Power Electronics 2 (ENG2045, SIT2004)
Tutorial sheet
Solutions

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2016 March 21

1 Fundamentals

1.1 The timebase of an oscilloscope is set to 10 µs per division and there are 10 divisions across the screen. Exactly four complete cycles of a signal can be seen. What are its period and (common) frequency?

Solution. The width of the screen is $10 \times 10\,\mu\text{s} = 100\,\mu\text{s}$. If four cycles can be seen, the period is a quarter of the width of the screen, 25 µs. The relation between period T and common frequency f is $f = 1/T$ so $f = 40\,\text{kHz}$. ❦

1.2 Write down the *general* relations between current and charge. The current flowing into a component is found to be $i(t) = I_0 \cos(\omega t)$ with $I_0 = 100\,\text{mA}$ and $f = 50\,\text{Hz}$. Derive an expression for the charge on the component if it is uncharged at $t = 0$.

Solution. The general relations are

$$q(t) = \int^t i(t')\,\mathrm{d}t' + \text{constant} \qquad \text{and} \qquad i(t) = \frac{\mathrm{d}q}{\mathrm{d}t}. \tag{1}$$

If $i(t) = I_0 \cos(\omega t)$, then

$$q(t) = \int^t I_0 \cos(\omega t')\,\mathrm{d}t' + \text{constant} = \frac{I_0}{\omega} \sin(\omega t) + \text{constant}. \tag{2}$$

Thus $q(t)$ oscillates as a sine wave, unlike the cosine for $i(t)$, and the peak value of the oscillations is $I_0/\omega = I_0/(2\pi f) = 318\,\mu\text{C}$. To find the constant, use $q = 0$ at $t = 0$, which shows that the constant is zero. Thus $q(t) = (I_0/\omega)\sin(\omega t)$. ❦

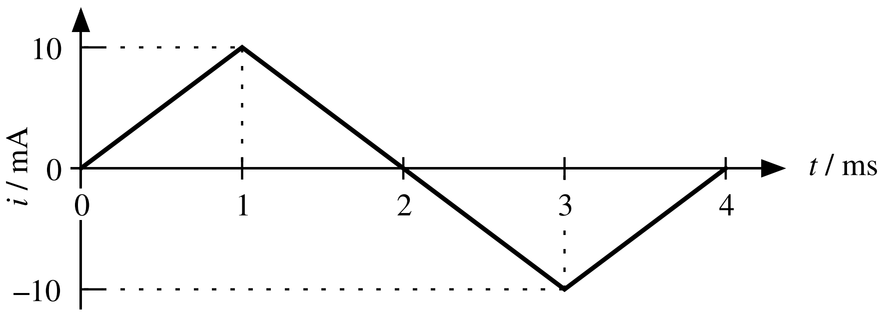

Figure 1. Current applied to a 10 mH inductor.

1.3 The current $i(t)$ shown in figure 1 is applied to a 10 mH inductor. Calculate the voltage $v(t)$ across the inductor as a function of time.

Solution. See figure 2. For the inductor, $v_L(t) = L\, di_L/dt$. Between $t = 0$ and 1 ms, the slope of $i_L(t)$ is a positive constant, $(10\,\text{mA})/(1\,\text{ms}) = 10\,\text{A s}^{-1}$. The voltage is therefore $v_L = 10\,\text{mH} \times 10\,\text{A s}^{-1} = 100\,\text{mV}$, constant throughout this period.

For $1\,\text{ms} < t < 3\,\text{ms}$, the slope has an equal but opposite value, so $v_L = -100\,\text{mV}$.

For $3\,\text{ms} < t < 4\,\text{ms}$ the voltage returns to $+100\,\text{mV}$. ❦

1.4 The current $i(t)$ shown in figure 3 is applied to a $10\,\mu\text{F}$ capacitor, which is initially uncharged. Calculate the voltage $v(t)$ across the capacitor as a function of time.

Solution. See figure 4. The voltage is given by $v(t) = (1/C) \int i(t)\, dt$. Between $t = 0$ and 0.5 ms the current is constant so the integral is linear in t. You can view it as the area under the curve. As an equation,

$$v(t) = \frac{1}{C} \int i(t)\, dt = \frac{(10\,\text{mA})\, t}{10\,\mu\text{F}} = (10^3\,\text{V s}^{-1})\, t. \tag{3}$$

When $t = 0.5\,\text{ms}$, $v = 0.5\,\text{V}$.

For $0.5\,\text{ms} < t < 1.5\,\text{ms}$, the current is again constant and the voltage is a linear function but this time the current is negative and so $v(t)$ decreases linearly with t. The slope has half the magnitude because the current is $-5\,\text{mA}$, compared with $+10\,\text{mA}$. The voltage starts from 0.5 V at 0.5 ms; it does not jump in any way. It falls linearly and reaches zero at 1.5 ms. ❦

*With grateful acknowledgment to Ravinder Dahiya, Vassilios Agelidis, Calum Cossar and Andrew Knox.

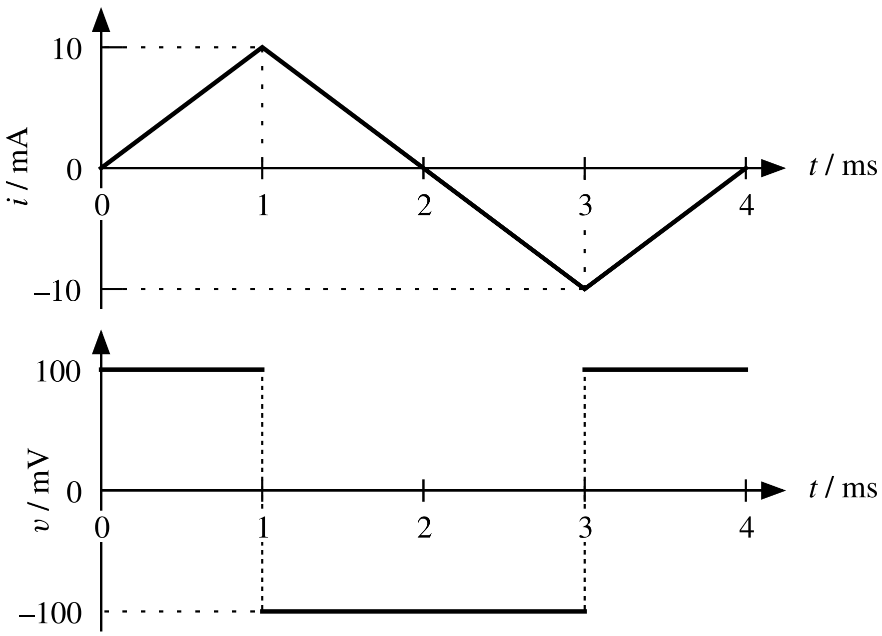

Figure 2. Voltage produced by current applied to a 10 mH inductor.

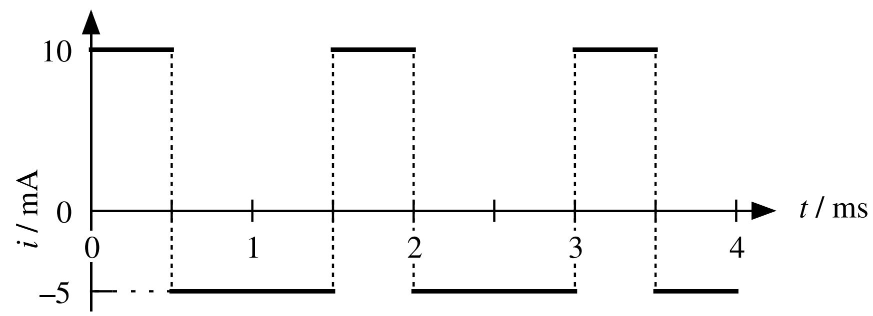

Figure 3. Current applied to a 10 μF capacitor, which is initially uncharged.

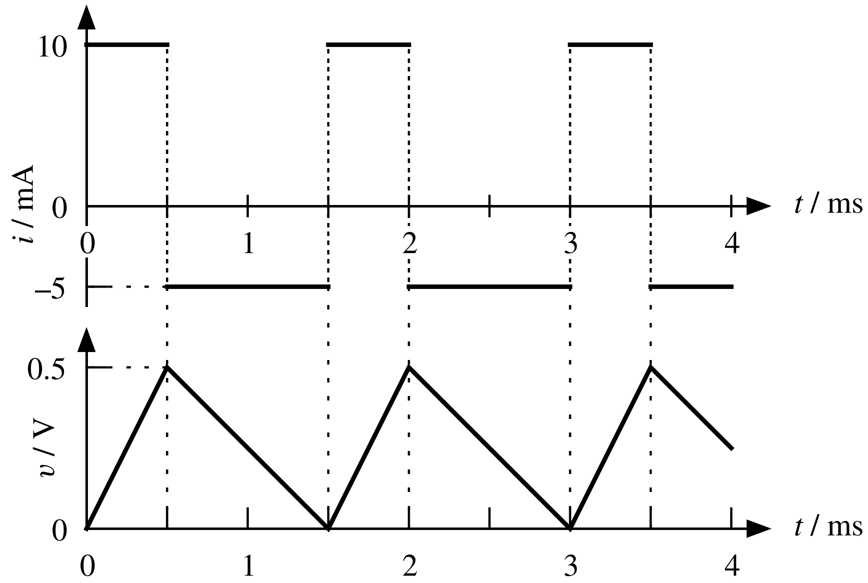

Figure 4. Voltage produced by current applied to a $10\,\mu F$ capacitor, which is initially uncharged.

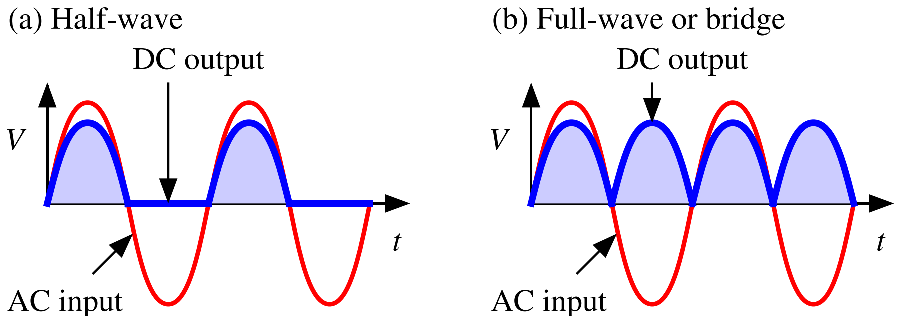

Figure 5. Half and full-wave or bridge rectification. Ignore the drop in voltage.

2 RMS values and and power

2.1 Consider the standard european AC mains, $v(t) = V_0 \sin(\omega t)$ with frequency 50 Hz and RMS voltage 230 V. First write down the peak value V_0. Then calculate the (i) average value, (ii) RMS value and (iii) form factor for

(a) the original sine wave

(b) the half-rectified wave

(c) the full-rectified wave.

The waves are shown in figure 5. Ignore losses in the diodes.

Solution. The peak value is $V_0 = \sqrt{2} V_{\text{RMS}} = \sqrt{2} \times 230\,\text{V} \approx 325\,\text{V}$.

For the original sine wave, the average value is zero because it is a symmetric function (pure AC). The RMS value is of course 230 V. For the form factor we need the average value of the *magnitude* of the wave, in other words the average of the full-wave rectified wave. This was covered in the lectures and repeated here for completeness. The full-wave rectified signal is $v_{\text{FW}}(t) = V_0 \left|\sin(\omega t)\right|$ and has double the frequency of the original wave. This means that we need integrate over only half the original period T. Remember that ω and T are related by $\omega = 2\pi f = 2\pi/T$ so $\omega T = 2\pi$.

$$\begin{aligned}
V_{\text{FW,ave}} &= \frac{1}{T/2} \int_0^{T/2} V_0 \sin \omega t \, \mathrm{d}t && (4) \\
&= \frac{2V_0}{T} \left[-\frac{\cos \omega t}{\omega} \right]_0^{T/2} && (5) \\
&= \frac{2V_0}{\omega T} \{-[\cos(\omega T/2)] + [\cos(0)]\} && (6) \\
&= \frac{2V_0}{2\pi} [-\cos(\pi) + \cos(0)] && (7) \\
&= \frac{V_0}{\pi} [-(-1) + 1] && (8) \\
&= \frac{2V_0}{\pi} \approx 207\,\text{V}. && (9)
\end{aligned}$$

The form factor is then

$$F_{\mathrm{AC}} = \frac{V_{\mathrm{RMS}}}{V_{\mathrm{ave}}} = \frac{V_0/\sqrt{2}}{2V_0/\pi} = \frac{\pi}{2\sqrt{2}} \approx 1.11\,. \tag{10}$$

That finishes the original wave. Note that the frequency cancels out of the averages after integration, as it should.

Now for the half-wave rectified wave. We can solve this without any further integration but it needs some thought. Remember that this has the positive half-cycle of the original sine wave but zero instead of the negative half-cycle. This means that the integrals for the RMS and average values are the same as the original wave for half the time but zero for the other half. The average is simply half the value that we have just calculated,

$$V_{\mathrm{HW,ave}} = \frac{V_0}{\pi} \approx 104\,\mathrm{V}. \tag{11}$$

The integral is halved for the RMS value as well but remember that the integral gives V_{RMS}^2, not V_{RMS} itself. This means that V_{RMS} is divided by $\sqrt{2}$, not 2. This shows that

$$V_{\mathrm{HW,RMS}} = \frac{V_0}{2} \approx 163\,\mathrm{V}. \tag{12}$$

A more physical way of thinking about this is that removing half of the cycles also halves the power delivered to a resistor. The power is proportional to V_{RMS}^2 so V_{RMS} must go down by a factor of $\sqrt{2}$. Finally, the form factor is

$$F_{\mathrm{HW}} = \frac{V_{\mathrm{RMS}}}{V_{\mathrm{ave}}} = \frac{V_0/2}{V_0/\pi} = \frac{\pi}{2} \approx 1.57\,. \tag{13}$$

The larger form factor means that the wave is more ‘spiky’ (loosely speaking).

Finally, the full-wave rectified wave needs almost no work at all. The sign of the voltages does not affect the power so the RMS value is the same as the original wave. The average value was calculated for the form factor of the original wave so we have that too. The form factor must therefore be the same as the original wave. Collecting these,

$$V_{\mathrm{FW,ave}} = \frac{2V_0}{\pi} \approx 207\,\mathrm{V}, \qquad V_{\mathrm{FW,RMS}} = \frac{V_0}{\sqrt{2}} = 230\,\mathrm{V}, \qquad F_{\mathrm{FW}} = \frac{\pi}{2\sqrt{2}} \approx 1.11\,. \tag{14}$$

❦

2.2 Find the average and RMS voltages and the form factor for the sawtooth function shown in figure 6, whose period is 10 ms and peak value is 200 V.

Solution. The average is clearly zero because the wave is symmetric. Figure 7 shows the waveforms involved in the solution.

The RMS value needs an integral and some thought saves a lot of work. In principle we should integrate from 0 to $T = 10\,\mathrm{ms}$. However, the symmetry of the wave means that

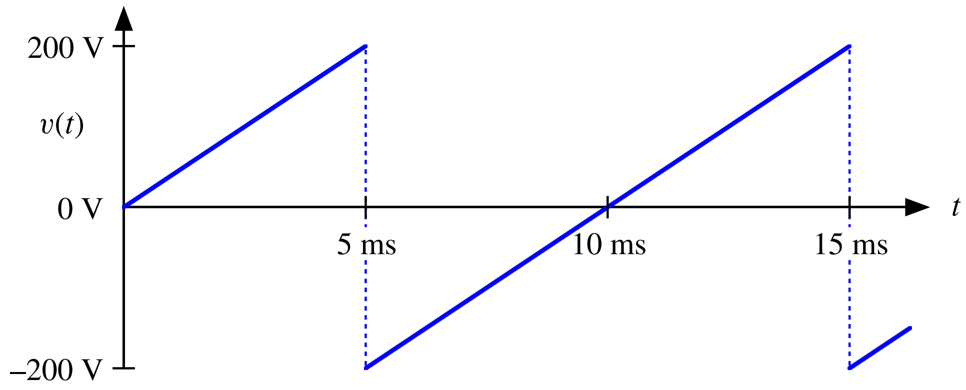

Figure 6. Sawtooth wave.

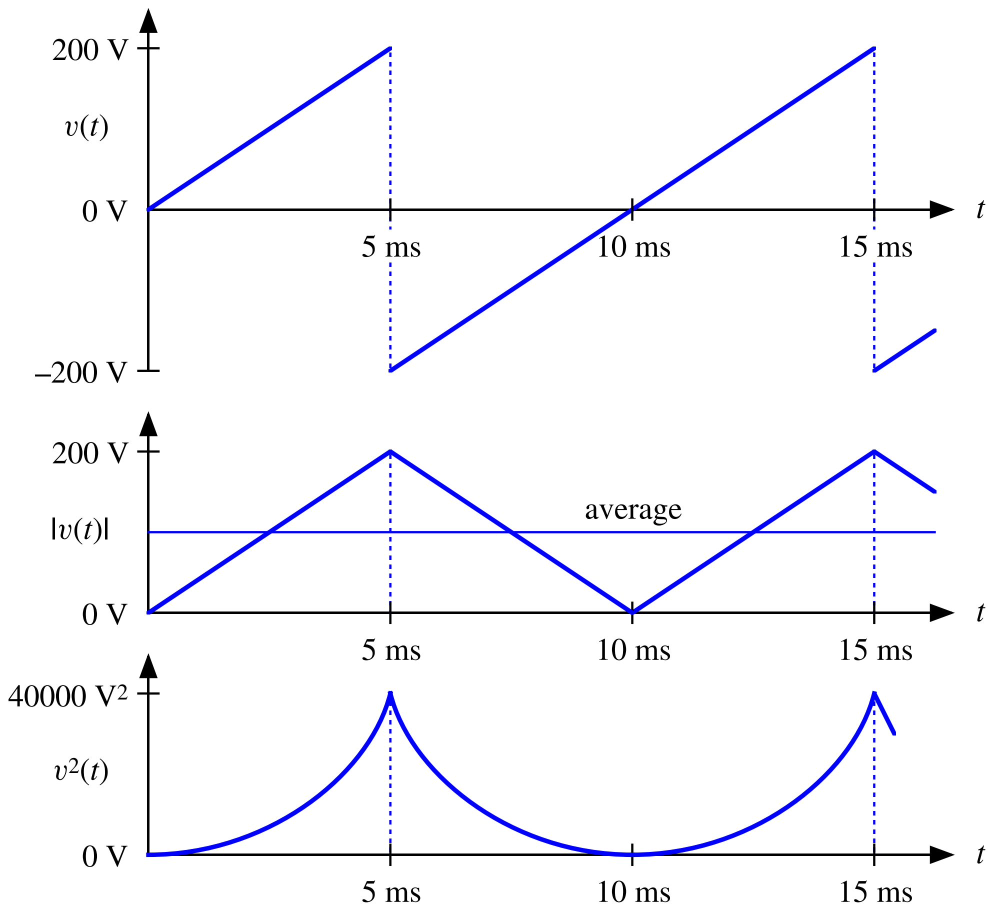

Figure 7. Sawtooth wave, its full-wave rectified form (magnitude) and the squared form, used for power and RMS.

we get the same result from $T/2$ to T as from 0 to $T/2$. We can therefore calculate the average by integrating only from 0 to $T/2$.

The wave has been chosen so that $v(0) = 0$ and the shape is a straight line. It goes through the peak value V_0 at $t = T/2$ so the slope is $2V_0/T$. Between these times we can write

$$v(t) = \frac{2V_0}{T}t. \tag{15}$$

I prefer to use symbols but if you prefer the numerical slope is $40\,\mathrm{V\,ms^{-1}} = 40\,\mathrm{kV\,s^{-1}}$. Now we can do the integral for the RMS value.

$$\begin{aligned}
V_{\mathrm{RMS}}^2 &= \frac{1}{T/2}\int_0^{T/2}\left[\frac{2V_0}{T}t\right]^2 \mathrm{d}t & (16)\\
&= \frac{8V_0^2}{T^3}\int_0^{T/2} t^2 \mathrm{d}t & (17)\\
&= \frac{8V_0^2}{T^3}\frac{(T/2)^3}{3} & (18)\\
&= \frac{8V_0^2}{T^3}\frac{T^3}{24} & (19)\\
&= \frac{V_0^2}{3} & (20)
\end{aligned}$$

so

$$V_{\mathrm{RMS}} = \frac{V_0}{\sqrt{3}} \approx 115\,\mathrm{V}. \tag{21}$$

For the form factor we need the average of the full-wave rectified wave (its magnitude). Again we can use symmetry to integrate this only from 0 to $T/2$.

$$\begin{aligned}
V_{\mathrm{FW,ave}} &= \frac{1}{T/2}\int_0^{T/2}\frac{2V_0}{T}t\,\mathrm{d}t & (22)\\
&= \frac{4V_0}{T^2}\int_0^{T/2} t\,\mathrm{d}t & (23)\\
&= \frac{4V_0}{T^2}\frac{(T/2)^2}{2} & (24)\\
&= \frac{4V_0}{T^2}\frac{T^2}{8} & (25)\\
&= \frac{V_0}{2} = 100\,\mathrm{V}. & (26)
\end{aligned}$$

This result should be obvious by inspection – don't forget that it is just an average, the result that you get by smoothing out the wave. Finally, the form factor is

$$F = \frac{V_{\mathrm{RMS}}}{V_{\mathrm{ave}}} = \frac{V_0/\sqrt{3}}{V_0/2} = \frac{2}{\sqrt{3}} \approx 1.15\,. \tag{27}$$

This is just a bit higher than the value for a sine wave. ❦

2.3 Show that the RMS value of a sinusoidal waveform with peak-to-peak amplitude $2A$ and DC offset B is given by

$$V_{\mathrm{RMS}} = \sqrt{\frac{A^2}{2} + B^2}. \tag{28}$$

Confirm that this is correct in the two limits $A = 0$ and $B = 0$.

Solution. It's easier to do the last part first. If $A = 0$ we are left with DC and $V_{\mathrm{RMS}} = |B|$, as it should be. If $B = 0$ the wave is a pure sine wave and $V_{\mathrm{RMS}} = |A|/\sqrt{2}$, again as it should be.

The general case has

$$v(t) = B + A\sin(\omega t). \tag{29}$$

The usual integral for RMS becomes

$$\begin{aligned} V_{\mathrm{RMS}}^2 &= \frac{1}{T}\int_0^T [v(t)]^2 \mathrm{d}t & (30)\\ &= \frac{1}{T}\int_0^T [B + A\sin(\omega t)]^2 \mathrm{d}t & (31)\\ &= \frac{1}{T}\int_0^T [B^2 + 2AB\sin(\omega t) + A^2\sin^2(\omega t)]\mathrm{d}t. & (32)\end{aligned}$$

It is straightforward to do this integral but we can be lazy and use well-known and previous results.

(a) The first term, B^2, is a constant so its average is also B^2.

(b) The middle term is an oscillating function and we are integrating over a complete cycle so its positive and negative parts cancel out to give zero.

(c) The third term is the usual integral for the RMS value of a pure sine wave so we have seen this before too.

This shows that the DC and sinusoidal parts make the same contributions to V_{RMS} as they would if they were by themselves. We therefore get

$$V_{\mathrm{RMS}}^2 = B^2 + \frac{A^2}{2}, \tag{33}$$

which gives the RMS value required. ❦

2.4 A square wave spends a fraction D of the time at voltage V_{on} and a fraction $(1 - D)$ at voltage V_{off}. Calculate its average and RMS voltages. You should not need any integrals. The duty cycle D lies between 0 and 1. Check your answer for the three cases $V_{\mathrm{off}} = V_{\mathrm{on}}$, $V_{\mathrm{off}} = -V_{\mathrm{on}}$ and $V_{\mathrm{off}} = 0$.

Solution. This time we can avoid integration! Figure 8 shows the wave assuming that $V_{\mathrm{off}} < 0$ and $V_{\mathrm{on}} > 0$ but this does not have to be the case.

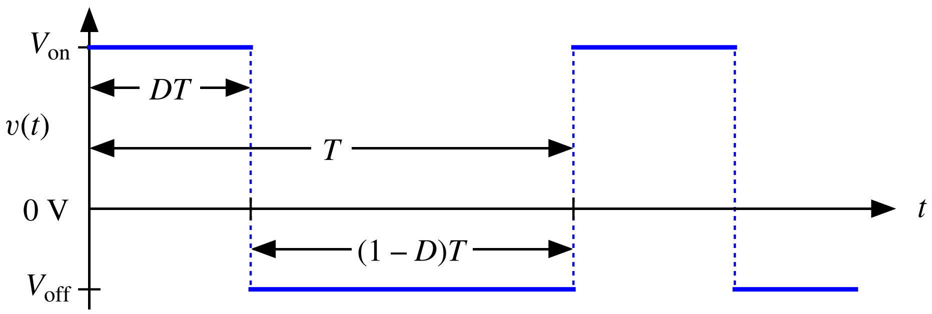

Figure 8. Square wave between V_{off} and V_{on} with duty cycle D.

The average value is just

$$V_{ave} = DV_{on} + (1 - D)V_{off}\,. \tag{34}$$

This is simple because D and $(1 - D)$ are the fractions of time spent at the two voltages. In the same way,

$$V_{RMS}^2 = DV_{on}^2 + (1 - D)V_{off}^2\,. \tag{35}$$

Remember that the integral gives V_{RMS}^2, not V_{RMS}. Taking the square root shows that

$$V_{RMS} = \sqrt{DV_{on}^2 + (1 - D)V_{off}^2}. \tag{36}$$

Now do the checks. Suppose that $V_{off} = V_{on}$. This is a complicated way of saying that $v(t) = V_{on}$ all the time so the voltage is DC. The equations given $V_{ave} = V_{RMS} = V_{on}$.

Now for $V_{off} = -V_{on}$. The RMS voltage is not affected because it depends on the *squared* voltage so the sign has no effect. The average value becomes

$$V_{ave} = DV_{on} - (1 - D)V_{on} = (2D - 1)V_{on}\,. \tag{37}$$

Finally, if $V_{off} = 0$ the voltages become $V_{ave} = DV_{on}$ and $V_{RMS} = \sqrt{D}V_{on}$. ❦

2.5 You need to buy power for a heater and a strange supplier offers you a choice of the following voltage waveforms at the same price. Which should you choose?

(a) A triangular wave, symmetric about zero, with a peak-to-peak voltage of 30 V.

(b) A sine wave, symmetric about zero, with a peak-to-peak voltage of 25 V.

(c) A square wave, symmetric about zero, with a peak-to-peak voltage of 15 V.

(d) A constant (DC) voltage of 10 V.

Solution. The power developed by the heater depends on the rms value of the voltage so we should compare these for the waveforms offered.

(a) The triangular wave has a peak voltage of 15 V (half the peak-to-peak voltage) and $V_{rms} = V_{pk}/\sqrt{3} = 8.7\,\text{V}$.

(b) The sine wave has a peak voltage of 12.5 V and $V_{rms} = V_{pk}/\sqrt{2} = 8.8\,\text{V}$.

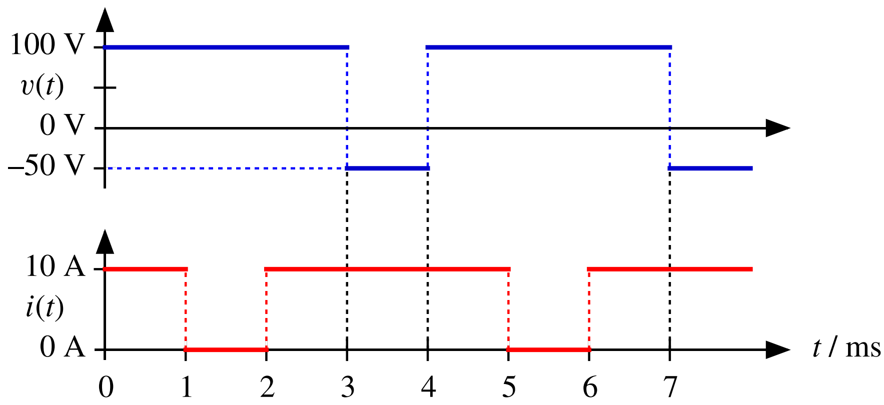

Figure 9. Voltage and current supplied to a component.

(c) The square wave has a peak voltage of 7.5 V and this is the same as the rms voltage.

(d) The rms voltage is the same as a constant (DC) voltage and is therefore 10 V.

The constant 10 V supply is clearly the best choice. ❦

2.6 The voltage $v(t)$ and current $i(t)$ shown in figure 9 are supplied to a component. Sketch the power $p(t)$ supplied to the component and calculate the average value over its 4 ms period.

Solution. The sketch is shown in figure 10. It is obtained using $p(t) = i(t)v(t)$. Power is absorbed by the component when $p(t) > 0$ and released by it when $p(t) < 0$. A resistor would have $p(t) \geq 0$ at all times while a capacitor or inductor would absorb and release equal energies over each cycle. This component is clearly none of these.

We must also find the average power. The period is 4 ms so we can calculate the total energy supplied over this period and divide by 4 ms to get the average power. In the first 1 ms the energy is power × time or $1000\,\text{W} \times 1\,\text{ms} = 1\,\text{J}$. It is zero in the second millisecond, 1 J again in the third millisecond and $-0.5\,\text{J}$ in the fourth millisecond because the current and voltage have opposite signs. The total energy is therefore 1.5 J in 4 ms so the average power is $P_{\text{ave}} = 1.5\,\text{J}/4\,\text{ms} = 375\,\text{W}$. ❦

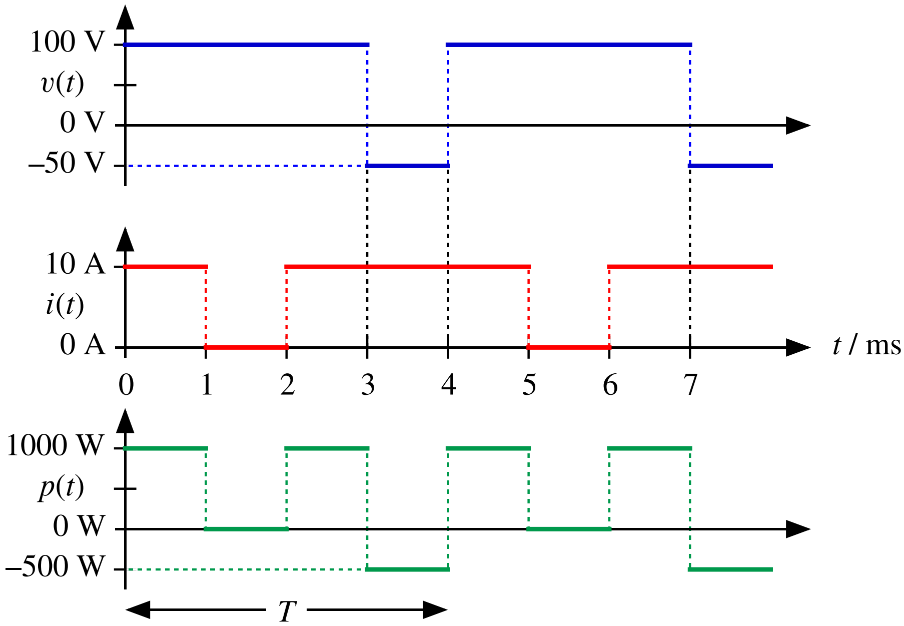

Figure 10. Voltage, current and resultant power supplied to a component.

Table 1. Comparison of half-wave, full-wave and bridge rectifiers.

	half-wave	full-wave	bridge
windings needed	one	two (centre-tapped)	one
utlilization	half	half (one winding at a time)	full
diodes needed	1	2	4
reverse rating	V_{peak}	$2V_{\text{peak}}$	V_{peak}
voltage drop	0.7 V	0.7 V	1.4 V
frequency of ripple	f_{input}	$2f_{\text{input}}$	$2f_{\text{input}}$
ease of smoothing	difficult (empty half cycles)	easier	easier

3 Rectifiers

The *regulation* of a power supply is often defined by

$$\text{regulation} = \frac{V_{\text{no load}} - V_{\text{full load}}}{V_{\text{no load}}}. \tag{38}$$

The regulation is often expressed as a percentage and a good power supply has a *small* value of regulation (an ideal power supply has zero regulation). Sometimes the denominator has $V_{\text{full load}}$ instead of $V_{\text{no load}}$ but it makes no significant difference for a good supply.

Alternatively, a power supply can be modelling with a Thévenin equivalent circuit comprising an ideal voltage supply in series with a Thévenin resistance R_s. In this case R_s measures the regulation because it absorbs 'lost volts' when a current is drawn from the supply. A good supply has a low value of R_s, ideally zero.

3.1 Draw up a table to compare half-wave, full-wave and bridge rectifiers with the headings: windings needed on transformer; utilization of transformer; number of diodes needed; reverse voltage rating of diodes; voltage drop across diodes (assuming typical silicon components); frequency of ripple; ease of smoothing. Assume that no smoothing capacitor is used.

Solution. See table 1. ❦

3.2 A bridge rectifier is required to supply an average current of 200 mA with ripple not to exceed 0.5 V. What size of smoothing capacitor is needed? The AC supply is at 50 Hz. [around 2400 μF]

How would your answer change if the supply were designed for North America rather than Europe?

Solution. A pessimistic approach is to assume that the capacitor must supply 200 mA for the full period of the ripple. This is half the period of the input or 10 ms in Europe. The charge required is $0.2 \times 0.01 = 0.002$ C. The change in voltage must not exceed 0.5 V and $Q = CV$ so $C = Q/V = 0.002/0.5 = 0.004$ F or 4000 μF.

It would be less pessimistic to assume that the capacitor supplies the current for only 60% of the time, which reduces C by the same factor to 2400 µF; the nearest conventional values are 2200 µF and 3300 µF.

The frequency of the mains (line) supply is 60 Hz rather than 50 Hz in North America, which reduces the period of each ripple and therefore the capacitance needed. The 'optimistic' estimate falls from 2400 µF to 2000 µF. Still big! ❦

3.3 Design a power supply with a transformer, bridge rectifier and smoothing capacitor to work from the AC mains in Europe and supply 9 V at 200 mA. Ripple on the output should not exceed 0.5 V. Assume that the secondary winding of the transformer has a resistance of 10 Ω. Calculate the value of smoothing capacitor and the (rms) output voltage of the transformer needed. What voltage rating should be specified for the capacitor, considering both no load and full load? Estimate the no-load voltage and regulation.

[Roughly 2400 µF, 12 V; 17 V; 40% or 30 Ω]

Solution. The 'AC mains in Europe' implies 230 VAC at 50 Hz. Calculate the smoothing capacitor needed first. This needs no effort at all because the current and ripple are the same as in the previous question. The average voltage has no effect on the value of capacitance; this depends only on the average current and tolerable ripple voltage. Thus $C = 2400\,\mu\text{F}$. For completeness we should also specify the voltage and ripple current ratings. The obvious peak voltage is the average output of 9 V plus half the ripple of 0.5 V (remember that this is a peak-to-peak value), giving 9.25 V. This is much too low, as we shall see shortly. The peak current into the capacitor every time it recharges is about 3 times the average current to the load or 600 mA and the ripple current rating must not be lower than this.

Now for the voltage required from the transformer. The peak value of output voltage is 9.25 V. Adding two diode drops from the bridge gives 10.8 V. The peak current from the transformer is around 600 mA, about 3 times the average output current, and the resistance of the transformer was given as 10 Ω, which yields a voltage drop of 6 V (*very* roughly). Adding this gives a peak voltage of 17 V to the nearest volt. Divide by $\sqrt{2}$ to find an rms voltage of 12 V. It is depressing that a 12 V transformer is needed for 9 V output.

Work forward from the input to find the no-load output voltage. The transformer provides 17 V peak without any drop in its winding because no current flows. The diodes reduce this to about 15.5 V. The smoothing capacitor will charge fully to the peak voltage and not discharge between cycles because no current is drawn from it. The no-load output voltage is therefore also 15.5 V. In fact it might well be higher because the diodes will drop a smaller voltage when so little current flows; the output could rise much closer to the peak voltage from the transformer of 17 V. The smoothing capacitor must therefore work safely at this voltage, much higher than the nominal 9 V output.

The regulation is clearly terrible, although not unrealistic for a cheap 'wall wart'. The usual definition in equation (38) with the no-load and full-load voltages of 15.5 V and 9.0 V, gives about 40%. Alternatively, the supply can be modelled with a Thévenin resistance of $R_\mathrm{s} = 32.5\,\Omega$. This seems a large resistance: where does it come from?

First, the imperfect smoothing by the capacitor causes the average voltage to drop when more current is drawn. This is just half of the (peak-to-peak) ripple voltage. The ripple is zero when no current flows and 0.5 V at 200 mA so the average voltage drops by 0.25 V at 200 mA, which is equivalent to a resistance of 1.25 Ω. The other obvious resistance is 10 Ω from the winding of the transformer. Unfortunately this is multiplied because of the peaked nature of the current, which rises to 600 mA rather than the average of 200 mA. It therefore gives a much higher voltage drop then might be expected, and the factor of 3 in current multiplies its effective resistance to 30 Ω. The sum of these is 31.25 Ω, which differs from 32.5 Ω because of rounding; the final estimate of the voltage required from the transformer can be trusted only to the nearest volt (if that). The important finding is that the transformer dominates the regulation of this supply and should be replaced by a superior component, with a lower resistance, if the performance of the power supply needs to be improved. ❦

4 Driving low-power loads

4.1 Design an 'active high' circuit for connecting a light-emitting diode (LED) to an output pin of a microcontroller. The LED is designed to operate at a current of 5 mA and drop 1.8 V. The circuit is supplied from 3.3 V.

Solution. This needs a resistor in series with the LED. The two components should be corrected from the output pin of the microcontroller to ground and the LED must be forward biassed.

If the supply is 3.3 V and the LED drops 1.8 V, this leaves 1.5 V across the resistor. The current is 5 mA. Dividing these gives a resistance of 300 Ω. (The nearest conventional values are 270 Ωand 330 Ω.) ❦

4.2 An n-channel MOSFET has threshold voltage $V_{\mathrm{T}} = 1.0\,\mathrm{V}$ and constant $K = 0.5\,\mathrm{A\,V^{-2}}$.

(a) Is this an enhancement-mode or depletion-mode device?

(b) Calculate the resistance of the channel R_{DS} for gate voltages of 3.3 V and 5.0 V.

(c) How much voltage is dropped across the MOSFET if it passes a drain current of $i_{\mathrm{D}} = 300\,\mathrm{mA}$ for the same two gate voltages?

(d) How much power P_{diss} is dissipated in the MOSFET for these two cases?

(e) What gate voltage produces a channel resistance $R_{\mathrm{DS}} = 1.0\,\Omega$?

Solution. This needs the relation

$$i_{\mathrm{D}} = K(v_{\mathrm{GS}} - V_{\mathrm{T}})v_{\mathrm{DS}}\,. \tag{39}$$

This holds only for small values of the drain–source voltage v_{DS}, in other words when the transistor is turned on as a switch rather than an amplifier. The resistance of the channel is

$$R_{\mathrm{DS}} = \frac{v_{\mathrm{DS}}}{i_{\mathrm{D}}} = \frac{1}{K(v_{\mathrm{GS}} - V_{\mathrm{T}})}. \tag{40}$$

(a) It is an enhancement-mode device because $V_{\mathrm{T}} > 0$.

(b) The equation above gives $R_{\mathrm{DS}} = 0.87\,\Omega$ and $0.50\,\Omega$.

(c) Multiplying R_{DS} by the current gives $v_{\mathrm{DS}} = 0.26\,\mathrm{V}$ and $0.15\,\mathrm{V}$.

(d) Multiplying the voltages by the current again gives $P_{\mathrm{diss}} = 78\,\mathrm{mW}$ and $45\,\mathrm{mW}$.

(e) The equation above gives $R_{\mathrm{DS}} = 1.0\,\Omega$ at $v_{\mathrm{GS}} = 3.0\,\mathrm{V}$.

❦

4.3 A digital system works from a supply at +5.0 V and its outputs can produce up to ±5 mA. It is required to control a load that draws 500 mA from a 12 V supply. Consider the following options for a switch for this load. Explain whether each is suitable and design the complete circuit, including the values of any other components required. All devices are rated to carry at least 500 mA and withstand at least 12 V.

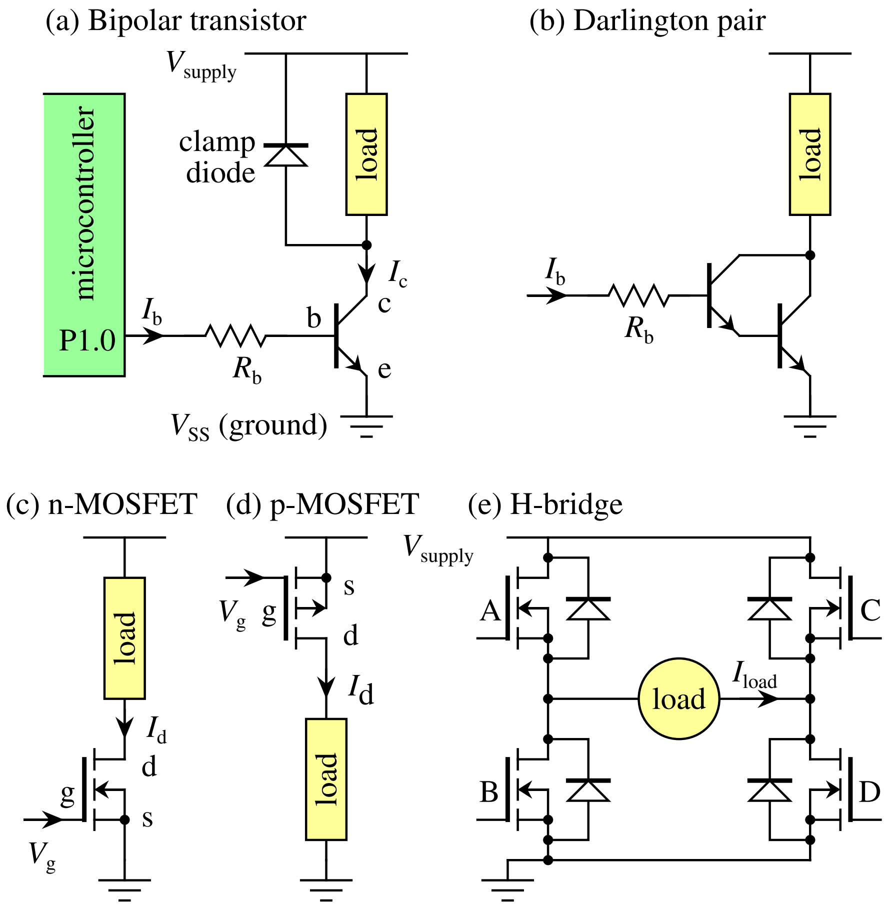

Figure 11. Standard circuits for switching a load on and off, controlled by a microcontroller or other digital system.

(a) Single BFY51 npn transistor with $\beta = 40$ and maximum power dissipation of 800 mW (without heatsink).

(b) Darlington pair with $\beta = 500$ and maximum power dissipation of 1.5 W.

(c) Single FDV301N n-channel MOSFET with threshold voltage $V_{\mathrm{T}} = 0.8\,\mathrm{V}$, $K = 0.07\,\mathrm{A\,V^{-2}}$ and maximum power dissipation of 0.35 W.

Solution. The general approach is to use a 'low-side' switch between ground and the 'bottom' of the load. This can be an npn bipolar transistor or an n-channel MOSFET. See figure 11 for standard circuits. We cannot use a 'high-side' switch between the load and its 12 V supply because the digital system would have to produce 12 V to turn the transistor off, which it cannot do with only a 5.0 V supply. (In practice you need to do this sometimes and can use another transistor as a level shifter. Special devices are available for when the supply is at a much higher voltage and might endanger the digital system.)

(a) The single transistor does not work. The load current gives $i_{\mathrm{C}} = 500\,\mathrm{mA}$ for the transistor, which requires a base current of at least $i_{\mathrm{B}} = i_{\mathrm{C}}/\beta = (500\,\mathrm{mA})/40 = 13\,\mathrm{mA}$. This is more than the digital system can provide. In practice a larger base current should be used so that the transistor is driven into saturation.

(The BFY51 is an elderly medium-power transistor, popular in the past.)

(b) The Darlington pair has a much higher value of $\beta = 500$. this gives a base current of $i_B = i_C/\beta = (500\,\text{mA})/500 = 1.0\,\text{mA}$. It is better to allow a larger current to ensure that the transistor is saturated, say a factor of 2, giving $i_B = 2.0\,\text{mA}$. (There is no fixed number for this factor.) The digital system has no problem providing this.

The circuit is shown in figure 11(b) and we need to calculate R_b. The voltage on the base of the Darlington pair is about 1.5 V, twice the normal value because the emitter–base junctions of the two transistors are connected in series. The output of the digital system is at 5.0 V, which leaves $(5.0 - 1.4) = 3.6\,\text{V}$ across the resistor. The value of resistance is therefore $R_b = (3.6\,\text{V})/(2.0\,\text{mA}) = 1.8\,\text{k}\Omega$. This is a standard value.

Finally we should check the power dissipation. The collector–emitter voltage of the Darlington pair is given by $v_{CE} = v_{CE1} + v_{BE2}$, where Q1 is the input transistor and Q2 carries the main current from the load. This gives $v_{CE} = (0.2 + 0.7)\,\text{V} = 0.9\,\text{V}$. The load current flows into the collector so $i_C = 0.5\,\text{A}$. Multiplying these gives the dissipated power $P_{diss} = i_C v_{CE} = (0.5\,\text{A}) \times (0.9\,\text{V}) = 0.45\,\text{W}$. This is safely below the limit of 1.5 W.

(c) The circuit for the n-channel MOSFET is shown in figure 11(c). The threshold voltage is positive, which is important or the transistor would not switch off when the output from digital system is at ground for logic 0. In other words, we must use an enhancement-mode transistor. No additional components are required but we must check the power dissipation.

Recall the usual equation for a MOSFET used as a switch (in its linear region)

$$i_D = K(v_{GS} - V_T)v_{DS}\,. \tag{41}$$

$$R_{DS} = \frac{v_{DS}}{i_D} = \frac{1}{K(v_{GS} - V_T)}. \tag{42}$$

Here we have $v_{GS} = 5.0\,\text{V}$, $V_T = 0.8\,\text{V}$ and $K = 0.07\,\text{A}\,\text{V}^{-2}$, which gives $R_{DS} = 3.4\,\Omega$. The voltage drop is therefore $v_{DS} = R_{DS} i_D = (3.4\,\Omega) \times (0.5\,\text{A}) = 1.7\,\text{V}$. The power dissipation is $P_{diss} = i_D v_{DS} = (0.5\,\text{A}) \times (1.7\,\text{V}) = 0.85\,\text{W}$. This exceeds the limit of 0.35 W so the circuit cannot be used.

(The FDV301N is a so-called 'digital FET' in a SOT-23 package intended for switching loads under the control of a digital system. I have cheated in this exercise because its maximum current rating is only 0.22 A, not 0.5 A, and I wanted an example that would not work correctly. This device used to be popular but is now a fairly old product and better devices are available.)

❦

4.4 A load requires 5 V and its negative terminal is connected to its metal case, which must be grounded to dissipate heat. Draw a suitable circuit for switching it on and off from a digital system that also operates from 5 V.

Would your approach work if the digital system worked at 3 V rather than 5 V?

Solution. This requires a high-side switch between the load and the 5 V supply. It could be a pnp transistor or a p-channel MOSFET as shown in figure 11(d). This would not work with a 3 V digital system because it would not be possible to turn the transistor off, unless it had an unusually high threshold voltage (the threshold voltage of an enhancement-mode p-MOSFET is negative and it would have to be more negative than -2 V). We would need to use another transistor to drive the p-MOSFET or something completely different, such as a relay. ❦

5 Rectifiers, smoothing capacitors and linear regulators

5.1 Draw up a table to compare half-wave, full-wave and bridge rectifiers with the headings: windings needed on transformer; utilization of transformer; number of diodes needed; reverse voltage rating of diodes; voltage drop across diodes (assuming typical silicon components); frequency of ripple; ease of smoothing.

Solution. See table 2.

❦

5.2 The secondary winding of a transformer is rated at $V_{\mathrm{AC}} = 12.0\,\mathrm{V}$ and runs at 60 Hz. It can be connected to a $100\,\Omega$ load through different rectifiers, without or with a smoothing capacitor. Ignore the voltage drop in the diodes and assume that the capacitor is so large that it reduces ripple to a negligible value. First, what sort of voltage is the '12 V' quoted for the transformer?

(a) The load is connected directly to the transformer. Calculate the average voltage V_{ave} and RMS voltage V_{RMS} across the load and the power P dissipated in it.

(b) The load is connected to the transformer through a half-wave rectifier (single diode). Repeat the calculations.

(c) A smoothing capacitor is now connected across the load. Repeat the calculations.

(d) The load is connected to the transformer through a bridge rectifier (four diodes) and the smoothing capacitor is removed. Repeat the calculations.

(e) A smoothing capacitor is now connected across the load. Repeat the calculations.

Comment on the power dissipated by the resistor for the five cases.

Solution. The voltage is RMS, which is always the case (by convention) for standard transformers.

Table 2. Table to compare half-wave, full-wave and bridge rectifiers.

	half-wave	full-wave	bridge
windings needed	one	two (centre-tapped)	one
utlilization	half	half (one winding at a time)	full
diodes needed	1	2	4
reverse rating	V_{peak}	$2V_{\mathrm{peak}}$	V_{peak}
voltage drop	0.8 V	0.8 V	1.6 V
frequency of ripple	f_{input}	$2f_{\mathrm{input}}$	$2f_{\mathrm{input}}$
ease of smoothing	difficult (empty half cycles)	easier	easier

(a) With the load is connected directly to the transformer, the current and voltage are sine waves so the average voltage $V_{\text{ave}} = 0$ and the RMS voltage is that from the transformer, $V_{\text{rms}} = 12.0\,\text{V}$. The power dissipated is $P = V_{\text{RMS}}^2/R = 12.0^2/100 = 1.44\,\text{W}$.

(b) With the half-wave rectifier it may be easier to do the calculations in the opposite order. The rectifier stops half of the half-cycles so the power dissipated in the load is halved to $P = 0.72\,\text{W}$. We can then use the general formula $P = V_{\text{rms}}^2/R$ backwards to see that if the power is divided by 2, the RMS voltage is divided by $\sqrt{2}$. Thus $V_{\text{RMS}} = 12.0/\sqrt{2} = 8.5\,\text{V}$.

The average value needs an integral to do it properly. This was covered in section 2 but here again is the expression for half-wave rectification:

$$V_{\text{ave}} = \frac{V_{\text{pk}}}{2\pi}\int_0^{\pi} \sin\theta\,\mathrm{d}\theta = \frac{V_{\text{pk}}}{\pi} = \frac{\sqrt{2}V_{\text{AC}}}{\pi} = 5.4\,\text{V}. \tag{43}$$

I have written the integral over θ rather than t because the frequency cancels out. The denominator has 2π because that is the full period of the wave but the integral goes only from 0 to π because of the half-wave rectification.

(c) The smoothing capacitor is so large that it charges to the peak voltage of the output from the rectifier, $V_{\text{pk}} = \sqrt{2}V_{\text{AC}} = 17.0\,\text{V}$, and stays there despite the current drawn by the load. The load therefore sees perfect DC so $V_{\text{DC}} = V_{\text{ave}} = V_{\text{RMS}} = V_{\text{pk}} = 17.0\,\text{V}$ and the power dissipated is $P = V_{\text{DC}}^2/R = 2.88\,\text{W}$.

(d) With the bridge (full-wave) rectifier but without the smoothing capacitor, the power dissipated is identical to the directly connected resistor. This is because the power does not depend on the direction of current. This shows immediately that $P = 1.44\,\text{W}$ and $V_{\text{rms}} = 12.0\,\text{V}$. The average voltage is double the value for the half-wave rectified case so $V_{\text{ave}} = (2\sqrt{2}/\pi)V_{\text{rms}} = 10.8\,\text{V}$.

(e) The output with the smoothing capacitor is the same as with the half-wave case because the capacitor is so large.

Perhaps the most striking result is the impact of the smoothing capacitor on the half-wave rectified output. The power was 0.72 W with the rectifier alone, rising to 2.88 W with the capacitor. This is a factor of 4 and the power with the smoothing capacitor is double what it would be if the resistor were connected directly to the transformer. The enhanced power requires a much larger current to be drawn from the transformer because of the smoothing. We saw the peaked current that results in the lectures.

The effect of smoothing would be less dramatic with a more realistic calculation that included the voltage drop in the diodes and particularly the resistance (regulation) of the transformer. ❦

5.3 A bridge rectifier is required to supply an average current of 200 mA with ripple not to exceed 0.5 V. What size of smoothing capacitor is needed? The AC supply is at 50 Hz. [around 2400 μF]

How would your answer change if the supply were designed for North America rather than Europe?

Solution. A pessimistic approach is to assume that the capacitor must supply 200 mA for the full period of the ripple. This is half the period of the input or 10 ms in Europe. The charge required is $0.2 \times 0.01 = 0.002\,\mathrm{C}$. The change in voltage must not exceed 0.5 V and $Q = CV$ so $C = Q/V = 0.002/0.5 = 0.004\,\mathrm{F}$ or 4000 µF.

It would be less pessimistic to assume that the capacitor supplies the current for only 60% of the time, which reduces C by the same factor to 2400 µF; the nearest conventional values are 2200 µF and 3300 µF.

The frequency of the mains (line) supply is 60 Hz rather than 50 Hz in North America, which reduces the period of each ripple and therefore the capacitance needed. The 'optimistic' estimate falls from 2400 µF to 2000 µF. Still big! ❦

5.4 Design a power supply with a transformer, bridge rectifier and smoothing capacitor to work from the AC mains in Europe and supply 9 V at 200 mA. Ripple on the output should not exceed 0.5 V. Assume that the secondary winding of the transformer has a resistance of 10 Ω. Calculate the value of smoothing capacitor and the (rms) output voltage of the transformer needed. What voltage rating should be specified for the capacitor, considering both no load and full load? Estimate the no-load voltage and regulation. [Roughly 2400 µF, 12 V; 17 V; 40% or 30 Ω]

Solution. The 'AC mains in Europe' implies 230 VAC at 50 Hz. Calculate the smoothing capacitor needed first. This needs no effort at all because the current and ripple are the same as in the previous question. The average voltage has no effect on the value of capacitance; this depends only on the average current and tolerable ripple voltage. Thus $C = 2400\,\mu\mathrm{F}$. For completeness we should also specify the voltage and ripple current ratings. The obvious peak voltage is the average output of 9 V plus half the ripple of 0.5 V (remember that this is a peak-to-peak value), giving 9.25 V. This is much too low, as we shall see shortly. The peak current into the capacitor every time it recharges is about 3 times the average current to the load or 600 mA and the ripple current rating must not be lower than this.

Now for the voltage required from the transformer. The *average* value of the output voltage should be 9.0 V and the ripple goes either side of this so the peak value of output voltage is 9.25 V, including only half of the peak-to-peak ripple voltage. Adding two diode drops from the bridge gives 10.8 V. This is a peak voltage and dividing by the usual factor of $\sqrt{2}$ for a sine wave gives an RMS voltage of 7.6 V. The voltage produced by the transformer is specified at its rated load and should therefore be this value. The steps are illustrated in figure 12.

If the transformer is modelled with a Thévenin equivalent circuit, its internal (source) voltage is considerably higher than 10.8 V peak that we have just calculated. The peak current from the transformer is around 600 mA, about 3 times the average output current, and the resistance of the transformer was given as 10 Ω, which yields a voltage drop of 6 V (*very* roughly). Adding this gives a peak voltage of 17 V to the nearest volt. Divide by $\sqrt{2}$ to find an rms voltage of 12 V. It is depressing that a transformer with a no-load output of 12 V is needed for a final output of 9 V.

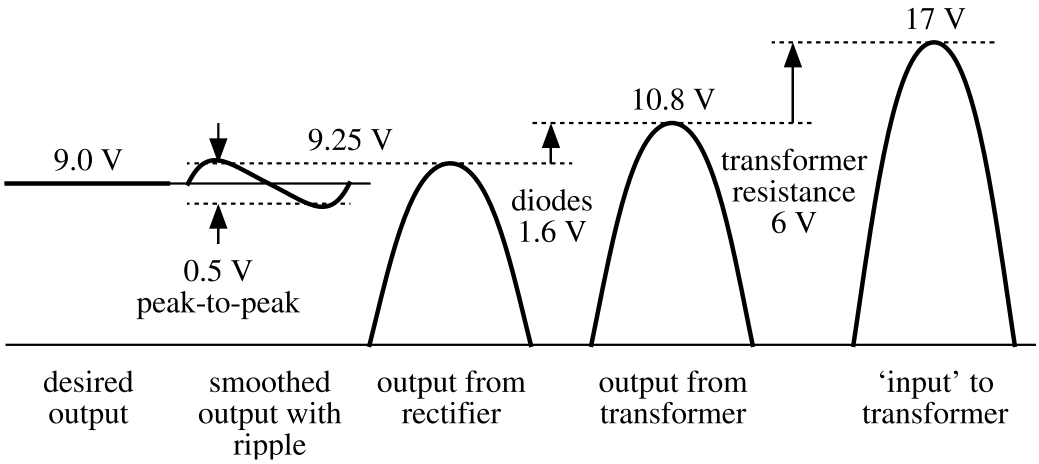

Figure 12. Voltage waveforms at different points in the power supply, working back from the output.

Work forward from the input to find the no-load output voltage. The transformer provides 17 V peak without any drop in its winding because no current flows. The diodes do not reduce this if there really is not current flowing. The smoothing capacitor will charge fully to the peak voltage and not discharge between cycles because no current is drawn from it. The no-load output voltage is therefore also 17 V. The smoothing capacitor must therefore work safely at this voltage, much higher than the nominal 9 V output.

The regulation is clearly terrible, although not unrealistic for a cheap 'wall wart'. The standard definition gives about 60% from the no-load and full-load voltages of 17 V and 9.0 V (depending on the exact definition).

[Harder] Alternatively, the supply can be represented as a Thévenin resistance with $R_s \approx 33\,\Omega$. This seems a large resistance: where does it come from?

First, the imperfect smoothing by the capacitor causes the average voltage to drop when more current is drawn. This is just half of the (peak-to-peak) ripple voltage. The ripple is zero when no current flows and 0.5 V at 200 mA so the average voltage drops by 0.25 V at 200 mA, which is equivalent to a resistance of about 1 Ω. The other obvious resistance is 10 Ω from the winding of the transformer. Unfortunately this is multiplied because of the peaked nature of the current, which rises to 600 mA rather than the average of 200 mA. It therefore gives a much higher voltage drop then might be expected, and the factor of 3 in current multiplies its effective resistance to 30 Ω. The sum of these is 31 Ω. This is still below 33 Ω because of the change in diode voltage drop. The final estimate of the voltage required from the transformer can be trusted only to the nearest volt (if that). The important finding is that the transformer dominates the regulation and should be replaced by a superior component, with a lower resistance, if the performance of the power supply needs to be improved. ❦

5.5 Suppose that you want an adjustable power supply with a linear regulator for an undergraduate electronics laboratory whose output is variable from 3 V for logic circuits to 20 V for analogue circuits, with a maximum current of 1 A. The input voltage is chosen to be 25 V to give plenty of headroom at all output voltages. Under what conditions is the maximum power dissipated in the regulator and what is its value? [22 W]

Solution. Maximum power is dissipated when the current and headroom voltage take their maximum values. The maximum headroom voltage implies the minimum output voltage of 3 V, when the headroom is $25 - 3 = 22$ V. The maximum current is 1 A, giving a power dissipation of 22 W. ❦

5.6 The bench PSUs in Rankine 709 have adjustable, bipolar, regulated outputs from 0 to ± 15 V at 200 mA. Estimate the maximum power dissipated. [7.2 W]

Why are these linear rather than switched regulators?

Solution. The input voltage to the regulators must be greater than 15 V to provide some headroom for regulation. Assume a value of 18 V for an old-fashioned regulator. The maximum dissipation is when the output voltage is very low, say zero. Then the dissipation is $18\,\text{V} \times 0.2\,\text{A} = 3.6\,\text{W}$ for each channel. This must be doubled for a bipolar supply, giving 7.2 W total.

The regulators are linear to avoid noise in their output, which would confuse your experiments! ❦

5.7 [Follows from lecture slides.] Design a power supply to deliver a current of 3.0 A at 15.0 V using a linear regulator. The regulator has a dropout voltage of 2.0 V and the mains input voltage is 110 V $\pm$ 8% at 60 Hz (North America, for example). The transformer has a regulation of 7% and is 90% efficient at full load. A bridge rectifier is used. Assume the diode conduction angle is 30° and that the diodes drop 1.0 V at full load. The smoothing capacitor is 10 mF.

(a) What is the minimum AC voltage for the secondary winding of the transformer?

(b) What is the power dissipation in the regulator under worst-case conditions?

(c) What voltage rating should the capacitor have?

(d) What PIV rating should the diodes have?

(e) What VA rating does the transformer require?

[Roughly 17 V, 24 W, 28 V, 28 V, somewhere between 150 W and 250 W depending on how you calculate it.]

Solution. This follows the example at the end of the lectures on linear regulators closely.

(a) As usual, work back from the output to the input. The voltages at different points are illustrated in figure 13.

- The final output is 15.0 V and the dropout voltate is 2.0 V so the minimum input to the regulator is the sum of these, 17.0 V. The bottom of the ripple on the output from the smoothing capacitor must stay above this.

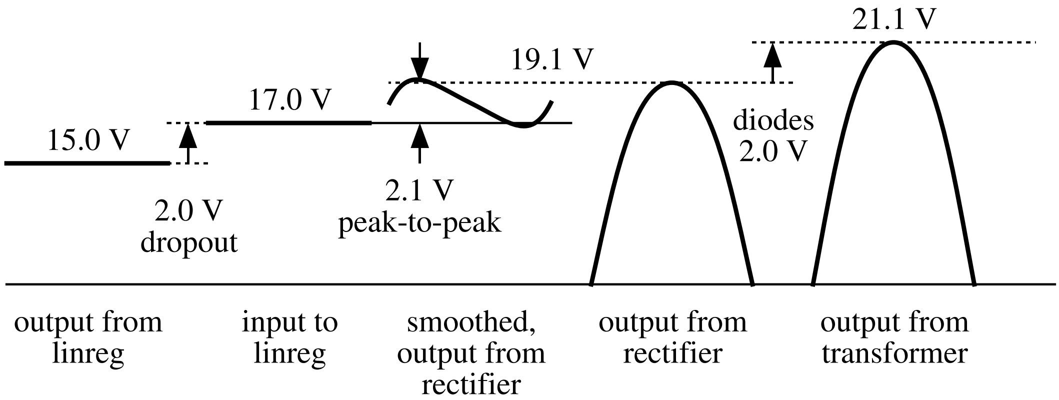

Figure 13. Voltage waveforms at different points in the power supply with linear regulator (linreg), working back from the output.

- If the diode conducts for 30°, the capacitor provides current to the load for 150° (they sum to 180° because of full-wave rectification). This is a time of $(150/360) \times (1/60) = 6.9\,\text{ms}$; remember that the AC is at 60 Hz. The peak-to-peak ripple voltage is therefore $V_{\text{ripple}} = \Delta q/C = I\delta t/C = (3.0\,A) \times (6.9\,\text{ms})/(10\,\text{mF}) = 2.1\,\text{V}$.
  Maximum current is the worst case for ripple; in the opposite case of zero load, there is no ripple at all (ideally) and the output would be at the peak voltage.
- The peak voltage from the rectifier should be $17.0 + 2.1 = 19.1\,\text{V}$. We have to add the full peak-to-peak ripple voltage to keep the output above 17.0 V.
- Adding the voltage drop of two diodes gives 21.1 V for the peak voltage from the secondary winding of the transformer. This is 14.9 V RMS.
- That would be the answer if we could assume that the transformer always provided its rated voltage. However, we must allow for the mains to go down in voltage by 8%, which means that we require a nominal value of $14.9/(1 - 0.08) = 16.2\,\text{V}$.

This should be rounded up to the nearest value available (unless you could order a special transformer). Assume that a 17 V transformer is used.

(b) Worst-case conditions for power dissipation in the regulator mean maximum current and maximum voltage drop across the regulator, which in turn means maximum input voltage.

- The transformer has nominal output 17.0 V RMS, which is 24.0 V peak, falling to 22.0 V because of the diodes. The ripple is 2.1 V peak-to-peak under full load so the average voltage going into the regulator is about 21.0 V.
- We want the worst case and should allow for a possible rise of 8% in mains voltage. This raises the 24.0 V peak to 26.0 V and the average input to the regulator becomes 23.0 V.
- This means that $23.0 - 15.0 = 8.0\,\text{V}$ is dropped across the regulator, which passes 3.0 A, so the power dissipated is $(8.0\,\text{V}) \times (3.0\,\text{A}) = 24\,\text{W}$.

For comparison, the load dissipates 45 W. At least the regulator dissipates less than the load!

(c) The capacitor should be rated for the maximum peak voltage that it can experience. This occurs when the mains goes up by 8% and no load is applied. Under these conditions no voltage is dropped in the diodes and the transformer regulation raises the output voltage by a further 7%. This gives $24.0 \times 1.08 \times 1.07 = 27.7$ V. In practice you would use the next value up from this; not many values are available but 33 V might be available.

(d) The diodes should have the same PIV rating as the capacitor.

(e) It is really difficult to estimate the VA rating. Following the method in the lecture notes, the form factor is 2.7 for a 30° conduction angle. We should also allow for the efficiency of 90%. This gives $(17.0\,\mathrm{V}) \times (3.0\,\mathrm{A}) \times 2.7/0.9 = 153$ V A. This is much larger than the useful power produced by the transformer, about 69 W (using the calculation above for the worst case), and 45 W delivered to the final load.

Another way of looking at the VA rating is to estimate the peak current that the transformer must produce. The conduction angle is 30° so the diodes conduct for $1/6$ of the time. The load draws a steady 3 A so the average current during the conduction angle is $6 \times 3 = 18$ A. If this were the peak of a sine wave, it would correspond to about 13 A RMS. The transformer produces 17.0 V RMS and multiplying these gives 216 V A. Dividing by the efficiency raises this further to 240 V A! ❦

5.8 A low-dropout regulator (LDO) has the following parameters: 2.5 V output voltage, 0.1 V dropout voltage, 6.0 V maximum input voltage, 150 mA maximum output current, 125°C maximum junction temperature, 250°C/W thermal resistance from junction to ambient.

(a) Define the terms *headroom voltage* and *dropout voltage* for a linear regulator and explain what is meant by a *low-dropout* regulator.

(b) Make an annotated sketch of the output voltage as a function of the input voltage from 0 to 6.0 V.

(c) What is the minimum input voltage required for a regulated output?

(d) How does the efficiency of the regulator depend of its input voltage V_{in}? Support your answer with calculations of the efficiency across the range of specified operation.

(e) What conditions across the full range of specified operation cause the maximum power dissipation in the regulator? What is the value of this power?

(f) Can the regulator safely dissipate this value of power under typical laboratory conditions?

Solution. This covers most aspects of linear regulators in practice.

(a) Headroom voltage: difference between input and output voltage dropped across regulator when regulating, or $V_{\mathrm{in}} - V_{\mathrm{out}}$. Dropout voltage: minimum headroom voltage at which regulation operates. Low-dropout regulator: has a low dropout voltage (!), typically meaning below 0.5 V; in terms of design the output is taken from the collector or drain of the pass transistor rather than the emitter or source.

(b) See the lecture notes for a sketch. The critical feature is that the output voltage should be flat at 2.5 V for input voltages of 2.6 V and above.

(c) Specified output voltage plus dropout voltage, giving 2.6 V. The input voltage is *higher* than the output.

(d) Efficiency η falls as the input voltage increases because the excess voltage is dissipated as heat. At minimum input voltage for regulation of 2.6 V, $\eta = V_{\text{out}}/V_{\text{in}} = (2.5\,\text{V})/(2.6\,\text{V}) = 96\%$; for maximum input voltage of 6.0 V, $\eta = 42\%$.

(e) The worst case is maximum input voltage (6.0 V, dropping 3.5 V across the regulator) and maximum current (150 mA), giving a dissipated power of $(3.5\,\text{V}) \times (0.15\,\text{A}) = 525\,\text{mW}$.

(f) The thermal resistance is 250°C/W so the maximum power gives a temperature difference of $\Delta T = P\theta = (525\,\text{mW}) \times (250°\text{C/W}) = 131°\text{C}$. Take the ambient temperature to be 25°C so the junction reaches $131 + 25 = 156°\text{C}$, exceeding the maximum permitted temperature of 125°C. This power cannot safely be dissipated.

Alternatively you can calculate the maximum permitted power but you must make it clear what you are calculating. The maximum junction temperature is 125°C and the ambient is at 25°C, giving a difference of $\Delta T = 100°\text{C}$. The maximum permitted power is $P_{\text{max}} = \Delta T/\theta = (100°\text{C})/(250°\text{C/W}) = 0.4\,\text{W}$. This is too small.

Power dissipation is covered in the notes on thermal management. ❦

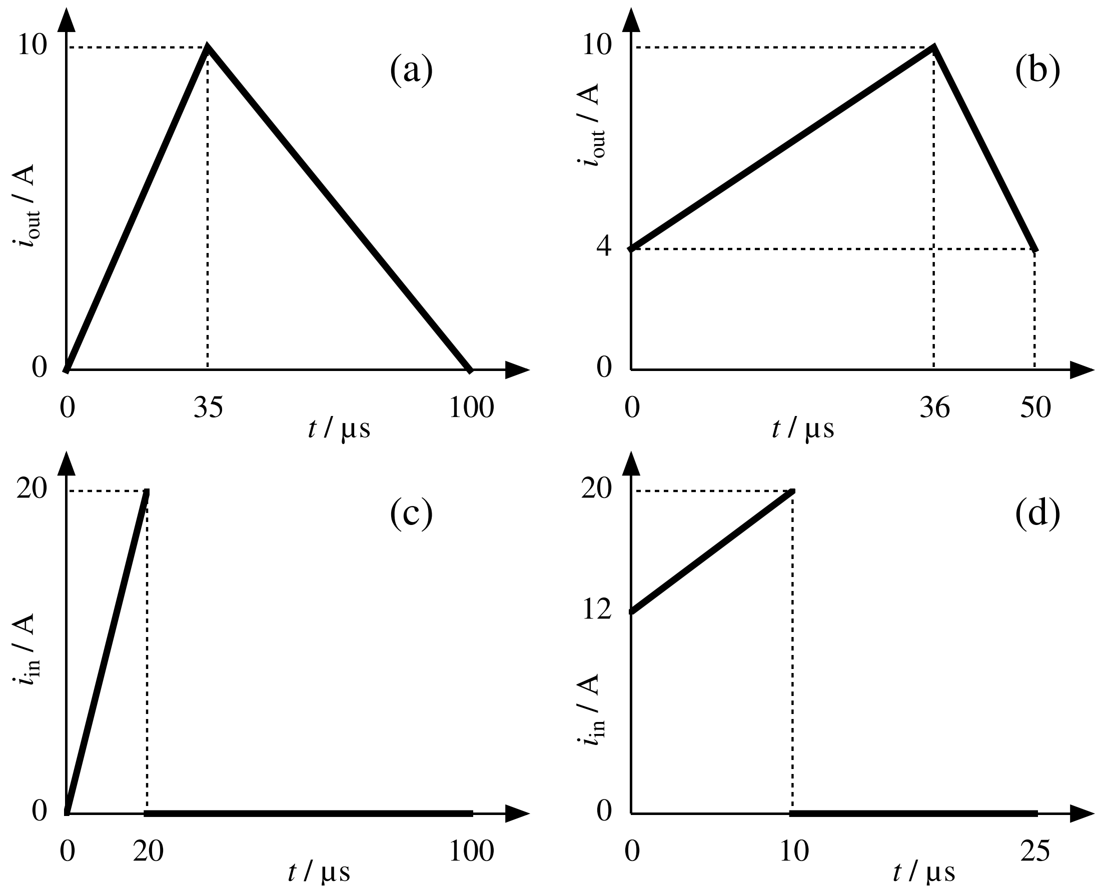

Figure 14. Input or output currents measured in four buck DC–DC converters.

6 Switch-mode DC–DC converters

6.1 *Buck regulators.* Figure 14 shows a single cycle of either the input current $i_{\text{in}}(t)$ or the output current $i_{\text{out}}(t)$ measured for four buck regulators, each of which has an input at $V_{\text{in}} = 100\,\text{V}$. For each of these calculate the duty factor D, output voltage V_{out}, average output current I_{out}, average input current I_{in}, average power P supplied and the value of the inductance L in the converter. Assume ideal components and operation. To help you get started, here is a walk through the first example.

(a) The current is rising for the first 35 µs so this must be the charging phase. The period is 100 µs so $D = T_{\text{charge}}/T_{\text{cycle}} = 0.35$.

(b) The output voltage is given by $V_{\text{out}} = DV_{\text{in}} = 35\,\text{V}$.

(c) From the plot of $i_{\text{out}}(t)$, the average value is $I_{\text{out}} = 5\,\text{A}$ (easy because it is triangular).

(d) The average input current is given by $I_{\text{in}} = DI_{\text{out}} = 1.75\,\text{A}$.v (The actual input current $i_{\text{in}}(t)$ is the same as $i_{\text{out}}(t)$ during charging, when the currents rise, and $i_{\text{in}}(t) = 0$ during discharging, when $i_{\text{out}}(t)$ falls.)

(e) The average power can be found from $P_{\text{in}} = V_{\text{in}}I_{\text{in}}$ or $P_{\text{out}} = V_{\text{out}}I_{\text{out}}$ because the voltages are assumed constant so the average power is proportional to the average

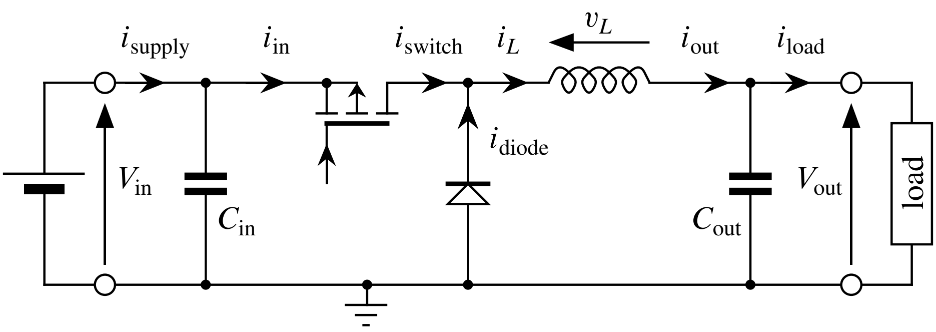

Figure 15. Circuit of a standard buck DC–DC converter.

current. Both give 175 W.

(f) The output current $i_{\text{out}}(t)$ is the same as the inductor current $i_L(t)$ for a buck converter. Use $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$ to find the inductance. In the charging phase, $v_L = V_{\text{in}} - V_{\text{out}} = 65\,\text{V}$ and $\mathrm{d}i_L/\mathrm{d}t = (10\,\text{A} - 0\,\text{A})/(35\,\mu\text{s}) = 0.29\ \text{A}\,\mu\text{s}^{-1}$ so $L = (65\,\text{V})/(0.29\ \text{A}\,\mu\text{s}^{-1}) = 227.5\,\mu\text{H}$. You should check that the discharging phase gives the same result.

Solution.

(a) Done in question.

(b) $D = 0.72$, $V_{\text{out}} = 72\,\text{V}$, $I_{\text{out}} = 7\,\text{A}$, $I_{\text{in}} = 5.04\,\text{A}$, $P = 504\,\text{W}$, $L = 168\,\mu\text{H}$.

(c) In this case the input current is shown. The current through the inductor is the same in the charging phase and follows a straight line from the peak (20 A for this example) returning to the starting value (0 A here). $D = 0.2$, $V_{\text{out}} = 20\,\text{V}$, $I_{\text{out}} = 10\,\text{A}$, $I_{\text{in}} = 2\,\text{A}$, $P = 200\,\text{W}$, $L = 80\,\mu\text{H}$. You could find I_{in} directly from the plot in the question: the average is 10 A while $i_{\text{in}}(t)$ is flowing, from 0 to 20 µs, and zero for the remaining 80 µs, so the overall average is 2 A.

(d) $D = 0.4$, $V_{\text{out}} = 40\,\text{V}$, $I_{\text{out}} = 16\,\text{A}$, $I_{\text{in}} = 6.4\,\text{A}$, $P = 640\,\text{W}$, $L = 75\,\mu\text{H}$.

❦

6.2 *Basic buck regulator.* A point-of-load (PoL) power supply is required to deliver 3.0 V at 100 mA from a 5.0 V bus. It is implemented as a buck converter with a 100 µH inductance running at 100 kHz in continuous conduction mode. Figure 15 shows the circuit. You may assume that all components are ideal and that the input and output are smoothed so effectively that their voltages may be treated as constant. Analyse the behaviour of the circuit as follows

(a) Show from the standard relations that the converter should use a duty cycle $D = 0.6$ and that the average input current is $I_{\text{in}} = 60\,\text{mA}$.

(b) Using the notation in figure 15, show that $v_L = +2\,\text{V}$ during charging and $v_L = -3\,\text{V}$ during discharging.

(c) Use $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$ to show that $\mathrm{d}i_L/\mathrm{d}t = +2\times10^4\ \mathrm{A\,s^{-1}} = +20\ \mathrm{mA\,\mu s^{-1}}$ during charging and $\mathrm{d}i_L/\mathrm{d}t = -3\times10^4\ \mathrm{A\,s^{-1}} = -30\ \mathrm{mA\,\mu s^{-1}}$ during discharging.

(d) Hence show that i_L rises by 120 mA during charging and falls by the same during discharging.

(e) We know that the average current through both the load and inductor is 100 mA. Hence show that the minimum value of i_L is 40 mA and its maximum is 160 mA.

(f) Now sketch $i_L(t)$, $i_{\mathrm{in}}(t)$, $i_{\mathrm{switch}}(t)$ and $i_{\mathrm{out}}(t)$. No further calculation is required. Show also $v_L(t)$.

(g) Suppose that a linear regulator was chosen for the supply instead. What would be its maximum efficiency?

This specification is a realistic starting point for many applications in consumer devices such as mobile phones and set-top boxes.

Solution. This runs through all the standard aspects of a buck converter.

(a) The standard relation between input and output voltages for a buck converter is $V_{\mathrm{out}} = DV_{\mathrm{in}}$ so $D = V_{\mathrm{out}}/V_{\mathrm{in}} = 3.0/5.0 = 0.6 = 60\%$. The average currents obey the inverse relation, $I_{\mathrm{in}} = DI_{\mathrm{out}} = 0.6\times100\ \mathrm{mA} = 60\ \mathrm{mA}$. This is definitely an average current because the input current to a buck converter is pulsed. Using the notation of figure 15, $i_{\mathrm{in}}(t)$ is pulsed but $i_{\mathrm{supply}}(t)$ is smoothed by C_{in}. Its average value is shown as I_{supply} on the plot.

(b) The p-MOSFET switch is turned on during charging and KVL gives $V_{\mathrm{in}} = v_L + V_{\mathrm{out}}$ so $v_L = V_{\mathrm{in}} - V_{\mathrm{out}} = 5.0 - 3.0 = 2.0\ \mathrm{V}$.

During discharging, the MOSFET is off and current flows through the diode, whose voltage drop is neglected, so KVL gives $v_L + V_{\mathrm{out}} = 0$ and $v_L = -V_{\mathrm{out}} = -3.0\ \mathrm{V}$.

(c) This is just a matter of substitution in the current–voltage relation for an inductor provided, $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$. When charging, $\mathrm{d}i_L/\mathrm{d}t = v_L/L = (2\ \mathrm{V})/(100\ \mathrm{\mu H}) = +2\times10^4\ \mathrm{A\,s^{-1}} = +20\ \mathrm{mA\,\mu s^{-1}}$. Similarly, when discharging, $\mathrm{d}i_L/\mathrm{d}t = v_L/L = (-3\ \mathrm{V})/(100\ \mathrm{\mu H}) = -3\times10^4\ \mathrm{A\,s^{-1}} = -30\ \mathrm{mA\,\mu s^{-1}}$ during discharging.

(d) The inductor charges for a fraction of the period given by $D = 0.6$. The period $T = 1/f = 1/(100\ \mathrm{kHz}) = 10^{-5}\ \mathrm{s} = 10\ \mathrm{\mu s}$. The charging time is therefore $0.6\times10 = 6\ \mathrm{\mu s}$. The change in current is given by $\Delta i_L = (\mathrm{d}i_L/\mathrm{d}t)(DT) = (20\ \mathrm{mA\,\mu s^{-1}})\times(6\ \mathrm{\mu s}) = 120\ \mathrm{mA}$.

The calculation for discharging is similar. This process lasts for $(1-D)T = 4\ \mathrm{\mu s}$ so $\Delta i_L = (\mathrm{d}i_L/\mathrm{d}t)[(1-D)T] = (-30\ \mathrm{mA\,\mu s^{-1}})\times(4\ \mathrm{\mu s}) = -120\ \mathrm{mA}$. This is equal but opposite, as it should be because the current must be period and return to its starting value after a cycle.

(e) The form of $i_L(t)$ is triangular so its average value lies halfway between its maximum and minimum values. The average is 100 mA because $i_L(t) = i_{\mathrm{out}}(t)$ for a buck converter and we know the average current through the load. This shows that $i_L(t)$ must vary through a peak-to-peak range of 120 mA about a mean of 100 mA, or can be written (100 ± 60) mA. The minimum value of i_L is therefore 40 mA and its maximum is 160 mA.

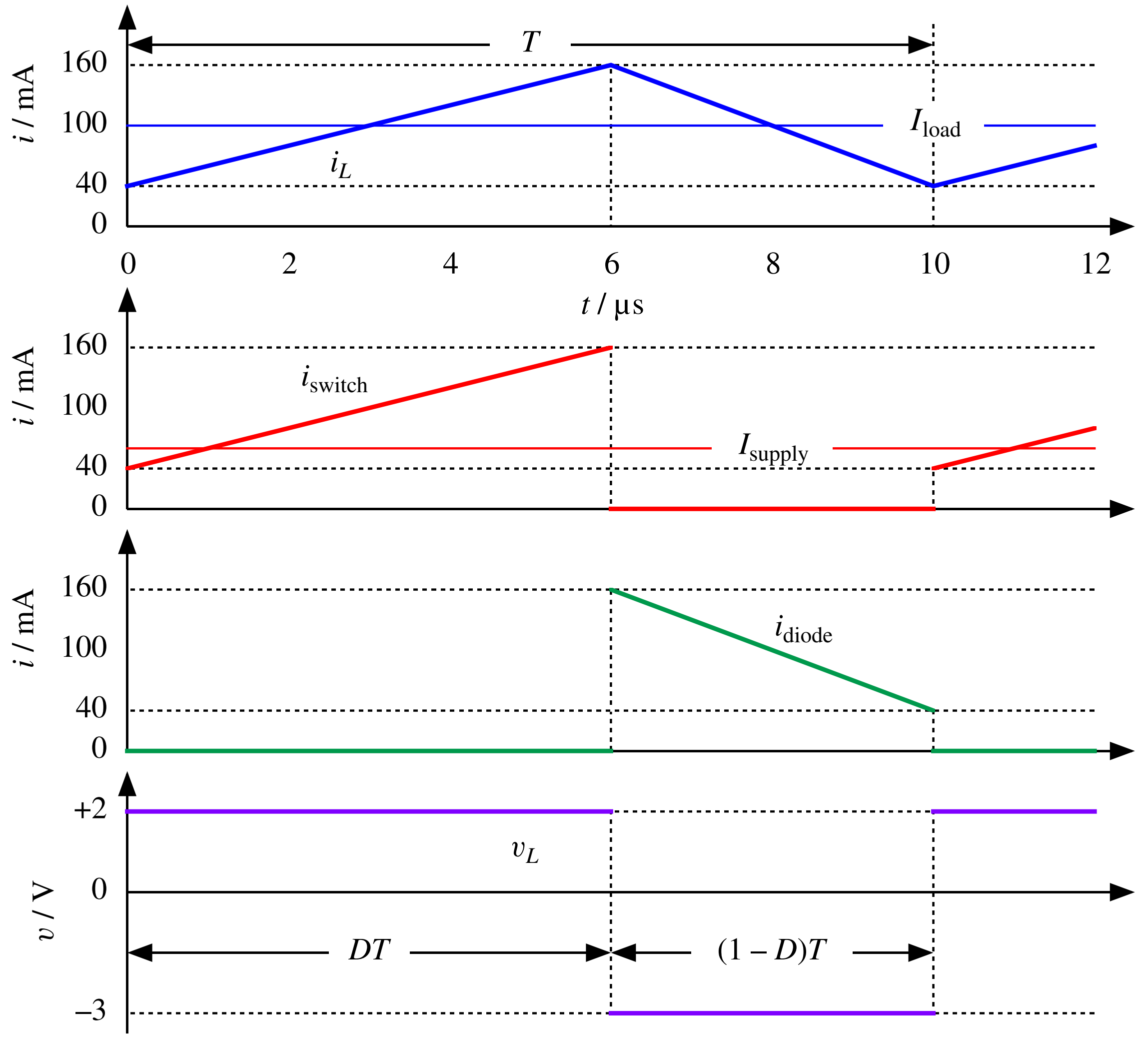

Figure 16. Currents and inductor voltage in a buck DC–DC converter.

(f) See figure 16 for the sketch. The curve for $i_L(t)$ follows from the calculations and $i_{\text{out}}(t) = i_L(t)$ for a buck converter. The average output current is shown as I_{load}. The current through the switch is the same as the input current, $i_{\text{switch}}(t) = i_{\text{in}}(t)$, and are the same as $i_L(t)$ while the switch is on to charge the inductor. These currents are zero when the transistor switches off and the current flows through the diode instead so $i_{\text{diode}}(t) = i_L(t)$ during discharge.

(g) The input is at 5.0 V and the output is at 3.0 V, the currents are the same for a linear regulator so the efficiency is $3.0/5.0 = 60\%$. It should be easy to do much better than this with a buck regulator; about 85% would be typical for this application.

❦

6.3 *Changing the load on a buck regulator.* [Harder.] Suppose that the load on the regulator in the previous question is changed. The voltages remain the same but I_{load} varies from 100 mA.

(a) How does a change in I_{load} affect the plot of $i_L(t)$? Remember that the voltages remain the same and that $\mathrm{d}i_L/\mathrm{d}t = v_L/L$, which means that the slopes of $i_L(t)$ remain the same too.

(b) Show that the minimum average current that this regulator can supply while remaining in continuous conduction mode is 60 mA.

(c) Suppose that the regulator must be redesigned so that it can supply 50 mA in continuous conduction mode. Does this require a larger or smaller value of inductance?

(d) Assume that the regulator runs at the boundary of continuous conduction mode, which means that the current just drops to zero between cycles. Show that this requires $\Delta i_L = 100\,\text{mA}$ in each cycle.

(e) Use Δi_L to work out $\mathrm{d}i_L/\mathrm{d}t$ and hence show that $L = 120\,\mu\text{H}$.

A 'rule of thumb' is to increase this value by 25% to allow for tolerances and fluctuations and ensure that the converter remains in continuous conduction mode. A designer would therefore specify around a 150 μH inductor.

Solution.

(a) A change in I_{load} causes the plot of $i_L(t)$ to slide rigidly up and down. As the hints in the question show, the slopes in the charging and discharging region remain the same because they are determined by the voltages and inductance, not the average current. The only thing that the curve can do is to slide up and down to give the correct average current.

(b) If the curve slides down, its minima will eventually hit zero current (the time axis). The system cannot be in continuous conduction mode if we try to slide it any further down. The minimum current therefore occurs at boundary mode, where the current just drops to zero between cycles. The minimum current with the original 100 mA load was 40 mA so we can slide the curve down by 40 mA, which reduces the average current by the same amount to 60 mA.

(c) The original circuit cannot supply 50 mA in continuous conduction mode; 60 mA is the minimum. We therefore need the change the inductance because it is the only parameter that can be varied. The curve for $i_L(t)$ must become flatter so that it does not reach down to zero. Now $\mathrm{d}i_L/\mathrm{d}t = v_L/L$ and v_L cannot be changed so the only way of reducing $\mathrm{d}i_L/\mathrm{d}t$ is to increase L. We need a larger value of inductance. This seems wrong and I'll say a bit at the end of the solution.

(d) If the regulator runs at the boundary of continuous conduction mode, the current just drops to zero between cycles. The average value of a triangular wave with a minimum of zero and a maximum of I_{pk} is $\frac{1}{2}I_{\text{pk}}$. We need the average of be 50 mA so the peak must be $I_{\text{pk}} = 100\,\text{mA}$. This is the same as the change in $i_L(t)$ during each cycle so $\Delta i_L = 100\,\text{mA}$ too.

(e) The slope during charging is found from $\mathrm{d}i_L/\mathrm{d}t = (\Delta i_L)/(DT) = (100\,\text{mA})/(6\,\mu s) = (100/6)\times 10^3\,\text{A}\,\text{s}^{-1}$. Then $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$ gives $L = v_L/(\mathrm{d}i_L/\mathrm{d}t) = (2\,\text{V})/((100/6)\times 10^3\,\text{A}\,\text{s}^{-1}) = 120\,\mu\text{H}$. You could use the discharging phase and get the same result.

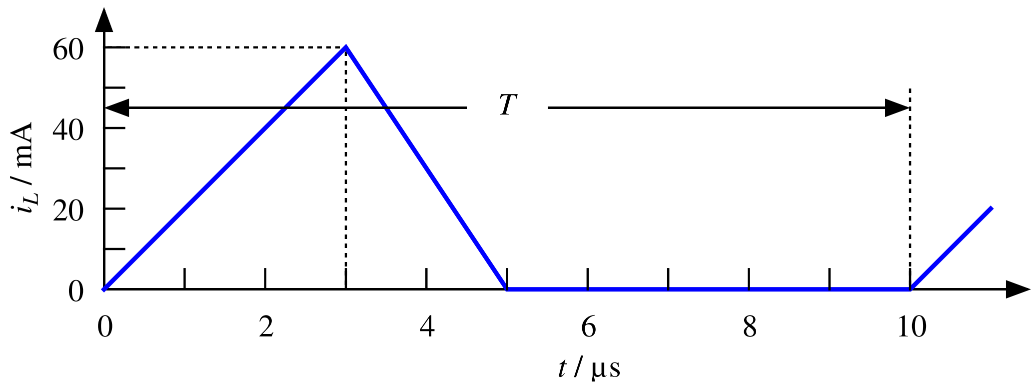

Figure 17. Inductor current for a buck DC–DC converter in discontinuous mode. It has $V_{\text{in}} = 5.0\,\text{V}$, $V_{\text{out}} = 3.0\,\text{V}$, a $100\,\mu\text{H}$ inductance and runs at $100\,\text{kHz}$.

As the question stated, this would typically be increased by around 25% to $150\,\mu\text{H}$. I designed the converter in the first question in this way but increased the inductance further to the next convenient value.

It is not obvious why a *reduction* in current requires an *increase* in inductance. One way follows from $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$; the voltage is the same but the current is reduced, so the derivative $\mathrm{d}i_L/\mathrm{d}t$ tends to reduce, so we need a larger inductance to compensate. Another way of thinking uses the energy stored in an inductor, $E = \frac{1}{2}Li_L^2$. The flow of power through a converter is proportional to the current (keeping voltages fixed). However, this equation shows that the energy stored in the inductor goes like i_L^2 and therefore reduces more than the power transferred if i_L goes down. We therefore need to raise L so that $\frac{1}{2}Li_L^2$ does not go down too much. This really isn't easy to understand! ❦

6.4 *Buck converter in discontinuous mode.* [Not examinable.] Suppose that the same buck converter has its original inductance of $100\,\mu\text{H}$ and operates at the same voltages but the duty cycle is reduced to $D = 0.3$. Show that the current behaves as shown in figure 17 and that the average output current is reduced to 15 mA.

What happens as D is increased, keeping the voltages the same?

Note that the usual relation $V_{\text{out}} = DV_{\text{in}}$ does *not* hold in discontinuous conduction mode! The required value of D depends on both the output voltage and the output current.

Solution. The slopes of $i_L(t)$ follow from the voltages and inductance, which are the same as in the original converter. The current therefore rises at $+20\,\text{mA}\,\mu\text{s}^{-1}$ for $3\,\mu\text{s}$ and reaches a peak of 60 mA. It then falls at $-30\,\text{mA}\,\mu\text{s}^{-1}$ and reaches zero after $2\,\mu\text{s}$. The inductor cannot recharge so its current then remains at zero (and so does the voltage across it).

As D is increased, the inductor is charged for longer and reaches a higher peak current. It takes longer to discharge so the time when there is no current is reduced from $5\,\mu\text{s}$. For example, if $D = 0.45$ the current rises to 90 mA after $4.5\,\mu\text{s}$ and falls to zero after a further $3\,\mu\text{s}$, leaving $2.5\,\mu\text{s}$ with no current flowing. The average current is proportional to

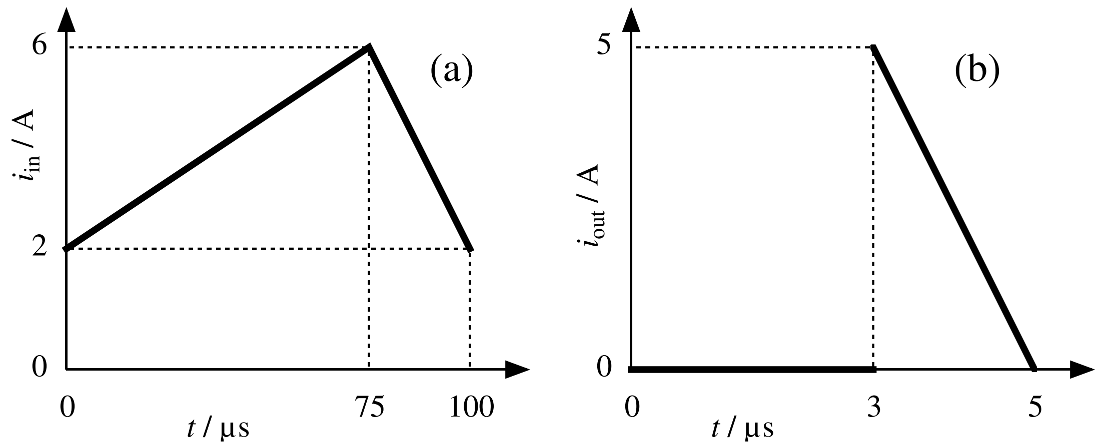

Figure 18. Currents measured in two boost DC–DC converters.

the area under $i_L(t)$ so it grows like D^2 and is 33.75 mA for $D = 0.45$. This continues until $D = 0.6$ when the time without current vanishes and we are back to continuous conduction mode. ❦

6.5 Figure 18 shows a single cycle of either the input current $i_{in}(t)$ or the output current $i_{out}(t)$ measured for two boost regulators, each of which has an input at $V_{in} = 10$ V. For each of these calculate the duty factor D, output voltage V_{out}, average output current I_{out}, average input current V_{in}, power P supplied and the value of the inductance L in the converter. Assume ideal components and operation. Remember that the input current is the same as the inductor current for a boost converter.

Solution.

(a) $D = 0.75$, $V_{out} = 40$ V, $I_{out} = 1$ A, $I_{in} = 4$ A, $P = 40$ W, $L = 187.5\,\mu H$.

(b) $D = 0.6$, $V_{out} = 25$ V, $I_{out} = 1$ A, $I_{in} = 2.5$ A, $P = 25$ W, $L = 6\,\mu H$.

❦

6.6 Design a boost converter to supply 500 mA at 12.0 V from 5.0 V. It should operate at 25 kHz and you may assume ideal components and operation. Calculate the inductance required for boundary mode and increase it by 25% for the final design to ensure the continuous mode of operation.

Solution. For a boost converter $V_{out}/V_{in} = 1/(1 - D)$ so $D = 7/12 = 0.58$. the charging time is $DT = 0.58 \times 40\,\mu s = 23.3\,\mu s$. The power supplied is $P_{out} = (500\,\text{mA}) \times (12.0\,\text{V}) = 6.0$ W so the average input current is given by $I_{in} = P_{in}/V_{in} = (6.0\,\text{W})/(5.0\,\text{V} = 1.2$ A.

The inductor current is the same as the input current and must therefore have an average value of 1.2 A. At boundary mode its minimum is 0 so its maximum must be 2.4 A. This

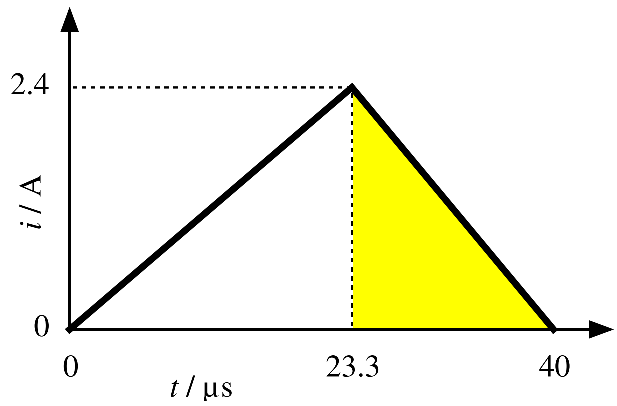

Figure 19. Current in a boost DC–DC converter to provide 12 V at 500 mA from a 5 V input, operating at boundary mode.

shown in figure 19. Using $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$ in the charging or discharging phase gives $L = 48/6\,\mu\mathrm{H}$. Scaling up by the usual 'rule of thumb' of 25% gives 60.8 μH. I'm not sure what the next standard value would be; possibly 100 μH. Inductors are not produced in a wide range of values. ❦

6.7 The previous question allowed you to assume that the output voltage was constant. However, the output current from a boost converter is pulsed and must be smoothed if the voltage and current supplied to the load are to be constant within limits. What capacitor is needed to reduce to ripple below 0.1 V? [Hint: this is very similar to the calculation of smoothing capacitor for a rectifier.] Suppose that the 12 V output was supplied by a classic bridge rectifier from the mains instead; how large a smoothing capacitor would needed in this case? [About 120 μF and 40 mF]

Solution. The smoothing capacitor has to provide all the current to the load while the inductor is being recharged from the input. This is a current of 500 mA for a time $DT = 23.3\,\mu\mathrm{s}$ so the charge is $\Delta q_C = (500\,\mathrm{mA}) \times (23.3\,\mu\mathrm{s}) = 12\,\mu\mathrm{C}$. The change in voltage is $\Delta v_C = \Delta q_C / C$ so we need $C > \Delta q_C / \Delta v_C$ with $\Delta v_C = 0.1\,\mathrm{V}$. This gives $C > (12\,\mu\mathrm{C})/(0.1\,\mathrm{V}) = 120\,\mu\mathrm{F}$.

If the supply worked at mains (line) frequency instead, the period would be 10 ms rather than 40 μs. The capacitor does not supply current for the whole period but only between the times when the diode conducts. The conduction angle is not specified but say that the capacitor must supply current for 8 ms in each cycle. Then $\Delta q_C = (500\,\mathrm{mA}) \times (8\,\mathrm{ms}) = 4\,\mathrm{mC}$ and $C > (4\,\mathrm{mC})/(0.1\,\mathrm{V}) = 40\,\mathrm{mF}$. The higher frequency of the switching supply permits a much smaller capacitor to be used. ❦

6.8 A switch-mode power supply can be modelled by the circuit shown in figure 20, which also shows a plot of the current through the inductor as a function of time for a single, complete period of switching. The switch alternates between positions (a) and (b) with corresponding labels on the plot.

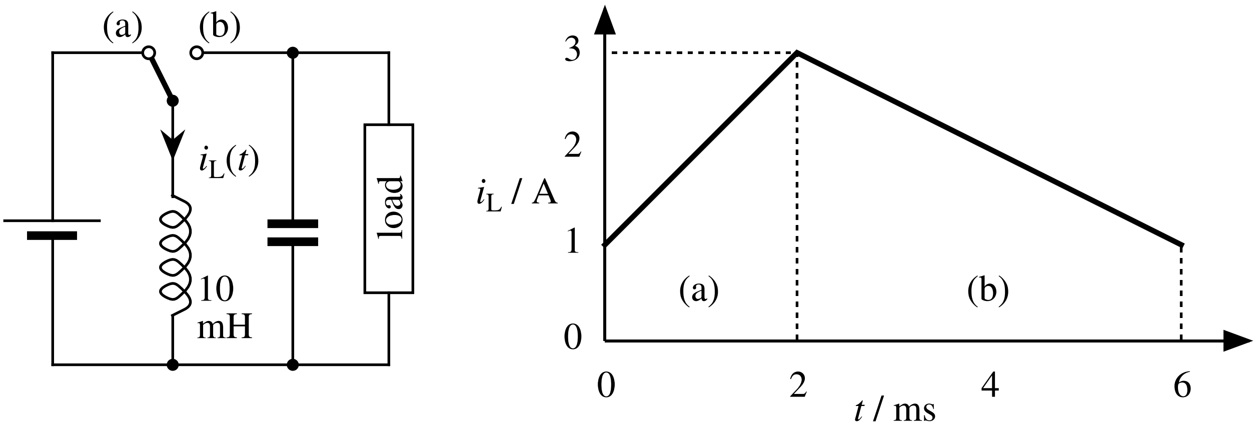

Figure 20. Switch-mode power supply.

(a) Write down the general relation between current and voltage as a function of time for (i) a capacitor and (ii) an inductor.

(b) What are the input and output voltages of this converter?

(c) How can the output voltage of this converter be varied and what range of output voltages can be produced?

Solution.

(a) You should know these!

(b) Deduce the input and output voltages from the plot of $i_L(t)$ and $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$. We are given $L = 10\,\mathrm{mH}$. The input voltage is given by phase (a) with $\mathrm{d}i_L/\mathrm{d}t = (2\,\mathrm{A})/(2\,\mathrm{ms}) = 1 \times 10^3\,\mathrm{A\,ws^{-1}}$ so $V_{\mathrm{in}} = 10\,\mathrm{V}$. The output voltage is in phase (b) with $\mathrm{d}i_L/\mathrm{d}t = (-2\,\mathrm{A})/(4\,\mathrm{ms}) = -0.5 \times 10^3\,\mathrm{A\,ws^{-1}}$ so $V_{\mathrm{out}} = -5\,\mathrm{V}$. The sign is critical.

(c) The output voltage can be varied by changing the fraction of time spent in modes (a) and (b), the duty cycle D in other words. Any negative output voltage is possible in principle, both larger and smaller in magnitude than the input voltage.

❦

6.9 A small wind turbine generates 1.5 V DC per mph of wind speed. It feeds a power electronic converter, which is required to deliver 200 W at 24 V DC to a load. For operation over the range 8 mph to 32 mph, determine the following:

(a) the type of converter to be used

(b) the required range of duty cycle for these wind speeds

(c) the average input current to the converter at 32 mph

(d) the peak input current at 32 mph assuming operation at the boundary condition.

Solution. For the range of wind speeds specified, the turbine delivers 12 V to 48 V. This goes both above and below the required output of 24 V, which determines the choice of converter.

(a) An inverting converter must be used because it is the only type whose output voltage can be both larger and smaller in magnitude than its input. The change in sign should not matter.

(b) The magnitudes of the input and output voltage are related by $V_{\text{out}}/V_{\text{in}} = D/(1-D)$. Turning this around gives $D = V_{\text{out}}/(V_{\text{in}} + V_{\text{out}})$. The duty cycle goes from $2/3$ to $1/3$ as the wind speed goes from 8 mph to 32 mph.

(c) At 32 mph the input voltage is 48 V so the *average* input current to supply a 200 W load is $I_{\text{in}} = (200\,\text{W})/(48\,\text{V}) = 4.2\,\text{A}$.

(d) The input current flows during the duty cycle of the converter, which is $1/3$ at this speed, so the average current *during the duty cycle* is $3 \times 4.2\,\text{A} = 12.5\,\text{A}$. The current is triangular at the boundary condition, starting from zero and going up to I_{pk}, and its average *during the duty cycle* is $I_{\text{pk}}/2$. Comparing the two expressions shows that $I_{\text{pk}} = 25.0\,\text{A}$.

The last result also shows that the peak (instantaneous) power output from the turbine is $P_{\text{pk}} = (48\,\text{V}) \times (25.0\,\text{A}) = 1200\,\text{W}$, much higher than the average output of 200 W. The factor of 6 between these is made up of a factor of 3 from the duty cycle and a factor of 2 from the triangular shape of the current at the boundary condition. A larger inductor in the converter would reduce the factor of 2 by allowing continuous operation but the factor of 3 from the duty cycle cannot be avoided. ❦

6.10 Suggest appropriate types of power supply for the following systems and justify your choice. Detailed circuits are not required.

(a) A portable digital system requires a 3.3 VDC supply from a lithium-ion battery, whose voltage drops from 3.6 V to 3.0 V as it runs down.

(b) An instrumentation amplifier requires a ± 15 VDC supply from the mains.

(c) A digital system has its main power supply at 3.3 VDC and includes a processor that requires a supply of 5 A at 1.8 VDC.

(d) A toy is powered from two AAA cells and includes an electroluminescent display that needs 20 VDC to illuminate it.

Solution.

(a) The output voltage may be higher or lower than the input so a buck/boost (inverting) converter is the most straightforward choice. The noise produced by the switcher is probably acceptable for a digital system. We have to hope that the change in polarity does not cause a problem; it should be fine if this is the only electronics powered by the battery.

(b) The instrumentation amplifier requires a quiet supply so a switching regulator is probably unacceptable. Instead we would have to use a classic transformer, rectifier, reservoir capacitor and linear regulator; one of these is needed for each polarity.

(c) A buck (step-down) switching supply should be used for efficiency. A linear regulator would waste $(3.3 - 1.8) = 1.5\,\mathrm{V}$ at 5 A, which means a power dissipation of 7.5 W. This is nearly the same power consumed by the processor. The noise produced by the switcher should not be a problem for the processor but this would need more care if the system includes any sensitive analogue electronics.

(d) Two AAA cells produce about 3.0 V when new. This voltage declines steadily as the cells run down but we'll ignore this complication. The voltage required is 20 V, which is higher than the supply, so a boost converter is the obvious choice. However, it must boost the voltage by a factor of nearly 7, which is ambitious. In practice a flyback converter might be required so that the transformer could provide some or all of the increase in voltage. (In practice a toy is so sensitive to cost that something more specialised but cheaper would probably be used.)

❦

7 Power switches (mainly SCRs and MOSFETs)

7.1 The power through a resistive load is phase-angle controlled by a pair of antiparallel SRCs (a TRIAC would work in the same way). The supply is standard, sinusoidal AC and the load dissipates $P_{\max}$ if it is connected directly to the supply. Show that the power dissipated as a function of delay angle α (in radians) is given by

$$\frac{P(\alpha)}{P_{\max}} = 1 - \frac{2\alpha - \sin(2\alpha)}{2\pi}. \tag{44}$$

This can be taken almost directly from the lecture notes but you are expected to be able to derive this sort of expression because it only requires a standard integral over a trigonometric function. You may need $\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

Solution. If the supply is $v(t) = V_{\text{pk}}\sin(\omega t)$, the RMS voltage of the supply is $V_{\text{pk}}/\sqrt{2}$ and the average power dissipated in a load of resistance R is $P_{\max} = V_{\text{rms}}^2/R = V_{\text{pk}}^2/(2R)$.

Now find the RMS value of the wave with phase angle control. The antiparallel SCRs allow conduction in both half-cycles and they each provide the same power so we need integrate only over one half-cycle. Save writing by using $\theta = \omega t$ and take the average from 0 to π.

$$V_{\text{rms}}^2(\alpha) = \frac{1}{\pi}\int_{\alpha}^{\pi} [V_{\text{pk}}\sin(\theta)]^2 \mathrm{d}\theta \tag{45}$$

This should be familiar as the integral to find the RMS value of a sine wave except that the lower limit is the delay angle α rather than 0. The power is related to $V_{\text{rms}}(\alpha)$ by $P(\alpha) = V_{\text{rms}}^2(\alpha)/R$ so

$$\frac{P(\alpha)}{P_{\max}} = \frac{V_{\text{rms}}^2(\alpha)/R}{V_{\text{pk}}^2/(2R)} = \frac{2}{\pi}\int_{\alpha}^{\pi} \sin^2(\theta)\mathrm{d}\theta. \tag{46}$$

The rest is mathematics. The identity provided shows that $2\sin^2\theta = 1 - \cos(2\theta)$ so we can take the factor of 2 from the fraction into the integral, which gives

$$\begin{aligned}
\frac{P(\alpha)}{P_{\max}} = \quad &= \frac{1}{\pi}\int_{\alpha}^{\pi} [1 - \cos(2\theta)]\mathrm{d}\theta \\
&= \frac{1}{\pi}\left[\theta - \frac{\sin(2\theta)}{2}\right]_{\alpha}^{\pi} \\
&= \frac{1}{\pi}\left[\pi - \alpha - \frac{\sin(2\pi) - \sin(2\alpha)}{2}\right] \\
&= 1 - \frac{2\alpha - \sin(2\alpha)}{2\pi}.
\end{aligned} \tag{47}$$

I have used $\sin(2\pi) = 0$ and have put the terms with α over the common denominator of 2π.

This holds for $0 \le \alpha \le \pi$. The angle α is the delay so the power decreases as α increases. Sometimes it is clearer to write the expression in terms of the conduction angle, which

I'll call β. It is given by $\beta = \pi - \alpha$ or $\alpha = \pi - \beta$. We need to use a trigonometric identity for the sine function:

$$\sin(2\alpha) = \sin(2\pi - 2\beta) = \sin(-2\beta) = -\sin(2\beta). \tag{48}$$

This uses the periodic nature of $\sin\theta$, which enables us to remove the 2π, and the odd nature, which allows the minus sign to be taken in front. Now put this in the expression for the power:

$$\frac{P(\beta)}{P_{\text{max}}} = 1 - \frac{2(\pi - \beta) + \sin(2\beta)}{2\pi} = \frac{2\beta - \sin(2\beta)}{2\pi}. \tag{49}$$

This is an increasing function of β. It goes to zero when the conduction angle $\beta = 0$ and to unity when $\beta = \pi$. ❦

7.2 What delay angles are needed to reduce the power in a resistive load to 75%, 50% and 25% of its maximum value using a TRIAC and phase-angle control? [Hint: one is easy but two are tricky.]

Solution. The easy one is 50% power, which needs an angle of 90°, halfway through each half-cycle. The others need a numerical solution. A power of 75% needs

$$0.75 = 1 - \frac{2\alpha - \sin(2\alpha)}{2\pi}, \tag{50}$$

which can be rearranged to

$$2\alpha - \sin(2\alpha) = 0.5\pi. \tag{51}$$

Perhaps your calculator has a numerical solver built into it. If not, you should recall standard methods such as Newton–Raphson or binary chopping from Engineering Mathematics 1. A graph is a useful way of estimating a value to start the search and the lecture notes include a plot. I estimated 65° or 1.1 rad to start and a few iterations homed in on 66.2°.

For 25% power we can use the symmetry of the sine wave and say immediately that the required angle is $\alpha = 180° - 66.2° = 113.8°$. ❦

7.3 A resistive heater dissipates 3.0 kW when connected directly to the 230 VAC mains. It is phase-angle controlled using a TRIAC, which has a forward voltage drop of 2.0 V.

(a) Calculate the power dissipated in the load for delay angles of 60° and 120°. [2.4 kW, 0.6 kW]

(b) Calculate the power dissipated in the TRIAC for the same delay angles. [Hint: the forward voltage across the TRIAC is assumed constant, so $P_{\text{ave}} = V_{\text{DC}} I_{\text{ave}}$.] [18 W, 6 W]

(c) Under what conditions does the power dissipated in the TRIAC reach a maximum and what is its value? [24 W]

(d) Is phase-angle control the most suitable method for controlling this load?

Solution.

(a) We can use the previous question for the power dissipated in the load,

$$P(\alpha) = P_{\max}\left[1 - \frac{2\alpha - \sin(2\alpha)}{2\pi}\right]. \tag{52}$$

Here $P_{\max} = 3.0\,\mathrm{kW}$ and we need only substitute the values of α, converting them first to radians. This gives 2.4 kW and 0.6 kW. These sum to 3.0 kW which they should by symmetry (the angles are supplementary, meaning that they sum to 180°).

(b) The power dissipated in the TRIAC needs more work. The general expression for power gives $p(t) = v(t)i(t)$ where $v(t)$ is the voltage dropped across the TRIAC and $i(t)$ is the current through it. Here we are told that the forward drop is $V_{\mathrm{TRIAC}} = 2.0\,\mathrm{V}$. Remember that the current changes sign every half-cycle because the supply is AC and so does the sign of the forward voltage drop (the TRIAC is like two SCRs in parallel but with opposite directions). The power dissipated can therefore be written as $p(t) = V_{\mathrm{TRIAC}}|i(t)|$ and its average is $P_{\mathrm{diss}} = V_{\mathrm{TRIAC}} I_{\mathrm{abs,ave}}$ where $I_{\mathrm{abs,ave}}$ is the average of the absolute value of the current (the average of the full-wave rectified current).

We haven't calculated $I_{\mathrm{abs,ave}}$ but we found the corresponding voltage in the lectures,

$$V_{\mathrm{abs,ave}} = \frac{2V_{\mathrm{pk}}}{\pi}\frac{1+\cos\alpha}{2}. \tag{53}$$

I have written it like this so that the fraction $(1+\cos\alpha)/2$ goes from 1 and 0 as α goes from 0 to 180°.

The heater is a resistor so we can find the current simply by dividing by the resistance of the heater, R_{heater}. It would be better to use the RMS voltage V_{AC} instead of the peak voltage V_{pk} as well. This gives

$$I_{\mathrm{abs,ave}} = \frac{2\sqrt{2}V_{\mathrm{AC}}}{\pi R_{\mathrm{heater}}}\frac{1+\cos\alpha}{2}. \tag{54}$$

The power dissipated is then

$$P_{\mathrm{diss}} = \frac{2\sqrt{2}V_{\mathrm{AC}}V_{\mathrm{TRIAC}}}{\pi R_{\mathrm{heater}}}\frac{1+\cos\alpha}{2}. \tag{55}$$

To find the resistance of the heater, use $P_{\mathrm{heater}} = V_{\mathrm{AC}}^2/R_{\mathrm{heater}}$ for when the heater is connected directly to the mains. This gives $R_{\mathrm{heater}} = V_{\mathrm{AC}}^2/P_{\mathrm{heater}} = (230\,\mathrm{V})^2/(3000\,\mathrm{W}) = 17.6\,\Omega$. Alternatively we could write

$$P_{\mathrm{diss}} = \frac{2\sqrt{2}}{\pi}\frac{V_{\mathrm{TRIAC}}}{V_{\mathrm{AC}}}\frac{1+\cos\alpha}{2}P_{\mathrm{heater}}. \tag{56}$$

This gives 17.6 W for $\alpha = 60°$ and 5.9 W for 120°. These powers are high enough that a substantial heatsink would be needed to protect the TRIAC. The factor of $V_{\mathrm{TRIAC}}/V_{\mathrm{AC}}$ makes the power dissipated in the TRIAC lower than that in the load by about a factor of 100.

(c) Maximum power dissipation occurs when the delay angle is zero so that the TRIAC conducts all the time. Putting $\alpha = 0°$ gives 23.5 W.

(d) A 3.0 kW heater is usually a fairly massive item and responds only slowly to changes in input power, much slower than the 100 Hz frequency of phase-angle control. This means that burst control may be more appropriate. The advantage is a reduction in radio-frequency interference (RFI or EMI). A possible disadvantage is that control can be only in integral numbers of cycles on and off, which may not allow fine enough adjustment of the power.

❦

7.4 A TRIAC is used to control the speed of a mains-powered electric drill. Would phase-angle or burst-mode control be more suitable?

Solution. Definitely phase-angle control; the drill would vibrate at the frequency of burst-mode control, which would be unpleasant and potentially dangerous. ❦

7.5 A DC motor is connected to the AC mains at V_{AC} through a bridge rectifier made of SCRs.

(a) Calculate the average voltage across the motor as a function of delay angle α. You may neglect the voltage drop across the SCRs. [Hint: the hard work has been done before: you can relate this to a result derived in the lectures.]

(b) If the product is used in the UK, what is the average voltage across the load at a firing angle of 90°? What is it if the SCRs are switched on continuously? [104 V, 207 V]

Solution.

(a) In the lectures we calculated the average of the magnitude of the voltage across a load controlled by two antiparallel SCRs or a TRIAC, using full-wave phase-angle control. The waveform from the controlled bridge rectifier is the same as this except that it is full-wave rectified: in other words it is the magnitude of the voltage, which is exactly what was calculated. The same result was used to calculate the power dissipated in the TRIAC.

$$V_{\mathrm{abs,ave}} = \frac{2\sqrt{2}V_{\mathrm{AC}}}{\pi}\frac{1+\cos\alpha}{2}. \tag{57}$$

The usual expression for $P(\alpha)$ is useless in this case because it applies only to a resistive load and a motor is highly inductive. The current has a quite different dependence on time from the voltage; they are not proportional.

(b) In the UK, $V_{\mathrm{AC}} = 230\,\mathrm{V}$ and we can just substitute in the expression above. At $\alpha = 90°$, $\cos\alpha = 0$ and $V_{\mathrm{abs,ave}} = V_{\mathrm{pk}}/\pi = \sqrt{2}V_{\mathrm{AC}}/\pi = 104\,\mathrm{V}$.

If the SCRs are switched on continuously, $\alpha = 0$, $\cos\alpha = 1$ and the voltage is doubled to $V_{\mathrm{abs,ave}} = 2V_{\mathrm{pk}}/\pi = 2\sqrt{2}V_{\mathrm{AC}}/\pi = 207\,\mathrm{V}$. You should recognise this as the average voltage of a full-wave rectified sine wave.

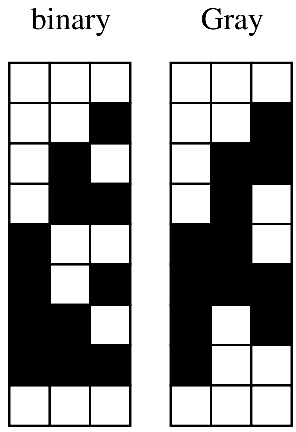

Figure 21. Binary and Gray codes for 3-bit binary numbers, counting downward.

❦

7.6 *Gray code.* Draw out a table of the binary codes for the sequence 0, 1, 2, 3, 0… and show that only one bit changes at two transitions but two bits change at the other two transitions. Two transitions are a problem in practice because they never occur simultaneously, which means that the output of a sensor that produces a binary sequence is briefly incorrect. Show that only one bit changes at each transition in the sequence 0, 1, 3, 2, 0… so that the problem of incorrect values cannot arise. This is the simplest example of a Gray code. Work out a 3-bit Gray code that cycles through the 8 possible values, changing only one bit every time.

Solution. The simplest extension of the 2-bit example is 0, 1, 3, 2, 6, 7, 5, 4, 0…. A systematic way of constructing Gray codes is called a binary-reflected Gray code. They are widely used where problems might arise when more than one bit changes simultaneously and Wikipedia has a good description. The binary and Gray sequences for 3 bits are compared in figure 21. ❦

7.7 A control knob on an electronic product is connected to an incremental encoder (shaft encoder) with two outputs A and B in quadrature. The outputs of the two sensors as a function of time are shown in figure 22. The 'click' (detent) on the knob is aligned with $A = B = 0$. Explain the output.

Could this sort of control be used for the knobs on the oscilloscopes in the laboratory?

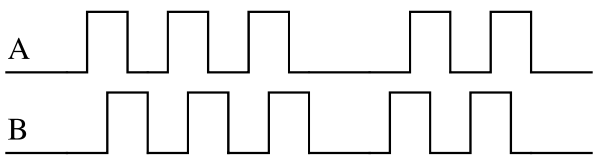

Figure 22. Outputs A and B from a quadrature encoder driven by a rotary control.

Solution. The knob was turned 3 clicks in one direction (we can't tell which without more information), then paused, followed by 2 clicks in the opposite direction.

The oscilloscopes in the laboratory almost certainly use this sort of sensor for the knobs that control the scales of voltage and time. These have a large angle between clicks. They are probably used as well for the knobs that shift the traces up and down, and left and right, but these do not have clicks and must respond to much smaller changes in angle. ❦

7.8 The brightness of an array of LEDs, which may be treated as a simple resistive load, is controlled by pulse-width modulation using a perfect switch. The LEDs draw 50 W when connected directly to a 12 V DC supply. Calculate the duty cycle D, average current I_{ave} and RMS current I_{rms} when the LEDs draw 10 W and 40 W. Comment on the differences between I_{ave} and I_{rms}.
[$D = 0.2$ and 0.8; $I_{\text{ave}} = 0.83$ A and 3.33 A; $I_{\text{rms}} = 1.86$ A and 3.73 A]

Solution. The duty cycle can be found directly from the average power using

$$\frac{P_{\text{ave}}(D)}{P_{\text{max}}} = D \tag{58}$$

because the LEDs dissipate $P_{\text{max}} = 50\,\text{W}$ for the fraction D of the time while they are alight. This gives $D = 0.2$ and 0.8 for 10 W and 40 W.

The average current can be found in the same way. The maximum current I_{max} flows when the LEDs are connected directly to the supply and follows from $P_{\text{max}} = V_{\text{DC}} I_{\text{max}}$, so $I_{\text{max}} = P_{\text{max}}/V_{\text{DC}} = (50\,\text{W})/(12\,\text{V}) = 4.2\,\text{A}$. The average current can then be found in the same way as the average power,

$$\frac{I_{\text{ave}}(D)}{I_{\text{max}}} = D, \qquad I_{\text{ave}}(D) = D I_{\text{max}}. \tag{59}$$

This gives $I_{\text{ave}}(0.2) = 0.83\,\text{A}$ and $I_{\text{ave}}(0.8) = 3.33\,\text{A}$.

The RMS current is a bit trickier and can be found in various ways. Remember the basic definition, that I_{rms} gives the average power dissipated in a resistive load:

$$P_{\text{ave}} = R I_{\text{rms}}^2. \tag{60}$$

We can find the resistance from the power of 50 W at 12 V using $P = V^2/R$ for pure DC, so $R_{\text{load}} = V^2/P = 2.88\,\Omega$. Then we can use $P_{\text{ave}} = R I_{\text{rms}}^2$ to find $I_{\text{rms}}(D)$ from $P_{\text{ave}}(D)$. This gives $I_{\text{rms}}(0.2) = 1.86\,\text{A}$ and $I_{\text{rms}}(0.8) = 3.73\,\text{A}$.

- For $D = 0.8$, $I_{\text{ave}} = 3.33\,\text{A}$ and $I_{\text{rms}} = 3.73\,\text{A}$, which are close. This is because the wave is not far from DC (it is zero for only 20% of the time) and $I_{\text{rms}} = I_{\text{ave}}$ for pure DC.
- For $D = 0.2$, the two values are very different, $I_{\text{ave}} = 0.83\,\text{A}$ and $I_{\text{rms}} = 1.86\,\text{A}$. This is because the current is a 'peaky' (or 'spiky') function of time and the two averages give significantly different values.

- The two values of duty cycle differ by a factor of 4. So do the values of I_{ave} but the values of I_{rms} differ only by a factor of $2 = \sqrt{4}$. This is because $P \propto I_{\text{rms}}^2$, the powers differ by a factor of 4 so the RMS values differ by the square root of this.

Another way of finding I_{rms} is to do the integral. This gives

$$I_{\text{rms}}^2 = \frac{1}{T}\int_0^T [i(t)]^2 \mathrm{d}t = \frac{1}{T}\int_0^{DT} [I_{\text{max}}]^2 \mathrm{d}t = D[I_{\text{max}}]^2 \tag{61}$$

so $I_{\text{rms}} = I_{\text{max}}\sqrt{D}$. ❦

7.9 The pulse-width modulation in the previous question is controlled by a MOSFET with $R_{\text{DS(on)}} = 0.8\,\Omega$, $T_{\text{on}} = 50\,\text{ns}$ and $T_{\text{off}} = 50\,\text{ns}$. The switching frequency is $f_{\text{sw}} = 20\,\text{kHz}$. Calculate the conduction and switching losses in the MOSFET for duty cycles of $D = 0.2$ and 0.8. [$P_{\text{cond}} = 2.8\,\text{W}$ and 11.1 W; $P_{\text{sw}} = 0.05\,\text{W}$ for both]

Solution. The conduction loss is due to the resistance of the channel of the MOSFET so the power is given by the usual expression in terms of the RMS value of the current, $P_{\text{cond}} = R_{\text{DS(on)}} I_{\text{rms}}^2$. The previous question showed that $I_{\text{rms}}(0.2) = 1.86\,\text{A}$, giving $P_{\text{cond}} = 2.8\,\text{W}$, and $I_{\text{rms}}(0.8) = 3.73\,\text{A}$, giving $P_{\text{cond}} = 11.1\,\text{W}$.

We can check these because the same current flows through the load and MOSFET, both of which are resistances. The equation $P_{\text{ave}} = RI_{\text{rms}}^2$ shows that power $\propto$ resistance so we can write

$$\frac{P_{\text{cond}}}{P_{\text{load}}} = \frac{R_{\text{DS(on)}}}{R_{\text{load}}} = \frac{0.8}{2.88} = 0.28. \tag{62}$$

For $D = 0.2$, $P_{\text{load}} = 10\,\text{W}$ so this gives $P_{\text{cond}} = 2.8\,\text{W}$ and for $D = 0.8$ it gives 11.1 W.

The switching loss is given by the expression derived in the lectures,

$$P_{\text{sw}} = \frac{f_{\text{sw}} V_{\text{DS(off)}}}{2}(T_{\text{on}} I_{\text{on}} + T_{\text{off}} I_{\text{off}}). \tag{63}$$

The switching frequency is the same for the two values of D and so is the current because I_{on} and I_{off} are the instantaneous values when the switch is turned on and off, not the average values, so these are both given by $I_{\text{max}} = 4.2\,\text{A}$. Then

$$P_{\text{sw}} = \frac{(20\,\text{kHz}) \times (12\,\text{V})}{2}[(50\,\text{ns})(4.2\,\text{A}) + (50\,\text{ns}) \times (4.2\,\text{A})] = 0.05\,\text{W}. \tag{64}$$

In this example the switching loss is *much* less than the conduction loss. The resistance of the MOSFET, 0.8 Ω, is relatively large compared with that of the load, 2.88 Ω, which causes the power dissipation in the MOSFET to be 28% that in the load. In fact the resistance of the MOSFET is large enough that it should be taken into account when calculating the current: it is not a good approximation to treat this MOSFET as a perfect switch. The designer should choose a device with a lower resistance. This might have larger switching losses but they are much less important. ❦

Table 3. Parameters for a load controlled by a MOSFET using PWM.

I_{on}	16 A
I_{off}	24 A
PWM duty cycle	50%
PWM frequency	50 kHz
Supply voltage	200 V
$R_{\mathrm{DS(on)}}$	50 mΩ
T_{on}	25 ns
T_{off}	30 ns

7.10 A MOSFET acts as a low-side switch for a load whose power is controlled by pulse-width modulation. The parameters of the system are given in table 3. Assume that components are ideal and that the current increases linearly while the MOSFET is on.

(a) Draw the current waveform. Clearly indicate the current and time values.

(b) Calculate the average and RMS value of the current from first principles. [10.0 A, 14.2 A]

(c) Determine the conduction loss in the MOSFET. [10.1 W]

(d) Determine the switching loss in the MOSFET. [5.6 W]

(e) Determine the total power dissipated in the MOSFET. [15.7 W]

(f) The power loss is found to be excessive so the designer suggest halving the switching frequency, keeping other parameters the same. Will this be effective?

[Adapted from 2015 May Q3.]

Solution.

(a) The current is shown in figure 23. The period is $1/(50\,\mathrm{kHz}) = 20\,\mu\mathrm{s}$ and the 50% duty cycle means that the MOSFET is on for 10 µs and off for 10 µs.

(b) The average current is given by the general expression

$$I_{\mathrm{ave}} = \frac{1}{T}\int_0^T i(t)\mathrm{d}t. \tag{65}$$

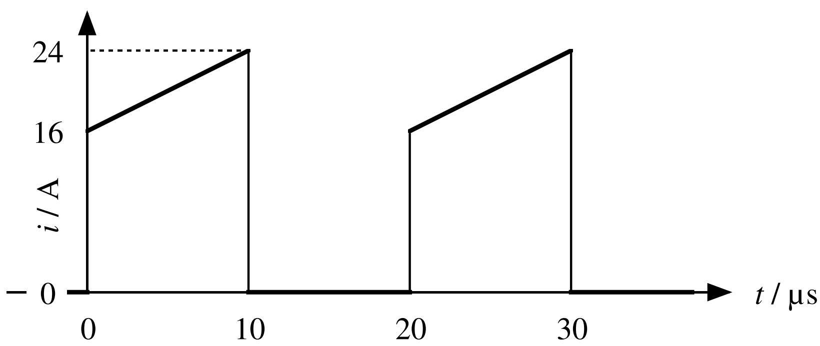

Figure 23. Current as a function of time through the load and MOSFET.

Here we can do this geometrically by interpreting the integral as the area under the curve. The average current for times from 0 to 10 μs is $(16 + 24)/2 = 20\,\mathrm{A}$ and this is half the total period, with no current flowing for the remaining part, so $I_{\mathrm{ave}} = 10\,\mathrm{A}$.

Alternatively we can do the integral algebraically. This can be done purely with symbols or with numbers. I suspect that most students will use numbers so I'll do this. First we need an expression for the current. Use units of A for the current and μs for time. The current rises 8 A in 10 μs so the slope is 0.8 and we can write

$$i(t) = 16 + 0.8\,t \tag{66}$$

for the current while the MOSFET is on. Then the integral is

$$\begin{aligned} I_{\mathrm{ave}} &= \frac{1}{20}\int_0^{10}(16 + 0.8\,t)\mathrm{d}t \\ &= \frac{1}{20}\left[16\,t + 0.8\,\frac{t^2}{2}\right]_0^{10} \\ &= \frac{1}{20}[160 + 0.8 \times 50] \\ &= 10\,\mathrm{A}. \end{aligned} \tag{67}$$

The RMS value definitely needs an integral.

$$\begin{aligned} I_{\mathrm{rms}}^2 &= \frac{1}{T}\int_0^{T}[i(t)]^2\mathrm{d}t \\ &= \frac{1}{20}\int_0^{10}[16 + 0.8\,t]^2\mathrm{d}t \\ &= \frac{1}{20}\int_0^{10}[256 + 2.56\,t + 0.64\,t^2]\mathrm{d}t \\ &= \frac{1}{20}\left[256\,t + 25.6\,\frac{t^2}{2} + 0.64\,\frac{t^3}{3}\right]_0^{10} \\ &= \frac{1}{20}[2560 + 1280 + 213.3] \\ &= 202.7\,\mathrm{A}^2. \end{aligned} \tag{68}$$

Taking the square root gives $I_{\mathrm{rms}} = 14.2\,\mathrm{A}$. This is larger than the average current, as it should be.

(c) The MOSFET behaves like a resistance so we can write $P_{\mathrm{ave}} = R_{\mathrm{DS(on)}} I_{\mathrm{rms}}^2$ and have done the hard work already.

$$P_{\mathrm{cond}} = (50\,\mathrm{m}\Omega) \times 202.7\,\mathrm{A}^2 = 10.1\,\mathrm{W}. \tag{69}$$

(d) The switching loss is given by

$$\begin{aligned} P_{\mathrm{sw}} &= \frac{f_{\mathrm{sw}} V_{\mathrm{DS(off)}}}{2}(T_{\mathrm{on}} I_{\mathrm{on}} + T_{\mathrm{off}} I_{\mathrm{off}}) \\ &= \frac{(50\,\mathrm{kHz}) \times (200\,\mathrm{V})}{2}[(25\,\mathrm{ns})(16\,\mathrm{A}) + (30\,\mathrm{ns}) \times (24\,\mathrm{A})] \\ &= 5.6\,\mathrm{W}. \end{aligned} \tag{70}$$

(e) The total power dissipated in the MOSFET is just the sum of these two powers, $P_{\text{diss}} = P_{\text{cond}} + P_{\text{sw}} = 15.7\,\text{W}$.

(f) Halving the switching frequency will halve the switching loss and have no effect on the conduction loss because the duty cycle is the same. Unfortunately the switching loss is the smaller of the two so this won't help a lot (although it does to some extent). It would be more effective to reduce the conduction loss.

❦

8 Inverters

8.1 An H-bridge inverter is fed from a 100 V DC supply and operates at 50 Hz. It produces a simple square wave output.

(a) What is the RMS output voltage?

(b) The inverter feeds a purely resistive load of 10 Ω. Calculate the form of the current $i(t)$ through the load, its RMS value and the average power dissipated in the load. [10 A, 1 kW]

(c) The inverter is connected instead to a purely inductive load of 50 mH. Calculate the form of the current $i(t)$ through the load, its RMS value and the average power delivered to the load. [5.8 A, zero]

Solution.

(a) The output voltage alternates between ± 100 V so its magnitude is always 100 V and the RMS voltage must have the same value.

(b) For a purely resistive load, $i(t) = v(t)/R$ so the current is a square wave of peak amplitude $(100\,\mathrm{V})/(10\,\Omega) = 10\,\mathrm{A}$. The RMS value has the identical value, 10 A, for the same reason as the previous part of the question. The power is given by $p(t) = v(t)i(t) = 1000\,\mathrm{W}$ and is always positive because the current and voltage are either both positive or both negative. The power is therefore constant and its (average) value is 1000 W or 1 kW.

(c) With a purely inductive load we need to use the current–voltage relation for an inductor, $v_L = L\,\mathrm{d}i_L/\mathrm{d}t$. When $v(t)$ is positive and constant, $i(t)$ must be a straight line with a positive slope. The opposite happens in the alternate half-cycles. The total change in current during each half-cycle can be found from $v_L = L\,\Delta i_L/\Delta t$ or $\Delta i_L = v_L \Delta t/L = (100\,\mathrm{V}) \times (10\,\mathrm{ms})/(50\,\mathrm{mH}) = 20\,\mathrm{A}$. In a steady state $i(t)$ must be symmetric about zero current so it is a triangular wave between ± 10 A.

The RMS value of a triangular wave is $I_{\mathrm{rms}} = I_{\mathrm{pk}}/\sqrt{3} = (10\,\mathrm{A})/\sqrt{3} = 5.8\,\mathrm{A}$.

The average power delivered to the load is zero because an inductor stores and releases energy, never dissipating it. In more detail, the inductor returns energy to the source during the first half of each half-cycle, while its current has the opposite sign to the voltage, and stores energy in the other half-cycles.

Plots of the current and power for the two types of load are shown in figure 24 on the next page. The voltage is not shown but it has the same shape as the current for the purely resistive load. ❦

8.2 [Harder, only for students who have taken Electrical Circuits 2, not examinable.] Suppose that the load on the inverter is a 10 Ω resistor and a 50 mH inductor in series. Derive an expression for $i(t)$ as follows.

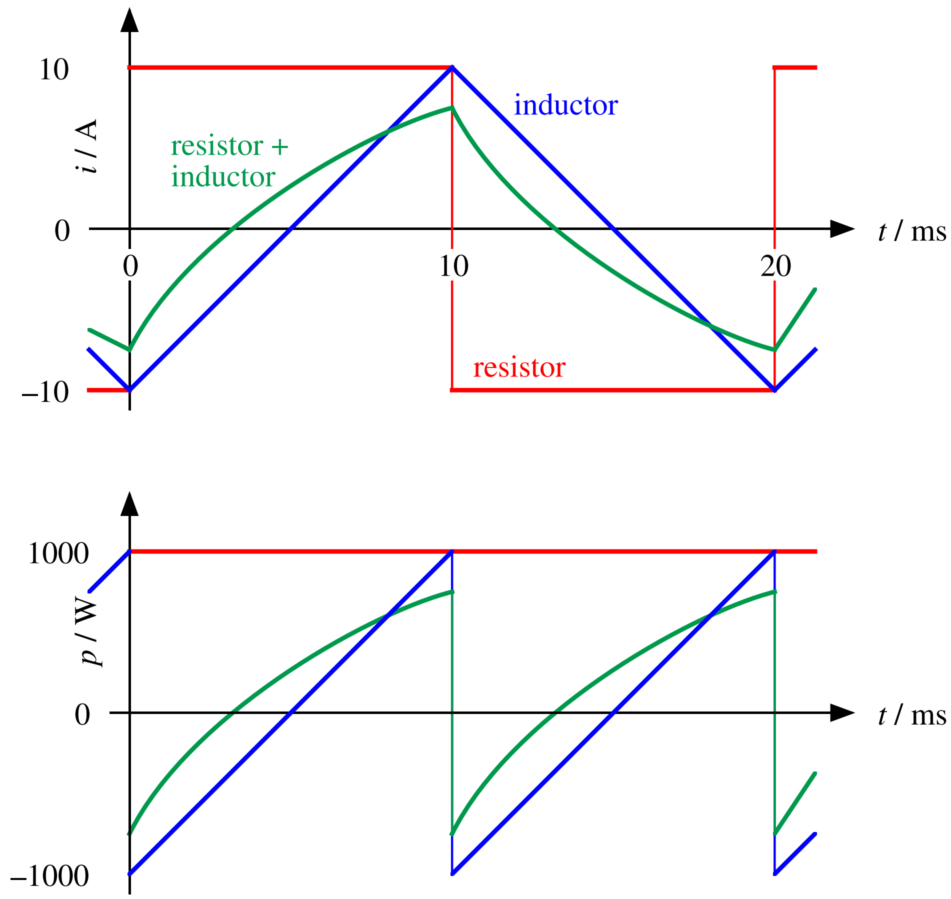

Figure 24. Current and power absorbed from inverter as a function of time for resistive, inductive and resistive–inductive loads.

(a) Focus on the first half-cycle, from 0 to 10 ms. The form of current must be a decaying exponential function because the RL circuit is a first-order system. It therefore has the general form

$$i(t) = I_{\text{final}} + (I_{\text{initial}} - I_{\text{final}}) \exp(-t/\tau). \tag{71}$$

What is the value of τ? [5 ms]

(b) The current I_{final} is the value that would be reached if the circuit was left for a long time so that the transient decays away and the circuit reaches a steady state. (This doesn't happen here because the voltage reverses every 10 ms but that doesn't affect the definition of I_{final}.) What is the value of I_{final}? [10 A]

(c) The final step is to find I_{initial} and this is the most complicated.

When the output of the inverter has reached a steady state, the current must be pure AC with an average value of zero. This means that successive half-cycles have the the same form but with their sign changed. In other words, if we know the form of $i(t)$ for $0 < t < 10$ ms, we can find its form for $10 < t < 20$ ms by sliding $i(t)$ along the time axis by 10 ms and flipping it vertically (changing the sign of $i(t)$).

The current through an inductor cannot change instantaneously so each half-cycle of $i(t)$ must join on to the previous one without any jumps. This means that the current must have the same value at the beginning and end of each half-cycle except that the sign changes. Mathematically, $i(\frac{1}{2}T) = -i(0)$ for the first half-cycle. Use this to show that

$$I_{\text{initial}} = -I_{\text{final}} \frac{1 - \exp(-\frac{1}{2}T/\tau)}{1 + \exp(-\frac{1}{2}T/\tau)} \equiv -I_{\text{final}} \tanh \frac{T}{4\tau}. \tag{72}$$

What is the numerical value of I_{initial}? [−7.6 A]

I won't ask you to find the average power but here is the result:

$$P_{\text{ave}} = \frac{V_{\text{DC}}^2}{R} \left[1 - \frac{4\tau}{T} \tanh \frac{T}{4\tau} \right]. \tag{73}$$

Numerically this gives $P_{\text{ave}} = 0.24\,\text{kW}$. The inductance reduces the power considerably from the value for the resistor alone.

This probably looks frightening but a course on power electronics for electrical engineers would need this level of analysis because many loads, particularly motors, behave like RL circuits. The current for AC phase control of an inductive load by an SCR can be analysed in a similar way.

Solution.

(a) For a first-order circuit with resistor and inductor, $\tau = L/R = (50\,\text{mH})/(10\,\Omega) = 5\,\text{ms}$.

(b) After a long time, when an LR circuit has reached a steady state, the inductor behaves like a short-circuit and only the resistor limits the current. This shows that $I_{\text{final}} = V_{\text{DC}}/R = (100\,\text{V})/(10\,\Omega) = 10\,\text{A}$. This is the same as was found in the previous question.

(c) We have to solve $i(\frac{1}{2}T) = -i(0)$ using the standard form of the transient given. Now $i(0) = I_{\text{initial}}$ so combining these gives

$$-i(0) = i(T/2) \tag{74}$$
$$-I_{\text{initial}} = I_{\text{final}} + (I_{\text{initial}} - I_{\text{final}}) \exp[-(T/2)/\tau]. \tag{75}$$

We know everything in this equation except I_{initial} so we have to rearrange it to make I_{initial} the subject.

$$\begin{aligned}
-I_{\text{initial}} &= I_{\text{final}} + I_{\text{initial}} \exp[-T/(2\tau)] - I_{\text{final}} \exp[-T/(2\tau)] \\
-I_{\text{initial}} - I_{\text{initial}} \exp[-T/(2\tau)] &= I_{\text{final}} + -I_{\text{final}} \exp[-T/(2\tau)] \\
-\{1 + \exp[-T/(2\tau)]\} I_{\text{initial}} &= I_{\text{final}} \{1 - \exp[-T/(2\tau)]\} \\
I_{\text{initial}} &= -I_{\text{final}} \frac{1 - \exp[-T/(2\tau)]}{1 + \exp[-T/(2\tau)]}. \qquad (76)
\end{aligned}$$

This is one of the expressions given in the question and is perfectly acceptable. You can make it neater by multiplying the top and bottom of the fraction by $\exp[+T/(4\tau)]$. This gives

$$I_{\text{initial}} = -I_{\text{final}} \frac{\exp[+T/(4\tau)] - \exp[-T/(4\tau)]}{\exp[+T/(4\tau)] + \exp[-T/(4\tau)]}. \tag{77}$$

The manipulation is useful because it matches the definition of the hyperbolic tangent function,

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}. \tag{78}$$

The fractions are identical if $x = T/(4\tau)$. This means that we can also write

$$I_{\text{initial}} = -I_{\text{final}} \tanh \frac{T}{4\tau}. \tag{79}$$

This form is useful only if you know how $\tanh x$ behaves. It is an odd function so it starts from 0 at $x = 0$, rises initially like x (as does $\tan x$) but then flattens out and approaches 1 as $x \to \infty$. Numerically, $T/(4\tau) = 1$ here and $I_{\text{final}} = 10\,\text{A}$, giving $I_{\text{initial}} = -7.6\,\text{A}$. Whew!

A plot of the current and power for the load with a resistor and inductor is shown in figure 24 on page 50. ❦

8.3 *Fourier analysis of voltage from inverter.* Find the Fourier series for the voltage produced by the inverter in question 1 of this section, which is a square wave between $\pm V_{\text{DC}} = \pm 100\,\text{V}$ at $f = 50\,\text{Hz}$. The series is a standard result that you should know from Engineering Mathematics 2.

Suppose that the square wave is filtered so well that only its fundamental Fourier component survives. What is the RMS voltage of this wave? [90 V]

Solution. You should remember finding the Fourier series of a symmetric square wave $v(t)$ in Engineering Mathematics 2 or Applicable Mathematics 2N. If the square wave oscillates between $\pm V_{\text{pk}}$ and has angular frequency ω, the result is

$$v(t) = \frac{4V_{\text{pk}}}{\pi}\left[\sin \omega t + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \cdots\right]. \tag{80}$$

Here $V_{\text{pk}} = V_{\text{DC}} = 100\,\text{V}$ and $f = 50\,\text{Hz}$ so $\omega = 100\pi\,\text{rad s}^{-1}$.

If all the higher harmonics are filtered away we are left with only $\sin \omega t$, whose peak amplitude is $4V_{\text{pk}}/\pi = 127\,\text{V}$. This is larger than the input to the inverter, which is possible because the filter that removes the higher harmonics stores energy at some times and releases it at others. The RMS value is found by dividing the peak amplitude by $\sqrt{2}$ because it is a sine wave, giving 90 V. ❦

8.4 The driver of the inverter in question 1 of this section is modified so that the H-bridge gives zero voltage on the output for angles of $\pm\alpha$ around the transitions between $\pm V_{\text{DC}}$ with $V_{\text{DC}} = 100\,\text{V}$ and $f = 50\,\text{Hz}$ as before. The waveform is shown in figure 25 on the next page.

(a) Find an expression for the RMS value of this voltage. What is its numerical value for $\alpha = 30°$? [82 V]

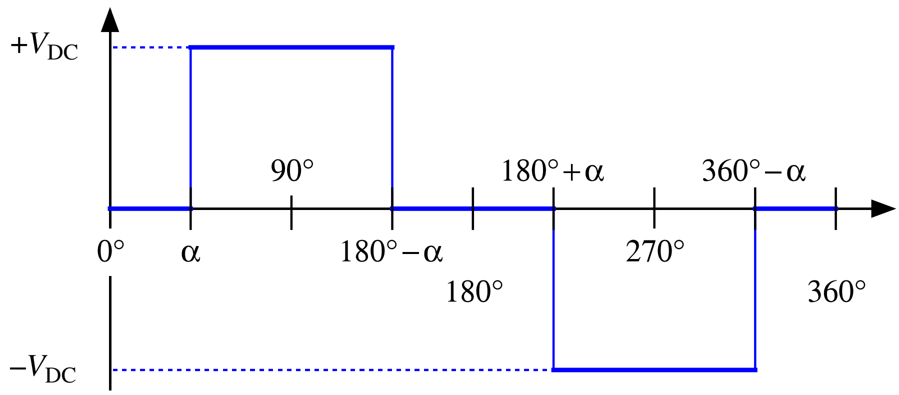

Figure 25. Voltage as a function of time (expressed as an angle) from inverter with output of zero volts for angles of $\pm\alpha$ about the transitions.

(b) [Harder] Derive expressions for the amplitude and RMS value of the fundamental component of this waveform. What is the RMS value of the fundamental for $\alpha = 30°$? [78 V]

This waveform has smaller amplitudes of higher harmonics, which is often important in real systems, but at the cost of a lower RMS voltage.

Solution.

(a) The RMS voltage is given by the general expression

$$V_{\mathrm{rms}} = \frac{1}{T}\int_0^T [v(t)]^2 \mathrm{d}t. \tag{81}$$

The integral should go over the whole period and it is easiest to integrate over angle here. By symmetry we need only integrate from 0 to $\pi/2$. Then

$$\begin{aligned} V_{\mathrm{rms}} &= \frac{1}{\pi/2}\int_\alpha^{\pi/2} V_{\mathrm{DC}}^2 \,\mathrm{d}\theta \\ &= \frac{2}{\pi} V_{\mathrm{DC}}^2 \left[\frac{\pi}{2} - \alpha\right] \\ &= V_{\mathrm{DC}}^2 \left[1 - \frac{2\alpha}{\pi}\right] \equiv V_{\mathrm{DC}}^2 \left[1 - \frac{\alpha}{90°}\right]. \end{aligned} \tag{82}$$

For $\alpha = 30°$, the factor in [] becomes $\frac{2}{3}$ so $V_{\mathrm{rms}} = \sqrt{2/3}V_{\mathrm{DC}} = 82\,\mathrm{V}$, reduced from 100 V by the intervals at zero voltage.

(b) The voltage $v(t)$ is an odd function of t so the Fourier series has only sine terms. The general expression for the coefficients (taken from the School of Engineering's *Table of mathematical formulae*) is

$$b_m = \frac{2}{T}\int_0^T f(t)\,\sin(m\omega t)\,\mathrm{d}t. \tag{83}$$

The even symmetry of $v(t)$ about 90° means that only terms with odd values of m survive the integral, as for the Fourier series for the square wave quoted above, but we only need the first term for this question. It is again easier to do the integral over the angle θ so the amplitude of the fundamental is

$$V_1 = \frac{2}{2\pi} \int_0^{2\pi} v(\theta)\ \sin(\theta)\, \mathrm{d}\theta. \tag{84}$$

Again we can reduce the range of the integral to its first quarter, 0 to $\pi/2$. The result must be multiplied by 4 to compensate for the change in range. The voltage $v(t)$ is zero for $\theta < \alpha$ and V_{DC} for $\theta > \alpha$. This gives

$$\begin{aligned} V_1 &= 4\frac{2}{2\pi} \int_\alpha^{\pi/2} V_{\mathrm{DC}}\ \sin(\theta)\, \mathrm{d}\theta \\ &= \frac{4V_{\mathrm{DC}}}{\pi} \int_\alpha^{\pi/2} \sin(\theta)\, \mathrm{d}\theta \\ &= \frac{4V_{\mathrm{DC}}}{\pi} [-\cos\theta]_\alpha^{\pi/2} \\ &= \frac{4V_{\mathrm{DC}}}{\pi}[\cos\alpha - \cos(\pi/2)] \\ &= \frac{4V_{\mathrm{DC}}}{\pi} \cos\alpha. \end{aligned} \tag{85}$$

Remember that $\cos(\pi/2) = 0$. The fraction is the usual result for a square wave, quoted above, and the factor of $\cos\alpha$ shows how the interval of zero voltage reduces the amplitude.

For $\alpha = 30°$, $\cos\alpha = \sqrt{3}/2$ and the amplitude of the fundamental is reduced from 127 V to 110 V. The RMS value of this is 78 V. This is only a little below the RMS voltage of the modified square wave from the inverter because it is closer to a sine wave than the normal square wave.

❦

9 Thermal management

9.1 A transistor dissipates 2 W and has a thermal resistance of 5°C/W between junction and case. Calculate the junction temperature when the ambient temperature is 50°C with a heat sink of thermal resistance (including the contact) (i) 50°C/W and (ii) 10°C/W. [80°C, 160°C]

Solution. $T_{\mathrm{j}} = T_{\mathrm{a}} + P(\theta_{\mathrm{jc}} + \theta_{\mathrm{ca}})$ where $T_{\mathrm{a}} = 50°\mathrm{C}$ and P = 2 W. Then (i) $T_{\mathrm{j}} = 50 + 2(5 + 50) = 160°\mathrm{C}$ and (ii) $T_{\mathrm{j}} = 50 + 2(5 + 10) = 80°\mathrm{C}$. ❦

9.2 A TIP120 transistor is required to dissipate 40 W in an ambient temperature of 30°C. Determine the thermal resistance of the heatsink required to keep the junction temperature below 150°C. There is an insulating washer of thermal resistance 0.5°C/W between the case and the heatsink. The transistor is specified for a dissipation of 65 W below 25°C, derated at 0.5 W/°C at higher temperatures. [Note that *derating* is the inverse of the thermal resistance and is an alternative specification.] [0.5°C/W]

What size of heatsink would be needed if a TIP3055 were used instead? This is a larger transistor with a maximum dissipation of 90 W derated at 0.7 W/°C.

Solution. The total thermal resistance must be less than $(T_{\mathrm{j}} - T_{\mathrm{a}})/P = (150 - 30)/40 = 3°\mathrm{C/W}$. The transistor is derated at 0.5 W/°C and its equivalent thermal resistance between junction and case is the reciprocal of this, 2.0°C/W. The overall thermal resistance $\theta_{\mathrm{ja}} = \theta_{\mathrm{jc}} + \theta_{\mathrm{ch}} + \theta_{\mathrm{ha}}$ where 'h' stands for heatsink and θ_{ch} is the thermal resistance of the washer between case and heatsink. Thus $2.0 + 0.5 + \theta_{\mathrm{ha}} < 3.0$ and we need $\theta_{\mathrm{ha}} < 0.5°\mathrm{C/W}$.

The TIP3055 is derated at 0.7 W/°C, equivalent to a thermal resistance of 1.4°C/W. Now we need $1.4 + 0.5 + \theta_{\mathrm{ha}} < 3.0$ so $\theta_{\mathrm{ha}} < 1.1°\mathrm{C/W}$. The transistor is more expensive but the heatsink can be smaller. ❦

9.3 Calculate T_{JD1}, T_{JD2} and θ_{SA} for the system shown in figure 26.

Solution. The strategy is to use the transistor to determine the heatsink temperature. Then divide the drop in temperature across the heatsink by the total power to get its thermal resistance. The diode junction temperatures can then offset from the calculated sink temperature accordingly.

In detail, the transistor dissipates 60 W, which passes through a thermal resistance of 0.27 °C/W from its case and mounting. This gives a temperature difference of $(0.27\,°\mathrm{C/W}) \times (60\,\mathrm{W}) = 16.2°\mathrm{C}$. The heatsink is therefore at $T_{\mathrm{S}} = 96.2 - 16.2 = 80.0°\mathrm{C}$.

The total power dissipated by the heatsink is $16 + 12 + 60 = 87\,\mathrm{W}$, which produces a change in temperature of $80.0 - 36.5 = 43.5°\mathrm{C}$. The thermal resistance is $\theta_{\mathrm{SA}} = (43.5°\mathrm{C})/(87\,\mathrm{W}) = 0.5\,°\mathrm{C/W}$.

Diode 1 dissipates 15 W, which passes through a total thermal resistance of 0.35 °C/W before it reaches the heatsink, giving a temperature difference of $(15\,\mathrm{W}) \times (0.35\,°\mathrm{C/W}) =$

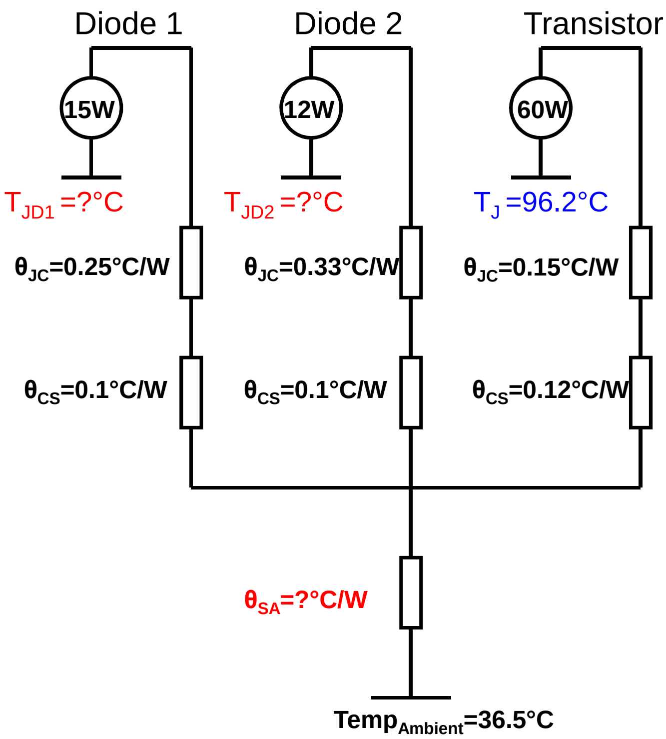

Figure 26. System comprising two diodes and a transistor on a heatsink.

5.3°C. Adding this to the temperature of the heatsink gives $T_{\mathrm{JD1}} = 85.3$°C. Similarly for diode 2, 12 W passing through 0.43 °C/W gives 5.2°C so $T_{\mathrm{JD2}} = 85.2$°C.

Some of these numbers are quoted to an excessive number of (in)significant figures. ❦

9.4 A switch-mode power supply contains two power semiconductor devices:

- A MOSFET switch, which dissipates 1.0 W
- A diode, which dissipates 2.0 W.

Both are in TO220 packages, which have thermal resistances $\theta_{\mathrm{jc}} = 3\,\mathrm{°C\,W^{-1}}$ and $\theta_{\mathrm{ca}} = 60\,\mathrm{°C\,W^{-1}}$. The temperature of their junctions should not exceed 125°C and the ambient temperature is 25°C.

(a) Is it possible to use either device safely without a heatsink?

(b) Despite the results of the previous analysis, the designer decides to mount both devices on a common heatsink. Specify the heatsink. Hint: which component is most at risk from overheating? [$\theta_{\mathrm{hs}} < 31\,\mathrm{°C\,W^{-1}}$]

(c) What temperatures will the junctions of the two devices reach, assuming that the heatsink has the maximum permitted thermal resistance?

(d) Is this a sensible design? If not, suggest how it could be improved.

Solution.

(a) If used without a heatsink, the total thermal resistance from junction to ambient is $\theta_{ja} = \theta_{jc} + \theta_{ca} = 63\,°\mathrm{C\,W^{-1}}$. The difference in temperature T_{ja} between the junction to ambient must not exceed $125 - 25 = 100°\mathrm{C}$.

- For the MOSFET, $T_{ja} = P\theta_{ja} = (1.0\,\mathrm{W})\times(63\,°\mathrm{C\,W^{-1}}) = 63\,°\mathrm{C}$, which is safe. The temperature of the junction would be $T_j = T_{ja} + T_a = 63 + 25 = 88°\mathrm{C}$.
- The diode dissipates twice the power so $T_{ja} = 126\,°\mathrm{C}$, which is excessive.

The MOSFET may be used without a heatsink but not the diode.

(b) The diode is most at risk from overheating so start from its junction, assuming that it is at the maximum temperature of $T_j = 125°\mathrm{C}$. It dissipates 2.0 W and its thermal resistance to the case is $\theta_{jc} = 3\,°\mathrm{C\,W^{-1}}$ so the temperature drop is $T_{jc} = P\theta_{jc} = (2.0\,\mathrm{W}) \times (3\,°\mathrm{C\,W^{-1}}) = 6°\mathrm{C}$. The heatsink is therefore at $T_{hs} = 125 - 6 = 119°\mathrm{C}$ (next to the diode). It must dissipate the total power of 3.0 W and has a temperature difference of $119 - 25 = 94°\mathrm{C}$. Its thermal resistance must therefore obey $\theta_{hs} < (94°\mathrm{C})/(3.0\,\mathrm{W}) = 31\,°\mathrm{C\,W^{-1}}$.

(c) With the heatsink on the limit of thermal resistance, the junction of the diode is at 125°C. The MOSFET dissipates half the power so $T_{jc} = 3°\mathrm{C}$ instead of 6°C for the diode and its junction reaches 122°C.

(d) This is a daft design. We have *increased* the temperature of the MOSFET by mounting it on the same heatsink as the diode. It would make more sense to leave the MOSFET freely mounted, when its junction would be at only 88°C. If we used the same heatsink but it now carries only the heat from the diode, its temperature drop would reduce from 94°C with 3 W to 63°C at 2 W. Adding $T_{jc} = 6°\mathrm{C}$ puts the junction of the diode at $6 + 63 + 25 = 94°\mathrm{C}$. This is better for both devices.

❦

Table 4. Parameters of critical components in LM3475 buck regulator evaluation board.

Parameter	Symbol	Value	Unit
Switching frequency	f_{sw}	200	kHz
Inductance of inductor	L	10	μH
DC resistance of inductor	R_L	0.13	Ω
Smoothing capacitance	C_{smooth}	100	μF
MOSFET channel resistance	$R_{\mathrm{ds,on}}$	50	mΩ
MOSFET switching times	$t_{\mathrm{on}} = t_{\mathrm{off}}$	20	ns
MOSFET thermal resistance to ambient	θ_{ja}	170	$^{\circ}\mathrm{C\,W^{-1}}$
MOSFET thermal resistance to leads	θ_{jl}	60	$^{\circ}\mathrm{C\,W^{-1}}$
Diode forward voltage drop	V_{f}	0.4	V
Diode thermal resistance to leads	θ_{jl}	25	$^{\circ}\mathrm{C\,W^{-1}}$

10 Putting it all together

These questions bring together topics from across the course and get much closer to engineering problems. They are more complex than exam questions.

10.1 *LM3475 buck regulator evaluation board.* This problem analyses an evaluation board for the Texas Instruments (formerly National Semiconductor) LM3475 controller for a buck DC–DC converter. The output of the board is rated at 2.5 V at 2.0 A and the input may lie between 5.0 V and 10.0 V. The parameters of the critical components are listed in table 4. The MOSFET is a p-channel SI2343CDS and the diode is a B220/A Schottky type to reduce forward voltage drop. Suppose initially that the board provides its full rated output and that the input is at 5.0 V.

(a) Calculate the duty cycle and average input current under these conditions, assuming that the converter runs in continuous conduction mode.

(b) Confirm that the assumption of continuous conduction mode is correct.

(c) Calculate the average power dissipated in the MOSFET. Is its temperature safe? Use the higher thermal resistance initially, which is for a device attached to narrow PCB tracks.

(d) If the temperature is too high, specify the heatsink needed, using the lower thermal resistance for the device. (This is specified as ‘to leads’ rather than ‘to case’ because it is a surface-mount device and heat is conducted through the leads.)

(e) Calculate the average power dissipated in the diode. Is its temperature safe?

Solution. Oh dear, a lot to write. ❦