

# **DSP UESTC 4005: Homework #chapter 6**


Due on: May 8 2019 at 23:59:59

Instructor: Wenhui Xiong

Jiayi Feng

## Problem 6.1

### Solution:


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6.1

a)  $g[n] = nr^n \cos(\omega_0 n) u[n]$

$$\therefore Z[g[n]] = -\frac{d}{dz} Z\{r^n \cos(\omega_0 n) u[n]\}$$

$$= -\frac{d}{dz} \left\{ \frac{(1 - r \cos \omega_0) z^{-1}}{1 - [2r \cos \omega_0] z^{-1} + r^2 z^{-2}} \right\} \quad \text{for } |z| > r$$

$$= (1) \frac{-r \cos \omega_0 [z^2 - (2r \cos \omega_0) z + r^2] - [1 - (r \cos \omega_0 / z)] [2z - 2r \cos \omega_0]}{[z^2 - (2r \cos \omega_0) z + r^2]^2}$$

$$= \frac{(-r \cos \omega_0) z^{-2} + (4r^2 \cos^2 \omega_0 + 2) z^{-3} + (r^2 r) \cos \omega_0 z^{-4}}{[1 - (2r \cos \omega_0) z^{-1} + r^2 z^{-2}]^2} \quad \text{for } |z| > r$$

b)  $g[n] = nr^n \sin(\omega_0 n) u[n]$

$$\therefore Z[g[n]] = -\frac{d}{dz} Z\left\{ \frac{[r \sin \omega_0] z^{-1}}{1 - [2r \cos \omega_0] z^{-1} + r^2 z^{-2}} \right\}$$


$$= -\frac{(r \sin \omega_0) z^{-2} + (4r^2 \sin \omega_0 \cos \omega_0) z^{-3} - r^3 \sin \omega_0 z^{-4}}{[1 - [2r \cos \omega_0] z^{-1} + r^2 z^{-2}]^2} \quad \text{for } |z| > r$$

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## Problem 6.3&6.5

Solution:



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6.3. from the definition.

$$Z\{x[n]\} = \sum_{n=-\infty}^{\infty} \frac{1}{n!} \cdot \mu[n] \cdot z^{-n} = \sum_{n=0}^{\infty} \frac{z^{-n}}{n!} = \exp(-z)$$

ROC:  $\{z \neq 0\}$

Taylor expansion:  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

6.5

a)  $Z\{\delta[n]\} = 1$       ROC: All values of  $z$

b)  $Z\{n\delta^n \mu[n]\} = \frac{dz^{-1}}{(1-dz^{-1})^2}$       ROC:  $|z| > 1$

c)  $Z\{(r^n \sin n\omega_0) \mu[n]\}$


$$= \frac{(r \sin \omega_0) z^{-1}}{1 - (2r \cos \omega_0) z^{-1} + r^2 z^{-2}}$$

ROC:  $|z| > r$



## Problem 6.7

Solution:



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6.7

a)  $Z\{x_1[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}$       ROC:  $|z| > 0.8$   
 $|z| > 0.6$        $|z| > 0.8$

b)  $Z\{x_2[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}$       ROC:  $0.6 < |z| < 0.8$   
 $|z| > 0.6$        $|z| < 0.8$


c)  $Z\{x_3[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}$       ROC:  $|z| < 0.6$   
 $|z| < 0.6$        $|z| < 0.8$

d)  $Z\{x_4[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}$       ROC: doesn't exist  
 $|z| < 0.6$        $|z| > 0.8$

As a result, none of these 4 sequences has the same Z-transform, owing to difference ROC, despite the same equation.

# Problem 6.10&6.13

Solution:



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6.10

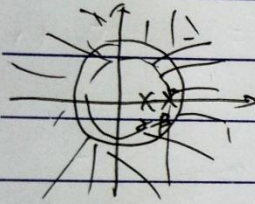
a)  $x_1[n] = \delta^{n+1} u[n+1] \cdot \delta^{-1} + \beta^{n+2} u[n+2] \beta^{-2}$

$\therefore Z\{x_1[n]\} = \delta^{-1} \cdot Z\left\{\frac{1}{1-\delta z^{-1}}\right\} + \beta^{-2} Z^2\left\{\frac{1}{1-\beta z^{-1}}\right\}$

$|z| > |\delta|$   $|z| > |\beta|$

$= \frac{(\delta z^{-1})^{-1}}{1-\delta z^{-1}} + \frac{(\beta z^{-1})^{-2}}{1-\beta z^{-1}}$

$\therefore Z\{x_1[n]\} = \frac{\beta^2 z^2 + (\delta^{-1} - \delta \beta^2) z - \delta^{-1}}{(1-\delta z^{-1})(1-\beta z^{-1})}$  Proc:  $|z| > |\beta|$



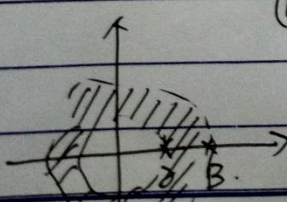
b)  $x_2[n] = \delta^{n-2} u[n-2] \cdot \delta^2 + (-1)^n u[-n-1] \cdot (-1)$

$\therefore Z\{x_2[n]\} = \delta^2 \cdot Z^2\left\{\frac{1}{1-\delta z^{-1}}\right\} + (-1) \left(\frac{1}{1-\beta z^{-1}}\right)$

$|z| > |\delta|$   $|z| < |\beta|$

$= \frac{(\delta z^{-1})^2}{1-\delta z^{-1}} - \frac{1}{1-\beta z^{-1}}$

$\therefore Z\{x_2[n]\} = \frac{-1 + \delta z^{-1} + \delta^2 z^{-2} - \delta^2 \beta z^{-3}}{(1-\delta z^{-1})(1-\beta z^{-1})}$  Proc:  $|\delta| < |z| < \beta$



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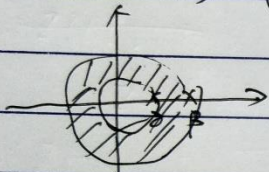
$$c) \quad x_3[n] = \delta^{n+2} u[n+2] \delta^{-2} + -\beta^n u[-n-1] (-1)$$

$$\therefore Z\{x_3[n]\} = \delta^{-2} Z^2 \left( \frac{1}{1-\delta z^{-1}} \right) - \left( \frac{1}{1-\beta z^{-1}} \right)$$

$|z| > |\delta| \quad |z| < |\beta|$

$$= \frac{\delta^{-2} z^2}{1-\delta z^{-1}} - \frac{1}{1-\beta z^{-1}}$$

$$\therefore Z\{x_3[n]\} = \frac{\delta^{-2} z^2 - \delta^{-2} \beta z - 1 + \delta z^{-1}}{(1-\delta z^{-1})(1-\beta z^{-1})} \quad \text{ROC: } |\delta| < |z| < \beta$$



$$1.13 a) \quad X_a(z) = \frac{2.6(z^{-1} + \frac{35}{18})}{(z^{-1} + \frac{10}{3})(z^{-1} + \frac{5}{3})} = \frac{\frac{50}{3}}{z^{-1} + \frac{10}{3}} + \frac{\frac{10}{3}}{z^{-1} + \frac{5}{3}}$$

$$= \frac{2}{1+0.6z^{-1}} + \frac{5}{1+0.3z^{-1}}$$

① ROC:  $|z| < 0.3$  then  $x_a[n] = (-2)(-0.6)^n u[-n-1] + (-5)(-0.3)^n u[-n-1]$

② ROC:  $|z| < 0.6$  then  $x_a[n] = (-2/0.6)^n u[-n-1] + 5(-0.3)^n u[n]$

③ ROC:  $|z| > 0.6$  then  $x_a[n] = 2(-0.6)^n u[n] + 5(-0.3)^n u[n]$

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$$b) \quad X_b(z) = \frac{\cancel{z^2}(-\frac{5}{2}) \cancel{z} - 3 - 2z^{-1}}{(z^2 - \frac{5}{2})(z - \frac{1}{5})} = \frac{-4}{1 - 0.4z^{-1}} + \frac{7}{1 - 0.2z^{-1}}$$

$$\therefore \textcircled{1} \text{ ROC: } |z| < 0.2 \quad x_b[n] = 4(0.4)^n u[-n-1] - 7(0.2)^n u[-n-1]$$

$$\textcircled{2} \text{ ROC: } 0.2 < |z| < 0.4 \quad x_b[n] = 4(0.4)^n u[-n-1] + 7(0.2)^n u[n]$$

$$\textcircled{3} \text{ ROC: } |z| > 0.4 \quad x_b[n] = (-4)(0.4)^n u[n] + 7(0.2)^n u[n]$$

$$c) \quad X_c(z) = \frac{2}{1 + 0.6z^{-1}} + \frac{3}{1 - 0.4z^{-1}} + \frac{-1}{(1 - 0.4z^{-1})^2}$$

$$\therefore \textcircled{1} \text{ ROC: } |z| < 0.4 \quad x_c[n] = [-2] \cdot (-0.6)^n u[-n-1] + (-3) 0.4^n u[-n-1] + (n+1)(0.4)^n u[-n-2]$$


$$\textcircled{2} \text{ ROC: } 0.4 < |z| < 0.6 \quad x_c[n] = (-2)(-0.6)^n u[-n-1] + 3 \cdot 0.4^n u[n] + (-1)(n+1)(0.4)^n u[n+1]$$

$$\textcircled{3} \text{ ROC: } |z| > 0.6 \quad x_c[n] = 2(-0.6)^n u[n] + 3 \cdot 0.4^n u[n] + (-1)(n+1)(0.4)^n u[n+1]$$



## Problem 6.81

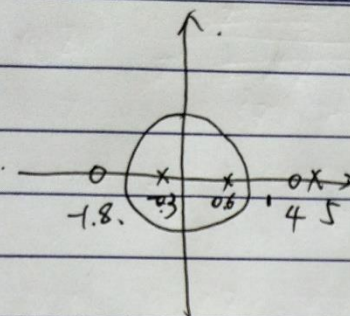
Solution:



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6.81



a) the frequency response exists if the ROC contains the unit circle.  
 If. So, the ROC:  $0.6 < |z| < 5$  would allow the existence of the frequency response

b) The system is stable if the ROC is  $0.6 < |z| < 5$ .  
 However, it cannot be causal since it is two-sided.

c) 
$$h[n] = A(-0.3)^n u[n] + B(0.6)^n u[n] + C(-5)^n u[n]$$



## Problem M6.1

### Solution:

a)

```
>>
Type in the numerator coefficients = [3 -2.4 15.36 3.84 9]
Type in the denominator coefficients = [5 -8.5 17.6 4.7 -6]

Numfactors =

    1.000000000000000    -1.200000000000000    5.000000000000000
    1.000000000000000     0.400000000000000    0.600000000000000

Denfactors =

    1.000000000000000    -1.800000000000000    4.000000000000000
    1.000000000000000     0.600000000000000     0
    1.000000000000000    -0.500000000000000     0

Numerator factors

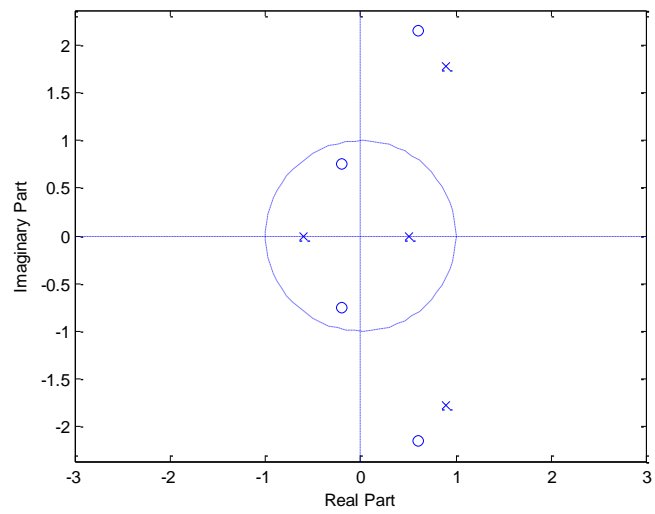
    1.000000000000000    -1.200000000000000    5.000000000000000
    1.000000000000000     0.400000000000000    0.600000000000000

Denominator factors

    1.000000000000000    -1.800000000000000    4.000000000000000
    1.000000000000000     0.600000000000000     0
    1.000000000000000    -0.500000000000000     0

Gain constant
    0.600000000000000

>> |
```





**b)**

```
Type in the numerator coefficients = [2 0.2 6.4 4.6 2.4]
Type in the denominator coefficients = [5 1 6.6 0.42 24]

Numfactors =

    1.000000000000000    -0.65920228694765    3.34132939149340
    1.000000000000000     0.75920228694765    0.35913849231837

Denfactors =

    1.000000000000000     1.85397919923719    2.29431858207741
    1.000000000000000    -1.65397919923719    2.09212444927931

Numerator factors

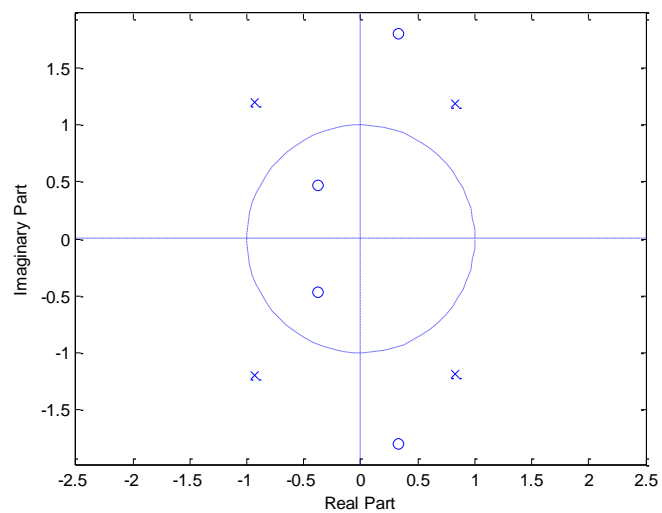
    1.000000000000000    -0.65920228694765    3.34132939149340
    1.000000000000000     0.75920228694765    0.35913849231837

Denominator factors

    1.000000000000000     1.85397919923719    2.29431858207741
    1.000000000000000    -1.65397919923719    2.09212444927931

Gain constant
    0.400000000000000

%% I
```





## Problem M6.2

### Solution:

a)

```
Type in numerator coefficients = [7 0 0]
Type in denominator coefficients = [1 0.3 -0.1]
Residues
      5      2

Poles
-0.5000000000000000  0.2000000000000000

Constants
      0
```

So, the z-transform will be:

$$X_a(Z) = \frac{5}{1+0.5z^{-1}} + \frac{5}{1-0.2z^{-1}}$$

The inverse z-transform is dependent on the ROC.

b)

```
Type in numerator coefficients = [0 3 1.8 1.28]
Type in denominator coefficients = [1 0.3 -0.24 -0.08]
Residues
      7.23456790123457  15.98765455604055  -7.22222245727512

Poles
      0.500000000000000  -0.40000000588086  -0.39999999411914

Constants
      -16
```

So, the z-transform will be:

$$X_b(Z) = -16 + \frac{7.2346}{1-0.5z^{-1}} + \frac{-7.2222}{1+0.4z^{-1}} + \frac{15.9877}{(1+0.4z^{-1})^2}$$

The inverse z-transform is dependent on the ROC.

## Problem M6.4

### Solution:

Choose a) as the example.

Type in the residues = [6/2 -12.5/2.5]

Type in the poles = [-1/2 0.4]

Type in the constants = [2]

Numerator polynomial coefficients

0 -3.5000 -0.4000

Denominator polynomial coefficients

1.0000 0.1000 -0.2000

Type in the length of output vector = 30

Type in the numerator coefficients = [0 -3.5 -0.4]

Type in the denominator coefficients = [1 0.1 -0.2]

Coefficients of the power series expansion

Columns 1 through 9

0 -3.5000 -0.0500 -0.6950 0.0595 -0.1450 0.0264 -0.0316 0.0084

Columns 10 through 18

-0.0072 0.0024 -0.0017 0.0006 -0.0004 0.0002 -0.0001 0.0000 -0.0000

Columns 19 through 27

0.0000 -0.0000 0.0000 -0.0000 0.0000 -0.0000 0.0000 -0.0000 0.0000

Columns 28 through 30

-0.0000 0.0000 -0.0000

... 1