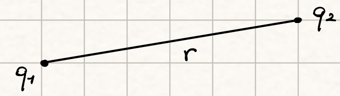
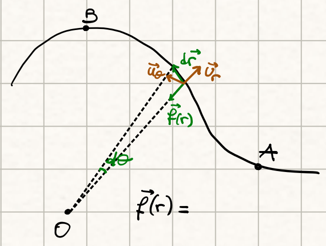
1) $\vec{f} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \vec{u}_r$ (FORZA DI COULOMB) $f = f(r)$



a) ? dimostrare che $\vec{f}$ è conservativa

b) ? $V(r)$ → potenziale

a)



$\vec{f}(r) =$

$$\mathcal{L}_{AB} = \int_A^B \vec{f} \cdot d\vec{r} = \int_A^B f(r) \vec{u}_r \cdot d\vec{r}$$

$$d\vec{r} = d(r\vec{u}_r) = dr\,\vec{u}_r + r\,d\vec{u}_r = dr\,\vec{u}_r + r\,d\theta\,\vec{u}_\theta$$

$$\Rightarrow \mathcal{L}_{AB} = \int f(r)\vec{u}_r \cdot (dr\,\vec{u}_r + r\,d\theta\,\vec{u}_\theta)$$

$$= \int \Big[f(r)\,dr\,\underbrace{(\vec{u}_r \cdot \vec{u}_r)}_{=1} + f(r)\,r\,d\theta\,\underbrace{(\vec{u}_r \cdot \vec{u}_\theta)}_{=0} \Big]$$

$$= \int_{r_A}^{r_B} f(r)\,dr$$

→ dipende solo da r → F CONSERVATIVA

b) $$\mathcal{L}_{AB} = V_A - V_B = \int_{r_A}^{r_B} f(r)\,dr = \frac{q_1 q_2}{4\pi\varepsilon_0} \int_{r_A}^{r_B} \frac{1}{r^2}\,dr = \frac{q_1 q_2}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_{r_A}^{r_B}$$

$$= \underbrace{\frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r_A}}_{V_A} - \underbrace{\frac{q_1 q_2}{4\pi\varepsilon_0} r_B}_{V_B}$$

$r_B \to +\infty \Rightarrow V_B \to 0$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

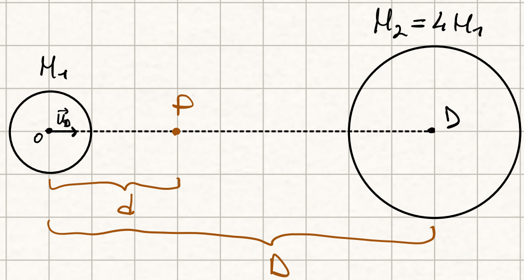
decresce come $\frac{1}{r}$

2) M_1

$M_2 = 4M_1$

D distanza

? $P \mid \vec{G}_{tot} = 0$



$$\vec{G} = -\gamma \frac{M}{r^2} \vec{u}_r$$

$$\vec{G}_1 = -\gamma \frac{M_1}{r_1^2} \vec{u}_D$$ verso M_1

$$\vec{G}_2 = +\gamma \frac{M_2}{r_2^2} \vec{u}_D$$ verso M_2

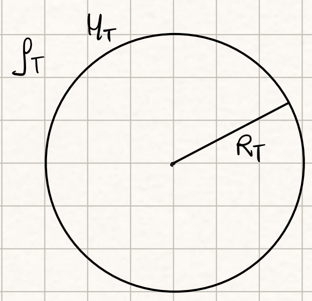
$$\vec{G}_{TOT} = \vec{G}_1 + \vec{G}_2 = -\gamma \left(\frac{M_1}{r_1^2} - \frac{M_2}{r_2^2} \right) \vec{u}_D = 0$$

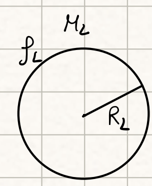
$r_1 = d \quad ; \quad r_2 = D - d$

$$\Rightarrow -\gamma M_1 \left(\frac{1}{d^2} - \frac{4}{(D-d)^2} \right) = 0$$ ← punto P

$$\Rightarrow d = \frac{-D \pm \sqrt{D^2 + 3D^2}}{3}$$ $\frac{D}{3}$; $-D$ NO

3) ? ρ_L





ρ_T

$g_L = \frac{1}{6} g$

$R_L = \frac{1}{4} R_T$

F_{G_T} : $\cancel{m} g = \gamma \frac{\cancel{m} M_T}{R_T^2}$

F_{G_L} : $\cancel{m} g_L = \gamma \frac{\cancel{m} M_L}{R_L^2}$

! volume sfera: $V = \frac{4}{3}\pi R^3$ $\rightarrow$ $M_T = V_T \rho_T = \frac{4}{3}\pi R_T^3 \rho_T$

$M_L = V_L \rho_L = \frac{4}{3}\pi R_L^3 \rho_L$

tutto in sistema

$\Rightarrow$ $g_L = \frac{1}{6} g = \frac{\gamma M_L}{R_L^2} = \gamma \rho_L \frac{4}{3}\pi \frac{R_T}{4}$

$\Rightarrow$ $\frac{\cancel{\gamma} \rho_T}{6} \frac{4}{\cancel{3}} \cancel{\pi} \cancel{R_T} = \cancel{\gamma} \rho_L \frac{\cancel{\pi}}{\cancel{3}} \cancel{R_T}$ $\rightarrow$ $\rho_L = \frac{2}{3}\rho_T$

4) m

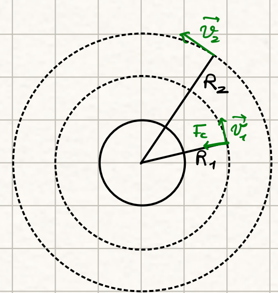
M_T

$R_1 = 2R_T$

$R_2 = 3R_T$

? a) $L_{1\to 2}$

b) v_2



traiettoria circolare

moto circolare uniforme

$F_c = m a_c = m \frac{v^2}{R}$

$a_c = \frac{v^2}{R}$

a) $\vec{f_c} = -\gamma \frac{m M_T}{r^2} \vec{u}_r = m \vec{a}_c$; $f_c = \gamma \frac{m M_T}{r^2} = m a_c = m \frac{v^2}{r}$

$\Rightarrow v^2 = \gamma \frac{M_T}{r}$

$E_{mecc} = E_k + E_p = \frac{1}{2} m v^2 - \gamma \frac{m M_T}{r} = \frac{1}{2}\gamma \frac{m M_T}{r} - \gamma \frac{m M_T}{r} = \frac{1}{2}\gamma \frac{m M_T}{r}$

$L_{1\to 2} = E_{mecc_2} - E_{mecc_1} = -\frac{1}{2}\gamma \frac{m M_T}{3R_T} + \frac{1}{2}\gamma \frac{m M_T}{2R_T} = -\frac{1}{2}\gamma m M_T \left(\frac{1}{3R_T} - \frac{1}{2R_T}\right) = \underline{\gamma \frac{m M_T}{12 R_T}}$

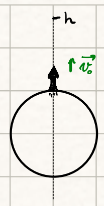
b) $v^2 = \gamma \frac{M_T}{r}$: $v_1^2 = \gamma \frac{M_T}{2R_T}$ $\Rightarrow v_1 = \sqrt{\frac{\gamma M_T}{2R_T}}$

$v_2^2 = \gamma \frac{M_T}{3R_T}$ $\Rightarrow v_2 = \sqrt{\frac{\gamma M_T}{3R_T}} = \sqrt{\frac{2}{3} \frac{\gamma M_T}{2R_T}} = \underline{\sqrt{\frac{2}{3}}\, v_1}$

5) a) ? $v_{0,min}$

b) $v_0 = v_{0,min}$

? $\Delta t \mid r = 2R_T$



- $v_0 < v_{0\,min}$ $\Rightarrow$ orbita chiusa (ritorno alla partenza)
- $v_0 \geq v_{0\,min}$ $\Rightarrow$ orbita aperta

↳ $E_{mec} \geq 0$

a) $$E_m = \frac{1}{2} m v_0^2 - \gamma \frac{m M_T}{R_T} \geq 0$$

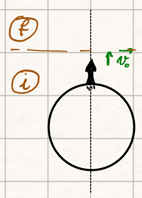
! dal momento di lancio la sola forza è la forza peso

$$\Rightarrow v_0 \geq \sqrt{2\gamma \frac{M_T}{R_T}}$$

non dipende dalla massa del corpo di lancio

oppure:

$\Delta E_m = 0$ (conservazione)



$$\Delta E_m = 0 \Rightarrow \cancel{E_{kf}} - E_{ki} + \cancel{E_{pf}} - E_{pi} = 0 \quad (r \to \infty)$$

$$\Rightarrow -\frac{1}{2} m v_0^2 + \gamma \frac{m M_T}{R_T} = 0$$

b) $$v_0 = v_{0\,min} = \sqrt{\frac{2\gamma M_T}{R_T}}$$

$\Delta t \mid h = R_T$ (dalla superficie)

! per grandi distanze $\vec{f}$ non è costante $\Rightarrow$ moto rettilineo vario

$$\vec{F} = -\gamma \frac{m M_T}{r^2} \vec{u}_r = m\vec{a}$$

$r = r(t)$, $a = a(t)$ $\Rightarrow$ dipendenti dal tempo

$E_{mec} = 0$: $$E_{mec} = \frac{1}{2} m v^2 - \gamma \frac{m M_T}{r} = 0$$

$$\Rightarrow v = \sqrt{\frac{2\gamma M_T}{r}}$$

$$v = \frac{dr}{dt} = \frac{\sqrt{2\gamma M_T}}{\sqrt{r}}$$

Eq. differenziale a var. separabili

γ

$$\Rightarrow \sqrt{r}\, dr = \sqrt{2\gamma M_T}\, dt$$

$$\Rightarrow \int_{r_0}^{r} \sqrt{r}\, dr = \int_0^t \sqrt{2\gamma M_T}\, dt$$

$$\Rightarrow \frac{2}{3}\left[\sqrt{r^3}\right]_{r_0}^{r} = \sqrt{2\gamma M_T}\,[t]_0^t \Rightarrow r = \sqrt[3]{\left(\frac{3}{2}\sqrt{2\gamma M_T}\, t + \frac{2}{3}\sqrt{r_0^3}\right)^2}$$

; $r_0 = R_T$

$\Delta t \to r = 2R_T$

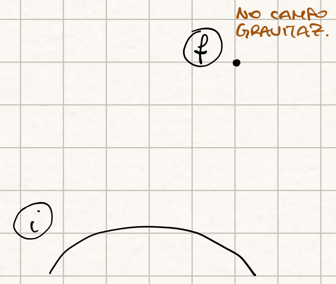
$\gamma = 6{,}67 \cdot 10^{-11}\ m^3/kg\,s^2$

$$\Rightarrow \Delta t = \frac{2}{3} \frac{\sqrt{8R_T^3} - \sqrt{R_T^3}}{\sqrt{2\gamma M_T}} = 658\ s$$

$R_T = 6{,}37 \cdot 10^6\ m$

$M_T = 5{,}97 \cdot 10^{24}\ kg$

1) $m = 1\,M_g$



conservazione dell' $E_{mecc.}$

$$\Delta E_m = 0 \Rightarrow E_f - E_i = 0 \Rightarrow E_f = E_i$$

$$\Rightarrow E_{K,f} + E_{P_f} = E_{K_i} + E_{P_i}$$

$$0 + 0 = \frac{1}{2}\cancel{m}\, v_{0,min}^2 - \frac{\gamma \cancel{m} M_T}{R_T}$$

$$\Rightarrow v_{0,min} = \sqrt{\frac{2\gamma M_T}{R_T}}$$ VELOCITÀ DI FUGA

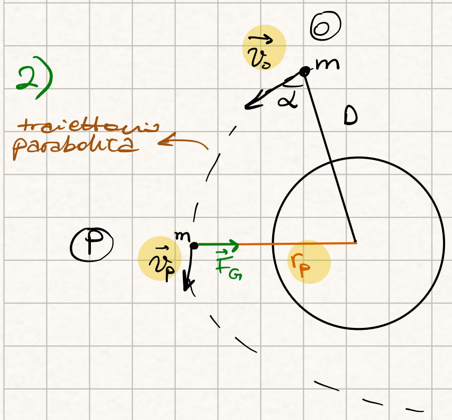
orbita chiusa circolare : $E < 0$
aperta - parabolica $E = 0$

$$\Rightarrow E_{K_i} = \frac{1}{2} m \frac{2\gamma M_T}{R_T} = \gamma \frac{m M_T}{R_T} = 6{,}4 \cdot 10^{10}\,J$$

$\Delta t = 4$ settimane

$$\overline{P} = \frac{E_K}{\Delta t} = \frac{6{,}4 \cdot 10^{10}\,J}{2419200\,s} = 2{,}65 \cdot 10^{4}\,W$$

2)



$D = 10^5\,km$
$\alpha = 30°$

? perigeo (p. più vicino) r_p

3 incognite

$\Delta E_m = 0$
$E_{m_P} = 0$
$E_{m_O} = 0$

$$\cancel{\vec{F}_G = m\vec{a}_c = m\frac{v_p^2}{r_p}}$$ NO perchè moto non circolare

SISTEMA ISOLATO

- $\Delta E_m = 0$ | FORZE CONSERVATIVE $(\vec{F}_p, \vec{F}_{el}, \vec{F}_G)$
- $\Delta(\underbrace{\vec{r} \times m\vec{v}}_{\vec{L}}) = 0$ | FORZE CENTRALI (F_G) ?$(\vec{r} \parallel \vec{v})$

(conservazione del momento angolare)

- $\Delta(m\vec{v}) = 0$ (solo in sistemi isolati)

1) $F = ma$
2) $\Delta E_m = 0$
3) $\Delta \vec{L} = 0$

- $\frac{d\vec{L}}{dt} = \vec{r} \times F(r)\vec{u}_r = 0$

$$\begin{cases} \Delta E_m = 0 \\ \Delta \vec{L} = 0 \\ E_{M,0} = 0 \\ E_{M,P} = 0 \end{cases}$$
mi bastano 3 (3 incognite)

$$\begin{cases} E_{m,0} = \frac{1}{2} \cancel{m} v_0^2 - \gamma \frac{\cancel{m} M}{D} = 0 \quad \Rightarrow \quad v_0 = \sqrt{\frac{2\gamma M_T}{D}} \\ E_{m,P} = 0 \quad \Rightarrow \quad v_P = \sqrt{\frac{2\gamma M_T}{r_P}} \\ \Delta \vec{L} = 0 \quad \Rightarrow \quad \vec{r}_P \times m \vec{v}_P = \vec{D} \times m \vec{v}_0 \end{cases}$$

$\Rightarrow r_P \cancel{m}\, v_P\, \vec{u}_z = D \cancel{m} v_0 \sin\alpha\, \vec{u}_z$

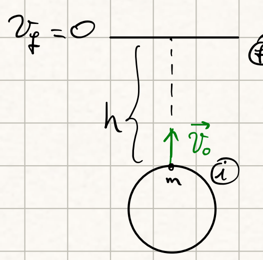
($\alpha = 90°$ al perigeo)

$\Rightarrow r_P v_P = D\, v_0 \sin\alpha$

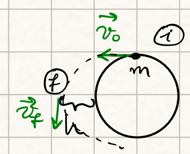
$\Rightarrow r_P \sqrt{\frac{2\gamma M_T}{r_P}} = D \sin\alpha \sqrt{\frac{2\gamma M_T}{D}} \quad \Rightarrow \quad r_P^{\cancel{2}} \frac{\cancel{2\gamma M_T}}{\cancel{r_P}} = D^{\cancel{2}} \sin^2\alpha \frac{\cancel{2\gamma M_T}}{\cancel{D}}$

$\Rightarrow \underline{r_P = D \sin^2\alpha}$

3)



$\Delta E = 0$

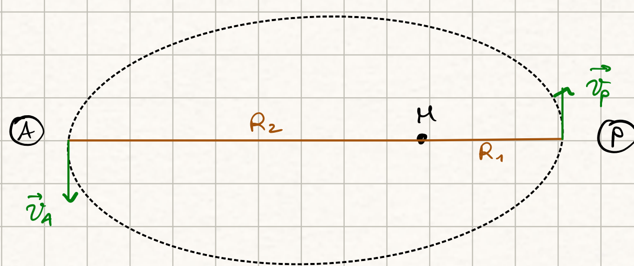


qui servono due equazioni

$\Delta E = 0$

$\Delta \vec{L} = \vec{0}$

4)



$$\begin{cases} \Delta E_m = 0 \\ \Delta \vec{L} = \vec{0} \end{cases} \quad \Rightarrow \quad \begin{cases} E_{M,A} = E_{M,P} \\ \vec{L}_A = \vec{L}_P \end{cases}$$

$$\begin{cases} \frac{1}{2} m v_A^2 - \frac{\gamma m M}{R_2} = \frac{1}{2} m v_P^2 - \frac{\gamma m M}{R_1} \\ \vec{R}_2 \times m \vec{v}_A = \vec{R}_1 \times m \vec{v}_P \end{cases} \quad \Rightarrow R_2 \cancel{m} v_A \vec{u}_z = R_1 \cancel{m} v_P \vec{u}_z$$

$\Rightarrow v_A = \frac{R_1}{R_2} v_P$

$\Rightarrow \frac{R_1^2}{R_2^2} v_P^2 - \frac{2\gamma M}{R_2} = v_P^2 - \frac{2\gamma M}{R_1}$

$\Rightarrow R_1^3 v_P^2 - 2\gamma M R_1 R_2 - v_P^2 R_2^2 R_1 + 2\gamma M R_2^2 = 0$

$\Rightarrow R_1 v_P^2 \underbrace{(R_1^2 - R_2^2)}_{(R_1+R_2)\cancel{(R_1-R_2)}} = 2\gamma M R_2 \cancel{(R_1 - R_2)}$

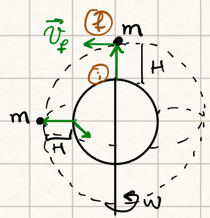
$\Rightarrow v_P = \sqrt{\frac{2\gamma M R_2}{R_1(R_1+R_2)}}$

$$L = L_A = L_P = \vec{R_1} \times m\vec{v_f} = m\sqrt{\frac{2\gamma M R_1 R_2}{R_1+R_2}}$$

$$\vec{L} = L\,\vec{u}_z$$

!

5)



$m,\ \gamma,\ M_T,\ R_T,\ T$

$$\Delta E_m = \mathcal{L}_{NC}$$

AL POLO:

$$\Delta E_m = E_{m_f} - E_{m_i} = E_{k_f} - E_{k_i} + E_{p_f} - E_{p_i}$$

$$\frac{1}{2}m v_f^2 - \underbrace{0}_{v_0=0} - \gamma\frac{m M_T}{R_T+H} + \gamma\frac{m M_T}{R_T}$$

$$\vec{F}_G = m\vec{a}_c$$

$$\Rightarrow \gamma\frac{\cancel{m} M_T}{(R_T+H)^2} = \cancel{m}\frac{v_f^2}{R_T+H}$$

$$\Rightarrow v_f^2 = \gamma\frac{M_T}{R_T+H}$$

$$\mathcal{L}_{NC} = -\frac{1}{2}\gamma m\frac{M_T}{R_T+H} + \gamma\frac{m M_T}{R_T}$$

$$\mathcal{L}_P = \gamma m M_T\left(\frac{R_T+2H}{2R_T(R_T+H)}\right)$$

EQUATORE:

$$v_0 = \omega R_T = \frac{2\pi}{T}R_T$$

$$\mathcal{L}_{NC} = \Delta E_m = E_{k_f} - E_{k_i} + E_{p,f} - E_{p_i}$$

$$\frac{1}{2}m v_f^2 - \frac{1}{2}m v_0^2 - \gamma\frac{m M_T}{R_T+H} + \gamma\frac{m M_T}{R_T}$$

$$\mathcal{L}_E = \underbrace{\gamma m M_T\left(\frac{R_T+2H}{2R_T(R_T+H)}\right)}_{\mathcal{L}_P} - \underline{\underline{\frac{2m\pi^2 R_T}{T}}}$$

lanciare il razzo all'equatore fa meno lavoro, perché all'equatore il razzo parte con $v \neq 0$