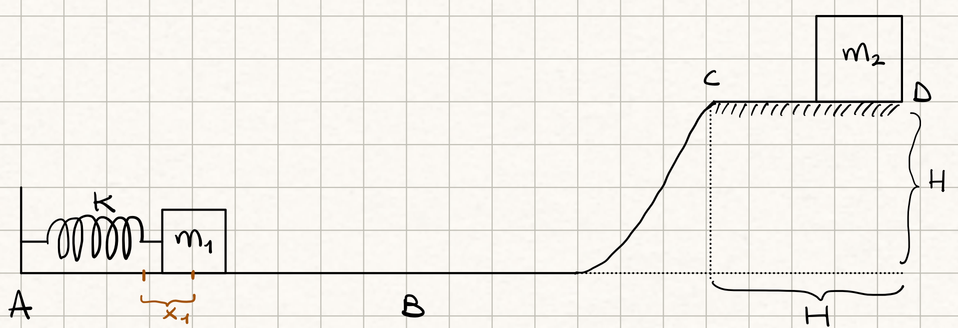
pre-urto

2



post-urto

3



· $\Delta E_m = 0$ tra 0 1 : $\quad \frac{1}{\cancel{2}} K x_0^2 = \frac{1}{\cancel{2}} m_1 v_1^2$

· URTO ELASTICO: $\Delta \vec{Q} = 0 \Rightarrow m_1 \vec{v}_1 = m_1 \vec{v}_1{}' + m_2 \vec{v}_2$ lungo asse x

$\Delta E_K = 0 \Rightarrow \frac{1}{\cancel{2}} m_1 v_1^2 = \frac{1}{\cancel{2}} m_1 v_1'^2 + \frac{1}{\cancel{2}} m_2 v_2^2$ $\}$ *

· $\Delta E_m = 0$ tra 2 3 : $\quad \frac{1}{\cancel{2}} m_1 v_1'^2 = \frac{1}{\cancel{2}} K x_1^2 \Rightarrow v_1'^2 = \frac{K}{m_1} x_1^2$

$$m_1 v_1 = m_1 v_1' + m_2 v_2 \Rightarrow v_2 = \frac{m_1}{m_2}(v_1 - v_1')$$

*

$$m_1 v_1^2 = m_1 v_1'^2 + m_2 v_2^2 \Rightarrow \cancel{m_1}(v_1^2 - v_1'^2) = \cancel{m_2} \frac{m_1^{\cancel{2}}}{m_2^{\cancel{2}}}(v_1 - v_1')^2 \Rightarrow (v_1 + v_1')\cancel{(v_1 - v_1')} = \frac{m_1}{m_2}(v_1 - v_1')^{\cancel{2}}$$

$$m_2 v_1 + m_2 v_1' = m_1 v_1 - m_1 v_1' \Rightarrow v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$\Rightarrow \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 v_1^2 = \frac{K}{m_1} x_1^2 \Rightarrow v_1^2 = \frac{(m_1 + m_2)^2}{(m_1 - m_2)^2} \frac{K}{m_1} x_1^2$$

$$\to x_0^2 = \frac{\cancel{m_1}(m_1+m_2)^2\,\cancel{K}}{\cancel{K}(m_1-m_2)^2\,\cancel{m_1}}\,x_1^2 = \left(\frac{m_1+2m_1}{m_1-2m_1}\right)^2 x_1^2 = 9x_1^2 \qquad \Rightarrow x_0 = 3x_1 = 3\cdot 3{,}5\cdot 10^{-2} = 10{,}5\cdot 10^{-2}\,m$$

$$v_2 = \frac{m_1}{m_2}(v_1-v_1') = \frac{m_1}{m_2}\,\frac{m_1+m_2}{m_1-m_2}\sqrt{\frac{K}{m_1}}\,x_1\left(1-\frac{m_1-m_2}{m_1+m_2}\right) = \frac{\cancel{m_1}}{2\cancel{m_1}}\,\frac{3\cancel{m_1}}{-\cancel{m_1}}\sqrt{\frac{K}{m_1}}\,x_1\left(1+\frac{1}{3}\right)$$

$$-\frac{\cancel{3}}{\cancel{2}}\sqrt{\frac{K}{m_1}}\,x_1\,\frac{\cancel{4}^{2}}{\cancel{3}} = -2\sqrt{\frac{K}{m_1}}\,x_1$$

$$\frac{1}{2}\cancel{m_2}v_2^2 = \cancel{m_2}gH + \frac{1}{2}\cancel{m_2}v_c^2 \quad\Rightarrow\quad v_c^2 = v_2^2 - 2gH$$

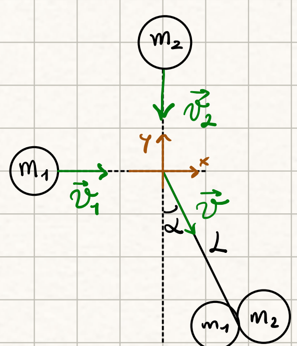
$$W_{NC} = \Delta E_m = \Delta E_K + \Delta E_p = -\frac{1}{2}m_2 v_c^2 = -\frac{1}{2}m_2\left(v_2^2-2gH\right)$$

$$= \int_C^D -\mu_d m_2 g\,dx = -\mu_d m_2 g H$$

$$\to \cancel{-}\frac{1}{2}\cancel{m_2}\left(v_2^2-2gH\right) = \cancel{-}\mu_d \cancel{m_2} g H \quad\Rightarrow\quad \frac{v_2^2}{2} - gH = \mu_d g H \quad\Rightarrow$$

$$H = \frac{1}{(1+\mu_d)g}\,\frac{v_2^2}{2} = \frac{1}{(1+\mu_d)g}\,\frac{\cancel{4}^{2}K}{\cancel{2}m_1}\,x_1^2 = \frac{2}{3}\,\frac{1}{9{,}8}\,2\,\frac{400}{0{,}2}\,0{,}035^2 = 0{,}33\,m$$

6)



$m_1 = 950\,kg$
$m_2 = 1100\,kg$
$L = 12\,m$
$\alpha = 30°$
$\mu_d = 0{,}4$

? $|\vec v_1|$, $|\vec v_2|$

? E_{diss}

URTO COMPLETAM. ANAELASTICO: $\quad m_1\vec v_1 + m_2\vec v_2 = (m_1+m_2)\vec v$

$$\begin{cases} \text{lungo } x: & m_1 v_1 = (m_1+m_2)\,v\sin\alpha \\ y: & m_2 v_2 = (m_1+m_2)\,v\cos\alpha \end{cases}$$

$$W_{NC} = \Delta E_m = \Delta E_K + \cancel{\Delta E_p} = -\frac{1}{2}(m_1+m_2)v^2$$

$$= -\mu_d(m_1+m_2)gL$$

$$\to \frac{1}{2}\cancel{(m_1+m_2)}v^2 = \mu_d\cancel{(m_1+m_2)}gL \quad\Rightarrow\quad v = \sqrt{2\mu_d g L}$$

$$\Rightarrow \begin{cases} m_1 v_1 = (m_1+m_2)\sqrt{2\mu_d g L}\,\sin\alpha \\ m_2 v_2 = (m_1+m_2)\sqrt{2\mu_d g L}\,\cos\alpha \end{cases}$$

$$\Rightarrow \begin{cases} v_1 = \dfrac{m_1+m_2}{m_1}\sqrt{2\mu_d g L}\,\sin\alpha = \dfrac{\overset{41}{\cancel{2050}}}{\underset{19}{\cancel{950}}}\sqrt{2\,\frac{2}{5}\,9.8\,12}\;\dfrac{1}{2} = \boxed{10{,}46\ m/s} \\[2ex] v_2 = \dfrac{m_1+m_2}{m_2}\sqrt{2\mu_d g L}\,\cos\alpha = \dfrac{\overset{41}{\cancel{2050}}}{\underset{22}{\cancel{1100}}}\sqrt{2\,\frac{2}{5}\,9.8\,12}\;\dfrac{\sqrt{3}}{2} = \boxed{15{,}65\ m/s} \end{cases}$$

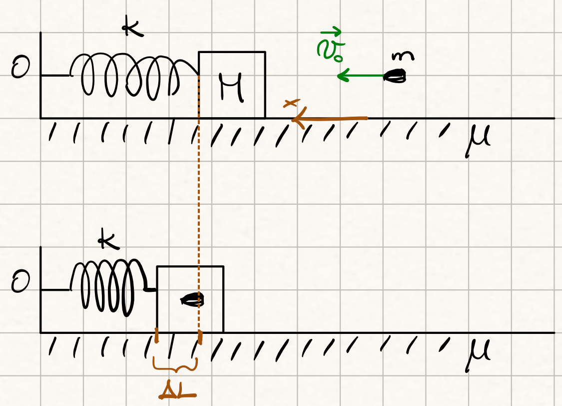
Durante l'urto:

$$E_{diss} = -\Delta E_K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}(m_1+m_2)\overset{\to\, 2\mu_d g L}{v^2} =$$

$$= \frac{\overset{480}{\cancel{950}}}{\cancel{2}}\,10{,}46^2 + \frac{\overset{550}{\cancel{1100}}}{\cancel{2}}\,15{,}65^2 - \frac{1}{\cancel{2}}\,2050\,\cancel{2}\,0{,}4\,9{,}8\,12 = \boxed{90{,}79\ kJ}$$

[LONGHI]

6.8



URTO COMPLETAMENTE ANAELASTICO

? v_0

- $m\vec{v}_0 = (m+M)\vec{V} \xrightarrow{\text{lungo } \hat{u}_x} m v_0 = (m+M)V$
- $\Delta E_m = W_{nc}$ (dopo l'urto):

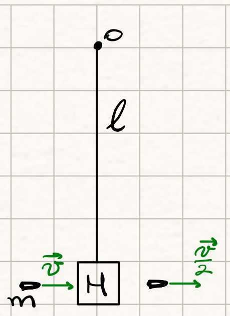
$$\frac{1}{2}k\,\Delta L^2 - \frac{1}{2}(m+M)V^2 = -\mu(m+M)g\,\Delta L$$

$$\begin{cases} V = \dfrac{m}{m+M}v_0 \\[2ex] \dfrac{1}{2}k\,\Delta L^2 + \mu(m+M)g\,\Delta L = \dfrac{1}{2}\cancel{(m+M)}\dfrac{m^2}{(m+M)^{\cancel{2}}}v_0^2 \end{cases}$$

$$v_0^2 = \frac{m+M}{m^2}k\,\Delta L^2 + 2\frac{(m+M)^2}{m^2}g\,\Delta L = \frac{m+M}{m^2}\Delta L\left(k\,\Delta L + 2\mu(m+M)g\right)$$

$$\rightarrow \boxed{v_0 = \sqrt{\frac{m+M}{m^2}\Delta L\left(k\,\Delta L + 2\mu(m+M)g\right)}}$$

6.8



?v_{min} | blocco compie un giro completo

1. $\Delta\vec{Q} = 0$: $m\vec{v} = M\vec{V} + m\frac{\vec{v}}{2}$

2. condiz. per compiere un giro intero: $T + Mg = M\frac{v_c^2}{\ell} \geq 0$

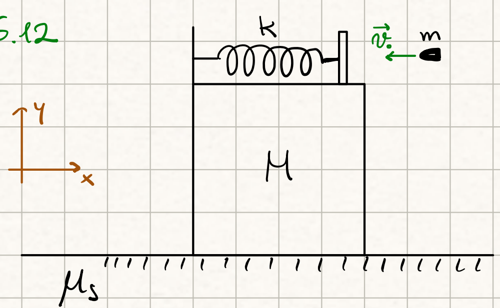
$\Rightarrow T = M\left(\frac{v_c^2}{\ell} - g\right) = 0 \Rightarrow v_c = \sqrt{g\ell}$

3. $\Delta E_m = 0$: $\cancel{M}g\overset{4}{2}\ell + \frac{1}{\cancel{2}}\cancel{M}v_c^2 = \frac{1}{\cancel{2}}\cancel{M}V^2$

$\Rightarrow 4g\ell + g\ell = V^2 \qquad \Rightarrow V = \sqrt{5g\ell}$

1. $mv = M\sqrt{5g\ell} + m\frac{v}{2} \qquad \Rightarrow v = 2\frac{M}{m}\sqrt{5g\ell}$

6.12



?$\vec{v}_0$ max | $\vec{v} \neq 0$

CONDIZIONE DI QUIETE:

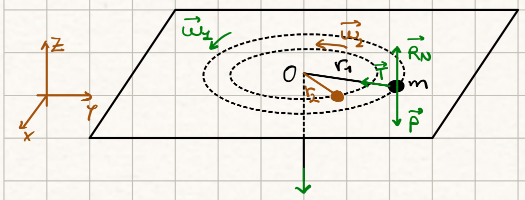
$F_{tot} \leq \mu_s(M+m)g \qquad \Rightarrow \cancel{(M+m)}a \leq \mu_s\cancel{(M+m)}g$

$\vec{F}_{el} + \vec{P} + \vec{R}_N = (m+M)\vec{a} \qquad \Rightarrow \begin{cases} -k\Delta x = (m+M)a_x \\ -P + R_N = (m+M)a_y = 0 \end{cases} \qquad \Rightarrow \begin{matrix} -k\Delta x \leq (m+M)\mu_s g \\ \Delta x \geq -\frac{(m+M)\mu_s g}{k} \end{matrix}$

condizione limite: $\Delta x = -\frac{(m+M)}{k}\mu_s g$

$\Delta E_m = 0$: $\frac{1}{\cancel{2}}k\Delta x^2 = \frac{1}{\cancel{2}}mv_0^2 \Rightarrow \cancel{k}\frac{(m+M)^2}{k^{\cancel{2}}}\mu_s^2 g^2 = mv_0^2 \rightarrow |\vec{v}_0| = (m+M)\mu_s g\sqrt{\frac{1}{mk}}$

6.18



$? \omega_2$

$$\vec{R}_N + \vec{P} + \vec{T} = m\vec{a} \quad \rightarrow \quad \begin{cases} z: & R_N - P = 0 \\ y: & T = ma \end{cases}$$

$$\vec{\tau}_o = \frac{d\vec{L}_o}{dt} \quad \Rightarrow \quad \vec{r}_1 \times \vec{F} = \vec{r}_1 \times (\cancel{\vec{P}} + \cancel{\vec{R}_N} + \vec{T}) = \vec{r}_1 \times \vec{T} = 0$$

$$\Rightarrow \frac{d\vec{L}_o}{dt} = 0 \quad \rightarrow \quad \vec{L}_o = \text{cost}$$

$$\vec{L}_{o_1} = \vec{L}_{o_2} : \quad \vec{r}_1 \times \vec{Q}_1 = \vec{r}_2 \times \vec{Q} \quad \rightarrow \quad \vec{r}_1 \times m\vec{v}_1 = \vec{r}_2 \times m\vec{v}_2$$

$$\Rightarrow r_1 \cancel{m} r_1 \omega_1 \vec{u}_z = r_2 \cancel{m} r_2 \omega_2 \vec{u}_z$$

$$\Rightarrow r_1^2 \omega_1 = r_2^2 \omega_2 \quad \Rightarrow \quad \omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1$$

6.19

$$\sum_{i=1}^{n} \vec{F}_{est} = 0 \quad ; \quad \vec{F}_i^{int} = 0$$

a) $\Delta \vec{q}_i = 0$ NO

$$\vec{F}_i^{est} = \frac{d\vec{q}_i}{dt} \neq 0 \quad \Rightarrow \quad \Delta\vec{q}_i \neq 0$$

b) $\Delta \vec{Q}_{CM} = 0$ sì

$$\sum_{i=1}^{n} \vec{F}_i^{est} = \frac{d\vec{Q}_{CM}}{dt} = 0$$

c) $\Delta E_{K tot} = 0$ ~~sì~~ NO

$$dE_{K tot} = \delta W_{TOT} = \sum_{i=1}^{n} \vec{F}_i^{est} \cdot d\vec{r}_i \quad ; \quad P_{tot} = \frac{\delta W_{TOT}}{dt}$$

$$\Rightarrow \frac{dE_{K tot}}{dt} = P_{TOT} = \sum_{i=1}^{n} \vec{F}_i^{est} \cdot \vec{v}_i = 0 \text{ se } \vec{v}_i = 0 \ \vee \ \sum_{i=1}^{n} \vec{F}_i^{est} \cdot \vec{v}_i = 0$$

d) $\Delta E_{K i} = 0$ NO

$$\Delta\left(\frac{1}{2} m_i v_i^2\right) = \Delta\left(\frac{1}{2} \frac{q_i^2}{m_i}\right) = \frac{1}{2m_i} \Delta q_i^2 \neq 0 \quad \text{poiché } \Delta\vec{q}_i \neq 0$$

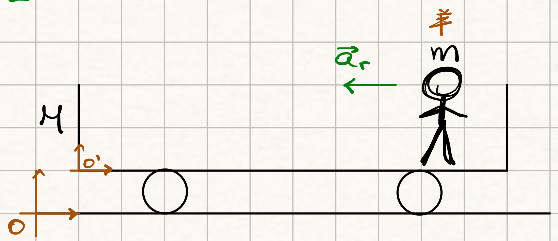
e) $\Delta \vec{L}_{tot} = 0$ ~~sì~~ NO

$$\vec{\tau}_{tot} = \frac{d\vec{L}_{tot}}{dt} = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i^{est} = 0 \quad \text{se} \quad \vec{r}_i = 0 \lor \vec{F}_i^{est} = 0 \rightarrow \text{NO}$$

f) $\Delta \vec{L}_i = 0$ ~~sì~~ NO

$$\vec{\tau}_i = \frac{d\vec{L}_i}{dt} = \vec{r}_i \times \vec{F}_i^{est} \neq 0$$

6.20



$M = 250$ kg
$m = 75$ kg
$a_r = 0{,}8\ m/s^2$

? $\vec{a}_c$? $\vec{a}_p$

$\Delta \vec{Q} = 0$; $\vec{Q}_i = 0 \Rightarrow \vec{Q}_f = 0$: $M\vec{a}_c \cancel{\Delta t} + m\vec{a}_p \cancel{\Delta t} = 0 \Rightarrow +Ma_c - ma_p = 0$

$$v_r = \int_i^f a_r\, dt = a_r \Delta t$$

$\Rightarrow Ma_c = ma_p \rightarrow a_c = \frac{m}{M} a_p$

$\vec{a}_p = \vec{a}_c + \vec{a}_r \Rightarrow -a_p = a_c - a_r = \frac{m}{M} a_p - a_r \Rightarrow$

$$a_p = a_r \left(\frac{1}{\frac{m}{M} + 1} \right) = a_r \left(\frac{M}{m+M} \right) = \boxed{0{,}615\ m/s^2}$$

$$\Rightarrow a_c = \frac{m}{\cancel{M}} \frac{\cancel{M}}{m+M} a_r = \boxed{0{,}185\ m/s^2}$$