

Average

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In colloquial language, an **average** is the sum of a list of numbers divided by the number of numbers in the list. In mathematics and statistics, this would be called the *arithmetic mean*. However, the word *average* may also refer to the median, mode, or other central or typical value. In statistics, these are all known as measures of central tendency.

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Calculation

Arithmetic mean

The most common type of average is the arithmetic mean. If n numbers are given, each number denoted by a_i , where $i = 1, \dots, n$, the arithmetic mean is the [sum] of the a_i 's divided by n or

$$AM = \frac{1}{n} \sum_{i=1}^n a_i = \frac{1}{n} (a_1 + a_2 + \cdots + a_n)$$

The arithmetic mean, often simply called the mean, of two numbers, such as 2 and 8, is obtained by finding a value A such that $2 + 8 = A + A$. One may find that $A = (2 + 8)/2 = 5$. Switching the order of 2 and 8 to read 8 and 2 does not change the resulting value obtained for A . The mean 5 is not less than the

minimum 2 nor greater than the maximum 8. If we increase the number of terms in the list to 2, 8, and 11, the arithmetic mean is found by solving for the value of A in the equation $2 + 8 + 11 = A + A + A$. One finds that $A = (2 + 8 + 11)/3 = 7$.

Pythagorean means

Along with the arithmetic mean above, the geometric mean and the harmonic mean are known collectively as the Pythagorean means.

Geometric mean

The geometric mean of n non-negative numbers is obtained by multiplying them all together and then taking the n th root. In algebraic terms, the geometric mean of a_1, a_2, \dots, a_n is defined as

$$GM = \sqrt[n]{\prod_{i=1}^n a_i} = \sqrt[n]{a_1 a_2 \cdots a_n}$$

Geometric mean can be thought of as the antilog of the arithmetic mean of the logs of the numbers.

Example: Geometric mean of 2 and 8 is $GM = \sqrt{2 \cdot 8} = 4$

Harmonic mean

Harmonic mean for a non-empty collection of numbers a_1, a_2, \dots, a_n , all different from 0, is defined as the reciprocal of the arithmetic mean of the reciprocals of the a_i 's:

$$HM = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{a_i}} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}}$$

One example where the harmonic mean is useful is when examining the speed for a number of fixed-distance trips. For example, if the speed for going from point A to B was 60 km/h, and the speed for returning from B to A was 40 km/h, then the harmonic mean speed is given by

$$\frac{2}{\frac{1}{60} + \frac{1}{40}} = 48$$

Inequality concerning AM, GM, and HM

A well known inequality concerning arithmetic, geometric, and harmonic means for any set of positive numbers is

$$AM \geq GM \geq HM$$

It is easy to remember noting that the alphabetical order of the letters A , G , and H is preserved in the inequality. See Inequality of arithmetic and geometric means.

Thus for the above harmonic mean example: $AM = 50$, $GM = 49$, and $HM = 48$ km/h.

Statistical location

In addition to the mean, the mode, the median, and the mid-range are often used in as estimates of central tendency in descriptive statistics.

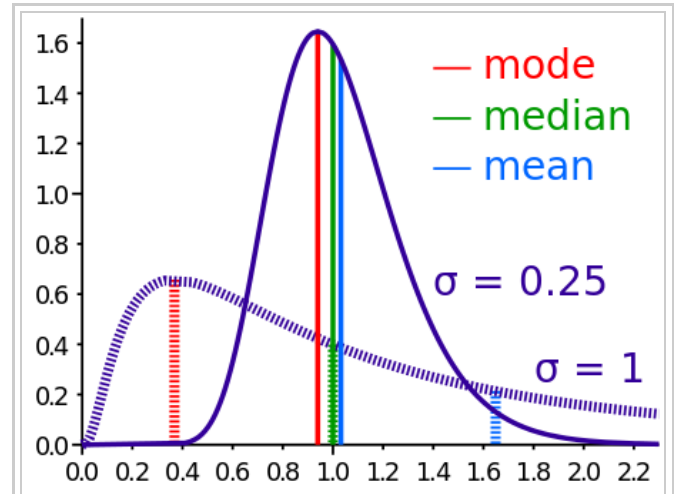
Mode

The most frequently occurring number in a list is called the mode. For example, the mode of the list (1, 2, 2, 3, 3, 3, 4) is 3. It may happen that there are two or more numbers which occur equally often and more often than any other number. In this case there is no agreed definition of mode. Some authors say they are all modes and some say there is no mode.

Median

The median is the middle number of the group when they are ranked in order. (If there are an even number of numbers, the mean of the middle two is taken.)

Thus to find the median, order the list according to its elements' magnitude and then repeatedly remove the pair consisting of the highest and lowest values until either one or two values are left. If exactly one value is left, it is the median; if two values, the median is the arithmetic mean of these two. This method takes the list 1, 7, 3, 13 and orders it to read 1, 3, 7, 13. Then the 1 and 13 are removed to obtain the list 3, 7. Since there are two elements in this remaining list, the median is their arithmetic mean, $(3 + 7)/2 = 5$.



Comparison of arithmetic mean, median and mode of two log-normal distributions with different skewness.

Summary of types

Name	Equation or description
Arithmetic mean	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{n} (x_1 + \cdots + x_n)$
Median	The middle value that separates the higher half from the lower half of the data set
Geometric median	A rotation invariant extension of the median for points in \mathbb{R}^n
Mode	The most frequent value in the data set
Geometric mean	$\left(\prod_{i=1}^n x_i \right)^{\frac{1}{n}} = \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$
Harmonic mean	$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$
Quadratic mean (or RMS)	$\sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} = \sqrt{\frac{1}{n} (x_1^2 + x_2^2 + \cdots + x_n^2)}$
Cubic mean	$\sqrt[3]{\frac{1}{n} \sum_{i=1}^n x_i^3} = \sqrt[3]{\frac{1}{n} (x_1^3 + x_2^3 + \cdots + x_n^3)}$
Generalized mean	$\sqrt[p]{\frac{1}{n} \cdot \sum_{i=1}^n x_i^p}$
Weighted mean	$\frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i} = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w_1 + w_2 + \cdots + w_n}$
Truncated mean	The arithmetic mean of data values after a certain number or proportion of the highest and lowest data values have been discarded
Interquartile mean	A special case of the truncated mean, using the interquartile range
Midrange	$\frac{1}{2} (\max x + \min x)$
Winsorized mean	Similar to the truncated mean, but, rather than deleting the extreme values, they are set equal to the largest and smallest values that remain

The table of mathematical symbols explains the symbols used below.

Miscellaneous types

Other more sophisticated averages are: trimean, trimedian, and normalized mean, with their generalizations.^[1]

One can create one's own average metric using the generalized *f*-mean:

$$y = f^{-1} \left(\frac{1}{n} [f(x_1) + f(x_2) + \cdots + f(x_n)] \right)$$

where f is any invertible function. The harmonic mean is an example of this using $f(x) = 1/x$, and the geometric mean is another, using $f(x) = \log x$.

However, this method for generating means is not general enough to capture all averages. A more general method^[2] for defining an average takes any function $g(x_1, x_2, \dots, x_n)$ of a list of arguments that is continuous, strictly increasing in each argument, and symmetric (invariant under permutation of the arguments). The average y is then the value that, when replacing each member of the list, results in the same function value: $g(y, y, \dots, y) = g(x_1, x_2, \dots, x_n)$. This most general definition still captures the important property of all averages that the average of a list of identical elements is that element itself. The function $g(x_1, x_2, \dots, x_n) = x_1 + x_2 + \dots + x_n$ provides the arithmetic mean. The function $g(x_1, x_2, \dots, x_n) = x_1 x_2 \cdots x_n$ (where the list elements are positive numbers) provides the geometric mean. The function $g(x_1, x_2, \dots, x_n) = -(x_1^{-1} + x_2^{-1} + \dots + x_n^{-1})$ (where the list elements are positive numbers) provides the harmonic mean.^[2]

Average percentage return and CAGR

A type of average used in finance is the average percentage return. It is an example of a geometric mean. When the returns are annual, it is called the Compound Annual Growth Rate (CAGR). For example, if we are considering a period of two years, and the investment return in the first year is -10% and the return in the second year is $+60\%$, then the average percentage return or CAGR, R , can be obtained by solving the equation: $(1 - 10\%) \times (1 + 60\%) = (1 - 0.1) \times (1 + 0.6) = (1 + R) \times (1 + R)$. The value of R that makes this equation true is 0.2 , or 20% . This means that the total return over the 2-year period is the same as if there had been 20% growth each year. Note that the order of the years makes no difference – the average percentage returns of $+60\%$ and -10% is the same result as that for -10% and $+60\%$.

This method can be generalized to examples in which the periods are not equal. For example, consider a period of a half of a year for which the return is -23% and a period of two and a half years for which the return is $+13\%$. The average percentage return for the combined period is the single year return, R , that is the solution of the following equation: $(1 - 0.23)^{0.5} \times (1 + 0.13)^{2.5} = (1 + R)^{0.5+2.5}$, giving an average percentage return R of 0.0600 or 6.00% .

Moving average

Given a time series such as daily stock market prices or yearly temperatures people often want to create a smoother series.^[3] This helps to show underlying trends or perhaps periodic behavior. An easy way to do this is to choose a number n and create a new series by taking the arithmetic mean of the first n values, then moving forward one place and so on. This is the simplest form of moving average. More complicated forms involve using a weighted average. The weighting can be used to enhance or suppress various periodic behavior and there is very extensive analysis of what weightings to use in the literature on filtering. In digital signal processing the term “moving average” is used even when the sum of the weights is not 1.0 (so the output series is a scaled version of the averages).^[4] The reason for this is that the analyst is usually interested only in the trend or the periodic behavior. A further generalization is an “autoregressive moving average”. In this case the average also includes some of the recently calculated outputs. This allows samples from further back in the history to affect the current output.

Etymology

"Few words have received more etymological investigation." ^[5] In the 16th century *average* meant a customs duty, or the like, and was used in the Mediterranean area. It came to mean the cost of damage sustained at sea. From that came an "average adjuster" who decided how to apportion a loss between the owners and insurers of a ship and cargo.

Marine damage is either *particular average*, which is borne only by the owner of the damaged property, or general average, where the owner can claim a proportional contribution from all the parties to the marine venture. The type of calculations used in adjusting general average gave rise to the use of "average" to mean "arithmetic mean".

The root is found in Arabic as *awar*, in Italian as *avaria*, in French as *avarie* and in Dutch as *averij*. It is unclear in which language the word first appeared.

There is earlier (from at least the 11th century), unrelated use of the word. It appears to be an old legal term for a tenant's day labour obligation to a sheriff, probably anglicised from "avera" found in the English Domesday Book (1085).

See also

- Law of averages
- Expected value

References

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- ↑ Box, George E.P.; Jenkins, Gwilym M. (1976). *Time Series Analysis: Forecasting and Control* (revised edition ed.). Holden-Day. ISBN 0816211043.
- ↑ Haykin, Simon (1986). *Adaptive Filter Theory*. Prentice-Hall. ISBN 0130040525.
- ↑ Oxford English Dictionary

External links

- Median as a weighted arithmetic mean of all Sample Observations

(<http://economicsbulletin.vanderbilt.edu/2004/volume3/EB-04C10011A.pdf>)

- Calculations and comparison between arithmetic and geometric mean of two values (<http://www.sengpielaudio.com/calculator-geommean.htm>)

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