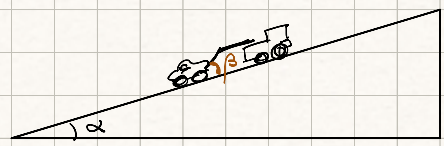
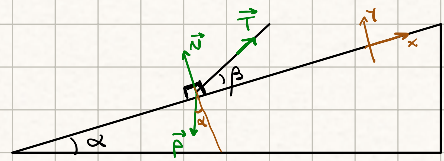
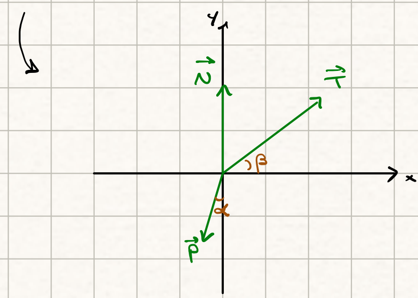
1) $m = 1200\ kg$
$\alpha = 18°$
$\beta = 27°$
$T_{max} = 4,6\ kN$

? Δs_{max}
$\Delta t = 7,5\ s$



→





$|\vec{P}| = P = mg$

$$\begin{cases} y: & \overset{?}{N} - mg\cos\alpha + T\sin\beta = 0 \\ x: & T\cos\beta - mg\sin\alpha = m\overset{?}{a} \end{cases}$$

$$\Rightarrow a_{max} = \frac{T_{max}\cos\beta}{m} - g\sin\alpha = 0,38\ m/s^2$$

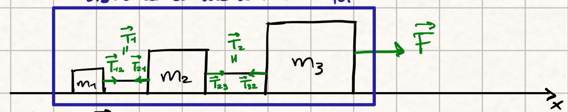
$$\Delta s_{max} = \frac{1}{2} a_{max} \Delta t^2 = 10,8\ m$$

2) $m_1 = 10\ kg$
$m_2 = 20\ kg$
$m_3 = 30\ kg$

$|\vec{F}| = 60N$

a) ? T funi
b) ? a

sistema di masse: $\vec{F} = m_{tot}\vec{a}$



$|\vec{T}_{12}| = |\vec{T}_{21}| = T_1$
$|\vec{T}_{23}| = |\vec{T}_{32}| = T_2$

$$\begin{cases} ① & T_1 = m_1 a_1 \\ ② & T_2 - T_1 = m_2 a_2 \\ ③ & F - T_2 = m_3 a_3 \end{cases}$$

i blocchi si muovono con la stessa accelerazione.

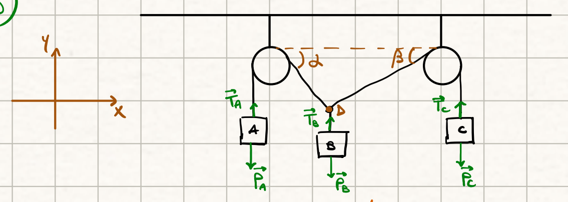
$$a_1 = a_2 = a_3 \Rightarrow \begin{cases} T_1 = m_1 a \\ T_2 - T_1 = m_2 a \\ F - T_2 = m_3 a \end{cases}$$

$$\begin{cases} T_1 = m_1 a \\ T_2 - m_1 a = m_2 a \\ F = T_2 + m_3 a \end{cases} \qquad \begin{cases} T_1 = m_1 a \\ T_2 = (m_1+m_2) a \\ F = (m_1+m_2+m_3) a \end{cases} \Rightarrow a = \frac{F}{m_1+m_2+m_3}$$

$$\begin{cases} T_1 = \dfrac{m_1}{m_1+m_2+m_3} F = 10\ N \\ T_2 = \dfrac{m_1+m_2}{m_1+m_2+m_3} F = 30\ N \\ a = \dfrac{F}{m_1+m_2+m_3} = 1\ m/s^2 \end{cases}$$

! $m_{tot} = m_1 + m_2 + m_3 \Rightarrow \vec{F} = m_{tot}\vec{a}$

3)



! è tutto in equilibrio

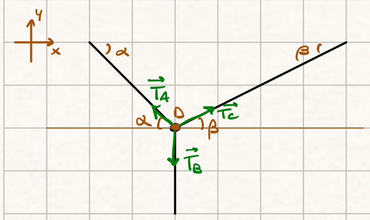
$\alpha = 60°$
$\beta = 30°$
$m_B = 2\,kg$

? m_A, m_C

$$\begin{array}{l} A \\ B \\ C \end{array}\begin{cases} T_A - m_A g = 0 \\ T_B - m_B g = 0 \\ T_C - m_C g = 0 \end{cases}$$

3 eq. , 5 incognite

mettendo insieme $\Rightarrow$ 5 eq. 5 incogn. ✓



$$\begin{array}{l} x: \\ y: \end{array}\begin{cases} T_C \cos\beta - T_A \cos\alpha = 0 \\ T_C \sin\beta + T_A \sin\alpha - T_B = 0 \end{cases}$$

2 eq. , 3 incognite

$$\begin{cases} T_A = m_A g \\ T_B = m_B g \\ T_C = m_C g \end{cases} \quad \text{Sostituendo} \quad \begin{cases} m_C \cos\beta - m_A \cos\alpha = 0 \\ m_C \sin\beta - m_A \sin\alpha - m_B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} m_A = m_C \dfrac{\cos\beta}{\cos\alpha} \\ m_C\left(\sin\beta + \dfrac{\sin\alpha\cos\beta}{\cos\alpha}\right) = m_B \end{cases} \Rightarrow m_C\left(\dfrac{\sin\beta\cos\alpha + \sin\alpha\cos\beta}{\cos\alpha}\right) = m_B \Rightarrow m_C = \dfrac{m_B}{\sin\beta + tg\alpha\cos\beta}$$

$$\dfrac{\sin\beta\cos\alpha + \sin\alpha\cos\beta}{\cos\alpha} = \sin\beta + tg\alpha \cos\beta$$

$$\Rightarrow \begin{cases} m_A = \dfrac{\cos\beta}{\cos\alpha} \dfrac{m_B}{\sin\beta + tg\alpha\cos\beta} = m_B \dfrac{\cos\beta}{\sin\alpha\cos\beta + \sin\beta\cos\alpha} = m_B \dfrac{\cos\beta}{\sin(\alpha+\beta)} \\ m_C = \dfrac{m_B}{\sin\beta + tg\alpha\cos\beta}\left(\dfrac{\cos\alpha}{\cos\alpha}\right) = m_B \dfrac{\cos\alpha}{\sin(\alpha+\beta)} \end{cases}$$

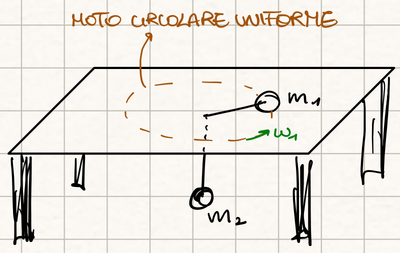
$$\Rightarrow \begin{cases} m_A = m_B \dfrac{\cos\beta}{\sin\frac{\pi}{2}} = m_B \dfrac{\sqrt{3}}{2} = 1{,}73\,kg \\ m_C = m_B \dfrac{\cos\alpha}{\sin\frac{\pi}{2}} = m_B \dfrac{1}{2} = 1\,kg \end{cases}$$

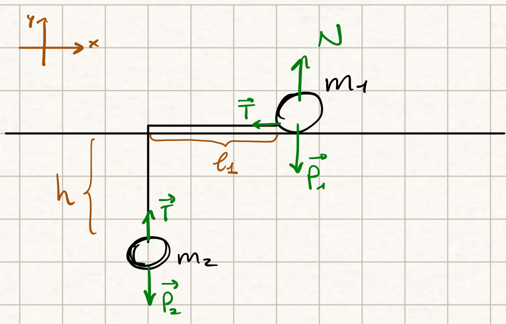
4)

lunghezza tot. filo = ℓ

m_1 , m_2

ω_1 cost





a) ? h

b) ? forze agenti

c) ? cosa succede se il filo viene tagliato

① $\begin{cases} N = m_1 g \\ T = m_1 a_c \end{cases}$

l'unica forza che agisce → mette in moto → centripeta

② $T = m_2 g$

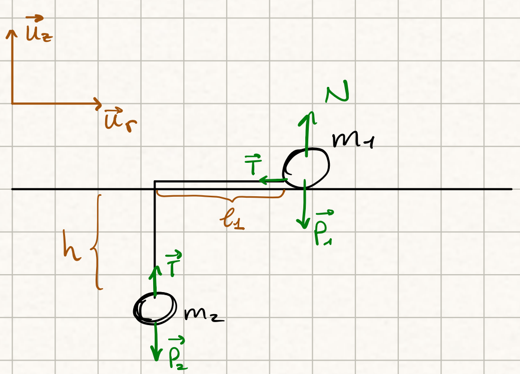
$a_c = \omega_1^2 \underline{\ell_1}$

raggio della circonferenza

$\Rightarrow a_c = \dfrac{m_2}{m_1} g = \omega_1^2 \ell_1 \quad \Rightarrow \ell_1 = \dfrac{m_2 g}{m_1 \omega_1^2} = \ell - h$

$\Rightarrow h = \ell - \dfrac{m_2 g}{m_1 \omega_1^2}$

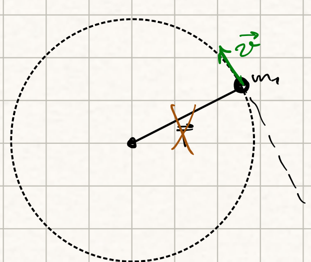
con l'analisi dimensionale si può verificare che il risultato è effettivamente una lunghezza (in m)



b) $\begin{cases} \vec{T} = -m_2 g \, \vec{u}_r \\ \vec{N} = m_1 g \, \vec{u}_z \\ \vec{P} = -m_1 g \, \vec{u}_z \end{cases}$

c) Se il filo viene taglia, T si annulla $\rightarrow a_c = 0$ $\Rightarrow$ il corpo m_1 si dirigerà verso la direzione della velocità in quell'istante in moto rettilineo uniforme

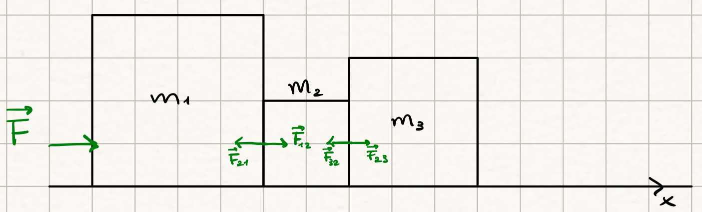
velocità:



$a_c = 0$

$|\vec{v}| = \omega_1 \ell_1 = \dfrac{m_2 g}{m_1 \omega_1}$

5)



$m_1 = 45,2$ kg
$m_2 = 22,8$ kg
$m_3 = 34,3$ kg

a) ? $\vec{F}$ | $a = 1,32$ m/s^2

(NO attrito)

b) ? $|\vec{F}_{23}| = F_2$

c) ? $|\vec{F}_{12}| = F_1$

$|\vec{F}_{21}| = |\vec{F}_{12}| = F_1$
$|\vec{F}_{23}| = |\vec{F}_{32}| = F_2$

a) $F = (m_1 + m_2 + m_3)\, a = 135$ N

$$\begin{cases} F - F_1 = m_1 a & (1) \\ F_1 - F_2 = m_2 a & (2) \\ F_2 = m_3 a & (3) \end{cases}$$

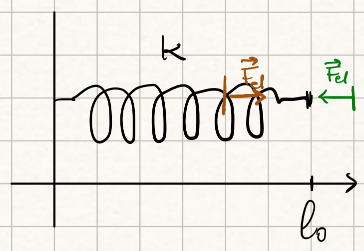
c) $\rightarrow F_1 = F - m_1 a = 75,4$ N

b) $F_2 = 45,3$ N

Attrito statico, dinamico
Molla ideale
⋮

17/03/2022

MOLLA IDEALE



Forza elastica = forza di richiamo

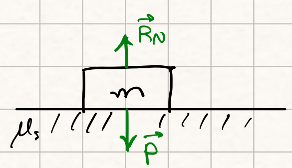
$|\vec{F}_{el}| = K(x - l_0)$

$\vec{F}_{el} = -K(x - l_0)\vec{u}_x$

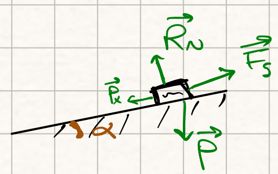
ATTRITO STATICO :

$0 \leq F_s \leq \mu_s R_N$ → se supera: il corpo inizia a muoversi

L COEFFICIENTE DI ATTRITO STATICO



$\vec{F}_s = 0$

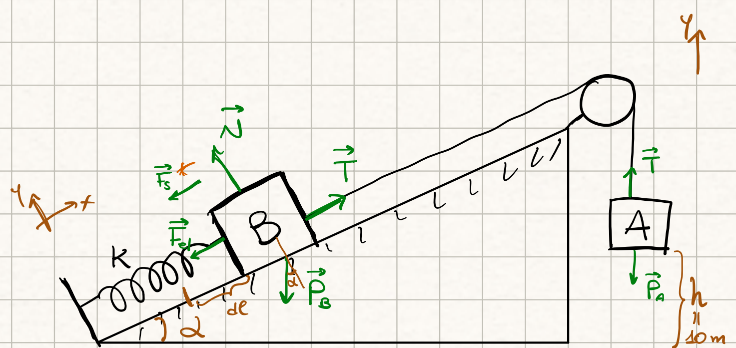


$|\vec{P}_x| \leq |\vec{F}_{s\,max}|$

$|\vec{F}_s| = |\vec{P}_x|$

→ se si supera un angolo limite α_c :
$|\vec{F}_d| < |\vec{F}_{s\,max}|$

1)



$m_A = 3\,kg$
$m_B = 2\,kg$
$\alpha = 30°$
$K = 30\,N/m$
$d\ell = 0,5\,m$

attrito statico
a) ? $|\vec{F}_s|$

b) ? Δt sapendo che
$\mu_d = 0,3$ (molla rotta)

! sistema in equilibrio

* Non sappiamo ancora dov'e' diretta F_s (se alla fine otteniamo $F_s > 0$ allora il verso scelto è giusto, altrimenti è nell'altro verso)

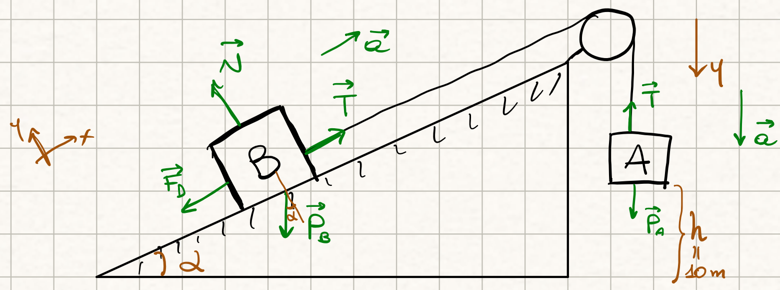
a) A : $T - m_A g = 0$

B : $\begin{cases} x: & T - P_{Bx} - F_s - F_{el} = 0 \\ y: & N - P_{By} = 0 \end{cases}$ $\Rightarrow \begin{cases} m_A g - m_B g \sin\alpha - \overset{\text{incognita}}{F_s} - K d\ell = 0 \\ N = m_B g \cos\alpha \end{cases}$

$\Rightarrow F_s = m_A g - m_B g \sin\alpha - K d\ell = 4,62\,N$

! versi in base ai sistemi cartesiani scelti

b) accelerazione dei due corpi hanno modulo uguale



A: $-T + m_A g = +m_A a$

B: $\begin{cases} x & T - F_D - m_B g \sin\alpha = m_B a \\ y & N = m_B g \cos\alpha \end{cases}$

A: $T = m_A (g - a)$

$F_D = \mu_d N = \mu_d m_B g \cos\alpha \quad \Rightarrow \quad B: \quad m_A(g-a) - \mu_d m_B g \cos\alpha - m_B g \sin\alpha = m_B a$

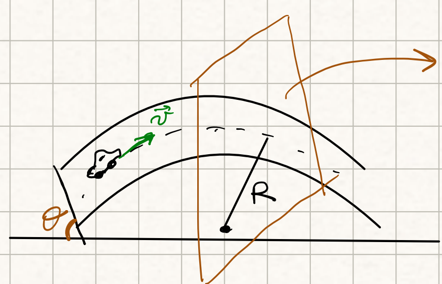
$a = \dfrac{m_A - m_B \sin\alpha - \mu_d m_B \cos\alpha}{m_A + m_B} g = 2{,}9 \ m/s^2$

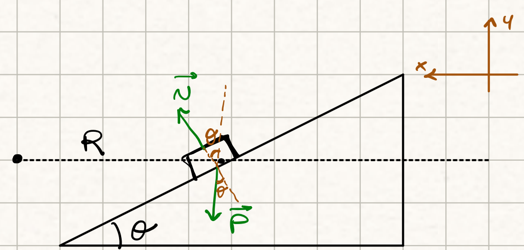
└ MOTO UNIFORMEMENTE ACCELERATO

legge oraria: $y(t) = \overset{0}{y_0} + \overset{0}{v_0} t + \frac{1}{2} a t^2 \Rightarrow y(\Delta t) = h = \frac{1}{2} a \Delta t^2$

$\Delta t = \sqrt{\dfrac{2h}{a}} = 2{,}63 \ s$

2) $\mu_s = 0$
$v = 50 \ km/h$ *
$R = 50 \ m$
?θ





→ ci dovrà essere una forza centripeta tale che la macchina non parta per la tangente

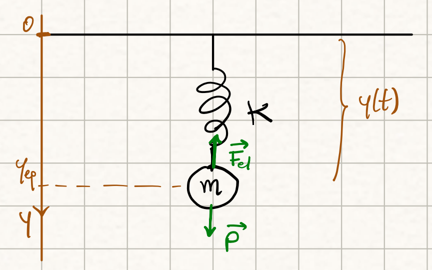
$\begin{cases} x & \boxed{N \sin\theta} = m a_c \\ y & N \cos\theta - mg = 0 \end{cases} \rightarrow \begin{cases} \cancel{m} g \, tg\theta = \cancel{m} a_c \\ N = \dfrac{mg}{\cos\theta} \end{cases} \rightarrow a_c = g \, tg\theta$

* $\vec{v}$ cost $\Rightarrow a_T = 0 \Rightarrow$ solo a_c

$a_c = \dfrac{v^2}{R} \Rightarrow g \, tg\theta = \dfrac{v^2}{R} \Rightarrow \theta = \arctg\left(\dfrac{v^2}{gR}\right) = 21{,}5°$

verifica dimensionale: $\left[\dfrac{m^2 s^2}{s^2 m \cdot m}\right]$ ⇒ adimension. ✓

3) m
k
$\ell_0 = 0$



a) ? $y_{equilibrio}$
nel caso statico

b) ? $y(t)$:

$\boxed{\begin{aligned} t &= 0\,s \\ y_0 &= 0 \\ v_{y_0} &= 0 \end{aligned}}$ condizioni iniziali

a) $P - F_{el} = 0$

$mg - k y_{eq} = 0 \quad \Rightarrow \quad y_{eq} = \dfrac{mg}{k}$

(la molla non è a riposo)

b) $mg - k y(t) = m a(t)$

$a = \dfrac{d^2 y}{dt^2} \quad \rightarrow \quad \dfrac{d^2 y}{dt^2} + \dfrac{k}{m} y = g$

eq. differenziale non omogenea

eq. moto armonico

soluz. dell'eq. differenziale omogenea

P.S. $\dfrac{d^2 x}{dt^2} + \underbrace{\omega^2}_{\text{pulsazione}} x = 0$

$\rightarrow \quad x(t) = A\cos(\omega t + \varphi)$

SOLUZ. EQ. DIFFERENZIALE NON OMOGENEA :

porto dentro g $\Rightarrow$ rendo l'eq. omogenea

$$\frac{d^2 y}{dt^2} + \frac{k}{m}\Big(y - \underbrace{\frac{gm}{k}}_{y_{eq}}\Big) = 0$$

$y \rightarrow y - y_{eq}$; $\dfrac{d^2(y - y_{eq})}{dt^2} = \dfrac{d^2 y}{dt^2}$

$b = y - y_{eq}$

$\dfrac{d^2 b}{dt^2} + \dfrac{k}{m} b = 0$! CASO OMOGENEO

$\rightarrow \quad b(t) = A\cos(\omega t + \varphi)$, $\omega = \sqrt{\dfrac{k}{m}}$

Dunque :

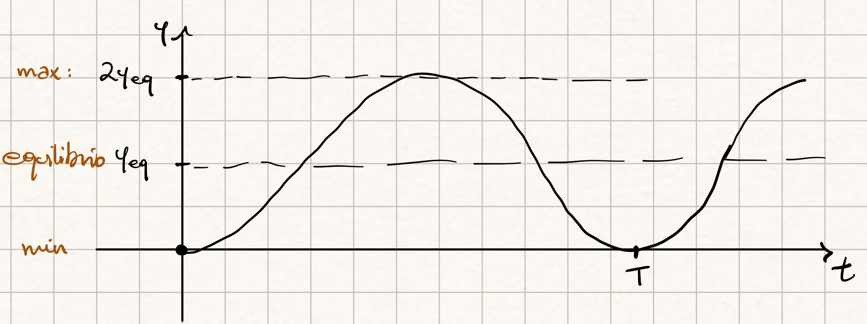
$b(t) = y(t) - y_{eq} = A\cos(\omega t + \varphi) \quad \Rightarrow \quad y(t) = y_{eq} + A\cos(\omega t + \varphi)$

$y(0) = y_0 = y_{eq} + A\cos(\varphi) = 0$

$v_{y_0} = \left.\dfrac{dy(t)}{dt}\right|_{t=0} = -\underset{\neq 0}{\omega}\underset{\neq 0}{A}\sin(\varphi) = 0 \quad \Rightarrow \quad \sin(\varphi) = 0 \quad \Rightarrow \quad \varphi = 0, \pm\pi, \pm 2\pi, \ldots$

$A = -y_{eq}$

$\rightarrow \quad y(t) = y_{eq}\left[1 - \cos(\omega t)\right]$ equazione del moto

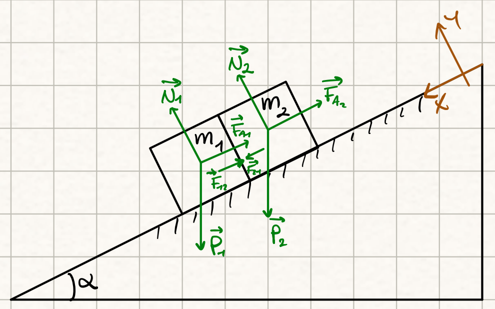


T : y si annulla $\Rightarrow \omega T = 2\pi$

$\Rightarrow \quad T = \dfrac{2\pi}{\omega}$ ↳ pulsazione

PROBLEMI DI DINAMICA $\Rightarrow$ BILANCIAMENTO DELLE FORZE

1)



$m_1 = m_2 = 65\,kg$
$\alpha = 30°$
$\mu_1 = 0{,}5$
$\mu_2 = 0{,}2$

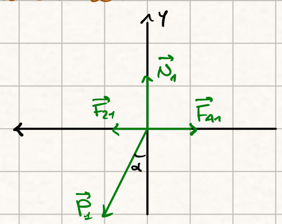
a) I due corpi rimangono attaccati durante il moto? perché? ? F_{21}

b) ? θ | v cost, $a=0$

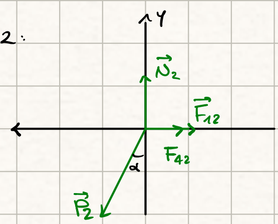
a) Sì, rimangono attaccati perché $\mu_1 > \mu_2$
$\Rightarrow F_{A_1} > F_{A_2}$

grafici delle forze:

CORPO 1



CORPO 2:



BILANCIO DI FORZE: $F = ma$

CORPO 1:
$$\begin{cases} x: & F_{21} + mg\sin\alpha - F_{A1} = m a_1 \\ y: & N_1 - mg\cos\alpha = 0 \end{cases}$$

CORPO 2:
$$\begin{cases} x: & mg\sin\alpha - F_{12} - F_{A2} = m a_2 \\ y: & N_2 - mg\cos\alpha = 0 \end{cases}$$

! $F_{12} = F_{21}$ (AZIONE E REAZIONE)

- $a_2 = a_1 = a$
- $N_1 = N_2$

$$\Rightarrow \begin{cases} N = mg\cos\alpha \\ mg\sin\alpha + F_{21} - \mu_1 N = ma \\ mg\sin\alpha - F_{21} - \mu_2 N = ma \end{cases} \Big\} \ominus$$

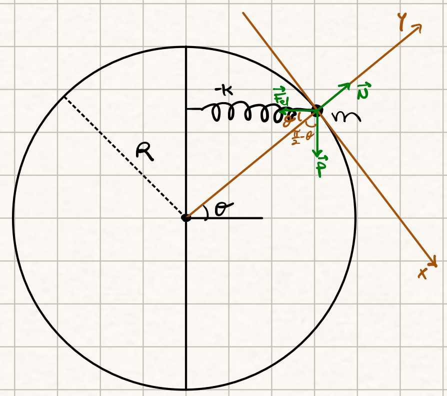
$$\diagup \quad 2F_{21} - \mu_1 N + \mu_2 N = 0 \quad \Rightarrow \quad F_{21} = \frac{(\mu_1 - \mu_2)}{2} N = 82{,}83\,N$$

b) $a = 0$:
$$\begin{cases} N = mg\cos\theta \\ mg\sin\theta + F_{21} - \mu_1 N = 0 \\ mg\sin\theta - F_{21} - \mu_2 N = 0 \end{cases} \Big\} +$$

$$2mg\sin\theta \;\diagup\; - \mu_1 N - \mu_2 N = 0 \quad \Rightarrow \quad 2mg\sin\theta - (\mu_1+\mu_2)mg\cos\theta = 0$$
$$\Rightarrow mg\left(2\sin\theta - (\mu_1+\mu_2)\cos\theta\right) = 0$$
$$\Rightarrow 2\sin\theta - (\mu_1+\mu_2)\cos\theta = 0$$
$$\Rightarrow 2\sin\theta = (\mu_1+\mu_2)\cos\theta \Rightarrow$$
$$\Rightarrow \tan\theta = \frac{\mu_1+\mu_2}{2} \Rightarrow \theta = \arctan\frac{\mu_1+\mu_2}{2} = 19{,}3°$$

! DINAMICA DEL P. MATERIALE → BILANCIAMENTO DI FORZE →
- PESO
- F. VINCOLARE
- TENSIONI / F. DI CONTATTO
- FORZE CENTRIPETE
- F. ELASTICA

2)



$m = 100g$
$R = 1m$
$k = 20\ N/m$

? posizioni di equilibrio ⇒ $\Sigma F = 0$

$F_{el} = -k\,\Delta\ell\,\vec{u}_x$

$$
\begin{cases}
y: \ N - F_{el}\cos\theta - mg\cos\left(\frac{\pi}{2}-\theta\right) = 0 \\
x: \ mg\sin\left(\frac{\pi}{2}-\theta\right) - F_{el}\sin\theta = 0 \\
F_{el} = k\,\underline{\underline{\Delta\ell}}
\end{cases}
$$

↳ in funzione del raggio: $F_{el} = kR\cos\theta$

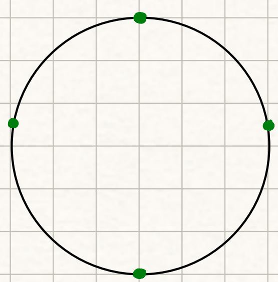
$$\Rightarrow \begin{cases} N - kR\cos^2\theta - mg\sin\theta = 0 \\ mg\cos\theta - kR\cos\theta\sin\theta = 0 \end{cases} \Rightarrow R\cos\theta = \frac{mg}{k}\frac{\cos\theta}{\sin\theta}$$

1) $\cos\theta = 0 \Rightarrow \underline{\theta_1 = 90°}$, $\underline{\theta_2 = 270°}$

2) $\theta \neq \theta_1, \theta_2$:

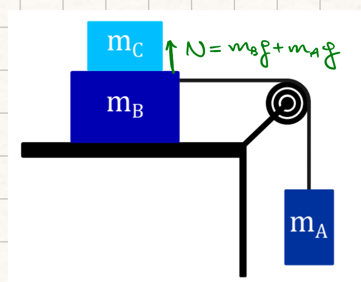
$$R = \frac{mg}{k\sin\theta} \Rightarrow \theta = \arcsin\left(\frac{mg}{Rk}\right) = \underline{2{,}8°}$$
$$\vee$$
$$\theta = \pi - \arcsin\left(\frac{mg}{Rk}\right) = \underline{177{,}2°}$$

Dunque ci sono 4 posizioni di equilibrio:



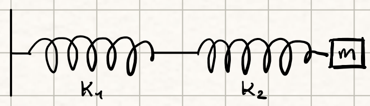
⇒ In questi punti le forze si bilanciano: $\Sigma F = 0$ ⇒ EQUILIBRIO

! se in equilibrio (fermo):



Invece quando si rompe l'equilibrio
$N = m_B g$

MOLLE IN SERIE :



Allungamento totale : $\boxed{x_{TOT} = x_1 + x_2}$

$F = K_1 x_1 \rightarrow x_1 = F/K_1$

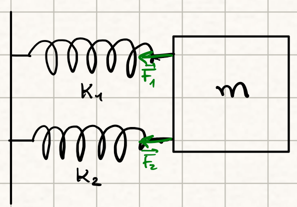
$F = K_2 x_2 \rightarrow x_2 = F/K_2$

$(F_1 = F_2 = F_{tot})$

$\rightarrow \boxed{F = K_{eq} x_{TOT} = K_{eq}(x_1 + x_2)} \rightarrow \cancel{F} = K_{eq}\left(\frac{\cancel{F}}{K_1} + \frac{\cancel{F}}{K_2}\right)$

$\rightarrow \boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} = \sum_{i=1}^{n} \frac{1}{K_i}}$

MOLLE IN PARALLELO

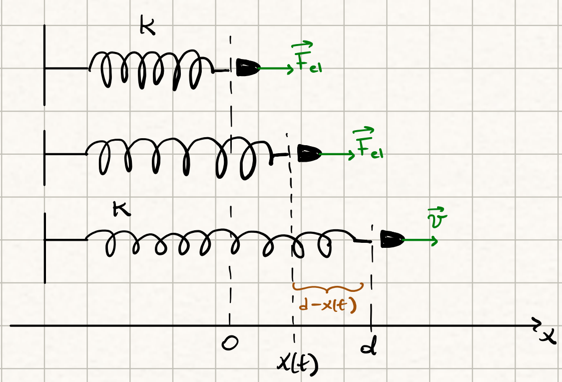


$x_{TOT} = x_1 = x_2$

$F_{eq} = F_1 + F_2 = K_1 x_1 + K_2 x_2 = \boxed{(K_1 + K_2)\, x_{TOT}}$

$\boxed{K_{eq} = K_1 + K_2 = \sum_{i=1}^{n} K_i}$

3)



MOLLA COMPRESSA

FASE DI DISTACCO

DISTACCO AVVENUTO

$$F = ma = m\frac{dv}{dt} = \frac{m\, d^2x}{dt^2} = m\ddot{x}$$

$\parallel$

F_{el}

$\parallel$

$K(d-x) = m\ddot{x} \rightarrow \ddot{x}(t) + \frac{K}{m}x(t) - \frac{K}{m}d = 0$ eq. differenziale

↳ evolvere del moto nel tempo

↳ MOTO ARMONICO con $\omega = \sqrt{\frac{K}{m}}$ (pulsazione)

soluzione

$\rightarrow \ddot{x} + \omega^2 x - \omega^2 d = 0 \rightarrow \boxed{x(t) = d\cos(\omega t) + d}$

$d = \frac{F}{K}$ $(F = Kd)$

$\rightarrow v = \frac{dx}{dt} = -\omega d \sin(\omega t)$; $v_{max} = \omega d = \sqrt{\frac{K}{m}}\, d = \sqrt{\frac{K}{m}}\,\frac{F}{K} = \sqrt{\frac{1}{mK}}\, F$

↳ F_{el}

1) Nel caso di due molle in serie : $\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \Rightarrow k_{eq} = \frac{k}{2} = 100\ N/m$

2) // : $k_{eq} = k + k = 400\ N/m$

$\Rightarrow$ La velocità è massima nel caso di molle in serie.

OPPURE usando energie :

$$E_K = \frac{1}{2} k_{eq} d^2 = \frac{1}{2} m v_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{k_{eq}}{m}}\, d$$