Root mean square

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In mathematics, the **root mean square** (abbreviated **RMS** or **rms**), also known as the **quadratic mean**, is a statistical measure of the magnitude of a varying quantity. It is especially useful when variates are positive and negative, e.g., sinusoids. In the field of electrical engineering, the RMS value of a periodic current is equal to the direct current (DC) that delivers the same average power to a resistor as the periodic current.[1]

It can be calculated for a series of discrete values or for a continuously varying function. Its name comes from its definition as the square root of the mean of the squares of the values. It is a special case of the generalized mean with the exponent $p = 2$.

Contents

Definition

The RMS value of a set of values (or a continuous-time waveform) is the square root of the arithmetic mean of the squares of the original values (or the square of the function that defines the continuous waveform).

In the case of a set of n values $\{x_1, x_2, \ldots, x_n\}$, the RMS

$$x_{\mathrm{rms}} = \sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)}.$$

The corresponding formula for a continuous function (or waveform) $f(t)$ defined over the interval $T_1 \leq t \leq T_2$ is

$$f_{\mathrm{rms}} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [f(t)]^2 \, dt},$$

and the RMS for a function over all time is

$$f_{\mathrm{rms}} = \lim_{T \to \infty} \sqrt{\frac{1}{T} \int_0^T [f(t)]^2 \, dt}.$$

The RMS over all time of a periodic function is equal to the RMS of one period of the function. The RMS value of a continuous function or signal can be approximated by taking the RMS of a series of equally spaced samples. Additionally, the RMS value of various waveforms can also be determined without calculus, as shown by Cartwright.[2]

In the case of the RMS statistic of a random process, the expected value is used instead of the mean.

RMS of common waveforms

Sine, square, triangle, and sawtooth waveforms
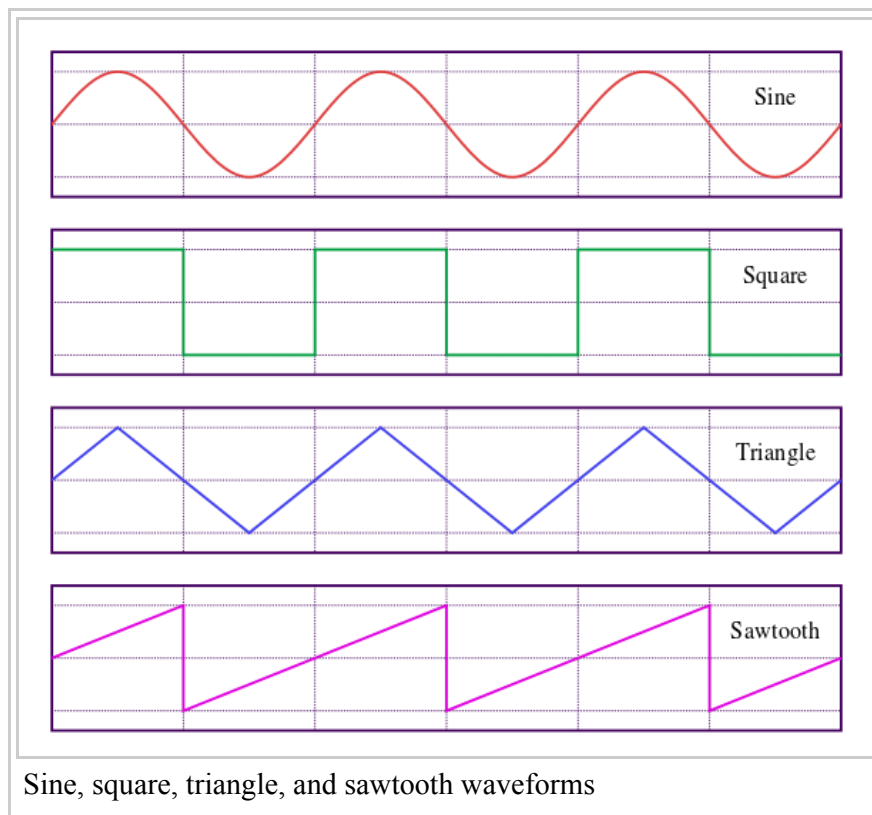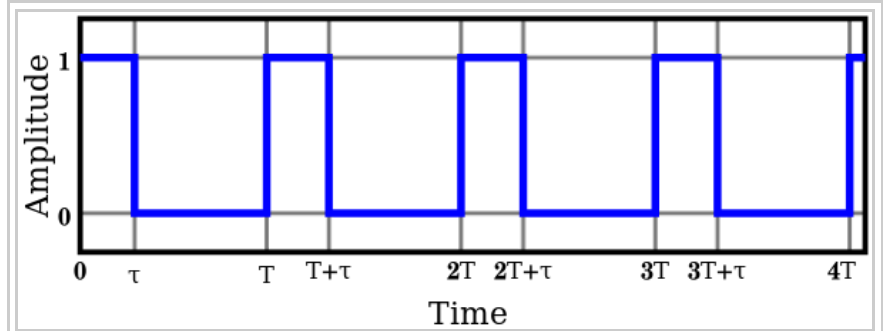
A rectangular pulse wave of duty cycle D, the ratio between the pulse duration (τ) and the period (T); illustrated here with $a = 1$
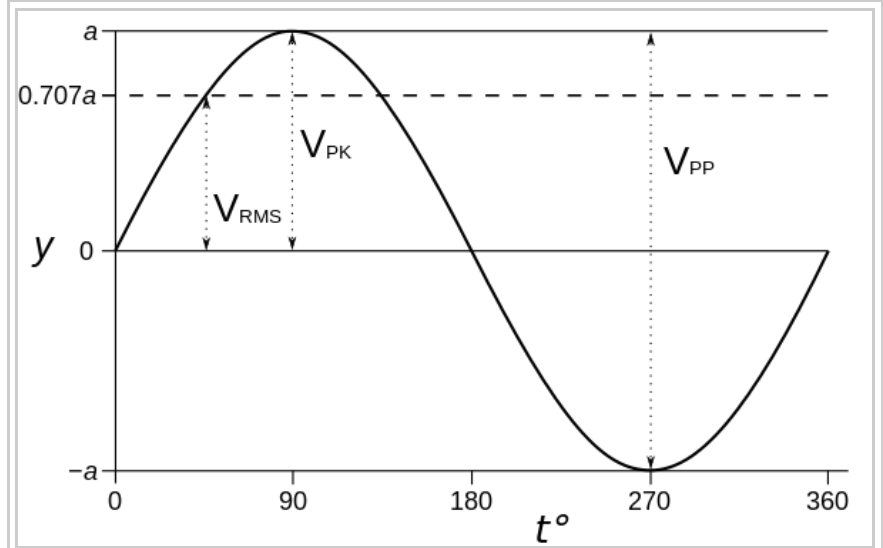
Graph of a sine wave's voltage vs. time (in degrees), showing RMS, peak, and peak-to-peak voltages

Waveform	Equation	RMS
DC, constant	$y = a$	a
Sine wave	$y = a\sin(2\pi f t)$	$\dfrac{a}{\sqrt{2}}$
Square wave	$y = \begin{cases} a & \{ft\} < 0.5 \\ -a & \{ft\} > 0.5 \end{cases}$	a
DC-shifted square wave	$y = \begin{cases} a + DC & \{ft\} < 0.5 \\ -a + DC & \{ft\} > 0.5 \end{cases}$	$\sqrt{DC^2 + a^2}$
Modified square wave	$y = \begin{cases} 0 & \{ft\} < 0.25 \\ a & 0.25 < \{ft\} < 0.5 \\ 0 & 0.5 < \{ft\} < 0.75 \\ -a & \{ft\} > 0.75 \end{cases}$	$\dfrac{a}{\sqrt{2}}$
Triangle wave	$y = \lvert 2a\{ft\} - a \rvert$	$\dfrac{a}{\sqrt{3}}$
Sawtooth wave	$y = 2a\{ft\} - a$	$\dfrac{a}{\sqrt{3}}$
Pulse train	$y = \begin{cases} a & \{ft\} < D \\ 0 & \{ft\} > D \end{cases}$	$a\sqrt{D}$
Phase-to-phase voltage	$y = a\sin(t) - a\sin(t - \dfrac{2\pi}{3})$	$a\sqrt{\dfrac{3}{2}}$

Notes:
t is time
f is frequency
a is amplitude (peak value)
D is the duty cycle or the percent(%) spent high of the period (1/*f*)
{r} is the fractional part of r

Waveforms made by summing known simple waveforms have an RMS that is the root of the sum of squares of the component RMS values, if the component waveforms are orthogonal (that is, if the average of the product of one simple waveform with another is zero for all pairs other than a waveform times itself).[3]

$$RMS_{Total} = \sqrt{RMS_1{}^2 + RMS_2{}^2 + \cdots + RMS_n{}^2}$$

A special case of this, particularly helpful in electrical engineering, is

$$RMS_{Total} = \sqrt{RMS_{DC}{}^2 + RMS_{AC}{}^2}$$

where RMS_{DC} refers to the DC component of the signal and RMS_{AC} is the AC component of the signal.

Uses

The RMS value of a function is often used in physics and electrical engineering.

Average electrical power

Electrical engineers often need to know the power, P, dissipated by an electrical resistance, R. It is easy to do the calculation when there is a constant current, I, through the resistance. For a load of R ohms, power is defined simply as:

$$P = I^2 R.$$

However, if the current is a time-varying function, $I(t)$, this formula must be extended to reflect the fact that the current (and thus the instantaneous power) is varying over time. If the function is periodic (such as household AC power), it is still meaningful to discuss the *average* power dissipated over time, which is calculated by taking the average power dissipation:

$$
\begin{aligned}
P_{\text{avg}} &= \langle I(t)^2 R \rangle \text{ (where } \langle \dots \rangle \text{ denotes the mean of a function)} \\
&= R \langle I(t)^2 \rangle \text{ (as } R \text{ does not vary over time, it can be factored out)} \\
&= (I_{\text{RMS}})^2 R \text{ (by definition of RMS)}
\end{aligned}
$$

So, the RMS value, I_{RMS}, of the function $I(t)$ is the constant current that yields the same power dissipation as the time-averaged power dissipation of the current $I(t)$.

Average power can also be found using the same method that in the case of a time-varying voltage, $V(t)$, with RMS value V_{RMS},

$$P_{\text{avg}} = \frac{(V_{\text{RMS}})^2}{R}.$$

This equation can be used for any periodic waveform, such as a sinusoidal or sawtooth waveform, allowing us to calculate the mean power delivered into a specified load.

By taking the square root of both these equations and multiplying them together, the power is found to be:

$$P_{\text{avg}} = V_{\text{RMS}} I_{\text{RMS}}.$$

Both derivations depend on voltage and current being proportional (i.e., the load, R, is purely resistive). Reactive loads (i.e., loads capable of not just dissipating energy but also storing it) are discussed under the topic of AC power.

In the common case of alternating current when $I(t)$ is a sinusoidal current, as is approximately true for mains power, the RMS value is easy to calculate from the continuous case equation above. If I_p is defined to be the peak current, then:

$$I_{\text{RMS}} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} (I_p \sin(\omega t))^2 dt}.$$

where t is time and ω is the angular frequency ($\omega = 2\pi/T$, where T is the period of the wave).

Since I_p is a positive constant:

$$I_{\mathrm{RMS}} = I_{\mathrm{P}} \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \sin^2(\omega t)\, dt}.$$

Using a trigonometric identity to eliminate squaring of trig function:

$$I_{\mathrm{RMS}} = I_{\mathrm{P}} \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \frac{1 - \cos(2\omega t)}{2}\, dt}$$

$$I_{\mathrm{RMS}} = I_{\mathrm{P}} \sqrt{\frac{1}{T_2 - T_1} \left[\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega} \right]_{T_1}^{T_2}}$$

but since the interval is a whole number of complete cycles (per definition of RMS), the *sin* terms will cancel out, leaving:

$$I_{\mathrm{RMS}} = I_{\mathrm{P}} \sqrt{\frac{1}{T_2 - T_1} \left[\frac{t}{2} \right]_{T_1}^{T_2}} = I_{\mathrm{P}} \sqrt{\frac{1}{T_2 - T_1} \frac{T_2 - T_1}{2}} = \frac{I_{\mathrm{P}}}{\sqrt{2}}.$$

A similar analysis leads to the analogous equation for sinusoidal voltage:

$$V_{\mathrm{RMS}} = \frac{V_{\mathrm{P}}}{\sqrt{2}}.$$

Where I_P represents the peak current and V_P represents the peak voltage.

Because of their usefulness in carrying out power calculations, listed voltages for power outlets, e.g. 120 V (USA) or 230 V (Europe), are almost always quoted in RMS values, and not peak values. Peak values can be calculated from RMS values from the above formula, which implies $V_p = V_{\mathrm{RMS}} \times \sqrt{2}$, assuming the source is a pure sine wave. Thus the peak value of the mains voltage in the USA is about $120 \times \sqrt{2}$, or about 170 volts. The peak-to-peak voltage, being twice this, is about 340 volts. A similar calculation indicates that the peak-to-peak mains voltage in Europe is about 650 volts.

RMS quantities such as electric current are usually calculated over one cycle. However for some purposes the RMS current over a longer period is required when calculating transmission power losses. The same principle applies, and (for example) a current of 10 amps used for 6 hours each day represents an RMS current of 5 amps in the long term.

The term "RMS power" is sometimes used in the audio industry as a synonym for "mean power" or "average power" (it is proportional to the square of the RMS voltage or RMS current in a resistive load). For a discussion of audio power measurements and their shortcomings, see Audio power.

Root-mean-square speed

In the physics of gas molecules, the **root-mean-square speed** is defined as the square root of the average squared-speed. The RMS speed of an ideal gas is calculated using the following equation:

$$v_{\mathrm{RMS}} = \sqrt{\frac{3RT}{M}}$$

where R represents the ideal gas constant, 8.314 J/(mol·K), T is the temperature of the gas in kelvins, and M is the molar mass of the gas in kilograms. The generally accepted terminology for speed as compared to velocity is that the former is the scalar magnitude of the latter. Therefore, although the average speed is between zero and the RMS speed, the average velocity for a stationary gas is zero.

Root-mean-square error

When two data sets—one set from theoretical prediction and the other from actual measurement of some physical variable, for instance—are compared, the RMS of the pairwise differences of the two data sets can serve as a measure how far on average the error is from 0.

The mean of the pairwise differences does not measure the variability of the difference, and the variability as indicated by the standard deviation is around the mean instead of 0. Therefore, the RMS of the differences is a meaningful measure of the error.

RMS in frequency domain

The RMS can be computed in the frequency domain, using Parseval's theorem. For a sampled signal,

$$\sum_n x^2(t) = \frac{\sum_n |X(f)|^2}{n},$$

where $X(f) = \mathrm{FFT}\{x(t)\}$ and n is number of $x(t)$ samples.

In this case, the RMS computed in the time domain is the same as in the frequency domain:

$$\mathrm{RMS} = \sqrt{\frac{1}{n}\sum_n x^2(t)} = \sqrt{\frac{1}{n^2}\sum_n |X(f)|^2} = \sqrt{\sum_n \left|\frac{X(f)}{n}\right|^2}.$$

Relationship to the arithmetic mean and the standard deviation

If $\bar{x}$ is the arithmetic mean and σ_x is the standard deviation of a population or a waveform then:[4]

$$x_{\mathrm{rms}}^2 = \bar{x}^2 + \sigma_x^2 = \overline{x^2}.$$

From this it is clear that the RMS value is always greater than or equal to the average, in that the RMS includes the "error" / square deviation as well.

Physical scientists often use the term "root mean square" as a synonym for standard deviation when referring to the square root of the mean squared deviation of a signal from a given baseline or fit.[5] [6] This is useful for electrical engineers in calculating the "AC only" RMS of a signal. Standard deviation

being the root mean square of a signal's variation about the mean, rather than about 0, the DC component is removed (i.e. RMS(signal) = Stdev(signal) if the mean signal is 0).

See also

- Central moment
- Geometric mean
- L2 norm
- Least squares
- Mean squared displacement
- Mean squared error
- Root mean square deviation
- Table of mathematical symbols
- True RMS converter

References

1. ^ *Fundamentals of Electric Circuits*. McGraw-Hill Science/Engineering/Math; 5 edition (January 12, 2012). p. 467.
2. ^ Cartwright, Kenneth V (Fall 2007). "Determining the Effective or RMS Voltage of Various Waveforms without Calculus" (http://technologyinterface.nmsu.edu/Fall07/). *Technology Interface* **8** (1): 20 pages.
3. ^ Nastase, Adrian S. "How to Derive the RMS Value of Pulse and Square Waveforms" (http://masteringelectronicsdesign.com/how-to-derive-the-rms-value-of-pulse-and-square-waveforms/). *MasteringElectronicsDesign.com*. Retrieved 21 January 2015.
4. ^ Chris C. Bissell and David A. Chapman (1992). *Digital signal transmission* (http://books.google.com/books?id=ItJoq36hCoYC&pg=PA64) (2nd ed.). Cambridge University Press. p. 64. ISBN 978-0-521-42557-5.
5. ^ "Root-Mean-Square" (http://mathworld.wolfram.com/Root-Mean-Square.html).
6. ^ "ROOT, TH1:GetRMS" (http://root.cern.ch/root/html/TH1.html#TH1:GetRMS).

External links

- A case for why RMS is a misnomer when applied to audio power (http://www.hifi-writer.com/he/misc/rmspower.htm)
- RMS, Peak and Average for some waveforms (http://www.opamp-electronics.com/tutorials/measurements_of_ac_magnitude_2_01_03.htm)
- A Java applet on learning RMS (http://phy.hk/wiki/englishhtm/Rms.htm)

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