Power Electronics

Magnetic Circuit and Inductor Design

磁路与电感设计

Magnetic Devices磁性器件

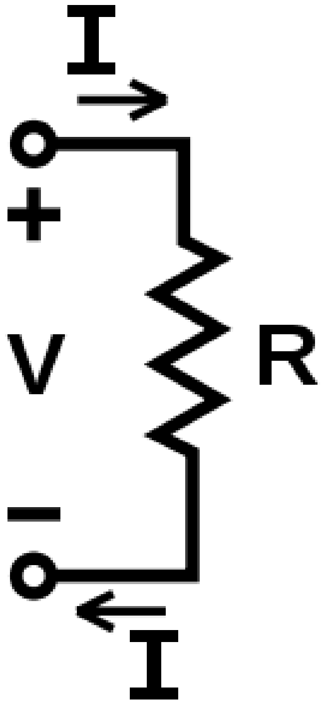
Inductors and **Transformers** built with magnetic materials are important components of most power electronic systems. Their design and application requires an understanding of basic magnetic circuits.

- Customized 客户定制 Device for Converter
- Magnetic Circuit Analysis
  - Electrical Models 电路模型 for Real Devices
- Device Design and Build
  - Nonlinear Saturation 非线性饱和 of Magnetic Materials
  - Design Method

DC Electric Circuit

Read <u>Chapter 3, Mohan etc.</u>

Ohm’s Law 欧姆定理



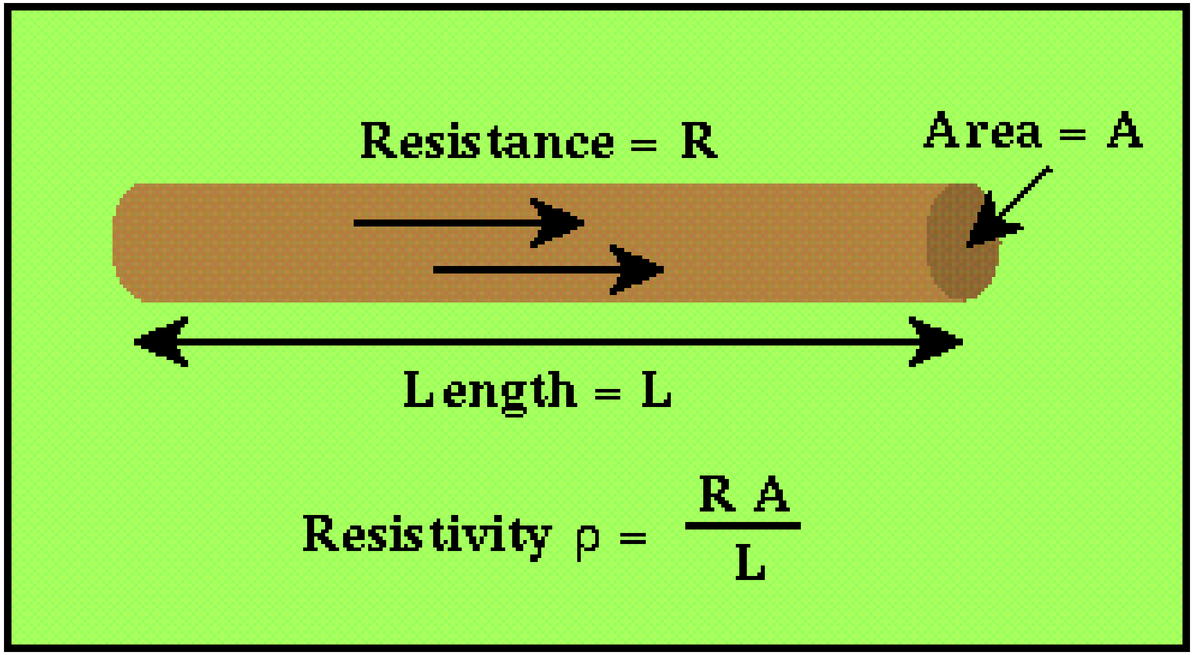
$$\boxed{\frac{V}{I} = R}$$

Voltage: ***V*** (Electric Potential)

Current: ***I*** (Electric Flux)

Resistance: ***R*** (Electric Reluctance)

Resistivity of Electric Medium介质



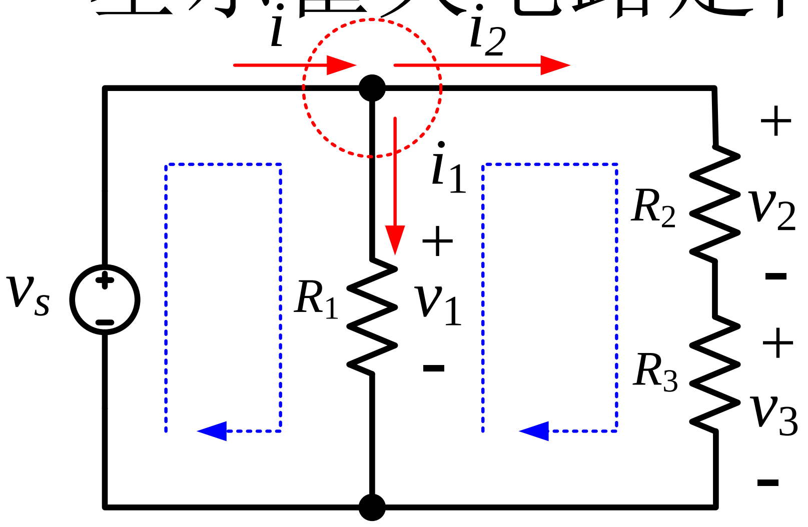
Physical Model

Resistance: $R = \dfrac{\rho l}{A}$ **How to design resistor R ?**

Resistivity电阻率 ρ is for conduction medium

Kirchhoff's Circuit Laws

基尔霍夫电路定律



$$\text{KVL}: \quad \sum_k v_k = 0, \quad \textit{for any loop}$$

$$\text{KCL}: \quad \sum_k i_k = 0, \quad \textit{for any node}$$

类比/模拟的磁路

Analogy Magnetic Circuit

Magnetic Analogues to Electric Circuits

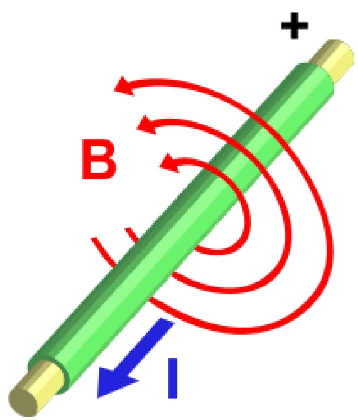
Electric Circuit	Magnetic Circuit
Electromotive force $-\int \mathbf{E} \cdot d\ell$	Magnetomotive force (MMF) $\int \mathbf{H} \cdot d\ell$
Voltage source $d\lambda/dt$	MMF source Ni
KVL, $\Sigma v_{\text{loop}} = 0$	MMF law, $\Sigma \text{MMF}_{\text{loop}} = 0$
KCL, $\Sigma i_{\text{node}} = 0$	Gauss's Law, $\Sigma \phi_{\text{node}} = 0$
Current	Magnetic flux
Resistance $R = \rho\ell/A$	Reluctance $\mathcal{R} = \ell/(\mu A)$
Conductance $G = 1/R$	Permeance $\mathcal{P} = 1/\mathcal{R}$
Conductivity $\sigma = 1/\rho$	Permeability μ
Conductor $\sigma \to \infty$	Ferromagnetic material $\mu \to \infty$
Insulator $\sigma \to 0$	Diamagnetic material μ small

Read Chapter 3, Mohan etc.

Basic Physics Laws

Ampere’s Law 安培定律

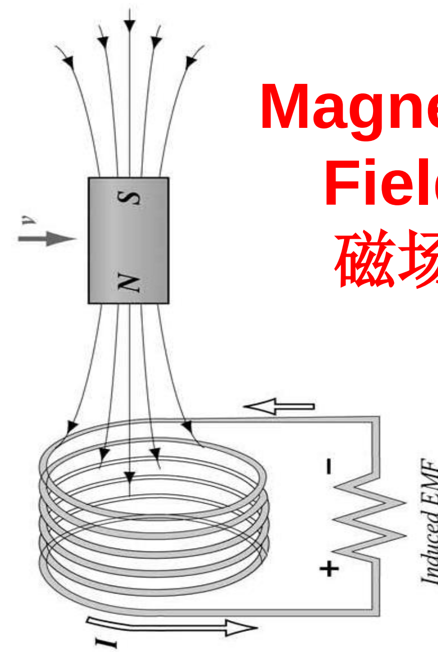
Electric current produces magnetic field



Electric Field
电场

Electro-Magnetic Transform

电磁转换



Magnetic Field
磁场

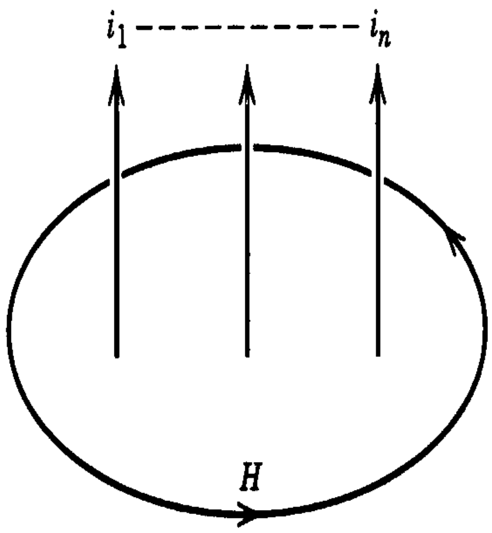
Faraday’s Law法拉第定律

A changing magnetic field gives rise to a voltage/*emf*

Magnetic "Voltage"

A current carrying conductor produces a magnetic field of intensity磁场强度 ***H*** (*amperes per meter* - A/m), and generate magnetic motive force (*mmf*) ***F*** as follows

Right-hand Rule 右手定则



$$\mathcal{F} = \oint H dl = \sum i$$

More generally, it can be written as

$$\mathcal{F} = \sum_k H_k l_k = \sum_k N_m i_m$$

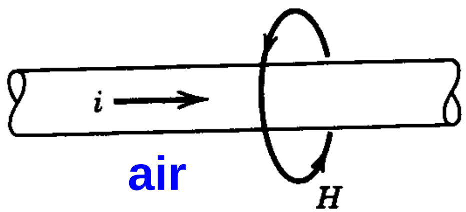
Magnetic "**Voltage**" ***F*** : *mmf*, magnetic motive potential磁动势

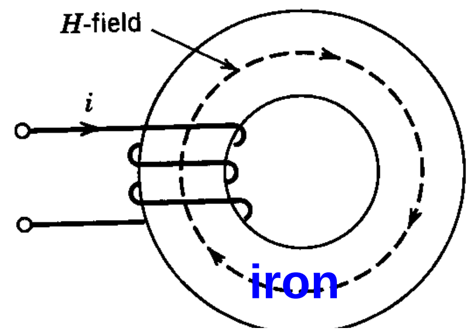
Magnetic Medium

The magnetic field intensity ***H*** is related to the magnetic flux density磁通密度 ***B*** (Wb/m^2, or tesla–*T*) by the property of the medium:

$$B = \mu H$$

$$\mu = \mu_0 \mu_r$$





Permeability磁导率 of the medium μ :

Permeability of the free space

μ_0: $=4\pi x10^{-7}$H/m

Relative Permeability相对磁导率:

μ_r =1.0 for air or nonmagnetic medium; μ_r = several thousand for iron

Material	μ_r
Air	1
Permalloy	100,000
Cast steel	1,000
Sheet steel	4,000
Iron	5,195

Permeability 磁导率 and permittivity 电容率. (Not examinable)

It is sometimes easier to think of permeability as a measure of the ease with which magnetic flux 磁通 lines can pass through a material.

The permeability of free space is μ_0 where $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A}^2$

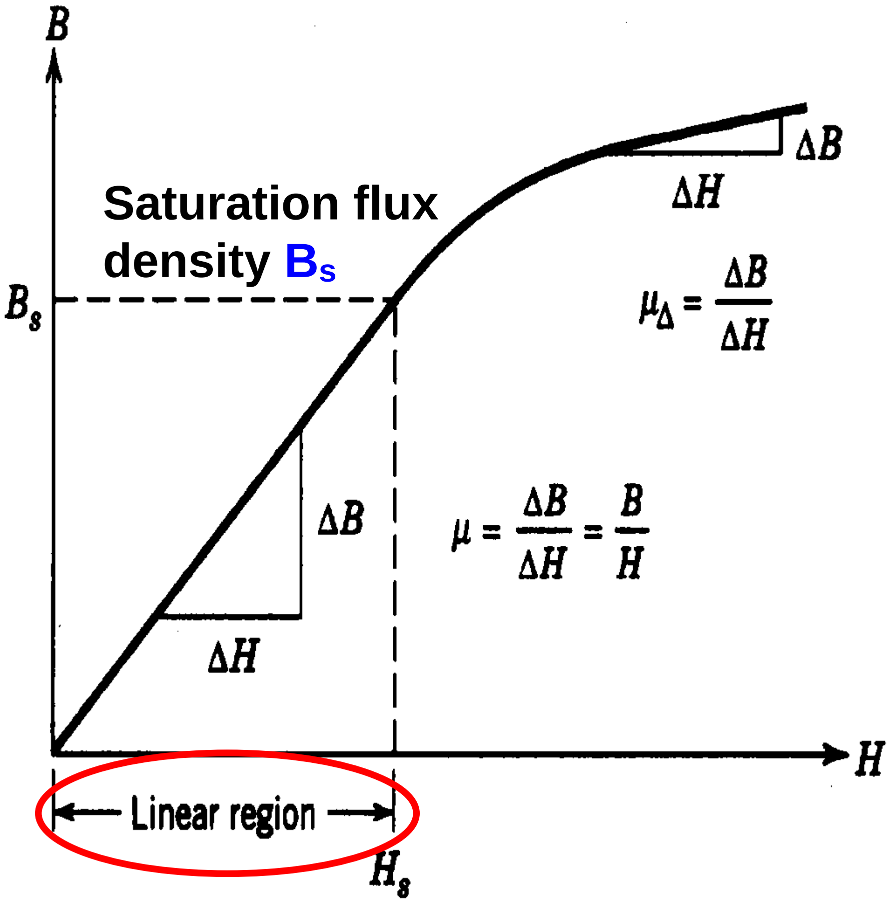
In general, the permeability $\mu = \mu_0 \mu_r$ where μ_r is a dimensionless constant and is called the "relative permeability". It is this value that is most often used.

For all practical purposes, the permeability of air is the same as that of a vacuum. High quality steel for transformers etc. typically has a relative permeability > 40,000.

An inductor can be made by winding a conductor around a former to make a coil. If the former is a hollow tube, the coil is said to be "air-cored". If a high permeability material is now placed inside the coil, the inductance will be much higher. In other words, a given value of inductor can be made with fewer turns of wire if it is wound on a high permeability core.

Similarly, adding a sheet of high permittivity 电容率 material (a "dielectric"电介质) between the plates of a capacitor increases the capacitance between the plates. Ceramics 陶瓷 and polymers 聚合物 are often used because of their superior insulating 绝缘 properties (the "dielectric strength").

Permeability of Magnetic Medium



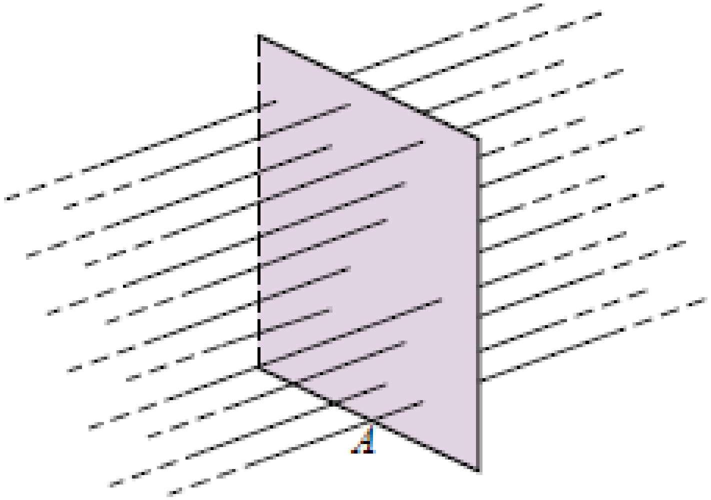
Below the saturation flux 饱和磁通 **B_s**, the medium operates in linear region, i.e. μ is constant;

Beyond ***B_s***, permeability μ can be much smaller than that in the linear region线性区域.

Magnetic “Current”

The magnetic flux Φ crossing an area can be obtained by the surface integral of magnetic flux density **B** normal to 法向 that area

$$\Phi = \iint_A B dA$$

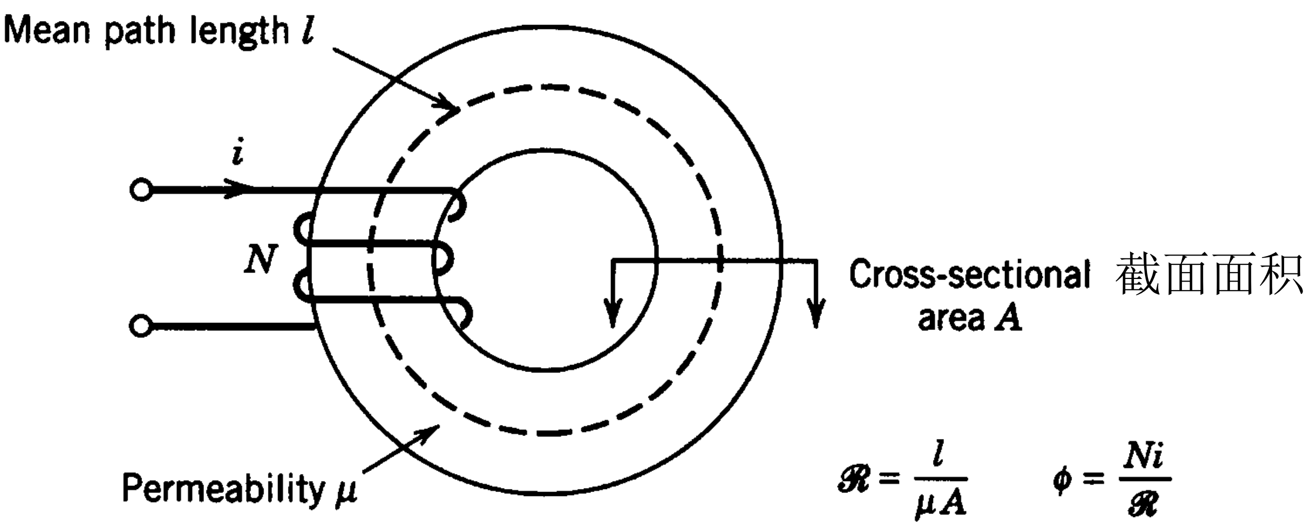


For most practical cases, it can be written as

$$\Phi = B \cdot A$$

Magnetic “Current” **: magnetic flux Φ**

Magnetic “Resistance”



Physical Model

Magnetic Reluctance 磁阻 **(“Resistance”):**

$$\boxed{\Re = \frac{l}{\mu A}}$$

Magnetic Permeance 磁导 **(“resistivity”):**

$$\boxed{\mathcal{P} = \frac{1}{\Re}}$$

Magnetic "Ohm's Law"

For a magnetic circuit, if reluctance 磁阻 and exciting current 激励电流 are given, the magnetic flux Φ can be calculated using **magnetic "Ohm's law"**:

$$\Phi = \frac{\mathcal{F}}{\Re}$$

or

$$\Phi = \frac{\sum_m N_m i_m}{\sum_k \Re_k}$$

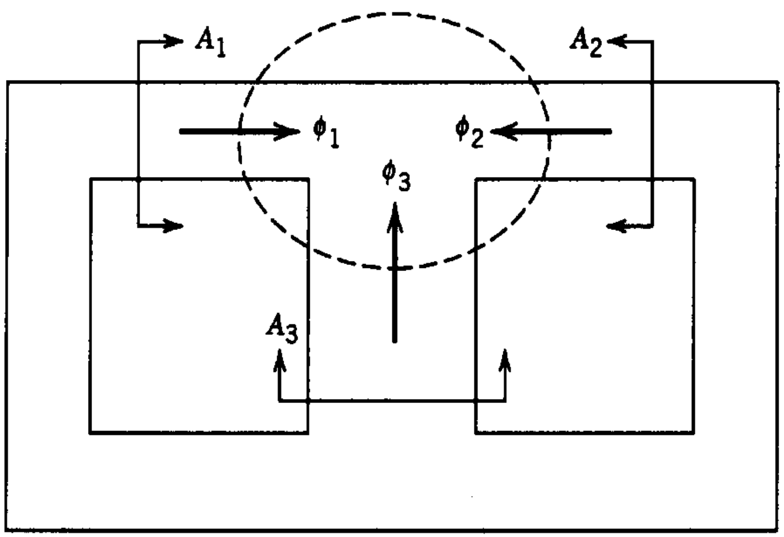
Analogy

$$I = \frac{V}{R}$$

Magnetic "KCL"

Continuity of Flux磁通连续性: magnetic flux lines forms closed loops, the flux lines entering a closed surface area must equal those leaving it. (i.e. Magnetic "**KCL**")

$$\boxed{\Phi = \iint_{A(closed\ surface)} BdA = 0}$$



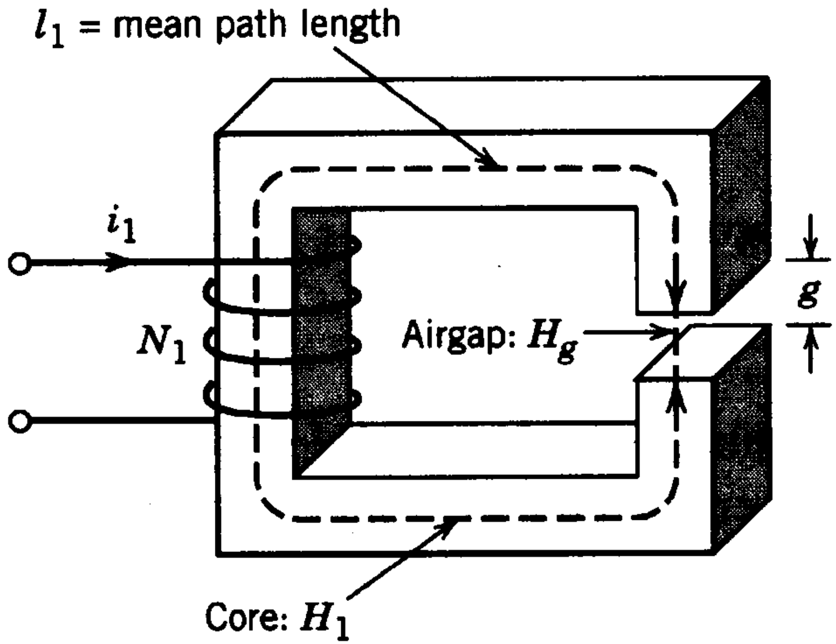
$$\Phi_1 + \Phi_2 + \Phi_3 = 0, \quad or$$

$$B_1A_1 + B_2A_2 + B_3A_3 = 0$$

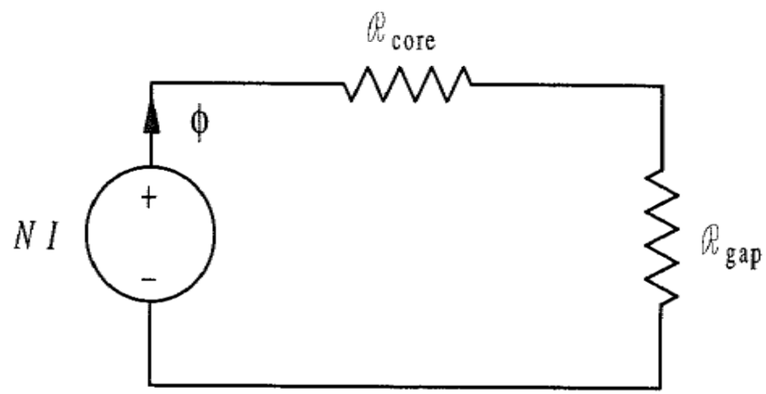
i.e.

$$\sum_k \Phi_k = 0$$

Magnetic "KVL"



$$\underbrace{\phi_{\text{core}}\mathcal{R}_{\text{core}}}_{\substack{\text{MMF}\\\text{drop}}} + \underbrace{\phi_{\text{air}}\mathcal{R}_{\text{air}}}_{\substack{\text{MMF}\\\text{drop}}} = \underbrace{\text{MMF}_{\text{in}}}_{\substack{\text{MMF}\\\text{source}}}$$



For above magnetic circuit, we can have circuit equation which follows magnetic **"KVL"**:

$$\boxed{\mathcal{F} = N_1 i_1 = H_1 l_1 + H_g l_g}$$

Customized Magnetic Device

用户定制的磁性器件

Inductor

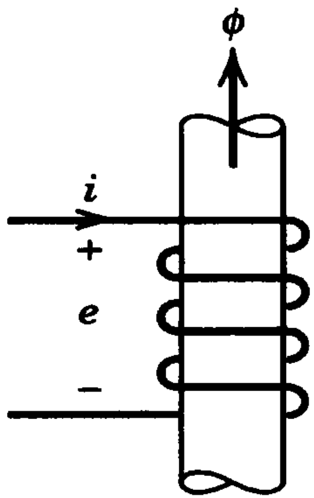
L: inductance

$$v_L = L\frac{di_L}{dt}$$

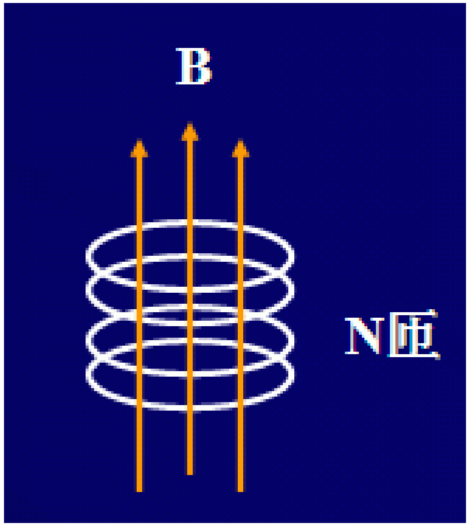
Electric Circuit Model

Read <u>Chapter 3, Mohan etc.</u>

Faraday's Induction Law
法拉第感应定律



A changing magnetic field gives rise to a voltage

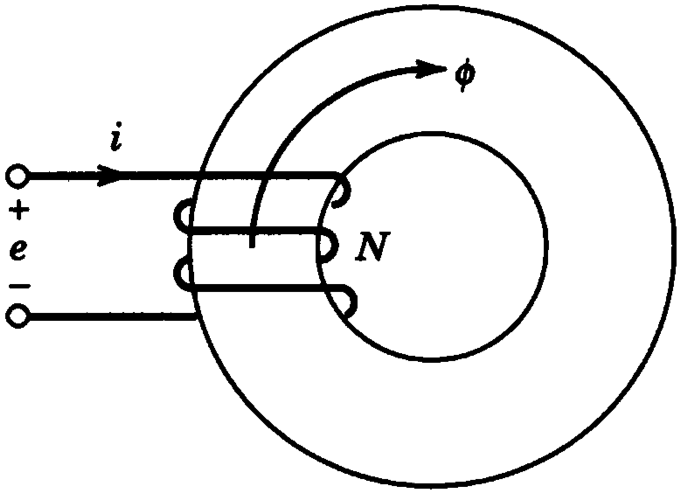


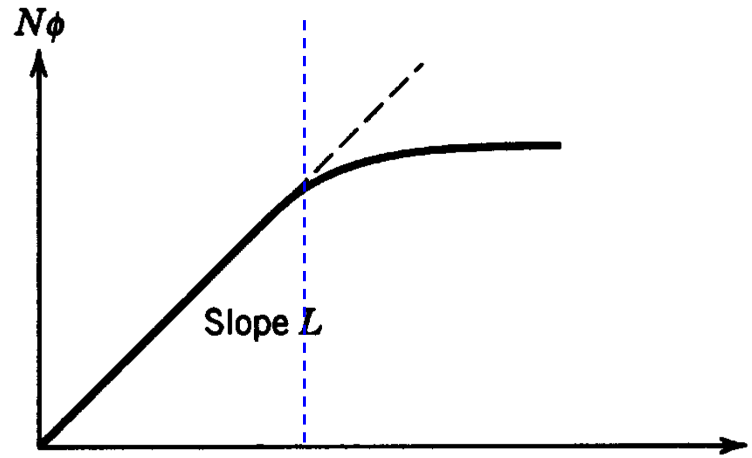
$$\Psi = N\Phi$$

$$e = \frac{d\Psi}{dt} = N\frac{d\Phi}{dt}$$

Electricity ⟷ **Magnetics**

Self-Inductance 自感 L





A Coil 线圈has a inductance ***L***, which is defined as

感应系数！

$$\boxed{L = \frac{\Psi}{i_L} = \frac{N\Phi}{i_L} \quad or \quad N\Phi = L\,i_L}$$

In linear region of the core material, ***L*** is constant and is independent of ***i***.

Inductor's Electric Circuit Model

$$L = \frac{\Psi}{i_L} = \frac{N\Phi}{i_L} \quad or \quad N\Phi = L\, i_L$$

$$e = N\frac{d\Phi}{dt} = L\frac{di_L}{dt} + i_L\frac{dL}{dt} \Rightarrow v_L = L\frac{di_L}{dt}$$

Inductor's Physical Model

$$L = \frac{\Psi}{i_L} = \frac{N\Phi}{i_L} \quad or \quad N\Phi = L\, i_L$$

$$\Phi = \frac{\mathcal{F}}{\Re}$$

$$\Re = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A}$$

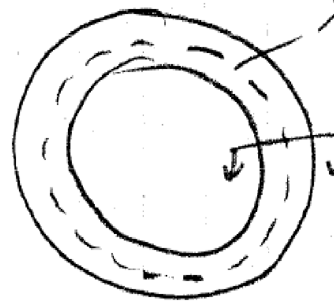
$$L = \frac{N}{i_L}\frac{\mathcal{F}}{\Re} = \frac{N^2}{\Re} = N^2 \frac{\mu_r \mu_0 A}{l}$$

How to design inductance L ?

Provided假定 magnetic saturation does not occur, coil inductance ***L*** is the property of the core material and is independent of ***i***.

An Example

Calculate the number of turns for the toroidal 环形的 core芯 obtain an inductance of 25μH.



$L = 25\mu H$

$\mu_r = 125$

$$L = N^2 P = N^2 \frac{A_c (\mu_o \mu_r)}{l_c}$$

$$\therefore N = \left[\frac{L\, l_c}{A_c\, \mu_o \mu_r}\right]^{1/2}$$

$\mu_o = 4\pi \times 10^{-7}$ H/m

$$= \left[\frac{25 \times 10^{-6} \times 3.12 \times 10^{-2}}{0.113 \times 10^{-4} \times 4\pi \times 10^{-7} \times 125}\right]^{1/2}$$

$\simeq 21$ turns

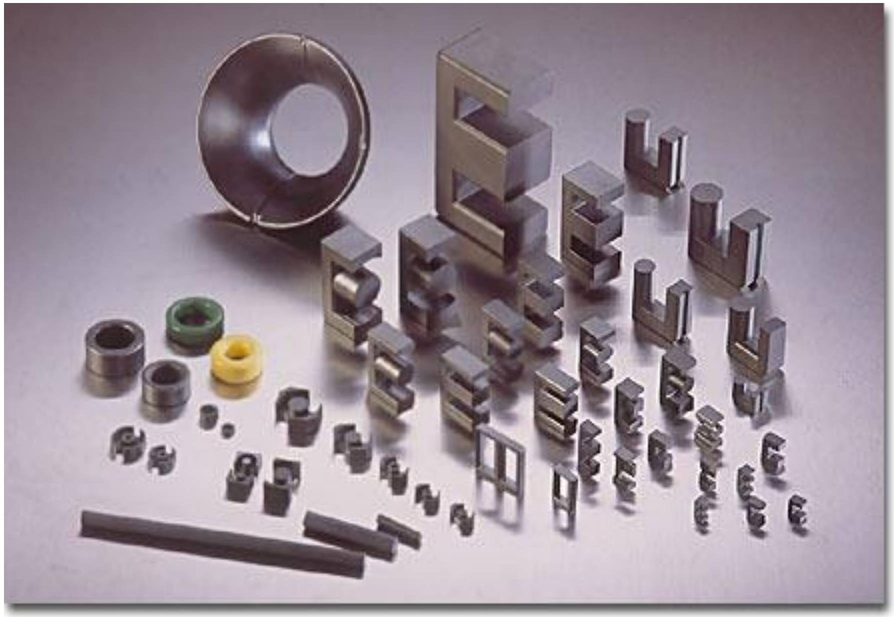
Inductor Design

Read Chapter 30, Mohan etc.

Two Groups of Components

Magnetic or Wound绕线的 components

- Inductors or Chokes扼流圈
- Transformers



Parts部件 and Materials

Wound components include three parts:

1) Cores磁芯, 2) Bobbin骨架, 3) Coil Windings线圈

Cores and bobbins/coil-formers: Toroids and U/I cores, Shell-type cores (E core, RM core etc),

Magnetic materials: Permeable materials are used to constrain the flux path and increase volumetric inductance. Materials include steel laminations 叠片(“iron”), ferrite铁氧体, iron powder铁粉芯, amorphous metal 非晶金属etc.

Winding materials: Enamel insulated copper conductors 漆包铜线 are generally used, although un-insulated foils箔, plastic triple-insulated copper, Litz利兹 wire are also employed.

Magnetic Circuit Equations磁路方程式

"Voltage"

$$\mathcal{F} = \sum_k H_k l_k = \sum_k N_m i_m$$

Medium

$$B = \mu H, \quad \mu = \mu_0 \mu_r$$

"Current"

$$\Phi = BA$$

"Resistance"

$$\Re = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$$

"Ohm's Law"

$$\Phi = \frac{\mathcal{F}}{\Re} = \frac{Ni}{\Re}$$

"KVL" & "KCL"

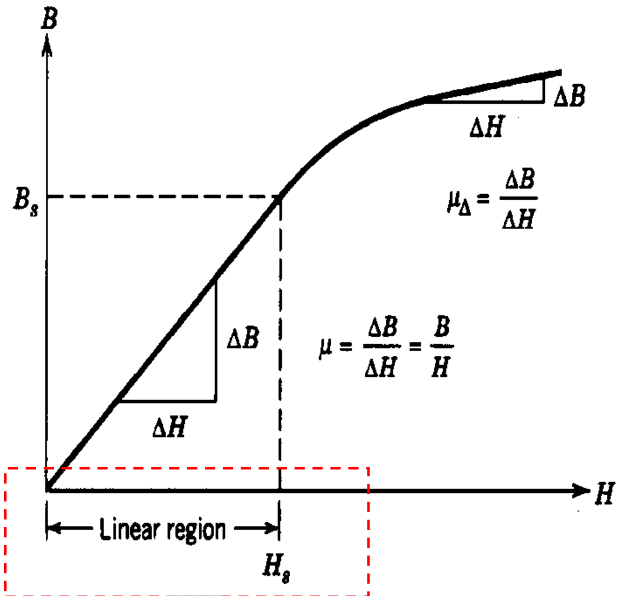
$$\mathcal{F} = N_1 i_1 = H_1 l_1 + H_g l_g$$

$$\sum_k \Phi_k = 0$$

Faraday's Law

$$e = \frac{d\Psi}{dt} = N \frac{d\Phi}{dt}$$

Inductor Design



Constant Permeability μ

Inductor's Physical Model

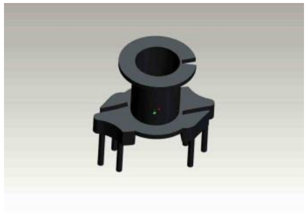
$$L = \frac{N\Phi}{i} = \frac{N^2}{\Re} = N^2 \frac{\mu_0 \mu_r A_e}{l_e}$$

Inductor Design is to choose

1) **Coil:** winding turns ***N***;

2) **Core:** μ_r, A_e, l_e

Inductor Design



"Area-Turns" Product

Assuming the flux is uniformly distributed

$$\boxed{L = \frac{N\Phi}{i} \Rightarrow Li = N\Phi = NBA_e}$$

$$\boxed{NA_e = \frac{Li_{L\max}}{B_{\max}}}$$

$$L,\ i_{\mathrm{Lmax}},\ B_{\max}$$

"Area-Turns" product : reduce the number of turns ***N*** will reduce the winding resistance and thus the copper loss, while increasing the area *Ae* will usually increase the volume and the core loss. Balancing these losses and designing the smallest, cheapest inductor is often the goal.

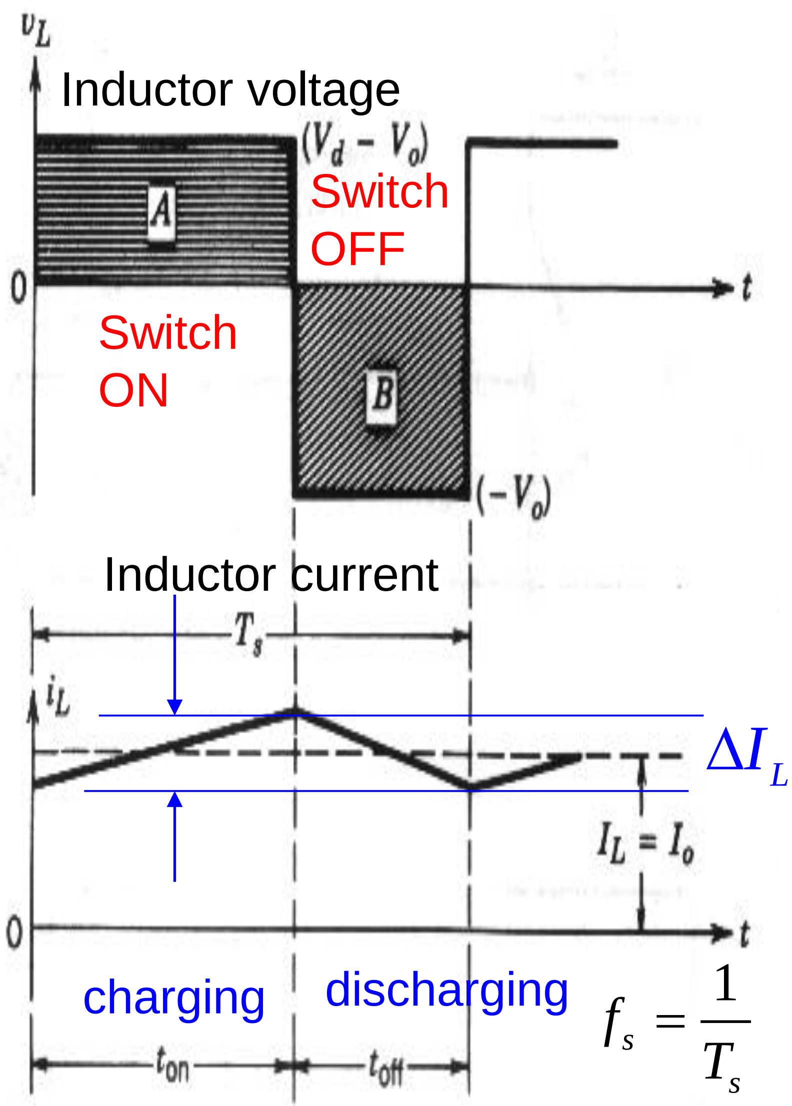
Setting Operation Conditions

From given converter specifications

1) Input voltage/current range
2) Output voltage/current range
3) Switching frequency

1. Determine the max inductor current i_{Lmax}
2. Set inductance value L to choose conduction mode: DCM or CCM
3. Choose desired maximum magnetic flux density B_{max} in the core (make full use of core)
4. Choose a core with $\mathrm{A_e}$, l_e and μ_r

Inductor Peak Current



$$f_s = \frac{1}{T_s}$$

$$i_{peak} = i_{L(\max)} = I_L + \frac{1}{2}\Delta I_L$$

$$\Delta I_L = \frac{v_d}{f_s L} k(1-k) = \frac{v_o}{f_s L}(1-k)$$

$$I_L = I_o$$

$$i_{L(\max)} = \frac{1}{2}\frac{v_d}{f_s L} k(1-k) + I_o$$

Switch Peak Current

Trial Design设计尝试

1. Calculate coil turns ***N***

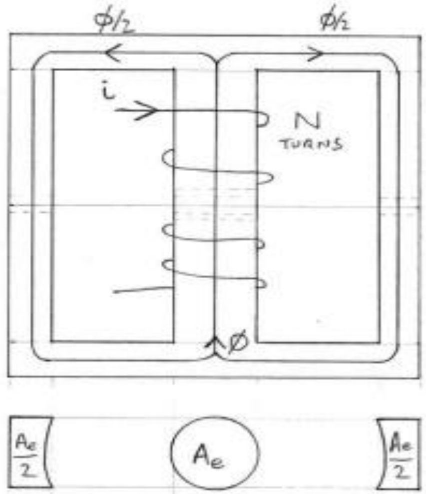
$$Li = N\Phi \Rightarrow NB_{\max}A_e = Li_{L\max} \Rightarrow N = \frac{Li_{L\max}}{B_{\max}A_e}$$

2. Verify inductance ***L*** and maximum flux density ***B***$_{max}$

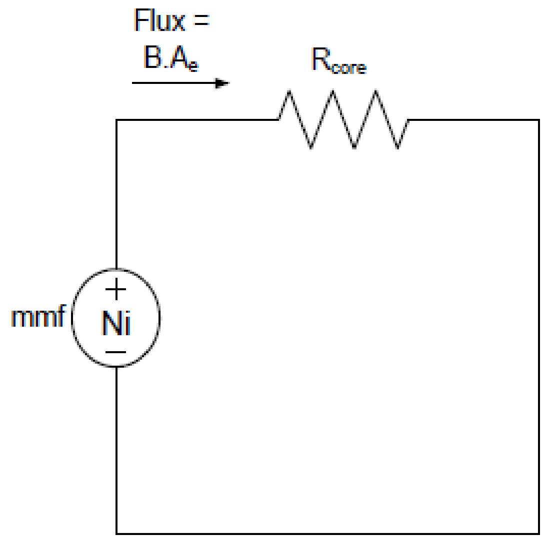
$$L^1 = \frac{N\Phi}{i} = \frac{N^2}{\Re_{core}} = \frac{\mu_o \mu_e A_e N^2}{l_e} \overset{?}{=} L$$

$$L = \frac{N\Phi}{i} = \frac{NBA_e}{i} = \frac{\mu_o \mu_e A_e N^2}{l_e} \Rightarrow B^1_{\max} = \frac{\mu_o \mu_e N}{l_e} i_{\max} \overset{?}{\le} B_{sat}$$

No Air Gap 气隙 Inductor



Inductor with no air gap



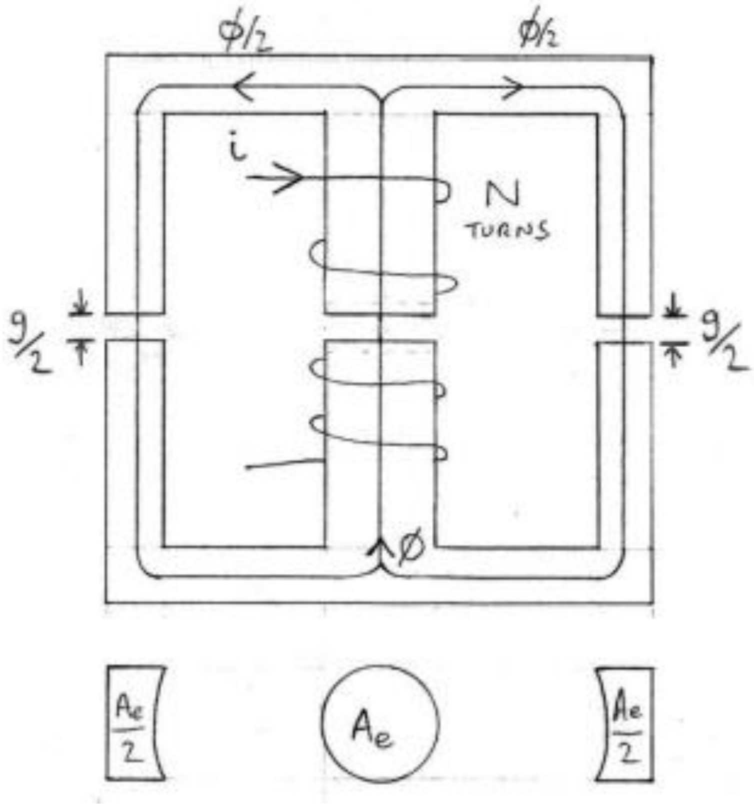
Magnetic circuit

$$\Re_{core} = \frac{l_e}{\mu_0 \mu_e A_e}$$

$$\mathcal{F} = Ni = \Phi \Re_{core} = BA_e \Re_{core} \Rightarrow B = \frac{Ni}{A_e \Re_{core}} = \frac{\mu_0 \mu_e Ni}{l_e}$$

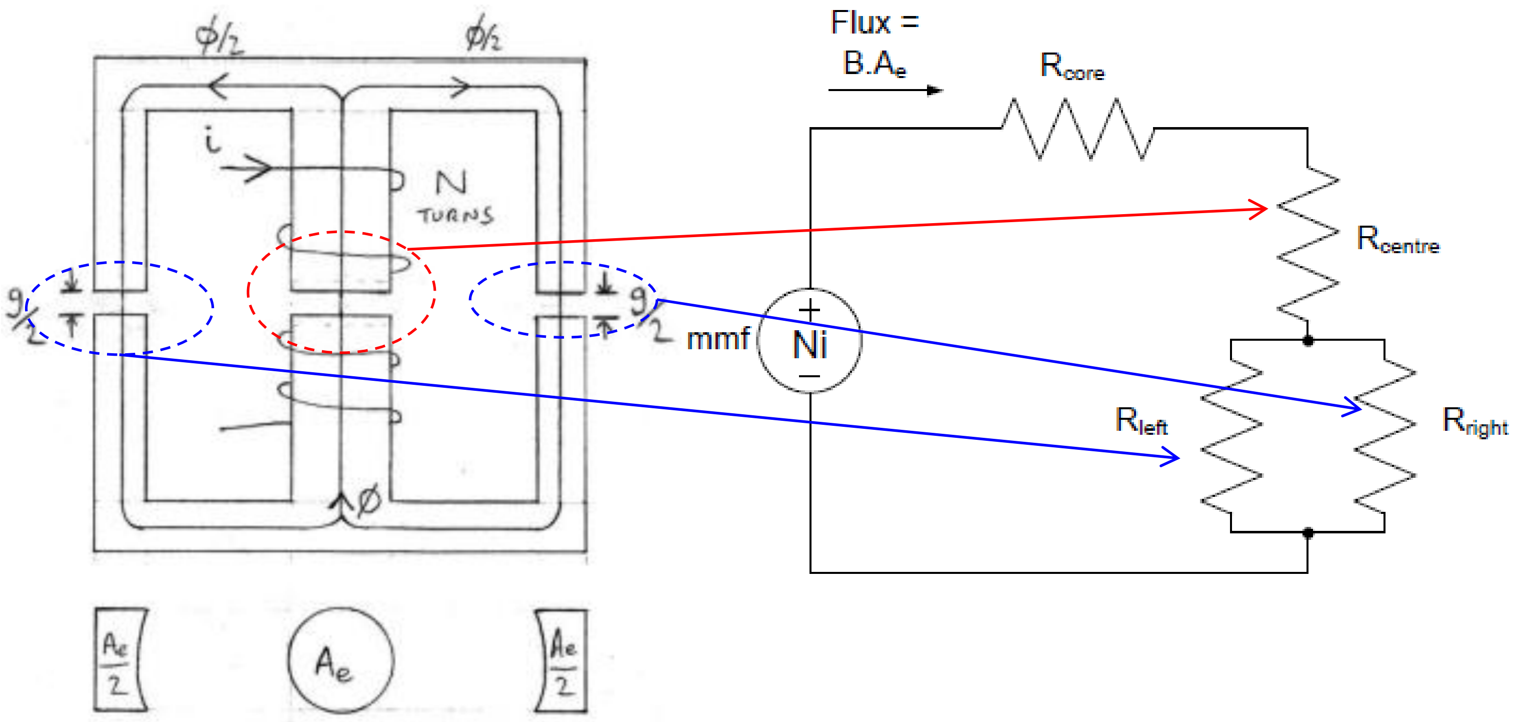
Since permeability μ_r of core is high, reluctance *R*core of the no-gap core may be so low that B_{max} exceeds B_{sat}.

Insert Air Gap



Insert airgap in the core is use to reduce relative permeability of the magnetic loop and then increase reluctance of the magnetic loop.

Airgap Reluctance气隙磁阻

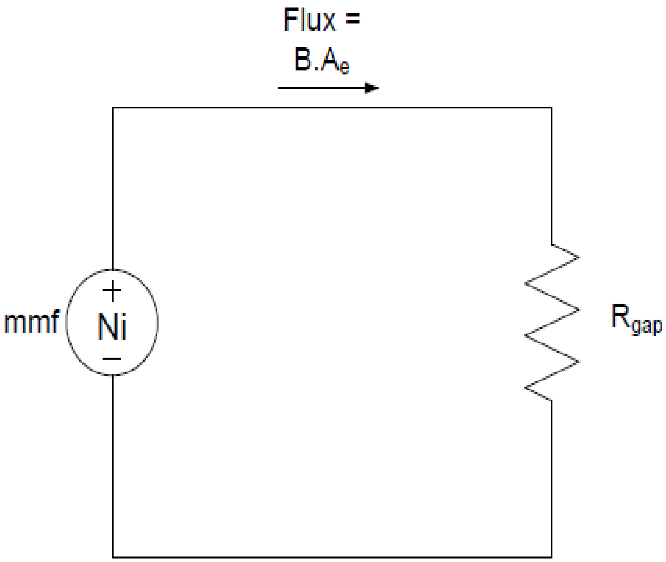


Airgap Reluctance

$$\Re_{left} = \Re_{right} = 2\Re_{centre} = \frac{g}{\mu_o A_e}$$

$$\Rightarrow \Re_{gap} = \left(\Re_{left} \,||\, \Re_{right}\right) + \Re_{centre} = 2\Re_{centre} = \frac{g}{\mu_o A_e}$$

Air Gap Length



Total Reluctance

$$\Re_{gap} = \frac{g}{\mu_o A_e} \gg \Re_{core} = \frac{l_e}{\mu_0 \mu_e A_e}$$

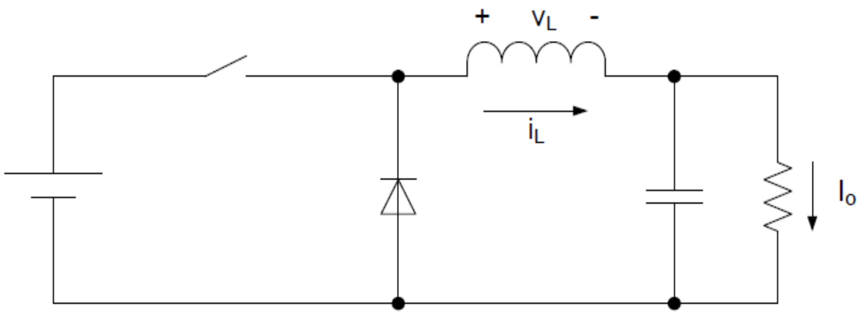
$$\Rightarrow \Re_{total} = \Re_{gap} + \Re_{core} \approx \Re_{gap}$$

$$\Re_{total} = \frac{\mathcal{F}_{\max}}{\Phi_{\max}} = \frac{Ni_{L\max}}{\Phi_{\max}} = \frac{Ni_{L\max}}{B_{\max} A_e}$$

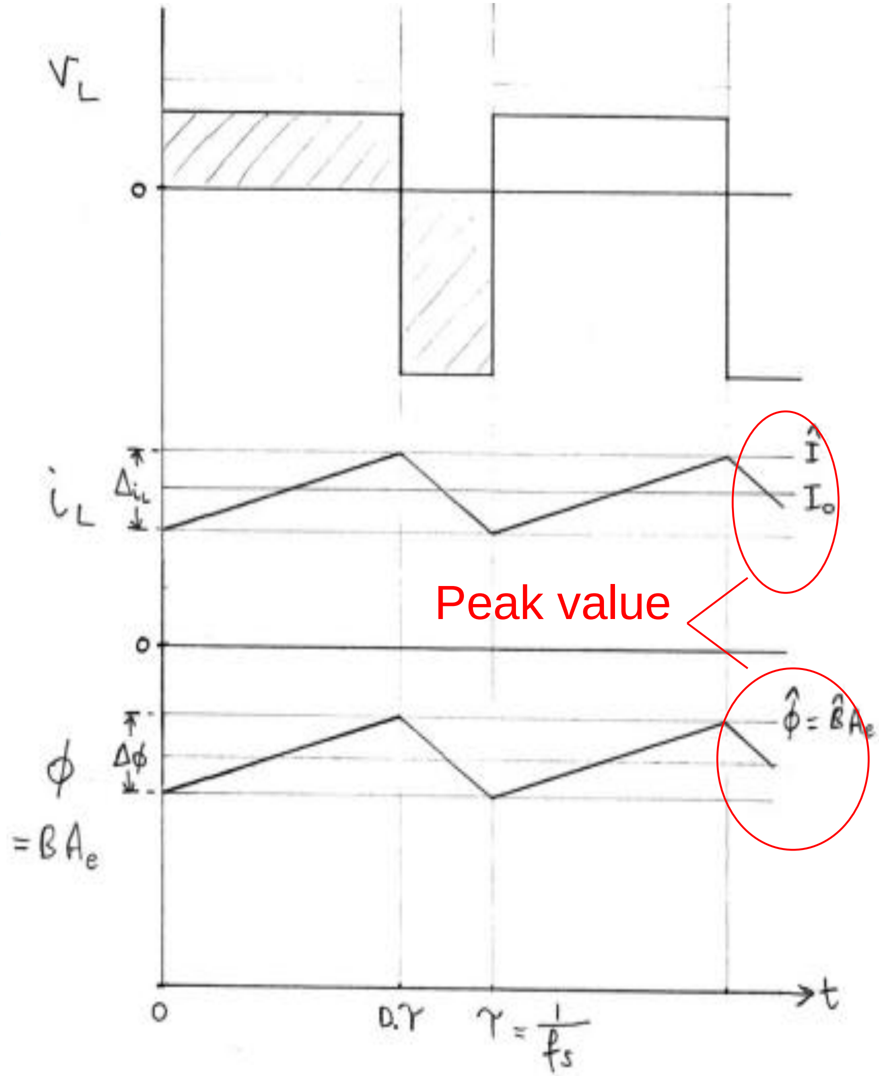
Gap Length

$$\Re_{total} = \frac{Ni_{L\max}}{B_{\max} A_e} \approx \frac{g}{\mu_o A_e} \Rightarrow g = \frac{\mu_o Ni_{L\max}}{B_{\max}}$$

Example: Buck Converter in CCM



Buck converter



$$i_{L\max} = 1\text{A}$$

$$L = 1\text{mH}$$

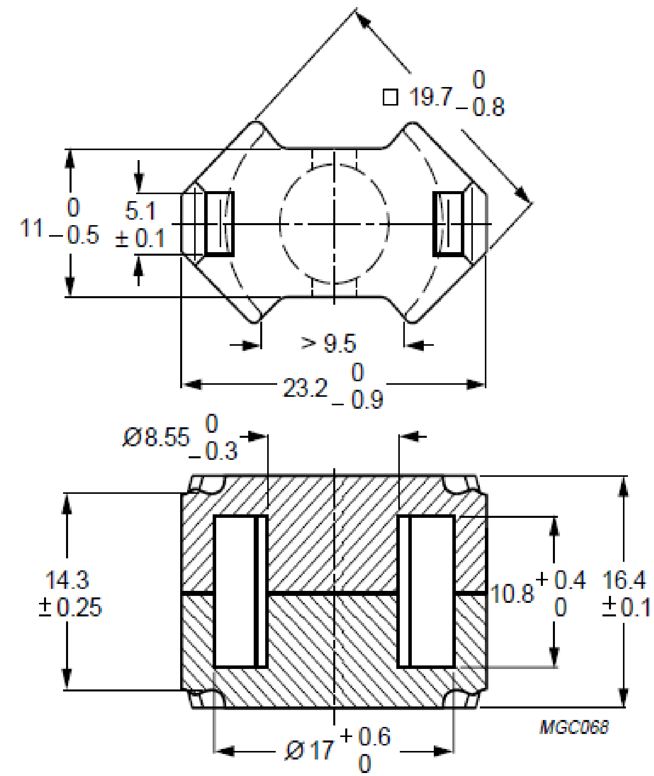
$$B_{\max} = 0.3T$$

Size of RM8/I-3C90 Core

For the solar car an **RM8/I** core in 3C90 (or 3C85) ferrite is chosen. Size, shape and material are already determined.

Effective core parameters

SYMBOL	PARAMETER	VALUE	UNIT
$\Sigma(I/A)$	core factor (C1)	0.604	mm^{-1}
V_e	effective volume	2440	mm^3
l_e	effective length	38.4	mm
A_e	effective area	63.0	mm^2
A_{min}	minimum area	55.4	mm^2
m	mass of set	≈12.0	g



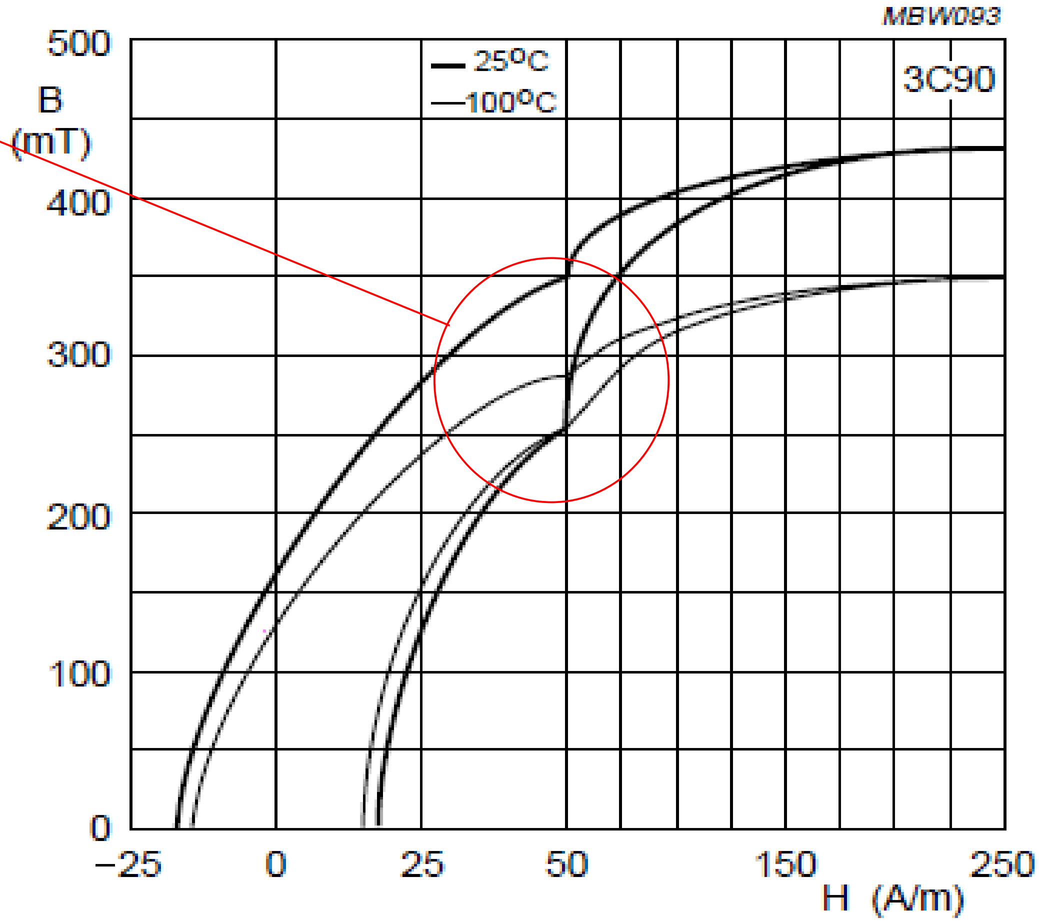
Dimensions in mm.

B-H of RM8/I-3C90 Core

Desired flux density

$$B_{\max} = 0.3T$$

Relative Permeability
μ_r=1600



B-H curve

No Airgap Trial

$$i_{L\max} = 1\text{A}; \quad L = 1\text{mH}; \quad B_{\max} = 0.3T$$

$$\mu_e = 1600, \quad \mu_0 = 4\pi \cdot 10^{-7}\ \text{H/ m}$$

no airgap

$$N = \frac{Li_{L\max}}{\Phi} = \frac{Li_{L\max}}{B_{\max} A_e} = \frac{10^{-3} \cdot 1}{0.3 \cdot 63 \cdot 10^{-6}} \approx 53\,\text{turns}$$

$$B^1_{\max} = \frac{\mu_0 \mu_e N i_{L\max}}{l_e} = \frac{4\pi \cdot 10^{-7} \cdot 1600 \cdot 53 \cdot 1}{38.4 \cdot 10^{-3}} \approx 2.7T \gg B_{\max} = 0.3T$$

$$L^1 = \frac{N\Phi_{\max}}{i_{L\max}} = \frac{NB_{\max} A_e}{i_{\max}} = \frac{53 \cdot 2.7 \cdot 63 \cdot 10^{-6}}{1} \approx 9\,\text{mH} \gg L = 1\text{mH}$$

Airgap Calculation Method I&II

Full Method I for Gap Length Calculation

$$\Re_{total} = \frac{\mathcal{F}_{\max}}{\Phi_{\max}} = \frac{Ni_{L\max}}{B_{\max}A_e} = \frac{53}{0.3\cdot\left(63\cdot10^{-6}\right)} = 2.804\cdot10^{6}\ \text{A/Web}$$

$$\Re_{core} = \frac{l_e}{\mu_r\mu_0 A_e} = \frac{38\cdot10^{-3}}{1600\cdot\left(4\pi\cdot10^{-7}\right)\cdot\left(63\cdot10^{-6}\right)} = 3.03\cdot10^{5}\ \text{A/Web}$$

$$\Re_{gap} = \Re_{total} - \Re_{core} = 2.501\cdot10^{6}\ \text{A/Web}$$

$$g = \mu_0 A_e \Re_{gap} = \left(4\pi\cdot10^{-7}\right)\left(63\cdot10^{-6}\right)\left(2.501\cdot10^{6}\right) = 198\mu\text{m}$$

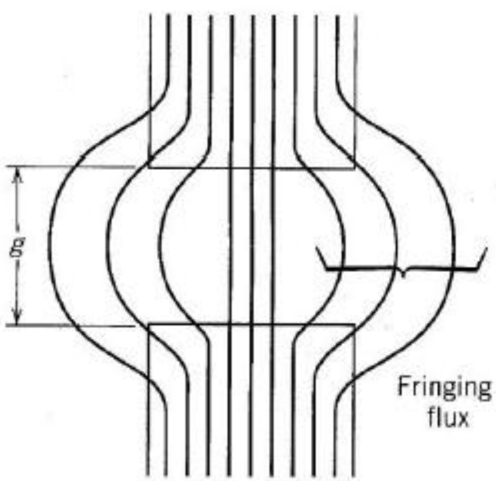
Simplified Method II for Gap Length Calculation

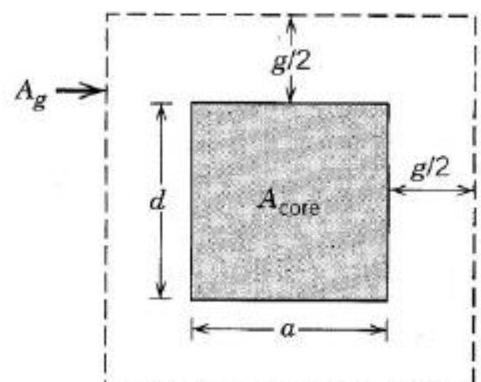
$$g = \mu_0 A_e \Re_{gap} \approx \mu_0 A_e \Re_{total} = \left(4\pi\cdot10^{-7}\right)\left(63\cdot10^{-6}\right)\left(2.804\cdot10^{6}\right) = 222\mu\text{m}$$

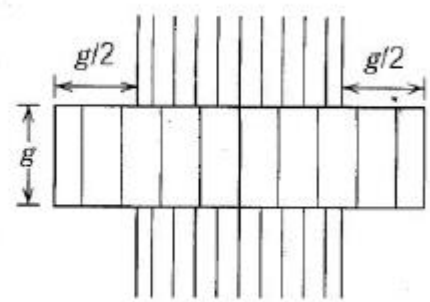
Shorter Gap in Practice

Note: In practice the full method underestimates the airgap length because it does not allow for flux fringing 边缘磁通 at the airgap. The fringing has the effect of increasing the effective area *A*e at the gap and hence reducing the gap reluctance.

$$\Re_{gap} = \frac{g}{\mu_o A_e}$$







How to Insert Airgap

An exact airgap length is not needed in practice.

In our case, insert A4 75g papers (about 100μm/per sheet) between cores, press the core and use LCR meter to measure the inductance for verification.

Distributed Airgap 分布式气隙

To reduce fringing flux, distributed airgap core material can be used, such as iron powder material. Such materials are often used for toroidal core inductors, for which air gapping of ferrite is not practical, and are also available for E cores. (e.g. Magnetics Kool Mu 铁硅铝,磁粉芯 material).

More Physical Restrictions

1. Core (in linear region):
   - Maximum flux density: B_{max} close to but $\leq B_{sat}$
   - Relative permeability: μ_r
   - Size (area, length): A_e, l_e
2. Coil Winding:
   - Area: cooper fill factor<0.5
   - Current Density: $\leq 5\mathrm{A/mm}^2$
   - Resistance: low copper Loss

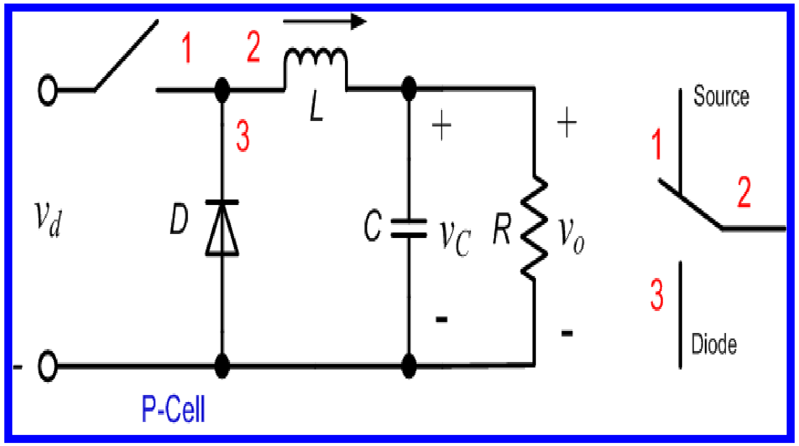
Lab Project: Buck Converter

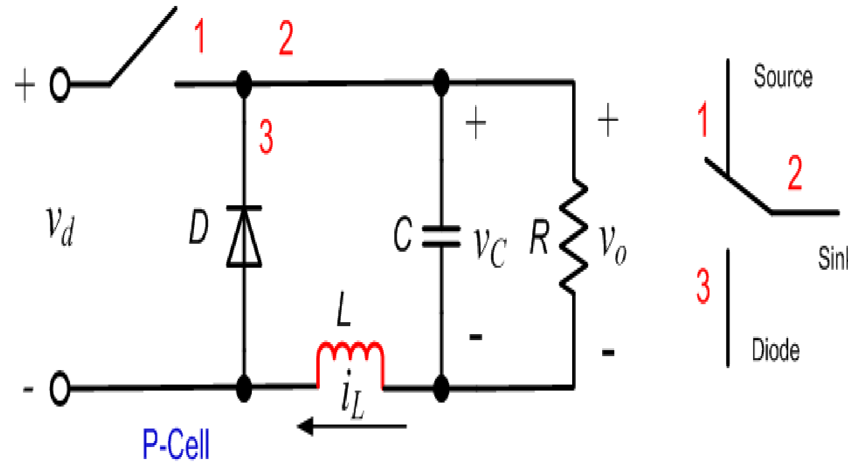
- Design and build an inductor for a continuous conduction Buck converter
- Buck Converter employs an equivalent circuit topology instead of typical one (why ???)
- Test and measure the operation waveforms of the self-built converter with self-wounded inductor and purchased inductor

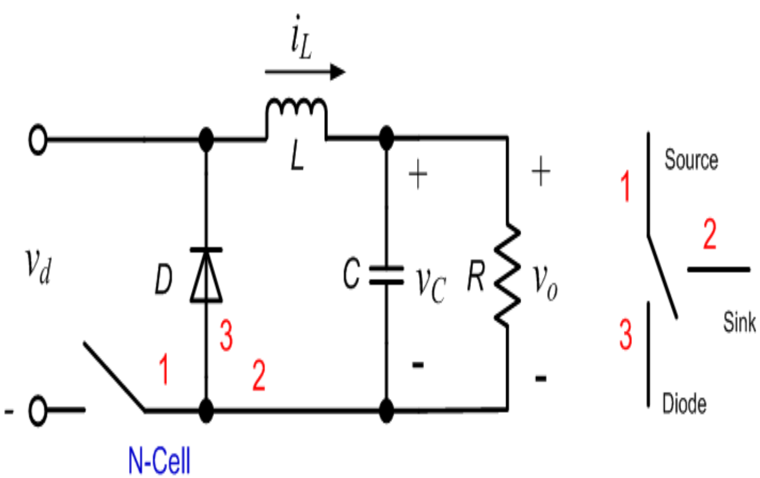
Buck Converter Topologies

You should know

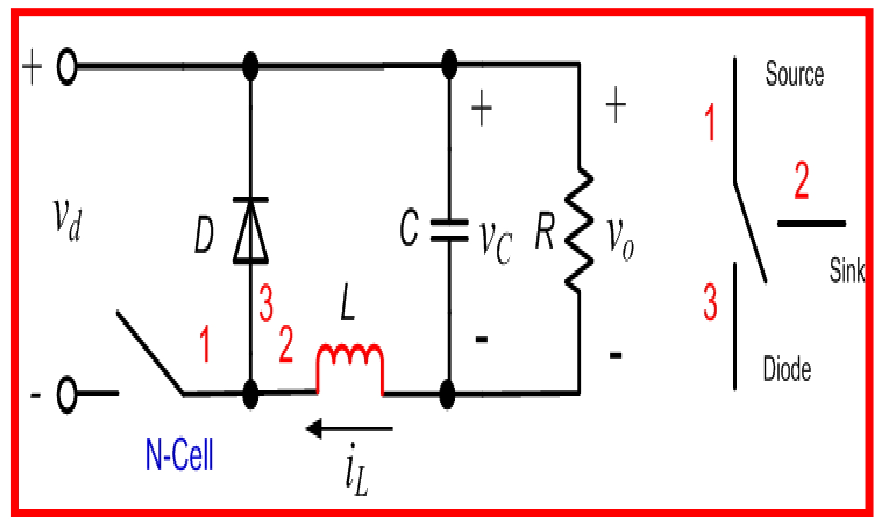
Textbook version







Project version



Why our project take above buck converter topology ?