Contents

Introduction

The students are exposed to the good learning of coding using MATLAB in I year and have got introduced to the coding part for the topics in the Mathematics syllabus of I and II semesters.

"The learning never stops, the more the better".

The department of Mathematics has taken all efforts to give best knowledge of Mathematics relevant to the programmes. In this direction the syllabus is framed in a branch specific way enabling the different programmes to take this as basis and develop the further theoretical knowledge.

To make the students more industry-ready, the MATLAB coding is included in the syllabus and the students are made to be aware of specific MATLAB functions.

This manual is made-to-order MATLAB content for the specific syllabus and number of application problems are included.

After going through the manual, the students would be well equipped to solve any such related problems.

1 Linear Algebra - I

Topic learning outcomes:

Student will be able to:

1. Find Four Fundamental Subspaces,
2. Basis for the Range and null space of a Linear Transformation.

Syntax and description:

- `sym()` -stores the values in symbolic math toolbox notation.
- `null(A)`–finds the basis for the null space of the matrix A.
- `colspace(A)`–finds the basis for the column space of the matrix A.
- `null(A')`–finds the basis for the left null space of the matrix A.
- `colspace(A')`– finds the basis for the row space of the matrix A.
- `B*inv(A)`– finds the matrix representation of Linear transformation, where A is the matrix having the basis vectors of the domain as its columns, B is the matrix having the images of the basis vectors as its columns.
- `null(LT)`– finds the basis for the null space of the Linear transformation, where LT is the matrix representation of the Linear transformation.
- `colspace(B)`– finds the basis for the column space of the Linear transformation, where B is the matrix having the images of the Linear transformation as its columns.
- `rank(colspace(B))`– finds the rank of the Linear transformation
- `rank(null(LT))`– Finds the nullity of the Linear transformation.

Example 1.1: Obtain the bases for the Four Fundamental Subspaces of the matrix A, by storing the matrix using symbolic math toolbox notation.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

```
A= sym([1,2,3,4;5,6,7,8;9,10,11,12;13,14,15,16])
```

A =

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

```
nsA=null(A)
```

nsA =

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
csA=colspace(A)
```

csA =

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

```
lnsA=null(A')
```

lnsA =

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
rsA=colspace(A')
```

rsA =

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

Note:

One can also find the Four Fundamental Subspaces using .m file.

(i) Create a function file with the name ffss.

```
function[fourfundamentalsubspaces]=ffss(A)
nsA=null(A)
csA=colspace(A)
lnsA=null(A')
rsA=colspace(A')
end
```

(ii)Enter the matrix A in command window using symbolic math toolbox notation as:

```
A= sym([1,2,3,4;5,6,7,8;9,10,11,12;13,14,15,16])
```

```
A =
```

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

(iii) call the m file in the command window as ffss(A)

```
ffss(A)
```

```
nsA =
```

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
csA =
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

```
rsA =
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

```
lnsA =
```

$$\begin{pmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Example 1.2: Find the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ such that $T(1,1) = (0,1,2), T(-1,1) = (2,1,0)$.

```
A=[1,-1;1,1]
```

```
A = 2×2

     1    -1
     1     1
```

```
B=[0,2;1,1;2,0]
```

```
B = 3×2

     0     2
     1     1
     2     0
```

```
LT=B*inv(A)
```

```
LT = 3×2

    -1     1
     0     1
     1     1
```

Example 1.3: Find the range space, null space, rank and nullity of T, where $T: V_3(R) \rightarrow V_4(R)$, defined by

$T(e_1) = (0, 1, 0, 2), T(e_2) = (0, 1, 1, 0), T(e_3) = (0, 1, -1, 4)$

```
A= sym([1,0,0;0,1,0;0,0,1])
```

A =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
B=sym([0,0,0;1,1,1;0,1,-1;2,0,4])
```

B =

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$$

```
LT=B*inv(A)
```

LT =

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$$

```
nsLT=null(LT)
```

nsLT =

$$\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

```
rsLT=colspace(B)
```

rsLT =

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 2 & -2 \end{pmatrix}$$

```
rLT=rank(rsLT)
```

rLT = 2

```
nLT=rank(nsLT)
```

nLT = 1

Exercise:

1. Compute the bases for the four fundamental subspace of the following matrix
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{pmatrix}.$$
2. Compute the bases for the four fundamental subspace of the following matrix
$$B = \begin{pmatrix} 3 & 4 & -2 & -5 \\ 4 & 3 & 2 & 4 \\ 2 & 5 & -6 & -14 \end{pmatrix}.$$
3. Find the bases for the range space and null space of the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x) = Ax$, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{pmatrix}$.
4. Find the bases for the range space and null space of the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x) = Ax$, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ -2 & 1 & -3 \end{bmatrix}$.

2 Linear Algebra-II

Topic learning outcomes:

Student will be able to:
1. Obtain orthonormal bases for vectors. Eigen values, Eigen vectors & diagonalization of a matrix.
2. Form QR-factorization and singular value decomposition (SVD) of a matrix.

Syntax and description:

- `[Q,R]=qr(A)`-returns an upper triangular matrix R and a unitary matrix Q such that A = Q*R.
- The orthogonal or QR, factorization expresses any rectangular matrix as the product of an orthogonal or unitary matrix and an upper triangular matrix. An orthogonal matrix or a matrix with orthonormal columns, is a real matrix whose columns all have unit length and are perpendicular to each other. If Q is orthogonal, then $Q^T\ Q = I$
- `e=eig(A)` -returns a column vector containing the eigenvalues of square matrix A.
- `[V,D]=eig(A)` –returns diagonal matrix D of eigenvalues and matrix V whose columns are the corresponding right eigenvectors, so that A*V = V*D.
- The eigenvalue problem is to determine the solution to the equation Av = λv, where A is an n-by-n matrix, v is a column vector of length n and λ is a scalar. The values of λ that satisfy the equation are the eigenvalues. The corresponding values of v that satisfy the equation are the right eigenvectors.
- `s=svd(A)` -returns the singular values of matrix A in descending order.
- `[U,S,V]=svd(A)` –performs a singular value decomposition of matrix A such that A = U*S*V.

Singular value decomposition is a process through which any m by n matrix A can be factored into A = U*S*V = (orthonormal matrix) (diagonal matrix) (orthonormal matrix). The columns of U (m by m) are eigenvectors of $A\,A^T$ and the columns of V (n by n) are eigenvectors of $A^T\,A$. The r singular values on the diagonal of S (m by n) are the square roots of the non-zero eigenvalues of both $A\,A^T$ and $A^T\,A$.

Example 2.1: If the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$, find Q and R matrices representing the QR decomposition of A.

```
A=[1,1,1;1,2,3;1,3,6]
```

```
A = 3×3

     1     1     1
     1     2     3
     1     3     6
```

```
[Q,R]=qr(A)
```

```
Q = 3×3

   -0.5774    0.7071    0.4082
   -0.5774   -0.0000   -0.8165
   -0.5774   -0.7071    0.4082
R = 3×3

   -1.7321   -3.4641   -5.7735
         0   -1.4142   -3.5355
         0         0    0.4082
```

Example 2.2: If the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$, find the characteristic equation, the eigenvalues and the eigenvectors of A.

```
A = [8 -6 2; -6 7 -4; 2 -4 3]
```

```
A = 3×3

     8    -6     2
    -6     7    -4
     2    -4     3
```

```
p=poly(A)
```

```
p = 1×4

1.0000  -18.0000   45.0000   -0.0000
```

```
e=eig(A)
```

```
e = 3×1

    0.0000
    3.0000
   15.0000
```

```
[V,D]=eig(A)
```

```
V = 3×3

    0.3333    0.6667   -0.6667
    0.6667    0.3333    0.6667
    0.6667   -0.6667   -0.3333
D = 3×3

    0.0000         0         0
         0    3.0000         0
         0         0   15.0000
```

Example 2.3: Compute the singular values of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$.

```
A = [1 0 1; -1 -2 0; 0 1 -1]
```

```
A = 3×3

     1     0     1
    -1    -2     0
     0     1    -1
```

```
s = svd(A)
```

```
s = 3×1

    2.4605
    1.6996
    0.2391
```

Example 2.4: Find the singular value decomposition of a rectangular matrix $A = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

```
A = [-1; 2; 2]
```

```
A = 3×1

    -1
     2
     2
```

```
[U,S,V] = svd(A)
```

```
U = 3×3

   -0.3333    0.6667    0.6667
    0.6667    0.6667   -0.3333
    0.6667   -0.3333    0.6667
```

```
S = 3×1

     3
     0
     0
V = 1
```

Example 2.5: Obtain an orthonormal basis of the range of the matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$.

```
A = sym([2 -3 -1; 1 1 -1; 0 1 -1])
B = orth(A)
```

B =

$$\begin{pmatrix} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{2}\,\sqrt{15}}{30} \\ \frac{\sqrt{5}}{5} & \frac{\sqrt{6}}{3} & \frac{\sqrt{2}\,\sqrt{15}}{15} \\ 0 & \frac{\sqrt{6}}{6} & -\frac{\sqrt{2}\,\sqrt{15}}{6} \end{pmatrix}$$

Exercise:

1. If $v_1 = (0, 1, 2), v_2 = (1, 1, 2), v_3 = (1, 0, 1)$ construct an orthonormal basis.
2. Find the QR factorization of $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
3. Consider the matrix $D = \begin{pmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$, find a factorization D = QR.
4. Find the characteristic equation, eigenvalues of the matrix $B = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Also diagonalize it.
5. Diagonalize the matrix $A = \begin{pmatrix} 5 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 9 \end{pmatrix}$. Also find its characteristic equation and its eigenvalues.
6. Obtain the SVD of matrix $A = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$.

3 Fourier Series

Topic learning outcomes:

Student will be able to:

1. Develop Fourier series for different types of functions.
2. Plot and analyse the partial sums of Fourier series.

The **Fourier series** of a $2l$ periodic function $f(x)$ is given by

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi}{l}\right)x+b_n\sin\left(\frac{n\pi}{l}\right)x,$$

where the **Fourier coefficients** a_0, a_n and b_n are given by

$$a_0=\frac{1}{l}\int_a^{a+2l}f(x)dx,$$

$$a_n=\frac{1}{l}\int_a^{a+2l}f(x)\cos\left(\frac{n\pi}{l}\right)xdx,\quad n=1,2,3,.....$$

$$b_n=\frac{1}{l}\int_a^{a+2l}f(x)\sin\left(\frac{n\pi}{l}\right)xdx,\quad n=1,2,3,......\ .$$

The n^{th} partial sum of the Fourier series is

$$s_n(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi}{l}\right)x+b_n\sin\left(\frac{n\pi}{l}\right)x.$$

MATLAB program to compute Fourier series and plot first n terms:

```
clear all;clc;
syms x
pi=3.14;   %required when l=pi
sum=0;
y=_____;   %function

a0=(1/l)*int(y,x,a,a+2*l);
sum=sum+a0/2

for n=1:___%specify number of terms here
 %finding the coefficients
an=(1/l)*int(y*cos(n*pi*x/l),x, a,a+2*l);
bn=(1/l)*int(y*sin(n*pi*x/l),x, a,a+2*l);
```

```
sum=sum+(an*cos(n*pi*x/l)+bn*sin(n*pi*x/l));

end
ezplot(x,y,[a,a+2*l])
grid on;hold on;
ezplot(x,(sum),[ a,a+2*l])
```

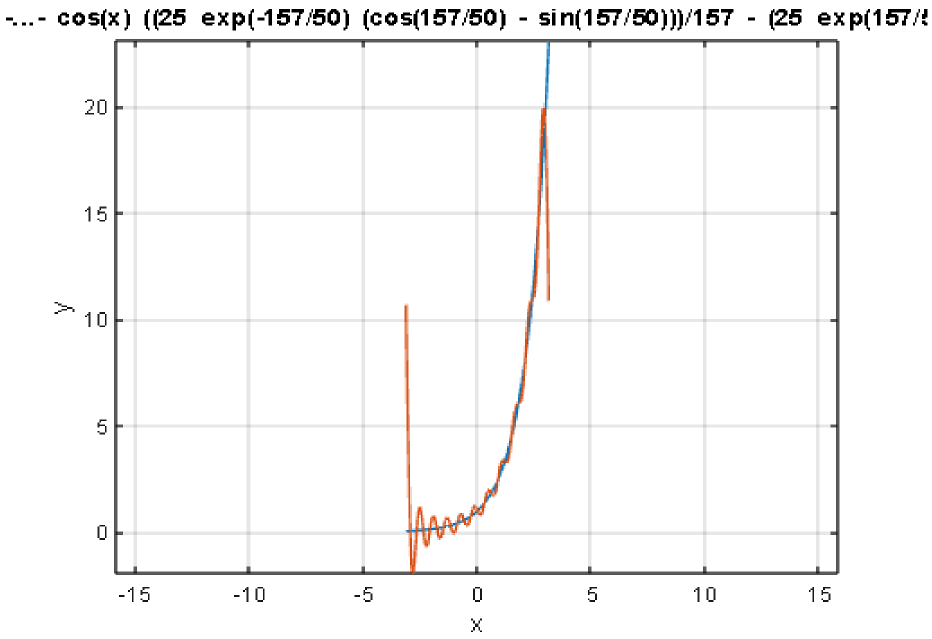
Example 3.1:.Compute Fourier series expansion of e^x in $(-\pi, \pi)$ up to 10 terms and plot.

```
clear all;clc;
figure
syms x
pi=3.14;
sum=0;
y=exp(x);    %function

a0=(1/pi)*int(y,x,-pi,pi);
sum=sum+a0/2;

for n=1:10
 %finding the coefficients
an=(1/pi)*int(y*cos(n*x),x,-pi,pi);
bn=(1/pi)*int(y*sin(n*x),x,-pi,pi);
sum=sum+(an*cos(n*x)+bn*sin(n*x));

end
ezplot(x,y,[-pi,pi])
grid on;hold on;
ezplot(x,(sum),[-pi,pi])
```



Example 3.2: Find Fourier series expansion of $y=x$ in $(-\pi, \pi)$ up to 10 terms and plot.

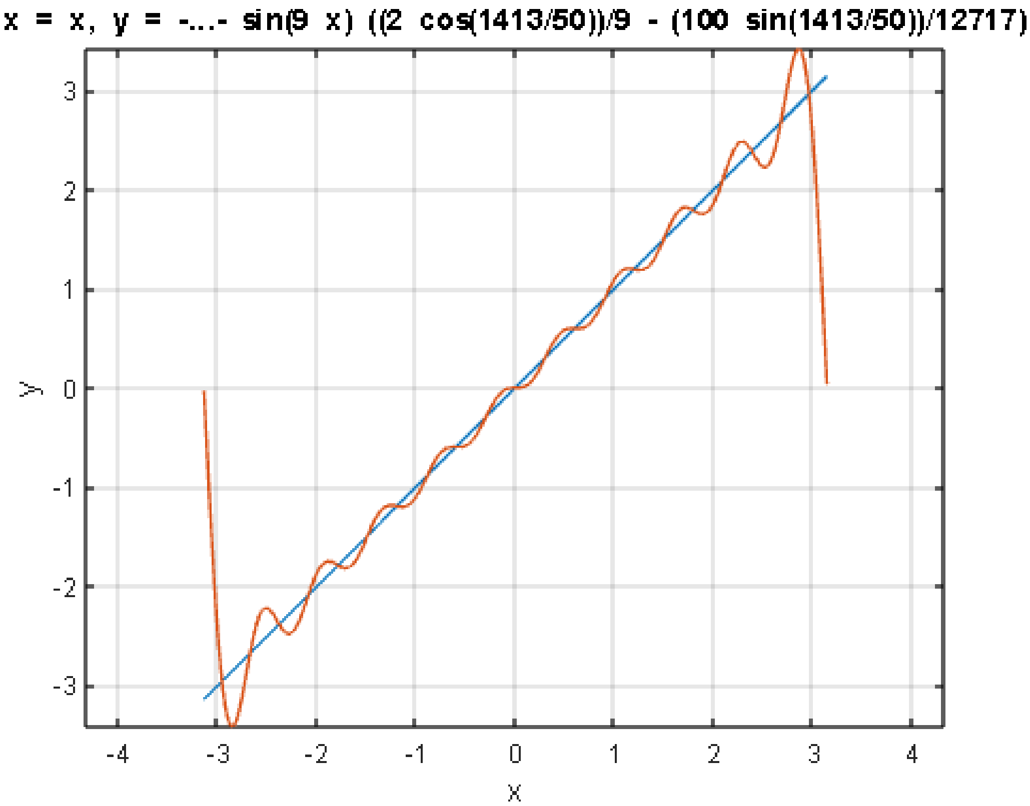
```
clear all;
clc;
figure

syms x
pi=3.14;
sum=0;
y=x;    %function

a0=(1/pi)*int(y,x,-pi,pi);
sum=sum+(a0/2);
for n=1:10
  %finding the coefficients

an=(1/pi)*int(y*cos(n*x),x,-pi,pi);
bn=(1/pi)*int(y*sin(n*x),x,-pi,pi);
sum=sum+(an*cos(n*x)+bn*sin(n*x));

end
ezplot(x,y,[-pi,pi])
grid on;hold on;
ezplot(x,(sum),[-pi,pi])
```



Example 3.3: Expand |x| as Fourier series over in (-1, 1) up to 10 terms and plot.

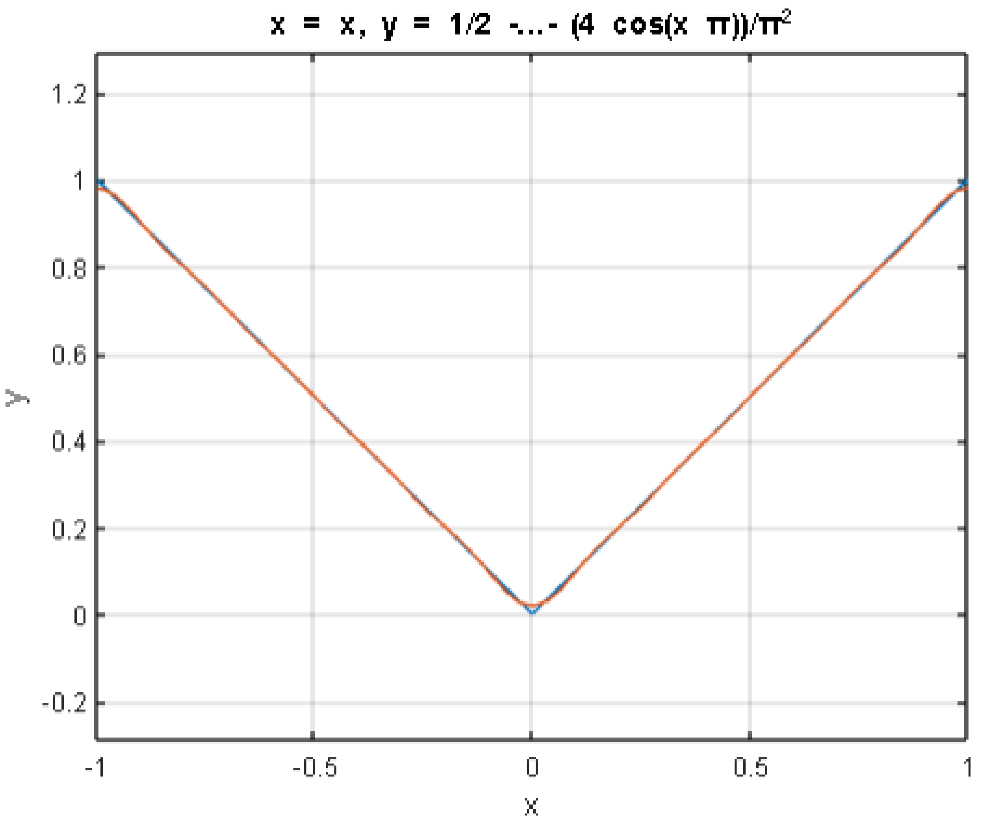
```
clear all;
clc;
figure

syms x
sum=0;
y=abs(x);    %function

a0=(1/1)*int(y,x,-1,1);
sum=sum+a0/2;

for n=1:10
  %finding the coefficients

an=(1/1)*int(y*cos(n*pi*x),x,-1,1);
bn=(1/pi)*int(y*sin(n*pi*x),x,-1,1);
sum=sum+(an*cos(n*pi*x)+bn*sin(n*pi*x));
end
ezplot(x,y,[-1,1])
grid on;hold on;
ezplot(x,(sum),[-1,1])
```



Exercise:

1. Obtain the Fourier series of the function $f(x) = x - x^2$ over ($-\pi$, π).
2. Obtain Fourier series expansion of $\cos x$ in (0, 4) up to 10 terms and plot.
3. Compute Fourier series expansion of the exponential signal e^{-x} in (-1,1) up to 12 terms and plot.
4. The temperature distribution $T(x)$ at a distance x measured from one end along a metal bar of length 2π is given by $T(x) = x \sin x$ over $(0, 2\pi)$. Express $T(x)$ as a Fourier series up to 15 terms and plot.

4 Fourier Transform

Topic learning outcomes:

Student will be able to:

1. Determine the Fourier transform of continuous time signals.
2. Evaluate the inverse Fourier Transform of the functions.

Syntax and description:

- `fourier(f)` –returns the Fourier transform of `f`. By default, the function `symvar` determines the independent variable, and `w` is the transformation variable.
- `fourier(f,transVar)` – uses the transformation variable `transVar` instead of w.
- `fourier(f,var,transVar)` – uses the independent variable var and the transformation variable `transVar` instead of `symvar` and w respectively.
- `ifourier(F)` returns the Inverse Fourier transform of `F`. By default, the independent variable is `w` and the transformation variable is `x`. If `F` does not contain `w`, `ifourier` uses the function `symvar`.
- `ifourier(F,transVar)` uses the transformation variable `transVar` instead of `x`.
- `ifourier(F,var,transVar)` uses the independent variable `var` and the transformation variable `transVar` instead of `w` and `x` respectively.

Example 4.1: Find Fourier transform of $f(x)=e^{-x^2}$.

```
syms x
f = exp(-x^2);
f_FT = fourier(f)
```

f_FT =

pi^(1/2)*exp(-w^2/4)

Example 4.2: Obtain the Fourier transform of the rectangular pulse

$$f(x)=\begin{cases}1, & |x|\le a\\ 0, & |x|>a\end{cases},$$

where ‘a’ is a positive constant.

```
syms a t
assume(a > 0)
f = rectangularPulse(-a, a, t);
f_FT = fourier(f)
```

```
f_FT =

(sin(a*w) + cos(a*w)*1i)/w - (cos(a*w)*1i - sin(a*w))/w
```

Note: This is nothing but 2*sin(a*w) /w.

Specify Independent Variable and Transformation Variable

Example 4.3: Compute the Fourier transform of $f(x) = e^{-t^2 - x^2}$.

By default, `symvar` determines the independent variable and `w` is the transformation variable. Here `symvar` chooses `x`.

```
syms t x
f = exp(-t^2-x^2);
fourier(f)
```

```
ans =

pi^(1/2)*exp(- t^2 - w^2/4)
```

Specify the transformation variable as `y`. If only one variable is specified, that variable is the transformation variable. `symvar` still determines the independent variable.

```
syms y
fourier(f,y)
```

```
ans =

pi^(1/2)*exp(- t^2 - y^2/4)
```

Specify both the independent and transformation variables as `t` and `y` in the second and third arguments respectively.

```
fourier(f,t,y)
```

```
ans =

pi^(1/2)*exp(- x^2 - y^2/4)
```

Example 4.4: Compute the inverse Fourier transform of $e^{-\frac{w^2}{4}}$. By default, the inverse transform is in terms of `x`.

```
syms w
F = exp(-w^2/4);
ifourier(F)
```

```
ans =

exp(-x^2)/pi^(1/2)
```

Default Independent Variable and Transformation Variable

Example 4.5: Compute the inverse Fourier transform of $e^{-w^2 - a^2}$.

By default, the independent and transformation variables are `w` and `x` respectively.

```
syms a w t
F = exp(-w^2-a^2);
ifourier(F)
```

```
ans =

exp(- a^2 - x^2/4)/(2*pi^(1/2))
```

Specify the transformation variable as `t`. If only one variable is specified, that variable is the transformation variable. The independent variable is still `w`.

```
ifourier(F,t)
```

```
ans =

exp(- a^2 - t^2/4)/(2*pi^(1/2))
```

Exercise:

1. Obtain the Fourier transform of the triangular pulse.
2. Compute the Fourier transform of te^{-t^2}.
3. Obtain the inverse Fourier transform of $2e^{-|w|-1}$.
4. Compute inverse Fourier transform of $e^{-\frac{w^2 - b^2}{2}}$, where b is a constant.

5 Statistics

Topic learning outcomes:

Student will be able to:

1. Find measures of central tendency: moments, skewness and kurtosis.
2. Fit data into several types of curves.
3. Obtain correlation coefficient and regression lines.

Syntax and description:

- `m = moment(X,order)` returns the central moment of X for the order specified by order.
- `y = skewness(X)` returns the sample skewness of X.
- `k = kurtosis(X)` returns the sample kurtosis of X.
- `R = corrcoef(A,B)` returns coefficients between two variables A and B.
- `fitobject = fit(x,y,fitType)` creates the fit to the data in x and y with the model specified by fitType.
- `R = corrcoef(A,B)` returns correlation coefficient between two random variables A and B.
- `[r,m,b] = regression(x,y)` returns correlation coefficient 'r' between two variables x and y, slope 'm' of the regression line of y on x, i.e., y= mx+ b and constant term 'b'.

Note: `fitType` can be

1. Linear model 'Poly1':
   `f(x) = p1*x + p2`
2. Quadratic model 'Poly2':
   `f(x) = p1*x^2 + p2*x + p3`
3. General model 'Exp1':
   `f(x) = a*exp(b*x)`
4. General model 'Power1':
   `f(x) = a*x^b`

Example 5.1 Wages of workers in a company are given in the following table:

Wages	50-70	70-90	90-110	110-130	130-150	150-170	170-190
No. of workers(f)	4	8	12	20	6	7	3

Calculate the first four central moments, skewness and kurtosis for the following distribution.

Note : 'x' is taken as the middle value of class interval, 'f' is no. of times x is repeated.

```
x=[60 60 60 60 80 80 80 80 80 80 80 80 100 100 100 100 100 100 100 100 100
100 100 100 120 120 120 120 120 120 120 120 120 120 120 120 120 120 120
120 120 120 120 120 140 140 140 140 140 140 160 160 160 160 160 160 160
180 180 180];
```

```
y1=moment(x,1)
y2=moment(x,2)
y3=moment(x,3)
y4=moment(x,4)
sk = skewness(x)
k = kurtosis(x)
```

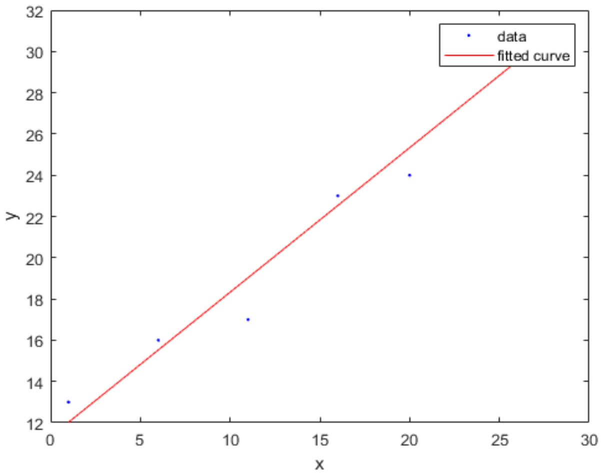
```
ans =
y1 = 0
y2 = 926.5556
y3 = 4.7747e+03
y4 = 2.1951e+06
sk = 0.1693
k = 2.5569
```

Example 5.2: Fit a straight line to the following data.

x	1	6	11	16	20	26
y	13	16	17	23	24	31

```
x = [1  6       11      16      20      26]';
y =[ 13   16       17       23       24       31]';
f=fit(x,y,'Poly1')
plot(f,x,y)
```

```
f =
     Linear model Poly1:
     f(x) = p1*x + p2
     Coefficients (with 95% confidence bounds):
       p1 =      0.7008  (0.4936, 0.908)
       p2 =       11.32  (8.057, 14.59)
```



Example 5.3: The following table gives the results of the measurements of train resistances; V is the velocity in mile per hour and R is the resistance in pound per ton.

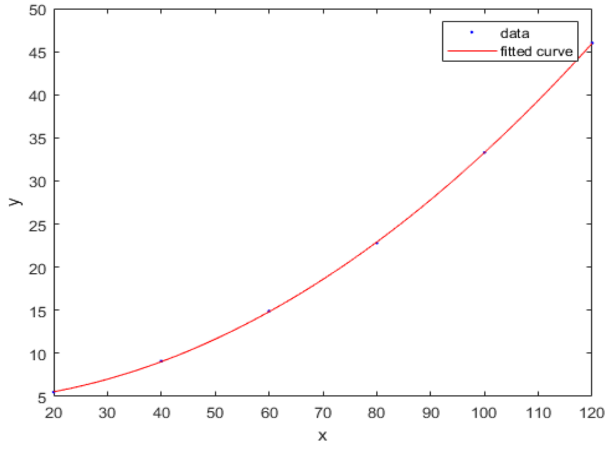
V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46

If R is related to V by the relation $R = a + bV + cV^2$.

```
x=[20   40      60      80      100     120]';
y=[5.5  9.1     14.9    22.8    33.3    46]';
f=fit(x,y,'Poly2')
plot(f,x,y)
```

```
f =

     Linear model Poly2:
     f(x) = p1*x^2 + p2*x + p3
     Coefficients (with 95% confidence bounds):
       p1 =     0.002871  (0.002753, 0.002989)
       p2 =     0.002411  (-0.01447, 0.01929)
       p3 =         4.35  (3.834, 4.866)
```



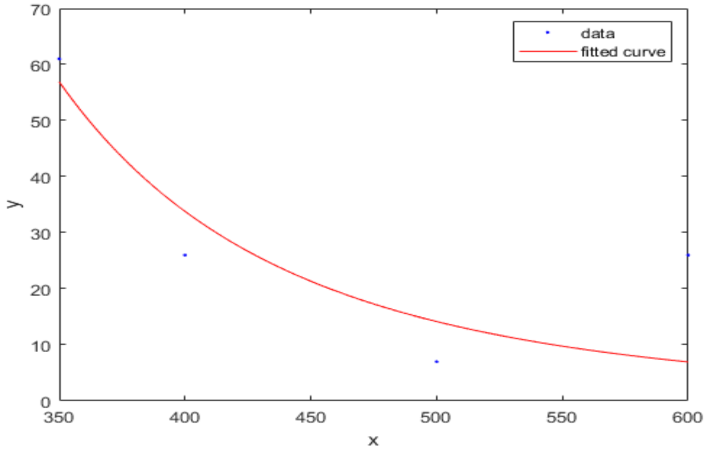
Example 5.4: An experiment gave the following values

v(ft/min)	350	400	500	600
t(min)	61	26	7	26

It is known that v and t are connected by the relation $v = at^b$.

```
v=[350   400    500     600]';
t=[61    26     7       26]';
f=fit(v,t,'Power1')
plot(f,v,t)
```

```
f =
     General model Power1:
     f(x) = a*x^b
     Coefficients (with 95% confidence bounds):
       a =    4.802e+11  (-2.764e+13, 2.86e+13)
       b =       -3.902  (-13.81, 6.005)
```

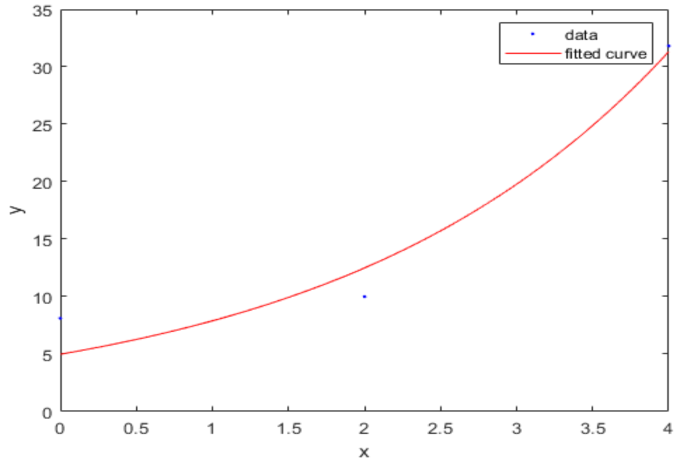


Example 5.5: Fit a curve of the form $y = a\,e^{b\,x}$ to the data by the method of least squares.

x	0	2	4
y	8.12	10	31.82

```
x=[0    2       4]';
y=[8.12 10      31.82]';
f=fit(x,y,'Exp1')
plot(f,x,y)
```

```
f =
     General model Exp1:
     f(x) = a*exp(b*x)
     Coefficients (with 95% confidence bounds):
       a =        4.989  (-27.4, 37.38)
       b =       0.4593  (-1.273, 2.192)
```



Example 5.6 The following table gives the stopping distance y in meters of a motor bike moving at a speed of x kms/hour when the breaks are applied.

x	16	24	32	40	48	56
y	0.39	0.75	1.23	1.91	2.77	3.81

Find the correlation coefficient between the speed and the stopping distance, and the equations of regression lines. Hence estimate the maximum speed at which the motor bike could be driven if the stopping distance is not to exceed 5 meters.

```
x=[16   24      32      40      48      56]';
y=[0.39 0.75    1.23    1.91    2.77    3.81]';
R = corrcoef(x,y)
[r,m,b] = regression(x,y)
```

```
ans =
R = 2×2
    1.0000    0.9827
    0.9827    1.0000

r = 0.9827
m = 0.0851
b = -1.2551
```

Exercise:

1. Find the second, third and fourth central moments of the frequency distribution given below. Hence, find (i) a measure of skewness and (ii) a measure of kurtosis.

Class limits	110.0 – 114.9	115.0 – 119.9	120.0 – 124.9	125.0 – 129.9	130.0 – 134.9	135.0 – 134.9	140.0 – 144.9
Frequency	5	15	20	35	10	10	5

2. Find the second, third and fourth central moments of the frequency distribution given below. Hence, find (i) a measure of skewness and (ii) a measure of kurtosis.

x	5	10	15	20	25	30	35
f	4	3	2	8	10	10	5

3. Estimate the mean radiation dose at an altitude of 3000 ft by fitting a curve y=ae^{x} to the given data:

Altitude(x)	50	450	780	1200	4400	4800	5300
Dose of radiation	28	30	32	36	51	58	69

4. The latent heat of vaporisation of steam r is given in the following table at different temperature t:

t	40	50	60	70	80	90	100	110
r	1069.1	1063.6	1058.2	1052.7	1049.3	1041.8	1036.3	1030.8

Fit a relation to the form r =a+bt.

5. The following table gives the result of the measurement of train resistances; V is the velocity in miles per hour; R is the resistance in pounds per ton:

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

6. The following data represent carbon dioxide(co_2) emission from coal-fluid boilers (In units of 1000 tons) over a period of years 2005 to 2017. The variable (year) has been standardized and yield following table.

Years(x)	0	5	8	9	10	11	12
co_2 emission (y)	910	680	520	450	370	350	340

Find regression model of y on x.

7. Obtain the lines of regression and hence find the coefficient of correlation for the following data

X	1	3	4	2	5	8	9	10	13	15
Y	8	6	10	8	12	16	16	10	32	32

****