Power Electronics 2 (ENG2045, SIT2004)
Tutorial sheet

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2016 March 14

1 Fundamentals

1.1 The timebase of an oscilloscope is set to 10 μs per division and there are 10 divisions across the screen. Exactly four complete cycles of a signal can be seen. What are its period and (common) frequency?

1.2 Write down the *general* relations between current and charge. The current flowing into a component is found to be $i(t) = I_0 \cos(\omega t)$ with $I_0 = 100\,\mathrm{mA}$ and $f = 50\,\mathrm{Hz}$. Derive an expression for the charge on the component if it is uncharged at $t = 0$.

1.3 The current $i(t)$ shown in figure 1 is applied to a 10 mH inductor. Calculate the voltage $v(t)$ across the inductor as a function of time.

1.4 The current $i(t)$ shown in figure 2 is applied to a 10 μF capacitor, which is initially uncharged. Calculate the voltage $v(t)$ across the capacitor as a function of time.

*With grateful acknowledgment to Ravinder Dahiya, Vassilios Agelidis, Calum Cossar and Andrew Knox.

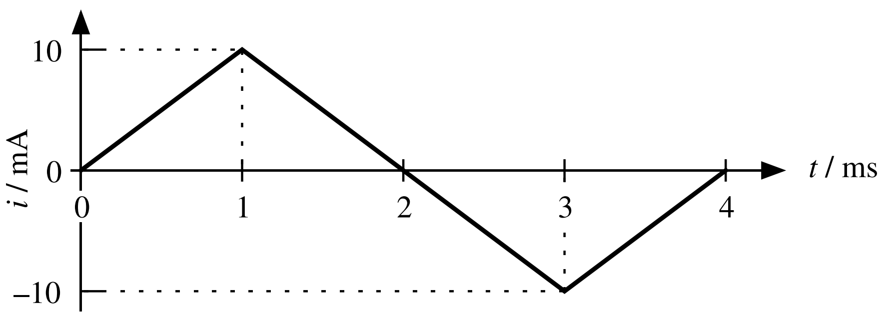

Figure 1. Current applied to a 10 mH inductor.

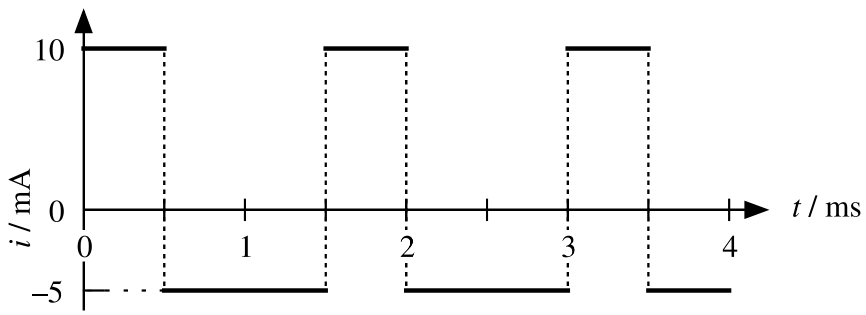

Figure 2. Current applied to a 10 μF capacitor, which is initially uncharged.

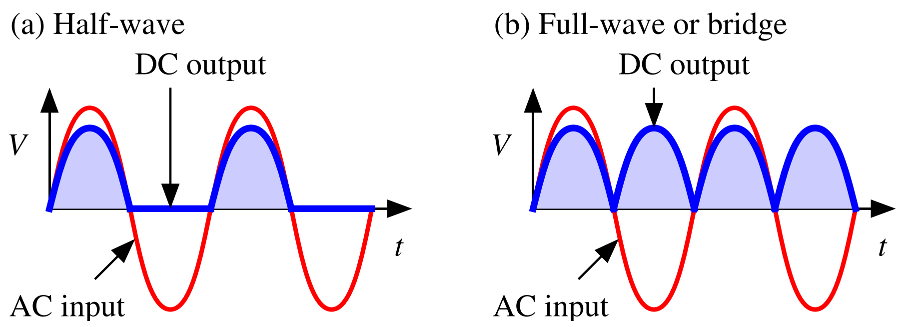

Figure 3. Half and full-wave or bridge rectification. Ignore the drop in voltage.

2 RMS values and and power

2.1 Consider the standard european AC mains, $v(t) = V_0 \sin(\omega t)$ with frequency 50 Hz and RMS voltage 230 V. First write down the peak value V_0. Then calculate the (i) average value, (ii) RMS value and (iii) form factor for

(a) the original sine wave

(b) the half-rectified wave

(c) the full-rectified wave.

The waves are shown in figure 3. Ignore losses in the diodes.

2.2 Find the average and RMS voltages and the form factor for the sawtooth function shown in figure 4, whose period is 10 ms and peak value is 200 V.

2.3 Show that the RMS value of a sinusoidal waveform with peak-to-peak amplitude $2A$ and DC offset B is given by

$$V_{\text{RMS}} = \sqrt{\frac{A^2}{2} + B^2}. \tag{1}$$

Confirm that this is correct in the two limits $A = 0$ and $B = 0$.

2.4 A square wave spends a fraction D of the time at voltage V_{on} and a fraction $(1 - D)$ at voltage V_{off}. Calculate its average and RMS voltages. You should not need any integrals.

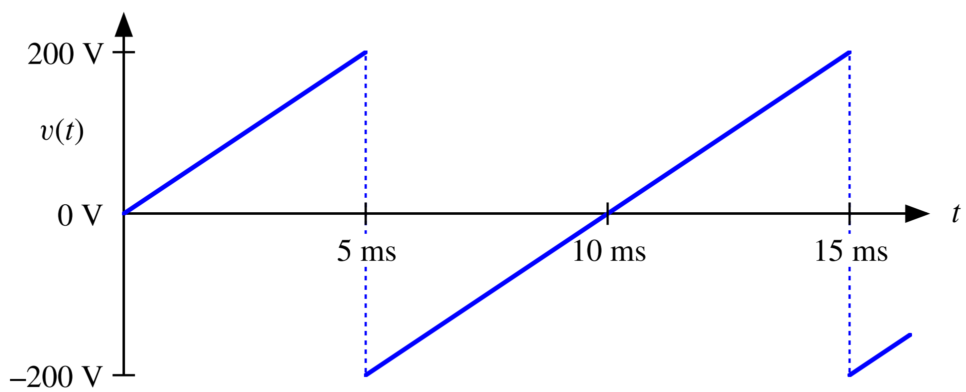

Figure 4. Sawtooth wave.

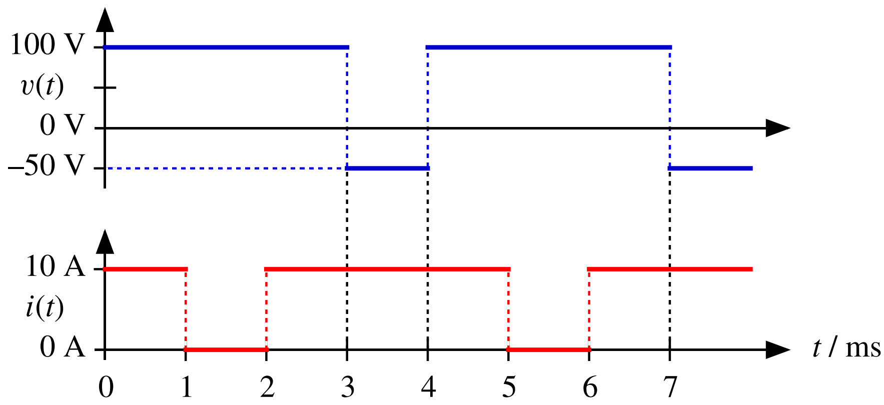

Figure 5. Voltage and current supplied to a component.

The duty cycle D lies between 0 and 1. Check your answer for the three cases $V_{\text{off}} = V_{\text{on}}$, $V_{\text{off}} = -V_{\text{on}}$ and $V_{\text{off}} = 0$.

2.5 You need to buy power for a heater and a strange supplier offers you a choice of the following voltage waveforms at the same price. Which should you choose?

(a) A triangular wave, symmetric about zero, with a peak-to-peak voltage of 30 V.

(b) A sine wave, symmetric about zero, with a peak-to-peak voltage of 25 V.

(c) A square wave, symmetric about zero, with a peak-to-peak voltage of 15 V.

(d) A constant (DC) voltage of 10 V.

2.6 The voltage $v(t)$ and current $i(t)$ shown in figure 5 are supplied to a component. Sketch the power $p(t)$ supplied to the component and calculate the average value over its 4 ms period.

3 Rectifiers

The *regulation* of a power supply is often defined by

$$\text{regulation} = \frac{V_{\text{no load}} - V_{\text{full load}}}{V_{\text{no load}}}. \tag{2}$$

The regulation is often expressed as a percentage and a good power supply has a *small* value of regulation (an ideal power supply has zero regulation). Sometimes the denominator has $V_{\text{full load}}$ instead of $V_{\text{no load}}$ but it makes no significant difference for a good supply.

Alternatively, a power supply can be modelling with a Thévenin equivalent circuit comprising an ideal voltage supply in series with a Thévenin resistance R_s. In this case R_s measures the regulation because it absorbs 'lost volts' when a current is drawn from the supply. A good supply has a low value of R_s, ideally zero.

3.1 Draw up a table to compare half-wave, full-wave and bridge rectifiers with the headings: windings needed on transformer; utilization of transformer; number of diodes needed; reverse voltage rating of diodes; voltage drop across diodes (assuming typical silicon components); frequency of ripple; ease of smoothing. Assume that no smoothing capacitor is used.

3.2 A bridge rectifier is required to supply an average current of 200 mA with ripple not to exceed 0.5 V. What size of smoothing capacitor is needed? The AC supply is at 50 Hz. [around 2400 μF]

How would your answer change if the supply were designed for North America rather than Europe?

3.3 Design a power supply with a transformer, bridge rectifier and smoothing capacitor to work from the AC mains in Europe and supply 9 V at 200 mA. Ripple on the output should not exceed 0.5 V. Assume that the secondary winding of the transformer has a resistance of 10 Ω. Calculate the value of smoothing capacitor and the (rms) output voltage of the transformer needed. What voltage rating should be specified for the capacitor, considering both no load and full load? Estimate the no-load voltage and regulation.

[Roughly 2400 μF, 12 V; 17 V; 40% or 30 Ω]

4 Driving low-power loads

4.1 Design an 'active high' circuit for connecting a light-emitting diode (LED) to an output pin of a microcontroller. The LED is designed to operate at a current of 5 mA and drop 1.8 V. The circuit is supplied from 3.3 V.

4.2 An n-channel MOSFET has threshold voltage $V_T = 1.0\,\text{V}$ and constant $K = 0.5\,\text{A}\,\text{V}^{-2}$.

(a) Is this an enhancement-mode or depletion-mode device?

(b) Calculate the resistance of the channel R_{DS} for gate voltages of 3.3 V and 5.0 V.

(c) How much voltage is dropped across the MOSFET if it passes a drain current of $i_D = 300\,\text{mA}$ for the same two gate voltages?

(d) How much power P_{diss} is dissipated in the MOSFET for these two cases?

(e) What gate voltage produces a channel resistance $R_{DS} = 1.0\,\Omega$?

4.3 A digital system works from a supply at $+5.0\,\text{V}$ and its outputs can produce up to $\pm 5\,\text{mA}$. It is required to control a load that draws 500 mA from a 12 V supply. Consider the following options for a switch for this load. Explain whether each is suitable and design the complete circuit, including the values of any other components required. All devices are rated to carry at least 500 mA and withstand at least 12 V.

(a) Single BFY51 npn transistor with $\beta = 40$ and maximum power dissipation of 800 mW (without heatsink).

(b) Darlington pair with $\beta = 500$ and maximum power dissipation of 1.5 W.

(c) Single FDV301N n-channel MOSFET with threshold voltage $V_T = 0.8\,\text{V}$, $K = 0.07\,\text{A}\,\text{V}^{-2}$ and maximum power dissipation of 0.35 W.

4.4 A load requires 5 V and its negative terminal is connected to its metal case, which must be grounded to dissipate heat. Draw a suitable circuit for switching it on and off from a digital system that also operates from 5 V.

Would your approach work if the digital system worked at 3 V rather than 5 V?

5 Rectifiers, smoothing capacitors and linear regulators

5.1 Draw up a table to compare half-wave, full-wave and bridge rectifiers with the headings: windings needed on transformer; utilization of transformer; number of diodes needed; reverse voltage rating of diodes; voltage drop across diodes (assuming typical silicon components); frequency of ripple; ease of smoothing.

5.2 The secondary winding of a transformer is rated at $V_{AC} = 12.0$ V and runs at 60 Hz. It can be connected to a 100 Ω load through different rectifiers, without or with a smoothing capacitor. Ignore the voltage drop in the diodes and assume that the capacitor is so large that it reduces ripple to a negligible value. First, what sort of voltage is the '12 V' quoted for the transformer?

(a) The load is connected directly to the transformer. Calculate the average voltage V_{ave} and RMS voltage V_{RMS} across the load and the power P dissipated in it.

(b) The load is connected to the transformer through a half-wave rectifier (single diode). Repeat the calculations.

(c) A smoothing capacitor is now connected across the load. Repeat the calculations.

(d) The load is connected to the transformer through a bridge rectifier (four diodes) and the smoothing capacitor is removed. Repeat the calculations.

(e) A smoothing capacitor is now connected across the load. Repeat the calculations.

Comment on the power dissipated by the resistor for the five cases.

5.3 A bridge rectifier is required to supply an average current of 200 mA with ripple not to exceed 0.5 V. What size of smoothing capacitor is needed? The AC supply is at 50 Hz. [around 2400 μF]

How would your answer change if the supply were designed for North America rather than Europe?

5.4 Design a power supply with a transformer, bridge rectifier and smoothing capacitor to work from the AC mains in Europe and supply 9 V at 200 mA. Ripple on the output should not exceed 0.5 V. Assume that the secondary winding of the transformer has a resistance of 10 Ω. Calculate the value of smoothing capacitor and the (rms) output voltage of the transformer needed. What voltage rating should be specified for the capacitor, considering both no load and full load? Estimate the no-load voltage and regulation. [Roughly 2400 μF, 12 V; 17 V; 40% or 30 Ω]

5.5 Suppose that you want an adjustable power supply with a linear regulator for an undergraduate electronics laboratory whose output is variable from 3 V for logic circuits to 20 V for analogue circuits, with a maximum current of 1 A. The input voltage is chosen to be 25 V to give plenty of headroom at all output voltages. Under what conditions is the maximum power dissipated in the regulator and what is its value? [22 W]

5.6 The bench PSUs in Rankine 709 have adjustable, bipolar, regulated outputs from 0 to ±15 V at 200 mA. Estimate the maximum power dissipated. [7.2 W]

Why are these linear rather than switched regulators?

5.7 [Follows from lecture slides.] Design a power supply to deliver a current of 3.0 A at 15.0 V using a linear regulator. The regulator has a dropout voltage of 2.0 V and the mains input voltage is 110 V $\pm$ 8% at 60 Hz (North America, for example). The transformer has a regulation of 7% and is 90% efficient at full load. A bridge rectifier is used. Assume the diode conduction angle is 30° and that the diodes drop 1.0 V at full load. The smoothing capacitor is 10 mF.

(a) What is the minimum AC voltage for the secondary winding of the transformer?

(b) What is the power dissipation in the regulator under worst-case conditions?

(c) What voltage rating should the capacitor have?

(d) What PIV rating should the diodes have?

(e) What VA rating does the transformer require?

[Roughly 17 V, 24 W, 28 V, 28 V, somewhere between 150 W and 250 W depending on how you calculate it.]

5.8 A low-dropout regulator (LDO) has the following parameters: 2.5 V output voltage, 0.1 V dropout voltage, 6.0 V maximum input voltage, 150 mA maximum output current, 125°C maximum junction temperature, 250°C/W thermal resistance from junction to ambient.

(a) Define the terms *headroom voltage* and *dropout voltage* for a linear regulator and explain what is meant by a *low-dropout* regulator.

(b) Make an annotated sketch of the output voltage as a function of the input voltage from 0 to 6.0 V.

(c) What is the minimum input voltage required for a regulated output?

(d) How does the efficiency of the regulator depend of its input voltage V_{in}? Support your answer with calculations of the efficiency across the range of specified operation.

(e) What conditions across the full range of specified operation cause the maximum power dissipation in the regulator? What is the value of this power?

(f) Can the regulator safely dissipate this value of power under typical laboratory conditions?

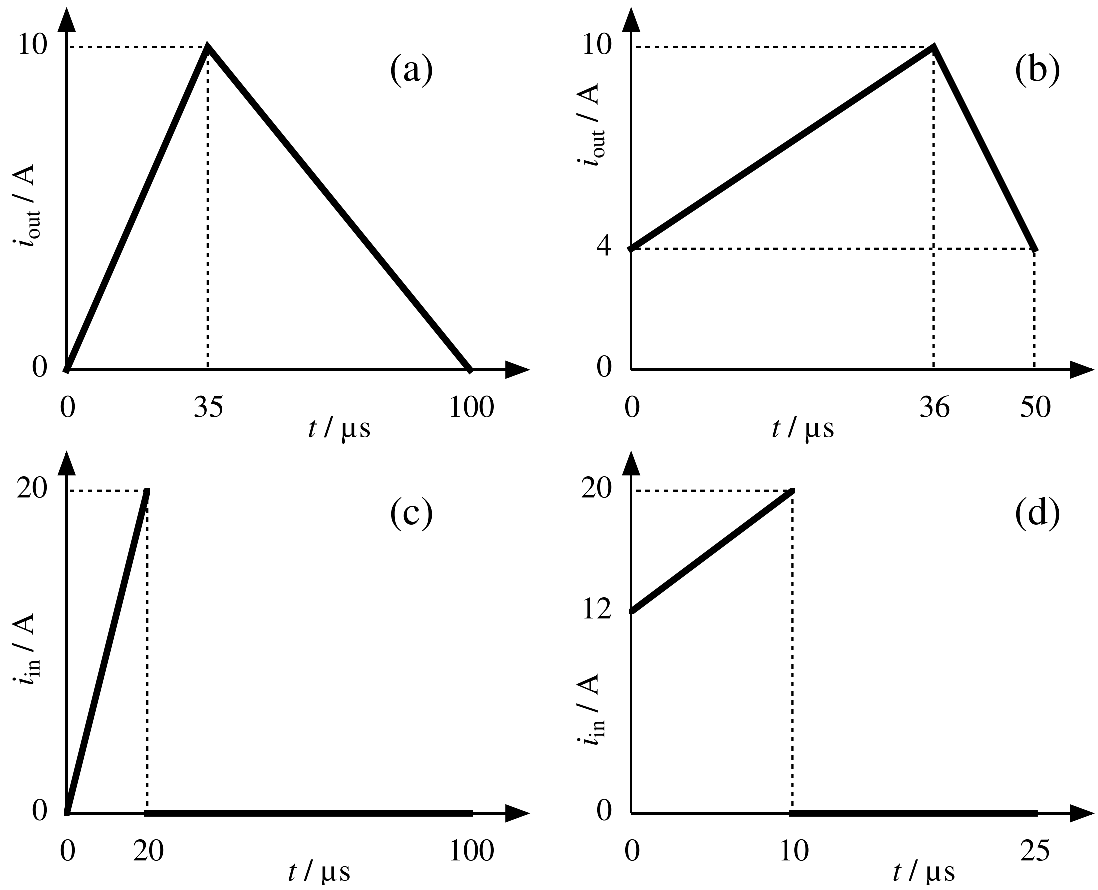

Figure 6. Input or output currents measured in four buck DC–DC converters.

6 Switch-mode DC–DC converters

6.1 *Buck regulators.* Figure 6 shows a single cycle of either the input current $i_{\text{in}}(t)$ or the output current $i_{\text{out}}(t)$ measured for four buck regulators, each of which has an input at $V_{\text{in}} = 100\,\text{V}$. For each of these calculate the duty factor D, output voltage V_{out}, average output current I_{out}, average input current I_{in}, average power P supplied and the value of the inductance L in the converter. Assume ideal components and operation. To help you get started, here is a walk through the first example.

(a) The current is rising for the first 35 µs so this must be the charging phase. The period is 100 µs so $D = T_{\text{charge}}/T_{\text{cycle}} = 0.35$.

(b) The output voltage is given by $V_{\text{out}} = DV_{\text{in}} = 35\,\text{V}$.

(c) From the plot of $i_{\text{out}}(t)$, the average value is $I_{\text{out}} = 5\,\text{A}$ (easy because it is triangular).

(d) The average input current is given by $I_{\text{in}} = DI_{\text{out}} = 1.75\,\text{A}$.v (The actual input current $i_{\text{in}}(t)$ is the same as $i_{\text{out}}(t)$ during charging, when the currents rise, and $i_{\text{in}}(t) = 0$ during discharging, when $i_{\text{out}}(t)$ falls.)

(e) The average power can be found from $P_{\text{in}} = V_{\text{in}}I_{\text{in}}$ or $P_{\text{out}} = V_{\text{out}}I_{\text{out}}$ because the voltages are assumed constant so the average power is proportional to the average

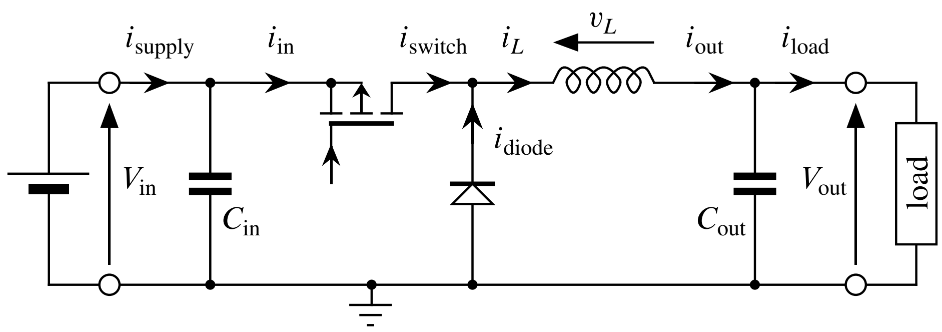

Figure 7. Circuit of a standard buck DC–DC converter.

current. Both give 175 W.

(f) The output current $i_{\text{out}}(t)$ is the same as the inductor current $i_L(t)$ for a buck converter. Use $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$ to find the inductance. In the charging phase, $v_L = V_{\text{in}} - V_{\text{out}} = 65\,\text{V}$ and $\mathrm{d}i_L/\mathrm{d}t = (10\,\text{A} - 0\,\text{A})/(35\,\mu\text{s}) = 0.29\,\text{A}\,\mu\text{s}^{-1}$ so $L = (65\,\text{V})/(0.29\,\text{A}\,\mu\text{s}^{-1}) = 227.5\,\mu\text{H}$. You should check that the discharging phase gives the same result.

6.2 *Basic buck regulator.* A point-of-load (PoL) power supply is required to deliver 3.0 V at 100 mA from a 5.0 V bus. It is implemented as a buck converter with a 100 µH inductance running at 100 kHz in continuous conduction mode. Figure 7 shows the circuit. You may assume that all components are ideal and that the input and output are smoothed so effectively that their voltages may be treated as constant. Analyse the behaviour of the circuit as follows

(a) Show from the standard relations that the converter should use a duty cycle $D = 0.6$ and that the average input current is $I_{\text{in}} = 60\,\text{mA}$.

(b) Using the notation in figure 7, show that $v_L = +2\,\text{V}$ during charging and $v_L = -3\,\text{V}$ during discharging.

(c) Use $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$ to show that $\mathrm{d}i_L/\mathrm{d}t = +2 \times 10^4\,\text{A}\,\text{s}^{-1} = +20\,\text{mA}\,\mu\text{s}^{-1}$ during charging and $\mathrm{d}i_L/\mathrm{d}t = -3 \times 10^4\,\text{A}\,\text{s}^{-1} = -30\,\text{mA}\,\mu\text{s}^{-1}$ during discharging.

(d) Hence show that i_L rises by 120 mA during charging and falls by the same during discharging.

(e) We know that the average current through both the load and inductor is 100 mA. Hence show that the minimum value of i_L is 40 mA and its maximum is 160 mA.

(f) Now sketch $i_L(t)$, $i_{\text{in}}(t)$, $i_{\text{switch}}(t)$ and $i_{\text{out}}(t)$. No further calculation is required. Show also $v_L(t)$.

(g) Suppose that a linear regulator was chosen for the supply instead. What would be its maximum efficiency?

This specification is a realistic starting point for many applications in consumer devices such as mobile phones and set-top boxes.

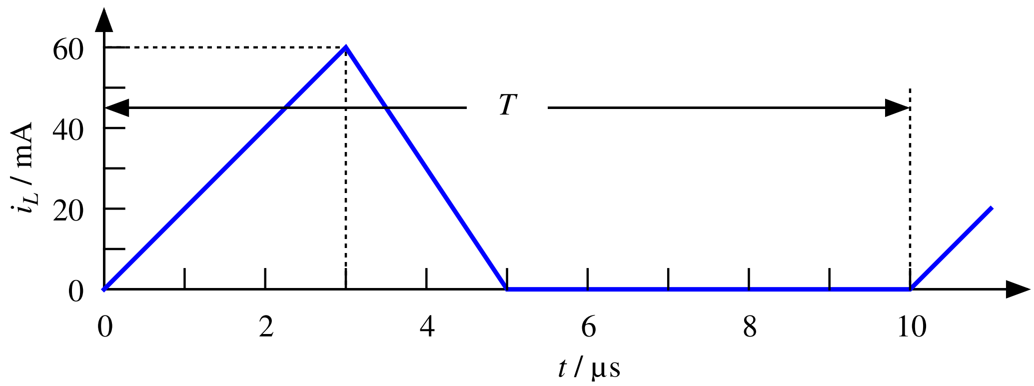

Figure 8. Inductor current for a buck DC–DC converter in discontinuous mode. It has $V_{\text{in}} = 5.0\,\text{V}$, $V_{\text{out}} = 3.0\,\text{V}$, a 100 µH inductance and runs at 100 kHz.

6.3 *Changing the load on a buck regulator.* [Harder.] Suppose that the load on the regulator in the previous question is changed. The voltages remain the same but I_{load} varies from 100 mA.

(a) How does a change in I_{load} affect the plot of $i_L(t)$? Remember that the voltages remain the same and that $\mathrm{d}i_L/\mathrm{d}t = v_L/L$, which means that the slopes of $i_L(t)$ remain the same too.

(b) Show that the minimum average current that this regulator can supply while remaining in continuous conduction mode is 60 mA.

(c) Suppose that the regulator must be redesigned so that it can supply 50 mA in continuous conduction mode. Does this require a larger or smaller value of inductance?

(d) Assume that the regulator runs at the boundary of continuous conduction mode, which means that the current just drops to zero between cycles. Show that this requires $\Delta i_L = 100\,\text{mA}$ in each cycle.

(e) Use Δi_L to work out $\mathrm{d}i_L/\mathrm{d}t$ and hence show that $L = 120\,\mu\text{H}$.

A 'rule of thumb' is to increase this value by 25% to allow for tolerances and fluctuations and ensure that the converter remains in continuous conduction mode. A designer would therefore specify around a 150 µH inductor.

6.4 *Buck converter in discontinuous mode.* [Not examinable.] Suppose that the same buck converter has its original inductance of 100 µH and operates at the same voltages but the duty cycle is reduced to $D = 0.3$. Show that the current behaves as shown in figure 8 and that the average output current is reduced to 15 mA.

What happens as D is increased, keeping the voltages the same?

Note that the usual relation $V_{\text{out}} = DV_{\text{in}}$ does *not* hold in discontinuous conduction mode! The required value of D depends on both the output voltage and the output current.

6.5 Figure 9 shows a single cycle of either the input current $i_{\text{in}}(t)$ or the output current $i_{\text{out}}(t)$ measured for two boost regulators, each of which has an input at $V_{\text{in}} = 10\,\text{V}$. For each of these calculate the duty factor D, output voltage V_{out}, average output current I_{out}, average

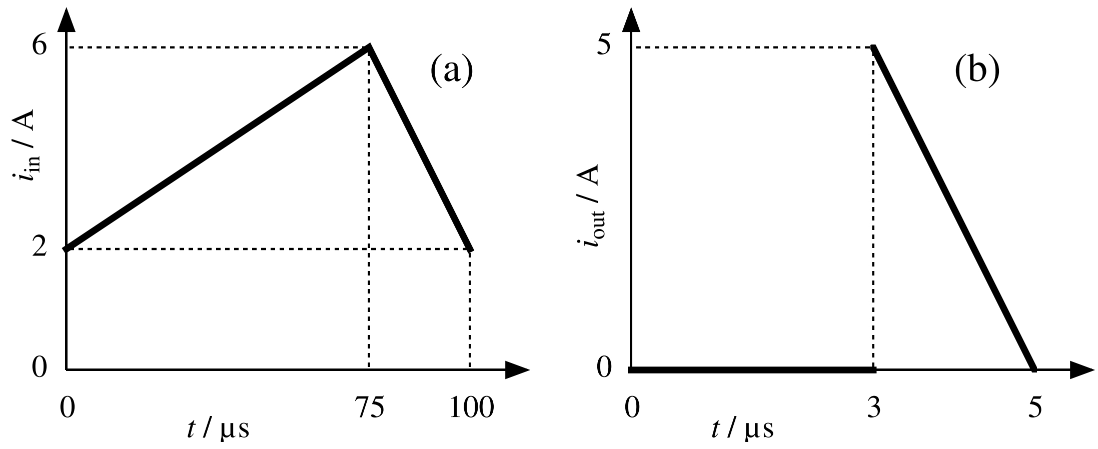

Figure 9. Currents measured in two boost DC–DC converters.

input current V_{in}, power P supplied and the value of the inductance L in the converter. Assume ideal components and operation. Remember that the input current is the same as the inductor current for a boost converter.

6.6 Design a boost converter to supply 500 mA at 12.0 V from 5.0 V. It should operate at 25 kHz and you may assume ideal components and operation. Calculate the inductance required for boundary mode and increase it by 25% for the final design to ensure the continuous mode of operation.

6.7 The previous question allowed you to assume that the output voltage was constant. However, the output current from a boost converter is pulsed and must be smoothed if the voltage and current supplied to the load are to be constant within limits. What capacitor is needed to reduce to ripple below 0.1 V? [Hint: this is very similar to the calculation of smoothing capacitor for a rectifier.] Suppose that the 12 V output was supplied by a classic bridge rectifier from the mains instead; how large a smoothing capacitor would needed in this case? [About 120 µF and 40 mF]

6.8 A switch-mode power supply can be modelled by the circuit shown in figure 10, which also shows a plot of the current through the inductor as a function of time for a single, complete period of switching. The switch alternates between positions (a) and (b) with corresponding labels on the plot.

(a) Write down the general relation between current and voltage as a function of time for (i) a capacitor and (ii) an inductor.

(b) What are the input and output voltages of this converter?

(c) How can the output voltage of this converter be varied and what range of output voltages can be produced?

6.9 A small wind turbine generates 1.5 V DC per mph of wind speed. It feeds a power electronic converter, which is required to deliver 200 W at 24 V DC to a load. For operation over the range 8 mph to 32 mph, determine the following:

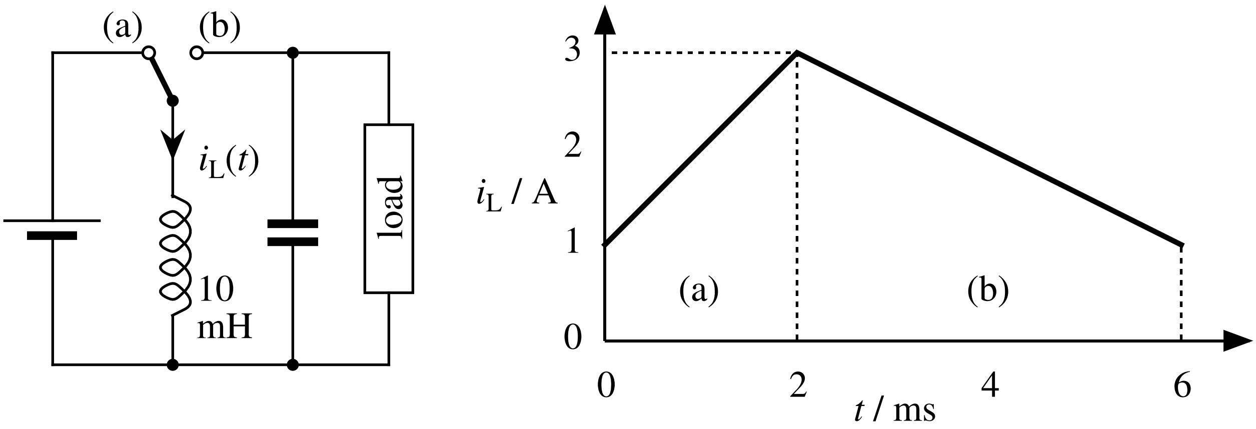

Figure 10. Switch-mode power supply.

(a) the type of converter to be used

(b) the required range of duty cycle for these wind speeds

(c) the average input current to the converter at 32 mph

(d) the peak input current at 32 mph assuming operation at the boundary condition.

6.10 Suggest appropriate types of power supply for the following systems and justify your choice. Detailed circuits are not required.

(a) A portable digital system requires a 3.3 VDC supply from a lithium-ion battery, whose voltage drops from 3.6 V to 3.0 V as it runs down.

(b) An instrumentation amplifier requires a ± 15 VDC supply from the mains.

(c) A digital system has its main power supply at 3.3 VDC and includes a processor that requires a supply of 5 A at 1.8 VDC.

(d) A toy is powered from two AAA cells and includes an electroluminescent display that needs 20 VDC to illuminate it.

7 Power switches (mainly SCRs and MOSFETs)

7.1 The power through a resistive load is phase-angle controlled by a pair of antiparallel SRCs (a TRIAC would work in the same way). The supply is standard, sinusoidal AC and the load dissipates $P_{\max}$ if it is connected directly to the supply. Show that the power dissipated as a function of delay angle α (in radians) is given by

$$\frac{P(\alpha)}{P_{\max}} = 1 - \frac{2\alpha - \sin(2\alpha)}{2\pi}. \tag{3}$$

This can be taken almost directly from the lecture notes but you are expected to be able to derive this sort of expression because it only requires a standard integral over a trigonometric function. You may need $\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

7.2 What delay angles are needed to reduce the power in a resistive load to 75%, 50% and 25% of its maximum value using a TRIAC and phase-angle control? [Hint: one is easy but two are tricky.]

7.3 A resistive heater dissipates 3.0 kW when connected directly to the 230 VAC mains. It is phase-angle controlled using a TRIAC, which has a forward voltage drop of 2.0 V.

(a) Calculate the power dissipated in the load for delay angles of 60° and 120°. [2.4 kW, 0.6 kW]

(b) Calculate the power dissipated in the TRIAC for the same delay angles. [Hint: the forward voltage across the TRIAC is assumed constant, so $P_{\mathrm{ave}} = V_{\mathrm{DC}} I_{\mathrm{ave}}$.] [18 W, 6 W]

(c) Under what conditions does the power dissipated in the TRIAC reach a maximum and what is its value? [24 W]

(d) Is phase-angle control the most suitable method for controlling this load?

7.4 A TRIAC is used to control the speed of a mains-powered electric drill. Would phase-angle or burst-mode control be more suitable?

7.5 A DC motor is connected to the AC mains at V_{AC} through a bridge rectifier made of SCRs.

(a) Calculate the average voltage across the motor as a function of delay angle α. You may neglect the voltage drop across the SCRs. [Hint: the hard work has been done before: you can relate this to a result derived in the lectures.]

(b) If the product is used in the UK, what is the average voltage across the load at a firing angle of 90°? What is it if the SCRs are switched on continuously? [104 V, 207 V]

7.6 *Gray code.* Draw out a table of the binary codes for the sequence 0, 1, 2, 3, 0… and show that only one bit changes at two transitions but two bits change at the other two transitions. Two transitions are a problem in practice because they never occur simultaneously, which means that the output of a sensor that produces a binary sequence is briefly incorrect. Show that only one bit changes at each transition in the sequence 0, 1, 3, 2, 0… so that the problem of incorrect values cannot arise. This is the simplest example of a Gray code. Work out a 3-bit Gray code that cycles through the 8 possible values, changing only one bit every time.

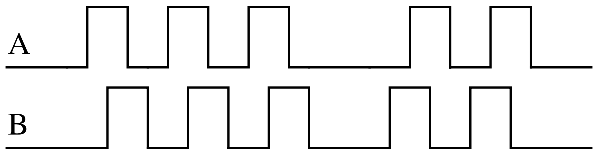

Figure 11. Outputs A and B from a quadrature encoder driven by a rotary control.

7.7 A control knob on an electronic product is connected to an incremental encoder (shaft encoder) with two outputs A and B in quadrature. The outputs of the two sensors as a function of time are shown in figure 11. The 'click' (detent) on the knob is aligned with $A = B = 0$. Explain the output.

Could this sort of control be used for the knobs on the oscilloscopes in the laboratory?

7.8 The brightness of an array of LEDs, which may be treated as a simple resistive load, is controlled by pulse-width modulation using a perfect switch. The LEDs draw 50 W when connected directly to a 12 V DC supply. Calculate the duty cycle D, average current I_{ave} and RMS current I_{rms} when the LEDs draw 10 W and 40 W. Comment on the differences between I_{ave} and I_{rms}.
[$D = 0.2$ and 0.8; $I_{\text{ave}} = 0.83$ A and 3.33 A; $I_{\text{rms}} = 1.86$ A and 3.73 A]

7.9 The pulse-width modulation in the previous question is controlled by a MOSFET with $R_{\text{DS(on)}} = 0.8\,\Omega$, $T_{\text{on}} = 50$ ns and $T_{\text{off}} = 50$ ns. The switching frequency is $f_{\text{sw}} =$ 20 kHz. Calculate the conduction and switching losses in the MOSFET for duty cycles of $D = 0.2$ and 0.8. [$P_{\text{cond}} = 2.8$ W and 11.1 W; $P_{\text{sw}} = 0.05$ W for both]

7.10 A MOSFET acts as a low-side switch for a load whose power is controlled by pulse-width modulation. The parameters of the system are given in table 1. Assume that components are ideal and that the current increases linearly while the MOSFET is on.

(a) Draw the current waveform. Clearly indicate the current and time values.

(b) Calculate the average and RMS value of the current from first principles.
[10.0 A, 14.2 A]

(c) Determine the conduction loss in the MOSFET. [10.1 W]

Table 1. Parameters for a load controlled by a MOSFET using PWM.

I_{on}	16 A
I_{off}	24 A
PWM duty cycle	50%
PWM frequency	50 kHz
Supply voltage	200 V
$R_{\text{DS(on)}}$	50 mΩ
T_{on}	25 ns
T_{off}	30 ns

(d) Determine the switching loss in the MOSFET. [5.6 W]

(e) Determine the total power dissipated in the MOSFET. [15.7 W]

(f) The power loss is found to be excessive so the designer suggest halving the switching frequency, keeping other parameters the same. Will this be effective?

[Adapted from 2015 May Q3.]

8 Inverters

8.1 An H-bridge inverter is fed from a 100 V DC supply and operates at 50 Hz. It produces a simple square wave output.

(a) What is the RMS output voltage?

(b) The inverter feeds a purely resistive load of 10 Ω. Calculate the form of the current $i(t)$ through the load, its RMS value and the average power dissipated in the load. [10 A, 1 kW]

(c) The inverter is connected instead to a purely inductive load of 50 mH. Calculate the form of the current $i(t)$ through the load, its RMS value and the average power delivered to the load. [5.8 A, zero]

8.2 [Harder, only for students who have taken Electrical Circuits 2, not examinable.] Suppose that the load on the inverter is a 10 Ω resistor and a 50 mH inductor in series. Derive an expression for $i(t)$ as follows.

(a) Focus on the first half-cycle, from 0 to 10 ms. The form of current must be a decaying exponential function because the RL circuit is a first-order system. It therefore has the general form

$$i(t) = I_{\text{final}} + (I_{\text{initial}} - I_{\text{final}}) \exp(-t/\tau). \tag{4}$$

What is the value of τ? [5 ms]

(b) The current I_{final} is the value that would be reached if the circuit was left for a long time so that the transient decays away and the circuit reaches a steady state. (This doesn't happen here because the voltage reverses every 10 ms but that doesn't affect the definition of I_{final}.) What is the value of I_{final}? [10 A]

(c) The final step is to find I_{initial} and this is the most complicated.

When the output of the inverter has reached a steady state, the current must be pure AC with an average value of zero. This means that successive half-cycles have the the same form but with their sign changed. In other words, if we know the form of $i(t)$ for $0 < t < 10$ ms, we can find its form for $10 < t < 20$ ms by sliding $i(t)$ along the time axis by 10 ms and flipping it vertically (changing the sign of $i(t)$).

The current through an inductor cannot change instantaneously so each half-cycle of $i(t)$ must join on to the previous one without any jumps. This means that the current must have the same value at the beginning and end of each half-cycle except that the sign changes. Mathematically, $i(\frac{1}{2}T) = -i(0)$ for the first half-cycle. Use this to show that

$$I_{\text{initial}} = -I_{\text{final}} \frac{1 - \exp(-\frac{1}{2}T/\tau)}{1 + \exp(-\frac{1}{2}T/\tau)} \equiv -I_{\text{final}} \tanh \frac{T}{4\tau}. \tag{5}$$

What is the numerical value of I_{initial}? [−7.6 A]

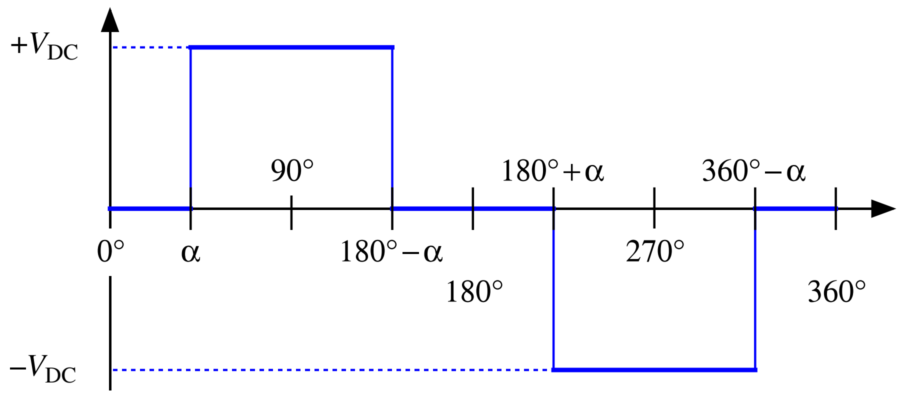

Figure 12. Voltage as a function of time (expressed as an angle) from inverter with output of zero volts for angles of $\pm\alpha$ about the transitions.

I won't ask you to find the average power but here is the result:

$$P_{\mathrm{ave}} = \frac{V_{\mathrm{DC}}^2}{R}\left[1 - \frac{4\tau}{T}\tanh\frac{T}{4\tau}\right]. \tag{6}$$

Numerically this gives $P_{\mathrm{ave}} = 0.24\,\mathrm{kW}$. The inductance reduces the power considerably from the value for the resistor alone.

This probably looks frightening but a course on power electronics for electrical engineers would need this level of analysis because many loads, particularly motors, behave like RL circuits. The current for AC phase control of an inductive load by an SCR can be analysed in a similar way.

8.3 *Fourier analysis of voltage from inverter.* Find the Fourier series for the voltage produced by the inverter in question 1 of this section, which is a square wave between $\pm V_{\mathrm{DC}} = \pm 100\,\mathrm{V}$ at $f = 50\,\mathrm{Hz}$. The series is a standard result that you should know from Engineering Mathematics 2.

Suppose that the square wave is filtered so well that only its fundamental Fourier component survives. What is the RMS voltage of this wave? [90 V]

8.4 The driver of the inverter in question 1 of this section is modified so that the H-bridge gives zero voltage on the output for angles of $\pm\alpha$ around the transitions between $\pm V_{\mathrm{DC}}$ with $V_{\mathrm{DC}} = 100\,\mathrm{V}$ and $f = 50\,\mathrm{Hz}$ as before. The waveform is shown in figure 12.

(a) Find an expression for the RMS value of this voltage. What is its numerical value for $\alpha = 30°$? [82 V]

(b) [Harder] Derive expressions for the amplitude and RMS value of the fundamental component of this waveform. What is the RMS value of the fundamental for $\alpha = 30°$? [78 V]

This waveform has smaller amplitudes of higher harmonics, which is often important in real systems, but at the cost of a lower RMS voltage.

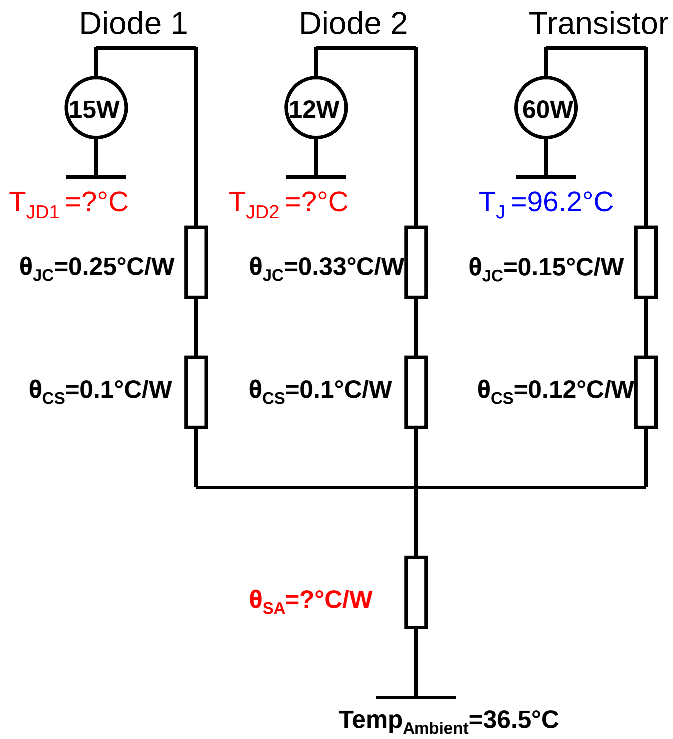

Figure 13. System comprising two diodes and a transistor on a heatsink.

9 Thermal management

9.1 A transistor dissipates 2 W and has a thermal resistance of 5°C/W between junction and case. Calculate the junction temperature when the ambient temperature is 50°C with a heat sink of thermal resistance (including the contact) (i) 50°C/W and (ii) 10°C/W. [80°C, 160°C]

9.2 A TIP120 transistor is required to dissipate 40 W in an ambient temperature of 30°C. Determine the thermal resistance of the heatsink required to keep the junction temperature below 150°C. There is an insulating washer of thermal resistance 0.5°C/W between the case and the heatsink. The transistor is specified for a dissipation of 65 W below 25°C, derated at 0.5 W/°C at higher temperatures. [Note that *derating* is the inverse of the thermal resistance and is an alternative specification.] [0.5°C/W]

What size of heatsink would be needed if a TIP3055 were used instead? This is a larger transistor with a maximum dissipation of 90 W derated at 0.7 W/°C.

9.3 Calculate T_{JD1}, T_{JD2} and θ_{SA} for the system shown in figure 13.

9.4 A switch-mode power supply contains two power semiconductor devices:

- A MOSFET switch, which dissipates 1.0 W
- A diode, which dissipates 2.0 W.

Both are in TO220 packages, which have thermal resistances $\theta_{\text{jc}} = 3\,°\text{C}\,\text{W}^{-1}$ and $\theta_{\text{ca}} = 60\,°\text{C}\,\text{W}^{-1}$. The temperature of their junctions should not exceed 125°C and the ambient temperature is 25°C.

(a) Is it possible to use either device safely without a heatsink?

(b) Despite the results of the previous analysis, the designer decides to mount both devices on a common heatsink. Specify the heatsink. Hint: which component is most at risk from overheating? [$\theta_{\text{hs}} < 31\,°\text{C}\,\text{W}^{-1}$]

(c) What temperatures will the junctions of the two devices reach, assuming that the heatsink has the maximum permitted thermal resistance?

(d) Is this a sensible design? If not, suggest how it could be improved.