# UESTC4004
# Digital Communications


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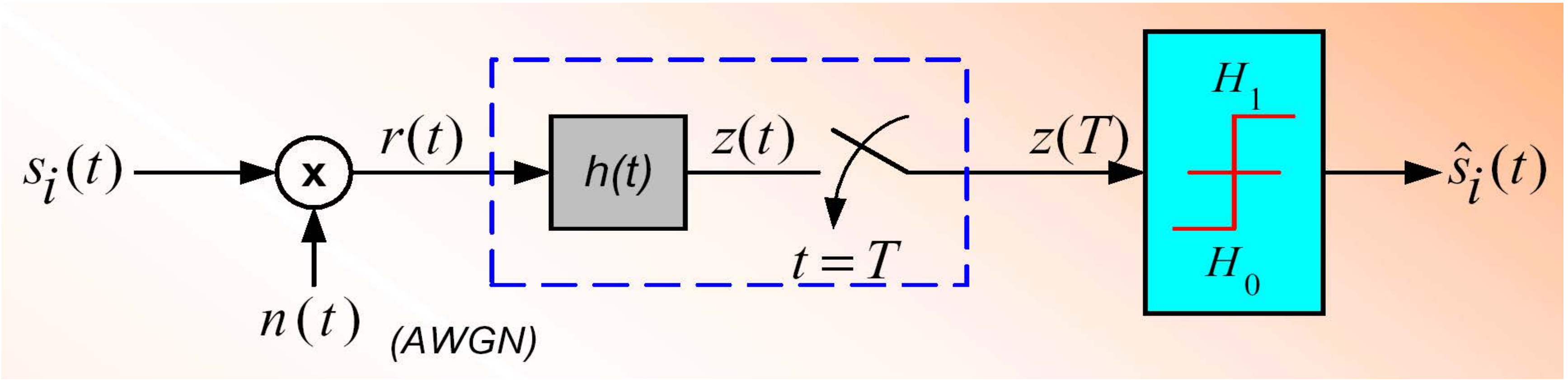
# Review Previous Lecture

- Binary Modulation (ASK, PSK, FSK)
- M-ary Modulation (QPSK)

# Lecture Preview

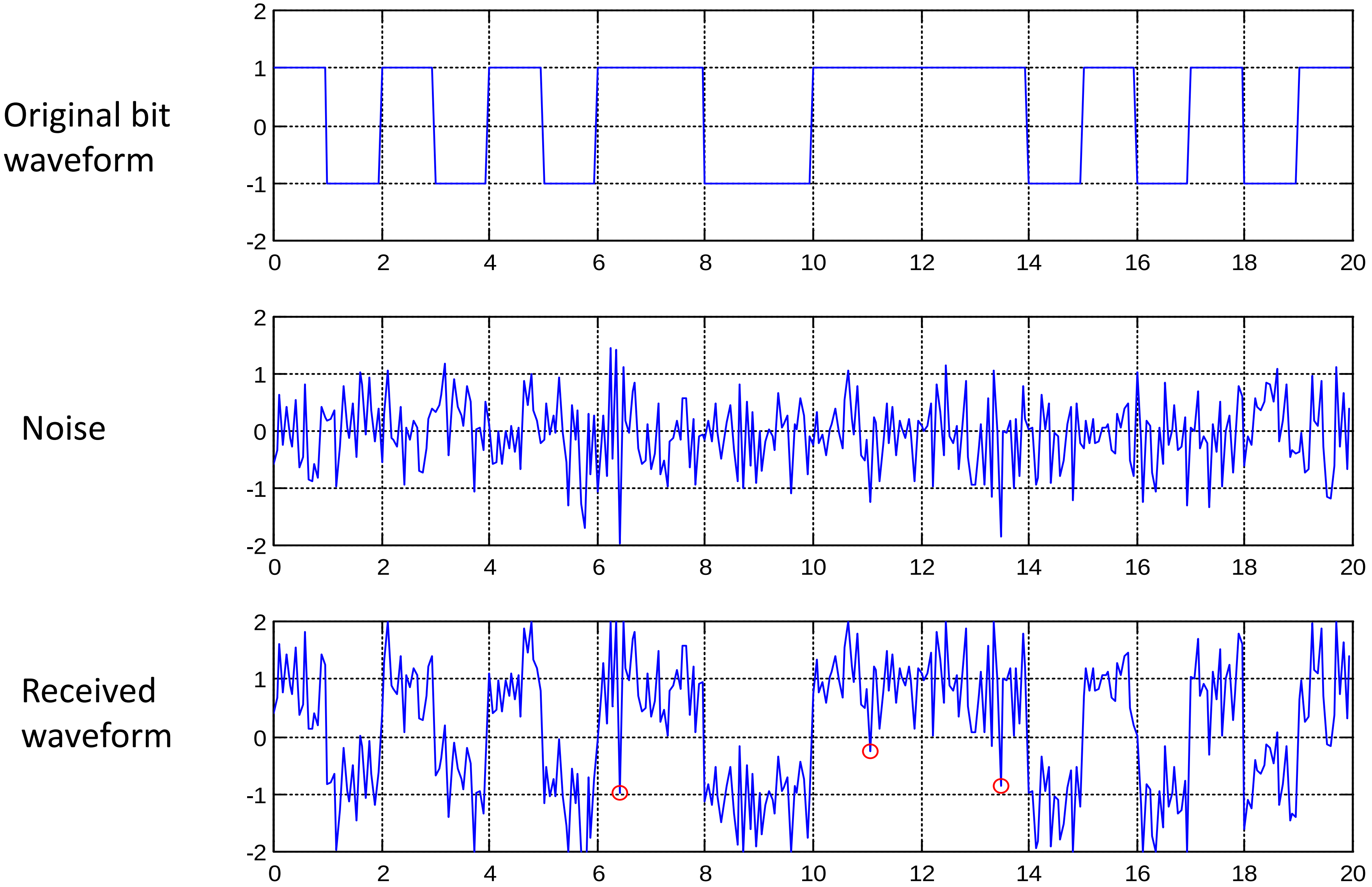
- Decoding the received signal
  - Demodulation
    - Matched Filter
  - Detection
    - Maximum likelihood detector

# Demodulation and Detection




- The digital receiver performs two basic functions:
  - Demodulation by using matched filter *h(t)*, to recover a waveform to be sampled at $t = nT$.
  - Detection, decision-making process of selecting possible digital symbol

# Detection of Binary Signal in Gaussian Noise



# Detection of Binary Signal in Gaussian Noise

- For any binary channel, the transmitted signal over a symbol interval (0,T) is:

$$s_i(t) = \begin{cases} s_0(t) \;\; 0 \le t \le T & \textit{for a binary } 0 \\ s_1(t) \;\; 0 \le t \le T & \textit{for a binary } 1 \end{cases}$$

- The received signal r(t) degraded by noise n(t) and possibly degraded by the impulse response of the channel $h_c(t)$, is

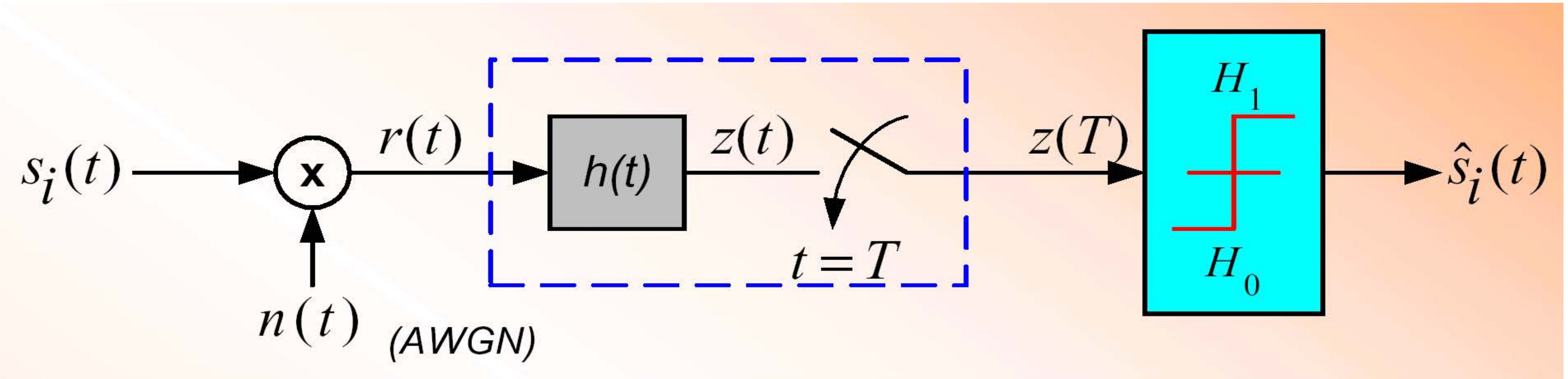
$$r(t) = s_i(t) * h_c(t) \; + \; n(t) \quad i = 0,1 \tag{3.1}$$

Where n(t) is assumed to be zero mean AWGN process

- For ideal distortionless channel where $h_c(t)$ is an impulse function and convolution with $h_c(t)$ produces no degradation, r(t) can be represented as:

$$r(t) = s_i(t) \; + \; n(t) \quad i = 0,1 \qquad 0 \le t \le T \tag{3.2}$$

# Detection of Binary Signal in Gaussian Noise




- The recovery of signal at the receiver consist of two parts
  - **Matched Filter h(t)**
    - Reduces the received signal to a single variable *z(T)*
    - *z(T)* is called the ***test statistics***
  - **Detector (or decision circuit)**
    - Compares the *z(T)* to some threshold level $\gamma_0$, i.e.,

$$z(T) \underset{H0}{\overset{H1}{\gtrless}} \gamma_0$$

where $H_1$ and $H_0$ are the two possible binary hypothesis

# Receiver Functionality

The recovery of signal at the receiver consist of two parts:

1. Waveform-to-sample transformation
   - Demodulator followed by a sampler
   - At the end of each symbol duration T, pre-detection point yields a sample z(T), called test statistic

$$z(t) = si(t) + n_0(t) \quad i = 0,1 \tag{3.3}$$

   Where $s_i(t)$ is the desired signal component,

   and $n_o(t)$ is the noise component

2. Detection of symbol
   - Assume that input noise is a Gaussian random process and receiving filter is linear

$$p(n_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2\right] \tag{3.4}$$

- Let $s_1(t)=a_1$ and $s_0(t)=a_o$
- Then output is another Gaussian random process

$$p(z|s_0)=\frac{1}{\sigma_0\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{z-a_0}{\sigma_0}\right)^2\right]$$

$$p(z|s_1)=\frac{1}{\sigma_0\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right]$$

Where $\sigma_0{}^2$ is the noise variance

- The ratio of instantaneous signal power to average noise power , $(S/N)_T$, at a time t=T, out of the sampler is:

$$\left(\frac{S}{N}\right)_T=\frac{a_i^2}{\sigma_0^2} \qquad (3.45)$$

- Need to achieve maximum $(S/N)_T$

- The impulse response of a filter producing maximum output signal-to-noise ratio is the shifted and inverted image of message signal s(t).
- The filter designed is called a MATCHED FILTER

$$\boxed{h(t) = \begin{cases} ks(T-t) & 0 \le t \le T \\ 0 & else\ where \end{cases}}$$

- Where k is some arbitrary constant
- Matching filter is defined as:

  a linear filter designed to provide the maximum

  signal-to-noise power ratio at its output for a given

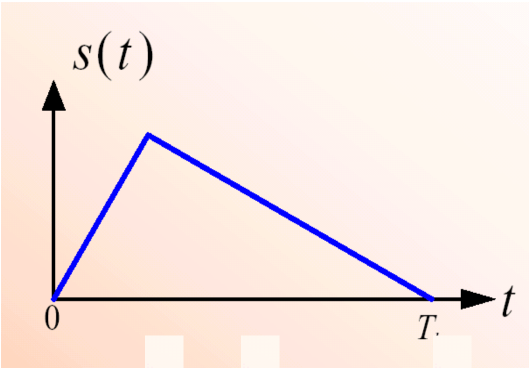
  transmitted symbol waveform

# Matched filter Summary

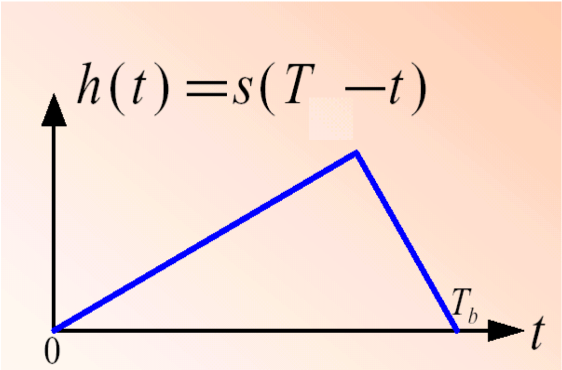
- A filter that is matched to the waveform *s(t)*, has an impulse response

$$h(t) = \begin{cases} ks(T-t) & 0 \le t \le T \\ 0 & else\ where \end{cases}$$

  - *h(t)* is a shifted and inverted version of the original signal waveform



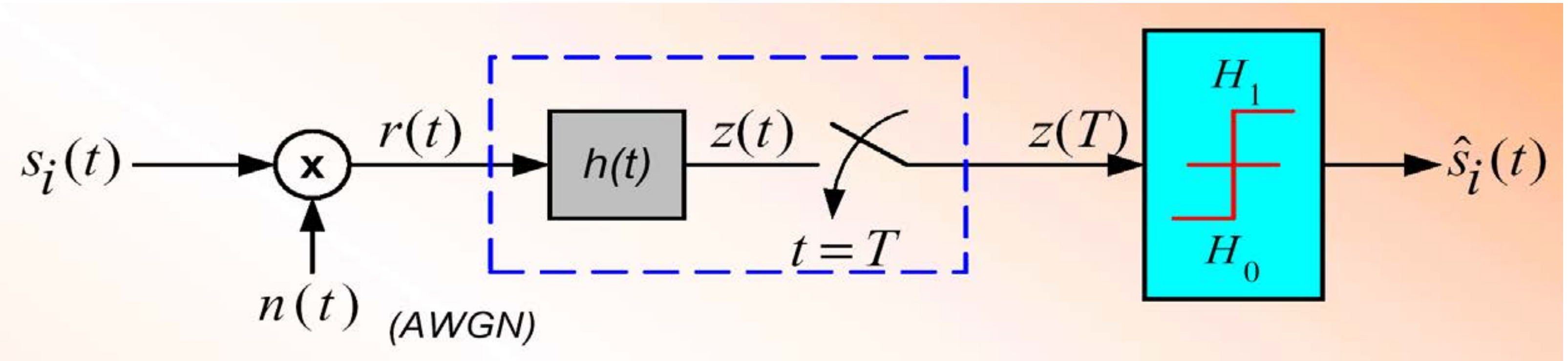

Signal Waveform




Impulse response of matched filter

# Detection

- Matched filter reduces the received signal to a single variable *z(T), after which the detection of symbol is carried out*
- The concept of ***maximum likelihood detector*** is based on Statistical Decision Theory
- It allows us to
  - formulate the decision rule that operates on the data
  - optimize the detection criterion




$$z(T) \underset{H0}{\overset{H1}{\gtrless}} \gamma_0$$

# Detection of symbol

Assume that input noise is a Gaussian random process and receiving filter is linear

$$p(n_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[ -\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2 \right] \qquad (3.4)$$

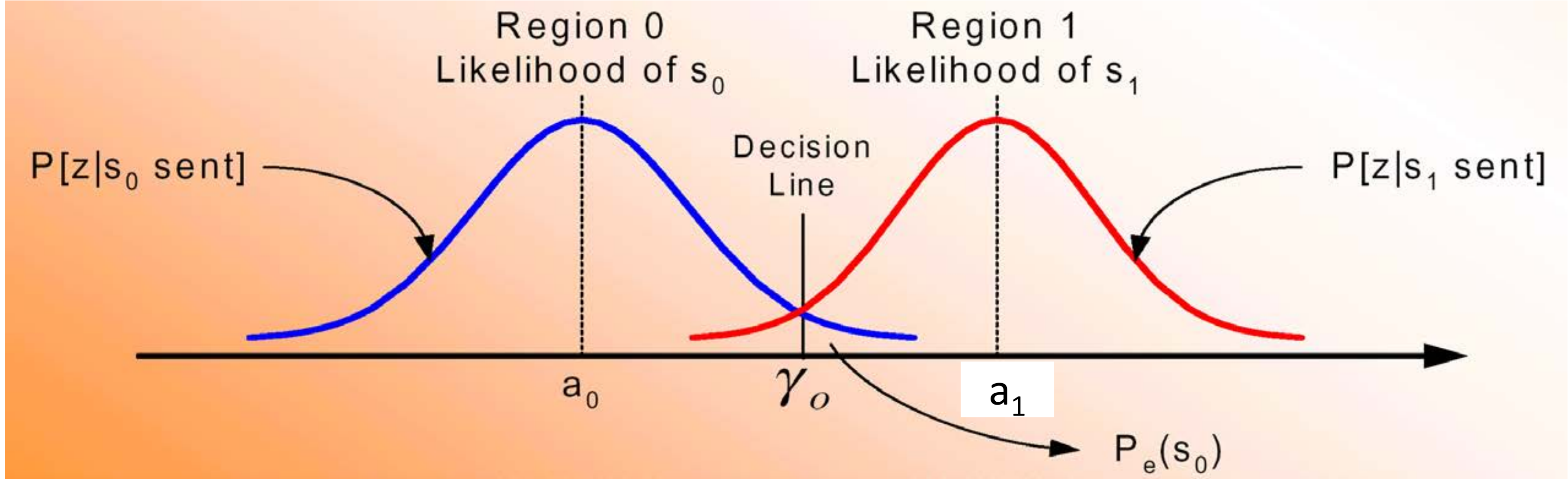
- Then output is another Gaussian random process

$$p(z \mid s_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[ -\frac{1}{2}\left(\frac{z-a_0}{\sigma_0}\right)^2 \right]$$

$$p(z \mid s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[ -\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2 \right]$$

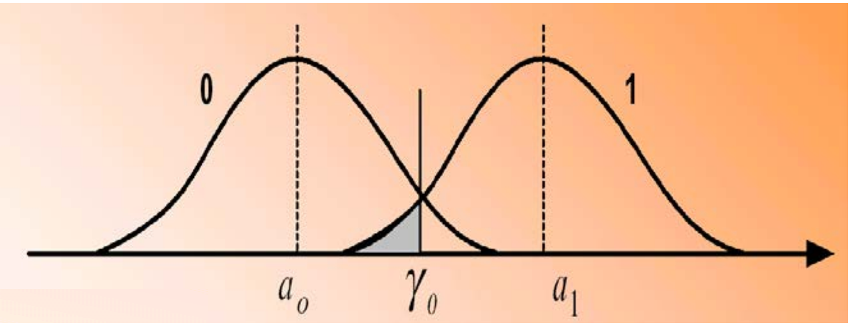
Where $\sigma_0^{\ 2}$ is the noise variance

■This means that if received signal was positive, $s_1$ *(t)* was sent, else $s_0$ *(t) was* sent

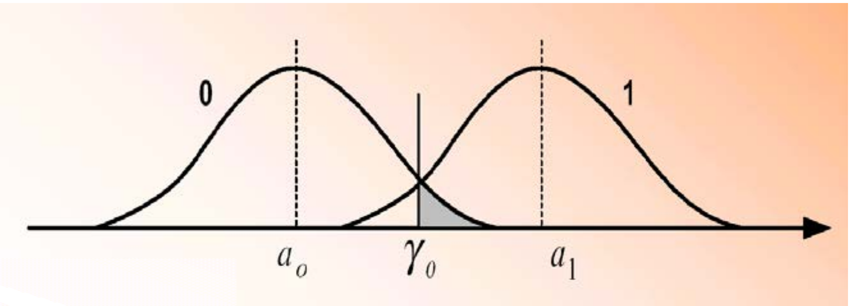


# Probability of Error

- Error will occur if
  - s1 is sent $\rightarrow$ s0 is received



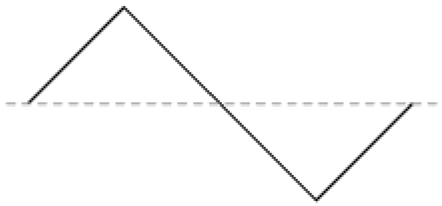

  - s0 is sent $\rightarrow$ s1 is received




- The total probability of error is the sum of the errors

# Review Questions

- What is the purpose of a matched filter?
- For a given waveform s(t), plot the matched filter impulse response h(t).



- Will the probability of error be more when the transmitted bits amplitudes $a_i(t)$ are ±5 instead of ±1? Justify please.