DSP UESTC 4005: Homework #chapter 4

Due on: April 17 2019 at 23:59:59

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Problem 4.1&4.3 a)

Solution:

DSP.

Problem 4.1.

The eq(2.7f) in the textbook is defined: $y[n] = x[Mn]$.

$y[n-n_0] = x[M(n-n_0)] = x[Mn - Mn_0]$

However, $x[n-n_0] = x[Mn - n_0]$

So. $y[n-n_0] \neq x[n-n_0]$

As a result, the down-sampler defined above is a time-varying system.

Problem 4.3

a) $y[n] = x[n+3]$

1) assume. $y_1[n] = x_1[n+3]$

$y_2[n] = x_2[n+3]$

$\because A x_1[n+3] + B x_2[n+3] = A y_1[n] + B y_2[n]$.

So. the system is linear.

2) assume $x[n] = \delta[n]$.

$x[n+3] = \delta[n+3] = y[n] \quad \therefore h[n] = \delta[n+3]$.

for $n < 0$, ~~$y[n]=0$~~. $h[n] \neq 0$

As a result, the system is casual

3) . assume $|x[n]| \leq C < \infty$.

then. $|y[n]| \leq B < \infty$.

So the system is Boundary Input Boundary Output stable

4) $y[n-n_0] = x[n+3-n_0]. = x[n-n_0+3]$

$y[n] = x[n+3]$

$y_1[n] = x_1[n+3]$

if $x_1[n] = x[n-n_0]$

then $y_1[n] = x_1[n+3] = x[n+3-n_0]$

$y[n-n_0] = x[n-n_0+3]$

$\therefore y[n-n_0] = y_1[n]$

so the system is time-invariant

Problem 4.3 b)

Solution:

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b) $y[n] = x[2-n] + \partial$, ∂ is a non zero constant.

1). $y_1[n] = x_1[2-n] + \partial$.
$y_2[n] = x_2[2-n] + \partial$.

$$Ay_1[n] + By_2[n] = Ax_1[2-n] + A\partial + Bx_2[2-n] + B\partial$$
$$= Ax_1[2-n] + Bx_2[2-n] + (A+B)\partial$$
$$\neq Ax_1[2-n] + Bx_2[2-n]$$

As a result, the system is non linear.

2). Assume. $x[n] = \delta[n]$.
$y[n] = \delta[2-n] + \partial = h[n]$.
$\therefore$ for $n<0$, $h[n] = \partial \neq 0$

As a result, the system is ~~anti quasi~~ non non-causal.

3). Assume $|x[n]| \le B < \infty$.
$|y[n]| \le B + \partial$.
since ∂ is ~~finite~~ constant
$\therefore |y[n]| \le B + \partial < \infty$.

As a result, the system is BIBO stable

4). $\cancel{y[n-n_0] = x[2-(n-n_0)]} + \partial$ Assume. $y[n] = x[2-n] + \partial$
$=$
$y_1[n] = x_1[2-n] + \partial$.
if $x_1[n] = x[n-n_0]$.
then. $y_1[n] = x_1[2-n] + \partial = x[n_0 - n + 2] + \partial$
$y[n-n_0] = x[2-(n-n_0)] + \partial$
$\therefore y_1[n] = y[n-n_0]$ $\therefore$ the system is time invariant

Problem 4.3 c)

Solution:

③. $y[n] = \ln(1-|x[n]|)$

1) $y_1[n] = \ln(1-|x_1[n]|)$

$y_2[n] = \ln(1-|x_2[n]|)$.

$Ay_1[n] + By_2[n] = A\ln(1-|x_1[n]|) + B\ln(1-|x_2[n]|)$

~~However~~, assume $x[n] = Ax_1[n] + Bx_2[n]$.

$y[n] = \ln(1-|Ax_1[n]+Bx_2[n]|) \neq Ay_1[n]+By_2[n]$.

As a result, the system is non linear

2). Assume $x[n] = \delta[n]$

$\therefore y[n] = h[n] = \ln(1-|\delta[n]|)$.

for $n<0$, $y[n]=0$.

As a result, the system is casual.

3). Assume $|x[n]| \le B < \infty$

then $|y[n]| \le \ln(1-B) < \infty$.

As a result, the system is BIBO stable

4) $y[n] = \ln(1-|x[n]|)$

$y_1[n] = \ln(1-|x_1[n]|)$.

if $x_1[n] = x[n-n_0]$.

then $y_1[n] = \ln(1-|x_1[n]|) = \ln(1-|x[n-n_0]|)$

$y[n-n_0] = \ln(1-|x[n-n_0]|) = y_1[n]$.

$\therefore$ the system is time-invariant

Problem 4.3 d)

Solution:

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d) $y[n] = \beta + \sum_{l=-1}^{3} x[n-l]$, β is a non zero constant

1) $y_1[n] = \beta + \sum_{l=-1}^{3} x_1[n-l]$

$y_2[n] = \beta + \sum_{l=-1}^{3} x_2[n-l]$

$Ay_1[n] + By_2[n] = (A+B)\beta + \cancel{(A+B)} A\sum_{l=-1}^{3} x_1[n-l] + B\sum_{l=-1}^{3} x_2[n-l]$

assume $x[n] = Ax_1[n] + Bx_2[n]$.

$y[n] = \beta + \sum_{l=-1}^{3} Ax_1[n-l] + Bx_2[n-l]$.

$= \beta + A\sum_{l=-1}^{3} x_1[n-l] + B\sum_{l=-1}^{3} x_2[n-l] \neq Ay_1[n] + By_2[n]$

As a result, the system is non linear.

2) Assume $x[n] = \delta[n]$.

$y[n] = h[n] = \beta + \sum_{l=-1}^{3} \delta[n-l]$.

for $n < 0$, $h[n] = \begin{cases} \beta, & \text{others} \\ 1+\beta, & n=-1 \end{cases} \neq 0$.

As a result, the system is not ~~stable~~ causal.

3) Assume $|x[n]| < B < \infty$.

$|y[n]| \leq \beta + 5B < \infty$.

As a result, the system is BIBO stable

4) $y[n] = \beta + \sum_{l=-1}^{3} x[n-l]$

$y_1[n] = \beta + \sum_{l=-1}^{3} x_1[n-l]$

if $x_1[n] = x[n-n_0]$.

$y_1[n] = \beta + \sum_{l=-1}^{3} x[n-n_0-l]$

$y[n-n_0] = \beta + \sum_{l=-1}^{3} x[n-n_0-l] = y_1[n]$

$\therefore$ the system is time-invariant.

Problem 4.9

Solution:

4.9 $y[n] = x^2[n] - x[n-1]x[n+1]$.

linearity: $y_1[n] \triangleq x_1^2[n] - x_1[n-1]x_1[n+1]$

$y_2[n] \triangleq x_2^2[n] - x_2[n-1]x_2[n+1]$

$Ay_1[n] + By_2[n] = x_1^2[n] + x_2^2[n] - x_1[n-1]x_1[n+1] - x_2[n-1]x_2[n+1]$

Assume $x[n] \triangleq Ax_1[n] + Bx_2[n]$

$y[n] = [Ax_1[n] + Bx_2[n]]^2 - (Ax_1[n-1] + Bx_2[n-1])(Ax_2[n+1] + Bx_2[n+1])$

$\neq Ay_1[n] + By_2[n]$

$\therefore$ the system is non-linear

Time-invariance: $y[n] = x^2[n] - x[n-1]x[n+1]$

$y_1[n] = x_1^2[n] - x_1[n-1]x_1[n+1]$

if $x_1[n] = x[n-n_0]$

$y_1[n] = x^2[n-n_0] - x[n-n_0-1]x[n-n_0+1]$

$y[n-n_0] = x^2[n-n_0] - x[n-n_0-1]x[n-n_0+1]$

$\therefore y[n-n_0] = y_1[n]$

$\therefore$ the system is time-invariancy.

Causal: Assume $x[n] = \delta[n]$

then $h[n] = y[n] = \delta^2[n] - \delta[n-1]\delta[n+1] = \begin{cases} 0 & n=-1 \\ 1 & n=0 \\ 0 & n=1 \end{cases} = \delta[n]$

for $n<0$, $h[n] = 0$.

$\therefore$ the system is causal.

Problem 4.30&4.44

Solution:

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4.30

a) $h[n] = h_1[n] * h_2[n] + (h_1[n] * h_5[n] + h_3[n]) * h_4[n]$.

b) $\{(x[n] + y[n] * h_5[n]) * h_3[n] + (x[n] + y[n] * h_5[n])\, h_1[n] * h_2[n]\} * h_4[n] = y[n]$

$$\therefore h[n] = \frac{(h_3[n] + h_1[n] * h_2[n]) * h_4[n]}{1 - (h_3[n] + h_1[n] * h_2[n]) * h_4[n] * h_5[n]}.$$

4-44. $y[n] - 0.3y[n-1] - 0.04y[n-2] = x[n] + 2x[n-1]$.

$\because x[n] = 3^n u[n]$, $y[-1] = 3$, $y[-2] = 0$

$y[n] = y_c[n] + y_p[n]$.

$y_c[n] - 0.3y_c[n-1] - 0.04y_c[n-2] = 0$.

suppose $y_c[n] = \lambda^n \Rightarrow \lambda^n - 0.3\lambda^{n-1} - 0.04\lambda^{n-2} = 0 \Rightarrow \lambda^2 - 0.3\lambda - 0.04 = 0$.

$(\lambda + 0.1)(\lambda - 0.4) = 0$

$\therefore y_c[n] = \alpha_1 (0.4)^n + \alpha_2 (-0.1)^n$.

$y_1[n] - 0.3y_1[n-1] - 0.04y_1[n-2] = 3^n u[n]$.

$y_2[n] - 0.3y_2[n-1] - 0.04y_2[n-2] = 2\cdot 3^{n-1} u[n-1]$

assume $y_{p1}[n] = \beta \cdot 3^n$. $y_{p2}[n] = 2\beta \cdot 3^{n-1}$

$y_{p1} + y_{p2}[n] = \beta \cdot 3^n + 2\beta 3^{n-1} = 3^n u[n] + 2\cdot 3^{n-1} u[n-1]$

when $n=1$ $3\beta + 2\beta = 3 + 2 \Rightarrow \beta = 1$

Problem 4.44&4.56

Solution:

$$\therefore y[n] = y_c[n] + y_p[n] = \alpha_1(0.4)^n + \alpha_2(0.1)^n + 3^n + 2\cdot 3^{n-1}$$

$$\begin{cases} y[-1] = (\alpha_1)0.4^{-1} + \alpha_2(-0.1)^{-1} + 3^{-1} + 2\cdot 3^{-2} = 3 \\ y[-2] = (\alpha_1)0.4^{-2} + \alpha_2(-0.1)^{-2} + 3^{-2} + 2\cdot 3^{-3} = 0 \end{cases}$$

$\therefore \quad \alpha_1 = 0.7763$

$\alpha_2 = -0.0504.$

4-56. $h[n] = \delta[n] - \delta[n-k]\cdot\alpha.$

$g[n] = h[n] * h[n] * h[n]$

$G(e^{jw}) = H(e^{jw})\,H(e^{jw})\,H(e^{jw})$

$H(e^{jw}) = 1 - \alpha e^{-kjw}$

$\therefore G(e^{jw}) = (1 - \alpha e^{-kjw})^3.$

Problem 4.79

Solution:

4.79

a) $H_a(e^{j\omega}) = \alpha + \beta\cos\omega - j\beta\sin\omega$

the phase $\theta_a(\omega) = \tan^{-1}\left(\frac{-\beta\sin\omega}{\alpha+\beta\cos\omega}\right)$

$$\tau_a(\omega) = -\frac{d\theta_a(\omega)}{d\omega} = \frac{\beta^2+\alpha\beta\cos\omega}{\alpha^2+\beta^2+2\alpha\beta\cos\omega}$$

b) $1+\gamma e^{-j\omega} = 1+\gamma\cos\omega - j\gamma\sin\omega$

$$\therefore H_b(e^{j\omega}) = \frac{1}{1+\gamma\cos\omega - j\gamma\sin\omega} =$$

$$\frac{1+\gamma\cos\omega}{(1+\gamma\cos\omega)^2+(\gamma\sin\omega)^2} + j\frac{\gamma\sin\omega}{(1+\gamma\cos\omega)^2+(\gamma\sin\omega)^2}.$$

$$\therefore \theta_b(\omega) = \arctan\left(\frac{\gamma\sin\omega}{1+\gamma\cos\omega}\right).$$

$$\therefore \tau_b(\omega) = -\frac{d\theta_b(\omega)}{d\omega} = -\frac{\gamma^2+\gamma\cos\omega}{1+\gamma^2+2\gamma\cos\omega}$$

c) $H_c(e^{j\omega}) = H_a(e^{j\omega})\, H_b(e^{j\omega})$

$\therefore \theta_c(\omega) = \theta_a(\omega) + \theta_b(\omega)$.

$$\therefore \tau_c(\omega) = \tau_a(\omega) + \tau_b(\omega) = \frac{\beta^2+\alpha\beta\cos\omega}{\alpha^2+\beta^2+2\alpha\beta\cos\omega} - \frac{\gamma^2+\gamma\cos\omega}{1+\gamma^2+2\gamma\cos\omega}$$

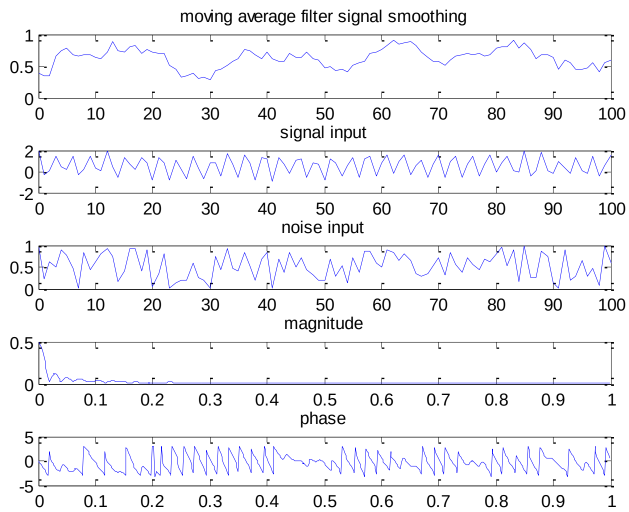
d) $H_d(e^{j\omega}) = H_b(e^{j\omega})\cdot\left(\frac{1}{1+\delta e^{-j\omega}}\right)$,

$$\therefore \tau_d(\omega) = \frac{\gamma^2+\gamma\cos\omega}{1+\gamma^2+2\gamma\cos\omega} + \frac{\delta^2+\delta\cos\omega}{1+\delta^2+2\delta\cos\omega}$$

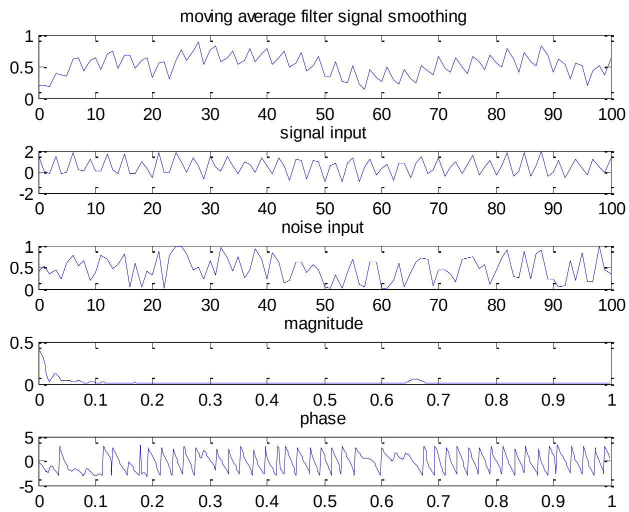
Problem M4.1

Solution:

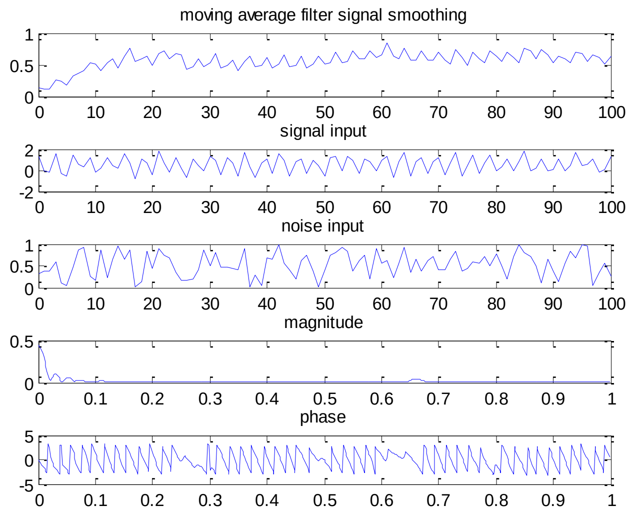
M=5



M=7


moving average filter signal smoothing
signal input
noise input
magnitude
phase

M=10


moving average filter signal smoothing
signal input
noise input
magnitude
phase

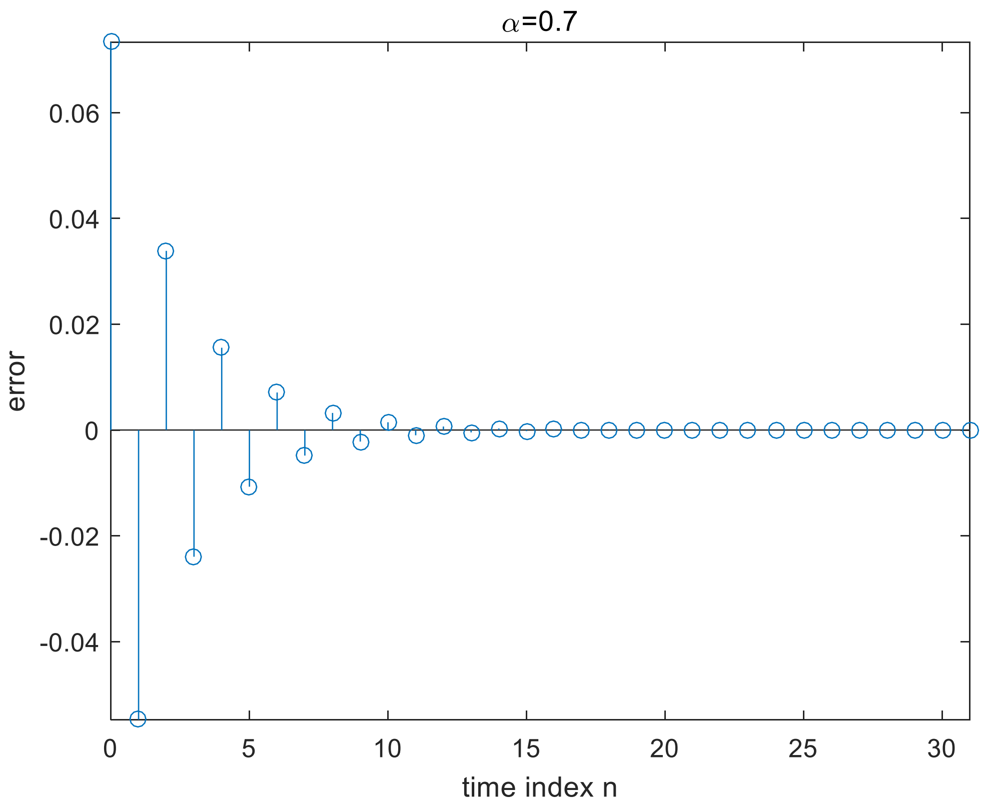
With the increased length, the signal smoothing improves.

However, the delay between input and output also increases.

Problem M4.3

Solution:

Alpha=0.7



Alpha=0.7

Square root of alpha is

0.836659663904726

Problem M4.5

Solution: