



POWER ENGINEERING
#05 THREE-PHASE AC POWER SYSTEMS
THREE-PHASE CIRCUITS

2018



University
of Glasgow

Three Phase AC Systems

3 Phase Power Networks are the de facto standard in the industrialised world. In today's lecture you will get an understanding of why 3 phase systems are used and the basics of this approach.

Today we will investigate:

- Why 3 Phase?
- 3 Phase Generator Voltages
- 3 Phase Transmission
- A (balanced) STAR connected 3 phase load
- A (balanced) DELTA connected 3 phase load

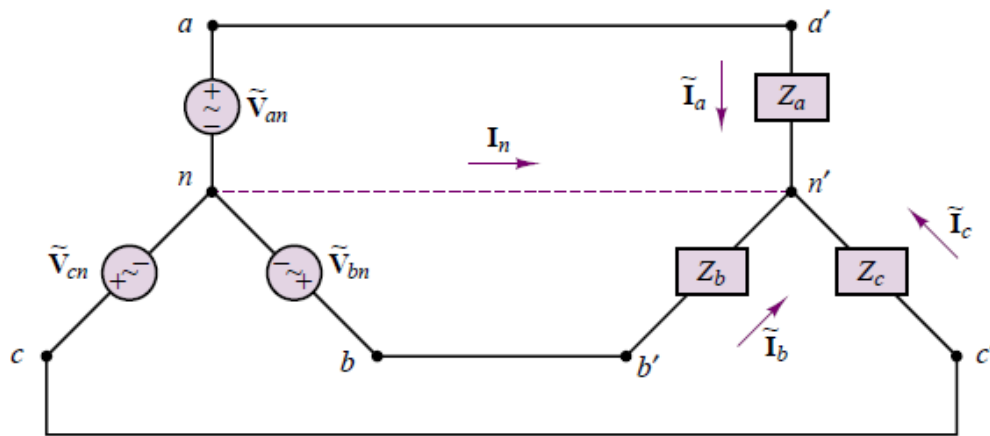
Why 3 Phase?

- The 'workhorse' of industry; the 3 phase induction motor requires a 3 phase AC power supply
- A balanced 3 phase system will lead to **constant instantaneous power demand** on the generator – results in a smoother running generator
- A 3 phase generator has far better energy density compared to a single phase machine, hence it is smaller for a given output power (see next slides)
- The weight of the conductors and other components in a three-phase system is much lower than in a single-phase system delivering the same amount of power.



BALANCED THREE-PHASE SYSTEM

Voltages of equal amplitude and frequency displaced in phase by 120° —has the property of delivering constant instantaneous power.



Balanced three-phase AC circuit
in **wye (or Y) configuration**

$$\tilde{V}_{an} = \tilde{V}_{an} \angle 0^\circ$$

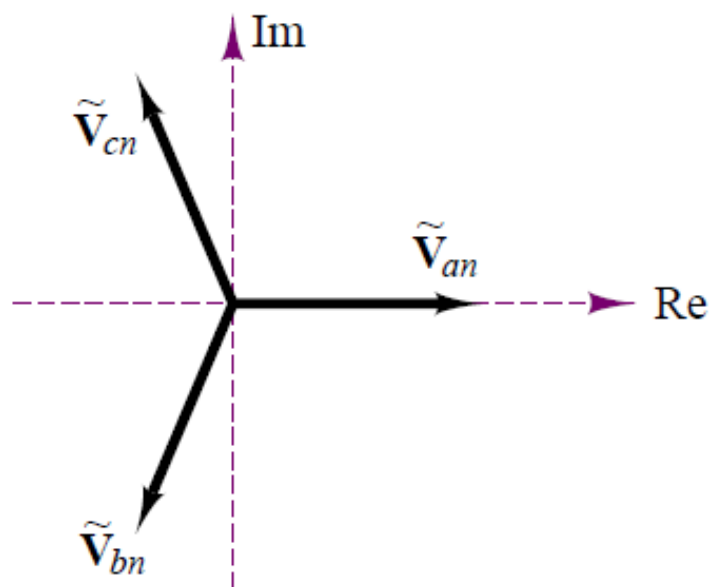
$$\tilde{V}_{bn} = \tilde{V}_{bn} \angle -120^\circ$$

$$\tilde{V}_{cn} = \tilde{V}_{cn} \angle -240^\circ = \tilde{V}_{cn} \angle 120^\circ$$

$$\tilde{V}_{an} = \tilde{V}_{bn} = \tilde{V}_{cn} = \tilde{V}$$



$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = 0$$



$$\tilde{\mathbf{V}}_{ab} = \tilde{\mathbf{V}}_{an} + \tilde{\mathbf{V}}_{nb} = \tilde{\mathbf{V}}_{an} - \tilde{\mathbf{V}}_{bn}$$

$$\tilde{\mathbf{V}}_{ab} = \tilde{V} \angle 0^\circ - \tilde{V} \angle -120^\circ = \sqrt{3} \tilde{V} \angle 30^\circ$$

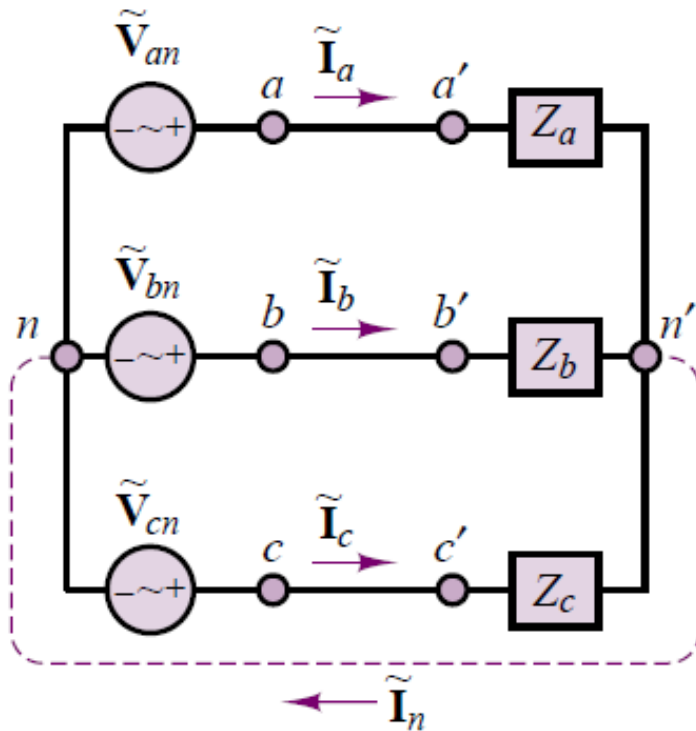
$$\tilde{\mathbf{V}}_{bc} = \tilde{V} \angle -120^\circ - \tilde{V} \angle 120^\circ = \sqrt{3} \tilde{V} \angle -90^\circ$$

$$\tilde{\mathbf{V}}_{ca} = \tilde{V} \angle 120^\circ - \tilde{V} \angle 0^\circ = \sqrt{3} \tilde{V} \angle 150^\circ$$

$$\tilde{\mathbf{I}}_n = (\tilde{\mathbf{I}}_a + \tilde{\mathbf{I}}_b + \tilde{\mathbf{I}}_c)$$

$$= \frac{1}{Z} (\tilde{\mathbf{V}}_{an} + \tilde{\mathbf{V}}_{bn} + \tilde{\mathbf{V}}_{cn})$$

$$= 0$$



$$p_a(t) = \frac{\tilde{V}^2}{R}(1 + \cos 2\omega t)$$

$$p_b(t) = \frac{\tilde{V}^2}{R}[1 + \cos(2\omega t - 120^\circ)]$$

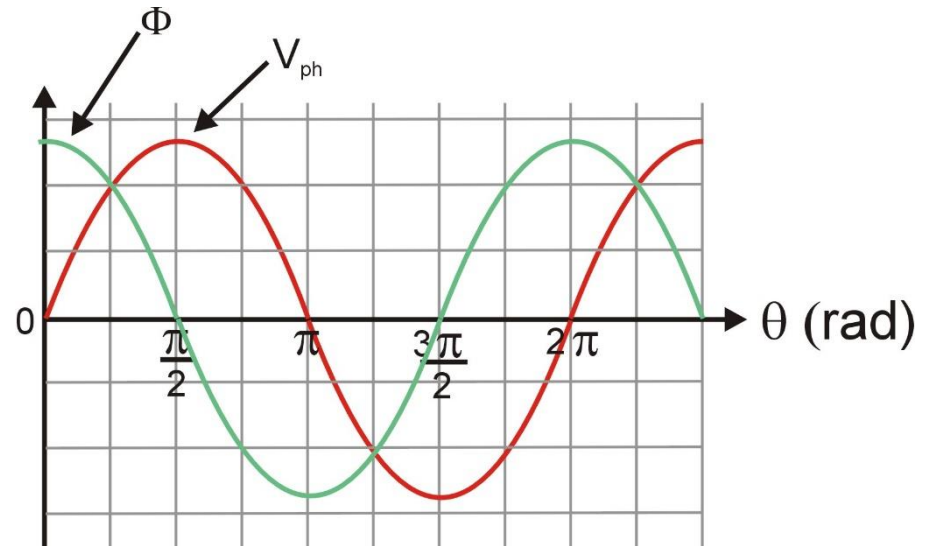
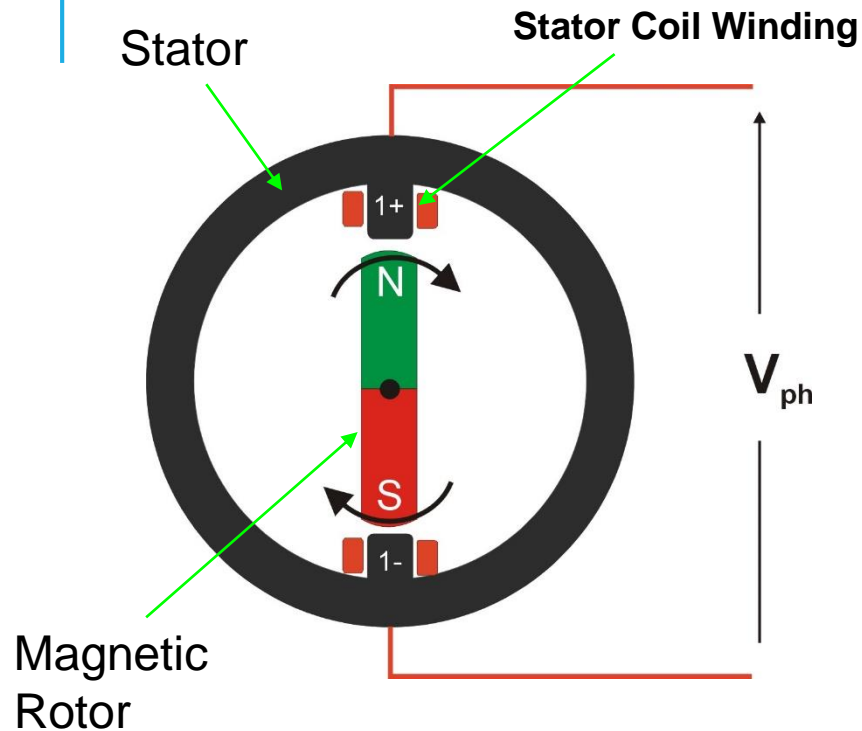
$$p_c(t) = \frac{\tilde{V}^2}{R}[1 + \cos(2\omega t + 120^\circ)]$$

$$p(t) = p_a(t) + p_b(t) + p_c(t)$$

$$= \frac{3\tilde{V}^2}{R} + \frac{\tilde{V}^2}{R}[\cos 2\omega t + \cos(2\omega t - 120^\circ) + \cos(2\omega t + 120^\circ)]$$

$$= \frac{3\tilde{V}^2}{R} = \text{a constant!}$$

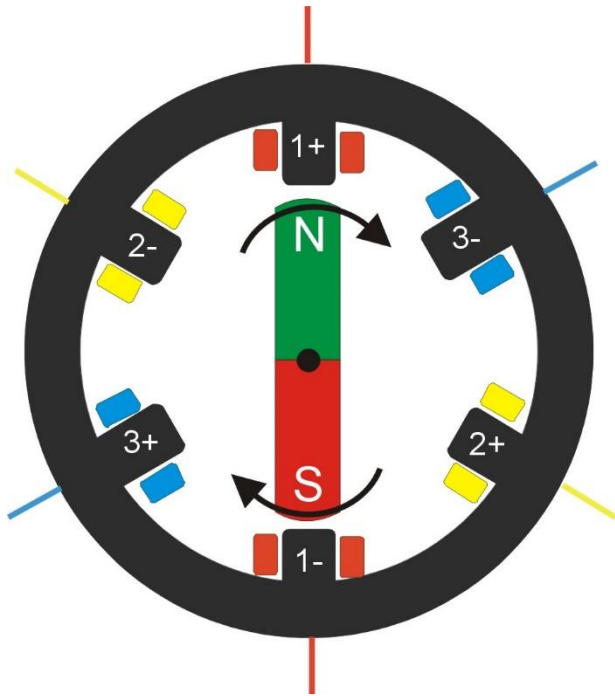
Single Phase AC Generator



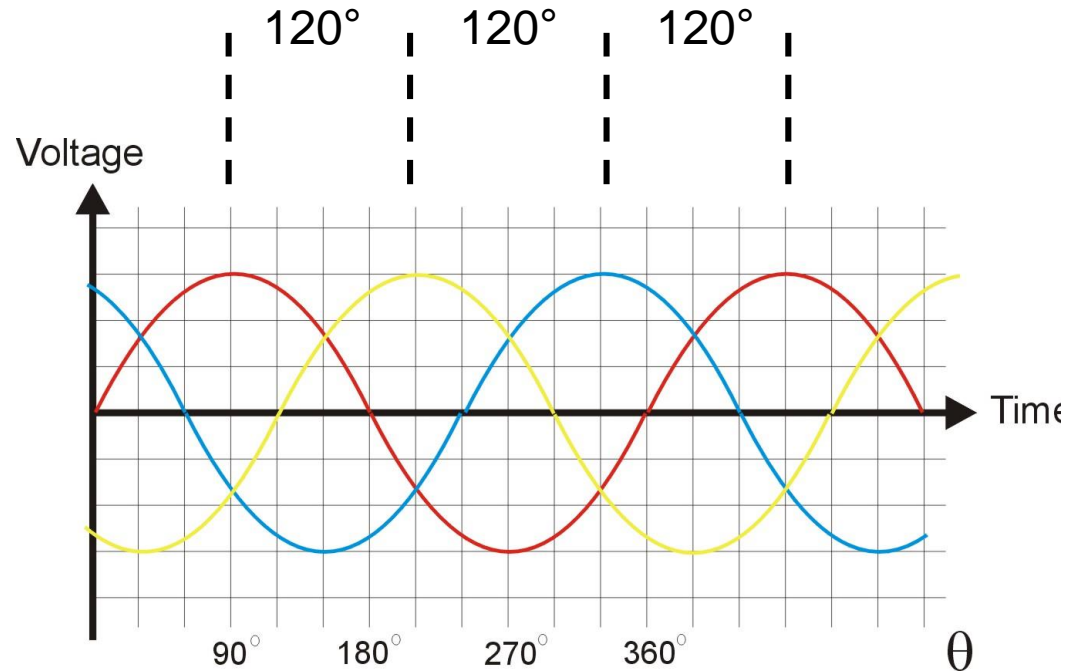
$$\text{Voltage } (V_{ph}) \text{ induced in Stator Winding Coils (1+, 1-) } = \frac{N \cdot d\phi}{dt}$$

Where N is the number of turns in the Stator coils and ϕ is the Rotor Magnet flux

3 Phase AC Generator



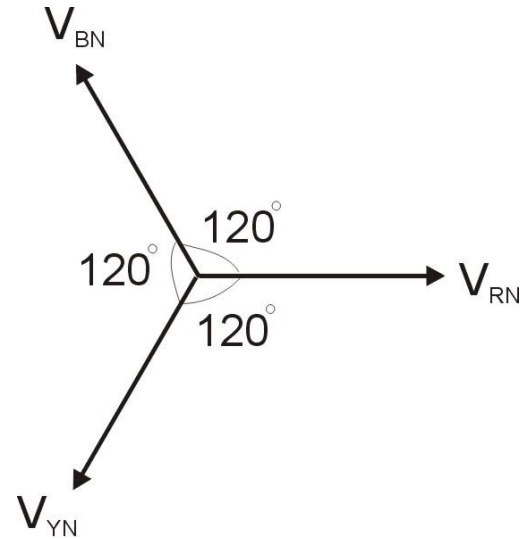
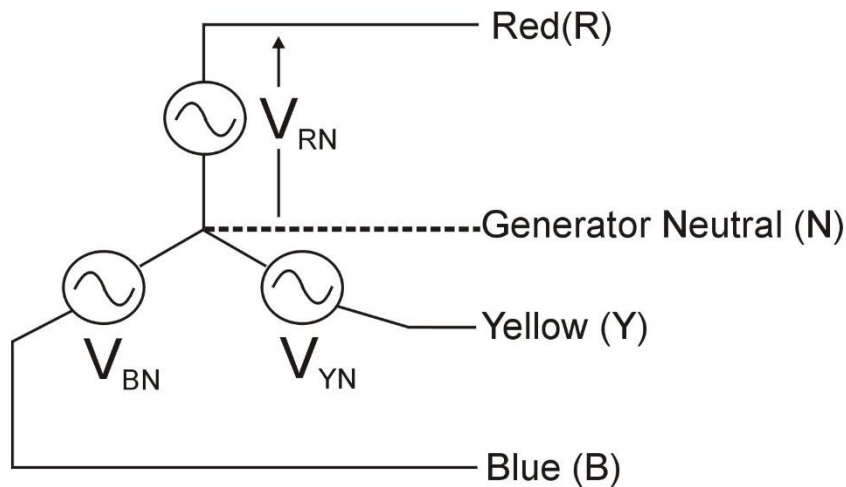
**Now have 3 Phase windings
on the Stator 120° apart**



**Countries use different conventions for
naming the 3 PHASE voltages. We will adopt
the (old!) UK convention of RED, YELLOW &
BLUE PHASES**

3 Phase Generator: Circuit Diagram & Phasor Diagram

3-phase 4-wire



$$V_{RN} = V_{pk} \cos \theta$$

$$V_{YN} = V_{pk} \cos \left(\theta - \frac{2\pi}{3} \right)$$

$$V_{BN} = V_{pk} \cos \left(\theta + \frac{2\pi}{3} \right)$$

Complex Form (Polar):

$$V_{RN} = V_{ph} \angle 0^\circ$$

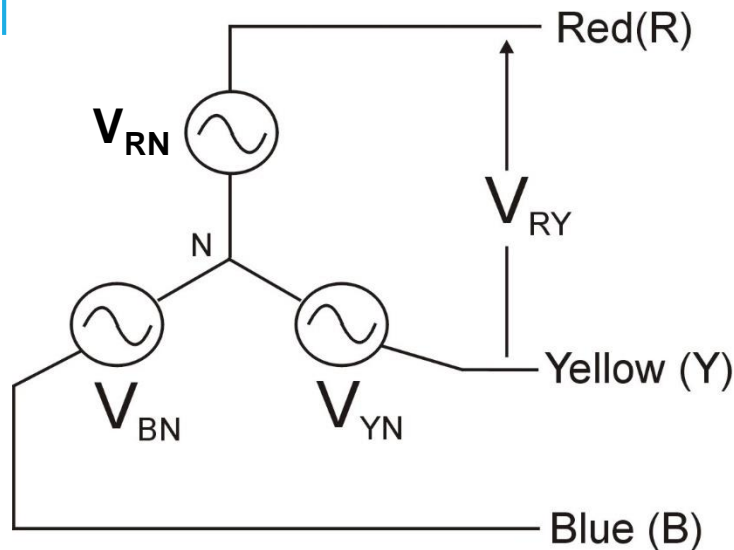
$$V_{YN} = V_{ph} \angle -120^\circ$$

$$V_{BN} = V_{ph} \angle 120^\circ$$

Note: all 3 Phase Voltages have the same rms magnitude V_{ph}

3 Phase Generator: Line Voltage V_{RY}

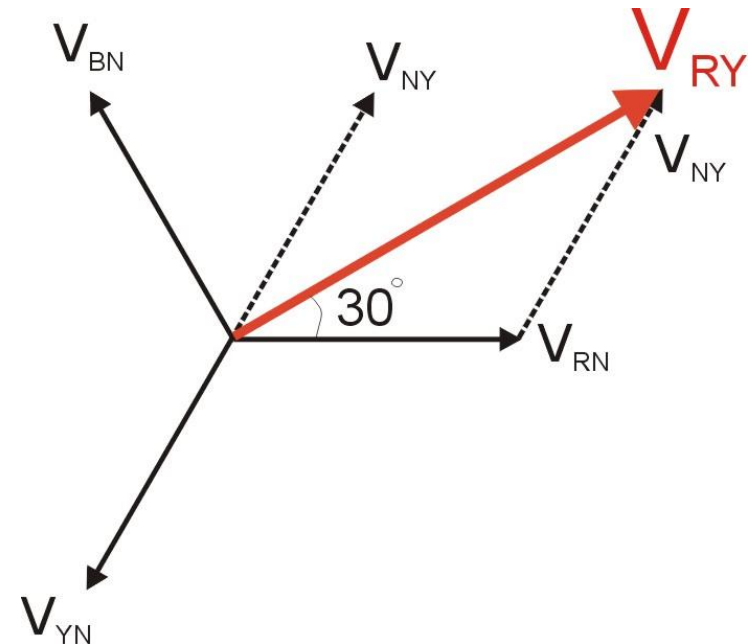
3-phase 3-wire



$$V_{RY} = V_{RN} + V_{NY} = V_{RN} - V_{YN}$$

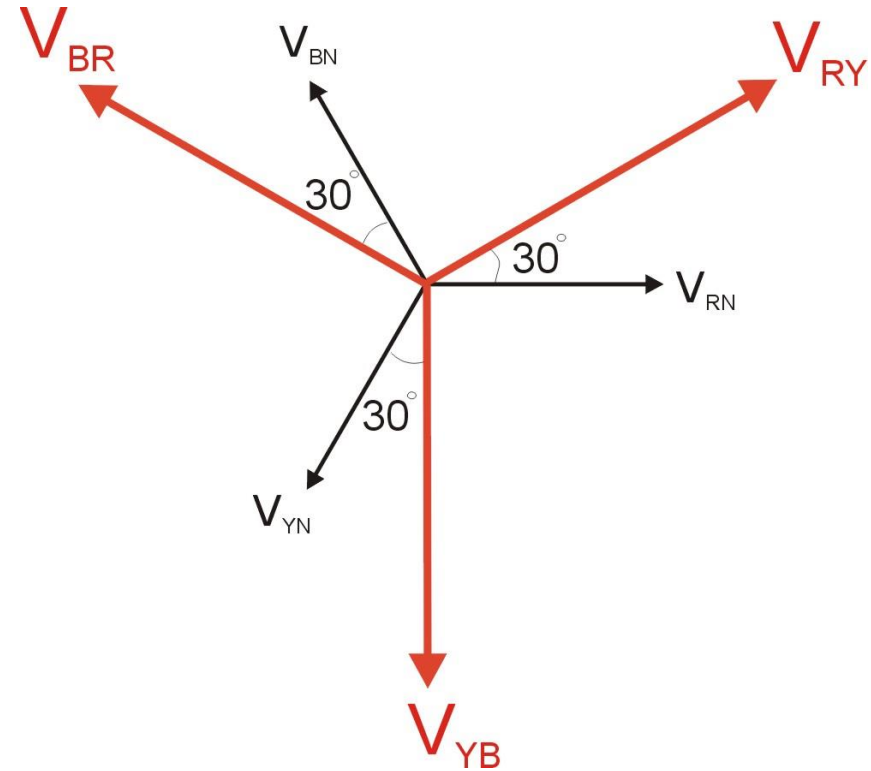
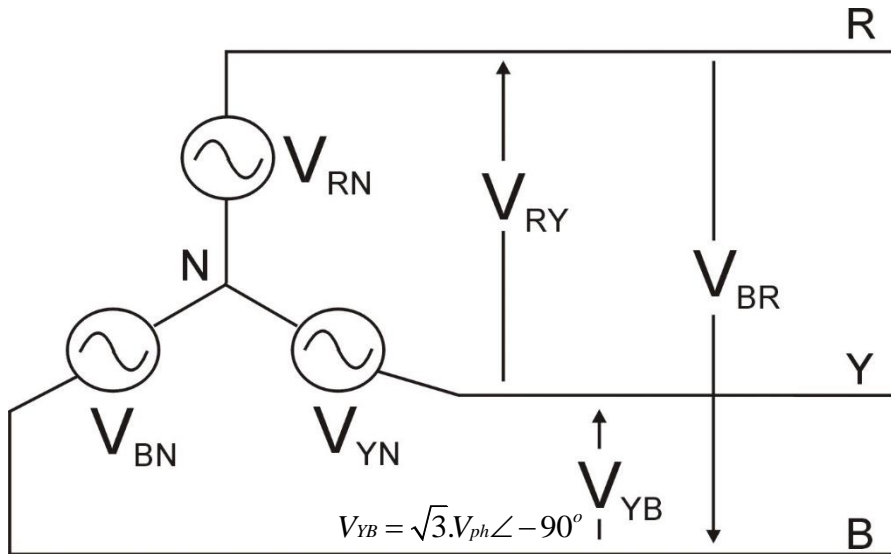
$$V_{RY} = \sqrt{3} \cdot V_{ph} \angle 30^0$$

Phasor Diagram



Proof: Convert V_{RN} and V_{NY} into Cartesian Form and ADD

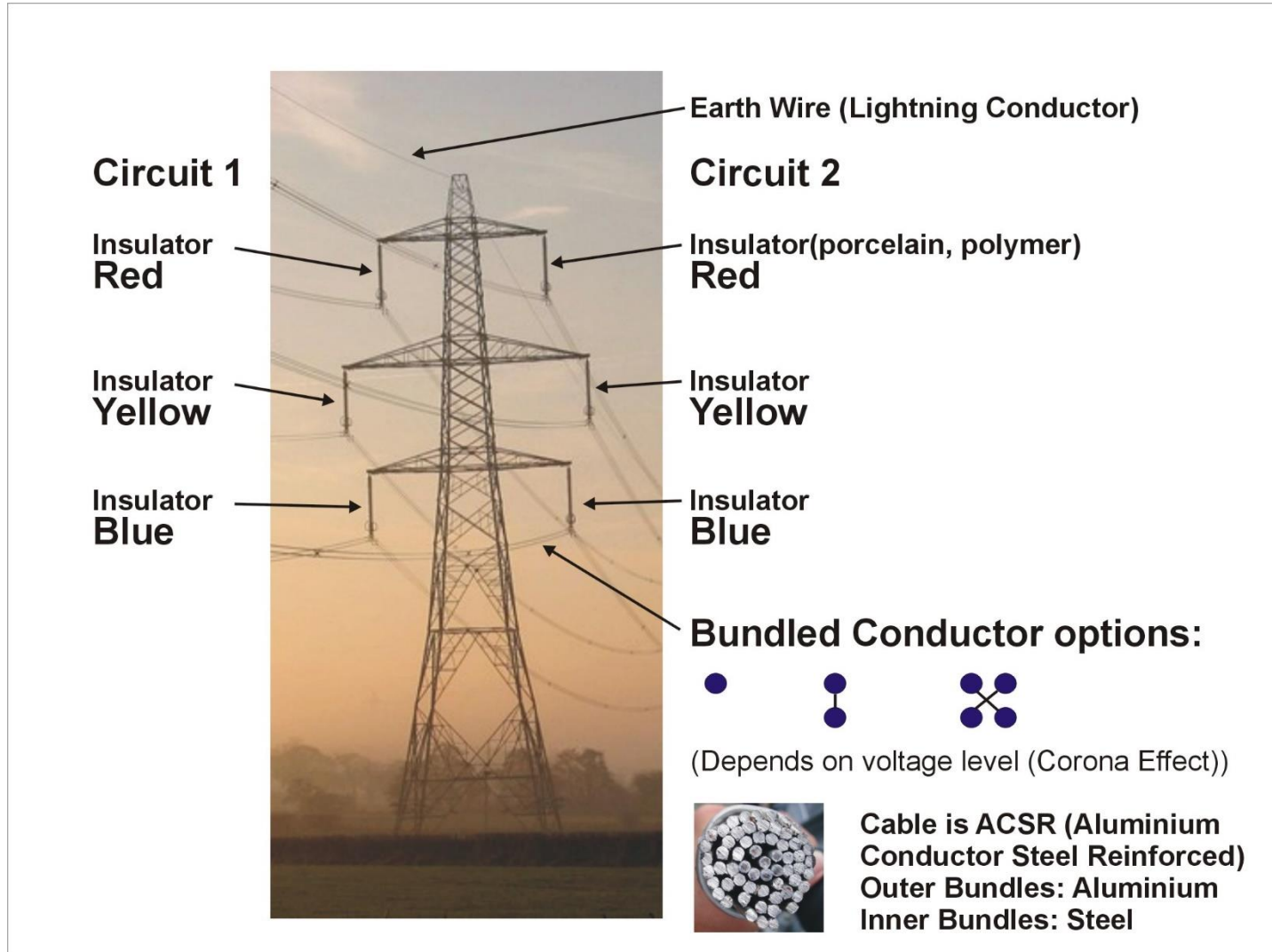
3 Phase Generator: Line Voltages



$$V_{RY} = \sqrt{3} \cdot V_{ph} \angle 30^\circ \quad V_{BR} = \sqrt{3} \cdot V_{ph} \angle 150^\circ \quad V_{YB} = \sqrt{3} \cdot V_{ph} \angle -90^\circ$$

Note: V_{RN} has been chosen to be on Reference axis

3 Phase Transmission Lines: Anatomy of a pylon

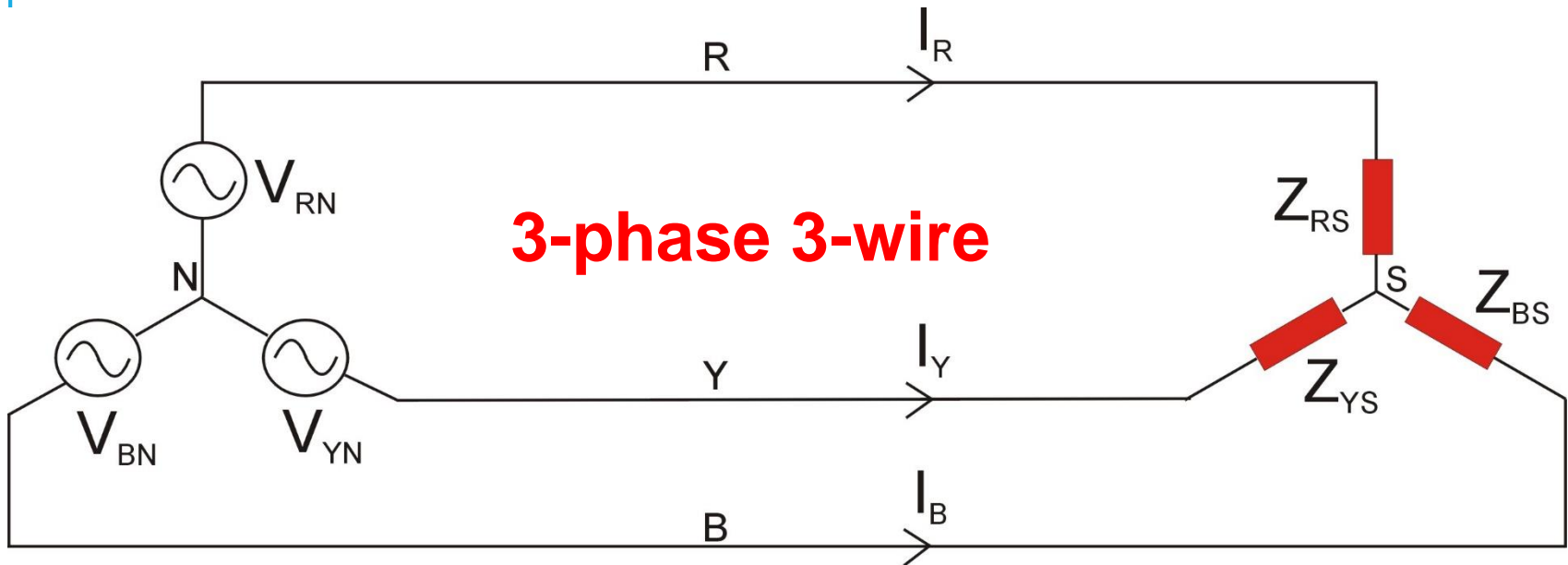


3 Phase Transmission Lines: some interesting facts!

If you want to find out more about pylons then check out the Pylon Appreciation Society at <http://www.pylons.org/>

- The 'standard' UK pylon was chosen by Sir Reginald Blomfield (a leading architect) in 1928
- The tallest pylon in the world is in China. The Yangtze River crossing pylon is 346.5m high
- The Beaulieu-Denny transmission line upgrade in Scotland consists of 600 towers with an average height of 53m
- The UK National Grid is made up of 440kV, 275kV, 132kV, 110kV, 33kV and 11kV transmission lines
- Q. Why do you never see birds standing on large transmission lines?

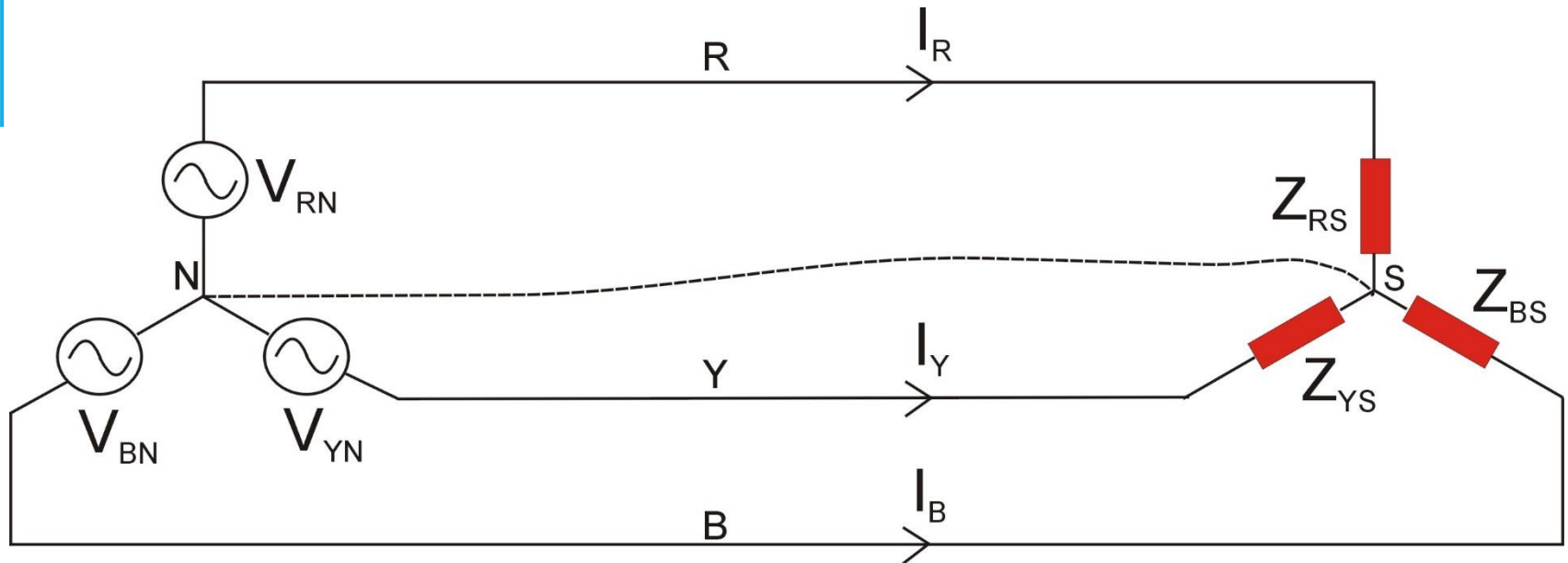
A (balanced) STAR connected 3 Phase Load



Notes:

1. 'Balanced' means that the load Impedances are equal: $Z_{RS} = Z_{YS} = Z_{BS}$
2. I_R , I_Y and I_B are termed the LINE currents
3. If the load is balanced then the load STAR point (S) is at the same voltage as the Generator Neutral (N): $V_{SN} = 0$

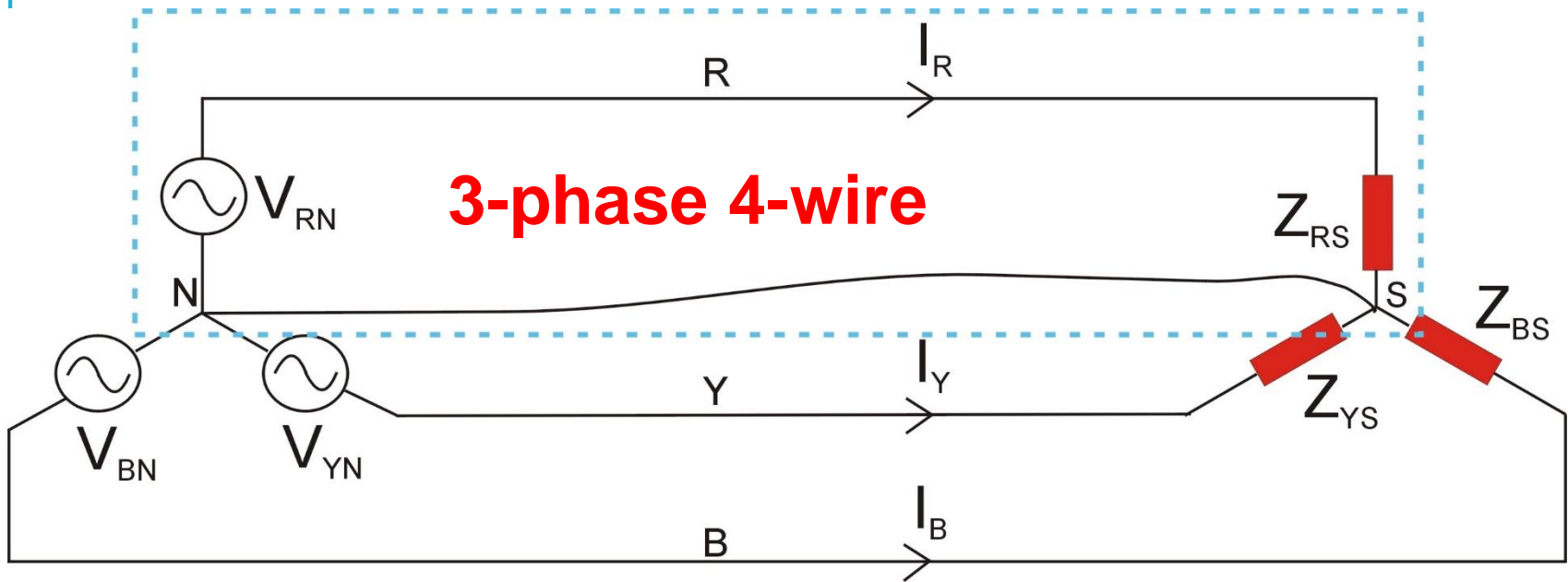
A (balanced) STAR connected 3 Phase Load



Notes:

1. 'Balanced' means that the load Impedances are equal: $Z_{RS} = Z_{YS} = Z_{BS}$
2. I_R , I_Y and I_B are termed the LINE currents
3. If the load is balanced then the load STAR point (S) is at the same voltage as the Generator Neutral (N): $V_{SN} = 0$

A (balanced) STAR connected 3 Phase Load



Observing the circuit within the blue dotted line we can see the following very simple relationship:

$$V_{RN} = Z_{RS} \cdot I_R$$

Therefore determining line currents (and subsequently power quantities) is relatively straight-forward for a given magnitude of phase voltage and load Impedance, the result being equal magnitude line currents 120° out of phase with one another.

Balanced STAR connected load: Line Currents

Notes:

1. Line Currents are 120° apart and equal in magnitude.

2. $V_{RN} = Z_{RS}.I_R$

$$V_{YN} = Z_{YS}.I_Y$$

$$V_{BN} = Z_{BS}.I_B$$

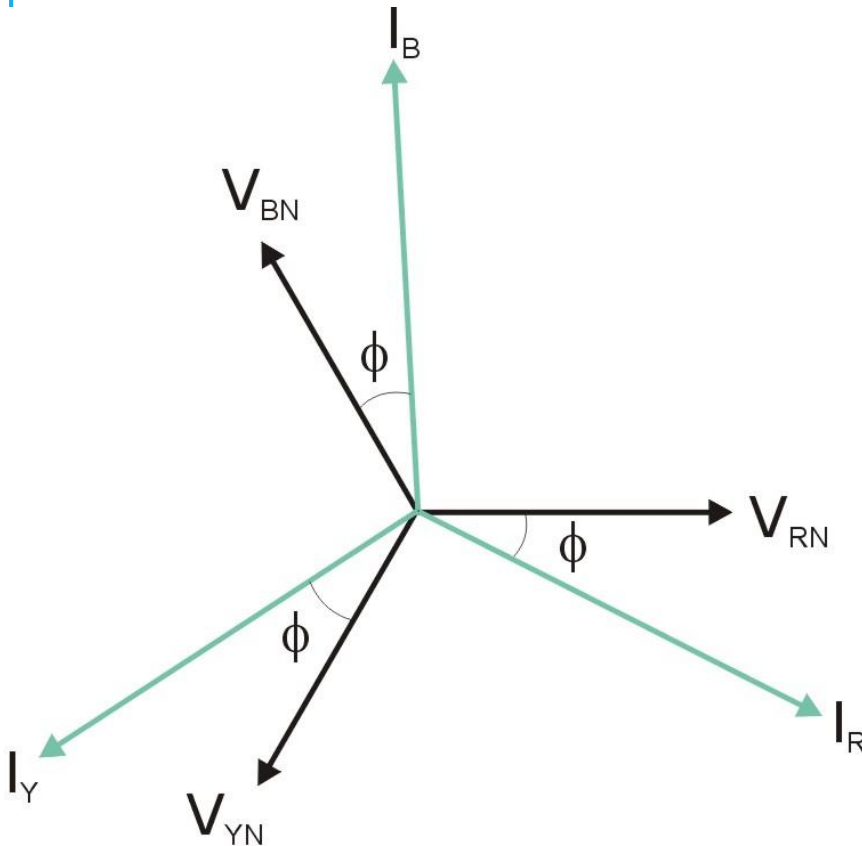
3. The power equations for any phase are:

$$S = V_{ph}.I_L$$

$$P = V_{ph}.I_L.\cos\phi$$

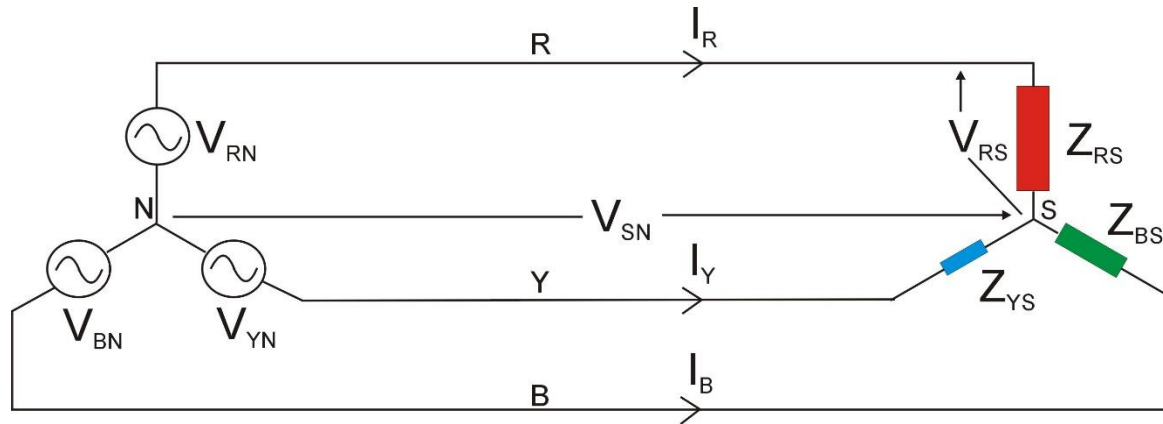
$$Q = V_{ph}.I_L.\sin\phi$$

Where V_{ph} is the rms magnitude of the phase voltage, and I_L is the rms line current and Φ is the angle between them



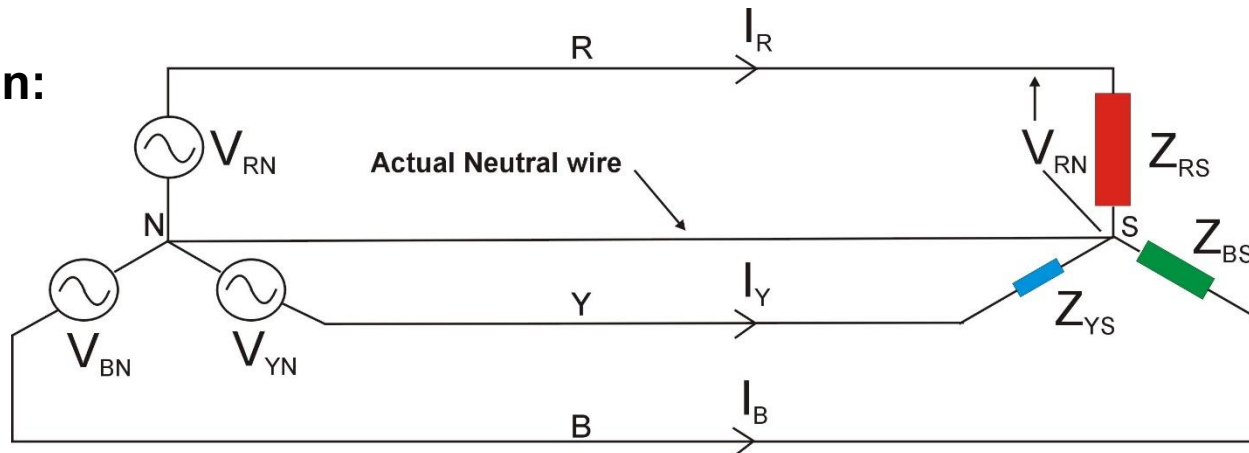
Note: Load Impedances shown here are Resistive + Inductive

The effects of an UNbalanced STAR connected 3 Phase Load:

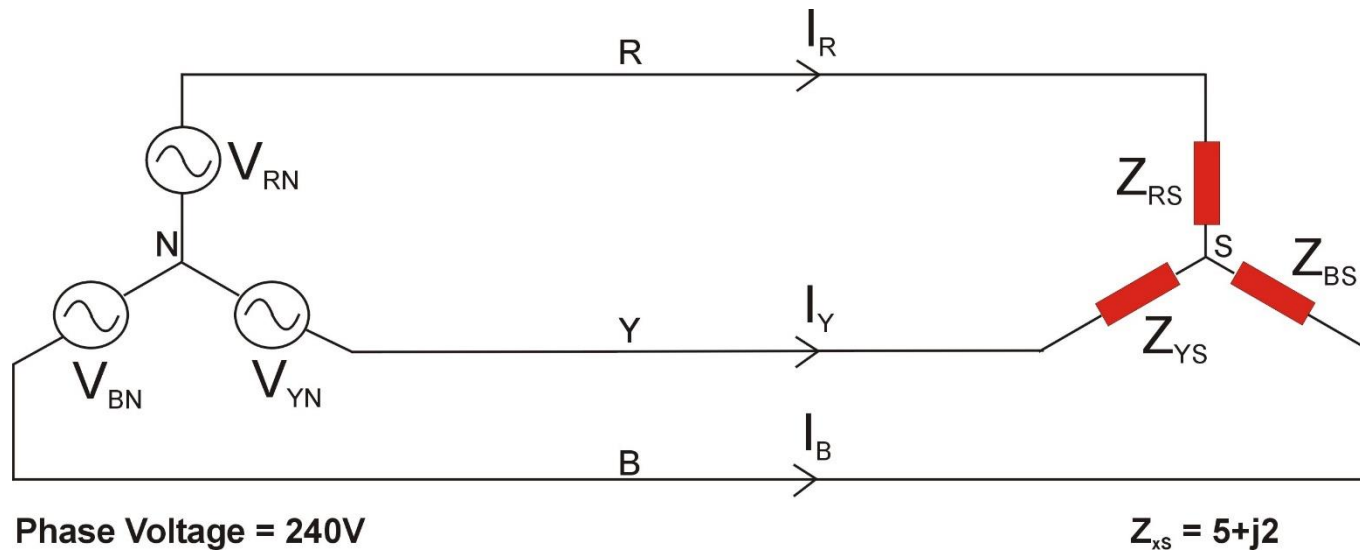


Now $V_{SN} \neq 0$, and therefore $V_{RS} \neq V_{RN}$, the result being potential over or under voltage conditions ie PROBLEMs for the end users.

'A' Solution:



Balanced STAR connected load EXAMPLE:



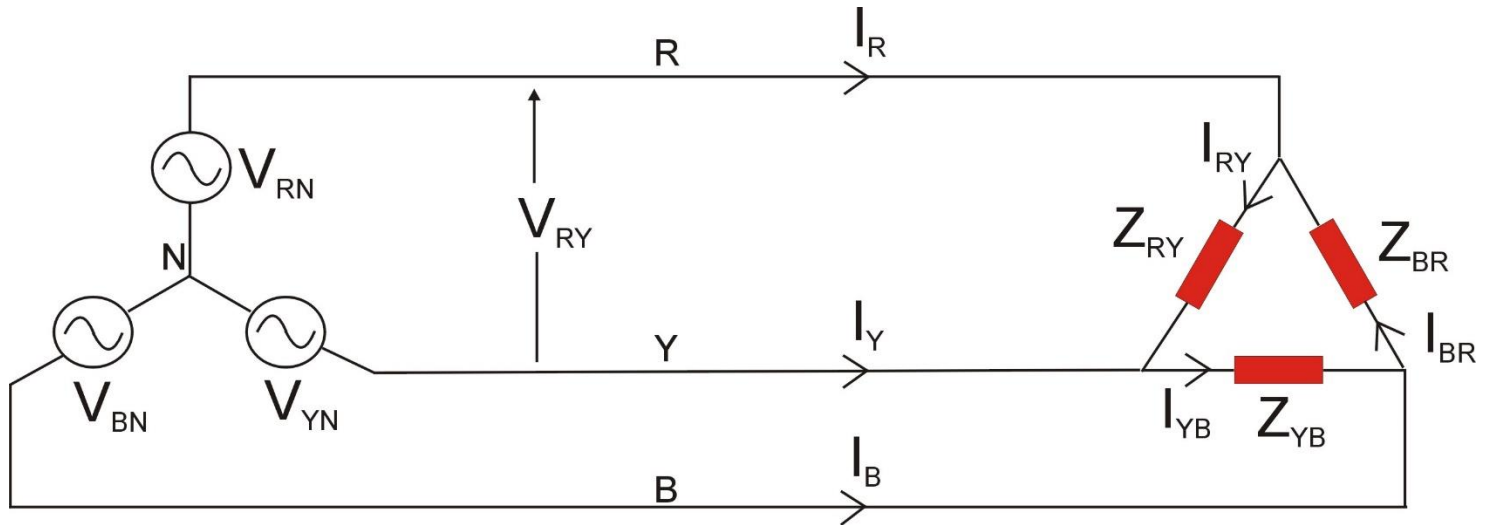
For the balanced 3 phase Star connected load determine the following:

1. The magnitude of the line voltages
2. The line currents I_R , I_Y and I_B
3. The phasor diagram showing all line currents and phase voltages
4. The TOTAL real power supplied by the 3 phase supply



Solution done on whiteboard during lecture

A (balanced) DELTA connected 3 Phase Load



For the balanced load:

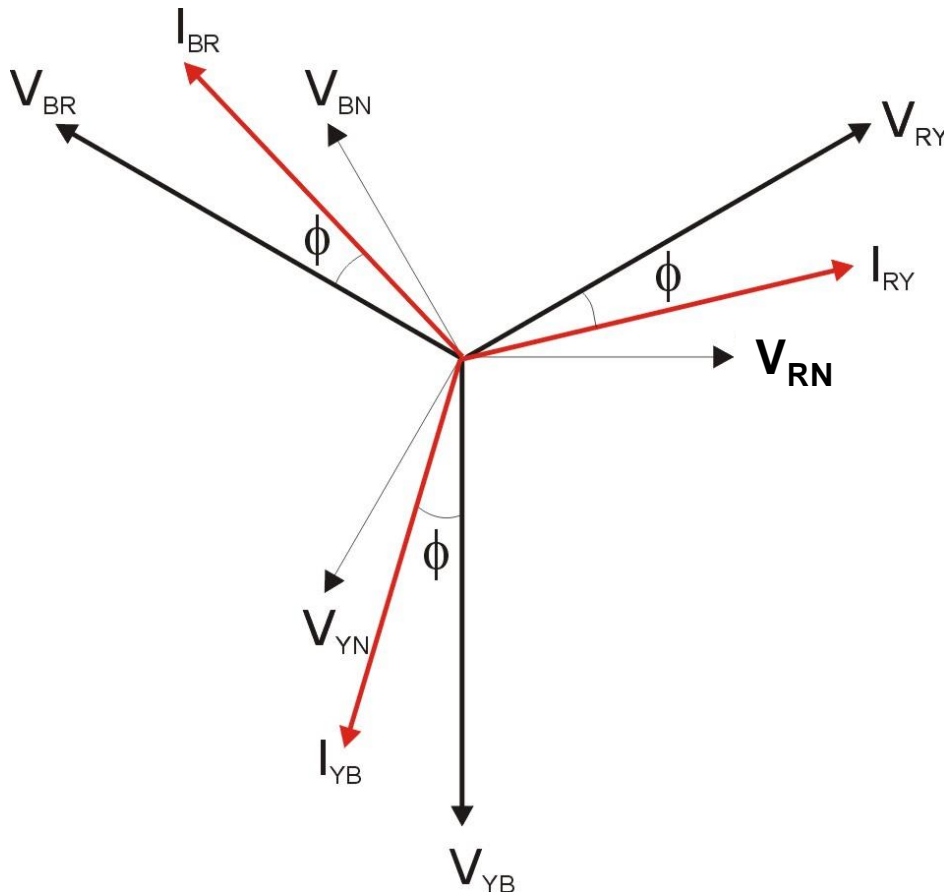
$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = I_{ph} \angle \phi$$

$$I_{YB} = I_{ph} \angle (\phi - 120^\circ)$$

$$I_{BR} = I_{ph} \angle (\phi + 120^\circ)$$

Where I_{ph} is rms magnitude of the load phase currents

Balanced DELTA connected load: Load Phase Currents (I_{RY} , I_{YB} , I_{BR})



Notes:

1. Load Phase Currents are 120° apart and equal in magnitude.

$$2. \quad I_{RY} = \frac{V_{RY}}{Z_{RY}}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}}$$

3. The power equations for any phase are:

$$S = V_L \cdot I_{ph}$$

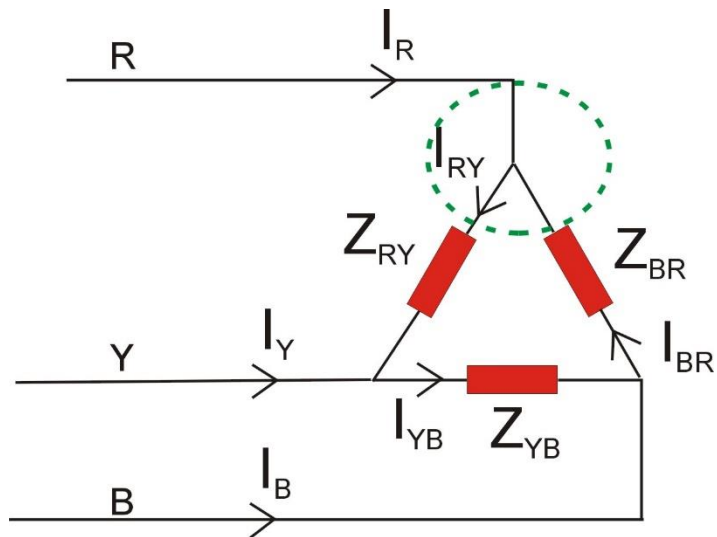
$$P = V_L \cdot I_{ph} \cdot \cos \phi$$

$$Q = V_L \cdot I_{ph} \cdot \sin \phi$$

Where V_L is the rms magnitude of the LINE voltage, I_{ph} is the rms load phase current and ϕ is the angle between them

Note Load Impedances shown here are Resistive + Inductive

Balanced DELTA connected load: Line Currents (I_R , I_Y , I_B)

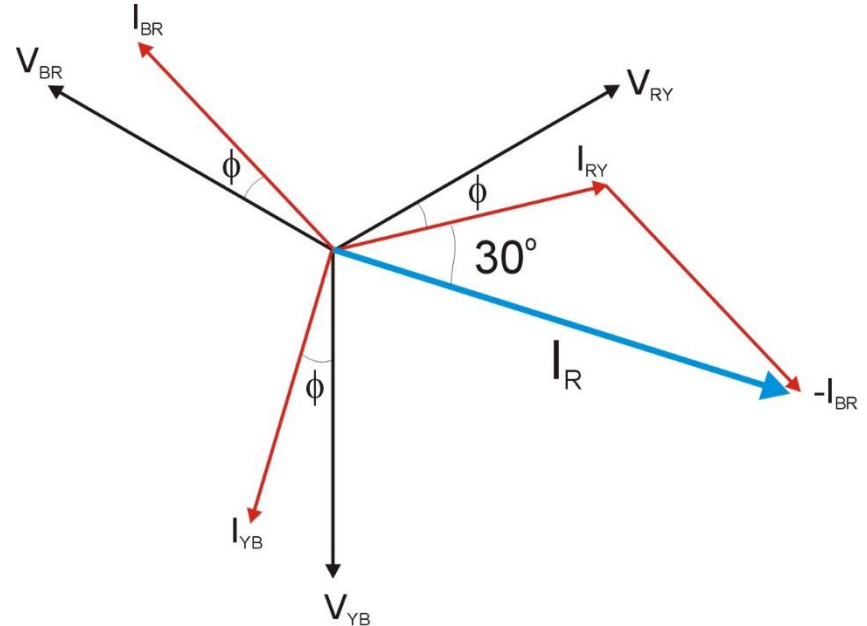


Currents in and out a junction must sum to zero:

$$I_R - I_{RY} + I_{BR} = 0$$

$$I_R = I_{RY} - I_{BR}$$

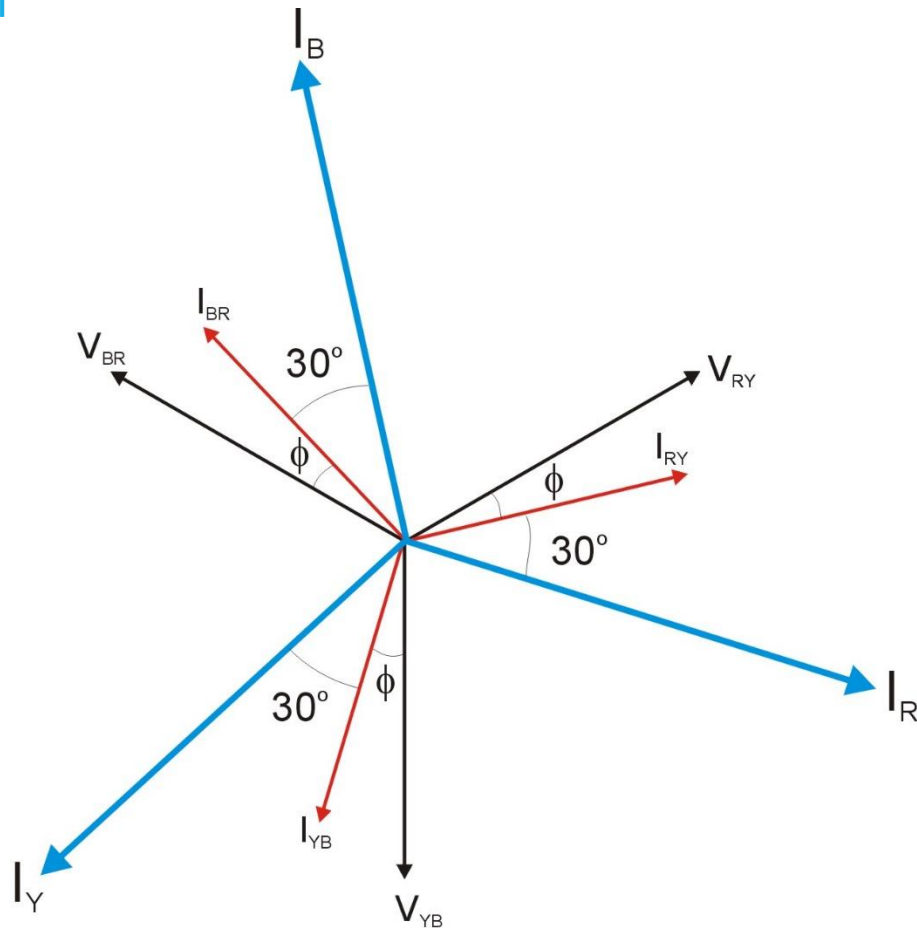
Where all the currents are phasors



Result:

The line current I_R LAGS I_{RY} by 30° and is $\sqrt{3}$ x magnitude of I_{RY}

Balanced DELTA connected load: Line Currents (I_R , I_Y , I_B)



Notes:

1. Line Currents are 120° apart and equal in magnitude.
2. Line Currents are $\sqrt{3}$ x magnitude of load phase currents
2. Line Currents lag the load phase currents by 30°