



University  
of Glasgow

# Power Electronics

## Switched Mode DC-DC Converters I: Concepts and Buck Converter



## Switched-mode Power Supplies (SMPS)开关电源

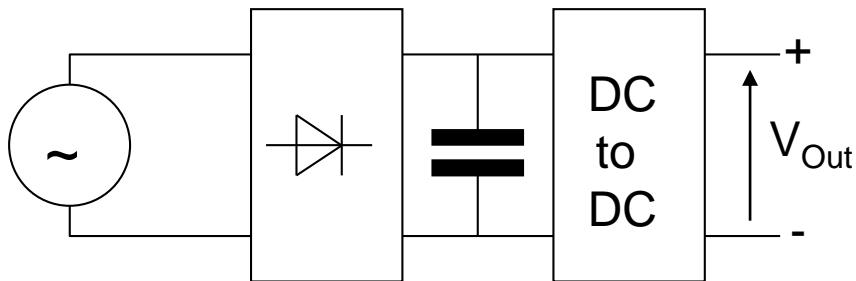
We saw how the linear power supply was very easy to build but suffered from relatively low efficiency. Where efficiency is an important consideration, the SMPS should be the supply of choice. For even moderately-sized power supplies the **advantages** of the SMPS efficiency can be considerable:

1. The transformer size is reduced;
2. Rectifier diodes have a smaller current rating;
3. Heatsinks 散热片 can be smaller;
4. A cooling fan is not usually needed;
5. The supply can have a wider input voltage range (no voltage selector needed);
6. The power density 功率密度(watts per cm<sup>3</sup>) is greater.
7. The weight is lower.

However, against these advantages, the following **disadvantages** must be considered:

1. The SMPS is electrically very noisy;
2. The filtering necessary to ensure feedback stability does make the transient response of the SMPS much slower (10 X) than an equivalent linear supply.

The principal function 主要功能 of an SMPS is to convert one DC voltage to another very efficiently. In the PC for example the 240V mains is rectified and used as the input to the computer's PSU which is invariably an SMPS.

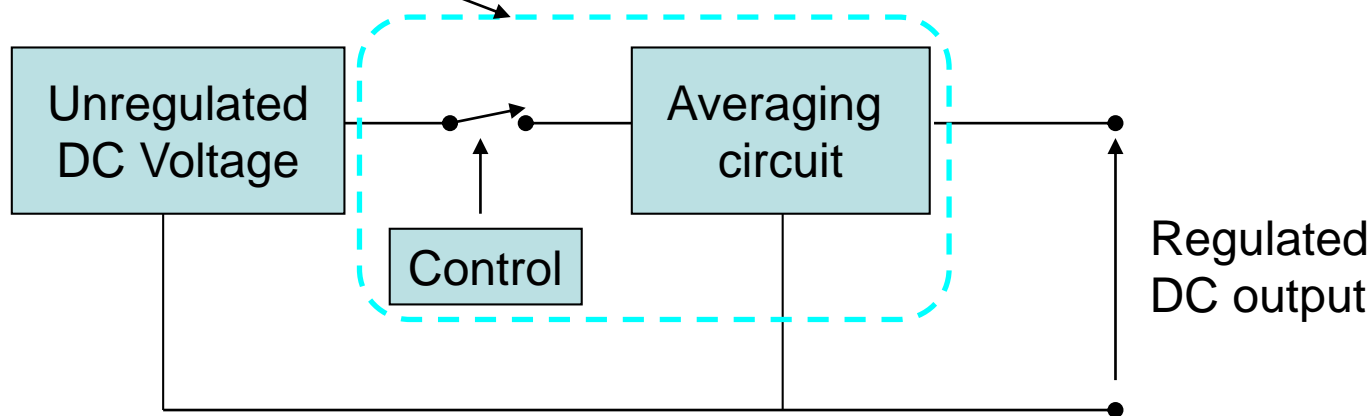


In a mobile phone a miniature SMPS is used to convert the battery voltage into multiple different DC voltages for the microprocessor, display, RF 射频 amplifier and camera.

SMPSs come in many different varieties. For the purposes of this course we will consider three of the most commonly used types:

1. The step-down or “buck” converter. [ $V_{\text{out}} < V_{\text{in}}$ ]
2. The step-up or “boost” converter. [ $V_{\text{out}} > V_{\text{in}}$ ]
3. The “buck-boost” or flyback converter. [ $V_{\text{out}} \lessgtr V_{\text{in}}$ ]

The basic SMPS block diagram is as follows:



The two important components here are the switch (usually a MOSFET) and the averaging circuit that will include an inductor.

# Power Supply Specifications 规格

## 1. Input/Output Voltage/Current

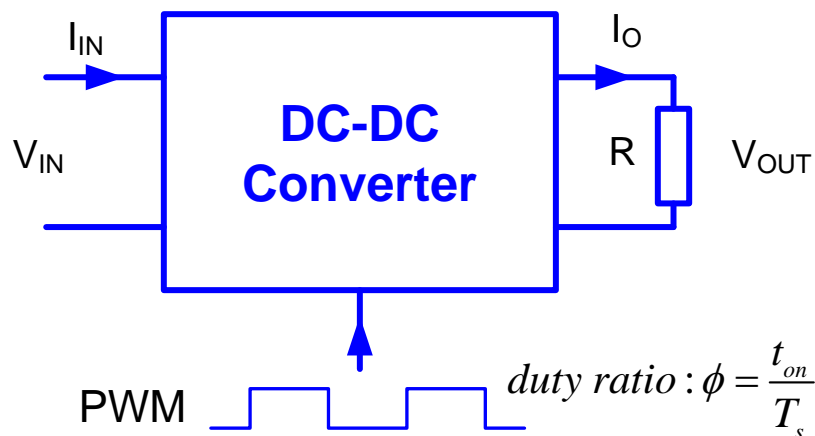
## 2. Line Regulation 线电压调整率 – the output voltage fluctuation under the input fluctuation

$$= \left( V_{out(\text{highest input})} - V_{out(\text{lowest input})} \right) / V_{out(\text{nominal})} \times 100\%$$

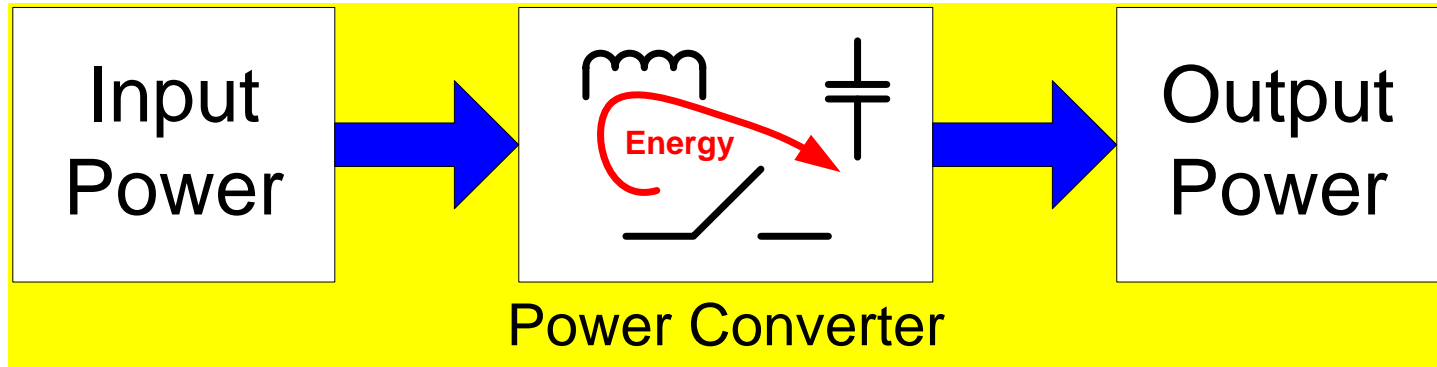
## 3. Load Regulation 负载电压调整率 - the output fluctuation under the load fluctuation

$$= \left( V_{out(\text{no load})} - V_{out(\text{full load})} \right) / V_{out(\text{nominal})} \times 100\%$$

## 4. Output Voltage Ripple 纹波



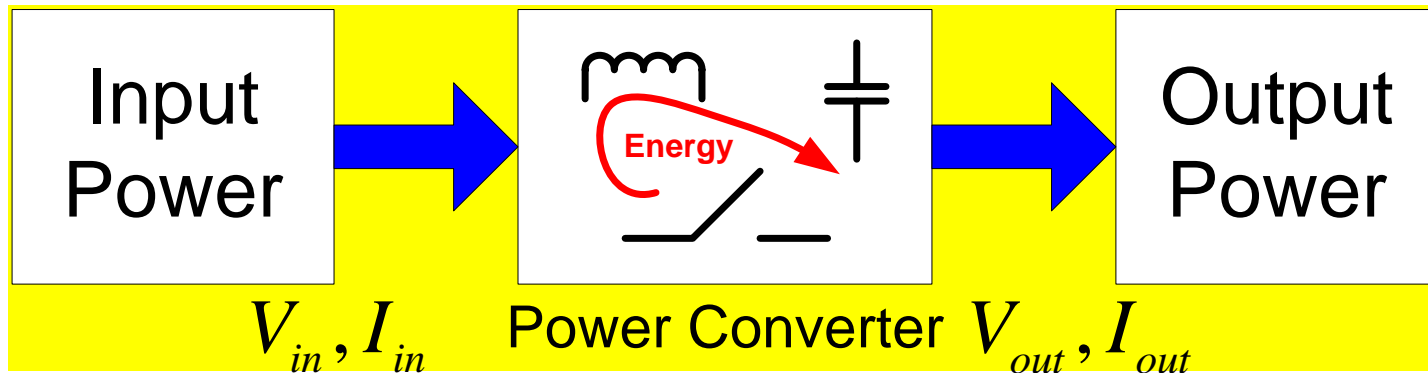
# Switch-Mode Power Converter



## Two Switch Modes: ON and OFF

- **Switch ON:**  
Charging the energy storage components
- **Switch OFF:**  
Discharging the energy storage components

# Role of Energy Storage Devices



Voltage/Current Conversion  $\Delta V = V_{out} - V_{in}$ ,  $\Delta I = I_{out} - I_{in}$

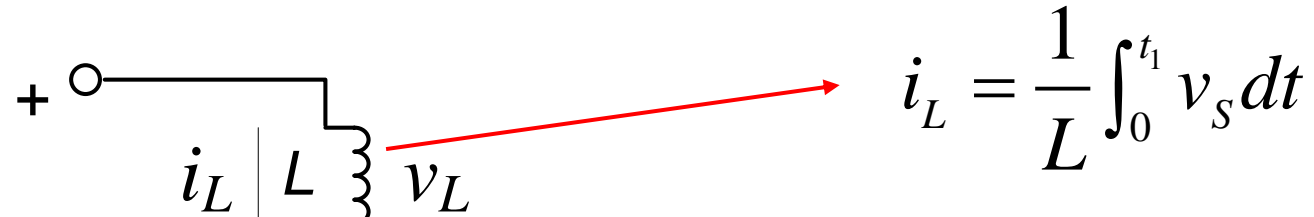
**Energy storage devices** - inductors, capacitors, transformers ...

$$i_C = C \frac{dv_C}{dt}, \quad E_C = \frac{1}{2} C v_C^2 \quad v_L = L \frac{di_L}{dt}, \quad E_L = \frac{1}{2} L i_L^2$$

Energy storage devices are employed to trap 捕获 or recover/release 释放 energy for filtering and buffering voltage/current fluctuations 波动.

**Switches is used to control the power flow by switching.** Only one uni-directional power switch is not enough for Energy Recovery in practice.

# Energy Trap (Charging)

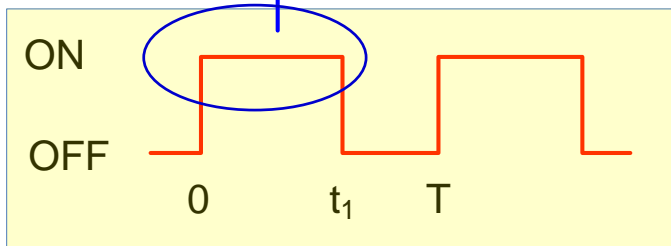


$$i_L = \frac{1}{L} \int_0^{t_1} v_s dt$$

**Switch is ON**, the inductor  $L$  will be charged (energized 蓄能)

Trapped Energy in the inductor will be

$$E_L = \frac{1}{2} L i_L^2$$



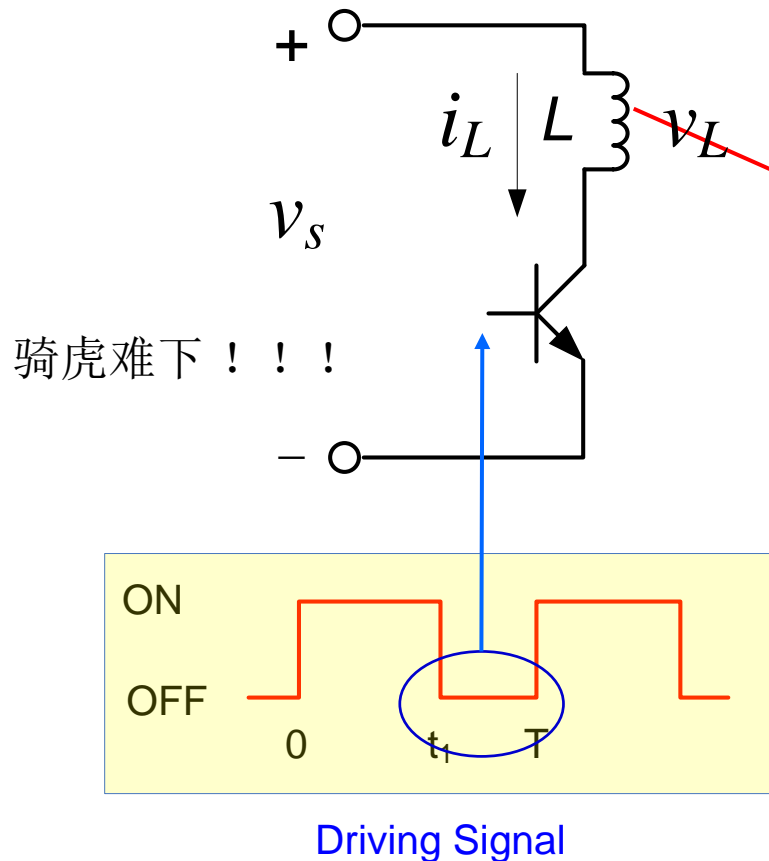
Driving Signal



# Energy Discharging Issue

In practice, we can't switch OFF the switch abruptly, because the inductor

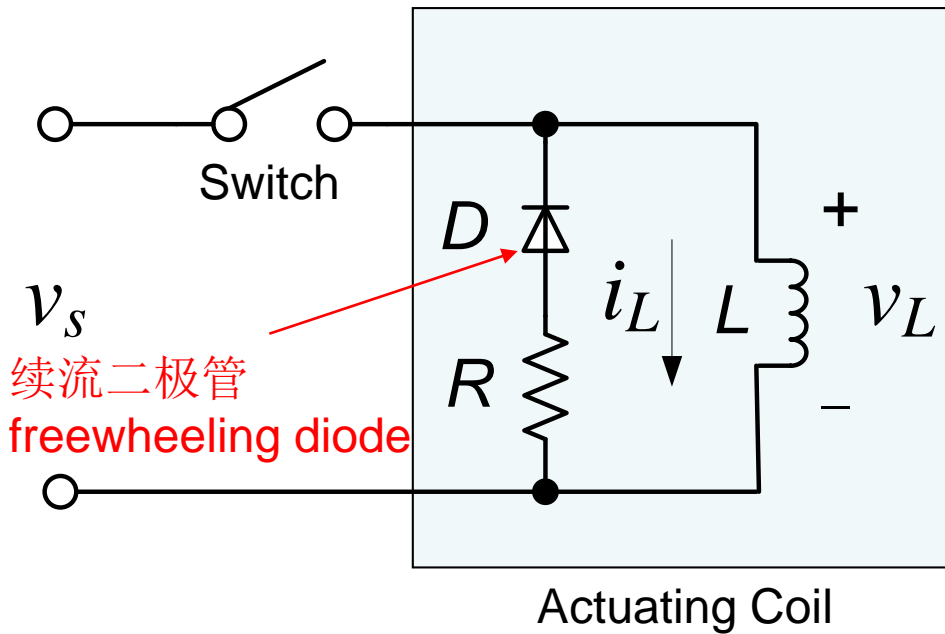
$$v_L = L \frac{di_L}{dt}$$



If the inductor current  $i_L$  drops to zero immediately, the inductor voltage  $v_L$  will become too large.  $v_L$  will be beyond devices' rating and destroy the switch and the whole circuit.

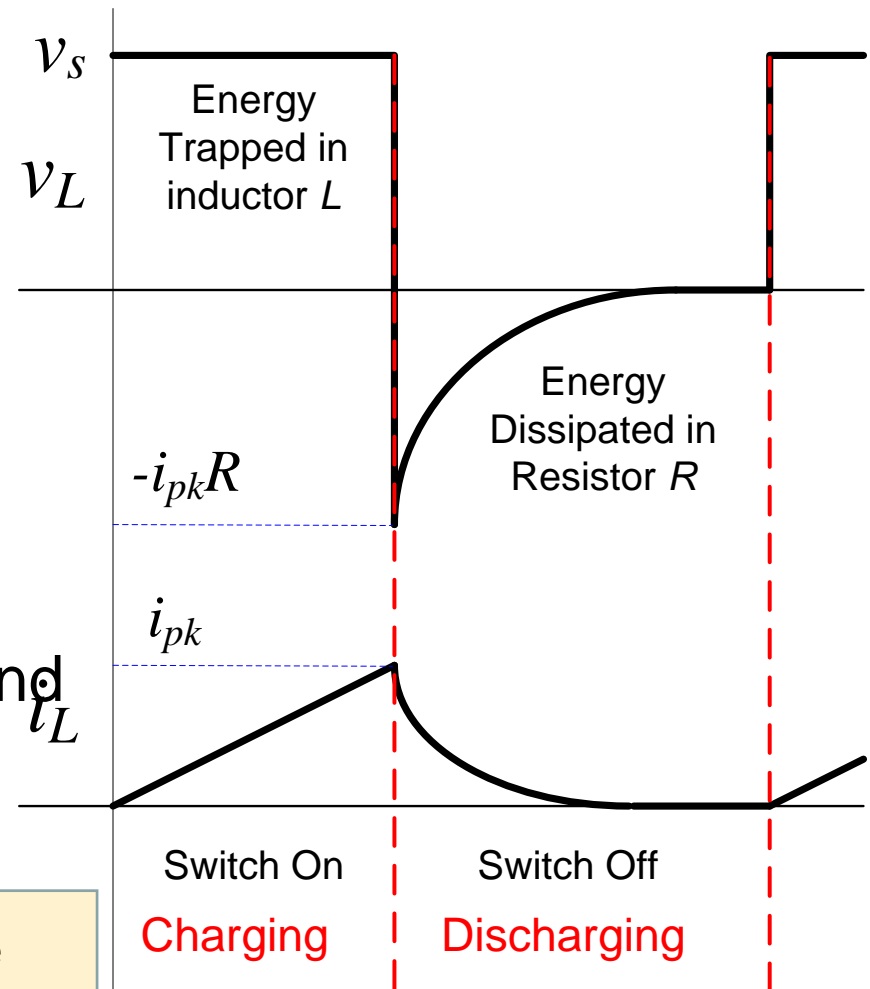
Above circuit is impractical for power conversion

# Dissipative Energy Recovery

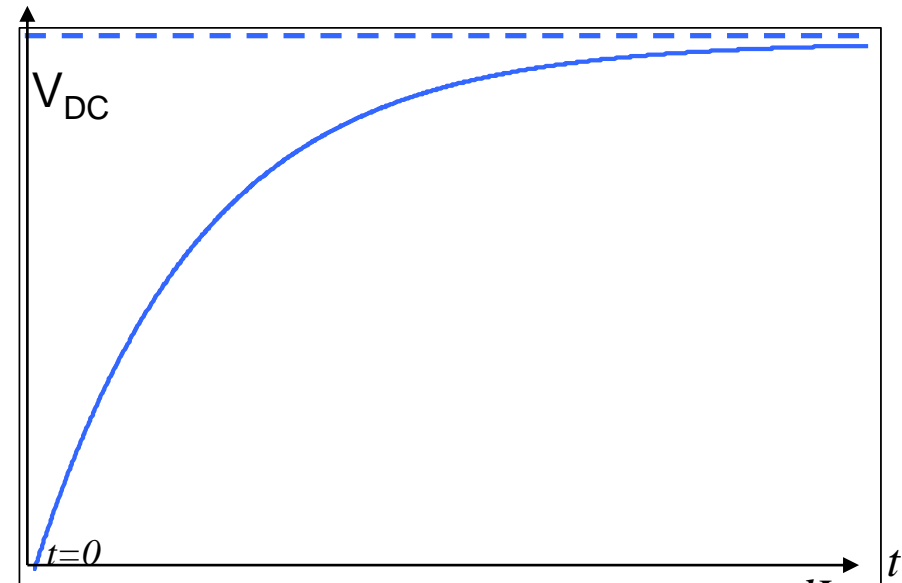
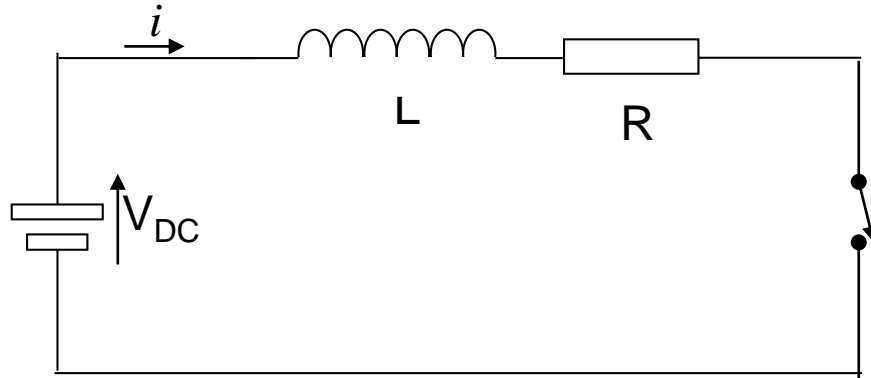


The coil inductor are energized and de-energized in above circuit by switching ON/OFF the switch.

**Diode**  $D$  is needed to provide the **freewheeling** path for discharging when the controllable switch is OFF



Recall the basic RL series circuit 阻感串联电路:

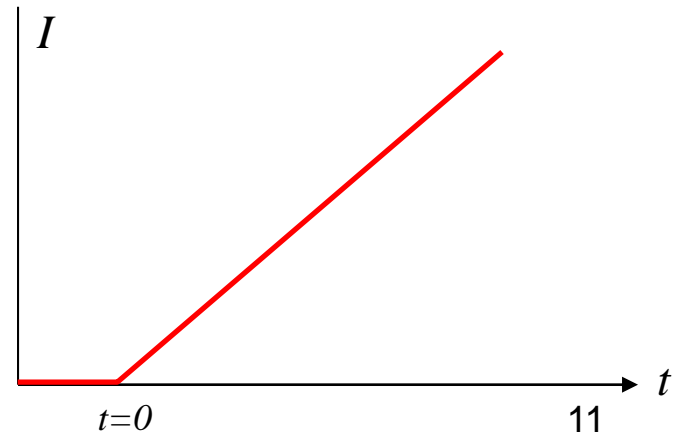


We can write the voltage equation for

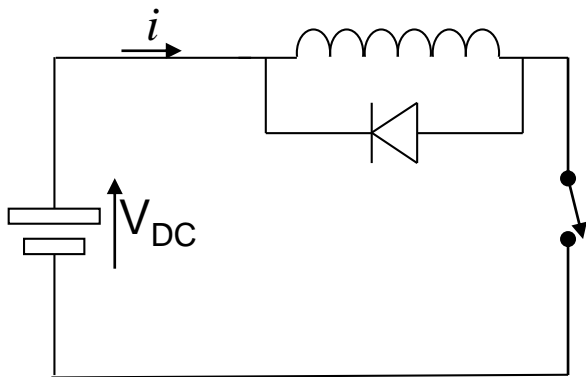
this circuit as 
$$V_{DC} = V_R + V_L = I_L R_L + L \frac{dI}{dt}$$

When the value of  $R$  is very small we can neglect the  $IR$  term and hence  $V_L = L \frac{dI}{dt}$

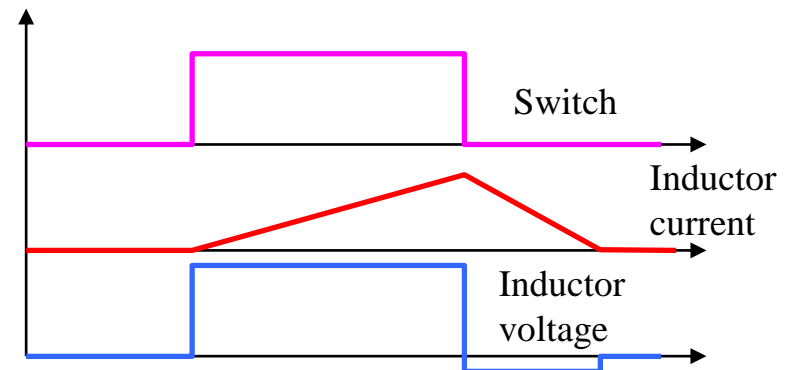
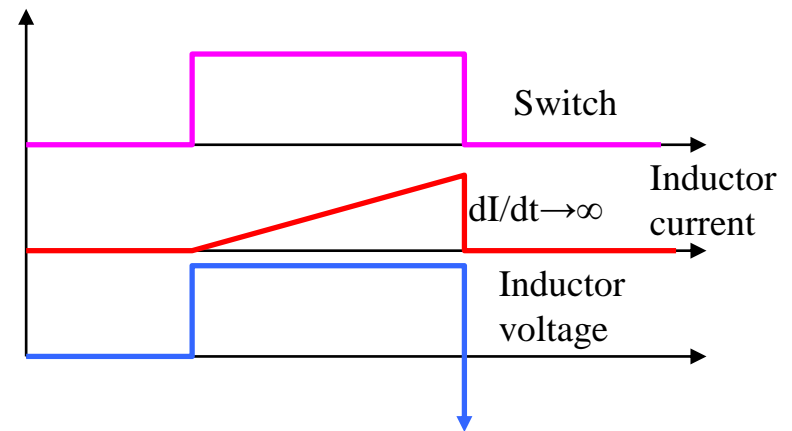
Hence we can approximate the voltage across the inductor to be a constant and therefore the current through the inductor to rise linearly when the switch is closed.



As previously discussed, as the current in the inductor rises, energy is stored in the magnetic field. When the switch is opened, the inductor tries to maintain the current flowing through it and thus the voltage across the inductor reverses. **It is necessary to provide a return path for the inductor current when the switch is opened** and this is usually done by adding a diode across the inductor.

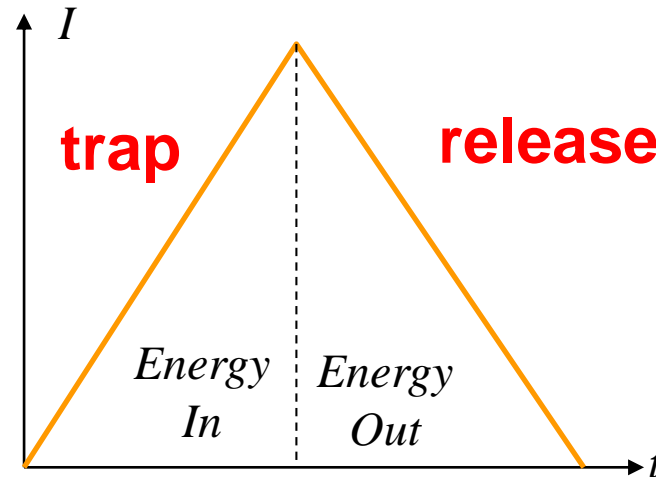


Non-Dissipative  
Recovery



-1V = diode  
conduction  
voltage drop

The energy stored in an inductor is  $\frac{1}{2}LI^2$ . When current through the inductor is increasing, additional energy is being stored in the magnetic field. If the current is decreasing then energy is being removed from the magnetic field. Assuming a perfect inductor, the energy (i.e. number of joules) returned to the circuit is exactly the same as was originally added.



For Power Conversion, released power from energy storage devices is expected to be delivered to the load as much as possible. Dissipative Energy Recovery is not acceptable.

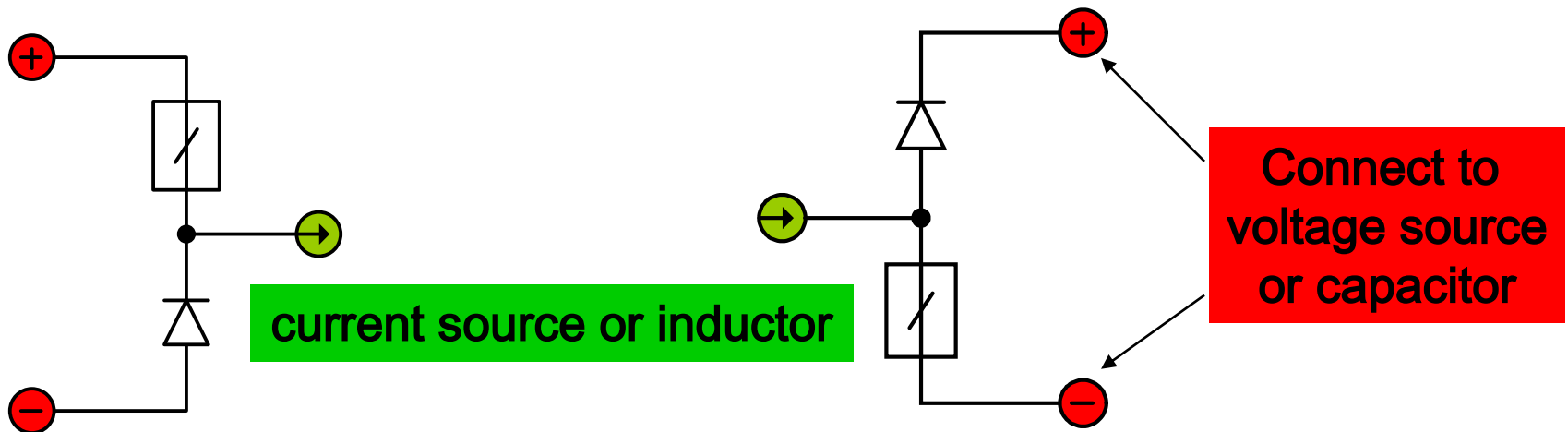
**How to recover the trapped energy in switch-mode power conversion?**

# Switch-Cells for Energy Trap/Recovery:

## Single-Switch Solution

开关单元

A switch-cell is a tri-port Single Pole Double Throw (SPDT单刀双掷) switch which consists of a power switch and a freewheeling diode



**P-cell 正单元**

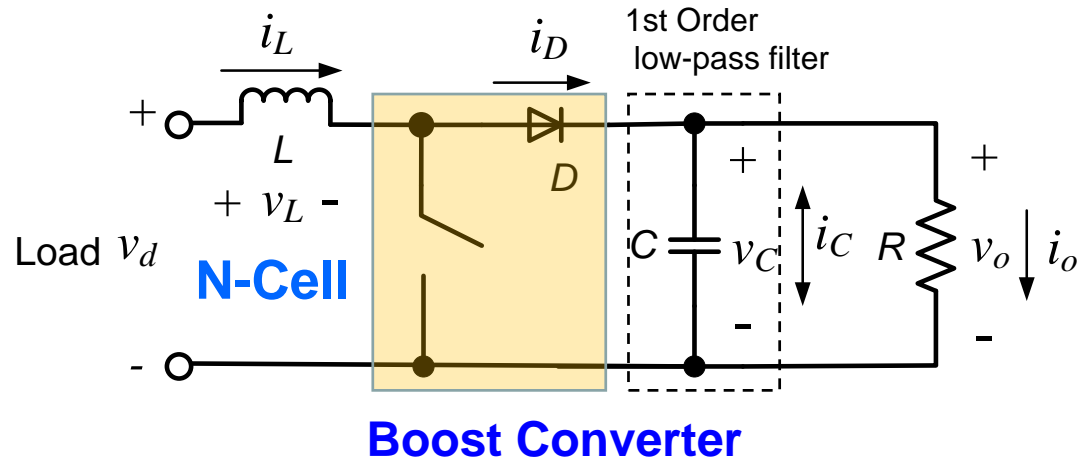
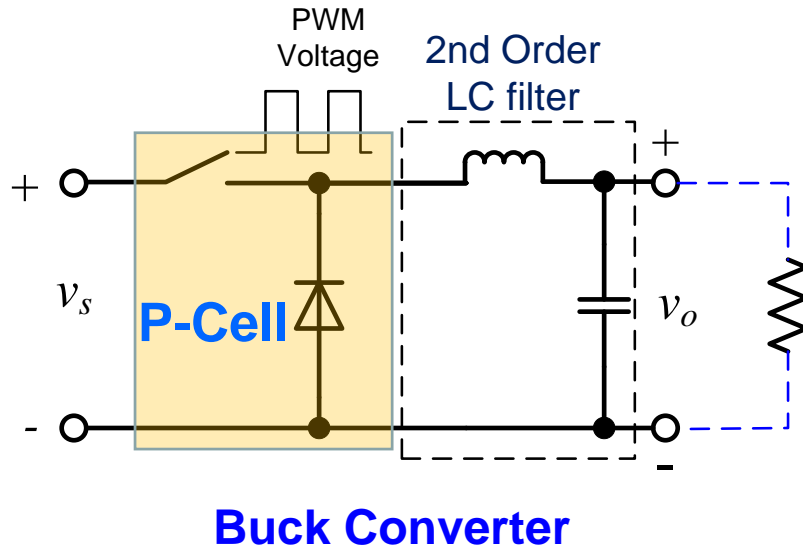
Controllable Switch is connected to **Positive** pole of voltage source

**N-cell 负单元**

Controllable Switch is connected to **Negative** pole of voltage source

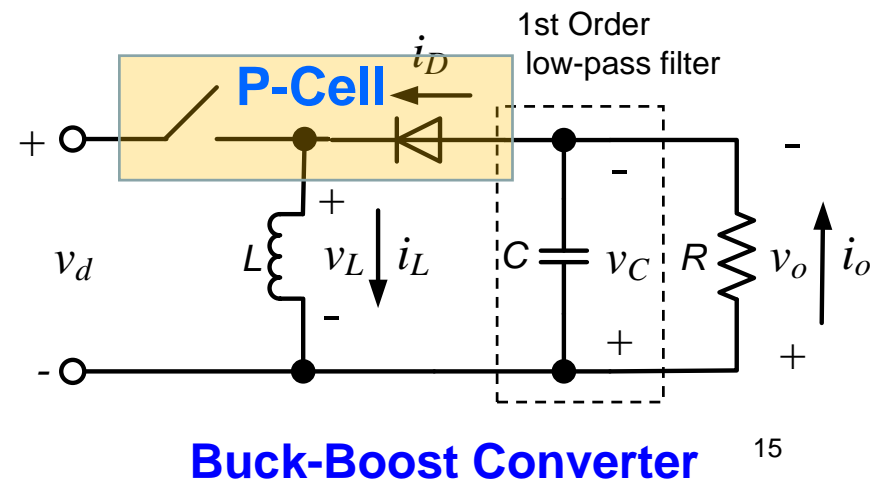
**Both the Switch and the diode will not be in conduction at the same time !!!**

# Single-Switch DC-DC Converters

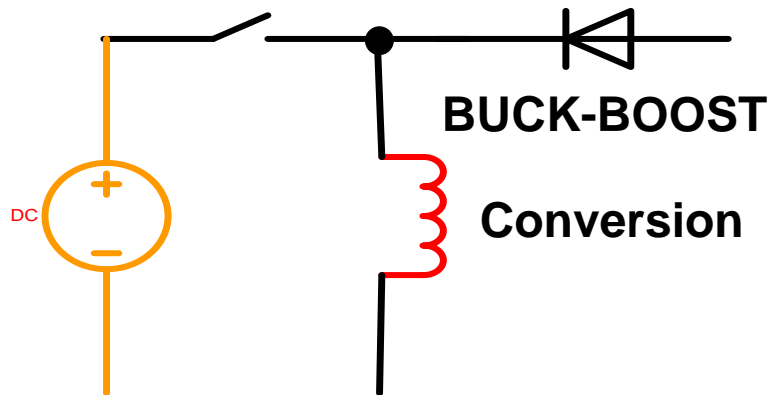
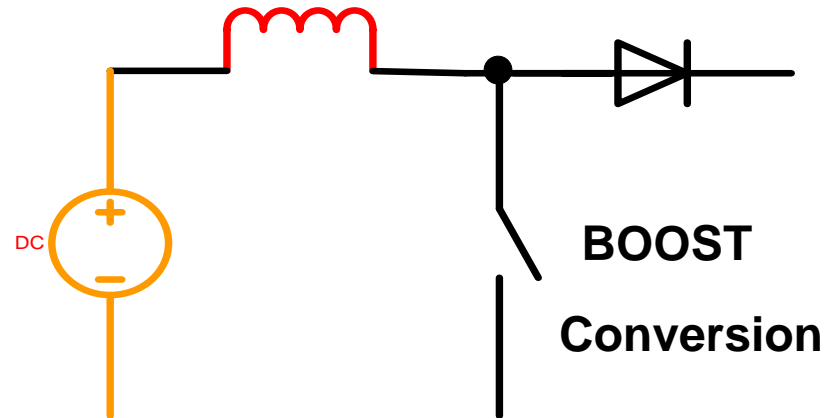
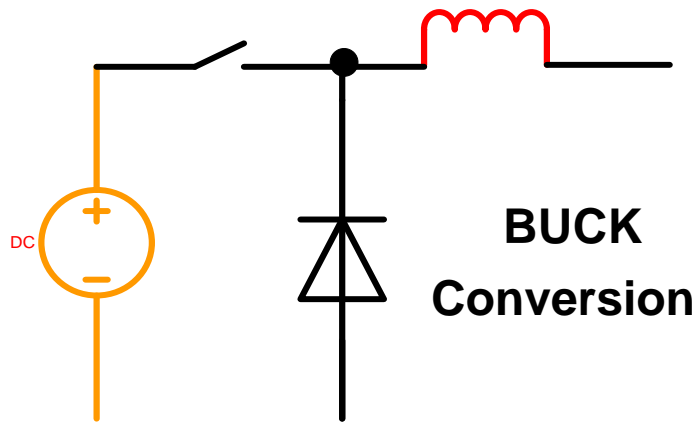


Identical components, different functions !!!

How do we know a circuit is a buck, boost or buck-boost converter ?!



# Inductors in Converters

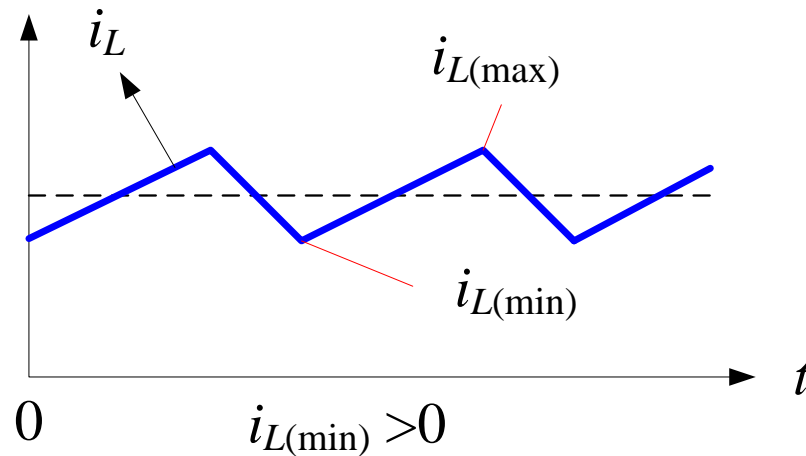


The Switch Cell is actually used to control the **charging** (**switch is on, diode is off**) and **discharging** (**diode is on, switch is off**) of the inductor in single-switch DC-DC converters.

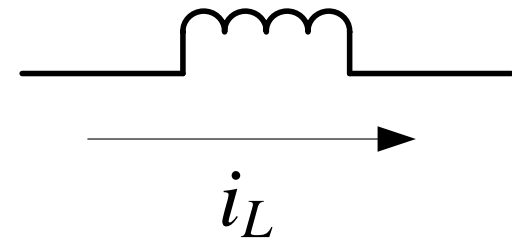


# Two Conduction Modes of Inductor

Continuous Conduction

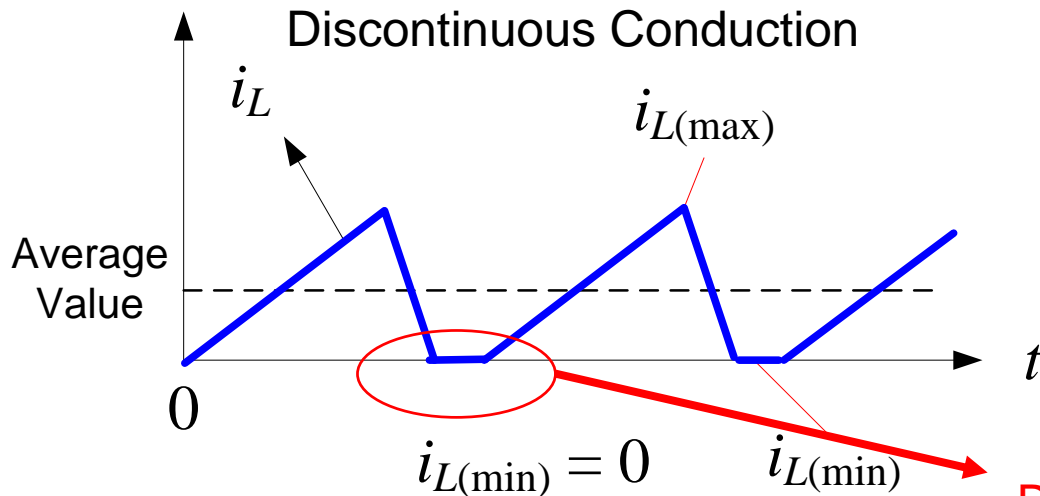


Inductor  $L$



**Inductor conduction mode (导通模式)** has significant impacts on the circuit properties of three DC-DC converters

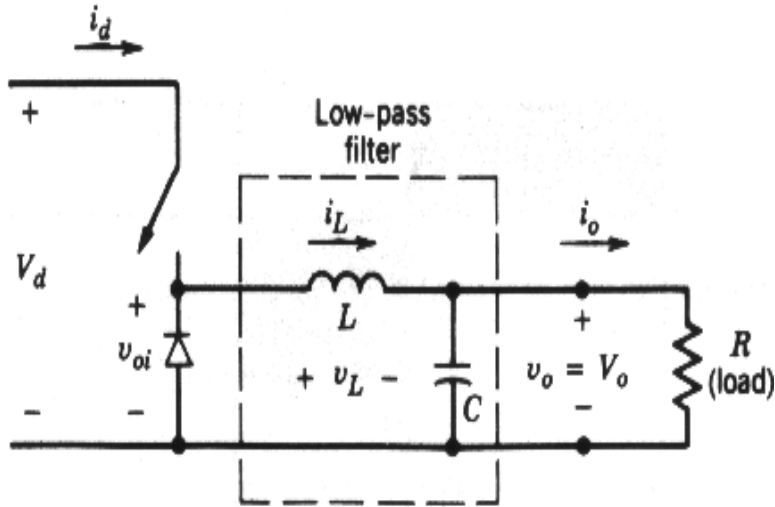
Discontinuous Conduction



Fully Discharged

# Example 1: Buck Converter

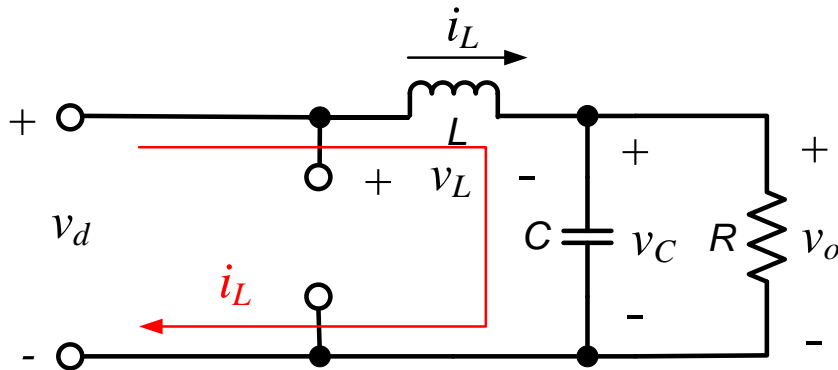
Continuous Conduction Mode (CCM)  $i_L > 0$



**Buck Converter**

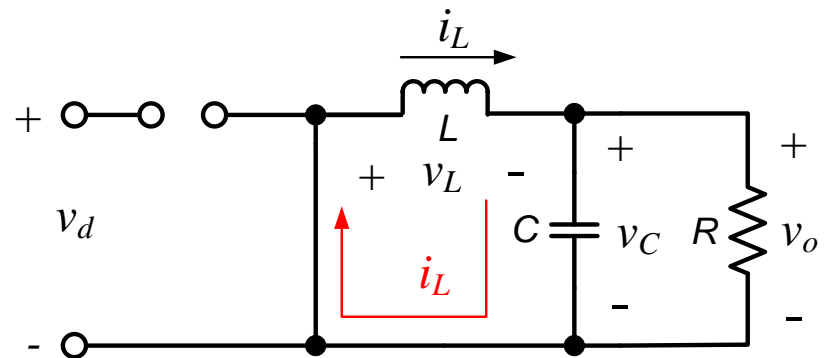
When the inductor current is continuous, the conduction statuses of the switch and diode are complementary, that is, **Switch is On, Diode is Off; or Switch is Off, Diode is on.**

Switching Buck Converters actually switch between **two circuit states** 电路状态 during their operation.



**Switch ON, Diode OFF**

**Circuit State 1 (Charging)**

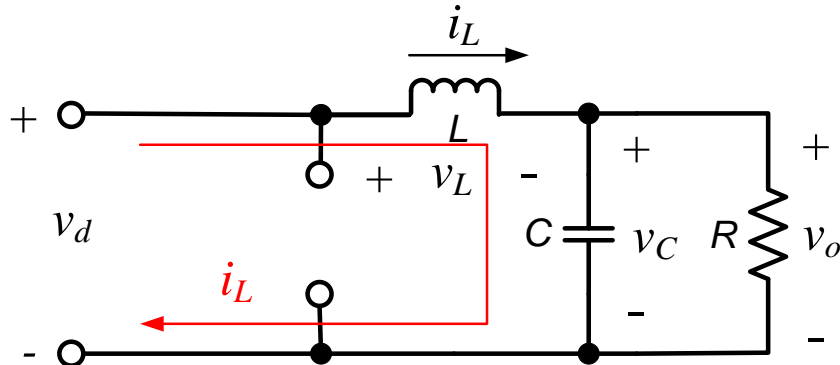


**Switch OFF, Diode ON**

**Circuit State 2 (Discharging)**

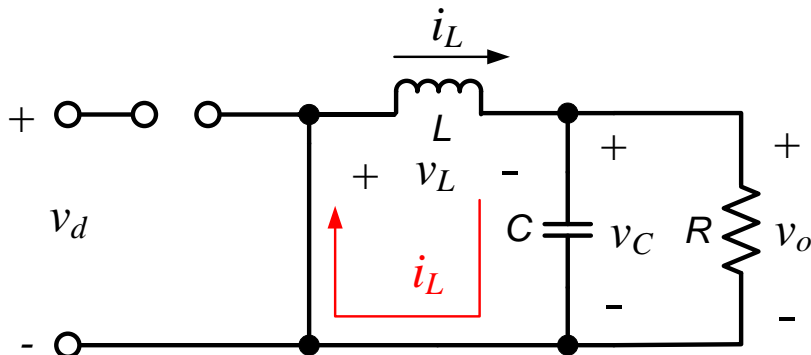
# Example 1: Buck Converter

## Discontinuous Conduction Mode



Switch ON, Diode OFF

Circuit State 1 (Charging 充电)

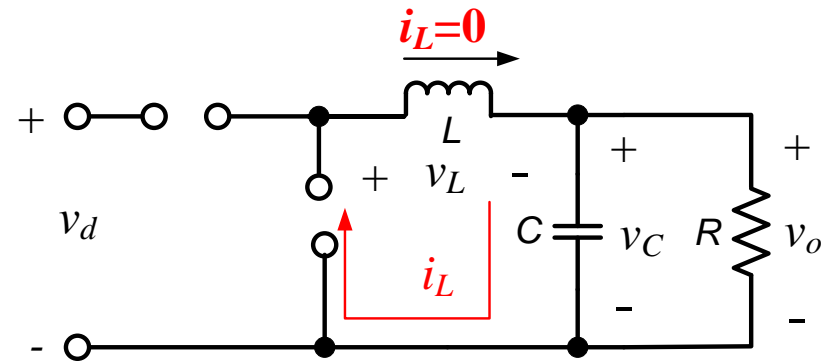


Switch OFF, Diode ON

Circuit State 2 (Discharging 放电)

When the inductor current is discontinuous, the conduction statuses of the switch and diode have 3 combinations: Switch is On, Diode is Off; Switch is Off, Diode is on; Switch is Off, diode is Off.

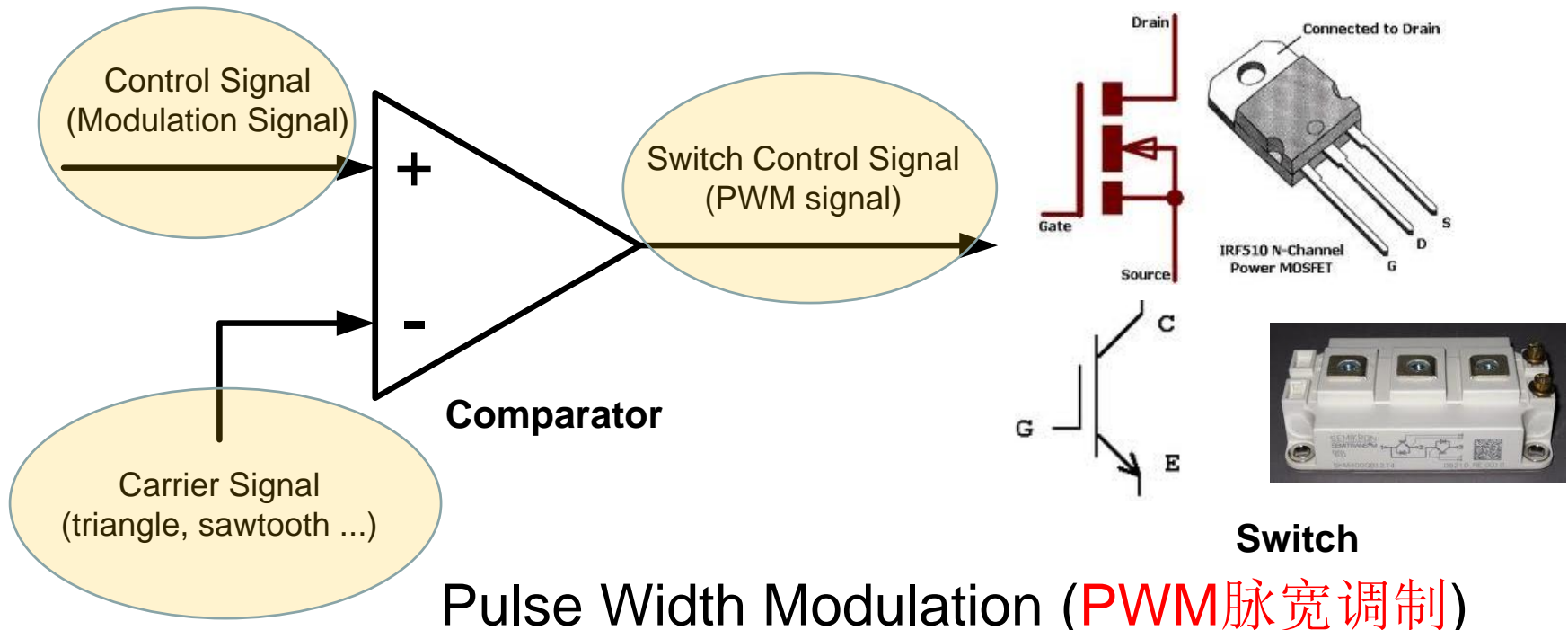
Switching Buck Converters actually switch between three circuit states during their operation.



Switch OFF, Diode OFF

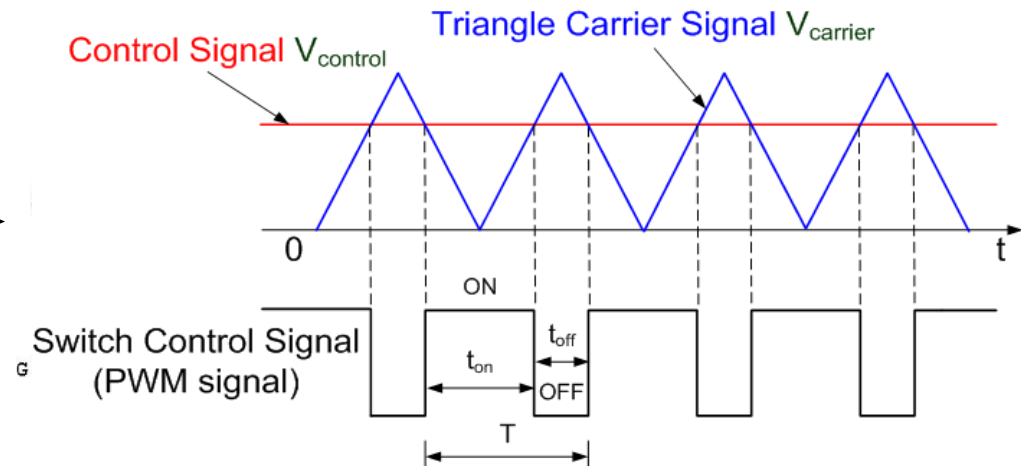
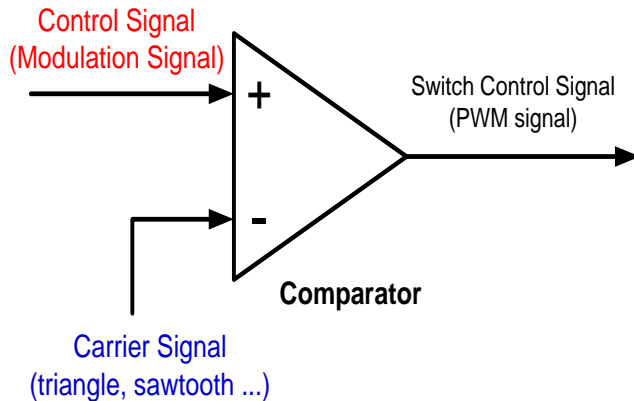
Circuit State 3 (Idle 空闲)

# Switching On/Off Methods: PWM Generation



1. Constant Switching (Carrier) Frequency
2. Carrier 载波 Frequency  $\gg$  Control Signal Frequency
3. Control Signal Peak  $\leq$  Carrier Signal Peak
4. Output pulse width is proportional to input control signal

# PWM Generation

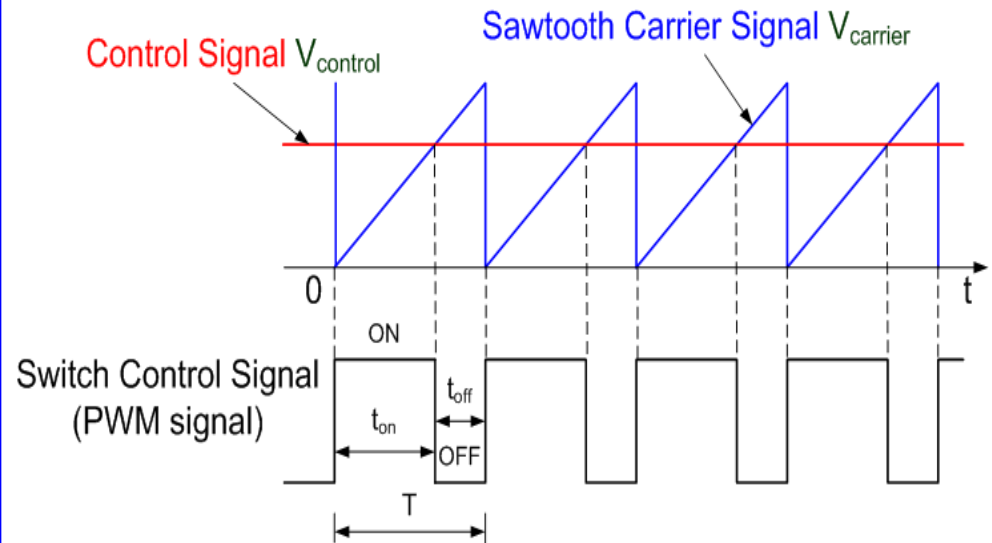


*switching frequency:*

$$f_s = \frac{1}{T} = \frac{1}{t_{on} + t_{off}}$$

*duty ratio:*

$$\phi = \frac{t_{on}}{T} = \frac{V_{control}}{V_{carrier(peak)}}, 0 \leq \phi \leq 1$$



# Switching Converter Circuit

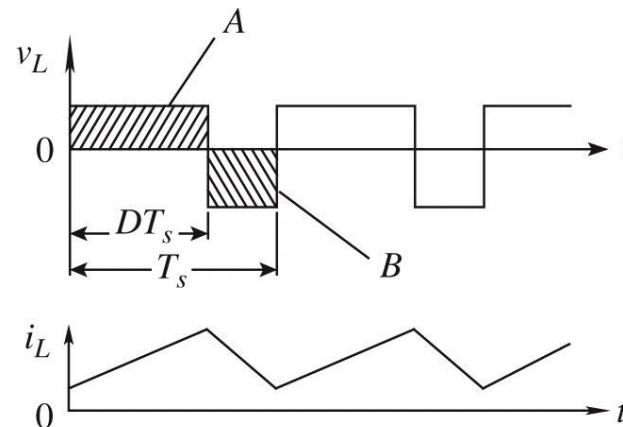
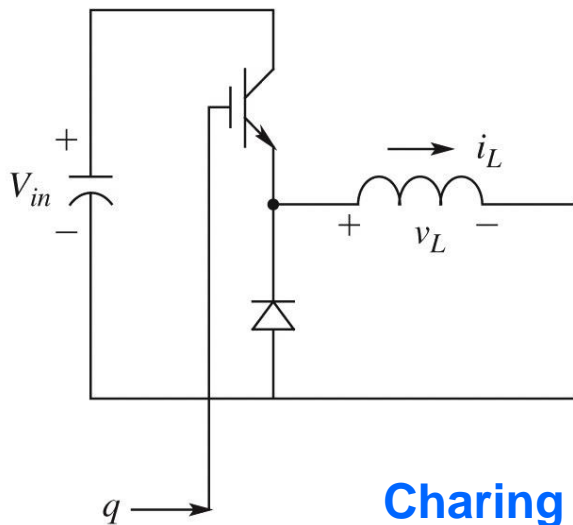
## Steady-State Analysis

- How do we know a circuit is a buck降压, boost升压 or buck-boost升降压 converter ?!
  - How do we analyse their circuit characteristics?

# Steady-State Circuit

All DC-DC converters discussed here, are in **steady state**. That means that

- All power waveforms repeats, unchanged from one switching cycle to the next
- The switching duty-ratio 占空比 remains constant.



**Charing Energy = Discharging Energy  
Over one Switching Cycle**

# Steady-State Inductor

For an **inductor**, steady state means that charged energy is equal to discharged energy in the inductor over one switching period.

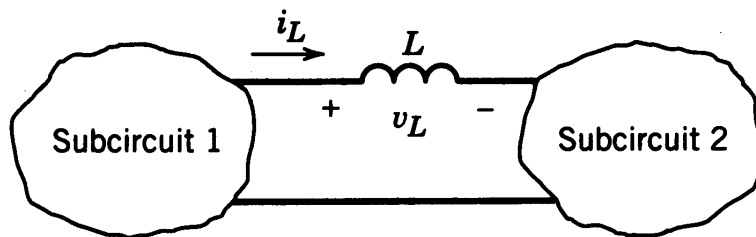
$$\left. \begin{array}{l} E_L = \frac{1}{2} L i_L^2 \\ \Delta E_L = 0 \end{array} \right\} \Rightarrow \Delta I = i_L(t+T) - i_L(t) = 0$$

$$\left. \begin{array}{l} L \frac{di_L}{dt} = v_L \\ i_L(t+T) = i_L(t) \end{array} \right\} \Rightarrow \frac{1}{L} \int_0^T v_L(t) dt = 0 \Rightarrow \int_0^T v_L(t) dt = 0$$

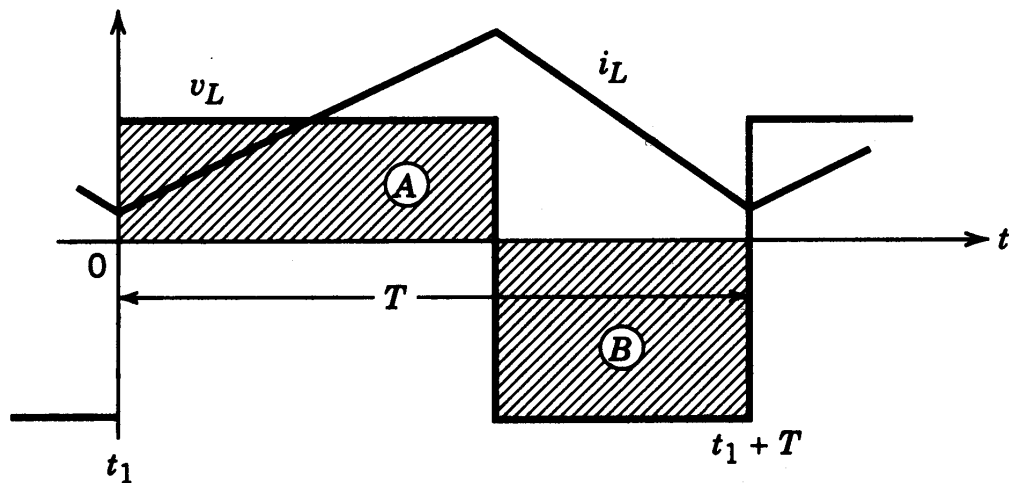
Its average voltage change per switching period = 0



# Inductor Voltage and Current in Steady State



(a)



(b)

$$\int_0^T v_L(t) dt = 0$$

- Volt-seconds 伏-秒乘积 over  $T$  equal zero.

# Steady-State Capacitor

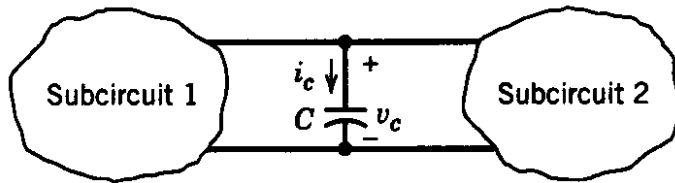
For a **capacitor** in DC-DC converters, steady state means that energy rise is equal to energy fall in the capacitor over one switching period.

$$\left. \begin{array}{l} E_C = \frac{1}{2} C v_C^2 \\ \Delta E_C = 0 \end{array} \right\} \Rightarrow \Delta v = v_C(t+T) - v_C(t) = 0$$

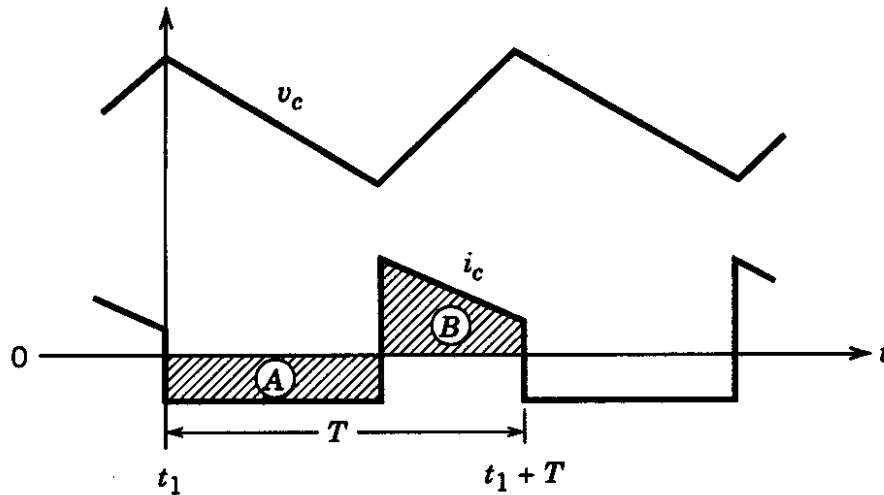
$$\left. \begin{array}{l} C \frac{dv_C}{dt} = i_C \\ v_C(t+T) = v_C(t) \end{array} \right\} \Rightarrow \frac{1}{C} \int_0^T i_C(t) dt = 0 \Rightarrow \int_0^T i_C(t) dt = 0$$

Its average current per switching period = 0

# Capacitor Voltage and Current in Steady State



(a)



(b)

$$\int_0^T i_c(t) dt = 0$$

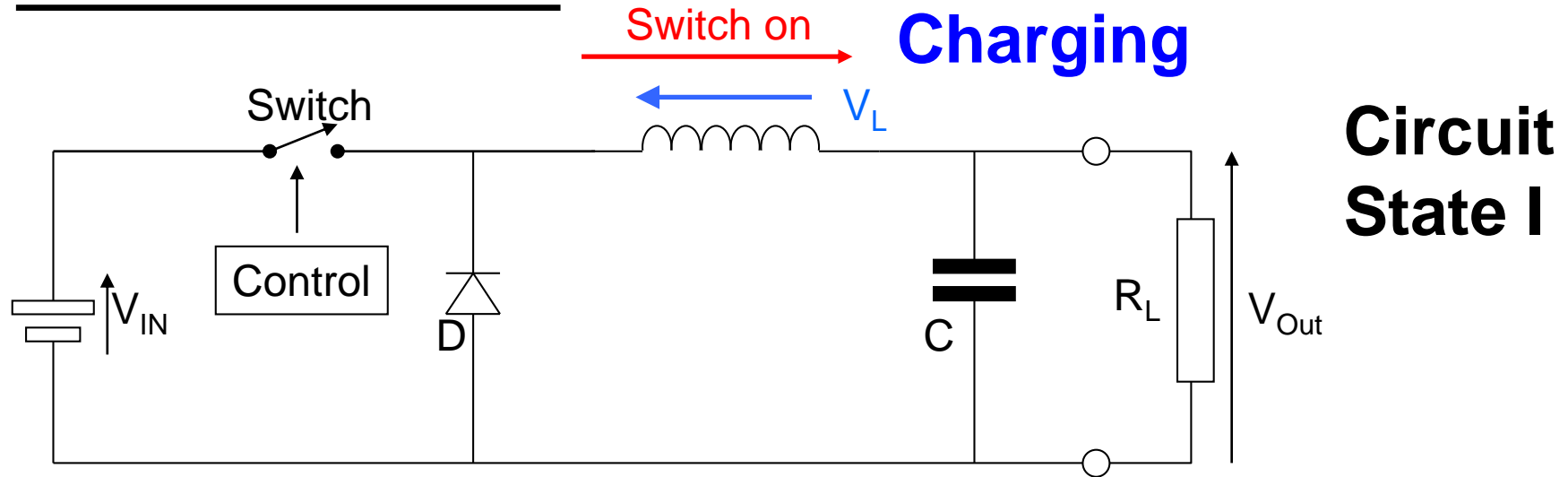
- Amp-seconds over  $T$  equal zero.

# **Buck Converter**

## **Continuous Conduction Mode**

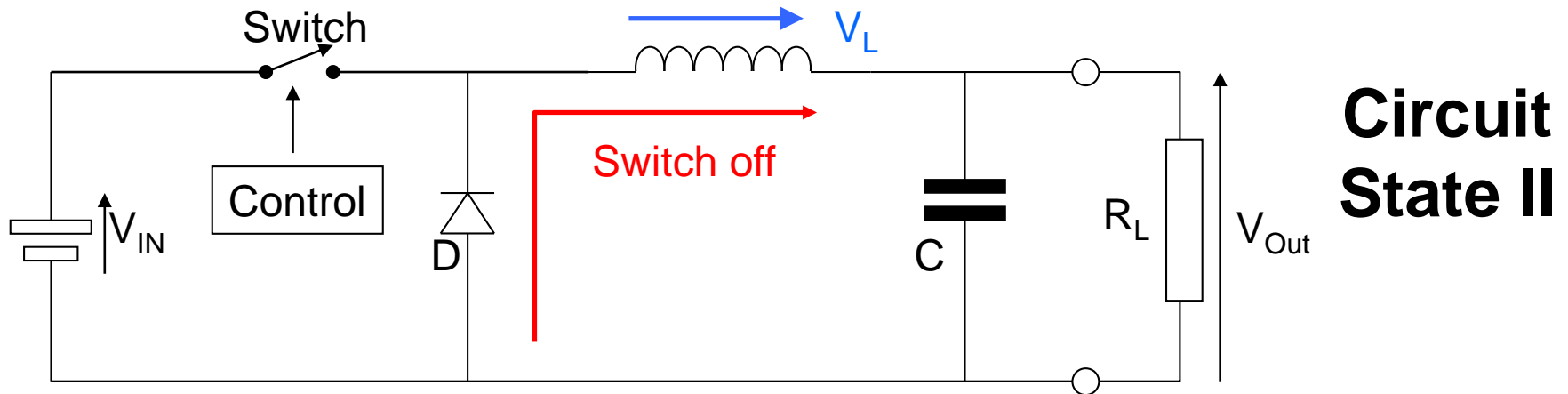
### **Steady-State Average Analysis**

# The Buck Converter



The circuit shows the SMPS circuit for a buck converter. When the switch is first closed, current starts to flow through the inductor and into the capacitor. The voltage across the inductor,  $V_L$ , is the difference between the input voltage  $V_{in}$  and the output voltage  $V_{Out}$ . The diode is reverse biased 偏置 ("off") by the supply voltage  $V_{in}$  and does not conduct any current. The rate of change of current into the capacitor is limited by the inductor ( $V = L \, di/dt$ ). At some point  $T_{off}$  the switch is opened. At this point the current through the inductor has risen to  $I_{Max}$  and the energy in the inductor ( $\frac{1}{2}LI^2$ ) is stored in its magnetic field.

## Discharging

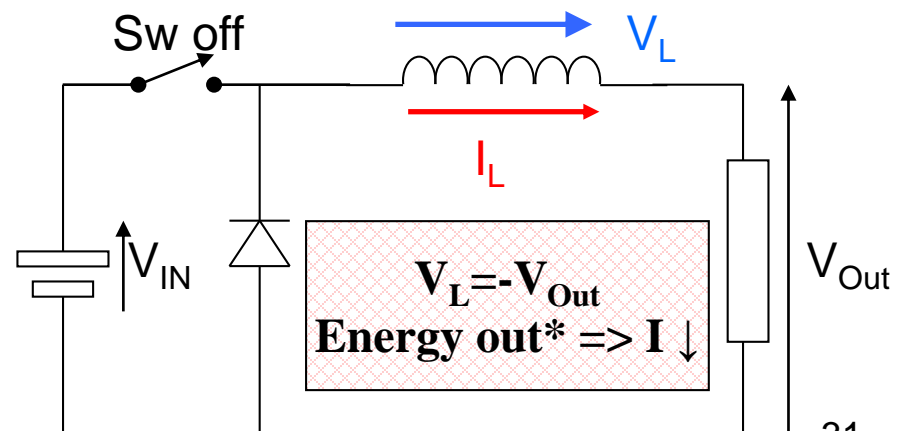
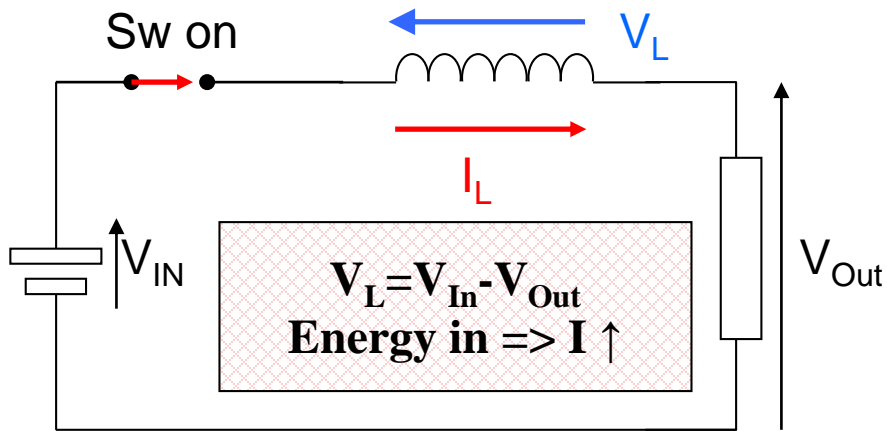
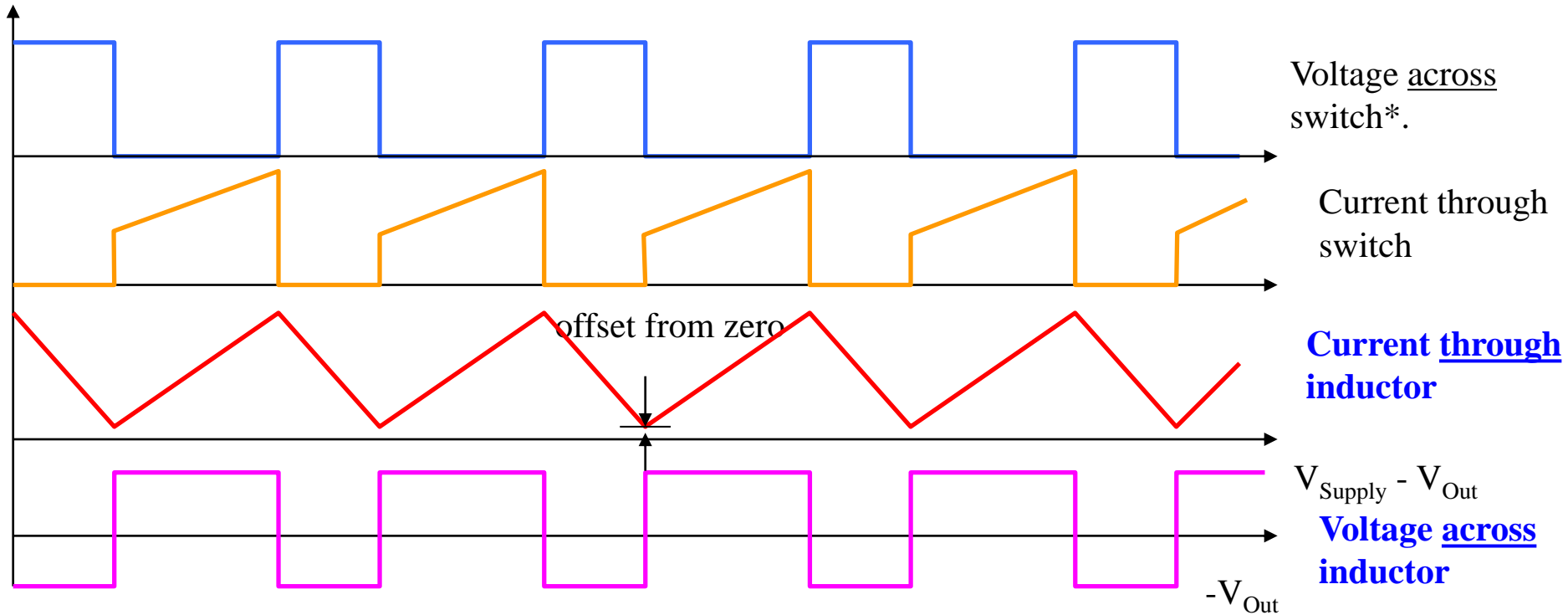


As soon as the switch is opened, the potential on the switched side of the inductor goes sharply negative (but the current continues to flow) as the energy in the magnetic field tries to maintain the current through the inductor. This has the effect of bringing the diode into conduction (hence  $V_L = -V_{Out} + 1V$ ), thereby completing the circuit and enabling the current to continue to flow into the capacitor. As the energy in the inductor falls so the current diminishes but the capacitor continues to be charged (more slowly). If we ignore the diode voltage drop, the voltage across the inductor is now  $-V_{Out}$ .

**The switching frequency, supply voltage and inductor value are all chosen such that there is always current flowing in the inductor.** This is called continuous mode, i.e., continuous current in the inductor.

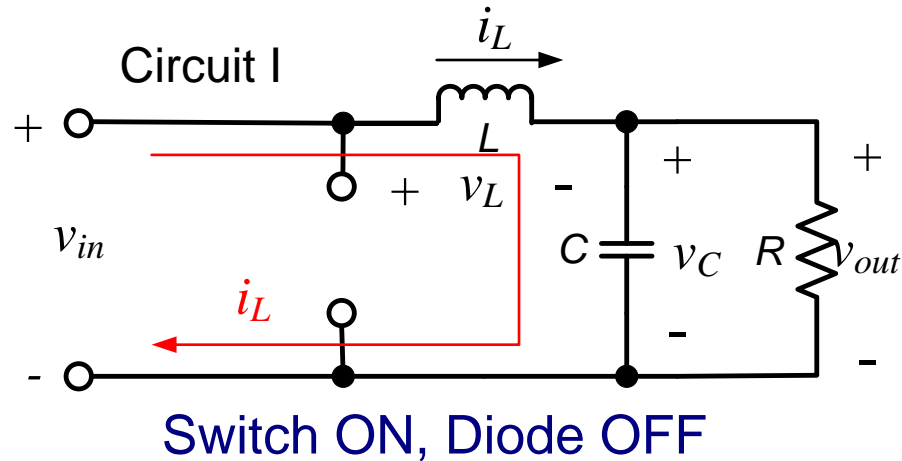
All of the circuitry necessary to implement the control function is available in an IC.

## Buck converter switching waveforms.



\* Ignoring the voltage drop across the diode, which we will do to simplify the analysis.

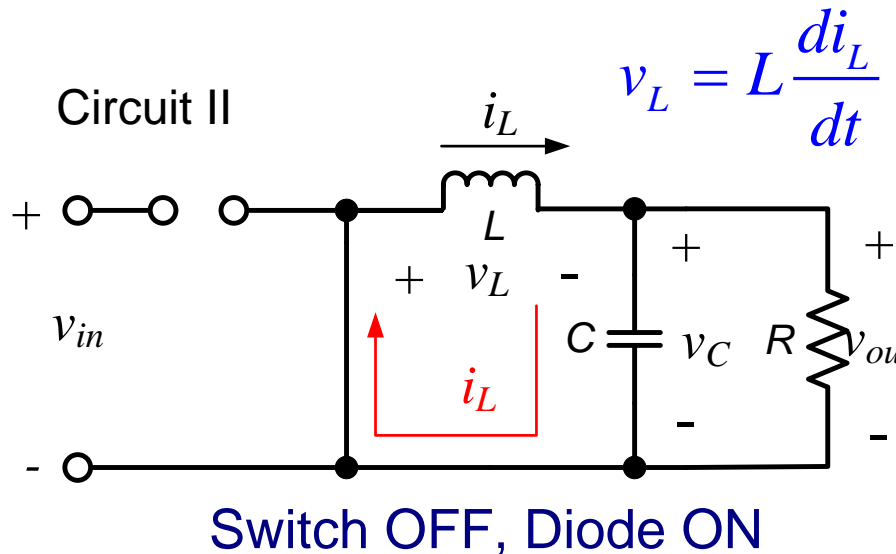
# Modeling: Two Switching Circuit States



**Charging inductor**

$$v_L = V_{IN} - V_{OUT} > 0$$

$$i_C = i_L - i_o$$



**Discharging inductor**

$$v_L = 0 - V_{OUT} = -V_{OUT} < 0$$

$$i_C = i_L - i_o$$

**Variable Structure Circuit**

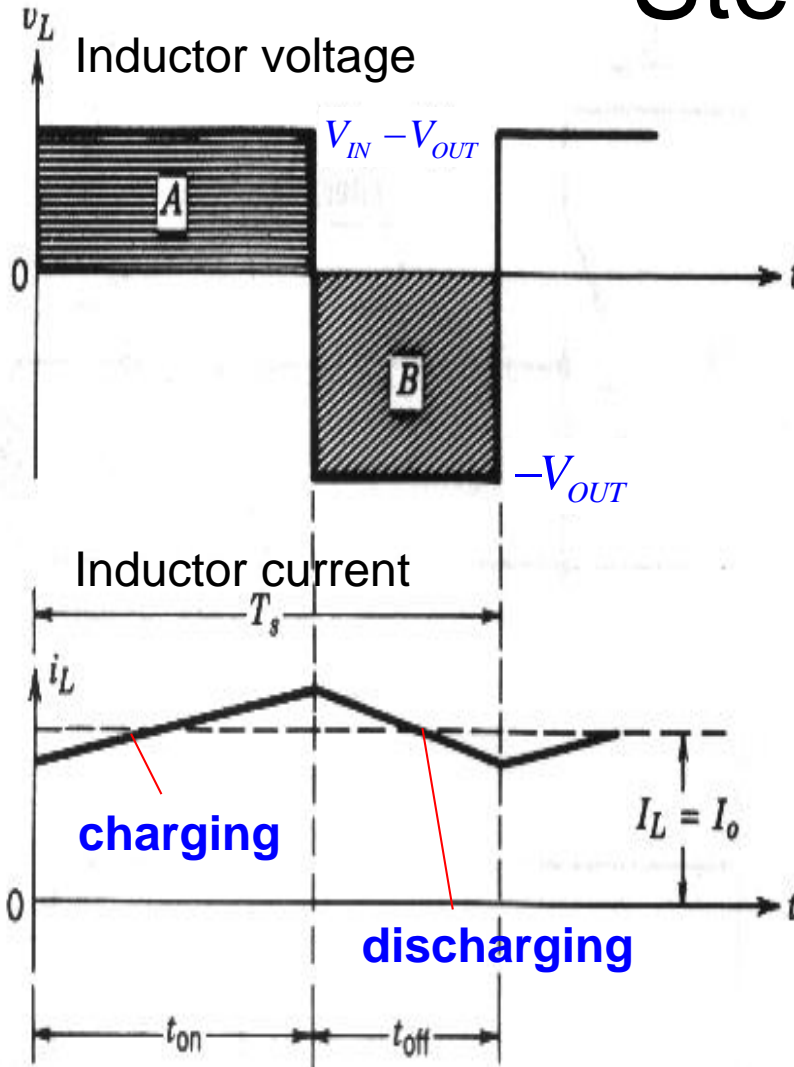
Circuit Equations



# Input-Output Voltage Conversion: Step-Down

降压

## Step-Down



$$\text{duty ratio : } 0 \leq \phi = \frac{t_{on}}{T_s} = \frac{t_{on}}{t_{on} + t_{off}} \leq 1$$

For the **inductor  $L$**  in steady state, its average voltage over one switching period

$$\int_0^T v_L(t) dt = 0$$

$$\Rightarrow \int_0^{t_{on}} (V_{IN} - V_{OUT}) dt + \int_{t_{on}}^{T_s} (-V_{OUT}) dt = 0$$

**Switch ON**                      **Switch OFF**

$$\Rightarrow (V_{IN} - V_{OUT}) \phi T_s + (-V_{OUT}) (1 - \phi) T_s = 0$$

$$\Rightarrow V_{OUT} = \phi V_{IN} \leq V_{IN}$$

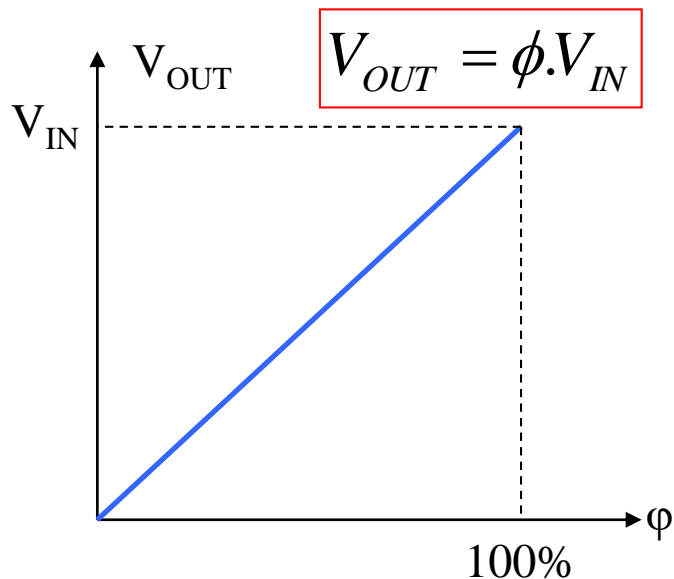
waveforms **Input-Output (average) Voltage Step-Down Converter**

## Buck converter in Continuous Conduction Mode.

1. The output voltage never exceeds the input voltage with  $0 \leq \Phi \leq 1$ .
2. **The output voltage is independent of the load on the output.** The power delivered to the load is proportional to the current in the inductor.

[Actually there is a small reduction due to losses in the system that have not been included in the analysis – but assume lossless for this course.)

Note that this relationship is true only if current is always flowing in the inductor, i.e. the SMPS is operating in continuous conduction mode.



Sticking with our assumption of a **lossless system**:

Output power = input power

$$\Rightarrow V_{OUT} I_{OUT(\text{Mean})} = V_{IN} I_{IN(\text{Mean})}$$

Setting  $V_{OUT} = \Phi V_{IN}$ , we have  $I_{OUT(\text{Mean})} = I_{IN(\text{Mean})} / \Phi$

(Remember for a DC voltage, average power =  $V_{DC} \times I_{\text{Mean}}$ )

Note that if the **smoothing capacitor** 平滑电容 on the output is sufficiently large then the output voltage is essentially constant. The typical PWM switching frequency is  $> 20\text{KHz}$  (above **audible frequency** 音频) so  $\Delta T$  on the capacitor is  $< 50\mu\text{S}$ .

We can use the **preceding** 先前的 current equation to make two very important **observations** 观察结果:

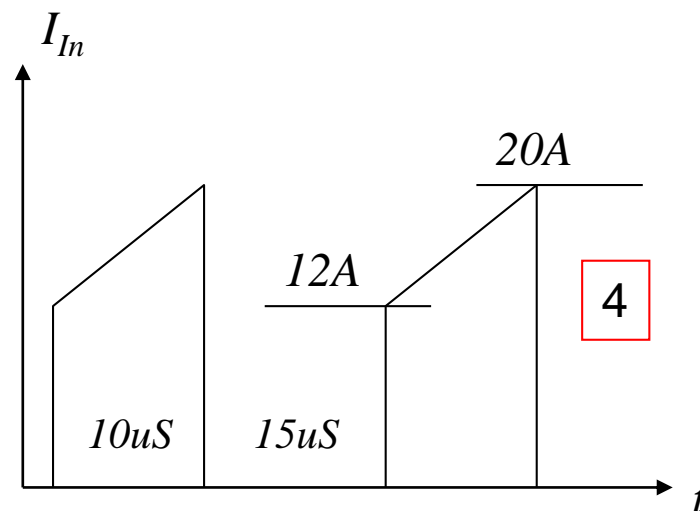
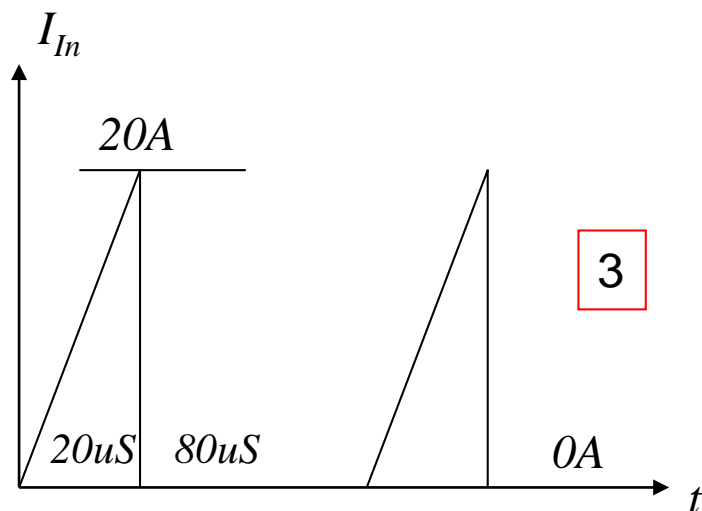
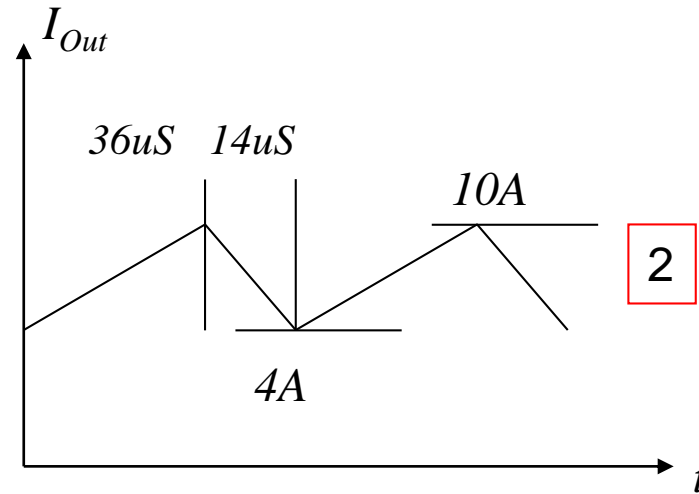
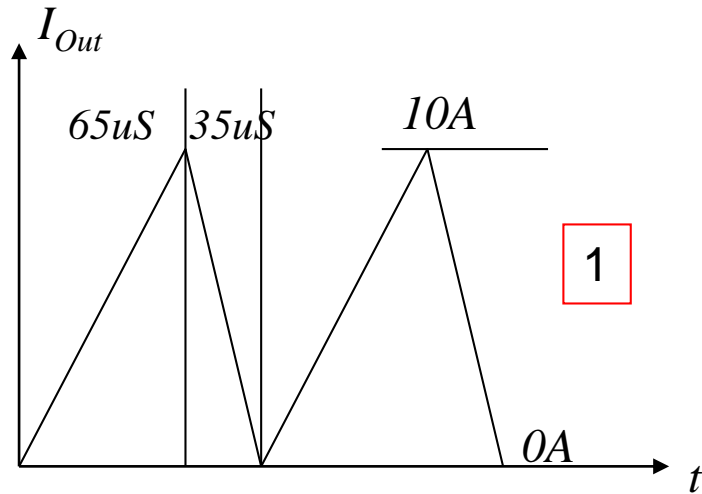
1. The **average** current in the inductor is the average load current.

$$\text{E.g. } P_{\text{Load}} = 100\text{W and } V_{\text{Load}} = 5\text{V} \Rightarrow I_{\text{Load}} = I_{\text{Inductor}} = 20\text{A}$$

2. The **average** input current is less than the output current for  $\phi < 100\%$ . In the linear power supply the output current could never exceed the input current.

## Buck tutorial.

Determine the mean output voltage, current and power for the following operating points. Assume 100% conversion efficiency and continuous mode operation.  $V_{in}=100V$



## Solutions to Buck tutorial questions

1.

$$V_{\text{In}} = 100\text{V}$$

$$V_{\text{Out}} = \Phi \cdot V_{\text{In}} = 65\text{V}$$

$$I_{\text{Out (Ave)}} = 5\text{A}$$

$$P_{\text{Out}} = 325\text{W}$$

2.

$$V_{\text{In}} = 100\text{V}$$

$$V_{\text{Out}} = \Phi \cdot V_{\text{In}} = 72\text{V}$$

$$I_{\text{Out (Ave)}} = 7\text{A}$$

$$P_{\text{Out}} = 504\text{W}$$

3.

$$V_{\text{In}} = 100\text{V}$$

$$V_{\text{Out}} = \Phi \cdot V_{\text{In}} = 20\text{V}$$

$$I_{\text{In(Ave)}} = 2\text{A}$$

$$I_{\text{Out (Ave)}} = 10\text{A}$$

$$P_{\text{Out}} = 200\text{W}$$

4.

$$V_{\text{In}} = 100\text{V}$$

$$V_{\text{Out}} = \Phi \cdot V_{\text{In}} = 40\text{V}$$

$$I_{\text{In(Ave)}} = 16\text{A} \times 40\% = 6.4\text{A}$$

$$I_{\text{Out (Ave)}} = 16\text{A}$$

$$P_{\text{Out}} = 640\text{W}$$

# Analysis and Design: Inductor Current Ripple

For the **inductor  $L$**  in steady state,

$$i_L(t + T_s) = i_L(t)$$

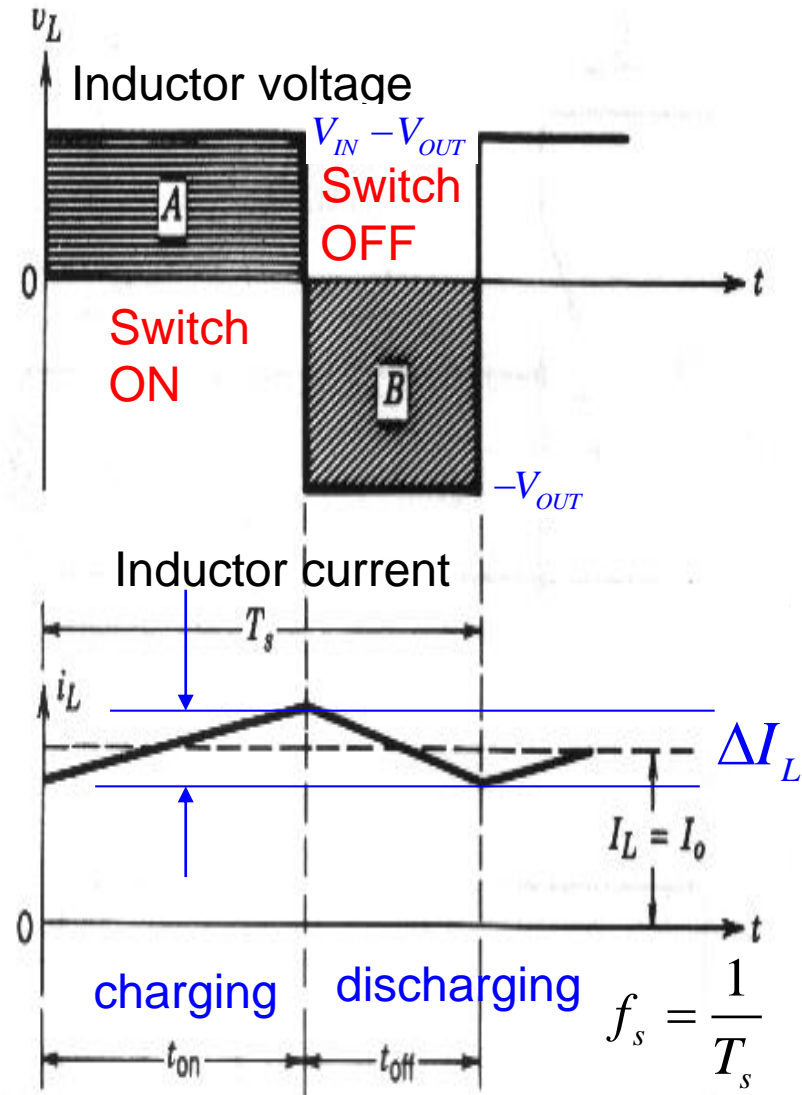
$$\Rightarrow \Delta I_L = |\Delta i_{L+}| = |\Delta i_{L-}| = I_{\max} - I_{\min}$$

Charging :

$$\Delta i_{L+} = \frac{1}{L} \int_0^{\phi T_s} (v_{IN} - v_{OUT}) dt = \frac{v_{IN} - v_{OUT}}{L} \phi T_s$$

Discharging :

$$\Delta i_{L-} = \frac{1}{L} \int_{\phi T_s}^{T_s} (-v_L) dt = \frac{-v_{OUT}}{L} (1 - \phi) T_s$$



## Absolute value

$$\begin{aligned}\because v_{OUT} &= \phi v_{IN} \\ \Delta I_L &= \frac{v_{IN}}{L} \phi (1 - \phi) T_s = \frac{v_{IN}}{f_s L} \phi (1 - \phi) \\ &= \frac{v_{OUT}}{L} (1 - \phi) T_s = \frac{v_{OUT}}{f_s L} (1 - \phi)\end{aligned}$$

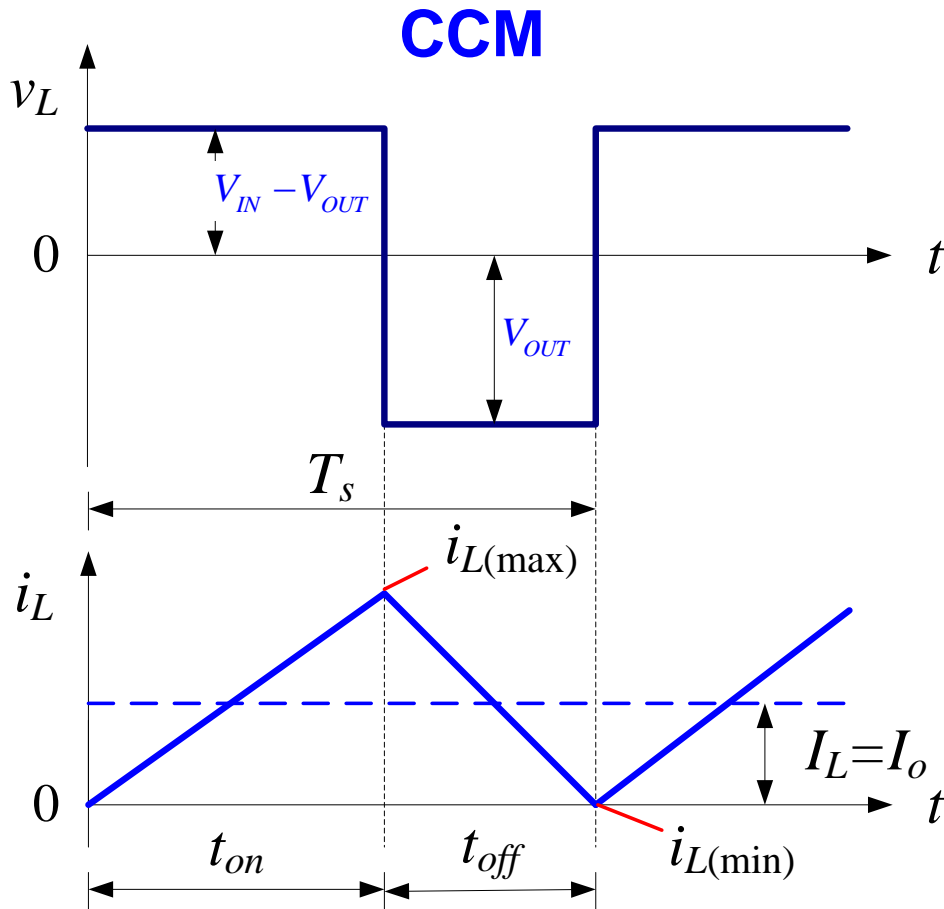
## Relative value

$$\begin{aligned}\because v_{OUT} &= i_R R \quad \text{and} \quad i_R = I_L \\ \Delta I_L &= \frac{v_{OUT}}{f_s L} (1 - \phi) = \frac{i_R R}{f_s L} (1 - \phi) \\ &= \frac{I_L R}{f_s L} (1 - \phi) \Rightarrow \frac{\Delta I_L}{I_L} = \frac{R(1 - \phi)}{f_s L}\end{aligned}$$

To increase switching frequency  $f_s$  and inductor value  $L$ , the inductor current ripple can be reduced;

Small duty ratio  $\phi$  and light load  $R$  will increase the inductor current ripple.

# Boundary Between CCM & DCM



$I_L$ : average inductor current  $> 0$

$I_o$ : average output/load current  $> 0$

When **inductor** current  $i_L$  goes to zero at the end of switch off period, circuit reaches the boundary between **CCM** and **DCM**.

Average inductor current at this boundary  $I_{LB}$  :

$$\begin{aligned} I_{LB} &= \frac{1}{2} \Delta i_L \\ &= \frac{1}{2} (i_{L(max)} - i_{L(min)}) = \frac{1}{2} i_{L(max)} \end{aligned}$$



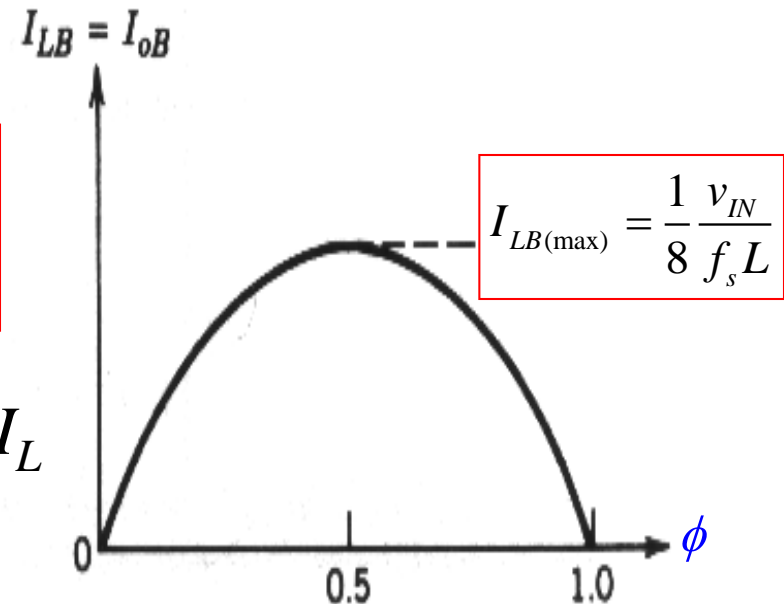
# Boundary Inductor Current

$$\left. \begin{aligned} \Delta I_L &= \frac{v_{IN}}{f_s L} \phi(1-\phi) = \frac{v_{OUT}}{f_s L} (1-\phi) \\ I_{LB} &= \frac{1}{2} i_{L(\max)} = \frac{1}{2} \Delta I_L \end{aligned} \right\} \Rightarrow I_{LB} = I_{OB} = \frac{1}{2} \frac{v_{IN}}{f_s L} \phi(1-\phi)$$



$$\frac{dI_{LB}}{d\phi} = 0 \Rightarrow \phi = 0.5, \quad I_{LB(\max)} = \frac{1}{8} \frac{v_{IN}}{f_s L}$$

For a given  $L$ , if  $I_{LB} < I_{LB, \max}$ , then  $I_L$  will become discontinuous.

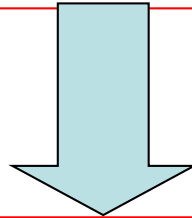


# CCM Inductor Value Selection

$$I_{LB} = I_{OB} = \frac{1}{2} \frac{v_{IN}}{f_s L} \phi (1 - \phi)$$

**CCM**

$$v_{OUT} = \phi v_{IN}$$



$$v_{OUT} = I_o R$$

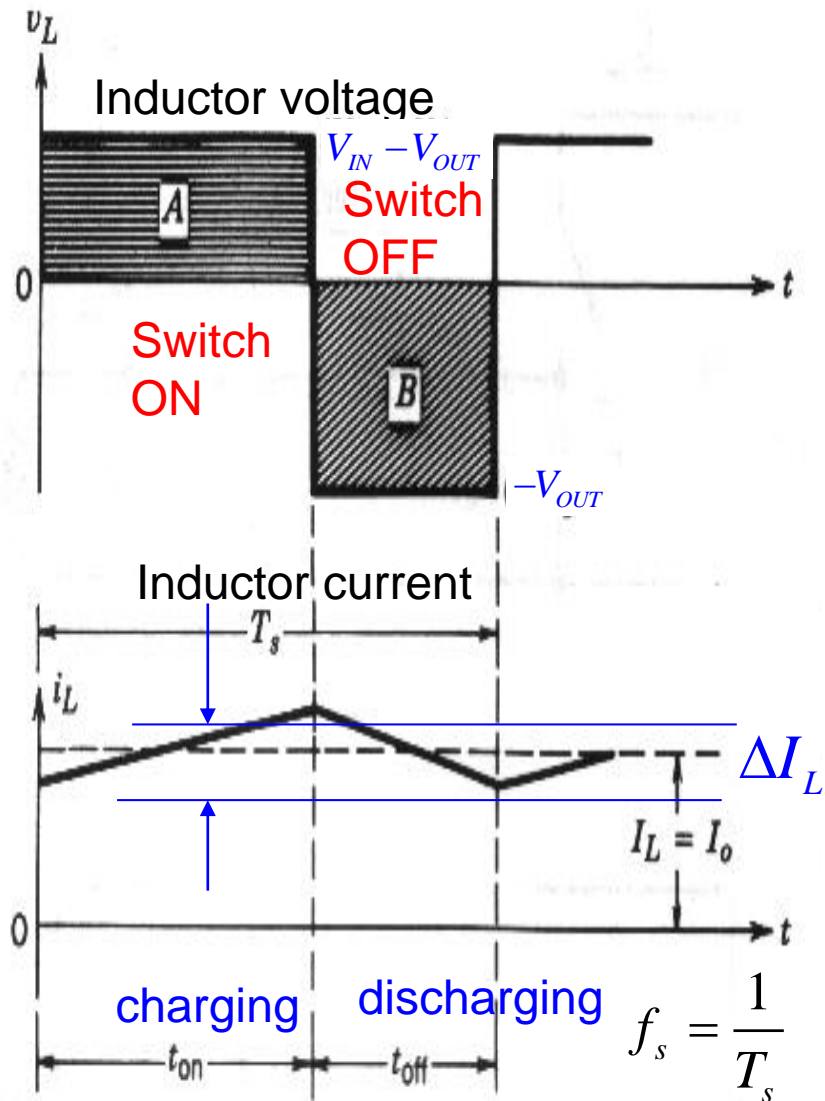
**Resistive  
Load**

$$L_{\min} = \frac{v_{IN}}{2 f_s I_o} \phi (1 - \phi) = \left( \frac{v_{OUT}}{I_o} \right) \frac{1}{2 f_s} (1 - \phi) = \frac{R}{2 f_s} (1 - \phi)$$

For a given load current  $I_o$ , if the inductor value  $L$  is less than  $L_{\min}$ ,  $i_L$  will become discontinuous.

# Analysis and Design:

## Switch Peak Current

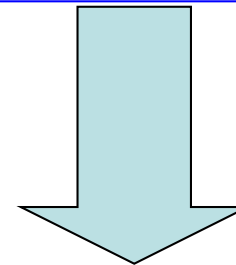


Device rating

$$i_{peak} = i_{L(max)} = I_L + \frac{1}{2} \Delta I_L$$

$$\Delta I_L = \frac{v_{IN}}{f_s L} \phi (1 - \phi) = \frac{v_{OUT}}{f_s L} (1 - \phi)$$

$$I_L = I_o$$

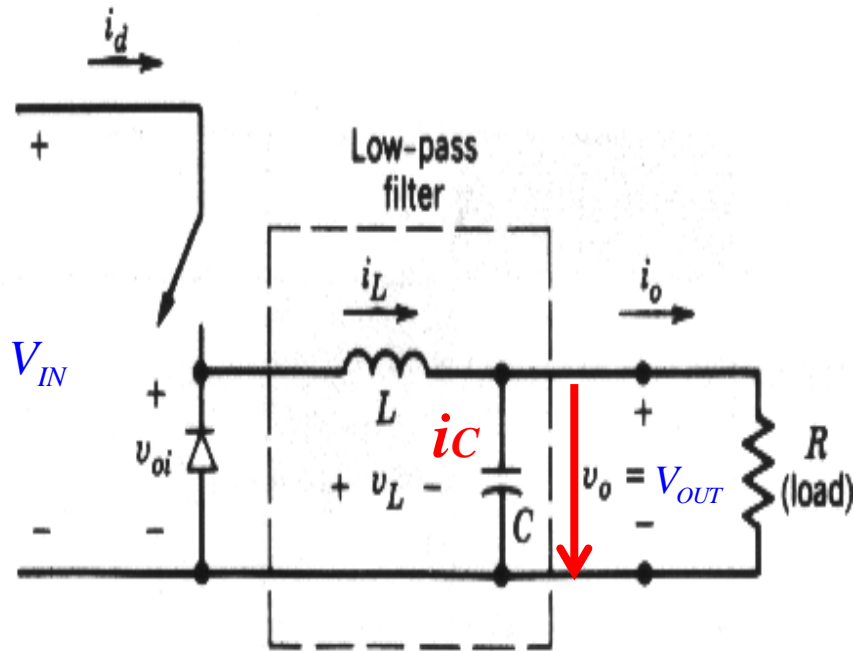


$$i_{L(max)} = \frac{1}{2} \frac{v_{OUT}}{f_s L} \phi (1 - \phi) + I_o$$

Switch Peak Current

# Analysis and Design:

## Capacitor Voltage Ripple



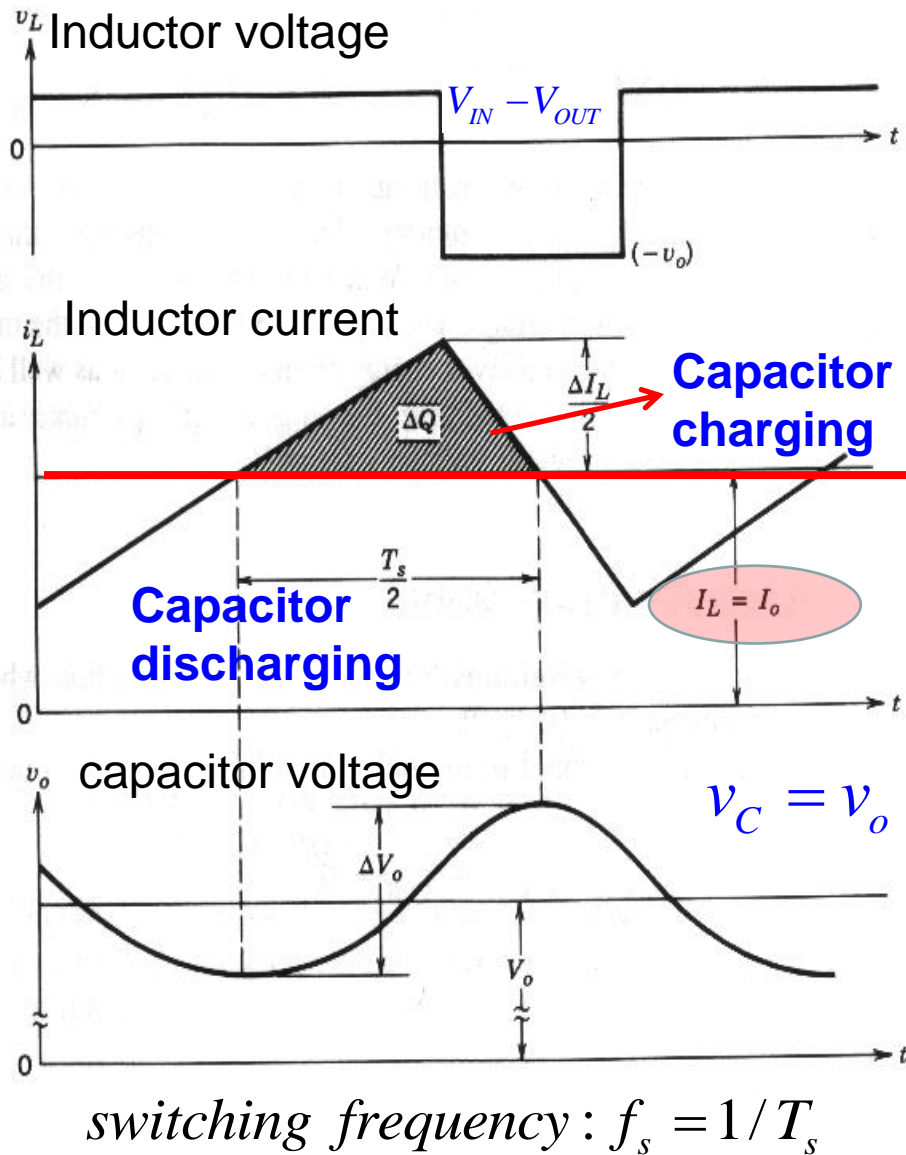
$$i_C = i_L - i_o = i_L - I_o$$

$i_C > 0 \Rightarrow$  capacitor charging

$$\Rightarrow v_o = v_C \quad \square$$

$i_C < 0 \Rightarrow$  capacitor discharging

$$\Rightarrow v_o = v_C \quad \square$$



## Output Voltage Ripples

$$i_c = i_L - I_o = C \frac{dv_c}{dt}$$


$$\Rightarrow \Delta v_c = \frac{1}{C} \left( \int i_c dt \right) = \frac{\Delta Q}{C}$$

$$\Delta v_c = \frac{\Delta Q}{C} = \frac{1}{C} \left[ \frac{1}{2} \left( \frac{\Delta I_L}{2} \right) \left( \frac{T_s}{2} \right) \right]$$

$$= \frac{\Delta I_L}{8} \cdot \frac{1}{f_s C}$$

$$\Rightarrow \Delta v_c \propto \Delta I_L$$

A function of inductor  
current ripple

LC filter cutoff frequency :  $f_c = \frac{1}{2\pi\sqrt{LC}}$   Filter bandwidth

### Absolute value

$$\therefore \Delta I_L = \frac{v_{IN}}{f_s L} \phi (1 - \phi) \quad v_{OUT} = \phi v_{IN}$$

$$\therefore \Delta v_C = \frac{v_{IN}}{8 f_s^2 LC} \phi (1 - \phi)$$

$$= \frac{v_{OUT}}{8 f_s^2 LC} (1 - \phi) = \frac{\pi^2}{2} \left( \frac{f_c}{f_s} \right)^2 (1 - \phi) v_C$$

### Relative value

$$\Delta v_C = \frac{v_C}{8 f_s^2 LC} (1 - \phi)$$

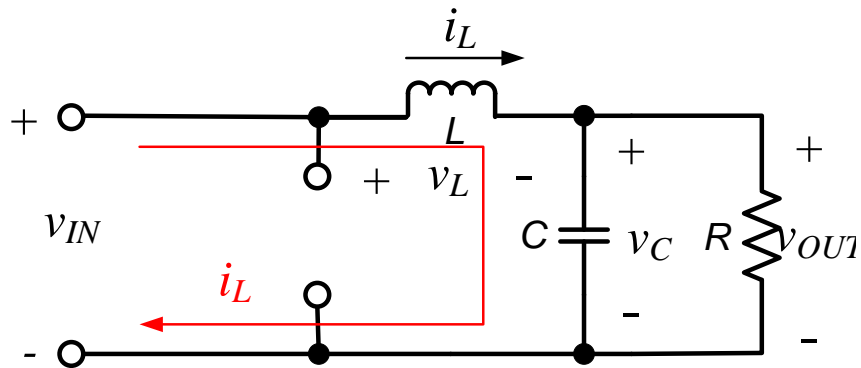
$$\Rightarrow \frac{\Delta v_C}{v_C} = \frac{\pi^2}{2} \left( \frac{f_c}{f_s} \right)^2 (1 - \phi)$$

Obviously, to increase switching frequency  $f_s$  or low LC bandwidth  $f_c$  can significantly reduce the capacitor voltage ripple, while high duty ratio  $\phi$  will reduce voltage ripple too.

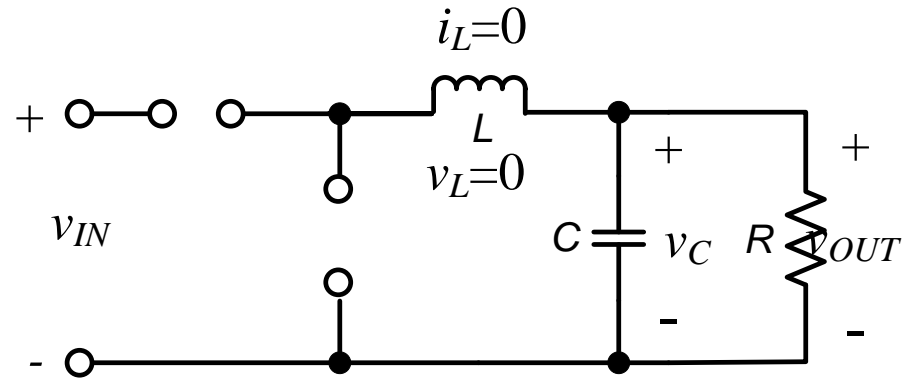
# **Buck Converter**

## **Discontinuous Conduction Mode (DCM)**

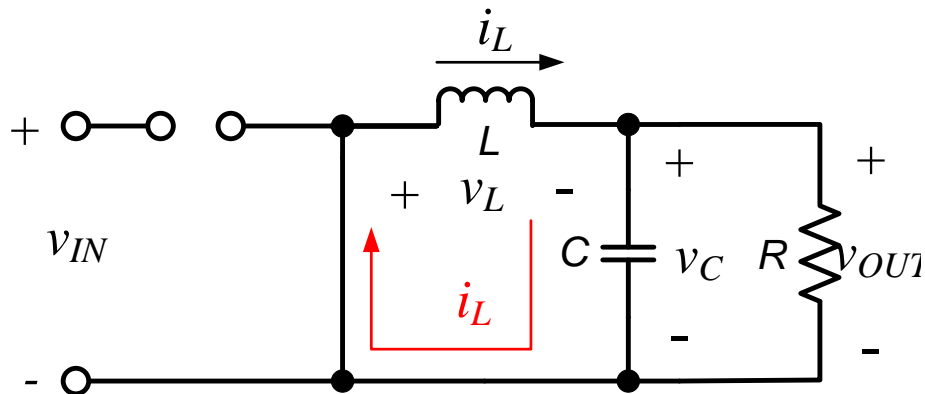
# Circuit States in DCM



**Charging:** Switch ON, Diode OFF



**Idle:** Switch OFF, Diode OFF



**Discharging:** Switch OFF, Diode ON

$$v_L = \begin{cases} v_{IN} - v_{OUT} & \text{charging} \\ -v_{OUT} & \text{discharging} \\ 0 & \text{idle} \end{cases}$$

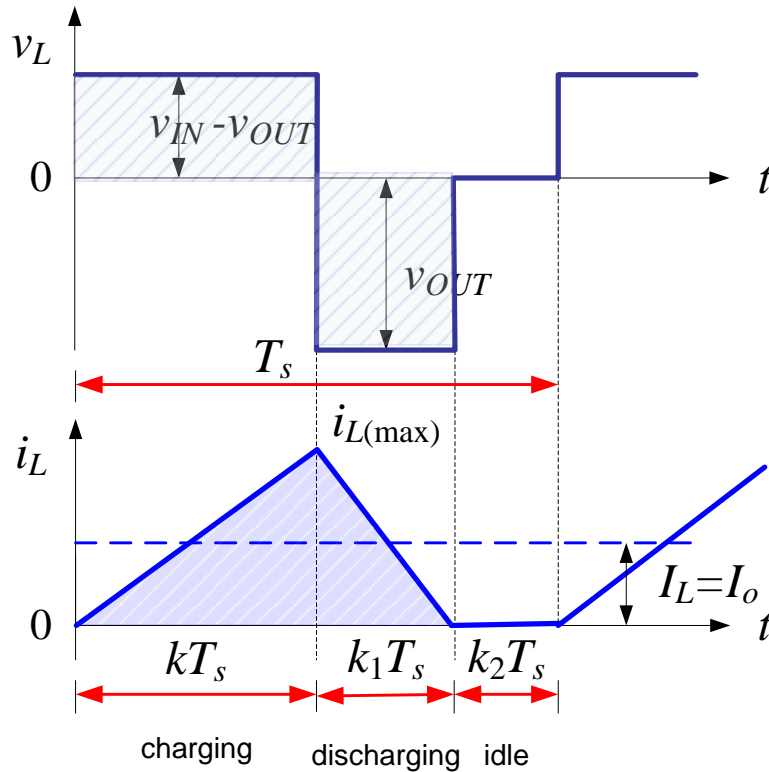
**Steady State**

$$\Rightarrow \int_0^T v_L(t) dt = 0, \quad \int_0^T i_C(t) dt = 0$$



# Voltage Conversion Ratio in DCM

Inductor  $L$  in steady state



$$\int_0^T v_L(t) dt = 0$$

$$\Rightarrow \int_0^{\phi T_s} (v_{IN} - v_{OUT}) dt + \int_{\phi T_s}^{(\phi + \phi_1) T_s} (-v_{OUT}) dt = 0$$

$$\Rightarrow \frac{v_{OUT}}{v_{IN}} = \frac{\phi}{\phi + \phi_1}, \quad 0 \leq \phi + \phi_1 < 1 \quad DCM$$

$$\frac{v_{OUT}}{v_{IN}} = \phi, \quad \phi + \phi_1 = 1 \quad CCM$$

Average output current in DCM :

$$\begin{aligned}
 I_o &= I_L = \frac{1}{T} \int_0^T i_L dt \\
 &= \frac{1}{T} \left( \frac{I_{L(\max)} \phi T_s}{2} + \frac{I_{L(\max)} \phi_1 T_s}{2} \right) \\
 &= \frac{1}{2} I_{L(\max)} (\phi + \phi_1) < \frac{1}{2} I_{L(\max)}
 \end{aligned}$$

Discharging inductor :

$$I_{L(\max)} = \frac{1}{L} \int_{\phi T}^{(\phi + \phi_1) T_s} v_{OUT} dt = \frac{v_{OUT} \phi_1 T_s}{L}$$

$$I_o = \frac{v_o \phi_1 (\phi + \phi_1)}{2 f_s L}$$

Discharging time  $\phi_1 T_s$  depends on the load current  $I_o$

$$I_o = \frac{v_{OUT} \phi_1 (\phi + \phi_1)}{2 f_s L}$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi}{\phi + \phi_1}$$

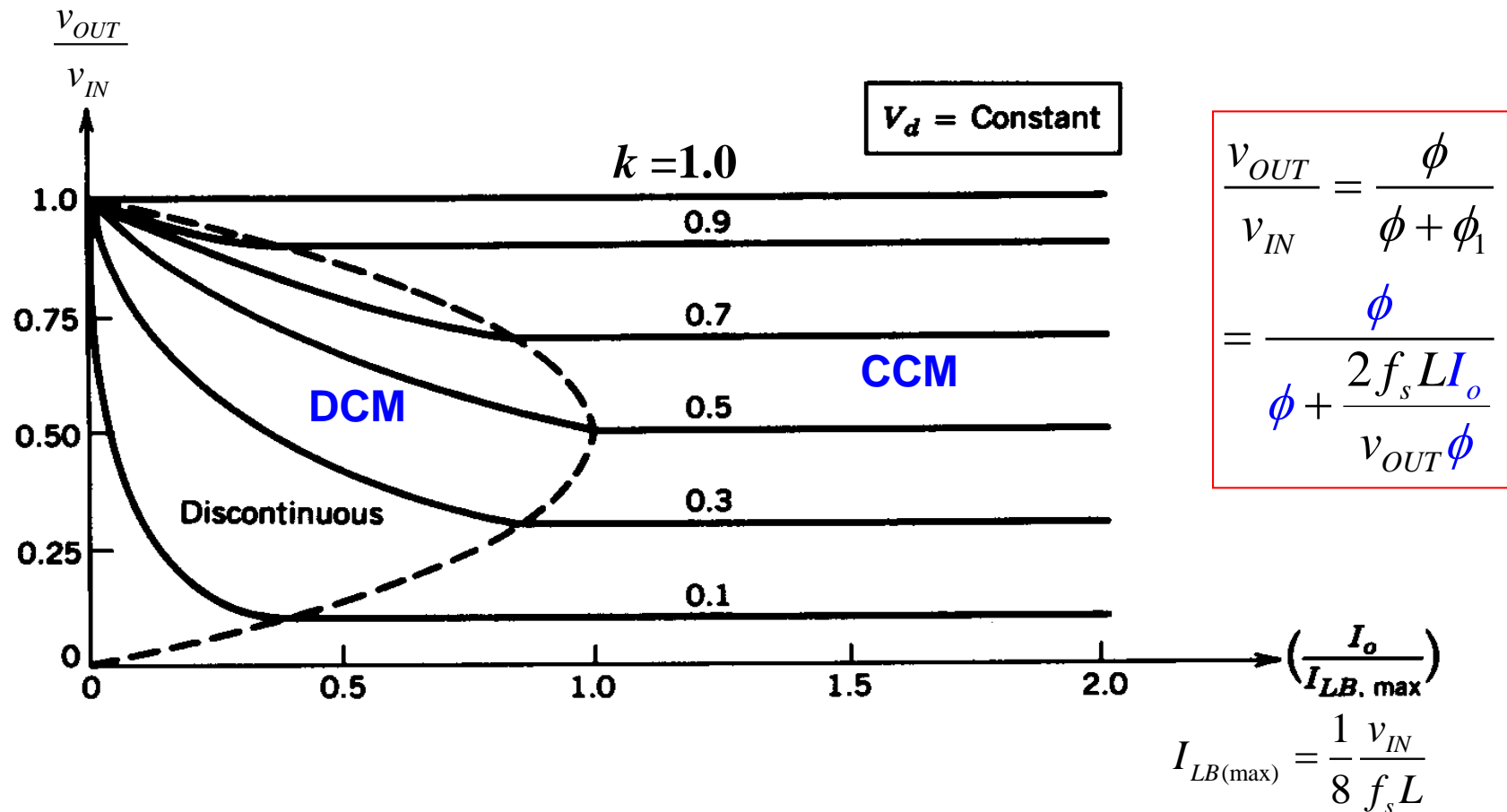
$$I_o = \frac{v_{OUT} \phi_1 (\phi + \phi_1)}{2 f_s L} = \frac{v_{IN} \phi_1 \phi}{2 f_s L}$$

$$\Rightarrow \phi_1 = \frac{2 f_s L I_o}{v_d \phi}$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{\phi}{\phi + \phi_1} = \frac{\phi}{\phi + \frac{2 f_s L I_o}{v_{OUT} \phi}}$$

**Conversion Ratio in DCM**

Voltage Conversion Ratio in CCM is only **dependent on the duty ratio**. It is easy to control the converter in CCM.



The duty-ratio of 0.5 has the highest value of the critical current  
Voltage Conversion Ratio in DCM is **dependent on both the duty ratio and the load**. It is not easy to control the converter in DCM.

## Typical DC to DC Converter applications.

- \* DC Motor drives (e.g. Robotics)
- \* Power Factor Correction (PFC功率因数校正) and **Active Filters**有源滤波器.
- \* **Photovoltaic**光伏 systems e.g. peak power tracking converters to transfer energy from the PV array to the load (satellites).
- \* **Automotive**汽车 applications – raising the battery voltage to another voltage e.g. High Intensity Discharge headlamps.
- \* Solar vehicles

