POWER ENGINEERING

#03 SINGLE-PHASE AC POWER SYSTEMS

AC CIRCUIT ANALYSIS

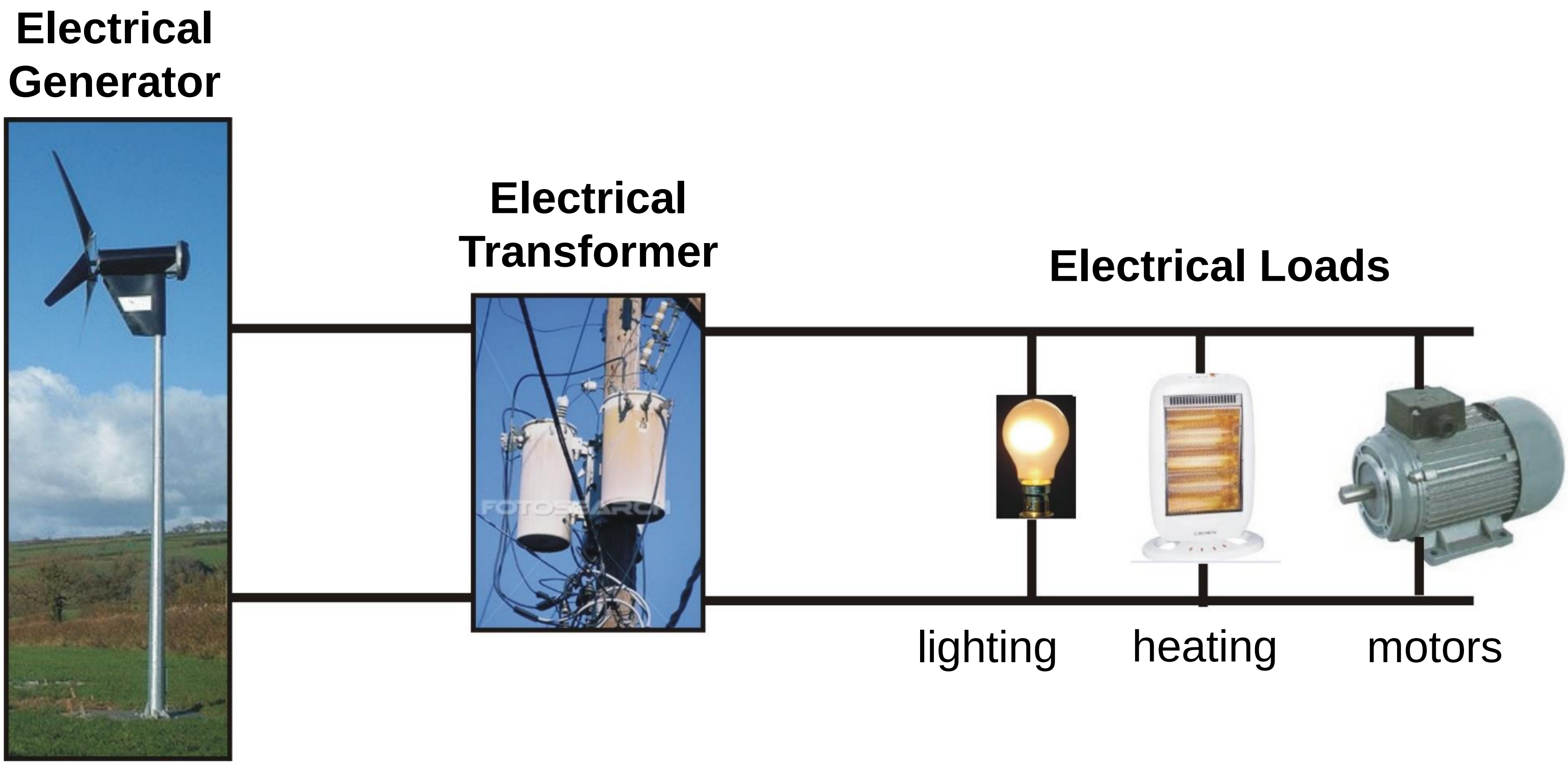
2018

University of Glasgow

Today we will investigate:

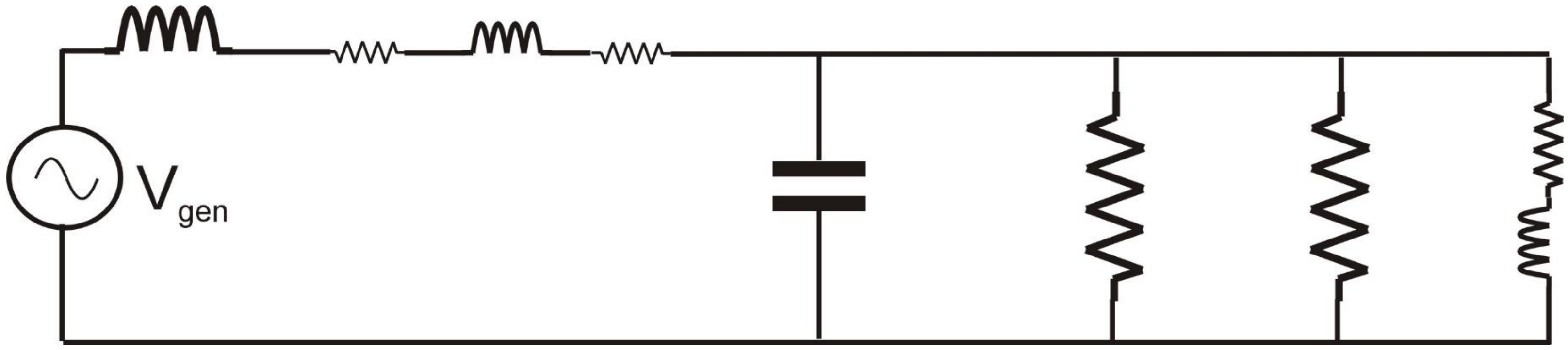
- Components of a simple Electrical Power System
- AC Voltage Source:
  - Instantaneous Voltage
  - Average & RMS Voltage
- Phasor Diagram Representation
- Resistive, Inductive and Capacitive loads:
  - Voltage v Current relationship
  - Impedance & Reactance
- Complex Loads (combination of R, L and C components)

A Simple Electrical Power System:



Each of these components can be represented by an equivalent circuit made up of a combination of simple electrical components: voltage source, resistance, inductance and capacitance

A Simple Electrical Power System (Equivalent Circuit):



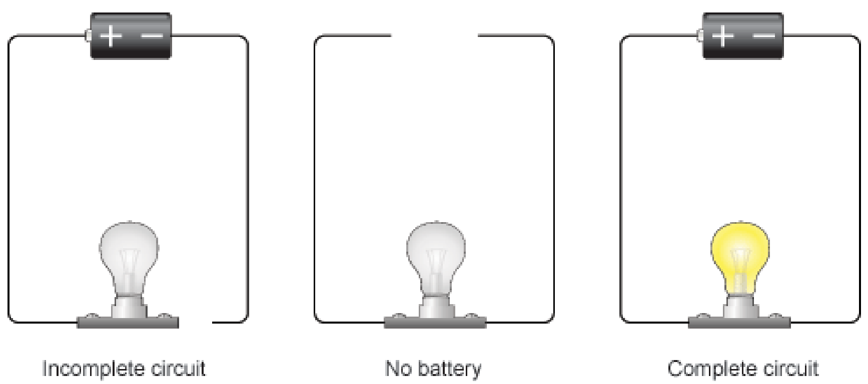
Once we have the equivalent circuit we can determine component voltages and currents, and then from this determine **POWER** related properties such as Real Power, Apparent Power, System Efficiency and Power Factor (we'll talk about these in detail on Thursday)

Before we move onto analysing equivalent circuits lets have a brief look at the properties of each circuit component

ELECTRICAL CIRCUIT

To have an electrical circuit with flowing current, we need

- (Power) Source 电源 : Voltage Source, Current Source, Dependant (Controlled) Source
- Complete （闭合） paths formed by conductors and components (Resistor, Inductor, Capacitor)



The bulb will only light if there is a battery and a complete circuit

We usually add in a **switch** （开关）to the circuit, so that we can break （关断） the circuit and stop （阻断） the electric current when we want to.

LINEAR ELECTRICAL CIRCUIT 线性电路

An equivalent definition of a linear circuit is that it obeys the Superposition Principle （叠加原理） in steady states. When a linear combination of signals **$ax_1(t) + bx_2(t)$** is applied to a linear circuit **F(x),** the output of **F(x)** is equal to the linear combination of the outputs due to the signals **$x_1(t)$** and **$x_2(t)$** applied separately:

$$F(a \cdot X_1(t)) = a \cdot F(X_1(t))$$
$$F(b \cdot X_2(t)) = b \cdot F(X_2(t))$$
$$F(a \cdot X_1(t) + b \cdot X_2(t)) = a \cdot F(X_1(t)) + b \cdot F(X_2(t))$$

Linear circuits are important because they can process analogue signals without introducing intermodulation distortion. This means that separate frequencies in the signal stay separate and do not mix, creating new frequencies. It is easy to analyse linear circuits using many mathematical techniques, e.g. Fourier analysis, Laplace Transform in frequency domain （频域）.

LINEAR ELECTRICAL CIRCUIT

A linear circuit is an electronic circuit in which, for a sinusoidal input voltage of frequency *f*, any steady-state output of the circuit (the current through any component, or the voltage between any two points) is also sinusoidal with frequency f. Note: a dc signal can also be treated as a sinusoidal input voltage of frequency *f=0*.

A linear circuit has no nonlinear electronic components in it.

- Examples of linear circuits are amplifiers比例放大器, differentiators微分器, and integrators积分电路, or any circuit composed exclusively of ideal passive 无源的 LCR (resistors, capacitors, inductors), active 有源的 op-amps (in the "non-saturated不饱和" regime), and other "linear" elements.
- Some examples of nonlinear (非线性) electronic components are: diodes, transistors, and inductors and transformers when the iron core is saturated, digital logic circuit, etc.

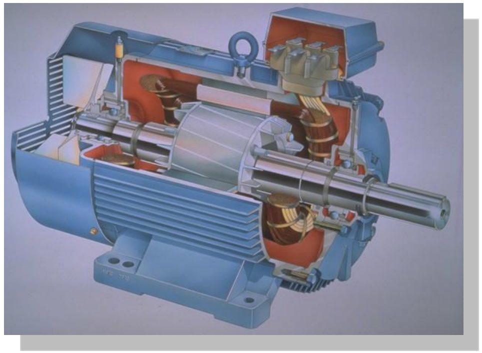
Resistive Elements

Examples:

- General Wiring
- Resistive loads (lighting, heating)
- Dump Resistors in power converters

Electrical Symbol	SI Units	Typical Values
	**Ohm (Ω)**	**mΩ - kΩ**

Inductive Elements



Examples:

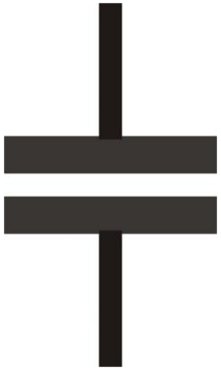
- General Wiring (stray inductance)
- Magnetic Components (Transformers, Electric Motors)
- Filters

Electrical Symbol	SI Units	Typical Values
	**Henry (H)**	**μH - mH**

Capacitive Elements

Examples:

- General Wiring (stray capacitance)
- Power Factor Correction
- Filters

Electrical Symbol	SI Units	Typical Values
	**Farad (F)**	**μF – 1000'sμF**

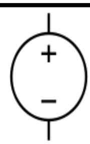
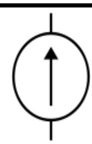
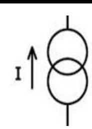
BASIC INDEPENDENT SOURCES

Voltage source

- Source of electrical potential difference. Voltage across it is independent of the current through it.

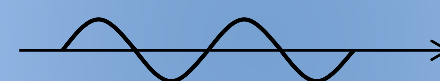
Current source

- Source of flow of electric charge. Current through it is independent of the voltage across it.

	DC	AC
Voltage Source	 or 	
Current Source	 or 	

<u>AC / DC</u>

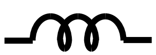
AC: flow of electrical charge alternates periodically



DC: flow of electrical charge is one direction & constant

BEHAVIOR OF BASIC PASSIVE (无源) ELEMENTS LCR

resistance (R) / conductance (G)
unit: ohm (Ω) / siemens (S)
Energy Dissipation Element
耗能元件



inductance (L)
unit: henry (H)
Energy Storage Element
储能元件

capacitance (C)
unit: farad (F)
Energy Storage Element
储能元件

Voltage-Current Relationship in Time Domain

	DC	AC
Resistor	$V = RI$ 欧姆定律	$v(t) = Ri(t)$
Inductor		$v(t) = L\dfrac{di(t)}{dt}$
Capacitor		$v(t) = \dfrac{1}{C}\int i(t)dt$

LCR元件时域特性

TECHNIQUES FOR LINEAR CIRCUIT ANALYSIS

Current Node method 电流节点法 and Voltage Mesh (Loop) method 电压网格/回路法

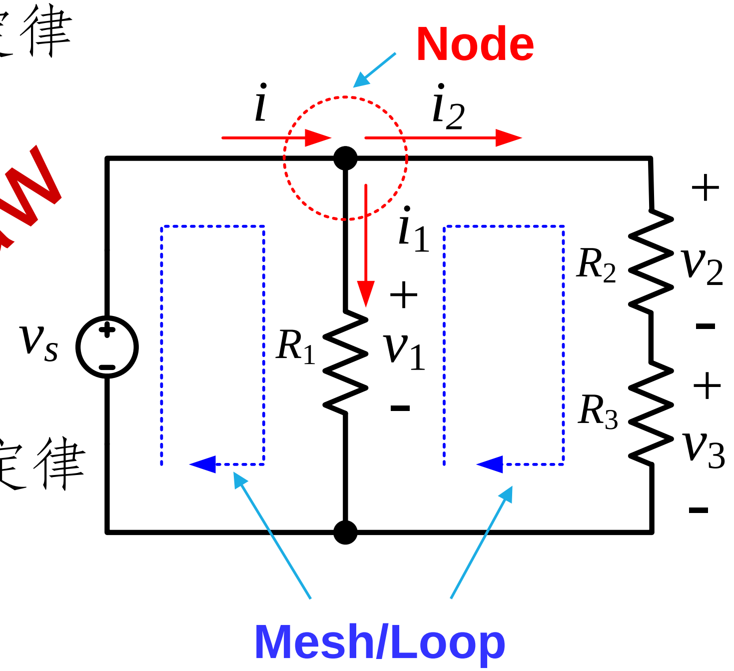
- Kirchoff's Current Law (KCL) 基氏电流定律

$$\text{KCL}: \quad \sum_k i_k = 0, \quad \textit{for any node}$$

- Kirchoff's Voltage Law (KVL) 基氏电压定律

$$\text{KVL}: \quad \sum_k v_k = 0, \quad \textit{for any loop}$$

conservation law



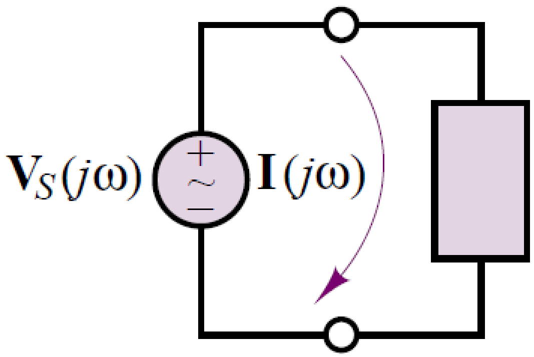
STEADY-STATE AND TRANSIENT RESPONSES

Steady state (稳态) is an equilibrium (平衡) condition of a circuit or network that occurs as the effects of transients are no longer important.

- All power waveforms repeat unchanged cylically with fixed AC inputs when t→∞;
- All power waveforms keep unchanged with constant DC inputs when t→∞.

Transient response (瞬态) is to describe the behaviour of a power variable (voltage or current) during the transition between two distinct steady-state (equilibrium) conditions

STEADY-STATE ANALYSIS OF AC CIRCUIT



What's AC Circuit ?

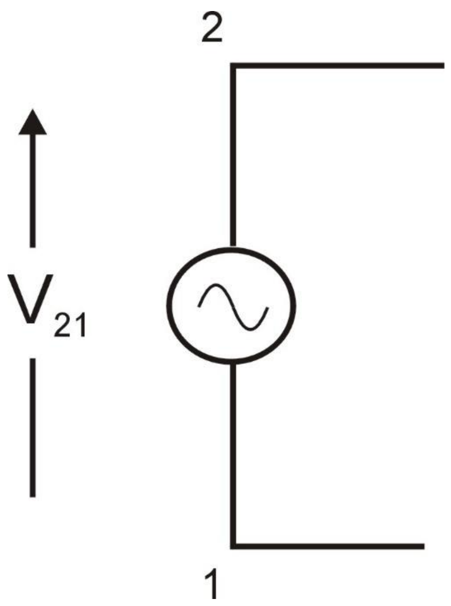
- Circuit in Steady State;
- Excitation Source V_s or I is a sinusoidal function with constant *amplitude* and constant *frequency*.

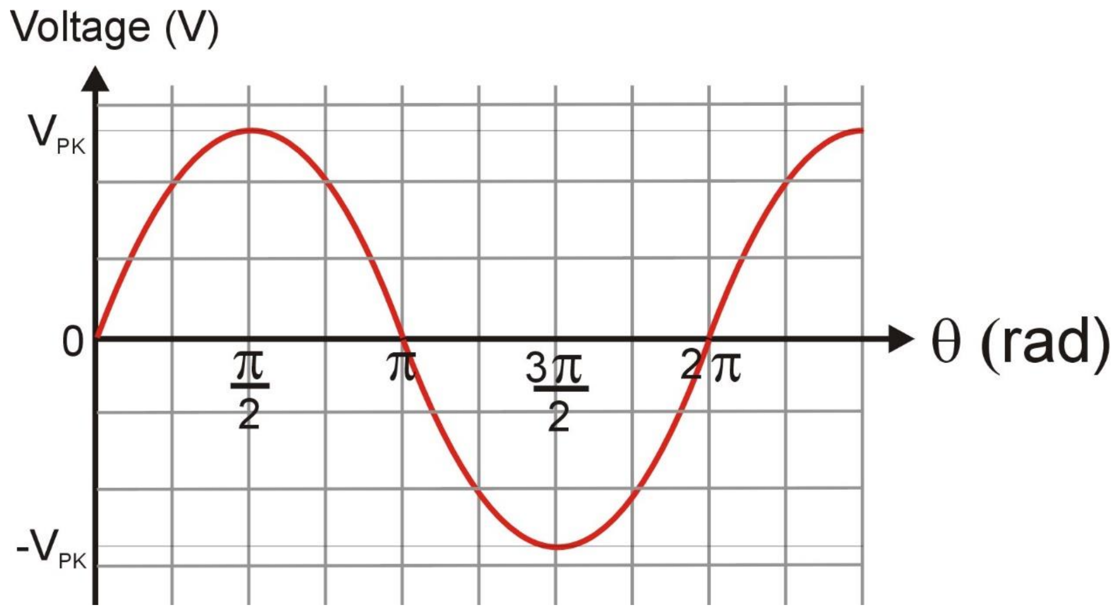
DC Circuit Analysis actually can be treated as a special case of steady-state AC Circuit Analysis in the case of frequency ω=0

Note: Steady State $\neq$ DC

Ideal Voltage Source:

Ideal **means that the sinusoidal voltage does NOT change when a load is applied across its terminals**





Expressions for Instantaneous Voltage: $$v(\theta) = V_{PK} Sin\theta$$

$$v(t) = V_{PK} Sin(\omega t)$$

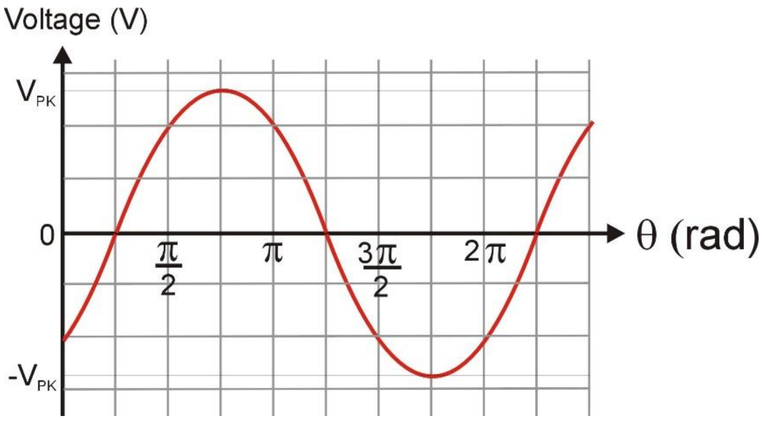
Example: 50Hz AC voltage

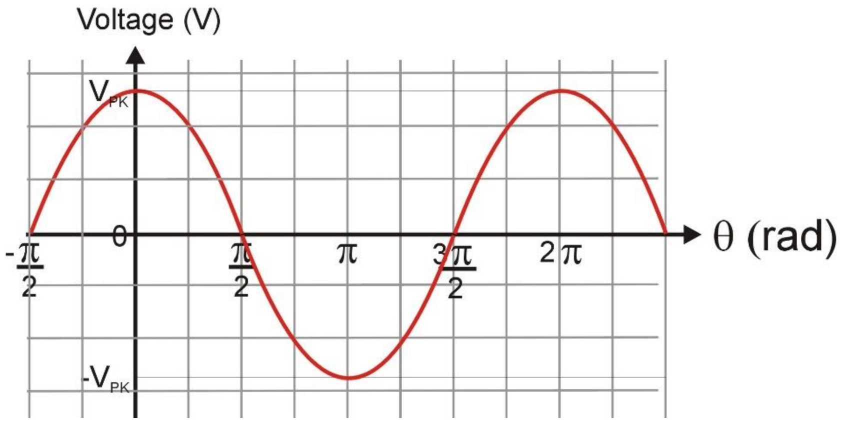
$\omega = 2\pi.f$

$\omega = 2\pi.50 = 314$ rad/sec

Where ω = Voltage frequency in rad/sec

Note: the time/angle reference for the previous voltage waveform has been chosen to (arbitrarily) start at time/angle = 0, here are a couple of other examples where this is not the case:





$$v(\theta) = V_{PK}Sin(\theta - \frac{\pi}{4})$$

$$v(\theta) = V_{PK}Sin(\theta + \frac{\pi}{2})$$

$$v(t) = V_{PK}Sin(\omega t - \frac{\pi}{4})$$

$$v(t) = V_{PK}Sin(\omega t + \frac{\pi}{2})$$

Phase Shift = -π/4 (delay)

Phase Shift = +π/2 (advance)

In time domain, it is quite easy to do DC circuit Analysis by using Ohm's Law

$$\boxed{R = \frac{V}{I}}$$

However, in time domain, AC circuit analysis with real number involves complicated calculus (differential equations) of trigonometric function. It is not easy !!!

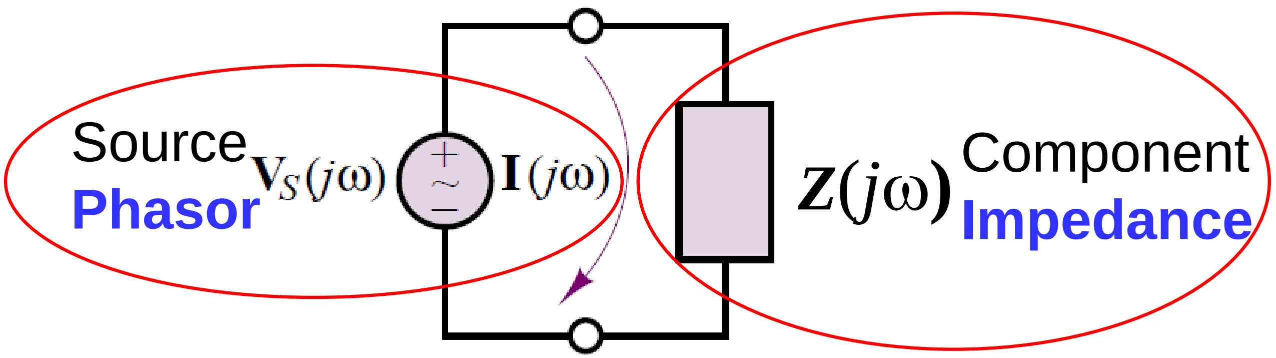
How to simplify AC Circuit ANALYSIS ???

Solving AC Circuits with Complex Number !!!

Create mathematical tools – phasor & impedance

工欲善其事，必先利其器

STEADY-STATE ANALYSIS OF AC CIRCUIT WITH COMPLEX NOTATION – PHASOR & IMPEDANCE



The principal difference between a resistive circuit and an AC circuit is that the latter is frequency-dependent.

A GENERALISED SINUSOID CAN BE EXPRESSED AS THE REAL PART OF A COMPLEX VECTOR

$$\boxed{\begin{aligned} v &= V_m \cos(\omega t+\phi) = \mathrm{Re}\left[V_m \cos(\omega t+\phi) + jV_m \sin(\omega t+\phi)\right] \\ &= \mathrm{Re}\left[V_m e^{j(\omega t+\phi)}\right] = \mathrm{Re}\left[V_m e^{j\phi} e^{j\omega t}\right] = \cancel{\mathrm{Re}}\left[\mathbf{V} \cancel{e^{j\omega t}}\right] \end{aligned}}$$

PHASOR REPRESENTATION OF THE GIVEN SINUSOIDAL FUNCTION CAN BE DEFINED (SIMPLIFIED) AS

$$\boxed{\mathbf{V}(j\omega) = \mathbf{V} = V_m e^{j\phi} \equiv V_m \angle \phi}$$

where **V(*jω*)** or **V** is "the phasor of cos(ωt+Φ), the $j\omega$ indicating the $e^{j\omega t}$ dependence of the phasor (or the representation in frequency domain).

PHASOR（相量）

Any sinusoidal signal may be mathematically represented in one of two ways:

- ❑ time-domain form: $v(t) = A\cos(\omega t + \varphi)$
- ❑ frequency-domain (or phasor) form: $\mathbf{V}(j\omega) = Ae^{j\varphi} = A\angle\varphi$

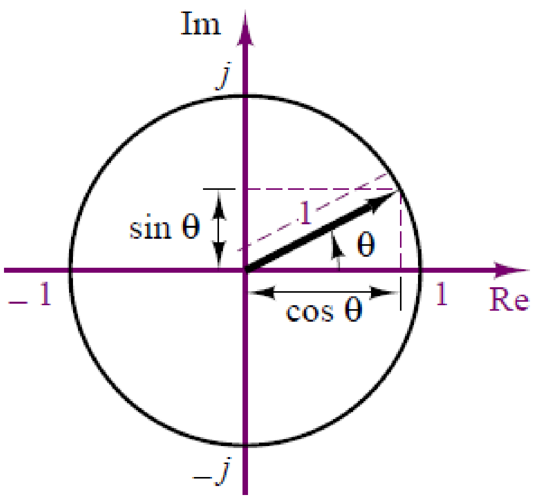
Note $j\omega$ in the notation $\mathbf{V}(j\omega)$, indicating $e^{j\omega t}$ dependence of the phasor.

Note:

- For a given frequency ω, ***Phasor*** *is only used to express* ***sinusoidal voltage*** *or* ***current*** *variables with complex number* !!!
- Phasors replaces time-varying sinusoidal functions with **constant complex numbers**, significantly simplify AC circuit analysis.
- *Simply Mathematical Convenience. No Real Physical Significance.*

PHASOR

The phasor concept is rooted in Euler's identity 欧拉等式, which relates the exponential function to the trigonometric function:



$e^{j\theta} = \cos\theta + j\sin\theta$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \mathrm{Re}\left[e^{j\theta}\right]$$

$$\sin\theta = \mathrm{Im}\left[e^{j\theta}\right]$$

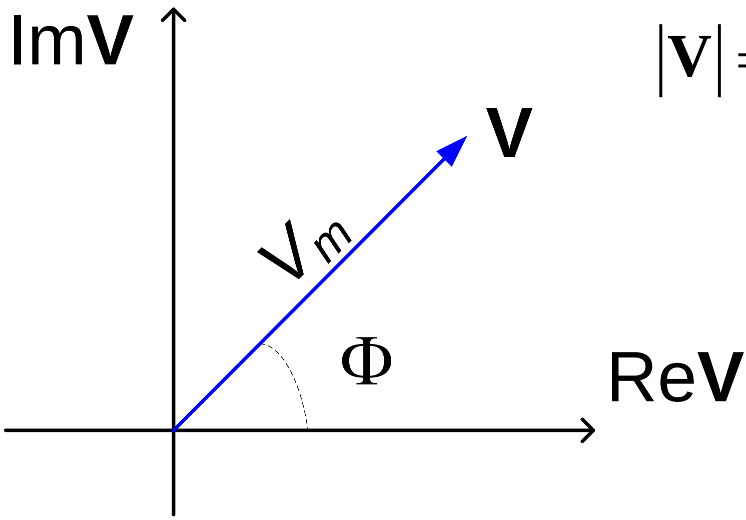
$$\left|e^{j\theta}\right| = \left|\cos\theta + j\sin\theta\right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

PHASOR

$$\mathbf{V} = V_m e^{j\phi} \equiv V_m \angle \phi$$

V_m - magnitude (modulus 模)
Φ - phase angle

Is a complex number.



Polar form — *Exponential form* — *Rectangular form*

$$\mathbf{V} = |\mathbf{V}|\angle\Phi = |\mathbf{V}|e^{j\Phi} = |\mathbf{V}|(\cos\Phi + j\sin\Phi)$$

$$|\mathbf{V}| = V_m$$

*The **phasor** is a complex number that carries the amplitude and phase angle information of a sinusoidal function.*

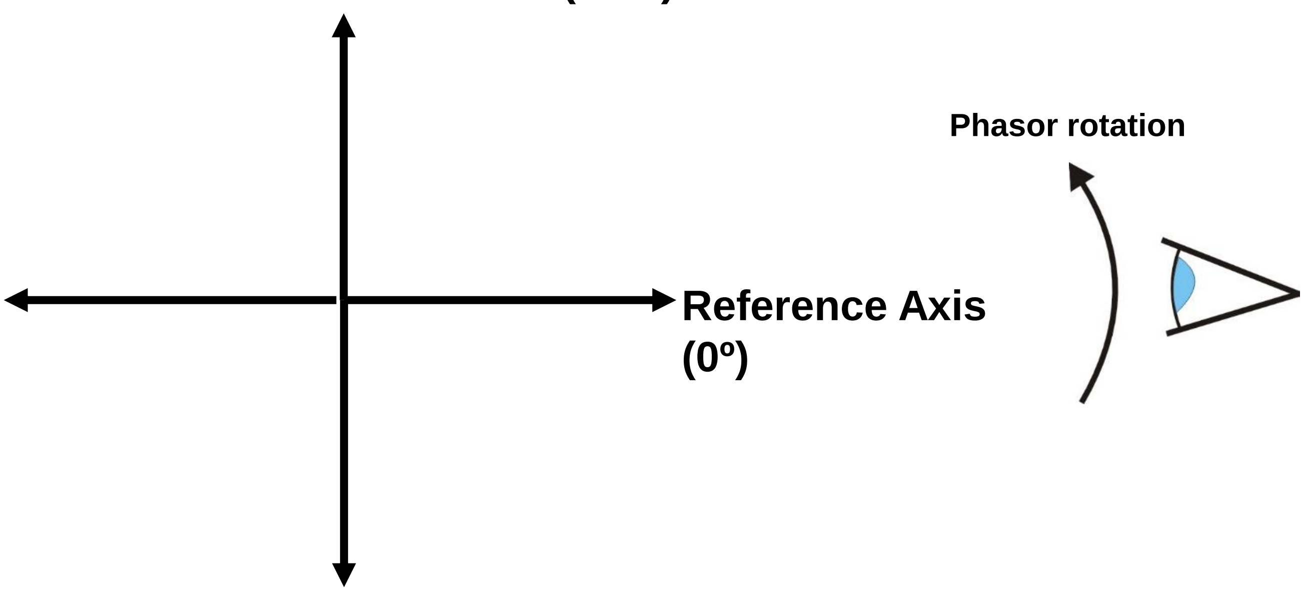
Phasor Diagram representation of Voltages (and Currents):

It is rather time consuming to draw sinusoidal waveforms, a quicker way of representing voltages and currents in in the form of a *PHASOR*. The phasor diagram very quickly illustrates:

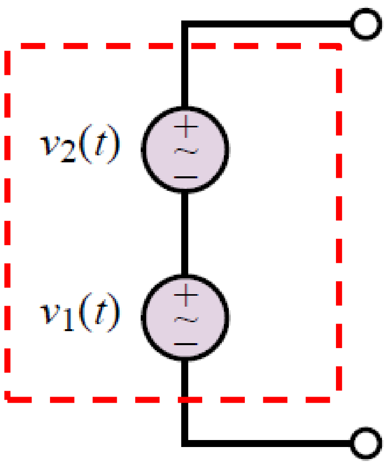
1] The RMS magnitude (see later slide for definition) of the voltage (length of the phasor)

2] The phase of the voltage (the angle of the phasor wrt a Reference (0º) axis)

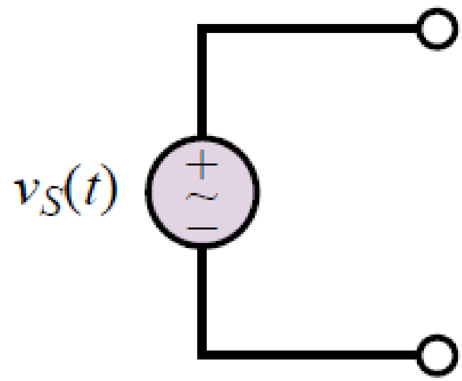
Phasor Diagram:



EXAMPLE – *ADDITION OF TWO SINUSOIDAL SOURCES IN PHASOR NOTATION*



$\equiv$



Equivalent Source

$$v_1(t) = 15\ \cos(377t + \pi/4)\ \text{V}$$

$$v_2(t) = 15\ \cos(377t + \pi/12)\ \text{V}$$

$$v_S(t) = v_1(t) + v_2(t)$$

Note: *$v_1(t)$ and $v_2(t)$ are sinusoids with identical frequency*

Time Domain Form :

$$v_1(t) = 15\ \cos(377t + \pi/4)\ \mathrm{V}$$
$$v_2(t) = 15\ \cos(377t + \pi/12)\ \mathrm{V}$$

Polar Form Phasor :

$$\mathbf{V}_1(j\omega) = 15\angle\pi/4\ \mathrm{V}$$
$$\mathbf{V}_2(j\omega) = 15e^{j\pi/12} = 15\angle\pi/12\ \mathrm{V}$$

Rectangular Form Phasor :

$$\mathbf{V}_1(j\omega) = 10.61 + j10.61\ \mathrm{V}$$
$$\mathbf{V}_2(j\omega) = 14.49 + j3.88$$

Equivalent Voltage Phasor:

$$\mathbf{V}_S(j\omega) = \mathbf{V}_1(j\omega) + \mathbf{V}_2(j\omega) = 25.10 + j14.49 = 28.98e^{j\pi/6} = 28.98\angle\pi/6\ \mathrm{V}$$

Convert $V_S(j\omega)$ to its time domain form:

$$v_S(t) = 28.98\ \cos(377t + \pi/6)\ \mathrm{V}$$

IMPEDANCE（阻抗）：
"A GENERALISED *COMPLEX RESISTANCE*"

Impedance: ***i*-*v* relationship** of three idea circuit elements **LCR** are described using complex number (frequency domain) in light of Voltage Phasor and Current Phasor

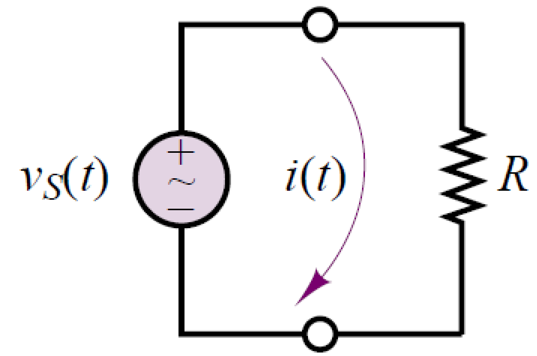
$$\mathbf{Z}(j\omega)=\mathbf{V}(j\omega)/\mathbf{I}(j\omega)$$

Note: *Impedance is just a complex number, but not a Phasor* !!!

IMPEDANCE OF A RESISTOR

Ohm's Law: *i*-*v* relationship of **Resistor** in time domain

$$\boxed{v(t) = i(t)R}$$



$$\because v_S(t) = V_m \cos(\omega t + \theta) \quad or \quad \mathbf{V}_S = V_m e^{j\theta} = V_m \angle\theta$$ ***Voltage Phasor***

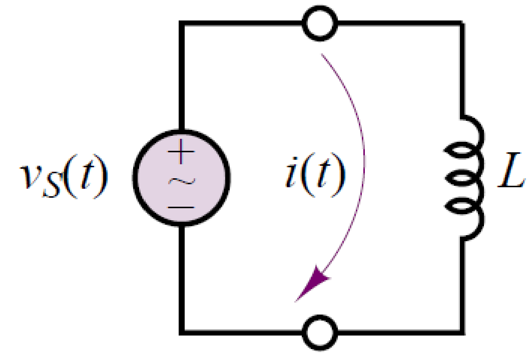
$$\Rightarrow i(t) = \frac{V_m}{R}\cos(\omega t + \theta) \quad or \quad \mathbf{I} = \frac{V_m}{R} e^{j\theta} = \frac{V_m}{R}\angle\theta$$ ***Current Phasor***

$$Z_R(j\omega) = \frac{\mathbf{V}_S(j\omega)}{\mathbf{I}(j\omega)} = R \qquad \text{Impedance of a resistor}$$

IMPEDANCE OF AN INDUCTOR

***i-v* relationship** of **Inductor** in time domain:

$$v_L(t) = L\frac{di_L(t)}{dt}$$

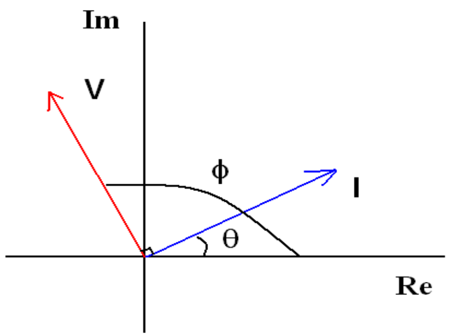


$$\because v_S(t) = V_m\cos(\omega t+\theta) \quad or \quad \mathbf{V}_S = V_m e^{j\theta} = V_m\angle\theta$$ ***Voltage Phasor***

$$\Rightarrow i(t) = \frac{1}{L}\int V_m\cos(\omega t+\theta)\,dt = \frac{V_m}{\omega L}\sin(\omega t+\theta) = \frac{V_m}{\omega L}\cos\left(\omega t+\theta-\frac{\pi}{2}\right) \quad or$$

$$\mathbf{I} = \frac{V_m}{\omega L}e^{j\left(\theta-\frac{\pi}{2}\right)} = \frac{V_m}{\omega L}\angle\left(\theta-\frac{\pi}{2}\right)$$ ***Current Phasor***

$$Z_L(j\omega) = \frac{\mathbf{V}_S(j\omega)}{\mathbf{I}(j\omega)} = \omega L\angle\pi/2 = j\omega L$$ Impedance of an inductor

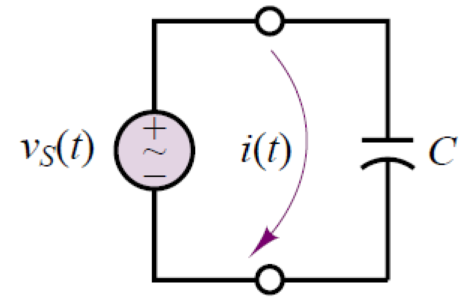


Note: $$j = e^{j\frac{\pi}{2}} = \angle\frac{\pi}{2} = \cos 90° + j\sin 90°$$

IMPEDANCE OF A CAPACITOR

***i-v* relationship** of **Capacitor** in time domain:

$$\boxed{i_C(t) = C\frac{dv_C(t)}{dt}}$$



$$\because v_S(t) = V_m\cos(\omega t+\theta) \quad or \quad \mathbf{V}_S = V_m e^{j\theta} = V_m\angle\theta$$ ***Voltage Phasor*** AC circuits

$$\Rightarrow i(t) = C\frac{d\left[V_m\cos(\omega t+\theta)\right]}{dt} = -\omega C V_m\sin(\omega t+\theta) = \omega C V_m\cos\left(\omega t+\theta+\frac{\pi}{2}\right) \quad or$$

$$\mathbf{I} = \omega C V_m e^{j\left(\theta+\frac{\pi}{2}\right)} = \omega C V_m\angle\left(\theta+\frac{\pi}{2}\right)$$ ***Current Phasor***

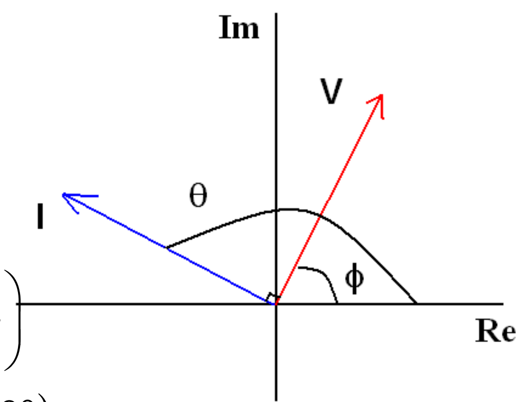
$$Z_C(j\omega) = \frac{\mathbf{V}_S(j\omega)}{\mathbf{I}(j\omega)} = \frac{1}{\omega C}\angle-\pi/2$$

$$= \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

Impedance of a capacitor

Note: $j = \frac{1}{j} = e^{-j\frac{\pi}{2}} = \angle\left(-\frac{\pi}{2}\right)$

$$= \cos(-90^\circ) + j\sin(-90^\circ)$$



IMPEDANCE IN AC CIRCUIT ANALYSIS – GENERISED COMPLEX "*RESISTANCE*" OF LCR

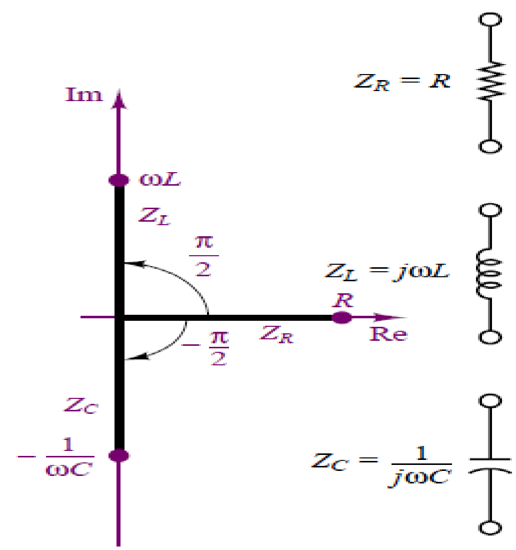
Generic "AC Ohm's Law": $\mathbf{V} = \mathbf{Z}\mathbf{I},$

$$Z_R(j\omega) = \frac{\mathbf{V}_S(j\omega)}{\mathbf{I}(j\omega)} = R$$ Impedance of a resistor

$$Z_L(j\omega) = \frac{\mathbf{V}_S(j\omega)}{\mathbf{I}(j\omega)} = \omega L \angle \pi/2 = j\omega L$$ Impedance of an inductor

$$Z_C(j\omega) = \frac{\mathbf{V}_S(j\omega)}{\mathbf{I}(j\omega)} = \frac{1}{\omega C} \angle -\pi/2$$

$$= \frac{-j}{\omega C} = \frac{1}{j\omega C}$$ Impedance of a capacitor



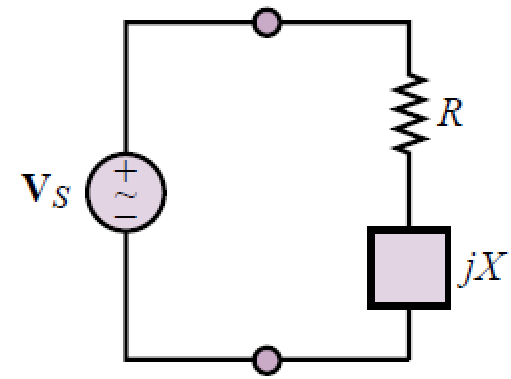
$$e^{j\frac{\pi}{2}} = e^{j90^\circ} = \cos 90^\circ + j \sin 90^\circ = j$$

Impedance of LCR

Component	Voltage/Current Phase Relationship	Complex Impedance (Cartesian)	Complex Impedance (Polar)
Resistance	**Voltage and Current are in phase**	$Z_R = R + j0$	$Z_R = R\angle 0^o$
Inductance	**Current lags the Voltage by 90º**	$Z_L = 0 + j\omega L$	$Z_L = \omega L\angle 90^o$
Capacitance	**Current leads the Voltage by 90º**	$Z_C = 0 - j/\omega C$	$Z_C = \frac{1}{\omega C}\angle -90^o$

Note The REACTANCE (X_L) of an Inductance = ωL and the REACTANCE (X_C) of a capacitance = 1/ωC

GENERIC COMPONENT IMPEDANCE

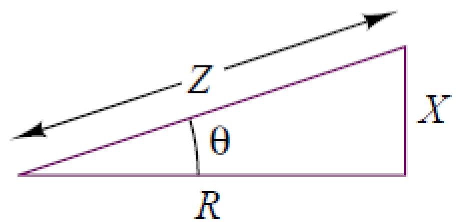


Generic component Impedance represents the **equivalent impedance** of a combination of LCR components

Impedance 阻抗 $Z = R + jX$

resistance 电阻

reactance 电抗（感抗）



Impedance Triangle

$\frac{1}{Z} = Y = G + jB$

admittance 导纳

conductance 电导

susceptance 电纳

$Z = R \qquad j\omega L \qquad \frac{1}{j\omega C} = \frac{-j}{\omega C}$

Impedance enables AC circuits to be analysed in the same way as DC ~~dc~~ circuits, but with complex numbers.

BASIC OPERATIONS + - × ÷ OF COMPLEX NUMBER

$$\mathbf{Z} = R + jX = |Z|\angle\varphi, \quad |Z| = \sqrt{R^2 + X^2}, \quad \varphi = \arctan(X/R), \quad R = |Z|\cos(\varphi) \quad X = |Z|\sin(\varphi)$$

Conjugate $\mathbf{Z}^* = R - jX, \quad ZZ^* = |Z|^2 = (R + jX)(R - jX) = R^2 + X^2, \quad j^2 = -1$

$|Z|$ - magnitude (modulus 模), Φ - phase angle

$$\mathbf{Z}_1 = a + jb = |Z_1|\angle\varphi_1, \quad \mathbf{Z}_2 = c + jd = |Z_2|\angle\varphi_2$$

$$\mathbf{Z}_1 \pm \mathbf{Z}_2 = (a \pm b) + j(c \pm d) = (|Z_1|\cos\varphi_1 \pm |Z_2|\cos\varphi_2) + j(|Z_1|\sin\varphi_1 \pm |Z_2|\sin\varphi_2)$$

$$\mathbf{Z_1 Z_2} = (a + jb)(c + jd) = (ac - bd) + j(ad + bc) = |Z_1||Z_2|\angle(\varphi_1 + \varphi_2)$$

$$\mathbf{Z_1/Z_2} = (a + jb)/(c + jd) = [(a + jb)(c - jd)]/(c^2 + d^2)$$

$$= [(ac + bd) + j(bc - ad)]/(c^2 + d^2) = (|Z_1|/|Z_2|)\angle(\varphi_1 - \varphi_2)$$

IMPEDANCE ENABLES AC CIRCUITS TO BE ANALYZED IN THE SAME WAY AS DC ("RESISTIVE") CIRCUITS, BUT WITH COMPLEX NUMBERS

Resistor

Series Resistance

$$R_{eq} = \sum_{s=1}^{S} R_s$$

Parallel Resistance

$$R_{eq} = \left[\sum_{p=1}^{P} \frac{1}{R_p} \right]^{-1}$$

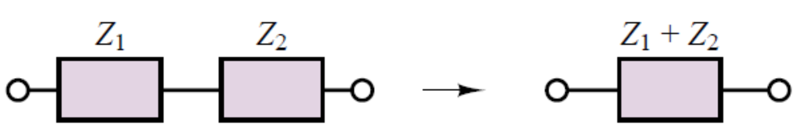
Impedance

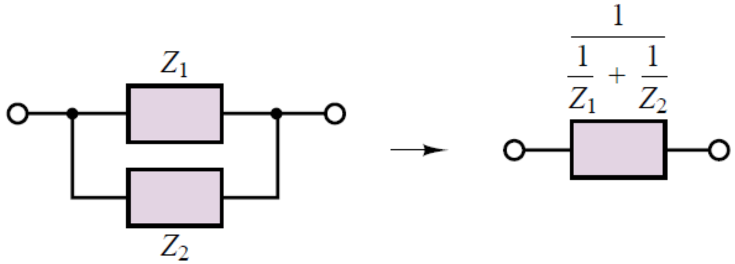
Series Impedance

$$Z_{eq} = \sum_{s=1}^{S} Z_s = Z_1 + Z_2 + \ldots + Z_S$$

Parallel Impedance

$$Z_{eq} = \left[\sum_{p=1}^{P} \frac{1}{Z_p} \right]^{-1} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_P}}$$





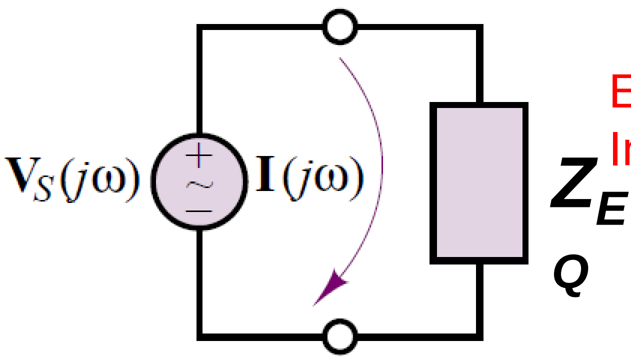
ADMITTANCE 导纳

For the analysis of certain circuits, such as circuits with many parallel elements, it is more easily to be handled in terms of Admittance Y. Just as the *reciprocal*(倒数) conductance(电导) **G=1/R** of a resistive element **R** in DC circuit analysis , Admittance **Y** is defined as the inverse of Impedance **Z** in AC circuit analysis.

I = Y V (Y = 1/Z), where **Y** is admittance of the component

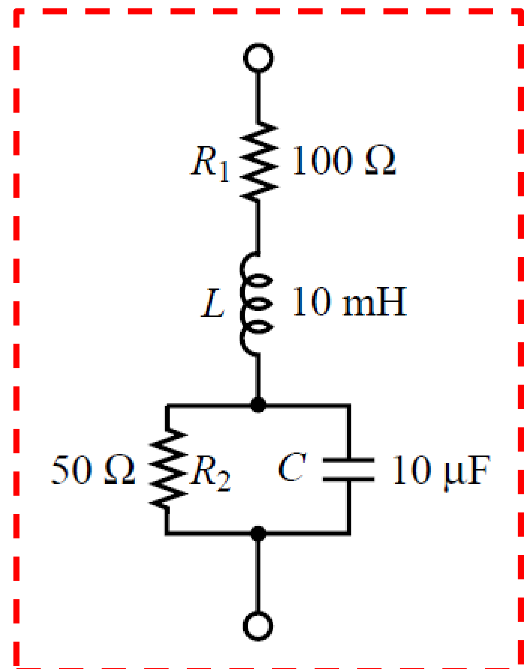
Resistor:	$\mathbf{Y_R} =$	$1/R = G$	$G\angle 0^o$
Capacitor:	$\mathbf{Y_C} =$	$j\omega C$	$\omega C\angle 90^o$
Inductor:	$\mathbf{Y_L} =$	$-j/(\omega L)$	$1/\omega L\angle -90^o$

EXAMPLE – *IMPEDANCE OF A GENERIC COMPONENT*

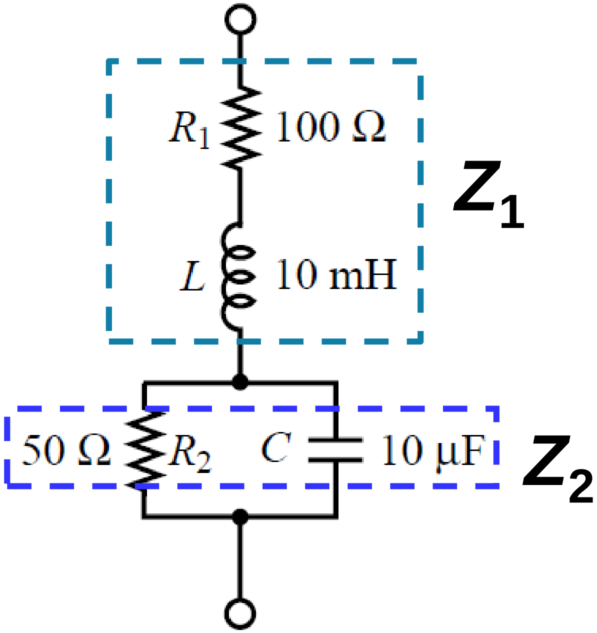


Equivalent Impedance

$\omega = 10^4$ rad/s



Z_{EQ}



$$Z_1 = Z_{R1} + Z_L = R_1 + j\omega L$$

$$Z_2 = Z_{R2} \parallel Z_C = \frac{Z_{R2} Z_C}{Z_{R2} + Z_C}$$

$$= \frac{R_2\left(1/\left(j\omega C\right)\right)}{R_2 + 1/\left(j\omega C\right)} = \frac{R_2}{1 + j\omega C R_2}$$

Equivalent Impedance

$$Z_{EQ} = Z_1 + Z_2 = R_1 + j\omega L + \frac{R_2}{1 + j\omega C R_2} = 100 + j10^4 \times \left(10 \times 10^{-3}\right) + \frac{50}{1 + j10^4 \times \left(10 \times 10^{-6}\right) \times 50}$$

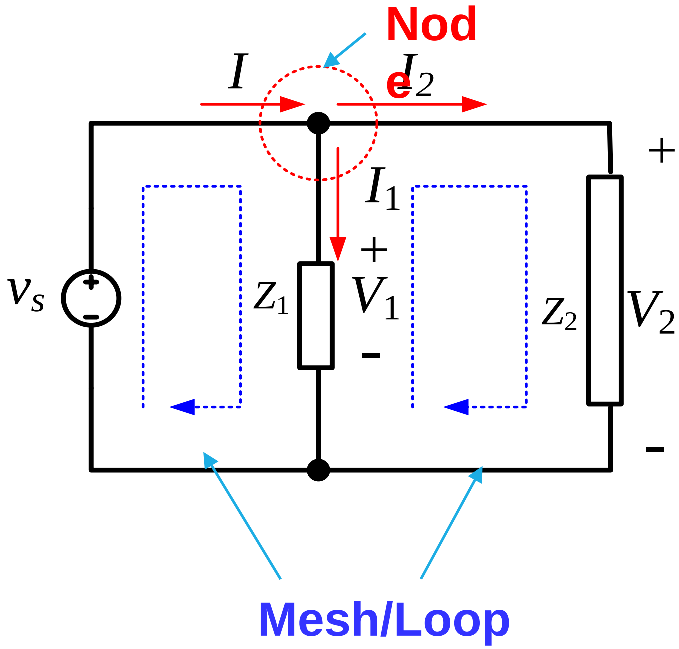
$$= 100 + j100 + \frac{50}{1 + j5} = 100 + j100 + 1.92 - j9.62 = 101.92 + j90.38 = 136.2\angle 0.723\Omega$$

PROCEDURE FOR AC CIRCUIT ANALYSIS

1. Identify the **sinusoidal source(s)** and note the **excitation frequency**.
2. Convert the **source(s)** to **phasor form.**
3. Represent each **circuit element** by its **impedance.**
4. Solve the resulting **phasor circuit,** using appropriate network analysis tools.
5. Convert the (phasor-form) answer to its **time-domain equivalent**

AC CIRCUIT ANALYSIS WITH *PHASOR AND IMPEDANCE*

Current Node method 电流节点法 and Voltage Mesh (Loop) method 电压网格/回路法 with *Voltage/Current* Phasors



- Kirchoff's Current Law (KCL)

$$\boxed{\text{KCL}: \quad \sum_k I_k = 0, \quad \textit{for any node}}$$

- Kirchoff's Voltage Law (KVL)

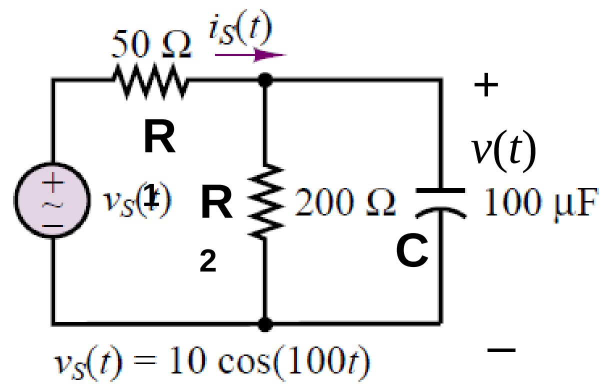
$$\boxed{\text{KVL}: \quad \sum_k V_k = 0, \quad \textit{for any loop}}$$

Example – A Simple AC Circuit

Source (Phasor)

$$v_S(t) = 10\ \cos(100t)\ \text{V};\ \omega = 100\ \text{rad/s}$$

$$\mathbf{V}_S(j\omega) = 10\angle 0\ \text{V}.$$



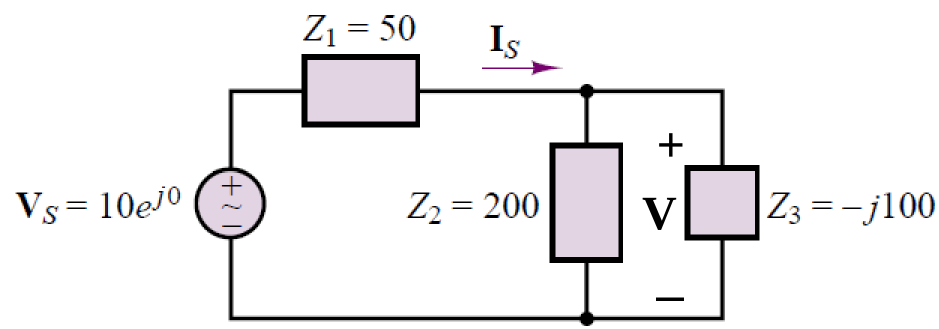
Circuit Element (Impedance)

$$Z_{R1} = 50\ \Omega,\ Z_{R2} = 200\ \Omega,\ Z_C = 1/(j100 \times 10^{-4}) = -j100\ \Omega$$

Voltage Mesh (Loop) method

$$\mathbf{I}_S = \frac{\mathbf{V}_S}{Z_{R1} + Z_{R2} \parallel Z_C} = \frac{\mathbf{V}_S}{R_1 + \dfrac{R_2}{1 + j\omega C R_2}}$$

$$= 0.083\angle 0.727\,\mathbf{A}$$

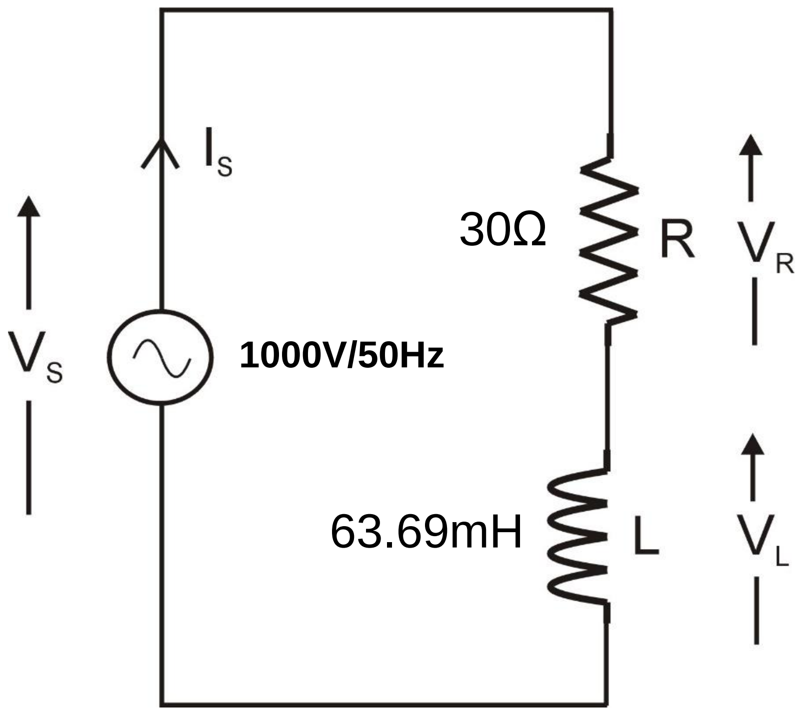


The same circuit in phasor form

Time Domain $\quad i_s(t) = 0.083\ \cos(100t + 0.727)\ \text{A}$

Example:

Solution done on whiteboard during lecture



Determine the following:

1] The Inductor Reactance X_L

2] The Supply Current I_S

3] Draw the phasor diagram for the supply voltage and current

4] Determine V_R and V_L and add these to the phasor diagram

Note: 1000V is RMS voltage!

TRIGONOMETRY 三角函数

$$\left(\sin\left(ax\right)\right)' = a\cos x, \quad \left(\cos\left(ax\right)\right)' = -a\sin x$$

$$\int \sin\left(ax\right)dx = -\frac{1}{a}\cos\left(ax\right), \quad \int \cos\left(ax\right)dx = \frac{1}{a}\sin\left(ax\right)$$

$$\cos\left(A \pm B\right) = \cos A\cos B \mp \sin A\sin B$$

$$\sin\left(A \pm B\right) = \sin A\cos B \pm \cos A\sin B$$

$$\cos x = \sin\left(x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) = -\cos\left(\frac{\pi}{2} + x\right)$$