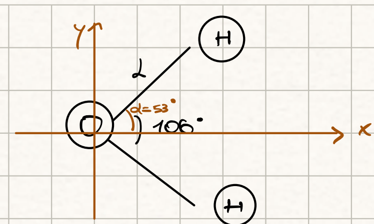
1) $L = 0{,}1 \text{ nm} = 0{,}1 \cdot 10^{-9} \text{ m}$

$m_O = 16\, m_H$



$\vec{r}_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{r}_i$; N corpi

$M = \sum_{i=1}^{N} m_i = \sum_{i=1}^{3} m_i = m_o + 2m_H = 18\, m_H$

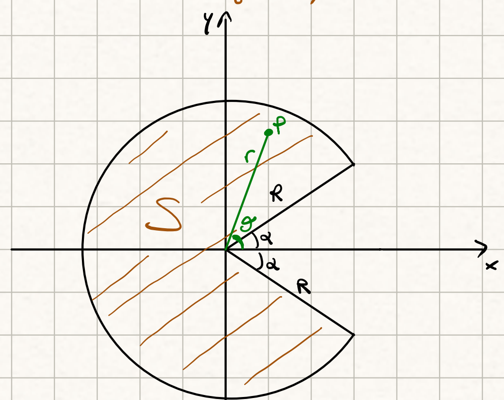
$$\Rightarrow \begin{cases} x_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i x_i \\ y_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i y_i \end{cases}$$

$$\Rightarrow \begin{cases} x_{CM} = \frac{1}{M}\left(m_O \underbrace{\cancel{x_O}}_{=0} + \underbrace{m_H x_{H1} + m_H x_{H2}}_{=}\right) = \frac{1}{M}\left(2\, m_H L\cos\alpha\right) = \frac{L\cos\alpha}{9} \\ y_{CM} = \frac{1}{M}\left(m_O \underbrace{\cancel{y_O}}_{=0} + \underbrace{m_H y_{H1} + m_H y_{H2}}_{\text{opposti}}\right) = 0 \end{cases} \Rightarrow \begin{cases} x_{CM} = 6{,}7 \text{ pm} \\ y_{CM} = 0 \end{cases}$$

asse di simmetria

2)

$\sigma = \text{cost}$ (è omogeneo)



$$M = \sigma \cdot S = \sigma\left(S_{cerchio} - S_{settore}\right) = \sigma\left(\pi R^2 - \alpha R^2\right)$$

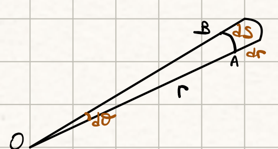
$$= \sigma R^2 (\pi - \alpha)$$

passando alle coordinate polari:

$$\begin{cases} x_P = r\cos\theta \\ y_P = r\sin\theta \end{cases}$$

con (dominio)

$$\begin{cases} 0 \leq r \leq R \\ \alpha \leq \theta \leq 2\pi - \alpha \end{cases}$$



$\widehat{AB} = r\,d\theta$

$dS = \widehat{AB} \cdot dr = r\,d\theta\,dr$

$dm = \sigma\, dS = \sigma\, r\, d\theta\, dr$

si può scomporlo in prodotto di integrali perchè le variabili non sono sovrapposte.

$$\Rightarrow x_{CM} = \frac{1}{M}\int_S x_P\, dm = \frac{1}{M}\int_S x_P\, \sigma\, r\, d\theta\, dr = \frac{1}{M}\int_0^R \int_\alpha^{2\pi-\alpha} r\cos\theta\, \sigma\, r\, d\theta\, dr = \frac{\sigma}{M}\int_0^R r^2 dr \int_\alpha^{2\pi-\alpha} \cos\theta\, d\theta =$$

$$= \frac{\sigma}{M}\left[\frac{r^3}{3}\right]_0^R \left[\sin\theta\right]_\alpha^{2\pi-\alpha} = \frac{\sigma}{M}\frac{R^3}{3}\left[\underbrace{\sin(2\pi-\alpha)}_{-\sin\alpha} - \sin\alpha\right] = \frac{\sigma}{M}\frac{R^3}{3}\left[-2\sin\alpha\right] = -\frac{2}{3}R^3\frac{\sigma}{M}\sin\alpha$$

$$\Rightarrow y_{CM} = \frac{1}{M}\int_S y_P\, dm = \ldots = 0$$ (perchè $y=0$ asse di simm.)

oppure

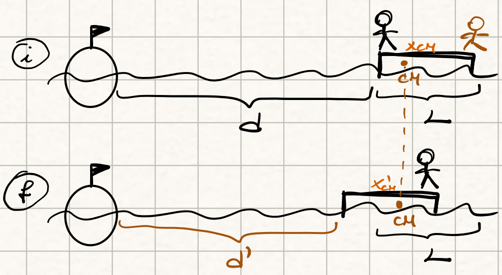
$$CM(B) = \underset{\text{di raggio } R}{CM(O)} - CM(\;)$$

3) $m = 50\,kg$

$M = 100\,kg$

$L = 3\,m$

$d = 2\,m$



$$\frac{d\vec{Q}}{dt} = \vec{F}_{est}^{\;=0} = 0$$

si conserva quant. di moto (CM fermo)

$$\Rightarrow \Delta\vec{Q} = 0$$

$$x_{CM} = \frac{1}{m+M}\sum_{i=1}^{2} m_i x_i = \frac{1}{m+M}\left[md + M\underline{\left(d+\frac{L}{2}\right)}\right]$$

considerando la zattera come un p. materiale con massa concentrata nel suo centro geometrico.

$$x'_{CM} = \frac{1}{m+M}\left[m(d'+L) + M\left(d'+\frac{L}{2}\right)\right] = \underline{x_{CM}}$$

CM rimane fermo

$$\Rightarrow md + M\left(d+\frac{L}{2}\right) = m(d'+L) + M\left(d'+\frac{L}{2}\right)$$

$$\Rightarrow (m+M)d + \cancel{M\frac{L}{2}} = (m+M)d' + mL + \cancel{M\frac{L}{2}}$$

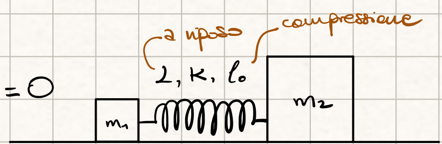
$$\Rightarrow d' = d - \frac{mL}{m+M} = \underline{1\,m}$$

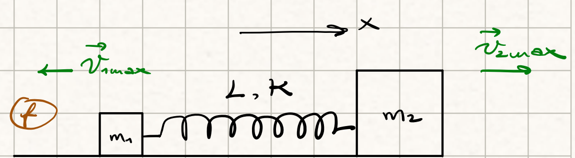
4) m_1

$m_2 = 2m_1$

? $\vec{v}_{1max}$

? $\vec{v}_{2max}$





le velocità sono max quando la lunghezza è L (a riposo)

- $\vec{F}_{est} = 0$ (risultante $= 0$)

$\rightarrow \Delta\vec{Q} = 0$ CONSERV. QUANT. MOTO

$\rightarrow \vec{Q}_f = \vec{Q}_i = 0$; $\vec{Q}_f = m_1\vec{v}_{1max} + m_2\vec{v}_{2max} = 0$

$$\Rightarrow -m_1 v_{1max}\,\vec{u}_x + m_2 v_{2max}\,\vec{u}_x = 0$$

$$\Rightarrow v_{1max} = \frac{m_2}{m_1} v_{2max} = 2v_{2max}$$

- CONSERV. E_m (solo forze conserv.) : $\Delta E_m = 0 \Rightarrow E_{k_f} - E_{k_i} + E_{p_f} - E_{p_i} = 0$

$$\rightarrow \frac{1}{2}m_1 v_{1max}^2 + \frac{1}{2}m_2 v_{2max}^2 - 0 - \frac{1}{2}k\,l_0^2 = 0$$

$$\Rightarrow m_1 (2 v_{2max}^2) + 2 m_1 v_{2Max}^2 = k \ell_0^2 \rightarrow 6 m_1 v_{2max}^2 = k \ell_0^2 \Rightarrow$$

$$\Rightarrow \vec{v}_{2Max} = + \ell_0 \sqrt{\frac{k}{6 m_1}} \, \vec{u}_x \quad ; \quad \vec{v}_{1max} = - \ell_0 \sqrt{\frac{2k}{3 m_1}} \, \vec{u}_x$$

6)

m

$t_0 = 0\,s$

$v_0 = 10\,m/s$

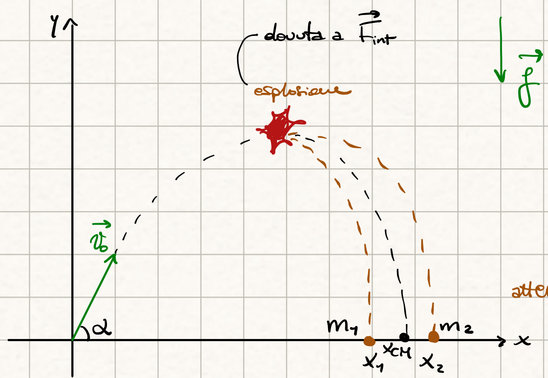
$\alpha = 60°$

$m_1 = \frac{2}{3} m$

$m_2 = \frac{1}{3} m$

$x_2 = 11\,m$

? x_1



$$\frac{d\vec{Q}}{dt} = \vec{F}_{est}$$

atterrano simultaneamente

- moto del CM:

$$\begin{cases} x_{CM}(t) = v_x t = v_0 \cos\alpha \, t \\ y_{CM}(t) = v_y t - \frac{1}{2} g t^2 = v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \end{cases}$$

$$\tau \rightarrow y_{CM}(\tau) = 0 \quad : \quad \tau = \frac{2 v_0 \sin\alpha}{g}$$

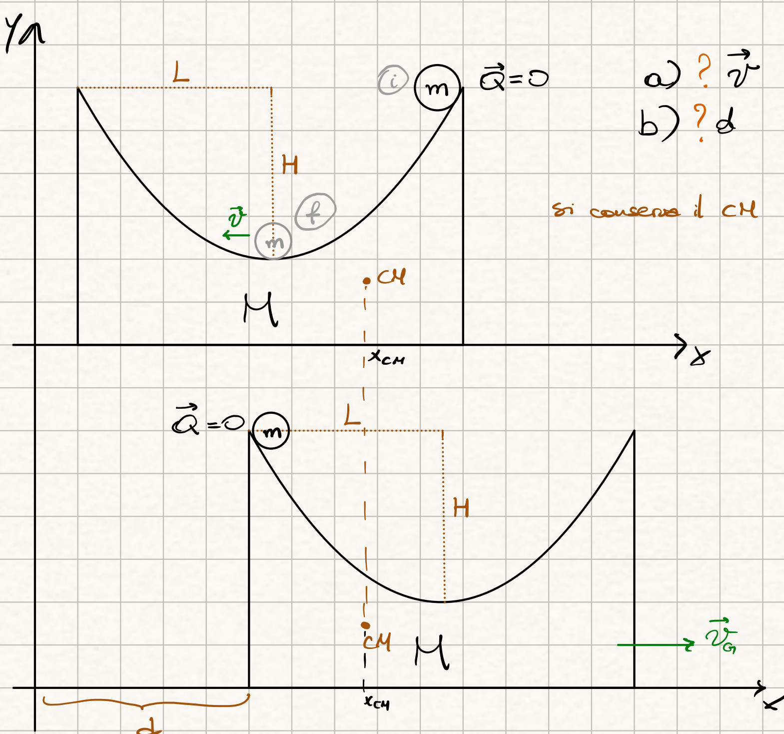
$$\Rightarrow x_{CM}(\tau) = \frac{v_0^2 \sin 2\alpha}{g}$$

$$x_{CM} = \frac{1}{m}(x_1 m_1 + x_2 m_2) = \frac{1}{\cancel{m}}\left(\frac{2}{3} x_1 \cancel{m} + \frac{1}{3} x_2 \cancel{m}\right) = \frac{2}{3} x_1 + \frac{1}{3} x_2$$

↳ ! si mantiene la proporzione

$$\Rightarrow x_1 = \frac{3}{2} \frac{v_0^2 \sin(2\alpha)}{g} - \frac{1}{2} x_2 \cong \underline{7{,}74\,m}$$

5)



a) ? $\vec{v}$

b) ? d

si conserva il CM

a) $\frac{d\vec{Q}}{dt} = \vec{F}_{est} = 0$

- $\Delta \vec{Q} = 0 \Rightarrow \vec{Q}_f - \cancel{\vec{Q}_i} = 0$

$$M v_{Guida} - m v = 0 \Rightarrow$$

$$\Rightarrow v_G = \frac{m}{M} v$$

- $\Delta E_m = 0$:

$$\frac{1}{2} m v^2 + \frac{1}{2} M v_G^2 - 0 + 0 - mgH = 0$$

$$\Rightarrow v = \sqrt{\frac{2gH}{1 + \frac{m}{M}}}$$

se $M \gg m$: $v \rightarrow \sqrt{2gH}$

b)

$$\begin{cases} x_{CM} = \dfrac{m\,2L}{m+M} = \dfrac{2Lm}{m+M} \\[2ex] x'_{CM} = \dfrac{Md + md}{m+M} = \dfrac{\cancel{(M+m)}d}{\cancel{M+m}} = d \end{cases}$$

$$\Rightarrow \Delta\vec{Q} = 0 \longrightarrow x_{CM} = x'_{CM} : \qquad d = \frac{2Lm}{M+m}$$