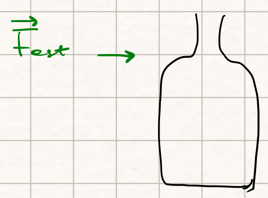
MODELLO PUNTO MATERIALE → limiti

⇒ MODELLO DI CORPO RIGIDO

↳ sistema di punti | distanza tra vari punti rimane uguale



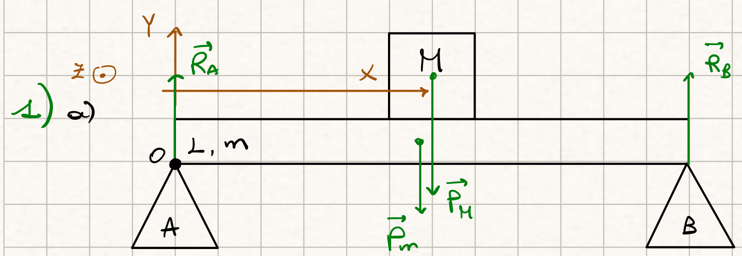
$$\vec F_{est} = m\vec a$$

↳ può essere applicata in punti diversi del c.r. ⇒ MOMENTI DI FORZA

$$\vec r \times \vec F_{est} = \vec \tau_{est} = \frac{d\vec L}{dt} \propto \alpha$$

Nel caso in cui $\begin{cases} \vec F_{est} = 0 \\ \tau_{est} = 0 \end{cases}$ → corpo fermo   STATICA DER CORPO RIGIDO

1) a)



$\begin{cases} \vec F_{est} = 0 & \text{BILANCIO FORZE ①} \\ \vec \tau_{est} = 0 & \text{BILANCIO MOMENTI ②} \end{cases}$

↳ scegliere un'origine (arbitrario se fermo)

$$\to \begin{cases} \vec R_A + \vec R_B + \vec P_M + \vec P_m = 0 \Rightarrow R_A + R_B - P_M - P_m = 0 \Rightarrow R_A + R_B = Mg + mg \\ \vec \tau_{R_A} + \underbrace{\vec \tau_{R_B}}_{\text{uscente}} + \underbrace{\vec \tau_{P_M} + \vec \tau_{P_m}}_{\text{entrante}} = 0 \Rightarrow 0 + \cancel{L} R_B - Mg\frac{x}{\cancel{L}} - mg\frac{\cancel{L}}{2} = 0 \end{cases}$$

$$\begin{cases} R_B = \frac{1}{2} mg + Mg\frac{x}{L} \\ R_A + \frac{1}{2}mg + Mg\frac{x}{L} = mg + Mg \Rightarrow R_A = \frac{1}{2} mg + Mg\left(1 - \frac{x}{L}\right) \end{cases}$$

se $x = \frac{L}{2}$ le forze R_A, R_B si distribuiscono equamente su A e B

b) $F_{max} = 600\ N$ → ? posizione per cui $R = F_{max}$

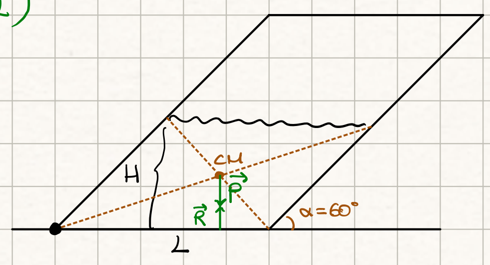
$$R_A = F_{max} = \frac{1}{2} mg + Mg\left(1 - \frac{x}{L}\right) \Rightarrow Mg\frac{x}{L} = \frac{1}{2}mg + Mg - F_{max} \Rightarrow x = \frac{2Mg + mg - 2F_{max}L}{2Mg}$$

$$= 0{,}23\ m$$

$$R_B = F_{max} = \frac{1}{2} mg + Mg\frac{x}{L} \Rightarrow x = \frac{2F_{max} - mg}{2Mg} = 0{,}77\ m$$

↳ $= 1 - 0{,}23$

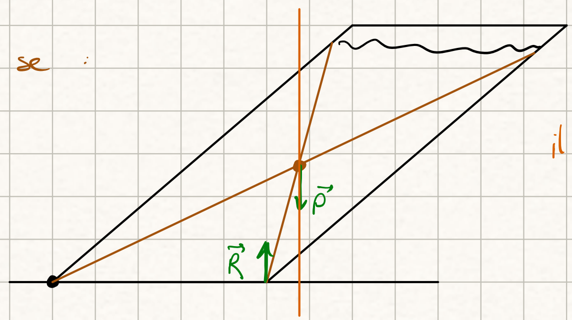
2)



$\vec{\tau}_{est} = 0$

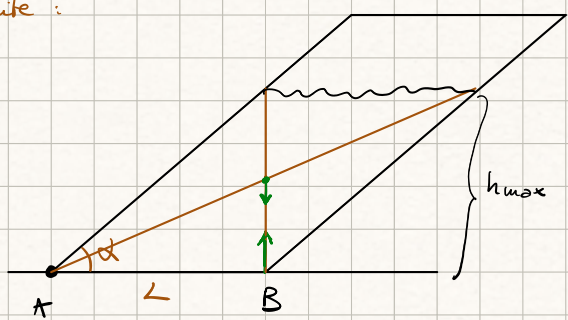
$|\vec{R}_N| = |\vec{P}|$

Si ribalta se :



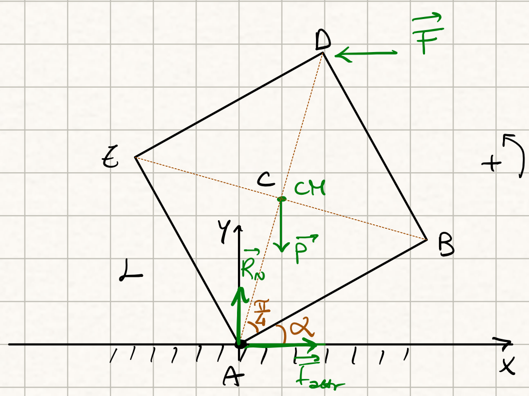
il baricentro esce dal corpo

condizione limite :



$h_{max} = L \, tg \, 60° = \sqrt{3} L$

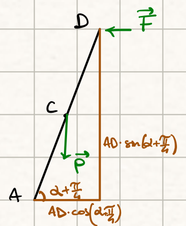
3) a)



$$\vec{\tau}_{est} = \sum_i \vec{\tau}_i = \vec{\tau}_F + \vec{\tau}_P = 0$$

$$\Rightarrow \vec{AD} \times \vec{F} + \vec{AC} \times \vec{P} = 0$$

$$\alpha + \frac{\pi}{4} < \frac{\pi}{2} \quad \rightarrow \quad \alpha < \frac{\pi}{4}$$



$$\underset{\sqrt{2}L}{\underline{AD}} \cdot F \sin\left(\alpha + \frac{\pi}{4}\right) - \underset{\frac{\sqrt{2}}{2}L}{\underline{AC}} \cdot \overset{Mg}{P} \cos\left(\alpha + \frac{\pi}{4}\right) = 0$$

$$\cancel{\sqrt{2}L} F \sin\left(\alpha + \frac{\pi}{4}\right) - \frac{\cancel{\sqrt{2}}}{2}\cancel{L} P \cos\left(\alpha + \frac{\pi}{4}\right) = 0$$

$$\Rightarrow F = \frac{1}{2} Mg \cot g\left(\alpha + \frac{\pi}{4}\right)$$

b) $0 \leq \alpha \leq \frac{\pi}{4}$

? μ_s

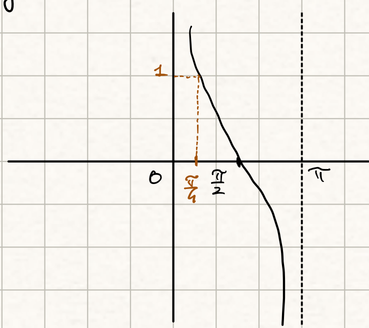
$\sum \vec{F} = 0$: $\quad$ Y $\begin{cases} R_N = P = Mg \\ F_{attr} = F \end{cases}$; $\quad F = F_{attr} \leq \mu_s R_N = \mu_s Mg$

$$\Rightarrow \frac{1}{2} \cancel{Mg} \cot g\left(\alpha + \frac{\pi}{4}\right) \leq \mu_s \cancel{Mg}$$

$$\Rightarrow \mu_s \geq \frac{1}{2} \cot g\left(\frac{\pi}{4} + \alpha\right)$$

se $\alpha = 0 \Rightarrow \cot g \frac{\pi}{4} = 1$

$$\Rightarrow \mu_{s_{min}} = \frac{1}{2}$$

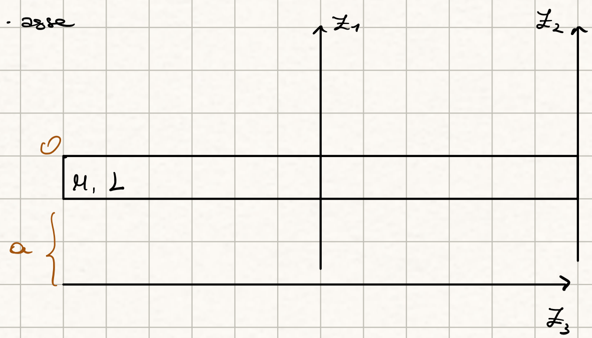


4) MOMENTO DI INERZIA:

$\vec{\tau} = I \vec{\alpha}$

tendenza di un corpo ad opporsi alla rotazione

dipende da
- massa (densità)
- forma
- asse



$$I = \int_V r^2 \, dm = \int_V r^2 \rho \, \underline{dV}$$

Nel caso di una sfera:

$dV = r^2 \sin\varphi \, dr \, d\varphi \, d\theta$

$$\Rightarrow I = \int_L r^2 \, dm = \int_L r^2 \lambda \, dx$$

1D) $dm = \lambda \, dx = \frac{m}{L} dx$

$\lambda \to \left[\frac{kg}{m}\right]$

2D) $dm = \sigma \, dS$

3D) $dm = \rho \, dV$

$$I_{z_1} = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \, dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{M}{L} \, dx$$

$$= \frac{M}{L} \left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{ML^2}{12}$$

$$I_{z_2} = \int_0^L x^2 \, dm = \int_0^L x^2 \frac{M}{L} \, dx = \frac{M}{L} \left[\frac{x^3}{3}\right]_0^L = \frac{ML^2}{3}$$

oppure con Steiner: $I_{z_2} = I_{z_1} + M\left(\frac{L}{2}\right)^2$

$$I_{z_3} = \int_0^L a^2 \, dm = \int_0^L a^2 \frac{M}{L} \, dx = a^2 \frac{M}{L} \int_0^L dx = a^2 M \frac{\cancel{L}}{\cancel{L}}$$

→ non dipende dalla lunghezza