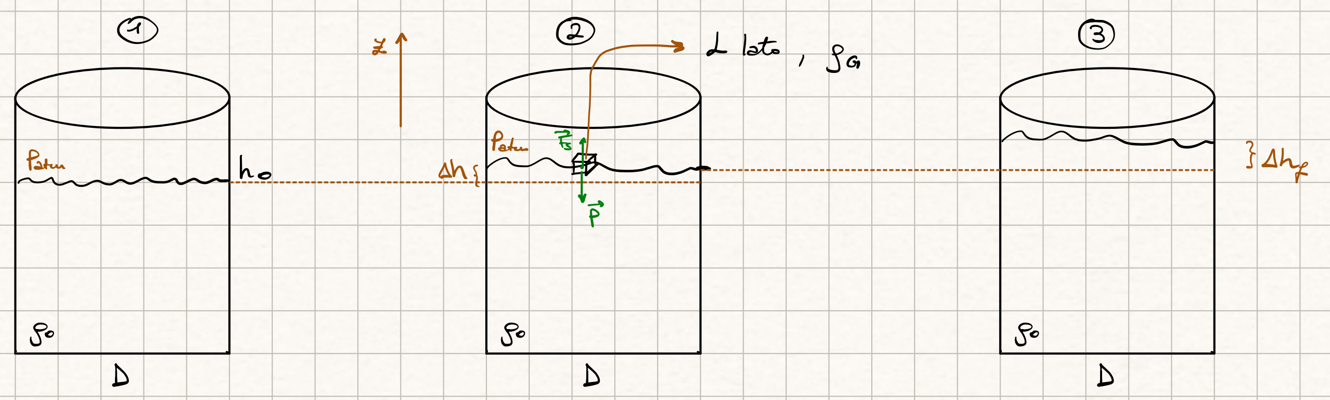
1)



a) ? Δh

Il sistema giunge all'equilibrio in 2 :

peso del cubetto di ghiaccio

$$\vec{P} + \vec{F}_A = 0$$

spinta di Archimede agente sul centro di massa del volume di liquido spostato

$$P = m_G g = \rho_G V_G g = \rho_G L^3 g$$

$$F_s = \rho_0 V_{imm} g$$

$\Rightarrow$ condizione di equilibrio : $-P + F_s = 0 \quad \Rightarrow \quad P = F_s$

$$\Rightarrow \rho_G L^3 \cancel{g} = \rho_0 V_{imm} \cancel{g} \quad \Rightarrow V_{imm} = \frac{\rho_G}{\rho_0} L^3$$

$$\Delta V = \left(\frac{D}{2}\right)^2 \pi \cdot \Delta h = V_{imm} \quad \Rightarrow \frac{D^2}{4} \pi \Delta h = \frac{\rho_G}{\rho_0} L^3 \quad \Rightarrow \boxed{\Delta h = \frac{\rho_G}{\rho_0} \frac{4L^3}{D^2 \pi}}$$

b) ? $V_{non\ imm}$

$$V_{non\ imm} = \frac{V - V_{imm}}{V} = \frac{1}{L^3}\left(L^3 - \frac{\rho_G}{\rho_0} L^3\right) = \boxed{1 - \frac{\rho_G}{\rho_0}}$$

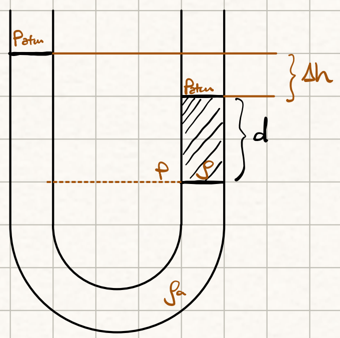
c) ? Δh_f

Si conserva la massa : $m_G = m_{\substack{ghiaccio \\ sciolto}\,(acqua)} \quad \Rightarrow \quad \rho_G V = \rho_0 V'$

$$\Rightarrow V' = \frac{\rho_G}{\rho_0} L^3 = V_{imm} \quad \Rightarrow \Delta V_f = 0 \quad \Rightarrow \boxed{\Delta h_f = 0}$$

$$\left(\Delta V = V' = \left(\frac{D}{2}\right)^2 \pi \, \Delta h_{3-1} \Rightarrow \Delta h_{3-1} = \frac{\rho_G}{\rho_0} \frac{4L^3}{D^2 \pi} = \Delta h_{2-1} \Rightarrow \Delta h_{3-2} = 0 \right)$$

2)



? ρ

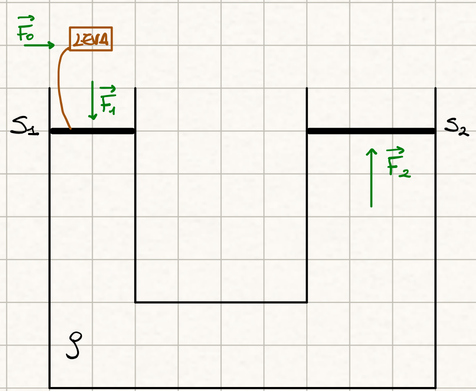
LEGGE DI STEVINO: $p(h) = \rho g h + p_0$

$p = \rho g d + p_{atm}$

$p = \rho_a g(\Delta h + d) + p_{atm}$

$\Rightarrow \rho \cancel{g} d = \rho_a \cancel{g}(\Delta h + d) \Rightarrow \rho = \dfrac{\rho_a(\Delta h + d)}{d}$

3)



$S_1 = 5\ cm^2$

→ LEVA DI 1° GENERE: $\dfrac{b_E}{b_R} = 10$ (< 1 ⇒ vantaggiosa)

$f = 400 \Rightarrow F_2 = 400 F_0 \Rightarrow \dfrac{F_2}{F_0} = 400$

? D_2

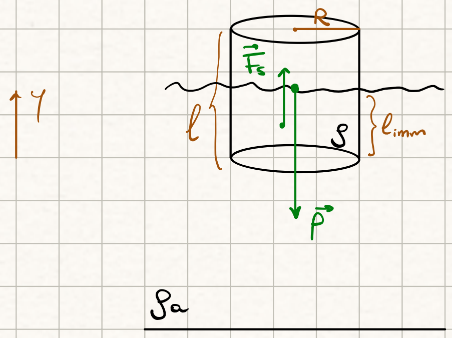
$F_1 = \dfrac{b_E}{b_R} F_0 = 10 F_0$

$p_1 = \dfrac{F_1}{S_1}$; $p_2 = \dfrac{F_2}{S_2}$; $p_1 = p_2$ (poichè $\begin{cases} \cancel{p_{atm}} = \cancel{p_0} + p_1 + \cancel{\rho g h} \\ \cancel{p_{atm}} = \cancel{p_0} + p_2 + \cancel{\rho g h} \end{cases}$)

$\Rightarrow \dfrac{F_1}{S_1} = \dfrac{F_2}{S_2} \Rightarrow \dfrac{10F_0}{S_1} = \dfrac{F_2}{S_2} \Rightarrow \dfrac{F_2}{10F_0} = \dfrac{S_2}{S_1} \Rightarrow \dfrac{400}{10} = \dfrac{S_2}{S_1} = 40$

$\Rightarrow \dfrac{\left(\frac{D_2}{2}\right)^2 \pi}{5 \cdot 10^{-4}} = 40 \Rightarrow D_2 = \sqrt{\dfrac{200 \cdot 10^{-4} \cdot 4}{\pi}} = 0,16\ m = 16\ cm$

4) i) ? V_{imm}



$\rho = 800\ kg/m^3$ $\qquad$ $\rho_a \cong 1000\ kg/m^3$

$R = 0,3\ m$

$\ell = 1\ m$

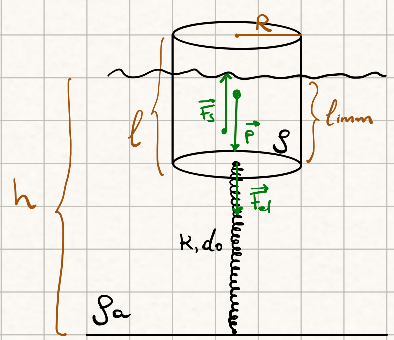
EQUILIBRIO : $\vec{P} + \vec{F}_s = 0 \quad \Rightarrow F_s = P = \rho V g$

Spinta di Archimede : $F_s = \rho_a V_{imm}\, g$

$P = F_s \quad \Rightarrow \quad \rho V \cancel{g} = \rho_a V_{imm} \cancel{g} \quad \Rightarrow$

$\Rightarrow \dfrac{V_{imm}}{V} = \dfrac{\rho}{\rho_a} = \dfrac{800}{1000} = 0,8 \quad \Rightarrow V_{imm}(\%) = 80\ \%$

ii) ? V'_{imm}



$k = 1000\, N/m$

$d_0 = 2,1\ m$

$h = 3\ m$

EQUILIBRIO : $\vec{P} + \vec{F}_s + \vec{F}_{el} = 0$

$$\begin{cases} -\rho V g + \rho_a V'_{imm}\, g - k\,\Delta d = 0 \\ \Delta d = d - d_0 = h - \ell_{imm} - d_0 = h - \dfrac{V'_{imm}}{R^2\pi} - d_0 \end{cases}$$

$$\Rightarrow -\rho V g + \rho_a g V'_{imm} - k\left(h - \frac{V'_{imm}}{R^2\pi} - d_0\right) = 0 \Rightarrow \left(\rho_a g + \frac{k}{R^2\pi}\right) V'_{imm} = \rho V g + kh - kd_0$$

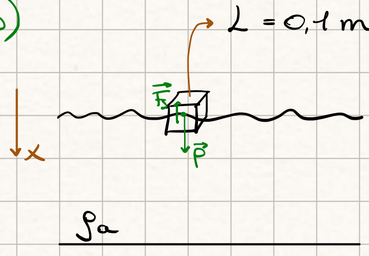
$$\Rightarrow \frac{V'_{imm}}{V} \cdot 100 = \left[\rho g + \frac{k(h-d_0)}{V}\right]\left(\frac{1}{\rho_a g + \frac{k}{R^2\pi}}\right) 100 = \left[800 \cdot 9,8 + \frac{1000(3-2,1)}{0,3^2\pi \cdot 1}\right]\left(\frac{1}{1000 \cdot 9,8 + \frac{1000}{0,3^2\pi}}\right) 100 =$$

$= 82,65\ \%$

iii) ? V''_{imm} ; $h' = 4m$

$$\frac{V'_{imm}}{V} \cdot 100 = \left[\rho g + \frac{k(h-d_0)}{V}\right]\left(\frac{1}{\rho_a g + \frac{k}{R^2\pi}}\right) 100 = \left[800 \cdot 9,8 + \frac{1000(4-2,1)}{0,3^2\pi \cdot 1}\right]\left(\frac{1}{1000 \cdot 9,8 + \frac{1000}{0,3^2\pi}}\right) 100 =$$

$= 108\ \% \quad \longrightarrow$ il bidone è completamente immerso nel lago $\Rightarrow$ 100%

5) 

$L = 0{,}1\,m$; $\rho_L = 0{,}5\,kg/dm^3 = 5\cdot 10^2\,kg/dm^3$

? moto del cubo | Δx rispetto posizione di equilibrio

$$\vec{F}_s + \vec{P} = m\vec{a} \quad \Rightarrow -F_s\vec{u}_x + mg\vec{u}_x = +ma\vec{u}_x$$

$$\Rightarrow -\rho_a(V_{imm} + \Delta V)g + \rho V g = +\rho V a \quad *$$

→ se EQUILIBRIO : $\vec{P} + \vec{F}_s = 0$

$$\Rightarrow \rho V \cancel{g} - \rho_a V_{imm} \cancel{g} = 0 \Rightarrow \rho L^{\cancel{3}} - \rho_a \cancel{L^2} x_{ep} = 0 \Rightarrow x_{ep} = \frac{\rho}{\rho_a} L$$

$$* \; -\rho_a(V_{imm\,ep} + \Delta V)g + \rho V g = +\rho V a \quad \Rightarrow -\rho_a\left(L^2 x_{ep} + L^2 \Delta x\right)g + \rho L^3 g = \rho L^3 a$$

$$\Rightarrow -\rho_a \cancel{L^2}\left(x_{ep} + \Delta x\right)g + \rho L^{\cancel{3}} g = \rho L^{\cancel{3}} a \quad \Rightarrow -\frac{\rho_a}{\rho L}\left(x_{ep} + \Delta x\right)g + \frac{\cancel{\rho L}}{\cancel{\rho L}} g = a$$

$$\Rightarrow \left[-\frac{\rho_a}{\rho L} x_{ep} - \frac{\rho_a}{\rho L}\Delta x + 1\right] g = a \Rightarrow \left[\cancel{-\frac{\rho_a}{\rho L}\frac{\rho L}{\rho_a}} - \frac{\rho_a}{\rho L}\Delta x \cancel{+1}\right] g = a$$

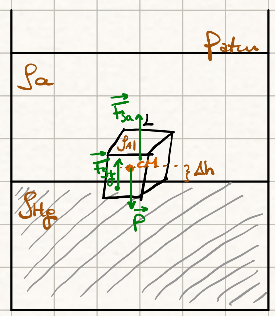
$$\Rightarrow \frac{d(\Delta x)^2}{dt^2} + \frac{\rho_a}{\rho L} g \Delta x = 0$$

MOTO ARMONICO CON $\omega = \sqrt{\frac{\rho_a g}{\rho L}}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\rho L}{\rho_a g}} = 2\pi\sqrt{\frac{5\cdot \cancel{10^2}\cdot 10^{-1}}{\cancel{10^3}\cdot 9{,}8}} = 0{,}45\,s$$

[LONGHI] CAP 8

8.6



EQUILIBRIO

$\rho_a = 1 \, g/cm^3$

$\rho_{Hg} = 13,6 \, g/cm^3$

$\rho_{Al} = 2,7 \, g/cm^3$

$L = 10 \, cm$

? Δh

EQUILIBRIO : $\vec{F}_{s_{Hg}} + \vec{F}_{sa} + \vec{P} = 0 \quad \Rightarrow \quad F_{s_{Hg}} \vec{u}_y + F_{sa} \vec{u}_y - m_{Al} g \, \vec{u}_y = 0$

$$\Rightarrow \rho_{Hg} V_{imm_{Hg}} \cancel{g} + \rho_a V_{imm\,a} \cancel{g} - \rho_{Al} V \cancel{g} = 0 \quad \Rightarrow \quad \rho_{Hg} \cancel{L^2} h_{imm_{Hg}} + \rho_a \cancel{L^2} h_{imm\,a} - \rho_{Al} L^{\cancel{3}} = 0$$

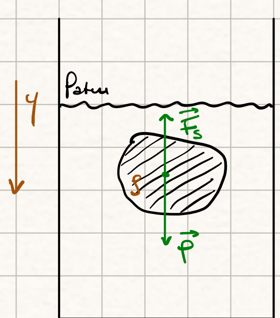
$h_{imm\,a} = L - h_{imm\,Hg} \quad ; \quad \rho_{Hg} h_{imm_{Hg}} + \rho_a (L - h_{imm_{Hg}}) - \rho_{Al} L = 0 \quad \Rightarrow (\rho_{Hg} - \rho_a) h_{imm_{Hg}} = L(\rho_{Al} - \rho_a)$

$$\Rightarrow h_{imm\,Hg} = L \frac{\rho_{Al} - \rho_a}{\rho_{Hg} - \rho_a}$$

$$y_{CM} = \frac{L}{2}$$

$$\Delta h = y_{CM} - h_{imm} = L\left(\frac{1}{2} - \frac{\rho_{Al} - \rho_a}{\rho_{Hg} - \rho_a}\right) = 3,65 \, cm$$

8.7



? ρ

$P = 5 N$

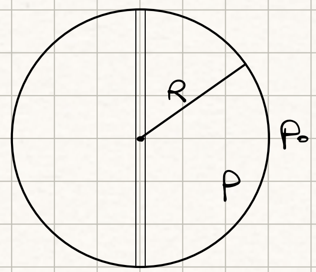
$F_{tot} = 3,5 N$

$$\begin{cases} \vec{F}_{tot} = \vec{F}_s + \vec{P} \\ P = mg = \rho V g \end{cases} \quad \Rightarrow \begin{cases} F_{tot} = P - F_s = 3,5 N \\ \rho = \dfrac{P}{Vg} \end{cases}$$

$$\Rightarrow -\rho_a V g + 5 = 3,5 \quad \Rightarrow V = \frac{1,5}{\rho_a g} = \frac{1,5}{1000 \cdot 9,8} = 1,53 \cdot 10^{-4} \, m^3$$

$$\Rightarrow \rho = \frac{5}{V \cdot 9,8} = 3333,33 \, Kg/m^3$$

8.8



Dimostrare che la forza richiesta per separare i due emisferi è:

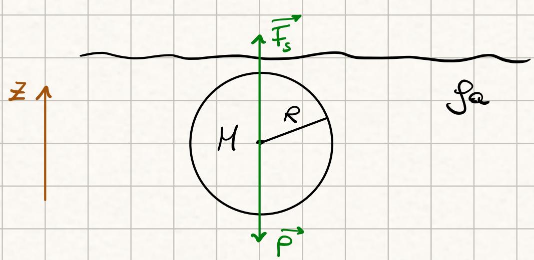
$$F = \pi R^2 (p_0 - p)$$

$$p_0 = \frac{F_{S_0}}{S} \quad ; \quad p = \frac{F_S}{S}$$

$\Rightarrow$ se

vedere libro

8.12



$R = 1m$
$M = 2500 \ kg$

? $m_{zavorra}$

$$\vec{P} + \vec{F_s} = 0 \Rightarrow (M+m)\cancel{g} - \rho_a V \cancel{g} = 0 \Rightarrow M + m = \rho_a \frac{4}{3} R^3 \pi$$

$$\Rightarrow m = \rho_a \frac{4}{3} R^3 \pi - M = 1688,8 \ kg$$

8.13

? $V_{He} \rightarrow h_0$
M
V_p , ρ_{He} cost
$\rho_{aria} = \rho_0 e^{-h/k}$

$\rho_{He} = 0,18 \ kg/m^3$
$h_0 = 10000 m$
$M = 250 \ kg$
$\rho_0 = 1,25 \ kg/m^3$
$K = 7500 m$

$$\Rightarrow \rho_{aria}(h_0) = \rho_0 e^{-h_0/k}$$

$$\Rightarrow F_s - P = 0 \Rightarrow \rho_{aria}(h_0) V_p \cancel{g} - (M + m_{He}) \cancel{g} = 0 \Rightarrow \rho_0 e^{-h_0/k} V_p - M - \rho_{He} V = 0$$

$$\Rightarrow (V_p = V_0) : \left(\rho_0 e^{\frac{-h_0}{k}} - \rho_{He}\right) V = M \Rightarrow V = \frac{M}{\rho_0 e^{\frac{-h_0}{k}} - \rho_{He}} = \frac{250}{1,25 e^{\frac{-10000}{7500}} - 0,18} = 1672,3 \ m^3$$