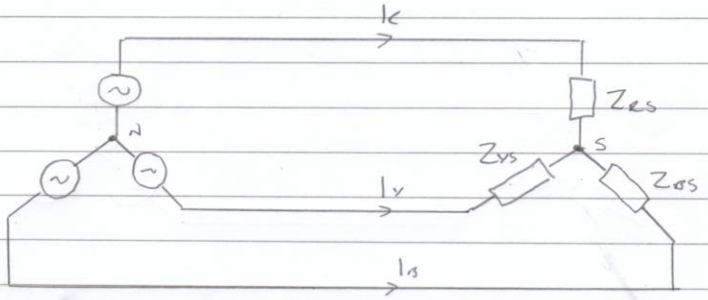
Power Eng 3

Tutorial 2 solutions

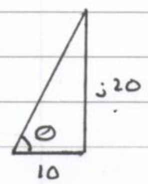
POWER ENG 3

TUTORIAL 2: 3 PHASE SYSTEMS

Q1



$$Z_{RS} = Z_{YS} = Z_{BS} = 10 + j20$$



$$\Rightarrow Z_{RS} \text{ etc} = 22.4 \angle 63.4^\circ$$

$$V_{RN} = 10{,}000 \angle 0^\circ$$

$$\Rightarrow \bar{I}_R = \frac{\bar{V}_{RN}}{\bar{Z}_{RS}} = \frac{10{,}000 \angle 0^\circ}{22.4 \angle 63.4^\circ} = 446 \angle -63.4^\circ$$

$$\Rightarrow \bar{I}_Y = 446 \angle (-63.4 - 120) = 446 \angle -183^\circ$$

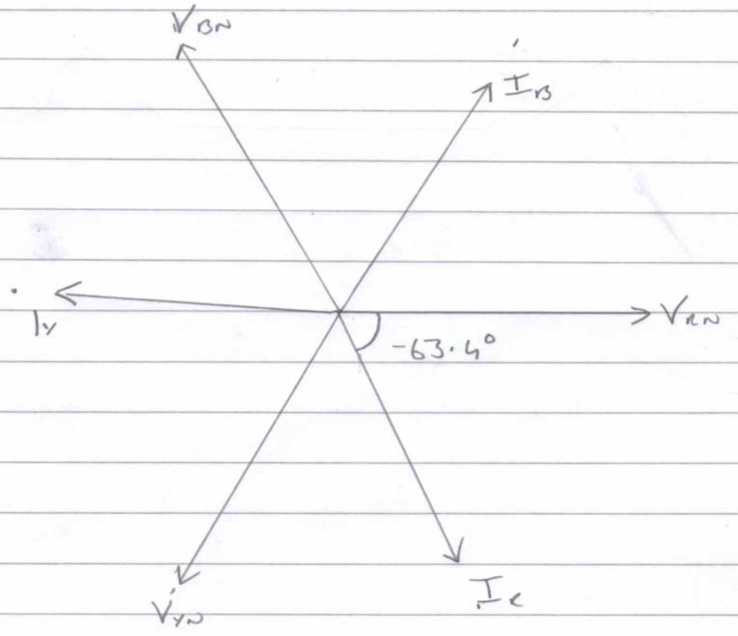
$$\Rightarrow \bar{I}_B = 446 \angle (-63.4 + 120) = 446 \angle 56.6^\circ$$

$$P_T = 3 |V_{RN}| |I_R| \cos \phi$$

$$\Rightarrow P_T = 3 \times 10{,}000 \times 446 \times \cos(-63.4)$$

$$\Rightarrow P_T = \underline{\underline{5.99 \text{ MW}}}$$

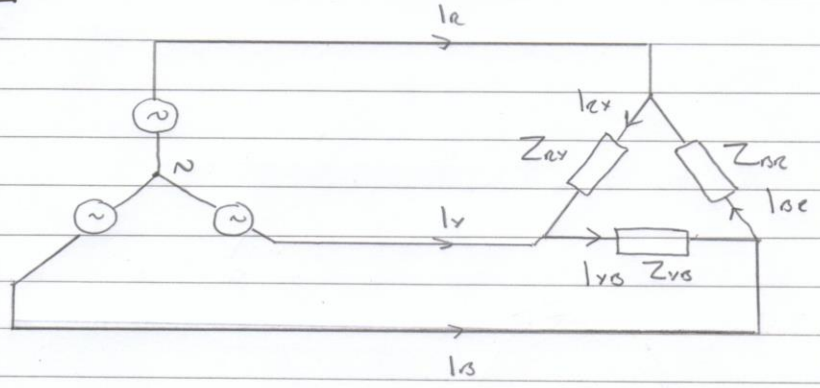
Q1 Phasor Diagram:



Scale: 1cm ≡ 2000 V

1cm ≡ 100 A

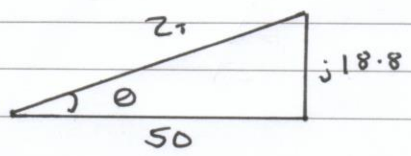
Q2



$$Z_{RY} = Z_{YB} = Z_{BR} = R + jX_L$$

$$X_L = \omega L = 2\pi \times 50 \times 60 \times 10^{-3} = 18.8\,\Omega$$

$$\Rightarrow Z_{RY} \text{ etc} = 50 + j18.8 = 53.4\angle 20.6^\circ$$



$$\Rightarrow \bar{I}_{RY} = \frac{\bar{V}_{RY}}{Z_{RY}} = \frac{440\angle 30^\circ}{53.4\angle 20.6^\circ} = 8.24\angle 9.4^\circ$$

$$\Rightarrow \bar{I}_{YB} = 8.24\angle(9.4-120) = 8.24\angle -110.6^\circ$$

$$\Rightarrow \bar{I}_{BR} = 8.24\angle(9.4+120) = 8.24\angle 129.4^\circ$$

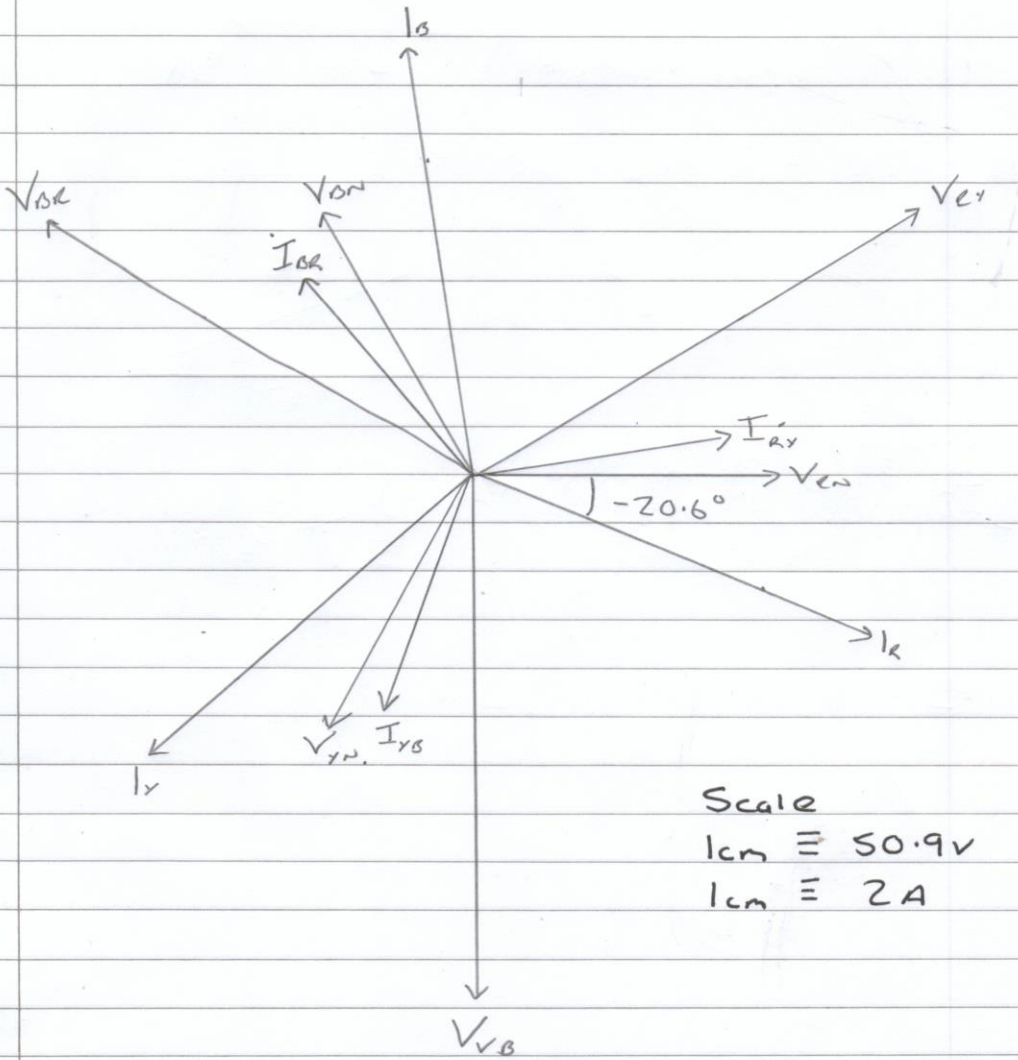
Line Currents (I_R etc) are $\sqrt{3}$ × Load Phase Currents (I_{RY} etc) and lag by 30°

$$\bar{I}_R = \sqrt{3} \times 8.24 \angle (9.4 - 30) = 14.2 \angle -20.6^\circ$$

$$\bar{I}_Y = 14.2 \angle (-20.6 - 120) = 14.2 \angle -140.6^\circ$$

$$\bar{I}_B = 14.2 \angle (-20.6 + 120) = 14.2 \angle 99.4^\circ$$

(Note alternatively I_R etc could be determined from scaled phasor diagram but for BALANCED case I would suggest doing the above)



Scale
1cm ≡ 50.9V
1cm ≡ 2A

(hint: Always make V_{RN} etc 5cm + V_{RY} etc 8.6cm)

APPARENT POWER (S) $= |V_{cN}||I_c|$

$= \frac{440}{\sqrt{3}} \times 14.2$

$= 254 \times 14.2$

$= \underline{\underline{3.6\ kVA}}$

REAL POWER (P) $= S.\cos\phi \leftarrow$ Angle between V_{cN} + I_c.

$= 3600 \times \cos(-20.6)$

$= \underline{\underline{3.37\ kW}}$

REACTIVE POWER (Q) $= -S.\sin\phi$

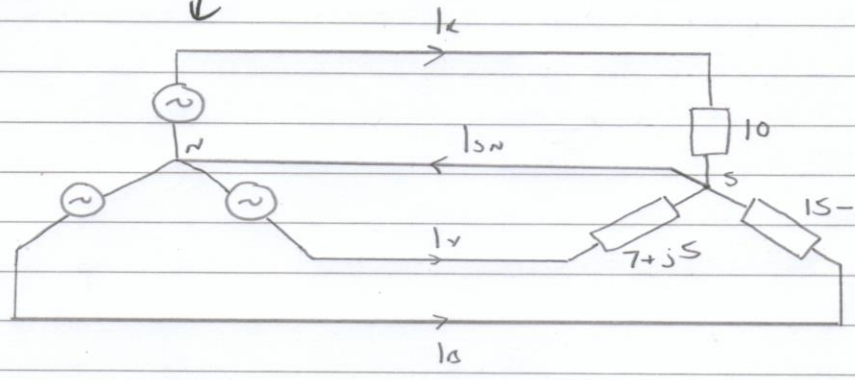
$= -3600 \times \sin(-20.6)$

$= \underline{\underline{1.27\ kVAr}}$

(note these are per phase values)

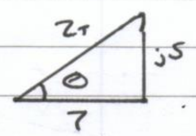
Q3

Phase Voltage $= \dfrac{400}{\sqrt{3}} = 231\,v$

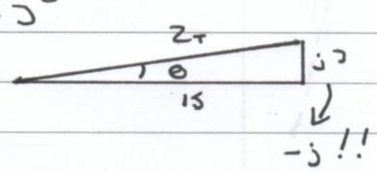


$Z_{RS} = 10 \equiv 10\angle 0°$

$Z_{YS} = 7 + j5 \equiv 8.6\angle 35.5°$



$Z_{BS} = 15 - j3 \equiv 15.3\angle -11.3°$



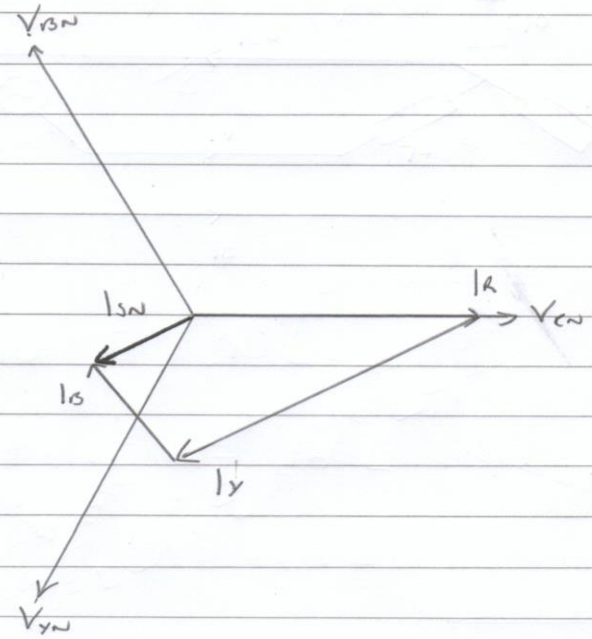
$$\bar{I}_R = \frac{\bar{V}_{RN}}{Z_{RS}} = \frac{231\angle 0°}{10\angle 0°} = 23.1\angle 0°$$

$$\bar{I}_Y = \frac{\bar{V}_{YN}}{Z_{YS}} = \frac{231\angle -120°}{8.6\angle 35.5°} = 26.8\angle -155°$$

$$\bar{I}_B = \frac{\bar{V}_{BN}}{Z_{BS}} = \frac{231\angle 120°}{15.3\angle -11.3°} = 15\angle 131°$$

Now draw Scaled Phasor Diagram

Scale
1cm ≡ 5A



measured I_{SN} = 1.8cm ≡ 9A

Based on 5A/mm² Current Density

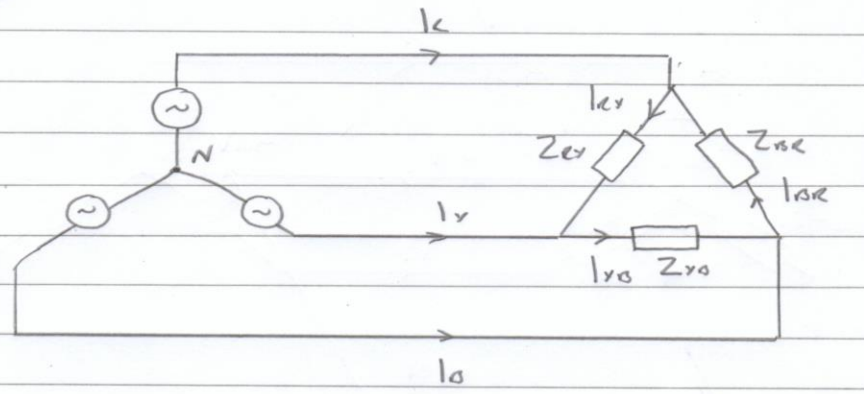
$\frac{9}{5} = 1.8\ mm^2$ Cross Sectional Area of Wire

$A = \pi r^2$

$\Rightarrow r = \sqrt{\left(\frac{A}{\pi}\right)} = \sqrt{\left(\frac{1.8}{\pi}\right)} = 0.76\ mm$

⇒ Wire Diameter = <u>1.5 mm</u>

Q4



V_{RY} etc $=$ 400 V

$Z_{RY} = 10\angle 0^\circ = 10 + j0$

$Z_{YB} = 8.6\angle 35.5^\circ = 7 + j5$

$Z_{BR} = 15.3\angle -11.3^\circ = 15 - j3$

$$\bar{I}_{RY} = \frac{\bar{V}_{RY}}{\bar{Z}_{RY}} = \frac{400\angle 30^\circ}{10\angle 0^\circ} = 40\angle 30^\circ$$

$$\bar{I}_{YB} = \frac{\bar{V}_{YB}}{\bar{Z}_{YB}} = \frac{400\angle -90^\circ}{8.6\angle 35.5^\circ} = 46.5\angle -125^\circ$$

$$\bar{I}_{BR} = \frac{\bar{V}_{BR}}{\bar{Z}_{BR}} = \frac{400\angle 150^\circ}{15.3\angle -11.3^\circ} = 26.1\angle 161^\circ$$

Knowing the load phase currents we can determine the total load power (W)

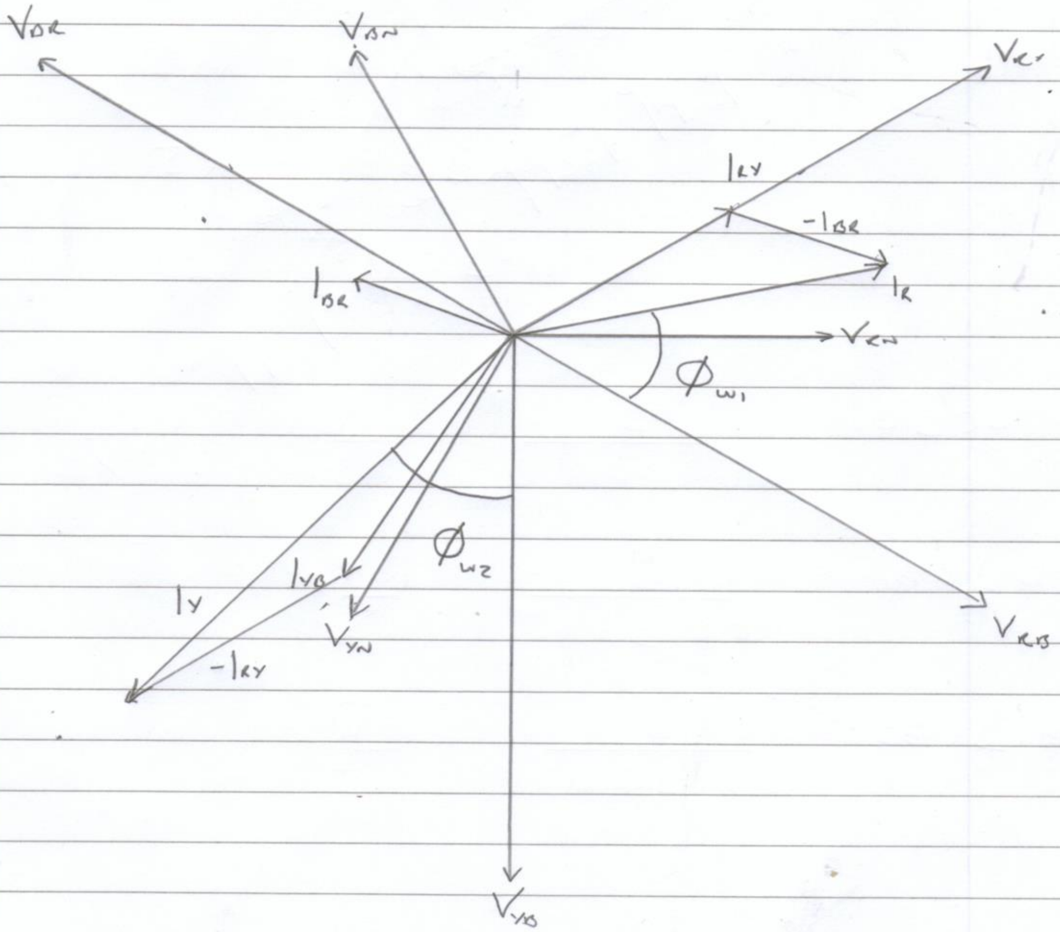
$$P_T = |I_{RY}|^2 . R_{RY} + |I_{YB}|^2 . R_{YB} + |I_{BR}|^2 . R_{BR}$$

$\Rightarrow P_T = 40^2 \times 10 + 46.5^2 \times 7 + 26.1^2 \times 15$

$\Rightarrow P_T = 16{,}000 + 15{,}100 + 10{,}200$

$\Rightarrow P_T = \underline{\underline{41.3\,kW}}$

Now draw Phasor Diagram to determine line Currents



Scale

1cm ≡ 10A

$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$

$I_Y = \bar{I}_{YB} - \bar{I}_{RY}$

2 WATTMETER METHOD:

$$W_1 = |V_{RO}|.|I_R|. \cos \phi_{W1}$$

angle between V_{RO} + I_R

$$W_2 = |V_{YO}|.|I_Y|. \cos \phi_{W2}$$

angle between V_{YO} + I_Y

measured from Phasor Diagram:

$|I_R| = 6cm \equiv 60A$

$|I_Y| = 8.4cm \equiv 84A$

$\phi_{W1} = 40°$

$\phi_{W2} = 46°$

$\Rightarrow W_1 = 400 \times 60 \times \cos 40°$
$\Rightarrow W_1 = 18.4 \text{ kW}$

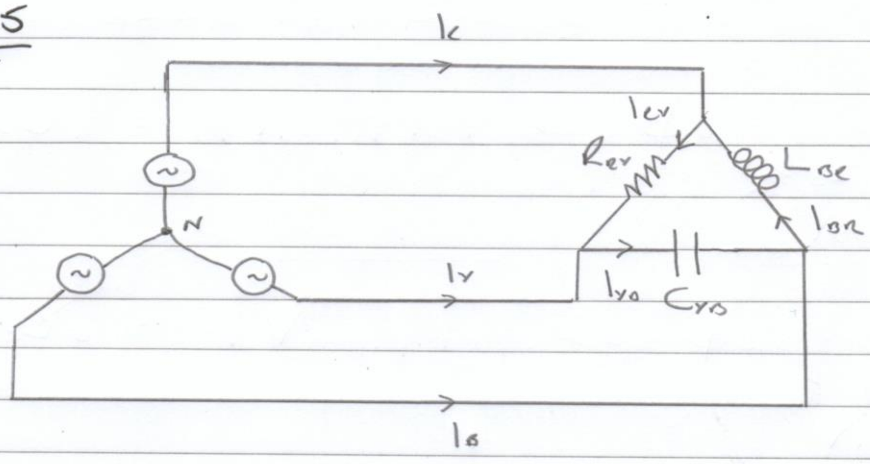
$\Rightarrow W_2 = 400 \times 84 \times \cos 46°$
$\Rightarrow W_2 = 23.3 \text{ kW}$

$\Rightarrow P_T = W_1 + W_2$

$\Rightarrow P_T = 18.4\text{kW} + 23.3\text{kW}$

$\Rightarrow P_T = \underline{\underline{41.7 \text{ kW}}}$ ✓ Allowing for estimation errors

Q5



$$X_C = \sqrt{3}.\ R = \sqrt{3} \times 0.3 = 0.52\ \Omega$$

$$X_L = \sqrt{3}.\ R = \sqrt{3} \times 0.3 = 0.52\ \Omega$$

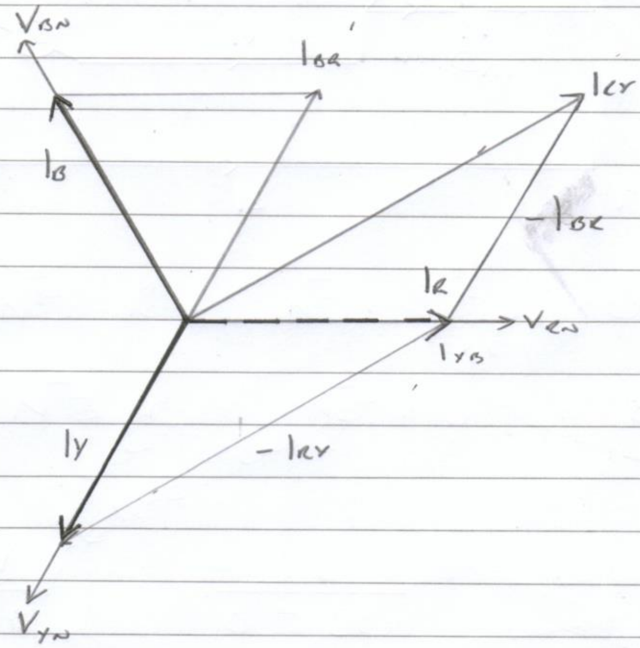
$$\bar{I}_{RY} = \frac{\bar{V}_{RY}}{R_{RY}} = \frac{415 \angle 30^\circ}{0.3 \angle 0^\circ} = 1380 \angle 30^\circ$$

$$\bar{I}_{YB} = \frac{\bar{V}_{YB}}{C_{YB}} = \frac{415 \angle -90^\circ}{0.52 \angle -90^\circ} = 798 \angle 0^\circ$$

$$\bar{I}_{BR} = \frac{\bar{V}_{BR}}{L_{BR}} = \frac{415 \angle 150^\circ}{0.52 \angle 90^\circ} = 798 \angle 60^\circ$$

Now draw scaled phasor diagram to determine line currents I_R, I_Y + I_B

Scale: 1cm ≡ 200 A



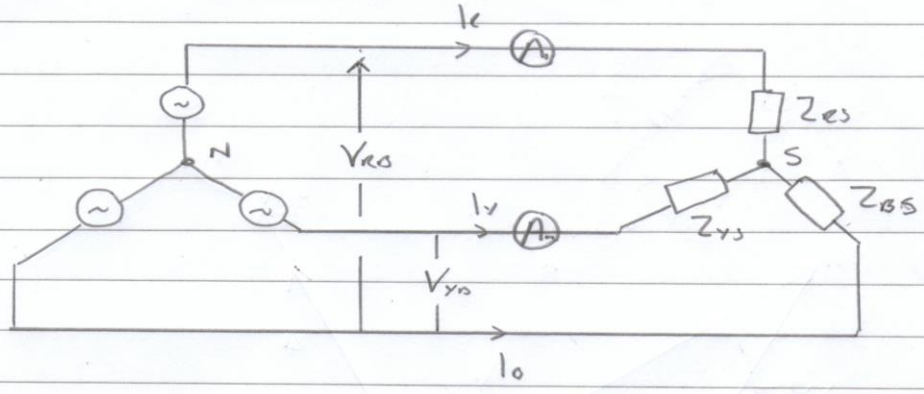
Measured from Phasor Diagram:

$\bar{I}_R = 800\angle 0°$

$\bar{I}_Y = 800\angle -120°$

$\bar{I}_B = 800\angle 120°$

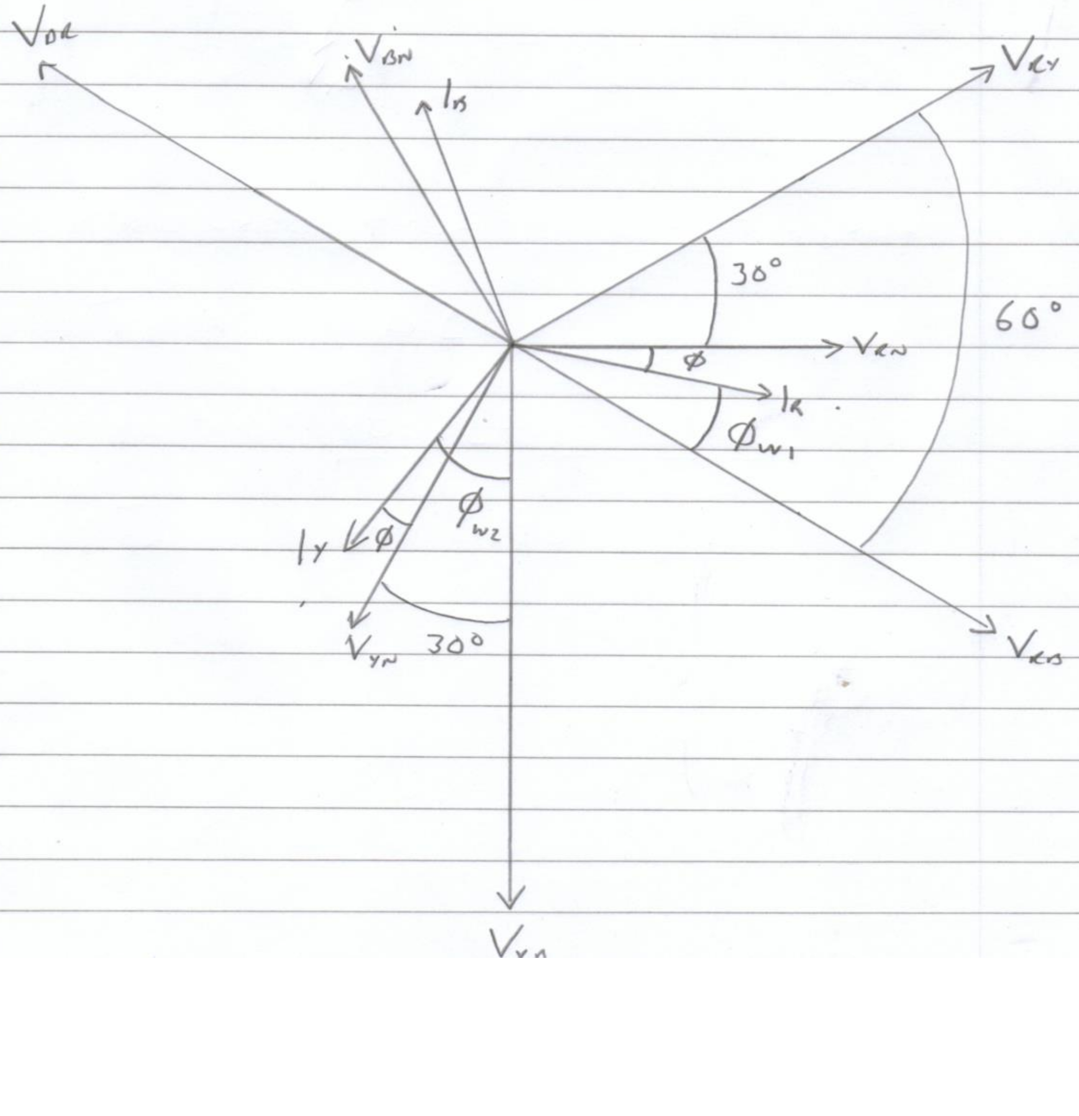
Q6



2 WATTMETER METHOD

$W_1 \rightarrow V_{RB} + I_R + \cos \phi_{W1}$

$W_2 \rightarrow V_{YB} + I_Y + \cos \phi_{W2}$



Prove that Total Real Power

$$P_T = 3.|V_{RN}|.|I_R|. \cos\phi$$

$|V_{RN}| = V_{ph}$, $|I_R| = I_L$, ϕ angle between V_{RN} + I_R

from Phasor Diagram:

$$\phi_{w1} + \phi + 30° = 60°$$

$$\Rightarrow \phi_{w1} = 30° - \phi \quad — \quad (1)$$

$$\phi_{w2} = 30° + \phi \quad — \quad (2)$$

$$W_1 = |V_{RB}|.|I_R|. \cos\phi_{w1}$$

$$\Rightarrow W_1 = |V_L|.|I_L|. \cos(30° - \phi)$$

$$W_2 = |V_{YB}|.|I_Y|. \cos\phi_{w2}$$

$$\Rightarrow W_2 = |V_L|.|I_L|. \cos(30° + \phi)$$

$$P_T = W_1 + W_2$$

$$\Rightarrow P_T = |V_L|.|I_L|. \cos(30° - \phi) + |V_L|.|I_L|. \cos(30° + \phi)$$

Subst

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\Rightarrow P_T = |V_L|.|I_L|.\left(\cos 30^\circ \cos\phi + \cos 30^\circ \cos\phi\right)$$

$$\Rightarrow P_T = |V_L|.|I_L|.\left(0{\cdot}866 \cos\phi + 0{\cdot}866 \cos\phi\right)$$

$$\Rightarrow P_T = \sqrt{3}.|V_L|.|I_L| \cos\phi$$

Subst $V_L = \sqrt{3}.V_{ph}$

$$\Rightarrow P_T = \underline{\underline{3|V_{ph}|.|I_A|.\cos\phi}}$$