Rashtreeya Sikshana Samithi Trust

RV COLLEGE OF ENGINEERING®

(Autonomous Institution Affiliated to VTU, Belagavi)

DEPARTMENT OF MATHEMATICS

MANUAL FOR

EXPERIENTIAL LEARNING

USING MATLAB

IV - SEMESTER

Contents

Introduction

Modules

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Introduction

The students are exposed to the good learning of coding using MATLAB in I year and have got introduced to the coding part for the topics in the Engineering Mathematics syllabus of I and II semesters.

“The learning never stops, the more the better”.

The department of Mathematics has taken all efforts to give best knowledge of Mathematics relevant to the programmes. In this direction the syllabus is framed in a branch specific way enabling the different programmes to take this as basis and develop the further theoretical knowledge.

To make the students more industry-ready, the MATLAB coding is included in the syllabus and the students are made to be aware of specific MATLAB functions.
This manual is made-to-order MATLAB content for the specific syllabus and number of application problems are included.

After going through the manual, the students would be well equipped to solve any such related problems.

1. Probability Distributions

Topic learning outcomes:

Student will be able to:

1. Generate probability distributions based on given parameters.
2. Plot the probability distribution functions.
3. Fit a probability distribution.

Syntax and description:

A Binomial Distribution object consists of parameters, a model description, and sample data for a binomial probability distribution

The binomial distribution models the total number of successes in repeated trials from an infinite population under the following conditions:

- Only two outcomes are possible for each of *n* trials.
- The probability of success for each trial is constant.
- All trials are independent of each other.

The binomial distribution uses the following parameters.

Parameter	Description	Support
N	Number of trials	positive integer
P	Probability of success	$0 \leq p \leq 1$

Distribution Parameters

N — Number of trials

positive integer value

p — Probability of success

positive scalar value in the range [0,1]

Creation

There are several ways to create a *Binomial Distribution* probability distribution object.

- Create a distribution with specified parameter values using *makedis*t.
- Fit a distribution to data using *fitdis*t.
- Interactively fit a distribution to data using the **Distribution Fitter** app.

'Name'	**Distribution**	**Input Parameter A**	**Input Parameter B**
'Binomial'	Binomial Distribution	n number of trials	p probability of success for each trial
'Chisquare'	Chi-Square Distribution	ν degrees of freedom	—
'Exponential'	Exponential Distribution	μ mean	—
'Normal'	Normal Distribution	μ mean	σ standard deviation
'Poisson'	Poisson Distribution	λ mean	—
'Webull'	Weibull Distribution	μ mean	—

Example 1.1 The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now aged 60,

(i) at most 6,

(ii) at least 7

will live to be 70?

```
pd = makedist('Binomial','N',10,'p',0.65)
x = [6];
y = cdf(pd,x)
z=1-y
```

pd = Binomial Distribution

Binomial distribution

N = 10

p = 0.65

y =

0.4862

z =

0.513

Example 1.2 Create a binomial distribution object by specifying the parameter values. Also compute the mean of the distribution.

```
pd = makedist('Binomial','N',30,'p',0.25)
m = mean(pd)
```

pd =

Binomial distribution

N = 30

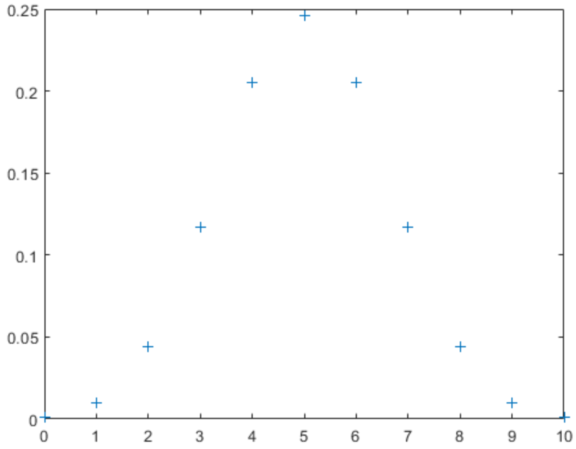
p = 0.25

m =

7.5000

Example 1.3 Generate a plot of the binomial pdf for n = 10 and p = 1/2.

```
x = 0:10;
y = binopdf(x,10,0.5);
plot(x,y,'+')
```



Example 1.4 Create a normal probability distribution object with mean 50 and SD 30. Generate a 2-by-3-by-2 array of random numbers from the distribution.

```
pd = makedist('Normal','mu',50,'sigma',30)
r = random(pd,[2,3,2])
```

```
pd =
NormalDistribution
Normal distribution
     mu = 50
  sigma = 30
  r =
  r(:,:,1) =
    31.4751   98.6682   42.3748
   122.9576   48.9985   25.4413
  r(:,:,2) =
   96.1240   24.0135   17.3124
    1.8054   36.8490   13.3768
```

Example 1.5 Create an exponential probability distribution object using the default parameter values, generate random numbers from the distribution. Construct a histogram using 100 bins with Exponential distribution fit.

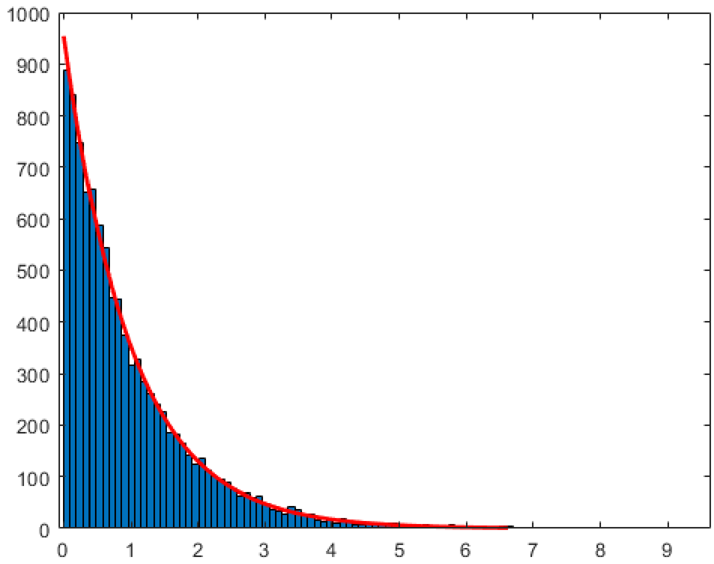
```
pd = makedist('Exponential')
rng('default')  % For reproducibility
r = random(pd,10000,1)
histfit(r,100,'Exponential')
```

```
pd =
Exponential Distribution
Exponential distribution
mu = 1
r = 10000×1
   0.2049
   0.0989
   2.0637
   0.0906
   0.4583
   2.3275
   1.2783
```

0.6035

0.0434

0.0357



Example 1.6 Create a Poisson distribution object with the rate parameter, λ, equal to 2. Compute the pdf and cdf values for the Poisson distribution at the values in the input vector x = [0,1,2,3,4].

```
lambda = 2;
pd = makedist('Poisson','lambda',lambda);
x = [0,1,2,3,4];
y = cdf(pd,x)
z = pdf(pd,x)
```

y = 1×5

0.1353 0.4060 0.6767 0.8571 0.9473

Z = 1×5

0.1353 0.2707 0.2707 0.1804 0.0902

Exercise:

1. Execute the example questions for different distributions with different parameter values.

2. The probability that an individual will suffer a bad reaction from an injection of a given serum is 0.001. Determine the probability that out of 2000 individuals

(i) exactly 3

(ii) more than 2

individuals will suffer a bad reaction. (use lambda = np)

3. The length of a telephone conversation on a cell phone has been an exponential distribution and found on an average to be 3 minutes. Find the probability that a random call made from this phone ends in less than 3 minutes.

4. In a test of 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for

(i) more than 1950 hours

(ii) more than 1920 and less than 2160 hours.

2. Sampling Distributions

Topic learning outcomes:

Student will be able to:

1. explore Chi square distribution
2. explore t-Distribution

Syntax and description:

- h = chi2gof(x)- returns a test decision for the null hypothesis that the data in vector x comes from a normal distribution with a mean and variance estimated from x, using the chi-square goodness-of-fit test. The alternative hypothesis is that the data does not come from such a distribution. The result h is 1 if the test rejects the null hypothesis at the 5% significance level, and 0 otherwise.
- h = chi2gof(x, Name, Value)- returns a test decision for the chi-square goodness-of-fit test with additional options specified by one or more name-value pair arguments. For example, you can test for a distribution other than normal, or change the significance level of the test.
- [h,p] = chi2gof(___)-also returns the *p*-value p of the hypothesis test, using any of the input arguments from the previous syntaxes.

  Input Arguments: x **-** Sample data for the hypothesis test, specified as a vector.

  Syntax and description:

- h = ttest(x) returns a test decision for the null hypothesis that the data in x comes from a normal distribution with mean equal to zero and unknown variance, using the one-sample *t*-test. The alternative hypothesis is that the population distribution does not have a mean equal to zero. The result h is 1 if the test rejects the null hypothesis at the 5% significance level, and 0 otherwise.
- h = ttest(x,y) returns a test decision for the null hypothesis that the data in x – y comes from a normal distribution with mean equal to zero and unknown variance, using the paired-sample *t*-test.

- h = ttest(x,y,Name,Value) returns a test decision for the paired-sample *t*-test with additional options specified by one or more name-value pair arguments. For example, you can change the significance level or conduct a one-sided test.
- h = ttest(x,m) returns a test decision for the null hypothesis that the data in x comes from a normal distribution with mean m and unknown variance. The alternative hypothesis is that the mean is not m.
- h = ttest(x,m,Name,Value) returns a test decision for the one-sample *t*-test with additional options specified by one or more name-value pair arguments. For example, you can change the significance level or conduct a one-sided test.
- [h,p] = ttest(___) also returns the *p*-value, p, of the test, using any of the input arguments from the previous syntax groups.
- [h,p,ci,stats] = ttest(___) also returns the confidence interval ci for the mean of x, or of x – y for the paired *t*-test, and the structure stats containing information about the test statistic.

1. Input Arguments:

- x — Sample data, specified as a vector, matrix, or multidimensional array. ttestperforms a separate *t*-test along each column and returns a vector of results. If y sample data is specified, x and y must be the same size.
- m — Hypothesized population mean, 0 (default) | scalar value, Hypothesized population mean, specified as a scalar value.
- 'Alpha' — Significance level, 0.05 (default) | scalar value in the range (0,1), Significance level of the hypothesis test, specified as the comma-separated pair consisting of 'Alpha' and a scalar value in the range (0,1).

2. 'Tail' — Type of alternative hypothesis, 'both' (default) | 'right' | 'left'

Type of alternative hypothesis to evaluate, specified as the comma-separated pair consisting of 'Tail' and one of the following.

- 'boht' Test the alternative hypothesis that the population mean is not m.
- 'right' Test the alternative hypothesis that the population mean is greater than m.
- 'left' Test the alternative hypothesis that the population mean is less than m.

3. Output Arguments

- h — Hypothesis test result1 | 0

4. Hypothesis test result, returned as 1 or 0.

- If h= 1, this indicates the rejection of the null hypothesis at the Alpha significance level.
- If h=0, this indicates a failure to reject the null hypothesis at the Alpha significance level.
- p — *p*-value scalar value in the range [0,1]
- *p*-value of the test, returned as a scalar value in the range [0,1]. p is the probability of observing a test statistic as extreme as, or more extreme than, the observed value under the null hypothesis. Small values of p cast doubt on the validity of the null hypothesis.
- ci — Confidence interval vector Confidence interval for the true population mean, returned as a two-element vector containing the lower and upper boundaries of the 100 ×(1 – Alpha)% confidence interval.
- stats — Test statistics structure

5. Test statistics, returned as a structure containing the following:

- tstat — Value of the test statistic.
- df — Degrees of freedom of the test.
- sd — Estimated population standard deviation. For a paired *t*-test, sd is the standard deviation of x – y.

Example 2.1 Create a standard normal probability distribution object. Generate a data vector x using random numbers from the distribution. Test the null hypothesis that the data in x comes from a population with a normal distribution.

```
pd = makedist('Normal');
rng default;  % for reproducibility
x = random(pd,100,1);
h = chi2gof(x)
```

```
ans =
h = 0
```

The returned value h = 0indicates thatchi2gofdoes not reject the null hypothesis at the default 5% significance level.

Example 2.2 Create a standard normal probability distribution object. Generate a data vector x using random numbers from the distribution. Test the null hypothesis that the data in x comes from a population with a normal distribution at the 1% significance level.

```
pd = makedist('Normal');
rng default;  % for reproducibility
x = random(pd,100,1);
[h,p] = chi2gof(x,'Alpha',0.01)
```

```
ans =
        h = 0
        p = 0.3775
```

The returned value h = 0indicates thatchi2gofdoes not reject the null hypothesis at the 1% significance level.

Example 2.3 Create six bins, numbered 0 through 5, to use for data pooling. Create a vector containing the observed counts for each bin and compute the total number of observations. Fit a Poisson probability distribution object to the data and compute the expected count for each bin. Use the transpose operator .' to transform bins and obs Counts from row vectors to column vectors. Test the null hypothesis that the data in obs Counts comes from a Poisson distribution with a lambda parameter equal to lambda Hat.

```
bins = 0:5;
obsCounts = [6 16 10 12 4 2];
n = sum(obsCounts);
pd = fitdist(bins','Poisson','Frequency',obsCounts');
expCounts = n * pdf(pd,bins);
[h,p,st] = chi2gof(bins,'Ctrs',bins,...
            'Frequency',obsCounts, ...
            'Expected',expCounts,...
            'NParams',1)
```

```
ans =
h = 0
```

p = 0.4654

st = *struct with fields:*

chi2stat: 2.5550

df: 3

edges: [-0.5000 0.5000 1.5000 2.5000 3.5000 5.5000]

O: [6 16 10 12 6]

E: [7.0429 13.8041 13.5280 8.8383 6.0284]

Example 2.4 Test the null hypothesis that the data 340, 220, 150, 270,370, 410, 240, 190, 260, 360, 300, 320, 228 comes from a population with a Weibull distribution. Use fitdist to create a probability distribution object with A and B parameters estimated from the data.

```
x = [340; 220; 150; 270; 370; 410; 240; 190; 260; 360; 300; 320; 228];
pd = fitdist(x,'Weibull');
h = chi2gof(x,'CDF',pd)
```

ans =

h = 0

Example 2.5 A research study was conducted to examine the differences between older and younger adults on perceived life satisfaction. A pilot study was conducted to examine this hypothesis. Ten older adults (over the age of 70) and ten younger adults (between 20 and 30) were give a life satisfaction test (known to have high reliability and validity). Scores on the measure range from 0 to 60 with high scores indicative of high life satisfaction; low scores indicative of low life satisfaction. The data are presented below. Compute the appropriate t-test.

Older Adults	Younger Adults
45	34
38	22
52	15
48	27
25	37
39	41
51	24
46	19
55	26
46	36

```
syms x y
x = [45 38 52 48 25 39 51 46 55 46];
y = [34 22 15 27 37 41 24 19 26 36];
[h,p,ci,stats] = ttest(x,y,'Alpha',0.01)
```

ans =

h = 1

p=0.0074

ci = 1×2

0.9110 31.8890

stats = *struct with fields:*

tstat: 3.4410

df: 9

sd: 15.0717

The returned value of h = 1 indicates that ttest rejects the null hypothesis that there are no significant differences between younger and older adults on life satisfaction at the 1% significance level.

Example 2.6 Test the null hypothesis that sample data 34, 22, 15, 27, 37, 41, 24, 19, 26, 36 comes from a distribution with mean m = 25.

```
x = [34 22 15 27 37 41 24 19 26 36];
h = ttest(x,25)
```

ans =

h = 0

The returned value of h = 0 indicates that ttest does not reject the null hypothesis at the 5% significance level.

Example 2.7 Test the null hypothesis that the data 45, 38, 52, 48, 25, 39, 51, 46, 55, 46 comes from a population with mean equal to 35, against the alternative that the mean is greater than 35.

```
y = [34 22 15 27 37 41 24 19 26 36];
[h,p] = ttest(x,35)
```

ans=

h = 1

p = 0.0072

The returned value of h = 1 indicates that t test rejects the null hypothesis at the 5% significance level, in favour of the alternate hypothesis that the data comes from a population with a mean greater than 35.

Exercise:

1. A researcher hypothesizes that electrical stimulation of the lateral habenula will result in a decrease in food intake (in this case, chocolate chips) in rats. Rats undergo stereotaxic surgery and an electrode is implanted in the right lateral habenula. Following a ten day recovery period, rats (kept at 80 percent body weight) are tested for the number of chocolate chips consumed during a 10 minute period of time both with and without electrical stimulation. The testing conditions are counter balanced. Compute the appropriate t-test for the data provided below.

Stimulation	No Stimulation
12	8
7	7
3	4
11	14
8	6
5	7
14	12
7	5
9	5
10	8

2. Generate 500 random samples from the standard normal distribution for sample sizes of n = 2, 15, and 45. At each sample size, calculate the sample mean for all 500 samples. How are the means distributed as n gets large? Look at a histogram of the sample means to help answer this question. What is the mean and variance of the sample means for each n? Is this what you would expect from the Central Limit Theorem? Here is some MATLAB code to get you started.

3. Generate a random sample of size 100 from a normal distribution with mean 10 and variance of 2 (use randn(1,100)*sqrt(2)+10). Plot the empirical cumulative distribution function. What is the value of the empirical distribution function evaluated at a point less than the smallest observation in your random sample? What is the value of the empirical cumulative distribution function evaluated at a point that is greater than the largest observation in your random sample?

3. Linear Programming Problems

Topic learning outcomes:

Student will be able to: Solve linear programming models.

Syntax and description:

- x = linprog(f,A,b) solves min f'*x such that A*x ≤ b.
- x= linprog(f,A,b,Aeq,beq) includes equality constraints Aeq*x = beq.

set A = [] and b = [] if no inequalities exist.

- x = linprog(f,A,b,Aeq,beq,lb,ub) defines a set of lower and upper bounds on the design variables, x, so that the solution is always in the range lb ≤ x ≤ ub. Set Aeq = [] and beq = [] if no equalities exist.

Example 3.1 Find the values of x_1, x_2, x_3 so that Minimize $z = x_1 - 3x_2 + 2x_3$ Subject to $3x_1 - x_2 + 3x_3 \leq 7$, $-2x_1 + 4x_2 \leq 12$, $-4x_1 + 3x_2 + 8x_3 \leq 10$, $x_1, x_2, x_3 \geq 0$.

```
f=[1 -3 2]; % defining the function to be maximized
A =[3 -1 3;-2 4 0;-4 3 8;-1 0 0;0 -1 0;0 0 -1]; % the constraints
b=[7 12 10 0 0 0];% lowerbound for the variable x=linprog(f,A,b)
minz=f*x
```

Optimization terminated x =
4.0000
5.0000
0.0000
z =
-11.0000

Example 3.2 Solve linear programming problem Maximize $z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 2, 5x_1 + 2x_2 \leq 10, 3x_1 + 8x_2 \leq 12, x_1, x_2 \geq 0$

```
f=[5 3]; % defining the function to be maximized
A =[1 1;5 2;3  ]; % the constraints
b=[2 10 12];
lb=[0 0];
x=linprog(-f,A,b,[ ],[ ],lb) % gives the point at which f attains maximum z=f*x
```

Optimization terminated x =

2.0000

0.0000

z = 10.0000

Example 3.3 Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of vitamin A per gram and 12 units of vitamin B and costs 20 paise per gram. The daily minimum requirements of vitamin A and vitamin B are 100 units and 120 units respectively. Find the minimum cost of product mix by simplex method.

Hint: Let x_1 grams of food X and x_2 grams of food Y be purchased. Then the problem can be formulated as:

Minimize $z = 12x_1 + 20x_2$

Subject to: $6x_1 + 8x_2 \geq 100, 7x_1 + 12x_3 \geq 120, x_1, x_2 \geq 0.$

```
f=[12 20]; % defining the function to be maximized
 A =[-6 -8;-7 -12]; % the constraints
b=[-100 -120];
lb=[0 0];
x=linprog(f,A,b,[],[],lb) % gives the point at which f attains minimum z=f*x
```

Optimization terminated. x =

15.0000

1.2500

z =

205.0000

Example 3.4 Solve LPP:

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15$, $2x_1 + x_2 + 5x_3 = 20$, $x_1 + 2x_2 + x_3 + x_4 = 10$,

$x_1, x_2, x_3, x_4 \geq 0$.

```
f=[1 2 3 -1]; % defining the function to be maximized
Aeq =[1 2 3 0;2 1 5 0;1 2 1 4]; % the constraints
beq=[15; 20; 10];
lb=[0 0 0 0];
x=linprog(-f, Aeq,beq,[],[],lb) % gives the point at which f attains maximum z=f*x
```

Optimization terminated. x =

2.5000

2.5000

2.5000

0.0000

z = 15.0000

Example 3.5 A manufacturer produces two models $M_1$and M_2 of a product. Each unit of model M_1requires 4 hours grinding and 2 hours polishing. Each unit of model M_2requires 2 hours grinding and 5 hours polishing. The manufacturer has 2 grinders each of which works for 40 hours a week. There are 3 polishers each of which works for 60 hours per week. Profit of model $M_1$is Rs.300 per unit and profit of model M_2 is Rs. 400 per unit. The manufacturer has to allocate his production capacity so as to maximize his profit. Formulate the LPP.

Hint: Maximize $= 300x_1 + 400x_2$

subject to $300\, x_1 + x_2 \leq 2$

$4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$

$x_1,\ x_2 \geq 0$

```
f=[300 400]; % defining the function to be maximized
A=[4 2;2 5]; % the constraints
b=[80 180];
lb=[0 0];            % lowerbound for the variable
x=linprog(f,A,b,[],[],lb) % gives the point at which z attains maximum z=f*x
```

x = 2.5000

35.0000

Exercise:

1. A firm produces an alloy having the following specifications.

(i) Specific gravity ≤ 0.98%

(ii) Chromium content ≥ 8%

(iii) Melting point ≥450°C

Raw materials A and B are used to produce the alloy. Raw material A has specific gravity 0.92, chromium content 7% and melting point 440°C. Raw material **B** has specific gravity 0.99, chromium content 13% and melting point 490°C. If raw material A costs Rs.90 per kg and raw material costs Rs.280 per kg . Find the rate at which the raw material should be blended keeping cost in mind.

2. Minimize $z = 2x_1 + x_2$

subject to $3x_1 + x_2 \le 3, 4x_1 + 3x_2 \le 6, x_1 + 2x_2 \le 3, x_1, x_2 \ge 0$

3. The following table gives the various vitamin contents of three types of food and daily requirements of vitamins along with cost per unit. Find the combination of food for minimum cost.

Vitamin (mg)	Food F	Food G	Food H	Minimum daily Requirement (mg)
A	1	1	10	1
C	100	10	10	50
D	10	100	10	10
Cost / unit(Rs)	10	15	5	

4. Minimize $z = x_1 - 3x_2 + 2x_3$

Subject to $3x_1 - x_2 + 2x_3 \le 7, -2x_1 + 4x_2 \le 12;\ -4x_1 + 3x_2 + 8x_3 \le 10, x_1, x_2, x_3 \ge 0$.