Power, Power Factor and Power Factor Correction

Power triangle for inductive (lagging) loads.

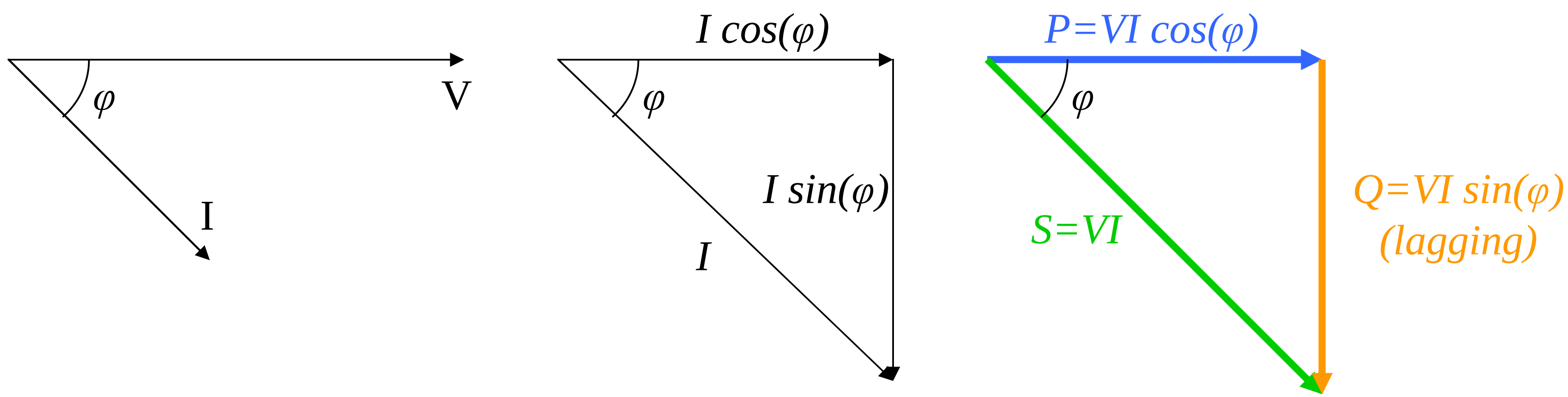
We have seen how the current and voltage in a circuit are not necessarily in phase. In power engineering, this phase difference is called the *Power Factor*.

For an inductive load, the current **lags** the voltage by some angle φ. Taking V as the reference, the in-phase component of the current is given by $I\cos(\varphi)$. The reactive or quadrature component of the current is given by $I\sin(\varphi)$.

The average ("real") power, P = Voltage x in-phase current = $VI\cos(\varphi)$ (units = watts).

The apparent power, S = Voltage x current = VI (units = VoltAmps, VAs)

The reactive power, Q = Voltage x quadrature current = $VI\sin(\varphi)$ (units = VARs)



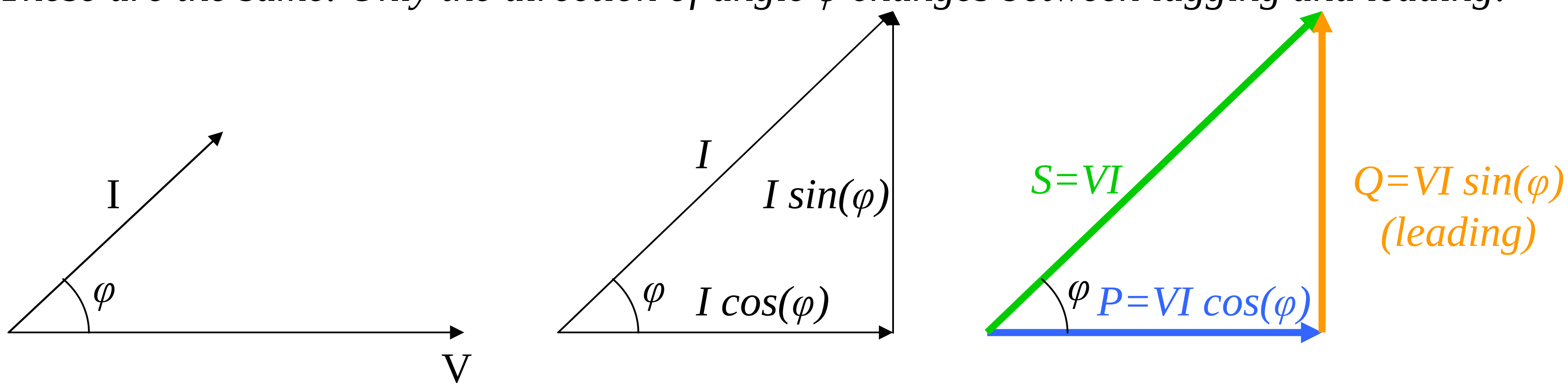
The reason the triangles appear upside down is that they are for phase, not -j.

Power triangle for capacitive (leading) loads.

For an capacitive load, the current **leads** the voltage by some angle φ. Taking V as the reference, the in-phase component of the current is given by $I \cos(\varphi)$. The reactive or quadrature component of the current is given by $I \sin(\varphi)$.

The average ("real") power, P = Voltage x in-phase current = $VI\cos(\varphi)$ (units = watts).
The apparent power, S = Voltage x current = VI (units = VoltAmps, VAs)
The reactive power, Q = Voltage x quadrature current = $VI\sin(\varphi)$ (units = VARs)

These are the same. Only the direction of angle φ changes between lagging and leading!



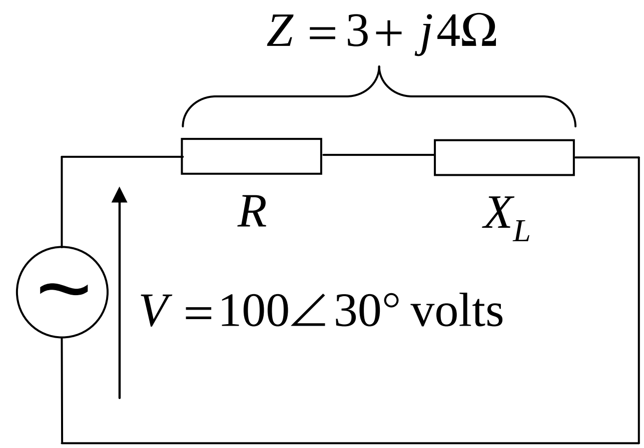
Power Factor.

For both inductive and capacitive loads, the $\boxed{\text{Power Factor (P.F.)} = \cos(\phi) = \frac{R}{Z} = \frac{P}{S}}$

A purely resistive load has a power factor of unity (no reactive component).

Example.

Determine the power triangle for this circuit.



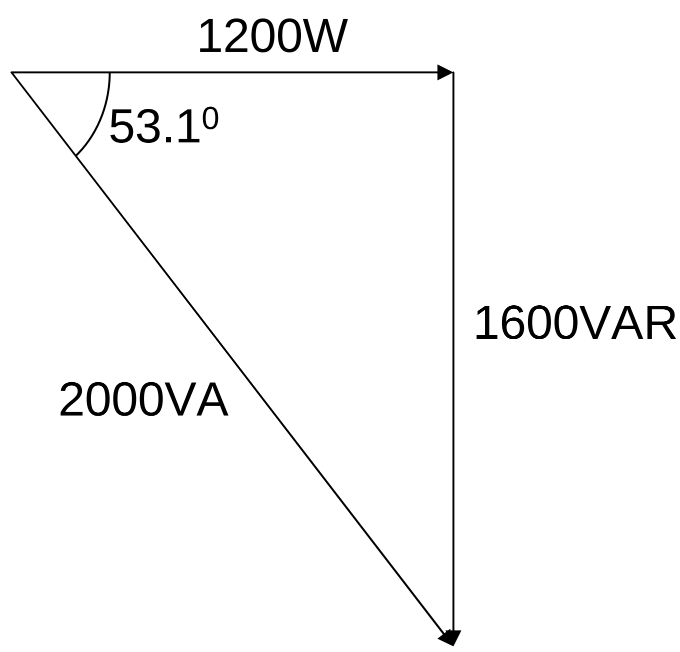
$$I = \frac{V}{Z} = \frac{(100\angle 30^\circ)}{5\angle 53.1^\circ} = 20\angle -23.1^\circ$$

$P = I^2R = 20^2 \times 3 = 1200W$

$Q = I^2X = 20^2 \times 4 = 1600VAR$ lagging

$S = I^2Z = 20^2 \times 5 = 2000VA$

P.F. = $\cos(53.1^0) = 0.6$ lag.



OR

$S = VI = 100 \times 20 = 2KVA$

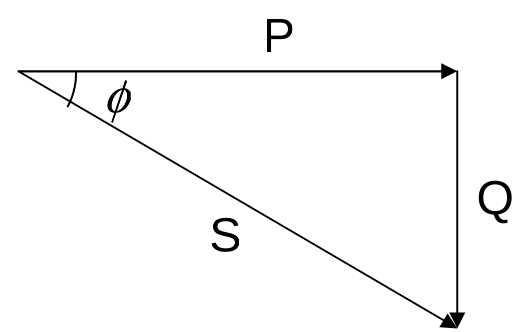
$P = VI\cos(\varphi) = 2000 \times \cos(53.1^0) = 1200W$

$Q = VI\sin(\varphi) = 2000 \times \sin(53.1^0) = 1600VAR$ lagging

P.F. = $\cos(\varphi) = \cos(53.1^0) = 0.6$ lagging

Complex Power

The three sides of the power triangle (S, P & Q) can be obtained from the product **VI*** where **I*** is the complex conjugate of **I**. The result of this product is a complex number called the complex power, S. The real part equals the average power P and the imaginary part is equal to the reactive power Q.



Consider $\mathbf{V} = Ve^{j\alpha}$ and $\mathbf{I} = Ie^{j(\alpha+\phi)}$, then

$$\mathbf{S} = \mathbf{VI}^* = Ve^{j\alpha} Ie^{-j(\alpha+\phi)}$$

$$= VIe^{-j\phi} = VI\cos(\phi) - jVI\sin(\phi)$$

$$= \mathrm{P - jQ.}$$

The absolute value of S is the apparent power S= VI. A leading phase angle (**I** leads **V**) determines a leading Q whereas a lagging phase angle (**I** lags **V**) indicates a lagging Q. Keep this in mind when constructing the power triangle.

$$\text{Average power P} = VI\cos(\phi) = I^2 R = \frac{V_R^{\ 2}}{R} = \mathrm{Re}(\mathbf{VI^*})$$

$$\text{Reactive power Q} = VI\sin(\phi) = I^2 X = \frac{V_X^{\ 2}}{R} = \mathrm{Im}(\mathbf{VI^*})$$

$$\text{Apparent power S} = VI = I^2 Z = \frac{V^2}{Z} = |\mathbf{VI^*}|$$

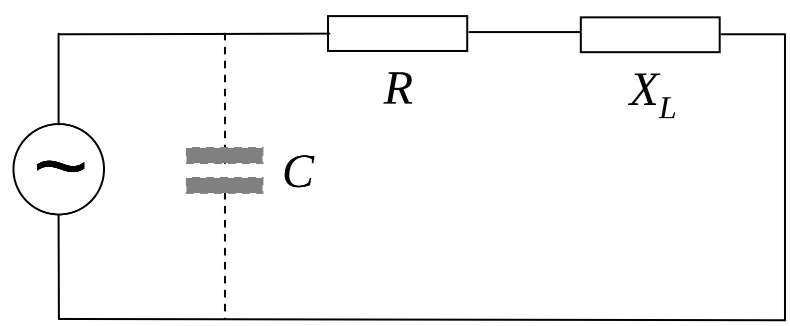
$$\mathrm{P.F.} = \cos(\phi) = \frac{R}{Z} = \frac{P}{S}$$

Power Factor Correction.

Most domestic and industrial loads are inductive and hence in the distribution system the current lags the voltage. Since the heating effect in distribution system components (wires, transformers, etc.) is proportional to I^2, non-unity P.F. means the system's ability to deliver power is reduced. This is an economically undesirable state of affairs, and utility companies charge consumers accordingly.

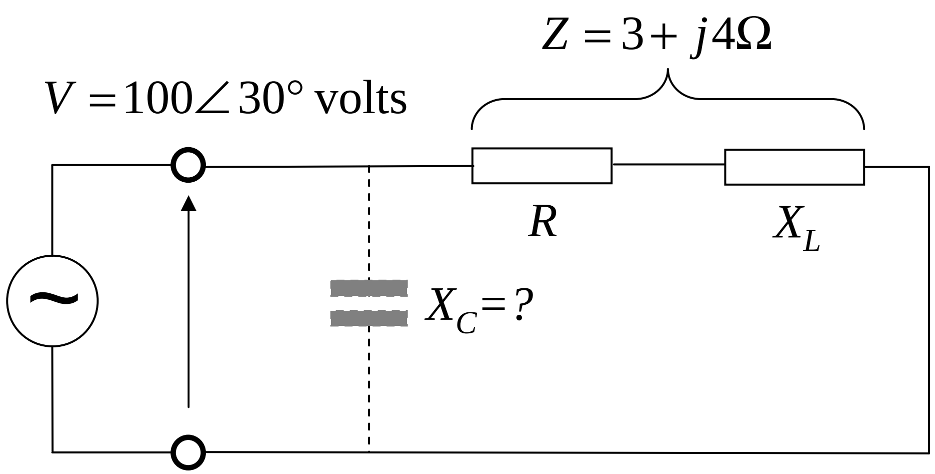
By way of illustration, if a pure inductance or pure capacitance was connected across the secondary winding of a transformer, the transformer could actually be fully loaded to the maximum permissible current in the winding without a single watt of useful power being delivered by the transformer.

For an inductive load (lagging P.F.), adding a capacitor in parallel with the load can bring the P.F. closer to unity, thereby improving the efficiency of the power distribution system. *Note that the capacitor is added in parallel with the load, hence the voltage across the load is unaltered, and hence the useful power (P) does not change.*



Example (contd)

Improve the power factor of the previous circuit to 0.9 lag and draw the new power triangle. What is X_C?



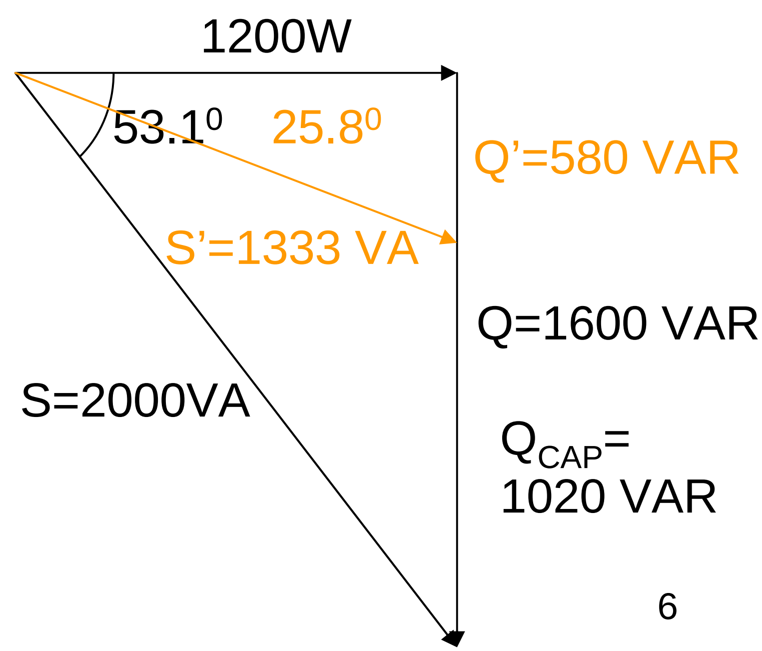
A 0.9 lag corresponds to a load angle of $\cos^{-1}(0.9) = 25.8^0$.

S'=P / cos(φ') = 1200 / 0.9 = <u>1333 VA</u>

and Q'=P tan(φ') = <u>580 VAR</u>

Note that the VA loading of the circuit has decreased from 2KVA to 1.333KVA.

$$Q_{CAP} = \frac{V_X^{\ 2}}{X} \Rightarrow X_{CAP} = \frac{V_X^{\ 2}}{Q} = \frac{10^4}{1020} = -j9.8\Omega$$



Check the power triangle for this circuit:

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{-j9.8(3+j4)}{-j9.8+3+j4}$$

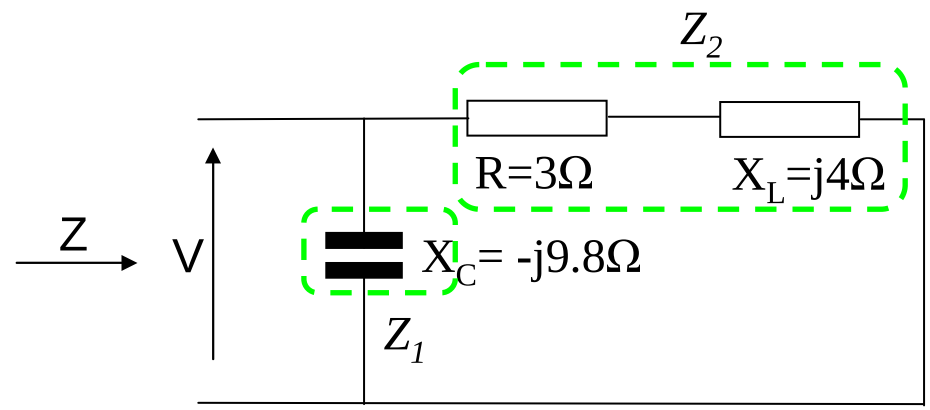
$$= \frac{-j29.4+39.2}{3-j5.8} = \frac{49\angle 36.9^\circ}{6.53\angle 62.65^\circ}$$

$$= 7.50\angle -25.75^\circ$$

$$|Z| = \sqrt{R^2 + X^2}$$

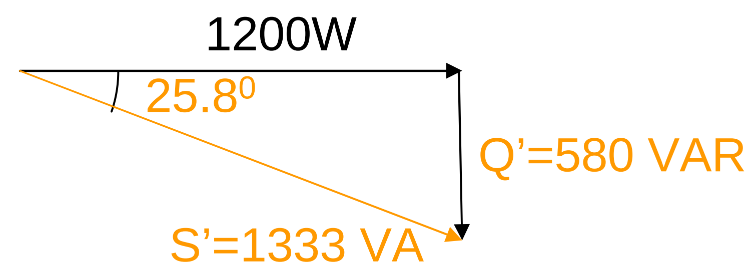
$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

This problem is most easily solved using the polar form of complex numbers now .



$$I = \frac{V}{Z} = \frac{100\angle 30^\circ}{7.50\angle -25.75^\circ} = 13.33\angle 55.75^\circ$$

$$S = \mathbf{VI^*} = (100\angle 30^\circ)(13.33\angle -55.75^\circ) = 1333\angle -25.75^\circ$$ <u>Q.E.D.</u>



Additional questions:

1. Given a circuit with an applied voltage $v=150\sin(\omega t+10°)$ volts and a resulting current of $i = 5\sin(\omega t-50°)$ amps, determine the power triangle.

2. A two element series circuit has a power of 940W and a power factor of 0.707 leading. If the applied voltage is $v=99\sin(6000t+30°)$ volts, determine the circuit component values.

3. An induction motor with a 2HP output has an efficiency of 85%. At this load the power factor is 0.8 lagging. Determine the complete input power information.
[1 horsepower = 746 watts]

4. A 25KVA transformer supplies a load of 12KW at a power factor of 0.6 lagging. Find the percentage of full load the transformer is carrying. If additional unity power factor loads are to be served with the same transformer, how many KW can be added before the transformer is at full load?