DSP UESTC 4005: Homework #chapter 8

Due on: June 12 2019 at 23:59:59

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Problem 8.1

Solution:

8.1 $\frac{Y(z)}{X(z)} = H(z) = \frac{G(z)}{1-G(z)C(z)}$

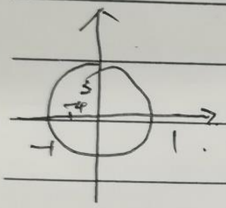
$G(z) = \frac{z^{-2}}{1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} = \frac{1}{z^2+\frac{3}{2}z+\frac{1}{2}}$

$C(z) = K$

$\therefore H(z) = \frac{1}{z^2+\frac{3}{2}z+(\frac{1}{2}-k)}$

$= \frac{1}{[z-(-\frac{3}{4}+\sqrt{k+\frac{1}{16}})][z-(-\frac{3}{4}-\sqrt{k+\frac{1}{16}})]}$

$= \frac{z^{-2}}{[1-z^{-1}(-\frac{3}{4}+\sqrt{k+\frac{1}{16}})][1-z^{-1}(-\frac{3}{4}-\sqrt{k+\frac{1}{16}})]}$.

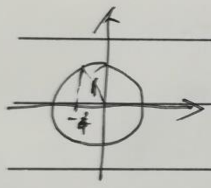


$\begin{cases} k>0 \\ \sqrt{k+\frac{1}{16}} < \frac{1}{4} \\ \sqrt{k+\frac{1}{16}} < 1+\frac{3}{4} \end{cases} \Rightarrow k \in \emptyset$

k doesn't exist in this part.

$k<0$.

$\sqrt{k+\frac{1}{16}} = j\sqrt{-(k+\frac{1}{16})}$



$\therefore \begin{cases} k<0 \\ \sqrt{-(k+\frac{1}{16})} < \sqrt{1^2-(\frac{3}{4})^2} \end{cases} \Rightarrow -\frac{1}{2}<k<0$.

$\therefore$ the overall structure is stable when $-\frac{1}{2}<k<0$.

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Problem 8.2

Solution:

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8.2. $\frac{Y(z)}{X(z)} = H(z) = \frac{G(z)}{1-G(z)C(z)}$.

$G(z) = \frac{z^{-1}}{1+1.5z^{-1}+0.5z^{-2}}$;

$C(z) = K$.

$\therefore H(z) = \frac{z^{-1}}{1+(1.5-K)z^{-1}+0.5z^{-2}}$.

$$\therefore H(z) = \frac{z^{-1}}{\{1-z^{-1}[(\frac{k}{2}-\frac{3}{4})+\sqrt{(\frac{3}{4}-\frac{k}{2})^2-\frac{1}{2}}]\}\{1-z^{-1}[(\frac{k}{2}-\frac{3}{4})-\sqrt{(\frac{3}{4}-\frac{k}{2})^2-\frac{1}{2}}]\}}$$

$\sqrt{(\frac{3}{4}-\frac{k}{2})^2-\frac{1}{2}} > 0$.

poles: $z = (\frac{k}{2}-\frac{3}{4}) \pm \sqrt{(\frac{k}{2}-\frac{3}{4})^2-\frac{1}{2}}$

poles must be in the circle.

$(\frac{k}{2}-\frac{3}{4})^2-\frac{1}{2} < 0$ ①

$\sqrt{(\frac{k}{2}-\frac{3}{4})^2-\frac{1}{2}} = j\sqrt{\frac{1}{2}-(\frac{k}{2}-\frac{3}{4})^2}$

$-1 < \frac{k}{2}-\frac{3}{4} < 1$ ②

$\sqrt{\frac{1}{2}-(\frac{k}{2}-\frac{3}{4})^2} < \sqrt{1-(\frac{k}{2}-\frac{3}{4})^2}$ ③.

$\therefore \frac{3}{2}-\sqrt{2} < k < \frac{3}{2}+\sqrt{2}$

$(\frac{k}{2}-\frac{3}{4})^2-\frac{1}{2} > 0$ ① $k > \frac{3}{2}+\sqrt{2}$ or $k < \frac{3}{2}-\sqrt{2}$

$(\frac{k}{2}-\frac{3}{4})+\sqrt{(\frac{k}{2}-\frac{3}{4})^2-\frac{1}{2}} < 1$ ② $\Rightarrow k < 3$

$(\frac{k}{2}-\frac{3}{4})-\sqrt{(\frac{k}{2}-\frac{3}{4})^2-\frac{1}{2}} > -1$ ③ $\Rightarrow k > 0$

$\frac{3}{2}+\sqrt{2} < k < 3$, $0 < k < \frac{3}{2}-\sqrt{2}$.

$\therefore$ overall, $0 < k < 3$

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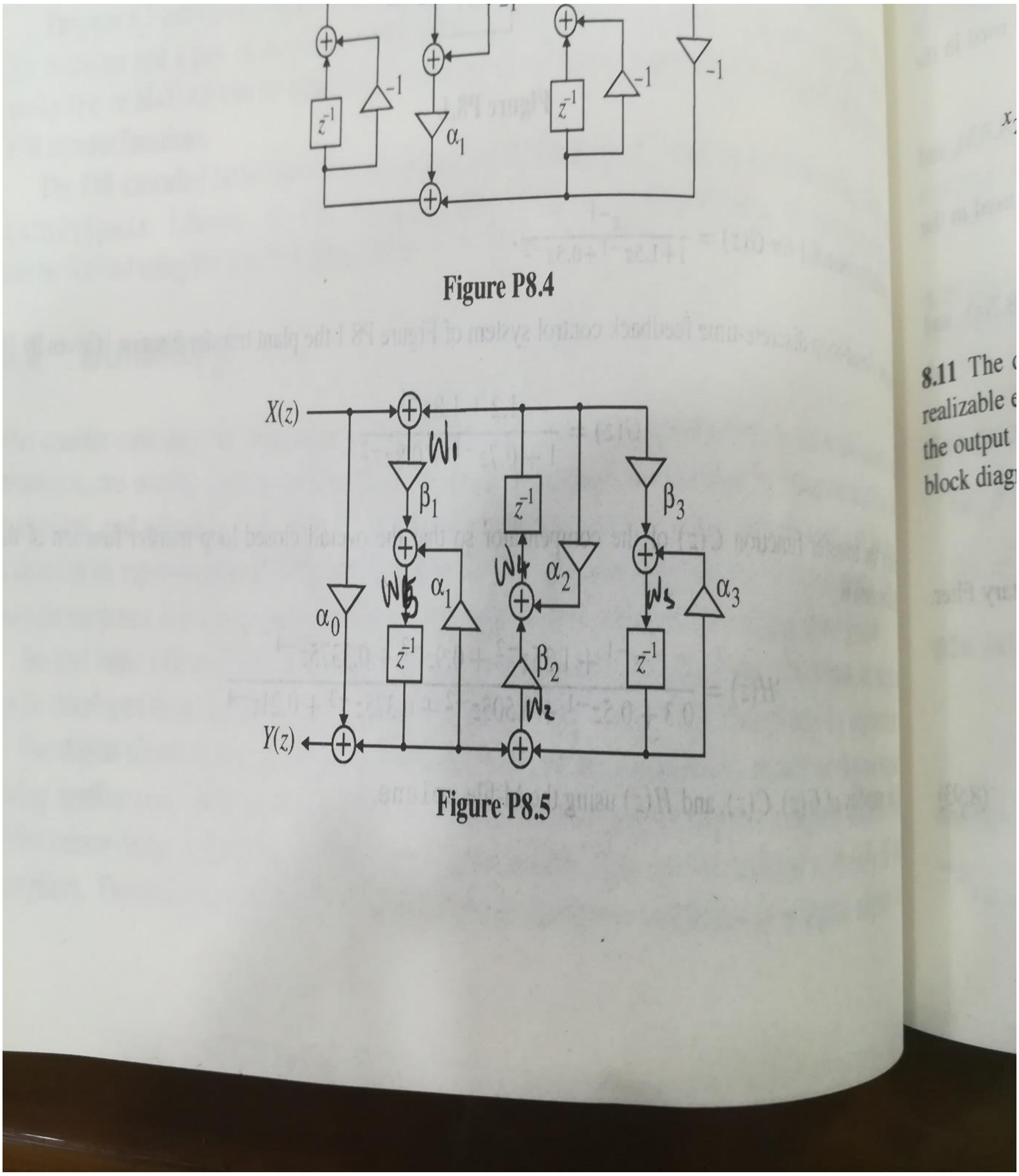
Problem 8.7

Solution:

Assumption:



8.7

$$W_1 = X(z) + W_4 \cdot z^{-1}$$

$$W_2 = W_5 z^{-1} + W_3 z^{-1}$$

$$W_3 = W_4 \cdot z^{-1} \cdot \beta_3 + W_3 z^{-1} \cdot \alpha_3 \Rightarrow W_3 = \left(\frac{z^{-1} \cdot \beta_3}{1-\alpha_3 z^{-1}}\right) W_4$$

$$W_4 = W_2 \beta_2 + W_4 z^{-1} \alpha_2 \Rightarrow W_4 = W_2 \left(\frac{\beta_2}{1-z^{-1}\alpha_2}\right)$$

$$W_5 = W_5 z^{-1} \alpha_1 + W_1 \cdot \beta_1 \Rightarrow W_5 = W_1 \left(\frac{\beta_1}{1-z^{-1}\alpha_1}\right)$$

$$Y(z) = X(z)\alpha_0 + W_5 z^{-1}$$

$$\therefore Y(z) = \alpha_0 X(z) + W_1 \left(\frac{\beta_1 z^{-1}}{1-\alpha_1 z^{-1}}\right)$$

$$W_1 = X(z) + W_2 \left(\frac{\beta_2 z^{-1}}{1-z^{-1}\alpha_2}\right)$$

$$W_2 = W_1 \left(\frac{\beta_1}{1-z^{-1}\alpha_1}\right) z^{-1} + z^{-1} \left(\frac{z^{-1}\beta_3}{1-\alpha_3 z^{-1}}\right)\left(\frac{\beta_2}{1-z^{-1}\alpha_2}\right) W_2$$

$$\Leftrightarrow \left[1 - \frac{z^{-2}\beta_2\beta_3}{(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1})}\right] W_2 = \left(\frac{\beta_1 z^{-1}}{1-\alpha_1 z^{-1}}\right) W_1$$

$$\Rightarrow W_2 = \frac{\beta_1 z^{-1}(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1})}{[(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1}) - \beta_2\beta_3 z^{-2}](1-\alpha_1 z^{-1})} W_1$$

$$\therefore \cancel{Y(z)}\ H(z) = \frac{Y(z)}{X(z)} = \alpha_0 + \frac{\beta_1 z^{-1}}{(1-\alpha_1 z^{-1})\left[1 - \left(\frac{\beta_2 z^{-1}}{1-\alpha_2 z^{-1}}\right)\frac{\beta_1 z^{-1}(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1})}{(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1}) - \beta_2\beta_3 z^{-2}}\left(\frac{1}{1-\alpha_1 z^{-1}}\right)\right]}$$

$$= \alpha_0 + \frac{\beta_1 z^{-1}}{1-\alpha_1 z^{-1} - \frac{\beta_1\beta_2 z^{-2}(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1})}{(1-\alpha_2 z^{-1})(1-\alpha_3 z^{-1})(1-\alpha_2 z^{-1}) - \beta_2\beta_3 z^{-2}}}$$

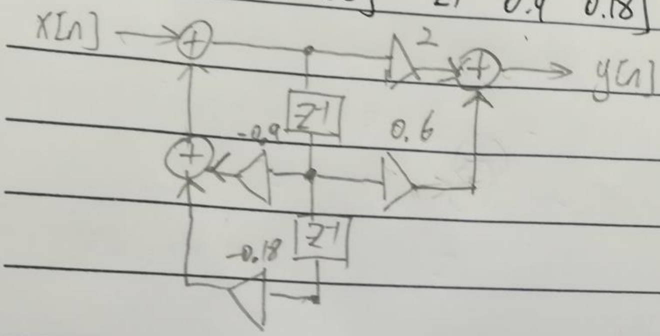
Problem 8.24

Solution:

8.24

a) $[p_0 \; p_1 \; p_2] = [2, \; 0.6 \; 0]$

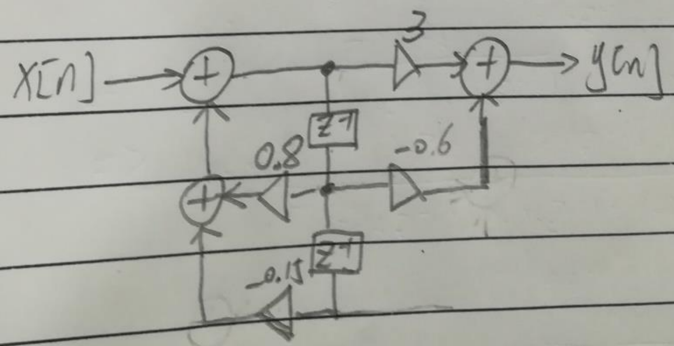
$[1 \; d_1 \; d_2] = [1 \; 0.9 \; 0.18]$



b) $H_2(z) = \dfrac{3-0.6z^{-1}}{1-0.8z^{-1}+0.15z^{-2}}$

$[p_0 \; p_1 \; p_2] = [3, \; -0.6 \; 0]$

$[1 \; d_1 \; d_2] = [1 \; -0.8 \; 0.15]$



Problem 8.61

Solution:

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8.61

a) $d_0 y[2l] + d_1 y[2l-1] + d_2 y[2l-2] + d_3 y[2l-3] + d_4 y[2l-4]$

$= p_0 x[2l] + p_1 x[2l-1] + p_2 x[2l-2] + p_3 x[2l-3] + d_4 x[2l-4]$

$\therefore d_0 y[2l+1] + d_1 y[2l] + d_2 y[2l-1] + d_3 y[2l-2] + d_4 y[2l-3]$

$= p_0 x[2l] + p_1 x[2l-1] + p_2 x[2l-2] + p_3 x[2l-3] + d_4 x[2l-4]$

$$\therefore \begin{bmatrix} d_0 & 0 \\ d_1 & d_0 \end{bmatrix} \begin{bmatrix} y[2l] \\ y[2l+1] \end{bmatrix} + \begin{bmatrix} d_2 & d_1 \\ d_3 & d_2 \end{bmatrix} \begin{bmatrix} y[2l-2] \\ y[2l-1] \end{bmatrix} + \begin{bmatrix} d_4 & d_3 \\ 0 & d_4 \end{bmatrix} \begin{bmatrix} y[2l-4] \\ y[2l-3] \end{bmatrix}$$

$$= \begin{bmatrix} p_0 & 0 \\ p_1 & p_0 \end{bmatrix} \begin{bmatrix} x[2l] \\ x[2l+1] \end{bmatrix} + \begin{bmatrix} p_2 & p_1 \\ p_3 & p_2 \end{bmatrix} \begin{bmatrix} x[2l-2] \\ x[2l-1] \end{bmatrix} + \begin{bmatrix} p_4 & p_3 \\ 0 & p_4 \end{bmatrix} \begin{bmatrix} x[2l-4] \\ x[2l-3] \end{bmatrix}$$

$\therefore D_0 = \begin{bmatrix} d_0 & 0 \\ d_1 & d_0 \end{bmatrix} \quad D_1 = \begin{bmatrix} d_2 & d_1 \\ d_3 & d_2 \end{bmatrix} \quad D_2 = \begin{bmatrix} d_4 & d_3 \\ 0 & d_0 \end{bmatrix}$

$P_0 = \begin{bmatrix} p_0 & 0 \\ p_1 & p_0 \end{bmatrix} \quad P_1 = \begin{bmatrix} p_2 & p_1 \\ p_3 & p_2 \end{bmatrix} \quad P_2 = \begin{bmatrix} p_4 & p_3 \\ p_0 & p_4 \end{bmatrix}$

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b) $D_0 = \begin{bmatrix} d_0 & 0 & 0 \\ d_1 & d_0 & 0 \\ d_2 & d_1 & d_0 \end{bmatrix}$ $P_0 = \begin{bmatrix} p_0 & 0 & 0 \\ p_1 & p_0 & 0 \\ p_2 & p_1 & p_0 \end{bmatrix}$

$D_1 = \begin{bmatrix} d_3 & d_2 & d_1 \\ d_4 & d_3 & d_2 \\ 0 & d_4 & d_3 \end{bmatrix}$ $P_1 = \begin{bmatrix} p_3 & p_2 & p_1 \\ p_4 & p_3 & p_2 \\ 0 & p_4 & p_3 \end{bmatrix}$

$D_2 = \begin{bmatrix} 0 & 0 & d_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $P_2 = \begin{bmatrix} 0 & 0 & p_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c) $D_0 = \begin{bmatrix} d_0 & 0 & 0 & 0 \\ d_1 & d_0 & 0 & 0 \\ d_2 & d_1 & d_0 & 0 \\ d_3 & d_2 & d_1 & d_0 \end{bmatrix}$ $P_0 = \begin{bmatrix} p_0 & 0 & 0 & 0 \\ p_1 & p_0 & 0 & 0 \\ p_2 & p_1 & p_0 & 0 \\ p_3 & p_2 & p_1 & p_0 \end{bmatrix}$

$D_1 = \begin{bmatrix} d_4 & d_3 & d_2 & d_1 \\ 0 & d_4 & d_3 & d_2 \\ 0 & 0 & d_4 & d_3 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$ $P_1 = \begin{bmatrix} p_4 & p_3 & p_2 & p_1 \\ 0 & p_4 & p_3 & p_2 \\ 0 & 0 & p_4 & p_3 \\ 0 & 0 & 0 & p_4 \end{bmatrix}$

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