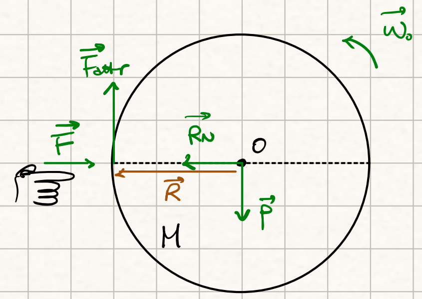
1) $M = 20\,kg$

$R = 0{,}1\,m$

$\omega_0 = 4\,rad/s$

$F = 1\,N$

$\mu_d = 0{,}5$



$+\circlearrowleft \neq \otimes$

condiz. che il corpo sia fermo:

$F - R_N = 0 \quad \rightarrow R_N = F$

? $\vec{\tau}_{attr}$

$$|\vec{F}_{attr}| = |\mu_d \cdot \vec{R}_N|$$

$$|\vec{\tau}_{attr}| = |\vec{R} \times \vec{F}_{attr}| = R\mu_d |\vec{R}_N| = \mu_d F R = 0{,}05\ N/m$$

(entrante)

? ΔE_{Diss}

? n_{giri}

$$E_{K,i} = \frac{1}{2}\overbrace{I}^{=\frac{1}{2}MR^2}\omega_0^2 + \cancel{\frac{1}{2}M v_{CM}^2}^{\text{No traslazione}} = \frac{1}{4}MR^2\omega_0^2 \quad ; \quad E_{K,f} = 0$$

$$\Rightarrow |E_{diss}| = |\underbrace{\Delta E_K}_{<0 \Rightarrow \text{dissipata}}| = \frac{1}{4}MR^2\omega_0^2 = 0{,}8\,J$$

$$\mathcal{L}^{est} = \Delta E_K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 \Rightarrow d\mathcal{L}^{est} = I\omega\, \overbrace{d\omega}^{=\alpha dt} = I\underbrace{\omega\alpha\, dt}_{d\theta} = I\alpha\, d\theta = \tau^{est} d\theta$$

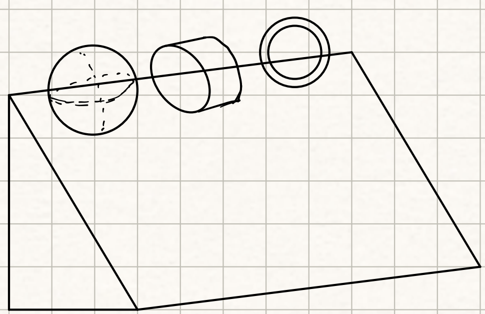
$$d\mathcal{L}^{est} = \tau^{est} d\theta \quad \Rightarrow \mathcal{L}^{est} = \int_{\theta_0}^{\theta_1} \underbrace{\tau^{est}}_{cost} d\theta = \tau^{est}\Delta\theta$$

$$\Delta\theta = \frac{\mathcal{L}^{est}}{\tau^{est}} = \frac{\Delta E_K}{\tau^{est}} = \frac{\frac{1}{4}MR^{\cancel{2}}\omega_0^2}{\mu_d \cancel{R} F} = 16\ rad$$

$$n_{giri} = \frac{\Delta\theta}{2\pi} = \frac{16}{2\pi} = 2{,}55\ giri$$

2) M, R

θ

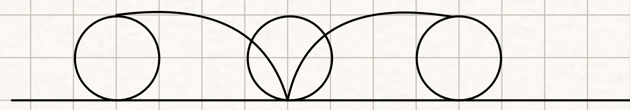


$\tau = I\alpha \quad ; \quad I = \int d^2\, dm$

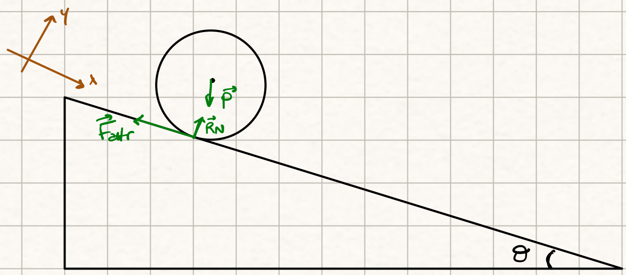
PURO ROTOLAMENTO :

$$\alpha = \frac{a_{CM}}{R} \longrightarrow \omega = \frac{v_{CM}}{R}$$

$|\vec{F}_{ATT}| \le \mu_s |\vec{R}_N| \rightarrow$ altrimenti striscia



CICLOIDE



BILANCIO :

$$\begin{cases} x: & Mg\sin\theta - F_{attr} = M\underbrace{a}_{\text{di CM}} \\ y: & R_N - Mg\cos\theta = 0 \end{cases}$$

L'unico momento di Forze è di ATTRITO : $\tau_{attr} = R F_{attr} = I\alpha = I\frac{a}{R} \Rightarrow F_{attr} = I\frac{a}{R^2} = \frac{Mg\sin\theta}{\frac{MR^2}{I}+1}$

$$\Rightarrow Mg\sin\theta - I\frac{a}{R^2} = Ma \Rightarrow a = \frac{Mg\sin\theta}{M+\frac{I}{R^2}} \rightarrow a \propto \frac{1}{I}$$

$$F_{attr.} = I\,\frac{Mg\sin\theta}{M+\frac{I}{R^2}}\,\frac{1}{R^2} = \frac{1}{\frac{1}{I}}\,\frac{Mg\sin\theta}{M+\frac{I}{R^2}}\,\frac{1}{R^2} = \frac{Mg\sin\theta}{\frac{MR^2}{I}+1}$$

SFERA : $I_S = \frac{2}{5}MR^2 \Rightarrow 1^{st}$

DISCO : $I_D = \frac{1}{2}MR^2 \Rightarrow 2^{nd}$

ANELLO : $I_A = MR^2 \Rightarrow 3^{rd}$

oppure : $\Delta E_m = 0 \rightarrow \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_f^2 + MgR - Mg(h+R) = 0$; $\omega = \frac{v}{R}$

$$\rightarrow v_f = \sqrt{\frac{Mgh}{M+\frac{I}{R^2}}}$$

$$\left.\begin{aligned} \rightarrow v_S &= \sqrt{\tfrac{10}{7}gh} \\ v_D &= \sqrt{\tfrac{4}{3}gh} \\ v_A &= \sqrt{gh} \end{aligned}\right\} < \sqrt{2gh}$$

$\rightarrow$ puro strisciamento senza attrito

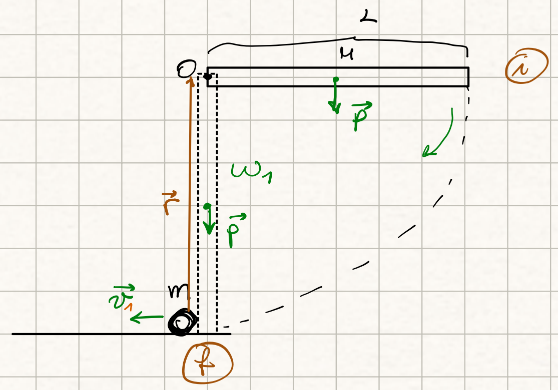
$$F_{att} \le \mu_s R_N = \mu_s Mg\cos\theta \Rightarrow \frac{\cancel{Mg}\sin\theta}{\frac{MR^2}{I}+1} \le \mu_s \cancel{Mg}\cos\theta \Rightarrow$$

$$\rightarrow \mu_s > \left(\frac{\tan\theta}{\frac{MR^2}{I}+1}\right)_{MAX}$$

$I \rightarrow$ MAX $\Rightarrow I_A$

$$\rightarrow \boxed{\mu_s \ge \frac{\tan\theta}{2}}$$

3) $M = 1\,kg$
$L = 0,5\,m$



? $m \mid v_{ast,f} = 0$

URTO ELASTICO

$\tau^{est} = \frac{dL_z}{dt}$ z fisso

DURANTE URTO: $\tau^{est} = \frac{dL_z}{dt} = 0 \Rightarrow \Delta L_z = 0$

$L_i = I_O\omega = \frac{1}{3}ML^2\omega$

$L_f = 0 + |\vec{r} \times m\vec{v}| = Lmv$ (M fermo)

$\Rightarrow v = \frac{ML}{3m}\omega$

$\Delta E_K = 0$ $\quad E_{K,i} = \frac{1}{2}I_O\omega^2 = \frac{1}{6}ML^2\omega^2$; $\quad E_{K,f} = \frac{1}{2}mv^2$

$\Delta E_K = 0 \rightarrow \frac{M}{3m} = 1 \Rightarrow m = \frac{M}{3} = 0,33\ kg$

? v_1 $\quad v_1 = \cancel{\frac{ML}{3m}}\omega_1 = L\omega_1 = \sqrt{3gL} = 3,83\ m/s$

$\Delta E_m = 0$: $\frac{1}{2}\underbrace{I_O}_{\frac{1}{3}ML^2}\omega_1^2 - 0 + \frac{MgL}{2} - MgL = 0 \Rightarrow \omega_1 = \sqrt{\frac{3g}{L}}$

? v_1 SE URTO COMPLETAM. ANAELASTICO
↳ del sistema

- $\Delta L = 0 \Rightarrow L_i = L_f \Rightarrow I_{ASTA}\,\omega_i = I_{ASTA+m}\,\omega_f$
- $\Delta E_K \neq 0$

$I_{ASTA+m} = I_A + mL^2 = \frac{1}{3}ML^2 + \frac{M}{3}L^2 = \frac{2}{3}ML^2 = 2I_A$

$\rightarrow \cancel{I_A}\,\omega_i = 2\cancel{I_A}\,\omega_f$

$\Rightarrow \omega_f = \frac{\omega_i}{2} = \frac{1}{2}\sqrt{\frac{3g}{L}}$

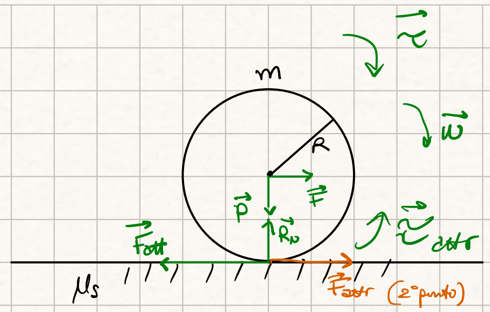
$\Rightarrow v_f = \omega_f L = \sqrt{\frac{3}{4}gL} = 1,92\ m/s$

4) $m = 5\,kg$

$R = 0{,}25\,m$

$\mu_s = 0{,}15$

? $\vec{F}_{max}$ | PURO ROTOLAM.



$\vec{\tau}^{ext}$ + ↻ ⊗ z

$$\begin{cases} x: & F - F_{attr} = ma \\ y: & R_N - mg = 0 \\ & R F_{attr} = I\alpha \end{cases}$$

$\rightarrow F = ma + F_{attr} = m\left(\frac{R^2}{I} + 1\right) F_{attr}$

$\le \mu_s mg$ *

$\rightarrow R_N = mg$

$I = \frac{1}{2} MR^2$

$\alpha = \frac{a}{R}$

$* \Rightarrow F \le \left(m\frac{R^2}{I} + 1\right)\mu_s mg = 3\mu_s mg = 22\,N$

2 −

$$\begin{cases} F_{attr} = ma \rightarrow a = \frac{F_{attr}}{m} \\ R_N = mg \\ \tau - R F_{attr} = I\frac{a}{R} = \frac{I F_{attr}}{mR} \end{cases}$$

$\frac{1}{2} mR^2$

$\rightarrow \tau = \left(\frac{I}{mR} + R\right) F_{attr} \le \left(\frac{I}{mR} + R\right)\mu_s mg = \frac{3}{2}\mu_s mg R = 2{,}76\,N\cdot m$