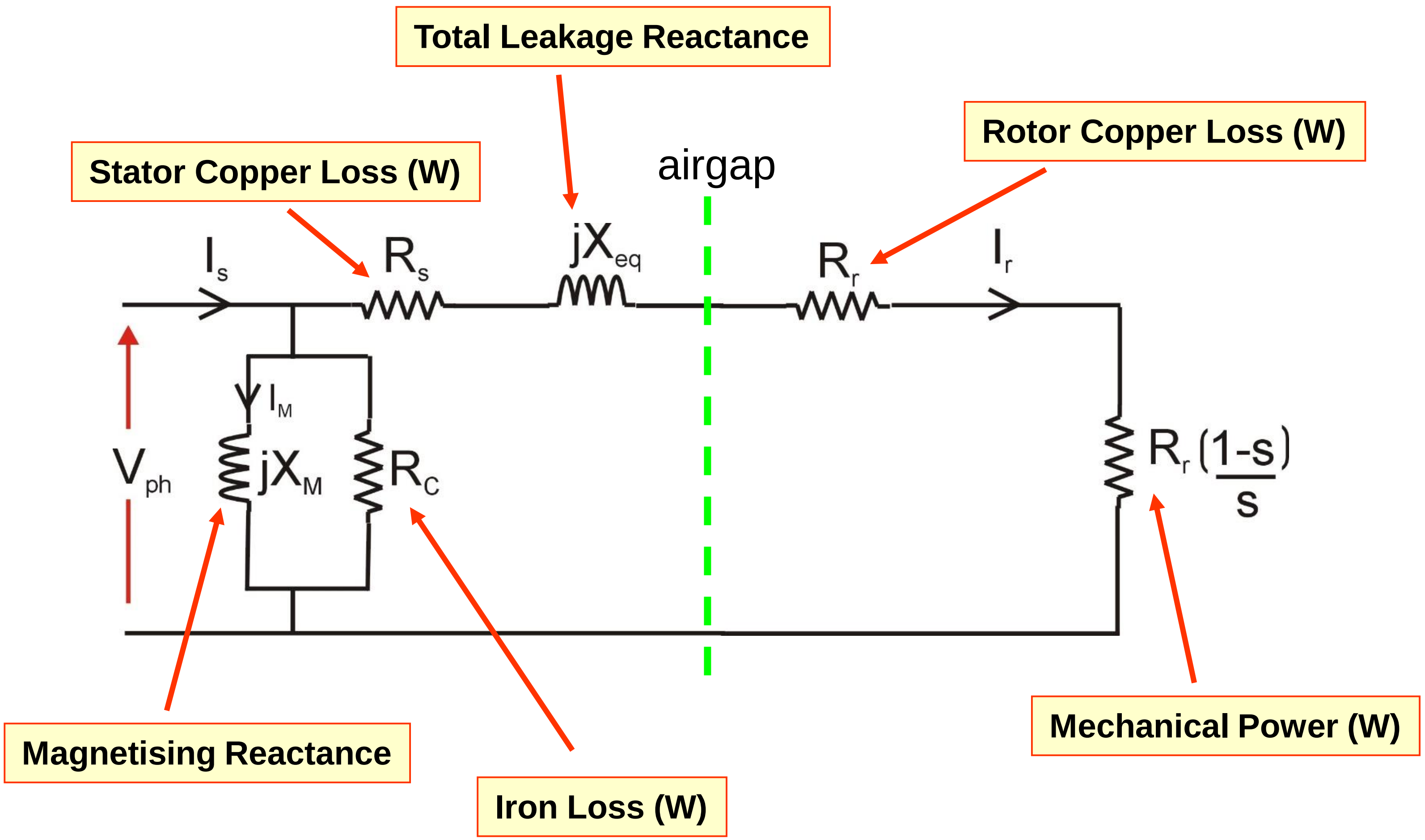
POWER ENGINEERING

#14 EQUIVALENT CIRCUIT OF INDUCTION MOTORS

2018

University of Glasgow

The Induction Motor (per phase) Equivalent Circuit



The plan for today:

- Induction Motor power flow and losses
- Using the Induction Motor equivalent circuit to determine machine performance Cont….
- Induction Motor torque v speed curves
- Torque Equation
- Wound Rotor + External Rotor Resistance
- Variable Speed Operation

Induction Motor Power Flow & Losses:

Electrical Input Power:

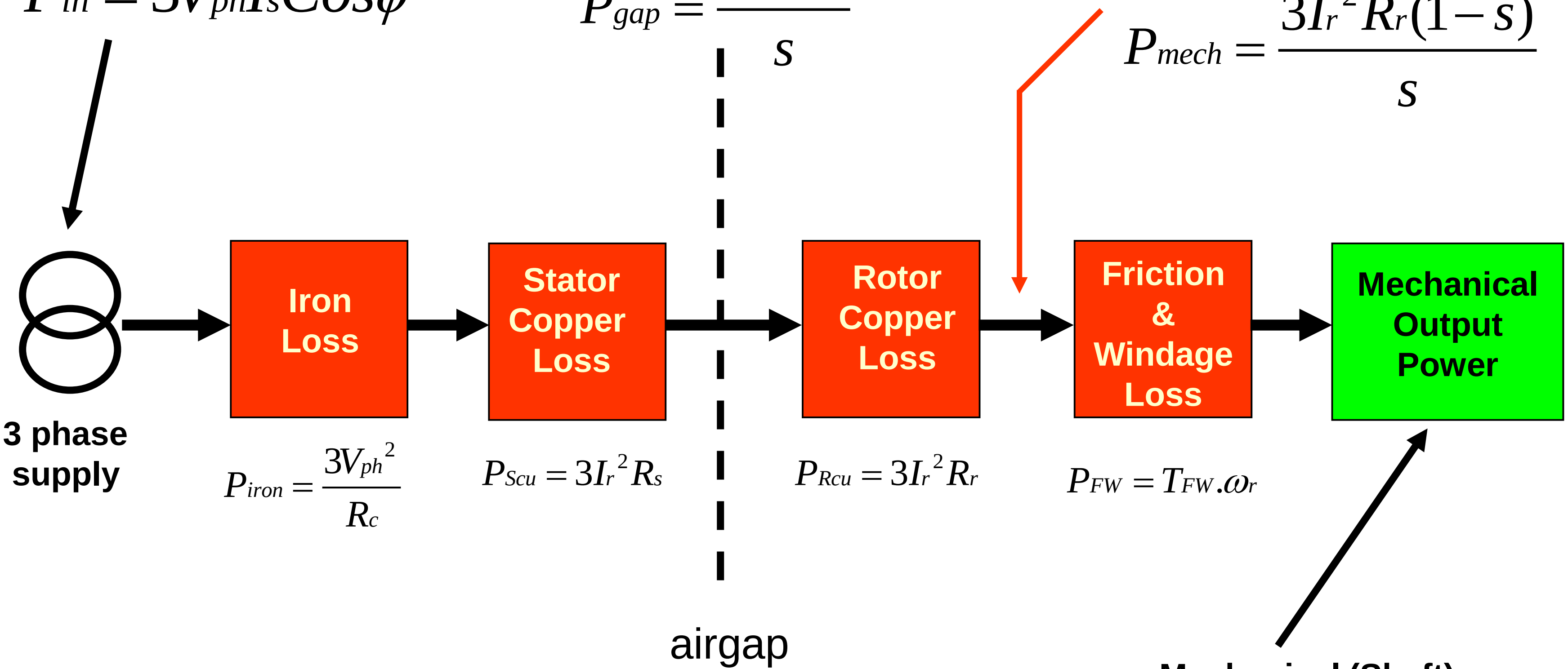
$$P_{in} = 3V_{ph}I_s Cos\phi$$

Power crossing Airgap:

$$P_{gap} = \frac{3I_r^{\,2}R_r}{s}$$

Equivalent circuit: Mechanical Power:

$$P_{mech} = \frac{3I_r^{\,2}R_r(1-s)}{s}$$



$$P_{iron} = \frac{3V_{ph}^{\,2}}{R_c}$$

$$P_{Scu} = 3I_r^{\,2}R_s$$

$$P_{Rcu} = 3I_r^{\,2}R_r$$

$$P_{FW} = T_{FW}.\omega_r$$

Mechanical (Shaft) Output Power:

$$P_{shaft} = T_{mech}.\omega_r$$

Note: ω_r is actual motor speed in rad/s

Power crossing the Airgap = $P_{gap} = \dfrac{3I_r^{\,2}R_r}{s}$

Mechanical Power = $P_{mech} = \dfrac{3I_r^{\,2}R_r(1-s)}{s}$ **= $\mathbf{P_{gap}.(1\text{-}s)}$**

Rotor Copper Loss = $P_{Rcu} = 3I_r^{\,2}R_r$ **= $\mathbf{P_{gap}.s}$**

So if for example we know the output power and the motor speed then we are able to determine the power crossing the airgap and the rotor copper loss

Note also: Pgap = Pin – Stator Copper Loss – Iron Loss

$P_{mech} = P_{FW} + P_{shaft}$

Example:

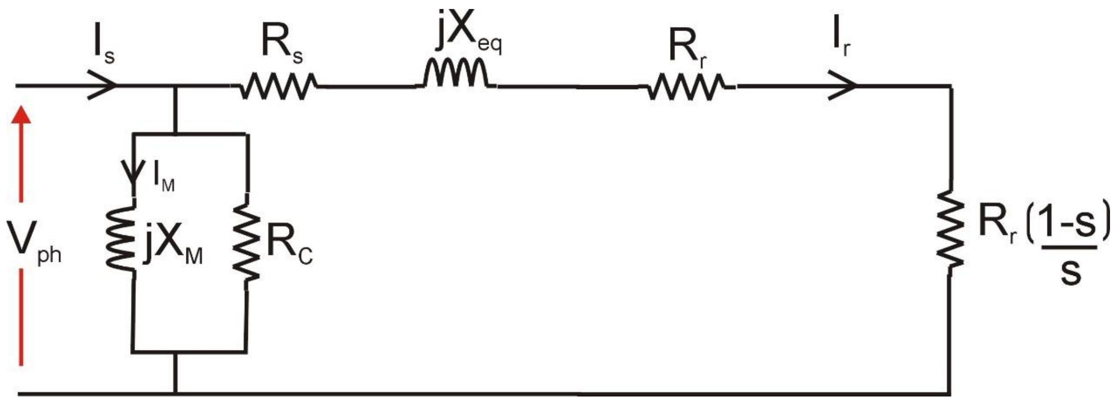


Table 1:

Parameter	Value
X_M	30Ω
R_c	500Ω
R_s	0.8Ω
X_{eq}	1.4Ω
R_r	0.3Ω

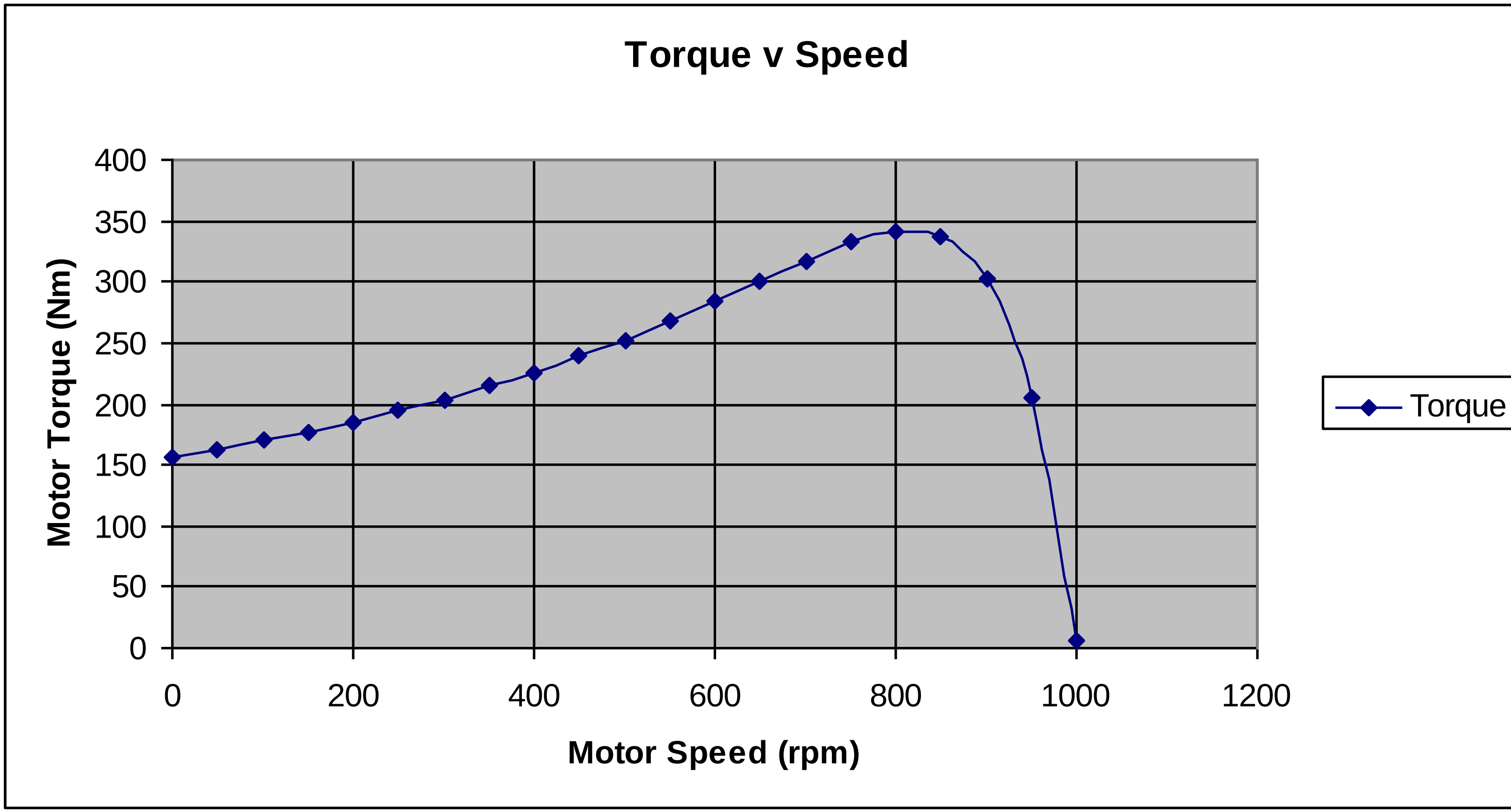
Given the equivalent circuit parameters shown on Table 1, determine the following for a 3 phase 6 pole machine rotating at 900rpm and operating off a 240V (phase voltage), 50Hz supply:

1. Slip (s)
2. Airgap Power (W)
3. Mechanical Output Power (W)
4. Motor Torque (Nm)
5. **Total Machine Losses (W)**
6. **Motor Efficiency (%)**

Note: Neglect Friction & Windage Loss

Solution done on whiteboard during lecture

A spreadsheet (Excel) calculation is useful here to determine torque (etc) as a function of speed (slip) from standstill to Synchronous Speed:



Q2 An 8 pole, 3 phase, 60Hz induction motor is operating at a speed of 870rpm. The input power is 33kW and the stator copper loss is 1200W. Friction and windage loss is 80W and the iron loss is negligible. Determine the following:

1] Slip (s)

2] Airgap Power (W)

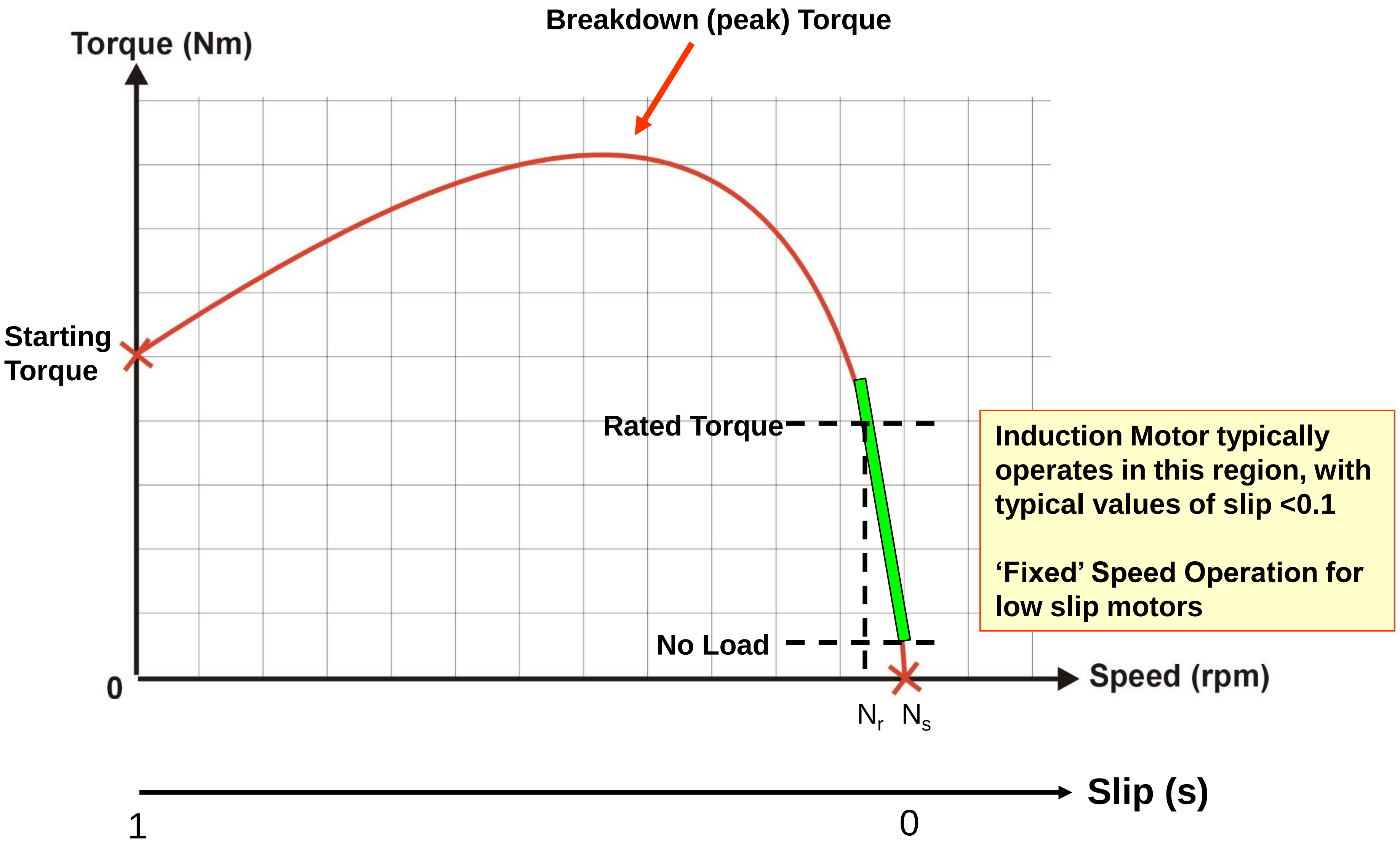
3] Rotor copper loss (W)

4] Mechanical output power (W)

5] Torque (Nm)

6] Efficiency (%)

Torque v Speed Curve:



Torque Equation:

Mechanical Power (W) = Torque (Nm) x Speed (rad/s)

$$P_{mech} = T_{mech} \cdot \omega_r$$

$$P_{gap}(1-s) = T_{mech}.\omega_r$$

$$T_{mech} = \frac{P_{gap}(1-s)}{\omega_r}$$

$$T_{mech} = \frac{P_{gap}(1-s)}{\omega_{sy}(1-s)} = \frac{P_{gap}}{\omega_{sy}}$$

$$T_{mech} = \frac{P_{gap}}{\omega_s.\frac{2}{P}}$$

Useful Relationships:

$$s = \frac{\omega_{sy} - \omega_r}{\omega_{sy}}$$

$$\omega_r = \omega_{sy}(1-s)$$

$$f_{sy} = f_s.\frac{2}{P}$$

$$\omega_{sy} = \omega_s.\frac{2}{P}$$

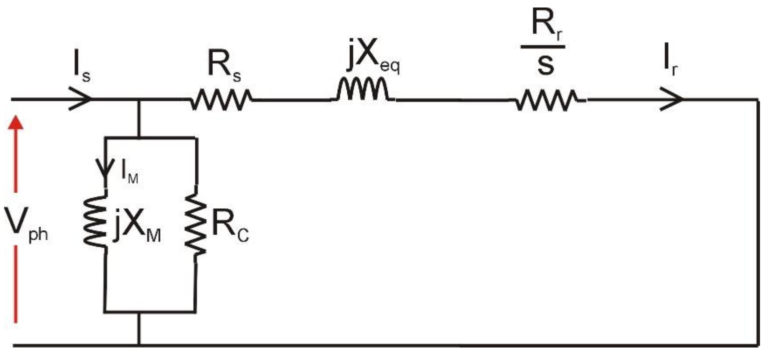
where:
f_{sy}=Synchronous frequency
f_s=electrical supply frequency
P= number of motor poles

$$P_{gap} = \frac{3.I_r^2.R_r}{s}$$

$$P_{mech} = \frac{3.I_r^2.R_r}{s}(1-s)$$

$$P_{mech} = P_{gap}(1-s)$$

Torque Equation cont. :



$$T_{mech} = \frac{1}{\omega_s} \cdot \frac{P}{2} \cdot P_{gap}$$

$$\Rightarrow \quad T_{mech} = \frac{1}{\omega_s} \cdot \frac{P}{2} \cdot \frac{3 I_r^{\,2} R_r}{s}$$

$$|V_s| = |I_r| \cdot \left[\left(R_s + \frac{R_r}{s} \right)^2 + X_{eq}^{\,2} \right]^{\frac{1}{2}}$$

$$\Rightarrow \quad T_{mech} = \frac{3}{\omega_s} \cdot \frac{P}{2} \cdot \frac{R_r}{s} \cdot I_r^{\,2}$$

$$\Rightarrow \quad T_{mech} = \frac{3}{\omega_s} \cdot \frac{P}{2} \cdot \frac{R_r}{s} \left[\frac{V_s^{\,2}}{(R_s + \frac{R_r}{s})^2 + X_{eq}^{\,2}} \right]$$

Torque Equation final :

$$T_{mech} = \frac{3}{\omega_s} . \frac{P}{2} . \frac{R_r}{s} \left[\frac{V_s^{\,2}}{(R_s + \frac{R_r}{s})^2 + X_{eq}^{\,2}} \right]$$

Notes:

1] ω_s is the electrical supply frequency in rad/s

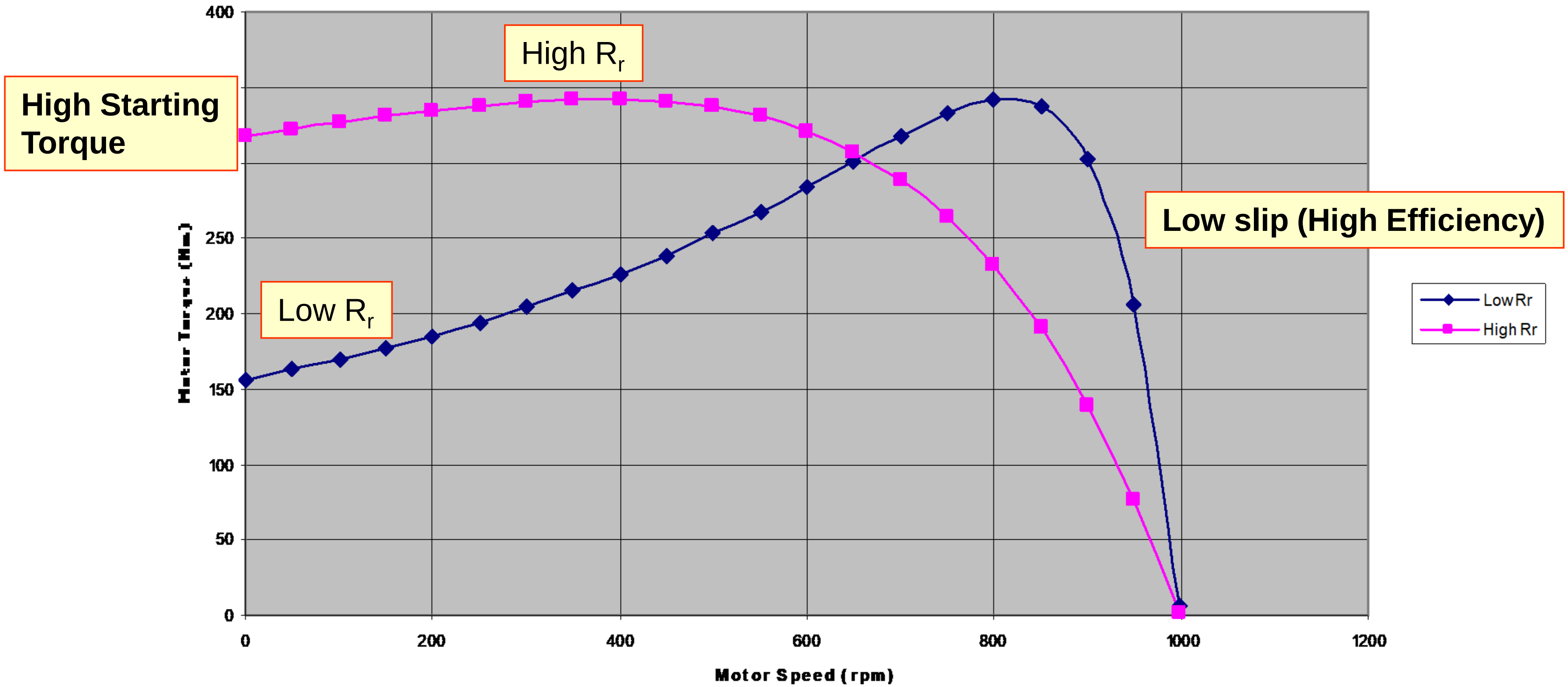
2] Torque is proportional to the square of the supply voltage (V_s)

3] Desire to keep R_s and X_{eq} to a minimum to maximise motor torque

4] There is a ‘complex’ relationship between Torque and Slip (s) hence the shape of the torque v speed (slip) curve

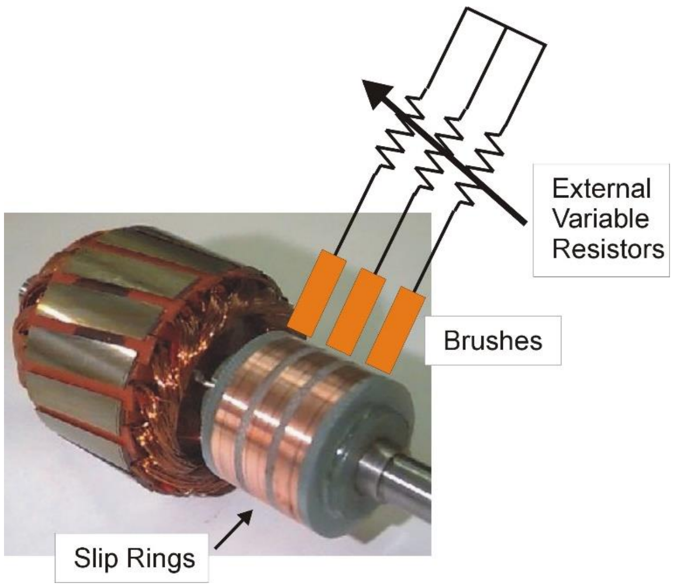
5] The magnitude of the Rotor Resistance (R_r) has opposite effects on the torque production at standstill and at rated speed (slip)

Effects of Rotor Resistance on Torque v Speed Curve

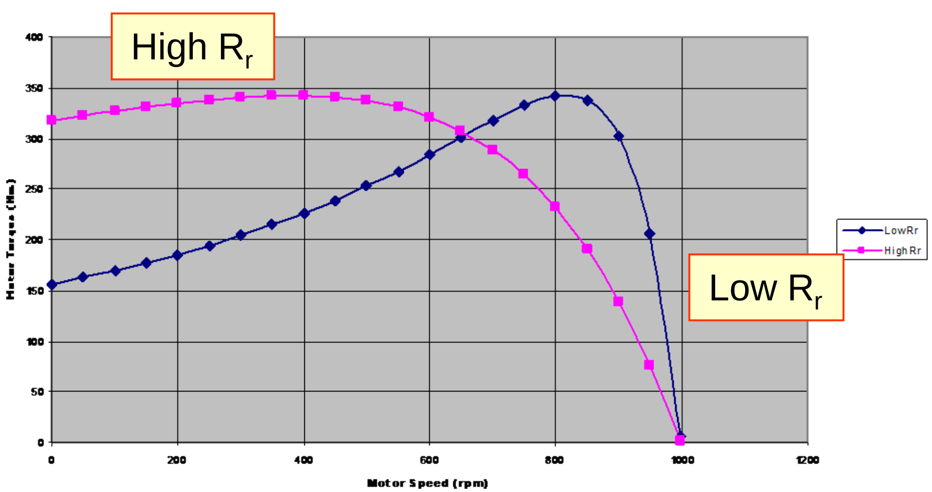


Result: a high Rotor Resistance gives higher starting torque, and a low Rotor Resistance gives high efficiency (low slip) at operating speed. A compromise is required or have Variable (External) Rotor Resistance

External Rotor Resistance: High Starting Torque



The Aluminium Rotor Cage is now replaced by a 3 phase wound rotor. The windings connect to external variable resistors via carbon brushes and Slip Rings



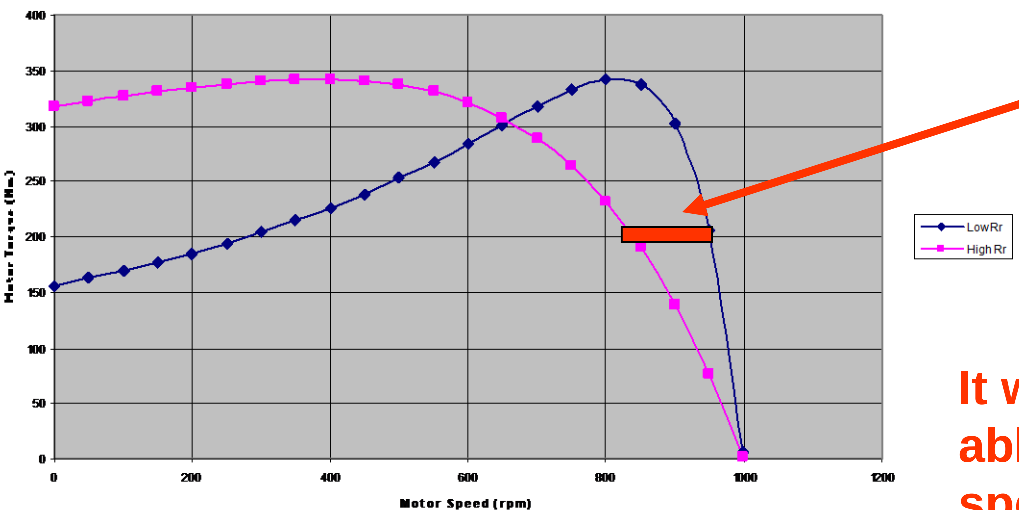
At Start-Up Select High R_r

At Operating Point Select Low R_r

External Rotor Resistance: (Limited) Variable Speed Operation

Many applications require a motor to operate over a range of speeds. In addition if we can control the speed of the motor then we can ‘match’ the input power to the demanded power and therefore waste less energy (this has ALOT more importance now that we are facing potential energy issues in the future!)

The inclusion of external rotor resistance allows the induction motor to operate over a *limited* speed range at a given load:



The red line indicates the limited range in speed the machine can now operate at for a given load

It would however be desirable to be able to operate over the complete speed range between zero speed and synchronous speed

Induction Motor: Modern Variable Speed Operation

Fixed Frequency Supply

High efficiency machine operates at low slip (<0.03%) therefore 'fixed speed'

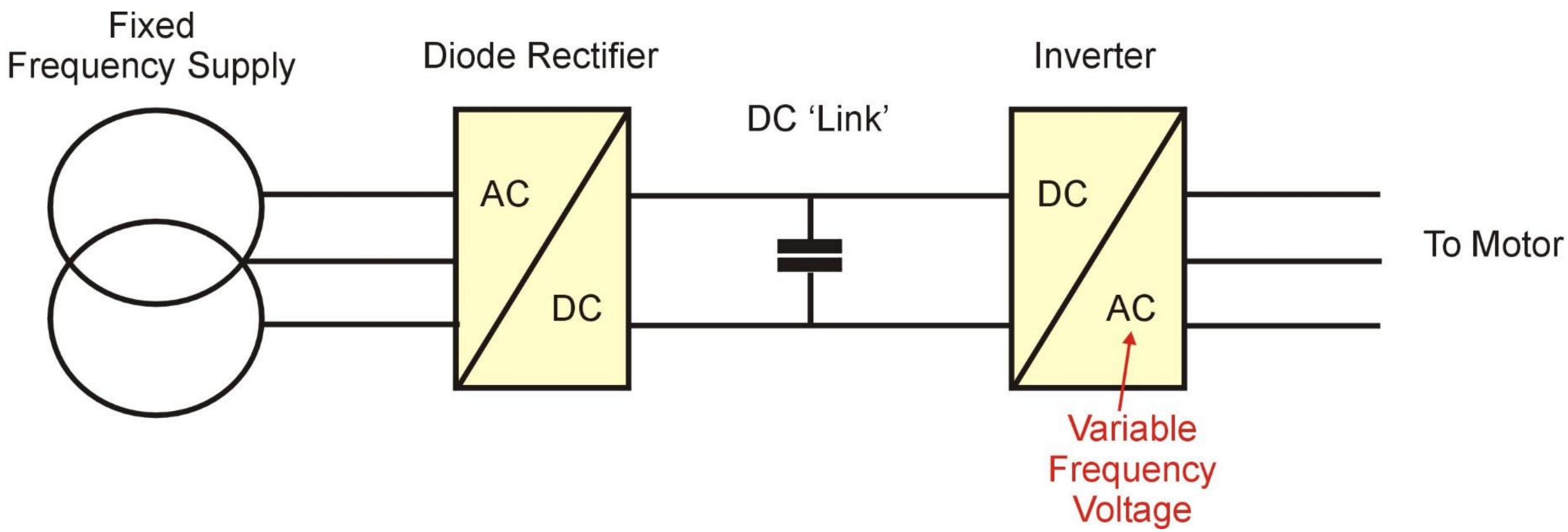
Variable Speed Solution #2:

Fixed Frequency Supply

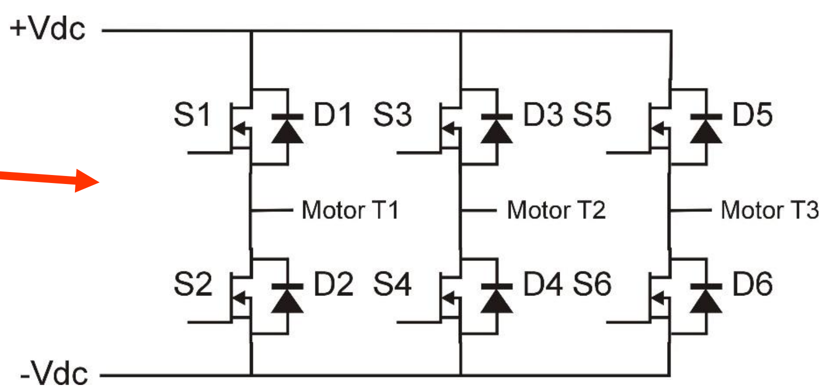
Power Electronic Converter

Controls voltage supply frequency to motor

Power Electronic Converter:



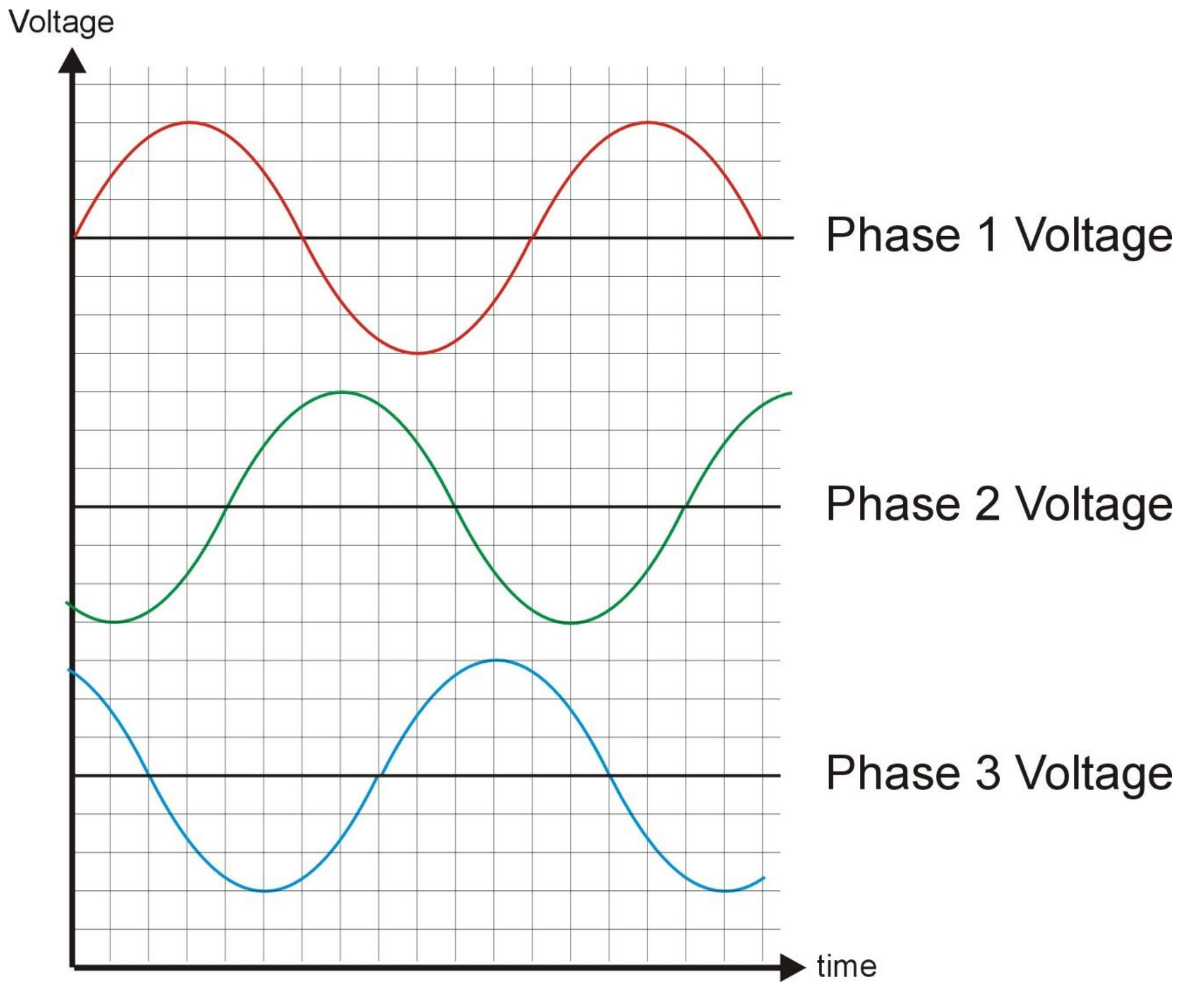
A microprocessor controls the electronic switches (S1-S6) to produce a variable frequency voltage supply (f_s) to the motor



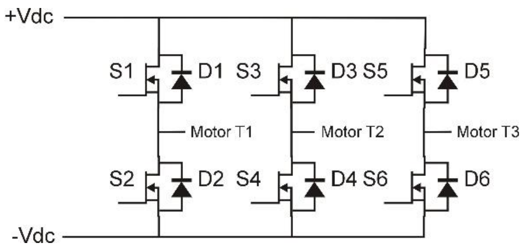
Note: when S1 is turned ON the Motor T1 is connected to +Vdc and when S2 is turned ON the Motor T1 is connected to –Vdc (same goes for other motor connections)

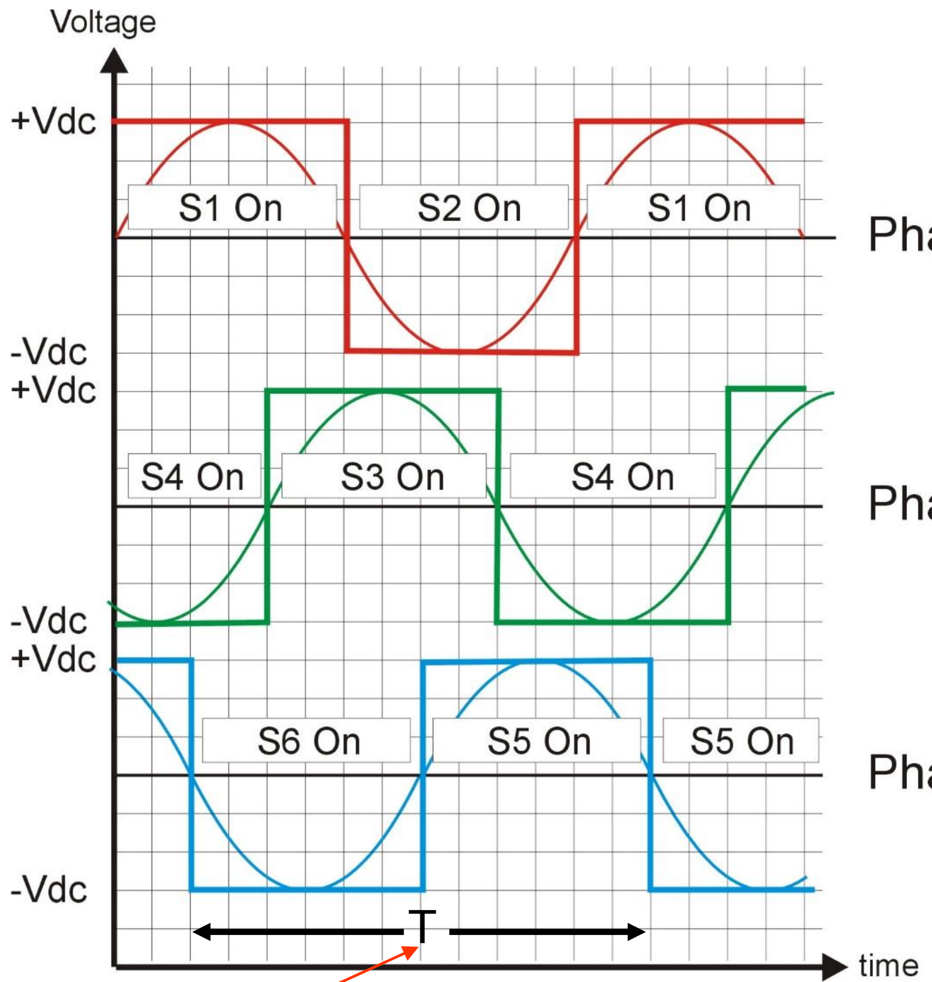
3 Phase Motor Voltage Requirements:

Voltages produced by 3 phase sinusoidal supply



3 Phase Motor Voltage Requirements:

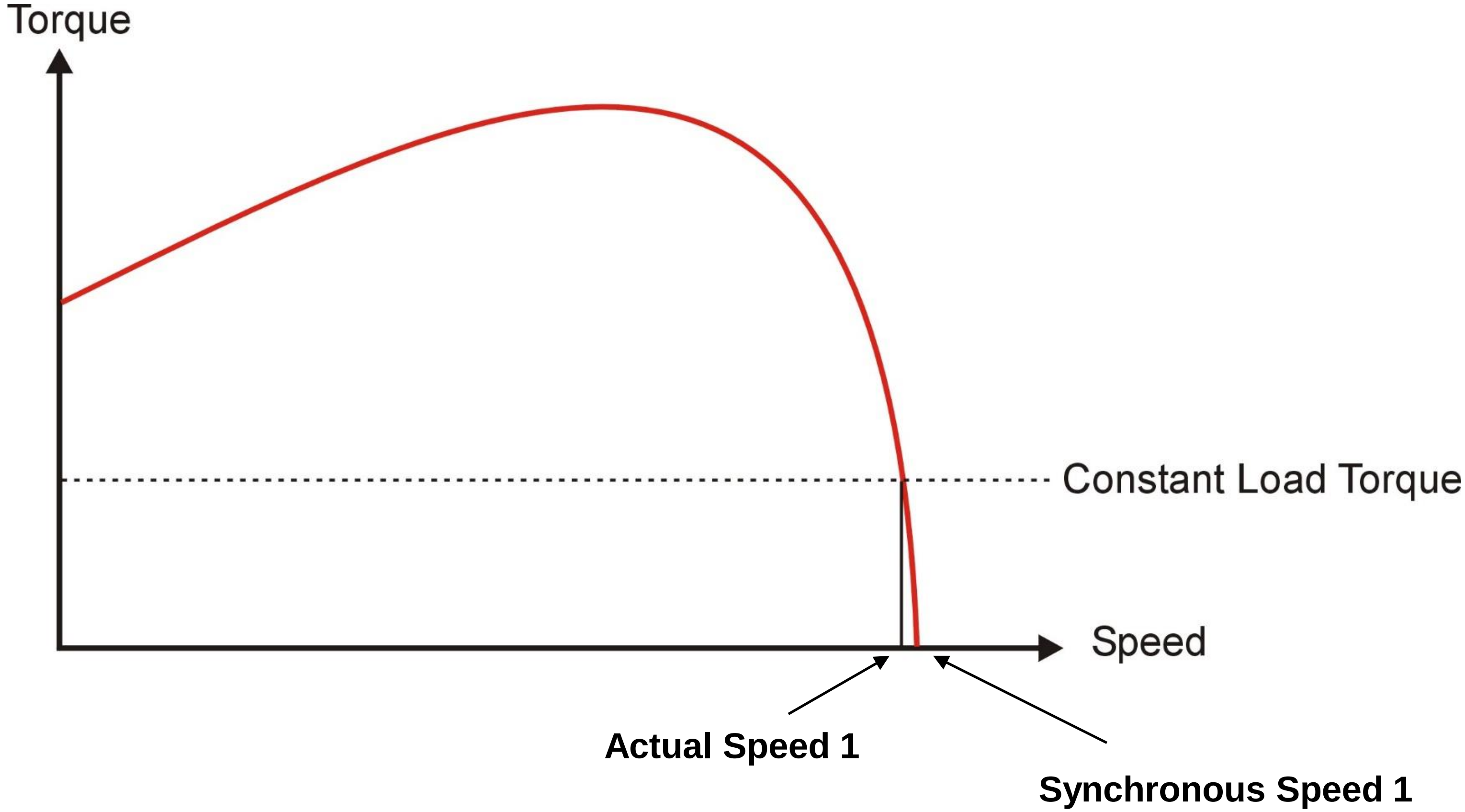




Period defines synchronous frequency

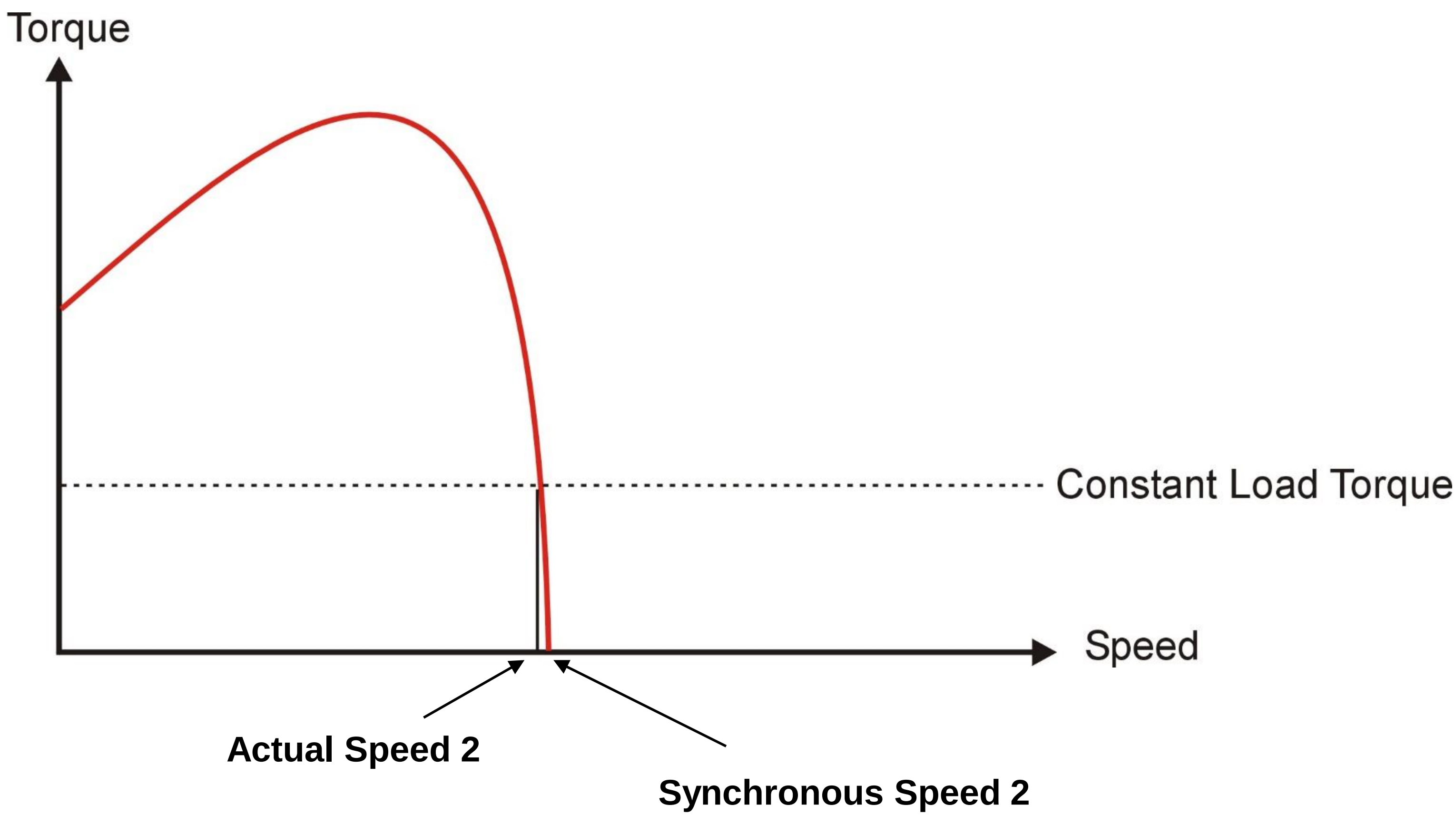
Torque v Speed curves for Variable Speed Operation

Speed 1:



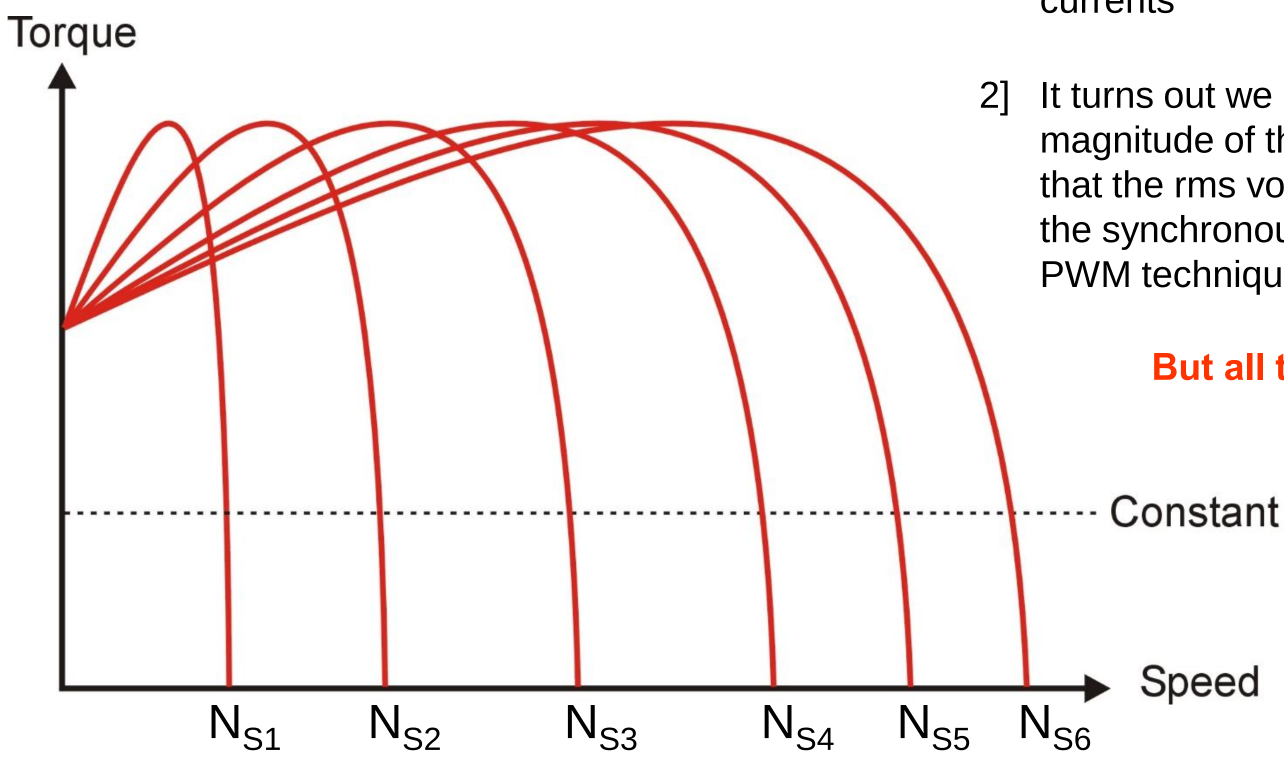
Torque v Speed curves for Variable Speed Operation

<u>Speed 2:</u>



Torque v Speed curves for Variable Speed Operation

$$V / f = \textbf{constant}$$



Speed Demand signal 'drives' synchronous frequency

Notes:

1] Sinusoidal Pulse Width Modulation (PWM) techniques improve the quality of the motor voltages and currents

2] It turns out we also need to control the magnitude of the motor voltages such that the rms voltage is proportional to the synchronous frequency – again PWM techniques take care of this.

But all this is another story….

With same slip ratio, the output torque will be same.