



University  
of Glasgow

# Power Electronics

## Lecture 4

### - Power Diode and Diode Rectifier (Lab 2)

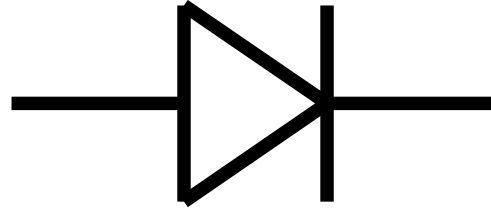
Please read

pages 16-31 in Chapter 2

pages 301-302 in chapter 10

Pages 524-546 Chapter 20

of the textbook



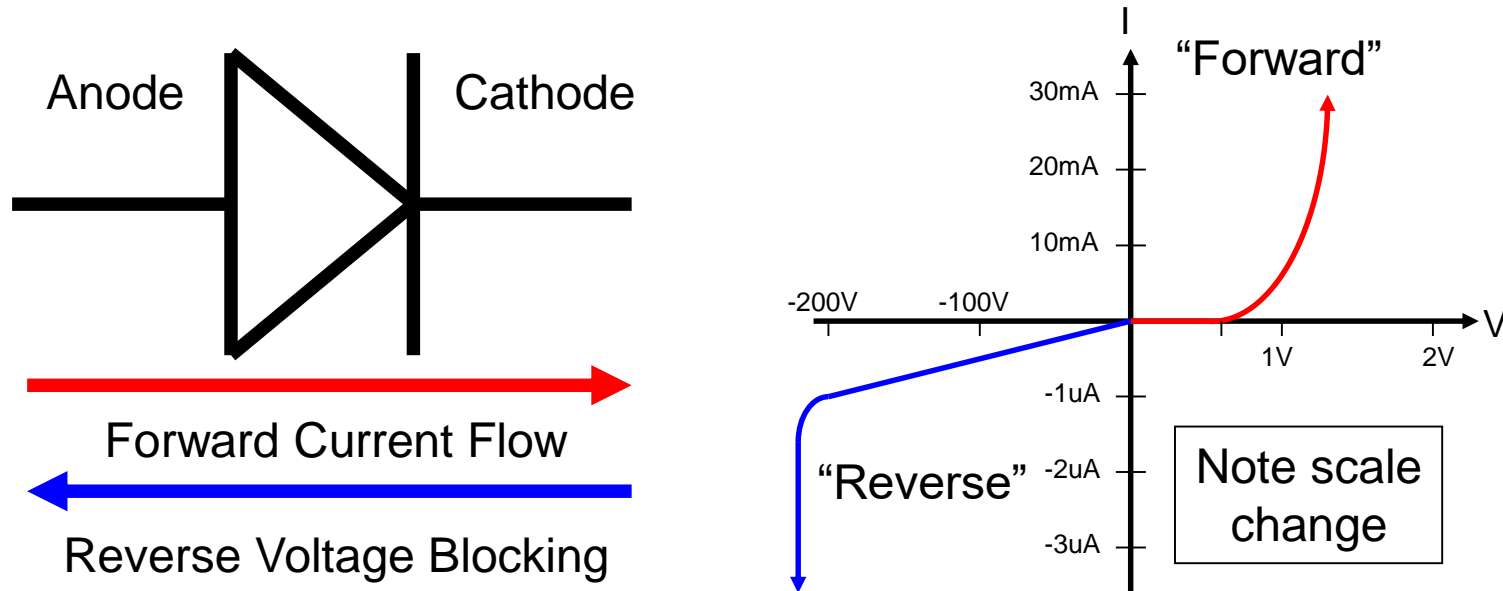
# Simplest Power Switch

## Power Diodes

Unidirectional power switch, nonlinear device,  
uncontrollable switch device ...

# Diodes

All the circuit elements we have considered so far have been linear – doubling the applied signal (voltage, say) doubles the response (a current, say). The diode is a non-linear semiconductor device with the V-I curve as shown in the diagram.



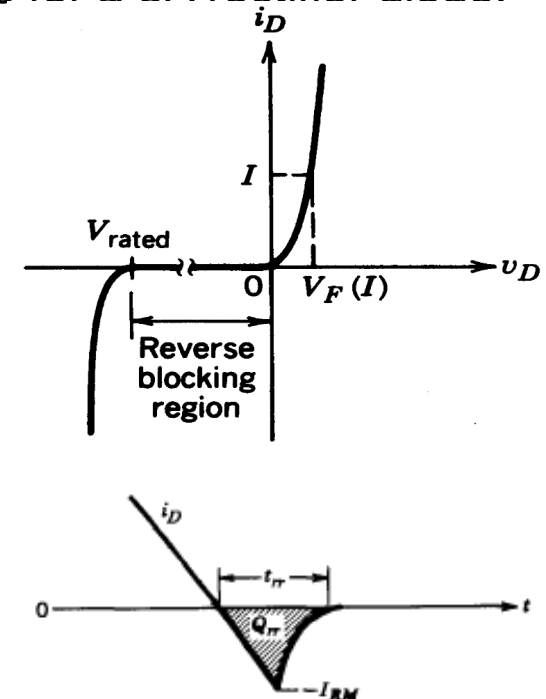
The diode's arrow indicates the normal current flow. In operation, the diode behaves like a voltage-sensitive switch. When the anode is more positive than the cathode current flows (quadrant 1 of diagram). When the anode is negative with respect to the cathode, current flow is blocked (quadrant 3 of diagram). This selective process of passage / blocking of current is called *rectification*.

Rectifier diodes typically have a “forward” (conduction ) voltage drop of between 0.6V (for small diodes) to 1.2V (for very large power diodes). This voltage is largely independent of the current flow once the diode is properly “on”. When the diode is “blocking”, the reverse current is very small, typically a few nanoamps for small diodes to a few microamps for power diodes. The maximum reverse voltage that can be applied without breakdown occurring (also known as the **Peak Inverse Voltage (反向电压峰值)**, PIV) will be specified for a particular device.

If this PIV is exceeded and the current is not limited in some way, reverse breakdown of the semiconductor junction will result in catastrophic and permanent failure of the diode. The following table summarises important parameters for a 5A rectifier diode.

<u>Parameter</u>	<u>Value</u>	<u>Units</u>
$V_{R(max)}$	400	Volts
$I_{R(max)}$	50	Microamps
$V_F @ I_{F(cont)}$	1.1V @ 5.0A	Volts, Amps
$V_F @ I_{F(peak)}$	<b>2.0V @ 50A</b>	Volts, Amps
$T_{rr}$ (reverse recovery)	2500	Nanoseconds
Capacitance	50	Picofarads
$Q_{rr}$ (reverse recovery)	2.5	Microcoulombs

Table: 5 Amp Rectifier diode characteristics.



## Diode Losses

Diodes are highly efficient devices. However, there are two important mechanisms by which a diode's efficiency is reduced.

$$\text{Losses: } P_{\text{Total}} = P_{\text{Conduction}} + P_{\text{Switching}}$$

### Conduction losses in a diode.

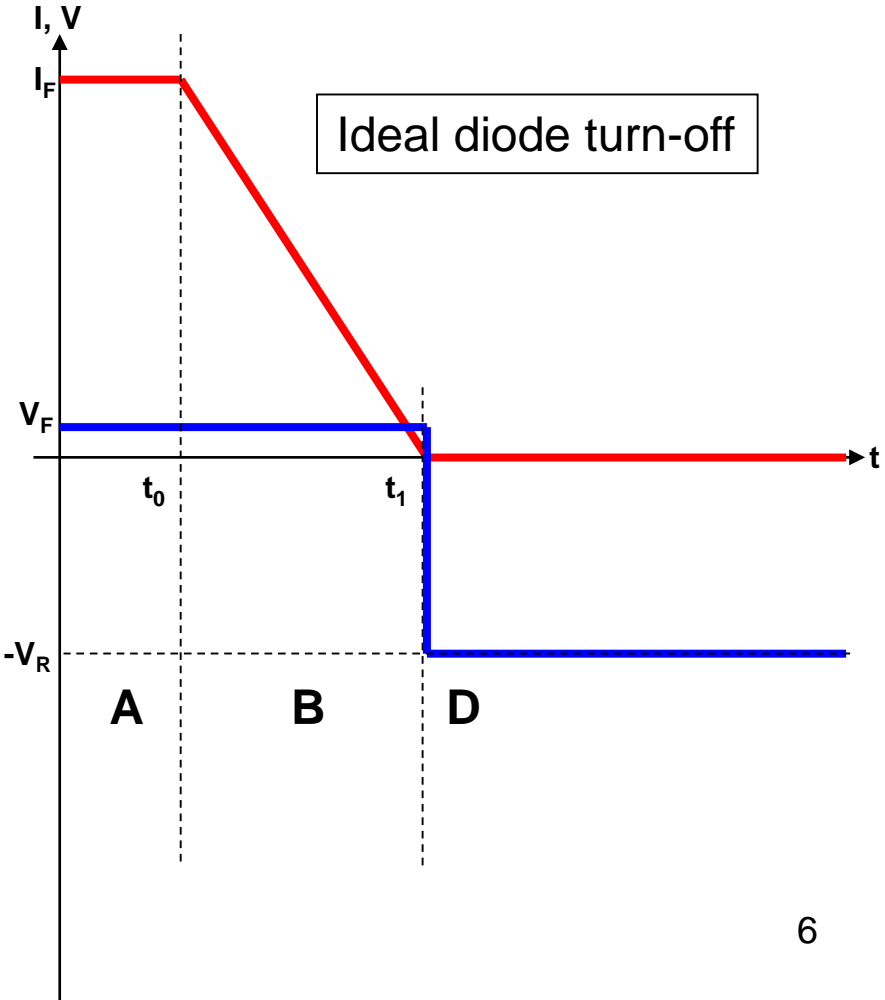
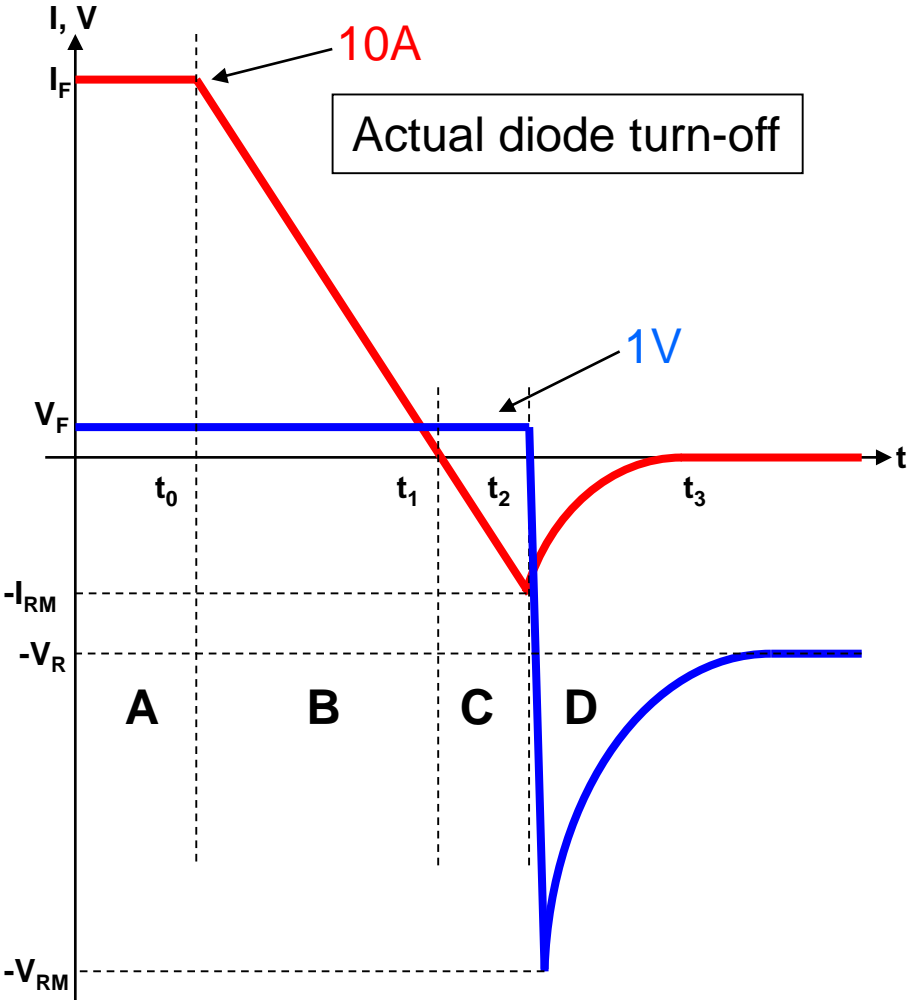
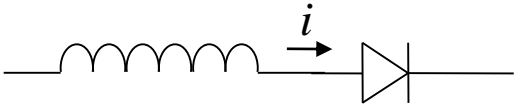
The diode's continuous rating is, as the name suggests, the maximum continuous current the diode can pass without overheating. The heat dissipated is just the product of the voltage drop across the diode multiplied by the current through it. In the case of the 5A rectifier diode example, 1.1V x 5.0A = 5.5W. This is called the *conduction loss*.

The diode junction has **a thermal inertia**, hence **it can withstand short overloads** (within reason). The peak current the diode can stand and the associated forward voltage drop for this device are 50A and 2V respectively; an instantaneous power dissipation of 100W. (Never design to the device's transient rating!!)

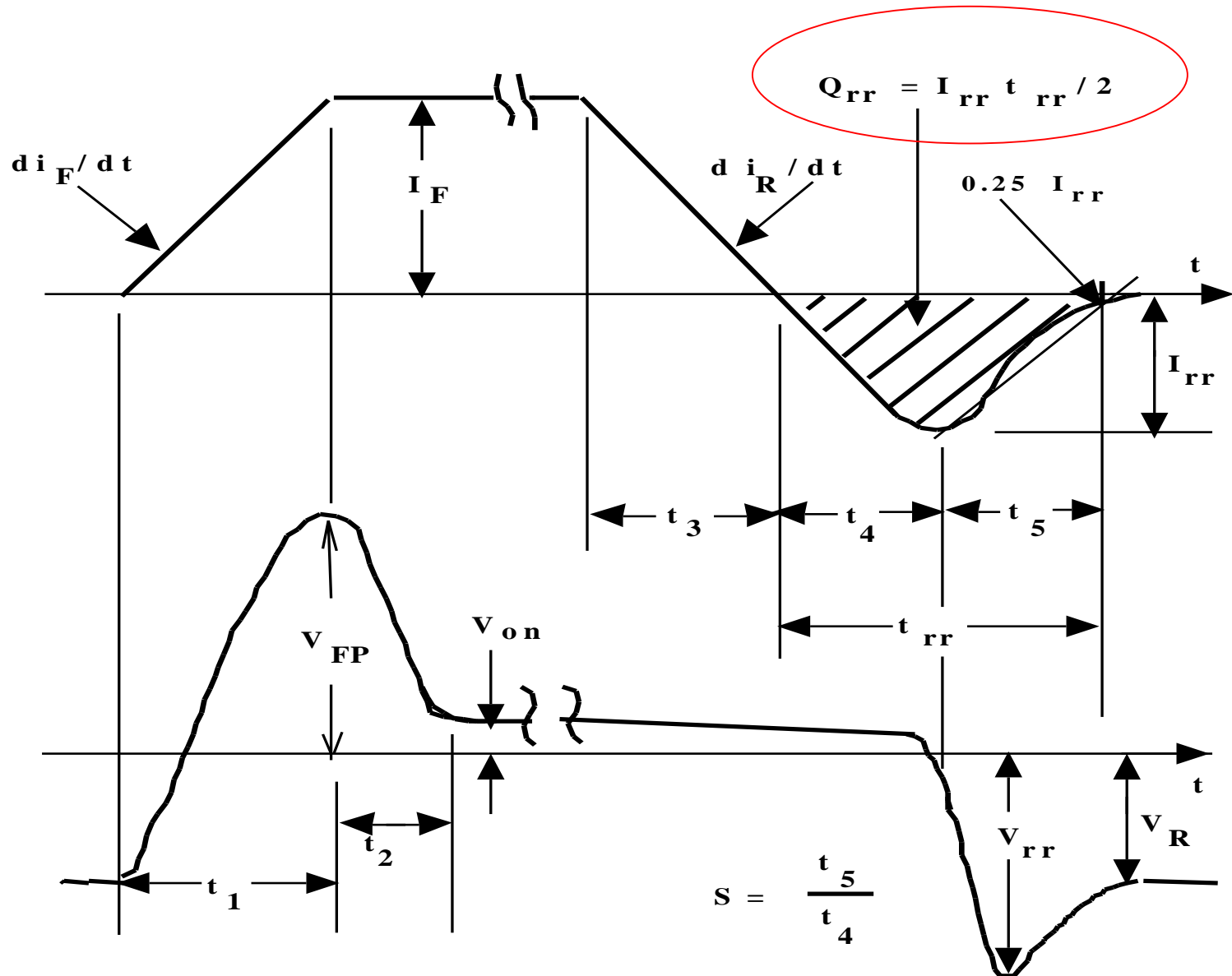
Switching losses in a diode.

Diodes switch from the “off” state to the “on” state very quickly. However, due to the way charge moves in the semiconductor junction, **turn-off can be relatively slow** (microseconds). Power diodes are particularly susceptible to **reverse recovery effect**.

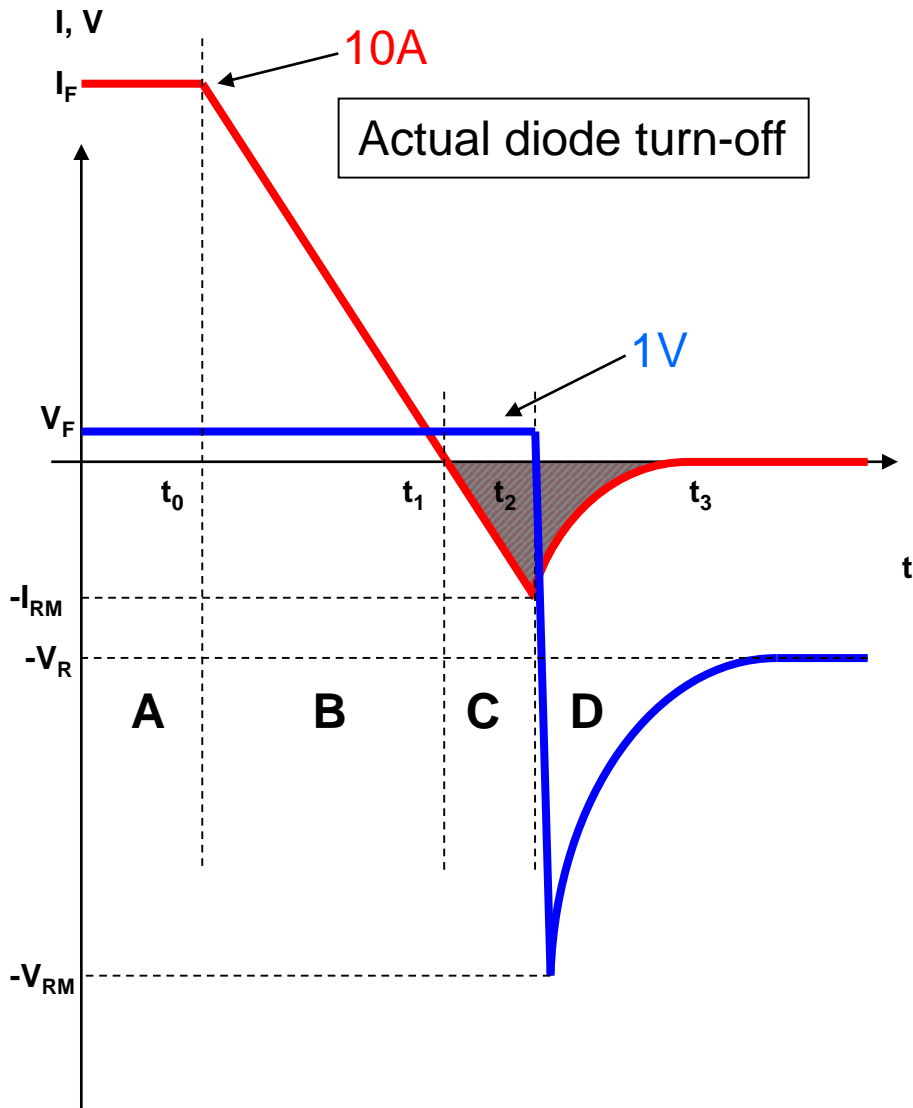
$$V = L \frac{di}{dt}$$



# Reverse Recovery Charge $Q_{rr}$



Reverse recovery effects (“switching losses”) in power diodes result in an unwanted current flow during phases ‘C’ and ‘D’. During phase ‘D’ in particular the combination of high voltage and high current causes considerable additional heating.



A simple formula that gives a good estimate of diode losses due to reverse recovery current is:

$$P_{rr} = Q_{rr} \times V_R \times F_{SW}$$

$P_{rr}$  = Reverse recovery power loss (W)

$Q_{rr}$  = Reverse recovery charge (C).

$V_R$  = Blocking voltage (V)

$F_{SW}$  = Switching Frequency (Hz)

At high frequencies, the reverse recovery losses can equal the forward conduction losses.



## Example:

A diode in a power supply circuit is operated with a forward current  $I_F = 10\text{A}$  and a forward voltage drop  $V_F$  at this current of  $1.1\text{V}$ . The power supply circuit is switching at a frequency of  $31.5\text{KHz}$  and the duty cycle is  $50\%$ . The steady-state reverse voltage across the diode is  $-50\text{V}$  and the reverse recovery charge  $Q_{rr}$  is  $2.5\mu\text{C}$ .

What power does the diode dissipate?

$$P_{\text{Total}} = P_{\text{Conduction}} + P_{\text{Switching}}$$

$$\text{Conduction loss} = V_F \times I_F \times 0.5 = 1.1\text{V} \times 10\text{A} \times 50\% = 5.5\text{W}.$$

$$\text{Switching loss} = Q_{rr} \times V_R \times F_{\text{SW}} = 2.5 \times 10^{-6}\text{C} \times 50\text{V} \times 31500 = 3.9\text{W}$$

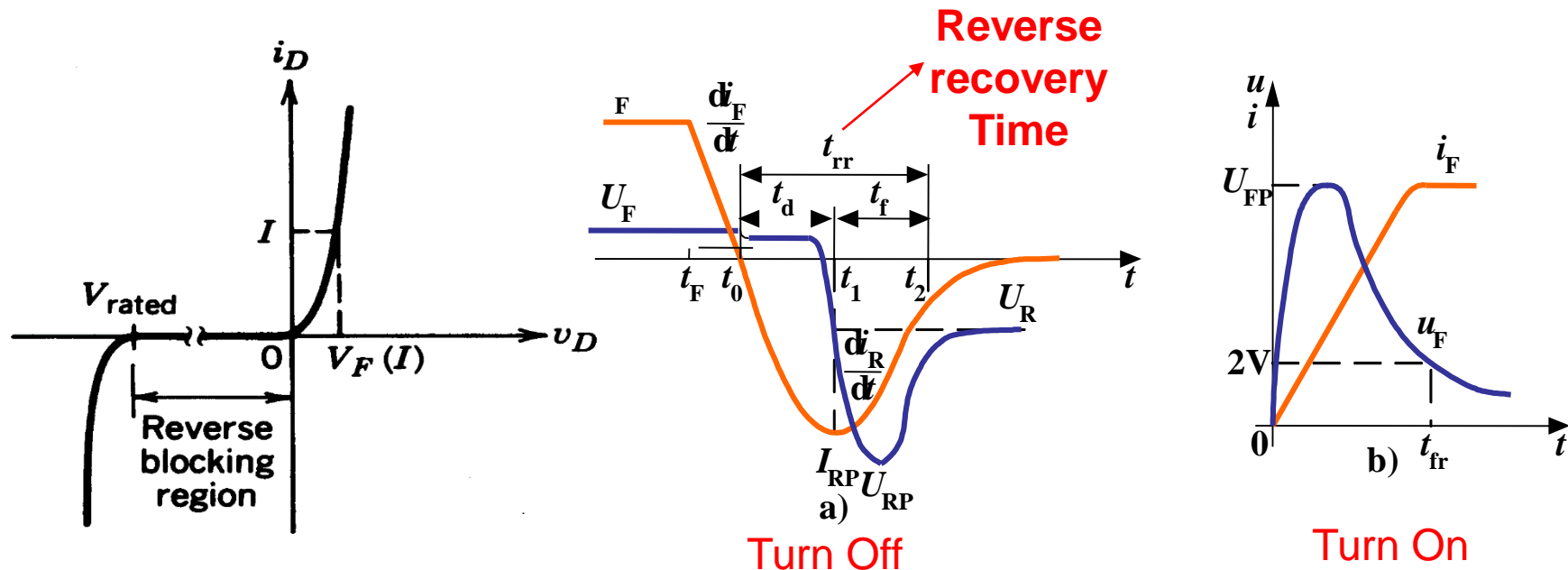
$$\Rightarrow \text{Total power loss} = 5.5\text{W} + 3.9\text{W} = \underline{9.4\text{W}}$$

### **Important:**

Switching losses vary directly with switching frequency.

Conduction losses vary directly with 'on' time (and current, obviously).

# Power Diodes



**Reverse-recovery effect(反向恢复效应):** Following the end of forward conduction in a p–n type diode, a reverse current can flow for a short time. The device does not attain its blocking capability until the mobile charge in the junction is depleted.

- Schottky (肖特基) diodes for low output voltage converter
  - Low forward voltage drop (about 0.15~0.45V), fastest ( $t_{rr} < 100\text{ns}$ )
- Fast-recovery (快恢复) diodes for up to several hundred kHz converter
  - reverse-recovery time  $t_{rr}$  (few  $\mu\text{s}$ ), ratings of several hundred volts/amperes
- Line frequency (工频) diodes (low switching frequency)
  - Long reverse-recovery time (few ms), several kilo volts/amperes

To overcome the reverse recovery problem when using diodes in power switching circuits, a special type of diode is available: the “**Fast Recovery Diode**” (FRD). Common diodes are also called “**Line Frequency Diodes**” because they can be used in applications at mains-voltage frequency (50Hz or 60Hz).

Typical parameters for a small FRD are:

Data for Rohm FRD type RF101L2S.

Maximum ratings @  $T_a = 25^\circ\text{C}$

Parameter	Symbol	Limits	Unit
Reverse voltage	$V_{RM}$	200	V
Average forward current	$I_O$	1	A
Peak surge current	$I_{FSM}$	20	A
Junction temperature	$T_j$	125	$^\circ\text{C}$

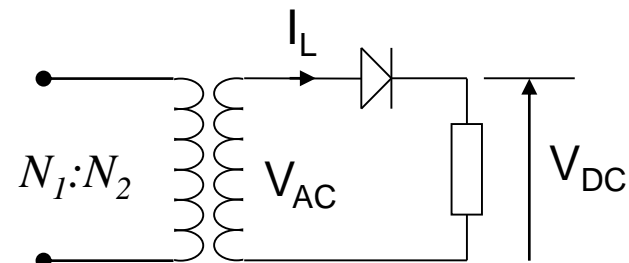
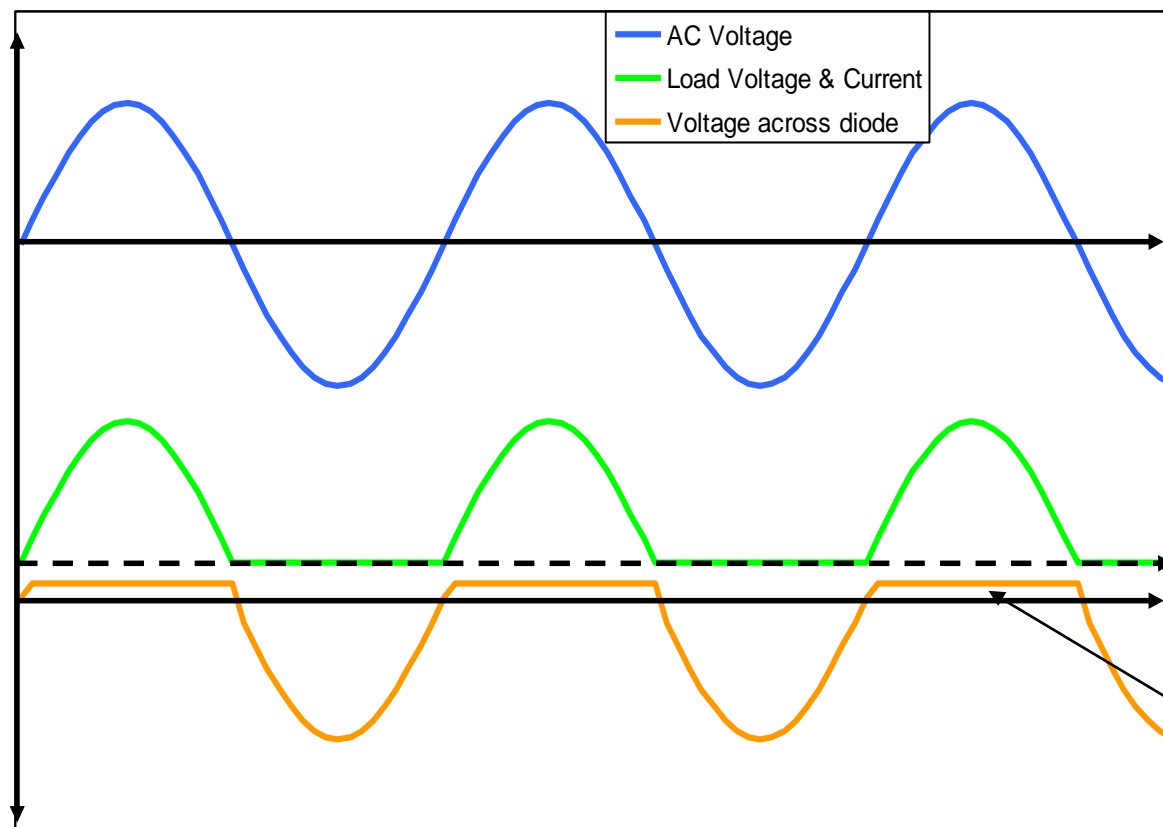
Electrical characteristics @  $T_a = 25^\circ\text{C}$

Parameter	Symbol	Min	Typ	Max	Unit	Condition
Forward voltage	$V_F$	-	0.8	0.9	V	$I_F=1.0\text{A}$
Reverse current	$I_R$	-	0.01	10	$\mu\text{A}$	$V_R=200\text{V}$
Reverse recovery time	$t_{rr}$	-	12	25	nS	$I_F=0.5\text{A}$ $I_R=1\text{A}$ $I_{rr}=1/4 I_R$

# Application of Diodes: 1 – Rectification.

Rectification is the process whereby AC is converted into DC. The most common way to accomplish this is by the use of a number of diodes. There are several methods of doing this:

## Half-wave rectification.



Forward conduction voltage drop

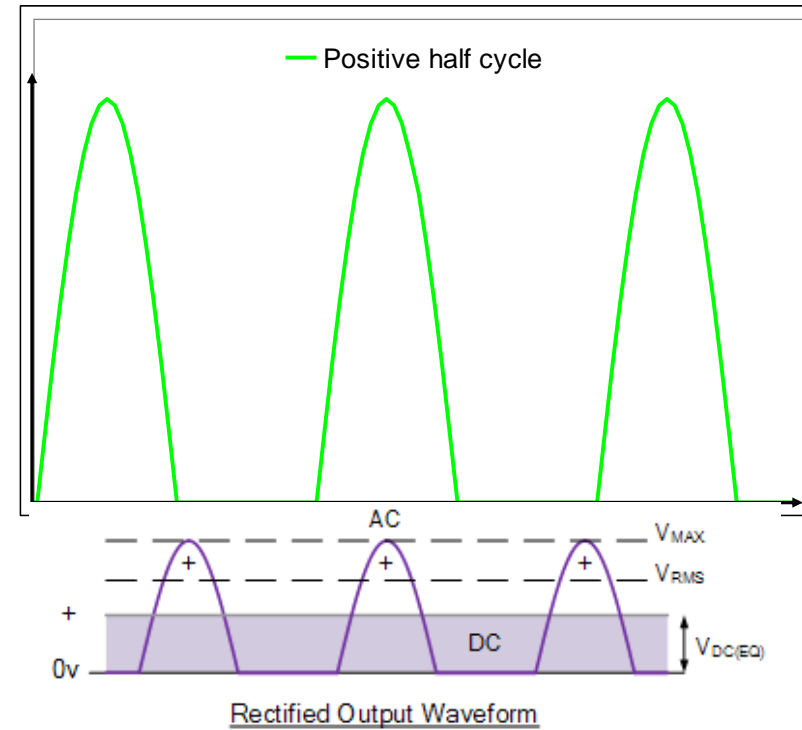
For this circuit,  $V_L = \hat{V}_{AC} \sin(\omega t), \quad 0 < \omega t < \pi.$   
 $V_L = 0, \quad \pi < \omega t < 2\pi.$

The average load voltage is therefore:

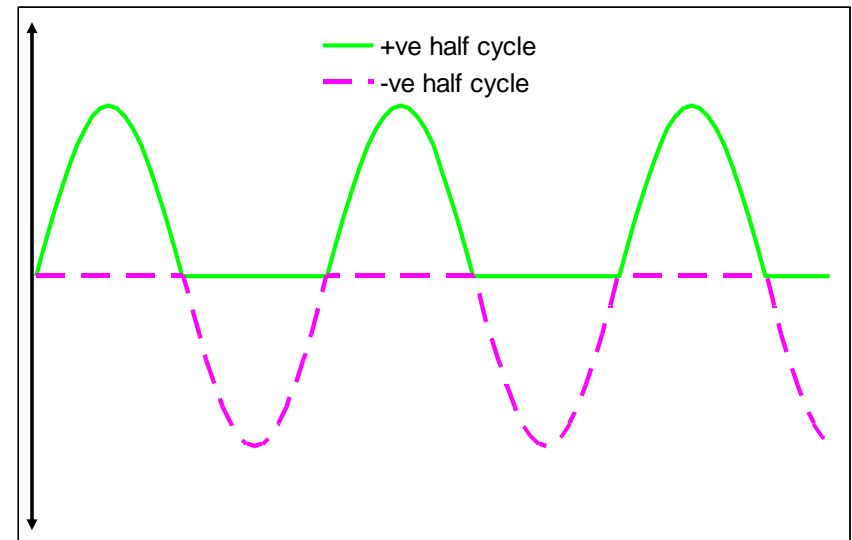
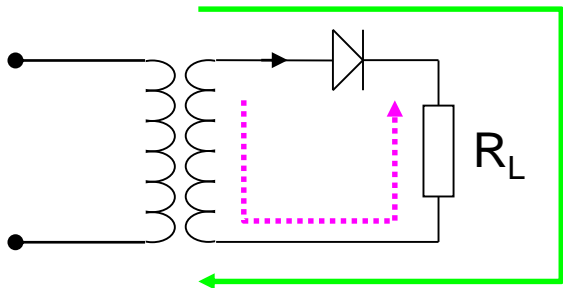
$$V_{DC} = \frac{1}{2\pi} \left[ \int_0^{\pi} \hat{V}_{AC} \sin(\omega t) d\omega t + \int_{\pi}^{2\pi} 0 d\omega t \right] = \frac{\hat{V}_{AC}}{\pi}$$

Normally we wish to use RMS values, so

$$V_{ACRMS} = \frac{\hat{V}_{AC}}{\sqrt{2}} \Rightarrow V_{DC} = \frac{1}{\pi} \times \sqrt{2} V_{ACRMS} = 0.45 V_{ACRMS}$$

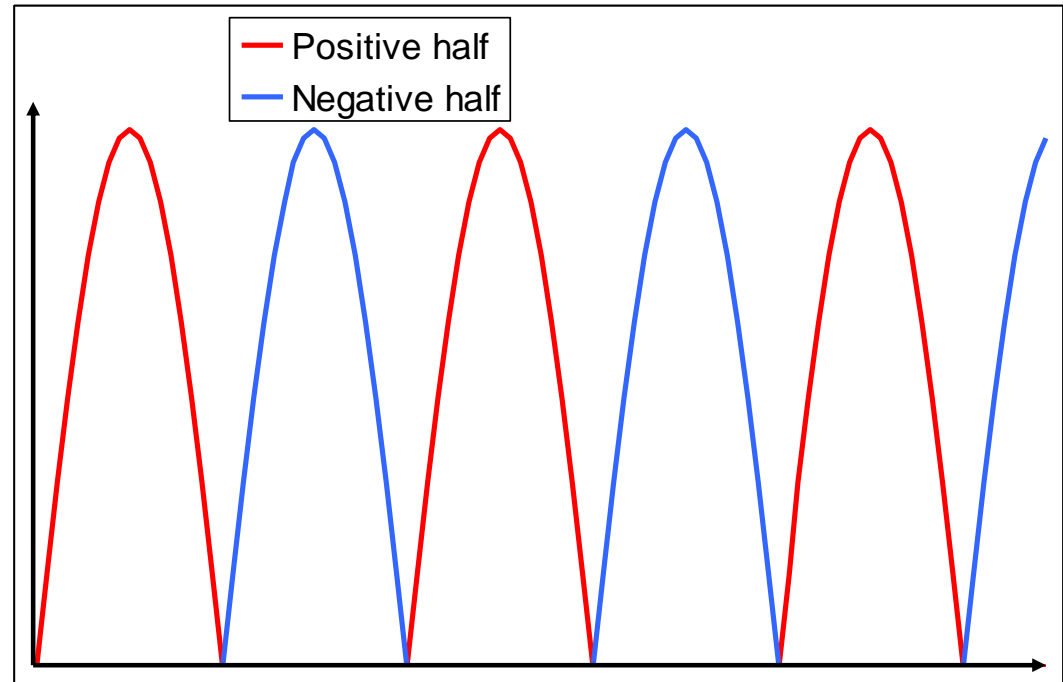
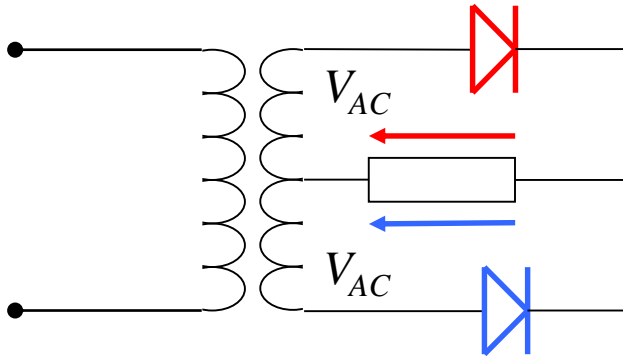


Note the maximum reverse voltage across the diode is the negative peak value of  $V_{AC}$ . This circuit is cheap and simple but it does not make good use of the transformer because current flows in the secondary winding for only 50% of the time.



If we add a centre-tap to the secondary winding of the transformer, we can improve on the previous circuit somewhat:

### Full-wave centre tap (中间抽头)



This circuit behaves as a pair of half-wave rectifiers that conduct on alternate half cycles of the AC waveform.

The load voltage  $V_L$  may be written as:

$$V_L = \hat{V}_{AC} \sin(\omega t), \quad 0 < \omega t < \pi.$$

$$V_L = -\hat{V}_{AC} \sin(\omega t), \quad \pi < \omega t < 2\pi.$$

The average load voltage is therefore:

$$V_{DC} = \frac{1}{2\pi} \left[ \int_0^{\pi} \hat{V}_{AC} \sin(\omega t) d\omega t + \int_{\pi}^{2\pi} -\hat{V}_{AC} \sin(\omega t) d\omega t \right]$$

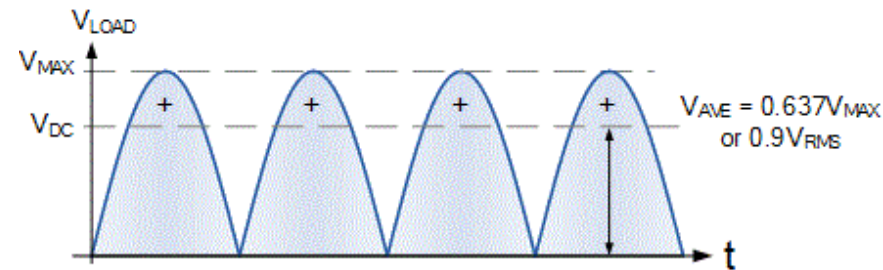
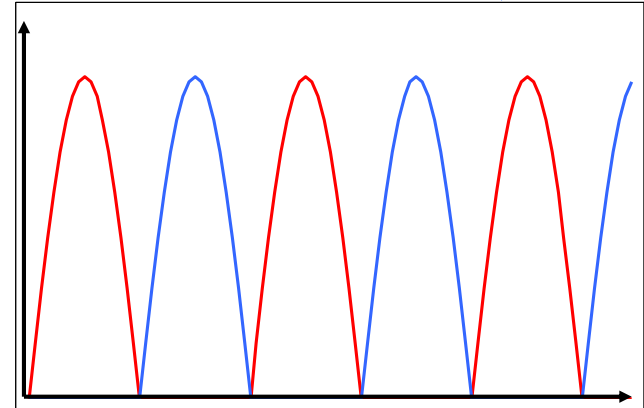
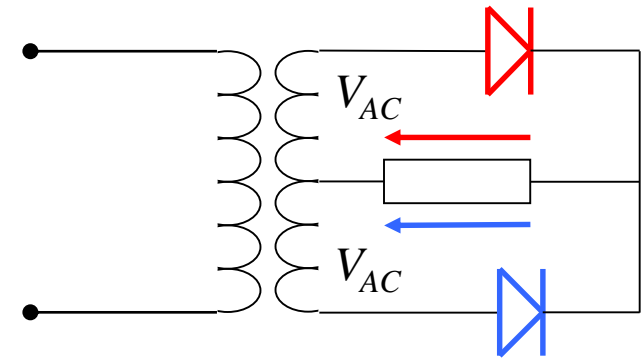
$$= \frac{2\hat{V}_{AC}}{\pi} = 0.637V_{AC \text{ MAX}}$$

Similarly, the direct current is

$$I_{DC} = \frac{V_{DC}}{R_L} = \frac{2\hat{V}}{\pi R_L}$$

The RMS voltage (and current) is found as previously by dividing by  $\sqrt{2}$ .

So  $V_{DC} = 0.90V_{AC \text{ RMS}}$

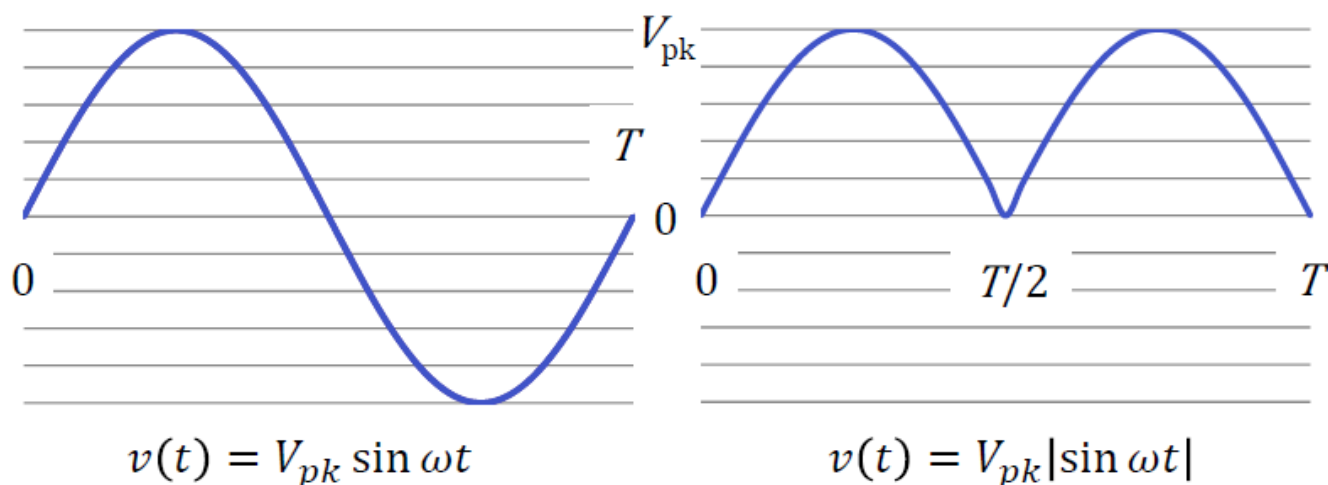


There are **three important points** to note about this circuit:

1. Only one half the transformer's secondary winding is in use at any instant;
2. The **maximum reverse voltage** across each diode has now **doubled**;
3. The **frequency** of the half-sinusoid has now **doubled**.

## Average and RMS values of *rectified* sine wave

The average value of a sine (or cosine) wave is zero but this changes when it is rectified (to be described shortly). Full-wave rectification is the same as taking the absolute value (magnitude). Look at the average value first.



The frequency is doubled by rectification so we need integrate only from 0 to  $T/2$ .

$$\begin{aligned} V_{ave} &= \frac{1}{\frac{1}{2}T} \int_0^{T/2} V_{pk} \sin \omega t \, dt \\ &= \frac{2V_{pk}}{T} \left[ -\frac{\cos \omega t}{\omega} \right]_0^{T/2} \end{aligned}$$



The frequency is doubled by rectification so we need integrate only from 0 to  $T/2$ . Remember that  $\omega T = 2\pi$ .

$$\begin{aligned} V_{\text{ave}} &= \frac{1}{\frac{1}{2}T} \int_0^{T/2} V_{\text{pk}} \sin \omega t \, dt \\ &= \frac{2V_{\text{pk}}}{T} \left[ -\frac{\cos \omega t}{\omega} \right]_0^{T/2} \\ &= \frac{2V_{\text{pk}}}{\omega T} [\cos 0 - \cos(\omega T/2)] \\ &= \frac{2V_{\text{pk}}}{2\pi} [\cos 0 - \cos \pi] \\ &= \frac{V_{\text{pk}}}{\pi} [1 - (-1)] \end{aligned}$$

Finally,

$$V_{\text{ave}} = \frac{2}{\pi} V_{\text{pk}} \approx 0.64 V_{\text{pk}}$$

Compare

$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\text{pk}} \approx 0.71 V_{\text{pk}}$$

Again the average does not depend on the frequency of the wave. It must always be true that  $V_{\text{rms}} \geq V_{\text{ave}}$ . These two values enter the *form factor*.

The RMS value is simple because of a general observation:

**Full-wave rectification has no effect on the RMS value**

Mathematically this is because the integral is over  $[v(t)]^2$ , which is unaffected if the sign of  $v(t)$  is changed.

Physically, RMS is associated with *power*, and the power dissipated in a resistor does not depend on the direction of the current through it.

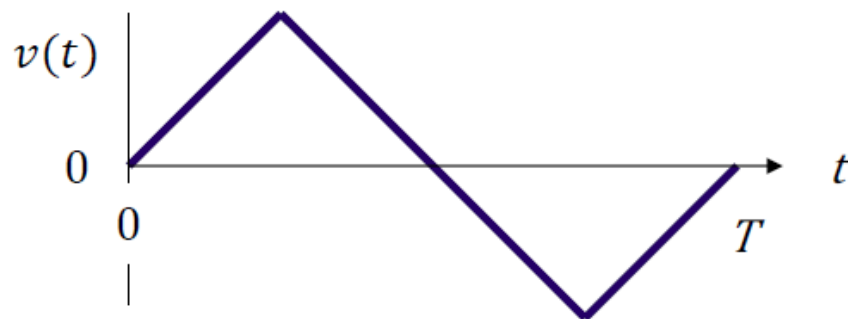
So  $V_{\text{rms}} = V_{\text{pk}}/\sqrt{2}$  for the full-wave rectified sine wave.

**Handy hint:** you can often save a lot of trouble when calculating average values, including RMS, with some thought at the beginning. Symmetry is particularly helpful.

For example, we could have taken the average from 0 to  $T/4$  instead of  $T/2$  for the full-wave rectified sine wave, although it would not have saved a lot of work here.

Another example: this triangular wave has period  $T$  but the integral for its RMS value need be taken only from 0 to  $T/4$ .

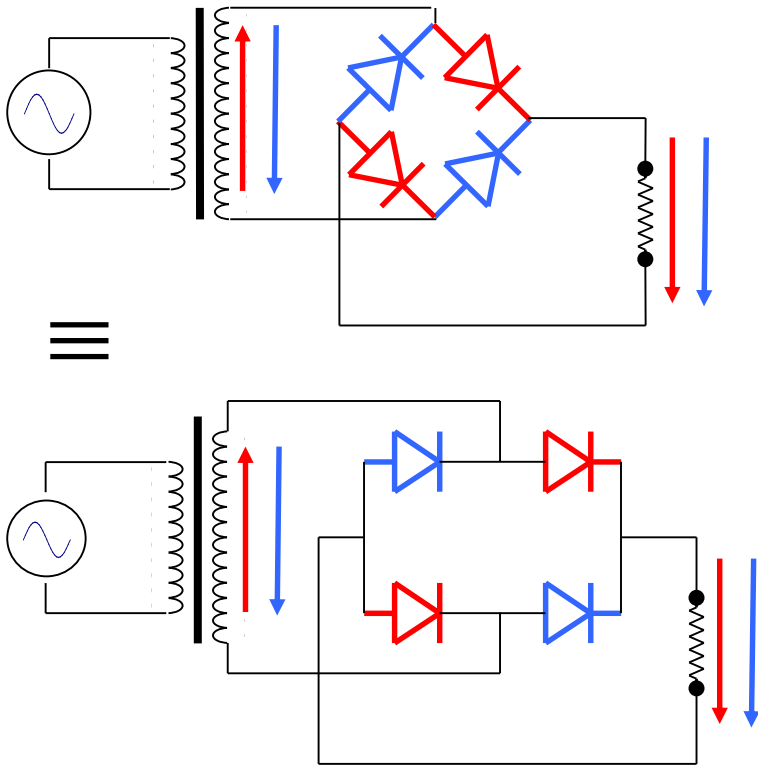
**Why?**



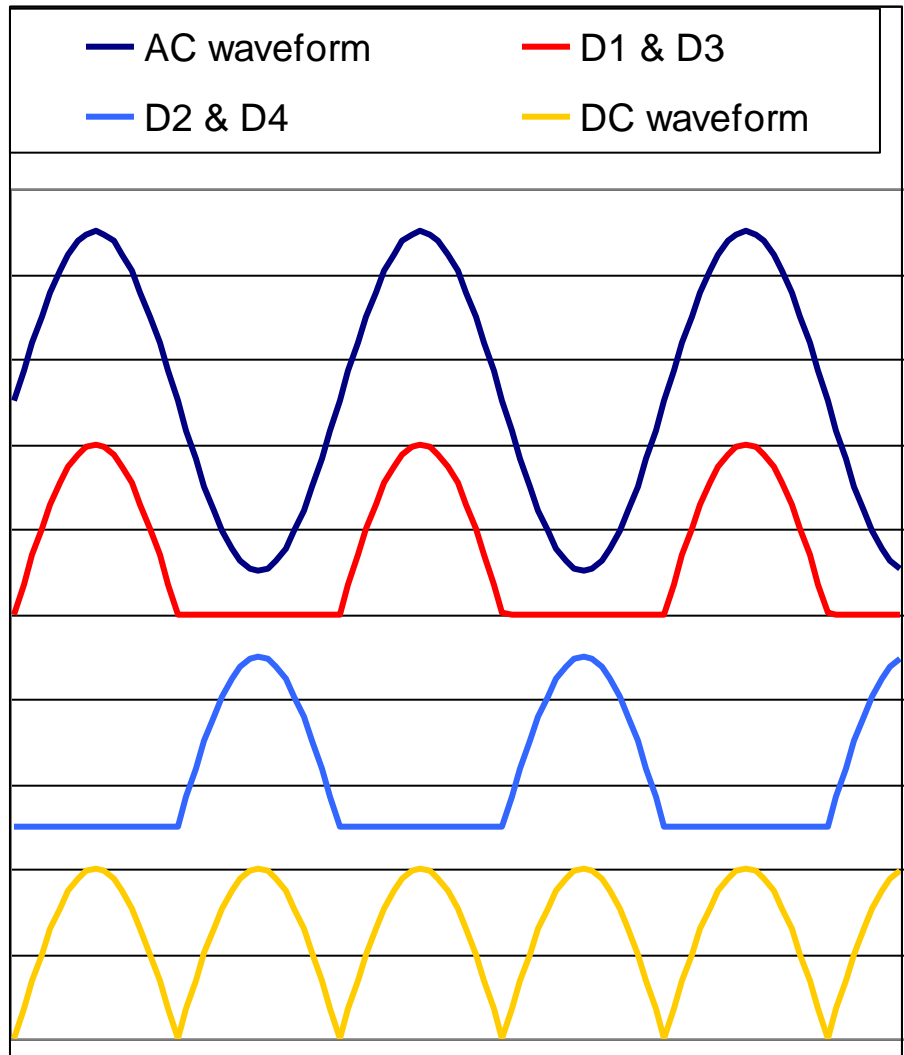
## Full-wave bridge rectifier circuits.

We can improve the previous circuit still further by the use of an additional pair of diodes. Now the transformer's secondary winding is fully utilised and hence a centre tap is not required. This is usually the most cost effective circuit to use.

### Single phase bridges.



Note diodes all point the same way!



The relationship between the bridge and full-wave rectifier circuits is the same:

$V_{DC}=0.9V_{AC\ RMS}$ . Because the peak inverse voltage is now shared between a pair of diodes, **each now needs to be able to withstand the peak value of  $V_{AC}$ .**

For applications where high currents and low voltages are required, the centre tap circuit is preferred because there is **one diode voltage drop** in the secondary circuit. However, for the same secondary power as a bridge, **the centre-tapped transformer must have the power rating increased by ~ 30%.**

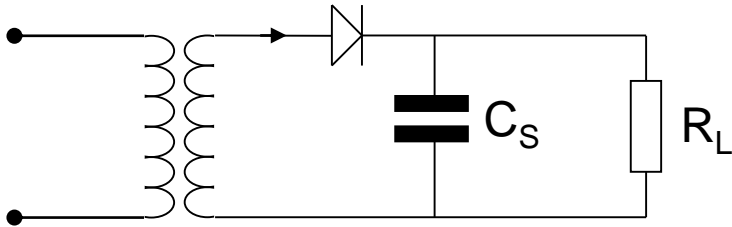
Finally, note that the secondary voltage of a transformer is always quoted at the full load current. For part-load conditions the secondary voltage will be higher. This is called the transformer's *regulation*, covered in a later lecture.

## Smoothing

The “raw” DC produced by a rectifier circuit is useful for items such as battery chargers. However, most electronic circuits require a **constant DC** supply. (Raw DC falls to zero 50 or 100 times per second.) To reduce the ripple voltage, a smoothing capacitor is added across the rectifier circuit output.

***The value of this capacitor should be such that  $R_{Load}C_{Smoothing} \gg 1/\text{Frequency}$ .***

## Half-wave rectification with capacitive smoothing

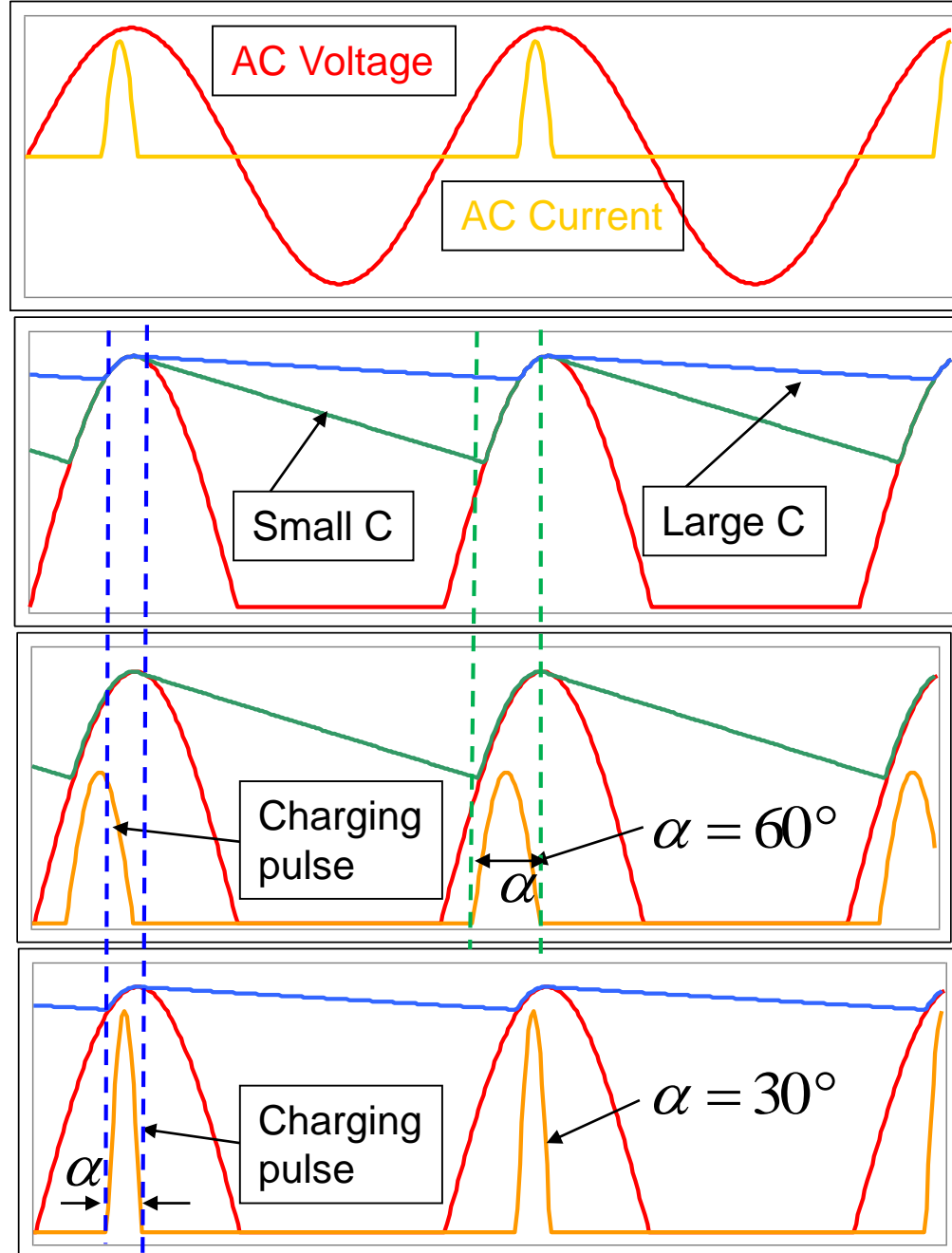


In this circuit the diode only conducts when the anode is more positive than the capacitor voltage.

For a given load power, a large capacitor results in a lower ripple voltage. However, this leads to a shorter charging current pulse and hence the peak current through the diode increases. The form factor also becomes higher (typically 1.8 to 2.2) and thus the transformer requires a larger wire diameter and higher VA rating to cope without overheating.

The diode conduction angle is  $\alpha$ .

Harmonic currents are high.

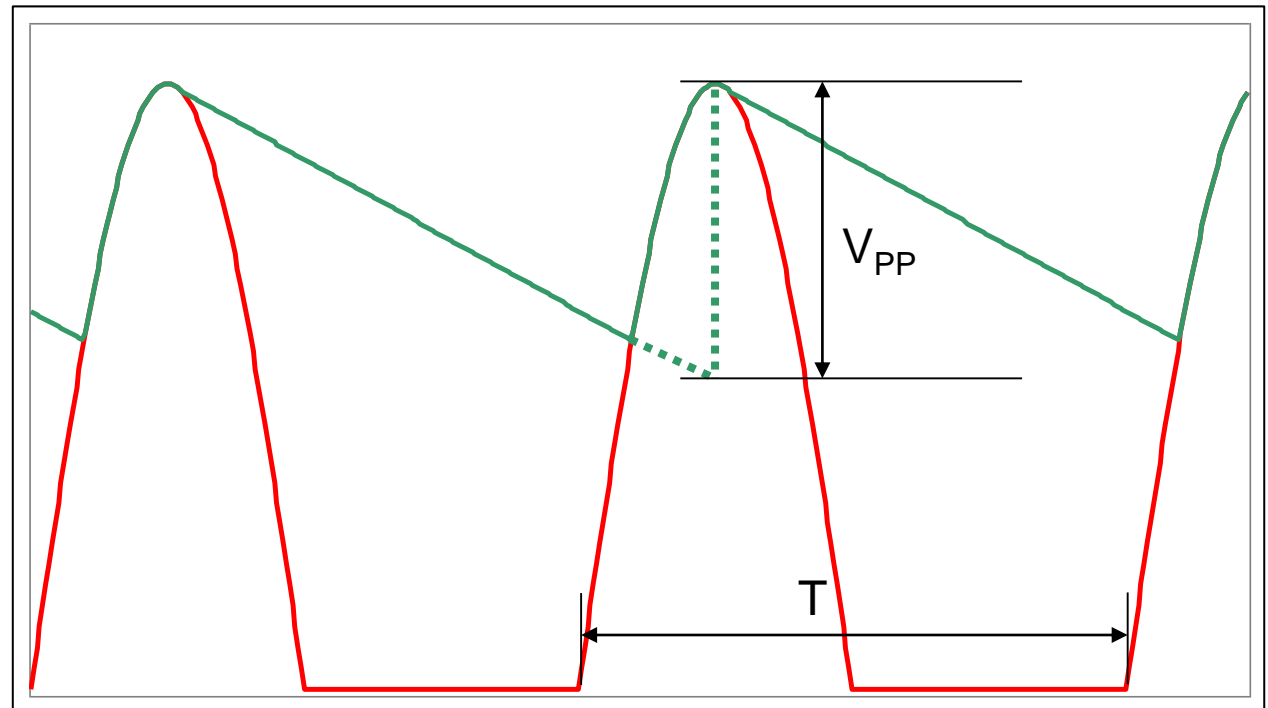


## Calculation of ripple (纹波) voltage.

For “sensible” values of smoothing capacitor (i.e.,  $RC \gg 1/F^*$ ) we can **approximate the exponential decay of voltage on the capacitor to a linear sawtooth waveform**. Similarly, since the charging ( $t \ll \tau$ ) period is short compared to the discharge period we can extend the duration of the discharge period to be the sawtooth period.

During the discharge period all the current is delivered by the capacitor, hence we can write:  $I_L = C \frac{dV_L}{dt}$  or  $\frac{dV_L}{dt} = \frac{I_L}{C}$

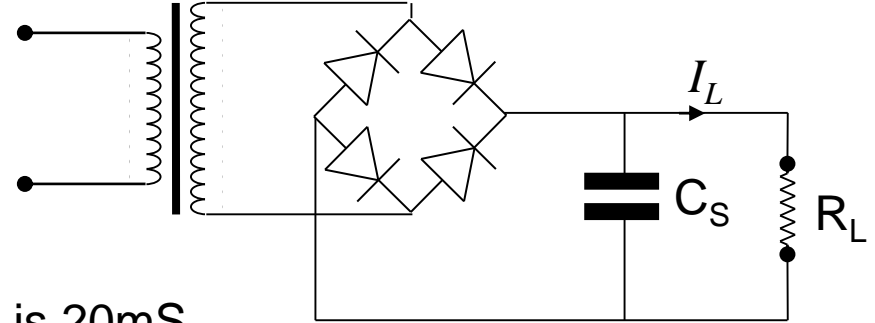
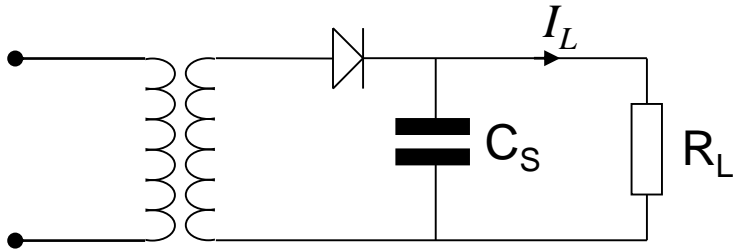
(These approximations are “safe”, in that they will yield a **value of C slightly larger than needed** because the discharge period ends in less than a half cycle.)



\* Where the RC *time constant*  $\tau$  is much longer than the period of the waveform.

### Example.

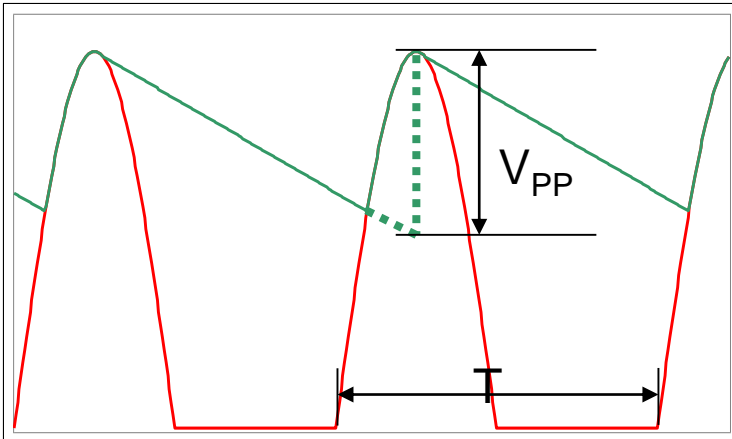
What is the peak to peak ripple voltage in the circuits shown for a load current of 5A and a capacitor of 15,000 $\mu$ F? Assume the mains frequency is 50Hz.



For the half-wave case, the discharge time is 20mS.

$$\frac{dV_{\text{Load}}}{dt} = \frac{I_{\text{Load}}}{C} = \frac{5}{15000 \times 10^{-6}} = 333 \text{ V s}^{-1}$$

$$\Rightarrow \Delta V_{\text{PP}} = \text{Discharge time} \times \frac{dV_L}{dt} = 20\text{mS} \times 333\text{V s}^{-1} = 6.67\text{V}$$



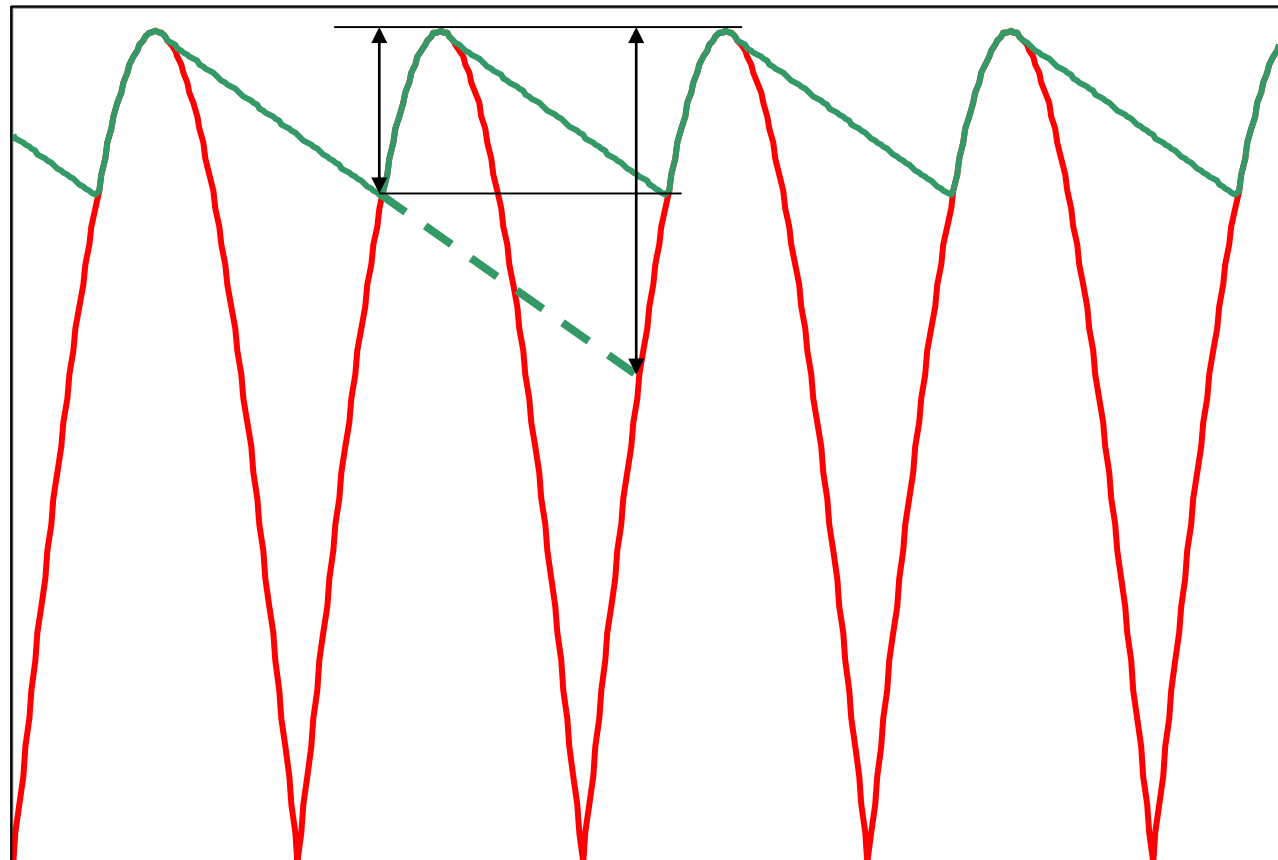
Half-wave case

For the full wave circuit the discharge period is 10mS. The rate of discharge of the capacitor will be the same ( $333\text{Vs}^{-1}$ ), hence the peak to peak ripple voltage is halved to 3.33V.

The average ripple voltage will be half this.

What is the RMS value of the ripple voltage?  $\left[ \frac{V_{PP}}{\sqrt{3}} \right]$

Full-wave case



Time between pulses halved; ripple voltage also halved.



We can also approximate the ripple voltage from energy considerations:

$$E = 0.5CV^2. \text{ If } V_{\text{Load}} = 40V, E = 0.5 \times 15 \times 10^{-3} \times 40^2 = \underline{12J}$$

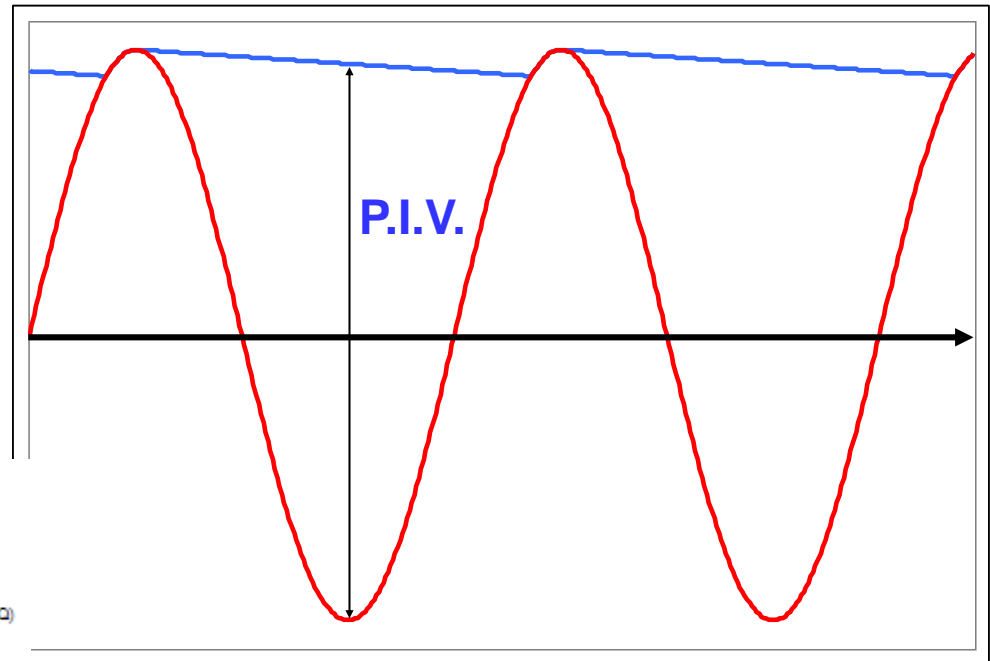
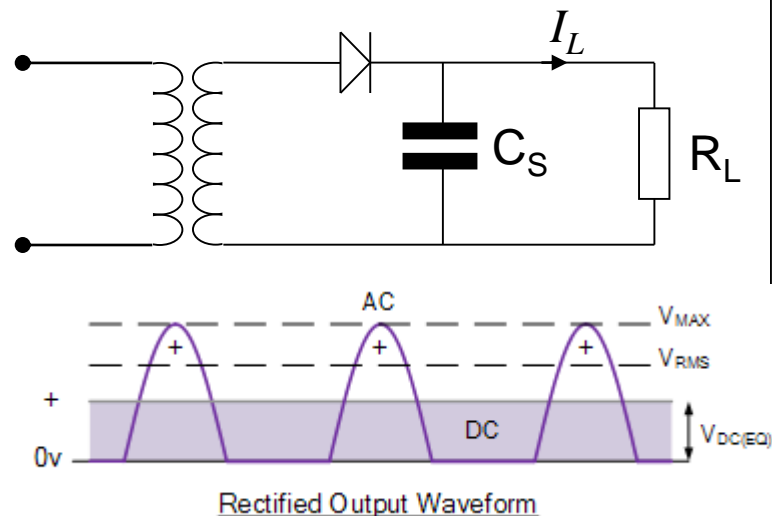
$$\Delta E = V \times A \times t = 40V \times 5A \times 10mS = 2J.$$

$$\Rightarrow V' = \sqrt{(2(E - \Delta E) / C)} = 36.52V.$$

$$\text{Hence } \Delta V = V - V' = 40V - 36.52V = 3.49V. \text{ (Other method was 3.33V)}$$

Note that when a capacitor is added to the half wave circuit the no-load DC voltage increases to the peak value of the AC voltage, so now  $V_{DC} = \sqrt{2}V_{AC}$  (Was  $0.45V_{AC \text{ RMS}}$ ). Also note that the smoothing capacitor maintains the voltage during the negative half cycles, thus the **PIV(peak inverse voltage)** rating of the diode must be doubled.

$$V_{d.c.} = \frac{V_{\max}}{\pi} = 0.318V_{\max} = 0.45V_{\text{RMS}}$$



## Form Factor considerations.

We have seen how a larger value of smoothing capacitor decreases the ripple voltage. However, seldom does something come for free, and in this case the larger smoothing capacitor adversely affects the form factor of the current waveform drawn from the transformer: a larger current pulse is drawn for a shorter period. **The Form Factor is directly related to the diode conduction angle  $\alpha$ .**

A  $60^\circ$  conduction angle equates to  $3\omega t$ .

$$V_{ave(\alpha=60)} = \frac{1}{\pi} \left[ \int_0^{\frac{\pi}{3}} \hat{V} \sin 3\omega t d\omega t + \int_{\frac{\pi}{3}}^{\pi} 0 d\omega t \right]$$
$$= \frac{\hat{V}}{\pi} \left[ -\frac{1}{3} \cos 3\omega t \right]_0^{\frac{\pi}{3}} = \frac{-\hat{V}}{3\pi} [-2] = \frac{2\hat{V}}{3\pi}$$

$$FF = \frac{V_{RMS}}{V_{AVE}} = \frac{\frac{\hat{V}}{\sqrt{6}}}{\frac{2\hat{V}}{3\pi}} = \frac{3\pi}{2\sqrt{6}} = 1.933$$

$$V_{RMS(\alpha=60)}^2 = \frac{1}{\pi} \left[ \int_0^{\frac{\pi}{3}} (\hat{V} \sin 3\omega t)^2 d\omega t + \int_{\frac{\pi}{3}}^{\pi} 0 d\omega t \right]$$

$$= \frac{\hat{V}^2}{\pi} \int_0^{\frac{\pi}{3}} \sin^2 3\omega t d\omega t = \frac{\hat{V}^2}{\pi} \int_0^{\frac{\pi}{3}} \frac{1}{2} [1 - \cos 6\omega t]$$
$$= \frac{\hat{V}^2}{2\pi} \left[ \omega t \right]_0^{\pi/3} + \left[ \sin 6\omega t \right]_0^{\pi/3} = \frac{\hat{V}^2}{2\pi} \times \frac{\pi}{3} = \frac{V^2}{6}$$

$$\Rightarrow V_{RMS(\alpha=60)} = \frac{\hat{V}}{\sqrt{6}}$$

Show that for  $\alpha = 30^\circ$  the form factor is 2.72.

## The effect of the form factor on the capacitor ripple voltage

So far we have approximated the charging pulse in the capacitor to be of such a short time that it doesn't affect the voltage waveform. In reality, the charging pulse affects the waveform sufficiently to warrant its inclusion. Whilst the diode is conducting, it is supplying the load AND re-charging the capacitor. Hence the capacitor discharge period is shortened by the diode's conduction period.

**Example.** If the input to a full-wave rectifier is 12V @ 50Hz,  $R_{Load}$  is  $8.5\Omega$  and  $C_{Smoothing}$   $10,000\mu F$ , what is the ripple voltage if the diode conduction angle is  $30^\circ$ ?

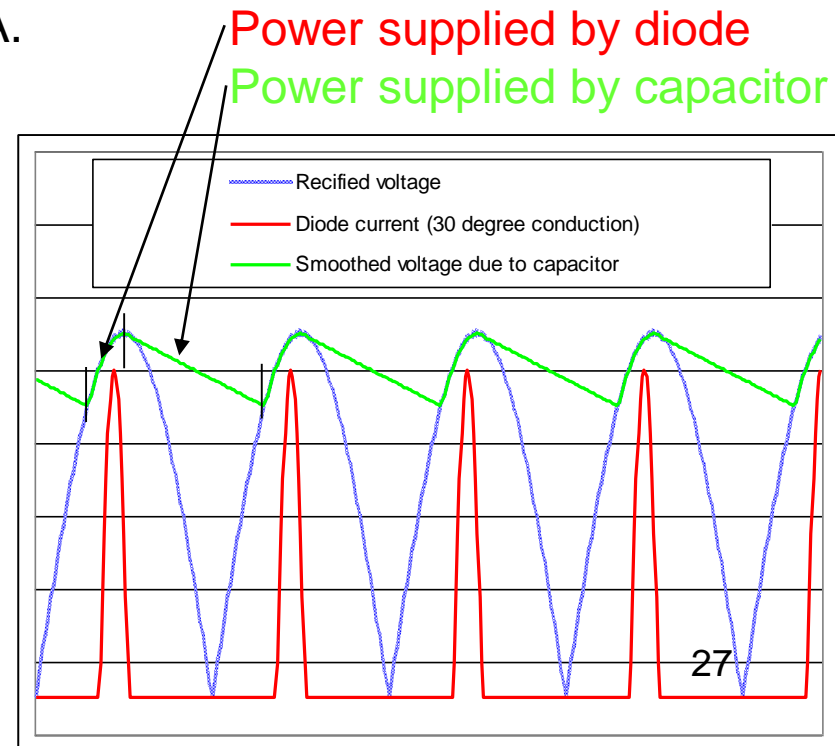
$f = 50\text{Hz} \Rightarrow$  full-wave rectified pulse freq is 100Hz. Period = 10ms, Conduction angle =  $30^\circ$  or 1.66mS, so capacitor discharge time is 8.33mS. Peak load voltage is  $\sqrt{2} \times 12\text{V} = 17\text{V}_{Peak}$ . Load =  $8.5\Omega$  so load current is 2A.

$$I_L = C \frac{dV_L}{dt} \quad \text{or} \quad dV_L = \frac{I_L \cdot dt}{C} = \frac{2\text{A} \times 8.33\text{mS}}{10\text{mF}} = 1.66\text{V}$$

### Notes.

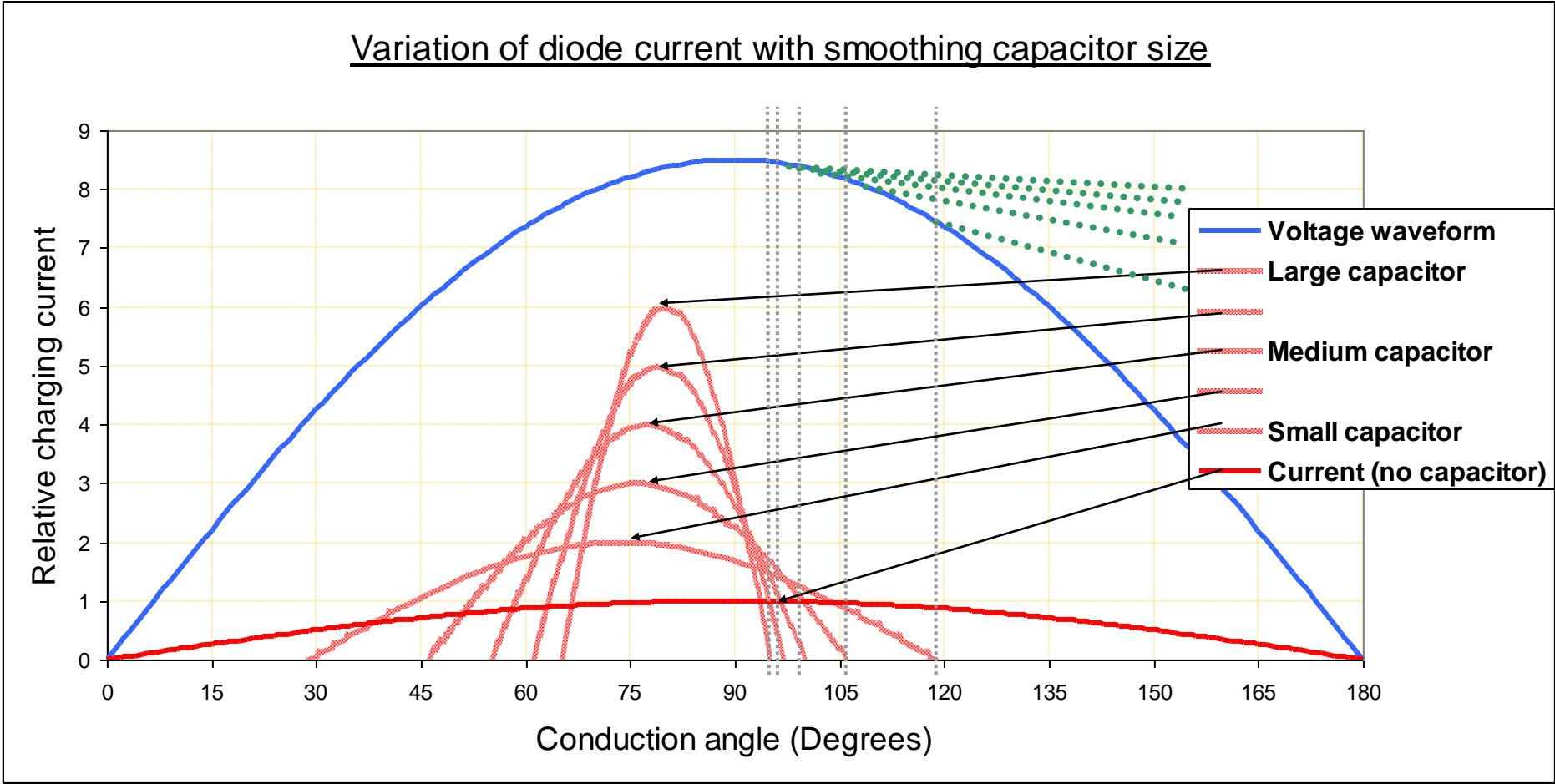
Assume load current doesn't vary during discharge period. Approximate to a linear discharge ramp. Peak to peak ripple is 1.66V and mean ripple is half this (0.83V) Mean output is 16.2V.

Larger cap: higher mean output voltage, reduced ripple, decreased diode conduction angle, higher diode current.



Energy per pulse considerations.

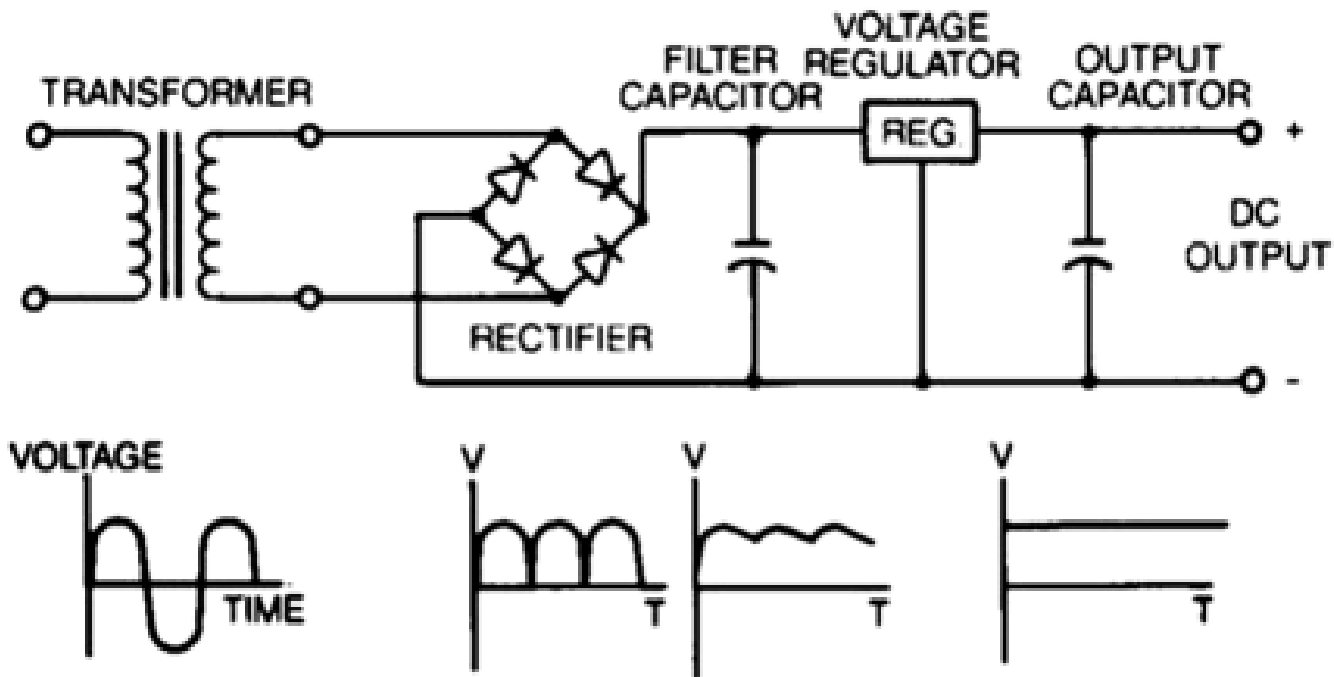
Variation of diode current with smoothing capacitor size



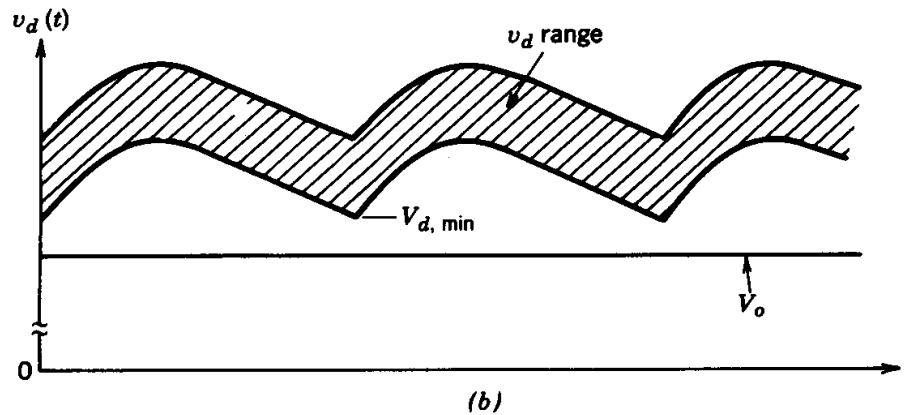
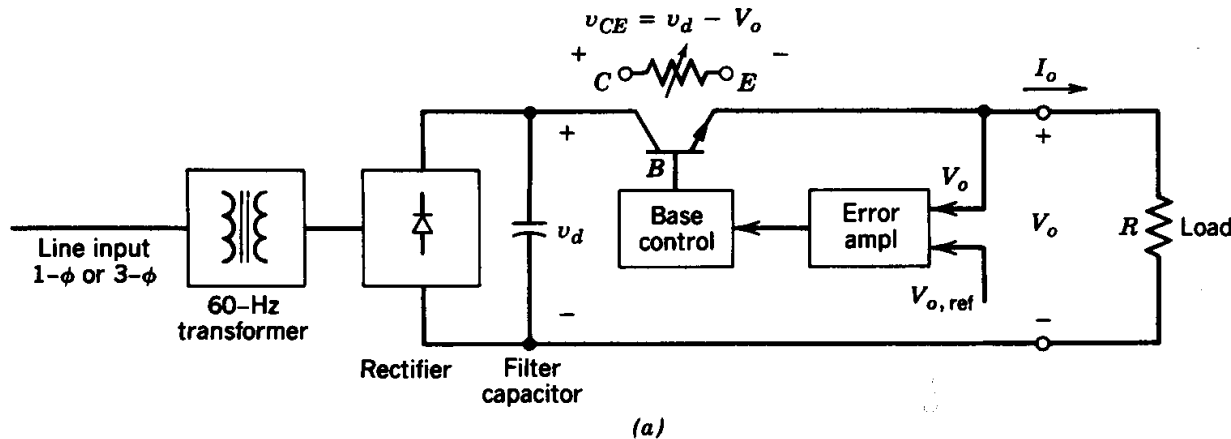
Let's say the power supply is delivering 100W of output power and the supply frequency is 50Hz. Then  $100\text{W} = 100\text{J/sec}$  and, neglecting losses, at 50Hz each pulse must have an "area" of 1 joule. As the duration of the charging pulse decreases, its amplitude must increase to maintain an equal area under the curve. This increase in current causes additional heating, given by  $I^2t$ : by shortening the time in which the energy is transferred to the capacitor, the amount of current required increases. Heating varies as the square of the current, but inversely to the time. **So a current of 10A for 10mS second yields an  $I^2t$  of 1 whereas 100A for 1mS gives  $I^2t$  of 10.**

**$I^2t$  is called the "let-through energy".** We will encounter it again when talking about semiconductor switches.

# Linear Power Supplies



# Linear Power Supplies



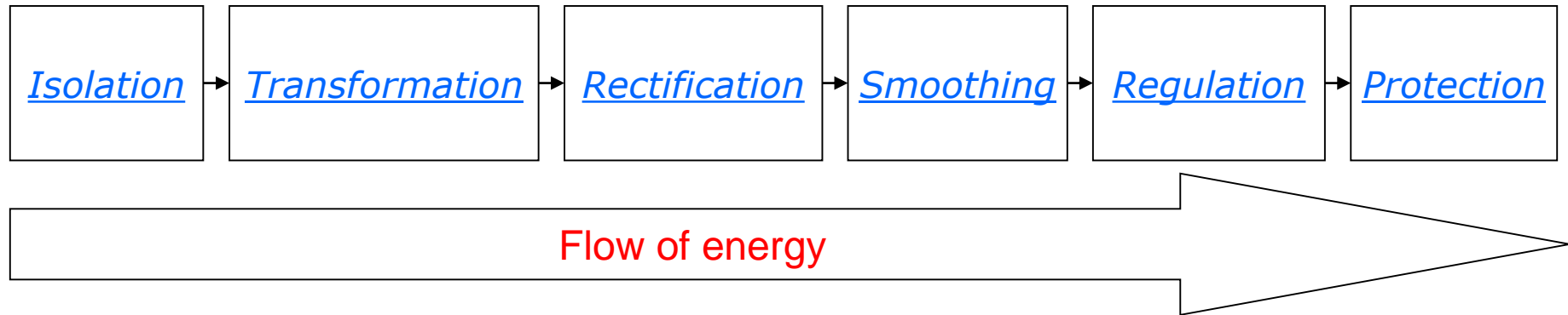
- Very poor efficiency (usually 30~60% and large weight and size)
- Low EMI, simple circuitry, may cost less in small power rating (<25W)

# Linear Power Supplies

Power supplies come in all shapes and sizes – from the tiny circuits used inside hearing aids to supplies housed in their own building for commercial data centres.

The primary function of a power supply is to provide the equipment attached to it with the power it needs at the correct voltage and current.

There are 6 basic stages in a linear power supply:



# 1. Isolation & Transformation

In the lecture on transformers the subjects of electrical isolation and transformation of voltage from the primary to the secondary side were dealt with. From a power supply design perspective, there are two important points to remember:

1. The transformer's regulation is given by:

$$\% \text{ Regulation} = \frac{\text{Voltage}_{\text{No load}} - \text{Voltage}_{\text{Fullload}}}{\text{Voltage}_{\text{No load}}} \times 100$$

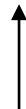
2. The secondary voltage rating of the transformer is quoted at the maximum load.

Also note that the mains voltage itself has a tolerance associated with it – usually in the range 230V (remember this is the AC RMS value) +10% - 6%.

So, if we have a transformer with a secondary voltage of 12V and a regulation of 8%, the range of secondary voltage that will be encountered during normal operation is between

$$V_{\text{Min}} = 12 \times (1 - 6\%) \equiv 12 \times 0.94 = 11.3\text{V} (16V_{\text{Peak}}) \text{ [Remember this is @ full load]}$$

$$V_{\text{Max}} = (12 \times (1 + 10\%)) \times 1.08 \equiv (12 \times 1.1) \times 1.08 = 14.3\text{V} (20.2V_{\text{Peak}})$$



Regulation component

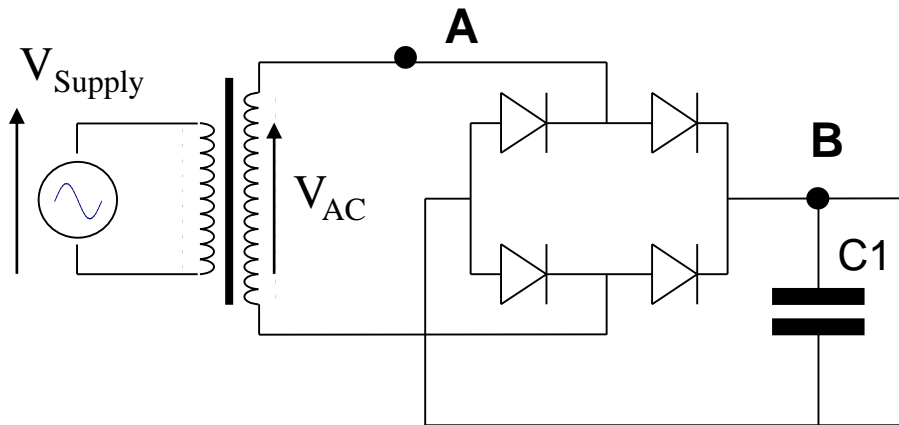


The VA (volt-amperes) rating of the transformer is given by the product of the secondary voltage at full load and the secondary current (also at full load). When the transformer is supplying a capacitive smoothing circuit, the form factor of the current waveform can be quite high, especially for large values of capacitor. If this is the case then the secondary rating of the transformer needs to be derated to allow for the heating effect in the transformer due to the high currents that flow during the diode conduction period.

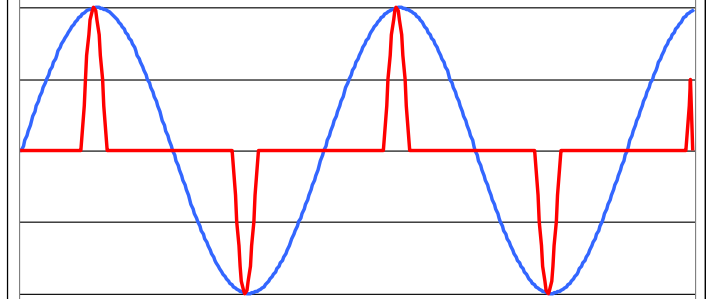
**Transformer derating:  $I_{AC} = I_{DC} \times \text{Form Factor}$ .**

$$VA_{LOAD} = V_{AC} \times I_{AC} \cong V_{AC} \times F^* \times I_{DC}; \quad F^* = \text{Form factor.}$$

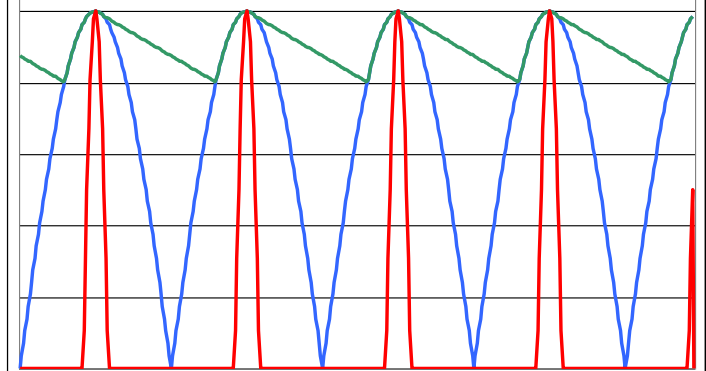
$$VA_{TRANSFORMER} = \frac{VA_{LOAD}}{\eta}; \quad \eta = \text{efficiency}$$



**Voltage & current at point A**



**Voltage & current at point B**



## 2. Rectification & Smoothing.

This was dealt with in the lecture on diodes and in the lab. To recap on the important points:

1. Assume the diodes have a 1 volt drop across them *when conducting current*.

Strictly speaking, if there is no load on the power supply then the diodes don't pass any current and hence the rectified voltage on the capacitor will rise to the peak value of the transformer's secondary  $\times \sqrt{2}$ . It must be assumed that at some point the power supply will be operated with no load, hence the diodes must be able to stand the peak inverse voltage that will be applied.

Under no-load conditions there is no voltage drop across the diodes.

2. The 4 diode full-wave bridge makes better use of the transformer winding than the 2 diode full wave centre-tap circuit. The peak inverse voltage for each diode in the bridge is half the full-wave CT case since, for the case of the bridge, **two diodes share the voltage**.

$$Total\ PIV = \hat{V}_{CAP} - \hat{V}_{AC(Min)}$$

Greatest positive voltage      Greatest negative voltage

3. Both full-wave rectifier circuits double the supply frequency on the DC side of the rectifier and this eases the smoothing problem (i.e. halves  $\Delta t$ ). The ripple voltage (i.e.  $\Delta V$  on the capacitor) may be estimated by considering the load current and/or the charge in the capacitor.

$$I = C \frac{dV}{dt} \Rightarrow \Delta V = \frac{I \cdot \Delta t}{C}; \text{ Similarly for } E = \frac{1}{2} CV^2 \text{ and } Q = CV.$$

If the diode conduction angle is known (say  $30^\circ$ ), subtract this from  $\Delta t$  for a better estimate. The rectified half sinusoid has a duration of  $180^\circ$ .

E.g. If we are told the diode conduction angle is  $36^\circ$  (one fifth of the half cycle), for a  $20,000\mu\text{F}$  capacitor on a  $50\text{Hz}$  supply with a load current of  $5\text{A}$  the the ripple voltage is:

$$I = C \frac{dV}{dt} : V_{\text{RIPPLE}} = \frac{I \cdot \Delta t}{C} = \frac{5 \times (10 - 2\text{ms})}{20\text{mF}} = 2\text{V}$$

4. Remember that a bigger value of smoothing capacitor will give a smaller ripple voltage for a given load current but the trade-off is that a bigger transformer will be required (because of derating due to the form factor of the current waveform) and the supply-line current harmonics will be worse. The load current determines the ripple voltage for constant capacitance according to the above relationships.

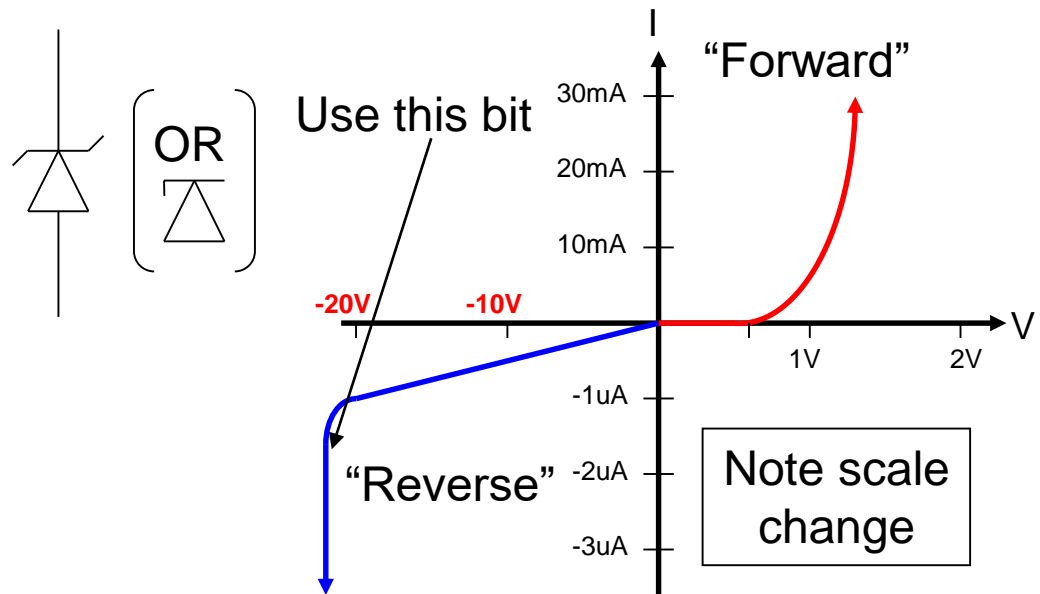
5. Also remember that the ripple frequency is a function of (2X) the supply frequency, not the value of the smoothing capacitor.

### 3. Regulation.

Regulation in a power supply context is a figure of merit describing how the output voltage changes in response to varying load current. A “good” power supply will have a very stable output voltage that ideally is independent of the load current being drawn. In reality this is extremely hard to achieve. Once again, in the lecture on diodes the use of a Zener diode to produce a fixed voltage was presented. This suffers from poor regulation since as the load current varies, so the bias current through the zener changes and the output voltage varies along the load curve for the device.

Things to remember about zeners are:

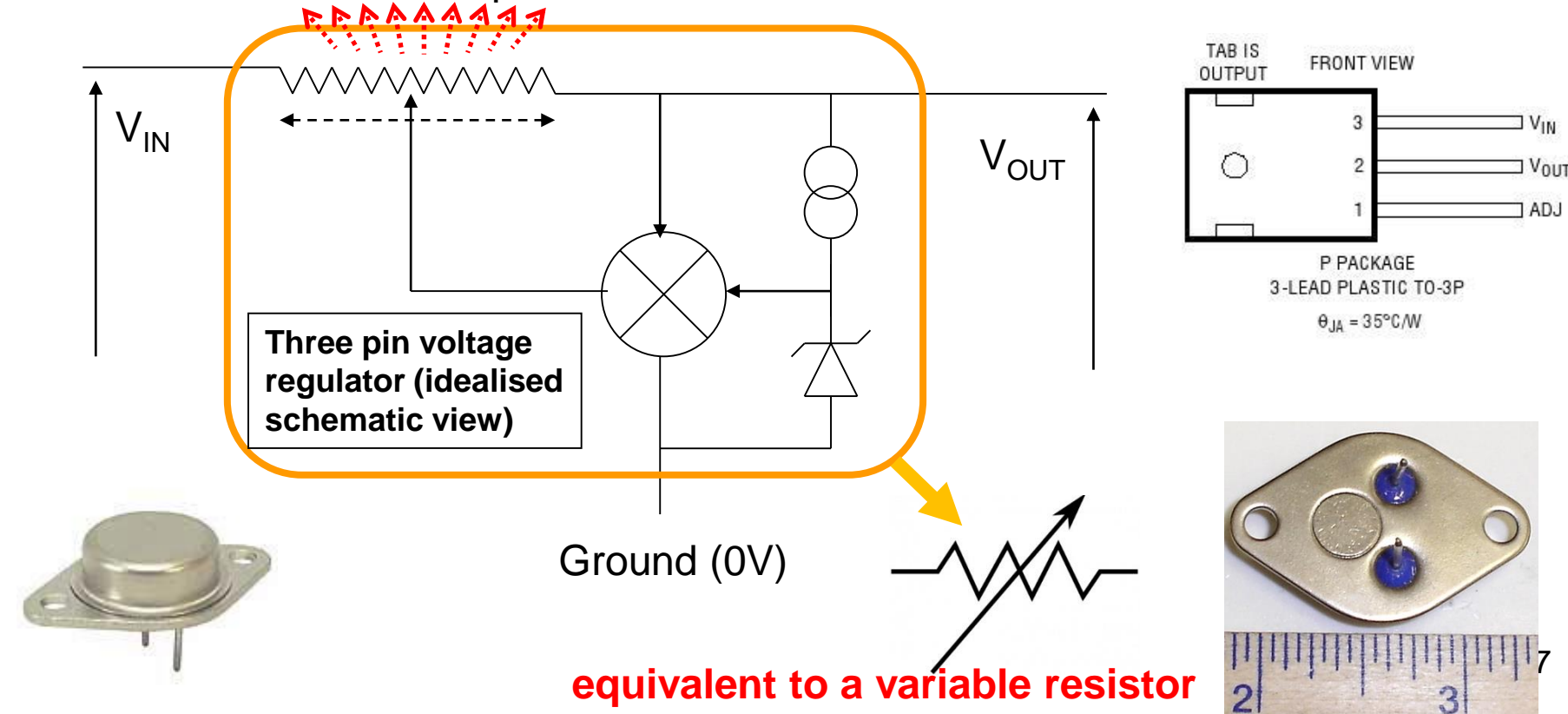
1. The regulation is quite poor;
2. The power dissipation is limited. They're OK for “signal” circuits but not much use for a large load;
3. The output voltage must always be less than the input voltage (Obviously!!)



# The linear voltage regulator.

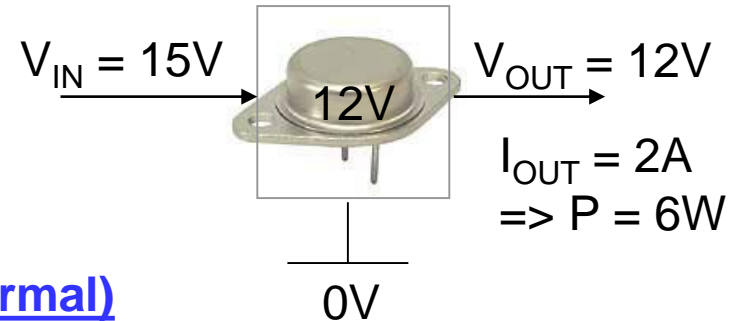
A very economical way to improve the regulation of a power supply is to use a linear voltage regulator. These are 3 pin semiconductor devices – input, output and ground.

So long as the input voltage remains a few volts higher than the output voltage (the regulator’s “headroom”) the electronics inside the device will maintain the output voltage at the specified value very accurately. In the lab experiment on rectification you (should have) found the DC voltage regulation was anywhere between 15% and 35%, depending on how the bulbs were wired up. The 3 pin regulator can easily achieve 0.1% - a 1500x improvement.



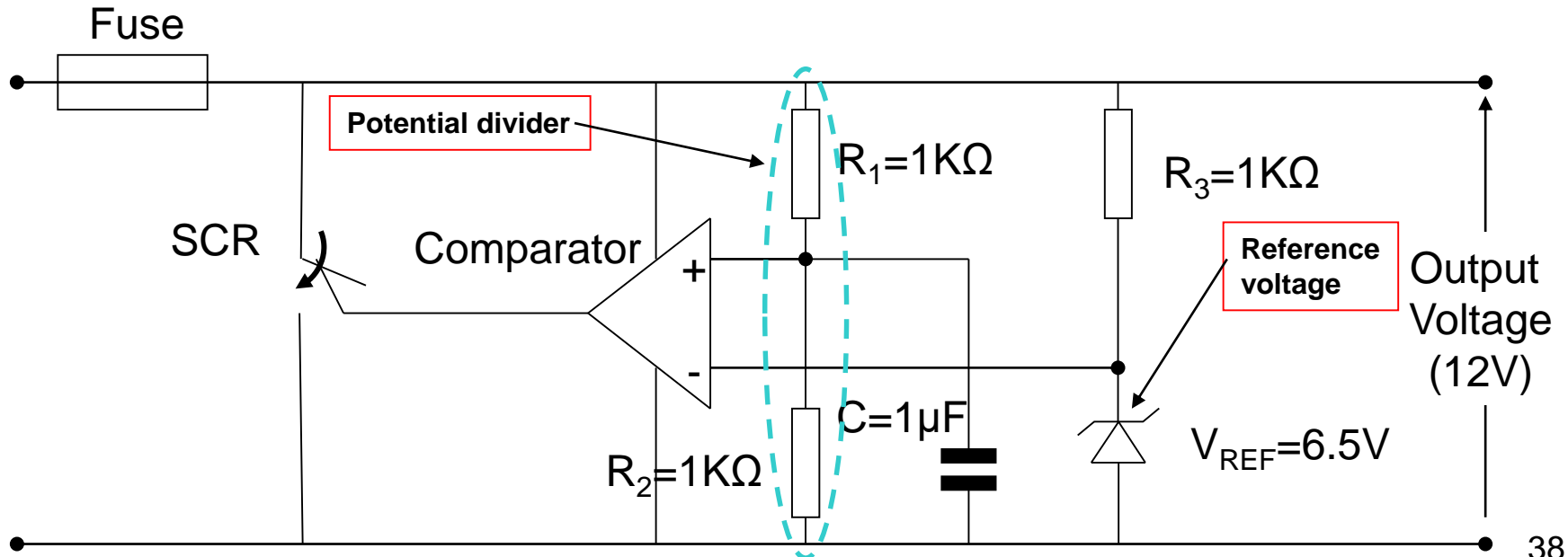
The power dissipation of the linear voltage regulator is the product of the voltage drop across the device multiplied by the current passing through it.  $P = I \times \Delta V$ .

The electronics in the device also takes care of self-protection due to thermal overload, short-circuits, and so on.



#### 4. Protection. (Overvoltage, Overcurrent, Thermal)

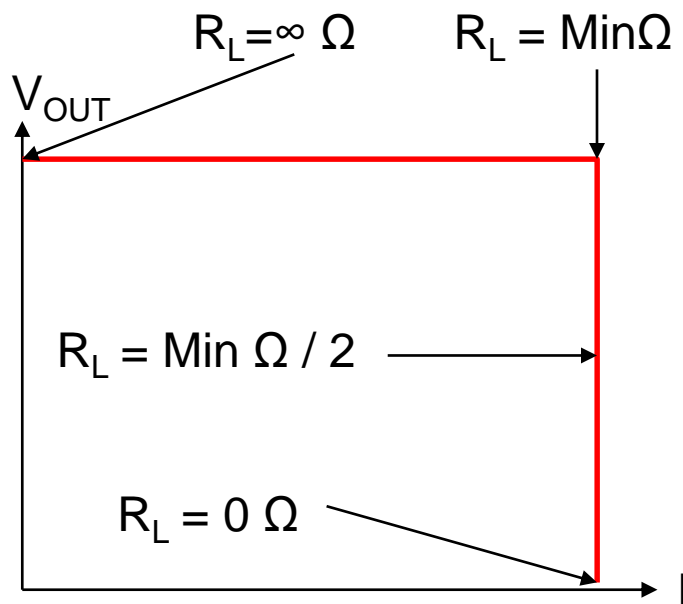
We saw in an earlier lecture how an SCR can be used as the switching element when implementing an over-voltage protection circuit. There are a couple of other protection measures that are provided by the linear voltage regulator.



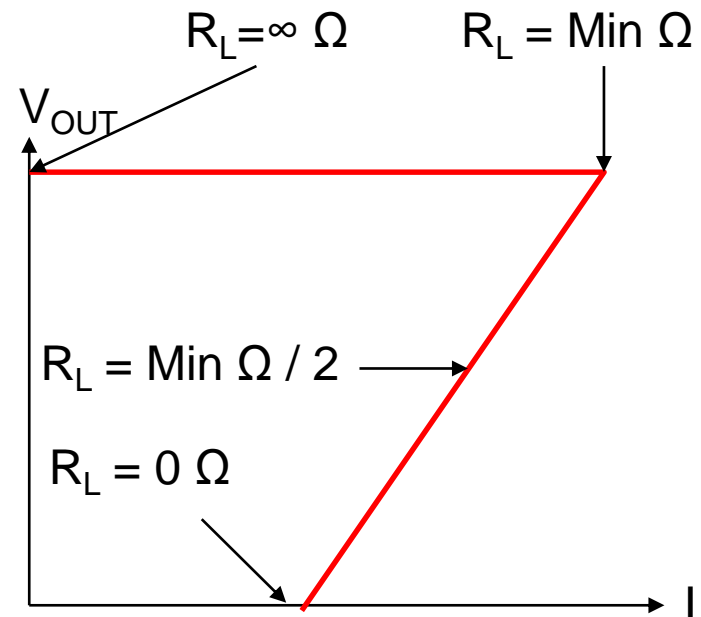
## Current limiting and thermal overload protection.

The linear regulator dissipates the “surplus” power between the input and output as power. *For the linear regulator the output current equals the input current.* When the voltage difference is high or the load current is high then this power can lead to excessive device temperatures. The problem is exacerbated when the device is overloaded or short-circuited.

With reference to the figures below, as the load current increases, the output voltage stays essentially flat. When the maximum output current is exceeded the regulator starts to reduce the output voltage.



Constant current limiting.



Current foldback limiting

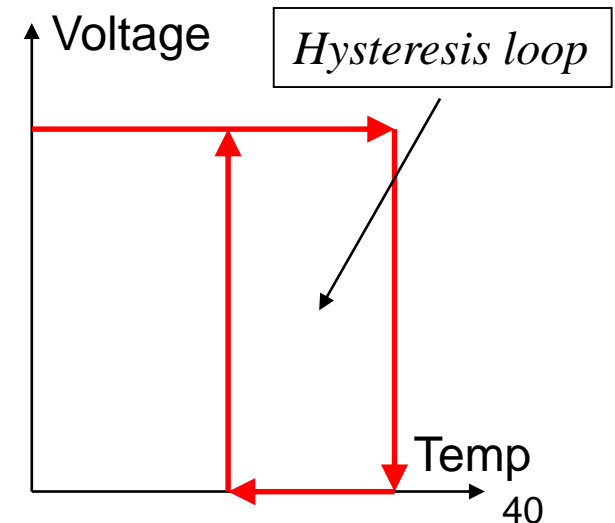
The first technique is called “constant current limiting”: the regulator decreases the output voltage whilst holding the output current constant at the maximum value. The problem with doing this is that  $\Delta V$  across the regulator starts to increase and this leads to even higher power dissipation.

To get around this problem, the second (more common) technique is to use “current foldback limiting”: both the output voltage AND output current are decreased simultaneously. This limits the power the regulator has to dissipate.

This allows a more economical design since the worst-case power is reduced. Despite these techniques, the regulator also incorporates **a comprehensive thermal protection circuit**. Because all the components are contained within a single package it is possible to accurately measure the temperature of the device and if the maximum allowable temperature is exceeded, the regulator starts to shut down to protect itself. Usually this is because of inadequate heatsinking.

Once the temperature has fallen to a safe value the regulator will re-activate its output. The figure opposite shows the hysteresis associated with this protection technique.

*Inefficiency in linear power supplies is a big problem.*



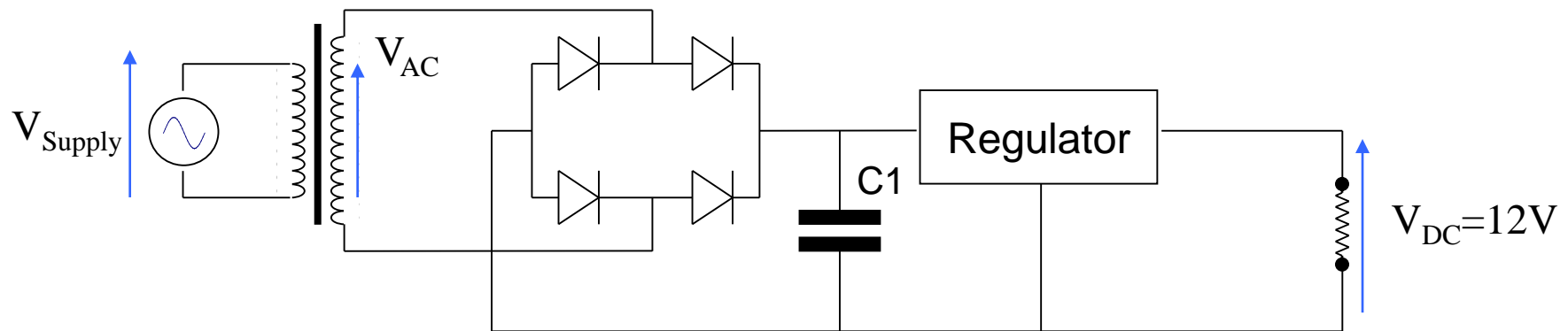


## Question

Design a power supply to deliver a current of 5A at 12V using a linear regulator. The regulator has a minimum input voltage of 14.5V and the mains input voltage is 230V  $+10\%$  /  $-6\%$  @ 50Hz. The transformer has a regulation of 5% and is 93% efficient on full load. Rectification should be by the use of a bridge rectifier. Assume the diode conduction angle at full load is  $36^\circ$ .

Answer the following questions:

- A. What is the minimum AC voltage the secondary winding should have if the smoothing capacitor is  $10,000\mu\text{F}$ ?
- B. What is the power dissipation in the regulator under worst-case conditions?
- C. What voltage rating should the capacitor have?
- D. What PIV rating should the diodes have?
- E. What VA rating does the transformer require?



Transformer regulation = 5%,  $\eta = 93\%$ ,  $V_{\text{Supply}} = 230\text{V AC } +10\%, -6\% @ 50\text{Hz}$   
 $C1 = 10,000\mu\text{F}$ , Regulator  $V_{\text{IN}} = 14.5\text{V} \rightarrow 40\text{V}$ ,  $I_{\text{Max}} = 5\text{A}$ ,  $V_{\text{OUT}} = 12\text{V}$

## Solution to A

*Start at the regulator and work back towards the mains supply.*

**Step 1.** Find out what the minimum DC voltage we need to drive the regulator is.

The regulator output voltage is 12V and it needs an extra 2.5V “headroom”. So not less than 14.5V input.

**Step 2.** Calculate the maximum ripple voltage (which depends on load current).

We know the maximum current is 5A and the capacitor is 10,000uF. We are also told the diode conduction angle is 36° (one fifth of the half cycle and the supply frequency is 50Hz. From this, we can calculate the peak to peak ripple voltage. We need to use the peak to peak rather than the average since the input to the regulator must always be kept above 14.5V.

$$I_{\text{Use}} = C \frac{dV}{dt} : V_{\text{RIPPLE}} = \frac{I \cdot \Delta t}{C} = \frac{5 \times (10 - 2\text{ms})}{10\text{mF}} = 4\text{V}$$

Thus the minimum value of the rectified voltage seen at the capacitor must be at least 18.5V to ensure the ripple voltage doesn't cause the regulator input to fall below 14.5V.

**Step 3.** Don't forget to add the voltage drop across the diodes. When the supply has no output current the voltage drop across each diode will be 0V. However, on load, assume each diode voltage drop is 1V.

Since we're using a bridge rectifier then there are two diodes in series (one in the +ve side & one in the -ve side). Thus the peak voltage needed is  $18.5\text{V} + 2\text{V} \Rightarrow V_{\text{Peak}} = 20.5\text{V}$ .

**Step 4.** We also need to take into account the variation in the supply voltage. The nominal is 230V but it can drop down as low as 230V – 6%. At this level, we still need to ensure the peak voltage is  $\geq 20.5\text{V}$ . Hence the peak secondary voltage needs to be divided by 0.94. So 20.5V minimum now becomes  $20.5\text{V} / 0.94 = 21.8\text{V}$ .

**Step 5.** We need to express this peak voltage requirement in terms of the nominal AC RMS secondary voltage since this is what transformer manufacturers (and everybody else for that matter) actually use. We worked this out for the lowest supply voltage (the worst-case condition), so we just need to convert to RMS: divide the peak by  $\sqrt{2}$  – so  $21.8\text{V}_{\text{peak}}$  becomes  $15.3\text{V}_{\text{RMS}}$ . This is the required voltage rating of the secondary winding of the transformer. In reality you would round it up to 16V AC.

**Finally,** also remember that the transformer's rated output voltage is delivered at maximum load, so don't include the effect of regulation for this calculation.

### **Solution to B - Power calculation**

The worst case power dissipation in the voltage regulator will occur when it is delivering maximum output current and the mains supply voltage is at its highest value.

The highest supply voltage is  $230V + 10\%$ , and this corresponds to a 10% increase in  $V_{AC}$  we just worked out. Since we specified a 16V transformer, use this figure.

We need to calculate the average voltage drop across the regulator to find the power dissipated. We know the peak to peak ripple voltage is 4V, so the average ripple voltage is half this value: 2V. We need to subtract the voltage drop across the diodes, too. Hence the average input voltage to the regulator is:

$$V_{AC(Max)} = \frac{230V + 10\%}{230V} \times 16V = 17.6 V_{RMS}$$

$$V_{REGINPUT} = (17.6 \times \sqrt{2}) - V_{DIODES} - \text{Average ripple (2V)} = 20.895V$$

$$I = 5A \Rightarrow P = I\Delta V = 5 \times (20.89 - 12V) = 44.45W$$

The power dissipated is the product of volts dropped across the regulator x the current flowing through it.

**Solution to C & D – Highest voltages**

The maximum capacitor voltage will be when there is no load on the power supply (i.e. the transformer regulation causes the output voltage to rise) and the supply voltage is at its highest : 230V + 10%. Also, at no load, the voltage drop across the diodes falls to zero (since no current is flowing).

$$V_{CAP(Max)} = V_{AC} \times \frac{230V + 10\%}{230V} \times 1.05 \times \sqrt{2} = 26.1V$$

This is what voltage rating the capacitor needs.

The lowest value of the secondary voltage is negative of this value, hence  $PIV = V_{CAP(Max)} - V_{AC(Peak)} = 52.2V$

However, for a bridge rectifier, there are two diodes in series, so the PIV rating for the diodes in this case is the same as for the capacitor since both diodes share the peak voltage. The peak inverse voltage across the diodes will occur when the capacitor voltage is at the highest value and the secondary voltage is at its lowest value in the sinusoid.

Diode voltage rating too.

**Solution to E - calculation of VA rating.**

A high form factor for the current waveform leads to additional heating in the transformer windings. To ensure the transformer temperature rise remains within the manufacturer’s specification, we need to derate it. The working approximation is to say the AC current equals the DC current times the Form Factor of the waveform.

A 36° conduction angle per half cycle equates to 5ωt over π radians.

The following page has the mean and RMS calculations for 5ωt to find the form factor, 2.48 in this case .

We need to multiply the secondary VA load by the transformer’s efficiency to get the total VA rating:

$$VA_{LOAD} = V_{AC} \times I_{AC} = V_{AC} \times F^* \times I_{DC} = 16 \times 2.48 \times 5 \Rightarrow VA_{LOAD} = 198.4VA$$

$$\Rightarrow VA_{TRANSFORMER} = \frac{VA_{LOAD}}{\eta} = \frac{198.4}{0.93} = 213.3VA$$

*Final remark: Note we have a transformer able to deliver over 200W into a resistive load and yet the linear power supply output is just 60W!*

## Average voltage

$$V_{ave(\alpha=36)} = \frac{1}{\pi} \left[ \int_0^{\frac{\pi}{5}} \hat{V} \sin 5\omega t d\omega t + \int_{\frac{\pi}{5}}^{\pi} \hat{V} \sin 5\omega t d\omega t \right]$$
$$= \frac{\hat{V}}{\pi} \left[ -\frac{1}{5} \cos 5\omega t \right]_0^{\frac{\pi}{5}} = \frac{-\hat{V}}{5\pi} [-2] = \boxed{\frac{2\hat{V}}{5\pi}}$$

## Form factor

$$FF = \frac{V_{RMS}}{V_{AVE}} = \frac{\frac{\hat{V}}{\sqrt{10}}}{\frac{2\hat{V}}{5\pi}} = \frac{5\pi}{2\sqrt{10}} = \boxed{2.48}$$

## RMS voltage

$$V_{RMS(\alpha=36)}^2 = \frac{1}{\pi} \left[ \int_0^{\frac{\pi}{5}} (\hat{V} \sin 5\omega t)^2 d\omega t + \int_{\frac{\pi}{5}}^{\pi} 0 d\omega t \right]$$
$$= \frac{\hat{V}^2}{\pi} \int_0^{\frac{\pi}{5}} \sin^2 5\omega t d\omega t = \frac{\hat{V}^2}{\pi} \int_0^{\frac{\pi}{5}} \frac{1}{2} [1 - \cos 10\omega t] d\omega t$$
$$= \frac{\hat{V}^2}{2\pi} \left[ \omega t \right]_0^{\pi/5} + \left[ \frac{1}{10} \sin 10\omega t \right]_0^{\pi/5} = \frac{\hat{V}^2}{2\pi} \times \frac{\pi}{5} = \frac{V^2}{10}$$
$$\Rightarrow V_{RMS(\alpha=36)} = \boxed{\frac{\hat{V}}{\sqrt{10}}}$$

## HomeWork 1-3

Repeat questions (slide 41) A to E of the worked example for the following conditions:

- Regulator output voltage: 15V
- Regulator output current: 3A
- Regulator headroom: 2V
- Transformer regulation: 7%
- Transformer efficiency: 90% @ full load.
- Rectification: full-wave bridge
- Diode conduction angle:  $30^\circ$
- Smoothing capacitor:  $8333\mu\text{F}$
- Supply voltage: 110V AC  $\pm 8\%$ .
- Supply frequency: 60Hz