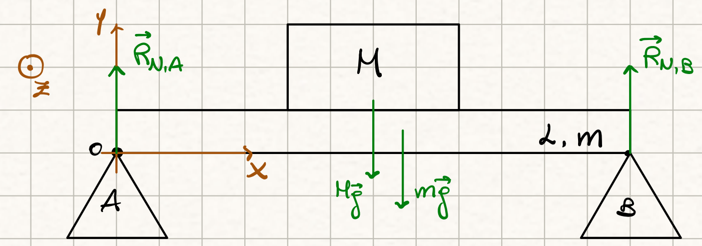
ESERCITAZIONE 1 :

1) $m = 30\,kg$
$l = 1\,m$
$M = 60\,kg$



a) ? $\vec{R}_{N,A}$? $\vec{R}_{N,B}$

$F_{max} = 600\,N$
b) ? x | uno dei sostegni cede

a) CONDIZIONI PER EQUILIBRIO STATICO DI UN CORPO RIGIDO :
$$\begin{cases} \vec{F}_{est} = 0 \\ \vec{\tau}_{est} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} M\vec{g} + m\vec{g} + \vec{R}_{N,A} + \vec{R}_{N,B} = 0 \\ \vec{\tau}_M + \vec{\tau}_m + \underbrace{\cancel{\vec{\tau}_A}}_{\vec{r}_A = 0} + \vec{\tau}_B = 0 \end{cases} \Rightarrow \begin{cases} -Mg - mg + R_{N,A} + R_{N,B} = 0 \\ \vec{x} \times M\vec{g} + \frac{\vec{l}}{2} \times m\vec{g} + \vec{l} \times \vec{R}_{N,B} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -Mg - mg + R_{N,A} + R_{N,B} = 0 \\ -xMg - \frac{l}{2} mg + l R_{N,B} = 0 \end{cases} \rightarrow R_{N,B} = \frac{1}{l}\left(xMg + \frac{l}{2} mg\right) = g\left(\frac{x}{l} M + \frac{1}{2} m\right) = \frac{60}{1}\,9{,}8\,x + \frac{\cancel{30}^{15}\cdot 9{,}8}{\cancel{2}} = 588\,x + 147 \text{ , N}$$

$$\Rightarrow -Mg - mg + R_{NA} + \frac{1}{2} mg + Mg\frac{x}{l} = 0 \Rightarrow R_{NA} = \frac{1}{2} mg + \left(1 - \frac{x}{l}\right) Mg = 147 + (1-x)\,588 \text{ , N}$$

$$\Rightarrow \boxed{\begin{cases} \vec{R}_{N,A} = \left[\frac{1}{2} mg + \left(1 - \frac{x}{l}\right) Mg\right] \vec{u}_y \\ \vec{R}_{N,B} = \left(\frac{1}{2} mg + \frac{x}{l} Mg\right) \vec{u}_y \end{cases}}$$

b) CASO SOSTEGNO A : se viene applicata la forza $\vec{F}$ sul sostegno A, si avrà che $R_{NA} = F$

riprendendo l'espressione di R_{NA} trovata prima : $\frac{1}{2} mg + \left(1 - \frac{x}{l}\right) Mg = F$

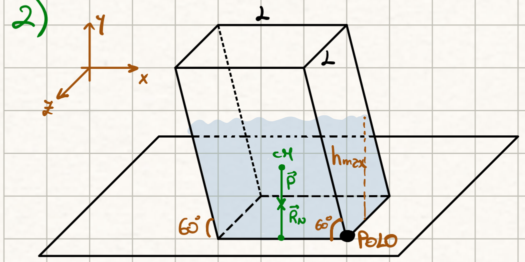
$$\Rightarrow \left(1 - \frac{x}{l}\right) Mg = F - \frac{1}{2} mg \Rightarrow \frac{x}{l} = 1 - \left(F - \frac{1}{2} mg\right)\frac{1}{Mg}$$

$$\Rightarrow x = l\left[1 - \left(F - \frac{1}{2} mg\right)\frac{1}{Mg}\right] = l\left[1 - \frac{F}{Mg} + \frac{m}{2M}\right] = 1 - \frac{\cancel{600}^{10}}{\cancel{60}\cdot 9{,}8} + \frac{\cancel{30}}{2\cdot \cancel{60}_2} = \boxed{0{,}23\,m}$$

CASO SOSTEGNO B : $R_{NB} = F \Rightarrow \frac{1}{2} mg + \frac{x}{l} Mg = F$

$$\Rightarrow x = \left(F - \frac{1}{2} mg\right)\frac{l}{Mg} = \left(600 - \frac{30}{2}\,9{,}8\right)\frac{1}{60\cdot 9{,}8} = \frac{\cancel{600}^{10}}{\cancel{60}\cdot 9{,}8} - \frac{\cancel{30}\cdot\cancel{9{,}8}}{2\cdot\cancel{60}_2\,\cancel{9{,}8}} = \boxed{0{,}77\,m}$$

2)



? h_{max}

Affinché non ribalti la verticale che passa per il centro di massa (→ baricentro) deve rientrare nella base del parallelepipedo.

$$\rightarrow \begin{cases} \vec{F}_{est} = 0 \\ \vec{\tau}_{est} = 0 \end{cases} \begin{matrix} \Rightarrow R_N - P = 0 \quad \Rightarrow P = R_N \\ \Rightarrow \vec{r}_R \times \vec{R}_N + \vec{r}_P \times \vec{P} = 0 \end{matrix}$$

→ la reazione vincolare è applicata sulla base di appoggio, mentre la forza peso nel CM.

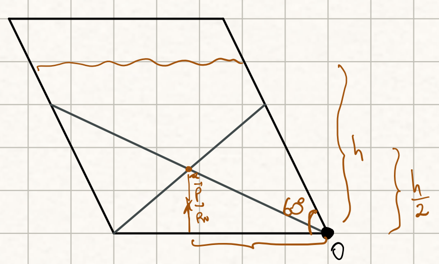
$$\rightarrow -r_R R_N + r_P P \sin\alpha = 0$$
$$\rightarrow -r_R P + r_P P \sin\alpha = 0$$
$$\rightarrow P(r_P \sin\alpha - r_R) = 0 \quad \Rightarrow r_P \sin\alpha - r_R = 0$$
$$\rightarrow r_R = r_P \sin\alpha$$

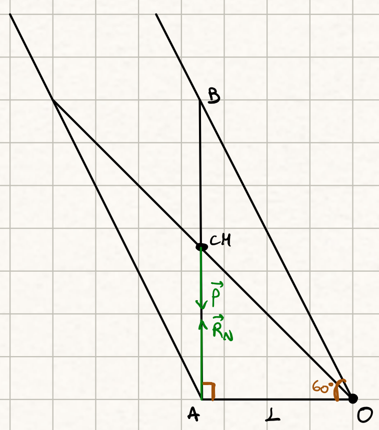
condizione $r_R \leq L$ ⇒ caso limite: $r_R = L$
(perché $\vec{R}_N$ applicato nella base)

$$\rightarrow L = r_P \sin\alpha$$

Bisogna trovare il centro di massa
→ ~~possiamo calcolarlo rispetto a un parallelogramma perché non ci interessa la sua coordinata z~~
→ ~~CM ≡ baricentro = intersezione diagonali~~
~~↳ punto medio delle due diagonali~~

$$~~= \left(\frac{h_{max}}{2\,tg\,60°}; \frac{h_{max}}{2}\right)~~$$
$$~~= \left(\frac{h_{max}}{2\sqrt{3}}; \frac{h_{max}}{2}\right)~~$$

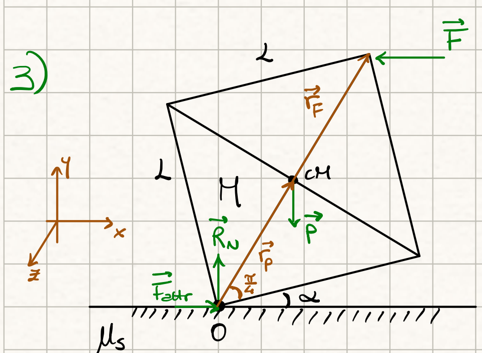




Poiché $\vec{P} \parallel \vec{R}_N$
$O\hat{A}B$ è tr. rettangolo

$$h_{max} = L\, tg\, 60° = \boxed{\sqrt{3}L}$$

3)



EQUILIBRIO STATICO

? F
? $\mu_{s,min}$ | $0 \leq \alpha \leq \frac{\pi}{4}$

$$\begin{cases} \vec{F}_{est} = 0 \\ \vec{\tau}_{est} = 0 \end{cases} \Rightarrow \begin{cases} \vec{R}_N + \vec{P} + \vec{F} + \vec{F}_{attr} = 0 \\ \underbrace{\cancel{\vec{\tau}_{R_N}}}_{r_R=0} + \vec{\tau}_P + \vec{\tau}_F + \underbrace{\cancel{\vec{\tau}_{attr}}}_{r_{attr}=0} = 0 \end{cases} \Rightarrow \begin{cases} R_N - P = 0 \\ F_{attr} - F = 0 \\ -\vec{r}_P \times \vec{P} + \vec{r}_F \times \vec{F} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} P = R_N \\ F = F_{attr} \\ -\frac{\cancel{\sqrt{2}L}}{2} Mg \sin\left(\frac{\pi}{4} - \alpha\right) + \cancel{\sqrt{2}L} F \sin\left(\frac{3}{4}\pi - \alpha\right) = 0 \end{cases} \Rightarrow -\frac{Mg}{2}\sin\left(\frac{\pi}{4} - \alpha\right) + F\sin\left(\frac{\pi}{4} + \alpha\right) = 0$$

$$\Rightarrow -\frac{Mg}{2}\cos\left(\frac{\pi}{4} + \alpha\right) + F\sin\left(\frac{\pi}{4} + \alpha\right) = 0 \Rightarrow F = \frac{Mg}{2}\cos\left(\frac{\pi}{4} + \alpha\right)\frac{1}{\sin\left(\frac{\pi}{4} + \alpha\right)} = \boxed{\frac{Mg}{2}\cot g\left(\frac{\pi}{4} + \alpha\right)}$$

per $0 \leq \alpha \leq \frac{\pi}{4}$

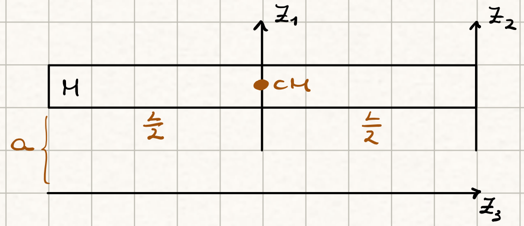
$$F_{attr} = F \quad \Rightarrow \quad F_{attr} \le \mu_s M g$$

$$\Rightarrow F \le \mu_s M g \quad \Rightarrow \quad \frac{\cancel{Mg}}{2}\cot\left(\frac{\pi}{4}+\alpha\right) \le \mu_s \cancel{Mg} \quad \Rightarrow \quad \mu_s \geq \frac{1}{2}\cot\left(\frac{\pi}{4}+\alpha\right) \quad \text{per} \quad 0 \le \alpha \le \frac{\pi}{4}$$

$$\mu_{s\,min} = \frac{1}{2}\cot\left(\frac{\pi}{4}+\alpha\right) \quad \text{per} \quad 0 \le \alpha \le \frac{\pi}{4} \quad \Rightarrow \quad \frac{1}{2}\cot\frac{\pi}{2} \le \mu_s \le \frac{1}{2}\cot\frac{\pi}{4}$$

$$\Rightarrow 0 \le \mu_s \le \frac{1}{2} \quad \Rightarrow \quad \boxed{\mu_{s\,min} = \frac{1}{2}}$$

4)



$$dm = \lambda\, dL = \frac{M}{L}dx$$

$$I_{z1} = \int_M \rho^2 dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{M}{L}dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{M}{L}\left[\frac{L^3}{24}+\frac{L^3}{24}\right]$$

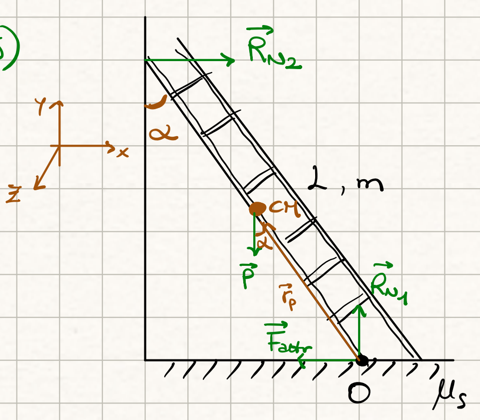
$$= \frac{M}{\cancel{L}}\frac{L^{\cancel{3}2}}{12} = \boxed{\frac{1}{12}ML^2}$$

TEOREMA DI HUYGENS-STEINER:

$$I_{z_2} = I_{CM} + M\left(\frac{L}{2}\right)^2 = I_{z1} + M\frac{L^2}{4} = \frac{1}{12}ML^2 + \frac{ML^2}{4} = \left(\frac{1}{12}+\frac{1}{4}\right)ML^2 = \boxed{\frac{1}{3}ML^2}$$

$$I_{z3} = \int \rho^2 dm = 0 + Ma^2 = \boxed{Ma^2}$$

5)



$?\alpha \mid$ equilibrio $\quad \left(0 \le \alpha \le \frac{\pi}{2}\right)$

$$\begin{cases} \vec{F}_{est} = 0 \\ \vec{\tau}_{est} = 0 \end{cases} \Rightarrow \begin{cases} \vec{P} + \vec{R}_{N1} + \vec{F}_{attr} + \vec{R}_{N2} = 0 \\ \vec{r}_p \times \vec{P} + \cancel{\vec{r}_1 \times \vec{R}_{N1}} + \cancel{\vec{r}_2 \times \vec{F}_{attr}} + \vec{r}_2 \times \vec{R}_{N2} = 0 \end{cases}$$

$$\begin{cases} P = R_{N1} \\ F_{attr} = R_{N2} \le \mu_s m g \\ r_p P \sin\alpha - r_2 R_{N2}\cos\alpha = 0 \end{cases} \Rightarrow \frac{\cancel{L}\cancel{mg}}{2}\sin\alpha_{max} - \cancel{L}\mu_s \cancel{mg}\cos\alpha_{max} = 0$$

$$\Rightarrow \frac{1}{2}\sin\alpha_{max} - \mu_s\cos\alpha_{max} = 0$$

$$\Rightarrow \frac{1}{2}\tan\alpha_{max} - \mu_s = 0 \quad \text{con } \cos\alpha_{max} \neq 0 \Rightarrow \alpha \neq \frac{\pi}{2}$$

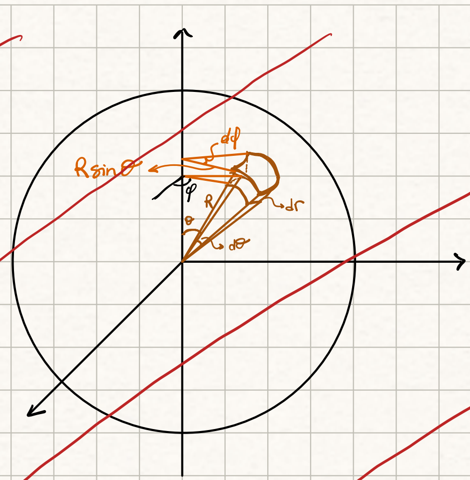
$$\Rightarrow \alpha_{max} = \boxed{\arctan 2\mu_s}$$

6) ? I_{sph}

Sfera omogenea : raggio R

$M = \delta V$

$dm = \delta dV = \frac{M}{V} dV$



$dV = (R d\vartheta)(R\sin\vartheta d\varphi) dr$
$= R^2 \sin\vartheta \, dr \, d\varphi \, d\vartheta$

$V = \frac{4}{3}\pi R^3$

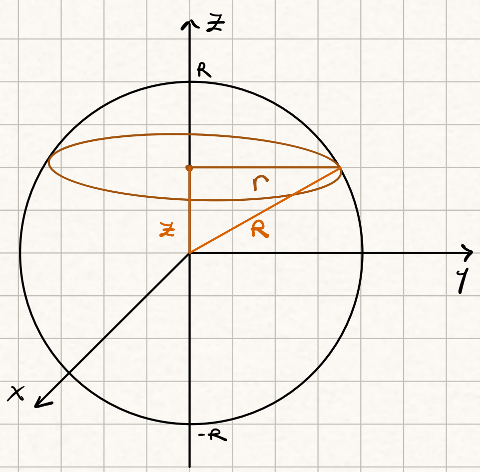
$$I_{sph} = \int_M r^2 dm = \int_V r^2 \frac{M}{V} dV =$$

$$= \int_V r^2 \frac{M}{\frac{4}{3}\pi R^3} dV = \int \frac{3}{4}\frac{M}{\pi R^3} r^2 \cancel{R^2} \sin\vartheta \, dr \, d\varphi \, d\vartheta$$

$$= \frac{3}{4}\frac{M}{\pi R} \int_0^R r^2 dr \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \sin\vartheta \, d\vartheta = \frac{3}{4}\frac{M}{\pi R}\left[\frac{r^3}{3}\right]_0^R \left[\varphi\right]_0^{2\pi} \left[-\cos\vartheta\right]_0^{\frac{\pi}{2}} = \frac{\cancel{3}}{\cancel{4}_2}\frac{M}{\cancel{\pi} R}\frac{R^{\cancel{3}\,2}}{\cancel{3}}\cancel{2}\cancel{\pi}(+1)$$

$M = \delta V \quad ; \quad dm = \delta dV = \frac{M}{V} dV$

$V = \frac{4}{3}\pi R^3 \quad ; \quad dV = \pi r^2 dz = \pi (R^2 - z^2) dz$



insieme di dischi

$$I_{sph} = \int_M \frac{1}{2} r^2 dm = \int_{-R}^{+R} \frac{\overset{(R^2-z^2)}{r^2}}{2}\frac{M}{\frac{4}{3}\cancel{\pi} R^3} \cancel{\pi}(R^2 - z^2) dz$$

$$= \frac{3}{8}\frac{M}{R^3}\int_{-R}^{+R} (R^2 - z^2)^2 dz = \frac{3}{8}\frac{M}{R^3}\int_{-R}^{+R} (R^4 + z^4 - 2R^2 z^2) dz$$

$(R^2 - z^2)^2 = R^4 + z^4 - 2R^2 z^2$

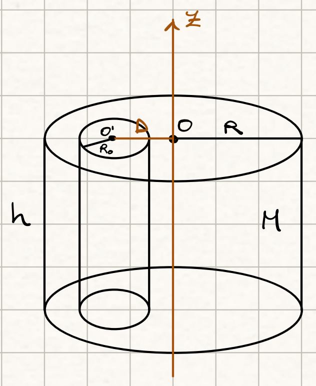
$$= \frac{3}{8}\frac{M}{R^3}\left\{ R^4 \int_{-R}^{+R} dz + \int_{-R}^{+R} z^4 dz - 2R^2 \int_{-R}^{+R} z^2 dz \right\}$$

$$= \frac{3}{8}\frac{M}{R^3}\left\{ R^4 \, 2R + \left[\frac{z^5}{5}\right]_{-R}^{+R} - 2R^2 \left[\frac{z^3}{3}\right]_{-R}^{+R} \right\}$$

$$= \frac{3}{8}\frac{M}{R^3}\left\{ 2R^5 + \frac{2}{5}R^5 - \frac{4}{3}R^5 \right\} = \frac{3}{\cancel{8}_4}\frac{M}{\cancel{R^3}} \cancel{2} R^{\cancel{5}\,2}\left\{ 1 + \frac{1}{5} - \frac{2}{3} \right\} = \frac{\cancel{3}}{4} M R^2 \, \frac{\cancel{8}^{\,2}}{\cancel{15}_5} = \frac{2}{5} M R^2$$

ESERCITAZIONE 2 :

1)



? I_z

Disco pieno : $I = \frac{1}{2} m r^2$

$$I_z = \int_M r^2 dm = I_R - I_{R_0} = \frac{1}{2}\left(M + M\frac{R_0^2}{R^2}\right)R^2 - I_{R_0}$$

$$\begin{cases} M = \rho V = \rho \pi (R^2 - R_0^2) h \\ M_0 = \rho V_0 = \rho \pi R_0^2 h \end{cases} \Rightarrow \frac{M}{\cancel{\pi}(R^2-R_0^2)\cancel{h}} = \frac{M_0}{\cancel{\pi} R_0^2 \cancel{h}} \Rightarrow M_0 = M\frac{R_0^2}{(R^2-R_0^2)} \; ; \; M + M_0 = M\left(1 + \frac{R_0^2}{R^2-R_0^2}\right) = M\frac{R^2}{R^2-R_0^2}$$

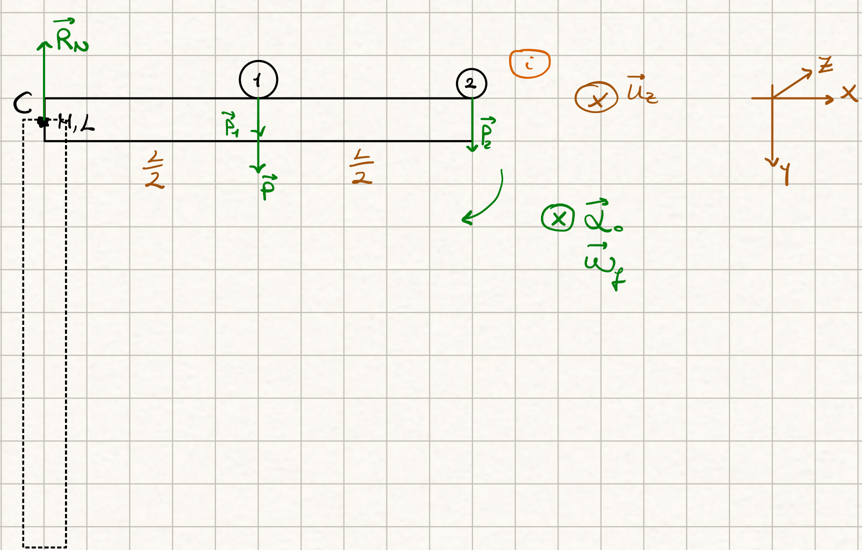
$$I_{O'} = \frac{1}{2}\left(M\frac{R_0^2}{R^2-R_0^2}\right)R_0^2$$

TEOREMA HUYGENS-STEINER : $I_O = I_{O'} + M\frac{R_0^2}{R^2-R_0^2} D^2$

$$\Rightarrow I_z = \frac{1}{2}\left(M\frac{R^2}{R^2-R_0^2}\right)R^2 - \frac{1}{2}\left(M\frac{R_0^2}{R^2-R_0^2}\right)R_0^2 - M\frac{R_0^2}{R^2-R_0^2}D^2 = \frac{1}{2}\frac{M}{R^2-R_0^2}\left(R^4 - R_0^4 - 2R_0^2 D^2\right)$$

$$= \frac{1}{2}\frac{M}{\cancel{R^2-R_0^2}}\cancel{(R^2-R_0^2)}(R^2+R_0^2) - \frac{M}{R^2-R_0^2}R_0^2 D^2 = \frac{1}{2}M\left(R^2 + R_0^2 - 2\frac{R_0^2 D^2}{R^2-R_0^2}\right)$$

2) $m_1 = 2\,kg$
$m_2 = 1\,kg$
$M = 3\,kg$
$L = 1\,m$



a) ? I_C
b) ? $\vec{\alpha}_i$; $\vec{\omega}_f$
c) ? T_θ

a)

Rispetto C :

$$I_1 = \left(\frac{L}{2}\right)^2 m_1 = \frac{L^2}{4} m_1$$

$$I_2 = L^2 m_2$$

$$I_{ASTA} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = ML^2\left(\frac{1}{12} + \frac{1}{4}\right) = ML^2\left(\frac{1}{3}\right) = \frac{1}{3}ML^2$$

$$\Rightarrow I_c = I_1 + I_2 + I_{ASTA} = m_1\frac{L^2}{4} + m_2L^2 + \frac{1}{3}ML^2 = L^2\left(\frac{3m_1+12m_2+4M}{12}\right) = \frac{6+12+12}{12}L^2 = \frac{5}{2}L^2$$

$$= \boxed{2{,}5\ kg\,m^2}$$

b)

$$O:\quad \vec{M}_c^{est} = I_c\vec{\alpha}_o \quad\Rightarrow\quad \cancel{\vec{r}_{R_N}\times\vec{R}_N}^{\ \vec{r}_{R_N}=0} + \vec{r}_1\times\vec{P}_1 + \vec{r}_A\times\vec{P} + \vec{r}_2\times\vec{P}_2 = I_c\,\alpha_o\,\vec{u}_z$$

$$\Rightarrow \frac{L}{2}m_1g + \frac{L}{2}Mg + Lm_2g = 2{,}5\ \alpha_o$$

$$\Rightarrow \alpha_o = \frac{2}{5}\left(g+\frac{3}{2}g+g\right) = \frac{\cancel{2}}{5}g\,\frac{7}{\cancel{2}} = \boxed{13{,}72\ rad/s^2}\ !!$$

NO forze dissipative : $\Delta E_m = 0$

E_K per rotaz. con asse fisso $= \frac{1}{2}I_o\omega^2$

E_p di m_2 finale $= 0\ (h=0)$

$$\Delta E_m = 0:\quad \frac{1}{2}I_c\omega_f^2 + (m_1+M)g\frac{L}{2} - (m_1+m_2+M)gL = 0$$

$$\Rightarrow \omega_f = \sqrt{\frac{2(m_1+m_2+M)gL-(m_1+M)g\frac{L}{2}}{I_o}} = \sqrt{\frac{2\,6g-\frac{5}{2}g}{\frac{5}{2}}} = \sqrt{7\cdot\frac{2}{5}g} = \boxed{5{,}24\ rad/s}$$

c) PENDOLO FISICO :

$$\vec{M}^{est} = I_c\vec{\alpha}\quad;\quad \vec{M}^{est} = \vec{M}_{P_1} + \vec{M}_{P_2} + \vec{M}_P = -\left(\frac{L}{2}m_1g\sin\theta + Lm_2g\sin\theta + \frac{L}{2}Mg\sin\theta\right)\vec{u}_z$$

$$\Rightarrow M^{est} = I_c\alpha = I_c\frac{d^2\theta}{dt^2} \quad\Rightarrow\quad Lg\sin\theta\left(\frac{1}{2}m_1+m_2+\frac{1}{2}M\right) = I_c\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{\left(\frac{m_1+2m_2+M}{2}\right)Lg}{I_c}\sin\theta \qquad \text{per } \theta \text{ piccolo :}$$

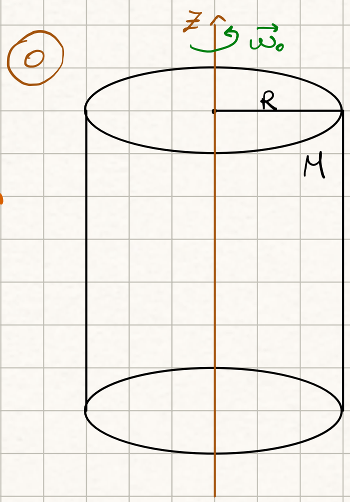
$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{(M+m_1+2m_2)gL}{2I_c}\theta = 0$$

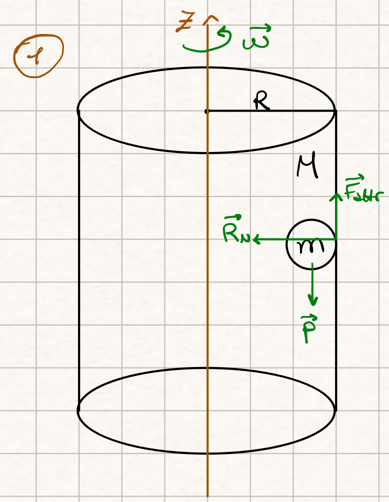
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2I_c}{gL(M+m_1+2m_2)}} = 2\pi\sqrt{\frac{5}{g\,7}} = \boxed{1{,}7\ s}$$

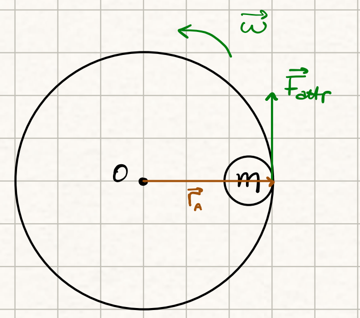
3)

? ω

? $\mu_{s\,min}$







$$\Delta \vec{L} = 0 \quad \Rightarrow \quad I_0 \vec{\omega}_0 = I_1 \vec{\omega} \quad \Rightarrow \quad I_0 \omega_0 \vec{u}_z = I_1 \omega \vec{u}_z \quad \Rightarrow \quad I_0 \omega_0 = I_1 \omega$$

$$I_0 = \int_M r^2 dm = R^2 \int_M dm = MR^2$$

$$I_1 = I_0 + I_m = I_0 + mR^2 = (M+m)R^2$$

$$\Rightarrow \quad M\cancel{R^2}\omega_0 = (M+m)\cancel{R^2}\omega \quad \Rightarrow \quad \omega = \frac{M}{M+m}\omega_0$$

$$\begin{cases} \vec{F}_{att} + \vec{P} = 0 \\ \vec{R}_N = m\vec{a}_c \end{cases} \Rightarrow \begin{cases} F_{att} = mg \leq \mu_s R_N \\ R_N = ma_c = m\omega^2 R \end{cases} \Rightarrow \quad \cancel{m}g \leq \mu_s \cancel{m}\omega^2 R \quad \Rightarrow \quad \mu_s \geq \frac{g}{\omega^2 R}$$

4) $m = 1\,kg$

$F_0 = 15\,N$

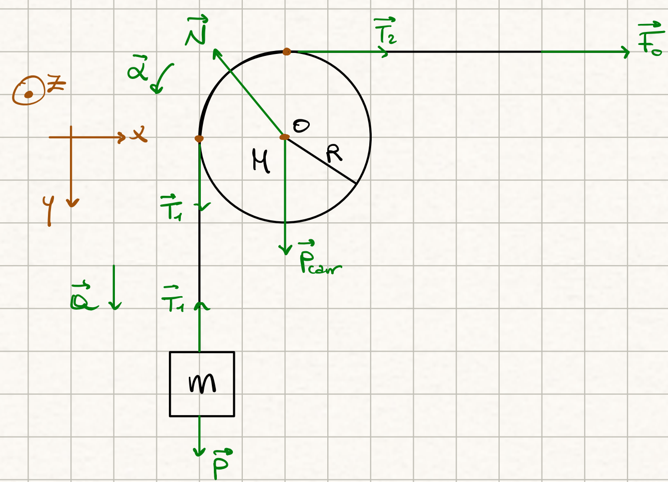
$R = 0{,}1\,m$

$M = 2\,kg$

a) $T_1 \overset{?}{=} T_2$

b) ? a

c) ? $W_{F_0}(\Delta t)$ $\quad \Delta t = 0{,}5\,s$



a) La carrucola non trasla ma ruota:

$$\begin{cases} \vec{F}^{est} = \vec{T}_1 + \vec{T}_2 + \vec{P}_{carr} + \vec{N} = 0 \\ \vec{\tau}^{est} = \vec{\tau}_1 + \vec{\tau}_2 + \underbrace{\cancel{\vec{\tau}_P}}_{=0} + \underbrace{\cancel{\vec{\tau}_N}}_{=0} = I_0 \vec{\alpha} \end{cases} \Rightarrow \quad \vec{R} \times \vec{T}_1 + \vec{R} \times \vec{T}_2 = \frac{1}{2}MR^2 \vec{\alpha}$$

$$\Rightarrow \quad RT_1 \vec{u}_z - RT_2 \vec{u}_z = \frac{1}{2}MR^2 \alpha \vec{u}_z \quad \Rightarrow \quad R(T_1 - T_2) = \frac{1}{2}MR^2 \alpha$$

$T_1 \neq T_2$ altrimenti il disco non ruoterebbe ($\alpha = 0$)

b) Massa m: $\vec{T}_1 + \vec{P} = m\vec{a} \Rightarrow -T_1 + mg = ma \Rightarrow T_1 = m(g-a)$

$\alpha = \frac{a}{R}$ (perché $Rd\theta = dy \Rightarrow R\omega = v \Rightarrow R\alpha = a$)

$\vec{F}_0 = \vec{T}_2$ poiché fune ideale $\Rightarrow F_0 = T_2$

$$\Rightarrow \cancel{R}(T_1 - T_2) = \frac{1}{2}MR^{\cancel{2}}\alpha \Rightarrow m(g-a) - F_0 = \frac{1}{2}M\cancel{R}\frac{a}{\cancel{R}} \Rightarrow$$

$$mg - ma - F_0 = \frac{1}{2}Ma \Rightarrow \left(\frac{1}{2}M + m\right)a = mg - F_0$$

$$\Rightarrow a = \frac{mg - F_0}{\frac{1}{2}M + m} = \frac{9,8 - 15}{1 + 1} = -2,6\ m/s^2$$

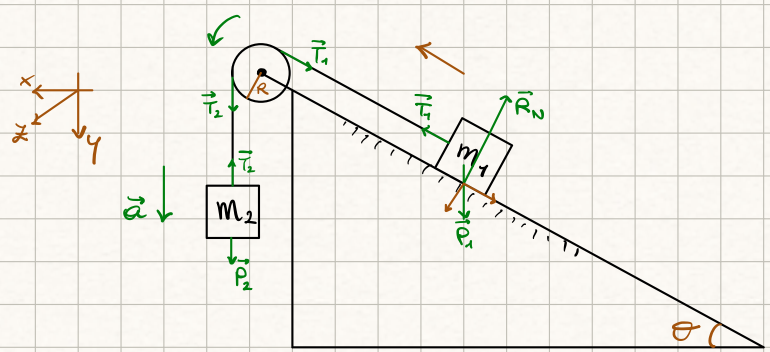
$$\Rightarrow \vec{a} = -2,6\,\vec{u}_y\ [m/s^2]$$

c) $$W_{F_0} = \int \vec{F}_0 \cdot d\vec{s} = F_0 \frac{1}{2} a \Delta t^2 = \frac{15}{2}\, 2,6\, \frac{1}{4} = 4,875\ J$$

$$s = \cancel{s_0} + \cancel{v_0 t} + \frac{1}{2}at^2$$

5) $m_1 = 15\ kg$
$m_2 = 20\ kg$
$\theta = 37°$
$R = 0,25\ m$
$a = 2\ m/s^2$

?I



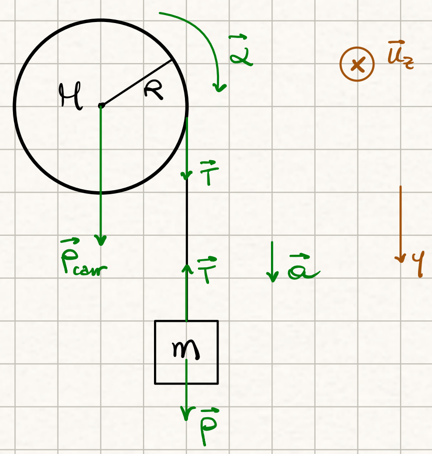
$$\vec{\tau}_{pul} = I\vec{\alpha} \rightarrow -\tau_{T_1}\vec{u}_z + \tau_{T_2}\vec{u}_z = I\alpha\vec{u}_z \quad ; \quad \alpha = \frac{a}{R}$$

$$\rightarrow -RT_1 + RT_2 = I\frac{a}{R} \Rightarrow I = \frac{R^2(-T_1 + T_2)}{a}$$

$$\begin{cases} P_2 - T_2 = m_2 a \\ T_1 - P_2 \sin\theta = m_1 a \end{cases} \Rightarrow \begin{cases} T_2 = m_2(g-a) \\ T_1 = m_1(a + g\sin\theta) \end{cases}$$

$$\Rightarrow I = \frac{R^2}{a}\left[-m_1(a + g\sin\theta) + m_2(g-a)\right] = \frac{1}{32}\left[-15(2 + 9,8 \cdot \sin 37°) + 20(9,8 - 2)\right] = 1,17\ kg\,m^2$$

6) $m = 1\,kg$
$M = 2\,kg$
R
? a



$$\begin{cases} \vec{P} + \vec{T} = m\vec{a} \\ \vec{\tau} = I\vec{\alpha} \end{cases}$$

$$\Rightarrow \begin{cases} mg - T = ma \\ RT = I\alpha \end{cases} \Rightarrow \cancel{R}T = \frac{1}{2}MR^{\cancel{2}}\frac{a}{\cancel{R}} \Rightarrow T = \frac{1}{2}Ma$$

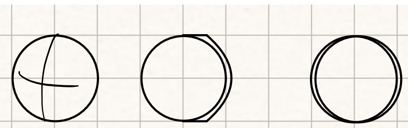
$$\Rightarrow mg - \frac{1}{2}Ma = ma \Rightarrow a = \frac{mg}{\frac{1}{2}M + m} = \frac{9{,}8}{1+1} = \boxed{4{,}9\ m/s^2}$$

ESERCITAZIONE 3 :

2) **2.** Una sfera, un disco pieno ed un anello omogenei, ciascuno di massa M e raggio R, vengono lasciati rotolare lungo un piano inclinato di un angolo θ rispetto all'orizzontale. Se i tre corpi partono da fermi e dalla stessa posizione, quale giungerà prima alla base del piano? Determinare inoltre il minimo valore del coefficiente di attrito statico μ_s affinché il loro moto sia di puro rotolamento.

$\left[\text{La sfera; } \mu_s > \frac{\text{tg}\,\theta}{2}\right]$

? quale primo
? $\mu_{s\,min}$

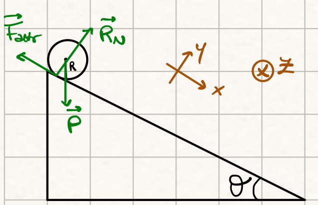


$M,\ R$

sfera : $I = \frac{2}{5}MR^2$

disco : $I = \frac{1}{2}MR^2$

anello : $I = MR^2$



$$\Delta E_m = 0 \Rightarrow \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 - Mg(h+R) = 0$$

$$\frac{1}{2}I\left(\frac{v}{R}\right)^2 + \frac{1}{2}Mv^2 - Mg(h+R) = 0 \Rightarrow v^2 = \frac{Mg(h+R)\cdot 2}{\frac{I}{R^2} + M}$$

→ La velocità è maggiore per il corpo che ha momento di inerzia minore

$$I_s < I_d < I_a \Rightarrow v_s > v_d > v_a$$

quindi il primo che giunge alla base è la sfera

$$\vec{P} + \vec{F}_{attr} + \vec{R}_N = 0 \Rightarrow \begin{cases} x: & F_{attr} = Mg\cancel{\cos\theta}\,\sin\theta + Ma \le \mu_s R_N = \mu_s Mg\cancel{\sin\theta}\,\cos\theta \\ y: & R_N - Mg\cancel{\sin\theta}\,\cos\theta = 0 \Rightarrow R_N = Mg\cancel{\sin\theta}\,\cos\theta \end{cases}$$

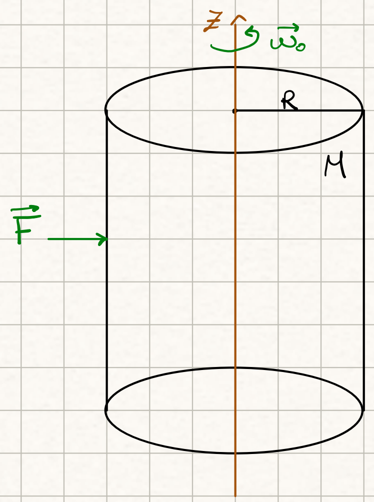
$$\Rightarrow \cancel{M}(g\cancel{\cos\theta}\,\sin\theta + a) \le \mu_s\cancel{M}g\cancel{\sin\theta}\,\cos\theta$$

$$\vec{\tau} = I\vec{\alpha} \Rightarrow \vec{R}\times\vec{F}_{attr} = I\vec{\alpha} \Rightarrow R\mu_s Mg\cancel{\sin\theta}\,\cos\theta = I\frac{a}{R} \Rightarrow a = \frac{R^2\mu_s Mg\cancel{\sin\theta}\,\cos\theta}{I}$$

$$\Rightarrow g\,\overset{\sin\theta}{\cancel{\cos\theta}} + \frac{R^2 M g\,\overset{\cos\theta}{\cancel{\sin\theta}}\,\mu_s}{I} \leq \mu_s g\,\overset{\cos\theta}{\cancel{\sin\theta}} \quad \Rightarrow \quad \cancel{g}\,\overset{\sin\theta}{\cancel{\cos\theta}} \leq \mu_s \cancel{g}\,\overset{\cos\theta}{\cancel{\sin\theta}}\left(1 + \frac{MR^2}{I}\right)$$

$$\Rightarrow \mu_s \geq tg\theta \Big/ \left(1 + \frac{MR^2}{I_{max}}\right) = tg\theta \Big/ \left(1 + \frac{\cancel{MR^2}}{\cancel{I_A}}\right) = \boxed{\frac{1}{2} tg\theta}$$

1)



$M = 20\,kg \quad ; \quad R = 0{,}1\,m$

$I = \frac{1}{2} MR^2$

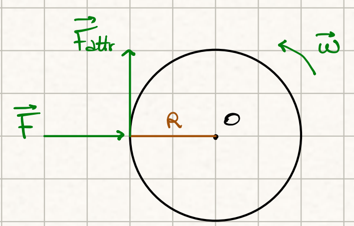
$\omega_0 = 4\,rad/s$

$F = 1N \quad \rightarrow \quad \omega = 0$

$\mu_d = 0{,}5$

a) ? $\vec{\tau}_{ATTR}$

b) ? E_{diss} ? n (giri prima di fermarsi)



$$\vec{\tau}_{attr} = \vec{R} \times \vec{F}_{attr} = -RF_{attr}\,\vec{u}_z = -R\mu_d F\,\vec{u}_z = \boxed{-0{,}05\,\vec{u}_z \;[N\cdot m]}$$

$$F_{attr} = \mu_d F$$

$$\Delta E_m = \Delta E_K = E_{K_f} - E_{K_i} = 0 - E_{K_i} = -E_{diss} \Rightarrow$$

$$E_{diss} = E_{K_i} = \frac{1}{2} I\omega_0^2 = \frac{1}{2}\frac{1}{2} MR^2\omega_0^2 = \frac{1}{4} MR^2\omega_0^2 = \boxed{0{,}8\,J}$$

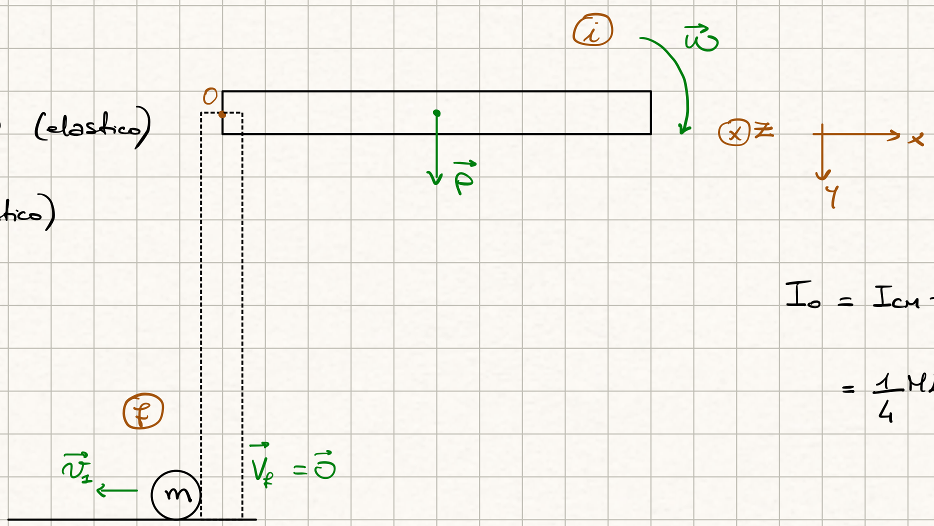
$$E_{diss} = W_{nc} = F_{attr}\,\Delta\theta\,R = \mu_d FR\,\Delta\theta = \mu_d FR\,n 2\pi$$

$$\frac{\Delta\theta}{2\pi} = n \Rightarrow \Delta\theta = n \cdot 2\pi$$

$$\Rightarrow n = \frac{E_{diss}}{\mu_d FR 2\pi} = \frac{0{,}8}{0{,}5 \cdot 1 \cdot 0{,}1 \cdot 2\pi} = \boxed{2{,}54 \text{ giri}}$$

3) $M = 1\,kg$
$L = 0{,}5\ m$

a) $?m \mid V_f = 0$ (elastico)
b) $?\vec{v}_1$
c) $?\vec{v}_1'$ (se anaelastico)



$$I_0 = I_{cm} + M\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + M\frac{L^2}{4} = \frac{1}{4}ML^2\left(\frac{1}{3}+1\right) = \frac{1}{3}ML^2$$

a) $\Delta E_m = 0$: $MgL = Mg\frac{L}{2} + \frac{1}{2}I_0\omega^2 \Rightarrow \cancel{M}g\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}\frac{1}{3}\cancel{M}L^2\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{L}}$

$\Rightarrow gL = \frac{1}{3}\cancel{L^2}\frac{V^2}{\cancel{L^2}} \Rightarrow V = \sqrt{3gL}$

URTO ELASTICO : • $\Delta E_K = 0 \Rightarrow \frac{1}{\cancel{2}}mv_1^2 - \frac{1}{\cancel{2}}I_0\omega^2 = 0 \Rightarrow v_1 = \sqrt{\frac{I_0}{m}}\,\omega$

• $\Delta L_z = 0 \Rightarrow Lmv_1 - I_0\omega = 0 \Rightarrow \cancel{L}m\sqrt{\frac{I_0}{m}}\cancel{\omega} = \frac{1}{3}ML^{\cancel{2}}\cancel{\omega}$

$\Rightarrow mI_0 = \frac{1}{9}ML \Rightarrow m\frac{1}{\cancel{3}}\cancel{M}\cancel{L^2} = \frac{1}{\cancel{9}_3}M^{\cancel{2}}\cancel{L^2} \Rightarrow \boxed{m = \frac{M}{3} = 0{,}33\,kg}$

b) $v_1 = \sqrt{\frac{I_0}{m}}\sqrt{\frac{3g}{L}} = \sqrt{\frac{1}{\cancel{3}}ML^{\cancel{2}}\frac{1}{m}\frac{\cancel{3}g}{\cancel{L}}} = \sqrt{\frac{MLg3}{\cancel{M}}} = \sqrt{3gL} = \sqrt{\frac{3}{2}9{,}8} = \boxed{3{,}83\ m/s}$

c) URTO ANAELASTICO :

$$I = I_0 + mL^2 = \frac{1}{3}ML^2 + \frac{M}{3}L^2 = \frac{2}{3}ML^2$$

$\Delta L_z = 0$: $I\omega_1' - I_0\omega_0 = 0 \Rightarrow \frac{2}{3}M\cancel{L^2}\frac{v_1'}{\cancel{L}} - \frac{1}{3}M\cancel{L^2}\frac{v_1}{\cancel{L}} = 0$

$\Rightarrow 2v_1' - v_1 = 0 \Rightarrow v_1' = \frac{v_1}{2} = \frac{\sqrt{3gL}}{2} = \sqrt{\frac{3}{4}gL} = \boxed{1{,}92\ m/s}$

4) $m = 5\,kg$

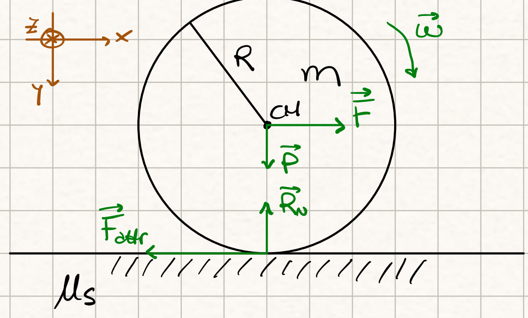
$R = 0{,}25\,m$

$\mu_s = 0{,}15$

a) ? $\vec{F}_{max}$ | $\vec{v}(P^*) = 0$

b) ? $\vec{\tau}_{max}$

a)



Puro rotolamento:

$v_{CM} = \omega R$

$a = \alpha R$

$$\vec{F} + \vec{F}_{attr} + \vec{P} + \vec{R}_N = m\vec{a}$$

$$\Rightarrow \begin{cases} F - F_{attr} = ma \\ P - R_N = 0 \end{cases} \Rightarrow \begin{cases} F_{attr} = F - ma \leq \mu_s R_N \\ R_N = mg \end{cases}$$

$$\Rightarrow F - ma \leq \mu_s mg \quad \Rightarrow F \leq m(\mu_s g + a) \quad ; \quad a = \alpha R$$

$$\vec{\tau}^{est} = I\vec{\alpha} \quad \Rightarrow \quad \vec{R} \times \vec{F}_{attr} = I\vec{\alpha} \quad \Rightarrow \quad RF_{attr} = I\alpha$$

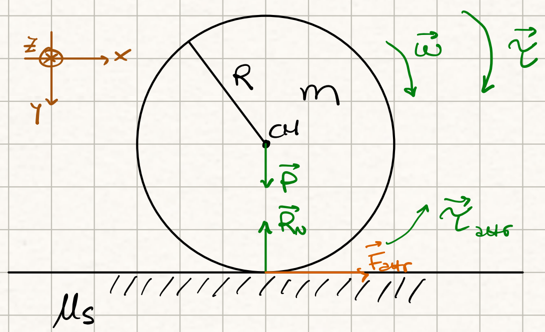
$$\alpha = \frac{R\mu_s mg}{I} = \frac{\cancel{R}\mu_s \cancel{m}g}{\frac{1}{2}\cancel{m}R^{\cancel{2}}} = \frac{2\mu_s g}{R}$$

$$\Rightarrow F \leq m(\mu_s g + 2\mu_s g) = 3\mu_s mg = 3 \cdot 0{,}15 \cdot 5 \cdot 9{,}8 = \boxed{22{,}05\,N}$$

b) $\cancel{\vec{F}_{attr} + \vec{P} + \vec{R}_N = m\vec{a}} \quad \Rightarrow \quad \cancel{F_{attr} = -ma \leq \mu_s mg} \quad \Rightarrow \quad \cancel{a \geq \mu_s g}$

$\cancel{\vec{\tau}_{attr} + \vec{\tau} = I\vec{\alpha}} \quad \Rightarrow \quad \cancel{RF_{attr} + \tau = -\frac{1}{2}mR^2 \frac{F_{attr}}{m}}$

$\Rightarrow \cancel{F_{attr}\left(R + \frac{1}{2}R^2\right) = -\tau} \quad \Rightarrow \quad \cancel{F_{attr} = -\tau \frac{2}{2R+R^2} \leq \mu_s mg}$



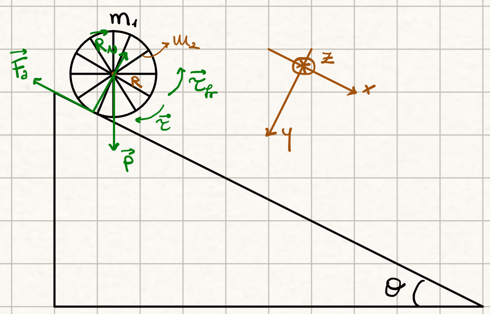
$$\begin{cases} P - R_N = 0 \\ F_{attr} = ma \leq \mu_s mg \end{cases}$$

$$\tau - \tau_{attr} = I\alpha = I\,a/R$$

$$\Rightarrow \tau = \frac{1}{2}\cancel{m}R^{\cancel{2}}\frac{F_{attr}}{\cancel{m}}\frac{1}{\cancel{R}} + RF_{attr}$$

$$\Rightarrow \tau \leq \frac{1}{2}R\,\mu_s mg + R\mu_s mg = \frac{3}{2}R\mu_s mg = \boxed{2{,}76\,Nm}$$

5)



$m_2 = \frac{m_1}{30}$

$\omega_0 \rightarrow$ cost $\rightarrow \alpha = 0, \quad a = 0$

MOTO DI PURO ROTOLAMENTO : $v_{CM} = \omega R$

a) ? I_{ruota}, asse $\parallel \vec{u}_z$ passante per CM

b) ? $\vec{\tau}_{freno}$

c) ? $W_f(\Delta t)$

a) $I_R = m_1 R^2 + 15\left[\frac{1}{12} m_2 R^2 + m_2\left(\frac{R}{2}\right)^2\right] = m_1 R^2 + 15\left[\frac{1}{12}\frac{m_1}{30}R^2 + \frac{m_1}{30}\frac{R^2}{4}\right] = m_1 R^2 + \frac{1}{24}m_1 R^2 + \frac{1}{8}m_1 R^2$

$= \frac{24+1+3}{24} m_1 R^2 = \frac{14}{12} m_1 R^2 = \frac{7}{6} m_1 R^2$

b) $\begin{cases} \vec{R}_N + \vec{P} + \vec{F}_a = 0 \\ \vec{\tau}_{fr} + \vec{\tau} = I_R \vec{\alpha} = 0 \end{cases}$

$\Rightarrow \begin{cases} P\cos\theta - R_N = 0 \\ P\sin\theta - F_a = 0 \\ -\tau_{fr} + \tau_a = 0 \end{cases} \rightarrow F_a = m_{tot}\, g\sin\theta = \left(m_1 + \cancel{15}\frac{m_1}{\cancel{30}_2}\right) g\sin\theta = \frac{3}{2} m_1 g \sin\theta$

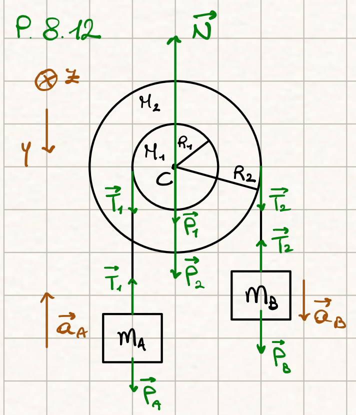
$\Rightarrow \tau_{fr} = \tau_a = \frac{3}{2} R m_1 g \sin\theta$

$\Rightarrow \vec{\tau}_{fr} = -\frac{3}{2} R m_1 g \sin\theta\, \vec{u}_z$

c) $W = \vec{\tau}_{fr} \cdot \vec{\omega}_0 \Delta t = -\tau_{fr}\omega_0 \Delta t$

ESERCIZI [LONGHI] CAP. 8 :

P.8.12



i) ? I_C
ii) ? $\vec{\alpha}$

i) $I_C = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2$

ii)
$$\begin{cases} \vec{\tau}_1 + \vec{\tau}_2 = I_C \vec{\alpha} \\ \vec{P}_A + \vec{T}_1 = m_A \vec{a}_A \\ \vec{P}_B + \vec{T}_2 = m_B \vec{a}_B \\ \vec{a}_A = R_1 \vec{\alpha} \\ \vec{a}_B = R_2 \vec{\alpha} \end{cases} \Rightarrow \begin{cases} -R_1 T_1 + R_2 T_2 = I_C \alpha \\ m_A g - T_1 = -m_A a_A \\ m_B g - T_2 = m_B a_B \\ a_A = R_1 \alpha \\ a_B = R_2 \alpha \end{cases}$$

$\Rightarrow T_1 = m_A(g + a_A) = m_A(g + R_1\alpha)$

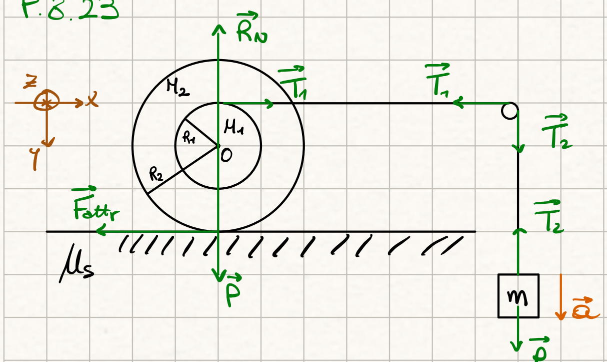
$\Rightarrow T_2 = m_B(g - a_B) = m_B(g - R_2\alpha)$

$$\Rightarrow -R_1 m_A (g + R_1\alpha) + R_2 m_B (g - R_2 \alpha) = I_C \alpha$$

$$\Rightarrow -R_1^2 m_A \alpha - R_2^2 m_B \alpha - I_C \alpha = -R_2 m_B g + R_1 m_A g$$

$$\Rightarrow \alpha = \frac{(R_2 m_B - R_1 m_A) g}{R_1^2 m_A + R_2^2 m_B + I_C}$$

P.8.23



MOTO DI PURO ROTOLAMENTO

i) ? I_O
ii) ? $\vec{a}$
iii) ? $\mu_{s,min}$ | $\vec{v}(P') = 0$

i) $I_O = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2 + (M_1 + M_2)$

! ASSE ISTANTANEA DI ROTAZIONE = PUNTO DI CONTATTO

! $T_1 = T_2$

; guardare soluzione sul libro

$$I_A = I_0 + MD^2 \Rightarrow I_0 = I_A - MD^2$$

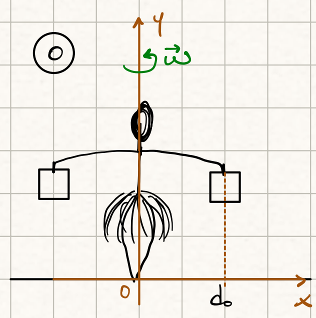
$$E_{k\,tot} = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_0 \omega^2$$

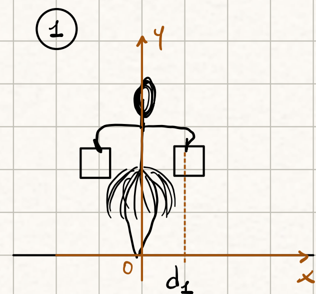
$$= \frac{1}{2} M \omega^2 R^2 + \frac{1}{2}(I_A - MR^2)\omega^2 = \cancel{\frac{1}{2} M\omega^2 R^2} + \frac{1}{2} I_A \omega^2 - \cancel{\frac{1}{2} M\omega^2 R^2} = \boxed{\frac{1}{2} I_A \omega^2}$$

ESERCITAZIONE 17 - PETTI

1) $f = 1\ Hz$
$d_0 = 0{,}6\ m$
$d_1 = 0{,}1\ m$

$?f_1$





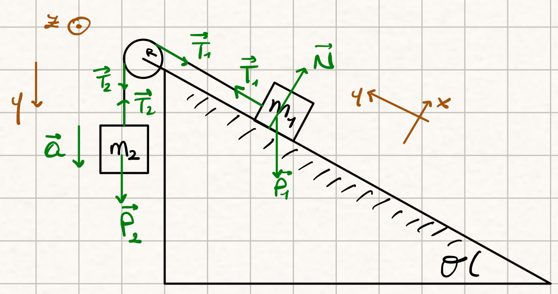
Conservazione momento angolare:

$$\Delta \vec{L} = 0 \Rightarrow L_0 \vec{u}_y = L_1 \vec{u}_y \Rightarrow I_0 \omega_0 = I_1 \omega_1$$

$$\Rightarrow \cancel{\frac{1}{12} M} d_0^2\, \cancel{2\pi} f_0 = \cancel{\frac{1}{12} M} d_1^2\, \cancel{2\pi} f_1 \Rightarrow f_1 = \frac{d_0^2}{d_1^2} f_0 = \frac{0{,}6^2}{0{,}1^2} \cdot 1 = 36\ Hz$$

2) $m_1 = 15\ kg$
$m_2 = 20\ kg$
$\theta = 37°$
m_p
$R = 0{,}25$
$a = 2\ m/s^2$

$?I$



$$\vec{\tau}^{\,est} = -RT_1 \vec{u}_z + RT_2 \vec{u}_z = I \alpha \vec{u}_z$$

$$\begin{cases} m_2 g - T_2 = m_2 a \\ T_1 - m_1 g \sin\theta = m_1 a \end{cases}$$

$$\alpha = \frac{a}{R}$$

$$\Rightarrow -Rm_1(g\sin\theta + a) + Rm_2(g-a) = I\frac{a}{R} \Rightarrow$$

$$I = \frac{R^2}{a}\left[-m_1(g\sin\theta + a) + m_2(g-a)\right] = 1{,}17\ kg\, m^2$$