POWER ENGINEERING

#04 SINGLE-PHASE AC POWER SYSTEMS

AC POWER

2018

University of Glasgow

Edison Quote of the Day:

" I have never failed, I've just found 10,000 ways that don't work"

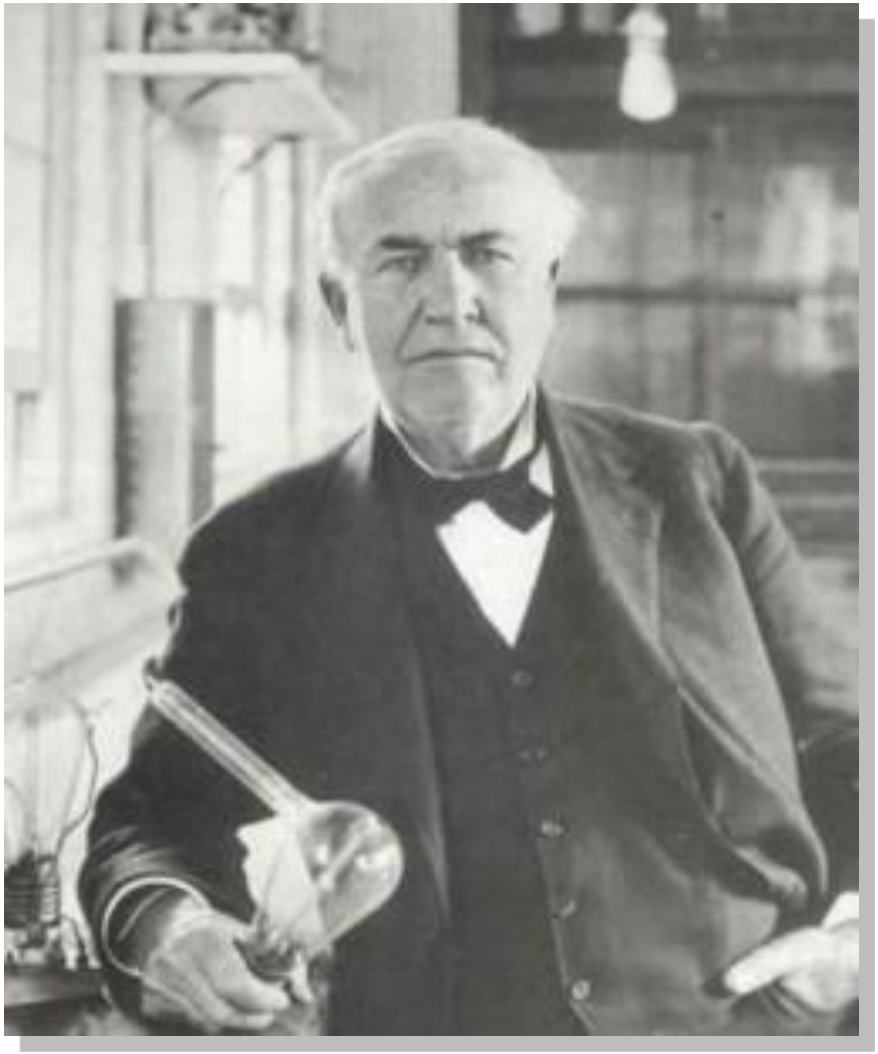
Stubborn

倔,轴,一根筋

Or

Persistence

执着



AC Power

Real Power (W), Apparent Power (VA), Reactive Power (VAr) & Power Factor

Today we will investigate:

- Instantaneous and Average Power for Resistive, Inductive and Capacitive Components
- The relationship between Real, Apparent and Reactive Powers in a circuit consisting of complex impedances
- Definition of Power Factor
- The Power Triangle
- System Efficiency

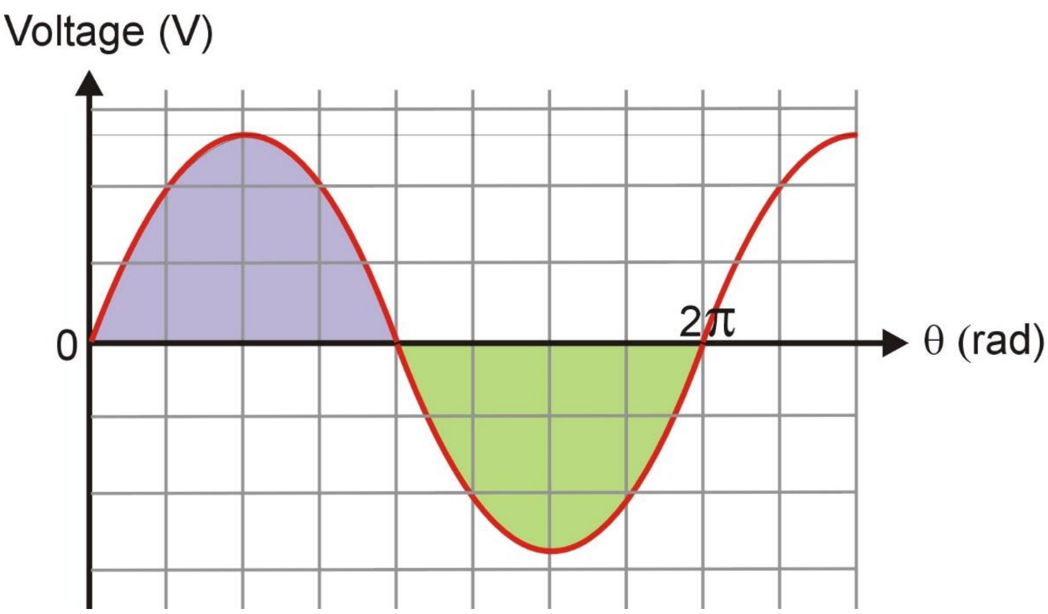
Real Time Domain

Average Values

A general periodic function $y(t)$, with period T, has an average or mean value Y_{av} given by:

$$\boxed{Y_{av} = \frac{1}{T}\int_0^T y(t)dt}$$

The mean value of sine and cosine functions *a* sin(ωt) and *a* cos(ωt) is 0.



$$V_{av} = \frac{1}{\tau}\int_0^{\tau} v(\theta).d\theta$$

$$V_{av} = \frac{1}{2\pi}\int_0^{2\pi} V_{pk}Sin\theta.d\theta$$

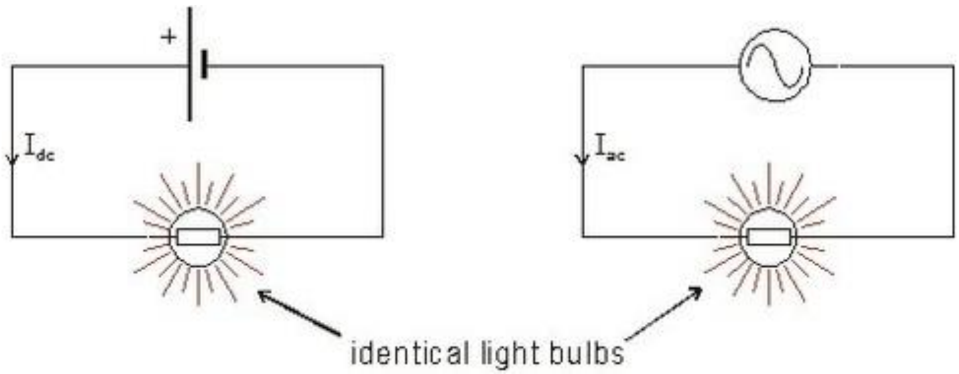
$$V_{av} = \frac{V_{pk}}{2\pi}\left[-Cos\theta\right]_0^{2\pi}$$

$$V_{av} = \frac{V_{pk}}{2\pi}\left[-1+1\right]$$

$$V_{av} = 0$$

Effective Values

The mean value of sine and cosine functions *are* 0, can not be used to measure the power of an AC circuit.



If the AC-powered bulbs light with the same brightness as the dc-powered one (that is, they are working at the same power), then it would be logical to regard the current I_{ac} as being equivalent to the current I_{dc}.

Effective Values

The Root Mean Square (RMS) or Effective value of a general periodic function $y(t)$, with period T, has an effective value Y_{rms} given by:

$$\boxed{Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2\,dt}}$$

The simple *mathematical average value* of I_{ac} (usually represented as sine and cosine functions *a* sin(ωt) and *a* cos(ωt) is equal to zero. This is why We need effective value of an alternating current (or voltage).

The RMS value is sometimes also referred to as the “heating” value since a current passing through a pure resistor results in power being dissipated. RMS value of the AC current equivalent of the DC current which need to pass through a resistor, to produce same heat as the AC !!!

Instantaneous Power: $p(t) = v(t)i(t)$

Average Power:

$$p_{av} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt \rightarrow$$

Dissipated power

For resistors:

$$p_{av} = \frac{1}{T}\int_0^T vidt = \frac{1}{T}\int_0^T \frac{v^2}{R}dt = \frac{1}{R}\left(\underbrace{\sqrt{\frac{1}{T}\int_0^T v^2 dt}}_{rms}\right)^2 = \frac{v_{rms}^2}{R} = i_{rms}^2 R$$

$$p_{av,dc} = \frac{V_{dc}^2}{R} = I_{dc}^2 R$$

Equivalent/Effective Values

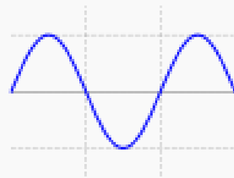
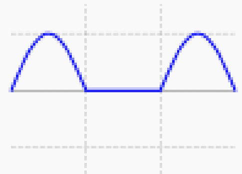
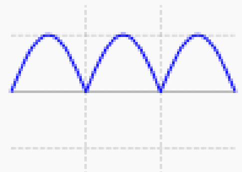
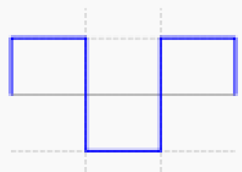
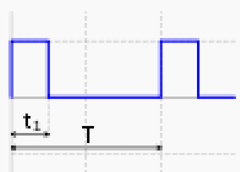
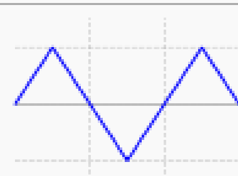
rms value of a Sinusoidal Waveform:

$$V_{rms} = \sqrt{\left[\frac{1}{\tau}\int_0^{\tau} v^2(\theta)d\theta\right]} = \sqrt{\left[\frac{1}{2\pi}\int_0^{2\pi} V_{pk}{}^2 Sin^2\theta.d\theta\right]}$$

$$= \sqrt{\left[\frac{V_{pk}{}^2}{2\pi}\int_0^{2\pi}\left[\frac{1}{2}\left(1-Cos2\theta\right)\right]d\theta\right]}$$

$$V_{rms} = \sqrt{\left[\frac{V_{pk}{}^2}{2\pi}\left[\frac{1}{2}\left(\theta-\frac{1}{2}Sin2\theta\right)\right]_0^{2\pi}\right]} = \sqrt{\left[\frac{V_{pk}{}^2}{2\pi}\left[\frac{2\pi}{2}-0-0+0\right]\right]} = \frac{|V_{pk}|}{\sqrt{2}}$$

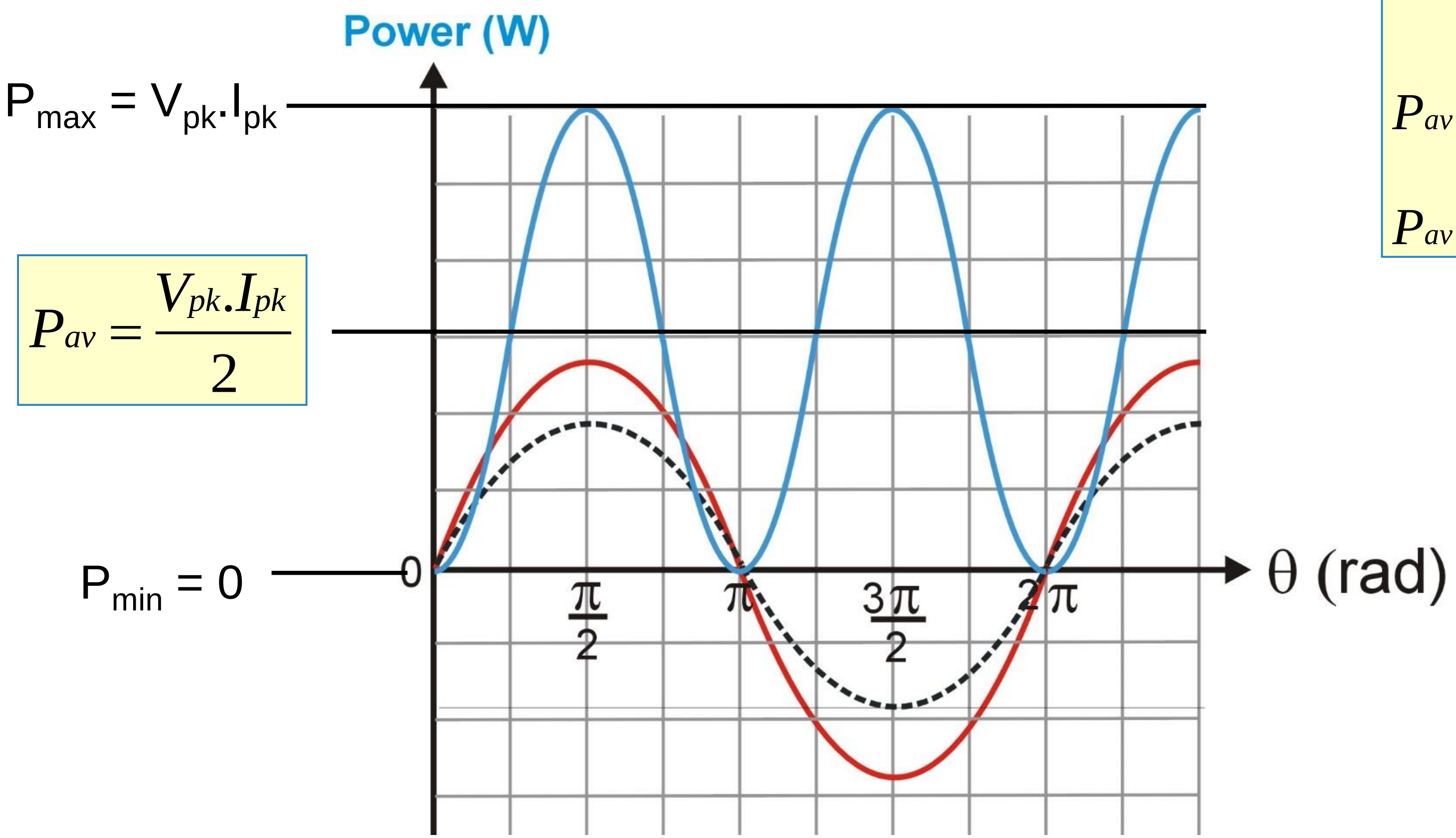
$$\boxed{V_{rms} = \frac{|V_{pk}|}{\sqrt{2}}}$$

Waveform	Image	RMS	ARV	Form Factor
Sine wave		$\frac{a}{\sqrt{2}}$ [2]	$a\frac{2}{\pi}$ [2]	$\frac{\pi}{2\sqrt{2}} \approx 1.11072073$ [3]
Half-wave rectified sine		$\frac{a}{2}$	$\frac{a}{\pi}$	$\frac{\pi}{2} \approx 1.5707963$
Full-wave rectified sine		$\frac{a}{\sqrt{2}}$	$a\frac{2}{\pi}$	$\frac{\pi}{2\sqrt{2}}$
Square wave, constant value		a	a	$\frac{a}{a} = 1$
Pulse wave		$a\sqrt{D}$ [7]	aD	$\frac{1}{\sqrt{D}} = \sqrt{\frac{T}{\tau}}$
Triangle wave		$\frac{a}{\sqrt{3}}$ [8]	$\frac{a}{2}$	$\frac{2}{\sqrt{3}} \approx 1.15470054$

a

Resistance

Instantaneous Power: **P(t) = v(t).i(t)**



$$P_{av} = \frac{V_{pk}.I_{pk}}{2}$$

$$P_{av} = \frac{V_{pk}.I_{pk}}{2}$$

$$P_{av} = \frac{V_{pk}}{\sqrt{2}}.\frac{I_{pk}}{\sqrt{2}}$$

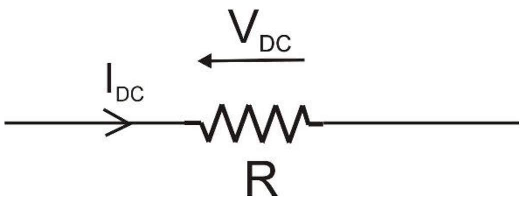
$$P_{av} = V_{rms}.I_{rms}$$

This is why rms values of voltage and current are important in AC systems

Voltage ········ Current Power

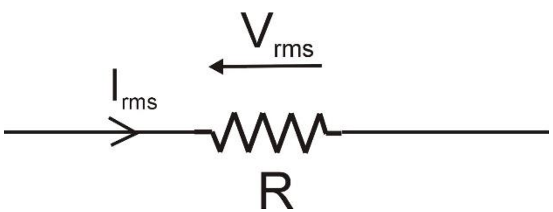
Average Power Loss in a Resistor:

DC System:



$P_{av} = V_{DC}.I_{DC}$

AC System:



$P_{av} = V_{rms}.I_{rms}$

$$P_{av} = \frac{1}{\tau}\int_0^{\tau} p_{inst}.d\theta$$

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} v_{inst}.i_{inst}.d\theta$$

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} V_{pk}S\,\text{in}\,\theta.I_{pk}S\,\text{in}\,\theta.d\theta$$

$$P_{av} = \frac{V_{pk}.I_{pk}}{\pi}\int_0^{\pi} S\,\text{in}^2\theta.d\theta$$

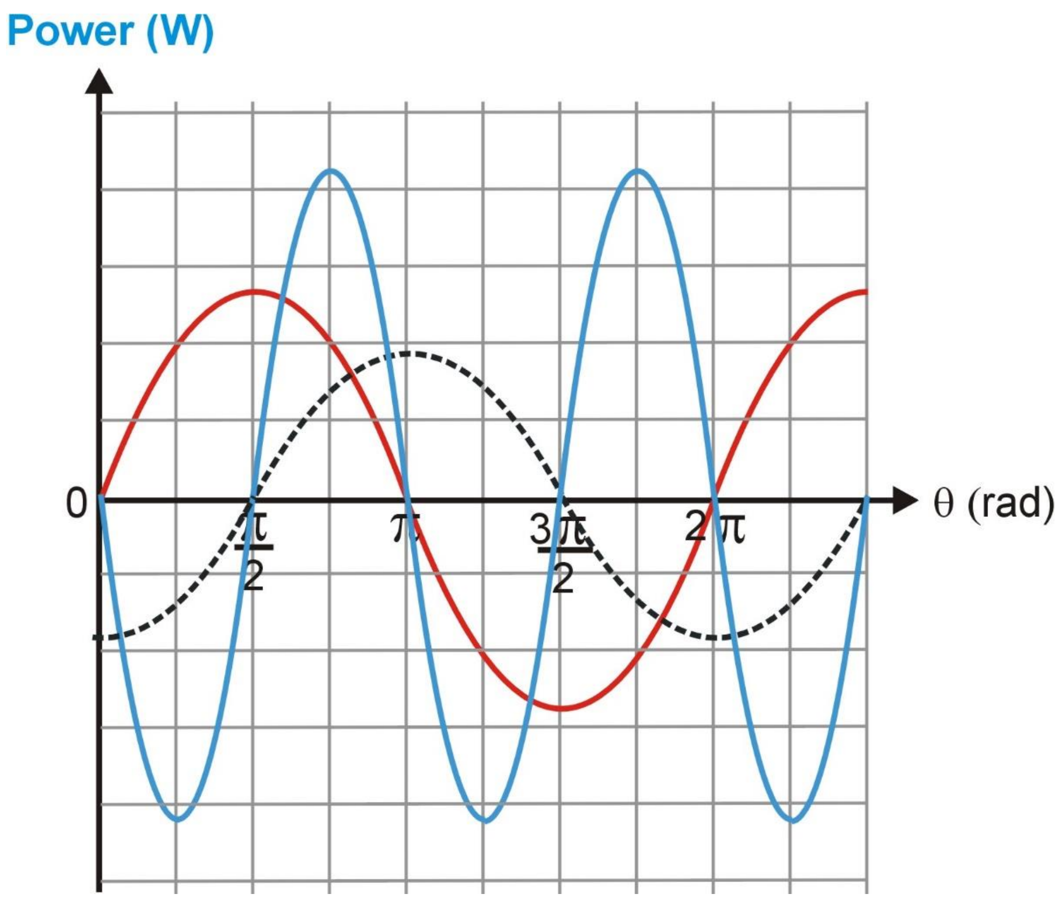
$$P_{av} = \frac{V_{pk}.I_{pk}}{\pi}\left[\frac{\pi}{2}\right] = \frac{V_{pk}}{\sqrt{2}}.\frac{I_{pk}}{\sqrt{2}}$$

In an AC system the rms values of voltage and current across a resistance results in the same power loss (HEAT) as that produced by the same values of voltage and current in a DC system – THAT'S WHY RMS IS IMPORTANT!

Inductance

Voltage
Current
Power

Instantaneous Power: **P(t) = v(t).i(t)**



BIG Result:

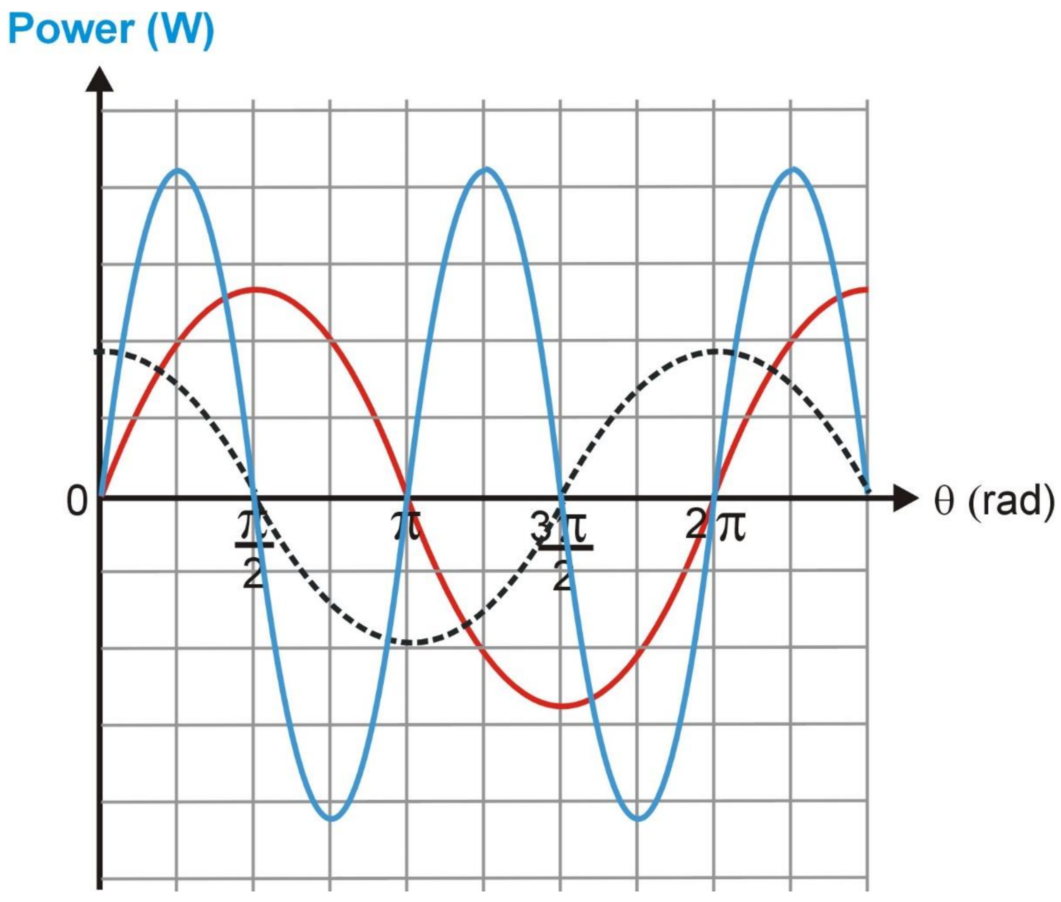
$$P_{av} = 0$$

The power simply oscillates between the inductor and the supply at twice the supply frequency

Note that since there is no average power loss an IDEAL inductor would not generate any heat

Capacitance

Instantaneous Power: **P(t) = v(t).i(t)**



Another BIG Result:

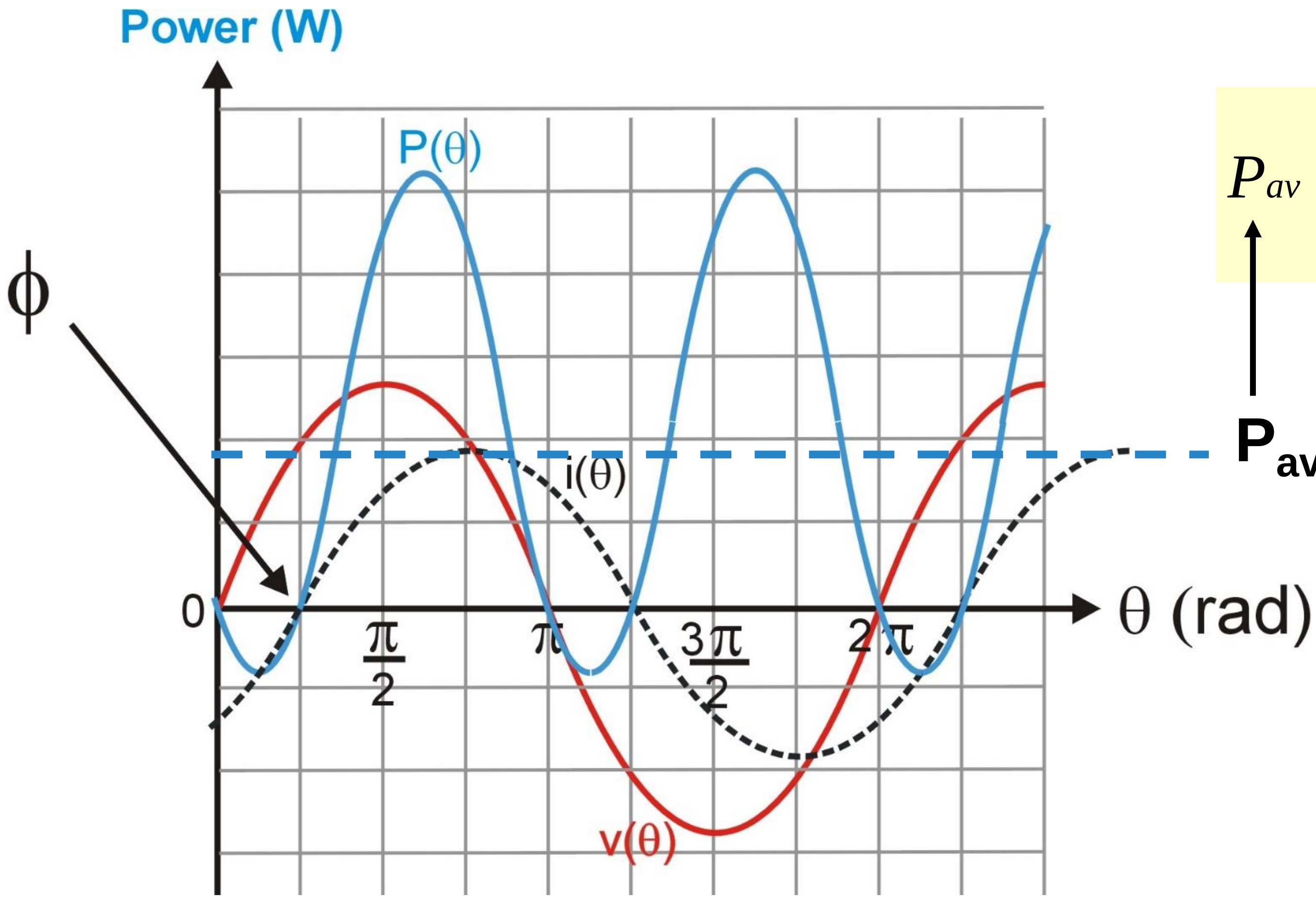
$$P_{av} = 0$$

Again the power simply oscillates between the capacitor and the supply at twice the supply frequency

Note that since there is no average power loss an IDEAL capacitor would not generate any heat

Complex Load (combination of R, L and C)

We need to derive a general expression for average power where the currents lags (or leads) the voltage by angle ϕ (where ϕ can be anywhere between 0 and 90°):



$$P_{av} = \frac{1}{\tau}\int_{0}^{\tau} v(\theta).i(\theta)d\theta$$

$$P_{av} = \frac{1}{\tau}\int_0^{\tau} v(\theta).i(\theta)d\theta$$

Replace Sin(θ-φ) with SinθCosφ - CosθSinφ

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} V_{pk}Sin\theta.I_{pk}Sin(\theta-\phi)d\theta$$

$$P_{av} = \frac{V_{pk}.I_{pk}}{\pi}\int_0^{\pi} Sin\theta.Sin(\theta-\phi)d\theta$$

$$P_{av} = \frac{V_{pk}.I_{pk}}{\pi}\int_0^{\pi} Sin\theta(Sin\theta Cos\phi - Cos\theta Sin\phi)d\theta$$

$$P_{av} = \frac{V_{pk}.I_{pk}}{\pi}\left[\int_0^{\pi} Sin^2\theta Cos\phi - Sin\theta Cos\theta Sin\phi)d\theta\right]$$

$$P_{av} = \left[\frac{Vpk.Ipk.Cos\phi}{\pi}\int_0^{\pi} Sin^2\theta.d\theta\right] - \left[\frac{V_{pk}.I_{pk}.Sin\phi}{\pi}\int_0^{\pi} Sin\theta Cos\theta.d\theta\right]$$

$$P_{av} = \left[\frac{Cos\phi . V_{pk} . I_{pk} .}{\pi} \int_0^{\pi} Sin^2\theta . d\theta \right] - \left[\frac{V_{pk} . I_{pk} . Sin\phi}{\pi} \int_0^{\pi} Sin\theta Cos\theta . d\theta \right]$$

From an ealier slide we saw that:

$$\frac{V_{pk} . I_{pk}}{\pi} \int_0^{\pi} Sin^2\theta . d\theta = V_{rms} . I_{rms}$$

$$\int_0^{\pi} Sin\theta Cos\theta . d\theta = \left[-Cos\theta . Sin\theta \right]_0^{\pi} = 0$$

Therefore first term becomes:

$$V_{rms} . I_{rms} . Cos\phi$$

Power Factor

$$P_{av} = V_{rms} . I_{rms} . \underline{Cos\phi}$$

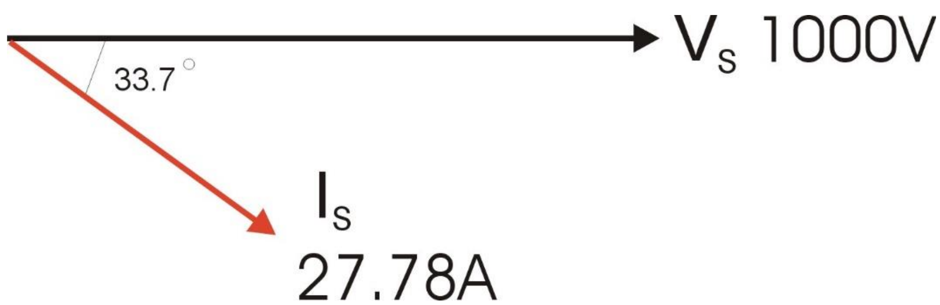
Where ϕ is the angle between the voltage and the current

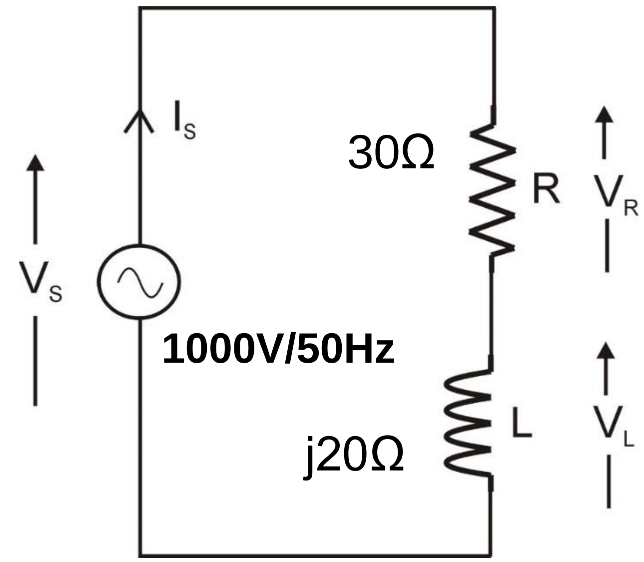
$$P_{out} = I_s^2 . R$$

$$P_{out} = 27.78^2 \times 30$$

$$P_{out} = 23152W$$

REAL power which is converted to heat or mechanical motion is not only a function of the rms voltage and current, it also depends on their phase relationship.





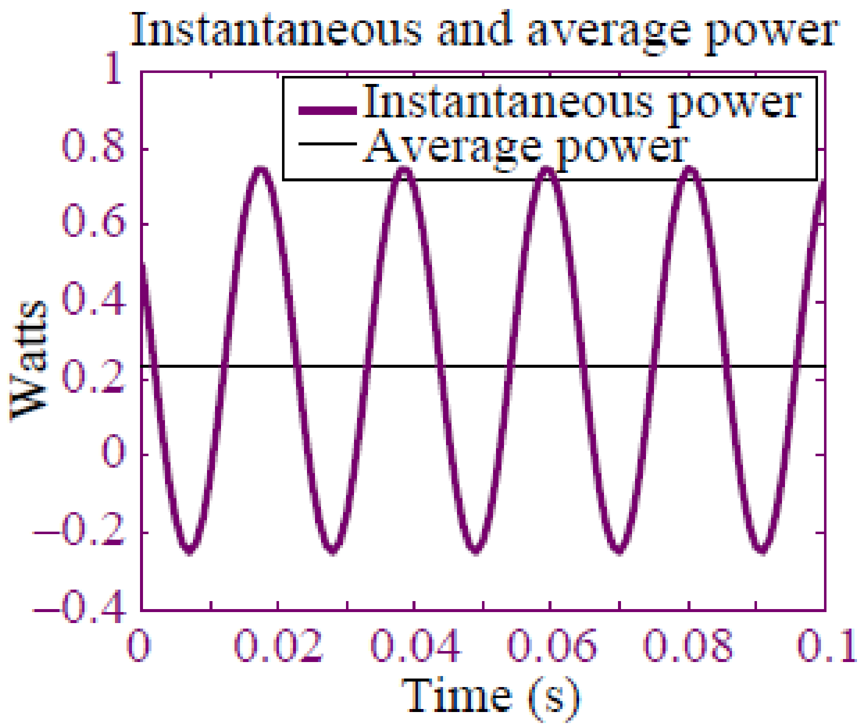
$$P_{av} = V_{rms}.I_{rms}.Cos\phi$$

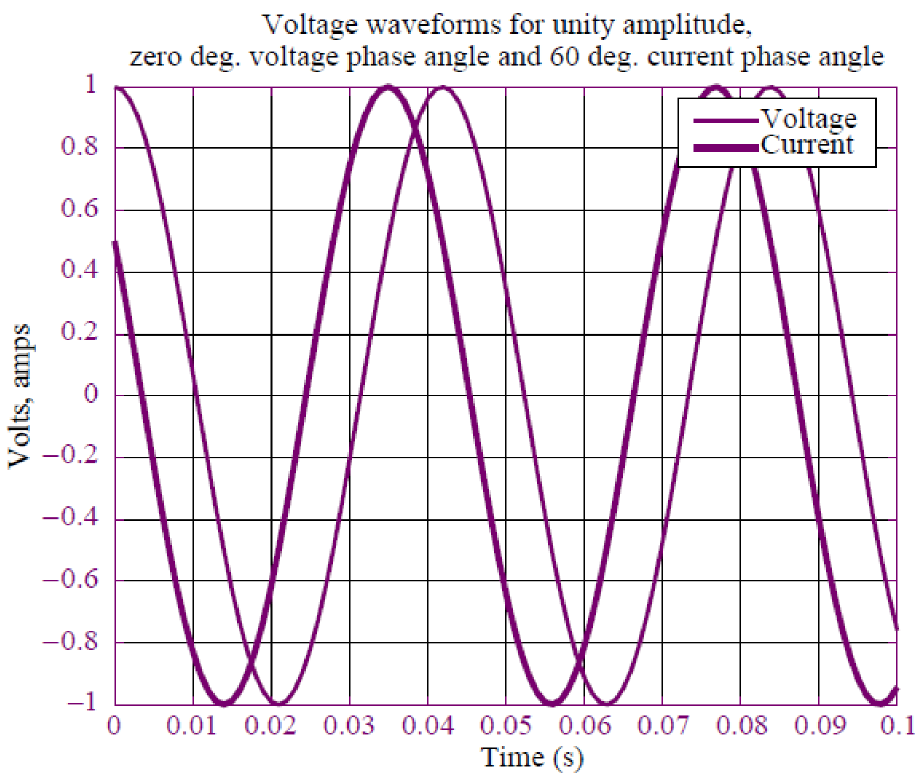
$$P_{av} = 1000 \times 27.78.Cos(33.7^o)$$

$$P_{av} = 23112W$$

Phew!, this agrees (allowing for rounding errors!) with the value calculated by I^2R calculation, and this is termed the **REAL** power.

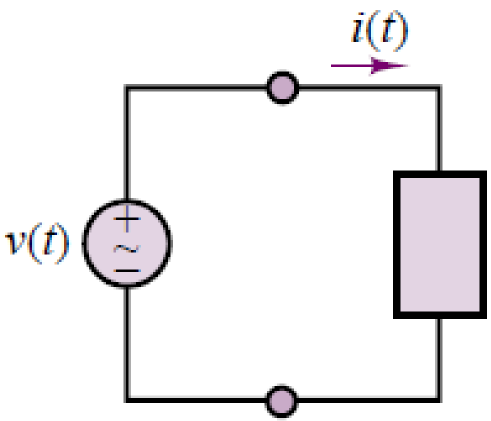
EXAMPLE


Instantaneous and average power
Instantaneous power
Average power
Watts
Time (s)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
0.02
0.04
0.06
0.08
0.1

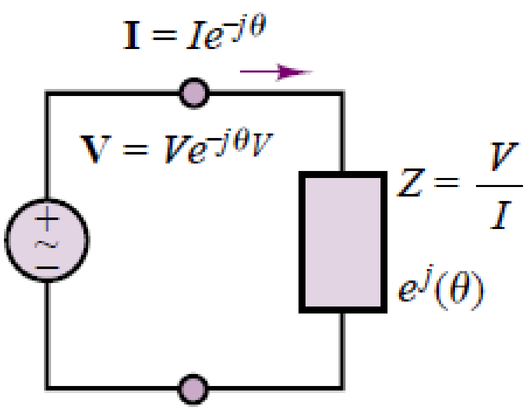

Voltage waveforms for unity amplitude,
zero deg. voltage phase angle and 60 deg. current phase angle
Voltage
Current
Volts, amps
Time (s)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

Complex Frequency Domain

RMS PHASORS FOR SINUSOIDAL SIGNALS



AC circuit
$v(t) = V\cos(\omega t - \theta_V)$
$i(t) = I\cos(\omega t - \theta_I)$



AC circuit
in phasor form

$$v(t) = V\cos\left(\omega t + \theta_V\right) = \sqrt{2}\tilde{V}\cos\left(\omega t + \theta_V\right)$$

$$V_{rms} = \tilde{V} = \frac{V}{\sqrt{2}}$$

$$\tilde{\mathbf{V}} = V_{\text{rms}} e^{j\theta_V} = \tilde{V} e^{j\theta_V} = \tilde{V}\angle\theta_V$$

$$i(t) = I\cos\left(\omega t + \theta_V\right) = \sqrt{2}\tilde{I}\cos\left(\omega t + \theta_I\right)$$

$$I_{rms} = \tilde{I} = \frac{I}{\sqrt{2}}$$

$$\tilde{\mathbf{I}} = I_{\text{rms}} e^{j\theta_I} = \tilde{I} e^{j\theta_I} = \tilde{I}\angle\theta_I$$

$$\theta = \theta_V - \theta_I$$

$$Z = \frac{V}{I} e^{j\theta} = |Z|\left(\cos\theta + j\sin\theta\right) = R + jX$$

POWER IN AC CIRCUITS (I)

Instantaneous Power

$$p(t) = v(t)i(t) = VI\cos\left(\omega t\right)\cos\left(\omega t - \theta\right)$$

$$= \frac{VI}{2}\cos\theta + \frac{VI}{2}\cos\left(2\omega t - \theta\right)$$

$$= \tilde{V}\tilde{I}\cos\theta\left(1 + \cos 2\omega t\right) + \tilde{V}\tilde{I}\sin\theta\sin 2\omega t$$

$$= \tilde{I}^2\left|Z\right|\cos\theta\left(1 + \cos 2\omega t\right) + \tilde{I}^2\left|Z\right|\sin\theta\sin 2\omega t$$

$$= \tilde{I}^2 R\left(1 + \cos 2\omega t\right) + \tilde{I}^2 X\sin 2\omega t$$

$$= P_{AV} + P_{AV}\cos 2\omega t + Q\sin 2\omega t$$

Average (Real) Power

$$P_{AV} = \int_0^T p(t)dt = V_{rms}I_{rms}\cos\theta$$

Reactive Power

$$Q = V_{rms}I_{rms}\sin\theta$$

$$Z = \left|Z\right|\left(\cos\theta + j\sin\theta\right) = R + jX$$

$$\cos\theta = \frac{R}{\sqrt{R^2 + X^2}}$$

$$\sin\theta = \frac{X}{\sqrt{R^2 + X^2}}$$

POWER IN AC CIRCUITS (II)

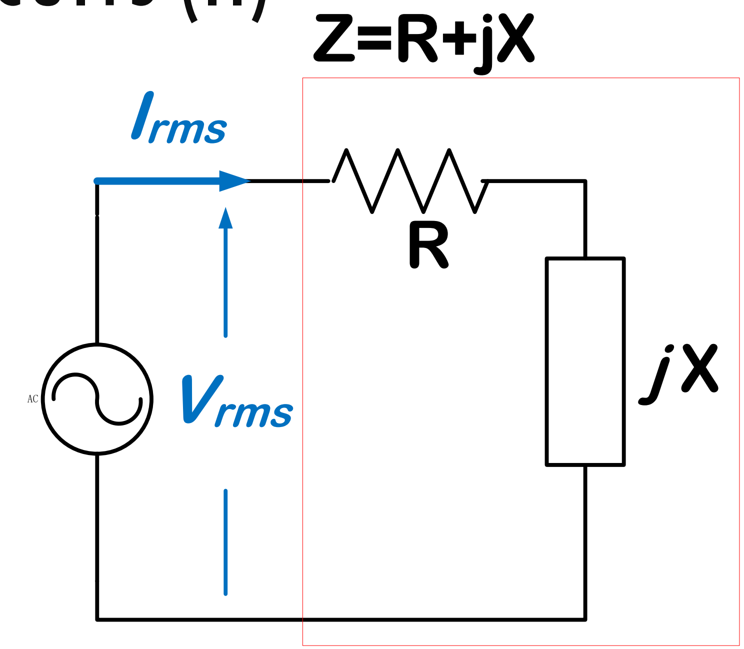
Average (Real) Power

$$P_{AV} = V_{rms} I_{rms} \cos\theta = I_{rms}^2 R$$

Reactive Power

$$Q = V_{rms} I_{rms} \sin\theta = I_{rms}^2 X$$

R, X in series



Load: $Z = |Z|(\cos\theta + j\sin\theta) = R + jX$

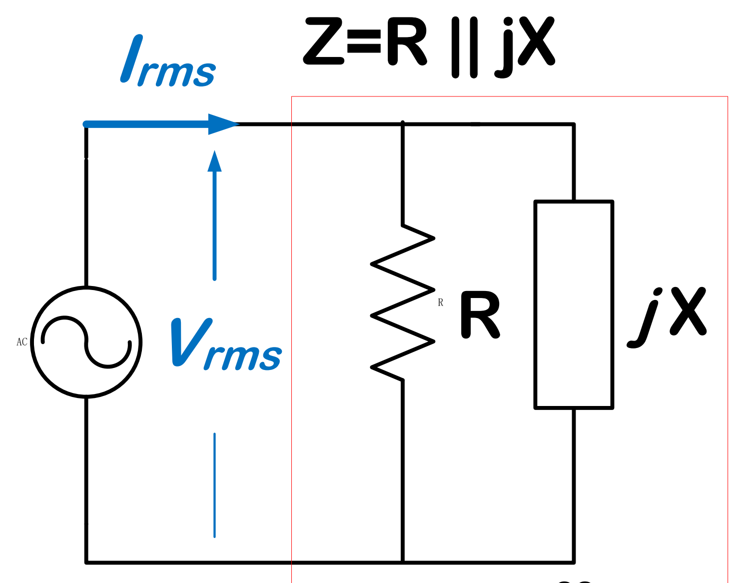
Average (Real) Power

$$P_{AV} = V_{rms} I_{rms} \cos\theta = V_{rms}^2 / R$$

Reactive Power

$$Q = V_{rms} I_{rms} \sin\theta = V_{rms}^2 / X$$

R, X in parallel



POWER FACTOR

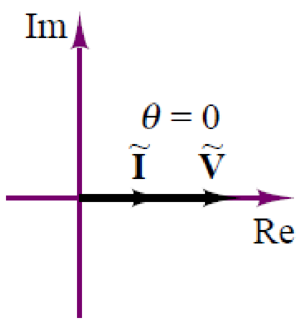
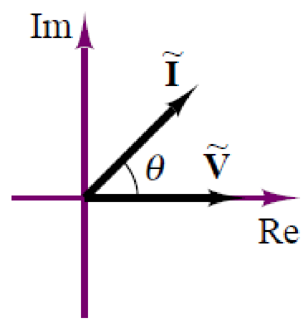
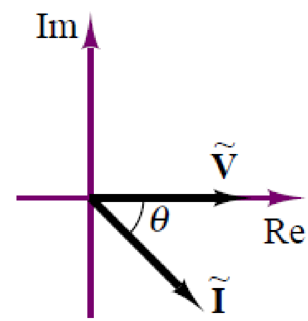
The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term cos*(θ)* is referred to as the power factor (pf). Note that the power factor is equal to 0 for a purely inductive or capacitive load and equal to 1 for a purely resistive load; in every other case,

$$0 < \text{pf} < 1$$

Two equivalent expressions for the power factor are given in the following:

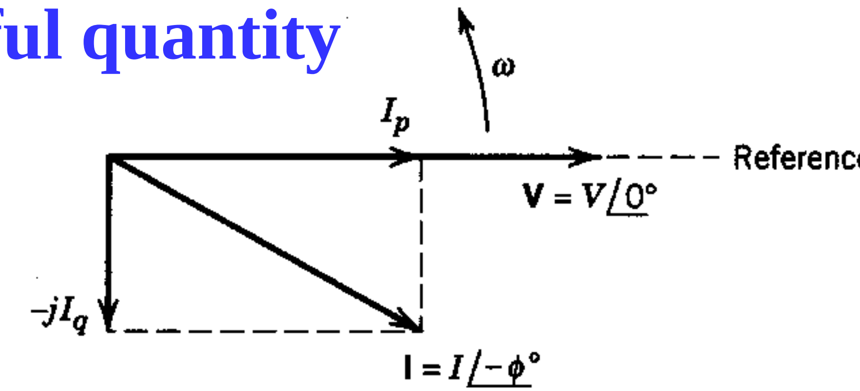
$$\mathbf{pf} = \cos(\theta) = \frac{P_{av}}{V_{rms} I_{rms}}$$

Power factor is dimensionless, a measure of how effectively the load draws the real power.

	Resistive load	**Capacitive load**	**Inductive load**
Ohm's law	$\tilde{\mathbf{V}}_L = Z_L \tilde{\mathbf{I}}_L$	$\tilde{\mathbf{V}}_L = Z_L \tilde{\mathbf{I}}_L$	$\tilde{\mathbf{V}}_L = Z_L \tilde{\mathbf{I}}_L$
Complex impedance	$Z_L = R_L$	$Z_L = R_L - jX_L$	$Z_L = R_L + jX_L$
Phase angle	$\theta = \theta$	$\theta < \mathbf{0}$	$\theta > \mathbf{0}$
Complex plane sketch			
Explanation	The current is in phase with the voltage.	The current "leads" the voltage.	The current "lags" the voltage.
Reactive power	0	Negative	Positive

COMPLEX POWER

a fictitious but very useful quantity



* Conjugate 共轭

$$\boxed{S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* \qquad \text{Complex power}}$$

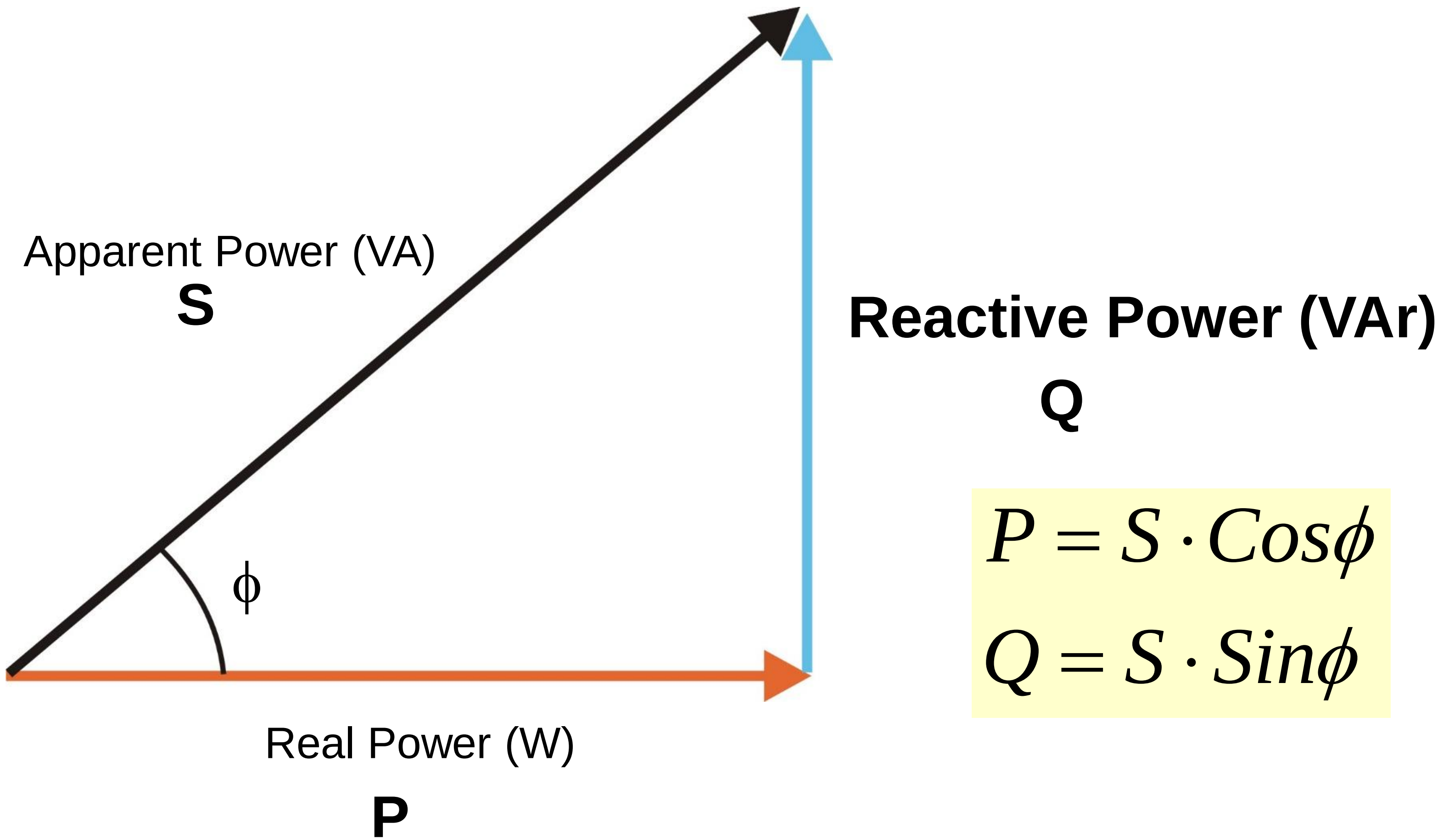
$$S = V_{rms} I_{rms} \cos\theta + jV_{rms} I_{rms} \sin\theta = I_{rms}^2 R + jI_{rms}^2 X = I_{rms}^2 Z$$
$$= P + jQ$$

$$\textit{apparent power}: |S| = \sqrt{P^2 + Q^2} = V_{rms} I_{rms}$$

$$\textit{real (average) power}: P = P_{av} = V_{rms} I_{rms} = V_{rms} I_p = |S|\cos\theta$$

$$\textit{reactive power}: Q = V_{rms} I_{rms} \sin\theta = V_{rms} I_q = |S|\sin\theta$$

The relationship between Real Power, Apparent Power and Power Factor angle ϕ is graphically represented by the **Power Triangle**:

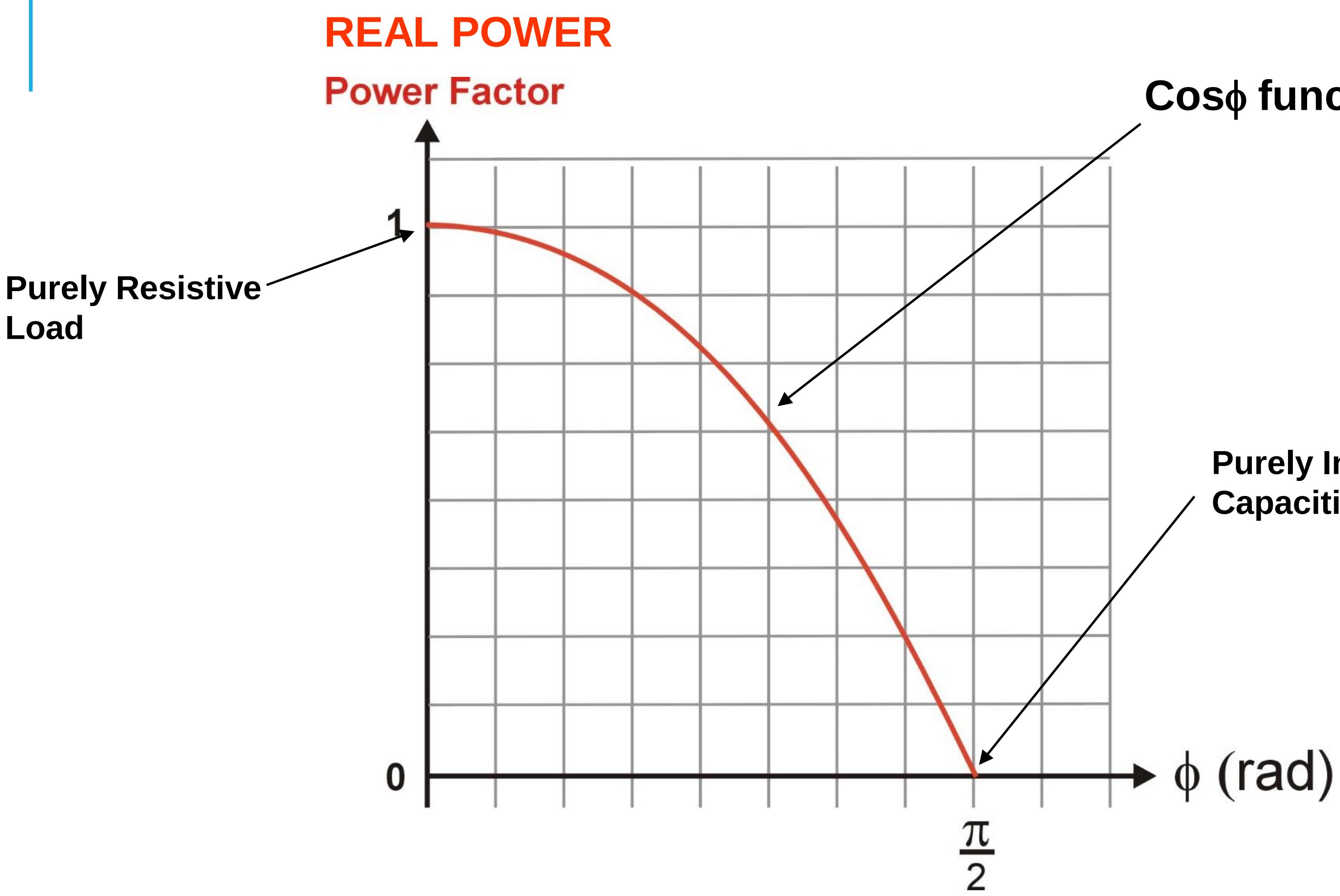


$$P = S \cdot Cos\phi$$

$$Q = S \cdot Sin\phi$$

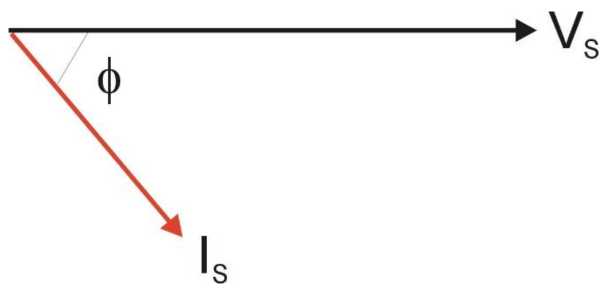
Where ϕ is the angle between the voltage and the current

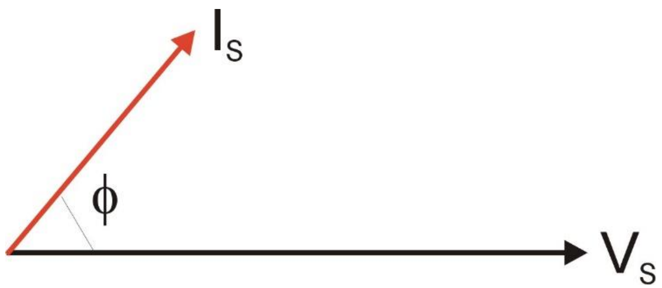
Power Factor (CosΦ) as a function of ϕ :



As we will see in future lectures/lab **Power Factor** is very important in Power Systems, the general aim being to keep it as high as possible (>0.9)

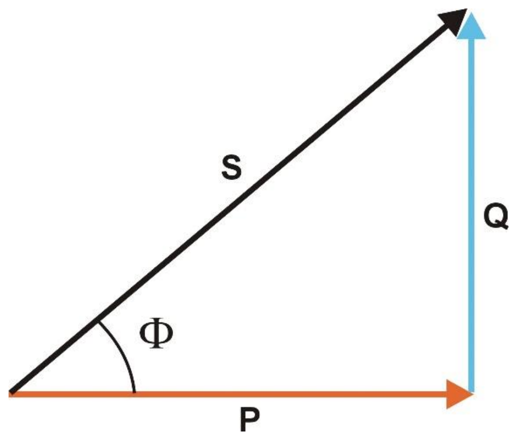
Rules:

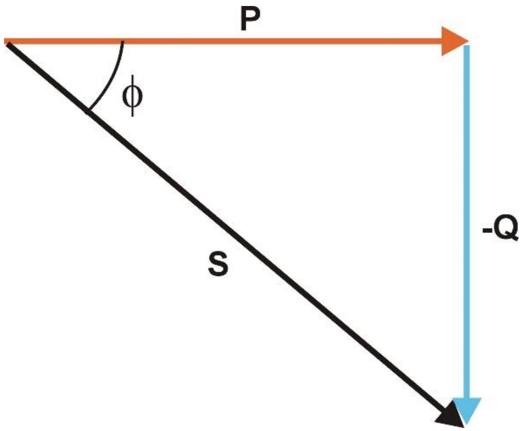




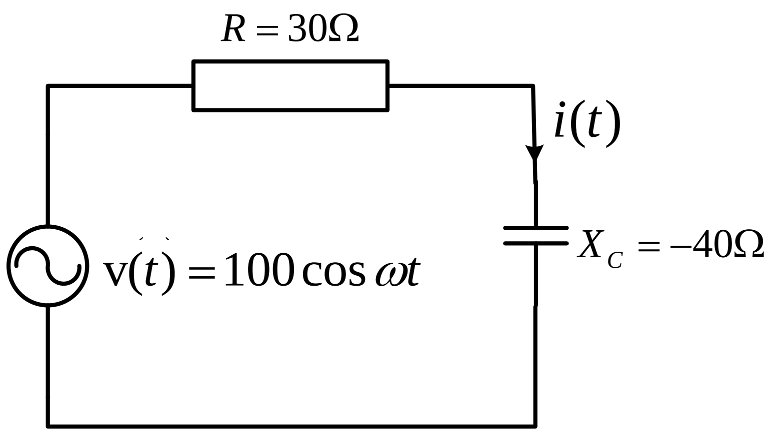
If the Current **LAGs** the voltage by angle ϕ (indicating that the load is inductive + resistive) then Reactive Power (Q) is deemed to be Positive and is termed **ABSORBING VAr's**

If the Current **LEADs** the voltage by angle ϕ (indicating that the load is capacitive + resistive) then Reactive Power (Q) is deemed to be Negative and is termed **GENERATING VAr's**





EXAMPLE



- **Determine the the values of the current *i*(*t*), write it in phasor form;**
- **Calculate the average power dissipation P_{av} in the circuit, and explain where the power is dissipated.**

$$Z = 30 - j40 = \sqrt{(-40)^2 + 30^2}\angle\tan^{-1}(-40/30)$$

$$= 50\angle - 53.13°$$

$$V = 100\angle 0°$$

$$I = \frac{V}{Z} = \frac{100\angle 0°}{50\angle - 53.13°} = 2\angle 53.13° = I_m\angle\varphi$$

$$i(t) = 2\cos\left(\omega t + 53.13°\right)$$

$$V = 100\angle 0 = \sqrt{2}V_{rms}\angle 0 = \sqrt{2}\left(\frac{100}{\sqrt{2}}\angle 0\right)$$

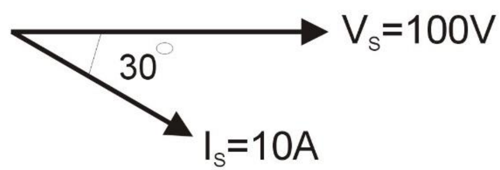
$$I = 2\angle 53.13° = \sqrt{2}I_{rms}\angle 53.13° = \sqrt{2}\left(\sqrt{2}\angle 53.13°\right)$$

$$\cos\varphi = 30/\sqrt{30^2 + (-40)^2} = 3/5 = 0.6$$

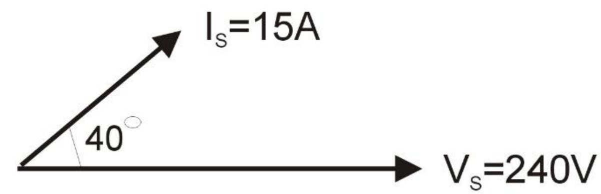
$$P_{avg} = V_{rms}I_{rms}\cos\varphi = I_{rms}^2 R = \left(\sqrt{2}\right)^2 \times 30 = 60\,\mathbf{W}$$

Calculate the Real Power, Apparent Power and Power Factor for the following 4 conditions:

1

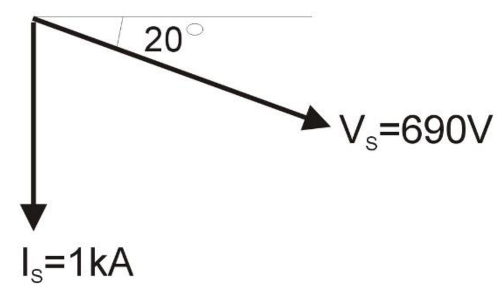


2



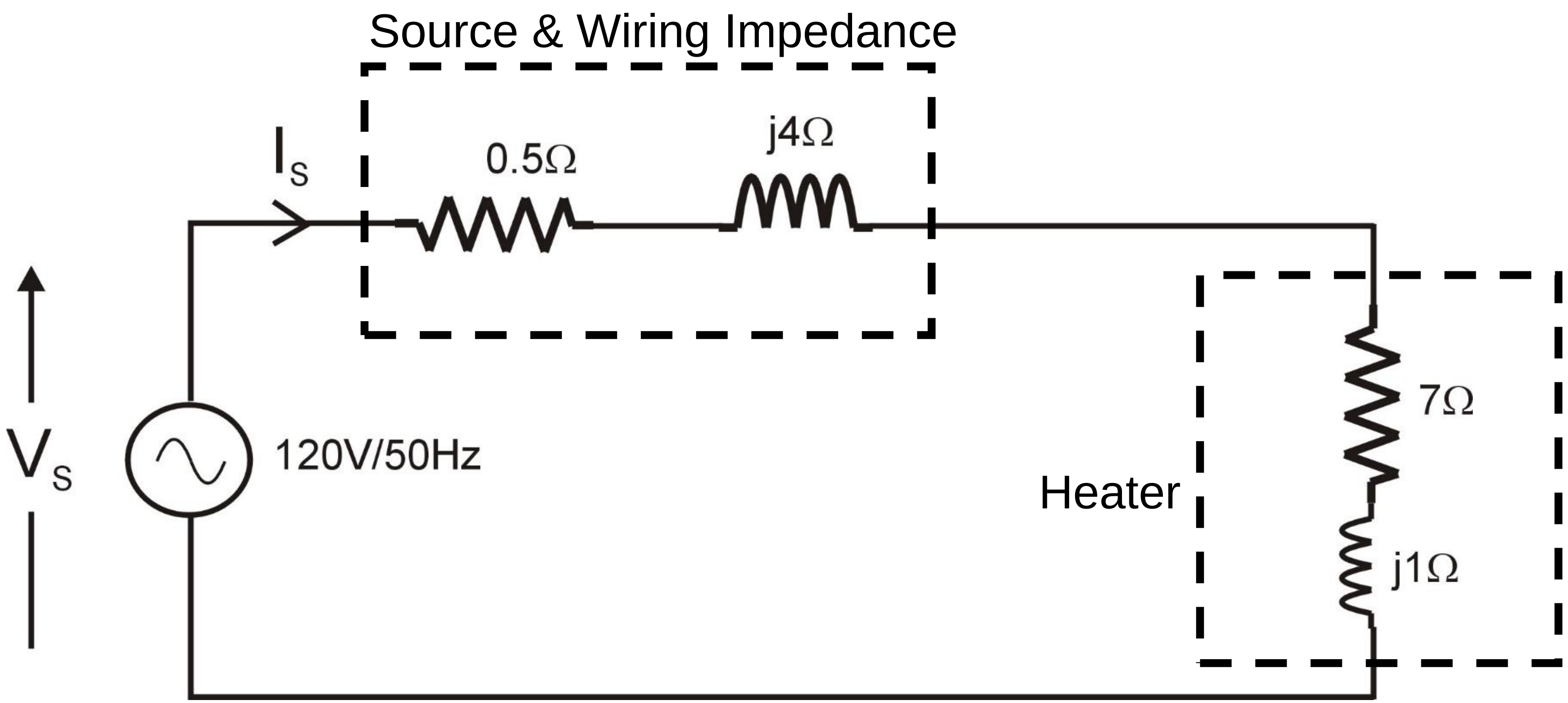
3

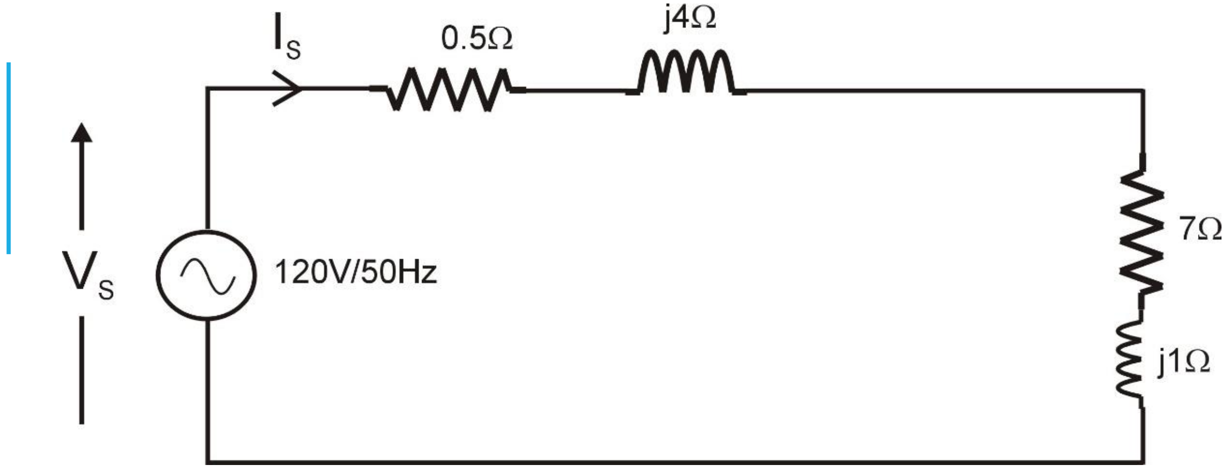
4



So lets look at an example:

The heating system for a remote monitoring station consists of a diesel generator which outputs 120V/50Hz connected to a Heater load. The heater is primarily resistive but has a small amount of inductance. The resistive wiring and inductive source impedance of the generator are also included in the equivalent circuit:





Solution done on whiteboard during lecture !!!

Determine the following:

1] The total circuit impedance Z_T

2] The supply current I_S

3] A phasor diagram indicating the supply voltage V_S and the supply current I_S

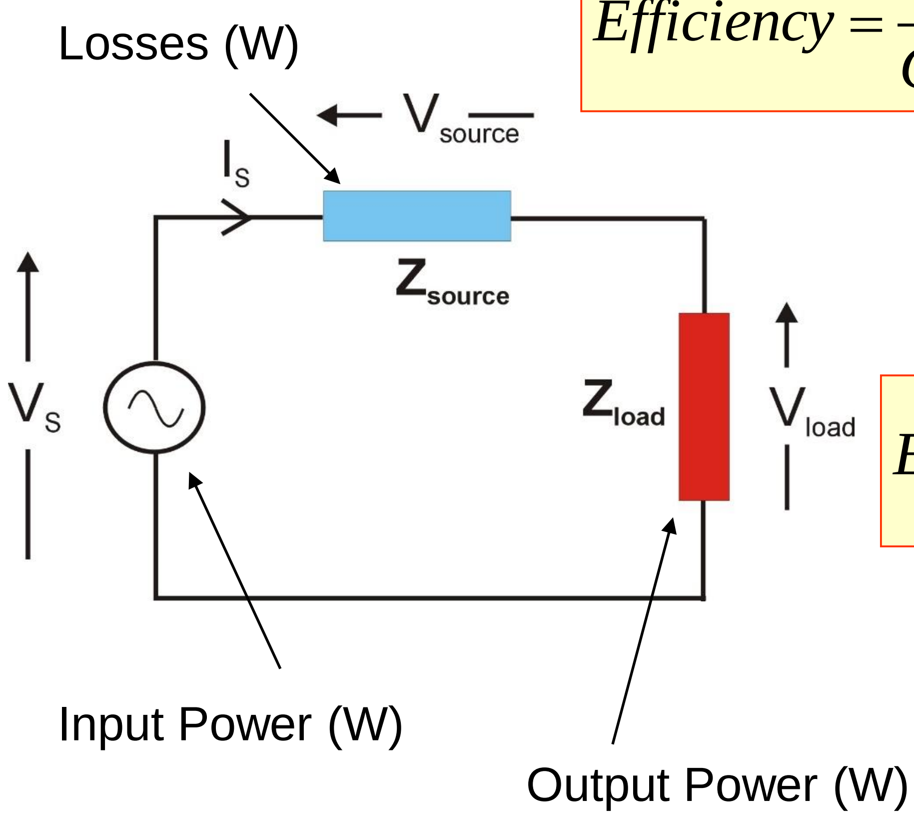
4] Apparent Power, Real Power, Reactive Power and Power Factor at the power supply

5] The Heater output power

Note that the (Real) power supplied by the generator is bigger than the (Real) power output by the heater. This is because we now have losses in the system due to the resistive element in the wiring (0.5Ω)

Definition of Efficiency in Electrical Power Systems:

$$Efficiency = \frac{OutputPower(W)}{OutputPower(W) + Losses(W)}.100\%$$



$$Efficiency = \frac{OutputPower(W)}{InputPower(W)}.100\%$$

Note that Input & Output Power are REAL Powers (W)