

离散数学

Discrete mathematics

1. 这题估计是被某种神秘的力量所吞噬了，所以请自己猜猜这道题到底考了什么。
2. [10 points] In the questions below, describe each sequence recursively. Include initial conditions and assume that the sequences begin with a_1 .
 - a) $a_n = 5^n$
 - b) 1, 101, 10101, 1010101
 - c) a_n = the number of bit strings of length n with an even number of 0s.
 - d) a_n = the number of ways to go down an n -step staircase if you go down 1, 2, or 3 step at a time.

3. [10 points] Suppose $A = \{2, 3, 6, 9, 10, 12, 14, 18, 20\}$ and R is the partial order relation balabalaba(原谅我, 这个地方我实在看不清).

A where xRy means x is a divisor of y .

- a) Draw the Hasse diagram for R .
- b) Find all maximal elements.
- c) Find all minimal elements.
- d) Find $\text{lub}(\{3, 10\})$
- e) Find $\text{glb}(\{14, 10\})$

4. [10 points] In the questions below give an example or else prove that there are none.

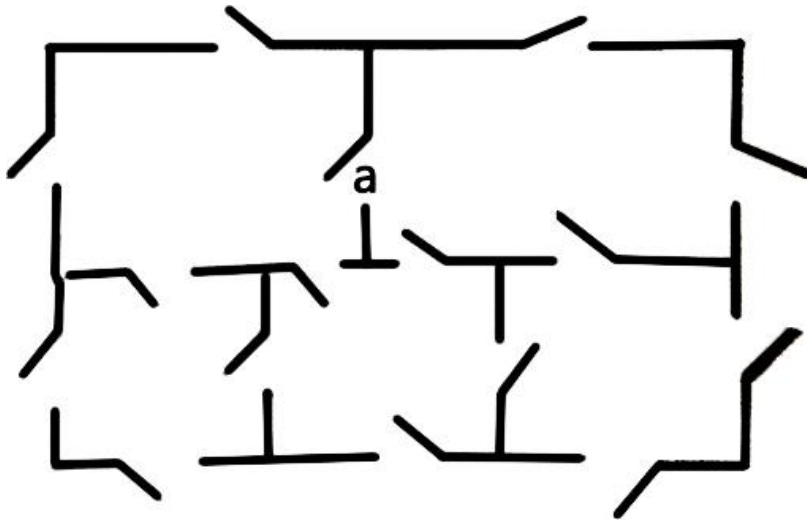
- a) A relation on $\{a, b, c\}$ that is reflexive and transitive, but not antisymmetric.
- b) A relation on $\{1, 2\}$ that is symmetric and transitive, but not reflexive.
- c) A relation on $\{1, 2, 3\}$ that is reflexive and transitive, but not symmetric.

5. [10 points] In the questions below fill in the blanks.

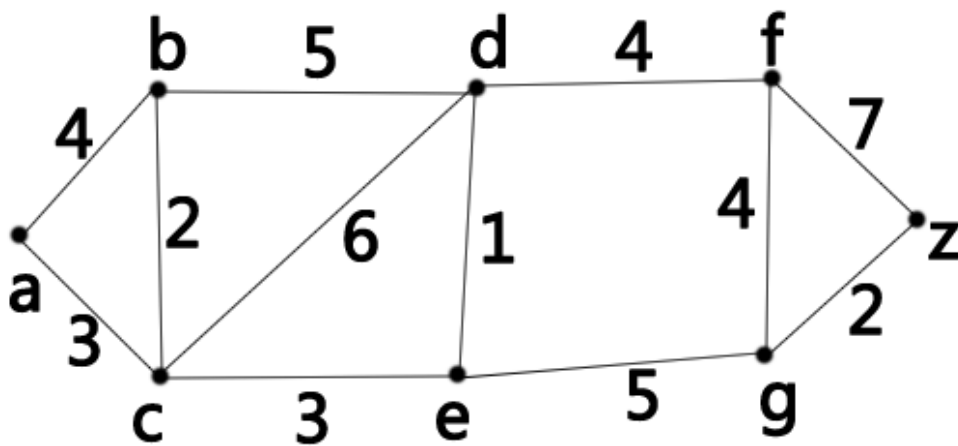
- a) W_n has _____ edges and _____ vertices.
- b) The adjacency matrix for K_n has _____ 1s and _____ 0s.
- c) If G is a connected graph with 12 regions and 20 edges, then G has _____ vertices.
- d) The vertex-chromatic number for $K_{7,7}$ = _____.
- e) If a regular graph G has 10 vertices and 45 edges, then each vertex of G has degree _____.

6. [10 points] An old puzzle presents a house with 5 rooms and 16 doors, as shown in the following figure. The problem is to figure out how to begin in a room or outside and take a walk that goes through each door exactly once.

- a) Is such a walk possible? Explain.
- b) How does your answer change if the door "a" adjoining the two large rooms is closed?



7.[10 points] Use Dijkstra's Algorithm to find the shortest path length between the vertices a and z in this weighted graph.(Please give the process!)

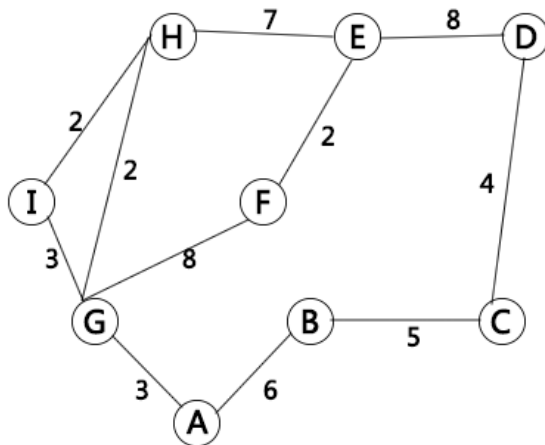


8.[10 points] Find a minimal spanning tree for the relation given by the graph .

a) Use Prim's algorithm, start from node H.(Write down

the detail process)

- b) Use Kruskal' s algorithm.List the edges in the order in which they are chosen.



9. [10 points] Let $(S,*)$ be the semigroup whose operation table is given below.Let R be the equivalence relation on S defined by the partition $\{\{x,y\},\{z,w\}\}$.Show that R is a congruence relation on $(S,*)$,and construct the operation table for quotient semigroup $(S/R, \odot)$.

$*$	x	y	z	w
x	x	y	z	w
y	y	z	w	z
z	z	z	z	z
w	w	w	w	w