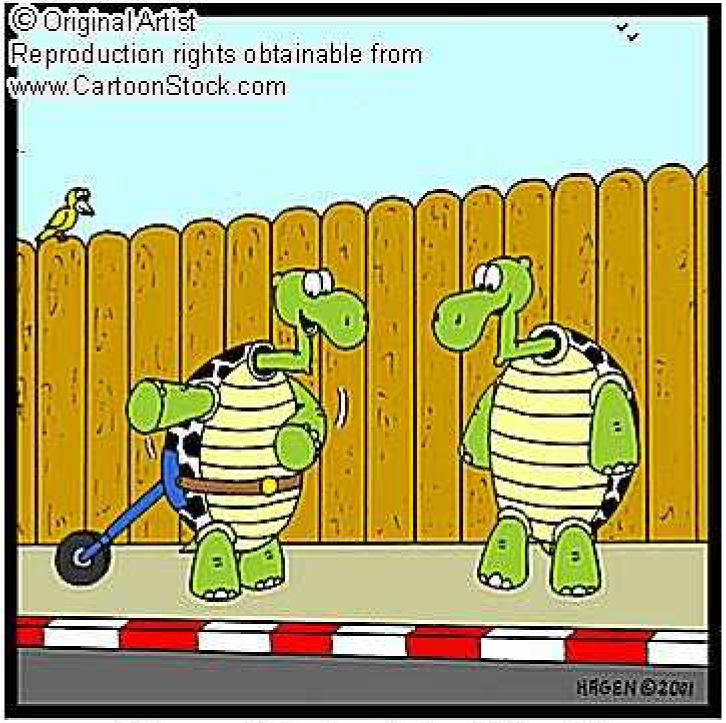
Electric Vehicles

Image courtesy of TreeHugger.Com

There are many design considerations, often contradictory, that need to be addressed when creating a new product. This lecture will provide a brief overview of the following:

- Rolling Resistance
- Drag & Friction
- Ultracapacitors
- Regeneration & Kinetic Energy
- Rechargeable Batteries
- Electric Motors
- Going Green



It's an anti-tipping device, I thought of it while watching Drag-Racing on TV...

Making sense of it all.

To be able to sensibly compare how much energy is "in" different substances, we need to have a common set of units. For example, oil is measured in units of litres (or "barrels" (205l), if there is a lot of it), gas is measured in therms or cubic feet, and central heating radiators are specified in terms of British Thermal Units (BTUs). The power of an electrical heater is measured in kWs and a car battery is measured in Ampere-hours (Ah). Logs come by the cord.

Reminder.

Energy is measured in joules (J).
1 joule is 1 Watt per second.
To raise 1 gram of water by 1°C takes 1 calorie or 4.184J.

The joule is small so we normally use the kilowatt hour (kWh) as a convenient unit.

$$1\text{kWh} = 1000\text{W} \times 3600 \text{ seconds} = 3.6\text{MJ}.$$

"We've sold out of unleaded petrol, but would a bag of charcoal be of any use?"

Rolling resistance.

Rolling resistance represents the sum of the various energies required to move a vehicle. It includes things like displacing water on a wet road, losses in the wheel bearings and transmission, tyre noise and heat, vibration of the ground, etc.

Rolling resistance is proportional only to the weight of the vehicle. The constant of proportionality is called the coefficient of rolling resistance, C_{rr}. [See table of values.]

$$F = C_{rr}N_f$$

F is the resisting force (Newtons)
C_{rr} is the (dimensionless) rolling resistance coefficient
N_f is the normal force (the weight of the object being supported)*

C_{rr} for a car tyre on smooth concrete is about 0.01. If the car weighs 1300kg,
$F = 0.01 \times 9.81\text{Nkg}^{-1} \times 1300\text{kg} = 127.5\text{N}$

At a speed of 70MPH (31ms^{-1}), the power required to overcome the rolling resistance is Force x velocity = $127.5\text{N} \times 31\text{ms}^{-1} \approx 4\text{kW} \approx 5$ h.p. [1 horsepower = 746 watts]

You need to divide this power by the efficiency to find the input power. A car (petrol) has $\eta \approx 25\%$, => $P_{In} = 16\text{kW}$ approx.

* 1 Newton is approximately the force exerted on William Tell's son's head, or Newton's, by 1 apple.

Description	C_{rr}
Steel wheel on railway track	0.0002
Bicycle tyre	0.0055
Car tyre (concrete)	0.01
Car tyre (tarmac)	0.03
Car tyre (sand)	0.3

Table of rolling resistances

General notes.

1. Doubling the number of wheels halves the normal force but the rolling resistance is still (to a first approximation) the same.

2. Mechanical compliance increases rolling resistance (think flat tyres), but for comfort some sort of shock absorption is required.

3. Dampers (“shock absorbers”) dissipate power.

4. Wide tyres have a lower rolling resistance than narrow tyres (for a given load & inflation pressure) because of reduced sidewall flex. They have higher aerodynamic drag, however.

5. Smaller wheels have a higher rolling resistance than those with a larger diameter.

6. A top of the range racing bicycle (no suspension; 70kg) has a rolling resistance of 3.2N.

Drag (Speed costs money – how fast do you want to go?)

A body moving at some velocity through a fluid experiences a retarding force that resists its motion. This is called drag.

$$F_d = -\frac{1}{2}\rho v^2 A C_d \quad \textit{(Approximation; - ve sign because retarding)}$$

$$\rho = \textit{density of fluid (Air} = 1.293 kg/m^3 \ @\ 0°C \ / \ 1 \ \textit{atmosphere)}$$

$$v = \textit{relative speed in } ms^{-1}$$

$$A = \textit{reference area, usually cross - sectional area or frontal area in } m^2$$

$$C_d = \textit{dimensionless drag coefficient}$$

$$Power = F_d \times v = \frac{1}{2}\rho v^3 A C_d \ \textit{(i.e., force} \times \textit{velocity)}$$

Example. Consider a cyclist (area = 0.75m^2) moving at 25 km/h (7ms^{-1})

P = ½ x 1.293 x 7^3 x 0.75 x 0.67 = 111W. (40km/h = 444W)

Somewhere between racing and sitting up

Vehicle	C_d
Cyclist (racing)	0.4
Cyclist (sitting up)	0.9
Car	0.3
4 x 4	0.5

Note that the power required varies as the cube of the velocity.

Dry Friction.

Friction (μ) is the force that resists the sliding motion of two contacting surfaces. This can be static friction (μ_s) for stationary surfaces or dynamic (or kinetic) friction (μ_k) for surfaces with relative motion. Static friction is usually higher than dynamic friction and must be overcome by an applied force before movement can start.

The coefficient of friction, μ, is a dimensionless scalar quantity that describes the ratio of the force between the bodies and the force pressing them together (the frictional force divided by the normal force). Teflon has a value of about 0.04 whereas a car tyre on a dry road may be as high as 1.5.

<u>Transmissions</u>

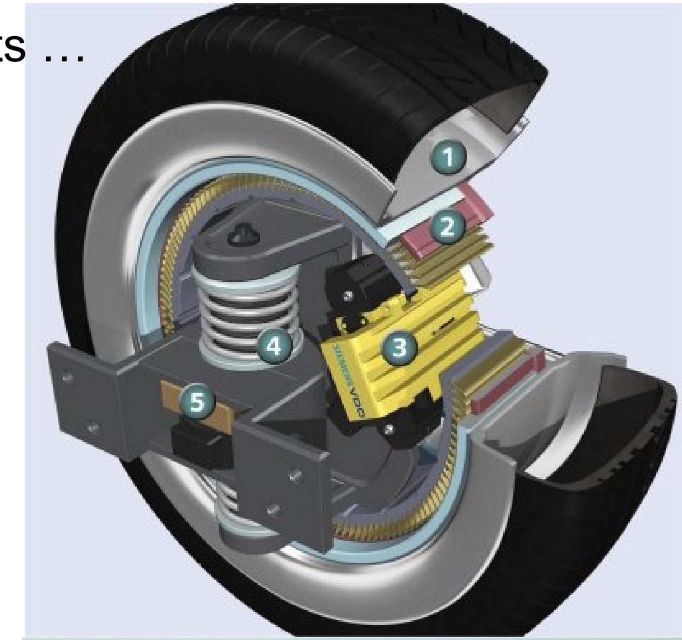
V belts – losses can be as high as 20%, slip, contaminants …

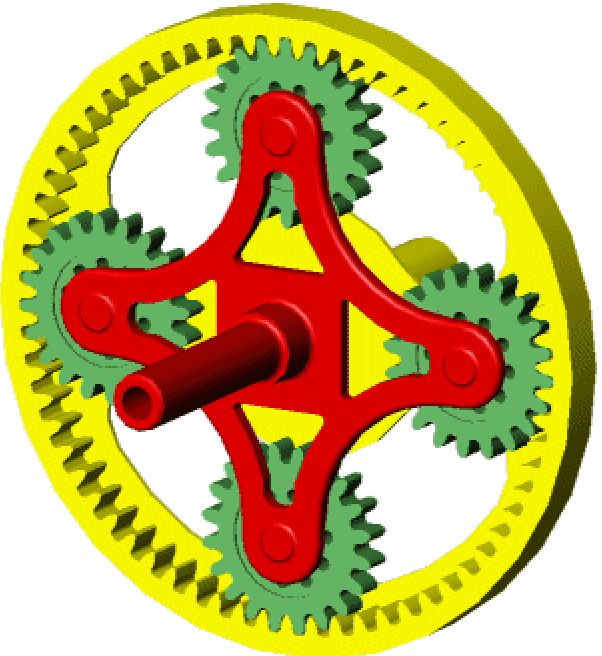
Synchronous (toothed) belts – 98% efficient, noise?

Roller Chains – 98%

Gears – 99%

Direct drive (motor inside the hub) – 100% (?)





Ultracapacitors.

Ultracapacitors (ultracaps) are a variation on a standard capacitor. They use a combination of very large surface area and very small plate spacing to store large amounts of energy. A typical ultracap is rated at a few thousand farads and a couple of volts. The small plate spacing puts great electrical stress on the dielectric material between the plates and this is why their operating voltage is so low.

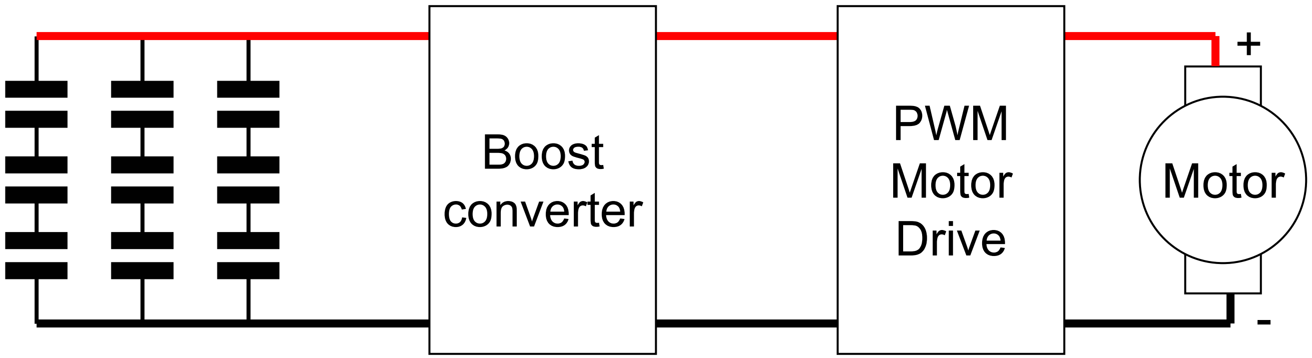
The (dis)charging of a capacitor is an electrical rather than a chemical process and thus they can be cycled many times with almost no deterioration – typically 1,000,000 cycles. Batteries use a (nearly) reversible chemical process to store charge and may manage only a few hundred cycles – less if the battery is abused.

Power density* is the ratio of the power delivery capacity to the weight of the device and a good ultracap may have a power density 10X that of a battery. Even though a battery will generally store more energy in a given volume, for mobile applications an ultracap may be a better solution.

* See slide 14

Charging, discharging and output voltage.

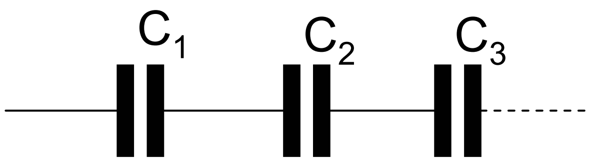
Capacitors and batteries differ greatly in these respects. A battery generally maintains its output voltage at a fairly constant level until it is almost exhausted. The capacitor's output voltage depends on the charge (or energy) stored according to $Q = CV$ (or $E=\frac{1}{2}CV^2$). Thus, whereas batteries can be thought of as a constant voltage supply, capacitors definitely are not. For the capacitor, it will probably be necessary to use a power converter to obtain a stable voltage that can in turn be used as the power source for a PWM motor speed controller.



A second consideration is the rate at which the capacitor can be charged or discharged. Capacitors are used in electronic circuits where they may be cycled many times per second. The limiting factor is often the current which can be allowed to flow without causing heating problems.

Example.

Suppose 10 ultracaps are connected in series. Each is rated at 3000F / 2.5V. If the capacitor string is to be fully charged in 10 seconds, what is the charging current?



$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...} = \frac{1}{\left(\frac{10}{3000}\right)} = 300F \quad (V = 25v)$$

$$I = C\frac{dv}{dt} = 300F \times \frac{25V}{10\sec} = 750A$$

Maximum battery charging currents are of the order of 100X lower than an ultracap for a given energy storage capacity.

A secondary consideration is whether to use constant current (CC) or constant voltage (CV) (dis)charging with the ultracaps. CV charging leads to high initial currents and an exponential waveform shape. CC yields constant current (as the name suggests) and the voltage profile is a uniform ramp (from Q = CV).

Regeneration and kinetic energy.

In a normal vehicle, when the brakes are applied the vehicle's KE is converted into heat. In theory, for an electric vehicle, we could use the motor as a generator and use this to apply a retarding torque (i.e. braking, going down hill, etc.) by converting the KE back to electrical energy. Trains, lifts, etc. often do this – but it only works because there is somewhere to dump large amounts of energy in a short time (the electricity grid). For a battery powered vehicle the battery cannot absorb the surplus electrical energy fast enough without being damaged (unless it is huge). However, if we add some ultracaps to the normal battery circuitry we can use these to absorb the surplus power during braking and then re-use it before drawing on the battery power again.

Example. An electric vehicle has a total mass of 400kg. If the vehicle is to be slowed from 60km/h to 30 km/h in 5 seconds using regenerative braking, what is the charging current and increase in the voltage on an ultracap of value 500F if it is initially at 10V? Assume 90% regeneration efficiency.

$$KE_{60} = \frac{1}{2}mv^2 = 0.5 \times 400 \times \left(\frac{60000}{3600}\right)^2 = 55{,}556J$$

$$KE_{30} = \frac{1}{2}mv^2 = 0.5 \times 400 \times \left(\frac{30000}{3600}\right)^2 = 13{,}8896J \Rightarrow \Delta E = 41667J$$

$$E = \frac{1}{2}CV^2.\ \ At\ V_{Cap(Start)} = 10V, E_{Start} = 0.5 \times 500 \times 10^2 = 25{,}000J$$

$$E_{End} = 25kJ + 41{,}667J \times (\eta =)0.9 = 62{,}500J \Rightarrow V_{Cap(End)} = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2 \times 62{,}500}{500}} = 15.81V$$

$$I = C\frac{dv}{dt} = 500 \times \frac{5.81V}{5} = 581A$$

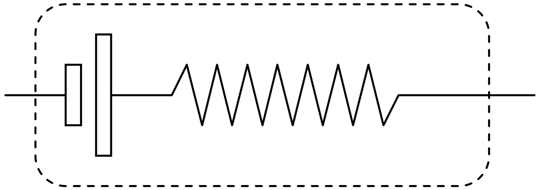
This is a really big current!

… and don't forget the P.E. – the world isn't flat

Rechargeable Batteries (An excellent reference is at http://batteryuniversity.com/index.htm)

Batteries come in all shapes and sizes, from the tiny devices used in earpieces and jewellery to batteries weighing many tonnes used in substations and wind farms for power backup. Rechargeable batteries come in 4 main types: Nickel-cadmium (NiCd), Nickel-metal-hydride (NiMH), Lead-acid and Lithium-ion (Li-ion). The table non page 14 summarises their important properties.

All batteries are DC devices and are modelled as a voltage source (i.e., able to deliver an infinite current) and a series resistor (which limits the current). Each battery is made up of one or more cells and these can be connected in series and parallel combinations.



Summary of characteristics of commonly used rechargeable batteries.

Parameter	Nickel-cadmium	Sealed Lead-acid	Nickel-metal-hydride	Lithium-ion
Energy density* (Wh/kg)	60	40	90	170
Internal resistance (mΩ)	30	15	50	15
Cycle life to 80% capacity	1500	Up to 1000	500	500
Fast charge time (hours)	1	10	3	2
Overcharge tolerance	Moderate	High	Low	None
Self-discharge / month	20%	5%	30%	<10%
Nominal cell voltage	1.2	2.0	1.2	3.6
Load current^ Peak Best	20C 1C	>5C >0.2C	5C 0.5C	>30C <10C
Safety	Thermally stable Use a fuse	Thermally stable Use a fuse	Thermally stable Use a fuse	Stable to ~200°C Use protection circuit

POWER DENSITY is the ratio of the available power from a cell (in Watts) to its volume or weight, measured in kW/l or kW/kg.

ENERGY DENSITY is the ratio of the available energy from a cell (in Watt-hours) to its volume or weight, measured in, e.g., Wh/kg.

E.g. A high performance petrol engine will give a power density of 75kW (100 horsepower) per litre.

For comparison, ultracaps may have a power density of > 21kW/kg but an energy density in the range 4.5→6 Wh/kg. The difference is due to the fact the ultracaps can absorb and release a lot of energy in a short time whereas a battery's charging and discharge rates are relatively slow (measured in minutes & hours, not seconds).

* Petrol has an energy density of 50MJ/kg (~14,000Wh/kg) and hydrogen has 121MJ/kg (~33,600Wh/kg). Dynamite has an energy density of 4.3MJ/kg (11x less than petrol), but the power output of dynamite is much higher because of the speed of combustion.

^ 'C' rating are discussed on slide 18.

The following discussion is for the lead-acid battery. Many of the principles are the same for virtually all battery types but there are a small number of very important differences.

Lead-acid batteries fall into two main categories:
* “SLI” or “starting, lighting and ignition” (i.e., what’s in your car)
* “Industrial” (i.e., what’s used in missiles, electric vehicles, uninterruptible PSUs, etc.)

The SLI and industrial batteries differ primarily in the thickness of the lead plates used in their construction. The former are designed to maximize the internal surface area and hence the current that can be produced. For example a car battery is used in short bursts to deliver high current to the vehicle’s starter motor (“CCA*” = cold cranking amps) and is designed for this purpose. SLIs do not like to be deeply discharged and their performance will be greatly reduced after a few discharge cycles.

The industrial battery is designed to give a lower peak current but to do so over a longer period of time. It achieves this by the use of thicker plates that are less prone to degradation due to deep discharge.

* **CCA** is a measurement of the number of amps a battery can deliver at 0° F for 30 seconds and not drop below 7.2 volts.

CA is cranking amps measured at 32° F. This rating is also called marine cranking amps **(MCA)**.

The “traditional” lead-acid battery used to have a number of chambers that held the liquid electrolyte (sulphuric acid) and care had to be exercised to ensure this didn’t leak. (Minis were notorious for the battery acid corroding the support brackets and dropping the battery out of the car when going over bumps.) These were called “vented” batteries and have largely been superseded by “valve regulated” designs. The valve-regulated lead-acid (VRLA) battery immobilises the electrolyte in a gel by adding a gelling agent or by absorbing the electrolyte in a fine glass mat used as a separator between plates in the cell. The absorbed glass mat VRLA designs are often referred to as AGM batteries.

VRLA batteries can be used in **<u>any</u>** orientation, unlike their vented counterparts. The valve operating pressure is typically in the range 1 -> 3 p.s.i.

Battery ratings

Industrial batteries are normally rated in ampere-hours (Ah) or watt-hours (Wh). This rating depends on some very specific conditions that will be discussed shortly. To convert from Ah or Wh to joules is straightforward:

$$1 \text{ Wh} = 1 \text{ Watt for } 3600 \text{ seconds} = 3600\text{J}$$

For the ampere-hour, we also need to know the nominal voltage of the battery. E.g., a car battery rated at 12V / 70Ah has an energy storage capacity of:

$$70\text{AH} @ 12\text{V} = 12\text{V} \times 70\text{A} \times 3600\text{seconds} = 3{,}024{,}000\text{J}$$

There are several factors that determine how much of the battery's capacity may be usefully used.

1. Discharge rate.

The quantity of energy a battery is able to deliver depends on how quickly the energy is transferred from it. For industrial VRLAs this is typically over a 10 hour period. So if a battery is rated at 100Ah, it is implicit that the battery is able to sustain a 10A output current for 10 hours. If the same battery is fully discharged in 1 hour, the discharge current will be 57A, not 100A. In other words, only 57% of the previous energy is available at the output terminals. Conversely, if the discharge period is longer then a greater Ah "rating" applies – the 100Ah battery discharged in 25 hours yields a discharge current of 4.4A, i.e., a 10% increase over the nominal rating.

A question that naturally arises is, “When is the battery discharged?”

This is determined by the voltage measured at the battery’s terminals during the discharge period. If the discharge period is 1 hour or longer, this is 1.80V per cell. If less than an hour, a value of 1.75V per cell is used. The reason for the difference lies in overexpansion of the cell’s plates, leading to permanent damage if the cell is too heavily discharged. The expansion is reduced if the battery is discharged quickly, hence why the lower voltage is allowed. (Completely discharging a car’s lead-acid battery a few times will effectively make it useless as a battery. Don’t confuse “discharged” with “flat” or “dead”.)

In order to overcome the confusion that can surround the performance of the battery under various conditions a convention to use the “C rate” has emerged. This is a term that clearly describes a cell’s (or battery’s) capacity. It is also used to express the charging and discharge rates of the battery (See battery summary table) In the case of the battery’s capacity, it will be also be followed by a number.

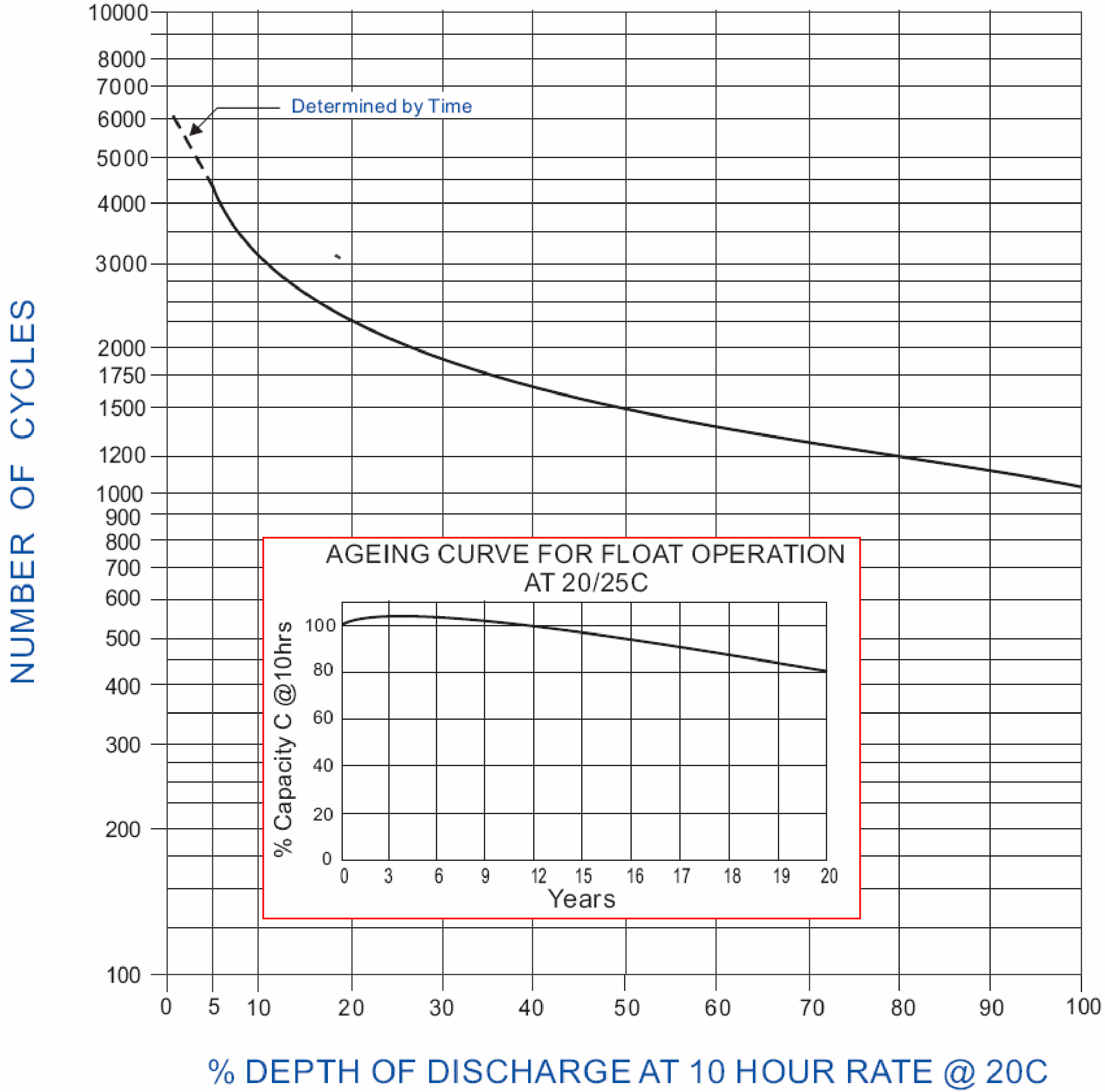
Example

Express the rate of a VRLA battery as a C rate assuming the battery is rated at 100Ah at the 10 hour rate to 1.80V per cell.

This is simply C with the suffix 10, i.e., “C_{10}”.

Similarly the 25-h, 3-h, 1-h and 15-min capacities for the same battery can be expressed as C_{25}, C_3, C_1 and $C_{0.25}$.

Battery life as a function of depth of discharge. (“Deep cycle” batteries)



20 years is an exceptionally long lifetime for a battery. End of life is defined as the point at which the battery can no longer deliver 80% of its rated capacity.

Graphs courtesy of SEC Industrial Battery Company

Example

The C_{10} rating of a battery is 250Ah at the 10-hr rate to 1.80V per cell. The battery is capable of delivering 25A for 10 hours at rated conditions. Express this discharge rate as a function of the C rate.

This is done by dividing the current by the C_{10} capacity in Ah, or 25 / 250 = 0.1. The rate as a C rate is $0.1C_{10}$. [i.e., current as a multiple of the C_{10} rate]

If the same cell was discharged at 400A or 50A, these rates would be $1.6C_{10}$ and $0.2C_{10}$.

Determine the charging current in amps if the battery manufacturer states the charging current should be limited to $0.25C_{10}$, for the 250Ah battery.

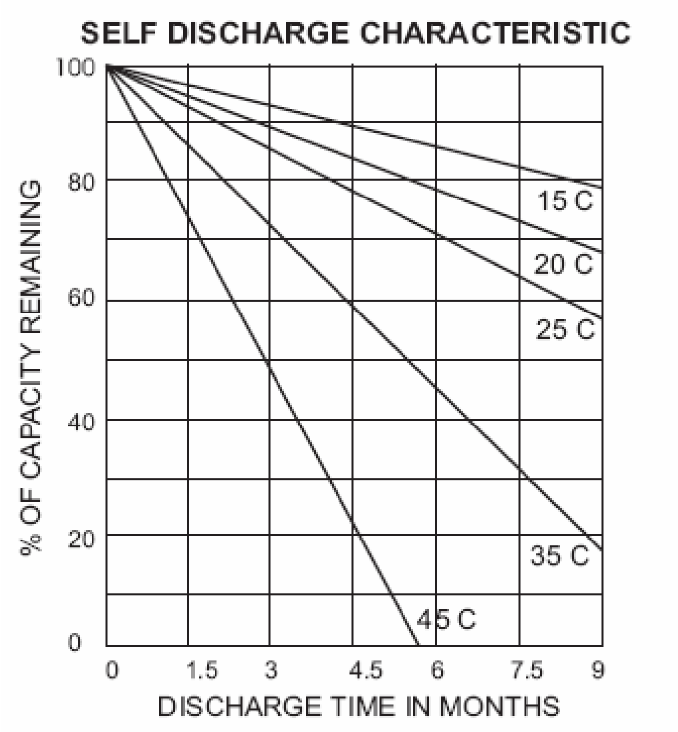
$0.25C_{10}$ = 0.25 x 250 = 62.5A

Application	Back-up time	Capacity range	Nominal voltage	Loads
Telecommunications	3 to 5 hours	3500Ah*	48*	Exchanges etc.
Industrial control	1 to 5 hours	2000Ah	48-240	Relays, SCADA, pumps etc.
Uninterruptible PSUs	5 to 15 mins	5 to 1500kW	120-520	Inverters
Emergency lighting	90 minutes	100Ah	12	Lamps

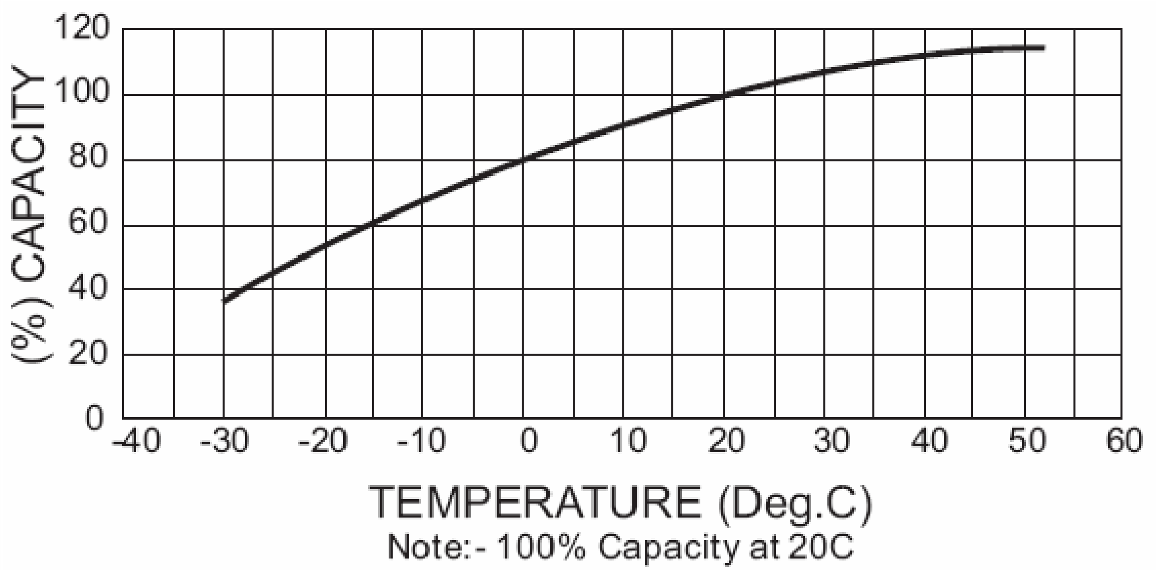
* 3500Ah x 48V = 168000Wh = 4200kg @ 40Wh/kg for lead-acid.

Temperature effects.

A VRLA battery is affected by temperature in a number of different ways. The battery will work over a wide range from below freezing to over 40°C. The nominal working temperature is 20°C, and this is optimal design point, but higher temperatures reduce working life.



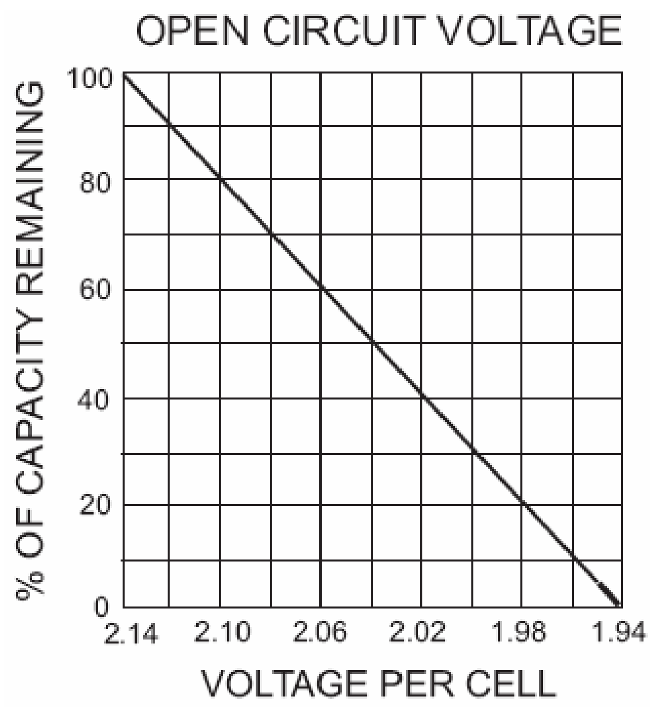
Note 'C' in this diagram is Celsius.



Graphs courtesy of SEC Industrial Battery Company

Variation of output voltage with state of charge

The open circuit (i.e., no load) voltage of a lead-acid cell can be approximated by the equation E_{OC} = 0.86 + specific gravity. When fully charged, the specific gravity varies from 1.215 to 1.280, giving a maximum open circuit voltage of between 2.05 and 2.14V. As the cell is discharged the specific gravity falls approximately linearly to slightly over 1.1.



Charging voltage.

Constant voltage charging is recommended for VRLAs. The level to be used depends on the application (float or cycle) and the temperature.

For float service, use 2.25V ± 1% @ 20°C per cell.

For cycle service, use 2.35V ± 1% @ 20°C per cell.

For temperatures other than 20°C use a correction factor of -0.003V per cell per °C.

So at 35°C the charging voltage should be reduced to 2.205V per cell for float applications.

Graph courtesy of SEC Industrial Battery Company

Three step battery charging.

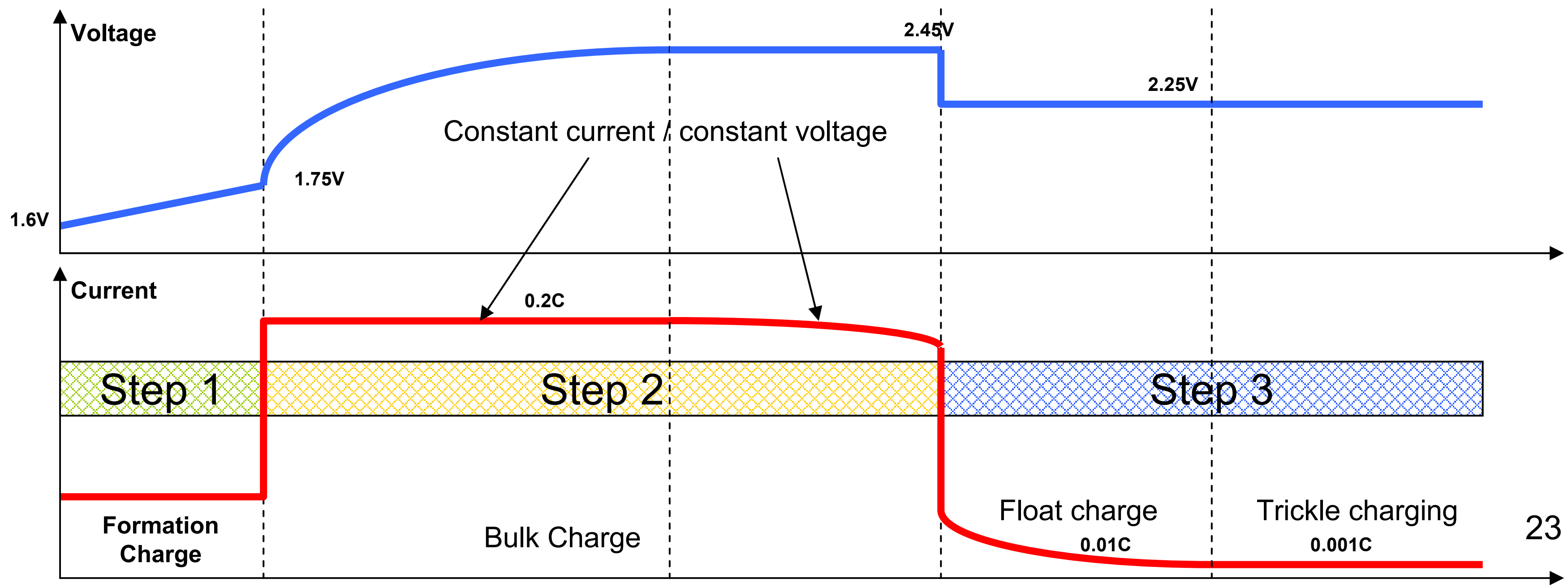
Cheap battery chargers can do more harm than good. It is worth spending a few extra £££ to get what's called a SMART CHARGER that adapts the way in which the battery is charged according to how much charge the battery has in it and the ambient temperature. "Proper" (as opposed to "fast") charging a lead-acid battery will take between 12 and 36 hours.

Step 1: If deeply discharged, use of a "forming current" in constant current mode until 1.75V per cell is reached.

Step 2: Bulk charging in constant current mode @ ~0.2C until 2.45V per cell is reached (about 80% charged)

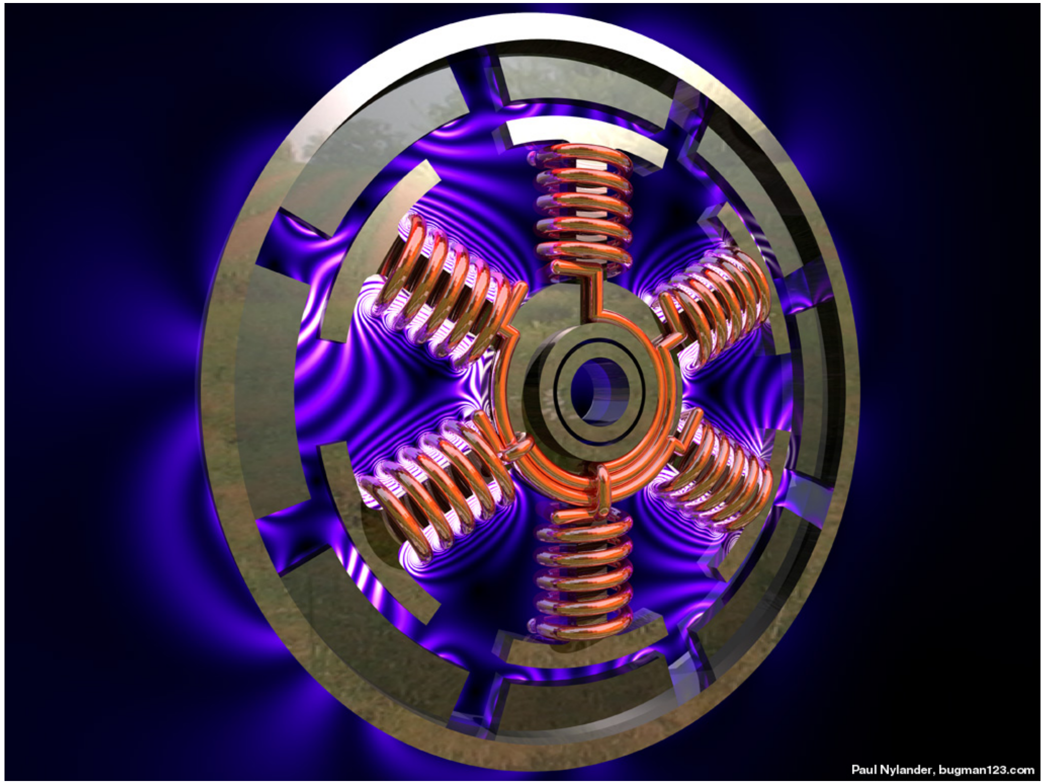
Step 3: Charging in constant voltage mode at 2.25V per cell. About 90% charged after 5 hours. When the current falls to below 0.01C the battery is fully charged. Thereafter, "trickle charging" at 2.25V can be maintained indefinitely. The current will gradually drop to about 0.001C – the "maintenance" level of the battery.

Different manufacturers make different claims about why their particular charger is "best", but they all basically follow the curves shown below. The threshold for transition from step to step will vary.



Electric Motors.

Think about the awful life the vehicle's motor will have to endure: getting hot, cold, dirty, overloaded, mechanically shocked, flooded and poor or no maintenance. The AC induction motor is just about the simplest, most-rugged design in existence so it suits the application very well. However, a Brushless DC (BLDC) motor is slightly more efficient. Both an induction motor and a BLDC motor need complex high-powered drive electronics to synthesize the power waveform for variable speed use. Design issues determine using the BLDC or the induction motor.



6 pole rotor / 8 pole stator BLDC

BLDC isn't really an accurate description because they need single or multiphase AC input power.
The electronics contains several subsystems:
* Bulk power storage in capacitors & batteries;
* A shaft encoder or EMF monitor and digital signal processing to track the rotation of the rotor;
* A power-stage using IGBTs or FREDFETs that inverts the bulk-DC into three-phase AC;
* An internal or external permanent magnet rotor.

These motors are used because they provide several advantages over the induction motor:
* Lowered power consumption;
* Simple variable-speed operation (which reduces the power consumption further);
* The possibility of direct-drive (rather than using belts or gears.

Motor Comparison table

Type	Advantages	Disadvantages	Applications	Drive
Induction	Cheap High power High starting torque Long life Low maintenance	Asynchronous (Slip) Can be heavy	Most things (inc. electric vehicles)	Needs drive for variable speed
Synchronous	High power Long life No slip	Low starting torque More costly	Things that depend on known RPM (e.g. mains clocks)	Needs drive for variable speed
Stepper	Precision positioning High holding torque	Needs controller Relatively low power	Positioning (e.g. printer, robotics, etc.)	Multiphase DC
Brushless DC	Long life Low maintenance High efficiency Lower weight Direct drive possible	Higher cost Needs controller	Electric vehicles Disk drives	Multiphase DC. Needs drive for variable speed.
Brushed DC / Universal	Cheap Simple speed control	Low lifetime High maintenance (brushes)	Starter motors Power tools	Direct (PWM for variable speed)

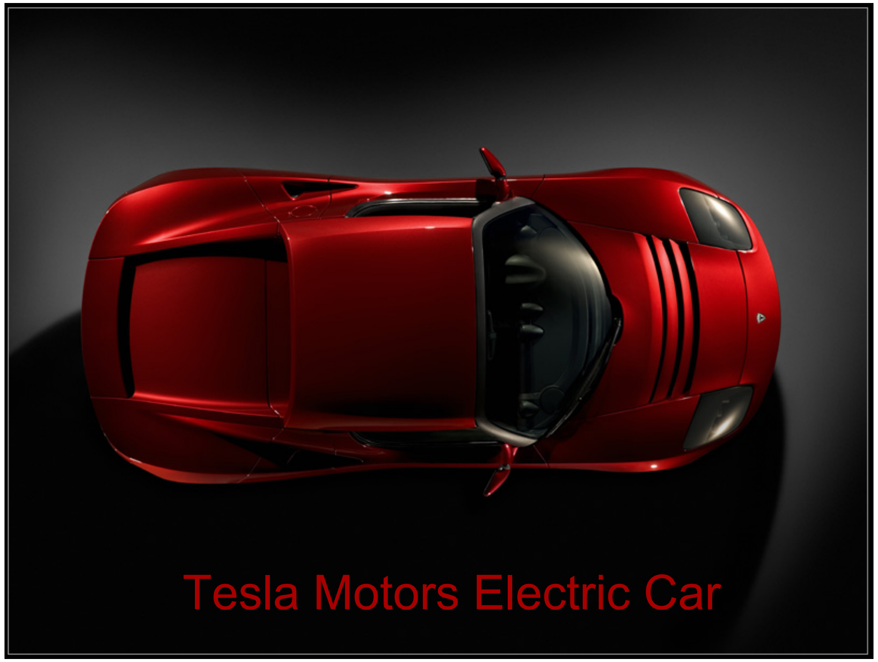
A small electric vehicle motor & drive electronics can probably manage >90% efficiency

Going Green …

Let’s suppose we’d like to use a domestic wind turbine to recharge our electric car.

Tesla Roadster Electric Car specs

ENGINE:	375v AC air-cooled induction motor with variable frequency drive. (115 lbs / 53 kg)
BATTERY:	Lithium-ion unit (992 lbs / 450kg) with 6,831 individual cells (50:50 Water / glycol cooled) ‘18650’ (AA) form factor cells
BATTERY CAPACITY:	140Ah / 375V / 53kWh. (Same as ~6 litres of petrol*)
POWER:	185kW / 248bhp @ 8000rpm
TORQUE:	276 lbft (375Nm) @ 0-4500rpm
REDLINE:	14,000 rpm
TRANSMISSION:	Two-speed manual
ACCELERATION:	0-60mph: 3.9sec
EFFICIENCY:	92% average / 85% @ peak power.
POWER CONSUMPTION:	110Wh/km / 2.18km/MJ
TOP SPEED:	130mph
PRICE:	$109,000 (£90,000)
ROAD TAX:	BAND A (free)


Tesla Motors Electric Car

Windsave (“B & Q”) 1000 wind turbine.
Rotor diameter = 1.75m

Average wind speed	Power Generated Annually (From HTTP://REUK.CO.UK)	Car recharge time
2ms^{-1}	6kWh	8 years 10 months
3ms^{-1} } Typical urban values	119kWh	163 days
4ms^{-1} } Typical urban values	443kWh	44 days
5ms^{-1}	990kWh	20 days

* But a petrol-engined car is only about 25% efficient. 53 kWh @ 15 pence per unit = £7.95.

Example.

a) An electric vehicle uses a 60V Li-ion battery with a C_1 capacity 300Ah. How much energy is stored in the battery when it is fully charged, and what is the maximum C_1 discharge current?

b) Using the data in the table, what is maximum range of the vehicle at 90km/h on a flat road with no headwind? By how much does the range change if there is a 5m/s headwind?

c) What is meant by regenerative braking? If the vehicle is slowed from 90km/h to 40km/h in 5 seconds using regenerative braking, what is the average current produced assuming the voltage is held constant at 60V? Neglect friction and drag and assume an electrical efficiency of 90%.

Parameter	Value
Mass of vehicle	1200kg
Rolling resistance coefficient	0.025
Density of air	1.293kg/m^3
Drag coefficient	0.3
Frontal area of vehicle	1.75m^2
Drive train efficiency	90%
'g'	10m/s^2

Solutions.

a) Energy in the battery is V x I x t = 60x300x3600 = **64.8MJ**

The C_1 rate is 300Ah / 1 hour = **300A.**

b) Rolling resistance: Calculate force $F_{rr} = C_{rr} \times F_n$ = 0.025 x 12000N = 300N (retarding)

Drag: $F = -½\rho v^2AC_d$ = -0.5 x 1.293 x 1.75 x 0.3 x v^2 = $0.339v^2$ (retarding)

@ 90km/h, v = 90,000 / 3,600 = 25m/s.

Power = force x velocity; for rolling resistance, power = 300N x 25m/s = **7500W**

For drag, power = $0.399v^2 \times v = 0.339 \times 25^3$ = **5297W** => total power = **12800W** approx.

Drive train is 90% efficient, so input power is 12800 / 0.9 = **14222W**

Energy = Power x time => t = E / P = 64.8MJ / 14.222kW = **4556 seconds**.

Distance = speed x time = 90km/h x 4556 / 3600 seconds = **113.9km**.

With a headwind, the rolling resistance is unchanged but the drag increases. The new drag velocity is 30m/s and the power required to overcome it is 0.339 x 30^3 = 9153W. Use the same method as before to find total power = 16.65kW, t = 3503 seconds, distance = 87.6km.

Hence the range is reduced by **26.3km**.

c) Regenerative braking is the conversion of the vehicle's kinetic energy back into electrical energy.

First, find the kinetic energy at 90km/h and 40km/h: K.E. = ½mv^2.

KE_{90} = 0.5 x 1200 x 25^2 = 375,000J. KE_{40} = 0.5 x 1200 x 11.1^2 = 74074J (40km/h = 11.1m/s)

=> Energy “lost” is KE_{90} – KE_{40} = 375,000J – 74,074J = 300,926J.

Drive train is 90% efficient, so 30,093J lost in transmission.

Electrical conversion efficiency also 90% => energy left = 300,926J x 0.9 x 0.9 = 243,750J.

Deceleration in 5 seconds so joules per second from regeneration is 243,750 / 5 = 48750W.

Voltage held constant @ 60V, so average current = Power / Voltage = 48750 x 60 = **812.5A**.

The current is well over the battery’s C1 rate, so in practise the vehicle would put 300A back into the battery and use the remainder to charge up a bank of ultracapacitors.