DSP UESTC 4005: Homework #2

Due on: March 31 2019 at 23:59:59

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Problem 3.2

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3.2 $X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$

a)

$$Y_a(j\Omega) = \int_{-\infty}^{\infty} \sin(\Omega_0 t)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{2j}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right) e^{-j\Omega t}\, dt$$

$$= \frac{1}{2j}\int_{-\infty}^{\infty} \left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right) e^{-j\Omega t}\, dt$$

$$= \frac{1}{2j}\left[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)\right]$$

b)

$$u_a(t) = e^{-\partial|t|} = e^{-\partial t}u(t) + e^{\partial t}u(-t)$$

$$U_a(j\Omega) = \int_{-\infty}^{\infty}\left[e^{-\partial t}u(t) + e^{\partial t}u(-t)\right] e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(\partial - j\Omega)t}\, dt + \int_{0}^{\infty} e^{-(\partial + j\Omega)t}\, dt$$

$$= \frac{1}{\partial - j\Omega} e^{(\partial - j\Omega)t}\Big]_{-\infty}^{0} + \frac{1}{-(\partial + j\Omega)} e^{-(\partial + j\Omega)t}\Big]_{0}^{\infty}$$

$$= \frac{1}{\partial - j\Omega}(1-0) + \frac{1}{-(\partial + j\Omega)}(0-1)$$

$$= \frac{1}{\partial - j\Omega} + \frac{1}{\partial + j\Omega}$$

c) $V_a(j\Omega) = \int_{-\infty}^{\infty} e^{j\Omega_0 t} e^{-j\Omega t} dt$

$= \int_{-\infty}^{\infty} e^{jt(\Omega_0-\Omega)} dt = \int_{-\infty}^{\infty} e^{-jt(\Omega-\Omega_0)} dt$

$\cancel{= \frac{1}{j(\Omega_0-\Omega)}} = \delta(\Omega-\Omega_0)$,

d). $P_a(j\Omega) = \int_{-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(t-lT) e^{-j\Omega t} dt$

$= \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-lT) e^{-j\Omega t} dt$.

$= \sum_{l=-\infty}^{\infty} e^{-j\Omega l T}$

e) $G_a(j\Omega) = \int_{-\infty}^{\infty} e^{-\alpha t^2} \cdot e^{-j\Omega t} d\tau$

不会

Problem 3.6

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3.2. ~~$Y(j\omega) = \sum_{n=-\infty}^{\infty} y t \int y(t) \cdot e^{-j\omega t} dt$~~

解. $Y(j\omega) = \int_{-\infty}^{\infty} y_a(t)\, e^{-j\Omega t}\, dt$

1). $y_a(t) = \sin(\Omega_0 t.)$

3.6 a)

$x_a(t - t_0)$

suppose $F\{x_a(t)\} = X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j\Omega t}\, dt$

$$\therefore \int_{-\infty}^{\infty} x_a(t-t_0)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x_a(t-t_0)\, e^{-j\Omega[(t-t_0)+t_0]}\, dt$$

$$= \int_{-\infty}^{\infty} x_a(t-t_0)\, e^{-j\Omega(t-t_0)} \cdot e^{-j\Omega t_0}\, dt$$

$$= e^{-j\Omega t_0} \int_{-\infty}^{\infty} x_a(t-t_0)\, e^{-j\Omega(t-t_0)}\, d(t-t_0)$$

$$= e^{-j\Omega t_0} F(x_a(t)) = e^{-j\Omega t_0} X_a(j\Omega).$$

b) $\int_{-\infty}^{\infty} x_a(t)\, e^{j\Omega_0 t} \cdot e^{-j\Omega t}\, dt$

$$= \int_{-\infty}^{\infty} x_a(t)\, e^{-j(\Omega-\Omega_0)t}\, dt.$$

$$= X_a(j(\Omega-\Omega_0))$$

c) $x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega$

$\therefore 2\pi x_a(-t) = \int_{-\infty}^{\infty} X_a(j\Omega) e^{-j\Omega t} d\Omega$

$\cancel{\therefore 2\pi x_a(\ ) = \int_{-\infty}^{\infty} X_a(t) e^{-j\Omega t} d\Omega}$

$\therefore 2\pi x_a(-\Omega) = \int_{-\infty}^{\infty} X_a(jt) e^{-j\Omega t}$

d) $\int_{-\infty}^{\infty} x_a(at) e^{-j\Omega t} dt$

$= \int_{-\infty}^{\infty} x_a(at) e^{-j\Omega (at) \cdot \frac{1}{a}} d(at) \frac{1}{|a|}$

$= \frac{1}{|a|}\int_{-\infty}^{\infty} x_a(at) e^{-j\Omega (at) \cdot \frac{1}{a}} d(at)$

$= \frac{1}{|a|} X_a\left(j\frac{\Omega}{a}\right).$

e) $x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega$

$\therefore \frac{d x_a(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\Omega\, X_a(j\Omega) e^{j\Omega t} d\Omega$

Problem 3.18

#) $$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] \cdot e^{-j\omega n}.$$

a) $$y_1[n] = \sum_{k=-N}^{N} \delta[n-k]$$

$$\therefore Y_1(e^{j\omega}) = F\left[\sum_{k=-N}^{N} \delta[n-k]\right]$$

$$= \sum_{k=-N}^{N} F[\delta[n-k]]$$

$$= \sum_{k=-N}^{N} e^{jk\omega} \cdot 1, \quad (F\{\delta[n]\} = 1)$$

$$= \sum_{k=-N}^{N} e^{-jk\omega} = \frac{e^{-j\omega N} - e^{j\omega(N+1)}}{1 - e^{j\omega}}$$

b) $$y_2[n] = \sum_{k=0}^{N} \delta[n-k]$$

$$\therefore Y_2(e^{j\omega}) = F\left[\sum_{k=0}^{N} \delta[n-k]\right]$$

$$= \sum_{k=0}^{N} e^{jk\omega} \cdot 1$$

$$= \sum_{k=0}^{N} e^{-jk\omega} = \frac{1 - e^{-j\omega(N+1)}}{1 - e^{-j\omega}}$$

c) $$y_3[n] = 1 - \frac{|n|}{N}, \quad -N \le n \le N$$

$$Y_3(e^{j\omega}) = \sum_{n=-N}^{N} \left(1 - \frac{|n|}{N}\right) e^{-j\omega n}.$$

$$= \sum_{n=-N}^{-1} \left(1 + \frac{n}{N}\right) e^{-j\omega n} + \sum_{n=0}^{N} \left(1 - \frac{n}{N}\right) e^{-j\omega n}$$

$$Y_3(e^{jw}) = \sum_{n=-N}^{N} 1 \cdot e^{-jwn} + \sum_{n=-N}^{-1} \frac{n}{N} e^{-jwn} + \sum_{n=0}^{N} \frac{-n}{N} e^{-jwn}$$

$$= \sum_{n=-N}^{N} e^{-jwn} + \frac{1}{N}\left[\sum_{n=-N}^{-1} n e^{-jwn} - \sum_{n=0}^{N} n e^{-jwn}\right]$$

$$= \frac{1}{N} \frac{\sin^2(wN/2)}{\sin^2(w/2)}$$

d) $y_4[n] = N+1 + \sum_{n=-N}^{N} -|n|$

$$= (N+1) - \sum_{n=-N}^{N} |n|$$

$$= (N+1) - \left[\sum_{n=-N}^{-1} (-n) + \sum_{n=0}^{N} n\right]$$

$$= (N+1) + \sum_{n=-N}^{-1} n - \sum_{n=0}^{N} n$$

$\therefore$ ~~$Y_4(e^{jw}) = (N+1) \sum_{n=-N}^{N} e^{jnw} +$~~

$$Y_4(e^{jw}) = Y_1(e^{jw}) + N \cdot Y_3(e^{jw})$$

$$= \frac{\sin w[N+\frac{1}{2}]}{\sin(w/2)} + \frac{\sin^2(wN/2)}{\sin^2(w/2)}$$

e) $y_5[n] = \frac{1}{2}\left(e^{j\left(\frac{\pi n}{2N}\right)} + e^{-j\frac{-\pi n}{2N}}\right), \quad n \in [-N, N]$

$$\therefore Y_5(e^{jw}) = \frac{1}{2}\sum_{n=-N}^{N} e^{-j(w+\frac{\pi}{2N})n} + \frac{1}{2}\sum_{n=-N}^{N} e^{-j(w-\frac{\pi}{2N})n}$$

$$= \frac{1}{2} \frac{\sin(w-\frac{\pi}{2N})(N+\frac{1}{2})}{\sin((w-\frac{\pi}{2N})/2))} + \frac{1}{2} \frac{\sin(w+\frac{\pi}{2N})(N+\frac{1}{2})}{\sin((w+\frac{\pi}{2N})/2)}$$

Problem 3.23

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3.23

a) $H_1(e^{j\omega}) = -4+3\cdot\frac{1}{2}(e^{j\omega}+e^{-j\omega}) + 4\cdot\frac{1}{2}(e^{j2\omega}+e^{-j2\omega})$

$= 2e^{-2j\omega} + \frac{3}{2}e^{-j\omega} + (-4) + \frac{3}{2}e^{j\omega} + 2e^{2j\omega}$

$\therefore h_1[n] = \{2, \frac{3}{2}, -4, \frac{3}{2}, 2\}, \ n\in[-2,2]$

b) $H_2(e^{j\omega}) = H_1(e^{j\omega})\cdot e^{-j\omega/2}\cdot\frac{1}{2}(e^{j\omega/2}+e^{-j\omega/2})$

$= H_1(e^{j\omega})(\frac{1}{2}+\frac{1}{2}e^{-j\omega})$

$\therefore H_1(e^{j\omega}) = [0, 2, \frac{3}{2}, -4, \frac{3}{2}, 2]$

$\frac{H_1}{2}(e^{j\omega}) = [0, 1, \frac{3}{4}, -2, \frac{3}{4}, 1]$

$e^{-j\omega}$ $\frac{H_1}{2}(e^{j\omega}) = [1, \frac{3}{4}, -2, \frac{3}{4}, 1, 0]$

$\therefore H_2(e^{j\omega}) = [1, \frac{11}{4}, -\frac{5}{4}, -\frac{5}{4}, \frac{7}{4}, 1]$

$\therefore h_2[n] = \{1, \frac{11}{4}, -\frac{5}{4}, -\frac{5}{4}, \frac{7}{4}, 1\}, \ n\in[-3,2]$

c) $H_3(e^{jw}) = H_1(e^{jw}) \cdot \frac{1}{2j} \cdot j(e^{jw} - e^{-jw})$

$= H_1(e^{jw}) \cdot \frac{1}{2}(e^{jw} - e^{-jw})$

$\therefore \frac{H_1}{2}(e^{jw}) = [0, 1, \frac{3}{4}, -2, \frac{3}{4}, 1, 0]$

$e^{jw} \cdot \frac{H_1}{2}(e^{jw}) = [0, 0, 1, \frac{3}{4}, -2, \frac{3}{4}, 1]$

$e^{-jw} \cdot \frac{H_1}{2}(e^{jw}) = [1, \frac{3}{4}, -2, \frac{3}{4}, 1, 0, 0]$

$\therefore H_3(e^{jw}) = [-1, -\frac{3}{4}, -3, 0, -3, \frac{3}{4}, 1]$

$\therefore h_3[n] = \{-1, -\frac{3}{4}, -3, 0, -3, \frac{3}{4}, 1\} \quad n \in [-3, 3]$

d) $H_4(e^{jw}) = H_1(e^{jw}) \cdot \frac{1}{2j} j(e^{j\frac{w}{2}} - e^{-j\frac{w}{2}}) \cdot e^{jw/2}$

$= H_1(e^{jw}) \cdot \frac{1}{2}(e^{jw} - 1)$

$\frac{H_1}{2}(e^{jw}) = [1, \frac{3}{4}, -2, \frac{3}{4}, 1, 0]$

$e^{jw} \frac{H_1}{2}(e^{jw}) = [0, 1, \frac{3}{4}, -2, \frac{3}{4}, 1]$

$\therefore H_4(e^{jw}) = [-1, \frac{1}{4}, -\frac{11}{4}, -\frac{11}{4}, -\frac{1}{4}, 1]$

Problem 3.43&3.53

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3.43

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = \sum_{n=-\infty}^{\infty}\left|x[n]\right|^2$$

for $x[n] = -a^n u[-n-1]$, $|a|>1$

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$$

$$\left|X(e^{j\omega})\right|^2 = \frac{1}{1+a^2-2a\cos\omega}$$

a). $\frac{1}{5+4\cos\omega} = \frac{1}{1+2^2-2(-2)\cos\omega}$ $\quad\therefore a=-2$

$\therefore x[n] = -(-2)^n u[-n-1]$

$$\therefore 4\int_0^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 2\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 4\pi\sum_{n=-\infty}^{\infty}\left|x[n]\right|^2$$

$$= 4\pi\sum_{n=-\infty}^{-1}\left|(-2)^n\right|^2 = 4\pi\sum_{n=-\infty}^{-1}4^n = 4\pi\left(\frac{0-\frac{1}{4}\cdot 4}{1-4}\right) = \frac{4\pi}{3}$$

b). $\frac{1}{3.25-3\cos\omega} = \frac{1}{1+1.5^2-2(1.5)\cos\omega}$ $\quad\therefore a=1.5=\frac{3}{2}$.

$\therefore x[n] = \cancel{-(\frac{3}{2})^n u[}\ -\left(\frac{3}{2}\right)^n u[-n-1]$

$$\therefore \int_0^{\pi}\left|\frac{1}{3.25-3\cos\omega}\right|^2 d\omega = \frac{1}{2}\int_{-\pi}^{\pi}\left|\frac{1}{3.25-3\cos\omega}\right|^2 d\omega = \pi\sum_{n=-\infty}^{\infty}\left|\left(\frac{3}{2}\right)^n u[-n-1]\right|$$

$$= \pi\sum_{n=-\infty}^{-1}\left(\frac{3}{2}\right)^{2n} = \pi\cdot\left(\frac{0-\frac{4}{9}\cdot\frac{9}{4}}{1-\frac{9}{4}}\right) = \frac{5}{4}\pi$$

c)

since $DTFT\{-n a^n u[-n-1]\} = \dfrac{a e^{-j\omega}}{(1-a e^{-j\omega})^2}$

$\cancel{\therefore DTFT\{a^n u[-n-1]\} =}$

$\therefore DTFT\{-(n+1)a^n u[-n-1]\} = \dfrac{1}{(1-a e^{-j\omega})^2}$.

$\therefore y[n] = -(n+1)2^n u[-n-1]$

$4\int_0^{\pi} |X(e^{j\omega})|^2 d\omega = 2\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 4\pi \sum_{n=-\infty}^{-1} (n+1)^2 \cdot 4^n$.

$= 4\pi$.

3.53 $G_1(e^{j\omega}) \triangleq F[g_1[n]]$

$\therefore G_2(e^{j\omega}) = e^{-4j\omega} G_1(e^{j\omega}) + G_1(e^{j\omega})$

$= (1+e^{-4j\omega}) G_1(e^{j\omega})$

$G_3(e^{j\omega}) = G_1(e^{j\omega}) + e^{-7j\omega} G_1(e^{-j\omega})$

$\cancel{= (1+e^{-7j\omega}) G_1(e^{j\omega})}$

$G_4(e^{j\omega}) = G_1(e^{-j\omega}) \cdot e^{-3j\omega} + e^{-4j\omega} G_1(e^{j\omega})$

Problem 3.61

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3.61

$F_n = \pm 3000n, \quad n \in (-\infty, +\infty)$

F_1	300	F_{1m}	300	2700	3300
F_2	500	F_{2m}	500	2500	3500
F_3	1200	F_{3m}	1200	1800	4200
F_4	2150	F_{4m}	850	2150	5150
F_5	3500	F_{5m}	500	3500	6500

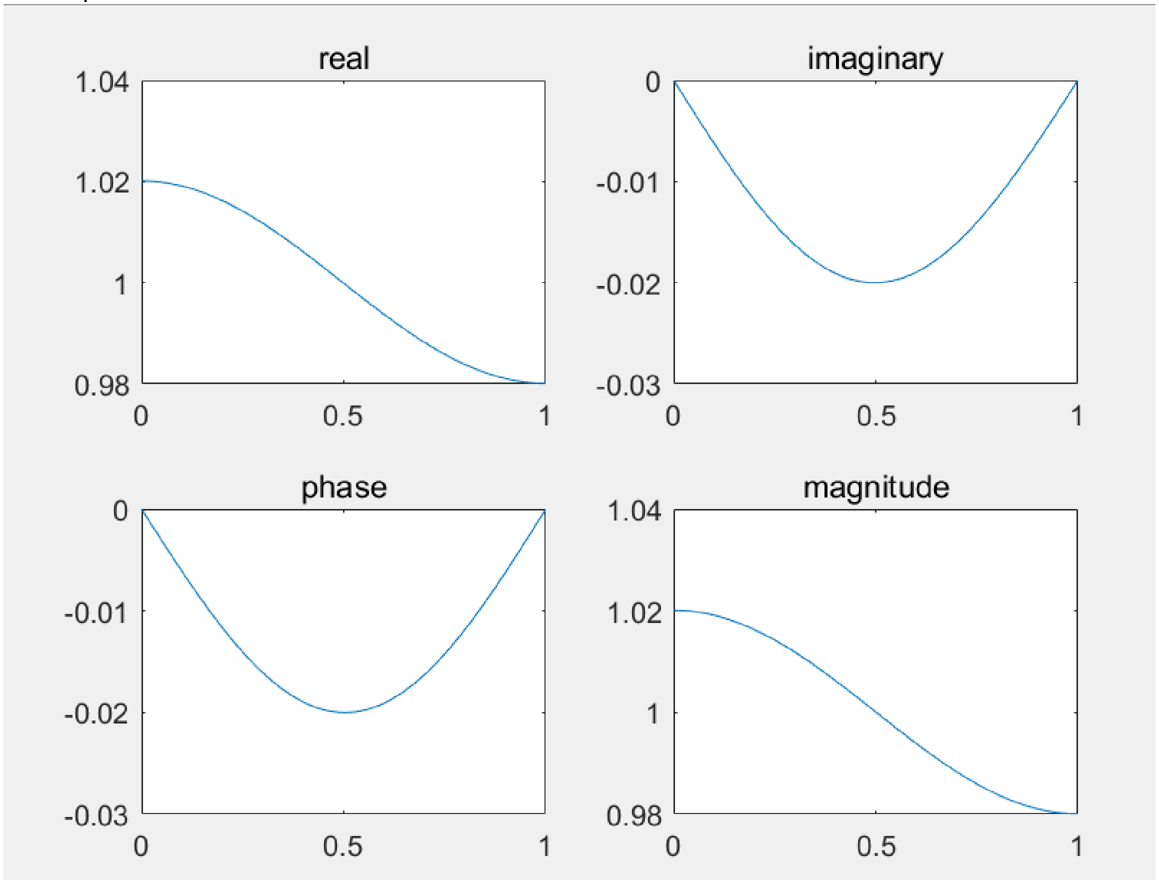
$\therefore$ after the LPF with cut-off frequency 900 HZ,

the remaining should be 300 HZ, 500 HZ, 850 HZ

Problem M 3.1

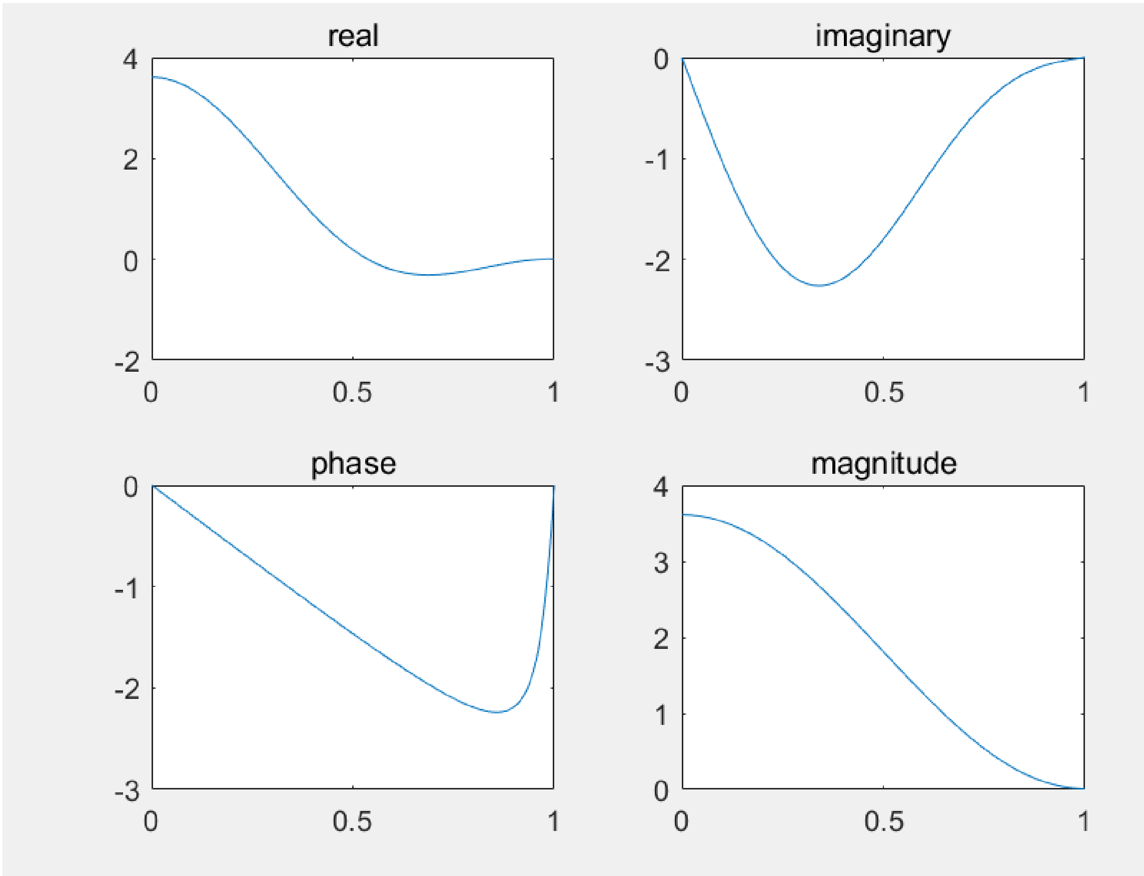
R=0.01

Theta=pi

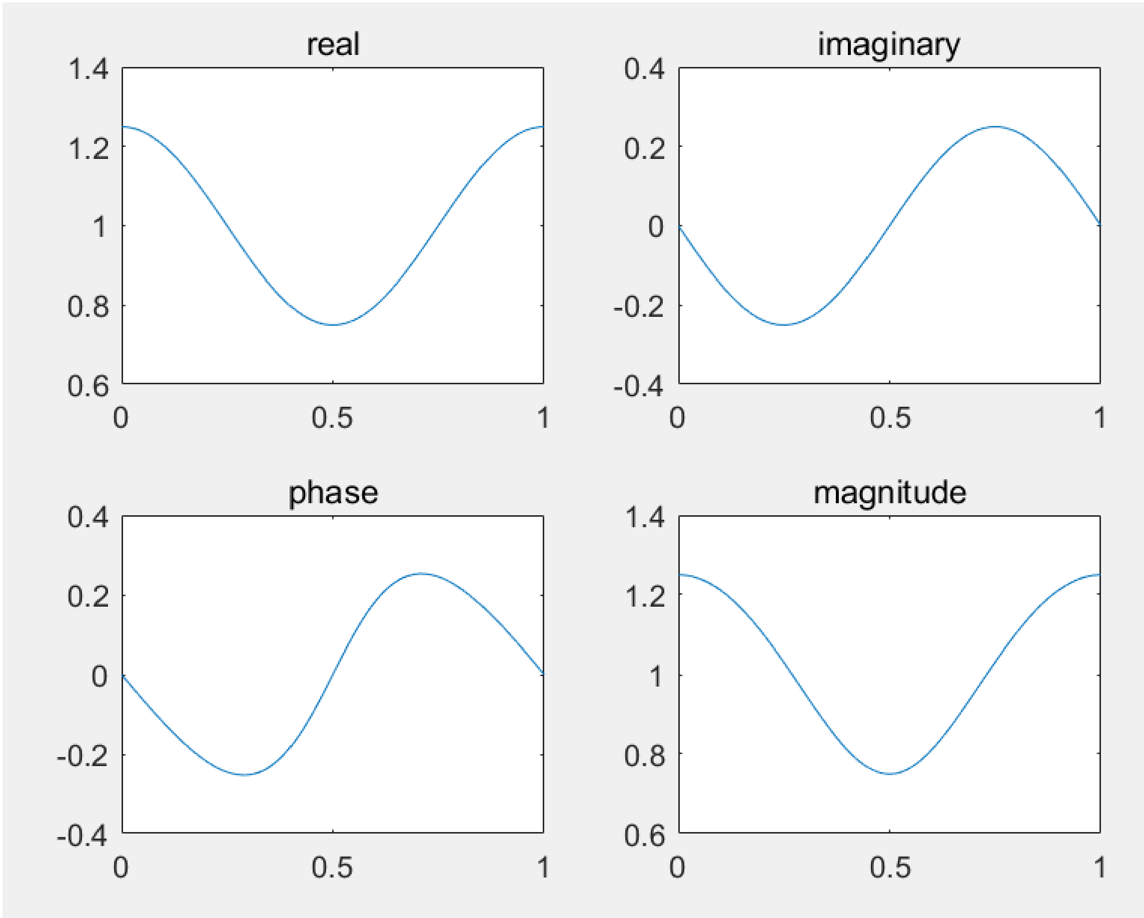


R=0.9

Theta=pi



R=0.5
Theta=pi/2


real
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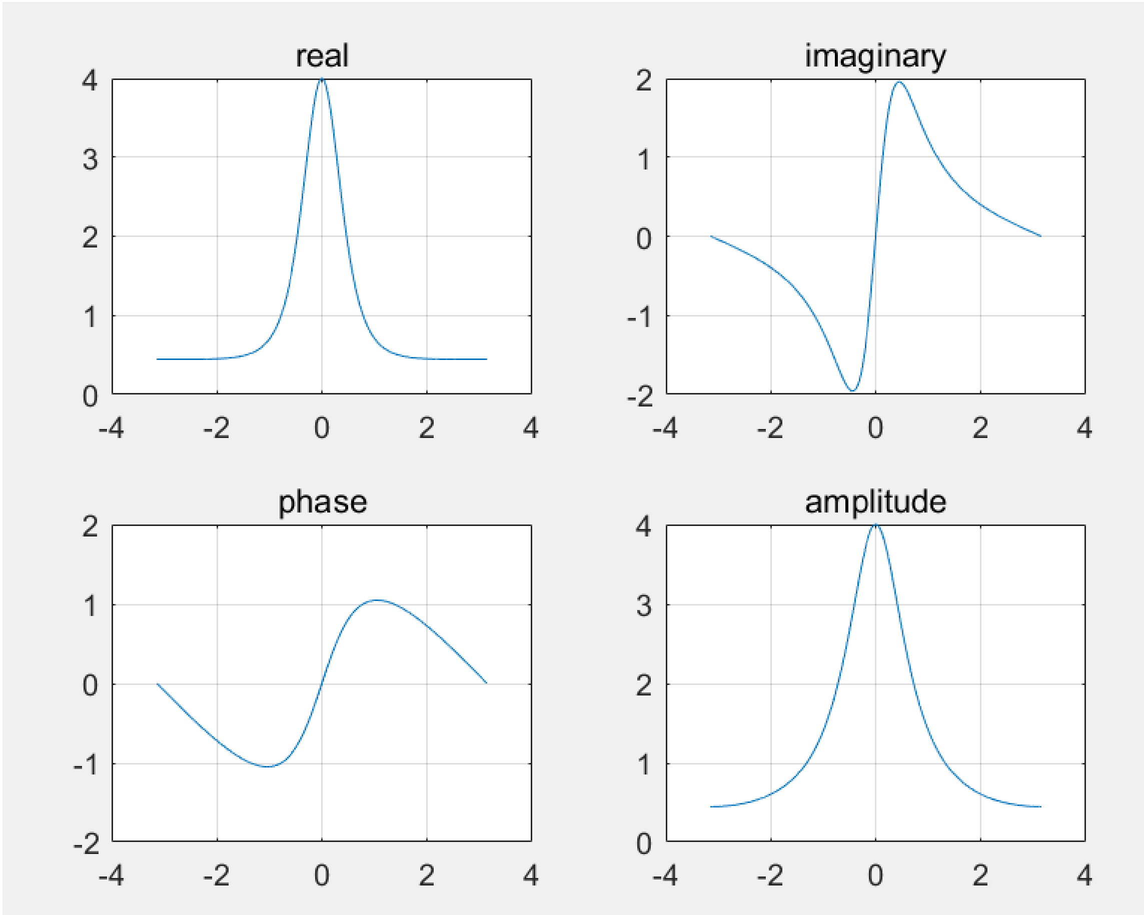
1.2
1
0.8
0.6
0
0.5
1
imaginary
0.4
0.2
0
-0.2
-0.4
0
0.5
1
phase
0.4
0.2
0
-0.2
-0.4
0
0.5
1
magnitude
1.4
1.2
1
0.8
0.6
0
0.5
1

Problem M 3.4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imaginary
phase
amplitude
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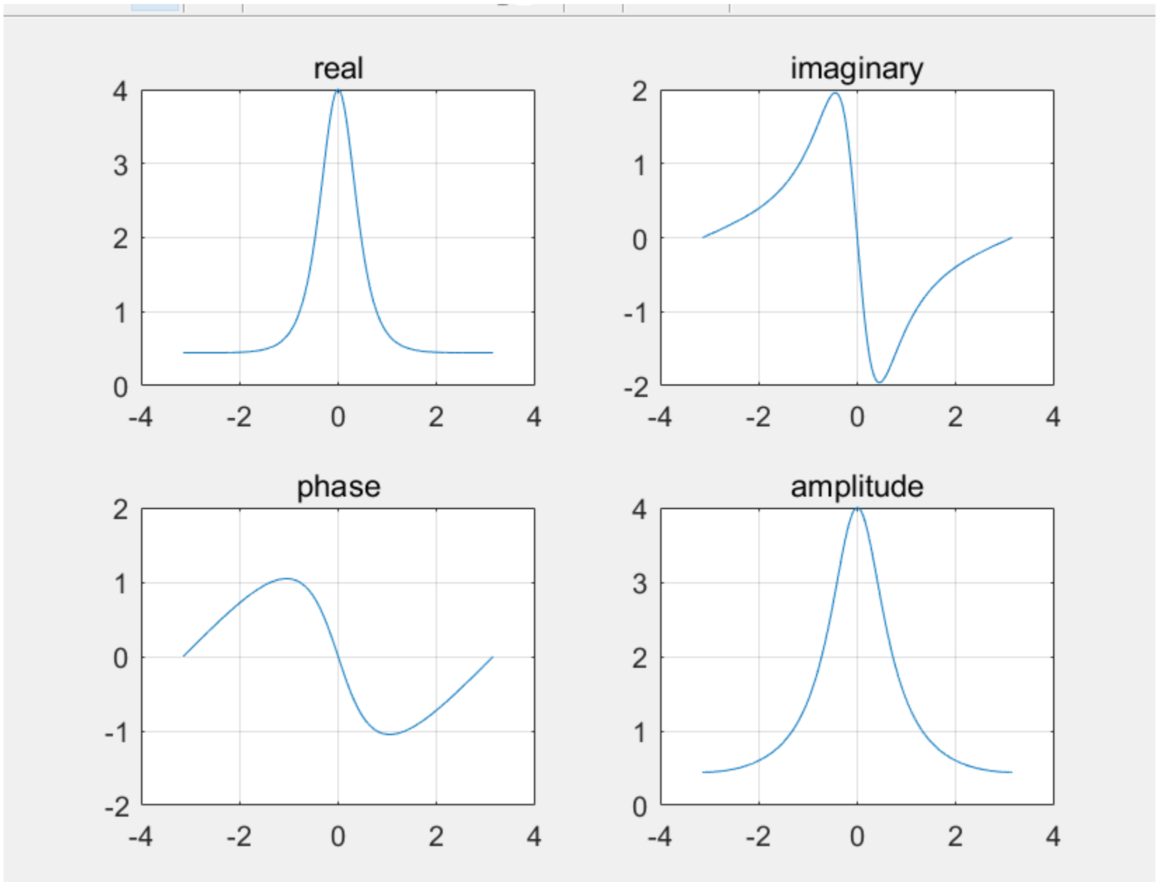
3
2
1
0
-1
-2
-4
-2
0
2
4



From these graphics, it is obvious that the real part and amplitude of a pair sequences could be the same, and the imaginary part and phase of a pair sequences could be the reverse of each other, when they are conjugate.

Eg:

```
x=1./((1-0.5.*exp(1i*omega)).^2);
y=1./((1-0.5.*exp(-1i*omega)).^2);
```

Problem M 3.5

The origin sequence is `x0=1+n+n.*1i;%xn`

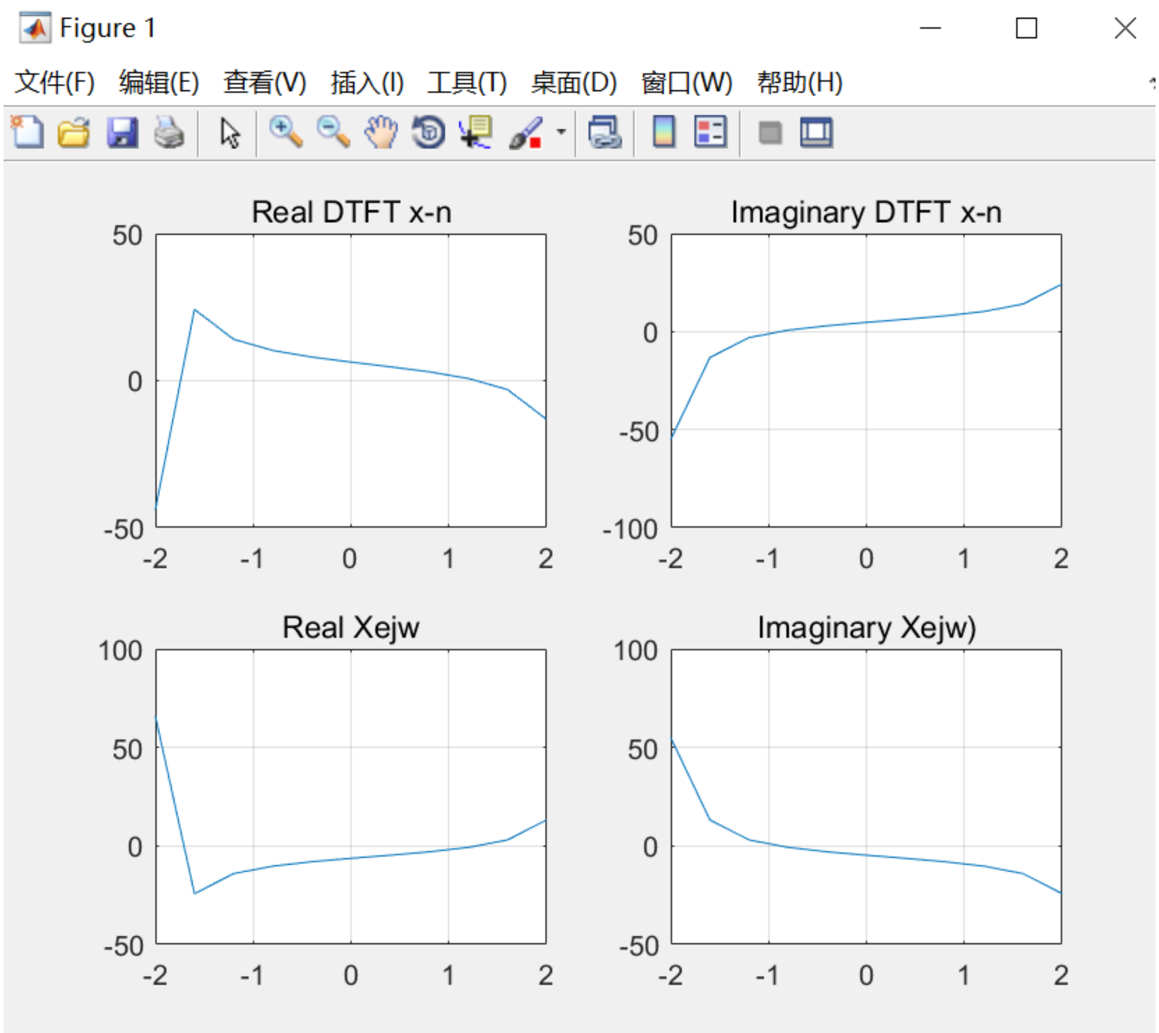


Figure 1
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Real DTFT x-n
Imaginary DTFT x-n
Real Xejw
Imaginary Xejw)
50
0
-50
-100
100
-2
-1
0
1
2

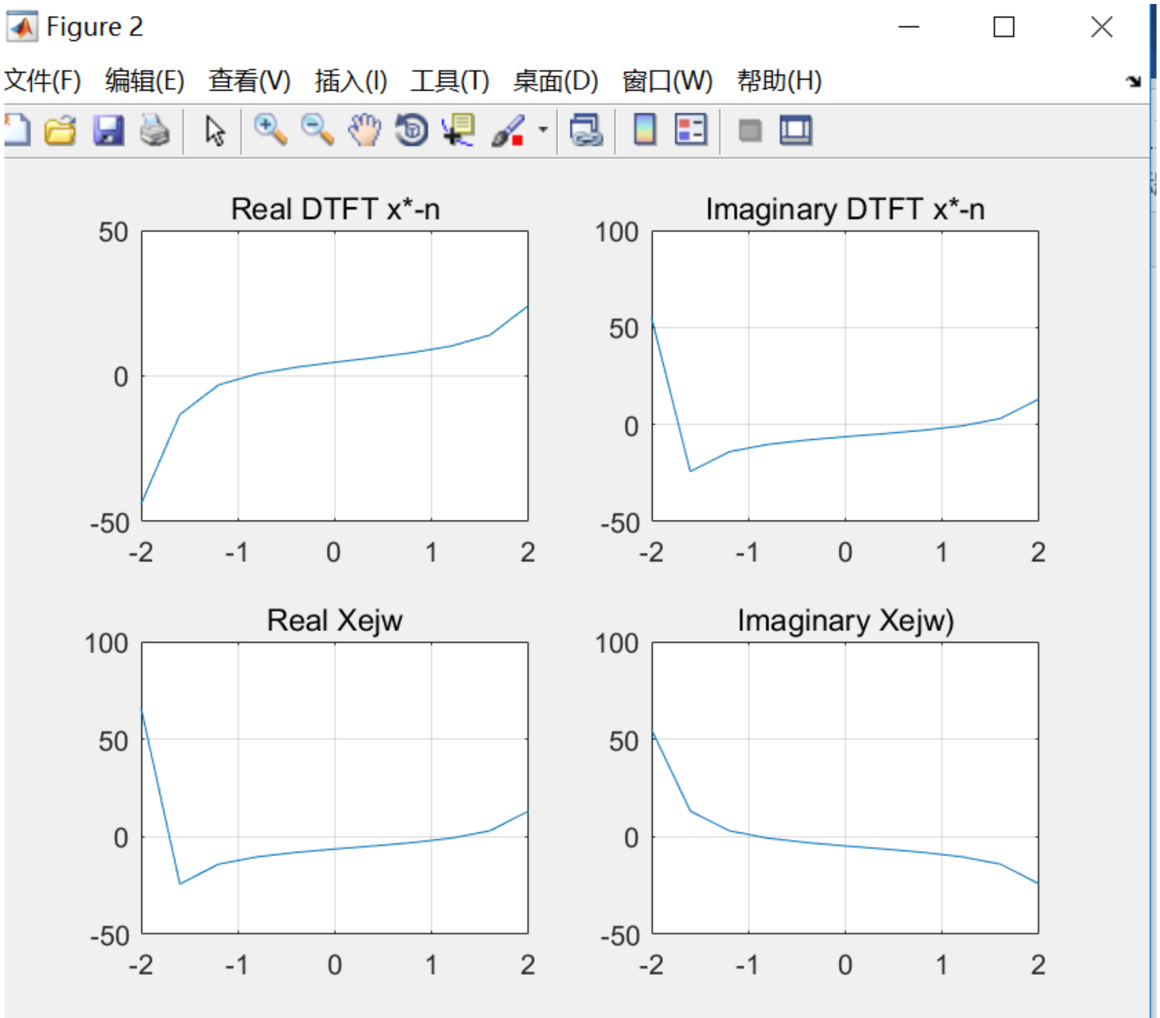


Figure 2
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Real DTFT x*-n
Imaginary DTFT x*-n
Real Xejw
Imaginary Xejw)
50
0
-50
100
-2
-1
0
1
2

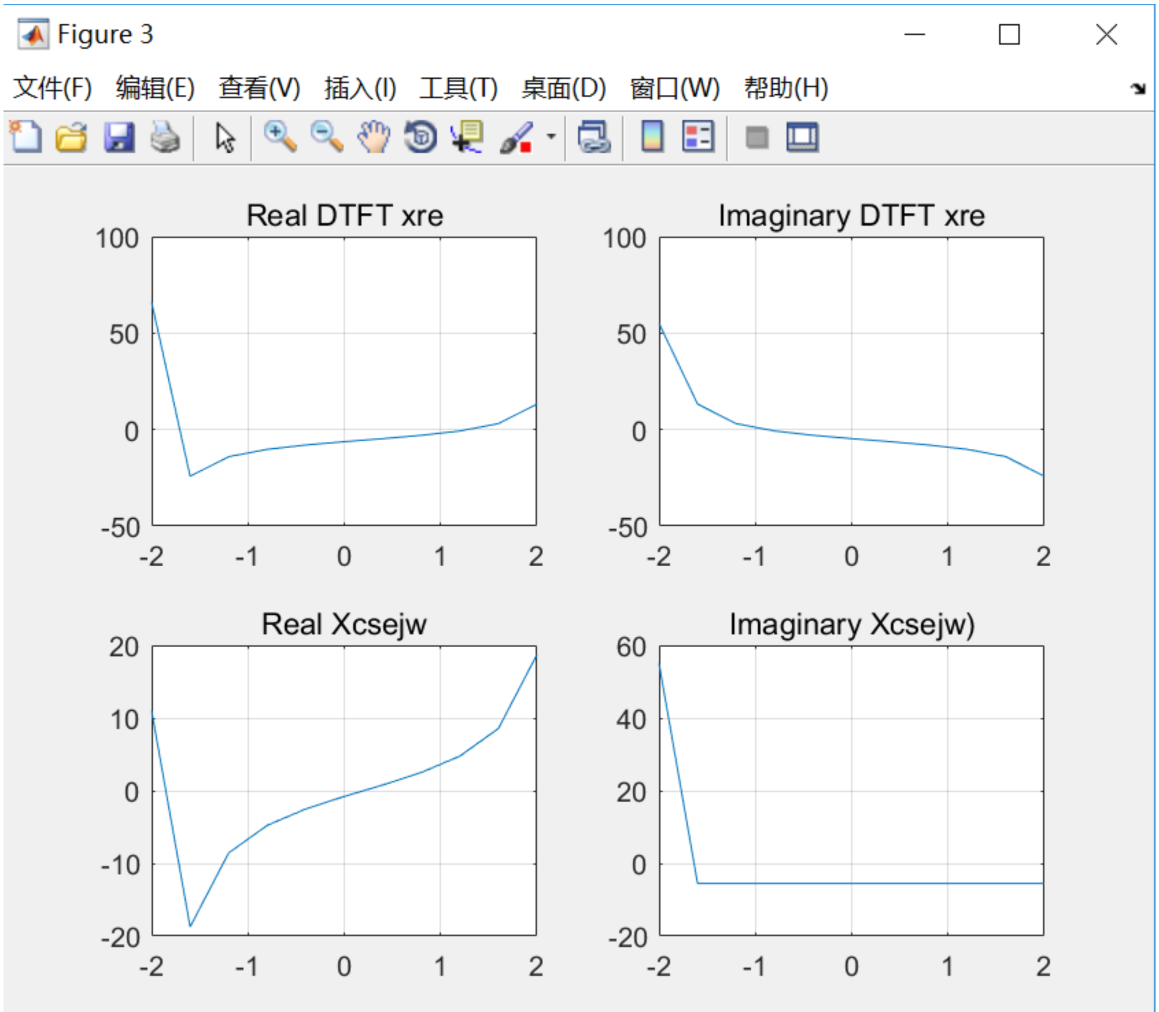


Figure 3
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Real DTFT xre
Imaginary DTFT xre
Real Xcsejw
Imaginary Xcsejw)

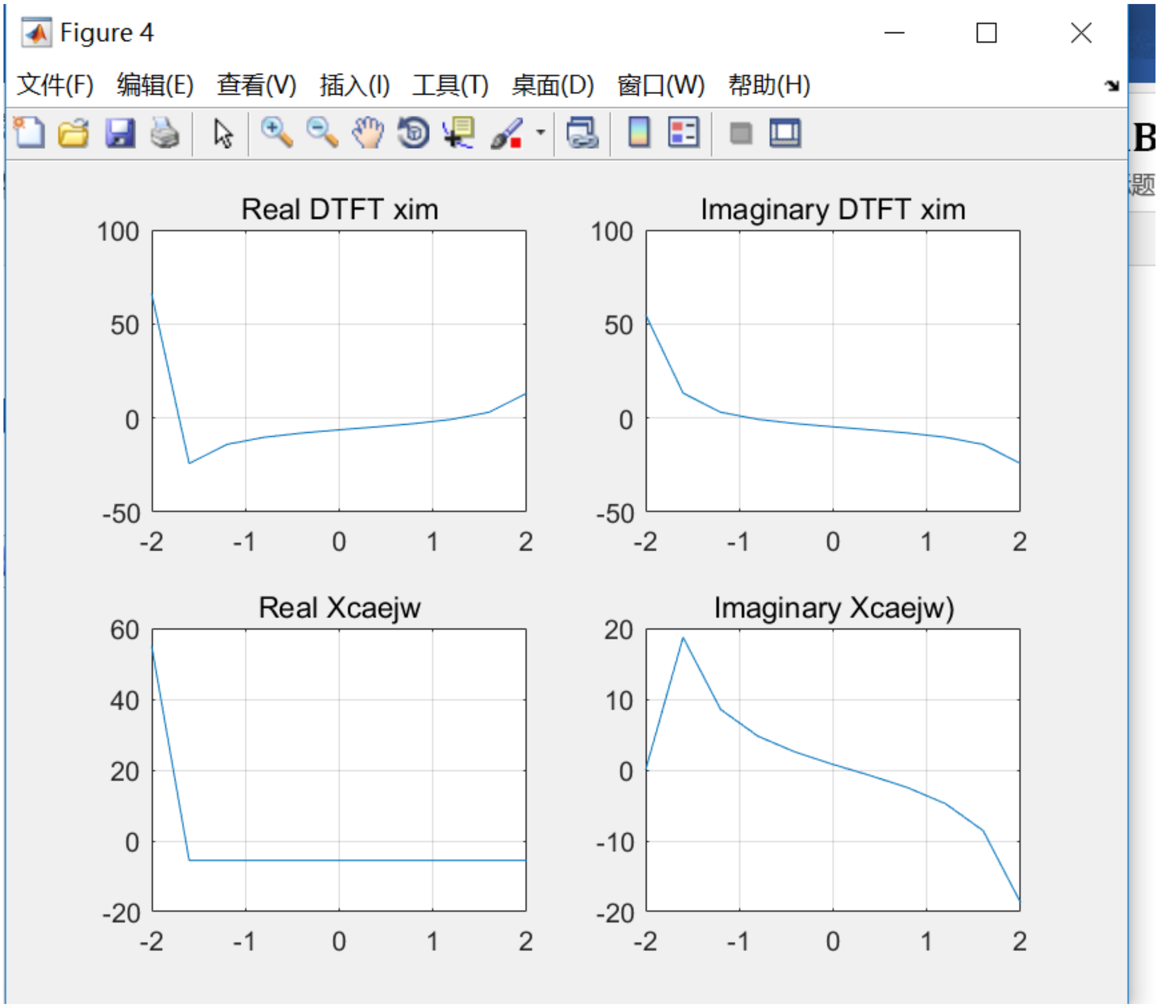


Figure 4
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Real DTFT xim
Imaginary DTFT xim
Real Xcaejw
Imaginary Xcaejw)
100
50
0
-50
60
40
20
-20
10
-10
-2
-1
1
2

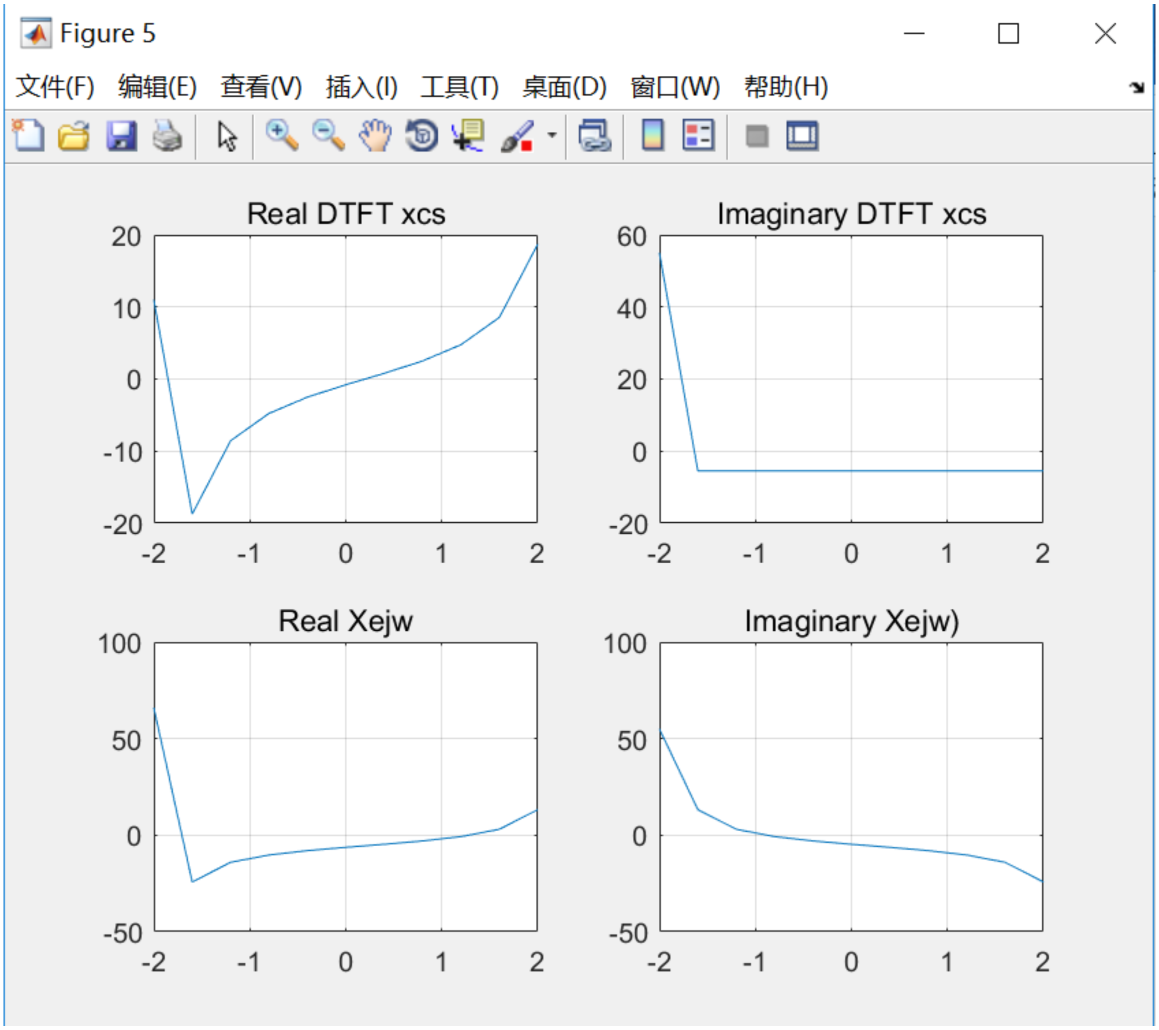
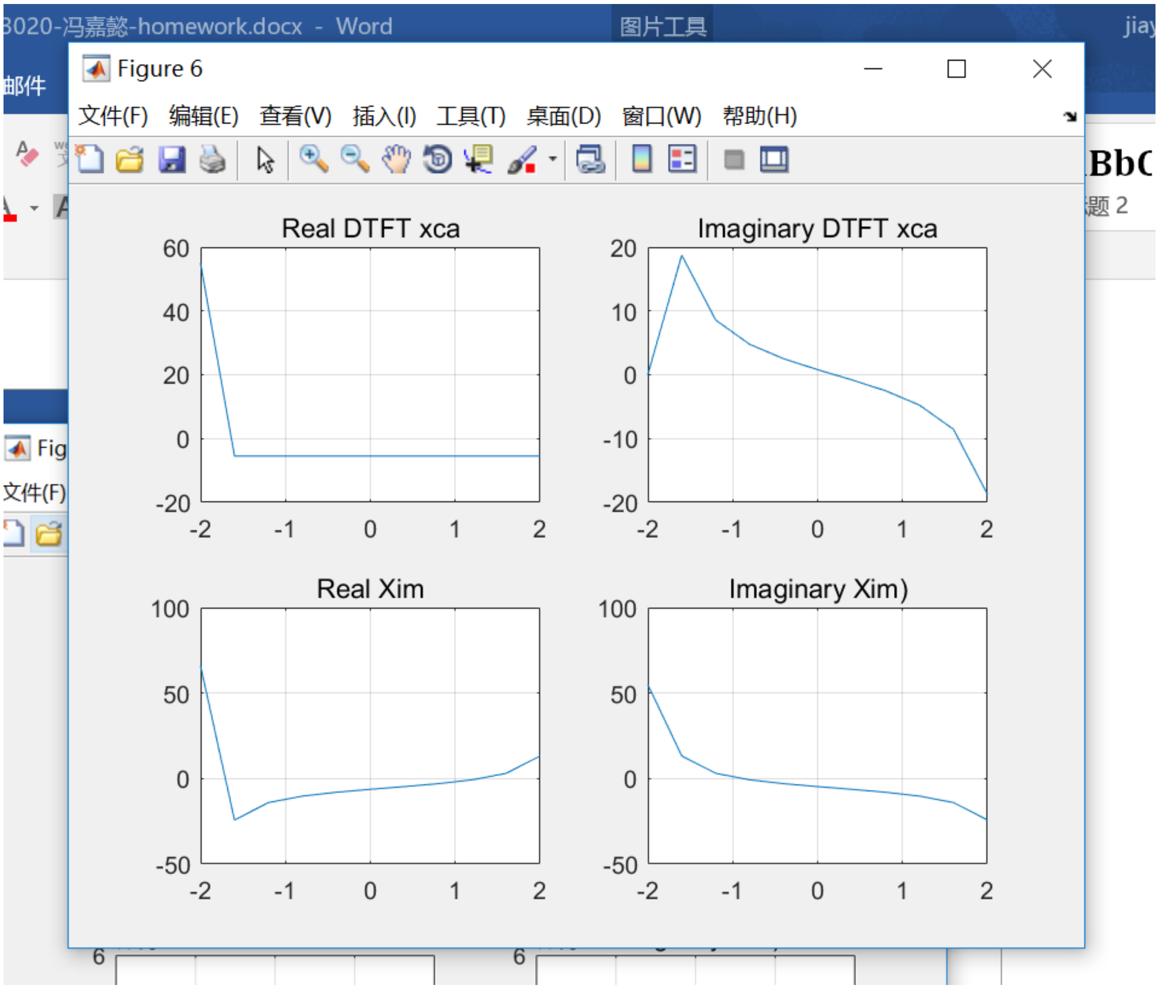


Figure 5
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Real DTFT xcs
Imaginary DTFT xcs
Real Xejw
Imaginary Xejw)


3020-冯嘉懿-homework.docx - Word
图片工具
Figure 6
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Real DTFT xca
Imaginary DTFT xca
Real Xim
Imaginary Xim)