# POWER ENGINEERING

## #09 TRANSFORMERS

## EQUIVALENT CIRCUIT (1)

2018

University of Glasgow

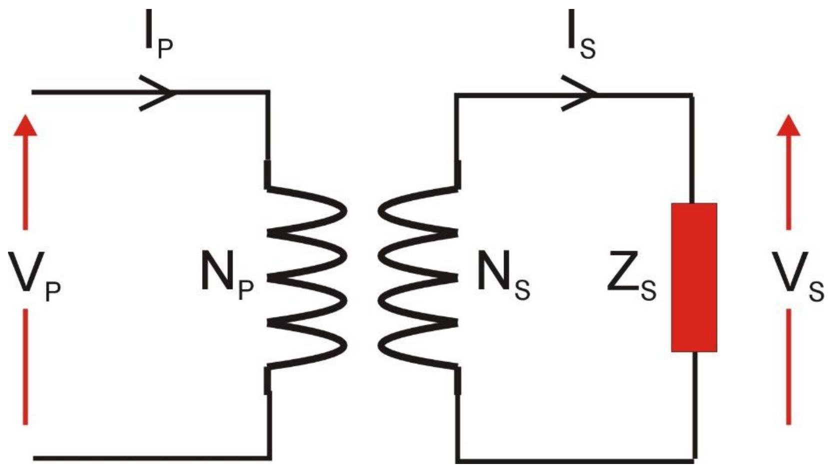
# Transformers

As we saw in lecture 2 the transformer is one of the principal reasons behind the adoption of AC power systems. Its ability to provide a (relatively!) cheap and reliable means of converting AC voltage levels results in high efficiency AC power transmission.

*Over the next 4 lectures we will investigate:*

- Types of Transformers
- The Ideal Transformer
- Basic Electromagnetics
- Transformer Limits
- The Equivalent Circuit for a practical Transformer **Today**
- Tests to determine the Equivalent Circuit
- Transformer performance under load: Regulation & Efficiency
- Three Phase Transformers

# Recap: The Ideal Transformer



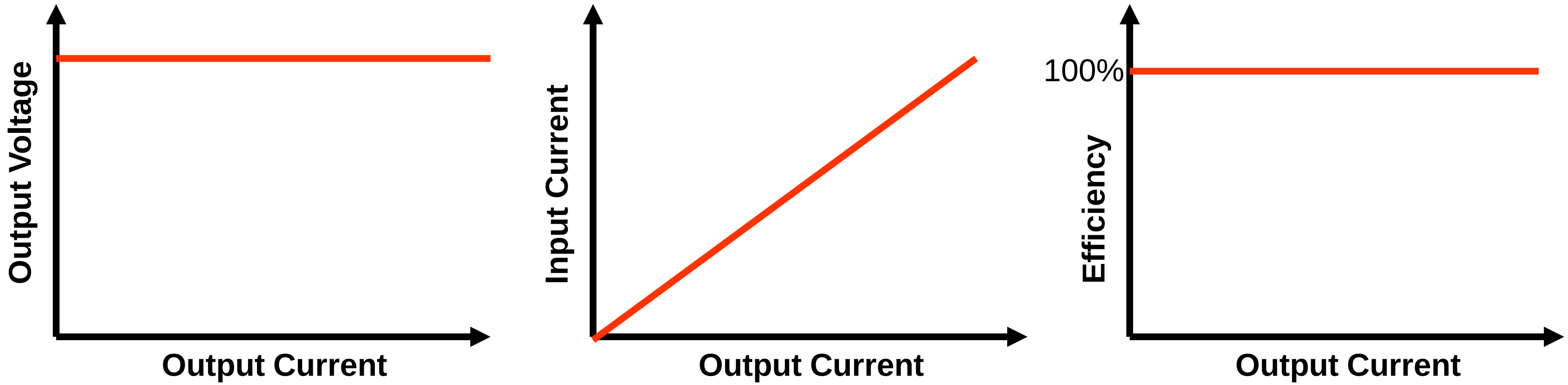

Voltage Ratio:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

Current Ratio:

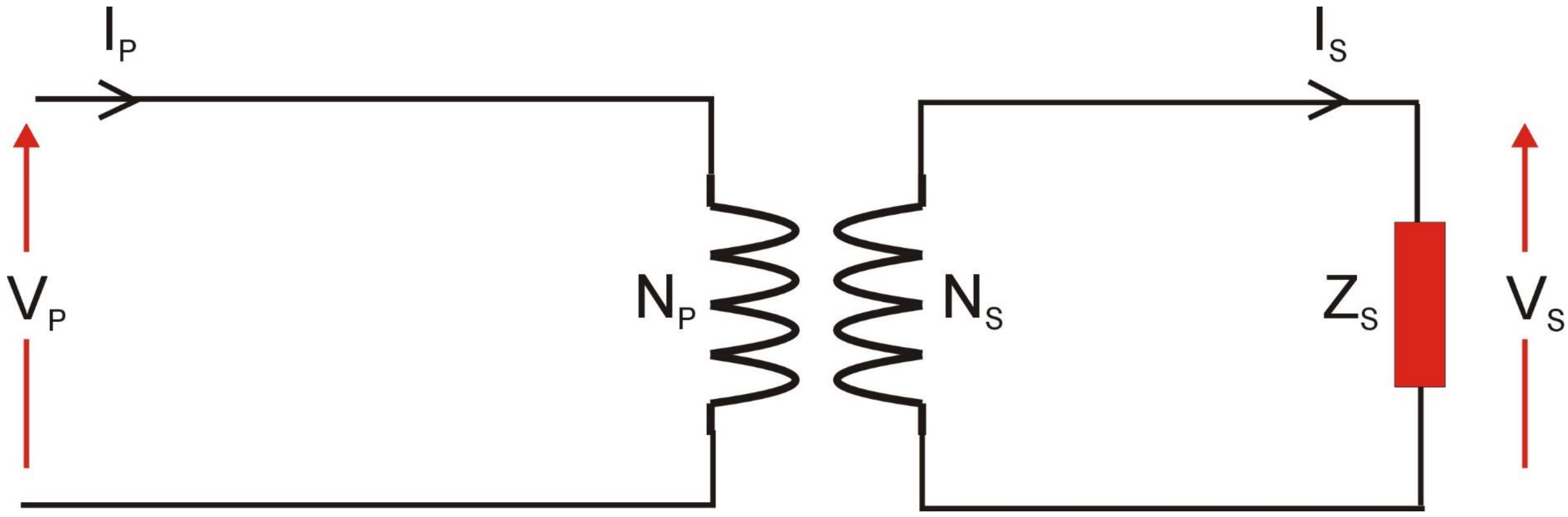
$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

- Transformer is 100% efficient
- No Input Current when Secondary is Open Circuit



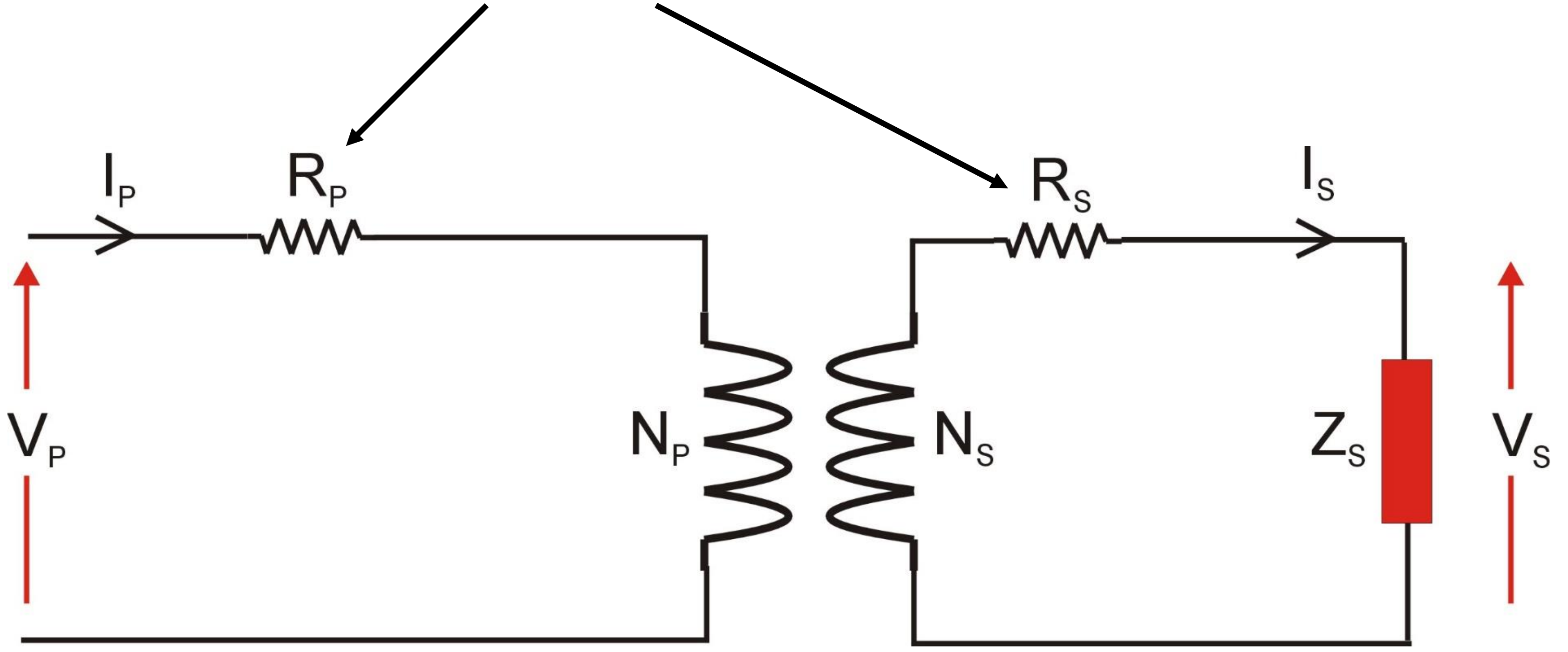
## Ideal Transformer:

# Time to get REAL!



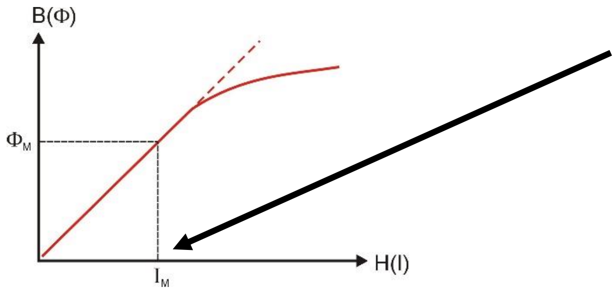
# A Practical Transformer #1: Winding Resistances

The copper wires which make up the Primary and Secondary windings have an associated resistance $R_P$ and $R_S$ respectively

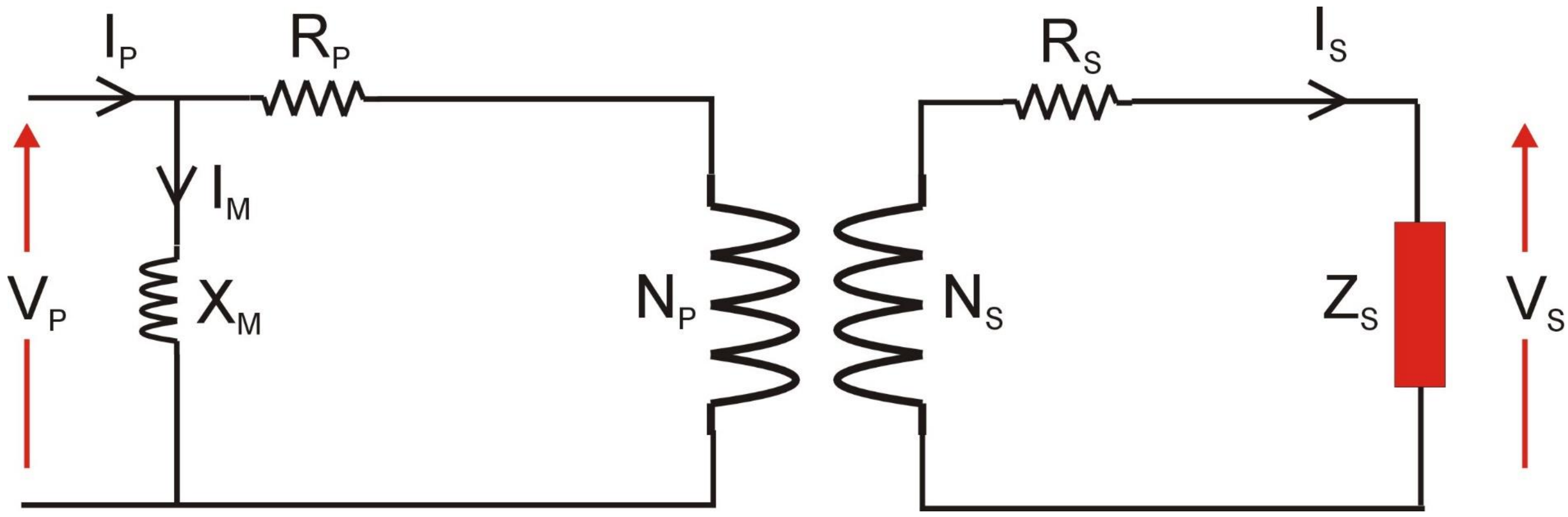



**Note: the wiring resistances will have an associated $I^2R$ power loss and therefore the transformer will no longer achieve 100% efficiency**

# A Practical Transformer #2: Magnetising Current




In a practical transformer made up of an Iron Core with **a finite permeability** we need a current $I_M$ to establish the Flux (Φ) in the Core. Given that this current is associated with a Magnetic component the circuit element ($X_M$) is purely Inductive




**Note: Given current $I_M$ flows through $X_M$ and primary voltage $V_P$ is across $X_M$; the current $I_M$ lags the primary voltage $V_P$ by 90°. Also note that current $I_M$ flows even when the secondary winding is *Open Circuit***

coefficient

# Definition of Inductance

$$L = \frac{\Psi}{i_L} = \frac{N\Phi}{i_L} = \frac{N^2}{\mathbb{R}} \quad or \quad N\Phi = L\, i_L$$

$$e = v_L = \frac{d\Psi}{dt} = L\frac{di_L}{dt}$$

coefficient

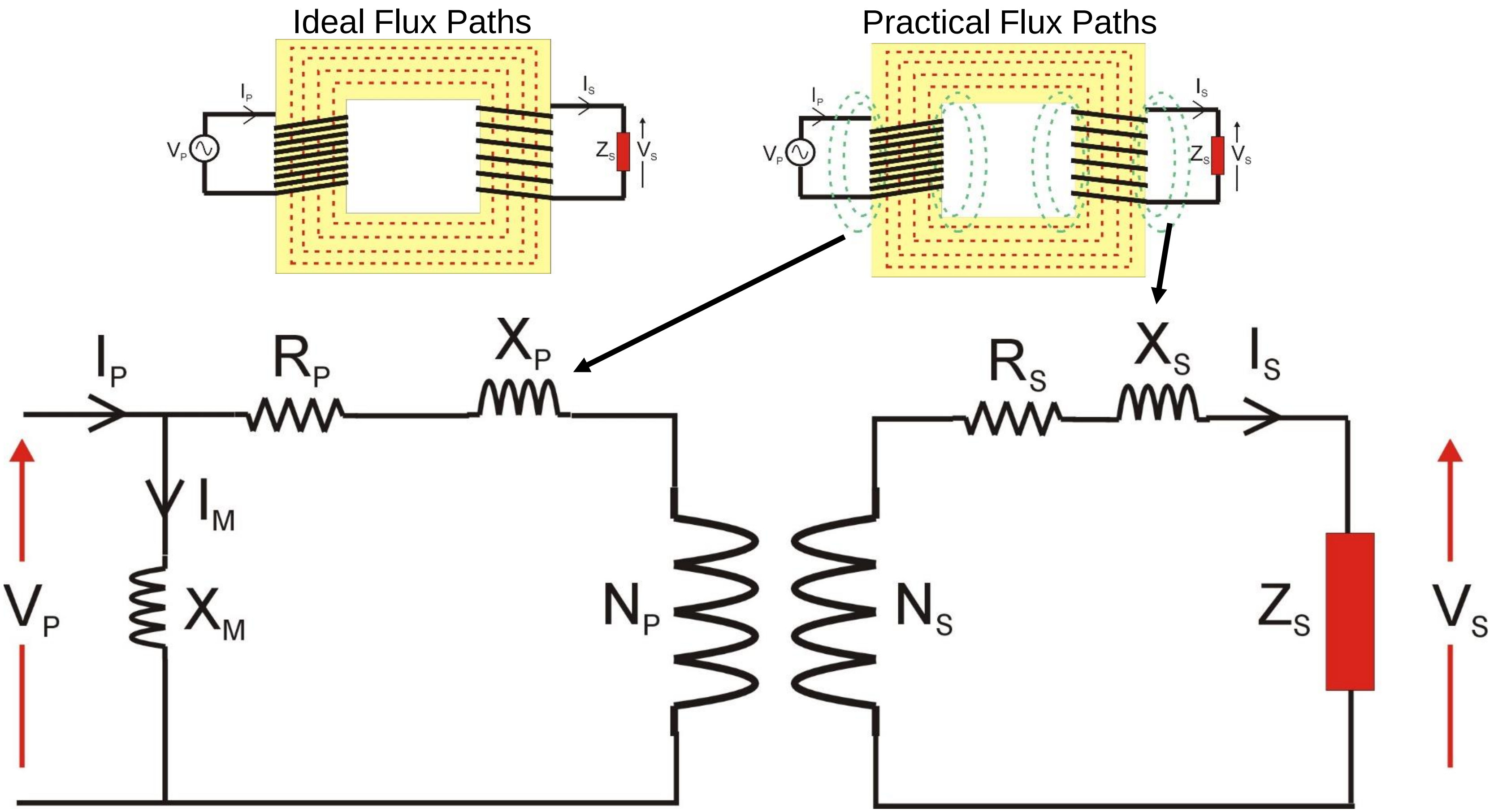
# Definition of Capacitance

$$C = \frac{q_C}{V_C}$$

charge

$$i_C = \frac{dq_C}{dt} = C\frac{dV_C}{dt}$$

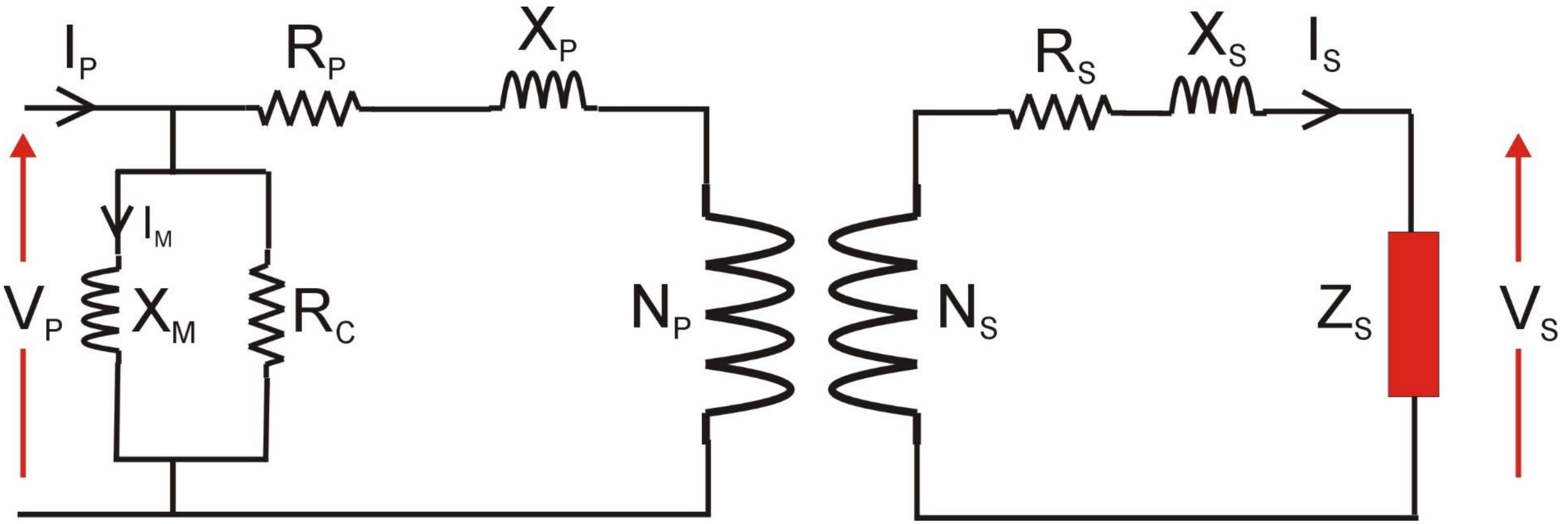
# A Practical Transformer #3: Leakage Inductances




**Note: again leakage flux is represented by inductive circuit elements ($X_P$ and $X_S$) but this time they are in series with the winding resistances as these elements are associated with load currents. Note also that practical transformers have primary and secondary windings interleaved to minimise the leakage components.**

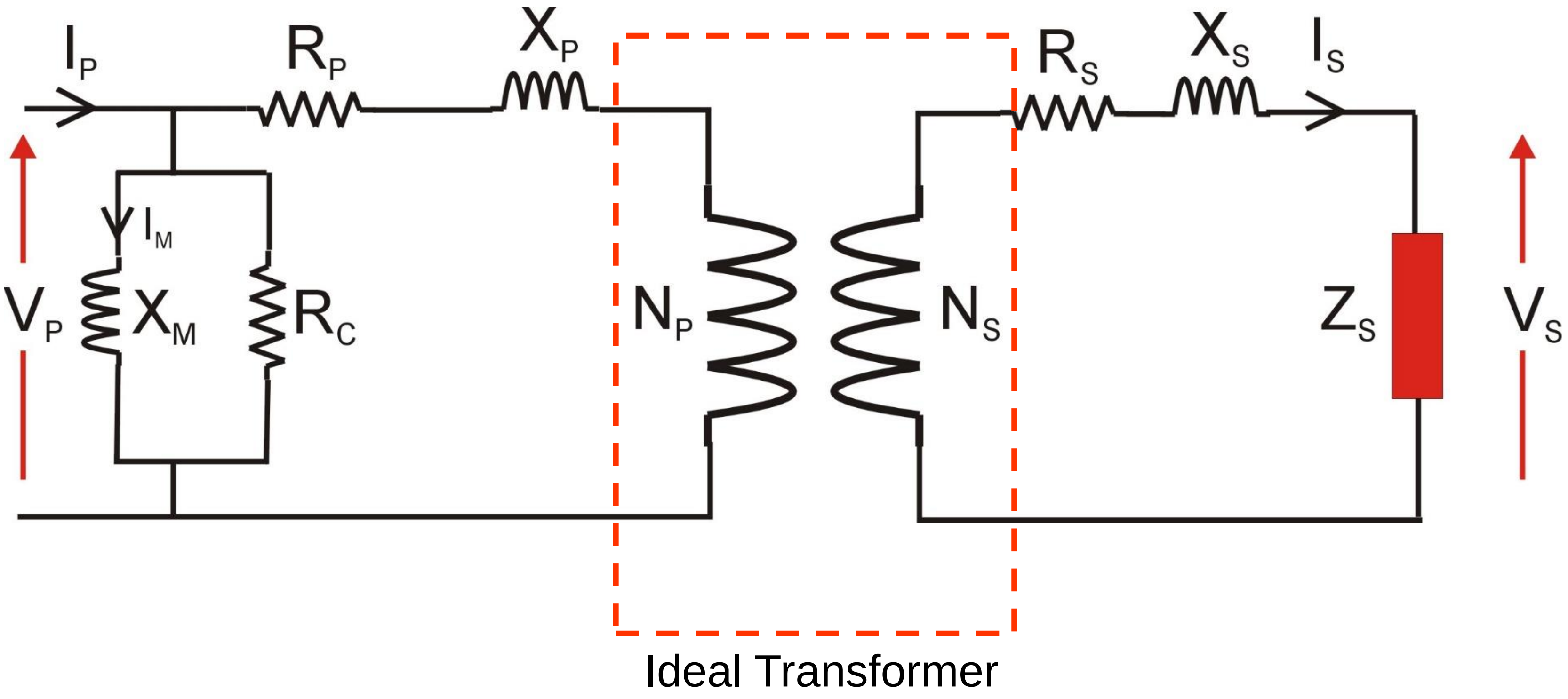
# A Practical Transformer #4: Iron Loss

In addition to the winding copper losses ($I^2R$), a transformer has an additional power loss associated with the Core – this is termed the ***IRON LOSS***. The loss is associated with eddy currents induced in the Iron core and also with hysteresis loss (you don't have to know any more than this!). Since this is a REAL power loss (W) causing heat and the fact that this loss is proportional to $V_P^2$, the component ($R_C$) representing this is resistive and in parallel with $V_P$.




**Note: To minimise the Iron Loss the Iron Core is made up of a stack of Iron *laminations*, typically around 0.5mm thick for 50Hz operation. The laminations have an oxide layer which acts as an insulation to the Eddy Currents and therefore reduces this loss.**

OK - That's all the Components we need to get a first approximation of the performance of a  practical transformer:




All that we will do now is rearrange the placement of the components (and eliminate the Ideal Transformer element) to make analysis a bit easier.

# Magnetising Current $I_m$

**Magnetic "Ohm's Law"**

$$\mathcal{F}_P = N_P i_P = \mathcal{F}_S + \Phi\Re_{core} = N_S I_S + \Phi\Re_{core}$$

$$\Rightarrow \Phi = \frac{N_P I_P - N_S I_S}{\Re_{core}} = \frac{N_1 I_m}{\Re_{core}}$$

## Ideal Transformer

$$V_P = e_P = N_P \frac{d\Phi}{dt}, \quad V_S = e_S = N_S \frac{d\Phi}{dt} \Rightarrow \quad \frac{V_P}{V_S} = \frac{N_P}{N_S}$$

$$\Re_{core}\Phi = N_P I_P - N_S I_S = 0, \quad when \quad \Re_{core} = 0$$

$$\Rightarrow N_P I_P = N_S I_S \quad or \quad \frac{I_P}{I_S} = \frac{N_S}{N_P} \quad and \quad I_m = 0$$

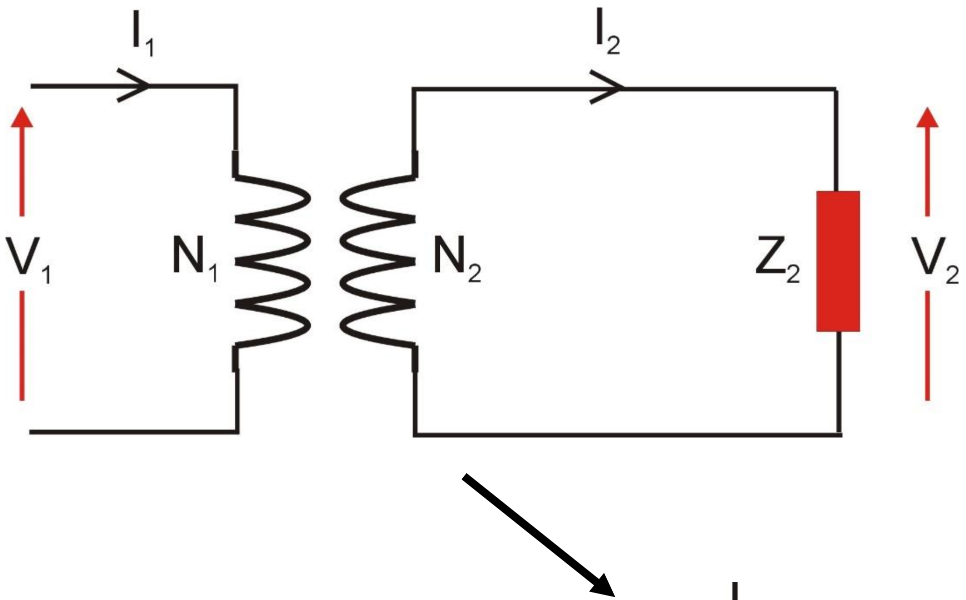
$$\Leftrightarrow p_{in} = V_P I_P = V_S I_S = p_{out} \qquad 100\%$$

**Assumptions**

- $R_P = R_S = 0$, no winding conduction losses
- Core reluctance $\Re_{core}=0$ (permeability $\mu=\infty$) and $X_m=\infty$
- Leakage reluctances $\Re_{li}=\infty$, thus $L_P=L_S=0$

## Equivalent circuit refer to primary side

**Consider the following Circuit:**




$$Z_2 = \frac{V_2}{I_2}$$

Substituting for $V_2$ and $I_2$ where:

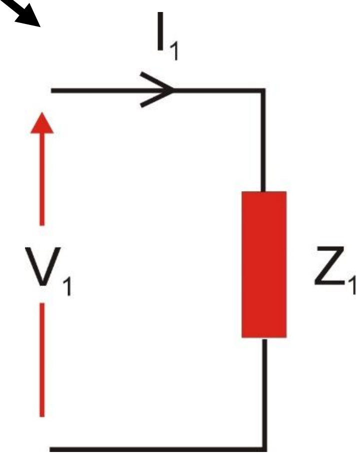
$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

We get:

$$Z_2 = \left(\frac{N_2}{N_1}\right)^2 \frac{V_1}{I_1} \qquad (1)$$

Rearranging (1):

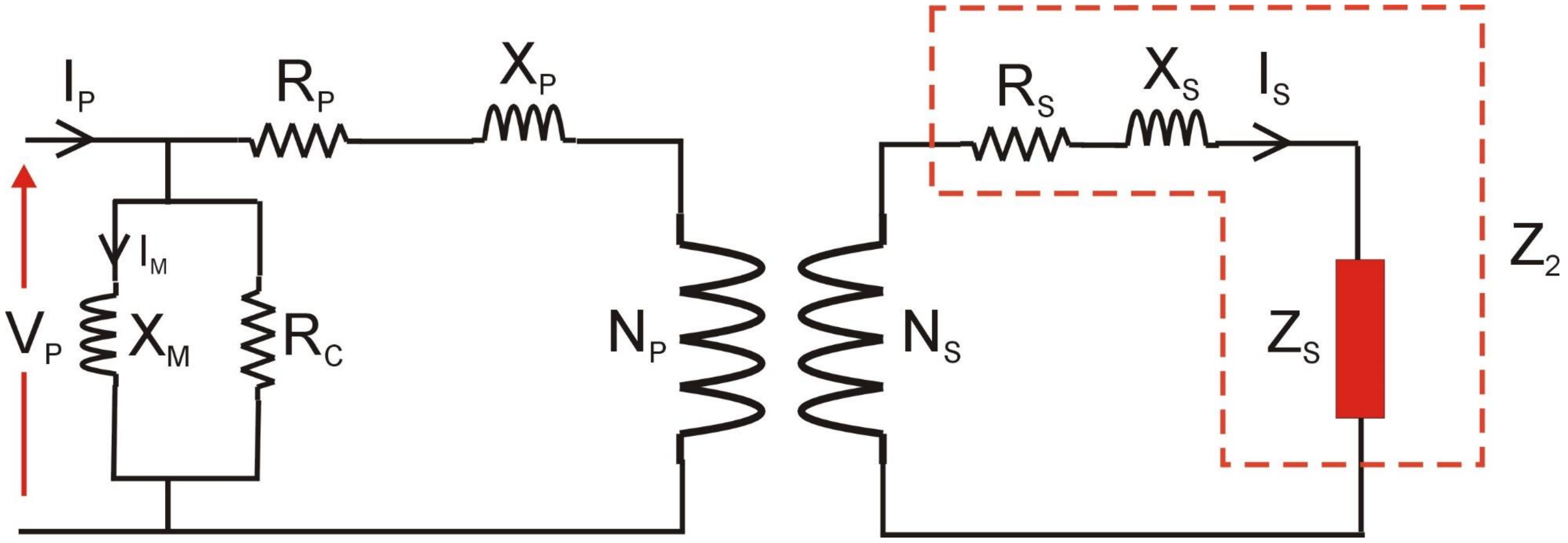
$$\left(\frac{N_1}{N_2}\right)^2 Z_2 = \frac{V_1}{I_1}$$




Where:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

**Using this principal on our equivalent circuit for the practical transformer:**



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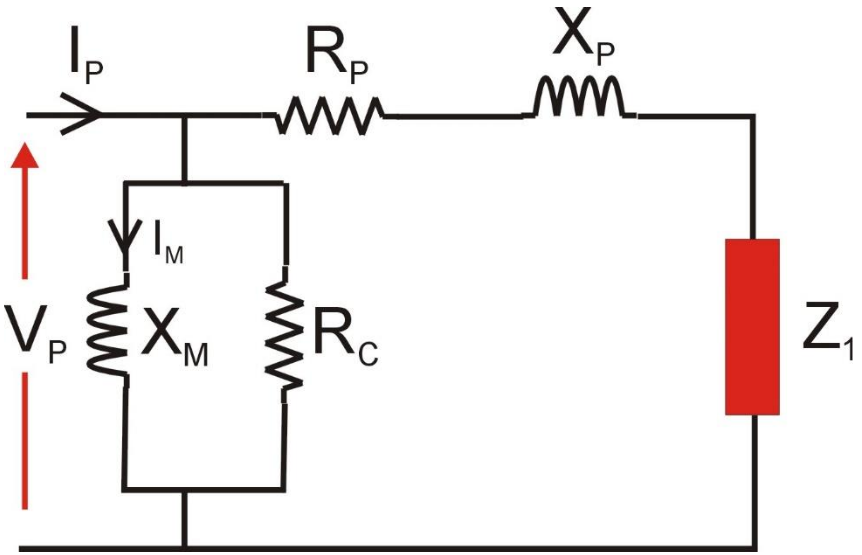
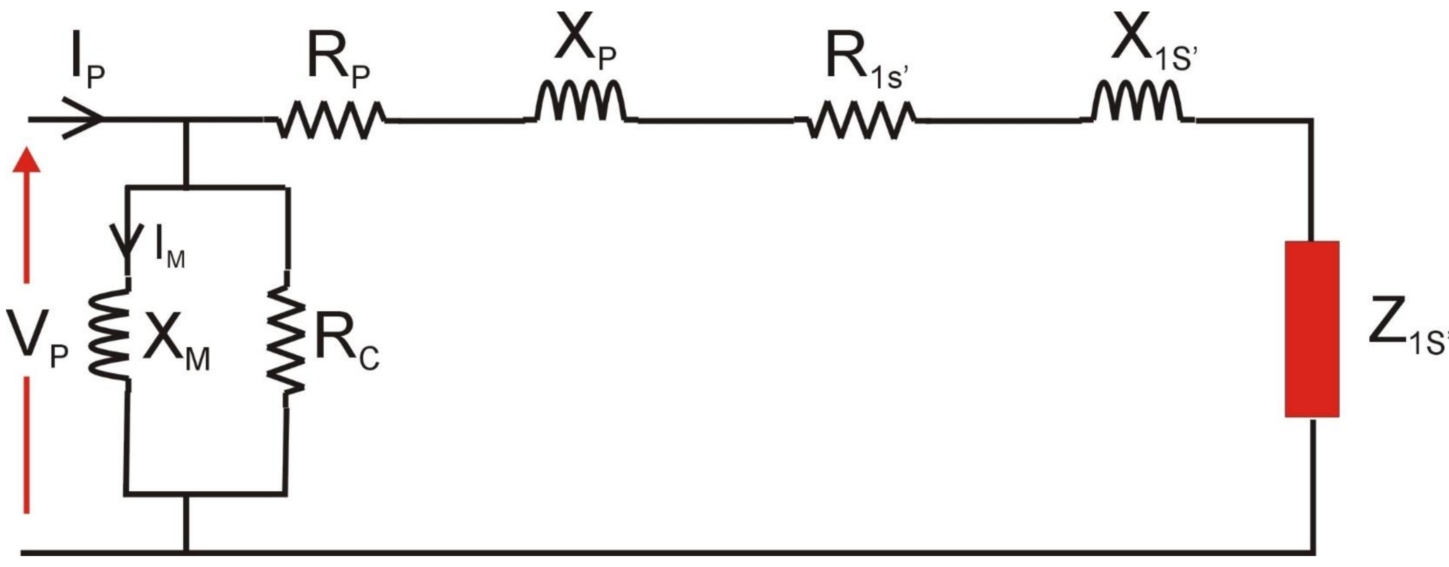



Where: $$Z_1 = \left(\frac{N_P}{N_S}\right)^2 Z_2$$

**Separating the various components which make up $Z_1$:**



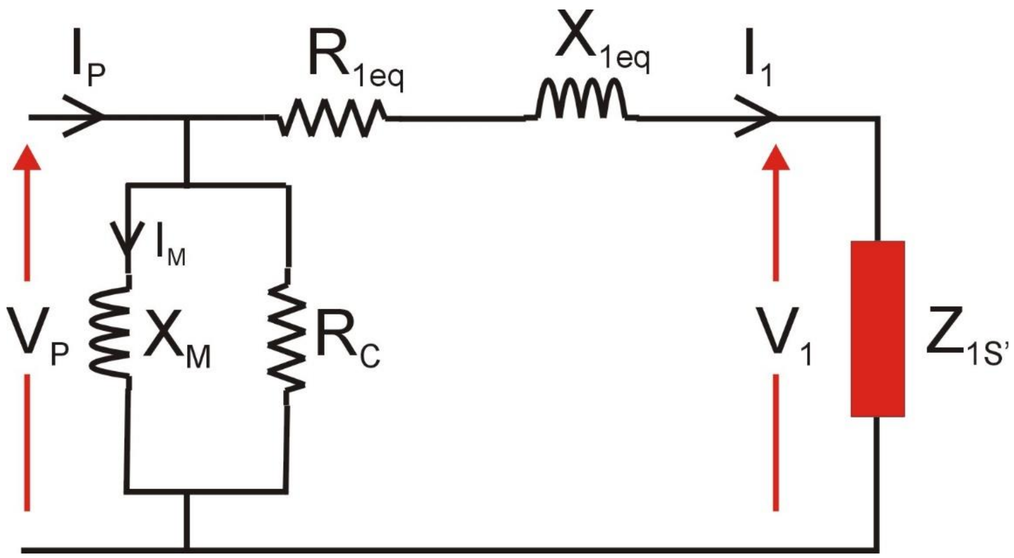

**Load referred to the primary side**

Where:

$$R_{1S'} = \left(\frac{N_P}{N_S}\right)^2 R_S \qquad X_{1S'} = \left(\frac{N_P}{N_S}\right)^2 X_S \qquad Z_{1S'} = \left(\frac{N_P}{N_S}\right)^2 Z_S$$

Note: remember $Z_S$ is the actual load connected to the Secondary Winding

**Combining wiring resistances and leakage reactances we get our FINAL equivalent circuit:**



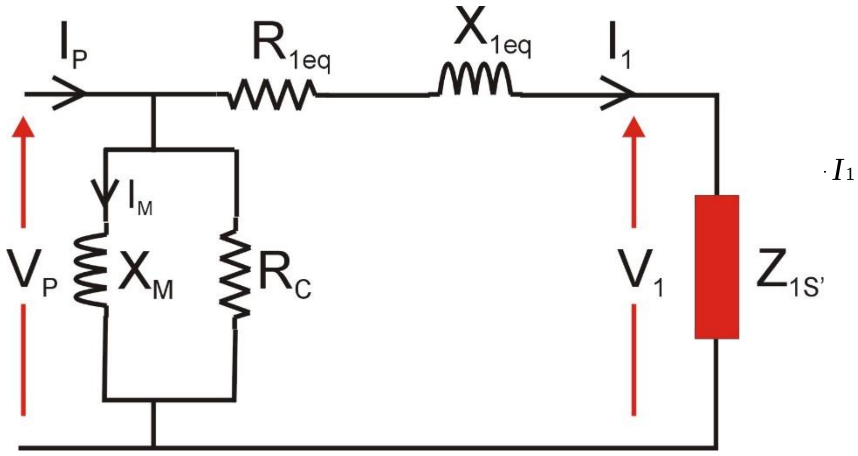

**Load referred to the primary side**

Where:

$$R_{1eq} = R_P + \left(\frac{N_P}{N_S}\right)^2 R_S$$

$$X_{1eq} = X_P + \left(\frac{N_P}{N_S}\right)^2 X_S$$

**To determine transformer performance we determine the various operating parameters such as $I_1$ and $V_1$ for a given equivalent circuit and referred load ($Z_{1S'}$) and then calculate the actual output voltage ($V_S$) and current ($I_S$) using the following relationships:**




Secondary Outputs:

$$V_S = \frac{N_S}{N_P} V_1$$

Notes:

1. $X_{1eq}$ and $X_M$ are REACTANCES the equivalent IMPEDANCES are $jX_{1eq}$ & $jX_M$

2. $V_S$ is *IN PHASE* with $V_1$, and $I_S$ is *IN PHASE* with $I_1$

## Example:

A 680Ω resistance is connected across the secondary winding of a 250V/500V transformer with the following equivalent circuit parameters:

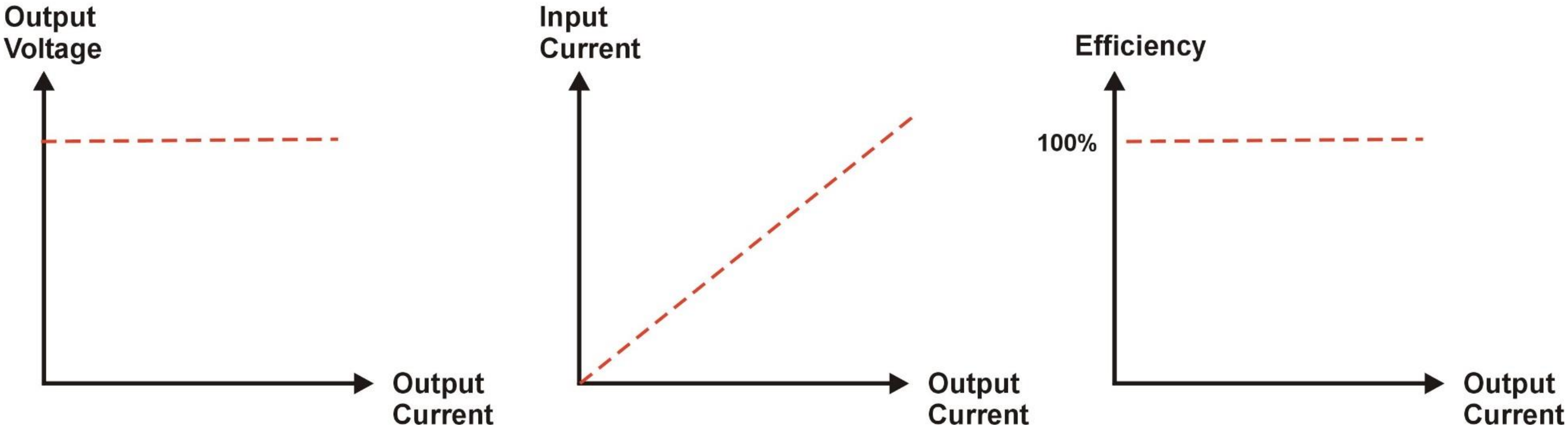
| | |
|---|---|
| $R_{1eq}$ | 30Ω |
| $X_{1eq}$ | 100Ω |
| $R_C$ | 8kΩ |
| $X_M$ | 1kΩ |

Determine the following:

1. The Output Current ($I_S$)
2. The Output Voltage ($V_S$)
3. The Magnetising Current ($I_M$)
4. The Transformer Efficiency (%)

Note: all voltages and currents are phasors

Solution done on whiteboard during lecture




**Draw the Graphs for a REAL transformer**