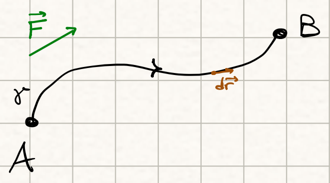
LAVORO E ENERGIA



$$\mathcal{L}_{AB} = \int_A^B \vec{F} \cdot d\vec{r} \qquad [J] = \left[kg\,\frac{m^2}{s^2}\right] = [N\,m]$$

$$E_{mecc} = E_K + E_P$$

princ. di conservaz. dell'en. meccan.

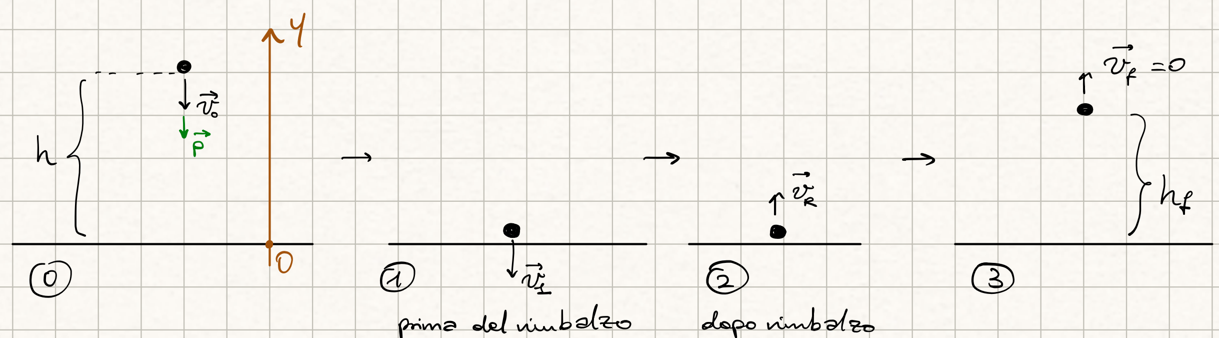
- $\Delta E_{mecc} = 0 \iff$ solo forze CONSERVATIVE
  - $\hookrightarrow \iff \mathcal{L}_\gamma = \mathcal{L}_{\gamma'} \quad \forall \gamma, \gamma'$ — non dipende dalla traiettoria
  - $\iff \oint \vec{F} \cdot d\vec{r} = 0 \quad \forall \gamma$
  - $\hookrightarrow$ circuitazione lungo un percorso chiuso
- ΔE_{mecc} = lavoro dissipato — se ci sono anche forze non conservative

1) h

v_0

rimbalzo $\to -\frac{1}{2} E_K$

$? v_0 \mid h_f = h$



non dipende dal sistema di riferim.

$$⓪ \to ① \quad \Delta E_{mecc} = \Delta E_K + \underline{\Delta E_P} = 0 \qquad \text{(solo forze conservative)}$$

$$= E_{K_f} - E_{K_i} + E_{P_f} - E_{P_i} = 0$$

$$= \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 + 0 - mgh = 0$$

$$\Rightarrow \frac{1}{2} m v_1^2 = \frac{1}{2} m v_0^2 + mgh$$

E_K prima del rimbalzo $E_{K,1}$

$$② \quad E_{K2} = \frac{1}{2} m v_R^2 = \frac{1}{2} E_{K,1} = \frac{1}{4} m v_0^2 + \frac{1}{2} mgh \qquad = E_{mecc}$$

$(E_P = 0 \iff h = 0)$

$$② \to ③ \quad \Delta E_{mecc} = 0 \quad \Rightarrow \quad \underbrace{E_{K_3}}_{0\ (v_f = 0)} - E_{K_2} + E_{P_3} - \underbrace{E_{P_2}}_{0\ (h_2 = 0)} = 0$$

$$0 - \frac{1}{4} m v_0^2 - \frac{1}{2} mgh + mgh_f = 0$$

$$h_f = h: \quad \frac{1}{4} \cancel{m} v_0^2 = mgh - \frac{1}{2} mgh = \frac{1}{2} \cancel{m} gh \quad \Rightarrow \quad v_0^2 = 2gh$$

$$\Rightarrow v_0 = \sqrt{2gh}$$

2) m

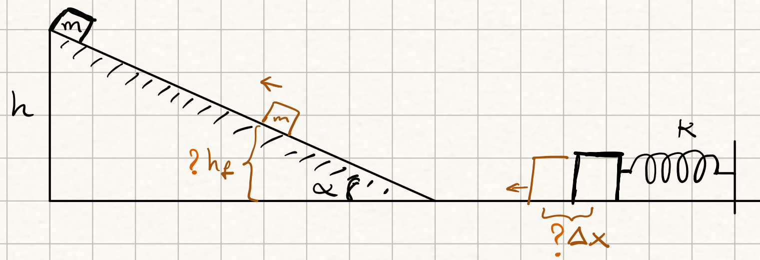
$v_0 = 0$

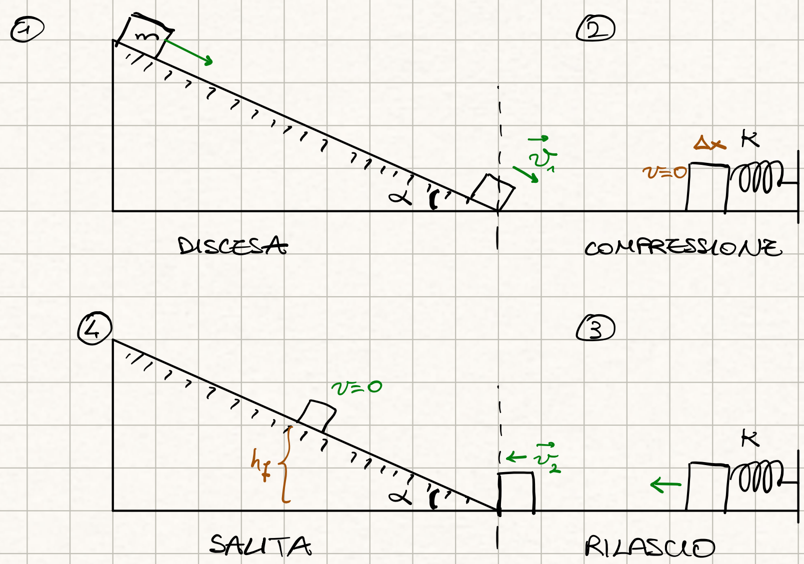
h

μ_d

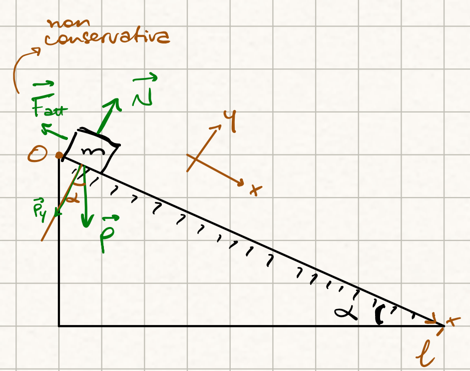
$h_f < h$

? h_f, Δx





1



$$F_{attr} = \mu_d N = \mu_d \, m g \cos\alpha$$

$$\vec{F}_{attr} = -\mu_d \, m g \cos\alpha \, \vec{u}_x + 0 \, \vec{u}_y$$

$d\vec{r}$ perchè lo spostamento è solo lungo l'asse x

$$\Delta E_{mecc} = L_{NC} = \int \vec{F}_{attr} \cdot \underbrace{d\vec{x}}_{dx\,\vec{u}_x} = \int_0^{\ell} -\mu_d \, m g \cos\alpha \, dx =$$

non conservativo

$$\parallel$$

$$\vec{F}_{attr} \cdot \Delta\vec{s}$$

$$= -\mu_d \, m g \cos\alpha \, [x]_0^{\ell} = -\mu_d \, m g \, \ell \cos\alpha = -\mu_d \, m g \frac{h}{\sin\alpha} \cos\alpha$$

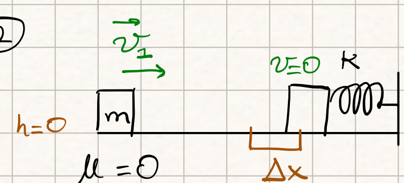
$$= -\mu_d \, m g h \, \mathrm{cotg}\,\alpha$$

↳ E_{mecc} diminuisce ($\text{Lavoro}_{NC} < 0$)

$$\Delta E_{mecc} = 0 - \frac{1}{2} m v_1^2 + 0 - mgh = -\mu_d \, m g h \, \mathrm{cotg}\,\alpha$$

$$\Rightarrow v_1 = \sqrt{2gh(1 - \mu_d \, \mathrm{cotg}\,\alpha)}$$

2



E_p gravitaz. = cost ; solo E_p elastica

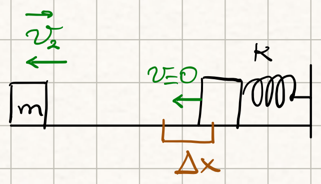
$\Delta E_{p,grav.} = 0$

$$\Delta E_{mecc} = \Delta E_k + \Delta E_p = 0 \quad \text{(solo forze conserv.)}$$

$$= 0 - \frac{1}{2} m v_1^2 + \frac{1}{2} k \Delta x^2 - 0 = 0$$

$$\rightarrow \Delta x = \sqrt{2 \frac{mgh}{k} (1 - \mu_d \, \mathrm{cotg}\,\alpha)}$$

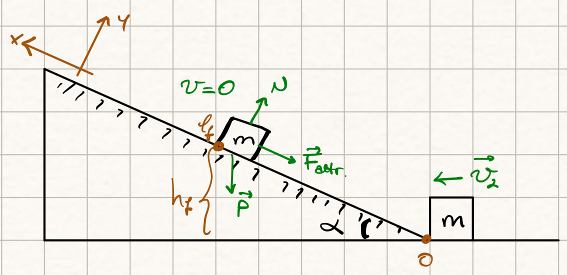
③



$$\Delta E_{mecc} = 0$$
$$= \frac{1}{2}mv_2^2 - 0 + 0 - \frac{1}{2}k\Delta x^2 = 0$$
$$\Rightarrow v_2 = \sqrt{2gh(1-\mu_d \cot g\alpha)} \quad = v_1 \;!!$$

conservazione energia cinetica (f. solo conservative)

④



$$\vec{F}_{attr} = \mu_d mg\cos\alpha$$

$$\Delta E_{mecc} = \mathcal{L}_{NC} = \int \vec{F}_{attr}\cdot d\vec{x} = -\mu_d mg\,\ell_f\cos\alpha$$
$$= -\mu_d mg h_f \cot g\alpha$$
↳ Em diminuisce

$$\Delta E_{mecc} = -\mu_d mg h_f \cot g\alpha = 0 - \frac{1}{2}mv_2^2 + mgh_f - 0$$

$$\Rightarrow h_f = h\,\underbrace{\frac{(1-\mu_d\cot g\alpha)}{(1+\mu_d\cot g\alpha)}}_{<1} \quad \Rightarrow h_f < h \quad \checkmark \text{finito}$$

UN ALTRO MODO : (più veloce)

→ si possono unire gli intervalli ①② e ③④

①② $\Delta E_K = 0$
$\Delta E_p = \frac{1}{2}k\Delta x^2 - mgh$

③④ $\Delta E_K = 0$
$\Delta E_p = mgh_f - \frac{1}{2}k\Delta x^2$

$$\Rightarrow \Delta E_{mec} = \Delta E_p = \frac{1}{2}k\Delta x^2 - mgh = \mathcal{L}_{NC}$$

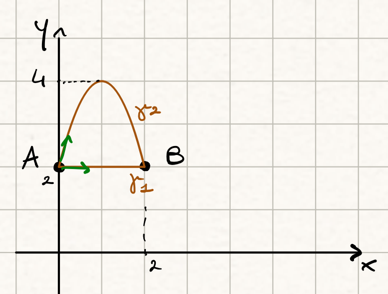
discesa : E_p convertita in E_K con dissipazione

② : $E_{p,grav.} = 0$; E_K convertita in $E_{p,elastica}$ (NO dissipaz. conservaz.)

③ : $E_K = 0$; poi $E_{p,elast.}$ convertita in E_K (NO dissipaz. conservaz.)

salita : E_K convertita in $E_{p,grav.}$ con dissipazione

3)



$$\vec{F} = y\,\vec{u}_x$$

la forza ha direzione dell'asse x, modulo = coordin. y
↳ ≠ es. forza peso : $\vec{P} = -mg\,\vec{u}_y$ ↳ dipende dalla posizione

A(0, 2)
B(2, 2)
γ_2 : $y = 2\left(1+\sin\left(\frac{\pi}{2}x\right)\right)$

? $\mathcal{L}_{\gamma_1}$, $\mathcal{L}_{\gamma_2}$
? Forza conservativa

↳ NO

$$\mathcal{L}_{\gamma_1} = \int_{\gamma_1}\vec{F}\cdot d\vec{r} = \int\left[y\vec{u}_x + 0\vec{u}_y\right]\cdot\left[dx\,\vec{u}_x + dy\,\vec{u}_y\right] = \int_0^2 y\,dx =$$
$$= y\left[x\right]_0^2 = 2y = 4\text{ J}$$

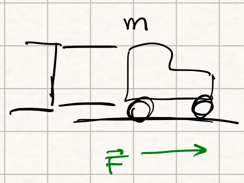
≠

$$\mathcal{L}_{\gamma_2} = \int_{\gamma_2}\vec{F}\cdot d\vec{r} = \int_{\gamma_2}y\,dx = \int_0^2 2\left(1+\sin\left(\frac{\pi}{2}x\right)\right)dx = \int_0^2 2\,dx + 2\int_0^2\sin\left(\frac{\pi}{2}x\right)dx$$
variano
$$= 4 + \frac{8}{\pi}\text{ J}$$

N.B.

$\Rightarrow$ Anche se $L_{\gamma_1} = L_{\gamma_2}$ non si potrebbe dire di certo che $\vec{F}$ sia conservativa perchè si dovrebbe dimostrare che L sia uguale per $\forall$ traiettoria, non basta che $L_{\gamma_1} = L_{\gamma_2}$.

4) a-



$P = 8\,MW$
$v_0 = 0$
$v_1 = 82\ m/s$
$\Delta t = 4\,min = 240\,s$
NO attrito

?m

$[W = J/s]$

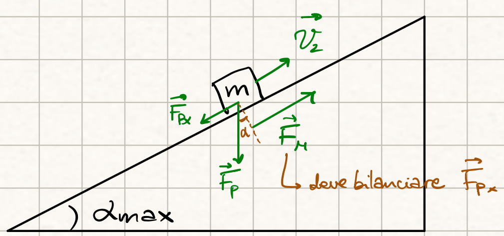
$P = \dfrac{L}{\Delta t}$ POTENZA MEDIA

$\Delta t \to 0 \Rightarrow P = \dfrac{dL}{dt}$ ISTANTANEA

$= \dfrac{\Delta E_k}{\Delta t} = \dfrac{\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2}{\Delta t}$

$\Rightarrow m = \dfrac{2P\Delta t}{v_1^2 - \cancel{v_0^2}} = 6{,}42 \cdot 10^5\ kg$

b-



v_2 cost $= 60\ Km/h$! va convertito in m/s $\Rightarrow$ moto rettil. uniforme

? α_{max}

$F_H = F_{P,x} = mg \sin\alpha_{max}$

! costante $\Rightarrow$ non dipende dal tempo

$P = \dfrac{dL}{dt} = \dfrac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F}_H \cdot \vec{v} = mg \sin\alpha_{max}\, v_2$

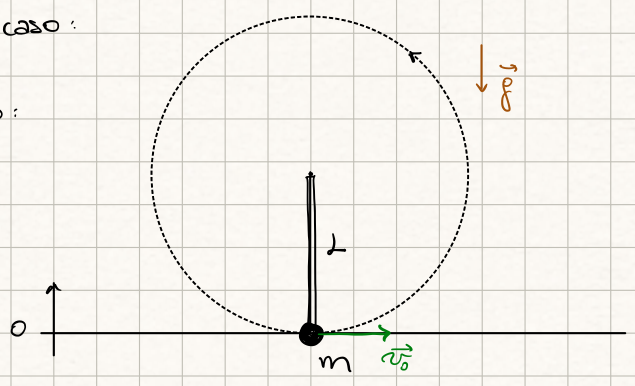
$\Rightarrow \alpha_{max} = \arcsin\left(\dfrac{P}{mg\,v_2}\right) = 4{,}67°$

VINCOLO UNILATERO / BILATERO

24/03

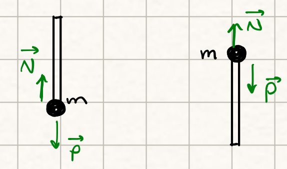
1) 1° caso:

inizio:

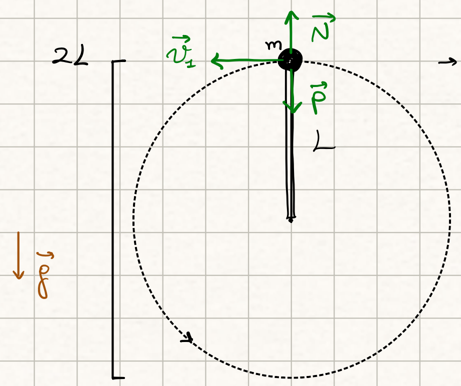


? $v_{0,min}$ per fare l'intero giro

! ASTA RIGIDA → pone un VINCOLO BILATERO



fine:



→ PUNTO DI EQ. INSTABILE

condizione per questo caso

$\boxed{\cdot \text{ se } v_{0,min} \Rightarrow v_1 = 0}$

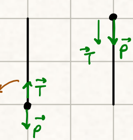
· se $v_{0,min} + v_\varepsilon \Rightarrow v_1 = 0 + \varepsilon$

$$\Delta E_m = 0 = \Delta E_k + \Delta E_p = E_{kf} - E_{ki} + E_{pf} - E_{pi} = 0$$

$$= 0 - \frac{1}{2} m v_{0min}^2 + mg(2L) - 0 = 0$$

$$\Rightarrow v_0 \geq 2\sqrt{gL}$$

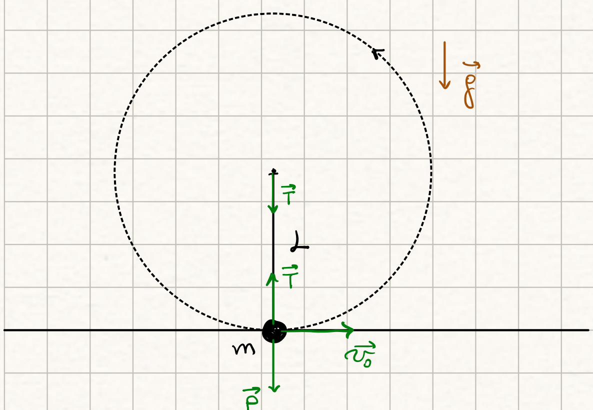
2° CASO: FUNE IDEALE → VINCOLO UNILATERO



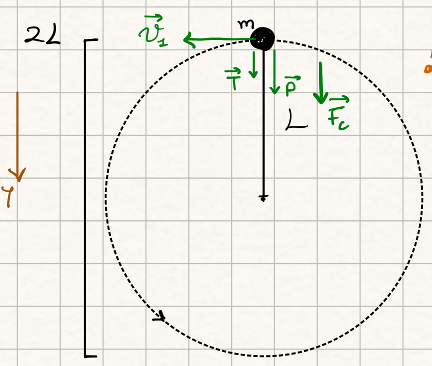
sempre verso l'interno

→ il filo non è in tensione!

inizio:



fine:



! non può essere in equilibrio in questo punto con v nulla

↳ $\vec{v_1} \neq 0$

$$F_c = m a_c = m \frac{v_1^2}{L}$$

(risultante di $\vec{T}$, $\vec{P}$)

$$T + mg = m a_c = m \frac{v_1^2}{L}$$

$$\Delta E_m = 0 = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 + mg(2L) - 0 = 0$$

$$\Rightarrow T = m\left(\frac{v_1^2}{L} - g\right) \geq 0$$

CASO LIMITE: $T = 0$

$$\Rightarrow v_0^2 = v_1^2 + 4gL \geq gL + 4gL$$

$$\Rightarrow v_1 \geq \sqrt{gL}$$

$$\Rightarrow v_0 \geq \sqrt{5gL}$$

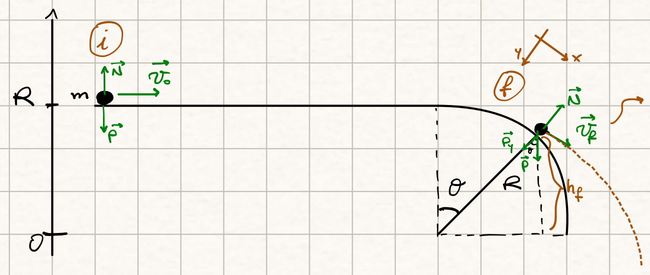
! TAVOLO = vincolo unilatero

solo verso l'alto

2) $m = 100\,g = 10^{-1}\,kg$

$v_0 = 3\ m/s$

$R = 5\,m$



si stacca dalla superficie :

DISTACCO ⇒ a θ_{max} $N = 0$

a) ? θ_{max}

b) ? $N(\theta)$

a) $*$ $\Delta E_m = 0 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 + mg(R\cos\theta) - mgR = 0$

BILANCIO DELLE FORZA

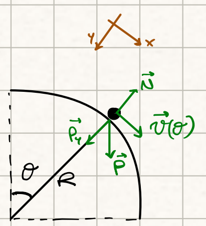
nel momento del distacco (lungo y) :

$$P_y - \underbrace{N}_{\hookrightarrow\, 0 \text{ con } \theta_{max}} = m a_c \Rightarrow mg\cos\theta_{max} = m a_c = m\frac{v_f^2}{R}$$

$$\Rightarrow v_f = \sqrt{gR\cos\theta_{max}}$$

$$* \Rightarrow \theta_{max} = \arccos\left(\frac{v_0^2}{3gR} + \frac{2}{3}\right) = 43{,}3^\circ$$

b)



lungo y : $mg\cos\theta - N = m a_c = m\frac{v^2(\theta)}{R}$

$$\Delta E_m = 0 \Rightarrow \frac{1}{2} m v^2(\theta) - \frac{1}{2} m v_0^2 + mgR\cos\theta - mgR = 0$$

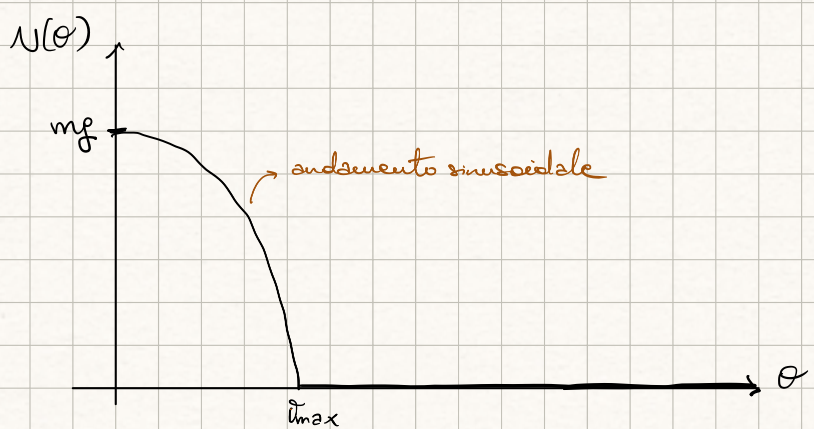
$$\Rightarrow v(\theta) = \sqrt{v_0^2 - 2gR(\cos\theta - 1)}$$

andamento PRIMA del distacco

⇒ perché poi il bilancio fatto non vale più

$$\Rightarrow N(\theta) = mg(3\cos\theta - 2) - m\frac{v_0^2}{R}$$

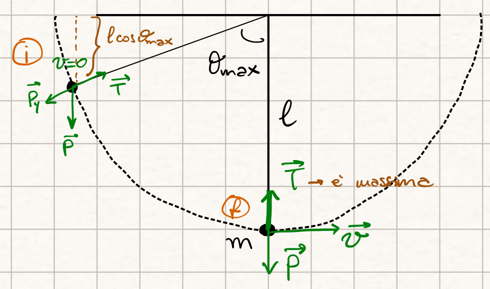
$(0 \leq \theta \leq \theta_{max})$

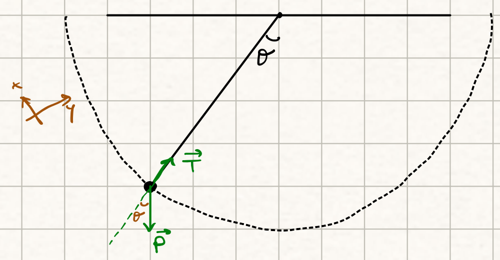


3) $m = 4\ kg$

$\theta_{max} = 77°$

? T_{max} $\Rightarrow$ bilancia P_y





$$y: \quad \begin{cases} T - mg\cos\theta = ma_c \\ -mg\sin\theta = ma_T \end{cases} \quad x: \quad \Rightarrow a_T = -g\sin\theta$$

$$s = \theta\ell \quad ; \quad a_T = \frac{d^2s}{dt^2} = \ell\frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{\ell}\sin\theta = 0$$

non approssimabile a θ perchè $\theta > 7°$

NON risolvibile

Un altro modo:

$$y: \quad T - mg\cos\theta = ma_c = m\frac{v^2}{\ell}$$

$$T = mg\cos\theta + m\frac{v^2}{\ell} = T(\theta)$$

$v(\theta) \Rightarrow$ da derivare !!

$$\Rightarrow \frac{dT}{d\theta} = 0 = -mg\sin\theta \quad \Rightarrow \quad \theta = n\pi \quad (n = 0, \cancel{\pm 1, \dots})$$

no perchè supera θ_{max}

L'angolo che massimizza T è 0

$$T(0) = T_{max}$$

*

$$\Rightarrow T_{max} = T(\theta = 0) = mg + m\frac{v^2}{\ell}$$

$$\Delta E_m = \frac{1}{2}mv^2 - 0 + mg(\ell - \ell\cos\theta) - 0 = 0$$

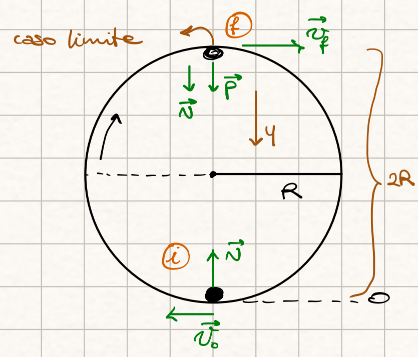
$$\Rightarrow v^2 = 2g\ell(1 - \cos\theta_{max})$$

$$* \Rightarrow T_{max} = mg(3 - 2\cos\theta_{max}) = 100\ N$$

! VINCOLO UNIDIREZIONALE

4) m, R, v_0

? $v_{0,min}$ | giro intero senza distacco



$$\Delta E_m = 0 \quad : \quad \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 + mg(2R) - 0 = 0 \quad *$$

$\downarrow y \quad : \quad N + mg = ma_c = m\frac{v_f^2}{R}$

CONDIZIONE : $N \geq 0$
(di contatto)

$$\rightarrow N = m\frac{v_f^2}{R} - mg \geq 0$$

$$\rightarrow v_f^2 \geq gR$$

$* \quad v_{0,min} = \sqrt{5gR}$

la guida
$\Rightarrow$ si comporta come il fune (VINCOLO UNILATER.)

5) $\Delta x = \frac{mg - F_{ATT} + \sqrt{\cdots}}{K} = 5{,}15\,m$