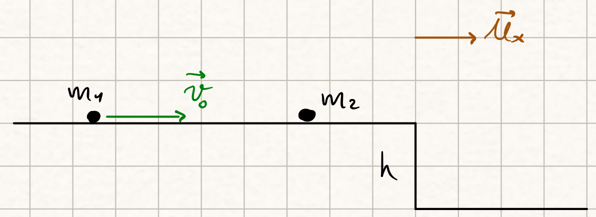
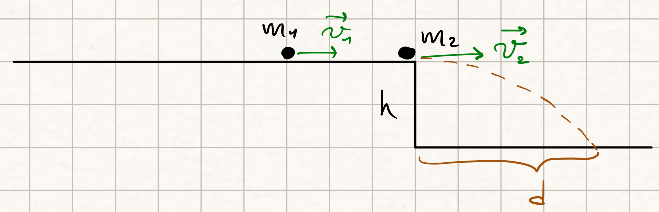
se $\vec{F}_{est} = 0$: $\Delta \vec{Q} = 0$ $\left(\vec{F}_{est} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{dm\vec{v}}{dt} = \frac{d\vec{Q}}{dt} \right)$

1)



$m_2 = 3m_1$
$h = 0,5\ m$
$d = 5\ m$
$v_0 = ?$



URTO $\Rightarrow$ CONSERVAZIONE $\vec{Q}$
↳ ELASTICO $\Rightarrow$ CONSERVAZIONE E_K e $\vec{Q}$
↳ ANAELASTICO $\Rightarrow$ solo $\vec{Q}$

$$\begin{cases} \vec{Q}_i = \vec{Q}_f \\ E_{Ki} = E_{Kf} \end{cases} \Rightarrow \begin{cases} m_1\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 \\ \cancel{\frac{1}{2}} m_1 v_0^2 = \cancel{\frac{1}{2}} m_1 v_1^2 + \cancel{\frac{1}{2}} m_2 v_2^2 \end{cases}$$

$$\Rightarrow \begin{cases} m_1 v_0 = m_1 v_1 + 3 m_2 v_2 \\ \cancel{m_1} v_0^2 = \cancel{m_1} v_1^2 + 3 \cancel{m_1} v_2^2 \end{cases} \Rightarrow \begin{cases} v_1 = v_0 - 3v_2 \\ v_0^2 = v_0^2 - 6v_0v_2 + 9v_2^2 + 3v_2^2 \Rightarrow \end{cases}$$

$$2v_2^2 - v_0 v_2 = 0 \Rightarrow v_2(2v_2 - v_0) = 0$$

$$\Rightarrow \cancel{\begin{cases} v_1 = v_0 \\ v_2 = 0 \end{cases}} \vee \begin{cases} v_1 = -v_0/2 \\ v_2 = \frac{v_0}{2} \end{cases}$$

non nel nostro caso

la 1° pallina torna indietro

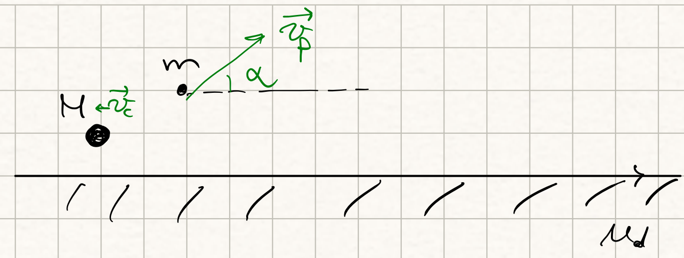
$$\begin{cases} x(t) = v_2 t \\ y(t) = h - \frac{1}{2} g t^2 \end{cases}$$ $\left[\text{MOTO} \right]$

$$\Rightarrow t_v : \begin{cases} x(t_v) = d = v_2 t_v \Rightarrow t_v = d/v_2 \\ y(t_v) = 0 = h - \frac{1}{2} g t_v^2 \Rightarrow h = \frac{1}{2} g \frac{d^2}{v_1^2} = \frac{1}{\cancel{2}} g \cancel{4}^{2} \frac{d^2}{v_0^2} \end{cases}$$

$$\Rightarrow v_0^2 = \frac{2gd^2}{h}$$

$$\Rightarrow v_0 = \sqrt{\frac{2g}{h}}\, d = 31,3\ m/s$$

2)



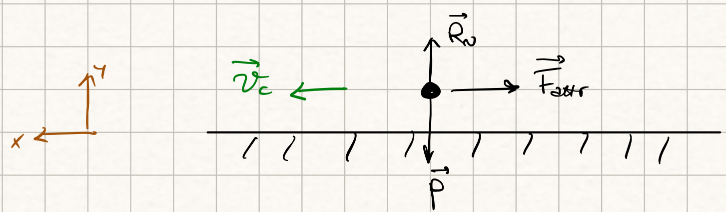
$\Delta x = ?$
$I_N = ?$ (vincolo)

~ esplosione

$Q_{x,f} = Q_{x,i}$:

$$-M v_c + m v_p \cos\alpha = 0 \quad \rightarrow \quad v_c = \frac{m}{M} v_p \cos\alpha$$

⇒ MOTO DEL CANNONE :



$$\begin{cases} R_N = Mg \\ -F_{attr} = -\mu_d R_N = -\mu_d M g = Ma \end{cases}$$

$$\Rightarrow \quad a = -\mu_d g$$

$$v(t) = v_c - \mu_d g t$$

$$x(t) = v_c t - \frac{1}{2}\mu_d g t^2$$

τ :
$$\begin{cases} x(\tau) = \Delta x = v_c \tau - \frac{1}{2}\mu_d g \tau^2 \\ v(\tau) = 0 = v_c - \mu_d g \tau \Rightarrow \tau = \frac{v_c}{\mu_d g} \end{cases}$$

$$\Delta x = v_c \frac{v_c}{\mu_d g} - \frac{1}{2}\cancel{\mu_d g}\frac{v_c^2}{(\mu_d g)^{\cancel{2}}} = \underline{\underline{\frac{1}{2}\frac{v_c^2}{\mu_d g}}}$$

TEOREMA DELL'IMPULSO :

$$I_N = Q_{y,f} - Q_{y,f}$$
$$= m v_p \sin\alpha - 0$$

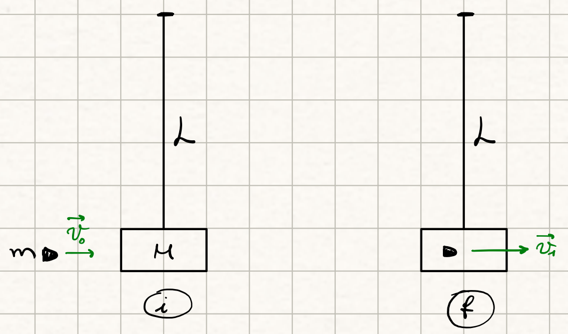
⇒ quando non si conserva la quantità di moto (una parte viene persa ed è $= I_N$)

modellizzare la realtà : punti (materia) + forze (interazione)

+ 4 leggi (Newton + 3 leggi di conserv.)

PENDOLO BALISTICO

3)



URTO COMPLETAMENTE ANAELASTICO

$$\vec{Q}_i = \vec{Q}_f$$

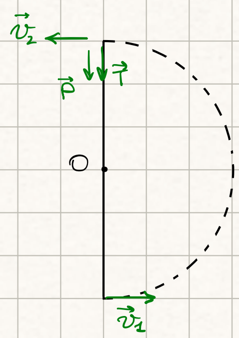
$$\Rightarrow m\vec{v}_0 = (m+M)\vec{v}_1$$

$$\Rightarrow m v_0 = (m+M) v_1 \quad \text{(lungo } x\text{)}$$

$$\Rightarrow v_1 = \frac{m}{m+M} v_0$$

• MOTO DEL PENDOLO :

se fune



$$T \geq 0$$

$$m_{tot} g + T = m_{tot} a_c = m_{tot} \frac{v_2^2}{L}$$

$$T = m_{tot} \frac{v_2^2}{L} - m_{tot} g \geq 0$$

$$\Rightarrow v_2^2 \geq gL$$

$\Delta E_m = 0$:

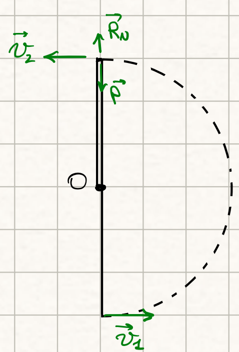
$$E_{kf} + E_{pf} = E_{ki} + E_{pi}$$

$$\cancel{\tfrac{1}{2}} \cancel{(m+M)} v_2^2 + \cancel{(m+M)} g \, \cancel{2}^{4} L = \cancel{\tfrac{1}{2}} \cancel{(m+M)} v_1^2 + 0$$

$$\Rightarrow v_1^2 = v_2^2 + 4gL \geq 5gL$$

$$\Rightarrow v_{0_{min}} = \frac{m+M}{m} v_{1\,min} = \frac{m+M}{m} \sqrt{5gL}$$

se asta rigida :



$$v_2 = 0$$

$$v_1^2 = 4gL$$

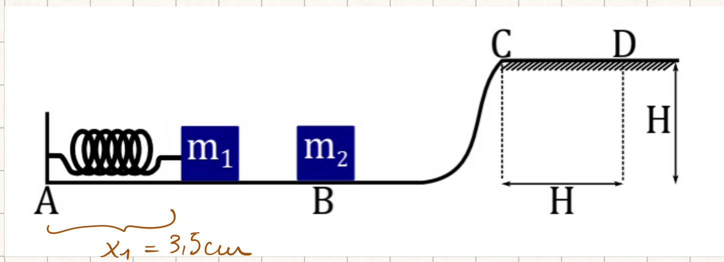
$$v_{0\,min} = \frac{m+M}{m} \sqrt{4gL} = \frac{2(m+M)}{m} \sqrt{gL}$$

? Energia dissipata :

$$E_{diss} = \Delta E_m = \overset{\text{perché } \Delta E_p = 0}{\Delta E_K} = E_{K,1} - E_{K,0} = \frac{1}{2}(m+M) v_1^2 - \frac{1}{2} m v_0^2$$

$$= \frac{1}{2} \cancel{(m+M)} \frac{m^2}{(m+M)^{\cancel{2}}} v_0^2 - \frac{1}{2} m v_0^2 = \frac{5}{2} \frac{M}{m} (m+M) g L$$

5)



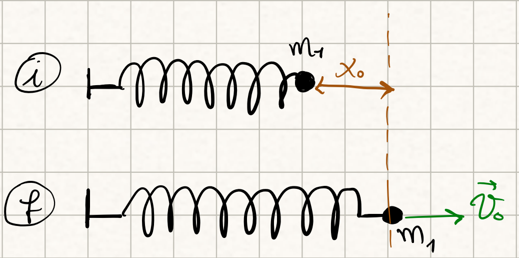
1° : molla compressa e allungata

2° : urto elastico

3° : compressione della molla

+ leggi di conservazione

1° : FASE DISTACCO

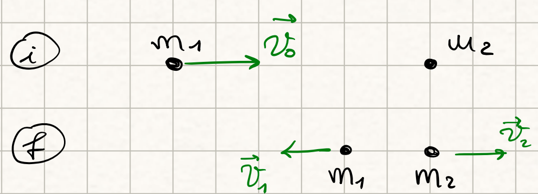


$\Delta E_m = 0$: $E_{kf} + E_{pf} = E_{ki} + E_{pi}$

$\Rightarrow \frac{1}{2} m_1 v_0^2 + 0 = 0 + \frac{1}{2} k x_0^2$

$\Rightarrow v_0 = \sqrt{\frac{k}{m}}\, x_0$

2° : URTO ELASTICO



$$\begin{cases} \vec{Q}_i = \vec{Q}_f \\ E_{Ki} = E_{Kf} \end{cases}$$

$$\Rightarrow \begin{cases} m_1 v_0 = -m_1 v_1 + m_2 v_2 \\ \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m v_2^2 \end{cases}$$

(nell'esame far vedere il passaggio da vettore a scalare)

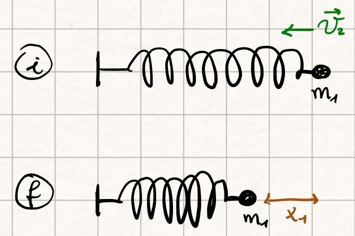
$$\Rightarrow \begin{cases} m_1 v_0 = -m_1 v_1 + 2 m_1 v_2 \\ v_0^2 = v_1^2 + 2 v_2^2 \end{cases}$$

$$\Rightarrow \begin{cases} v_1 = 2 v_2 - v_0 \\ \cancel{v_0^2} = 4 v_2^2 - 4 v_2 v_0 + \cancel{v_0^2} + 2 v_2^2 \end{cases}$$

$$\Rightarrow 6 v_2^2 - 4 v_2 v_0 = 0$$
$$v_2 (3 v_2 - 2 v_0) = 0$$

$$\Rightarrow \begin{cases} v_1 = \frac{1}{3} v_0 \\ v_2 = \frac{2}{3} v_0 \end{cases}$$

3° : COMPRESSIONE



$$\Delta E_m = 0 \quad : \quad E_{k_f} + E_{p_f} = E_{k_i} + E_{p_i}$$

$$0 + \frac{1}{2} k x_1^2 = \frac{1}{2} m_1 v_1^2 + 0$$

$$\rightarrow \quad x_1 = \sqrt{\frac{m_1}{k}}\, v_1$$

$$x_1 = \sqrt{\frac{\cancel{m}}{\cancel{k}}}\ \frac{1}{3} \sqrt{\frac{\cancel{k}}{\cancel{m}}}\ x_0 = 3 x_1$$