

# Convergence :

(Example quoted =  $x - \cos(x)$ ; for bisection, False position, Fixed point , N-R, Secant)

**Bisection** : It has a linear rate of convergence , we can predict the number of iterations required (*no. of iteration : 14*)

**False position** : It's rate of convergence is 1.618 (approximately) though for some problems it's rate might fall below bisection. (*no. of iteration : 5*)

**Fixed Point** : It also has a linear rate of convergence though the method may not always converge , when  $|g'(x)| < 1$  the method definitely converges. (*no. of iteration : 25*)

**Newton Raphson** : It has quadratic rate of convergence, it is one of the fastest methods. (*no. of iteration : 4*)

**Secant** : It's rate of convergence is between 1 and 2 (1.62) approx. (*no. of iteration : 4*)

**Muller**: It's rate of convergence is 1.82 it is faster than secant but slower than newton Rapshon.

**Bairstow**: Bairstow's algorithm inherits the local quadratic convergence of Newton's method, except in the case of quadratic factors of multiplicity higher than 1, when convergence to that factor is linear.

# Stability

(Example quoted =  $x - \cos(x)$ ; for bisection, False position, Fixed point, N-R, Secant)

**Bisection** : Bisection is a very stable method it always converges to a solution irrespective of initial guess though the method might be relatively slow (  $error = 0.0661$  )

**False position** : This is also a stable method as it always converges towards the solution . Its error is also very less than the bisection method . (  $error = 8.9997e-04$  )

**Fixed Point** : It is not always stable it guarantees convergence only when  $|g'(x)| < 1$  in other cases it may or may not converge. (  $error = 0.0069$  )

**Newton Raphson** : Newton-Raphson converges extremely rapidly. The method suffers from the fact that it can be wildly unstable if we start far from the root. If it happens to land near a turning point, where  $g'(z) = 0$ , the method will fail badly. Therefore Newton Raphson is unstable in some cases and doesn't always guarantee a solution . (  $error = 0.0038$  )

**Secant** : With the secant scheme, the root may not remain bracketed, and there is always the possibility of an instability causing the method to fail. This method works best when we are already close to the solution. (  $error = 0.0046$  )

**Muller:** Like the secant method, muller is also not always convergent and therefore unstable .

**Bairstow:** It is particularly unstable when the polynomial has an odd degree and only one real root. Quadratic factors that have a small value at this real root tend to diverge to infinity.