POWER ENGINEERING

#08 FUNDAMENTALS OF TRANSFORMERS

2018

University of Glasgow

Transformers

As we saw in lecture 2 the transformer is one of the principal reasons behind the adoption of AC power systems. Its ability to provide a (relatively!) cheap and reliable means of converting AC voltage levels results in high efficiency AC power transmission.

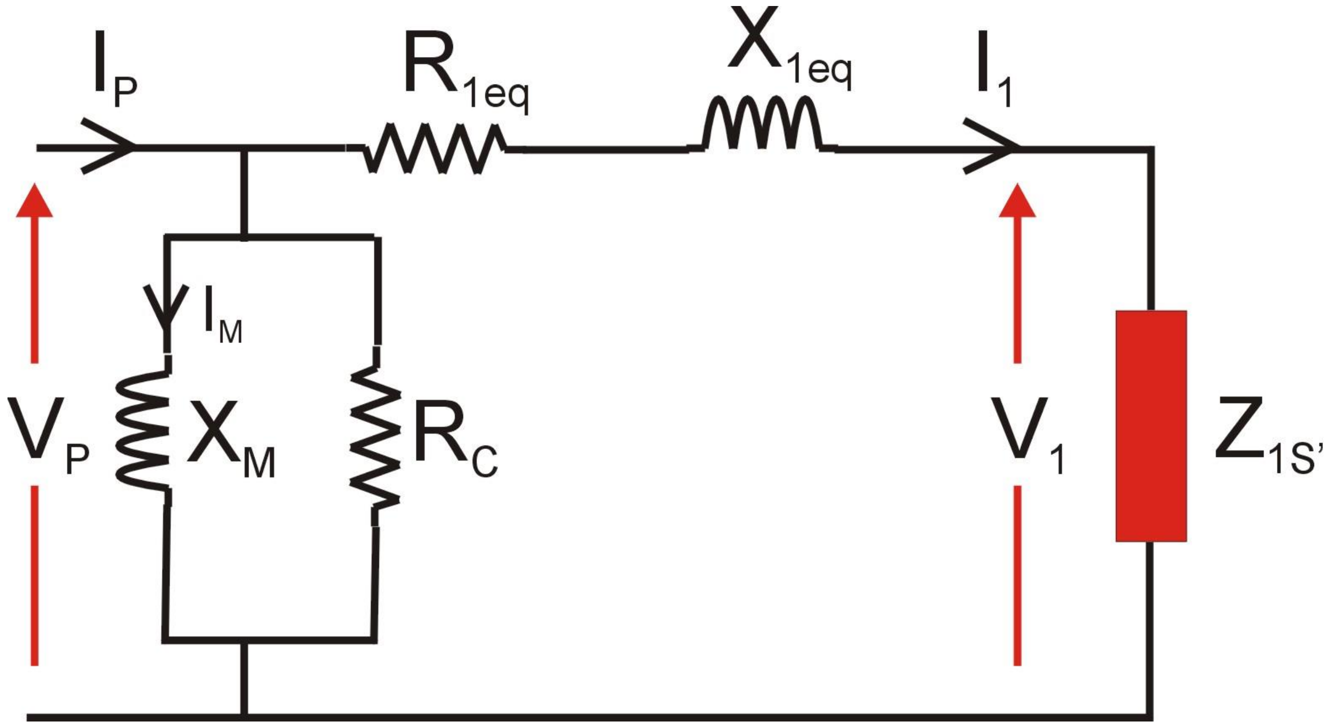
Over the next 4 lectures we will investigate:

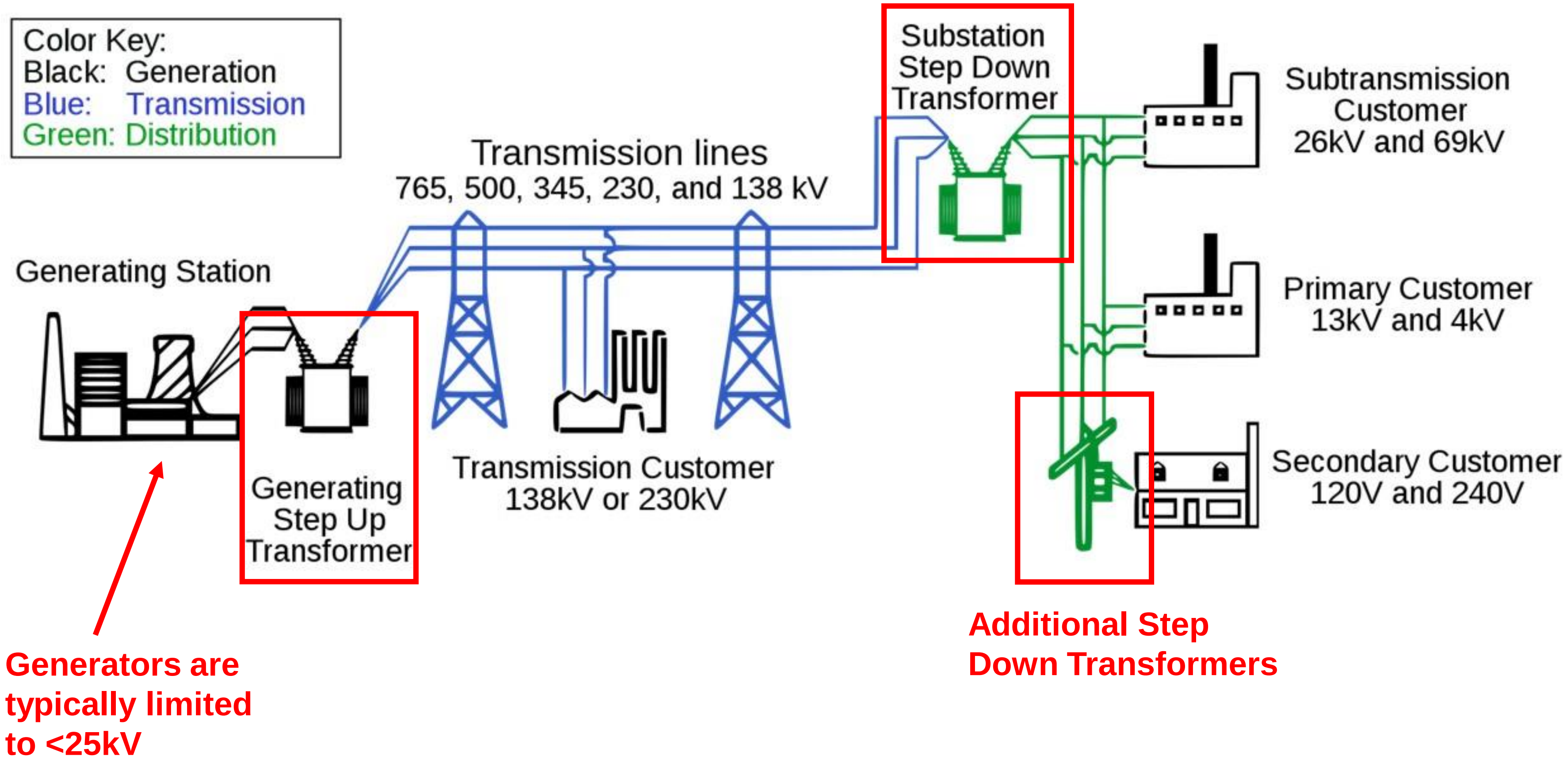
- Types of Transformers
- The Ideal Transformer
- Basic Electromagnetics
- Transformer Limits

Today

- The Equivalent Circuit for a practical Transformer
- Tests to determine the Equivalent Circuit
- Transformer performance under load: Regulation & Efficiency
- Three Phase Transformers

Our main aim is to determine the **EQUIVALENT CIRCUIT** for transformers and then use this to calculate/predict transformer performance under load (output voltage, output current, efficiency, regulation)

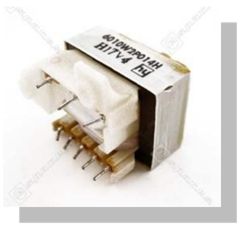




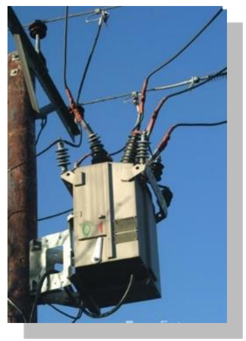
Transformers in Modern AC Power Generation/Transmission/Distribution System

Power Transformers come in all shapes and sizes:

pcb Mounted



Pole Mounted



Sub-Station

Other example of Transformers:

Valve Amp

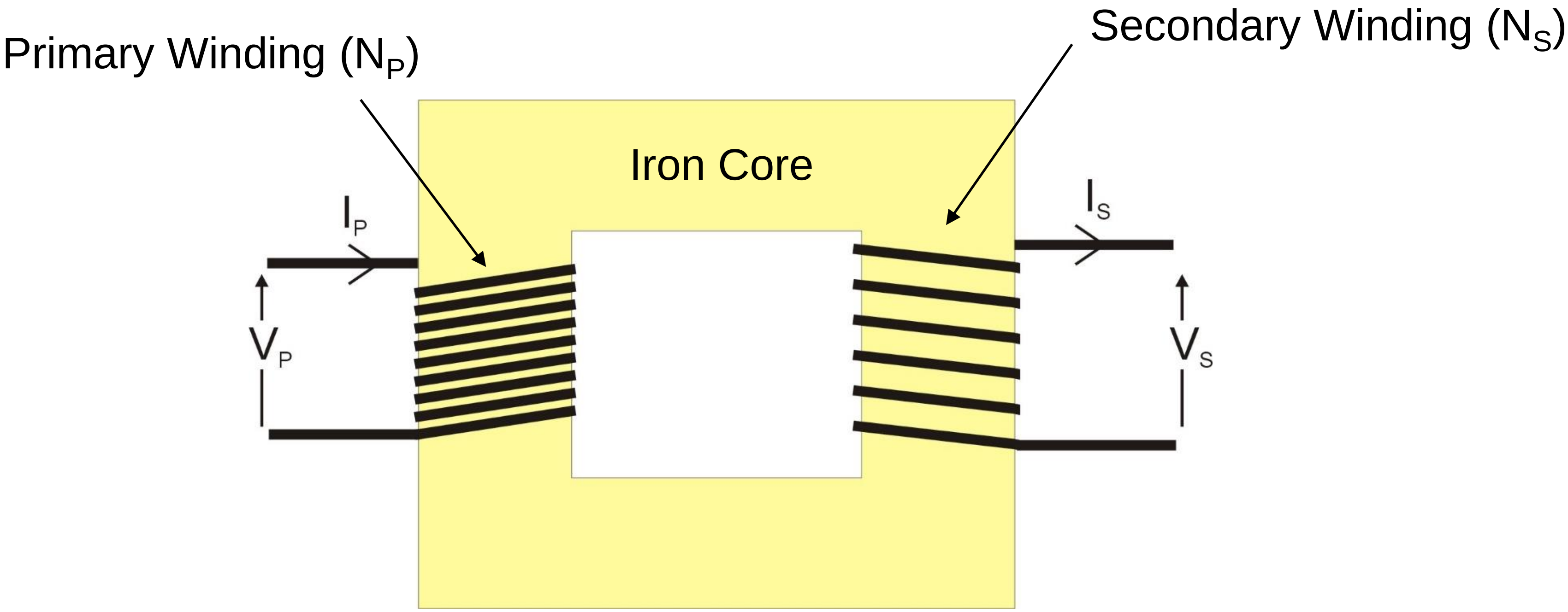
Transformer is required to impedance match the speaker to the Valve Amp

Speakers

A Typical 3 Phase Power (MVA) Transformer:

The Ideal Transformer

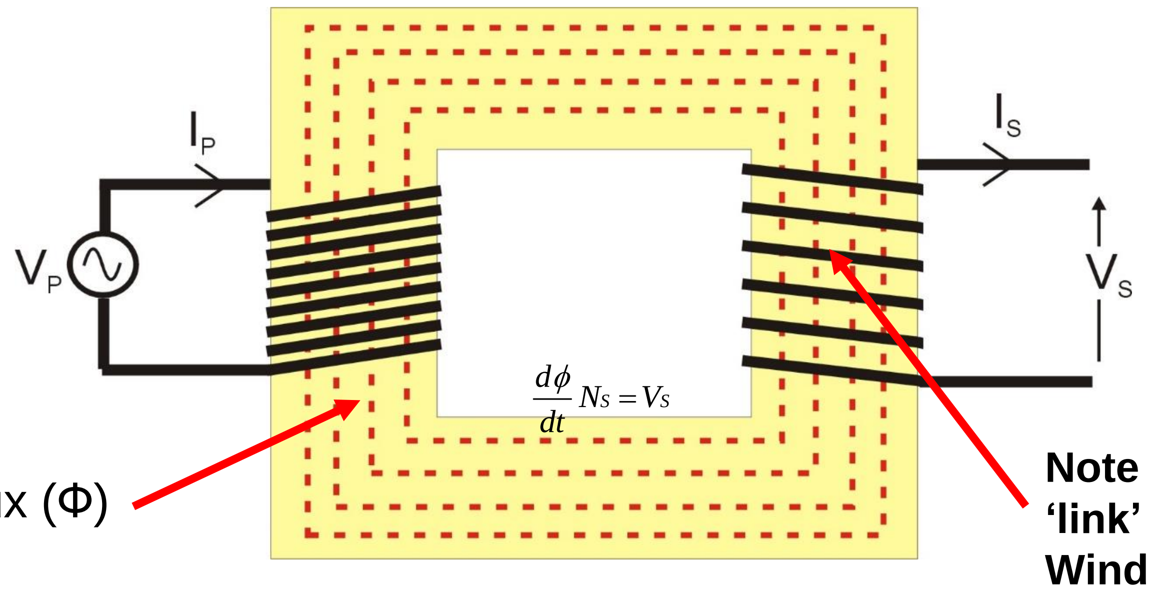
- No leakage flux
- Reluctance=0



The Ideal Transformer consists of a Primary winding of N_P turns and a Secondary Winding of N_S turns wound round a common Iron Core

An AC Voltage Source is connected to the Primary Winding and this results in an AC Magnetic Flux (Φ) in the Iron Core

The Ideal Transformer: Primary & Secondary Voltage Ratio



Magnetic Flux (Φ)

Note Flux Lines also 'link' with the Secondary Winding!

The Flux and the Voltage are related by Faraday's Law:

Same Flux

$$V_P = N_P \frac{d\phi}{dt}$$

Same Flux

$$V_P = N_P \frac{d\phi}{dt} \qquad \frac{d\phi}{dt} N_S = V_S$$

Rearranging these equations:

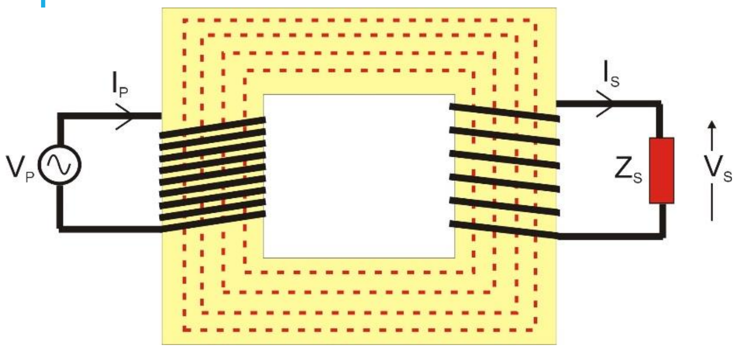
$$\frac{d\phi}{dt} = \frac{V_P}{N_P} = \frac{V_S}{N_S}$$

Which results in the 'classic' Transformer relationship:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

This states that the ratio of Primary and Secondary voltage is determined by the TURNS RATIO (N_P/N_S) of the transformer

The Ideal Transformer: Primary & Secondary Current Ratio



Consider connecting an Impedance Z_S across the secondary winding

This results in current flowing I_S in the Secondary winding, and an associated current I_P flowing in the Primary winding (power has to come from the primary source)

Consider for simplicity that Z_S is purely resistive:

Secondary Power = $V_S.I_S$

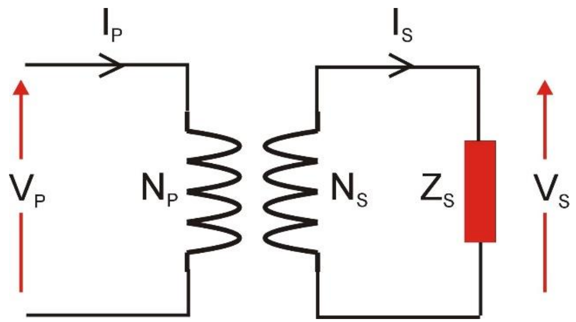
Primary Power = $V_P.I_P$

An Ideal transformer is 100% efficient, therefore:

Inverse Turns Ratio!

$$V_S.I_S = V_P.I_P \longrightarrow \boxed{\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}}$$

The Ideal Transformer: A Summary



- The ratio of input voltage and output voltage is equal to the transformer turns ratio, irrespective of the value of Z_S:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

- The ratio of input current and output current is equal to the inverse of the transformer turns ratio:

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

- The ideal transformer is 100% efficient

- No primary current flows when there is no load connected to the secondary

A transformer with 120 primary turns and 10 secondary turns is connected to a 240V AC supply and supplies a purely resistive load connected to the secondary winding. The input current is measured at 5A. Using the Ideal transformer equations calculate the following:

1] The secondary voltage (A)

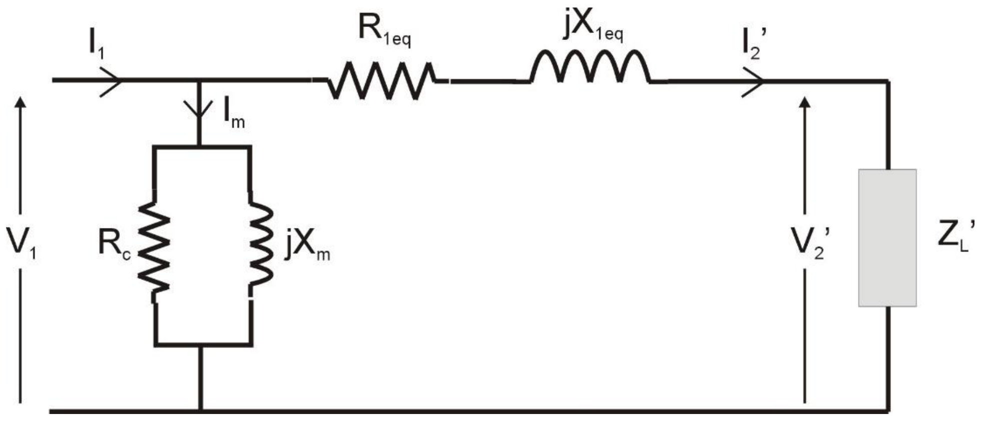
2] The input power (W)

3] The output power (W)

4] The output current (A)

Also draw a graph of transformer efficiency (y axis) against output current (x axis) over the range from zero to maximum output current

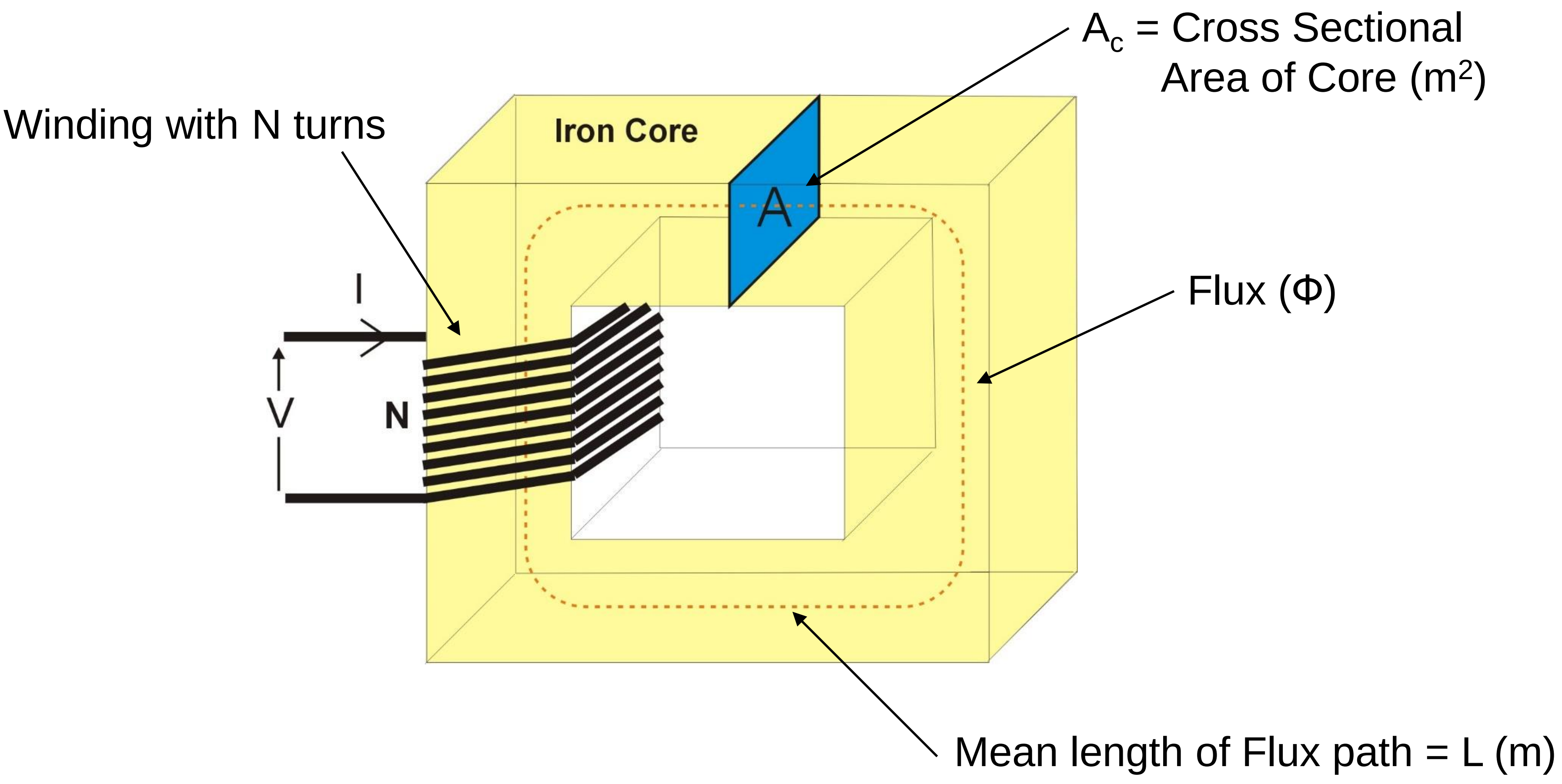
Time to Get Real!



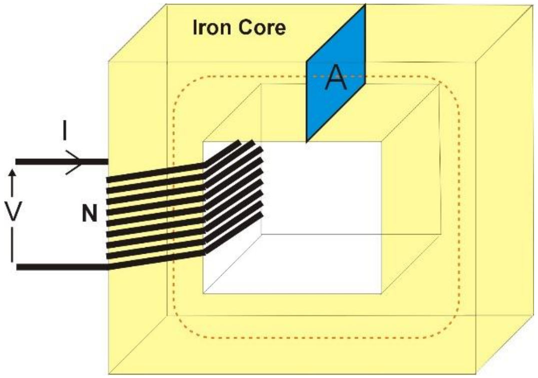
Unfortunately transformers are not ideal and during the next lecture we will develop the equivalent circuit for a real transformer.

Before this though it would be useful to have a very brief look at basic Electromagnetic Theory as some of the transformer 'imperfections' and limitations are as a result of the electromagnetic coupling between the primary and secondary electric circuits.

Basic Electromagnetics in 24 minutes!



- The current (I) in the windings produces a Magnetomotive Force (mmf), denoted by $\mathbb{F}$:



Magnetic 'voltage'

$$\mathbb{F} = N \times I$$

- The mmf produces a Magnetic Flux (Φ) in the Core whose magnitude is dependant on the RELUCTANCE ($\mathbb{R}$) of the Core:

Magnetic 'current'

$$\Phi = \frac{\mathbb{F}}{\mathbb{R}}$$

- The Reluctance ($\mathbb{R}$) of the core is dependant on the physical dimensions of the Core and its magnetic property, called PERMEABILITY (μ)

Magnetic 'resistance'

$$\mathbb{R} = \frac{L}{A.\mu}$$

Two other useful quantities to introduce are **Magnetic Flux Density** (B), and Magnetic Field Intensity (H):

$$B = \frac{\phi}{A_c}$$

$$H = \frac{N.I}{L}$$

Where A_c is the cross-sectional Area of the Core

The units of B is TESLA

Where L is the mean path length of the Flux

The units of H is A/m

Magnetic Field Intensity is related to Flux Density by the PERMEABILITY of the material. Permeability is a measure of how easy it is to set up magnetic flux in a material:

$$B = \mu.H$$

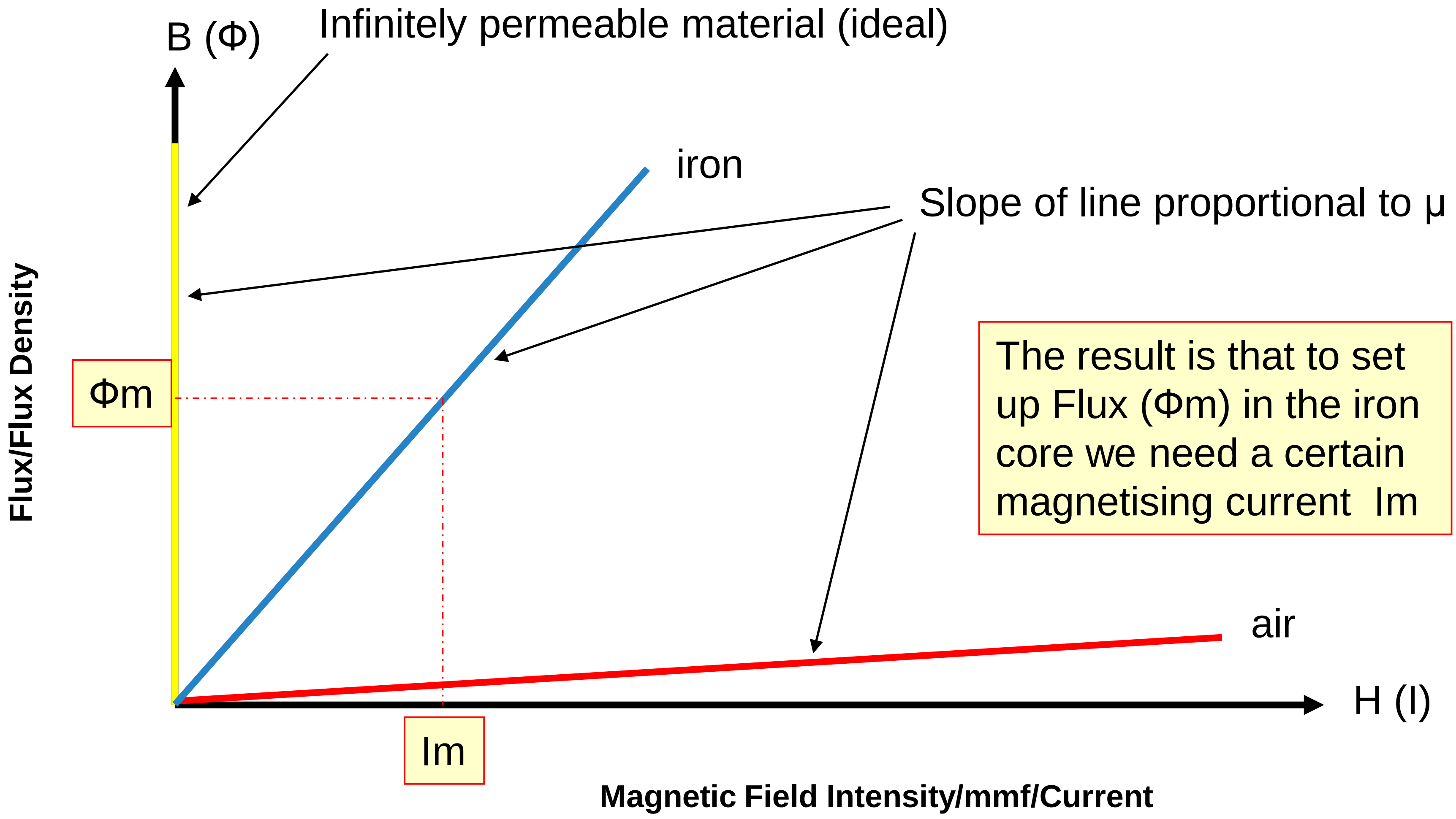
or

$$B = \mu_r.\mu_o.H$$

Note: most materials have μr = 1, but ferromagnetic materials have μr = 20,000 – 80,000

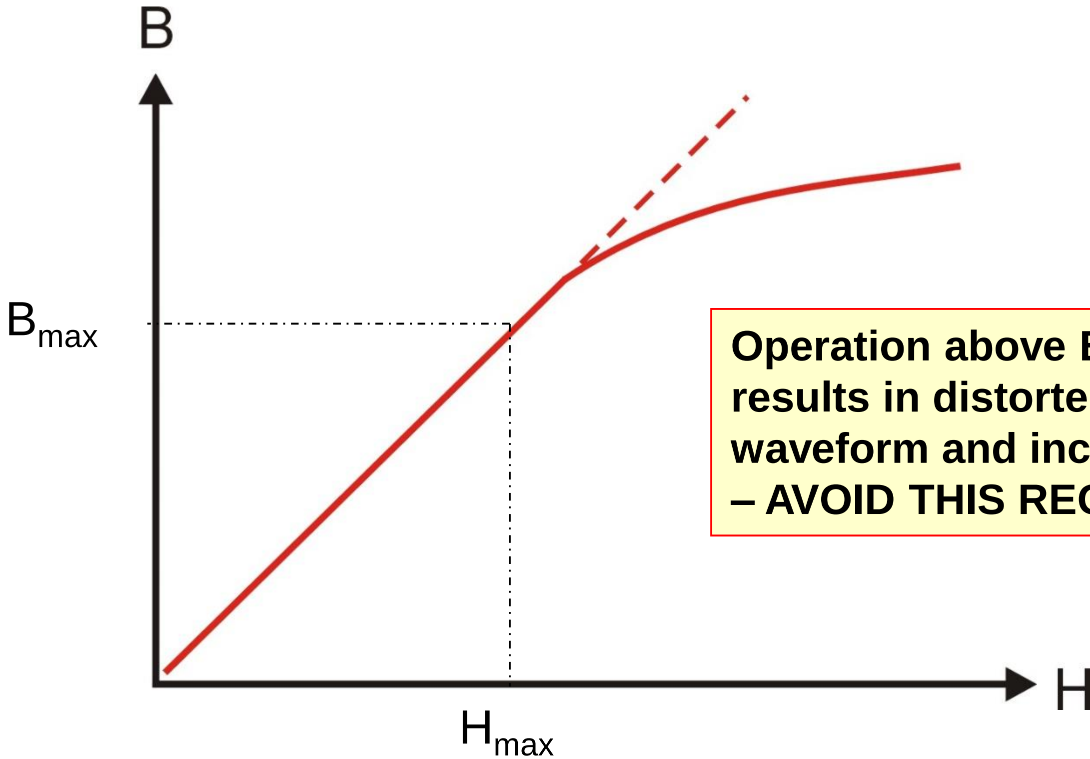
Where μo is the permeability of free space (air) = $4.\pi \times 10^{-7}$, and μr is the relative permeability of the material

BH Curves for different materials:



BH Curve for Iron: Magnetic Saturation

The final phenomenon we need to understand is that of Magnetic Saturation. This is evident in ferromagnetic materials (iron, nickel, cobalt, steel) only and occurs at higher values of mmf/H where the flux no longer increases linearly with H:



Operation above B_{max}/H_{max} results in distorted current waveform and increased losses – AVOID THIS REGION!

Typical values for Bmax are in the region of 0.9 – 1.3 Tesla

MAGNETIC "OHM'S LAW"

For a magnetic circuit, if reluctance and exciting current are given, the magnetic flux Φ can be calculated using **magnetic "Ohm's law"**:

$$\boxed{\Phi = \frac{\mathcal{F}}{\mathfrak{R}} \quad or \quad \Phi = \frac{\sum_m N_m i_m}{\sum_k \mathfrak{R}_k}}$$

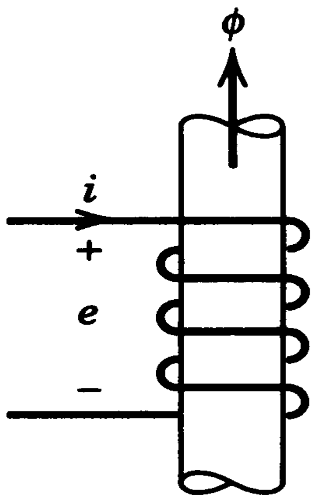
Analogy

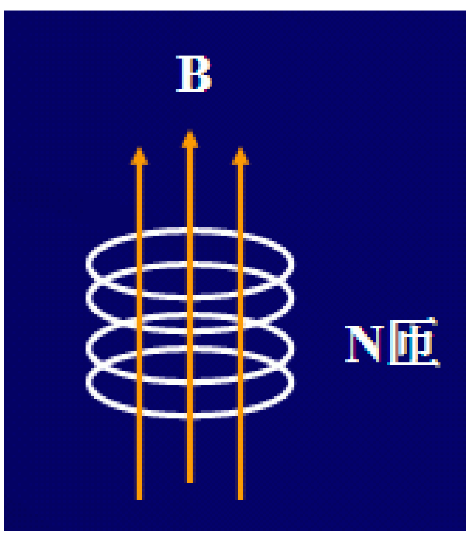
Magnetic	Electrical (dc)
$\frac{Ni}{\phi} = \mathfrak{R} = \frac{l}{\mu A}$	Ohm's law: $\frac{v}{i} = R = \frac{l}{A/\rho}$
$\phi \sum_k \mathfrak{R}_k = \sum_m N_m i_m$	Kirchhoff's voltage law: $i \sum_k R_k = \sum_m v_m$
$\sum \phi_k = 0$	Kirchhoff's current law: $\sum_k i_k = 0$

Basic Electromagnetics Summary:

Parameter	Symbol	Units	Equation
Magnetic Flux	Φ	Webers	
Magnetomotive Force (mmf)	$\mathbb{F}$	Ampere turns	$\mathbb{F} = N.I$
Magnetic Field Intensity	H	Amps/m	$H = N.I/L$
Magnetic Flux Density	B	Tesla	$B = \Phi/A_c$
Magnetic Reluctance	$\mathbb{R}$	Amps/Weber	$\mathbb{R} = L/\mu A$
Magnetic Permeability	μ ($\mu_r\mu_o$)	Weber/Amp.m	$B = \mu H$

FARADAY'S INDUCTION LAW





A changing magnetic field gives rise to a voltage

$$\boxed{\begin{aligned} &\Psi = N\Phi \\ &e = \frac{d\Psi}{dt} = N\frac{d\Phi}{dt} \end{aligned}}$$

Q. So what limitations does B_{max} place on the operation of the transformer?

The answer is found in Faraday's Equation: $$v = N\frac{\partial \phi}{\partial t}$$

Consider sinusoidal voltage: $v(\omega t) = V_{pk} Sin\omega t$

Supply Frequency (eg 2π.50)

Rearranging Faraday's Equation:

$$\phi = \frac{1}{N}\int v.\partial t$$

Solving:

$$\phi = \frac{1}{N}\int V_{PK}.Sin\omega t.\partial t$$

$$\phi = \frac{V_{PK}}{N}.\frac{1}{\omega}Cos\omega t$$

$$(given\_\phi = \phi_{PK}.Cos\omega t)$$

$$\phi_{PK} = \frac{V_{PK}}{N.\omega}$$

$$V_{PK} = \phi_{PK}.N.\omega$$

$$V_{rms} = \frac{\phi_{PK}.N.\omega}{\sqrt{2}}$$

Given: $B = \frac{\phi}{A_c}$

$$V_{rms} = \frac{N\omega B_{PK}A_c}{\sqrt{2}}$$

$$V_{rms} = \frac{N2\pi fB_{PK}A_c}{\sqrt{2}}$$

$$V_{rms} = 4.44NfB_{PK}A_c$$

So at a given Maximum Flux Density B_{max}:

$$\boxed{V_{rms\_max} = 4.44NfB_{max}A_c}$$

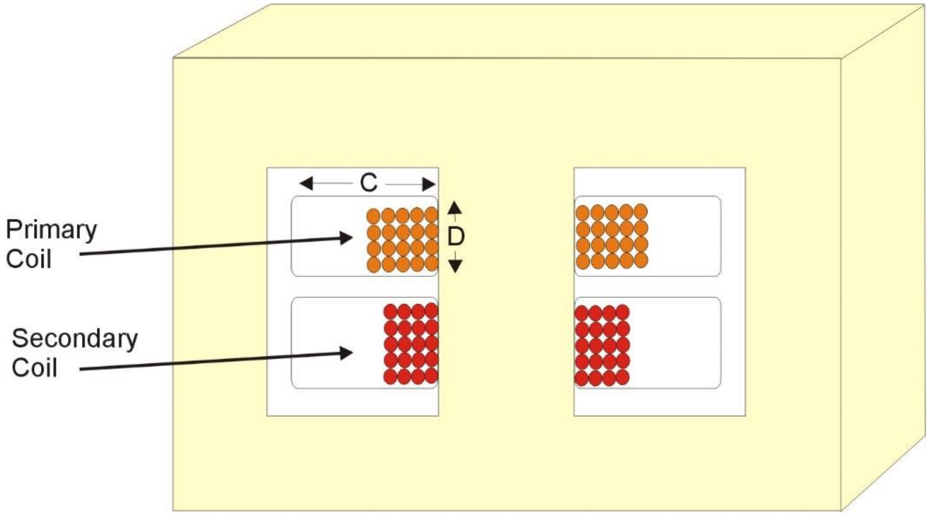
Note: typical value for B_{max} in the region 0.9 – 1.3T for iron

Q. Is there any other limit on transformer operation?

A. Yes, the maximum rms current in the windings

Current in a transformer winding leads to copper (I^2R) loss which causes the temperature of the winding to increase. If this temperature exceeds the winding insulation limit (Class F insulation has a 155°C limit) then the winding will get damaged.

Cooling systems (oil) are therefore generally used in large transformers to reduce the temperature rise for a given current, and therefore increase the maximum safe operating current



Current limit equates to a Current Density Limit (typical values in range 3 – 20A/mm^2 dependant on cooling)

$$\text{Current Limit} = \frac{\text{Current Density Limit x Coil Area x Fill Factor}}{\text{Number of Turns}}$$

Notes:

1. In the diagram the coil area for Primary coil is C(mm) x D(mm)

2. Fill Factor is the reduction in area due to the fact that it is not all filled with copper due to insulation thickness and gaps between turns. Typical value >0.9