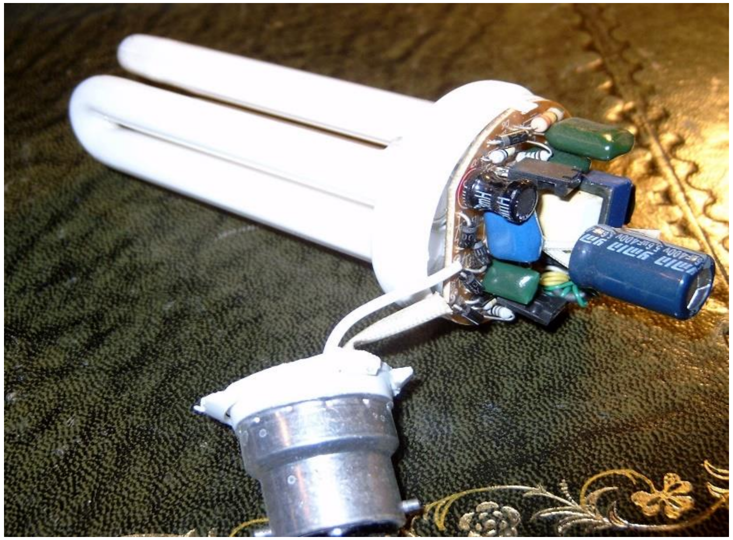
Power Electronics

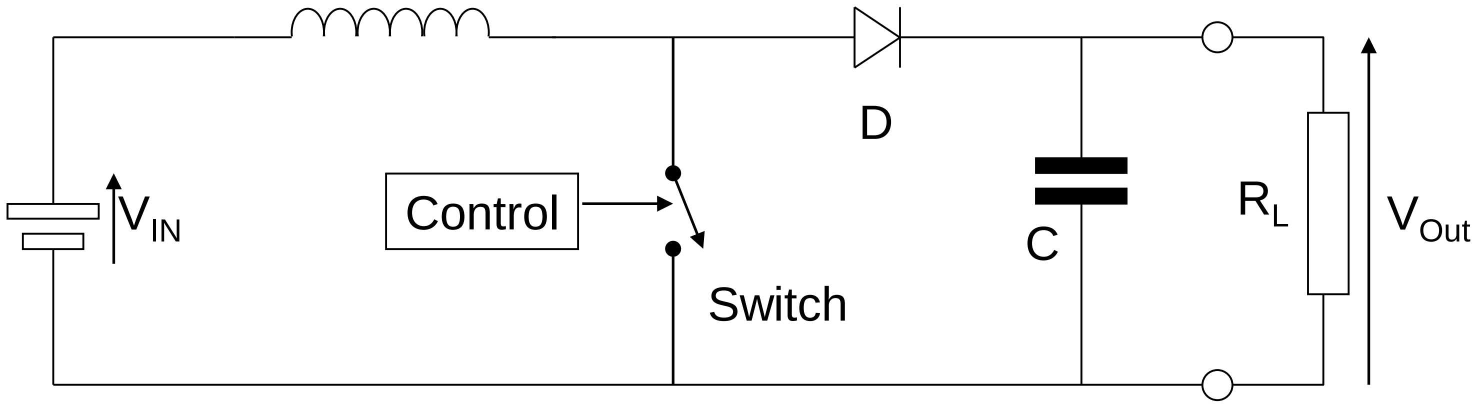
Switching Mode DC-DC Converters II:

Boost Converter and Buck-Boost Converter



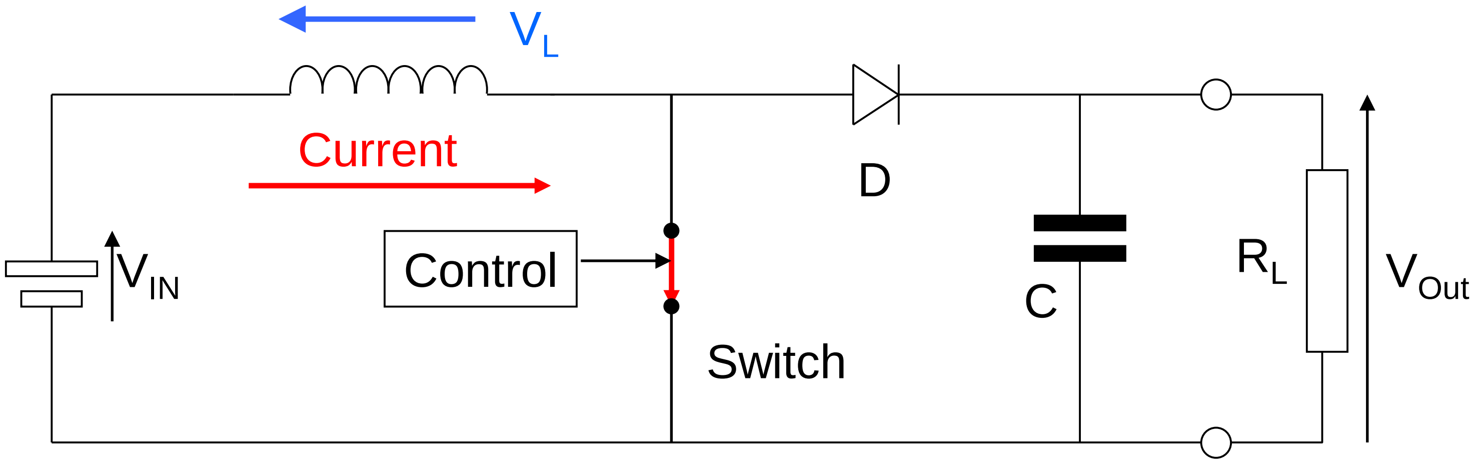
Boost (step-up) Converter
CCM
Steady-State Average Analysis

The boost (step-up) converter. ($V_{Out} > V_{In}$)



The boost converter is used to increase the output voltage (V_{Out}) above the input voltage (V_{In}). As in the case of the buck converter we will assume the circuit operates in **continuous mode**, i.e., current is always flowing in the inductor.

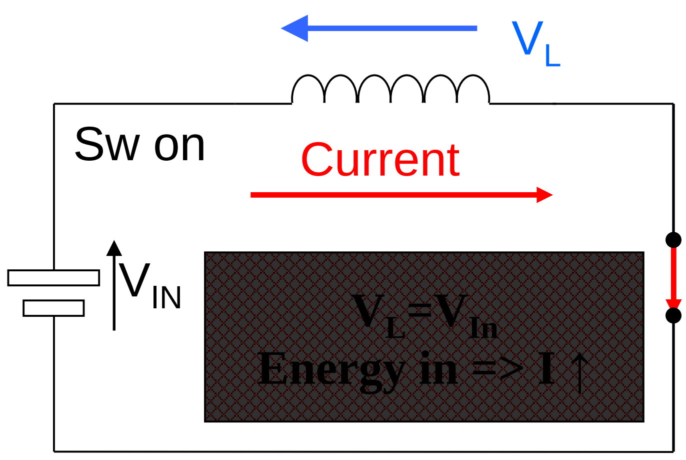
If we disregard the switch for a moment and examine the circuit, we see that in the steady state the output voltage is connected to the input voltage via an inductor (assume zero resistance) and a diode. Hence the output voltage will be one diode drop (1V) beneath the input voltage. As with the buck, ignore the voltage drop across the diode.

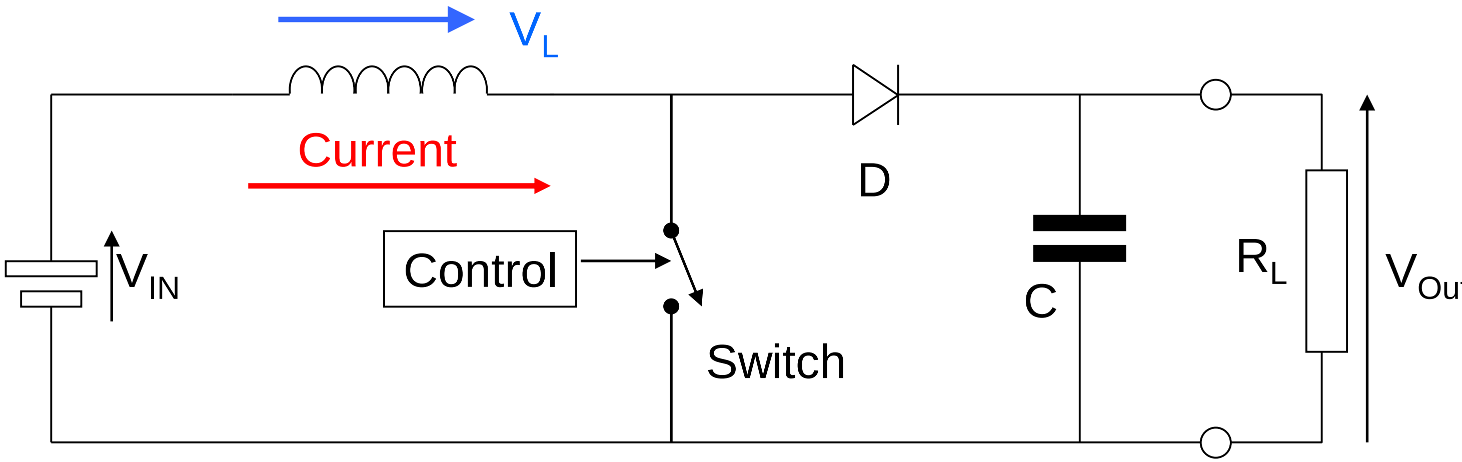


Now consider the circuit when the switch is closed. Any voltage on capacitor C will ensure the diode is reverse biased and therefore the output circuit is “disconnected” from the input. [Because the diode’s anode is connected to ground via the switch.]

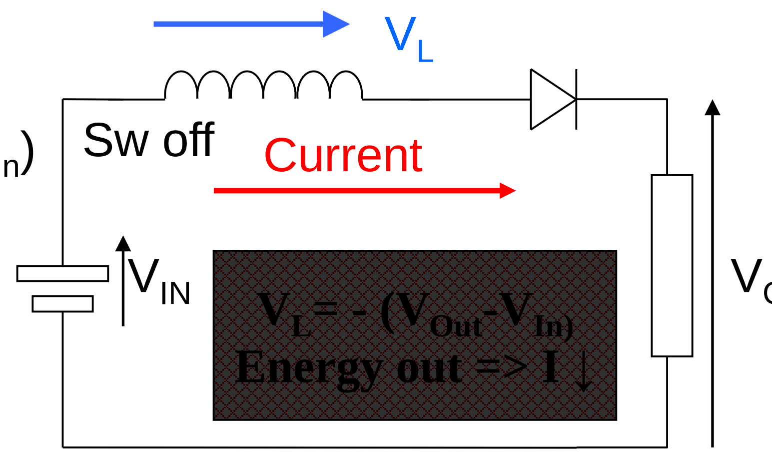
The inductor is connected across the supply (V_{In}) and hence current will start to build through it with the energy being stored in the magnetic field. During this phase, $V_L = V_{In}$.

Switch ON: Charging the inductor !!!





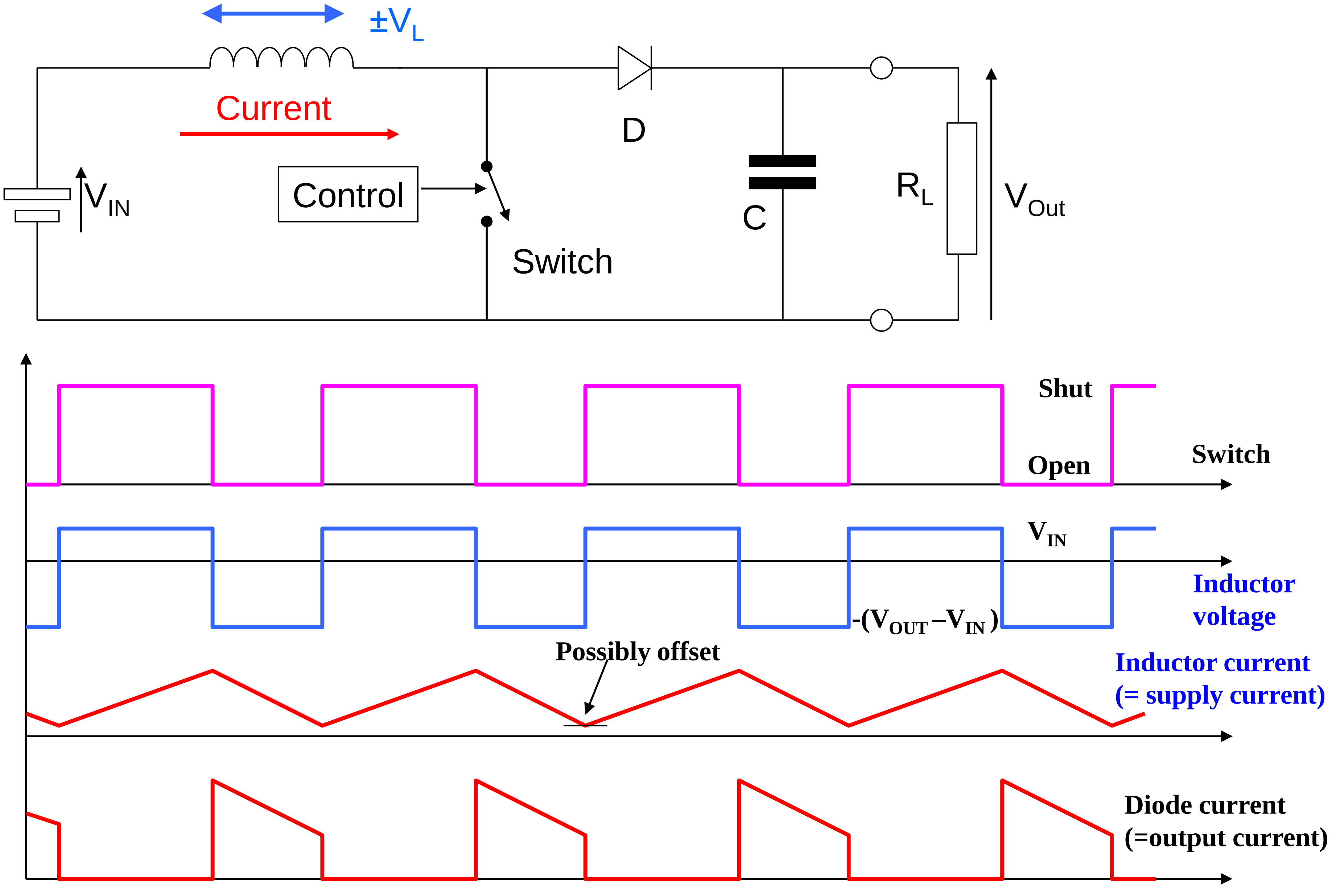
When the switch is opened, the inductor wants to maintain the current flow and with the supply side of the inductor still held at V_{In}, the potential at the opposite side of the inductor rises until the diode starts to conduct. The inductor voltage during this phase is the difference between the input voltage (V_{In}) and the output voltage (V_{Out}). Note also that the sign of the voltage has changed when compared with the inductor charging period.



So $V_{Inductor} = -(V_{Out} - V_{In}) = V_{In} - V_{Out}$.

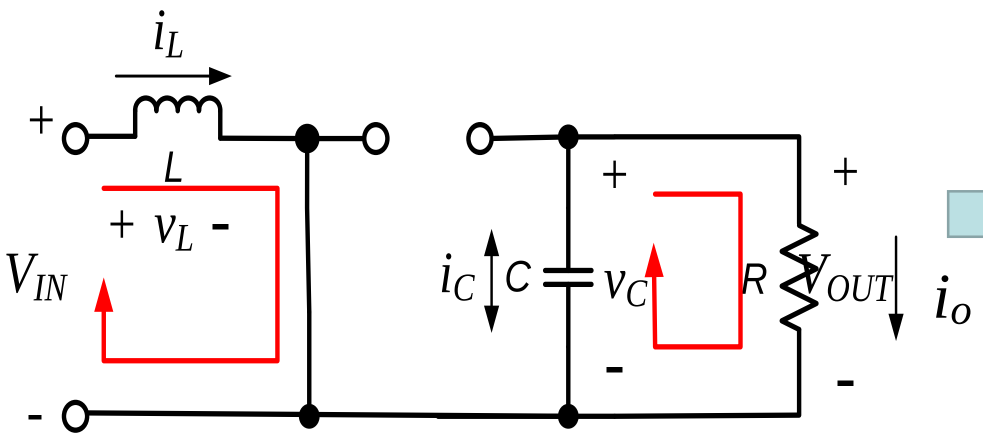
As energy is transferred from the inductor's magnetic field so the current flowing through the inductor and diode falls.

Switch OFF: Discharging the inductor !!!



Note that current always flows in the inductor (continuous mode again)

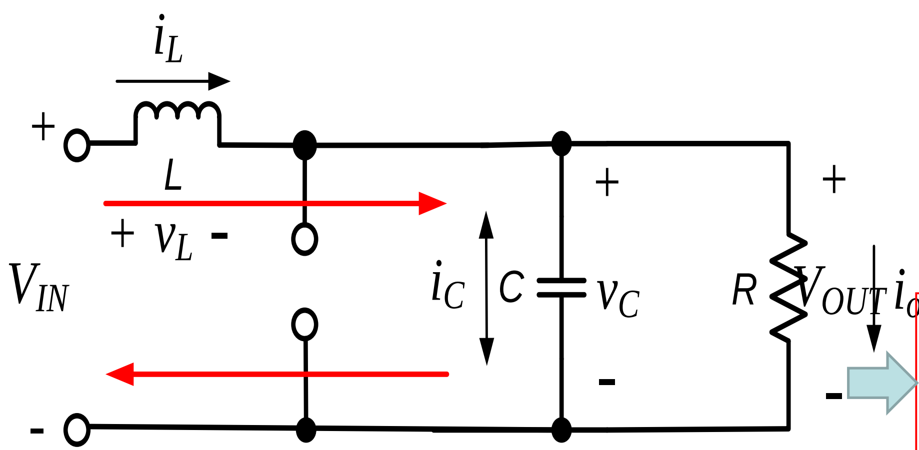
Modeling: Two Switching Circuit States



$$v_L = V_{IN} > 0 \Rightarrow \text{charging L}$$

$$i_C = -i_o < 0 \Rightarrow \text{discharging C}$$

Charging the inductor L: Switch ON, Diode OFF



$$v_L = V_{IN} - V_{OUT} > 0 \Rightarrow \text{discharging L}$$

$$i_C = i_L - i_o > 0 \Rightarrow \text{charging } C$$

Discharging the inductor L: Switch OFF, Diode ON

Input-Output: Step-up Voltage Conversion

In Steady State, for the inductor *L*

$$0 \le \phi = \frac{t_{on}}{T_s} = \frac{t_{on}}{t_{on} + t_{off}} < 1$$

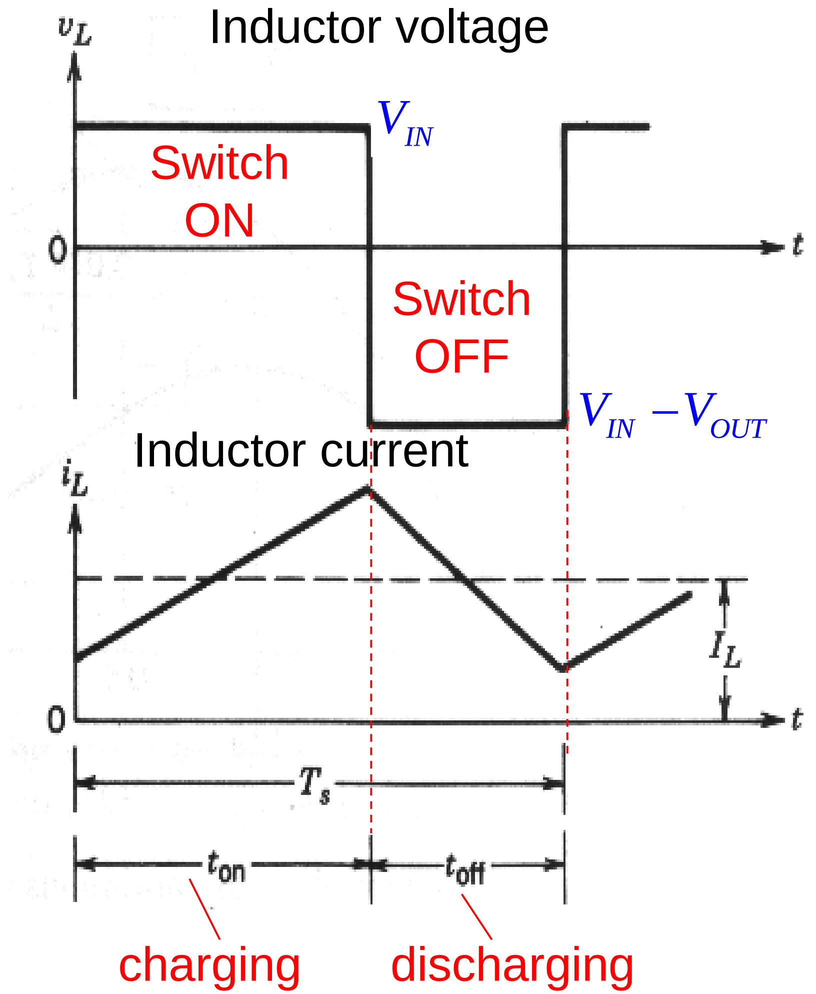
$$\int_0^T v_L(t)\,dt = 0$$

Switch ON   Switch OFF

$$\Rightarrow \int_0^{t_{on}} V_{IN}\,dt + \int_{t_{on}}^{T_s} (V_{OUT} - V_{IN})\,dt = 0$$

$$\Rightarrow V_{IN}\phi T_s + (V_{IN} - V_{OUT})(1-\phi)T_s = 0$$

$$\Rightarrow V_{OUT} = \frac{1}{1-\phi} V_{IN}, \quad \phi < 1$$



$P_{IN} = P_{out}$ $\quad \Rightarrow \dfrac{I_L}{I_o} = \dfrac{V_{OUT}}{V_{IN}} = \dfrac{1}{1-\phi}$

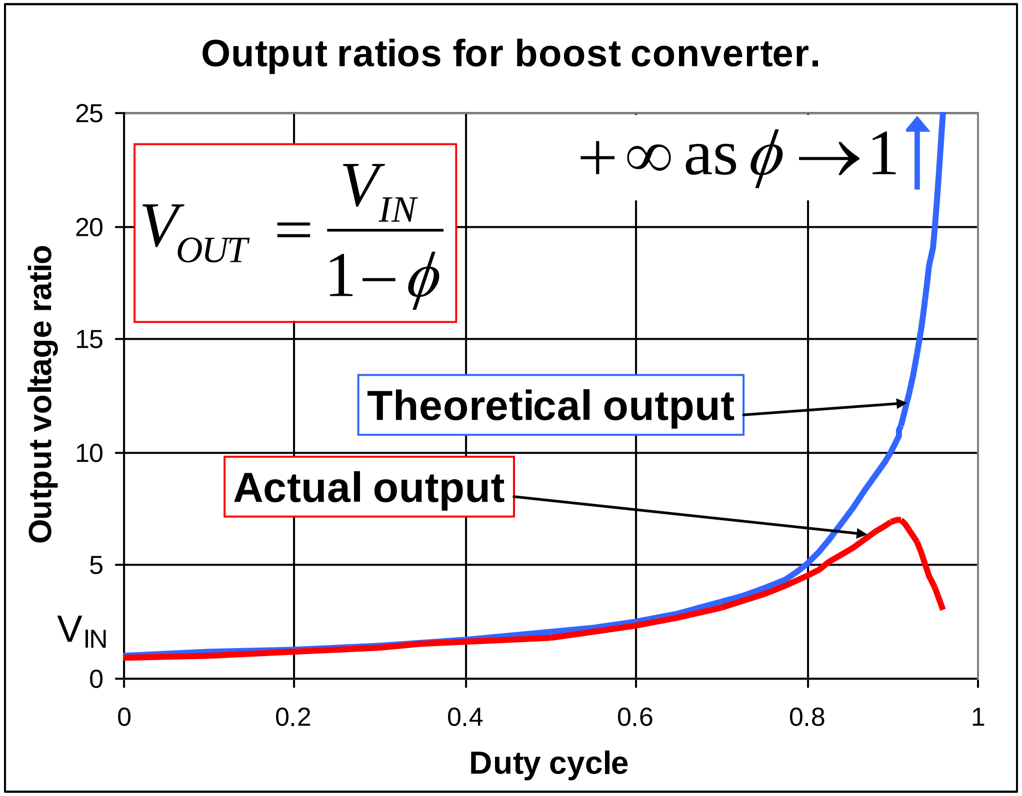
lossless system

If we plot this relationship, we have:

Some important observations are:

1. The output voltage is always greater than the input voltage.

2. Assuming a lossless system, the output voltage is independent of the load on the output. The power delivered is proportional to the current flowing in the inductor.



In reality, as φ approaches 1, the output voltage is limited due to loses in the system. If the switch were constantly closed the input would be short-circuited and no energy could be transferred to the output.

Note this relationship is true only if current is always flowing in the inductor, i.e. the converter is operating in **continuous mode**.

Design: Inductor Current Ripple

In Steady State, for the inductor *L*

$$v_L = L\frac{di_L}{dt}$$

Inductor voltage

$$i_L(t+T_s) = i_L(t)$$

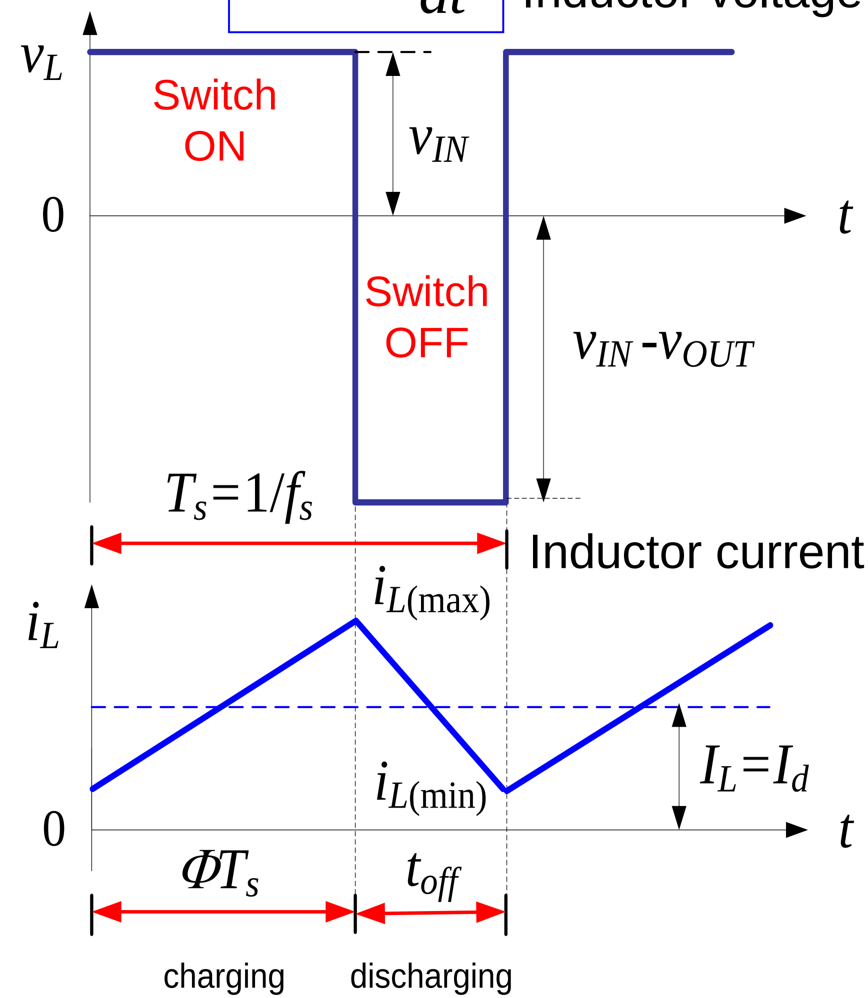
$$\Rightarrow \Delta I_L = \left|\Delta i_{L+}\right| = \left|\Delta i_{L-}\right| = I_{L(\max)} - I_{L(\min)}$$

$$\text{charging: } I_{L(\max)} = \frac{1}{L}\int_o^{\phi T_s} v_{IN}\,dt + I_{L(\min)}$$

$$\Rightarrow \Delta I_L = \frac{v_{IN}}{L}\phi T_s = \frac{\phi v_{IN}}{L f_s}$$

$$v_{OUT} = \frac{1}{1-\phi}v_{IN} \Rightarrow$$

$$\Delta I_L = \frac{\phi v_{IN}}{L f_s} = \frac{\phi(1-\phi)v_{OUT}}{L f_s}$$



Absolute value

$$\Delta I_L = \frac{\phi v_{IN}}{L f_s} = \frac{\phi(1-\phi) v_{OUT}}{L f_s}$$

$$\frac{I_L}{I_o} = \frac{v_{OUT}}{v_{IN}} = \frac{1}{1-\phi}$$

Relative value

$$\text{Resistive Load R}: v_{OUT} = I_o R$$

$$\Delta I_L = \frac{\phi(1-\phi) v_{OUT}}{L f_s}$$

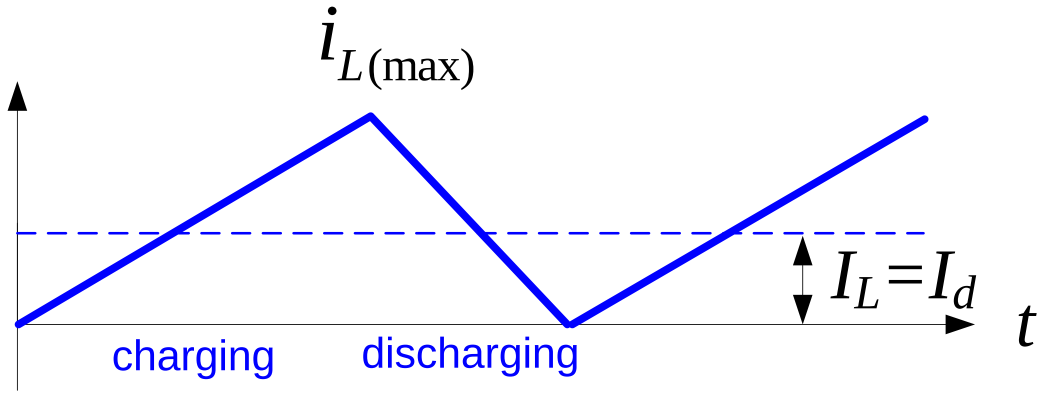
$$= \frac{\phi(1-\phi) R}{L f_s} I_o = \frac{\phi(1-\phi)^2 R}{L f_s} I_L$$

$$\Rightarrow \frac{\Delta I_L}{I_L} = \frac{\phi(1-\phi)^2 R}{L f_s}$$

Obviously, to increase switching frequency and inductor value, the inductor current ripple can be reduced; light load will increase inductor current ripple.

Operation at the Boundary Condition.

If the inductor current falls to zero for a vanishingly short time at the end of the PWM period, this is called the *"boundary condition".* It is useful in that this permits the value of the inductance to be calculated for a given switching frequency and current.

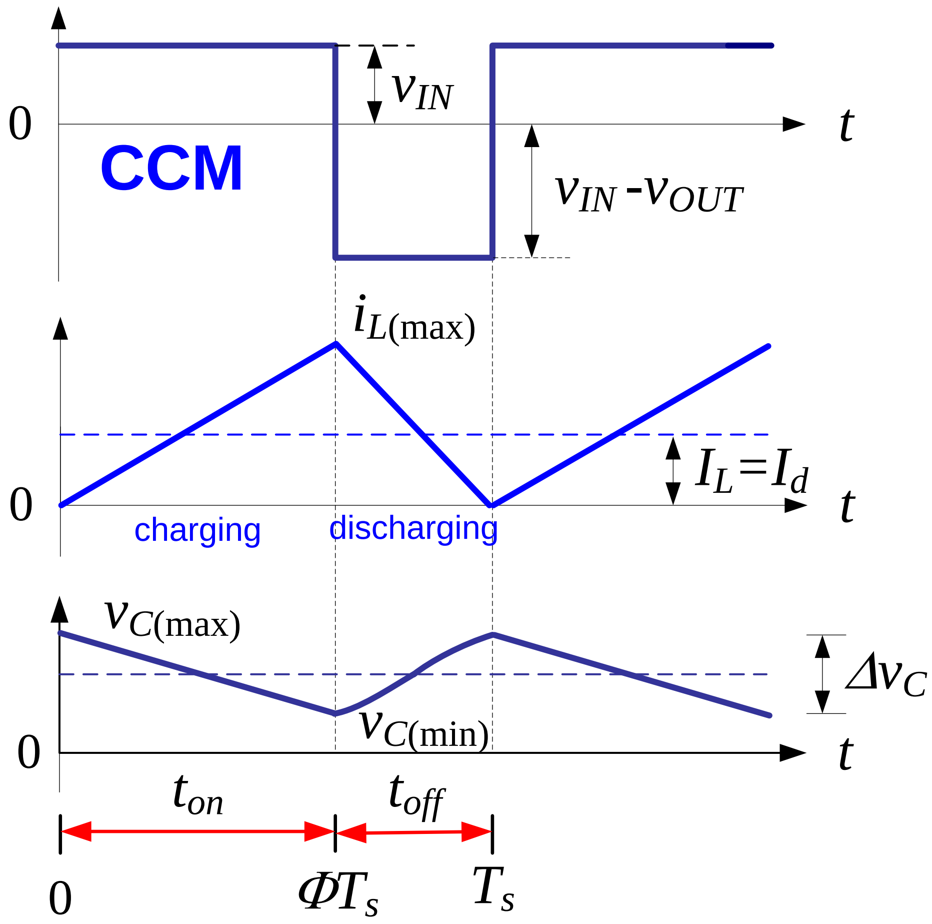


Average inductor current I_{LB} at this boundary :

$$\boxed{I_{LB} = \frac{1}{2} i_{L(\max)}}$$

I_L: average inductor current

Boundary btw CCM and DCM



I_L: average inductor current

When inductor current i_L goes to zero at the end of switch off period, circuit reaches the boundary between **CCM** and **DCM**.

Average inductor current at this boundary I_{LB} :

$$\boxed{I_{LB} = \frac{1}{2}\Delta i_L = \frac{1}{2} i_{L(\max)}}$$

Boundary Inductor Currents

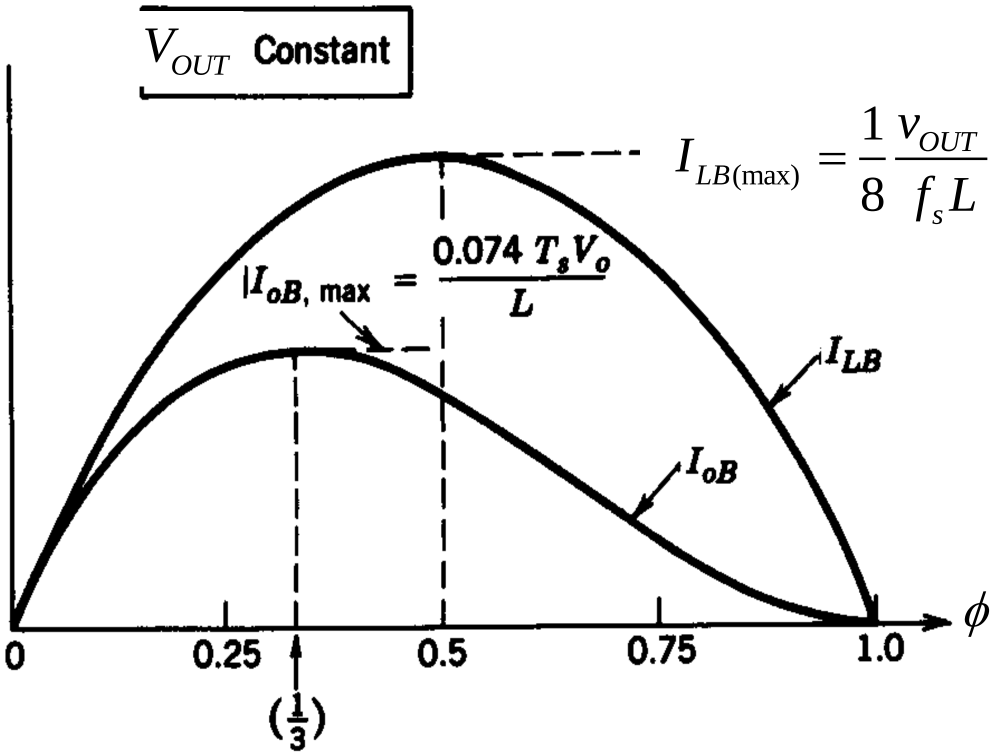
Boundary Inductor Current

$$\left.\begin{aligned}\Delta I_L &= \frac{\phi v_{IN}}{L f_s} = \frac{v_{OUT}}{f_s L}\phi\left(1-\phi\right) \\ I_{LB} &= \frac{1}{2} i_{L(\max)} = \frac{1}{2}\Delta I_L\end{aligned}\right\} \Rightarrow I_{LB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}\phi\left(1-\phi\right)$$

Boundary Output Current

$$\left.\begin{aligned}I_L &= I_{IN} = \frac{1}{\left(1-\phi\right)} I_o \\ I_{LB} &= \frac{1}{2}\frac{v_{OUT}}{f_s L}\phi\left(1-\phi\right)\end{aligned}\right\} \Rightarrow I_{oB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}\phi\left(1-\phi\right)^2$$

With given inductor L, if average inductor current $I_L < I_{LB}$ (or load current $I_o < I_{oB}$), then i_L will become discontinuous.



$$\frac{dI_{LB}}{d\phi} = 0 \Rightarrow$$

$$\phi = 0.5,\ \ I_{LB(\max)} = \frac{1}{8}\frac{v_{OUT}}{f_s L}$$

$$\frac{dI_{oB}}{d\phi} = 0 \Rightarrow$$

$$\phi = \frac{1}{3},\ \ I_{oB(\max)} = \frac{2}{27}\frac{v_{OUT}}{f_s L}$$

Boundary Inductance

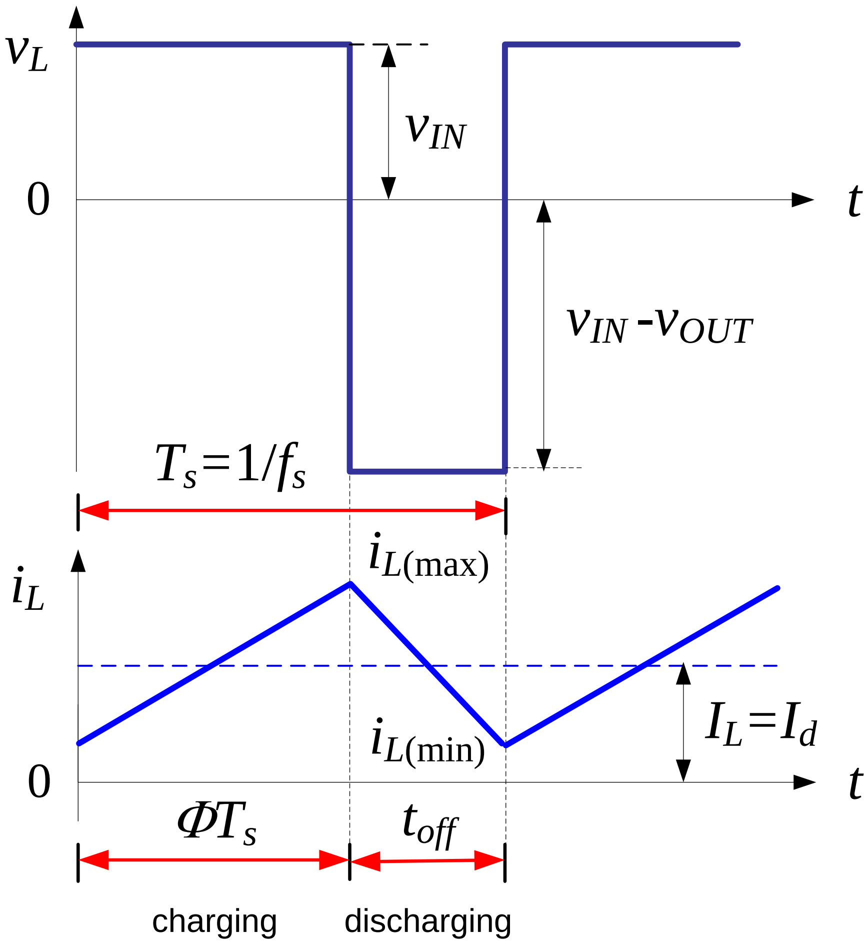
With given I_O, if the inductor value L is less than L_{min}, i_L will become discontinuous.

$$\boxed{I_{oB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}\phi(1-\phi)^2}$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{1}{1-\phi} \qquad \Downarrow \qquad v_{OUT} = I_o R$$

$$\boxed{L_{\min} = \frac{v_{OUT}}{2 f_s I_o}\phi(1-\phi)^2 = \frac{R}{2 f_s}\phi(1-\phi)^2 = \frac{v_{IN}}{2 f_s I_o}\phi(1-\phi)}$$

Design: Switch Peak Current



$$i_{peak} = i_{L(\max)} = I_L + \frac{1}{2}\Delta I_L$$

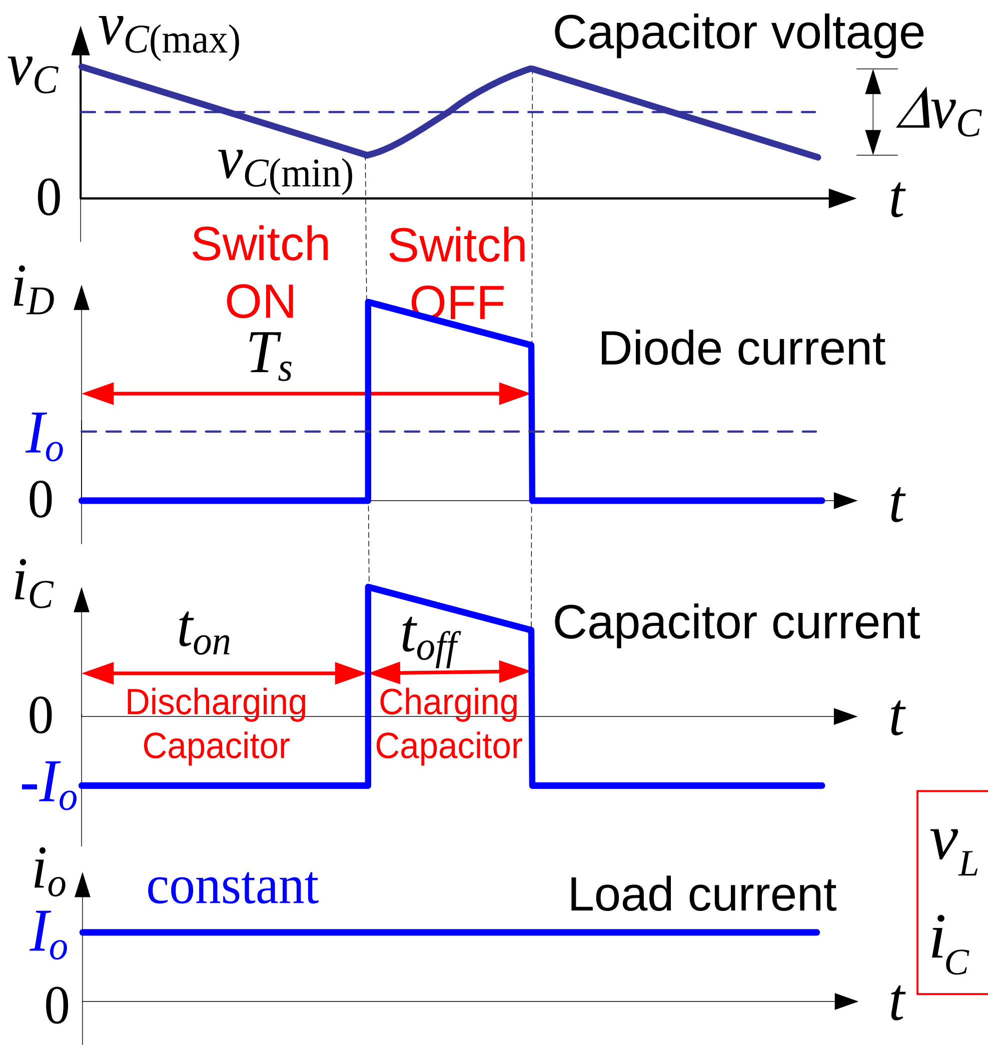
$$\Delta I_L = \frac{\phi v_{IN}}{L f_s} = \frac{\phi(1-\phi)v_{OUT}}{L f_s}$$

$$I_L = \frac{I_o}{1-\phi}$$

$$i_{L(\max)} = \frac{1}{2}\frac{\phi v_{IN}}{L f_s} + \frac{I_o}{1-\phi}$$

Switch Peak Current

Design: Capacitor Voltage Ripple



$$switching\ \ frequency: f_s = 1/T_s$$

Switch ON, Diode OFF

$$v_L = v_{IN} > 0 \Rightarrow \text{charging L}$$
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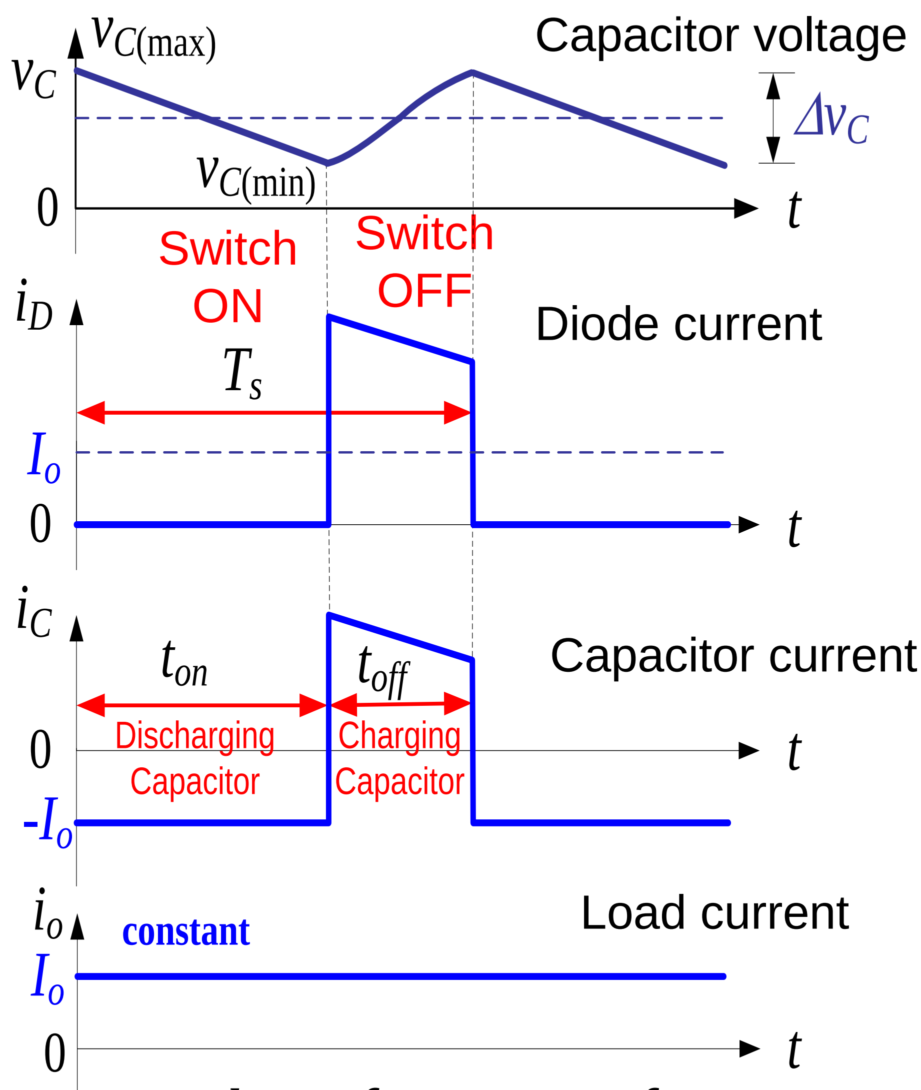
Switch OFF, Diode ON

$$v_L = v_{IN} - v_{OUT} > 0 \Rightarrow \text{discharging L}$$
$$i_C = i_L - i_o > 0 \Rightarrow \text{charging } C$$



$$switching\ \ frequency: f_s = 1/T_s$$

$$i_C = C\frac{dv_C}{dt} \quad and \quad \Delta v_c = \frac{\Delta Q_C}{C}$$

$$\Rightarrow \Delta v_c = v_C(t) - v_c(0) = \frac{1}{C}\left(\int_0^t i_C\,dt\right)$$

Switch ON Period :

$$\Rightarrow v_{C(\min)} = \frac{1}{C}\int_0^{\phi T_S}(-I_o)\,dt + v_{C(\max)}$$

$$\Rightarrow \Delta v_C = v_{C(\max)} - v_{C(\min)} = \frac{\phi I_o}{f_s C}$$

Absolute value

$$\boxed{\Delta v_C = \frac{\phi I_o}{f_s C}}$$

Relative value

Resistance Load R

$$v_C = v_{OUT}, \quad v_{OUT} = I_o R$$

$$\Rightarrow \Delta v_C = \frac{\phi I_o}{f_s C} = \frac{\phi}{f_s C}\frac{v_C}{R}$$

$$\Rightarrow \frac{\Delta v_C}{v_C} = \frac{\phi}{f_s C R}$$

$\longrightarrow$ 1st order low pass filter

Obviously, to increase switching frequency f_s can reduce the capacitor voltage ripple; low duty ratio Φ and capacitor value C will reduce capacitor voltage ripple too.

Example.

A boost converter is required to provide a constant 24V / 100W output from an input voltage that varies between 9V and 15V. For a switching frequency of 20KHz and assuming 100% efficiency and operation in continuous mode, determine the following:

1. The required PWM duty cycle range to maintain a 24V DC output.

2. The average input current when the input voltage is 15V.

3. The value of the inductor required if the instantaneous current through the inductor at the end of the PWM period is zero. Calculate the value for both 9V and 15V input.

Example.

A boost converter is required to provide a constant 24V / 100W output from an input voltage that varies between 9V and 15V. For a switching frequency of 20KHz and assuming 100% efficiency and operation in continuous mode, determine the following:

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3. The value of the inductor required if the instantaneous current through the inductor at the end of the PWM period is zero. Calculate the value for both 9V and 15V input.

Solution.

1. For continuous mode boost converter we have:

$$V_{OUT} = \frac{V_{IN}}{1-\phi} \Rightarrow \phi = 1 - \frac{V_{IN}}{V_{OUT}}$$

$$For\ V_{IN} = 9V, \phi = 1 - \frac{9}{24} = 62.5\%$$

$$For\ V_{IN} = 15V, \phi = 1 - \frac{15}{24} = 37.5\%$$

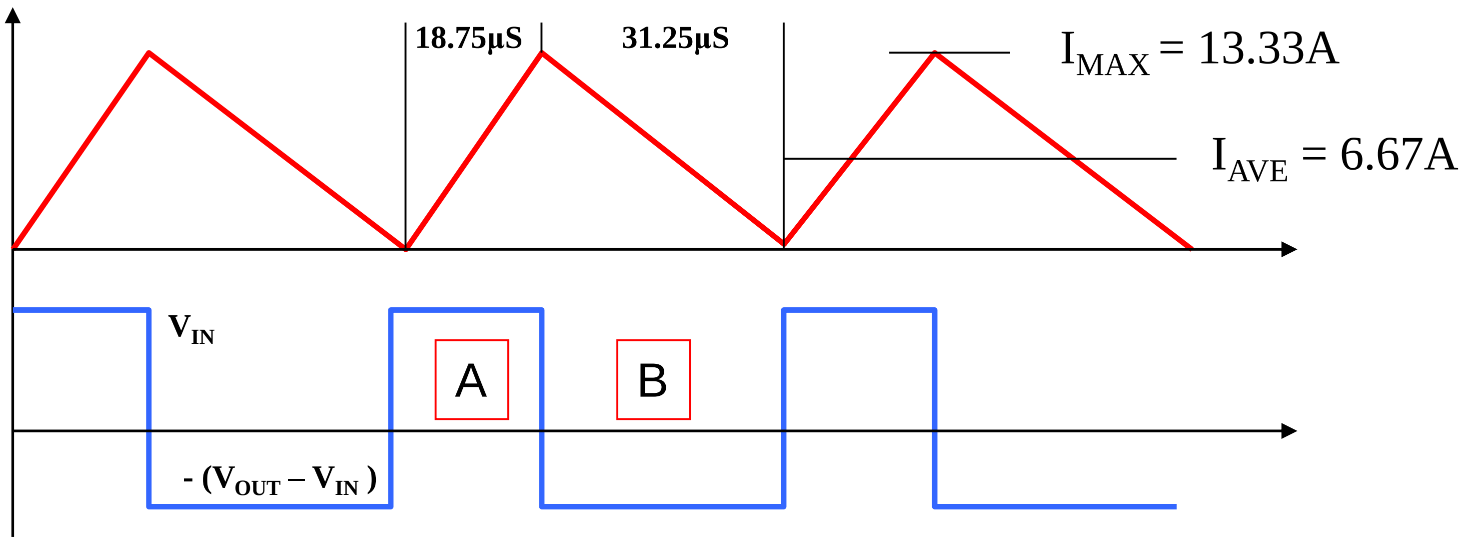
2. Assuming 100% efficiency:

$$V_{IN} I_{IN(MEAN)} = V_{OUT} I_{OUT(MEAN)} = 100W$$

$$For\ V_{IN} = 15V, I_{IN(MEAN)} = \frac{100W}{15V} = 6.67A$$

3. For a boost converter. the inductor current = input current. At 15V, φ=37.5% and $I_{IN(AVE)}$=6.67A. We know that at the end of the PWM period the inductor current is zero, hence we can construct the current waveform:

$I_{MAX} = 2 \text{ x } I_{AVE} = 13.33\text{A}$



F_{SWITCH} = 20KHz, φ = 37.5% => T_{ON} = 18.75µS

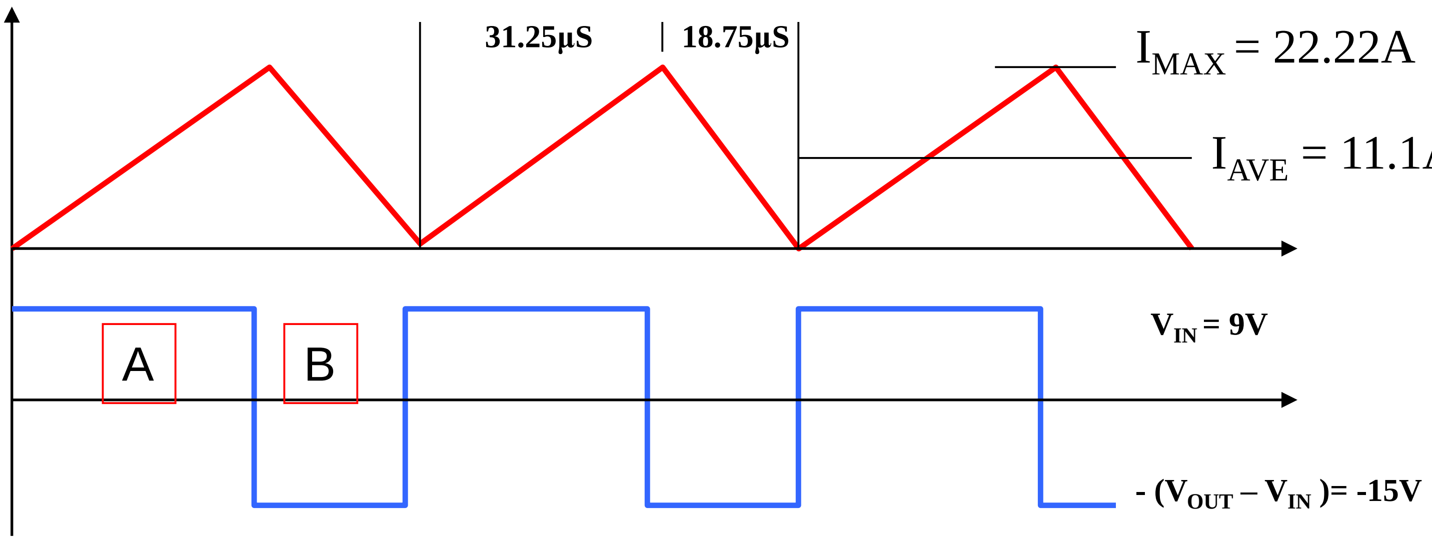
$$V = L\frac{di}{dt} \Rightarrow L = V\frac{\Delta t}{\Delta i}$$

$$For \quad A : L = \frac{15V \times 18.75\mu S}{13.\dot{3}A} = \boxed{21\mu H}$$

$$For \quad B : L = \frac{-(24V - 15V)\times 31.25\mu S}{-13.\dot{3}A} = \boxed{21\mu H}$$

Now we need to work out the average input current when the input voltage is 9V. At 9V, φ=62.5% and $I_{IN(AVE)}$=11.1A.

$I_{MAX} = 2 \text{ x } I_{AVE} = 22.22\text{A}$



$F_{SWITCH} = 20\text{KHz}$, φ = 62.5% => $T_{ON} = 31.25\mu S$

$$V = L\frac{di}{dt} \Rightarrow L = V\frac{\Delta t}{\Delta i}$$

$$For \quad A: L = \frac{9V \times 31.25\mu S}{22.\dot{2}A} \boxed{= 12.66\mu H}$$

$$For \quad B: L = \frac{-15V \times 18.75\mu S}{-22.\dot{2}A} \boxed{= 12.66\mu H}$$

If we compare these values with the case for 15V we see the inductor value is smaller – this is expected since ΔI is greater because of the lower input voltage and the reduced input voltage able to establish the current in the inductor.

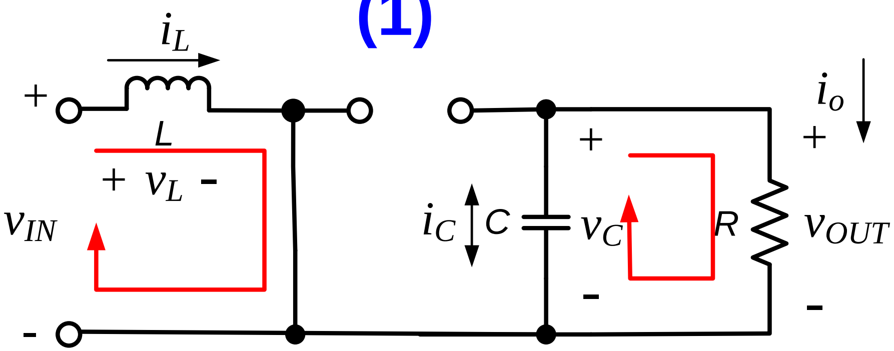
Boost (step-up) Converter

DCM

Steady-State Average Analysis

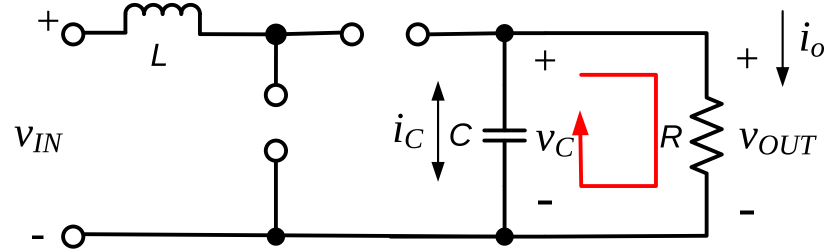
Circuit States in DCM

(1)



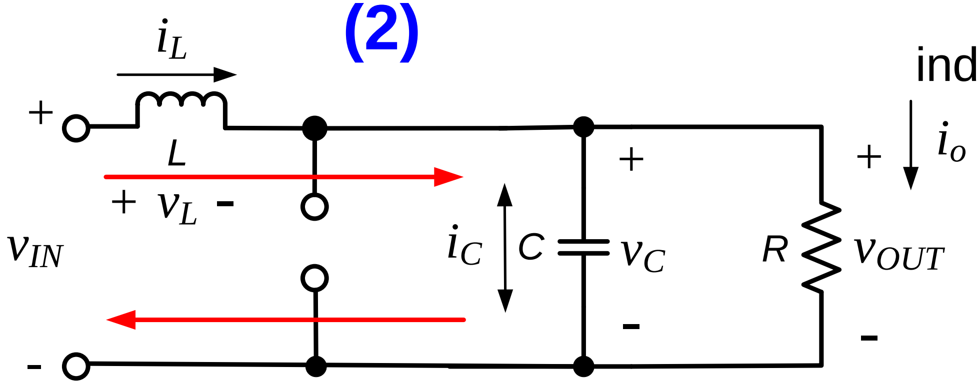
Charging the inductor L: Switch ON, Diode OFF

(3)



Idle: Switch OFF, Diode OFF

(2)



Discharging the inductor L: Switch OFF, Diode ON

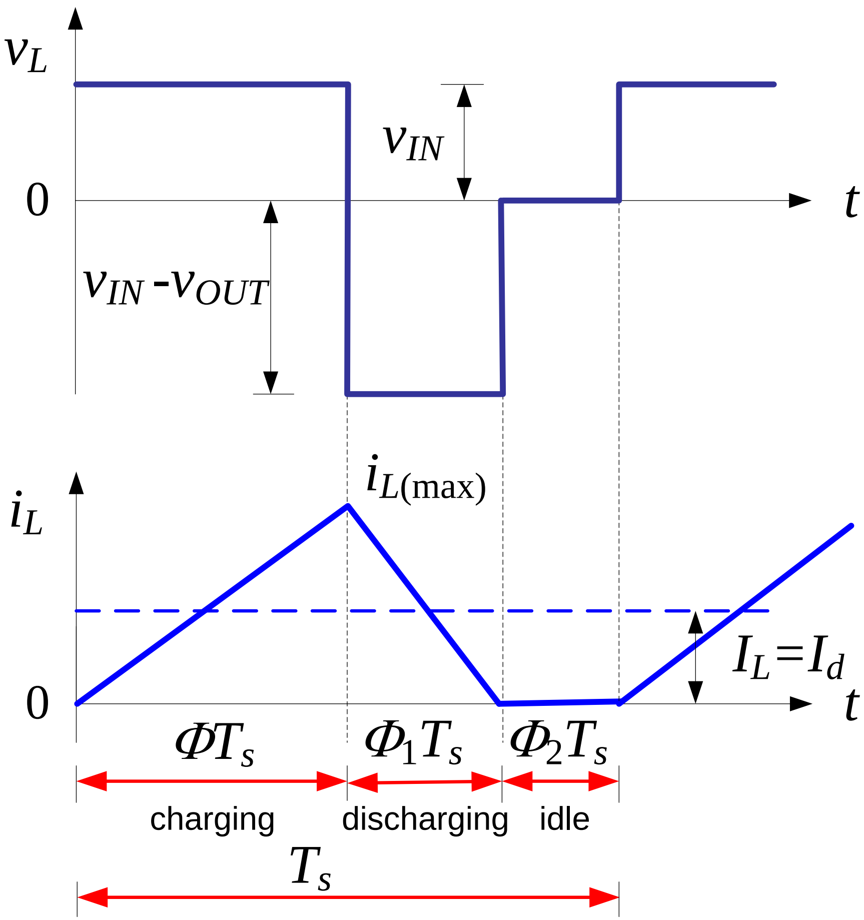
inductor

$$v_L = \begin{cases} v_{IN} & Circuit-(1) \\ v_{IN} - v_{OUT} & Circuit-(2) \\ 0 & Circuit-(3) \end{cases}$$

capacitor

$$i_C = \begin{cases} -I_o & Circuit-(1) \\ i_L - I_o & Circuit-(2) \\ -I_o & Circuit-(3) \end{cases}$$

Voltage Conversion in DCM



For the inductor in steady state

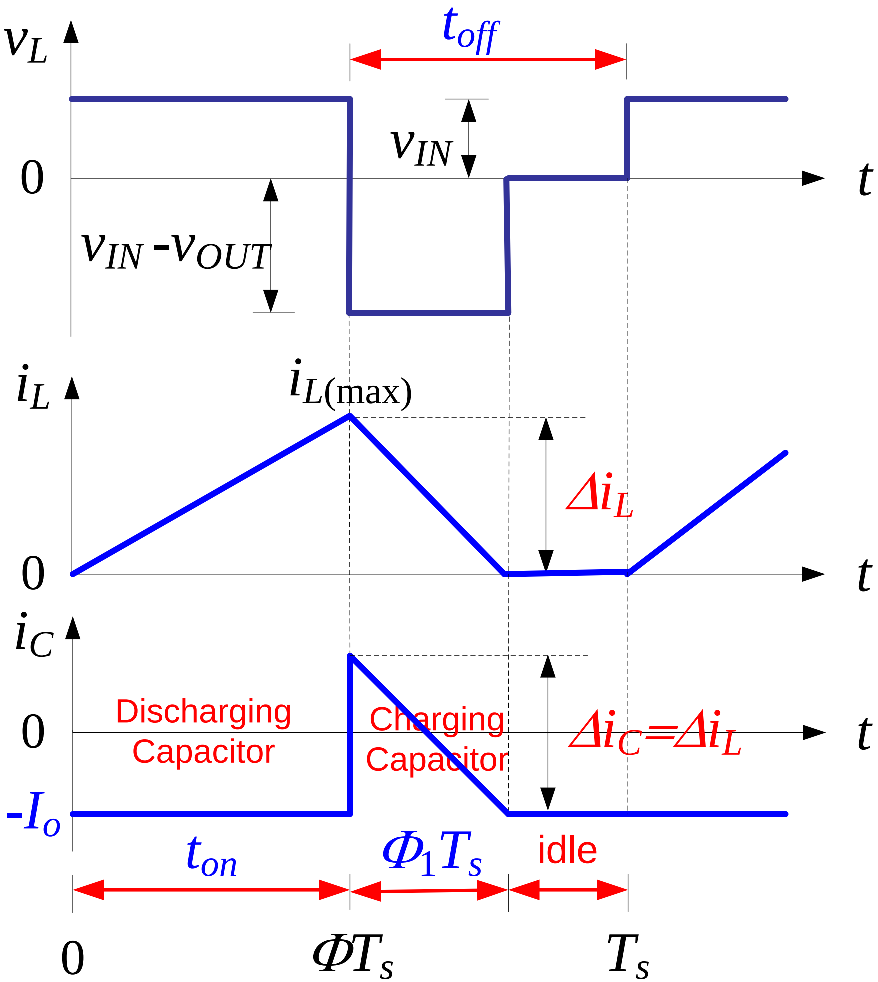
$$\int_0^T v_L(t)\,dt = 0 \qquad DCM$$

$$\Rightarrow \int_0^{\phi T_s} v_{IN}\,dt + \int_{\phi T_s}^{(\phi+\phi_1)T_s} \left(v_{IN} - v_{OUT}\right)dt = 0$$

$$\Rightarrow \frac{v_{OUT}}{v_{IN}} = \frac{\phi + \phi_1}{\phi_1}, \quad 0 < \phi + \phi_1 < 1$$

$$\boxed{I_L = \frac{1}{2} I_{L(\max)}\left(\phi + \phi_1\right)} \qquad DCM$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{1}{1-\phi}, \quad \phi + \phi_1 = 1 \quad CCM$$



$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi + \phi_1}{\phi_1} = \frac{\phi + \dfrac{I_o}{v_{IN}} \dfrac{2 f_s L}{\phi}}{\dfrac{I_o}{v_{IN}} \dfrac{2 f_s L}{\phi}}$$

For the Capacitor in steady state

$$\frac{1}{T_s}\int_0^{T_s} i_C\, dt = 0 \Rightarrow$$

$$\int_{\phi T_s}^{(\phi+\phi_1)T_s} \left(i_L - I_o\right) dt + \int_{(\phi+\phi_1)T_s}^{(1+\phi)T_s} \left(-I_o\right) dt = 0$$

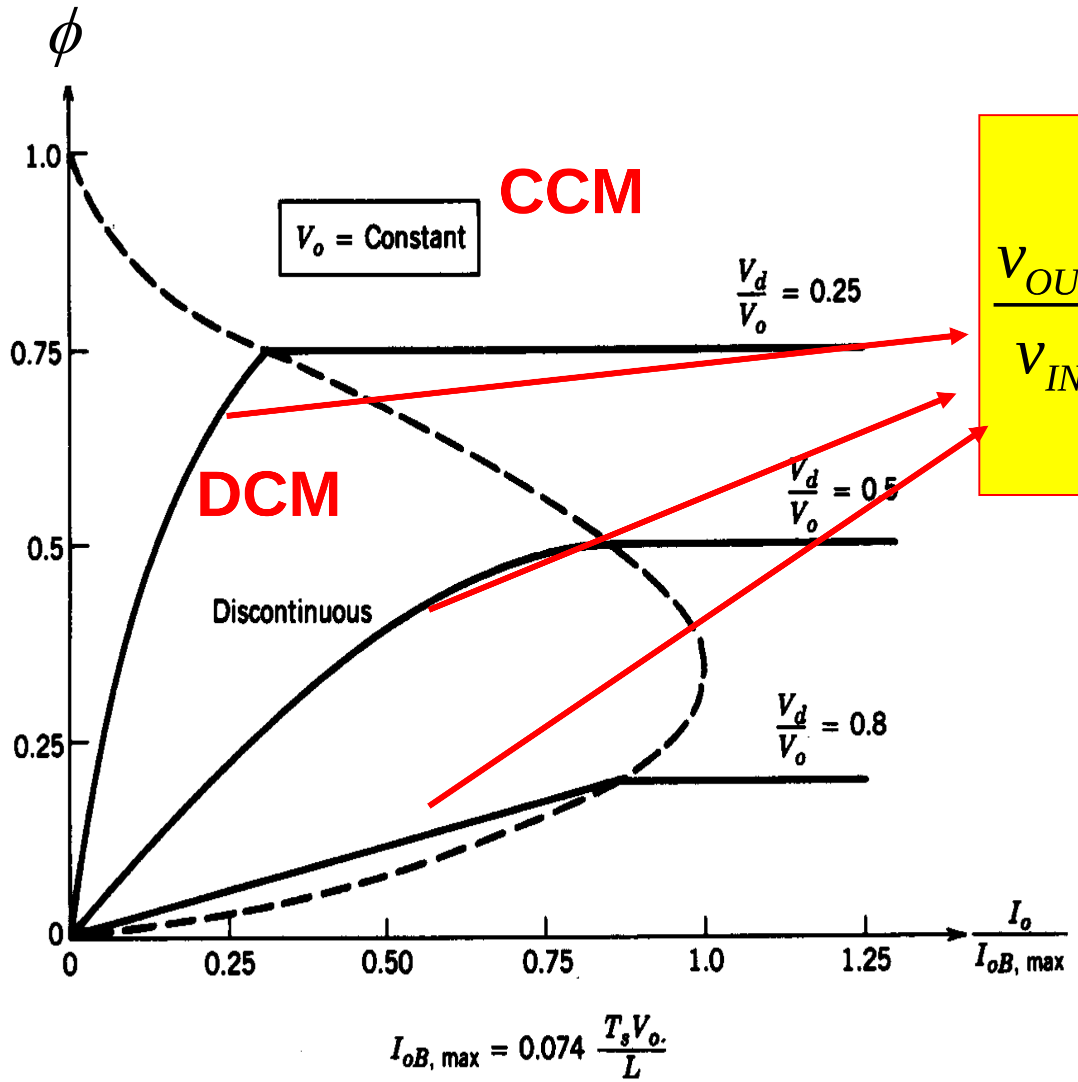
$$\Rightarrow \phi_1 = \frac{2I_o}{\Delta i_L} = \frac{2I_o}{I_{L(\max)}} = \frac{2LI_o f_s}{v_{IN}\phi}$$

$$I_L = \frac{1}{2} I_{L(\max)} \left(\phi + \phi_1\right)$$

$$I_{L(\max)} = \frac{1}{L}\int_0^{\phi T_s} v_{IN}\, dt = \frac{v_{IN}\phi T_s}{L}$$

Voltage Conversion in DCM

Voltage conversion ratio is dependent on both duty ratio Φ and the output current I_o in DCM



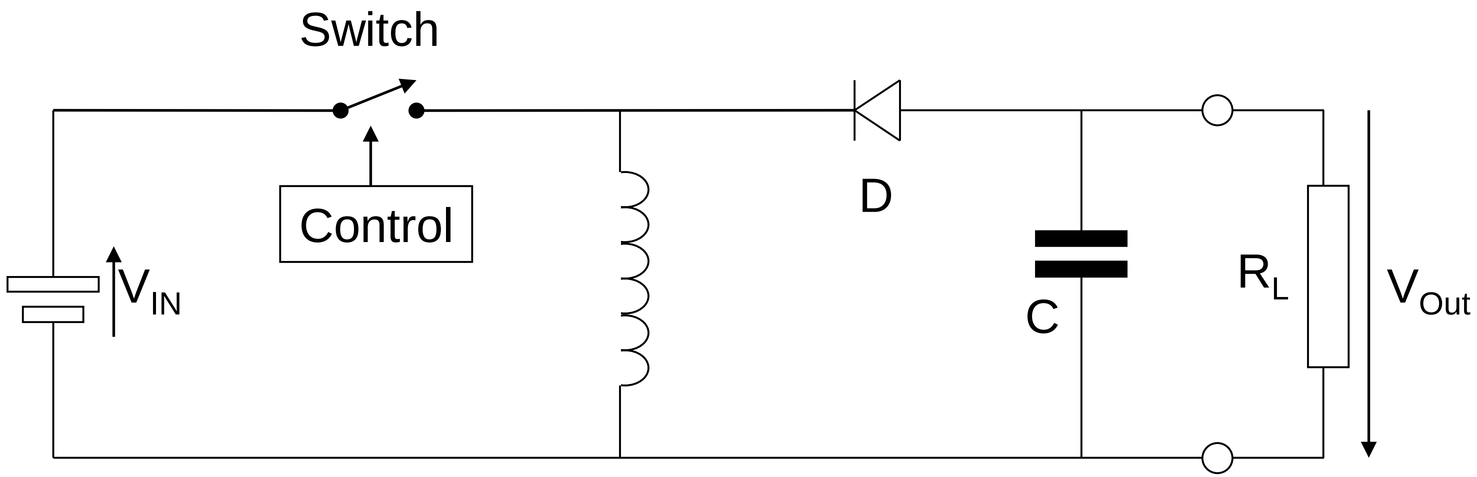
$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi + \phi_1}{\phi_1} = \frac{\phi + \dfrac{I_o}{v_{IN}}\dfrac{2 f_s L}{\phi}}{\dfrac{I_o}{v_{IN}}\dfrac{2 f_s L}{\phi}}$$

Buck-Boost Converter

Continuous Conduction Mode (CCM)

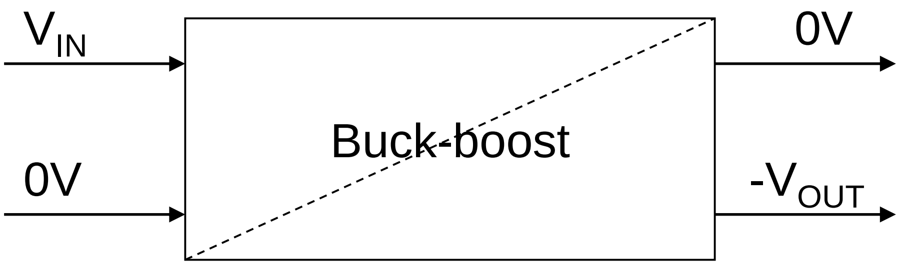
Read Chapter 7, Mohan etc.

The buck-boost (FLYBACK) converter (V_{OUT} <> V_{IN}).



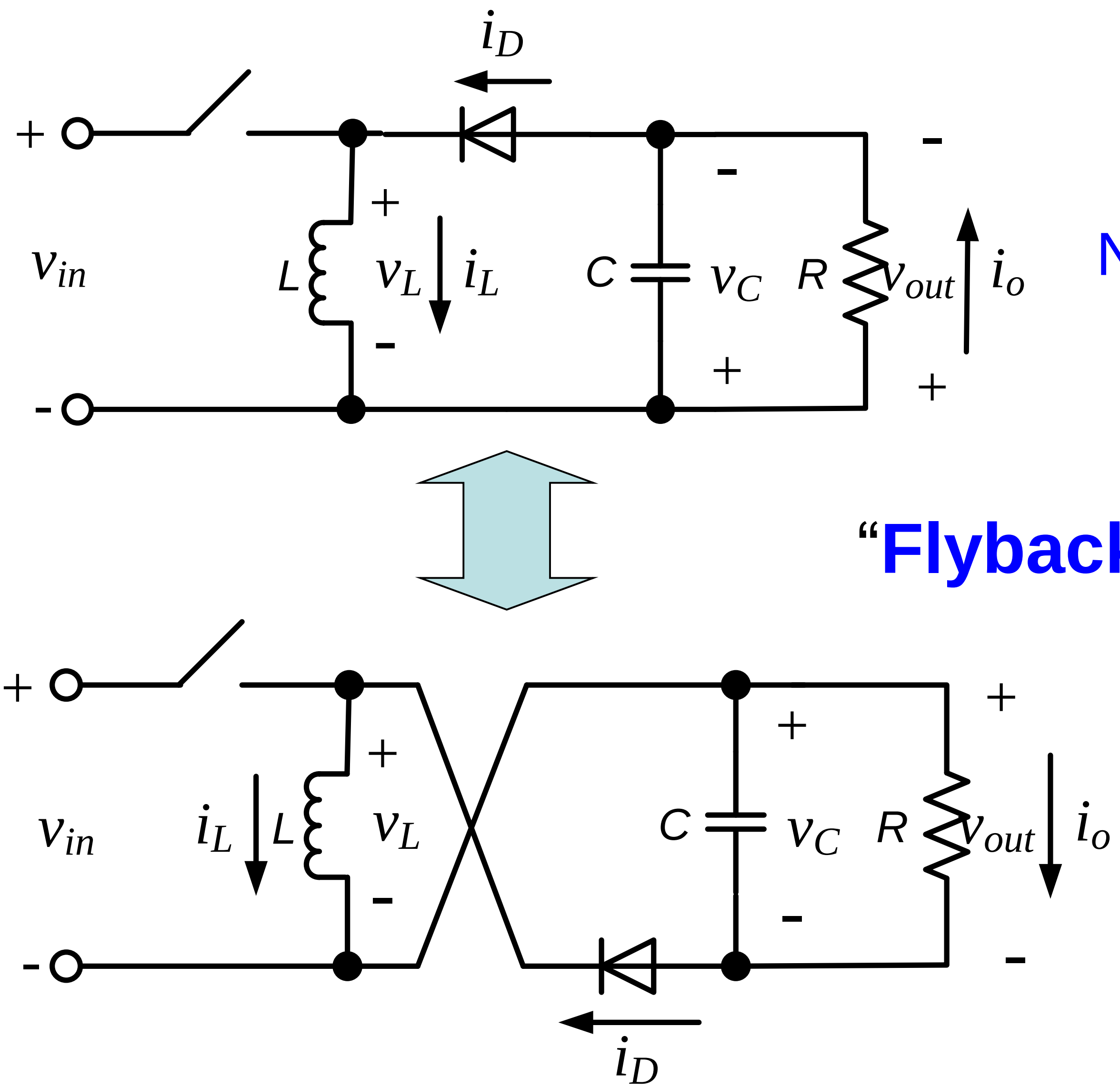
Note polarity of output voltage!

The Buck-Boost converter is able to produce output voltages that are higher or lower than the input voltage. Once again we will consider the continuous mode system. Note that the output is negative with respect to the 0V side of the input voltage.



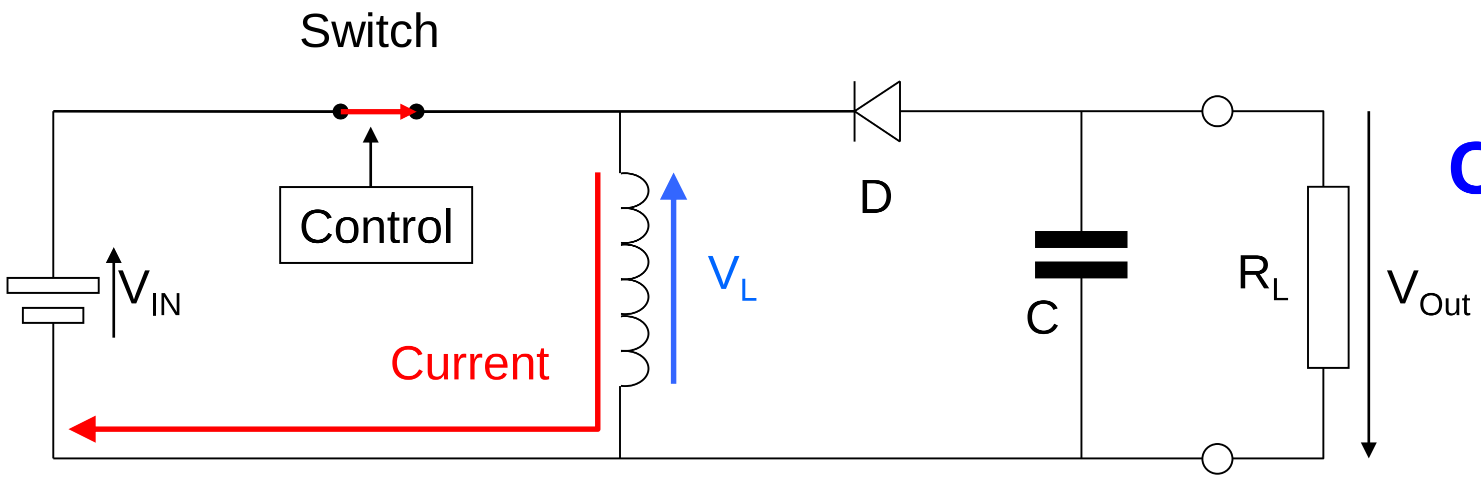
“Flyback” Topology

Positive input



Negative output

“**Flyback**”



Charging

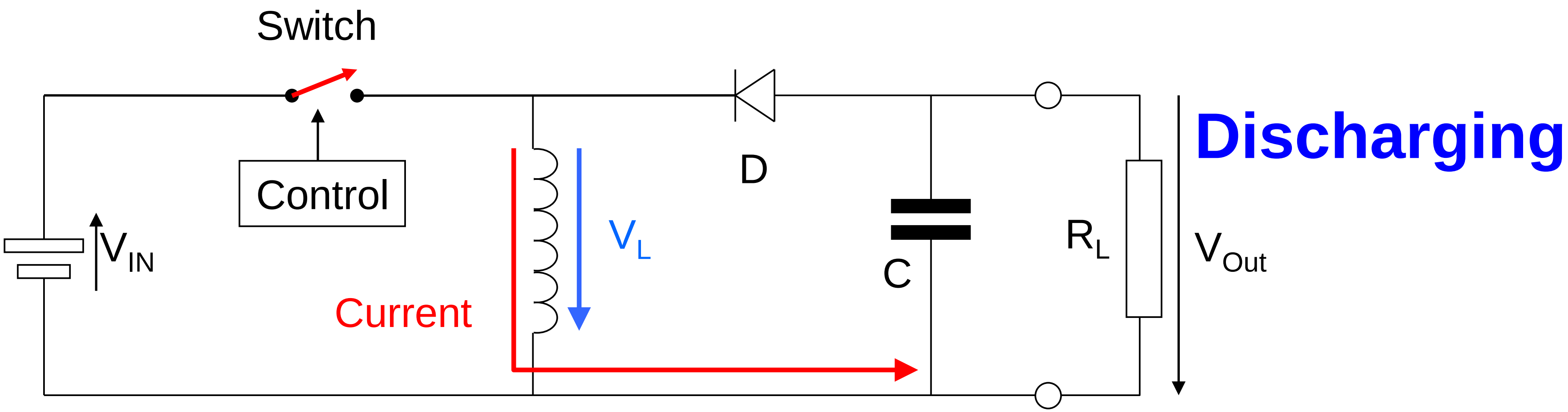
Before starting …

… Notice that when the switch is open there is no connection between the input to the supply and its output. Hence we can conclude that the supply is capable of delivering 0V output regardless of the input voltage for a 0% duty cycle on the switch.

With the switch closed, the input voltage is imposed across the inductor and the current through the inductor starts to rise. The diode is “off” since the cathode is more positive than the anode.

Hence for the “on” period, $V_L = V_{IN}$.

Note that with the diode reverse biased, the only route for the current to take that completes the circuit is back through the supply to the converter.



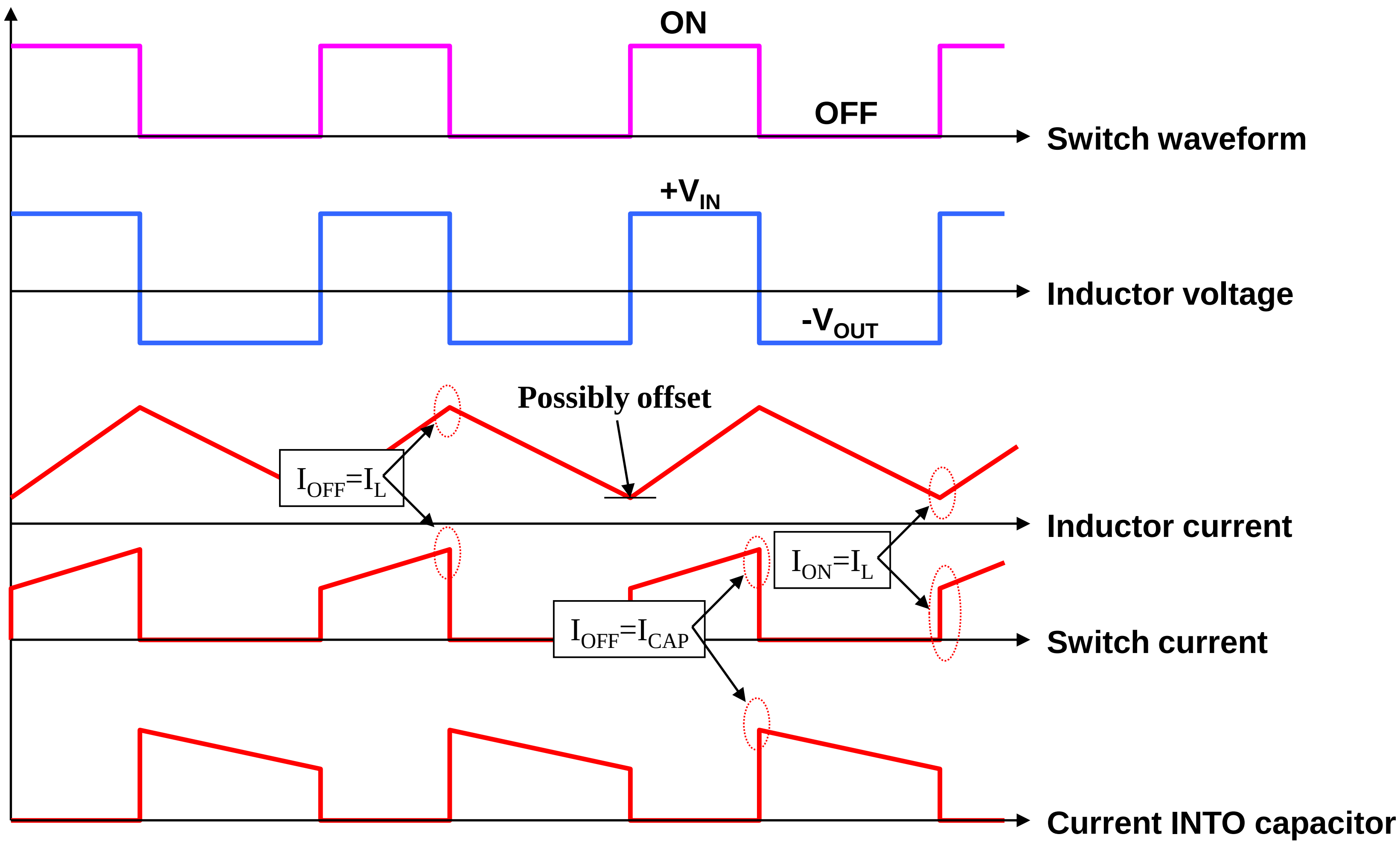
When the switch is opened the voltage on the inductor reverses polarity, the diode starts to conduct and hence the inductor current flows into the capacitor.

During this “off” period, $V_L = -V_{OUT}$ and this is maintained as the current in the inductor falls to its value at the end of the PWM period.

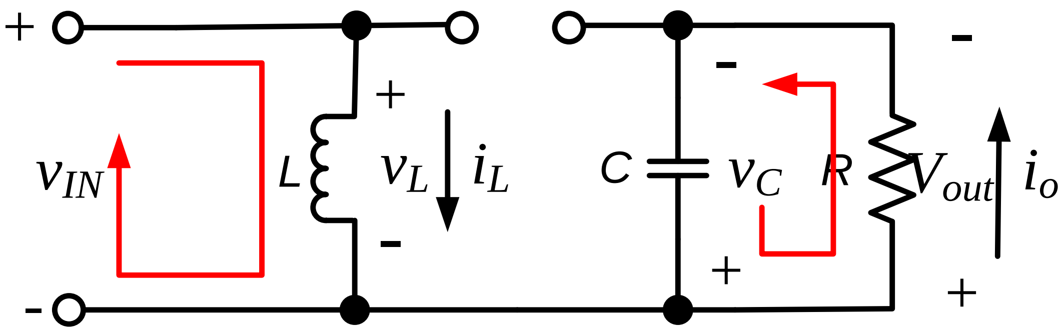
Note that with the switch opened there is no return path for current through the input to the converter and hence the only route the inductor current can follow is into the capacitor and load.

Also note that the capacitor will supply current to the load when there is no current contribution from the inductor.

Waveforms for the buck-boost converter.

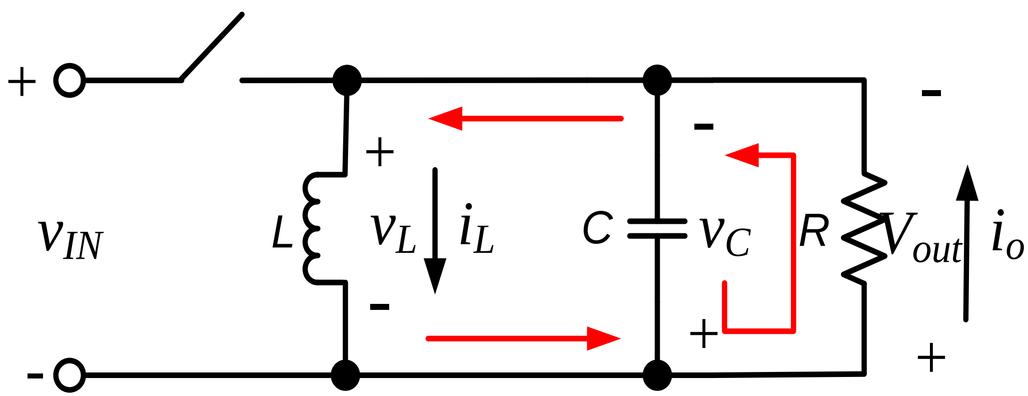


Two Circuit States



$$v_L = V_{IN} \qquad \text{charging}$$

Charging: Switch ON, Diode OFF



$$v_L = -V_{OUT} \qquad \text{discharging}$$

Discharging: Switch OFF, Diode ON

Note: Output voltage is negative polarity voltage

Voltage Conversion Ratio

In Steady State, for the **inductor *L***

$$\int_0^{T_s} v_L(t)\,dt = 0$$

$$0 \le \phi = \frac{t_{on}}{T_s} = \frac{t_{on}}{t_{on} + t_{off}} \le 1$$

$$\int_0^{T} v_L(t)\,dt = 0$$

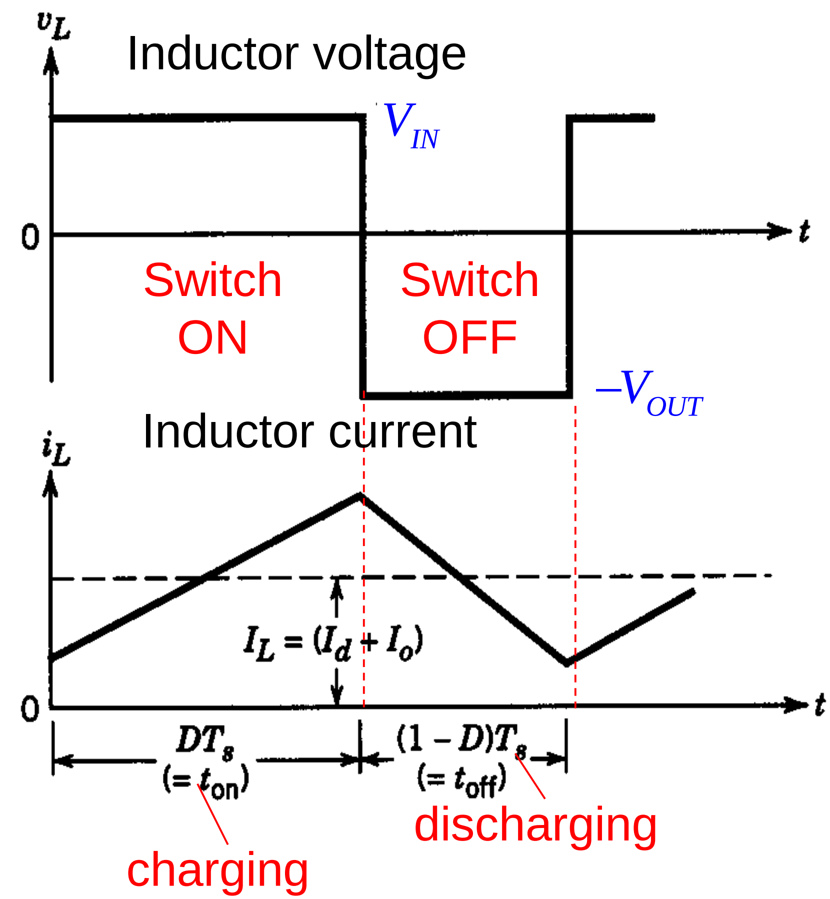
Switch ON

Switch OFF

$$\Rightarrow \int_0^{t_{on}} V_{IN}\,dt + \int_{t_{on}}^{T_s} \left(-V_{OUT}\right) dt = 0$$

$$\Rightarrow V_{IN}\phi T_s + \left(-V_{OUT}\right)\left(1-\phi\right) T_s = 0$$

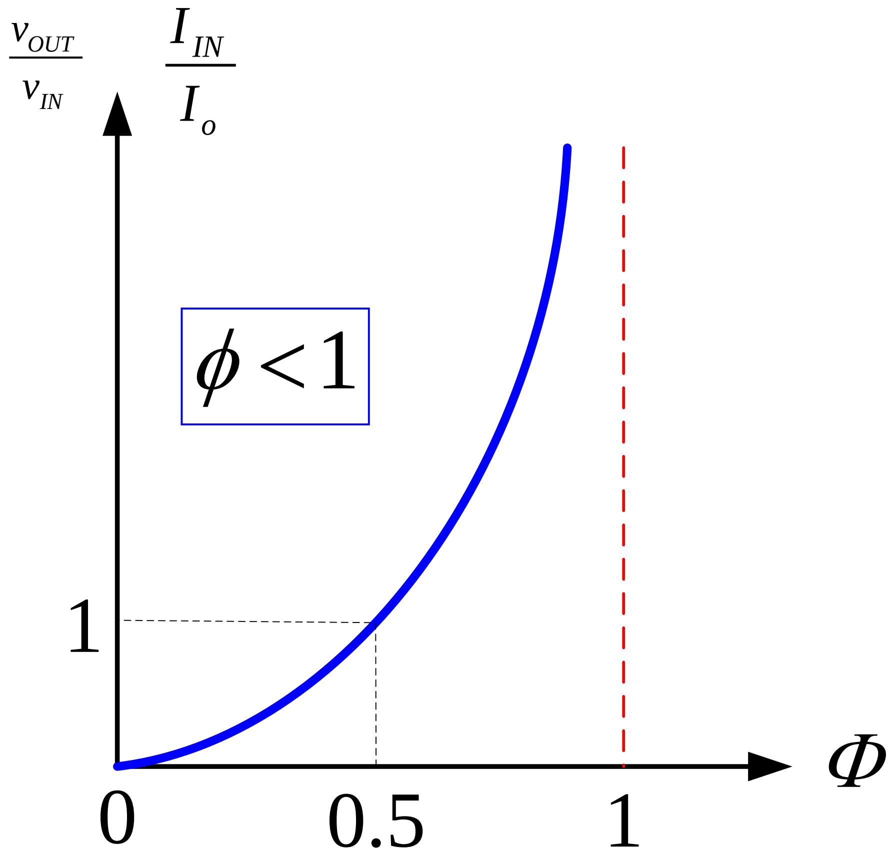
$$\Rightarrow V_{OUT} = \frac{\phi}{1-\phi} V_{IN}, \quad \phi < 1$$



Assuming power lossless

$$P_{OUT} = v_{OUT} I_{OUT} = v_{IN} I_{IN} = P_{IN}$$

$$\boxed{\frac{I_{OUT}}{I_{IN}} = \frac{v_{IN}}{v_{OUT}} = \frac{1-\phi}{\phi}}$$



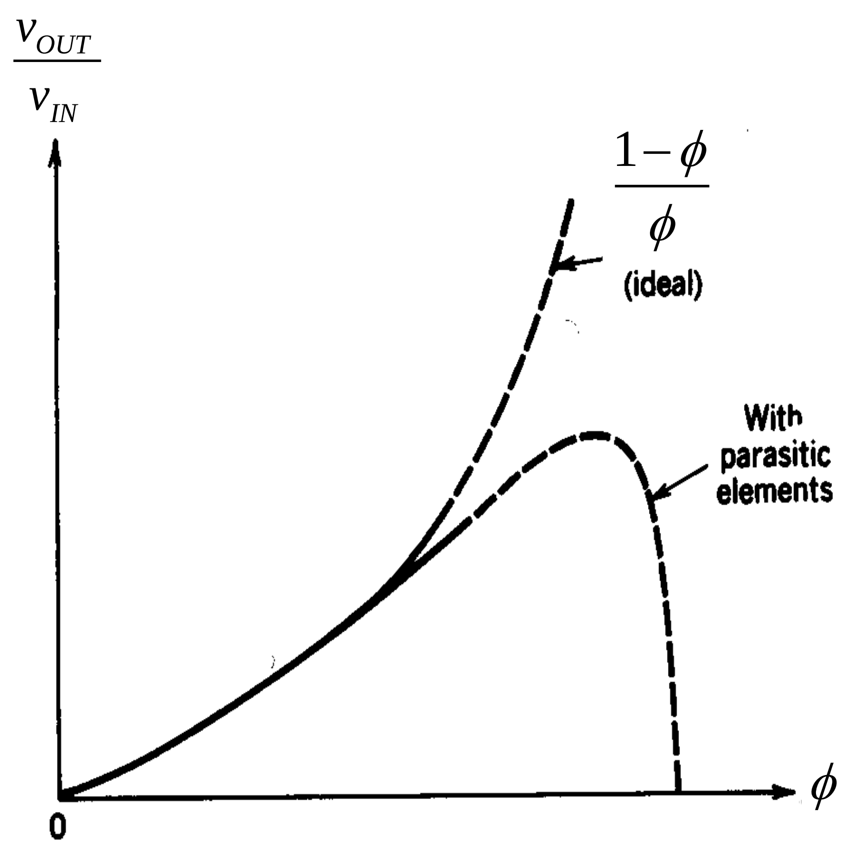
I_{IN}: average value of source current i_{in}

I_o: average value of load current i_o

I_L: average value of load current i_L

Buck-Boost: $\Phi<0.5$, step-down voltage; $0.5<\Phi<1$, step-up voltage. Nonlinear, hard to control

If we plot this relationship, we have:



Some important observations are:

1. The output voltage can be higher or lower than the input voltage.

2. Assuming a lossless system, the output voltage is independent of the load on the output. The power delivered is proportional to the current flowing in the inductor.

3. At φ=50%, $V_{OUT} = -V_{IN}$

$$\boxed{V_{OUT} = \frac{\phi . V_{IN}}{1-\phi}} \quad \rightarrow \infty \text{ as } \varphi \rightarrow 1$$

In reality, as φ approaches 1, the output voltage is limited due to loses in the system. If the switch were constantly closed the diode would be constantly reverse biased and no energy could be transferred to the output.

Note this relationship is true only if current is always flowing in the inductor, i.e. the converter is operating in continuous mode.

Design: Inductor Current Ripple in CCM

In Steady State, for the inductor L

$$i_L(t+T_s)=i_L(t)$$

$$\Rightarrow \Delta I_L = \left|\Delta i_{L+}\right| = \left|\Delta i_{L-}\right| = I_{L(\max)} - I_{L(\min)}$$

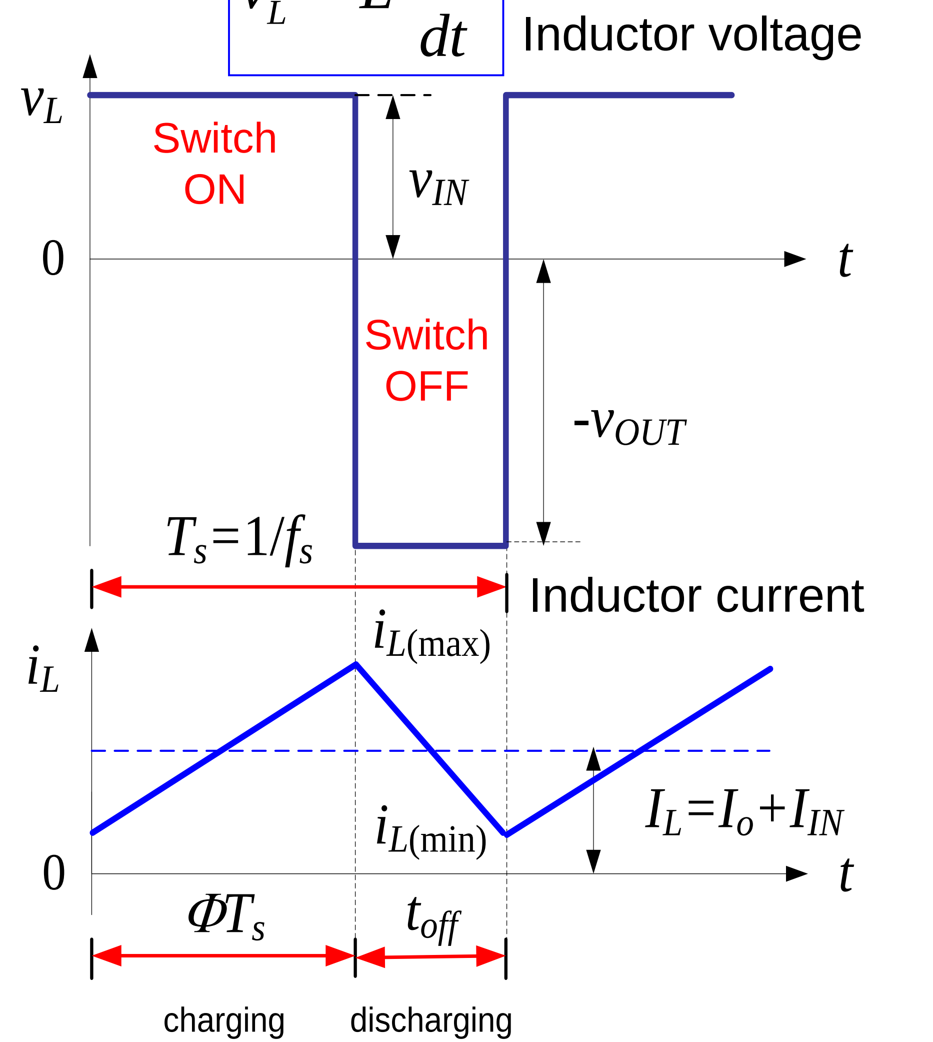
$$\text{charge}: I_{L(\max)} = \frac{1}{L}\int_o^{\phi T_s} v_{IN}\,dt + I_{L(\min)}$$

$$\Rightarrow \Delta I_L = \frac{v_{IN}}{L}\phi T_s = \frac{\phi v_{IN}}{L f_s}$$

$$v_{OUT} = \frac{\phi}{1-\phi} v_{IN} \Rightarrow$$

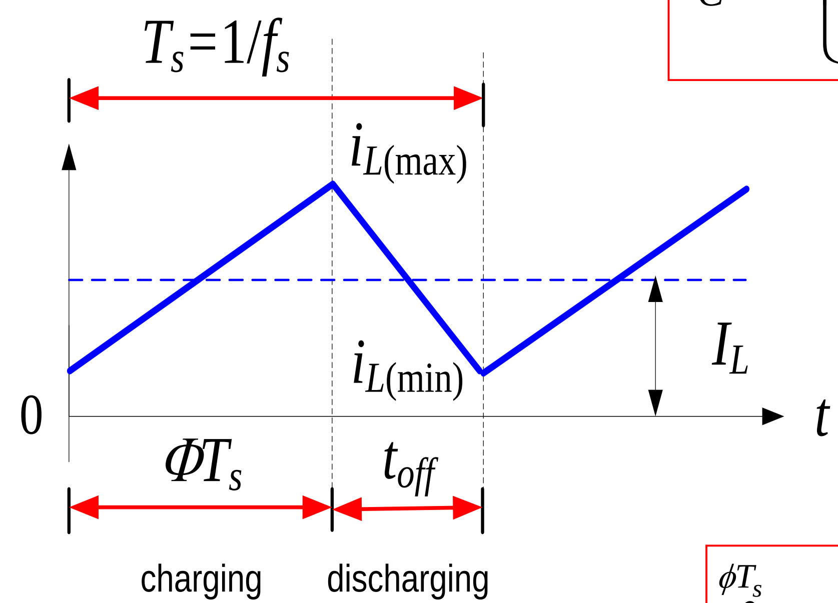
$$\Delta I_L = \frac{\phi v_{IN}}{L f_s} = \frac{v_{OUT}}{L f_s}(1-\phi)$$

$$v_L = L\frac{di_L}{dt}$$



Inductor Current Ripple in CCM

$$i_C = \begin{cases} I_o & L-\text{charging} \\ i_L - I_o & L-\text{discharging} \end{cases}$$



$$\int_0^{T_s} i_C\,dt = 0$$

$$\int_0^{\phi T_s} (-I_o)dt + \int_{\phi T_s}^{T_s} (i_L - I_o)dt = (1-\phi) I_L - I_o = 0$$

$$\Rightarrow I_L = \frac{1}{1-\phi} I_o$$

t_{off} *period :*

$$\frac{1}{T_s} \int_{\phi T_s}^{T_s} i_L\,dt = (1-\phi) I_L$$

Inductor Current Ripple in CCM

Absolute value

$$\Delta I_L = \frac{\phi v_{IN}}{Lf_s} = \frac{(1-\phi)v_{OUT}}{Lf_s}$$

Relative value

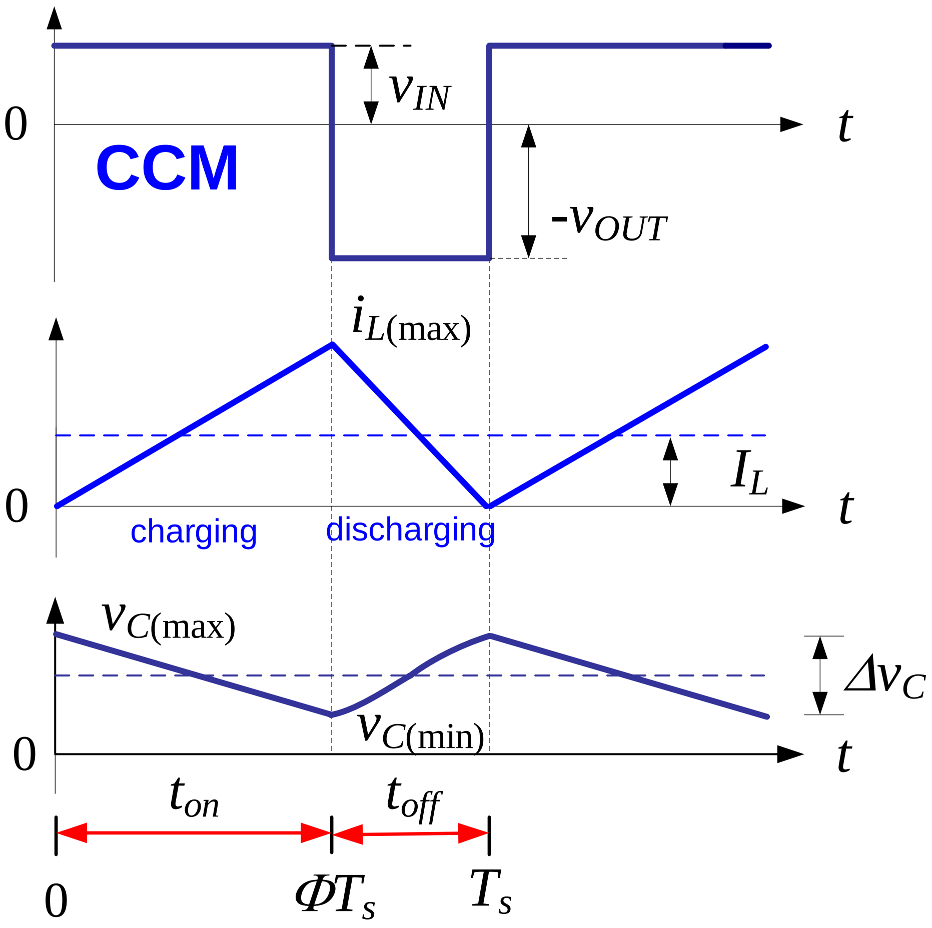
$$\text{Resistive Load R}: v_{OUT} = i_o R = I_o R$$

$$\Delta I_L = \frac{(1-\phi)v_{OUT}}{Lf_s} = \frac{(1-\phi)R}{Lf_s} I_o = \frac{(1-\phi)^2 R}{Lf_s} I_L$$

$$\Rightarrow \frac{\Delta I_L}{I_L} = \frac{(1-\phi)^2 R}{Lf_s}$$

Obviously, to increase switching frequency and inductor value, the inductor current ripple can be reduced; light load will increase inductor current ripple.

Boundary btw CCM and DCM



I_L: average inductor current

When inductor current i_L goes to zero at the end of switch off period, circuit reaches the boundary between **CCM** and **DCM**.

Average inductor current at this boundary I_{LB} :

$$\boxed{I_{LB} = \frac{1}{2}\Delta i_L = \frac{1}{2} i_{L(\max)}}$$

Boundary btw CCM and DCM

At the boundary

Inductor Current

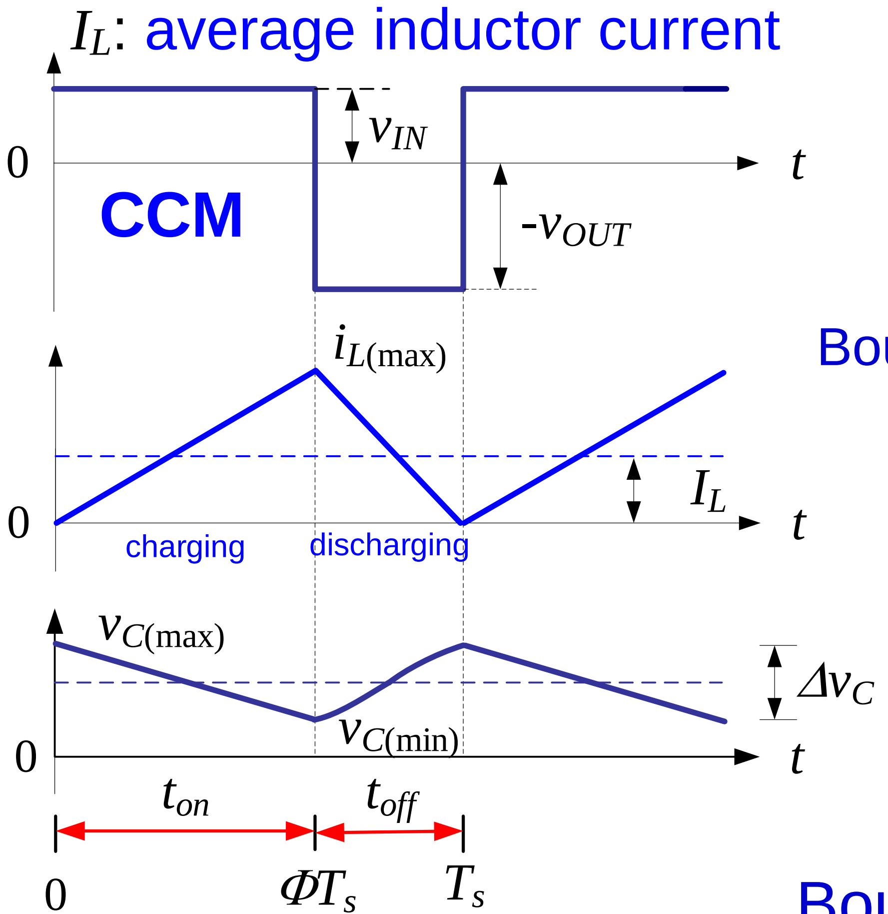
$$\left.\begin{aligned} \Delta I_L &= \frac{\phi v_{IN}}{Lf_s} = \frac{(1-\phi)v_{OUT}}{Lf_s} \\ I_{LB} &= \frac{1}{2} i_{L(\max)} = \frac{1}{2}\Delta I_L \end{aligned}\right\} \Rightarrow I_{LB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}(1-\phi)$$

Output Current

$$\left.\begin{aligned} I_o &= (1-\phi) I_L \\ I_{LB} &= \frac{1}{2}\frac{v_{OUT}}{f_s L}(1-\phi) \end{aligned}\right\} \Rightarrow I_{oB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}(1-\phi)^2$$

See page 180 in the Chapter 7 of the Textbook by Mohan

Boundary btw CCM and DCM



$$I_{LB} = \frac{1}{2}\Delta i_L = \frac{1}{2} i_{L(\max)}$$

Boundary Inductor/Output Current

$$I_{LB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}(1-\phi)$$

$$I_{oB} = \frac{1}{2}\frac{v_{OUT}}{f_s L}(1-\phi)^2$$

Boundary Inductance

$$L_{\min} = \frac{v_{OUT}}{2 f_s I_o}(1-\phi)^2 = \frac{R}{2 f_s}(1-\phi)^2 = \frac{v_{IN}}{2 f_s I_o}\phi(1-\phi)$$

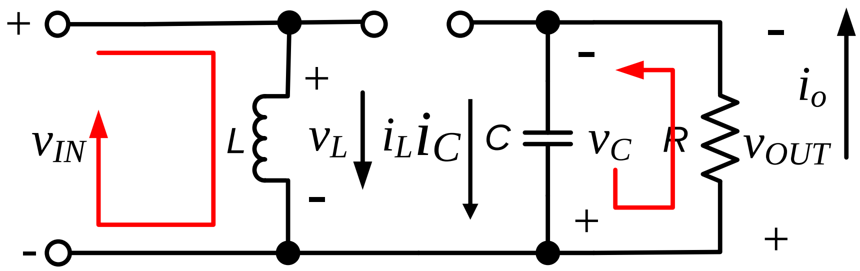
Buck-Boost Converter

Discontinuous Conduction Mode (DCM)

Read Chapter 7, Mohan etc.

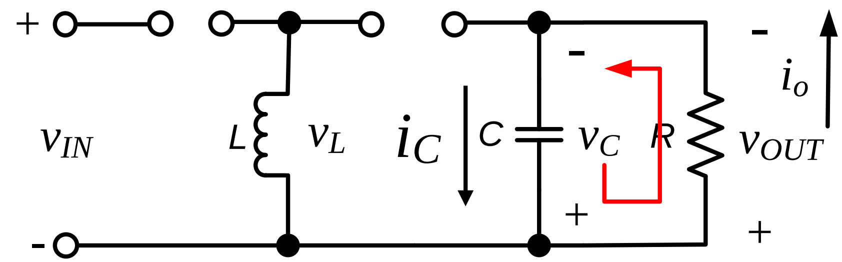
Circuit States in DCM

(1)



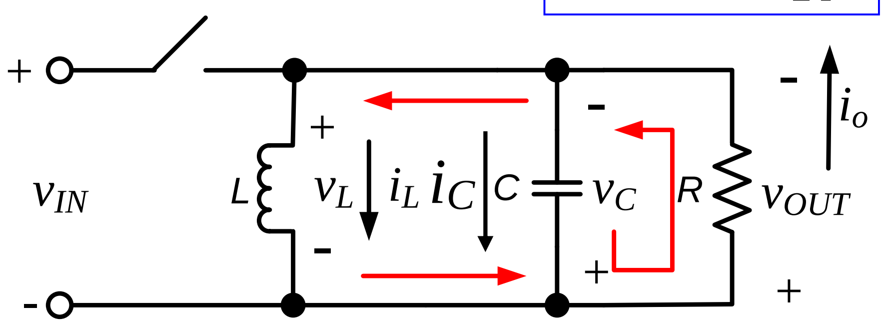
Charging: Switch ON, Diode OFF

(3)



Idle: Switch OFF, Diode OFF

(2)



Discharging: Switch OFF, Diode ON

$$i_o = I_o = \frac{v_o}{R},$$

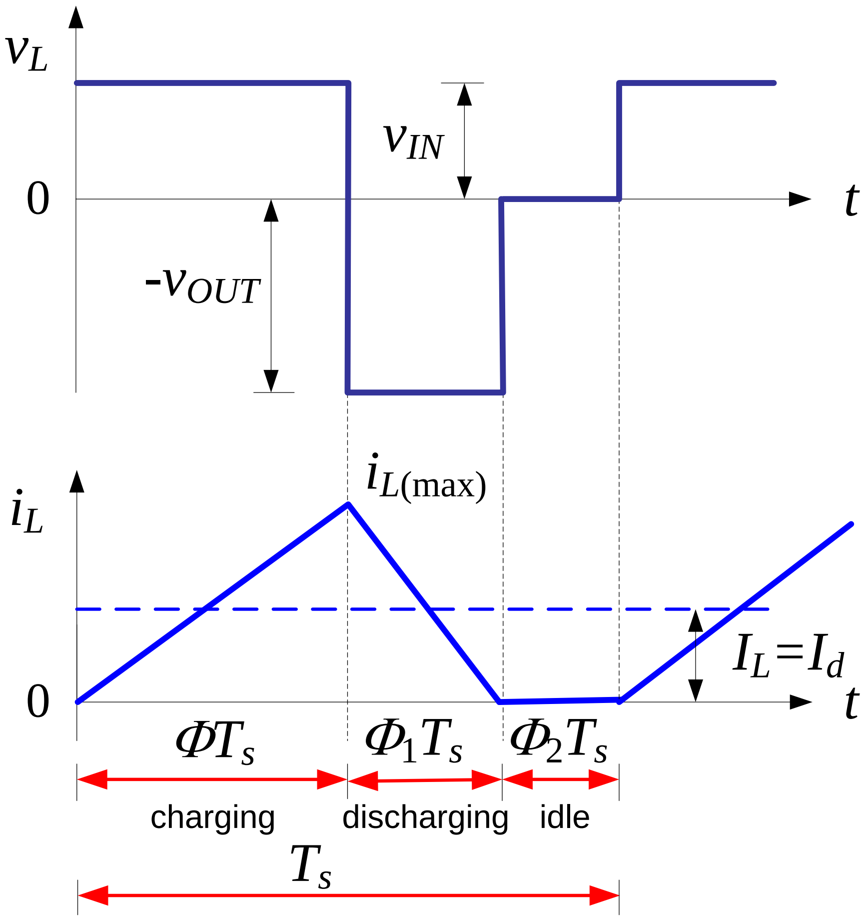
KVL

$$v_L = \begin{cases} v_{IN} & Circuit-(1) \\ -v_{OUT} & Circuit-(2) \\ 0 & Circuit-(3) \end{cases}$$

KCL

$$i_C = \begin{cases} -I_o & Circuit-(1) \\ i_L - I_o & Circuit-(2) \\ -I_o & Circuit-(3) \end{cases}$$

Voltage Conversion Ratio in DCM

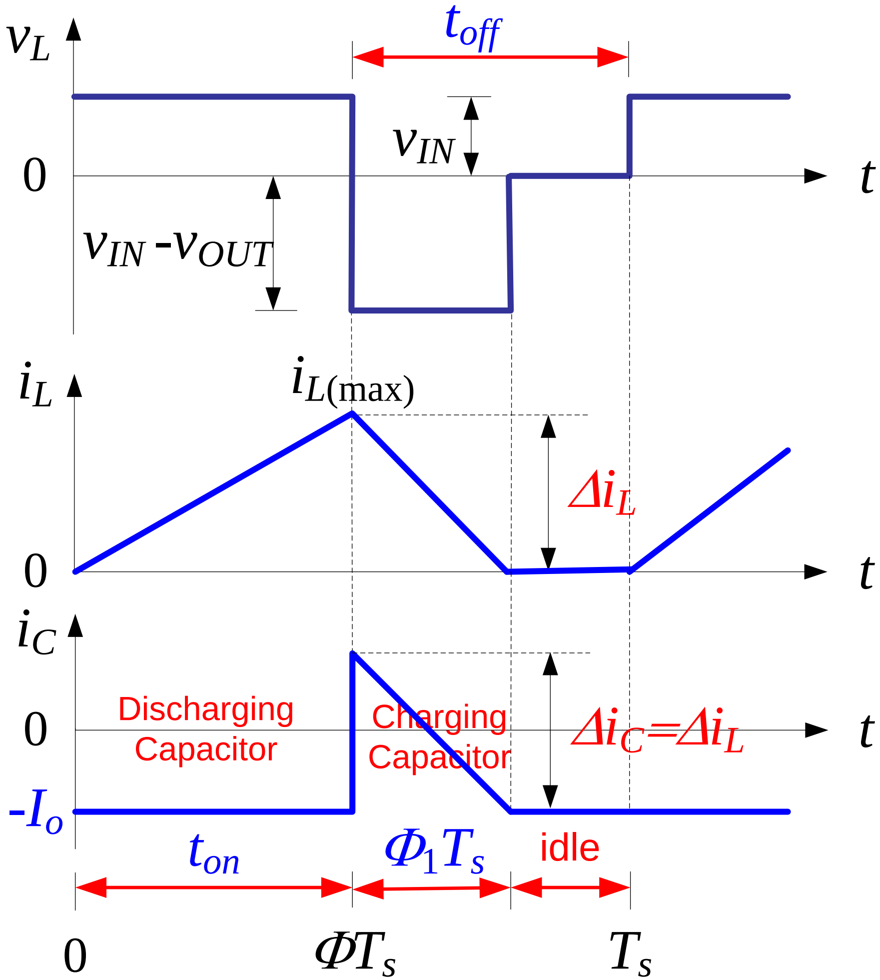


$$\int_0^T v_L(t)\,dt = 0 \qquad DCM$$

$$\Rightarrow \int_0^{\phi T_s} v_{IN}\,dt + \int_{\phi T_s}^{(\phi+\phi_1)T_s} \left(-v_{OUT}\right) dt = 0$$

$$\Rightarrow \frac{v_{OUT}}{v_{IN}} = \frac{\phi}{\phi_1}, \quad 0 < \phi + \phi_1 < 1$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi}{1-\phi}, \quad \phi + \phi_1 = 1 \quad CCM$$



$$\frac{1}{T_s}\int_0^{T_s} i_C\,dt = 0$$

$$i_C = \begin{cases} -I_o & 0 \le t < \phi T_s \\ i_L - I_o & \phi T_s \le t < (\phi + \phi_1) T_s \\ -I_o & (\phi + \phi_1) T_s \le t < T_s \end{cases}$$

$$\int_{\phi T_s}^{(\phi+\phi_1)T_s} (i_L - I_o)\,dt + \int_{(\phi+\phi_1)T_s}^{(1+\phi)T_s} (-I_o)\,dt = 0$$

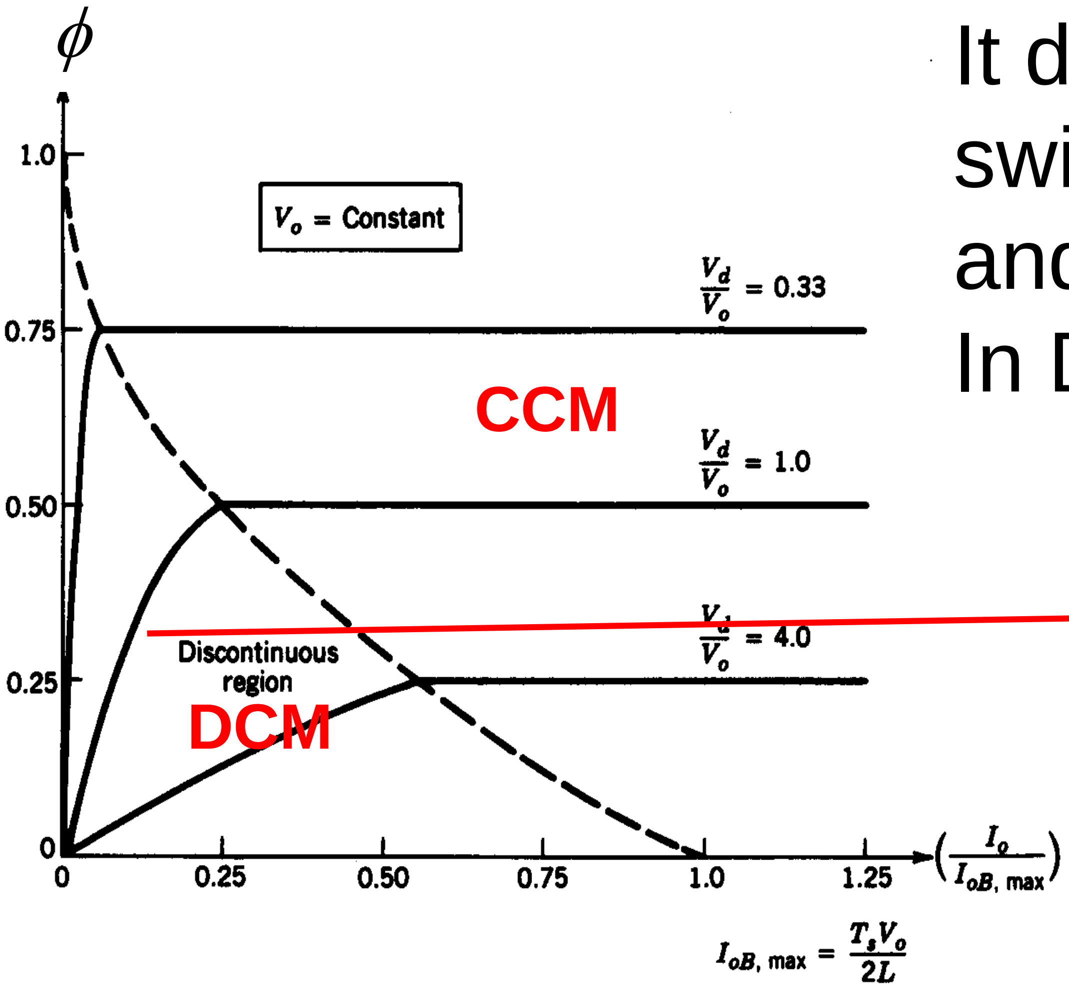
$$\Rightarrow -I_o T_s + \frac{\Delta i_L}{2}\phi_1 T_s = 0 \Rightarrow \phi_1 = \frac{2I_o}{\Delta i_L}$$

$$t_{on}\ \ period: \Delta i_L = i_{L(\max)} = \frac{1}{L}\int_0^{\phi T_s} v_L\,dt$$

$$\Rightarrow \Delta i_L = \frac{v_{IN}}{L}\phi T_s = \frac{v_{IN}}{f_s L}\phi$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi}{\phi_1} = \frac{\phi^2 v_{IN}}{2 f_s L I_o}$$

Voltage Conversion ratio in DCM



It depends on both switching duty ratio Φ and the load (I_o or R) In DCM

$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi}{\phi_1} = \frac{\phi^2 v_{IN}}{2 f_s L I_o}$$

CCM Converter summary

$$\boxed{\text{For all converters, assume } V_{IN} I_{IN(Ave)} = V_{OUT} I_{OUT(Ave)}}$$

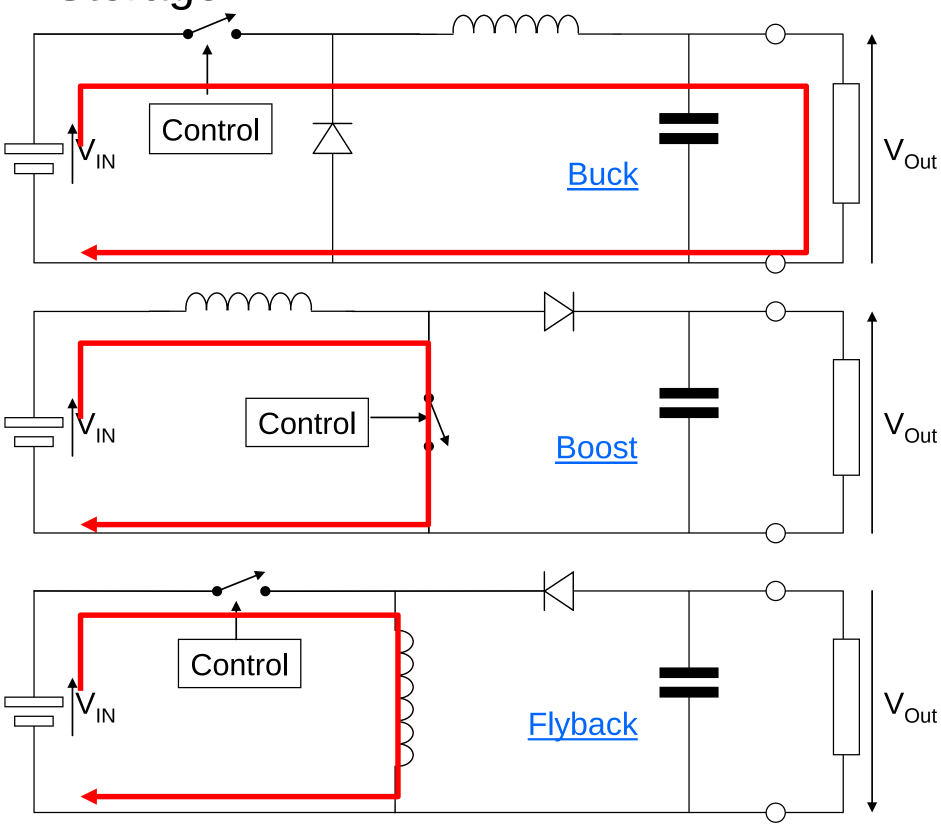
Type	Voltage Equation	Current Equation
Buck ($V_{OUT} < V_{IN}$)	$\boxed{V_{OUT} = \phi . V_{IN}}$	$\boxed{I_{OUT(Ave)} = \dfrac{I_{IN(Ave)}}{\phi}}$
Boost ($V_{OUT} > V_{IN}$)	$\boxed{V_{OUT} = \dfrac{V_{IN}}{1-\phi}}$	$\boxed{I_{OUT(Ave)} = (1-\phi).I_{IN(Ave)}}$
Buck-boost ($V_{OUT} <> V_{IN}$)	$\boxed{V_{OUT} = \dfrac{\phi . V_{IN}}{1-\phi}}$	$\boxed{I_{OUT(Ave)} = \dfrac{(1-\phi).I_{IN(Ave)}}{\phi}}$

The following diagrams summarise the energy flow in the circuit for each converter type during the storage and release phases of operation.

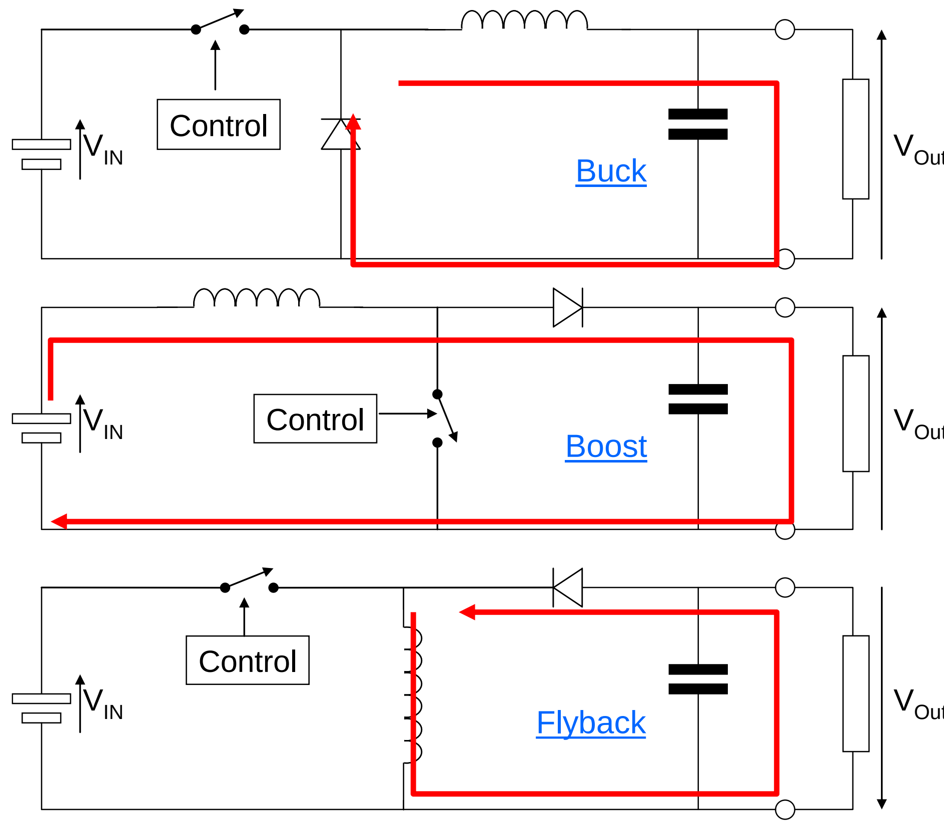
Charging the Inductor

Discharging the Inductor

Storage



Release



Switch ON

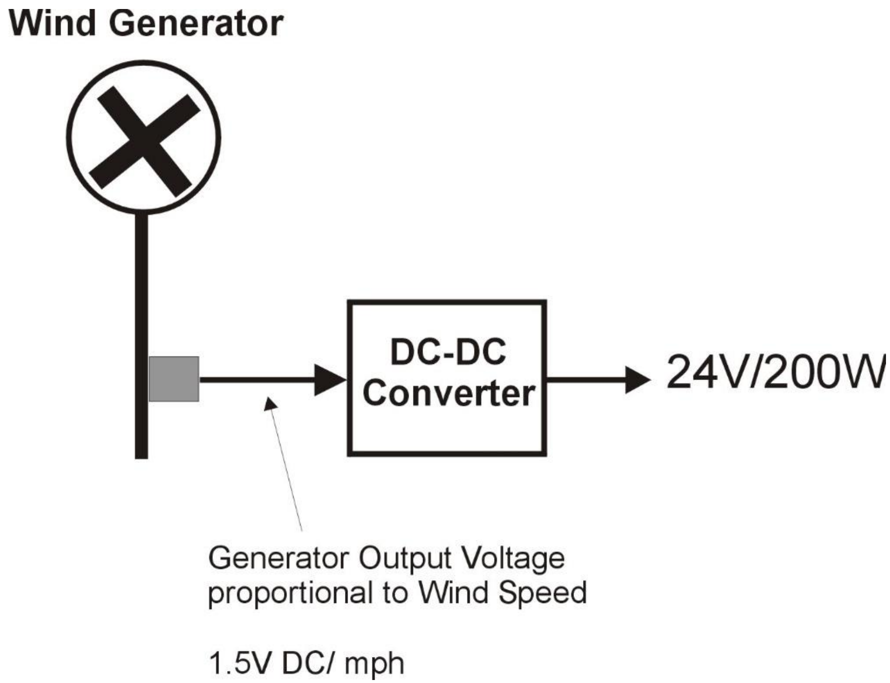
Switch OFF

Question.

A small wind turbine generates 1.5V DC per MPH of wind speed.

For operation over the range 8 MPH to 32 MPH, determine the following:

1. The type of converter to be used;
2. The required range of duty cycle for these wind speeds;
3. The average input current to the converter at 32MPH;
4. The peak input current @ 32 MPH assuming operation at the boundary condition.



Solution.

For a wind speed of 8MPH, the turbine output voltage is 8 x 1.5 = 12v. At a wind speed of 32 MPH the output voltage is 48V. Hence use a buck-boost converter since V_{IN} can be less than or greater than the output voltage.

For the BB, $$V_{OUT} = \frac{\phi . V_{IN}}{1-\phi} => \phi = \frac{V_{OUT}}{V_{IN} + V_{OUT}}$$

For 8MPH, φ = 24 / (12 + 24) = 66.6%

For 32 MPH, φ = 24 / (48 + 24) = 33.3%

Assuming 100% efficiency, for P_{OUT} = 200W, $_{IIN\ (Ave)}$ = 200 / 48 = 4.16A (ave) and $I_{IN\ (Peak)}$ = 2 x $I_{IN(ave)}$ = 8.33A (since @ boundary condition => initial current is zero).

Don’t forget to divide the peak input current by the duty cycle according to which value of φ you’re working with!