DSP UESTC 4005: Homework #chapter 5

Due on: April 28 2019 at 23:59:59

Instructor: Wenhui Xiong

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Problem 5.1

Solution:

5.1 $\tilde{y}[n] = \sum_{r=0}^{N-1} \tilde{x}[r]\,\tilde{h}[n-r]$

$\tilde{x}[n] = \tilde{x}[n+kN] \quad k \in \mathbb{Z}.$

$\tilde{h}[n] = \tilde{h}[n+kN].$

$\therefore \tilde{y}[n+kN] = \sum_{r=0}^{N-1} \tilde{x}[r] \cdot \tilde{h}[n+kN-r]$

since $\tilde{h}[n]$ is period sequence of period N

$\therefore \cancel{\tilde{h}[n]=}\ \tilde{h}[n-r] = \tilde{h}[n-r+kN]$

$\therefore \tilde{y}[n+kN] = \sum_{r=0}^{N-1} \tilde{x}[r]\,\tilde{h}[n-r] = \tilde{y}[n]$

Hence, $\tilde{y}[n]$ is also periodic in n with a period N

Problem 5.8&5.23

Solution:

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5.8 (5) a) $x_a[n] = \cos(2\pi n/N) = \frac{1}{2}(e^{j2\pi n/N} + e^{-j2\pi n/N})$

$\therefore X_a[k] = \sum_{n=0}^{N-1} x_a[n] e^{-j2\pi kn/N}$

$= \frac{1}{2}\sum_{n=0}^{N-1} e^{-j2\pi(k-1)n/N} - \frac{1}{2}\sum_{n=0}^{N-1} e^{j2\pi n(k+1)/N}$

eq (5-10): $\sum_{n=0}^{N-1} e^{j2\pi(k-l)n/N} = \frac{1-e^{j2\pi(k-l)}}{1-e^{j2\pi(k-l)/N}}$

5-11 : $\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi(k-l)n/N} = \begin{cases} 1, & k=l+rN \\ 0, & k\neq l \end{cases}$

$\therefore \cancel{X_a[k] =}$

$\therefore \sum_{n=0}^{N-1} e^{-j2\pi(k-1)n/N} = N, \quad k=1+rN$

$\sum_{n=0}^{N-1} e^{j2\pi n(k+1)/N} = N, \quad k=-1+rN$

$\therefore X_a[k] = \begin{cases} N/2, & k=1 \\ -N/2, & k\neq N-1 \\ 0, & \text{others} \end{cases}$

b) $x_b[n] = \sin^2(2\pi n/N) = \frac{1-\cos(4\pi n/N)}{2} = \frac{1}{2} - \frac{1}{2}\cos(4\pi n/N)$

$= \frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2}(e^{j4\pi n/N} + e^{-j4\pi n/N})$

$\therefore X_b[k] = \sum_{n=0}^{N-1} x_b[n]\cdot e^{-j2\pi kn/N}$

$= \frac{1}{2}\sum_{n=0}^{N-1} e^{-j2\pi kn/N} - \frac{1}{4}\sum_{n=0}^{N-1} e^{-j2\pi n(2-k)/N} - \frac{1}{4}\sum_{n=0}^{N-1} e^{-j2\pi n(2+k)/N}$

$\sum_{n=0}^{N-1} e^{-j2\pi kn/N} = N, \quad k=0+rN$

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$\sum_{n=0}^{N-1} e^{-j2\pi n(2-k)/N} = N, \quad k=2+rN$

$\sum_{n=0}^{N-1} e^{-j2\pi n(2+k)/N} = N, \quad k=-2+rN$

$\therefore \; X_b[k] = \begin{cases} \frac{N}{2}, & k=0 \\ -\frac{N}{4}, & k=2,\ k=-2. \\ 0, & \text{others}. \end{cases}$

c) $x_c[n] = \sin^3(2\pi n/N) = \frac{3}{4}\sin\left(\frac{2\pi n}{N}\right) - \frac{1}{4}\sin\left(\frac{6\pi n}{N}\right)$

$= \frac{3}{4}\frac{1}{2j}\left(e^{j2\pi n/N} - e^{-j2\pi n/N}\right) - \frac{1}{4}\frac{1}{2j}\left(e^{j6\pi n/N} - e^{-j6\pi n/N}\right)$

$X_c[k] = \sum_{n=0}^{N-1} x_c[n]\, e^{-j2\pi kn/N}$

$\therefore \; X_c[k] = \begin{cases} 3N/8j, & k=0, N \\ -N/8j, & k=3, N-3 \\ 0, & \text{others}. \end{cases}$

5-23 $\begin{bmatrix} p[0] \\ p[1] \\ p[2] \\ p[3] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 3.5 \\ -0.5-4.5j \\ 2.5 \\ -0.5+4.5j \end{bmatrix} = \begin{bmatrix} 1.25 \\ -4.5 \\ 1.75 \\ 5 \end{bmatrix}$

$\begin{bmatrix} d[0] \\ d[1] \\ d[2] \\ d[3] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 17 \\ 7.4+12j \\ 17.8 \\ 7.4-12j \end{bmatrix} = \begin{bmatrix} 12.4 \\ 5.8 \\ 5 \\ -6.2 \end{bmatrix}$

$\therefore \; X(e^{j\omega}) = 1.25 - 4.5e^{-j\omega} + 1.75e^{-j2\omega} + 5e^{-j3\omega}$

$12.4 + 5.8e^{-j\omega} + 5e^{-j2\omega} - 6.2e^{-j3\omega}$

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Problem 5.29&5.39&5.60

Solution:

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5-29.

$$y[3] = \sum_{l=0}^{5} x[l]\, h[\langle n-l\rangle_6] = x[0]h[3] + x[1]h[2] + x[2]h[1] + x[3]h[0] + x[4]h[5] + x[5]h[4] = 35$$

5-39

a) $X[\langle N-k\rangle_N] = \sum_{n=0}^{N-1} x[n] W_N^{(N-k)n} = \sum_{n=0}^{N-1} x[n] W_N^{-kn} = X^*[k]$

b) $X[0] = \sum_{n=0}^{N-1} x[n] W_N^0 = \sum_{n=0}^{N-1} x[n]$. which is real

c) $X[\frac{N}{2}] = \sum_{n=0}^{N-1} x[n] W_N^{(N/2)n} = \sum_{n=0}^{N-1} (-1)^n x[n]$, which is real

5-60

a) $X[0] = \sum_{n=0}^{9} x[n] = 6.44$

b) $X[5] = \sum_{n=0}^{9} (-1)^n x[n] = -30.66$

c) $\sum_{k=0}^{9} X[k] = 10 \cdot x[0] = 62.9$

d) $\sum_{k=0}^{9} e^{-j3\pi k/5} X[k] = 10\, x[7] = 9.3$

e) $\sum_{k=0}^{9} |X[k]|^2 = 10 \sum_{n=0}^{9} |x[n]|^2 = 4913.4$

Problem 5.79

Solution:

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5.79

a) overlap-add method:

Since the impulse response is of length 110 and the DFT size to be used is 128, the number of data samples required for each convolution will be $128-109=19$. Thus the total number of DFTs required for the length-1300 data sequence is $\lceil \frac{1300}{19} \rceil = 69$

Also, the DFT of the impulse response needs to be computed once.

Hence, the total number of DFTs used are $69+1=70$. The total number of IDFTs is 6

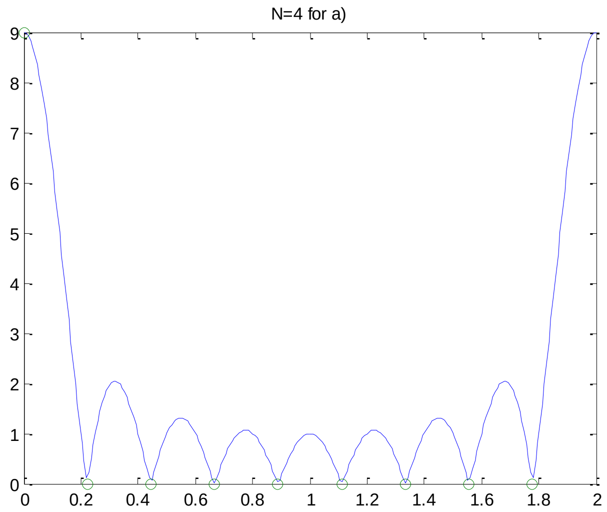
b) overlap-save method:

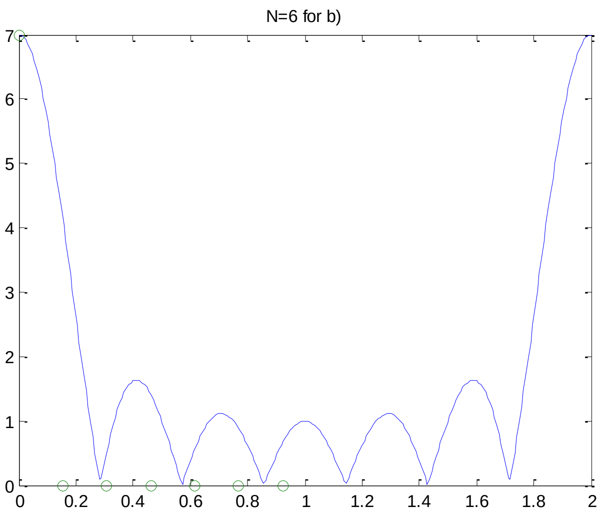
In this case, since the first $110-1=109$ points are lost, we need to pad the data sequence with 109 zeros for a total length of ~~[illegible]~~ 1409. Again, each convolution will result in $128-109=19$ values. Thus the total number of DFTs required for the data is $\lceil \frac{1409}{19} \rceil = 75$.

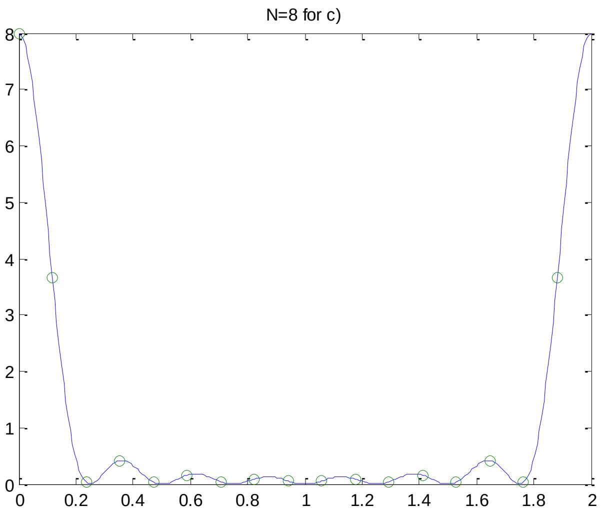
Again, 1 DFT is required for the impulse response. The total number of DFTs used is $75+1=76$. The total number of IDFTs used is 75.

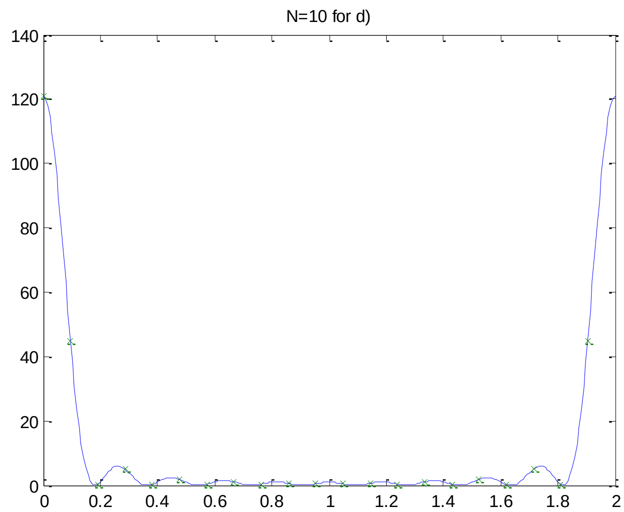
Problem M5.1

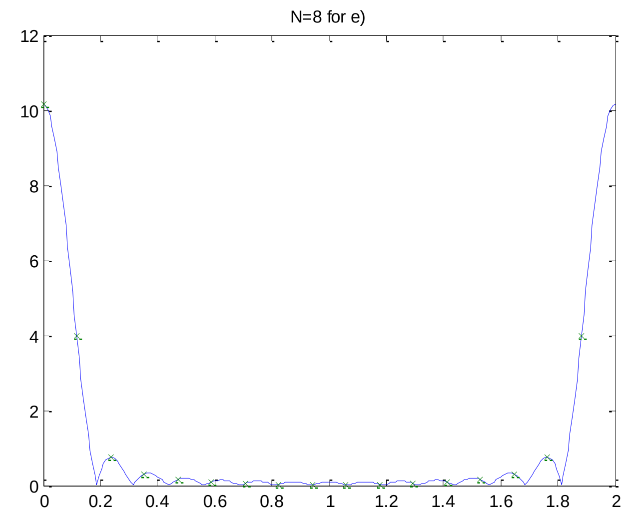
Solution:




N=6 for b)
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

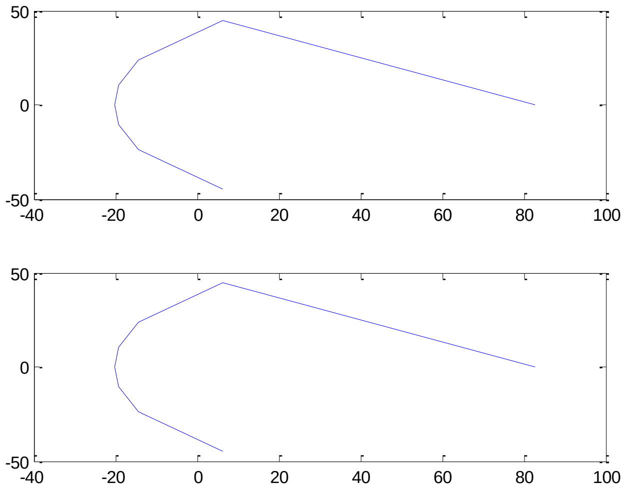

N=8 for c)
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

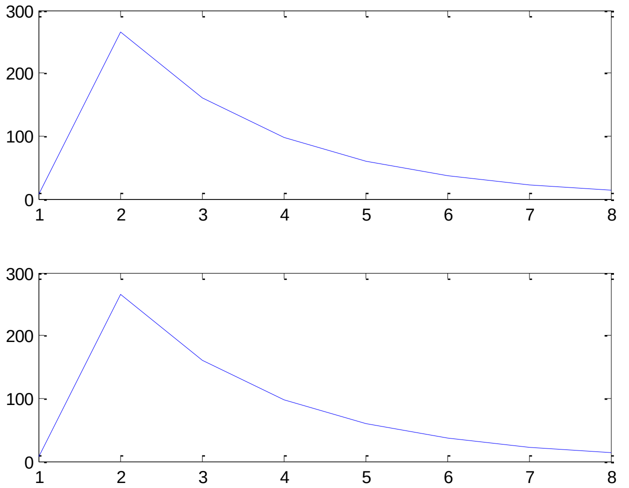

N=10 for d)
140
120
100
80
60
40
20
0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

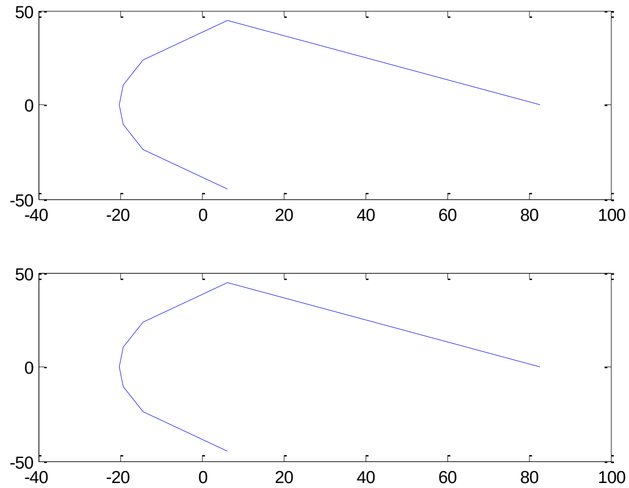

N=8 for e)
12
10
8
6
4
2
0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

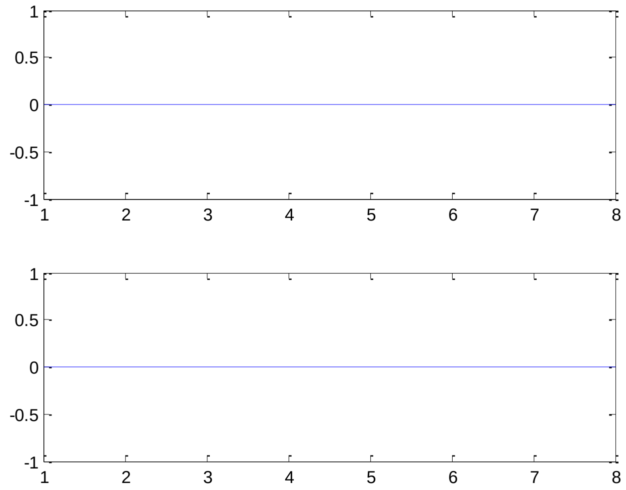
Problem M5.3

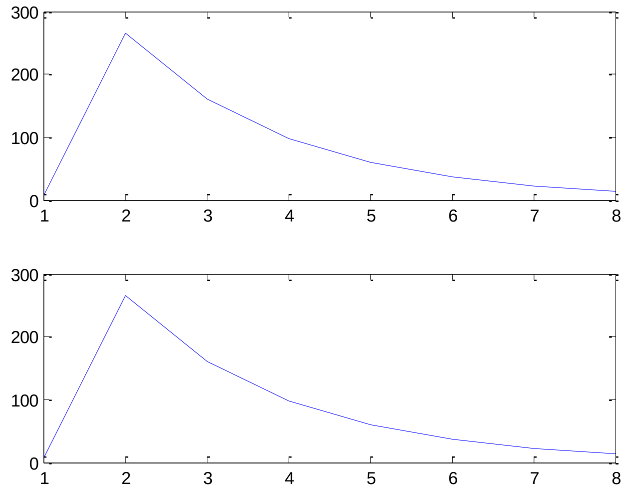
Solution:

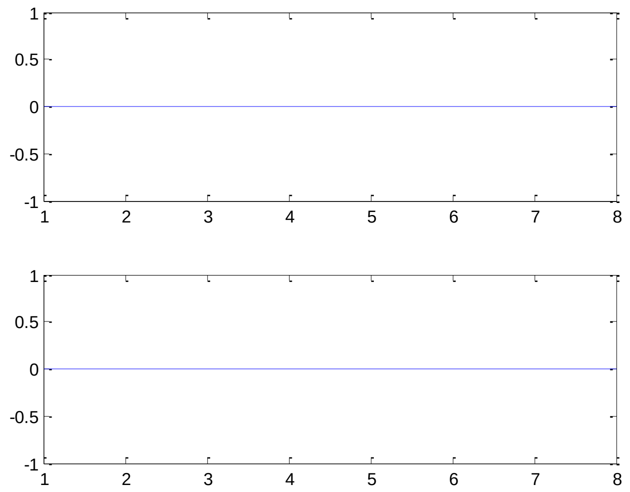



300
200
100
0
1
2
3
4
5
6
7
8
300
200
100
0
1
2
3
4
5
6
7
8


50
0
-50
-40
-20
0
20
40
60
80
100
50
0
-50
-40
-20
0
20
40
60
80
100


1
0.5
0
-0.5
-1
1 2 3 4 5 6 7 8
1
0.5
0
-0.5
-1
1 2 3 4 5 6 7 8


300
200
100
0
1 2 3 4 5 6 7 8
300
200
100
0
1 2 3 4 5 6 7 8



Problem M5.8

Solution:

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6.4400          -30.6600          62.9000          -91.5000 - 0.0000i

4.9134e+003
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