DSP UESTC 4005: Homework #chapter 7

Due on: May 26 2019 at 23:59:59

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Problem 7.1

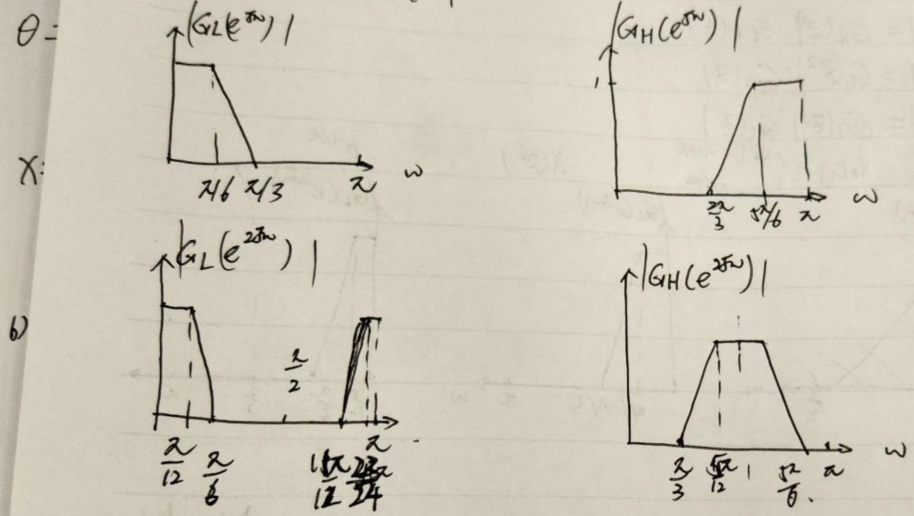
Solution:

$$H_0(z) = G_L(z)\,G_H(z^2)$$

$$H_1(z) = G_H(z)\,G_H(z^2)$$

$$H_2(z) = G_H(z)\,G_L(z^2)$$

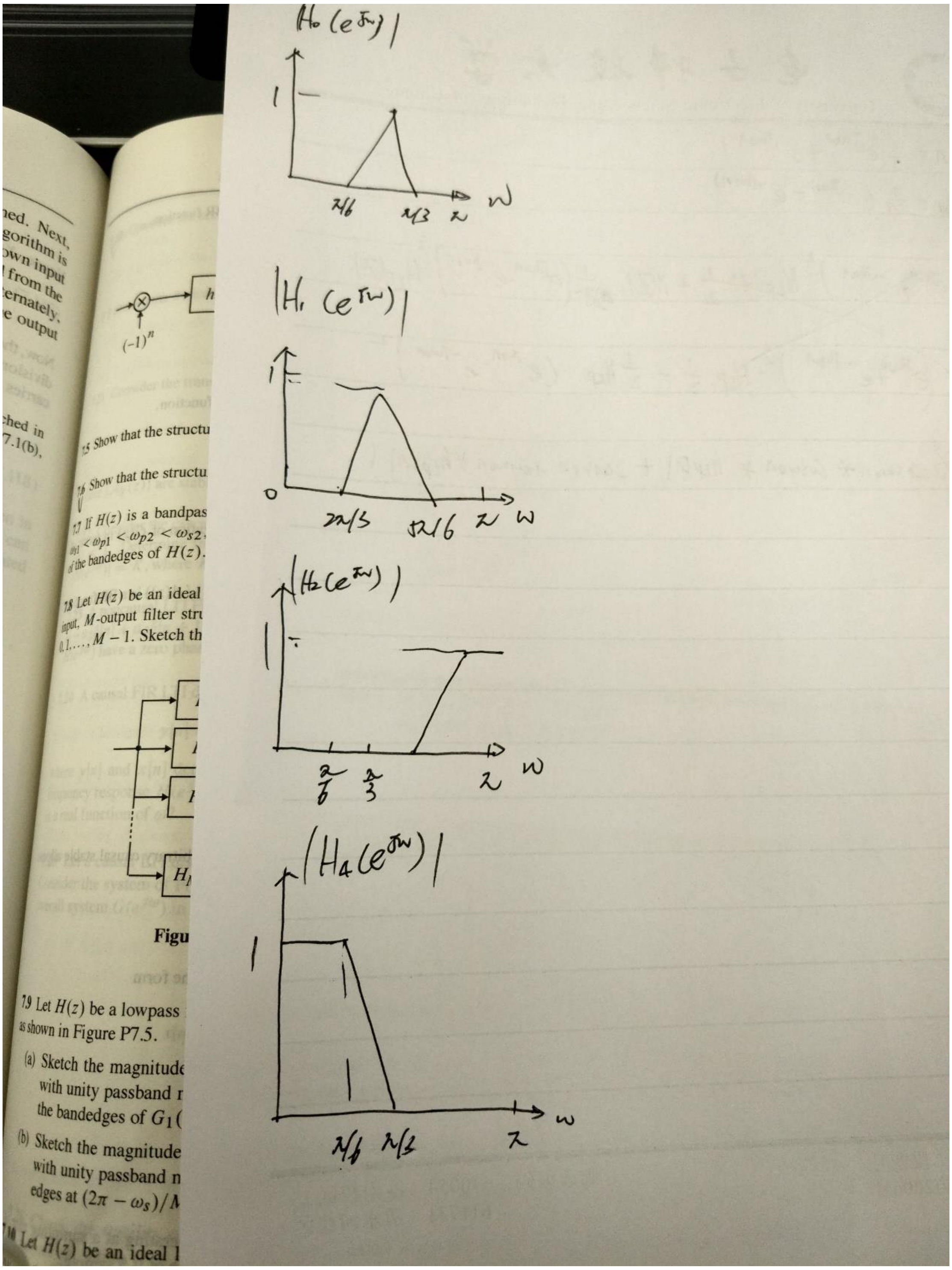
$$H_3(z) = G_L(z)\cdot G_L(z^2)$$



$$|G_L(e^{j2\frac{23\pi}{24}})| = |G_L(e^{j(2\pi-\frac{\pi}{12})})| = |G_L(e^{-\frac{\pi}{12}j})| = |G_L(e^{\frac{\pi}{12}j})|$$

$$|G_L(e^{j\cdot 2\cdot\frac{11}{12}\pi})| = |G_L(e^{j(2\pi-\frac{1}{6}\pi)})| = |G_L(e^{-\frac{j}{6}\pi})| = |G_L(e^{\frac{j}{6}\pi})|$$

$$|G_H(e^{j2\frac{5\pi}{6}})| = |G_H(e^{j(2\pi-\frac{\pi}{3})})| = G_H\, e^{j2\pi}\cdot e^{j(\frac{\pi}{3})}$$



Problem 7.6

Solution:

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7.6 $\cos\omega_0 n = \frac{1}{2}(e^{j\omega_0 n} + e^{-j\omega_0 n})$

~~y[n] = x~~ $\sin\omega_0 n = \frac{1}{2j}(e^{j\omega_0 n} - e^{-j\omega_0 n})$

解 $y[n] = x[n]\{2\cos^2\omega_0 n\, h_{LP}[n] + 2\sin^2\omega_0 n\, h_{LP}[n]\}$

$= x[n]\cdot 2h_{LP}[n]$

$$Y(z) = \frac{1}{4}\{H_{LP}(z\cdot e^{j\omega_0})X(ze^{j\omega_0}) + H_{LP}(z\cdot e^{j\omega_0})\cdot X(z) + H_{LP}(ze^{-j\omega_0})X(z) + H_{LP}(ze^{-j\omega_0})X(ze^{-j2\omega_0})\}$$
$$-\frac{1}{4}\{H_{LP}(z\cdot e^{j\omega_0})X(ze^{j2\omega_0}) - H_{LP}(ze^{j\omega_0})X(z) + H_{LP}(z\cdot e^{-j\omega_0})X(z) + H_{LP}(z\cdot e^{-j\omega_0})X(z\cdot e^{-j2\omega_0})\}$$

$$\therefore H(z) = \frac{1}{2}\{H_{LP}(ze^{j\omega_0}) + H_{LP}(ze^{-j\omega_0})\}$$

Problem 7.14

Solution:

$$F(z) = z\left(\frac{G(z)+\partial}{1+\partial G(z)}\right)$$

7.14

$$\therefore F(e^{j\omega}) = e^{j\omega}\left(\frac{G(e^{j\omega})+\partial}{1+\partial G(e^{j\omega})}\right)$$

$$= e^{j\omega}\left(\frac{e^{j\phi(\omega)}+\partial}{1+\partial e^{j\phi(\omega)}}\right)$$

$$\therefore \left|F(e^{j\omega})\right|^2 = \left|\frac{e^{j\phi(\omega)}+\partial}{1+\partial e^{j\phi(\omega)}}\right|^2$$

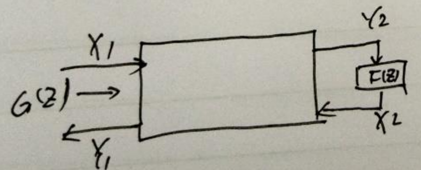
$$= \frac{1+2\partial\cos(\phi(\omega))+\partial^2}{1+2\partial\sin(\phi(\omega))+\partial^2} = 1$$

Let $z=\lambda$ be a pole of $F(z)$

$$G(z)\Big|_{z=\lambda} = \frac{F(z)-\partial z}{z-\partial F(z)}\Big|_{z=\lambda} = -\frac{1}{\partial}$$

Hence, $F(z)$ is LBR.

The order of $F(z)$ is the same as that of $G(z)$



$$G(z) = \frac{-\partial+z^{-1}F(z)}{1-\partial z^{-1}F(z)} = \frac{C+DF(z)}{A+BF(z)}$$

$A=1$

$B=-\partial z^{-1}$

$C=-\partial$

$D=z^{-1}$

Problem 7.20

Solution:

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7.20 $$A_2(e^{j\omega}) = \frac{d_2 + d_1 e^{-j\omega} + e^{-2j\omega}}{1 + d_1 e^{-j\omega} + d_2 e^{-2j\omega}} = \frac{d_2 e^{j\omega} + d_1 + e^{-j\omega}}{e^{j\omega} + d_1 + d_2 e^{-j\omega}}$$

$$= \frac{d_1 + (d_2+1)\cos\omega + j(d_2-1)\sin\omega}{d_1 + (d_2+1)\cos\omega - j(d_2-1)\sin\omega}$$

$$\therefore \theta(\omega) = 2\tan^{-1}\left(\frac{(d_2-1)\sin\omega}{d_1+(d_2+1)\cos\omega}\right)$$

$$\therefore \tau_p(\omega) = \frac{-\theta(\omega)}{\omega} = \frac{-2}{\omega}\tan^{-1}\left(\frac{(d_2-1)\sin\omega}{d_1+(d_2+1)\cos\omega}\right)$$

$\begin{cases} \sin\omega = \omega \\ \cos\omega = 1 \end{cases}$ when ω is very small

$$\therefore \tau_p(\omega) = \delta = -\frac{2(d_2-1)}{d_1+d_2+1}$$

$$d_1 = 2\left(\frac{2-\delta}{1+\delta}\right)$$

$$d_2 = \frac{(2-\delta)(1-\delta)}{(2+\delta)(1+\delta)}$$

$$\therefore \delta = -\frac{2(d_2-1)}{d_1+d_2+1}$$

Problem 7.27

Solution:

7.27.

$$(1+\alpha z^{-1})(1+\alpha z)\Big|_{z=e^{j\omega}} = (1+\alpha^2)+2\alpha\cos\omega$$

$$|H(e^{j\omega})|^2 = H(z)H(z^{-1})\Big|_{z=e^{j\omega}}$$

$$= \frac{4[1.25+0.5(z+z^{-1})][1.36-0.6(z+z^{-1})]}{[1.36+0.6(z+z^{-1})][1.64+0.8(z+z^{-1})]}\Bigg|_{z=e^{j\omega}}$$

$$\therefore H(z)H(z^{-1}) = \frac{4(1+0.5z)(1+0.5z^{-1})(1-0.6z)(1-0.6z^{-1})}{(1+0.6z)(1+0.6z^{-1})(1+0.8z)(1+0.8z^{-1})}$$

$\therefore$ i) $$H(z) = \frac{2(1+0.5z^{-1})(1-0.6z^{-1})}{(1+0.6z^{-1})(1+0.8z^{-1})}$$

ii) $$H(z) = \frac{2(1+0.5z)(1-0.6z^{-1})}{(1+0.6z^{-1})(1+0.8z^{-1})}$$

iii) $$H(z) = \frac{2(1+0.5z)(1-0.6z)}{(1+0.6z^{-1})(1+0.8z^{-1})}$$

iv) $$H(z) = \frac{2(1+0.5z^{-1})(1-0.6z)}{(1+0.6z^{-1})(1+0.8z^{-1})}$$