Power Electronics 2 (ENG2045, SIT2004)
Power dissipation in common components

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1 Fundamentals

The basic expression for instantaneous power is $p(t) = v(t)i(t)$ so the average power absorbed by (dissipated in) a component over time T is

$$P_{\text{ave}} = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t = \frac{1}{T}\int_0^T v(t)\,i(t)\,\mathrm{d}t. \tag{1}$$

In general you have to do this integral, analytically or numerically. Fortunately it can be simplified in many common situations.

2 Steady state DC

If the current and voltage are constant in time, which means pure DC, then

$$P_{\text{ave}} = V_{\text{DC}} I_{\text{DC}}\,. \tag{2}$$

This applies to a circuit fed from a DC supply after it has reached a steady state and all transients have died away.

3 Resistive load, including MOSFET devices

If the load is resistive then $v(t) = R\,i(t)$ and we can simplify equation (1) to

$$P_{\text{ave}} = R\left\{\frac{1}{T}\int_0^T [i(t)]^2\,\mathrm{d}t\right\} \equiv R I_{\text{rms}}^2 \tag{3}$$

or

$$P_{\text{ave}} = \frac{1}{R}\left\{\frac{1}{T}\int_0^T [v(t)]^2\,\mathrm{d}t\right\} \equiv \frac{V_{\text{rms}}^2}{R}. \tag{4}$$

The expressions with root-mean-square (rms) values look pretty but are only a shorthand way of writing the integral. You still have to do the integral to find the rms values. The only special case that you are expected to know by heart is $V_{\text{rms}} = V_{\text{pk}}/\sqrt{2}$ for a sine wave, where V_{pk} is the peak value. Here are a few tips.

- The rms value is an integral over $[v(t)]^2$ so it does not depend on the sign of $v(t)$. A full-wave rectified wave has the same rms value as the unrectified wave.
- You can often change the sign of different parts of $v(t)$ to give something whose rms value is obvious, or at least easier to find. This is useful for square waves.
- Many common waves, such as triangular waves, are piecewise continuous, which means that you need different mathematical expressions for $v(t)$ in different intervals of time.
- Remember the geometric interpretation of an integral as the area under a curve.
- Symmetry is often helpful to reduce the range of integration. Often you need only calculate half a cycle, sometimes less.
- The integral should be taken over one full period but it needn't be from 0 to T, although we usually write that down. It could be from $-T/2$ to $+T/2$ or any other interval of T that makes the calculation simpler.
- The rms value must be positive (or zero); something has gone badly wrong if you find a negative value!

You are expected to be able to calculate rms values of common functions from first principles. This includes power-law, trigonometric and exponential functions. For example, the current through the switch in a switch-mode power supply rises linearly from I_{on} at t_{on} to I_{off} at t_{off} and drops to zero for the remainder of the cycle. You need to be able to find an expression for $i(t)$ and calculate its rms value by integration. Do not attempt to memorise expressions for this sort of result; you will get no credit for it.

The channel of a MOSFET behaves like a resistor $R_{\text{DS,on}}$ when it is turned on so these expressions can be used to calculate the power dissipated in the MOSFET. You need to be a little bit careful because the model of a resistor applies only when the MOSFET is turned on. However, no current flows through the MOSFET when it is turned off so this shouldn't be a problem. You can just calculate I_{rms} following the tips above.

4 Steady state AC (not part of this course)

If the current and voltage are sine waves in a steady state, then

$$P_{\text{ave}} = V_{\text{rms}} I_{\text{rms}} \cos\phi, \tag{5}$$

where ϕ is the difference in phase between the voltage and current, and $\cos\phi$ is called the power factor. There are two special cases.

- For a DC circuit, $\phi = 0$, $\cos\phi = 1$ and $P_{\text{ave}} = V_{\text{rms}} I_{\text{rms}}$.
- For an inductor or capacitor, $\phi = \pm 90°$, $\cos\phi = 0$ and $P_{\text{ave}} = 0$. These components store and release energy but do not dissipate it.

5 Load with constant voltage, typically bipolar semiconductor devices

Suppose that the voltage across a component is constant at V_{DC}. Then we can simplify equation (1) to

$$P_{\mathrm{ave}} = \frac{1}{T}\int_0^T v(t)\, i(t)\, \mathrm{d}t = V_{\mathrm{DC}} \left\{ \frac{1}{T}\int_0^T i(t)\, \mathrm{d}t \right\} \equiv V_{\mathrm{DC}} I_{\mathrm{ave}} \,. \tag{6}$$

This has the simple average of the current, not the rms value.

The main application of this result is to semiconductor devices that have a roughly constant voltage across them when they are conducting. These include diodes, SCRs, TRIACs, bipolar junction transistors (BJTs) and insulated-gate bipolar transistors (IGBTs). You need to be careful to use the equation only for the times when the device is conducting, when a diode is forward biassed for example. Usually this is not a problem because no current flows through the device when it is turned off.

TRIACs are a little tricky because they conduct in both directions. However, the sign of the voltage across the TRIAC also changes when the current changes direction so the product $v(t)i(t)$ is always positive. It is easiest to handle this by taking the integral over the positive half-cycle rather than the full cycle because the negative half-cycle must give the same result.

6 Load with constant current, such as a linear regulator

This is similar to the previous section but the current is constant instead of the voltage. It is less common but a linear regulator is a good example. The current is constant because it is determined by the load, which sees a constant voltage from a regulator. The voltage across the regulator usually fluctuates because of the ripple voltage from the rectifier and smoothing capacitor. The average power is given by

$$P_{\mathrm{ave}} = I_{\mathrm{DC}} V_{\mathrm{ave}} \,. \tag{7}$$

7 Why do some results have the average current and some the rms current?

This is because semiconductor devices behave in different ways. (Electronics students should know this and other students need not worry.)

- The channel of a MOSFET behaves like a resistance with a low value when it is turned on. In other words, the voltage across the channel is proportional to the current that flows through it. The power dissipated is given by the usual expression for a resistance, which depends on the rms value of current.

- Other devices, such as diodes, behave differently and have a roughly constant voltage across them, which does not alter very much as the current changes. We model these with a constant voltage, which means that the power depends on the average current instead of the rms value.

These are only rough models but are good enough for estimating the power dissipation. A better calculation would need numerical methods; SPICE can do this, for example.