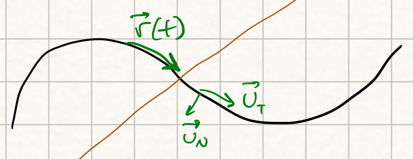
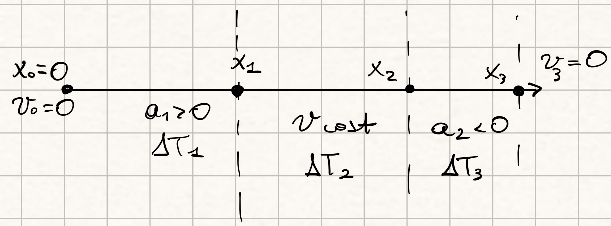


$\vec{v}(t)$
$\vec{a}(t) \rightarrow$ ACCELERAZIONE

$r(t) = r_0 + v_0 t$ moto rettilineo uniforme

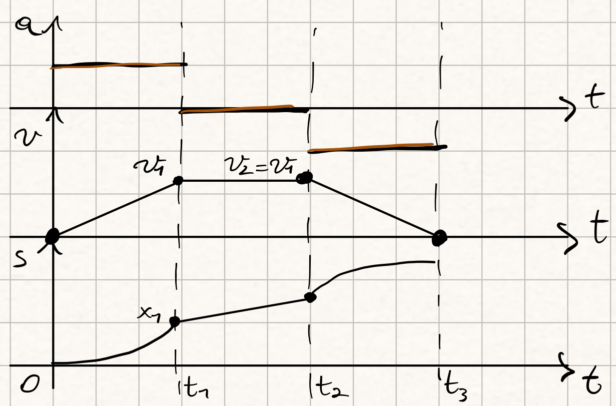
Es. 1



$v(t=0\,s) = v_0 = 0\ m/s$

ΔT_1 $\quad \vec{a}_1 = a_1 \vec{u}_x$
ΔT_2 $\quad$ velocità costante
ΔT_3 $\quad \vec{a}_2 = -a_2 \vec{u}_x$

? ΔT_3
? disegnare traiettoria, velocità, accelerazioni.

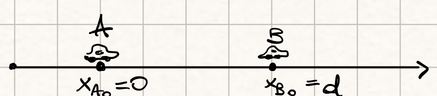


$v_0 = 0\,m/s$
$v_1 = a_1 \Delta T_1$ moto uniform. acceler.
$v_2 = v_1$ moto rettilin. uniform.
$v_3 = v_2 + a_2 \Delta T_3 = 0$

$$\Rightarrow \Delta T_3 = -\frac{v_2}{a_2} = -\frac{a_1}{a_2}\Delta T_1 = \frac{a_1}{|a_2|}\Delta T_1$$

giusto perchè $a_2 < 0$

Es 2 :



$v_{A_0} = 70\ Km/h$
$v_{B_0} = 90\ Km/h$
$a = 1{,}5\ m/s^2$

$t_0 = 0$ istante di sorpasso
$d = 60\ m$ a t_0

? $v_A(t_s)$
? t_s sorpasso

! unità di misura (SI) :

$v_{A_0} = 70\ km/h$
$= 70 \cdot \frac{10^3}{3{,}6 \cdot 10^3}\frac{m}{s} = 19{,}44\ m/s$

$v_{B_0} = 25\ m/s$

$$\begin{cases} x_A(t) = x_{A_0} + v_{A_0}t + \frac{1}{2}at^2 \\ x_B(t) = x_{B_0} + v_{B_0}t = d + v_{B_0}t \end{cases}$$

moto uniform. accelerato
moto uniforme

sorpasso : $\boxed{x_A(t_s) = x_B(t_s)}$

$\Rightarrow v_{A_0}t_s + \frac{1}{2}at_s^2 = d + v_{B_0}t_s$

$\Rightarrow at_s^2 - 2(v_{B_0} - v_{A_0})t_s - 2d = 0$

$$t_s = \frac{(v_{B_0}-v_{A_0}) \pm \sqrt{(v_{B_0}-v_{A_0})^2 + 2da}}{a} = \begin{cases} \cancel{-5{,}86\,s} \\ 13{,}42\,s \end{cases}$$

ci interessa ciò che succede dopo l'istante $t=0$

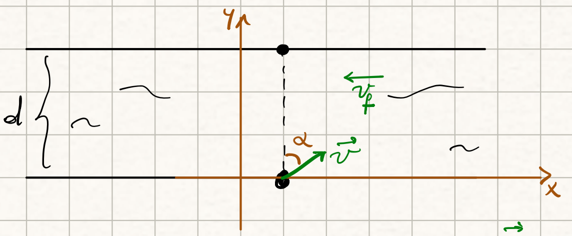
$v_A(t_s) = v_{A_0} + at_s \cong 38{,}5\ m/s$
$= 142{,}3\ km/h$

Es 3 :

$v = 20\ \text{Km/h}$ $\left(\frac{20}{3,6}\ \text{m/s}\right)$!

$d = 50\ \underline{\text{m}}$

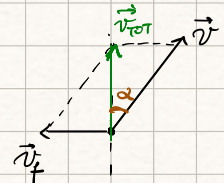
$v_f = 1\ \text{m/s}$ (fiume)



a) ? α

b) ? t_A (attraversamento)

a) $\vec{v}_{TOT} = \vec{v} + \vec{v}_f$

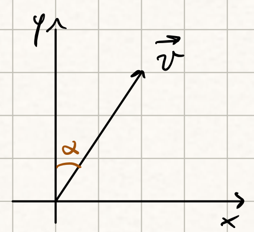


$\vec{v} = v_x \vec{u}_x + v_y \vec{u}_y$

$\vec{v}_f = -v_f \vec{u}_x$

$\vec{u}_y \parallel \vec{v}_{TOT} = \underbrace{(v_x - v_f)}_{0} \vec{u}_x + v_y \vec{u}_y$

$\Rightarrow v_x = v_f$



$v_x = |\vec{v}| \sin\alpha = v_f$

$\Rightarrow \sin\alpha = \frac{v_f}{v} \quad \Rightarrow \quad \alpha = \arcsin\left(\frac{v_f}{v}\right) \simeq 10,37°$

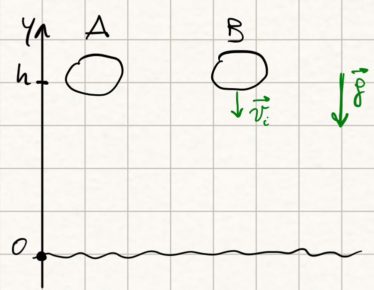
b) $v_{TOT,y} = \vec{v}_y = v\cos\alpha$ (non dipende dal tempo, costante $\Rightarrow$ moto rett. unif.)

$$\begin{cases} y(t) = \underset{0}{y_0} + v_y t \\ y(t_A) = d \end{cases} \Rightarrow \quad d = v_y t_A \quad \Rightarrow \quad t_A = \frac{d}{v\cos\alpha} \simeq 9,15\ \text{s}$$

Es 4 :

$h = 44\ \text{m}$

$\Delta t = 1\ \text{s}$



ACCELERAZIONE GRAVITAZIONALE $\rightarrow$ MOTO RETTIL. UNIF. ACCELER.

$\vec{g} = -g\vec{u}_y \qquad g = 9,81\ \text{m/s}^2$

$y_{A_0} = y(t_A = 0) = h$

$v_{A_0} = 0$

? $v_{B_0} \neq 0$

condizioni iniziali di B :

! $\int_{t_0+\Delta t}^{t} \Rightarrow \int_{\Delta t}^{t} \Rightarrow \int_{1}^{t}$

(A) $y_A(t) = \underset{h}{y_{A_0}} + \cancel{v_{A_0} t} - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$

$y_A(t_c) = 0 \quad \Rightarrow \quad h - \frac{1}{2}gt_c^2 = 0$

$t_c = \sqrt{\frac{2h}{g}}$ non dipende dalla massa (considerandoli come punti materiali) senza resistenza dell'aria

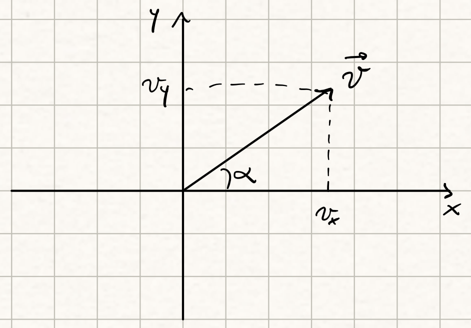
(B) $y_B(t) = \underset{h}{y_{B_0}} + v_{B_0}(t - \Delta t) - \frac{1}{2}g(t - \Delta t)^2$

$\Rightarrow y_B(t_c) = h + v_{B_0}(t_c - \Delta t) - \frac{1}{2}g(t_c - \Delta t)^2 \underline{\underline{= 0}}$

$\Rightarrow v_{B_0} = \frac{1}{2}g\left(\sqrt{\frac{2h}{g}} - \Delta t\right) - \frac{h}{\sqrt{\frac{2h}{g}} - \Delta t} = -12,27\ \text{m/s}$ (secondo il sistema)

$y_A(t_c) = y_B(t_c) = 0$

Es 5 :



$$\begin{cases} x = at \\ y = bt^2 + y_0 \end{cases}$$

$a = 2\ m/s$
$b = 2,5\ m/s^2$
$y_0 = 0,5\ m$

a) ? eq. della traiettoria $y(x)$

b) ? v (M, D, V) al tempo $t_1 = 1s$

a) $$\begin{cases} t = \frac{x}{a} \\ y = b\frac{x^2}{a^2} + y_0 \end{cases} \Rightarrow y(x) = \frac{b}{a^2}x^2 + y_0$$

b) $\vec{v}(t_1 = 1s)$:

$$\begin{cases} v_x = \frac{dx}{dt} = a \\ v_y = \frac{dy}{dt} = 2bt \end{cases}$$

velocità costante (MOTO RETT. UNIFOR.)

velocità dipendente dal tempo (MOTO UNIFORM. ACCELERATO)

$$|\vec{v}(t_1)| = \sqrt{a^2 + 4b^2t_1^2} \simeq 5,4\ m/s$$

$$\tan\alpha = \frac{v_y}{v_x} \Rightarrow \alpha = \arctan\left(\frac{v_y}{v_x}\right)$$

$$\alpha(t_1) = \arctan\left(\frac{2bt_1}{a}\right) \simeq 68,2^\circ$$

1) $\begin{cases} x = at^2 \\ y = 2bt \end{cases}$

a) ? eq. traiettoria
b) ? $\vec{a}_T$
c) ? $\vec{a}_N$
d) $\rho(t)$

a) $\begin{cases} x = \frac{a}{2b^2}y^2 \\ t = \frac{y}{2b} \end{cases} \Rightarrow x = \frac{a}{2b^2}y^2$

b) $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\vec{u}_T + v\frac{d\vec{u}_T}{dt} = \frac{dv}{dt}\vec{u}_T + \frac{v^2}{\rho}\vec{u}_N = \vec{a}_T + \vec{a}_N$$

$\begin{cases} v_x = \frac{dx}{dt} = 2at \\ v_y = \frac{dy}{dt} = 2b \end{cases} \Rightarrow \vec{v} = 2at\,\vec{u}_x + 2b\,\vec{u}_y$

$|\vec{v}| = v = 2\sqrt{a^2t^2+b^2}$

$\vec{v} = v\,\vec{u}_T \Rightarrow \vec{u}_T = \frac{\vec{v}}{v} = \frac{1}{\sqrt{a^2t^2+b^2}}(at\vec{u}_x + b\vec{u}_y)$

$$\vec{a}_T = \frac{dv}{dt}\vec{u}_T = \frac{d}{dt}\left(2\sqrt{a^2t^2+b^2}\right)\vec{u}_T = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\sqrt{a^2t^2+b^2}}(2a^2t)\frac{1}{\sqrt{a^2t^2+b^2}}(at\vec{u}_x + b\vec{u}_y)$$

$$\vec{a}_T = \frac{2a^2t}{a^2t^2+b^2}(at\vec{u}_x + b\vec{u}_y)$$

c) $$\vec{a}_N = v\frac{d\vec{u}_T}{dt} = 2\sqrt{a^2t^2+b^2}\cdot\left\{\left[\frac{-\frac{1}{2}\cdot\cancel{2}a^2t}{(a^2t^2+b^2)^{3/2}} + \frac{a}{\sqrt{a^2t^2+b}}\right]\vec{u}_x + \left[\frac{-\frac{1}{2}\cdot 2a^2t\; b}{(a^2t^2+b^2)^{3/2}}\right]\vec{u}_y\right\} =$$

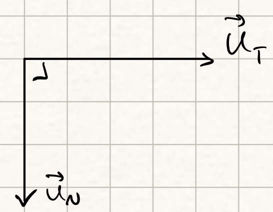
$$= 2\cancel{\sqrt{a^2t^2+b^2}}\,\frac{ab}{(a^2t^2+b^2)^{\cancel{3/2}}}(b\vec{u}_x - at\,\vec{u}_y) = \frac{2ab}{(a^2t^2+b^2)}(b\vec{u}_x - at\,\vec{u}_y)$$

d) $\vec{a}_N = \frac{v^2}{\rho}\vec{u}_N \quad ; \quad \rho = \frac{v^2}{a_N}$

$|\vec{a}_N| = a_N = \frac{2ab}{(a^2t^2+b^2)}\sqrt{b^2+a^2t^2}$

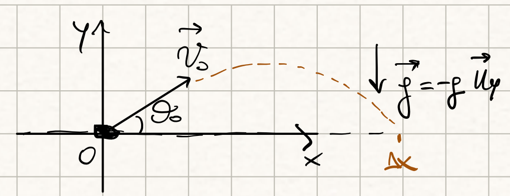
$\rho = \frac{2}{ab}(a^2t^2+b^2)^{3/2}$

$\vec{u}_T$: $\begin{cases} \vec{u}_T\cdot\vec{u}_N = 0 \\ |\vec{u}_N|^2 = 1 \end{cases} \Rightarrow \vec{u}_N = A\vec{u}_x + B\vec{u}_y$



$\begin{cases} A\vec{u}_x + B\vec{u}_y = 0 \\ A^2 + B^2 = 1 \end{cases}$

2) $\vec{v}_0$ ϑ_0



a) ? gittata $\Delta x(v_0, \vartheta_0)$

b) ? $\vartheta_0 \Rightarrow \Delta x_{max}$

a) $\begin{cases} x(t) = v_{0x}\, t & = v_0 \cos\vartheta_0\, t \\ y(t) = v_{0y} t - \frac{1}{2} g t^2 & = v_0 \sin\vartheta_0\, t - \frac{1}{2} g t^2 \end{cases}$

GITTATA: $\underline{y(t_v)} = y_0 = 0$

$\Rightarrow \begin{cases} x(t_v) = \Delta x = v_0 \cos\vartheta_0\, t_v \\ y(t_v) = 0 = v_0 \sin\vartheta_0\, t_v - \frac{1}{2} g t_v^2 \end{cases} \Rightarrow t_v\left(v_0 \sin\vartheta_0 - \frac{1}{2} g t_v\right) = 0 \Rightarrow$

$\cancel{t_v = 0\,s} \quad \vee \quad t_v = \frac{2 v_0 \sin\vartheta_0}{g}$

$\Rightarrow \Delta x = v_0 \cos\vartheta_0 \frac{2 v_0 \sin\vartheta_0}{g} = \frac{2 v_0^2 \sin\vartheta_0 \cos\vartheta_0}{g}$

$= \frac{v_0^2 \sin 2\vartheta_0}{g}$

Δx max $\rightarrow$ derivata Δx rispetto $\vartheta_0 = 0$ $\Rightarrow$ punto di massimo

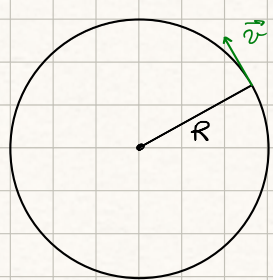
$k = 1, 2, 3, \ldots$

b) $\frac{d\Delta x}{d\vartheta_0} = \frac{2 v_0^2 \cos(2\vartheta_0)}{g} = 0 \Rightarrow \cos(2\vartheta_0) = 0 \Rightarrow 2\vartheta_0 = \frac{\pi}{2} + k\pi \Rightarrow \vartheta_0 = \frac{\pi}{4} + \frac{k\pi}{2}$

$\in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \vartheta_0 = \frac{\pi}{4} = 45°$ angolo che massimizza la gittata

3)



$s(t) = k_1 t^3 + k_2 t^2$

? R

$k_1 = 1\, m/s^3$

$k_2 = 2\, m/s^2$

$|\vec{a}| = a = 16\sqrt{2}\, m/s^2$ a $t = 2s$

$v(t) = \frac{ds}{dt} = 3k_1 t^2 + 2k_2 t$

$\vec{v}(t) = (3k_1 t^2 + 2k_2 t)\,\vec{u}_T$

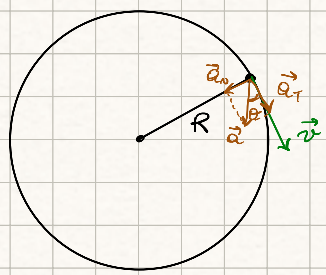
$\vec{a}_T = \frac{dv}{dt}\,\vec{u}_T = (6k_1 t + 2k_2)\,\vec{u}_T$

$\vec{a}_N = \frac{v^2}{R}\,\vec{u}_N = \frac{(3k_1 t^2 + 2k_2 t)^2}{R}\,\vec{u}_N$

$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(6k_1 t + 2k_2)^2 + \frac{(3k_1 t^2 + 2k_2 t)^4}{R^2}}$

$R^2 = \frac{(3k_1 t^2 + 2k_2 t)^4}{a^2 - (6k_1 t + 2k_2)^2} \Rightarrow R = 25\, m$

4)



$R = 150\ m$

$v_0 = 0\ m$

$a_T \begin{cases} > 0 & 0 \le t < t_1 \\ 0 & t \geq t_1 \end{cases}$ (cost.) $\Rightarrow$ moto circol. unif. acceler.

$\Rightarrow$ uniforme

$v_1 = v(t_1)$

$T = 2\ min$ (1 giro)

$\theta(t_1) = 45°$

a) ? v_1

$$v_1 = \frac{2\pi R}{T} = 7{,}85\ m/s$$

b) ? a_T $\quad \vec{v} = v\vec{u}_T \parallel \vec{a}_T$

$\theta(t_1) = 45°$

$$\left.\begin{aligned} a_T &= a\cos 45° = \frac{\sqrt{2}}{2}a \\ a_N &= a\sin 45° = \frac{\sqrt{2}}{2}a \end{aligned}\right\} a_N = a_T \quad a\ t_1$$

$t = t_1$: $\quad a_T = a_N = \frac{v_1^2}{R} = 0{,}41\ m/s^2$

c) ? t_1

$v(t) = a_T t$

$$v_1 = a_T t_1 \Rightarrow t_1 = \frac{v_1}{a_T} = \frac{v_1}{a_N} = \frac{v_1}{v_1^2/R} = \frac{R}{v_1} = \frac{T}{2\pi} = 19{,}1\ s$$

d) ? $s_1 = s(t_1)$

$$s(t) = \frac{1}{2}a_T t^2 \Rightarrow s_1 = \frac{1}{2}a_T t_1^2 = \frac{1}{2}a_N t_1^2 = \frac{1}{2}\frac{\cancel{v_1^2}}{\cancel{R}}\frac{R^2}{\cancel{v_1^2}} = \frac{R}{2} = 75\ m$$

e) ? $\theta(t)$

$$\theta(t) = \arctan\left(\frac{a_N}{a_T}\right) \quad ; \quad a_T = \frac{v_1^2}{R} \quad ; \quad a_N = \frac{v^2}{R} = \frac{a_T^2 t^2}{R}$$

$$\Rightarrow \theta(t) = \arctan\left(\frac{a_T t^2}{R}\right) = \arctan\left(\frac{v_1^2 t^2}{R^2}\right) = \arctan\left(\frac{t^2}{t_1^2}\right) \quad \text{per} \quad 0 \le t \le t_1$$

$$\Rightarrow \theta(t) = 90° \quad \text{per} \quad t > t_1$$