POWER ENGINEERING

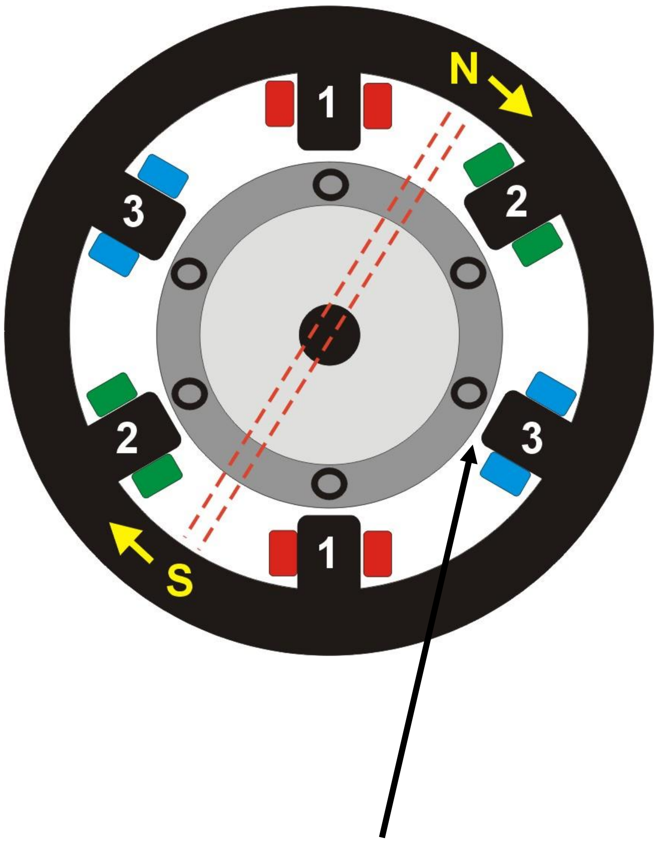
#13 3-PHASE INDUCTION MOTORS (II)

2018

University of Glasgow

3 Phase Induction Motors

The story so far:



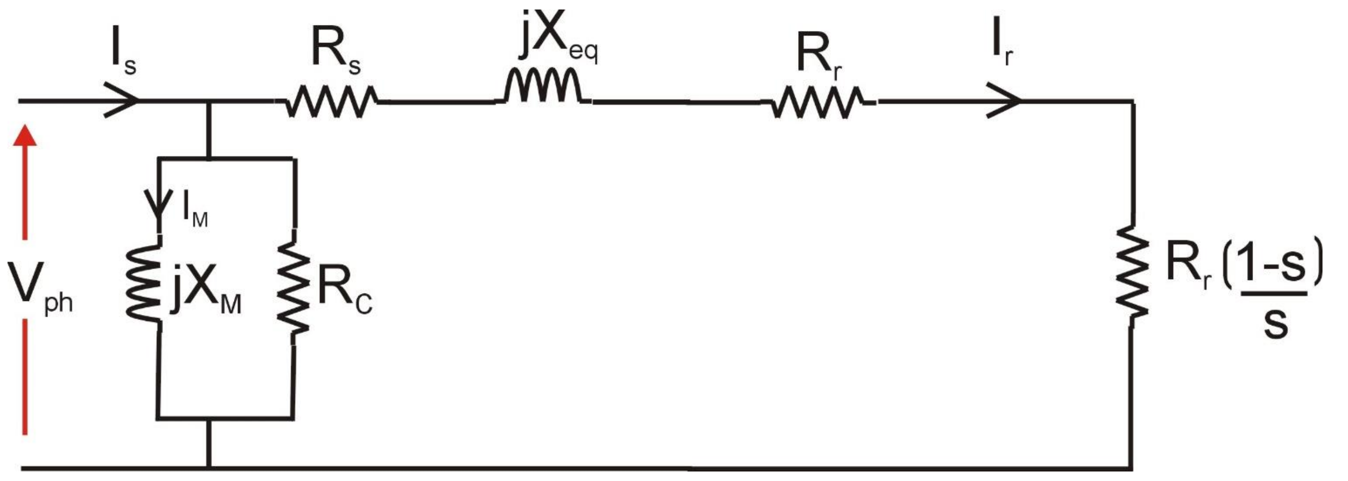
motor airgap

- The three phase voltages input to the three phase stator windings produces a rotating stator flux (magnet) in the motor airgap

- The stator flux rotates at SYNCHRONOUS SPEED (N_s) which is set by the electrical supply frequency (f_s) and the number of poles (P) in the machine:

$$N_S \text{ (rpm)} = \frac{120 \times f_s}{P}$$

The goal for today is to introduce the following equivalent circuit for the Induction Motor:



Hopefully you will recognise that it is quite similar to the equivalent circuit for a transformer, so that will be our starting point. BUT there are a couple of significant differences brought about by the fact that the secondary (rotor) circuit is rotating at a different speed to the stator magnetic field. Today's lecture will attempt to give you an understanding of how these differences affect the equivalent circuit.

The plan for today:

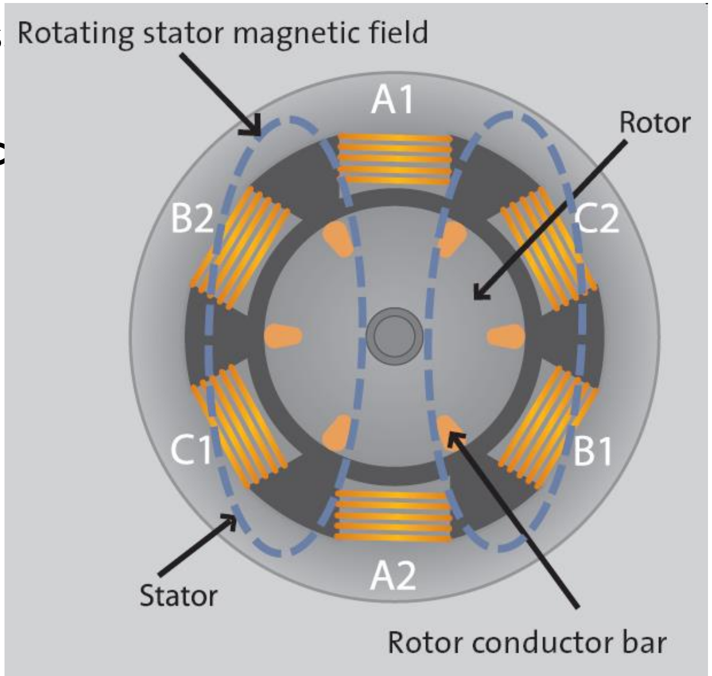
- Induction Motor rotor construction
- Induced currents in rotor cage
- Operation at standstill (zero speed) and at **Synchronous Speed (N_S)**
- Example Torque v Speed Curve
- Induction Motor rotation and definition of **SLIP** (s)
- Derivation of the electrical equivalent circuit for the Induction Motor
- Determining the MECHANICAL output power from the ELECTRICAL equivalent circuit

INDUCTION

When three phase power is applied to the stator, it generates a rotating field in the stator that cuts acros the rotor conductor bars and induce a rotor current. The relative motion of the conductor and the magnetic field causes an electric current in the conductor - induced current. (**Faraday's induction law**).

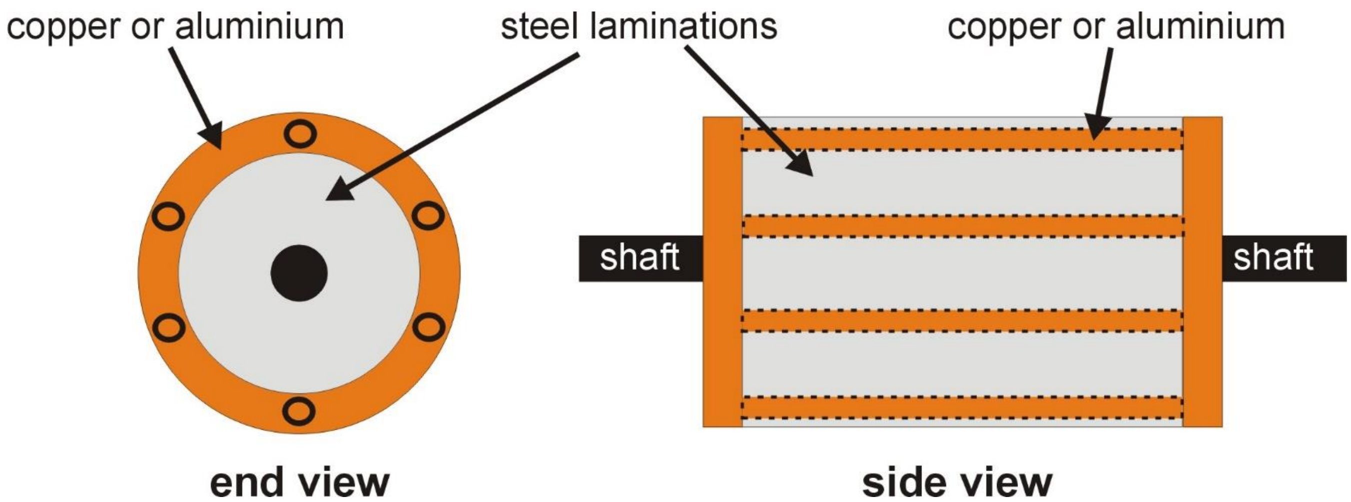
This induced current circulates through the bars in the rotor, and creates a magnetic field around each rotor conductor bar **– rotor magnetic field.** As the three-phase AC power supply makes the magnetic field of the stator rotate, the induced magnetic field of the rotor will follow this rotation.

This interaction is what causes the rotor to move. This is why AC motors are often called AC induction motors or IM (induction motors).

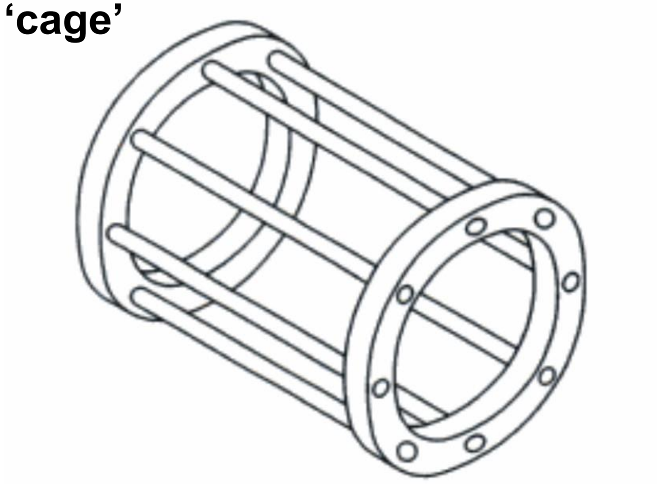


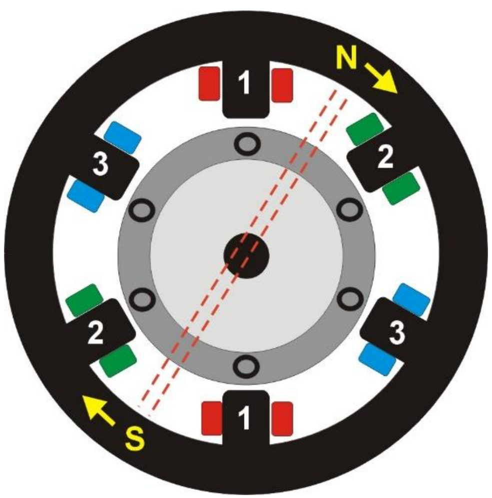
2-Poles, 3-Phase: The phase windings A, B and C are placed 120 degrees apart.

Induction Motor: Rotor Construction

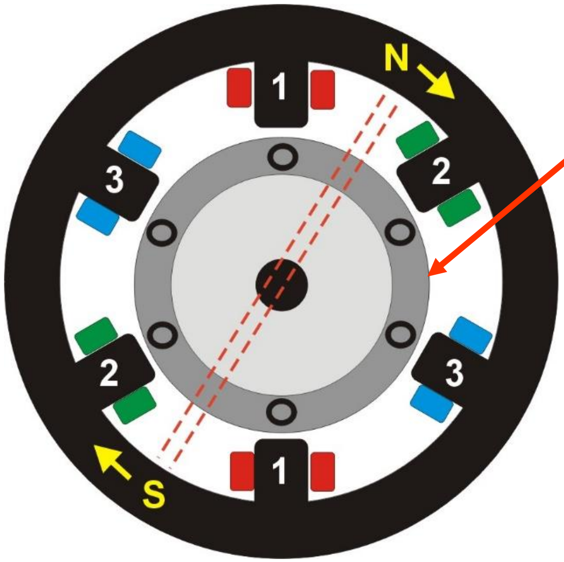


Aluminium or Copper ‘cage’

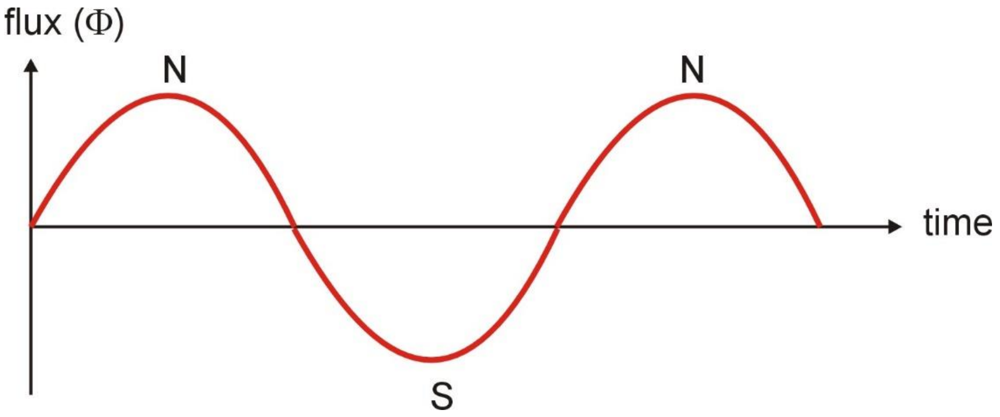




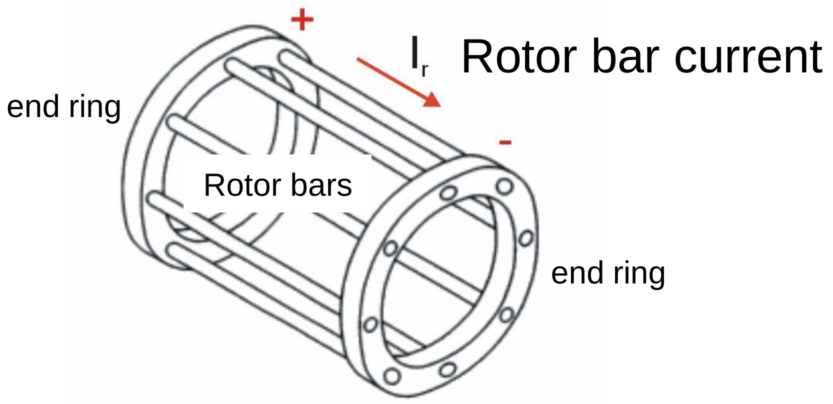
Induced Rotor Bar Currents:



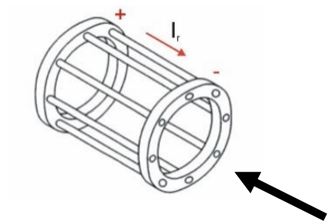
First of consider a single point on the rotor and initially that the rotor is stationary. What flux does this point 'see':

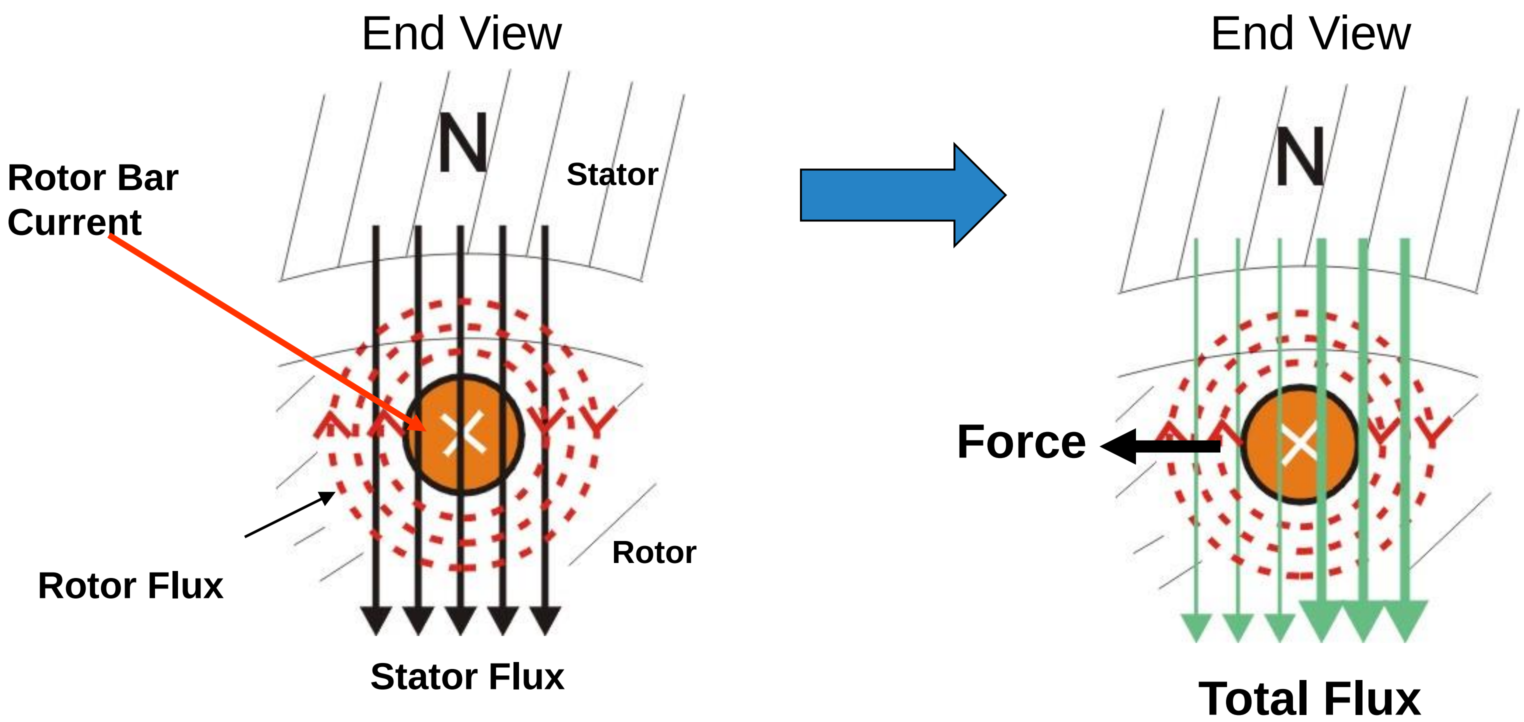


This sinusoidal flux (Φ) will induce a sinusoidal voltage across the rotor bars (Faraday's Law $V = d\Phi/dt$) which will cause sinusoidal currents to flow along the rotor bars given that they are short circuited by the end rings:



Rotor Flux:

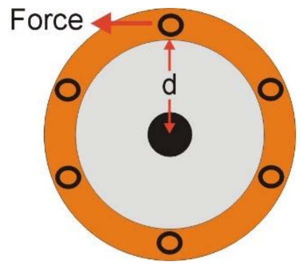




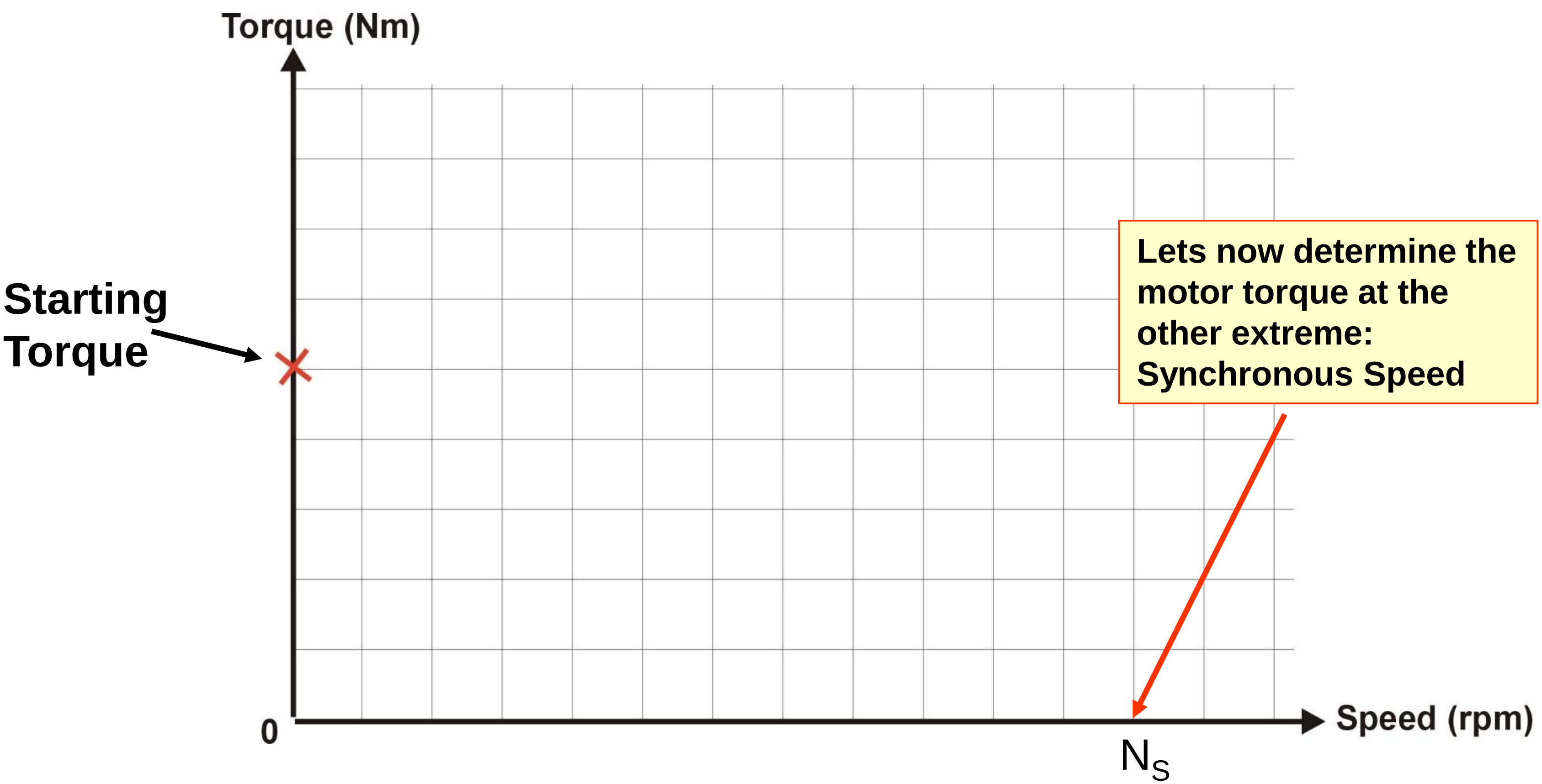
$$\boxed{\text{Force} = B.I_r.L}$$

Where:
B = Stator Flux Density (T)
I_r = Rotor Current (A)
L = Length of Rotor bar (m)

Rotor Torque @ Standstill:

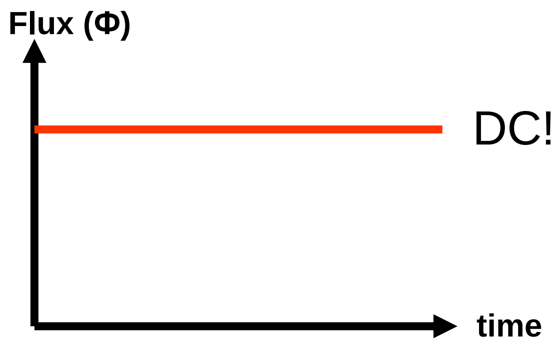


Torque (Nm) = Force (N) x d (m)



Rotor Torque @ Synchronous Speed (N_S):

If the rotor was rotating at Synchronous Speed then the rotor would be travelling at the same speed as the Stator Magnetic Field and therefore the Flux at any point on the rotor would be constant (DC):



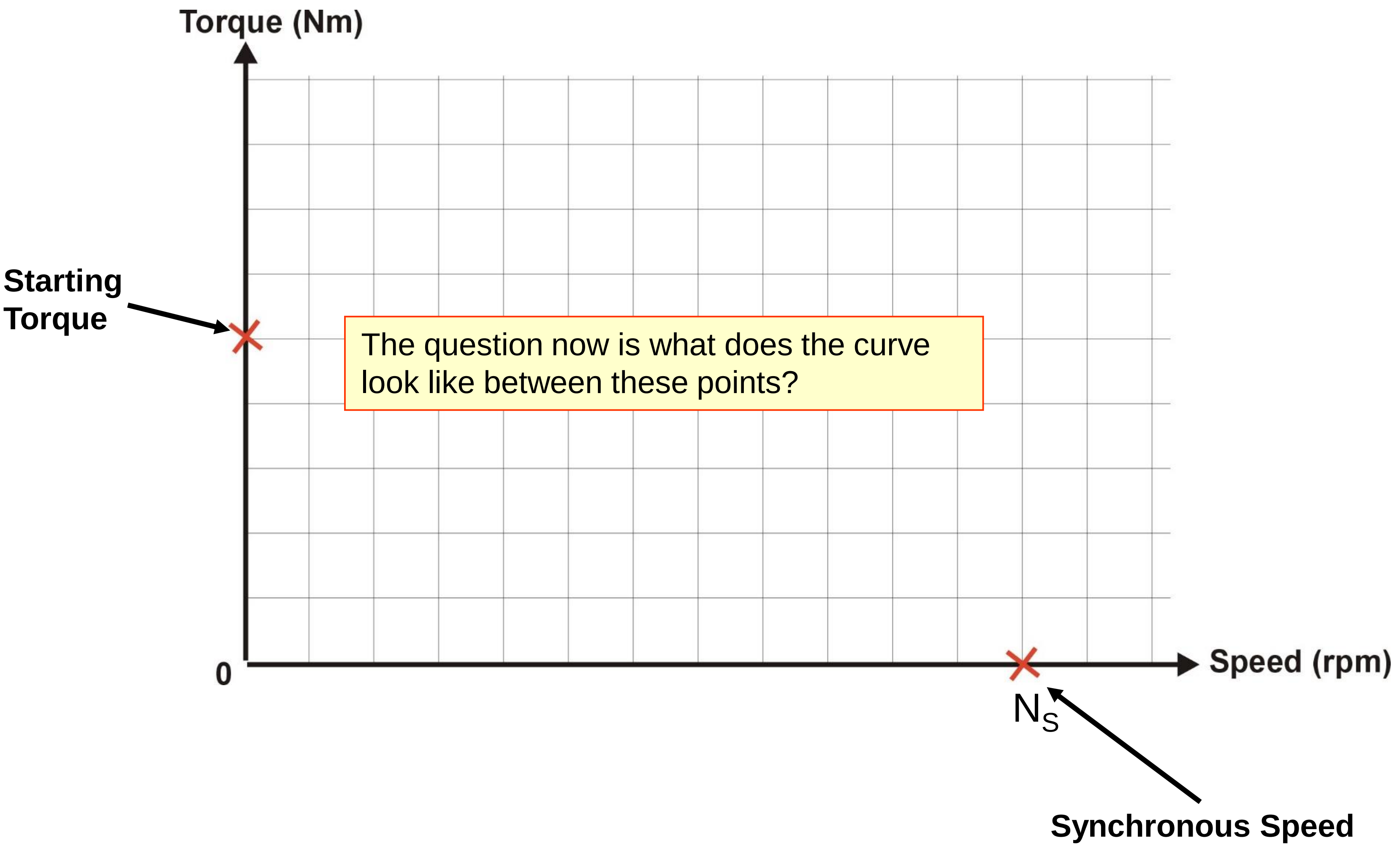
$$\frac{d\phi}{dt} = 0$$

Induced Voltage on Rotor = 0

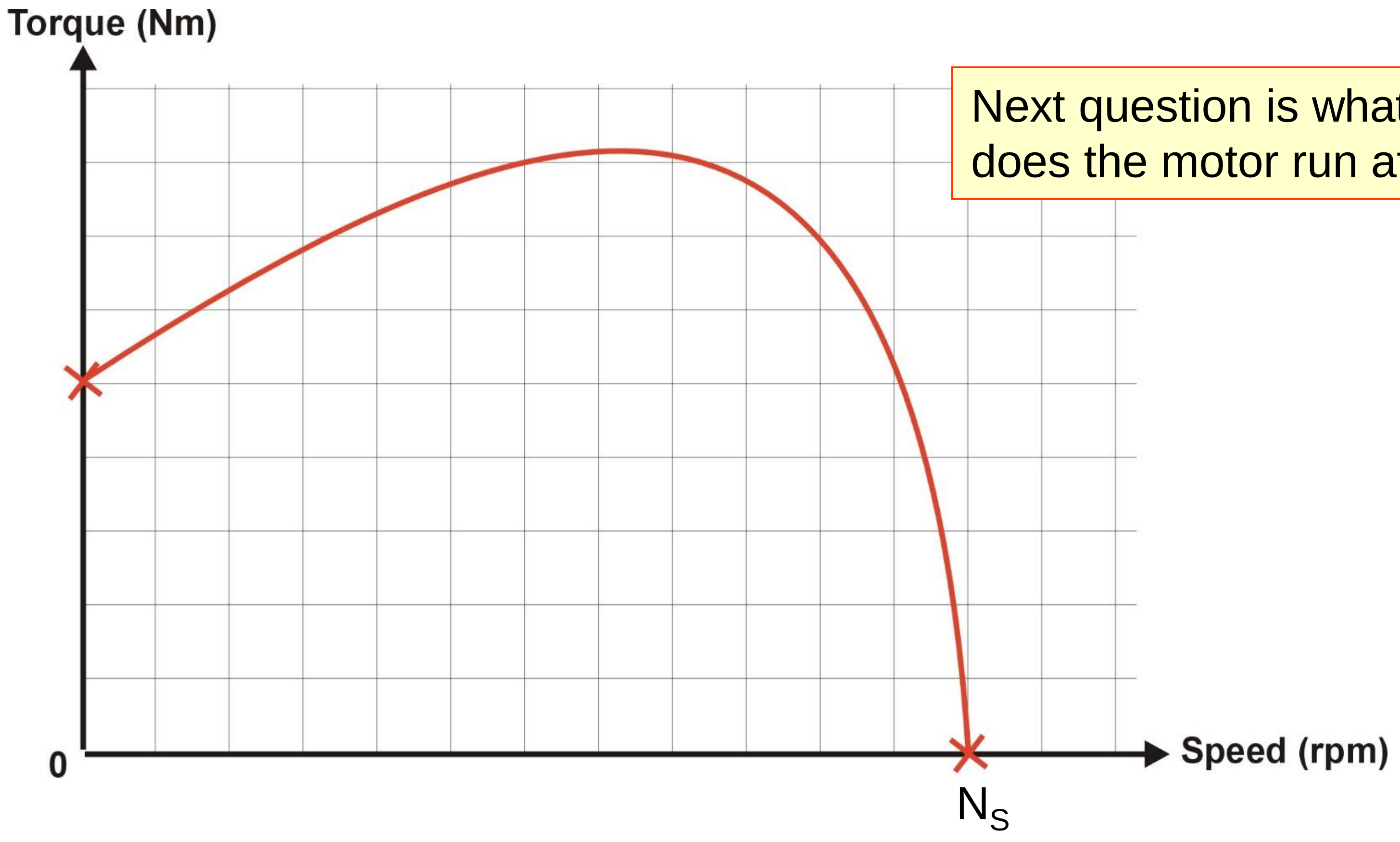
Induced Current in rotor bars = 0

$$\boxed{\textbf{Torque = 0}}$$

‘Extreme points on Torque v Speed Curve:



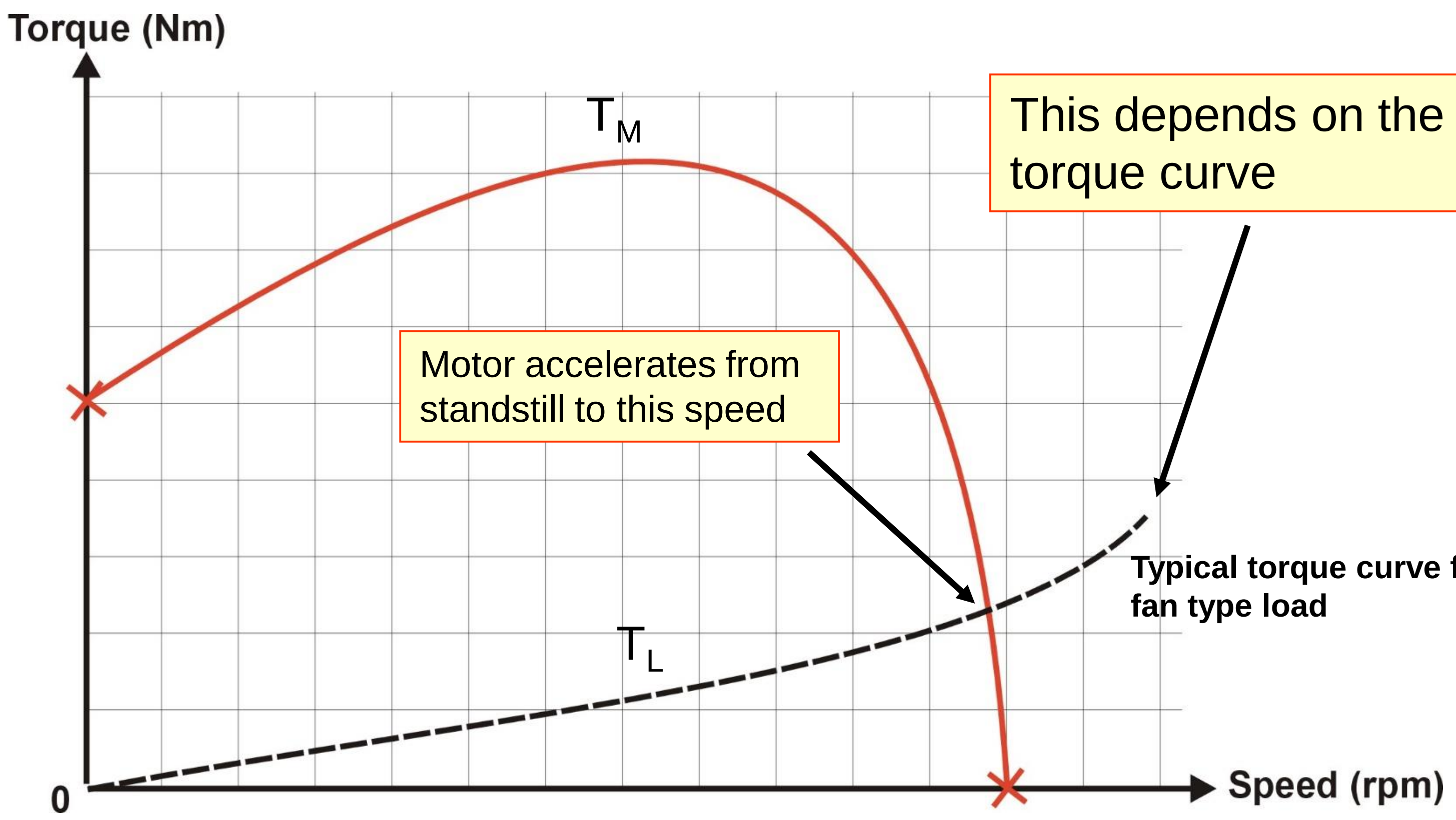
Typical Torque v Speed Curve:



Next question is what speed does the motor run at?

Note: we will derive an equation for this curve later using the equivalent circuit model

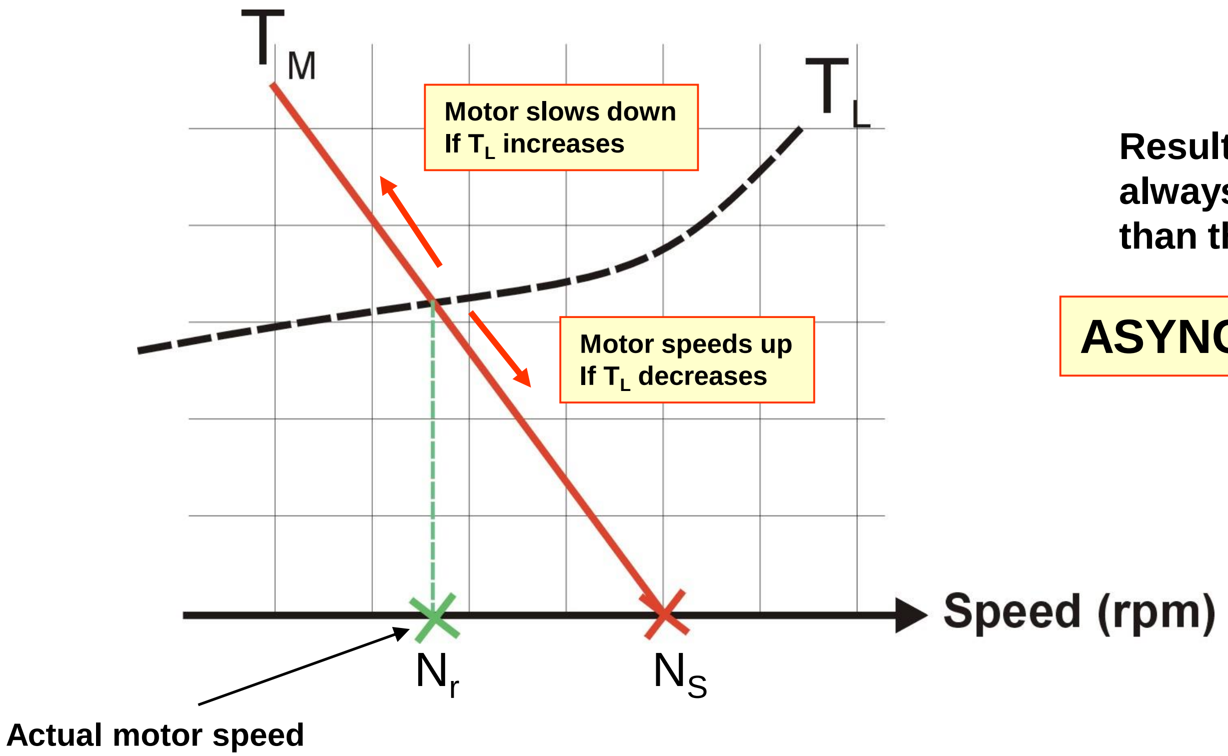
Typical Torque v Speed Curve:



$$T_M = T_L + J\alpha$$

where:
J = moment of inertia of load (kgm^2)
α = acceleration (rad/s^2)

Actual Motor Speed (N_r) and the definition of SLIP (s):



Result: The Induction Motor always runs at a lower speed than the Synchronous Speed

ASYNCHRONOUS MOTOR

Definition of SLIP (s):

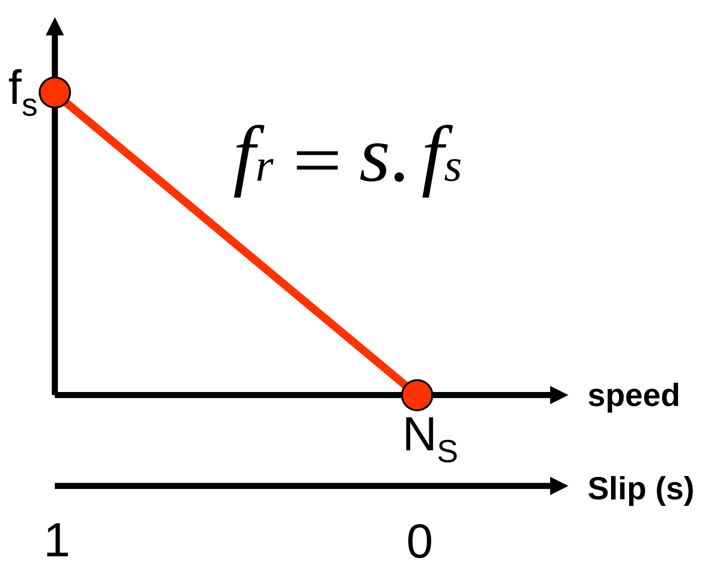
$$Slip(s) = \frac{N_s - N_r}{N_s}$$

VERY Important relationships as a function of Slip (s):

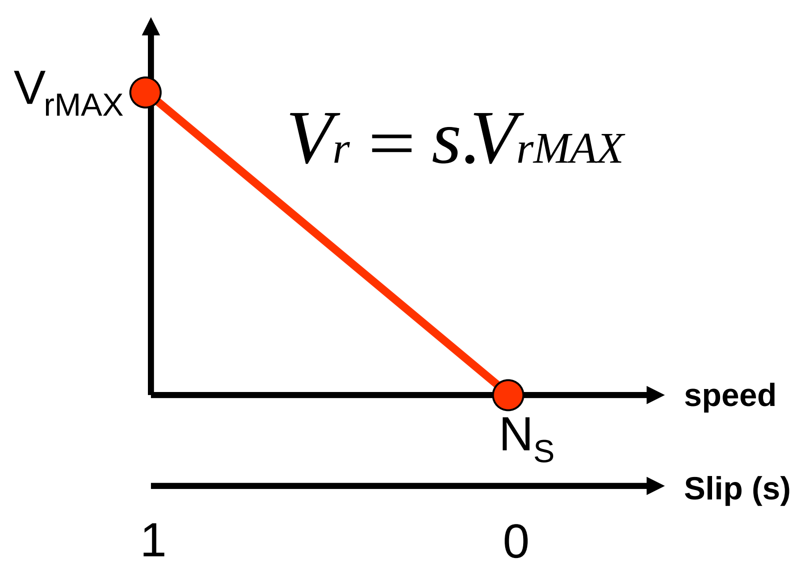
Rotor Speed	Slip (s)	Rotor Voltage frequency	Rotor Voltage Magnitude
0	1	f_s	V_{rMAX}
N_S	0	0	0

where fs is the electrical supply frequency (eg 50Hz)

Rotor Voltage frequency (f_r)



Rotor Voltage Magnitude (V_r)

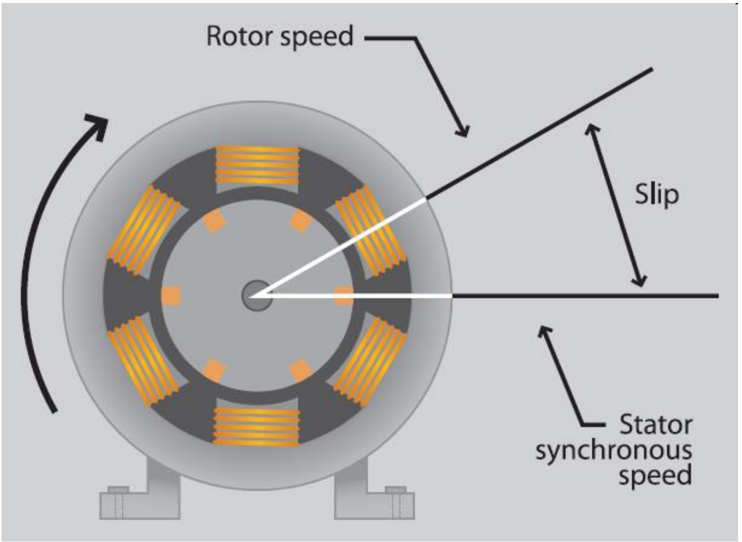


ASYNCHRONOUS SPEED

IF the rotor speed is identical to the electrical rotation speed of the stator field >> no relative motion >> no induced current in the rotor >> no torque.

During actual operation, the rotor mechanical speed ALWAYS lags behind the rotating speed of magnetic field of the stator. This allows the rotor's magnetic field to cut the stator's magnetic field and thereby produce torque. AC induction motors are known as **asynchronous motors.**

This difference in speed between rotor mechanic speed and stator magnetic field speed, is called **slip** and is measured in %. Slip is a key factor and is necessary to produce torque.

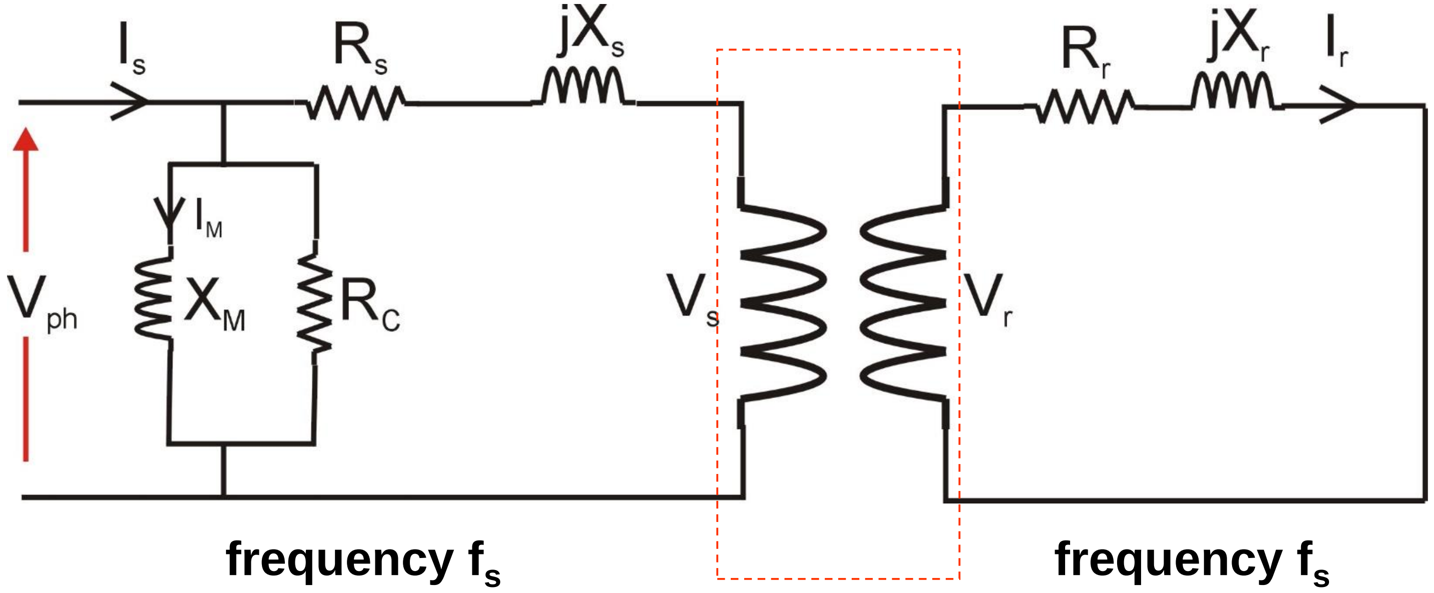


Note: The rotor magnetic field speed is still identical to the stator magnetic field speed. It is necessary to keep synchronization between stator "magnet" and rotor "magnet" for producing stable net torque

The Induction Motor (per phase) Equivalent Circuit

It would be very useful to derive an equivalent circuit for the Induction Motor much in the same way we did for the transformer. As we will find out this equivalent circuit is extremely useful as it allows us to determine MECHANICAL performance from a completely electrical equivalent circuit.

The starting point is the realisation that AT STANDSTILL each phase of the induction motor is equivalent to a single phase transformer with its secondary winding short circuited:



frequency f_s **frequency f_s**

Ideal transformer

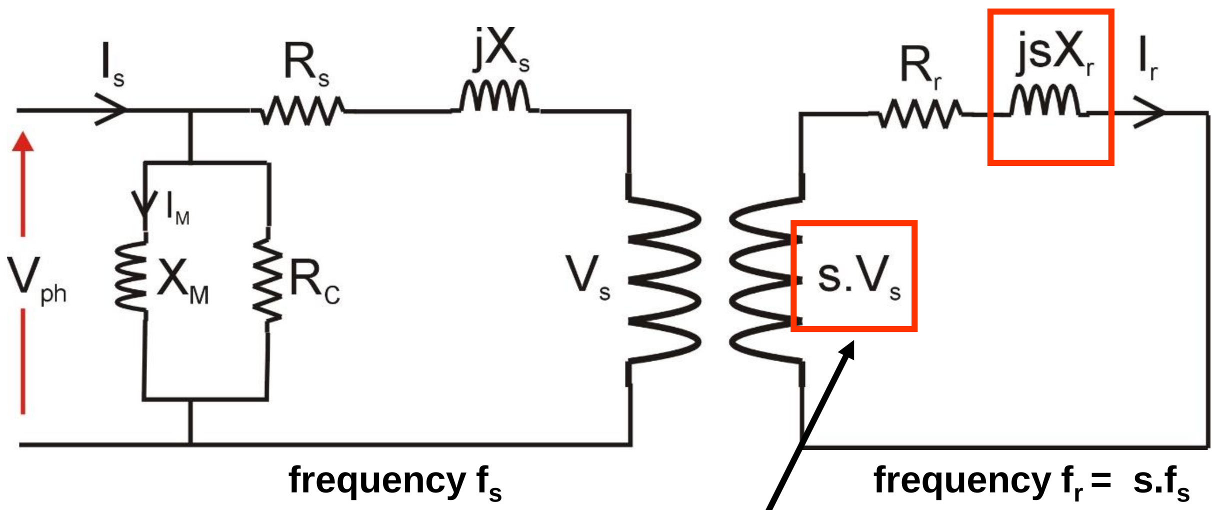
assume (for simplicity) Turns Ratio = 1 ($V_s=V_r$)

The Induction Motor (per phase) Equivalent Circuit

Now consider that the rotor is rotating at a given slip (s):

Due to relationship between induced rotor voltage frequency and slip shown on Slide 12, and where X_r is reactance measured at f_s

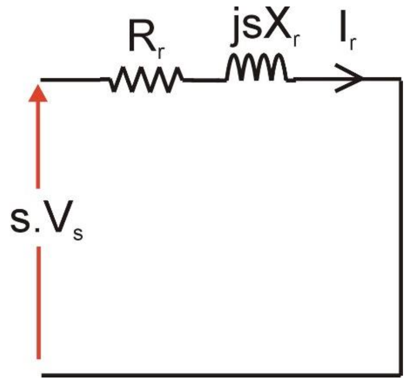
remember $X = 2\pi.f$



Due to relationship between induced rotor voltage magnitude and slip shown on Slide 12

The Induction Motor (per phase) Equivalent Circuit

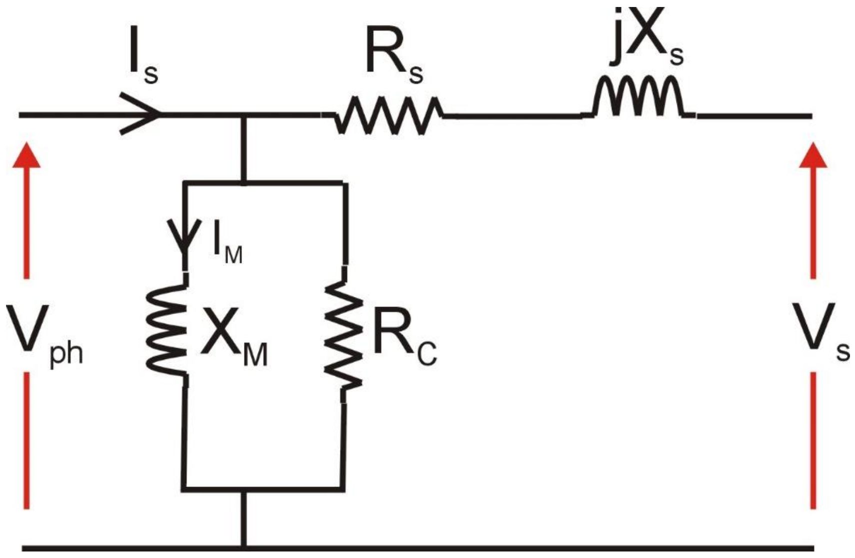
Rotor:



$$s.V_s = I_r(R_r + js.X_r)$$

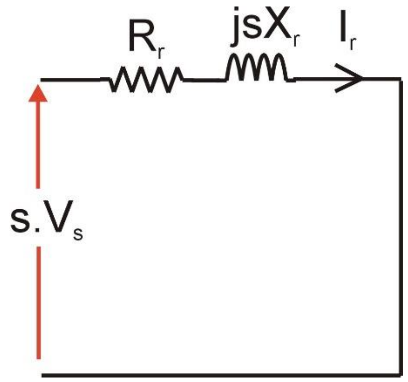
$$V_s = I_r\left(\frac{R_r}{s} + jX_r\right)$$

Transfer the rotor circuit to the primary side:



The Induction Motor (per phase) Equivalent Circuit

Rotor:



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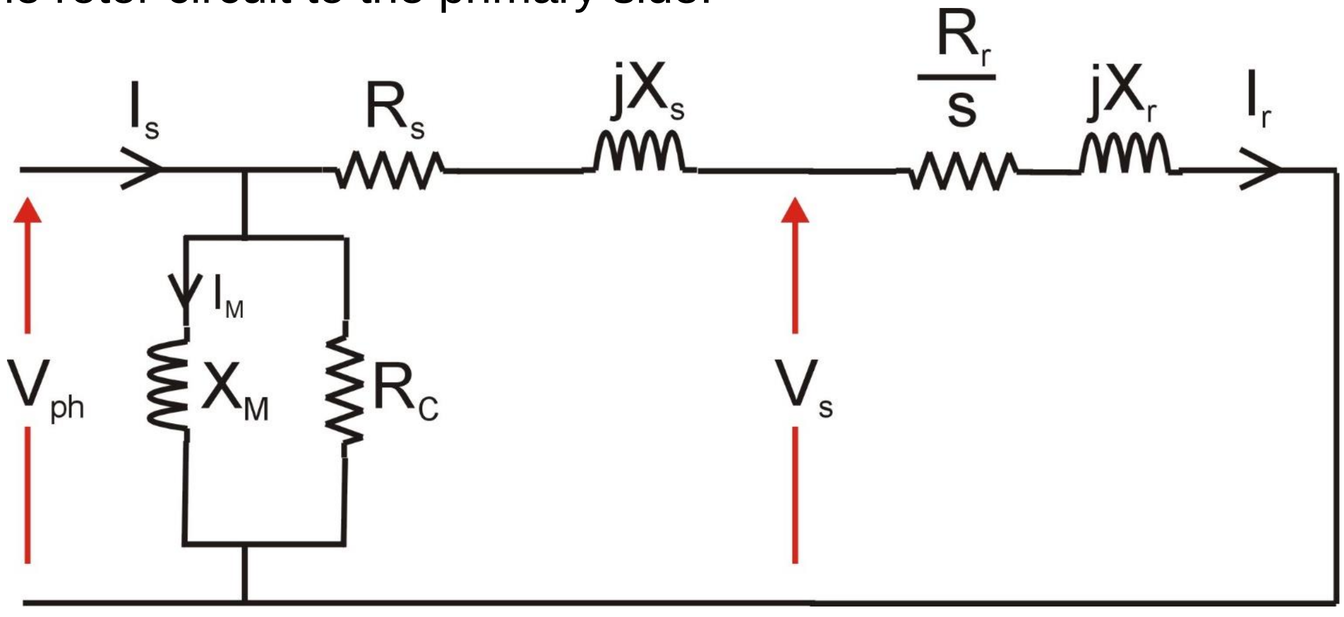
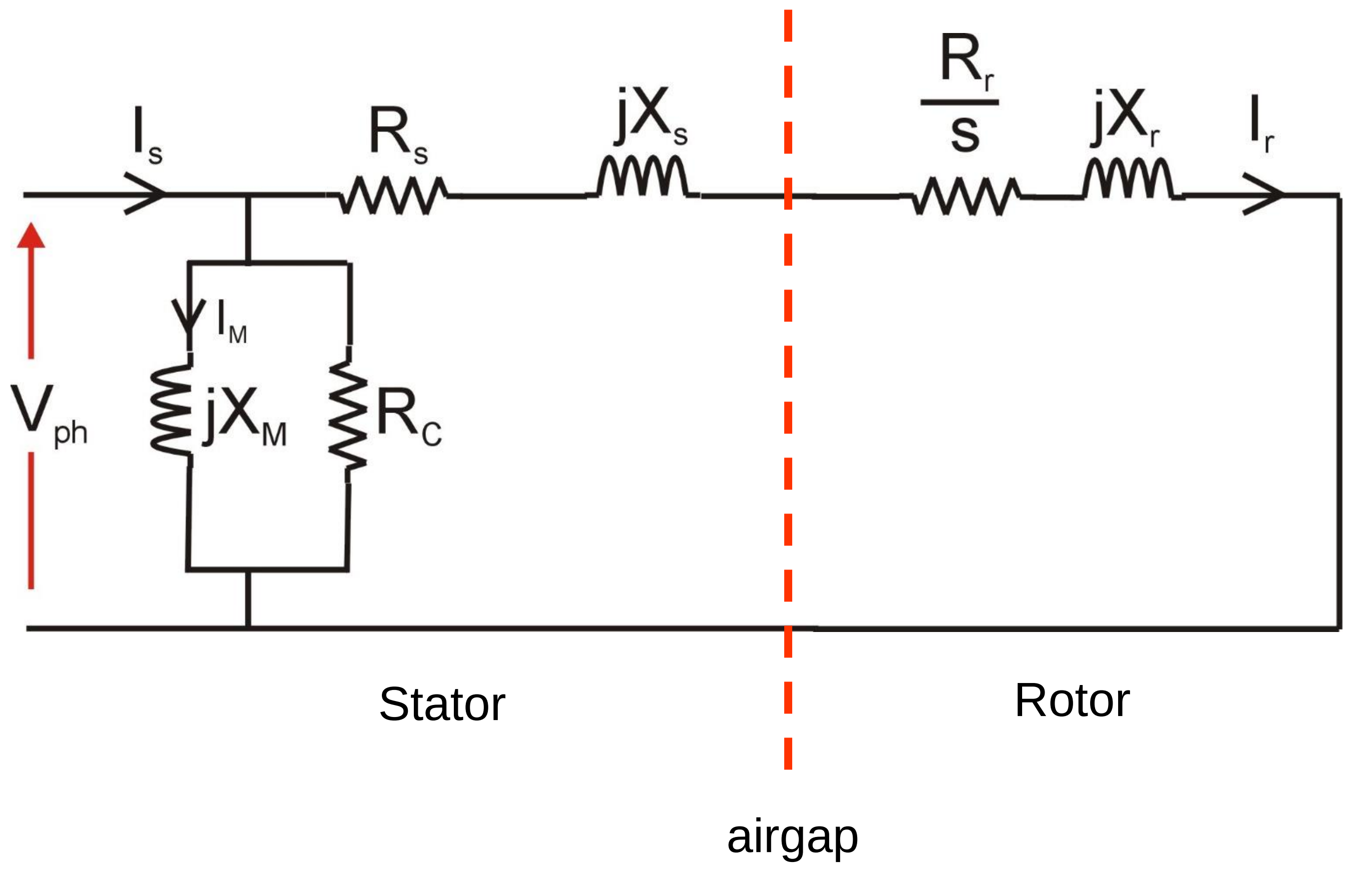
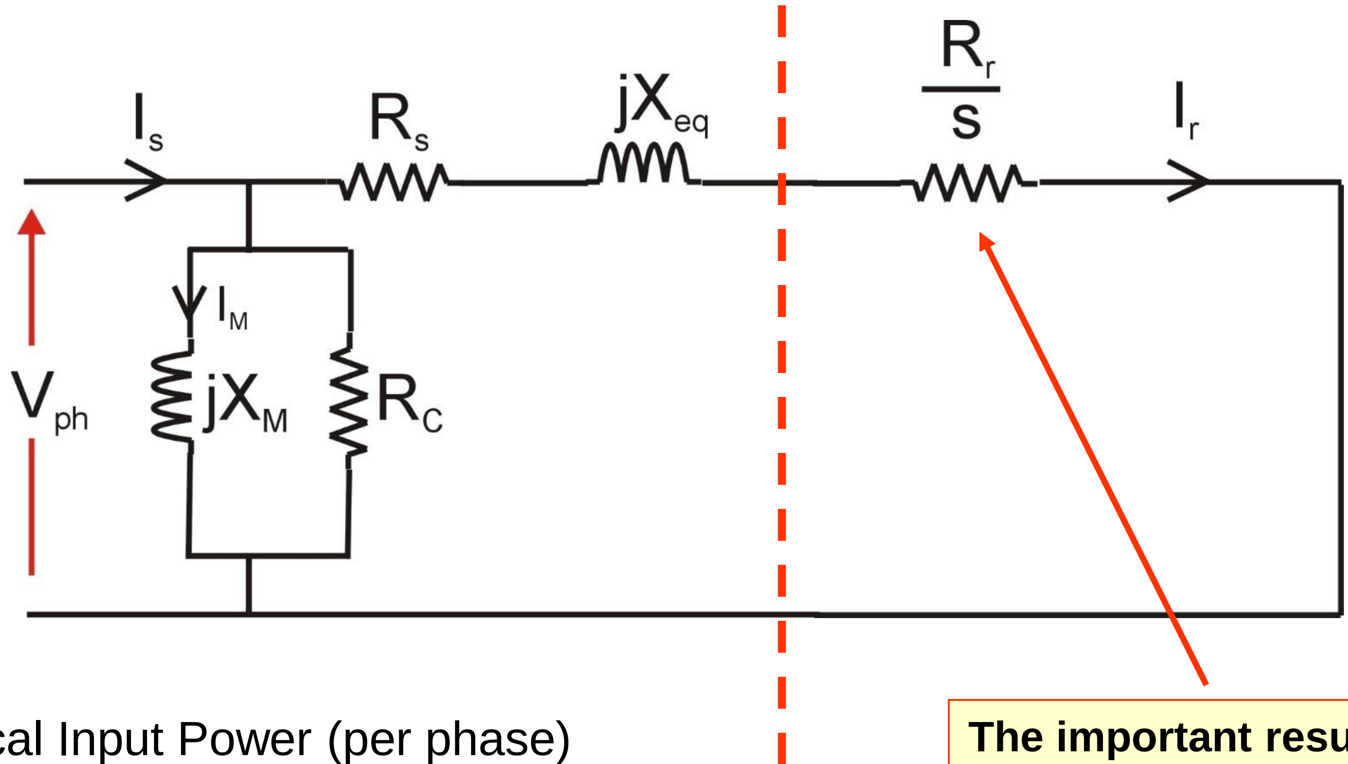


The Induction Motor (per phase) Equivalent Circuit



Combine X_s and X_r reactances into X_{eq}:

The Induction Motor (per phase) Equivalent Circuit



The important result of the equivalent circuit is that the TOTAL (per phase) power which crosses from the Stator to the Rotor is represented by this R_r/s component.

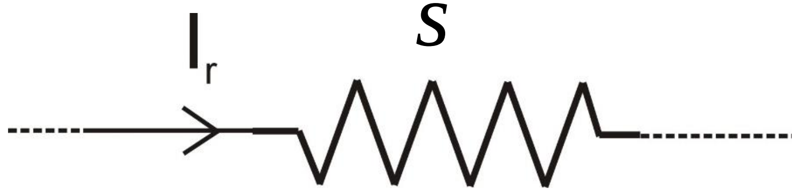
Electrical Input Power (per phase)

$$\boxed{P_{ph} = V_{ph} I_s Cos\phi}$$

Where Φ is the angle between V_{ph} and I_s

Replace:

$$\frac{R_r}{s}$$



$$\frac{R_r}{s} = R_r + \frac{R_r(1-s)}{s}$$

With:

$$R_r \qquad\qquad \frac{R_r(1-s)}{s}$$

Rotor Copper Loss (W) **Mechanical Power (W)**

$$P_{cu} = I_r^{\,2}.R_r \qquad\qquad P_{mech} = I_r^{\,2}.\frac{R_r(1-s)}{s}$$

Note: these powers are per phase – total power is 3 times this for 3 phase motor (see last slide)

Determining Mechanical Output Power from Equivalent Circuit:

The TOTAL per phase power which crosses the airgap = $I_r^{2}\dfrac{R_r}{s}$

➡ The TOTAL 3 phase power which crosses the airgap P_{gap} = $3I_r^{2}\dfrac{R_r}{s}$

ALSO 3 phase Rotor Copper Loss P_{cu} = $3I_r^{2}R_r$

$$P_{gap} - P_{cu} = 3I_r^{2}\frac{R_r}{s} - 3I_r^{2}R_r$$

$$P_{gap} - P_{cu} = 3I_r^{2}\frac{R_r}{s}(1-s)$$

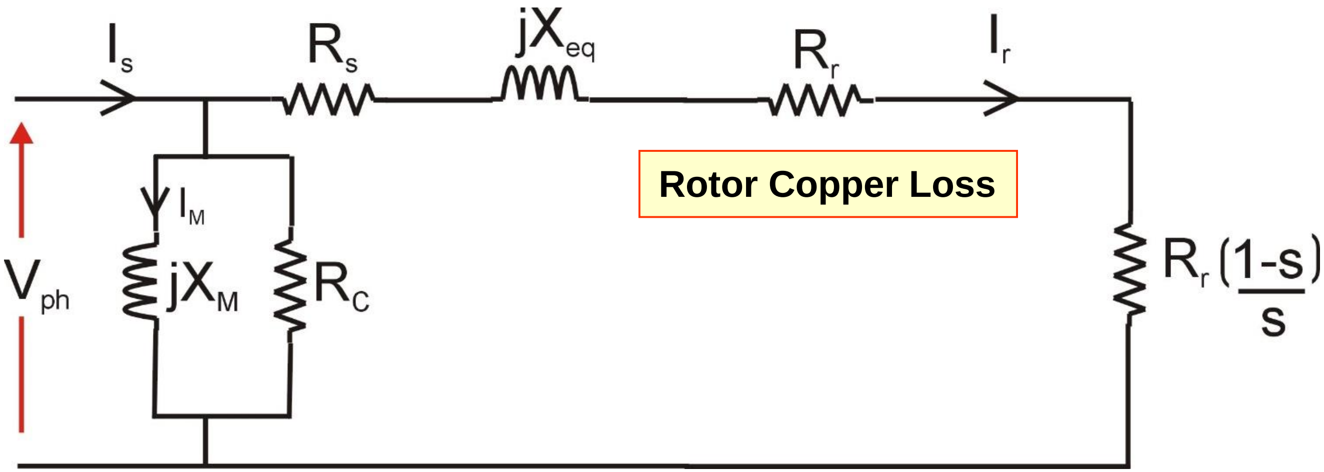
What is this power?

$$P_{mech} = 3I_r^{2}\frac{R_r}{s}(1-s)$$

answer:

MECHANICAL POWER

The Induction Motor (per phase) Equivalent Circuit



Mechanical Power

Note:

$$R_r + R_r\left(\frac{1-s}{s}\right) = \frac{R_r}{s}$$

Result:

If we have values for the equivalent circuit parameters then we can calculate **Mechanical** Output Power (and Torque) and Efficiency at any given speed (slip) and supply voltage.

Example:

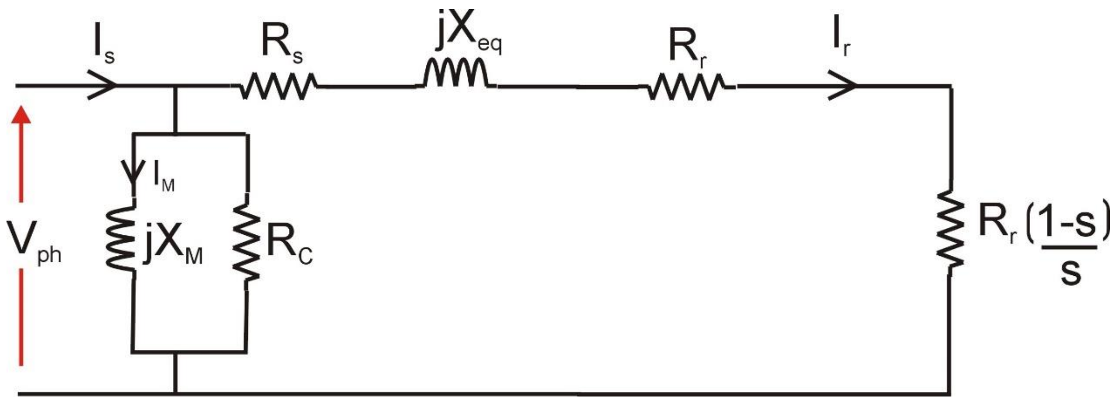


Table 1:

Parameter	Value
X_M	30Ω
R_c	500Ω
R_s	0.8Ω
X_{eq}	1.4Ω
R_r	0.3Ω

Given the equivalent circuit parameters shown on Table 1, determine the following for a 3 phase 6 pole machine rotating at 900rpm and operating off a 240V (phase voltage), 50Hz supply:

1. Slip (s)
2. Airgap Power (W)
3. Mechanical Output Power (W)
4. Motor Torque (Nm)

Note: Neglect Friction & Windage Loss

Solution done on whiteboard during lecture