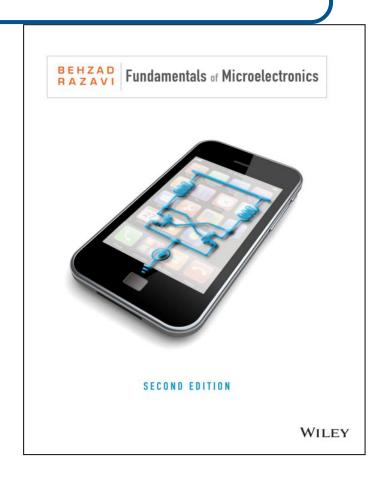
Frequency Response

- Fundamental Concepts
- High-Frequency Models of Transistors
- Analysis Procedure
- Frequency Response of CS Stages
- Frequency Response of CG Stages
- Frequency Response of Followers
- Frequency Response of Cascode Stage
- Frequency Response of Differential Pairs
- Additional Examples



Ref books:

Behzad Razavi, "Fundamentals of Microelectronics, 2nd Edition", Chapter 11 Behzad Razavi, "Design of Analog CMOS Integrated Circuits, 2nd Edition", Chapter 6

Outline

Fundamental Concepts

- Bode's Rules
- Association of Poles with Nodes
- Miller's Theorem



High-Frequency Models of Transistors

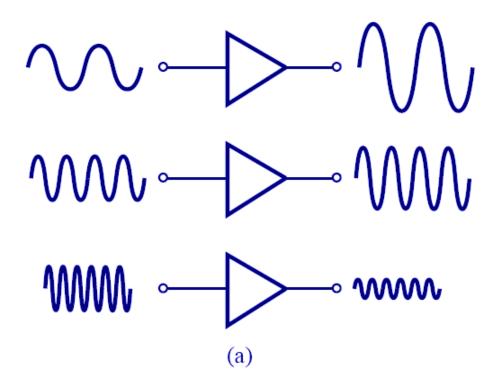
- MOS Model
- Transit Frequency

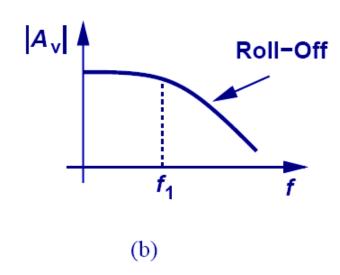


Frequency Response of Circuits

- CS Stages
- CG Stages
- Followers
- Cascode Stage
- Differential Pair

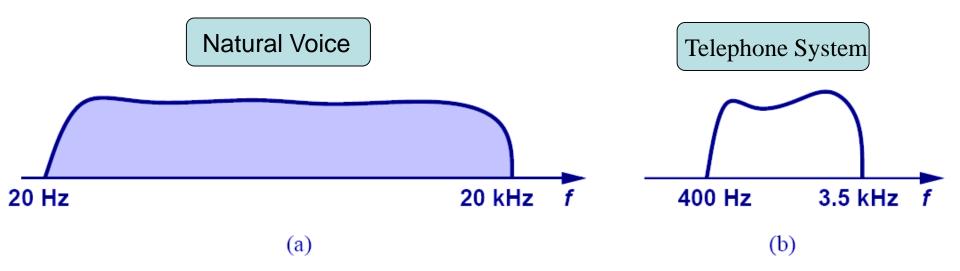
High Frequency Roll-off of Amplifier





> As frequency of operation increases, the gain of amplifier decreases. This chapter analyzes this problem.

Example: Human Voice I



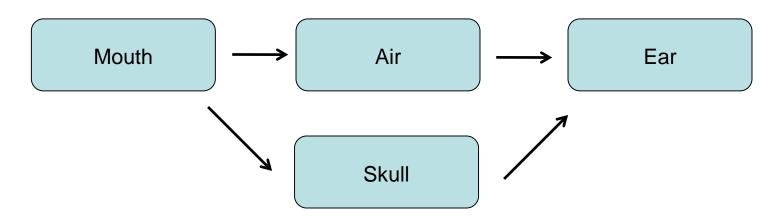
Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.

Example: Human Voice II

Path traveled by the human voice to the voice recorder

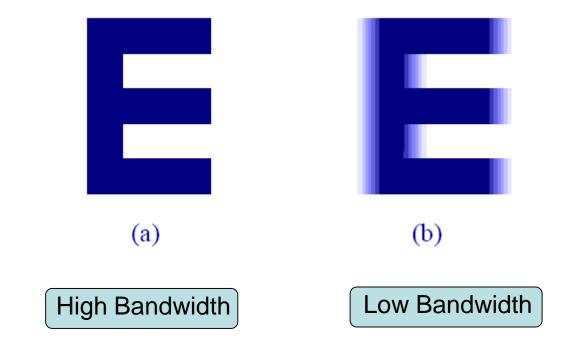


Path traveled by the human voice to the human ear



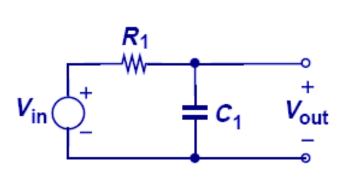
> Since the paths are different, the results will also be different.

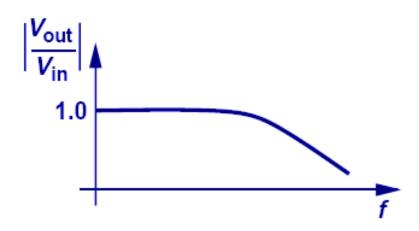
Example: Video Signal



Video signals without sufficient bandwidth become fuzzy as they fail to abruptly change the contrast of pictures from complete white into complete black.

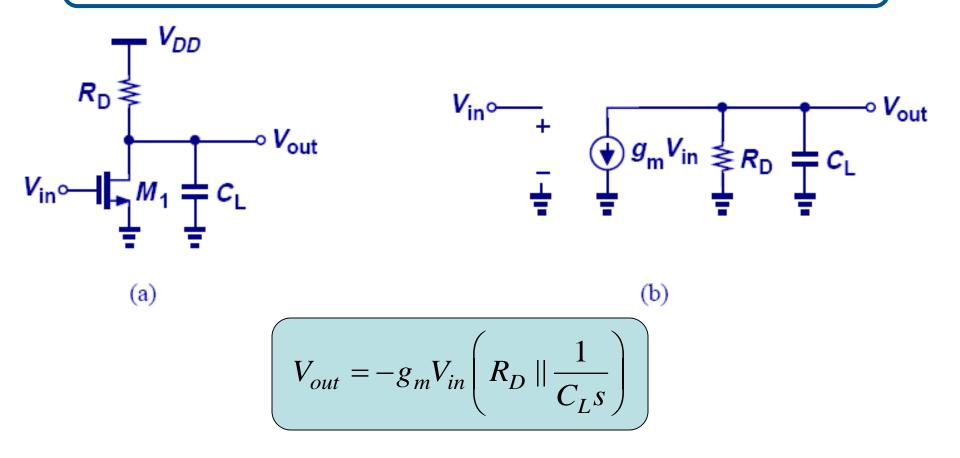
Gain Roll-off: Simple Low-pass Filter





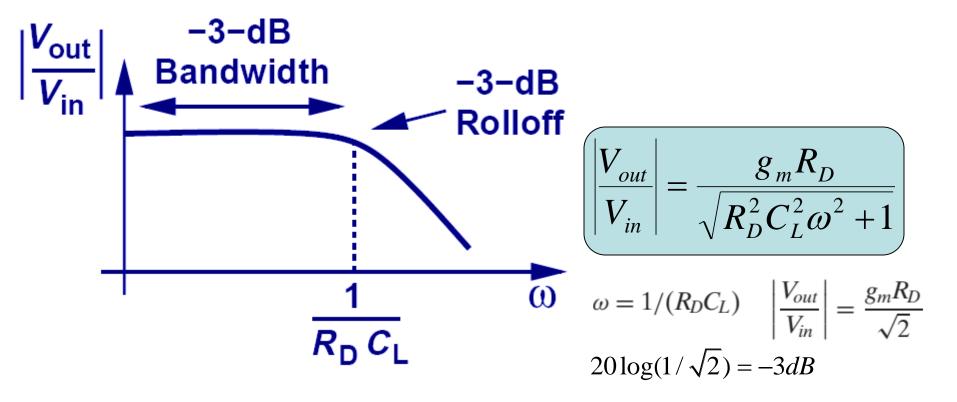
➤ In this simple example, as frequency increases the impedance of C₁ decreases and the voltage divider consists of C₁ and R₁ attenuates Vᵢn to a greater extent at the output.

Gain Roll-off: Common Source



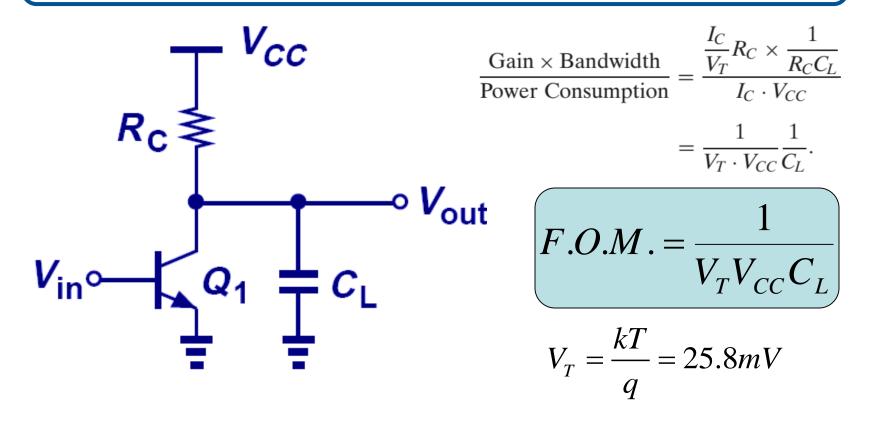
➤ The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will "steal" away some signal current and shunt it to ground.

Frequency Response of the CS Stage



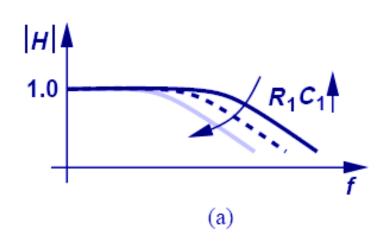
At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease. A special frequency is $ω=1/(R_DC_L)$, where the gain drops by 3dB.

Example: Figure of Merit



This metric quantifies a circuit's gain, bandwidth, and power dissipation. In the bipolar case, low temperature, supply, and load capacitance mark a superior figure of merit.

Example: Relationship between Frequency Response and Step Response



$$V_{\text{out}}$$
 t
(b)

$$\left|H\left(s=j\omega\right)\right| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0 \left(1 - \exp \frac{-t}{R_1 C_1} \right) u(t)$$

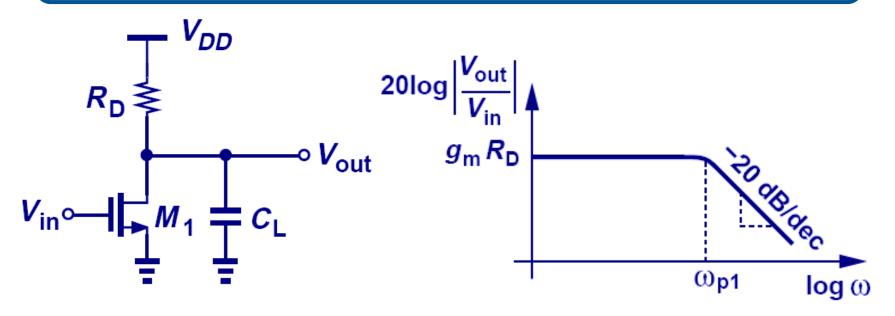
 \triangleright The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower*.

Bode Plot

$$H(s) = A_0 \frac{\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)\cdots}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\cdots}$$

- \triangleright When we hit a zero, $ω_{zj}$, the Bode magnitude rises with a slope of +20dB/dec.
- > When we hit a pole, ω_{pj} , the Bode magnitude falls with a slope of -20dB/dec

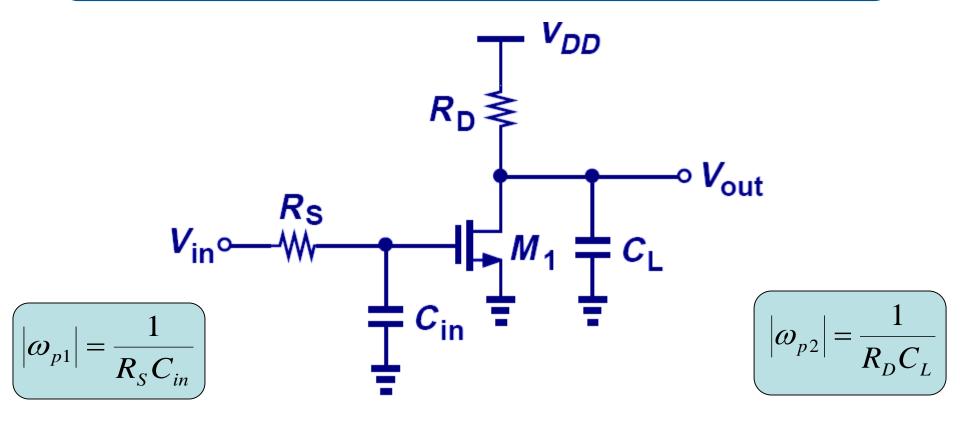
Example: Bode Plot



$$\left|\omega_{p1}\right| = \frac{1}{R_D C_L}$$

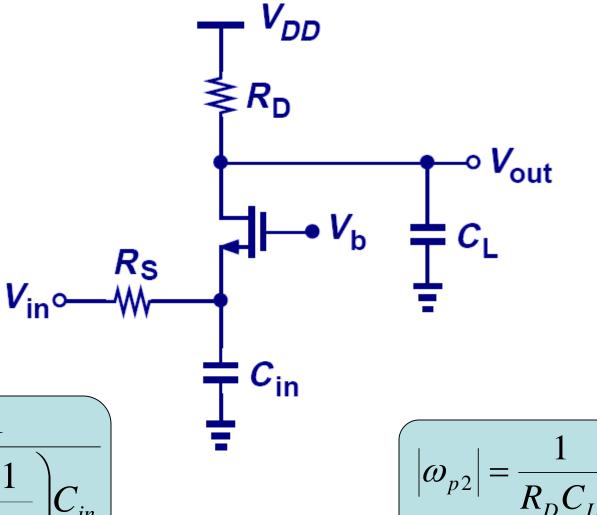
The circuit only has one pole (no zero) at $1/(R_DC_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1} .

Pole Identification Example I



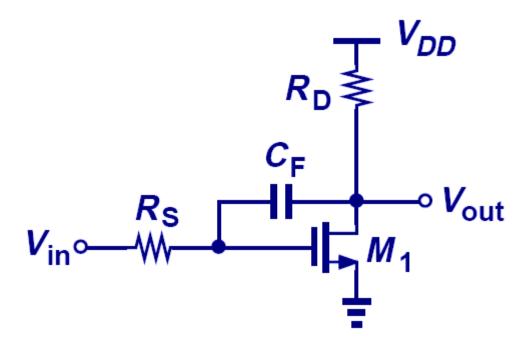
$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{\left(1 + \omega^2 / \omega_{p1}^2\right) \left(1 + \omega^2 / \omega_{p2}^2\right)}}$$

Pole Identification Example II



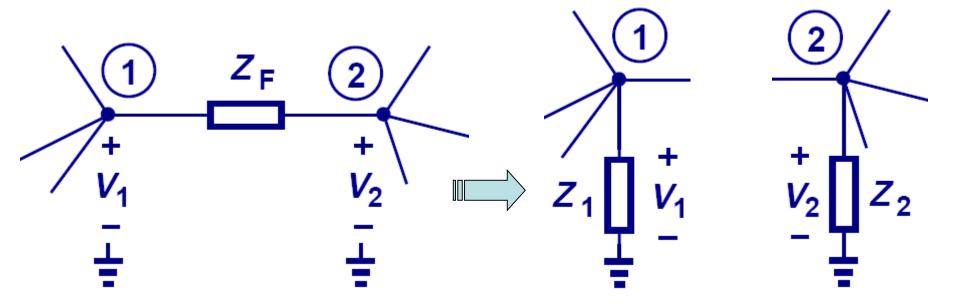
$$\left| |\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m} \right) C_{in}} \right|$$

Circuit with Floating Capacitor



- ➤ The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.

Miller's Theorem

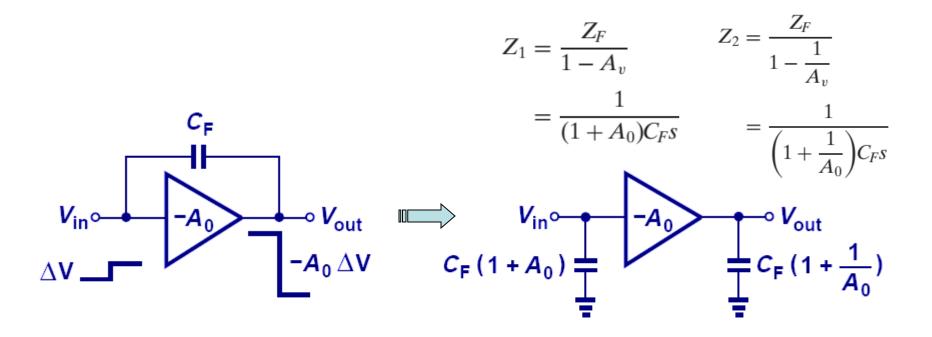


$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

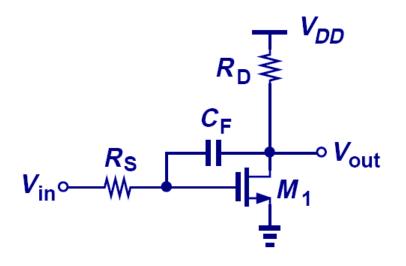
If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2 .

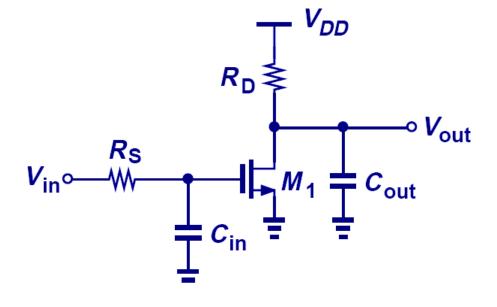
Miller Multiplication



➤ With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Example: Miller Theorem

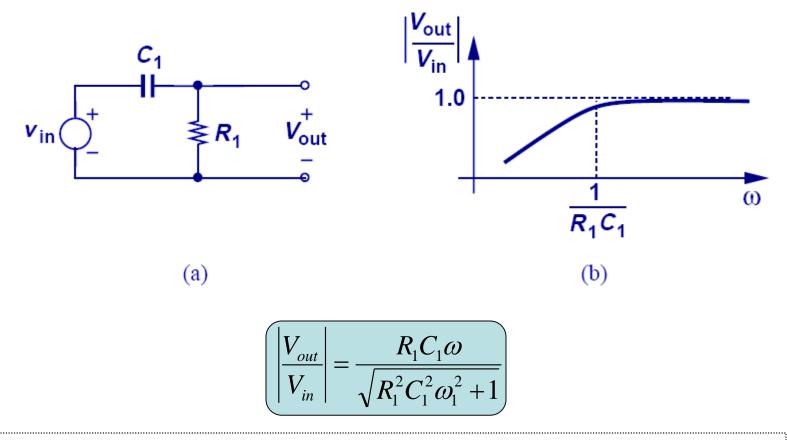




$$\omega_{in} = \frac{1}{R_S (1 + g_m R_D) C_F}$$

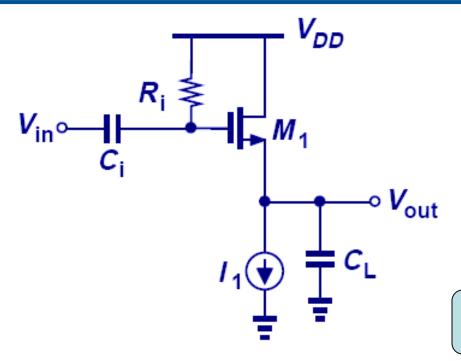
$$\omega_{out} = \frac{1}{R_D \left(1 + \frac{1}{g_m R_D}\right) C_F}$$

High-Pass Filter Response



➤ The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.

Example: Audio Amplifier



$$C_i = 79.6nF$$

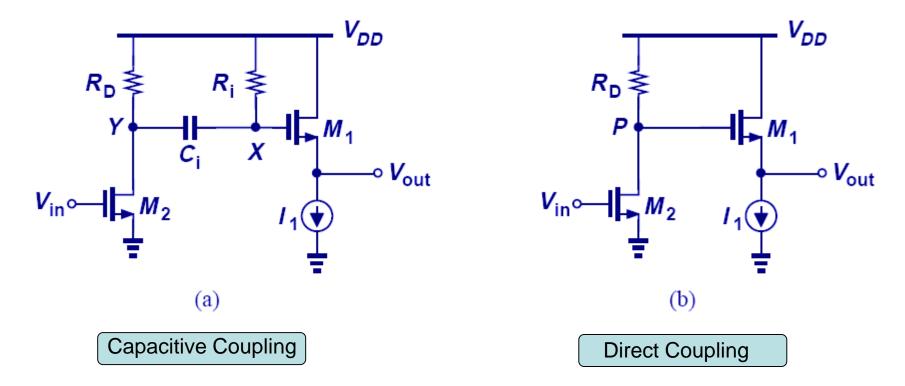
$$C_L = 39.8nF$$

$$R_i = 100K\Omega$$

$$g_m = 1/200\Omega$$

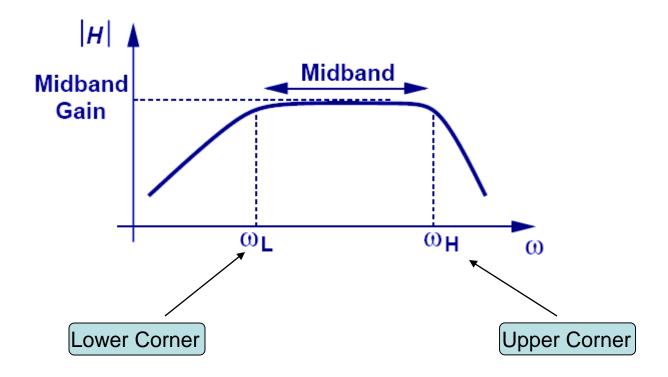
➤ In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and output capacitances are needed.

Capacitive Coupling vs. Direct Coupling

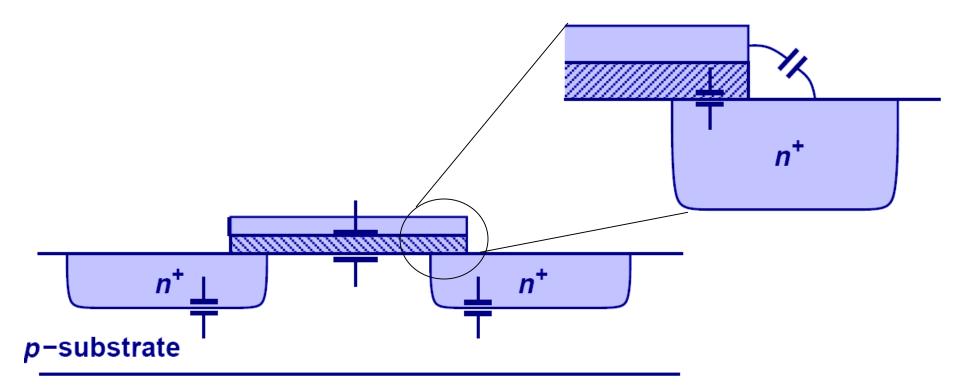


- ➤ Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.
- This technique allows independent bias conditions between stages. Direct coupling does not.

Typical Frequency Response

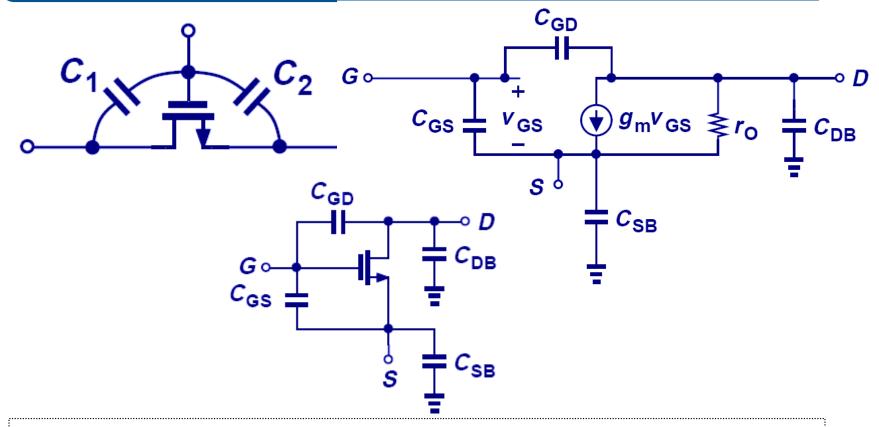


MOS Intrinsic Capacitances



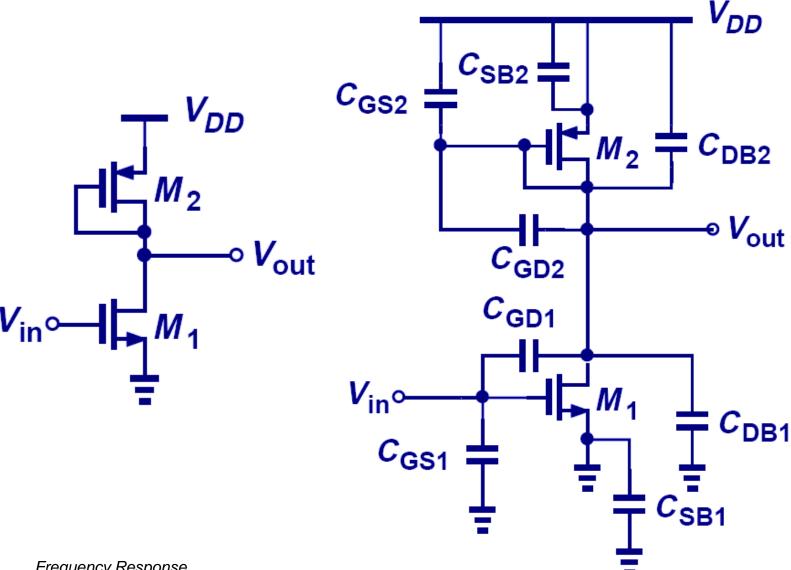
For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model

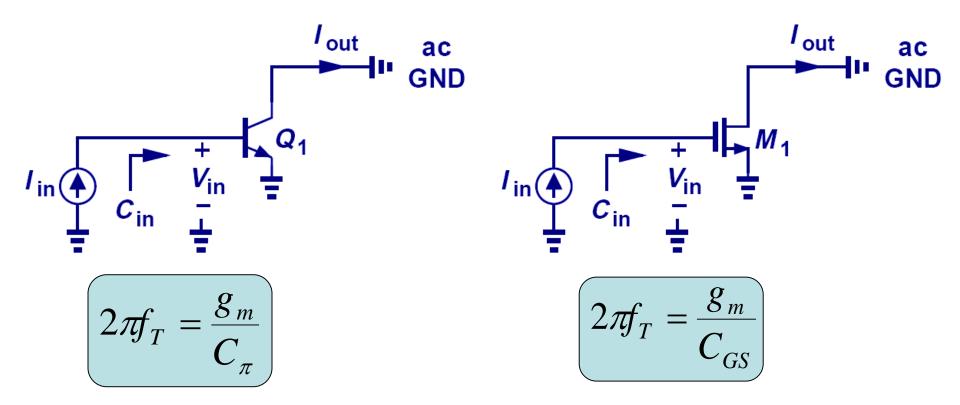


The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_2 \sim C_{gate}$, and $C_1 \sim 0$. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD} .

Example: Capacitance Identification

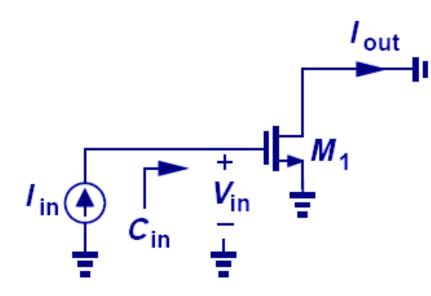


Transit Frequency



➤ Transit frequency, f_T, is defined as the frequency where the current gain from input to output drops to 1.

Example: Transit Frequency Calculation



$$L = 65nm$$

$$V_{GS} - V_{TH} = 100mV$$

$$\mu_n = 400cm^2 / (V.s)$$

$$f_T = 226GHz$$

It can be shown that $C_{GS} \approx (2/3)WLC_{ox}$ for a MOSFET operating in saturation.

prove that:

$$2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$$

Note that f increases with the overdrive voltage and the $\frac{1}{I^2}$

Example: Transit Frequency Calculation

The minimum channel length of MOSFETs has been scaled from 1 μ m in the late 1980s to 65 nm today. Also, the inevitable reduction of the supply voltage has reduced the gate-source overdive voltage from about 400 mV to 100 mV. By what factor has the f_T of MOSFETs increased?

Solution:

It can be proved that
$$2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$$

Thus, the transit frequency has increased by approximately a factor of 59. For example, if $\mu_{T} = 400 \text{cm/(V \cdot s)}$, then 65 nm devices having an overdrive of 100 mV exhibit an f_{T} of 226 GHz.

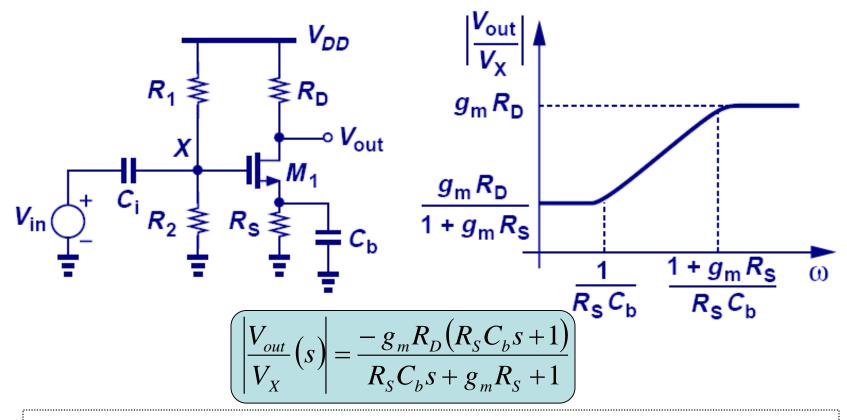
Analysis Summary

- The frequency response refers to the magnitude of the transfer function.
- ➤ Bode's approximation simplifies the plotting of the frequency response if poles and zeros are known.
- In general, it is possible to associate a pole with each node in the signal path.
- Miller's theorem helps to decompose floating capacitors into grounded elements.
- ➤ Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.

High Frequency Circuit Analysis Procedure

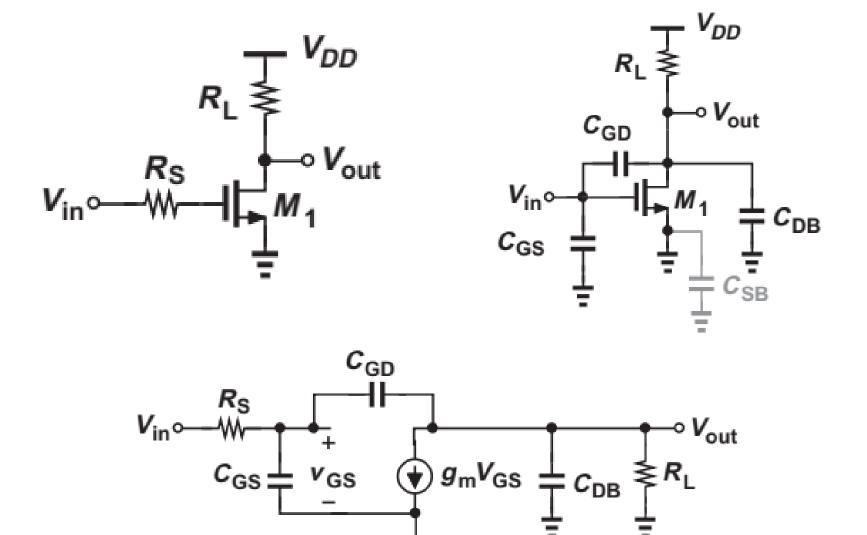
- Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).
- Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).
- Include transistor capacitances.
- Merge capacitors connected to AC grounds and omit those that play no role in the circuit.
- Determine the high-frequency poles and zeros.
- Plot the frequency response using Bode's rules or exact analysis.

Frequency Response of CS Stage with Bypassed Degeneration

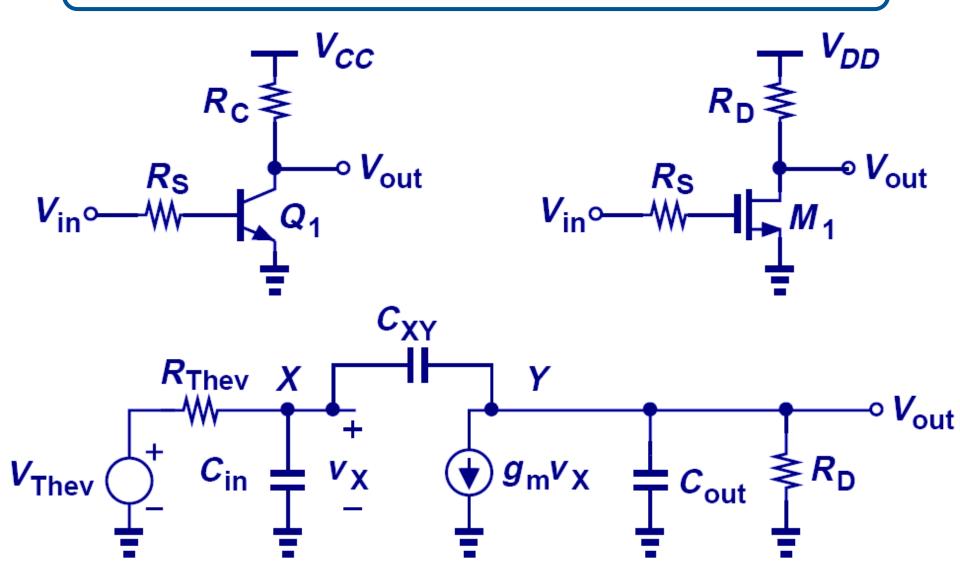


- ▶ In order to increase the midband gain, a capacitor C_b is placed in parallel with R_s.
- The pole frequency must be well below the lowest signal frequency to avoid the effect of degeneration.

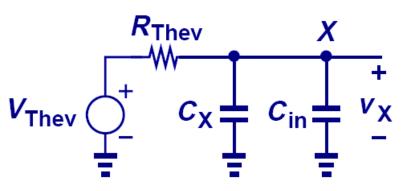
Small signal equivalents of CS stage

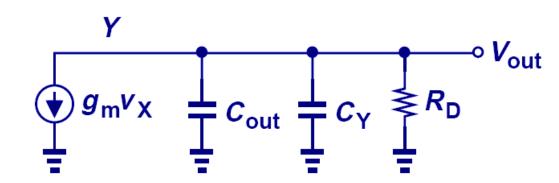


Unified Model for CE and CS Stages



Unified Model Using Miller's Theorem





CS Stage

$$V_{\text{Thev}} = V_{\text{in}}$$

$$R_{\text{Thev}} = R_{\text{S}}$$

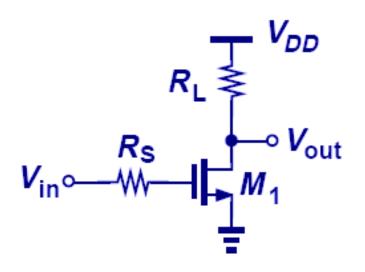
$$c_{\rm X} = c_{\rm GD} \left(1 + g_{\rm m} R_{\rm D}\right)$$

$$C_{\rm Y} = C_{\rm GD} \, (1 + \frac{1}{g_{\rm m} R_{\rm D}})$$

$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + (1 + g_m R_L)C_{XY}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L \left[C_{out} + \left(1 + \frac{1}{g_m R_L} \right) C_{XY} \right]}$$

Example: Half Width CS Stage



$$W \downarrow 2X$$

All capacitors are halved

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}.$$

$$\left|\omega_{p,in}\right| = \frac{1}{R_{S}\left[\frac{C_{in}}{2} + \left(1 + \frac{g_{m}R_{L}}{2}\right)\frac{C_{XY}}{2}\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_{L}\left[\frac{C_{out}}{2} + \left(1 + \frac{2}{g_{m}R_{L}}\right)\frac{C_{XY}}{2}\right]}$$

Direct Analysis of CS Stages

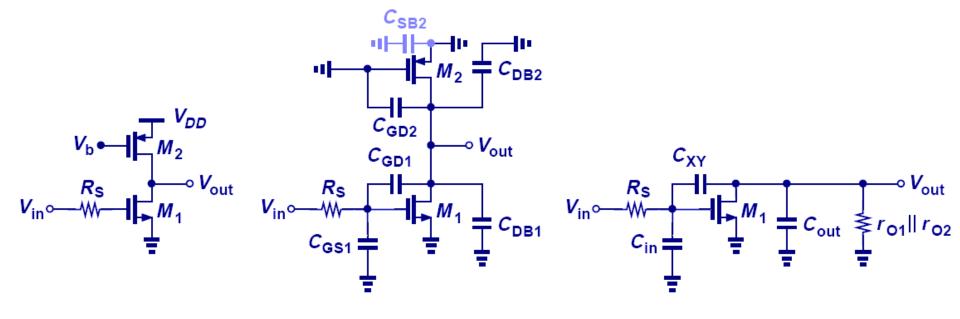
$$|\omega_{z}| = \frac{g_{m}}{C_{XY}}$$

$$|\omega_{p1}| = \frac{1}{(1+g_{m}R_{L})C_{XY}R_{Thev} + R_{Thev}C_{in} + R_{L}(C_{XY} + C_{out})}$$

$$|\omega_{p2}| = \frac{(1+g_{m}R_{L})C_{XY}R_{Thev} + R_{Thev}C_{in} + R_{L}(C_{XY} + C_{out})}{R_{Thev}R_{L}(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

Direct analysis yields different pole locations and an extra zero.

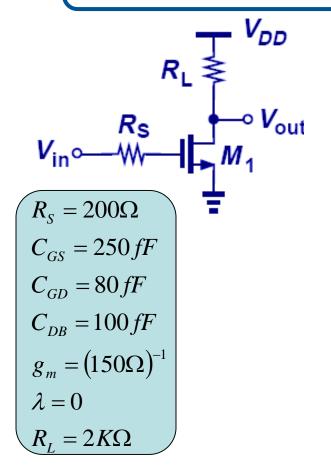
Example: CS Direct Analysis

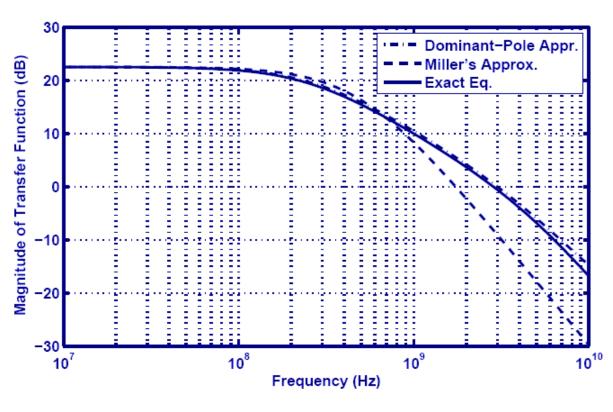


$$\omega_{p1} \approx \frac{1}{\left[1 + g_{m1}(r_{O1} \parallel r_{O2})\right]C_{XY}R_S + R_SC_{in} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{out})}$$

$$\omega_{p2} \approx \frac{\left[1 + g_{m1}(r_{O1} \parallel r_{O2})\right]C_{XY}R_S + R_SC_{in} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{out})}{R_S(r_{O1} \parallel r_{O2})(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

Example: Comparison Between Different Methods





Miller's

$$\begin{vmatrix} |\omega_{p,in}| = 2\pi \times (571MHz) \\ |\omega_{p,out}| = 2\pi \times (428MHz) \end{vmatrix}$$

Frequency Response

Exact

$$\left|\omega_{p,in}\right| = 2\pi \times (264MHz)$$

$$\left|\omega_{p,out}\right| = 2\pi \times (4.53GHz)$$

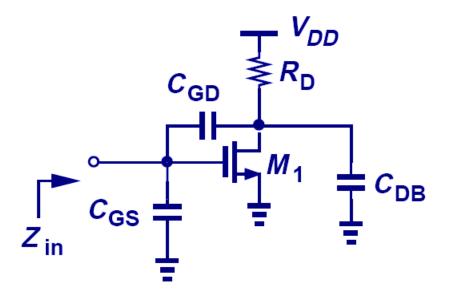
Dominant Pole

$$\left|\omega_{p,in}\right| = 2\pi \times (249MHz)$$

 $\left|\omega_{p,out}\right| = 2\pi \times (4.79GHz)$

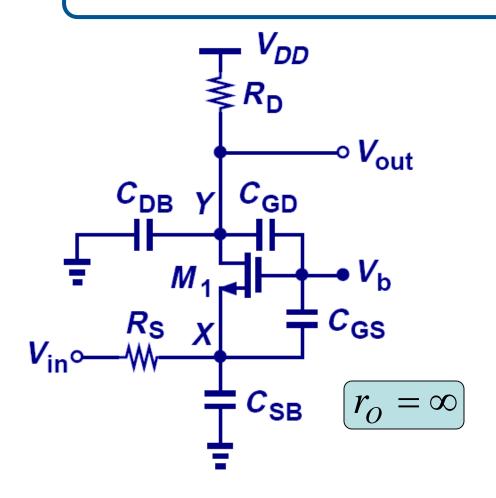
39

Input Impedance of CS Stages



$$Z_{in} \approx \frac{1}{\left[C_{GS} + (1 + g_m R_D)C_{GD}\right]s}$$

Frequency Response of CG Stage



$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right)C_X}$$

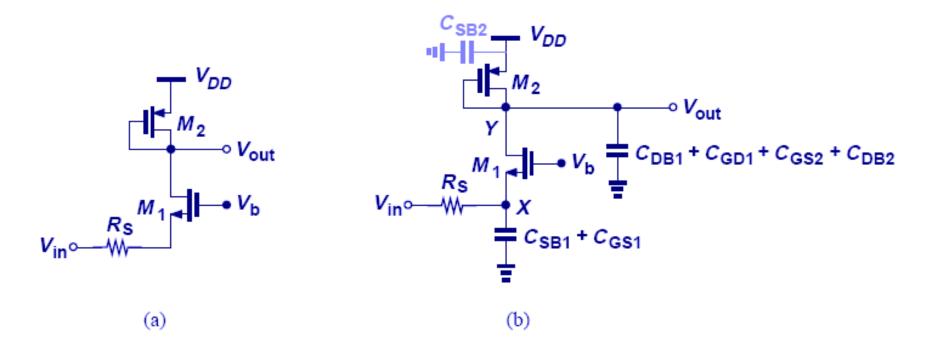
$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_D C_Y}$$

$$C_Y = C_{GD} + C_{DB}$$

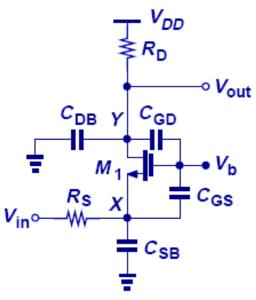
 \triangleright Similar to a CB stage, the input pole is on the order of f_T , so rarely a speed bottleneck.

Example: CG Stage Pole Identification

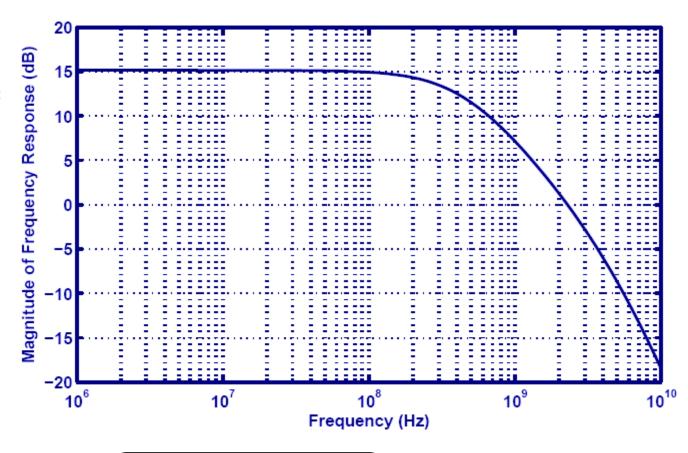


$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_{m1}}\right) \left(C_{SB1} + C_{GS1}\right)} \qquad \omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}} \left(C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2}\right)}$$

Example: Frequency Response of CG Stage

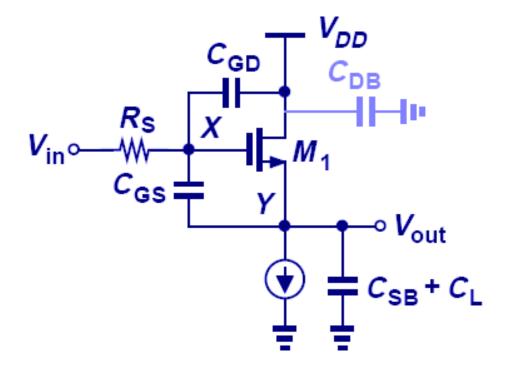


$$egin{aligned} R_S &= 200\Omega \ C_{GS} &= 250 fF \ C_{GD} &= 80 fF \ C_{DB} &= 100 fF \ g_m &= \left(150\Omega\right)^{-1} \ \lambda &= 0 \ R_d &= 2K\Omega \end{aligned}$$



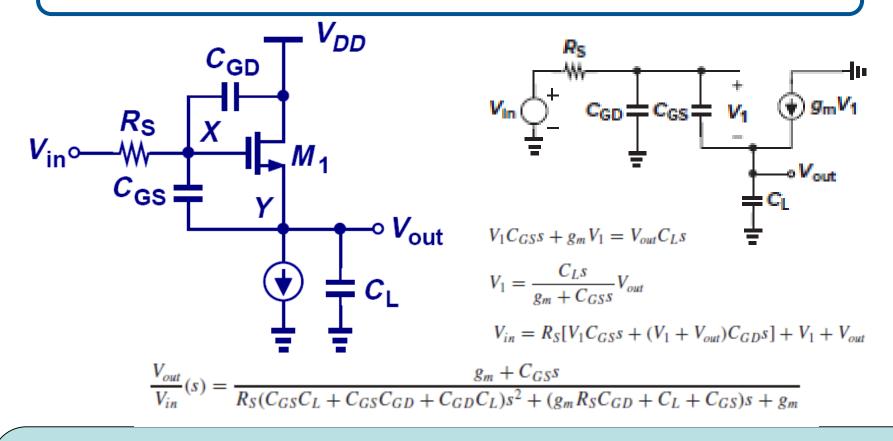
$$\left|\omega_{p,X}\right| = 2\pi \times (5.31GHz)$$
$$\left|\omega_{p,Y}\right| = 2\pi \times (442MHz)$$

Source Followers



> The following will discuss the frequency response of source followers using direct analysis.

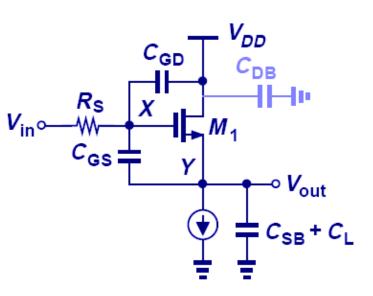
Direct Analysis of Source Follower Stage

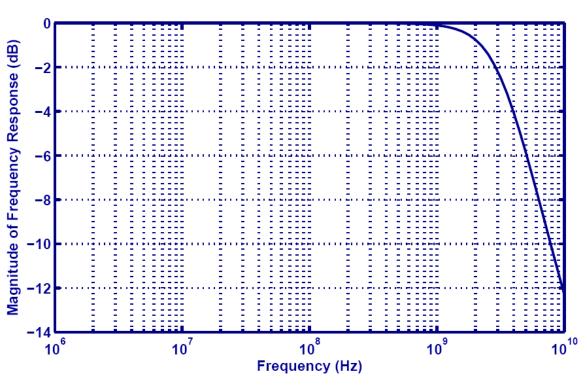


$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m}s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_m} \left(C_{GD} C_{GS} + C_{GD} C_L + C_{GS} C_L \right)$$
$$b = R_S C_{GD} + \frac{C_{GS} + C_L}{g_m}$$

Example: Frequency Response of Source Follower





$$R_{S} = 200\Omega$$

$$C_{L} = 100 fF$$

$$C_{GS} = 250 fF$$

$$C_{GD} = 80 fF$$

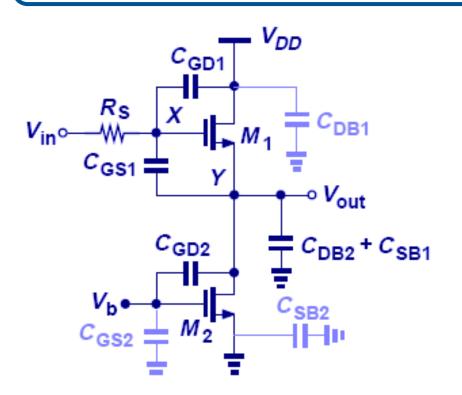
$$C_{DB} = 100 fF$$

$$g_{m} = (150\Omega)^{-1}$$

$$\lambda = 0$$

$$\omega_{p1} = 2\pi [-1.79GHz + j(2.57GHz)]$$
 $\omega_{p2} = 2\pi [-1.79GHz - j(2.57GHz)]$

Example: Source Follower



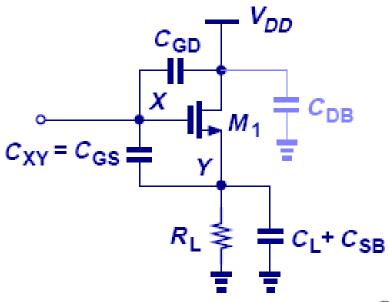
$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m}s}{as^2 + bs + 1}$$

$$C_L = C_{DB2} + C_{SB1} + C_{GD2}$$

$$a = \frac{R_S}{g_{m1}} \left[C_{GD1} C_{GS1} + (C_{GD1} + C_{GS1}) (C_{SB1} + C_{GD2} + C_{DB2}) \right]$$

$$b = R_S C_{GD1} + \frac{C_{GS1} + C_{SB1} + C_{GD2} + C_{DB2}}{g_{m1}}$$

Input Capacitance of Source Follower

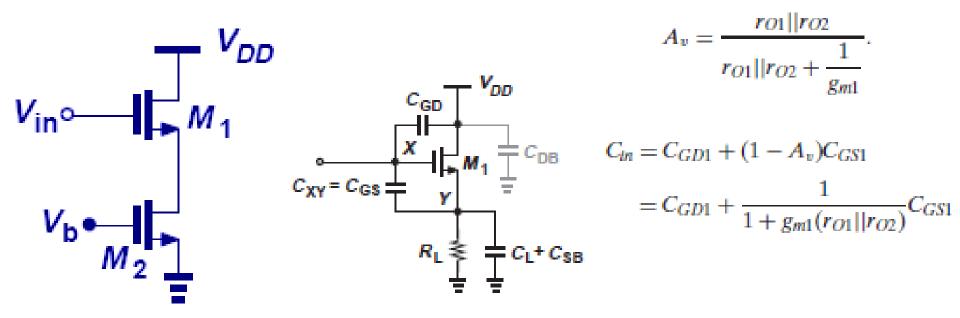


$$A_{v} = \frac{R_{L}}{R_{L} + \frac{1}{g_{m}}}$$

$$C_X = (1 - A_v)C_{XY}$$
$$= \frac{1}{1 + g_m R_L} C_{XY}$$

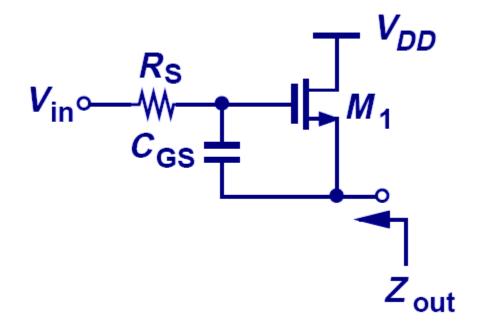
$$C_{in} = C_{GD} + \frac{C_{GS}}{1 + g_m R_L}$$

Example: Source Follower Input Capacitance



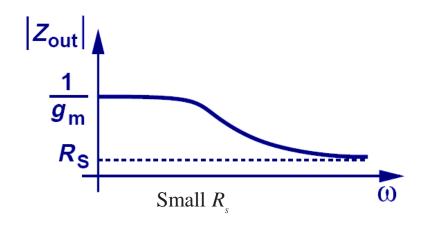
$$C_{in} = C_{GD1} + \frac{1}{1 + g_{m1}(r_{O1} \parallel r_{O2})} C_{GS1}$$

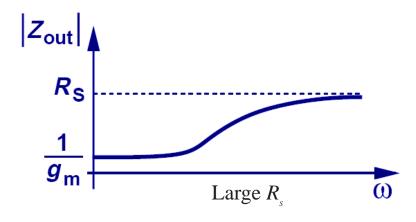
Output Impedance of Source Follower



$$\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

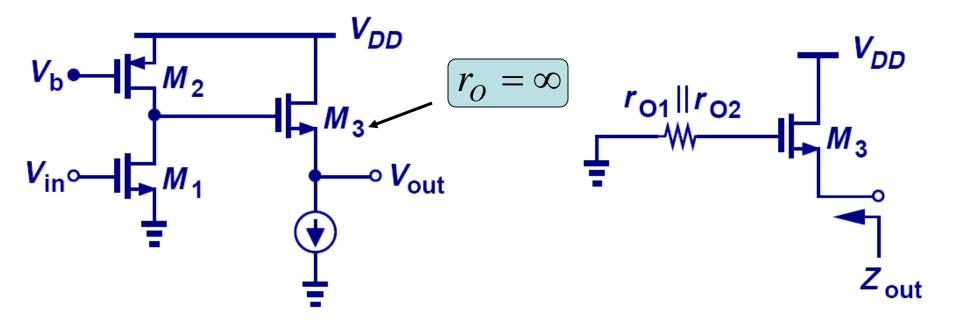
Active Inductor





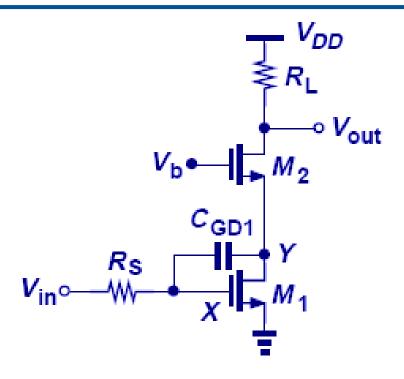
➤ The plot above shows the output impedance of source followers. Since a follower's primary duty is to lower the driving impedance (R_s>1/g_m), the "active inductor" characteristic on the right is usually observed.

Example: Output Impedance



$$\frac{V_X}{I_X} = \frac{(r_{O1} || r_{O2})C_{GS3}s + 1}{C_{GS3}s + g_{m3}}$$

Frequency Response of Cascode Stage



$$A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1 \qquad C_x \approx 2C_{XY}$$

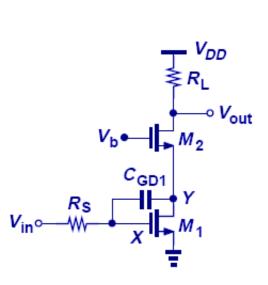
For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.

Poles of MOS Cascode

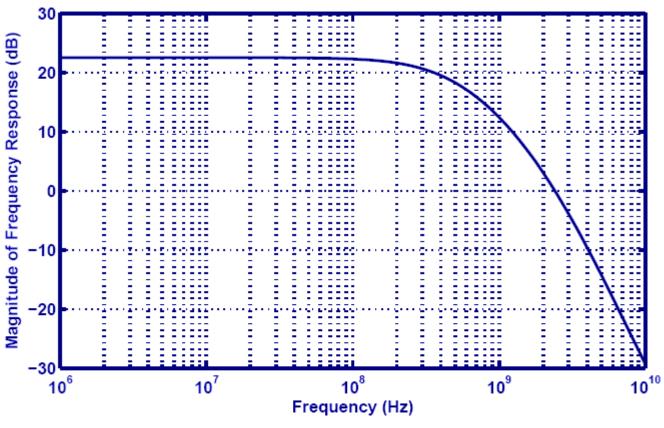
$$\omega_{p,X} = \frac{1}{R_{S} \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}} \right) C_{GD1} \right]} \qquad \omega_{p,out} = \frac{1}{R_{L} \left(C_{DB2} + C_{CO} \right)}$$

$$V_{b} \qquad V_{DD} \qquad V_{OU} \qquad V_$$

Example: Frequency Response of Cascode



$$R_S = 200\Omega$$
 $C_{GS} = 250 \, fF$
 $C_{GD} = 80 \, fF$
 $C_{DB} = 100 \, fF$
 $g_m = (150\Omega)^{-1}$
 $\lambda = 0$
 $R_L = 2K\Omega$
Frequency Response



$$\left|\omega_{p,X}\right| = 2\pi \times (1.95GHz)$$

$$\left|\omega_{p,Y}\right| = 2\pi \times (1.73GHz)$$

$$\left|\omega_{p,out}\right| = 2\pi \times (442MHz)$$

MOS Cascode Example

$$\omega_{p,X} = \frac{1}{R_S \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}} \right) C_{GD1} \right]} V_{DD}$$

$$V_{b2} \longrightarrow M_3$$

$$V_{b1} \longrightarrow M_2$$

$$V_{in} \longrightarrow M_1$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}} \left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}} \right) C_{GD1} + C_{GD3} + C_{DB3} \right]}$$

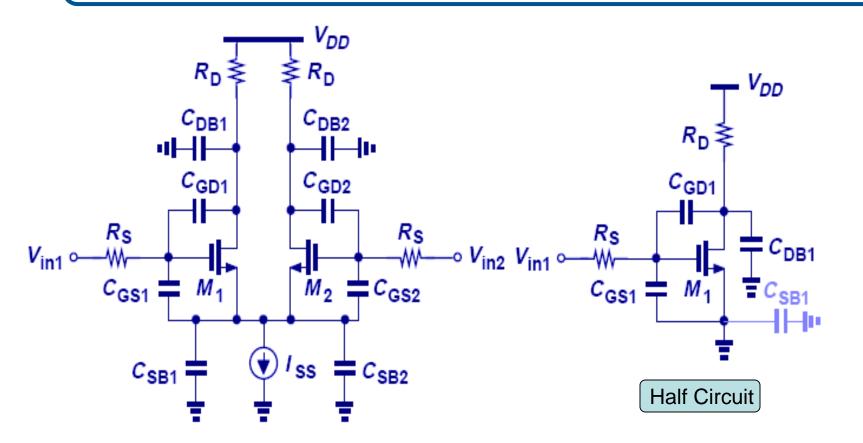
I/O Impedance of MOS Cascode

$$V_{DD}$$
 V_{DD}
 V_{D

$$Z_{in} = \frac{1}{\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]s}$$

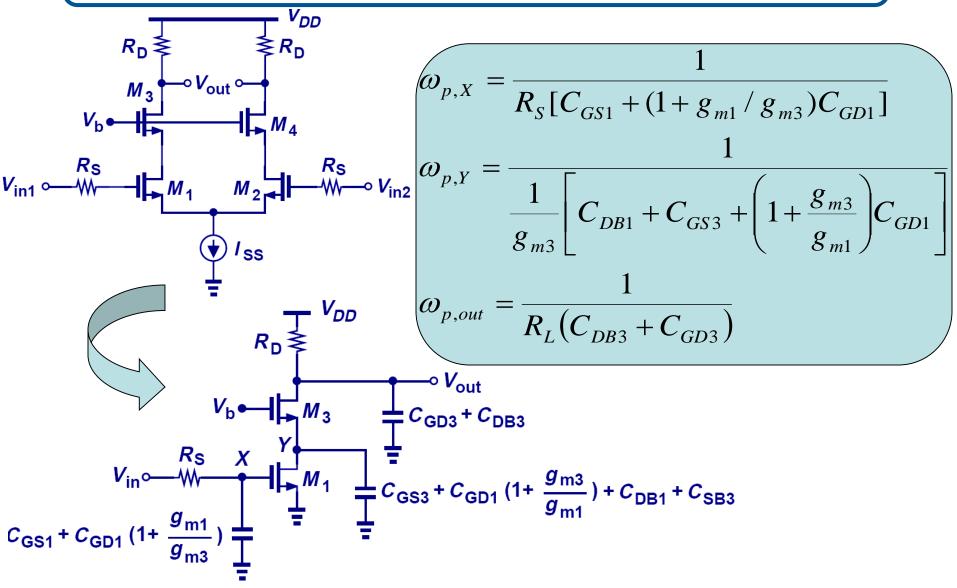
$$Z_{out} = R_L \parallel \frac{1}{(C_{GD2} + C_{DB2})s}$$

MOS Differential Pair Frequency Response

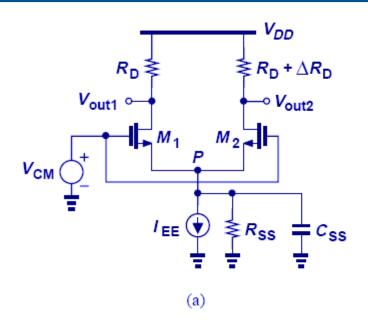


Since MOS differential pair can be analyzed using halfcircuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

Example: MOS Differential Pair



Common Mode Frequency Response



$$\begin{array}{c}
CM \\
Gain \\
g_{m} \Delta R_{D} \\
\hline
2R_{SS} + \frac{1}{g_{m}} \\
\hline
\frac{1}{R_{SS}C_{SS}} \frac{2g_{m}}{C_{SS}}
\end{array}$$

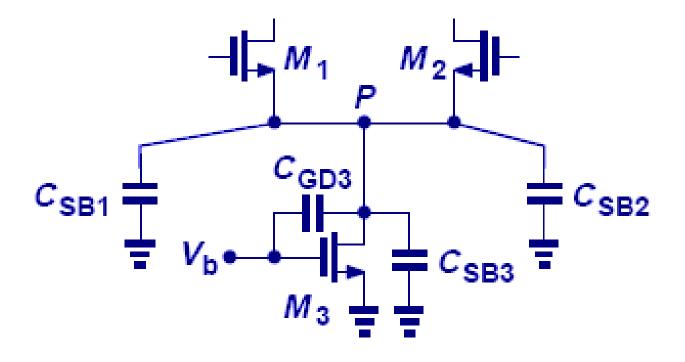
$$\left| \frac{\Delta V_{out}}{\Delta V_{CM}} \right| = \frac{g_m \Delta R_D (R_{SS} C_{SS} + 1)}{R_{SS} C_{SS} s + 2g_m R_{SS} + 1}$$

$$|\omega_z| = \frac{1}{R_{SS}C_{SS}}$$
 $|\omega_p| = \frac{2g_m}{C_{SS}}$

 $2g_m R_{SS} \gg 1$

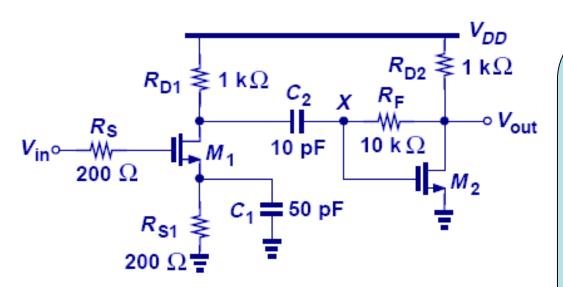
▶ C_{ss} will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.

Tail Node Capacitance Contribution



- Source-Body Capacitance of M₁, M₂ and M₃
- Gate-Drain Capacitance of M₃

Example: IC Amplifier – Low Frequency Design

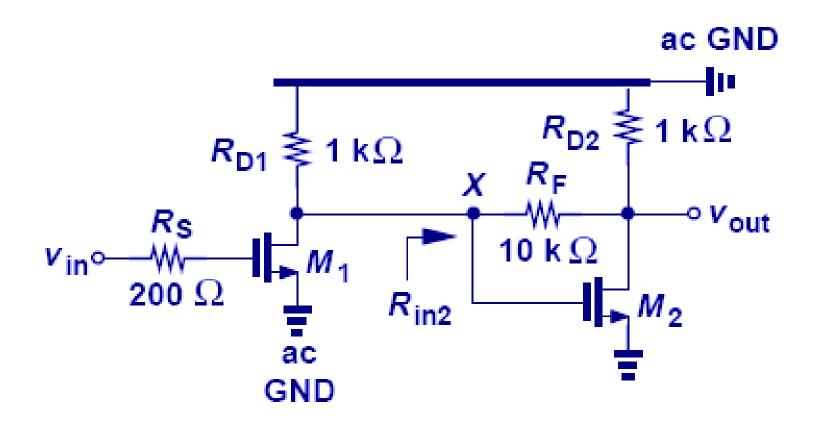


$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$\omega_{L1} = \frac{g_{m1}R_{S1} + 1}{R_{S1}C_1} = 2\pi \times (42.4MHz)$$

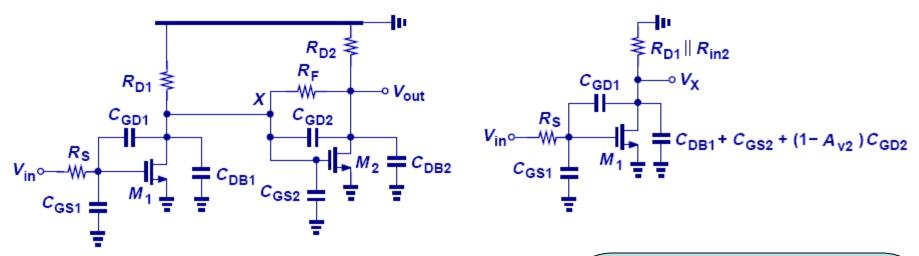
$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2})C_2} = 2\pi \times (6.92MHz)$$

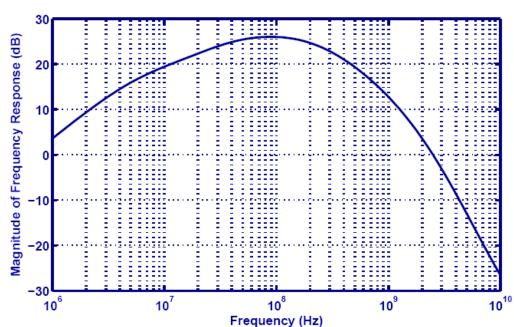
Example: IC Amplifier – Midband Design



$$\left(\frac{v_X}{v_{in}} = -g_{m1}(R_{D1} \parallel R_{in2}) = -3.77\right)$$

Example: IC Amplifier – High Frequency Design





$$\left|\omega_{p1}\right| = 2\pi \times (242 \text{ MHz})$$
 $\left|\omega_{p2}\right| = 2\pi \times (2.74 \text{ GHz})$
 $\left|\omega_{p3}\right| = \frac{1}{R_{L2}(1.15C_{GD2} + C_{DB2})}$
 $= 2\pi \times (0.829 \text{ GHz})$