



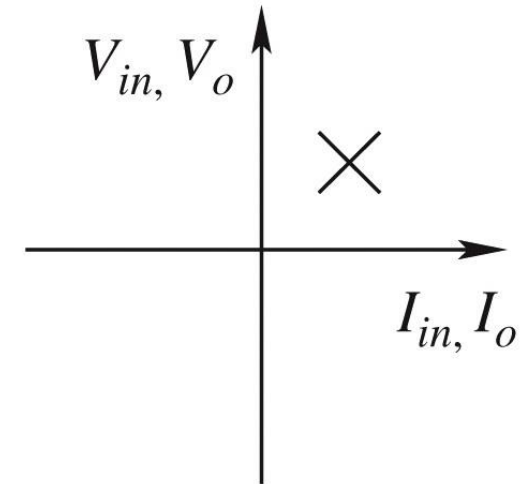
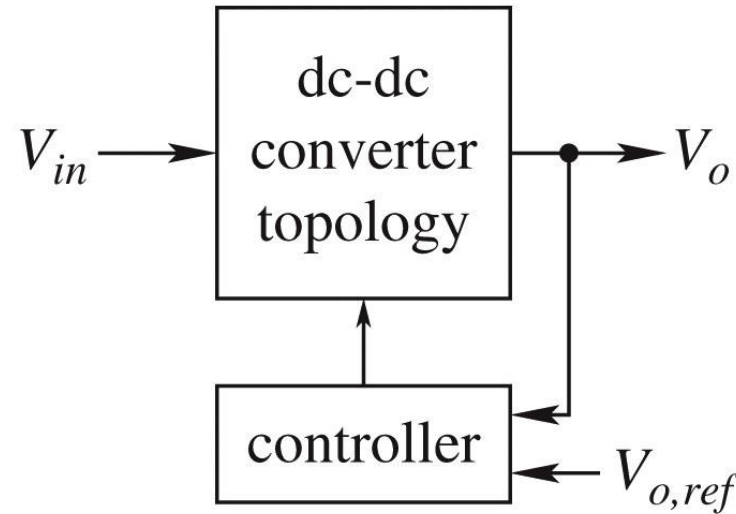
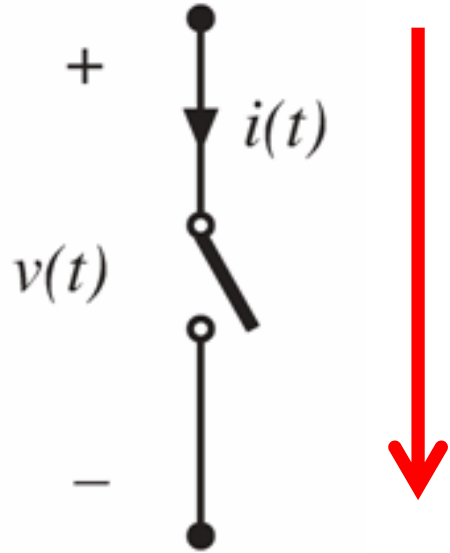
University  
of Glasgow

# Power Electronics

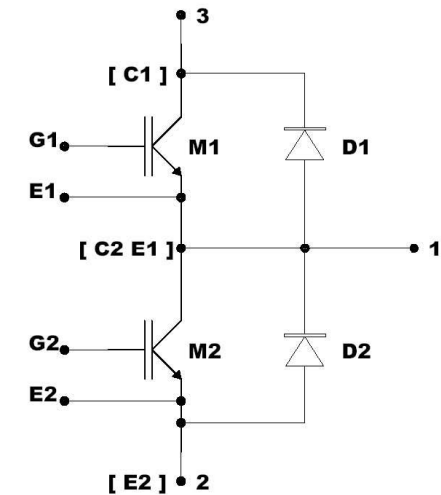
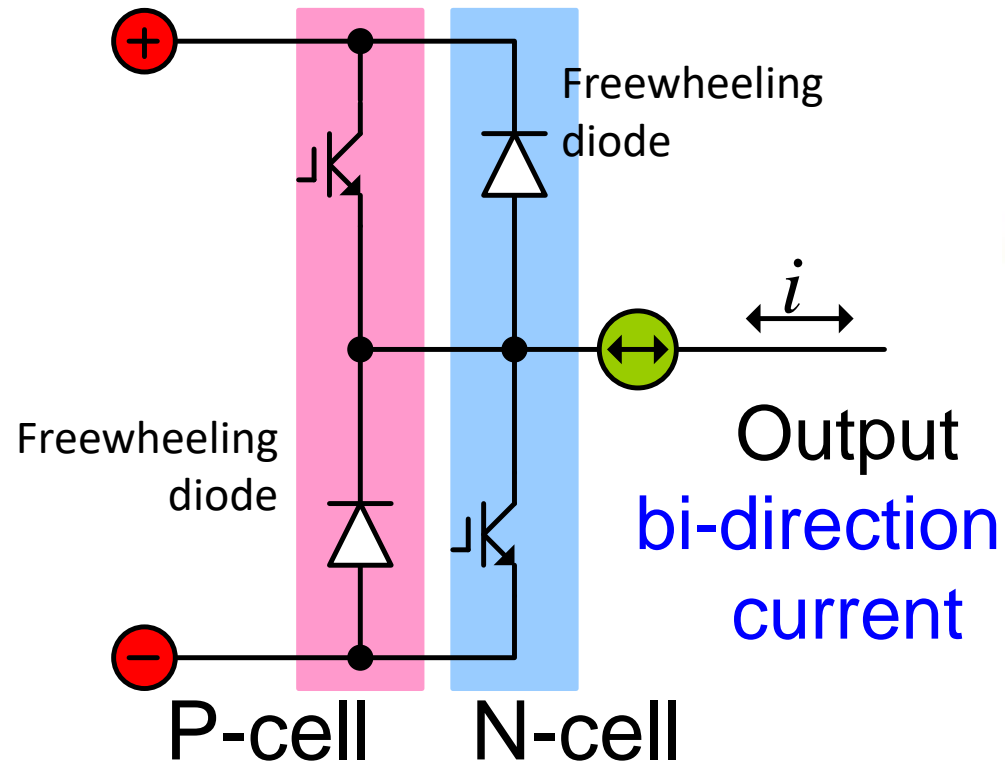
**Multi-Switch PWM Converters:**  
**DC-DC Choppers and DC-AC Inverters**

# Single-Switch Converter

**Single-switch** only allows unidirectional current flow. **Single-switch** (Buck, Boost, Buck-Boost) converters can be only used for DC-DC conversion in one quadrant of  $v$ - $i$  plane.

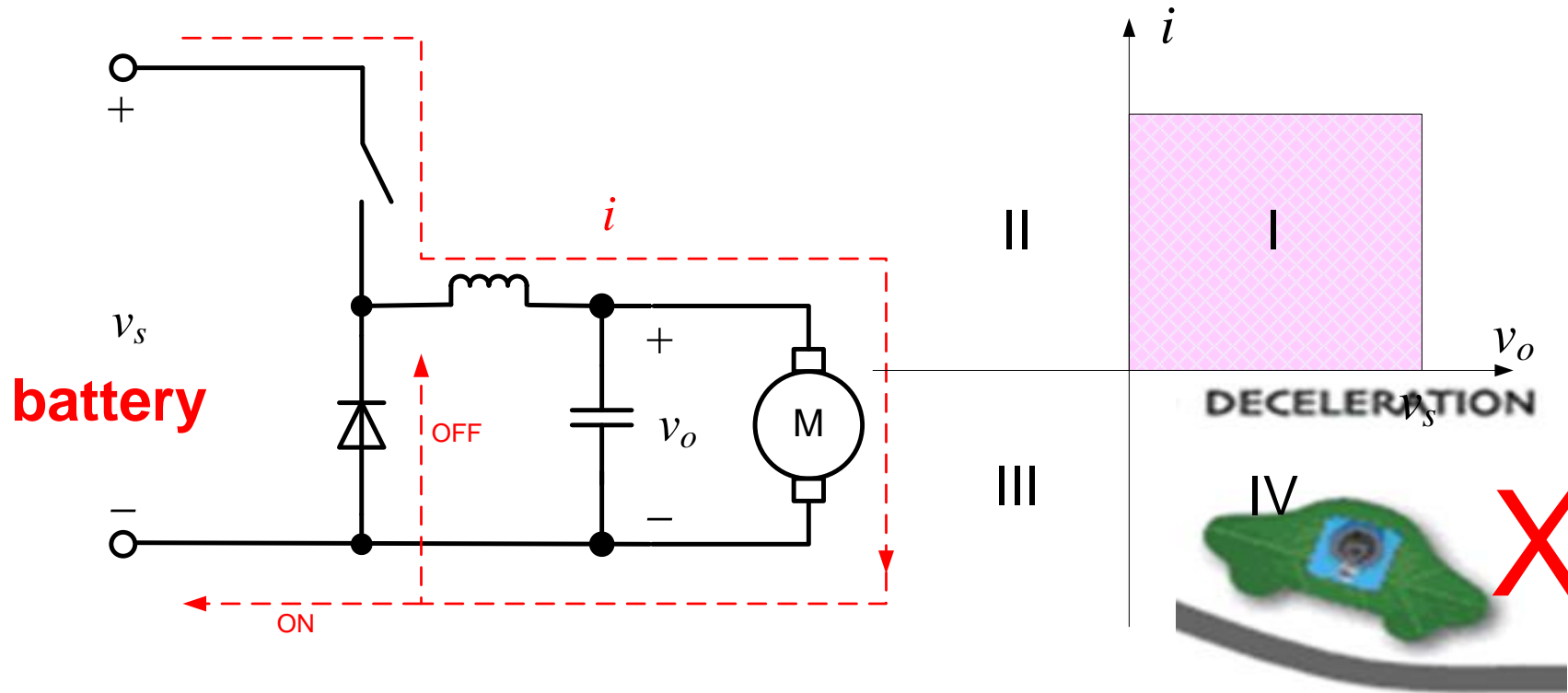


# Bi-directional Switch Cell



**Bi-directional Switch Cell** (Combination of two switch cell, Bridge-Leg Module) can be used to process both **DC-AC** and **DC-DC** conversion. That's why many commercial switch device modules include a anti-paralleled Fast Recovery Diode (FRD) for each switch. And commercial bridge-leg modules are very popular.

# Single Switch Buck Converter for DC Motor

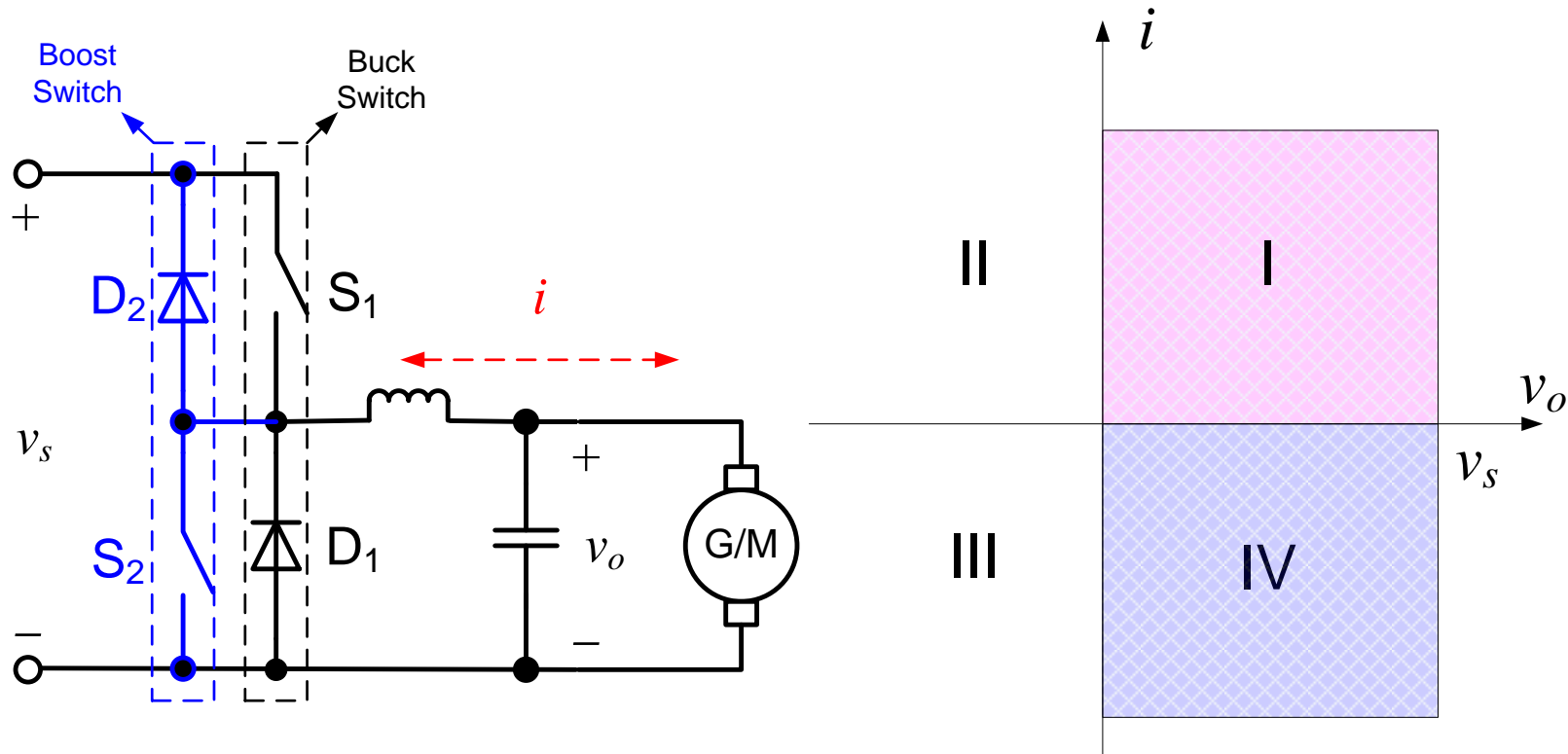


Buck Converter operate in a single quadrant of  $v_o$ - $i$  plane.

**DC** Motor only can rotate in positive direction.

In downhill deceleration, regenerative energy of **DC** motor can not feed back to the power source.

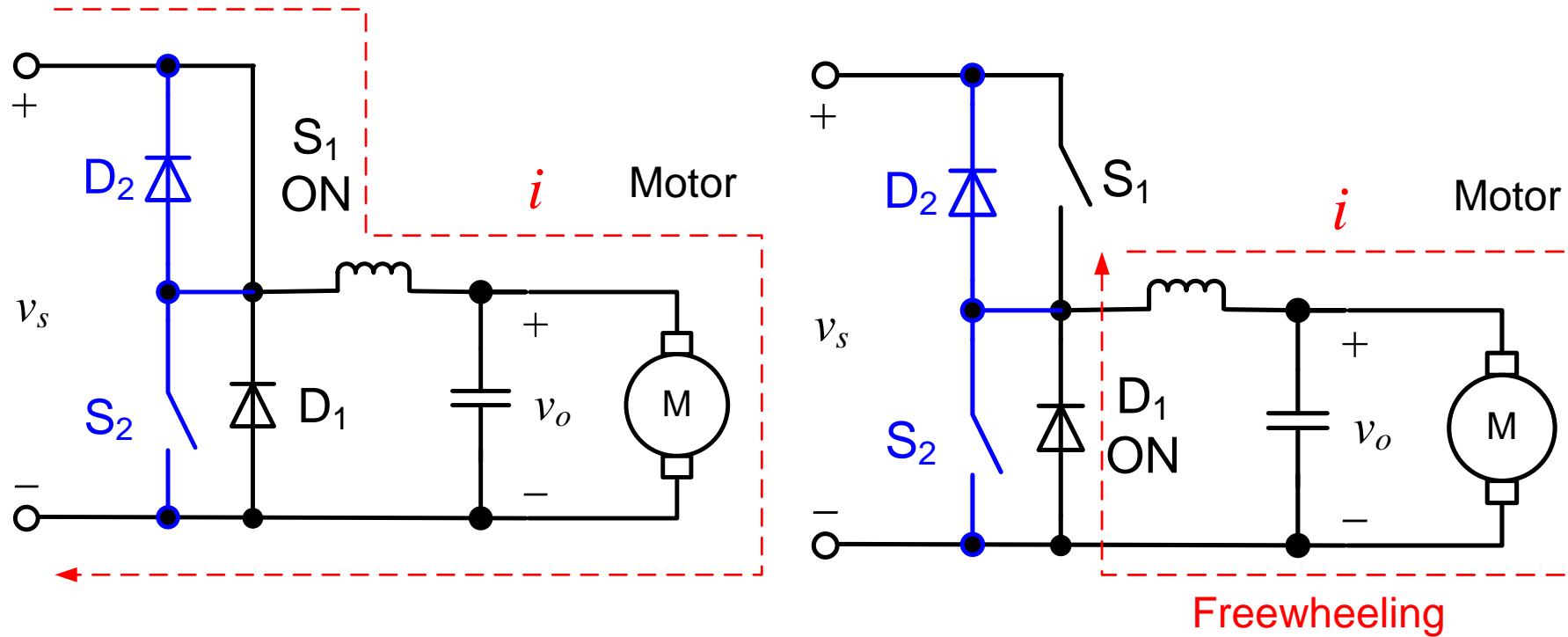
# One Bridge-leg Buck Converter for DC Motor



One bridge-leg module (bidirectional switch cell) allows bidirectional load flow (i.e. **Two Quadrants Operation** ).

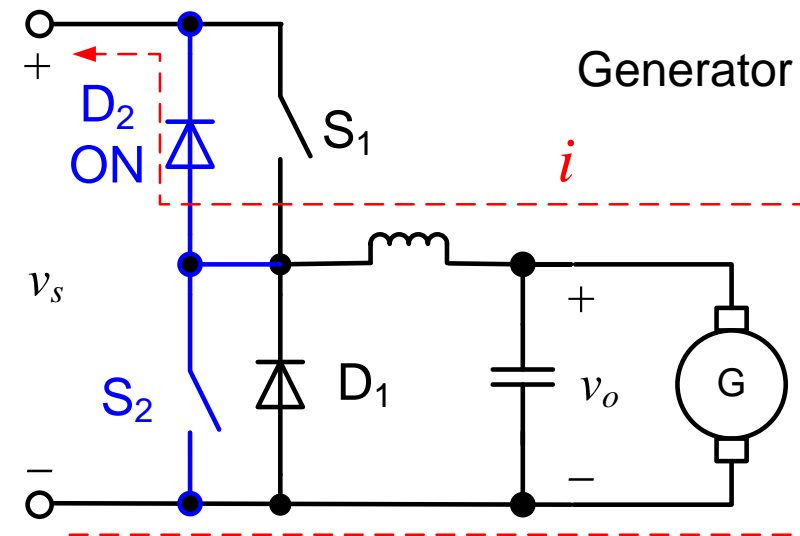
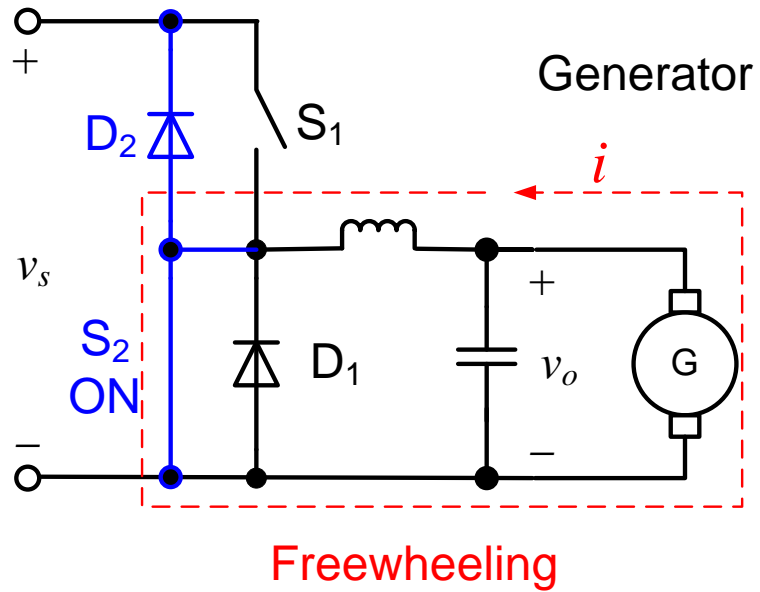
Regenerative energy of DC motor can feed back to the power source for energy saving.

# Driving Mode



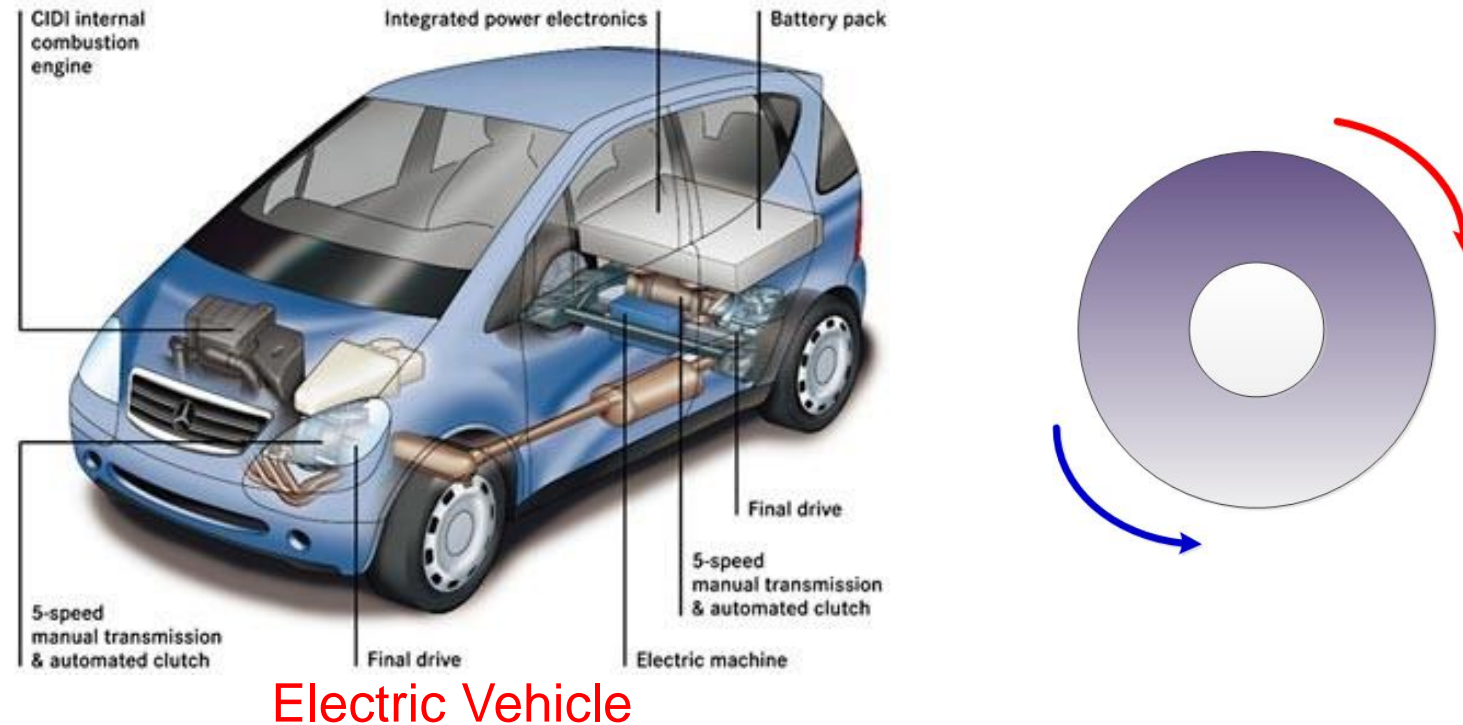
**Buck Conversion Mode:**  
Switching  $S_1$  ON/OFF  
 $S_2$   $D_2$  OFF

# Regeneration Mode



**BOOST Conversion Mode:**  
Switching  $S_2$  ON/OFF  
 $S_1$   $D_1$  OFF

# Four Quadrant Operation



However, two quadrant operated electric car can't reversely drive since output voltage is uni-directional. How to have **Four Quadrant Operation** converters?

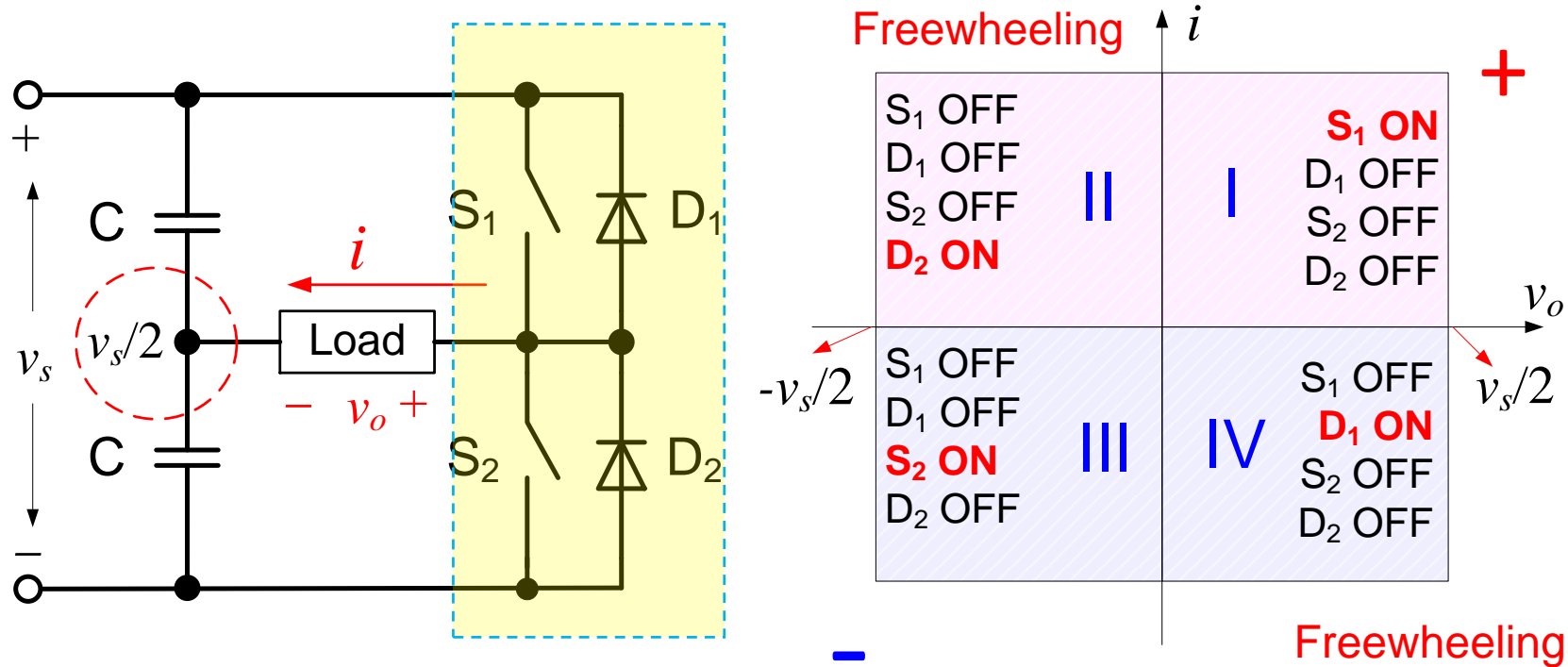


Multi-Switch

Single-Phase Converters

Four Quadrant Operation

# Half Bridge Converter



One-Leg Half Bridge Converter operate in a Four Quadrant of  $v_o$ - $i$  plane. Bi-directional power flow between power source and load.

Output voltage ranges from  $-V_s/2$  to  $+V_s/2$ .

# Half Bridge Converter Operation

## Instantaneous Value–

Quadrant I:  $v_o = +\frac{v_s}{2}, i > 0$

Quadrant II:  $v_o = -\frac{v_s}{2}, i > 0$

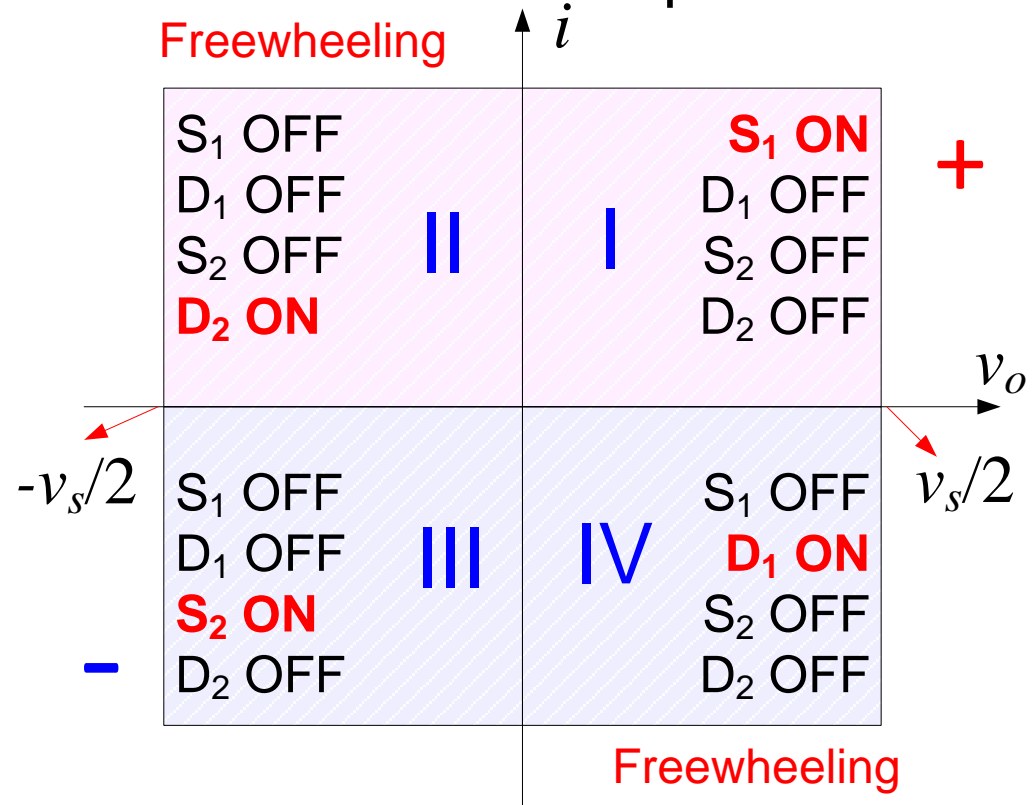
Quadrant III:  $v_o = -\frac{v_s}{2}, i < 0$

Quadrant IV:  $v_o = +\frac{v_s}{2}, i < 0$

## Output Average Value–

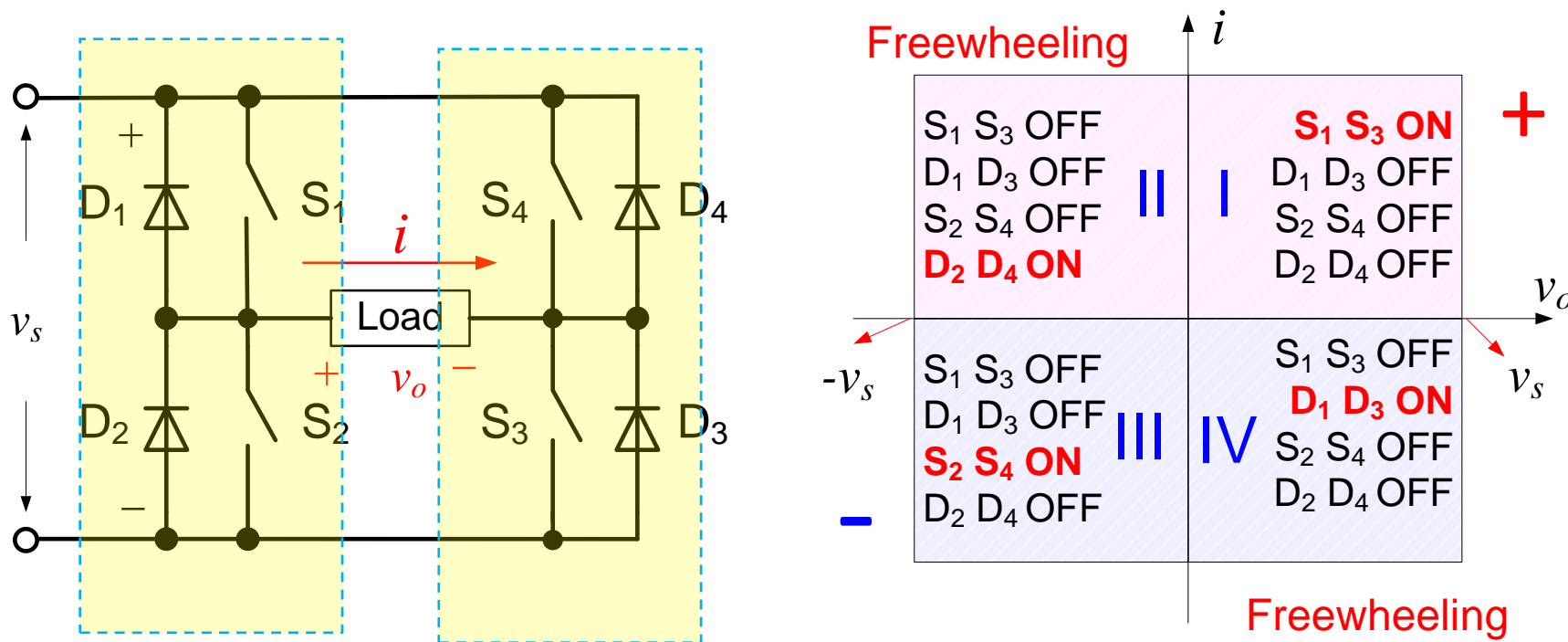
$$v_{o(average)} \in \left[ -\frac{v_s}{2}, +\frac{v_s}{2} \right]$$

## Four Quadrant Operation



With appropriate switching methods, it can convert DC voltage into another DC voltage or AC voltage.

# Full Bridge Converter



Dual-Leg Full Bridge Converter can operate in a Four Quadrant of  $v_o-i$  plane. Bi-directional power can flow between power source and load.

Output voltage ranges from  $-V_s$  to  $+V_s$ . Its range is two times of that of a half-bridge converter.

Average Value :  $v_{o(average)} \in [-v_s, +v_s]$

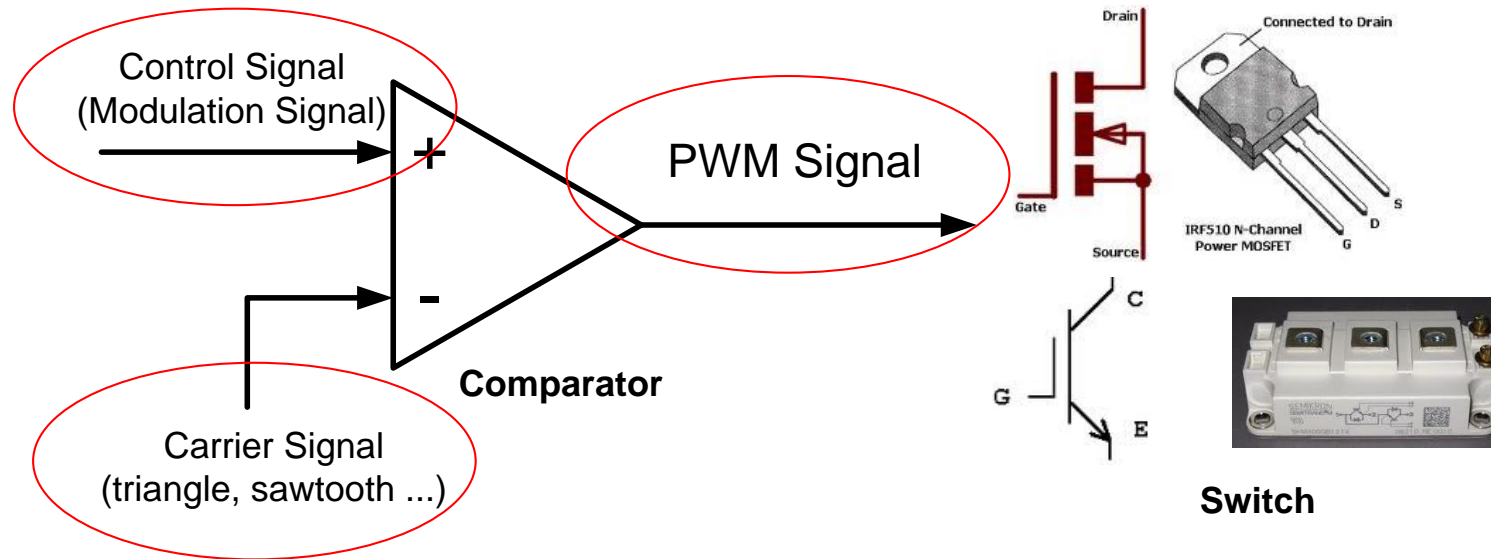
# Four Quadrant Operation

- If **DC-DC converters** with Four Quadrant Operation, can be used to drive **DC motors** to rotate in both directions (positive and reverse).
- Converters with Four Quadrant Operation, can convert DC power into AC power for AC loads (e.g. **AC motors**) , i.e. **DC-AC conversion**.

## Switching Strategy

How to switch bridge converters to provide desired DC-DC and DC-AC Conversion?

# PWM Signal Generation

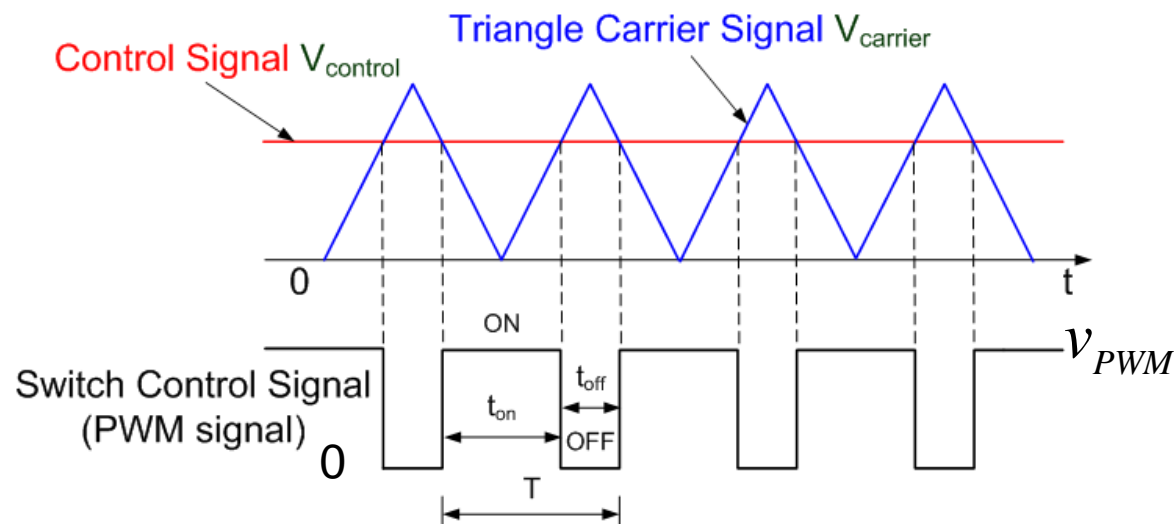
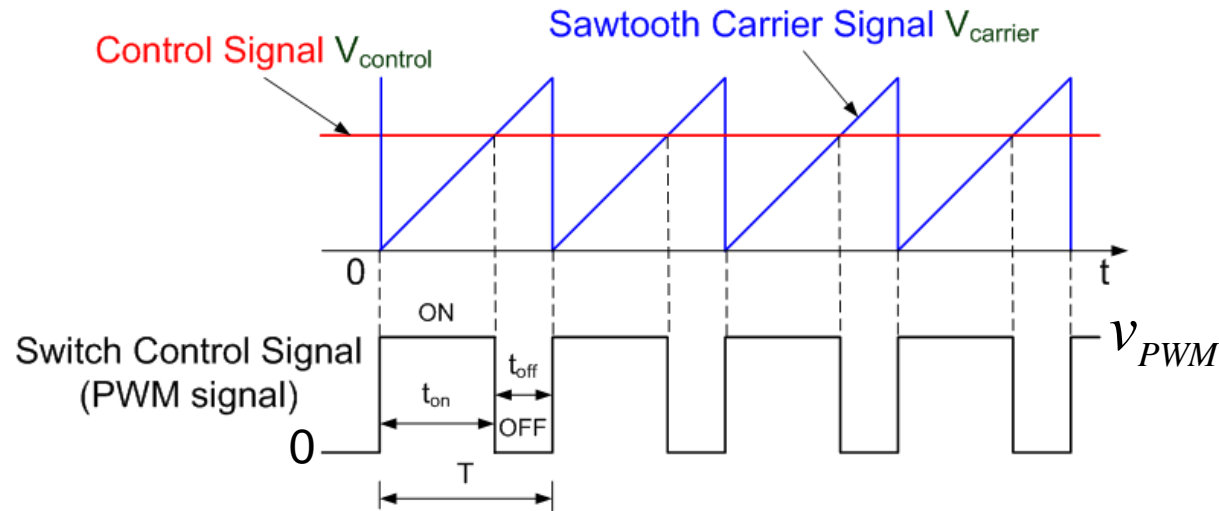


## Pulse Width Modulation (PWM)

1. Constant (**high**) Switching (Carrier) Frequency
2. Carrier Frequency  $\gg$  Control Signal Frequency
3. Control Signal Peak  $\leq$  Carrier Signal Peak
4. Output Pulse Width is proportional to input control signal

# Unipolar DC-DC PWM Signal

Within one switching period, PWM signal is either positive or zero; or PWM is either negative or zero.



*switching frequency:*

$$f_s = \frac{1}{T_s};$$

*duty cycle:*

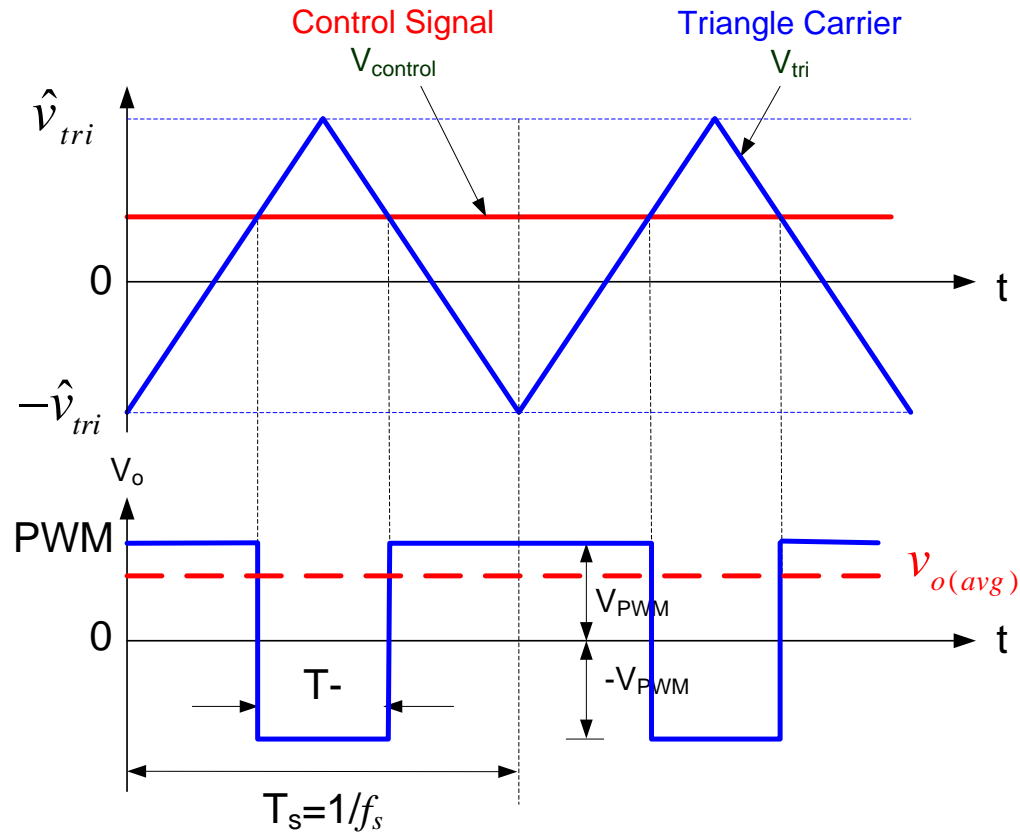
$$k = \frac{t_{on}}{T_s} = \frac{V_{control}}{\hat{V}_{carrier}}$$

*average output:*

$$v_{o(avg)} = k v_{PWM}$$

# Bipolar DC-DC PWM Signal

Within one switching period, PWM signal is either positive or negative.



$$T_s = \frac{1}{f_{tri}}, T_+ + T_- = T_s$$

$$V_{o(avg)} = \frac{T_+}{T_s} V_{PWM} - \frac{T_-}{T_s} V_{PWM}$$

$$= \frac{2T_+ - T_s}{T_s} V_{PWM} = \frac{V_{control}}{\hat{V}_{tri}} V_{PWM}$$

$$= k V_{PWM}$$

*Duty Cycle :*

$$k = \frac{2T_+ - T_s}{T_s} = \frac{V_{control}}{\hat{V}_{tri}}$$



# PWM Amplifier

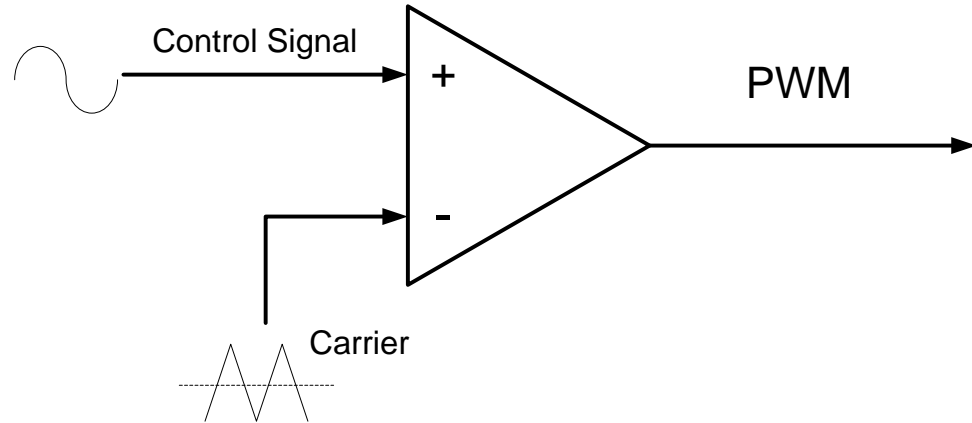
$$v_{o(avg)} = k v_{PWM} = \left( \frac{V_{control}}{\hat{V}_{carrier}} \right) v_{PWM} = \left( \frac{v_{PWM}}{\hat{V}_{carrier}} \right) V_{control}$$

1. Neglecting high order harmonics, PWM conversion is a linear amplifier. The **mean value of output PWM signal** for each switching period is equivalent to an amplified control signal with **amplifier gain** of  $v_{PWM}/V_{carrier(peak)}$ .
2. In fact, control signal  $V_{control}$  can be either a **sinusoidal signal** for DC-AC PWM conversion and a **dc signal** for DC-DC PWM conversion.

## Sinusoidal control signal

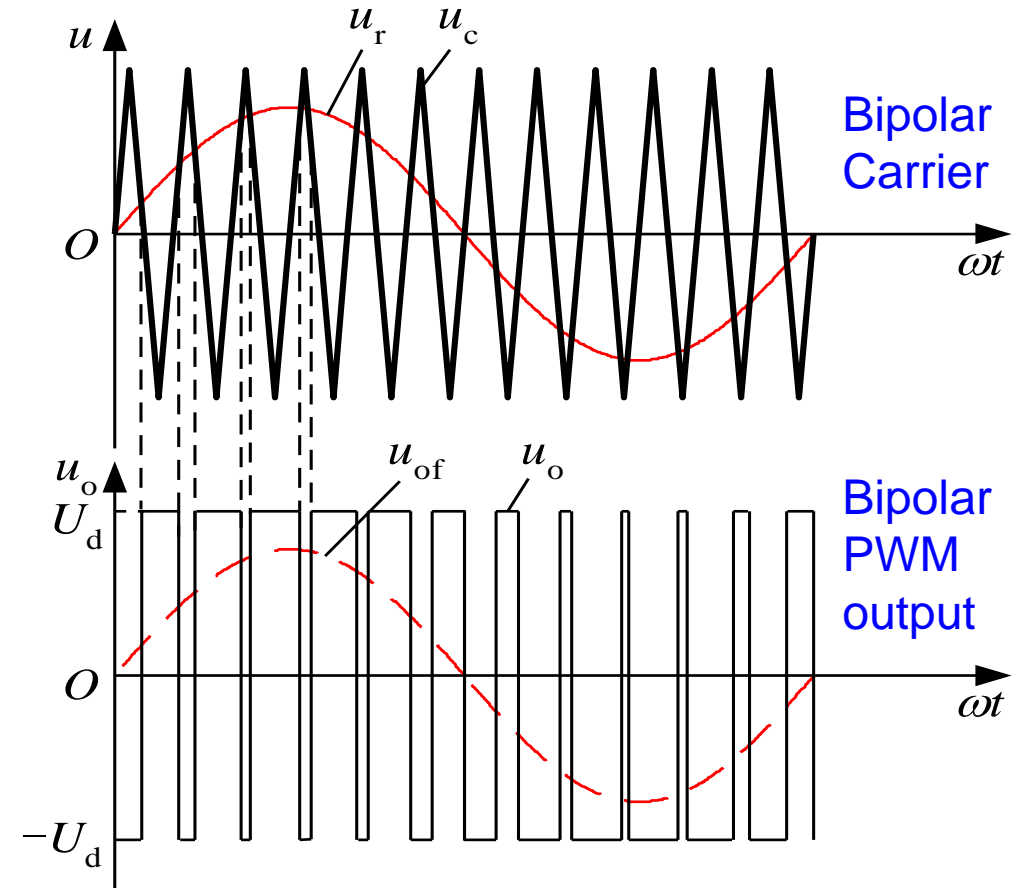
$$v_{control} = \hat{v}_{control} \sin(2\pi f_1 t)$$

carrier frequency:  $f_s$



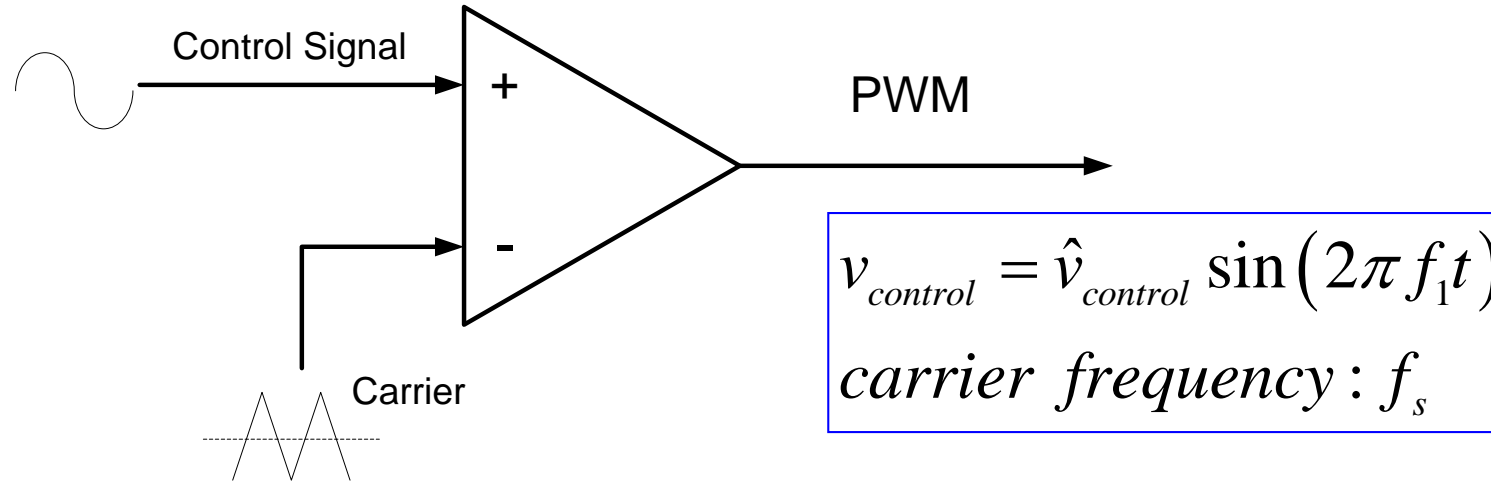
1. PWM is a **linear amplifier** for control signals. To produce **sinusoidal AC voltage**, control signal for PWM will be **sinusoidal AC signal** with **controllable magnitude and frequency**
2. Constant carrier (switching) frequency. Carrier signal frequency  $\gg$  Control signal frequency

## Sinusoidal PWM (SPWM)



**SPWM:** Within each one switching period, mean value of the output PWM signal is proportional to the sine control signals.

# Magnitude Modulation Ratio (SPWM)

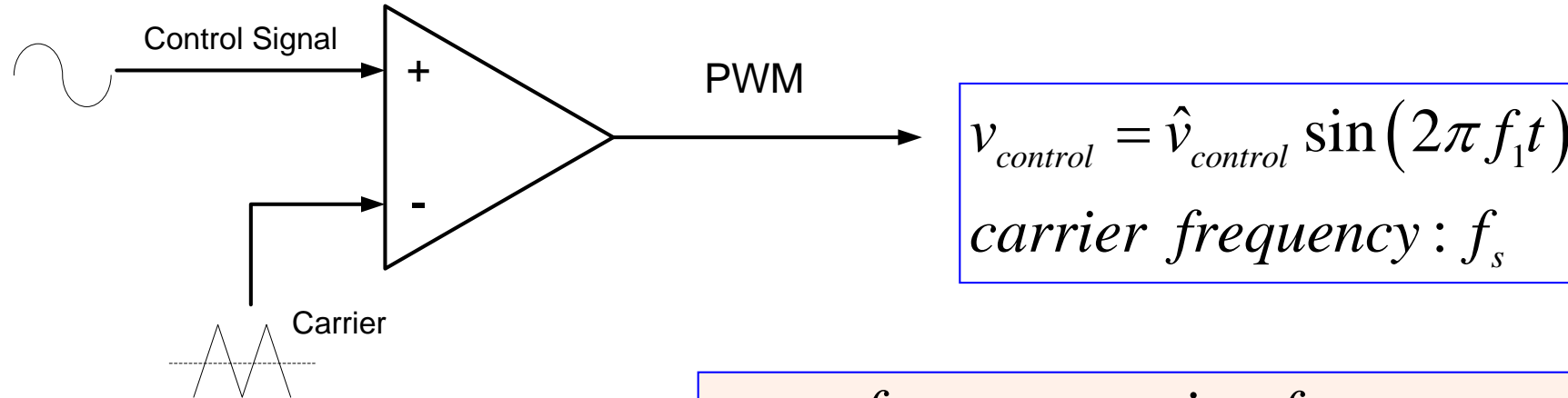


**Magnitude modulation ratio:**

$$m_a = \frac{\hat{v}_{control}}{\hat{v}_{tri}} = \frac{\text{peak of control}}{\text{peak of carrier}}$$

1. If  $m_a \leq 1$ , SPWM is in its linear modulation region.
2. If  $m_a > 1$ , SPWM is in its over-modulation (saturation) region.

# Frequency Modulation Ratio



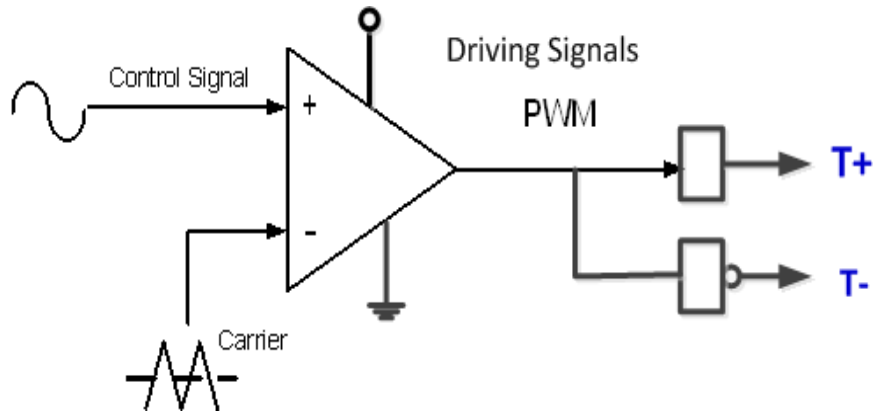
**Frequency modulation ratio:**

$$m_f = \frac{f_s}{f_c} = \frac{\text{carrier frequency}}{\text{control signal frequency}}$$

1. Synchronous PWM.  $m_f$  must be an integer. It will result identical PWM waveforms in every period  $1/f_s$ .
2. Asynchronous PWM.  $m_f$  is not an integer. Asynchronous PWM will result in undesirable low frequency subharmonics (of fundamental frequency  $f_s$ ) in the output.

# Half-Bridge Single-Phase SPWM Inverter

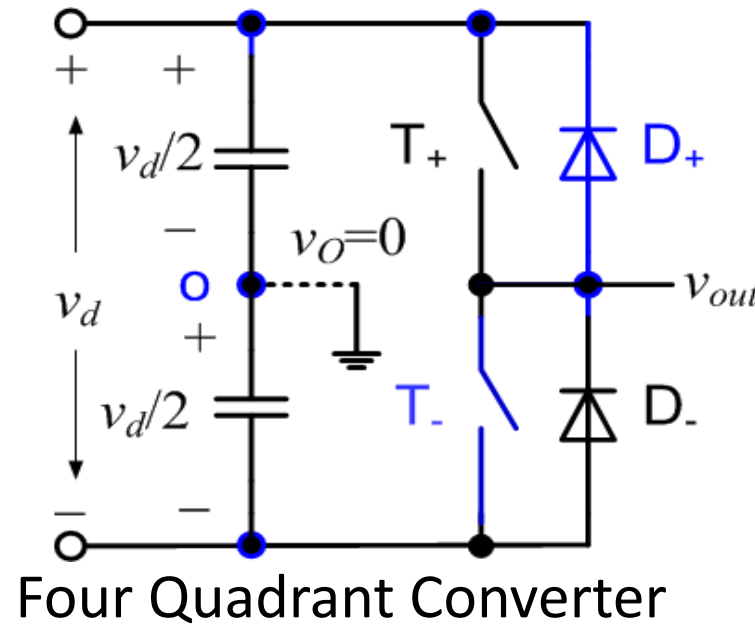
**T+ and T- are Complementary**



**Bridge Leg has two PWM**

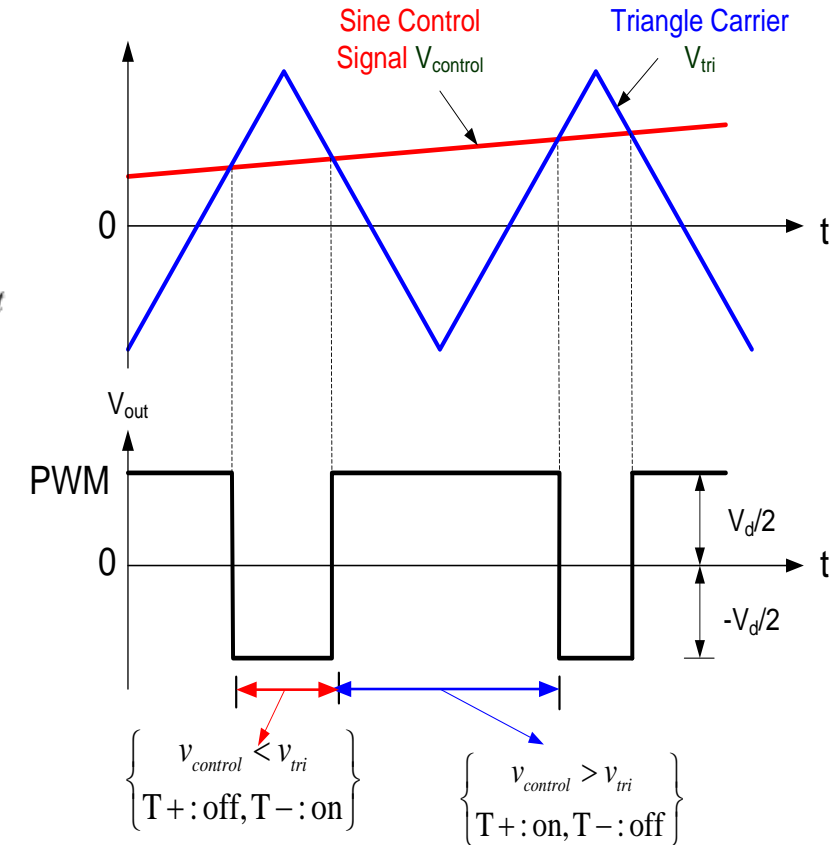
**Switching States:**

- (1) T+ on, T- off
- (2) T+ off, T- on



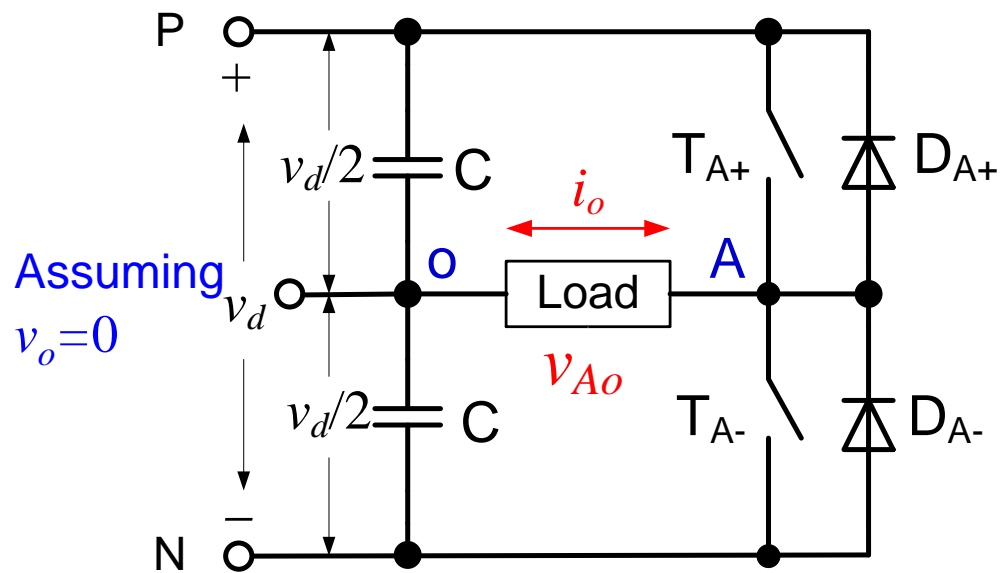
**Mean value of SPWM signal within any one switching period**

$$(v_{out})_{av} = \left( \frac{v_d}{2} \frac{1}{V_{carrier(peak)}} \right) V_{control}$$



**SPWM Switching**

**Bipolar PWM Output Voltage With Magnitude of  $V_{PWM}=V_d/2$**

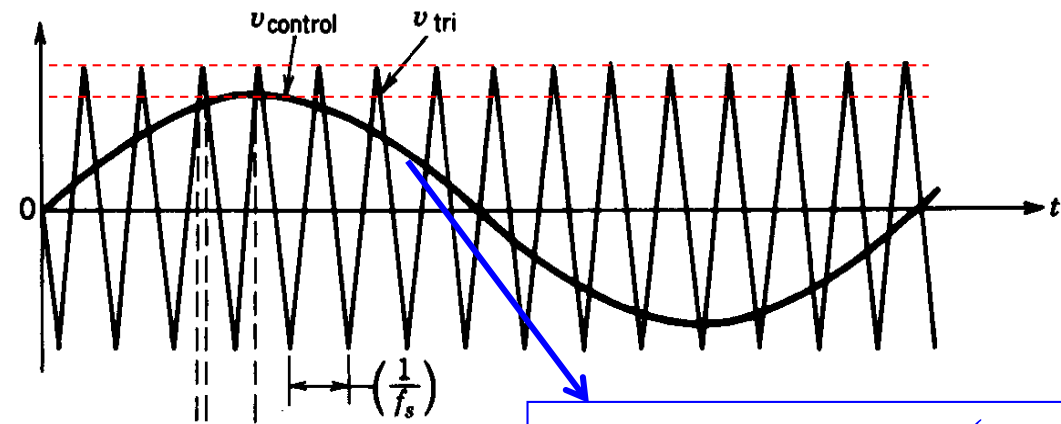


$$v_{control} = \hat{v}_{control} \sin(2\pi f_1 t)$$

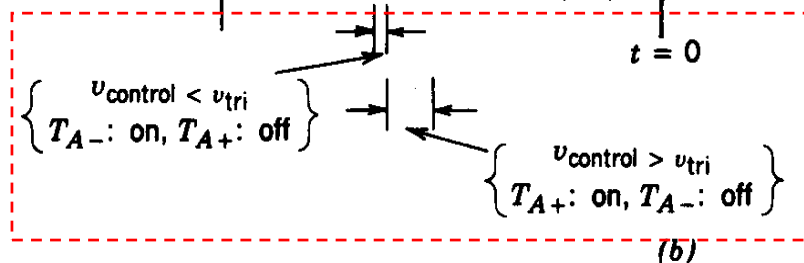
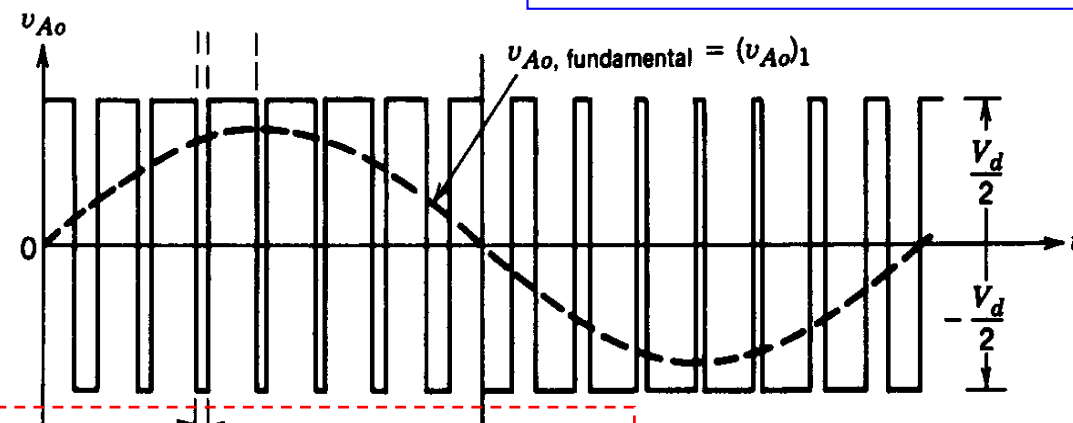
Neglecting harmonics

$$v_{Ao(avg)} \approx \left( \frac{v_d}{2} \frac{1}{V_{carrier(peak)}} \right) V_{control}$$

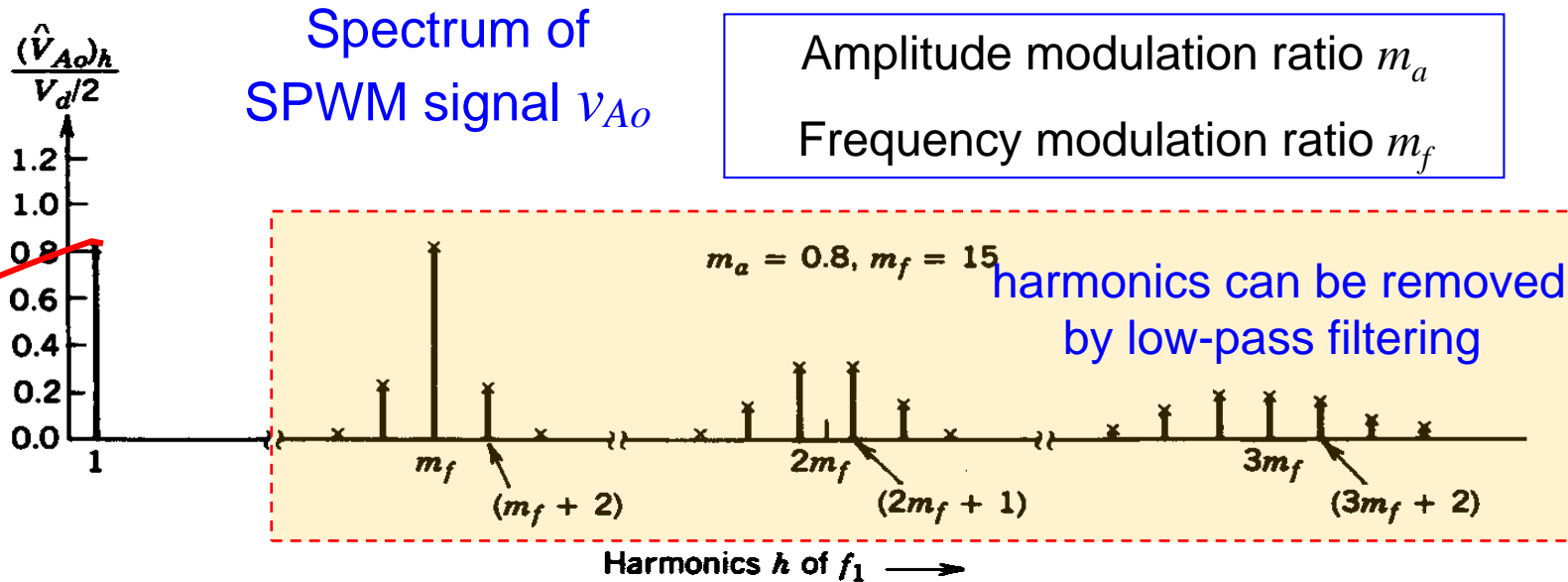
$$= \frac{v_d}{2} m_a \sin(2\pi f_1 t)$$



$$v_{control} = \hat{v}_{control} \sin(2\pi f_1 t)$$



# SPWM in Linear Modulation Region



1. Harmonics in SPWM voltage  $v_{Ao}$  appear as sidebands centered around

$$f_h = (jnm_f \pm k)f_1$$

2.  $m_a < 1$  (linear modulation), fundamental frequency component  $(v_{Ao})_1$

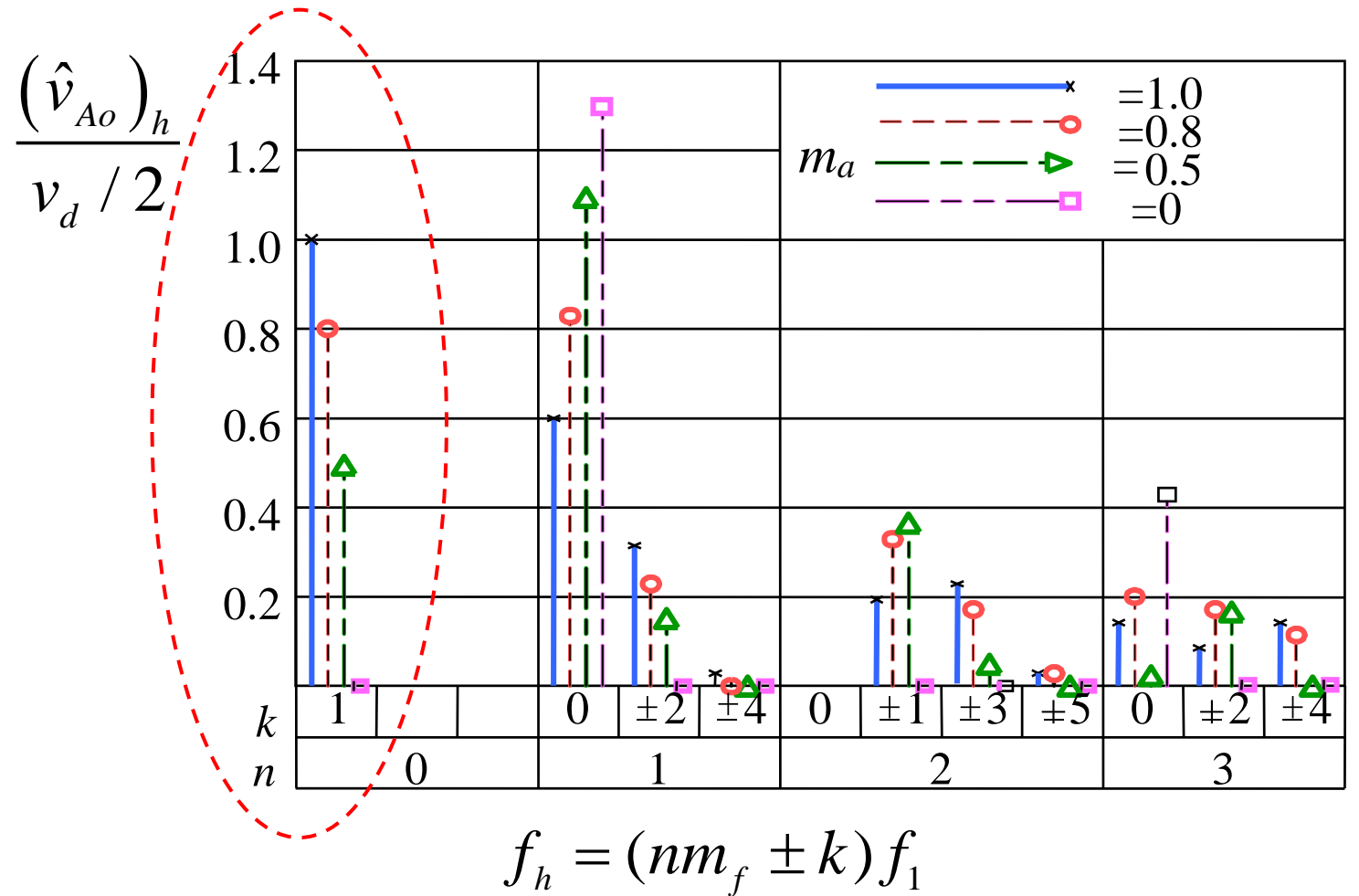
$$(v_{Ao})_1 \approx (v_{Ao})_{av} = \left( \frac{v_d}{2} \frac{1}{\hat{v}_{tri}} \right) \hat{v}_{control} \sin(2\pi f_1 t) = \frac{v_d}{2} m_a \sin(2\pi f_1 t)$$

normalized control signal

Fundamental Frequency Magnitude

$$\left(\hat{v}_{Ao}\right)_1 \approx m_a \cdot \frac{v_d}{2}$$

Normalized amplifier  
gain for half-bridge  
inverter is  $v_d/2$



Half-bridge single-phase SPWM inverter can be treated as linear amplifier for sinusoidal signal in linear modulation region ( $m_a \leq 1$ ).

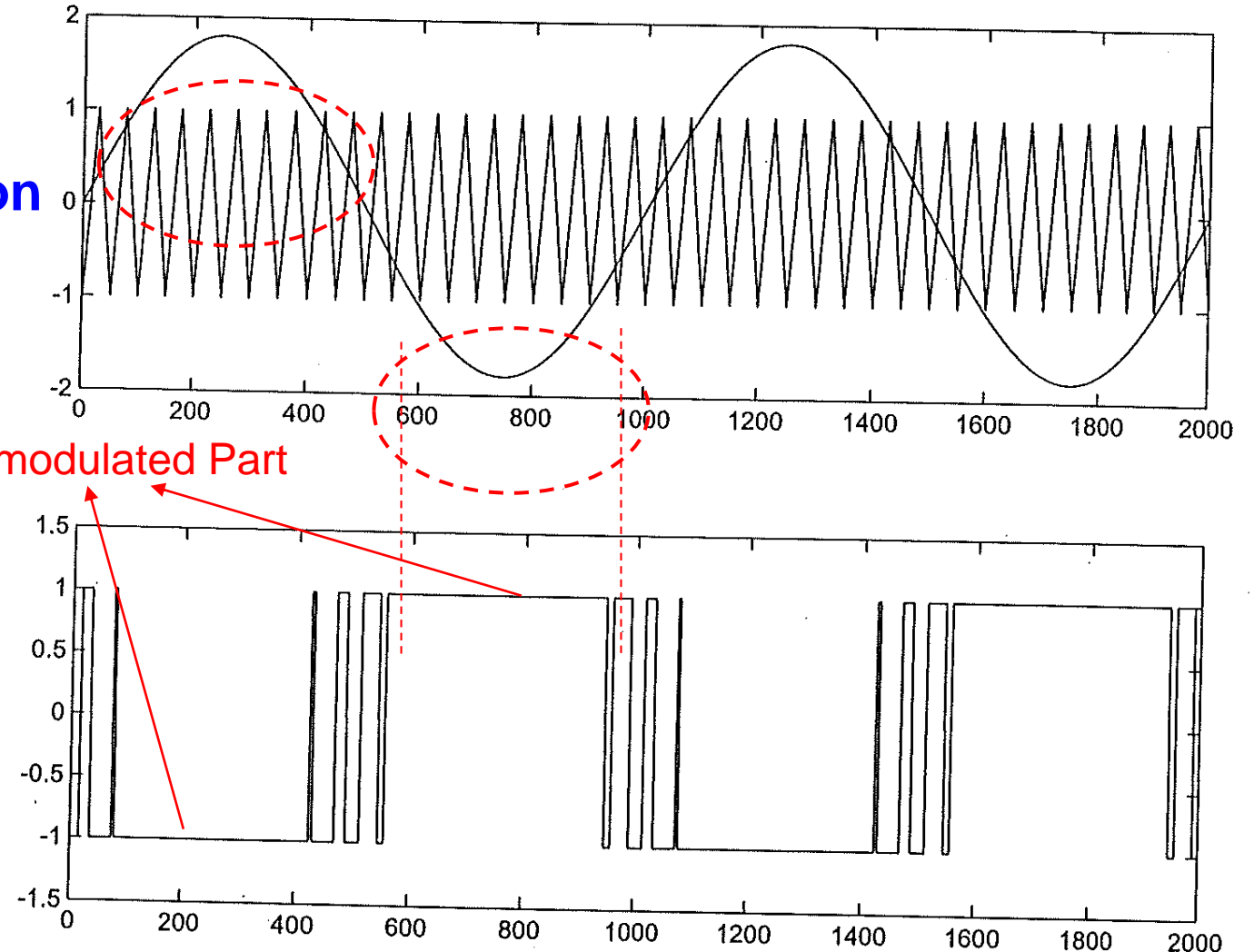


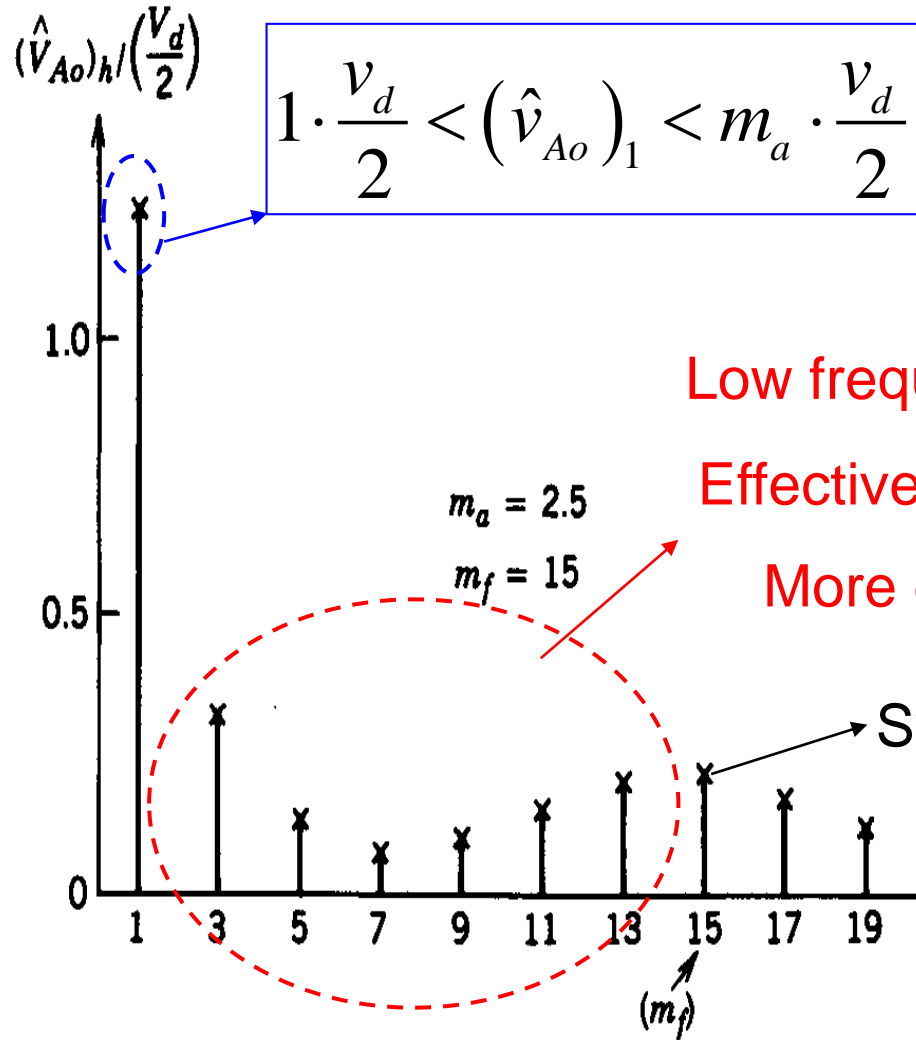
# SPWM in Overmodulation Region ( $m_a > 1$ )

**Overmodulation**  
( $m_a > 1$ )

Unmodulated Part

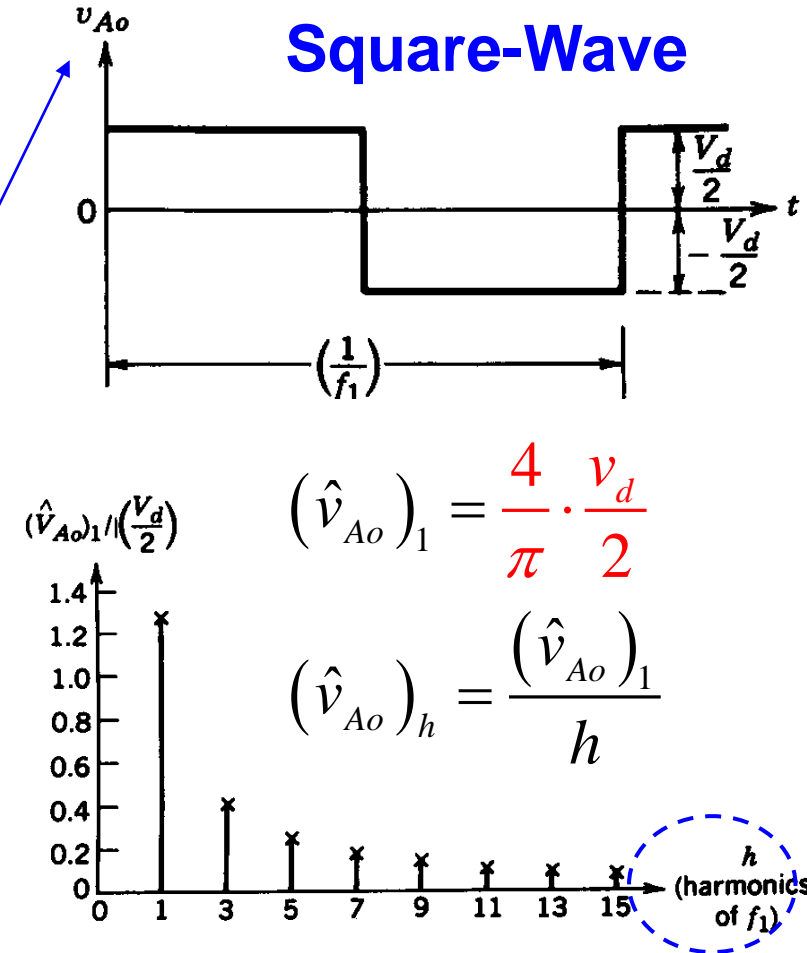
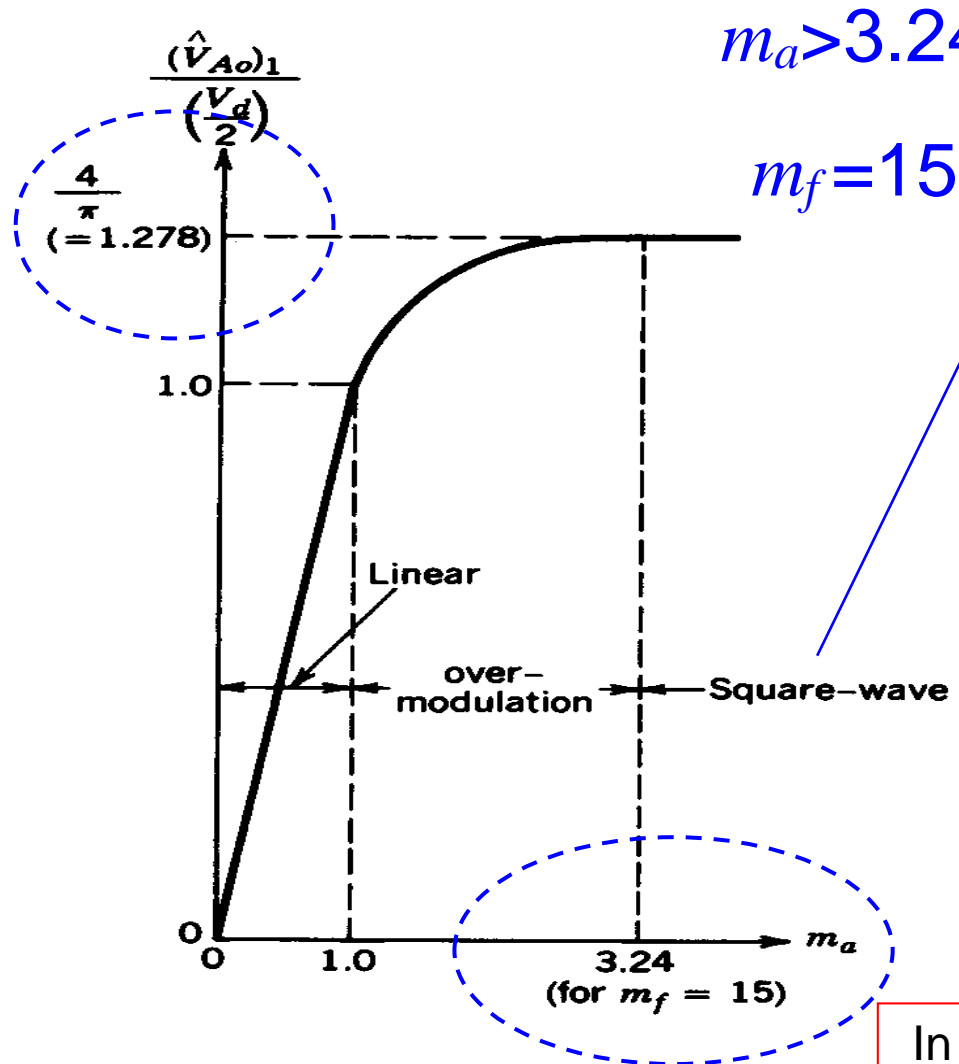
Output pulse width will not be proportional to the amplitude of the control signal during overmodulated switching periods. Effective Switching frequency reduced. PWM is not a linear amplifier in this region.





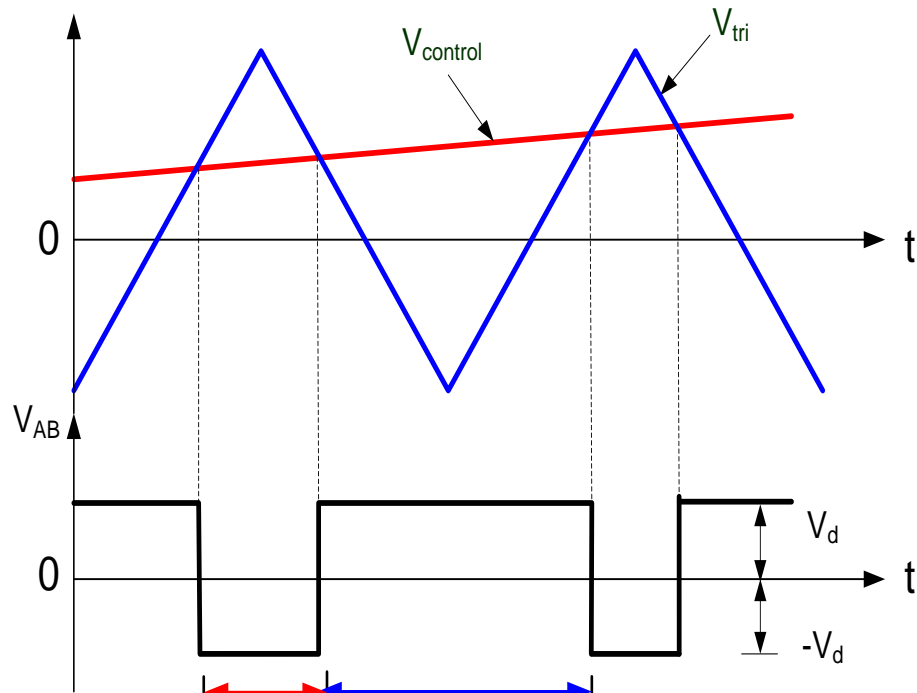
Increased fundamental frequency amplitude with reduced nonlinear gain ( $< v_d/2$ ).

# Square-Wave Mode

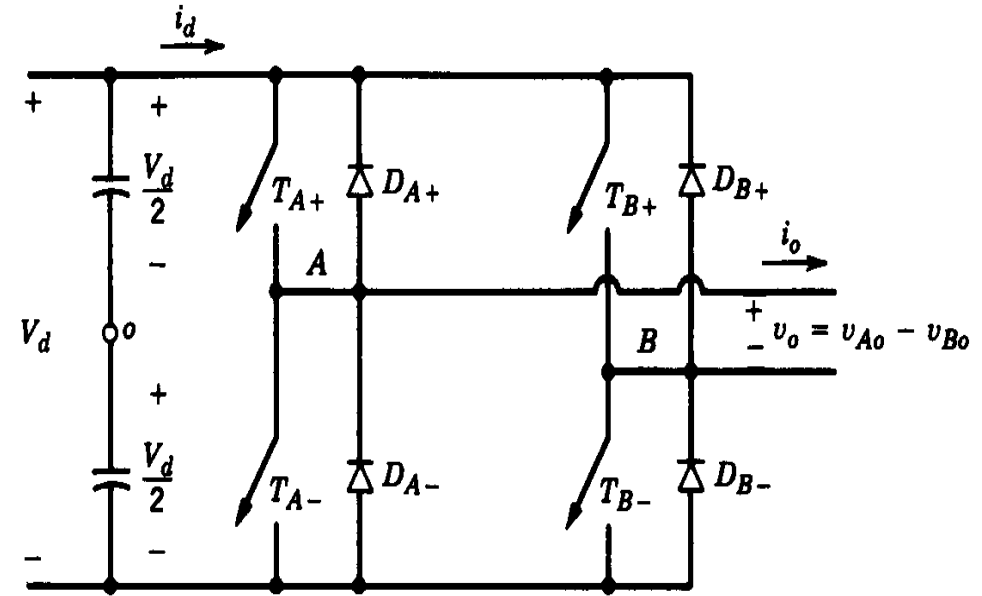


In Square-wave case, switching frequency is equal to control signal frequency  $f_1$

# Full Bridge SPWM Inverter



$$\left\{ \begin{array}{l} v_{control} < v_{tri} : \\ T_{A+} : \text{off}, T_{A-} : \text{on} \\ T_{B+} : \text{on}, T_{B-} : \text{off} \end{array} \right\} \quad \left\{ \begin{array}{l} v_{control} > v_{tri} : \\ T_{A+} : \text{on}, T_{A-} : \text{off} \\ T_{B+} : \text{off}, T_{B-} : \text{on} \end{array} \right\}$$

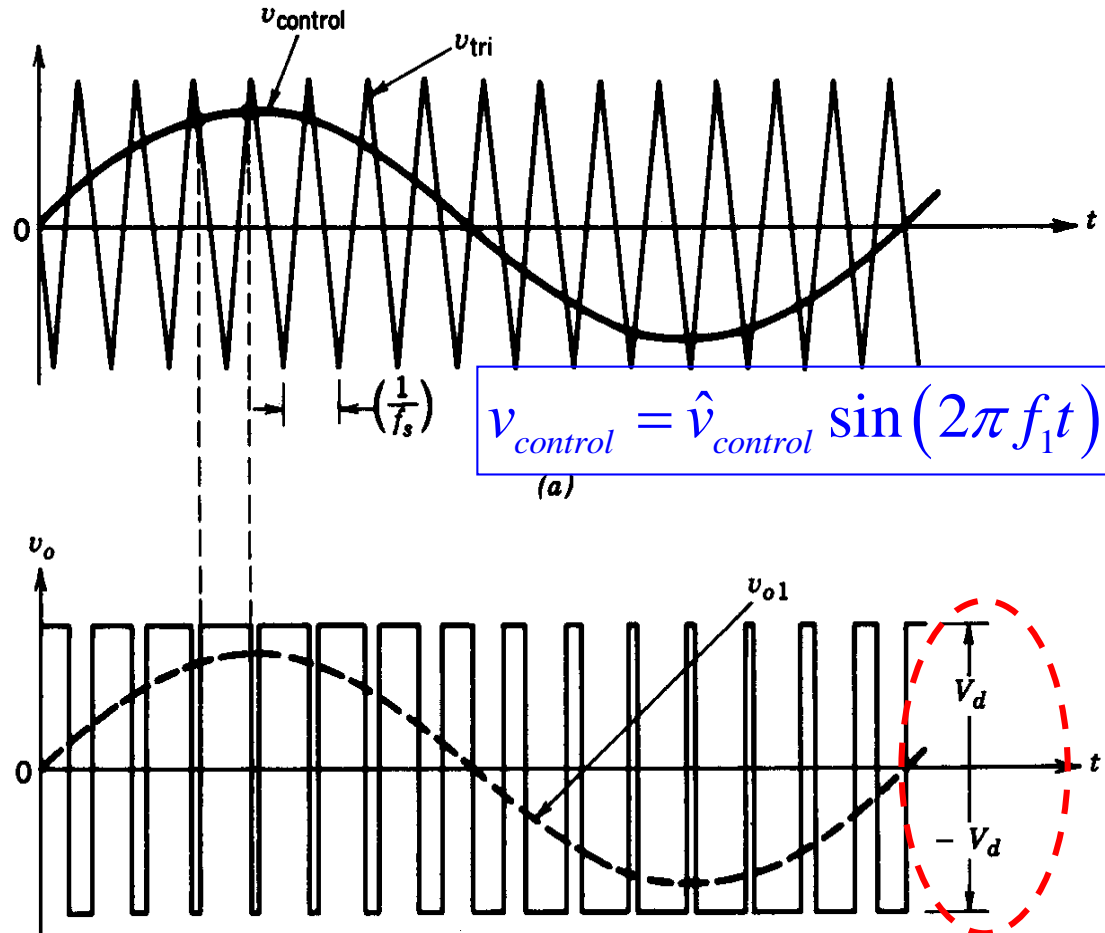


Bipolar PWM Output Voltage

$$v_o = v_{AB} = v_A - v_B = \{v_d, -v_d\}$$

$$\left\{ \begin{array}{l} v_{AB} = v_d : \\ T_{A+} : \text{off}, T_{A-} : \text{on} \\ T_{B+} : \text{on}, T_{B-} : \text{off} \end{array} \right\} \quad \left\{ \begin{array}{l} v_{AB} = -v_d : \\ T_{A+} : \text{on}, T_{A-} : \text{off} \\ T_{B+} : \text{off}, T_{B-} : \text{on} \end{array} \right\}$$

# Full-Bridge Bipolar SPWM Inversion



1. Sinusoidal control signal  $v_{control}$  with controllable magnitude and frequency
2. Bipolar PWM signal  $v_{Ao}$  with values  $(\pm V_d)$  over one switching period

## Linear modulation

$$m_a \leq 1.0:$$

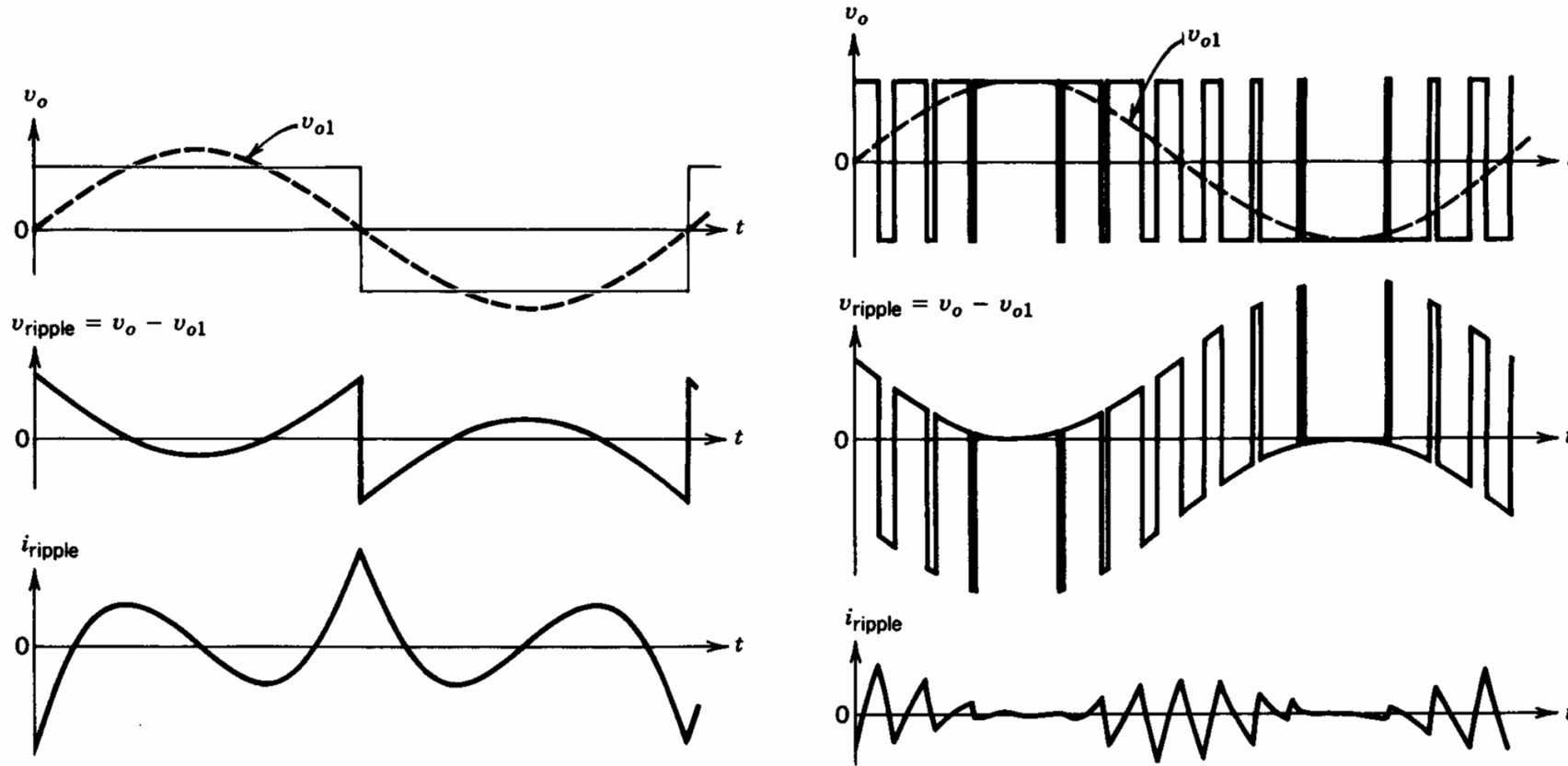
$$(v_o)_1 \approx \left( v_d \frac{1}{\hat{v}_{tri}} \right) v_{control} = v_d m_a \sin(2\pi f_1 t)$$

## Overmodulation

$$m_a > 1.0: v_d < \hat{v}_{o1} < \frac{4}{\pi} v_d$$

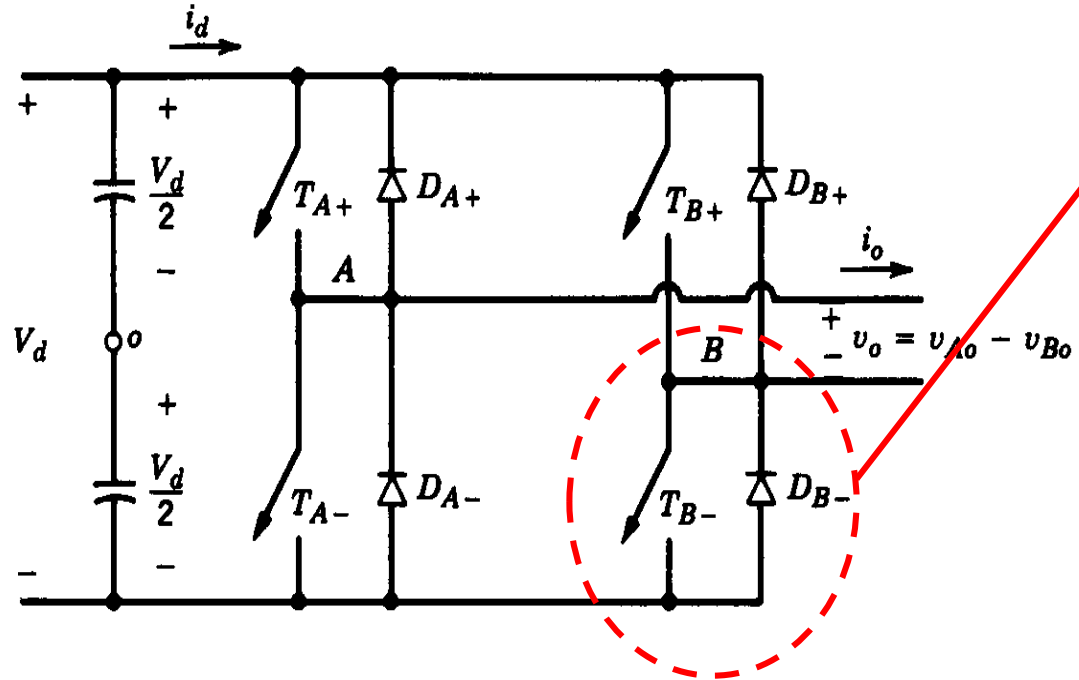
The maximum output voltage of full bridge inverter is twice that of half bridge inverter. It can offer twice power output.

# SPWM Inverter for AC Motor Drive



Linear modulation leads to smaller current (torque) ripple; over-modulation results in higher amplitude fundamental frequency voltage but higher current (torque) ripple

# Voltage Source Inverter (VSI)



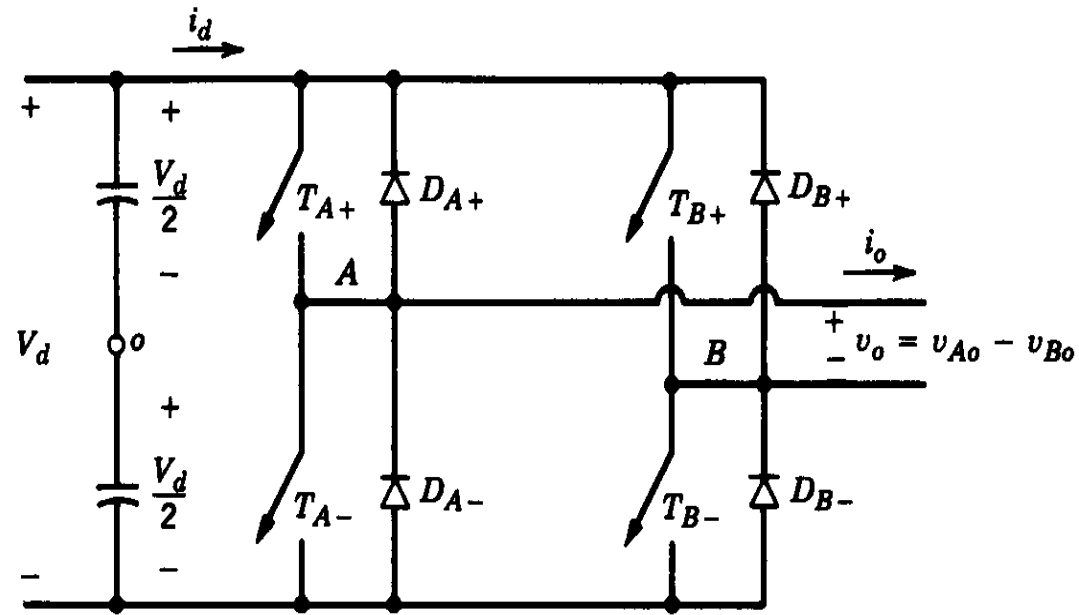
## Feature of VSI:

Power switches with anti-paralleled diodes

Diodes are for current freewheeling

Any Inverter with **DC side voltage source** (e.g. battery, large value capacitors, ...) is called **Voltage Source Inverter (VSI)**

# Shoot-Through of VSI



Two switches ( $T_+$ ,  $T_-$ ) of any inverter leg have complementary switching states: (On, Off) or (Off, On)

$T_{A+}, T_{A-}$  can't be on at same time

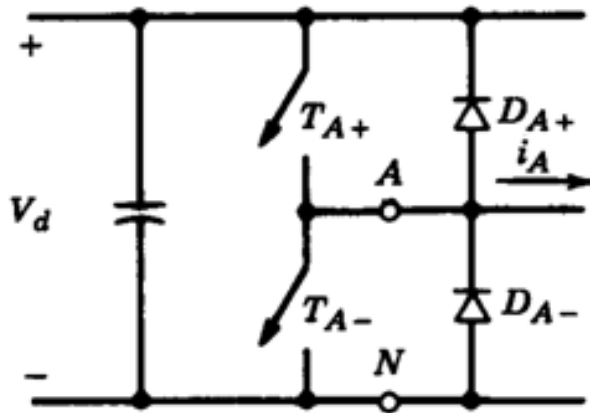
$T_{B+}, T_{B-}$  can't be on at same time

Two switches ( $T_+$ ,  $T_-$ ) of any inverter leg are not allowed to be turned on at the same time, otherwise dc side voltage source will be shorted (Shoot-Through).

However, it will take some time for the switches to switching on/off. Therefore blank time (deadtime) must be inserted between the ON/OFF of  $T_+$  and  $T_-$  to avoid shoot-through during the switching procedure.

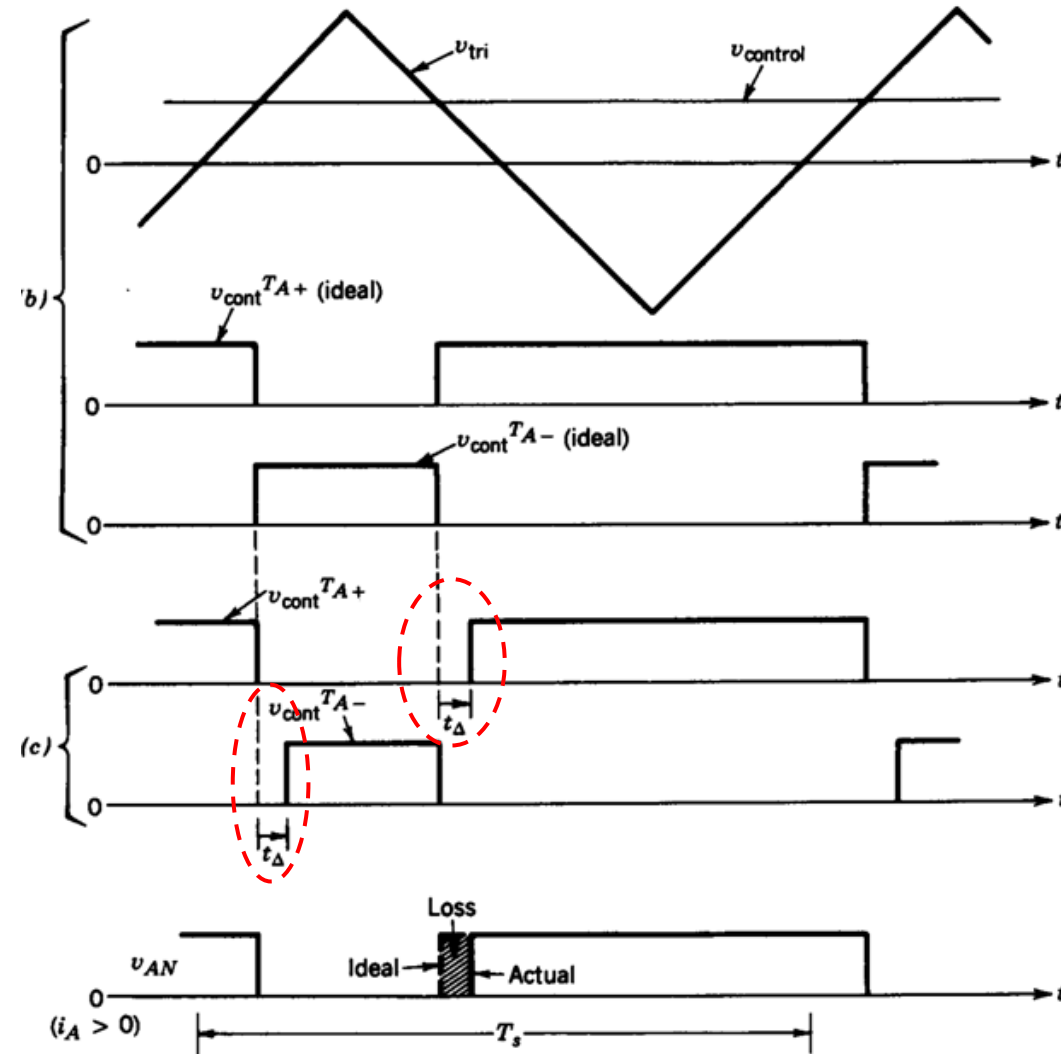


# Deadtime for VSI



Insert **deadtime** (*i.e.* both  $T_+$  and  $T_-$  are off) into the switching transient procedure.

**Deadtime** guarantee safe operation, but lose some PWM gain and cause distortion



# Three-Phase SPWM Inverter

Three(or multiple)-phase Converters can be used to deliver higher power than single-phase ones

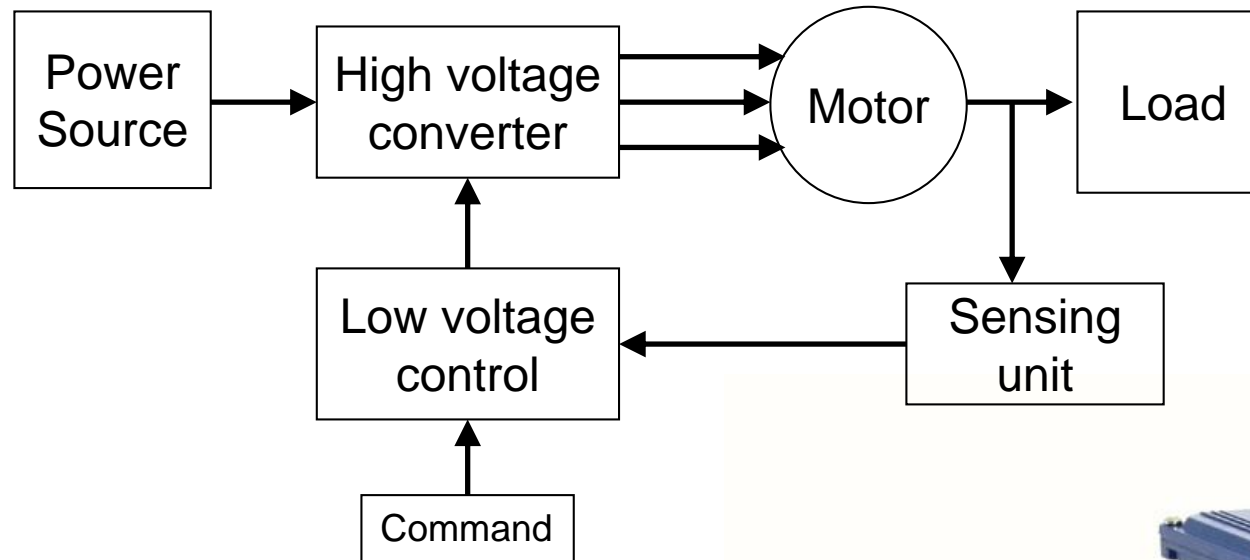
The DC side voltage or current ripples caused by converter commutation of three-phase converters can be much less than those of single-phase ones







## Variable speed motor drives.



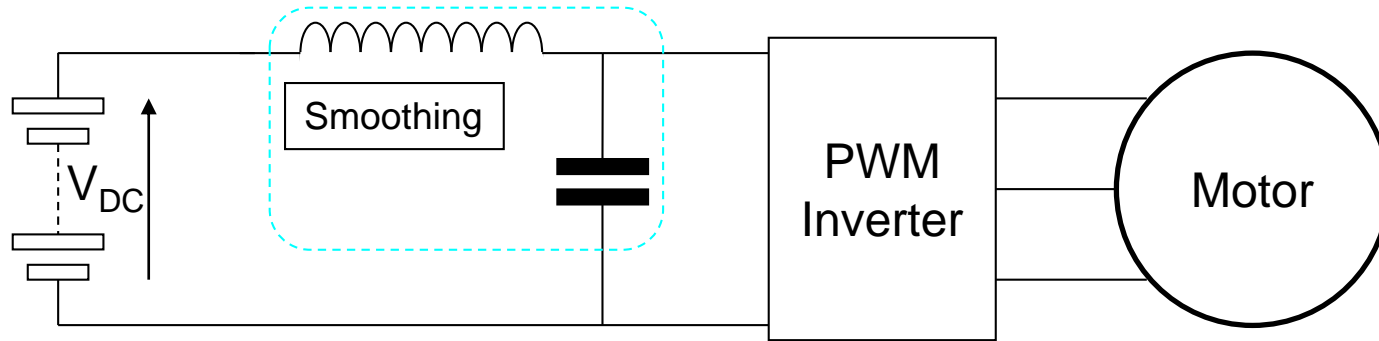
To effectively control the speed of an induction motor we need to be able to vary the frequency of the supply voltage to it. To do this we can use **a H bridge to synthesise a sine wave from a DC supply voltage. This process is called “inversion” and the circuitry to do it is called an “inverter”.**

Often the inverter is to be connected to an AC power source of fixed frequency. In this case the first step is therefore to obtain the necessary DC voltage to power the inverter.

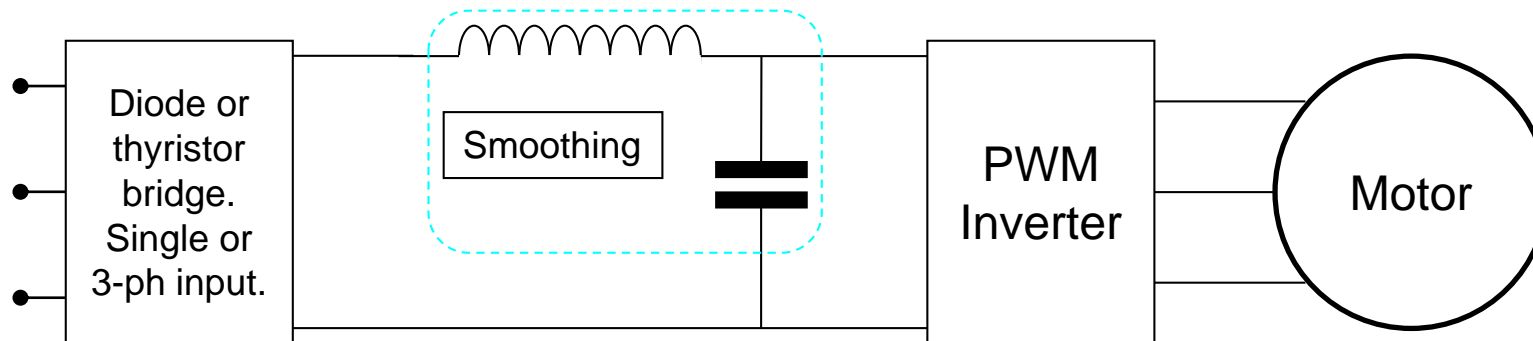


## PWM Inverter drives – power source.

For operation from a DC voltage, use:

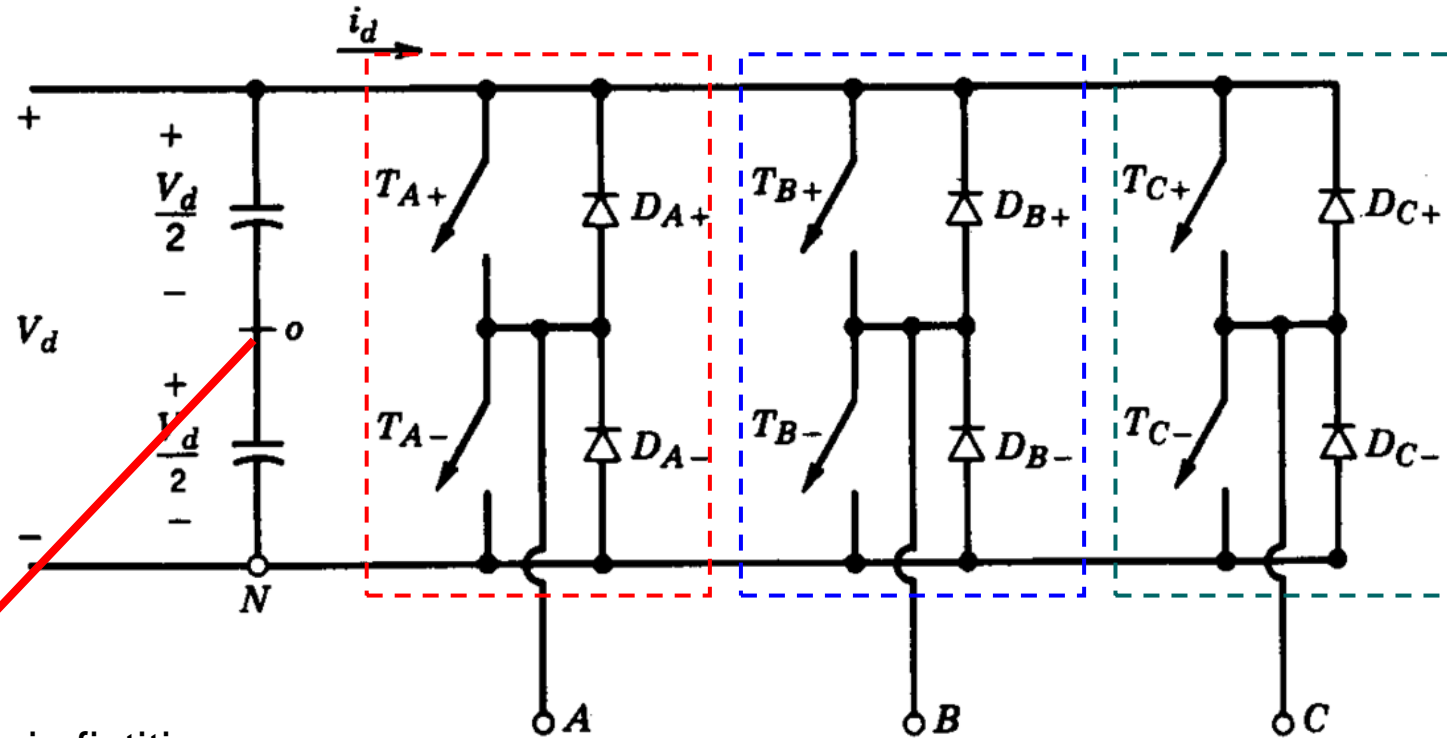


For operation from an AC voltage, use:



Regardless of the power source (AC or DC), the inverter uses what is called a DC Link and hence are known as "*DC Link Inverters*".

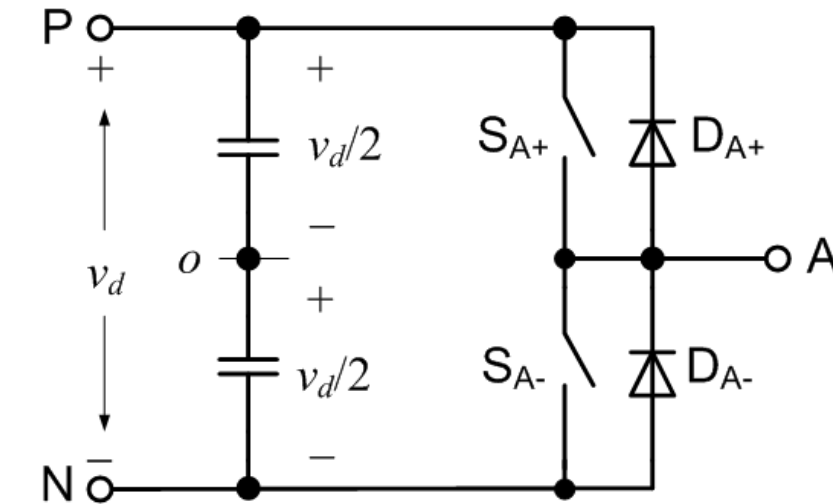
# Three-Phase SPWM Inverter: DC $\rightarrow$ 3-Phase AC



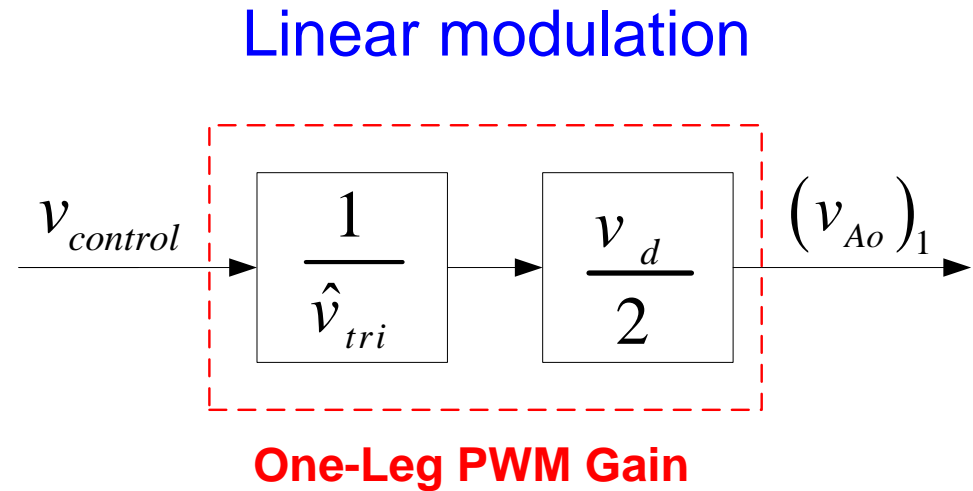
Capacitor mid-point  $o$  is fictitious  
reference point  $v_o=0$

**Three-Leg Inverter**

# One-Bridge-Leg PWM



Capacitor mid-point **o** is fictitious  
reference point  $v_o=0$



Neglecting Harmonic Ripples

$$(v_{Ao})_1 = \left( \frac{v_d}{2} \frac{1}{\hat{v}_{tri}} \right) v_{control} = \frac{v_d}{2} \left( \frac{v_{control}}{\hat{v}_{tri}} \right)$$

# One-Leg SPWM

$$v_{control} = \hat{v}_{control} \sin(\omega_1 t)$$

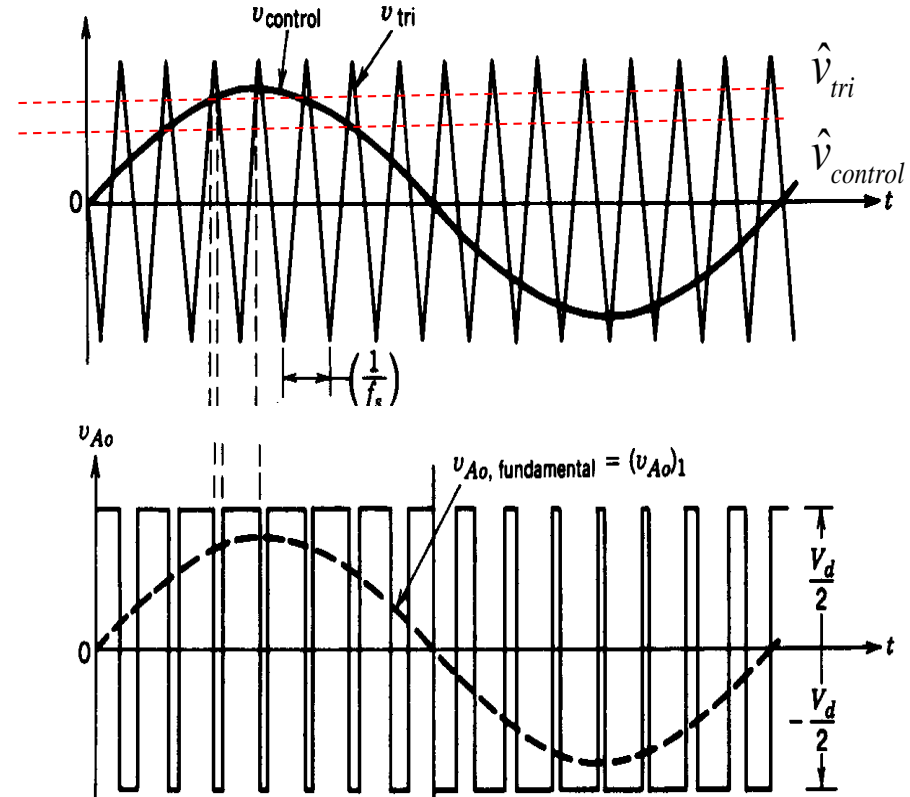
Normalized control signal

$$v_{na} = v_{control} / \hat{v}_{tri} = m_a \sin(\omega_1 t)$$

$$m_a = \hat{v}_{control} / \hat{v}_{tri},$$

Linear modulation ( $m_a \leq 1$ )

$$\begin{aligned} (v_{Ao})_1 &= \left( \frac{v_d}{2} \frac{1}{\hat{v}_{tri}} \right) v_{control} \\ &= \frac{v_d}{2} v_{na} = \frac{v_d}{2} m_a \sin(\omega_1 t) \end{aligned}$$



Neglecting  
Harmonic Ripples



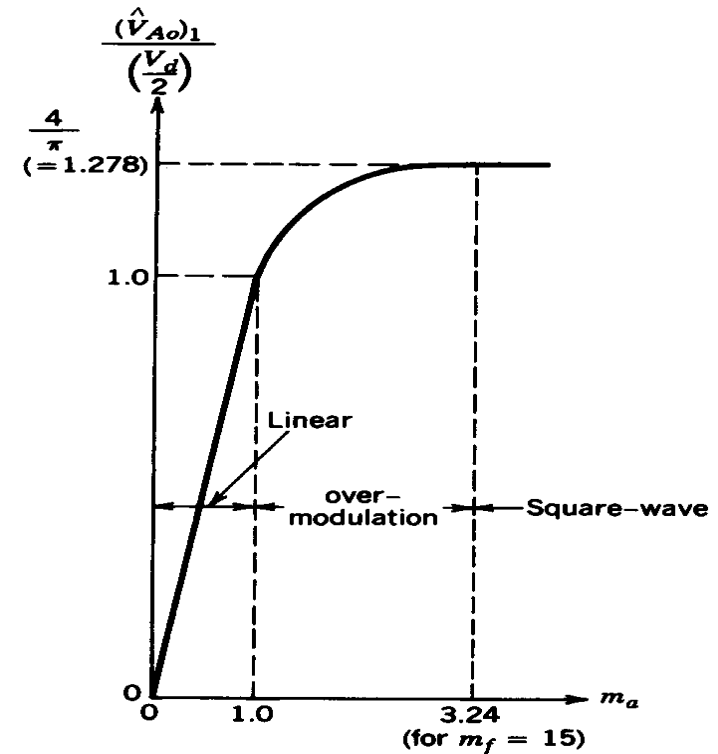
## Overmodulation ( $m_a > 1$ )

$$\frac{v_d}{2} \sin(\omega_1 t) < (v_{Ao})_1 \leq \frac{v_d}{2} \frac{4}{\pi} \sin(\omega_1 t)$$

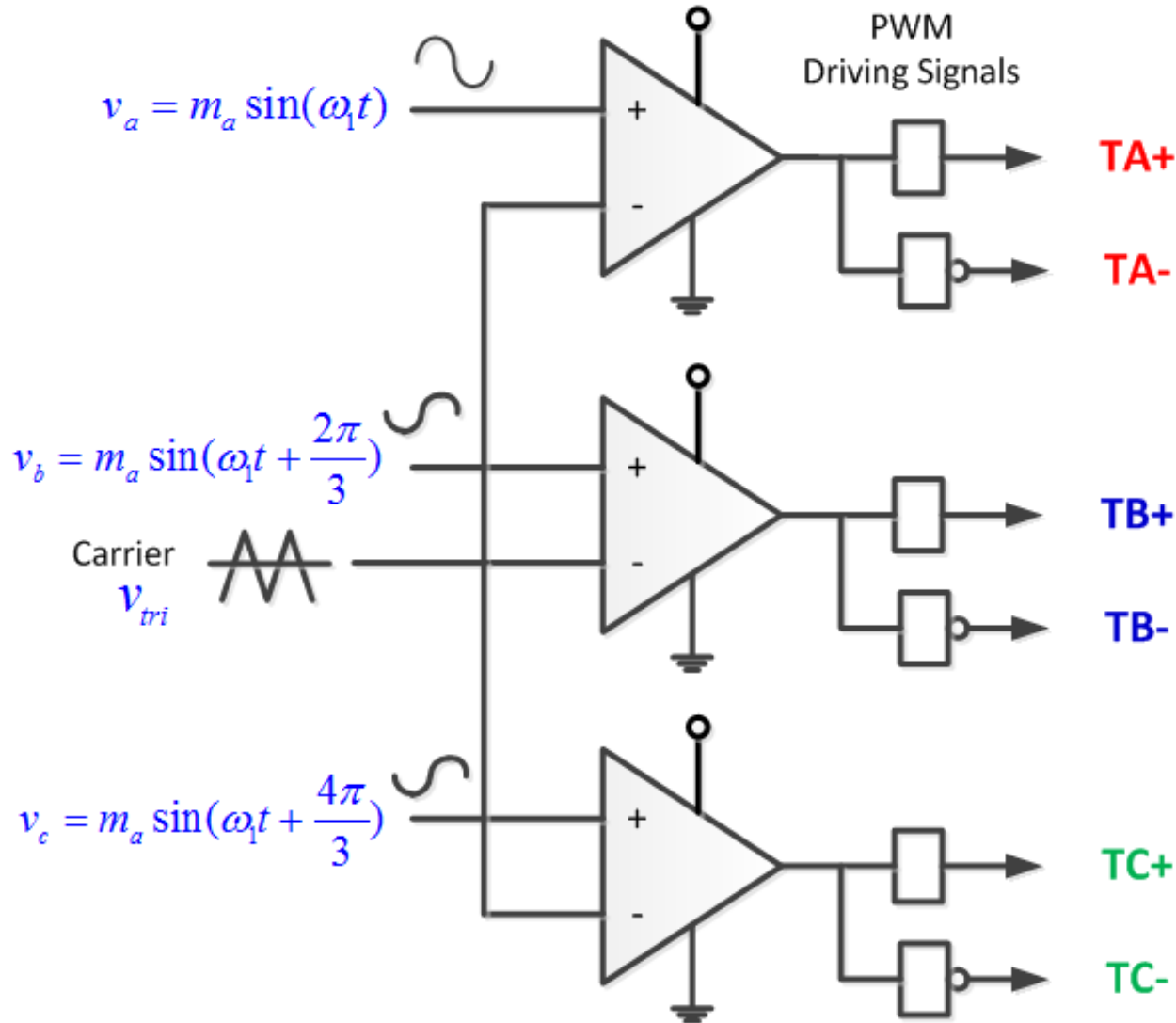
*square wave*

## Square-wave operation

$$(v_{Ao})_1 = \frac{v_d}{2} \frac{4}{\pi} \sin(\omega_1 t)$$



# Three-Leg SPWM Driving

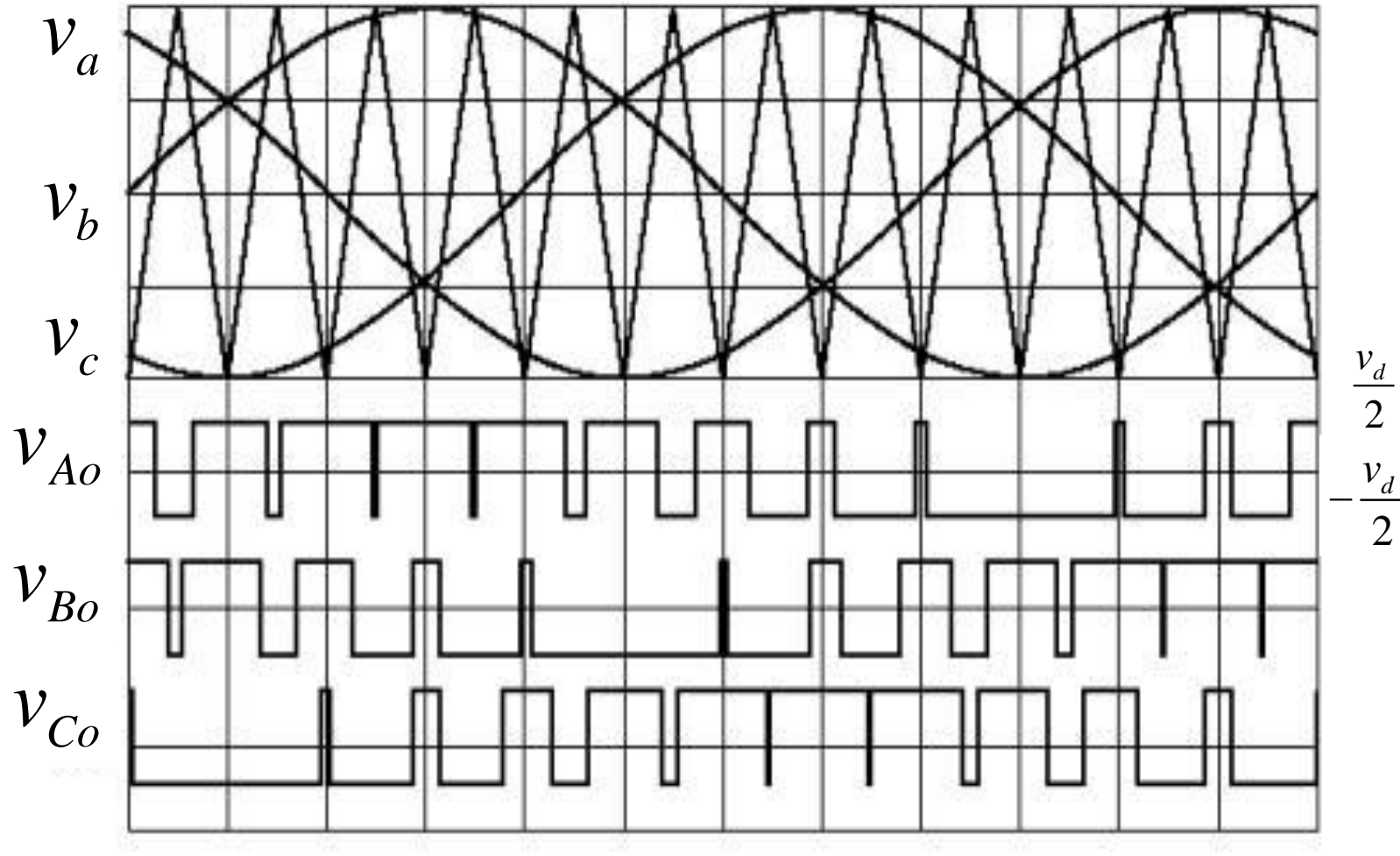


Unipolar full-bridge inverter:  
Phase shift between control signals for leg A and leg B is **180°**.

Phase shift between control signals for any two legs of the three-phase SPWM inverter is **120°**.

# Three-Phase SPWM

Use of Sawtooth Wave to Generate PWM Sine Waveform



**linear modulation**  
 **$(m_a \leq 1)$**

Three Phase Voltages

$$\begin{aligned} (v_{Ao})_1 &= \left( \frac{v_d}{2} \right) m_a \sin(\omega_1 t) \\ (v_{Bo})_1 &= \left( \frac{v_d}{2} \right) m_a \sin\left(\omega_1 t + \frac{2\pi}{3}\right) \\ (v_{Co})_1 &= \left( \frac{v_d}{2} \right) m_a \sin\left(\omega_1 t + \frac{4\pi}{3}\right) \end{aligned}$$

**Three Leg SPWM**

# Line-to-Line Voltages

$$\begin{aligned}(v_{AB})_1 &= (v_{Ao})_1 - (v_{Bo})_1 = \left(\frac{\sqrt{3}v_d}{2}\right)m_a \sin\left(\omega_1 t + \frac{\pi}{6}\right) \\(v_{BC})_1 &= (v_{Bo})_1 - (v_{Co})_1 = \left(\frac{\sqrt{3}v_d}{2}\right)m_a \sin\left(\omega_1 t + \frac{5\pi}{6}\right) \\(v_{CA})_1 &= (v_{Co})_1 - (v_{Ao})_1 = \left(\frac{\sqrt{3}v_d}{2}\right)m_a \sin\left(\omega_1 t + \frac{3\pi}{2}\right)\end{aligned}$$

Phase shift between  
any two line-to-line  
voltages is  $120^\circ$

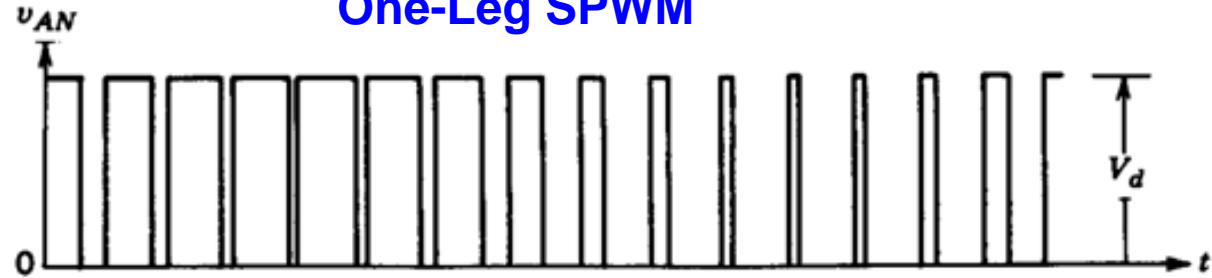
Since  $m_a \leq 1$  in linear modulation region, the  
maximum line-to-line voltage magnitude is

$$(\hat{v}_{AB})_1 = (\hat{v}_{BC})_1 = (\hat{v}_{CA})_1 = \frac{\sqrt{3}}{2}v_d < v_d$$

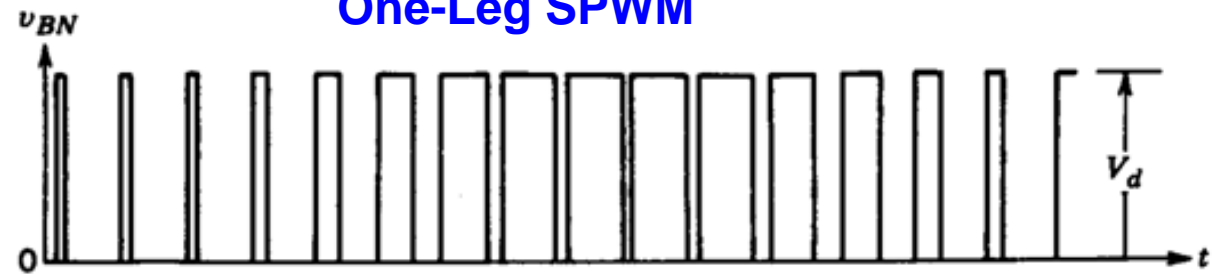
# Unipolar Line-to-Line SPWM

Three-phase  
SPWM inverter:  
phase shift  
between two leg A  
and leg B is  $120^\circ$

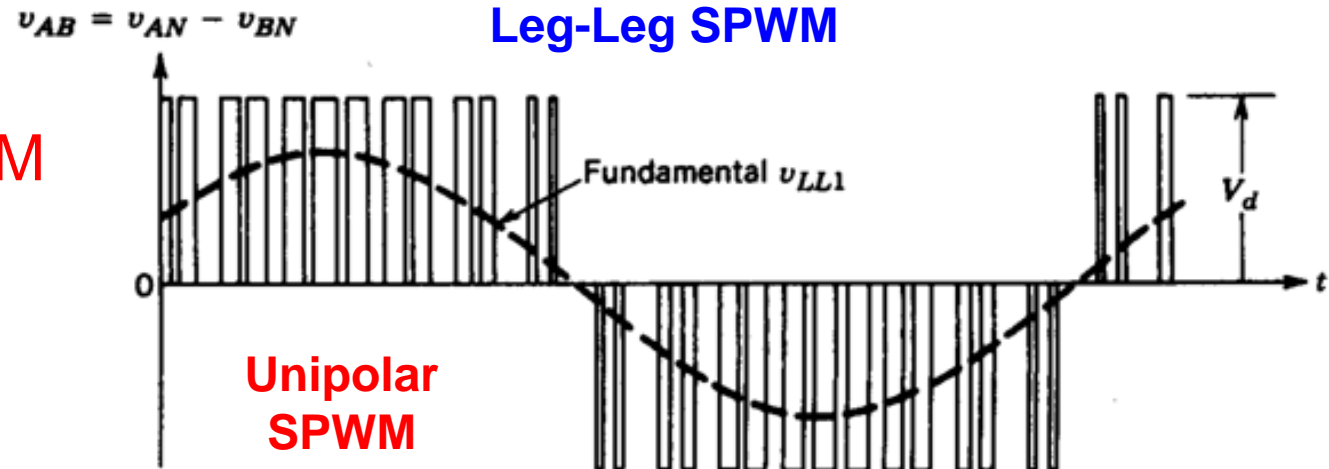
One-Leg SPWM



One-Leg SPWM



Leg-Leg SPWM



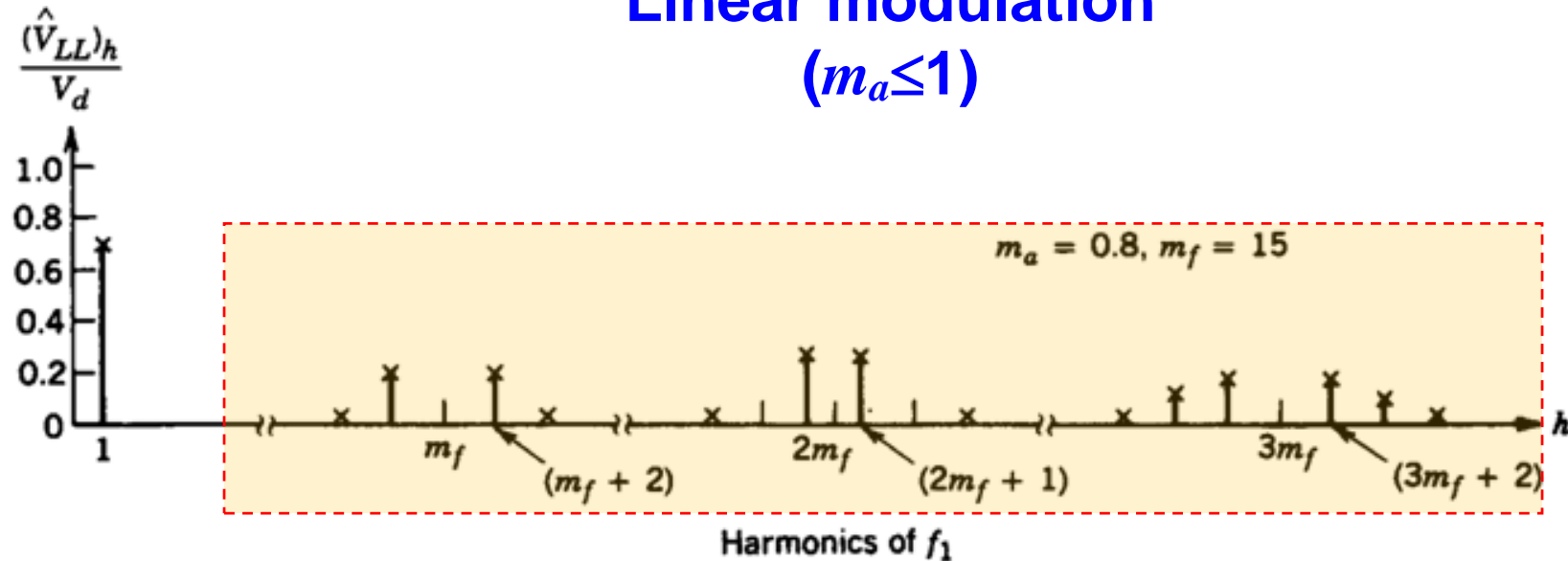
Line-Line SPWM  
voltage v<sub>AB</sub>

Switching  
Pattern

Unipolar  
SPWM

# Three-Phase SPWM Spectrum

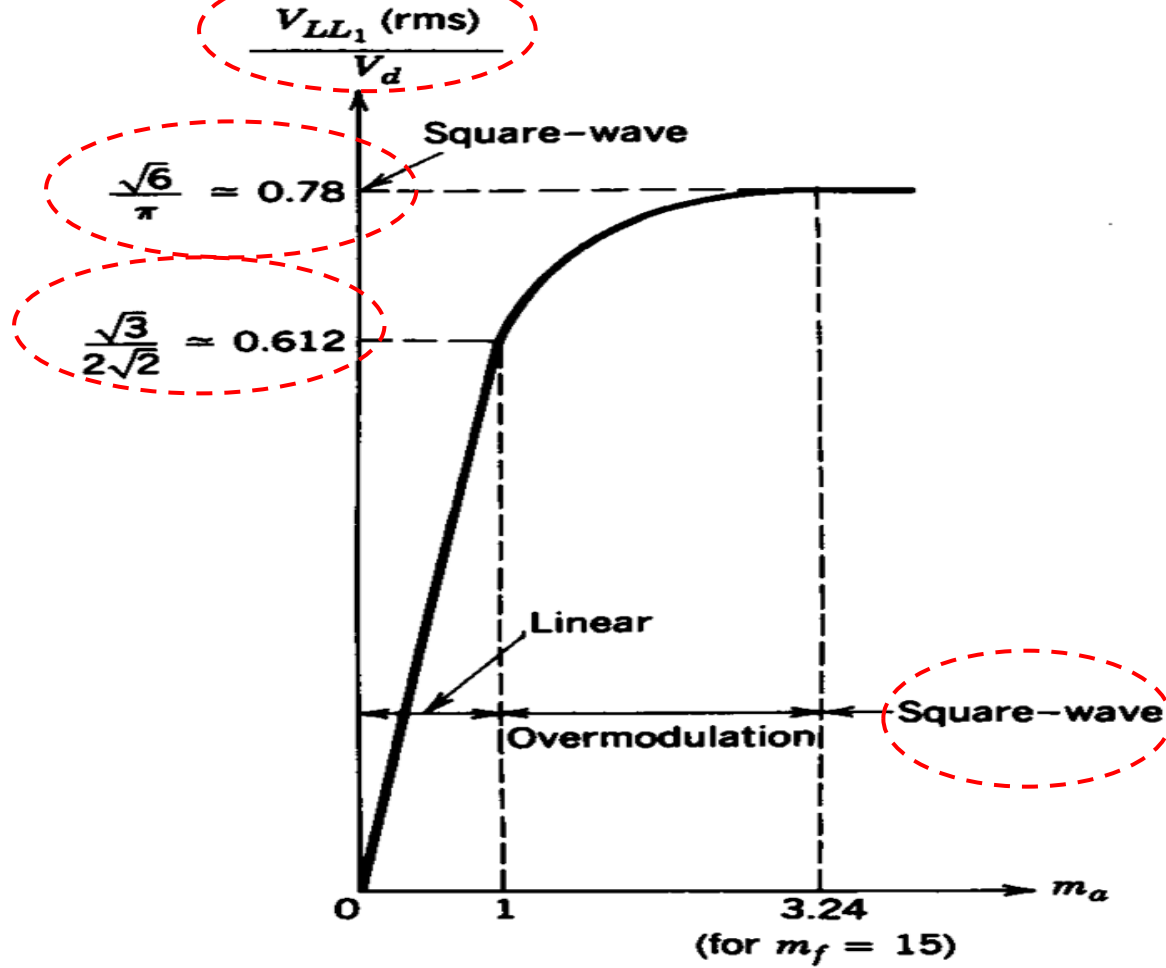
Linear modulation  
( $m_a \leq 1$ )



Harmonics still appear at frequencies of  $(nm_f \pm k)f_1$ , but their magnitude much less than bipolar one-leg SPWM.

Unlike unipolar SPWM of full bridge inverter, its switching frequency is not totally doubled.

# Three-Phase SPWM Gain



Very similar to single-phase SPWM

## Square-Wave Output

