Assignment 2

Module 2 – The LP Model

Quantitative Management Modeling (BA-64018-001), Fall 2025

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1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

1a) In above scenario, the decision variables are: 1) the quantity of Collegiate backpacks to produce per week and 2) the quantity of Mini backpacks to produce per week

1b) The objective function is to maximize the total profit. In this case I can express like below equation.

Max = $32xC+ $24xM

xC represents the weekly production quantity of the Collegiate backpacks

xM represents the weekly production quantity of the Mini backpacks

1c) In above case, constraints are the available amount of the materials, demand limits, and labor hours limits.

The available size of the nylon is 5,000 square-foot which Collegiate backpack requires 3 square feet while each Mini requires 2 square feet. We can express as $3xC + 2xM \leq 5{,}000$.

The demand for Collegiate backpacks is 1,000 that we can express as $xC \leq 1{,}000$ and the demand for Mini backpacks is 1,200 which can be expressed as $xM \leq 1{,}200$.

The labor time limits are $45 \text{ X } xC + 40 \text{ X } xM \leq 84{,}000$.

1d) $\max = 32xC + 24xM$

Subject to $3xC + 2xM \leq 5{,}000$ (fabric)

$45xC + 40xM \leq 84{,}000$ (labor)

$xC \leq 1{,}000$ (demand for Collegiate)

$xM \leq 1{,}200$ (demand for Mini)

$xC \geq 0,\ xM \geq 0$

When I use R to solve this problem, the optimal solution for xC is 1000 and xM is 975 with the maximum profit of $55,400.

```
title: "Assignment 2"
author: "Brian Yim"
date: "2025-09-06"
output: html_document
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```{r}
# install.packages("lpSolve")  # uncomment if not installed
library(lpSolve)

# Objective coefficients (maximize)
obj <- c(32, 24)

# Constraints (left-hand side)
const_mat <- matrix(c(
  3, 2,         # fabric
  45, 40,       # labor
  1, 0,         # xC <= 1000
  0, 1          # xM <= 1200
), nrow = 4, byrow = TRUE)

const_dir <- c("<=", "<=", "<=", "<=")
const_rhs <- c(5000, 84000, 1000, 1200)

sol <- lp("max", obj, const_mat, const_dir, const_rhs, compute.sens = TRUE)
sol$solution   # xC, xM
sol$objval     # maximum profit
```
```

```
Warning: package 'lpSolve' was built under R version 4.4.2
[1] 1000  975
[1] 55400
```

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

2a) Define the decision variables

In above scenario, the decision variables are the daily production quantities of large, medium, and small products at plant 1, 2, and 3.

2b) Formulate a linear programming model for this problem

First, I came up with below formula for maximizing the total profit

Max total profit = $\$420x1 + \$420x2 + \$420x3 + \$360x4 + \$360x5 + \$360x6 + \$300x7 + \$300x8 + \$300x9$

Where

$x1$ is the daily production of large products at plant 1

$x2$ is the daily production of large products at plant 2

$x3$ is the daily production of large products at plant 3

$x4$ is the daily production of medium products at plant 1

$x5$ is the daily production of medium products at plant 2

$x6$ is the daily production of medium products at plant 3

$x7$ is the daily production of small products at plant 1

$x8$ is the daily production of small products at plant 2

$x9$ is the daily production of small products at plant 3

Subject to

$x1 + x4 + x7 \leq 750$ (daily production capacity of plant 1)

$x2 + x5 + x8 \leq 900$ (daily production capacity of plant 2)

$x3 + x6 + x9 \leq 450$ (daily production capacity of plant 3)

$20x1 + 15x4 + 12x7 \leq 900$ (daily in-process storage capacity of plant 1)

$20x2 + 15x5 + 12x8 \leq 1200$ (daily in-process storage capacity of plant 2)

$20x3 + 15x6 + 12x9 \leq 750$ (daily in-process storage capacity of plant 3)

$x1 + x2 + x3 \leq 900$ (daily sales capacity of large products)

$x4 + x5 + x6 \leq 1200$ (daily sales capacity of large products)

$x7 + x8 + x9 \leq 750$ (daily sales capacity of large products)

$(x1 + x4 + x7)/750 = (x2 + x5 + x8)/900$ (excess capacity between plant 1 and 2 regardless the size)

$(x1 + x4 + x7)/750 = (x3 + x6 + x9)/900$ (excess capacity between plant 1 and 3 regardless the size)

$x1 \geq 0$, $x2 \geq 0$, $x3 \geq 0$, $x4 \geq 0$, $x5 \geq 0$, $x6 \geq 0$, $x7 \geq 0$, $x8 \geq 0$, $x\ 9 \geq 0$

Below is the R code and the solution results.

##Case 2

````
```{r}

# install.packages("lpSolve")

library(lpSolve)


# Objective coefficients (profits for x1..x9)

obj <- c(420, 420, 420,   # x1,x2,x3 (Large)

         360, 360, 360,   # x4,x5,x6 (Medium)

         300, 300, 300)   # x7,x8,x9 (Small)


# Constraints matrix

A <- matrix(0, nrow=11, ncol=9)


# 1) Plant capacities

A[1, c(1,4,7)] <- 1   # x1+x4+x7 ≤ 750


A[2, c(2,5,8)] <- 1   # x2+x5+x8 ≤ 900


A[3, c(3,6,9)] <- 1   # x3+x6+x9 ≤ 450



# 2) Storage

A[4, c(1,4,7)] <- c(20,15,12)   # Plant 1 ≤ 13000


A[5, c(2,5,8)] <- c(20,15,12)   # Plant 2 ≤ 12000


A[6, c(3,6,9)] <- c(20,15,12)   # Plant 3 ≤ 5000
````
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```
# 3) Market demand

A[7, c(1,2,3)] <- 1           # Large ≤ 900

A[8, c(4,5,6)] <- 1           # Medium ≤ 1200

A[9, c(7,8,9)] <- 1           # Small ≤ 750


# 4) Equal capacity usage (cross-multiplied equations)
A[10, c(1,4,7)] <- 900
A[10, c(2,5,8)] <- -750

A[11, c(1,4,7)] <- 450
A[11, c(3,6,9)] <- -750

# Directions
dir <- c("<=", "<=", "<=",
         "<=", "<=", "<=",
         "<=", "<=", "<=",
         "=", "=")

# RHS values
rhs <- c(750, 900, 450,
         13000, 12000, 5000,
```

```
        900, 1200, 750,
        0, 0)

# Solve LP
solution <- lp("max", obj, A, dir, rhs)

# Output
print(solution$status)     # 0 = optimal
print(solution$objval)     # max profit
print(solution$solution)   # values of x1..x9

```

Solution:

Max profit = 696000

$x1$ = 516.6667

$x2$ = 0

$x3$ = 0

$x4$ = 177.7778

$x5$ = 666.6667

$x6$ = 0

$x7$ = 0

$x8$ = 166.6667

x 9 = 416.6667