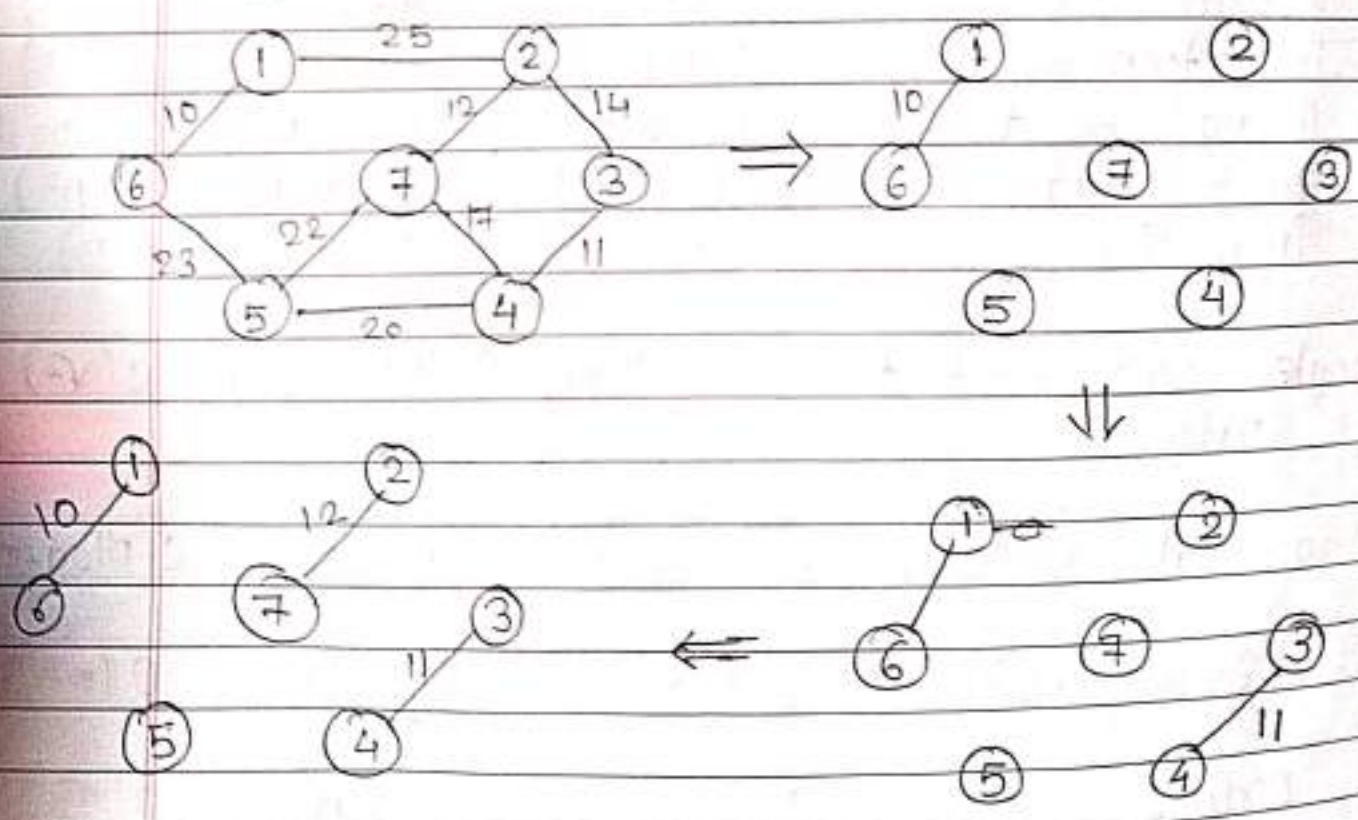


* Kruskal Algorithm:-

- Select the edges is minimum weight it is not necessary that Selected optimum edge is adjacent.

- i) Solve all the edges in decreasing order of there weight.
- ii) Pick the Smallest edge and check if it forms the cycle. if the cycle is not form include this edge else discard it.
- iii) Repeat Step ii) Until there are $V-1$ edges in the Spanning tree.

Qo 1] Find the Cost of minimum spanning tree by using Kruskal Algorithm.

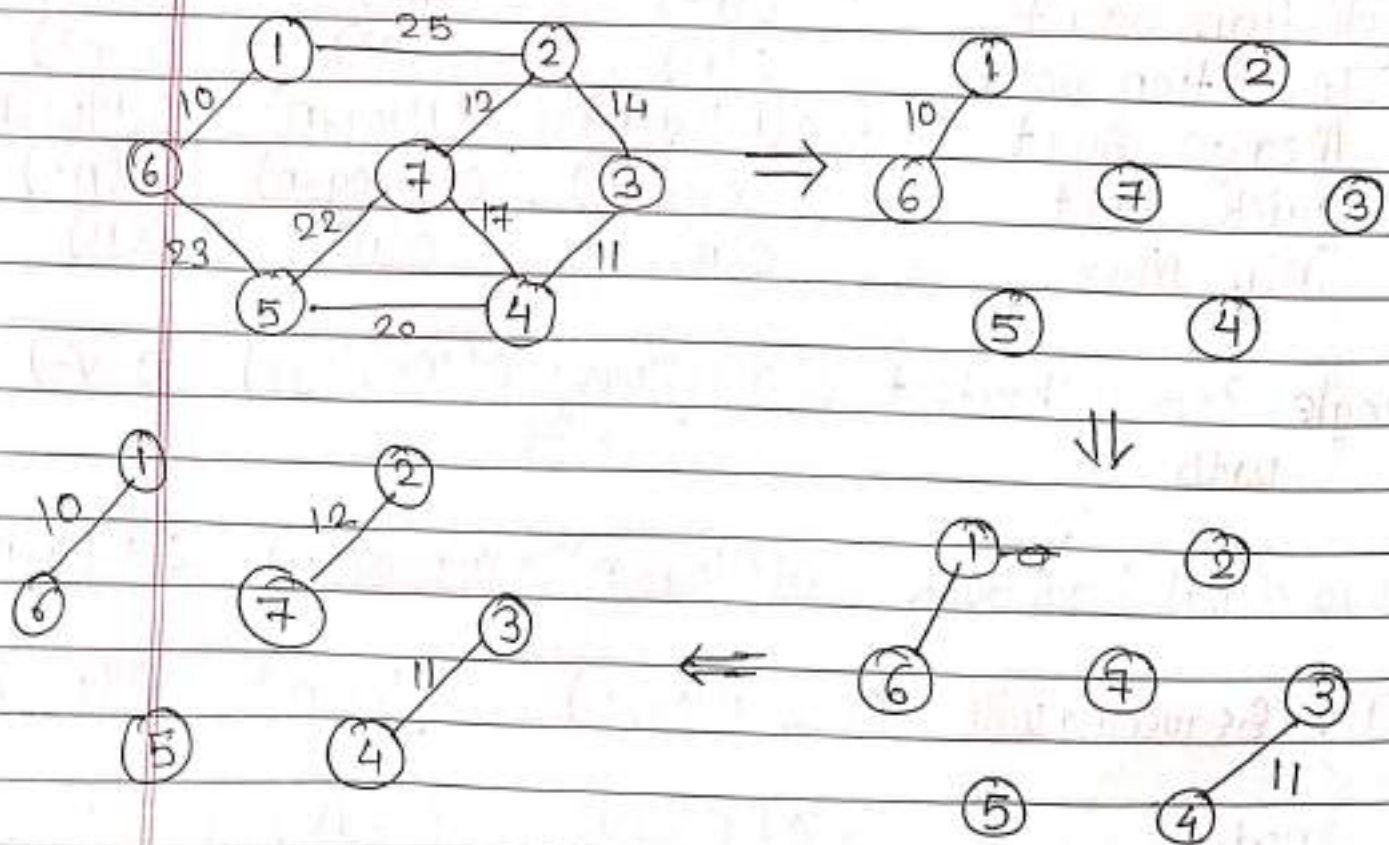


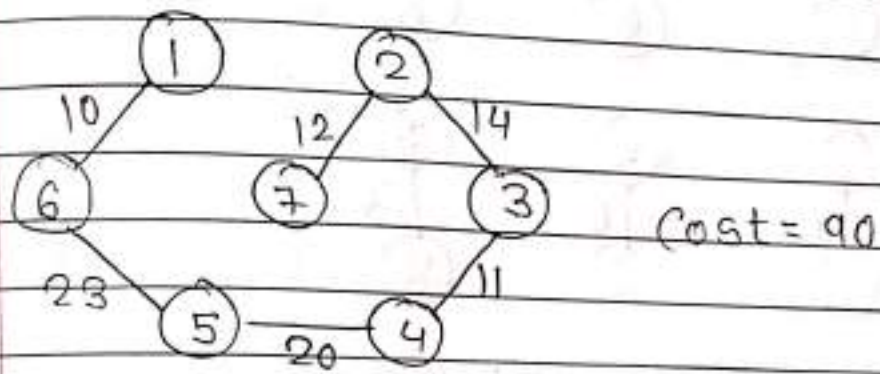
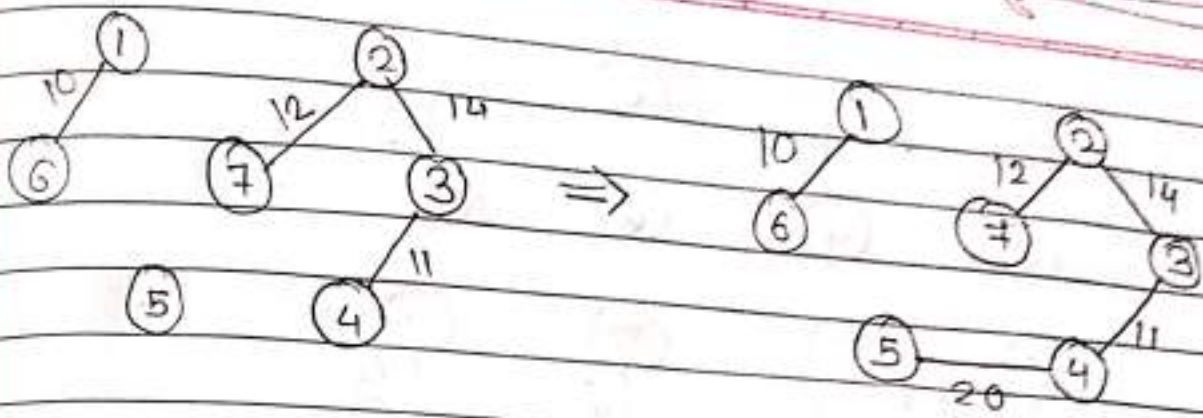
* Kruskal Algorithm:-

- Select the edges is minimum weight it is not necessary that Selected optimum edge is adjacent.

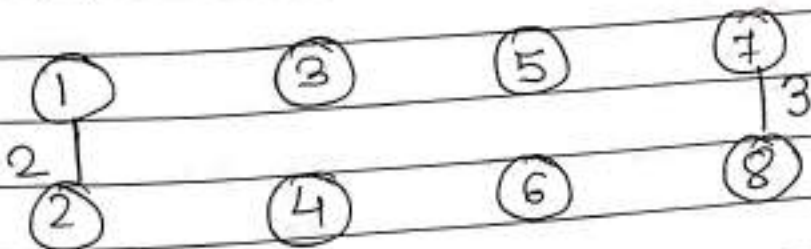
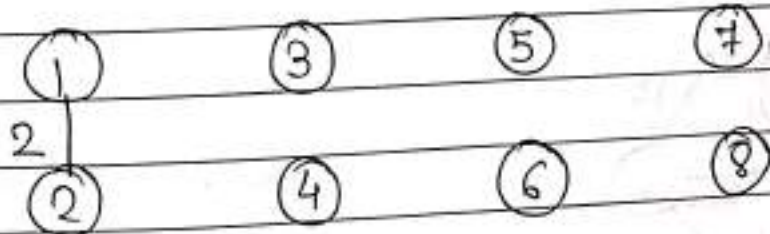
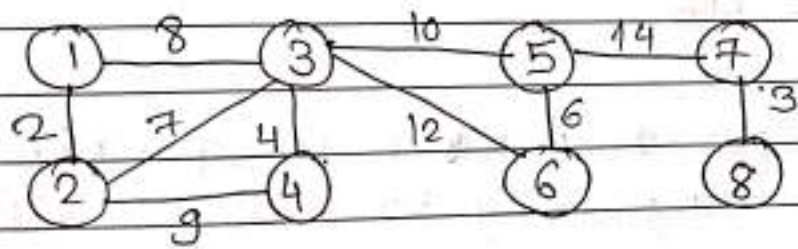
- (i) Solve all the edges in decreasing order of there weight.
- (ii) Pick the Smallest edge and check if it forms the cycle. if the cycle is not form include this edge else discard it.
- (iii) Repeat Step (ii) Until there are $V-1$ edges in the Spanning tree.

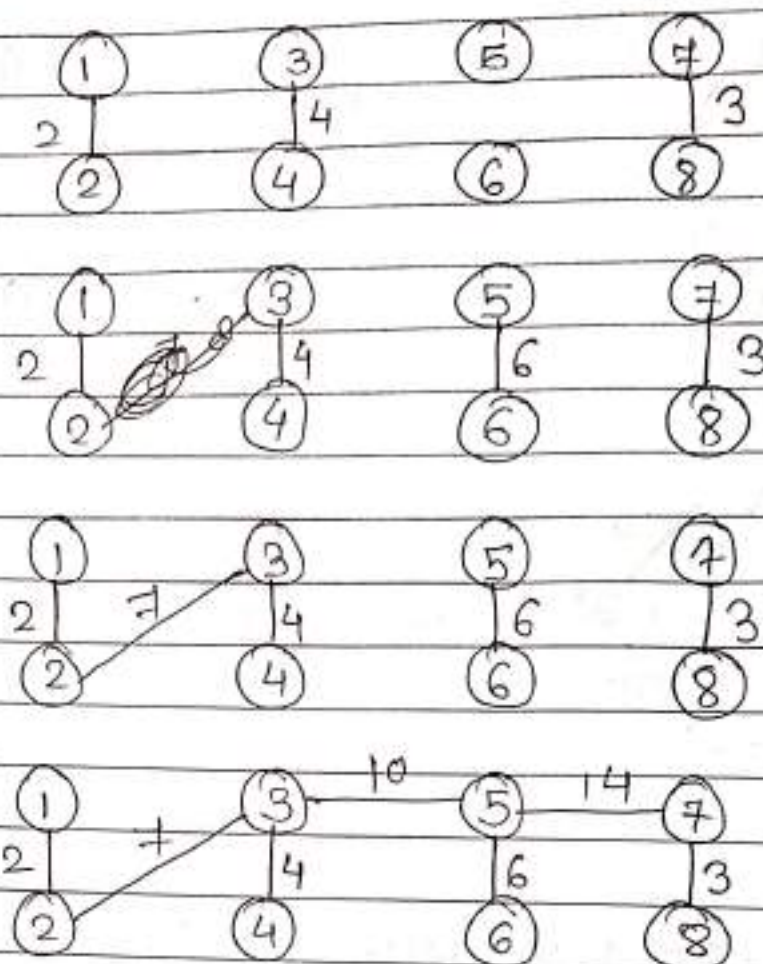
Qo 1] Find the Cost of minimum spanning tree by using Kruskal Algorithm.





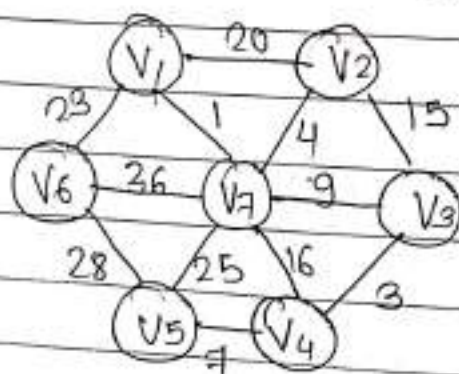
Q.2] Find the cost of minimum spanning tree for the following graph by using Kruskal Algorithm.

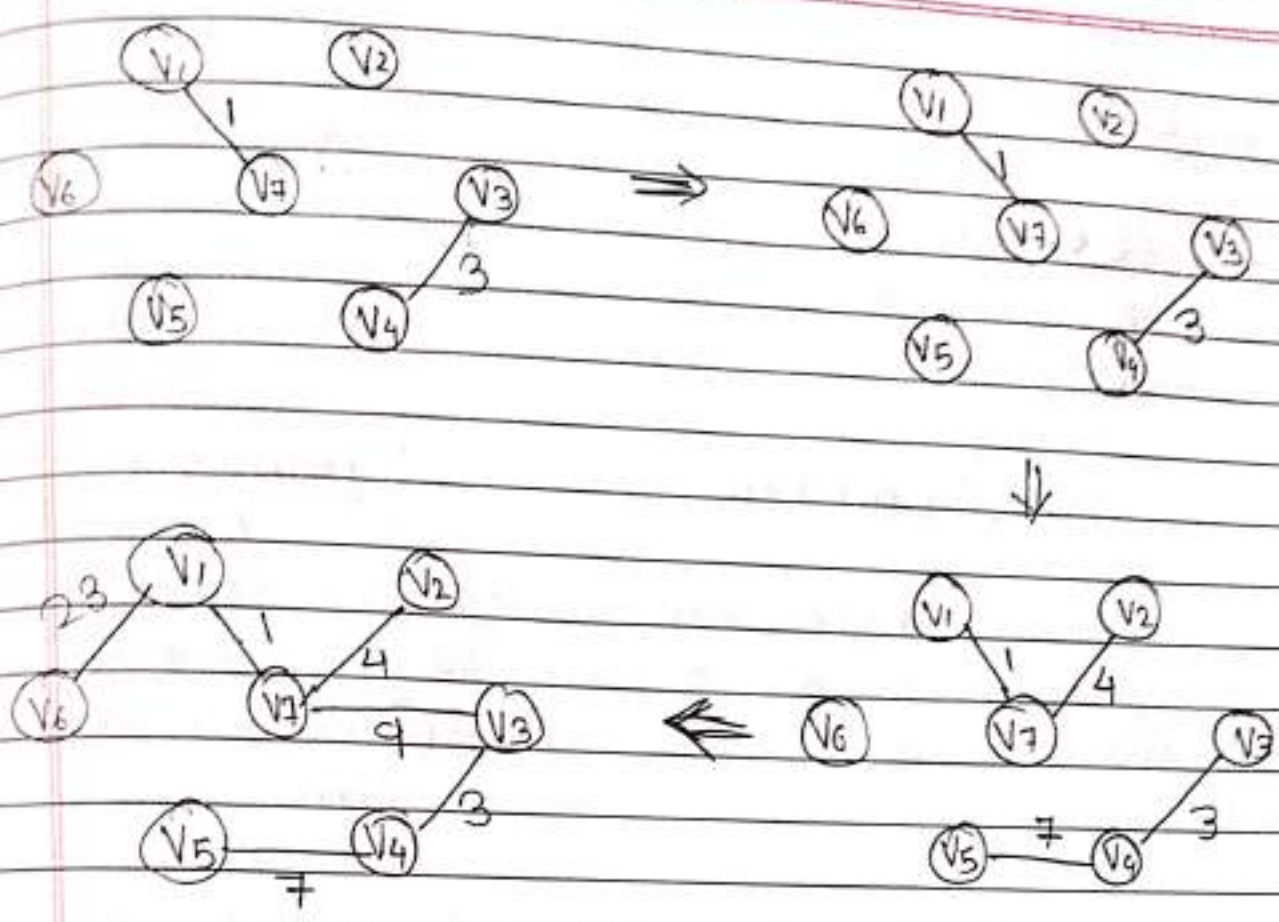




$$\text{Cost} = 46$$

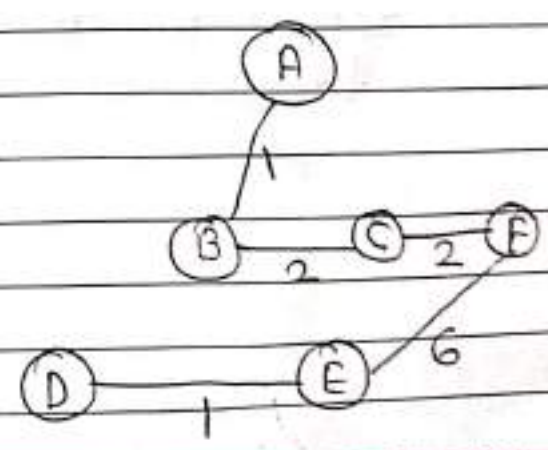
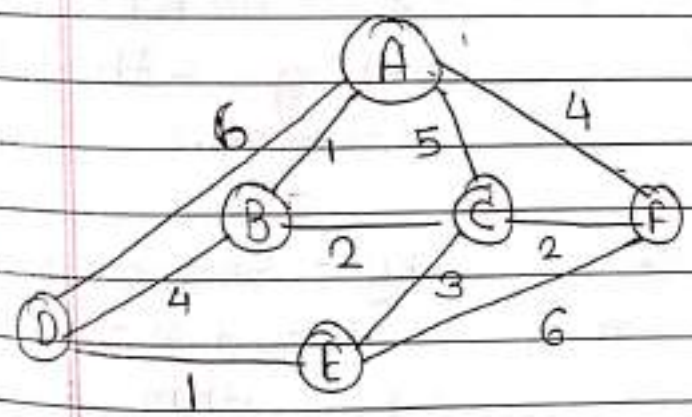
Q. 3] Find the Cost of minimum Spanning tree for the following using Kruskal Algorithm.



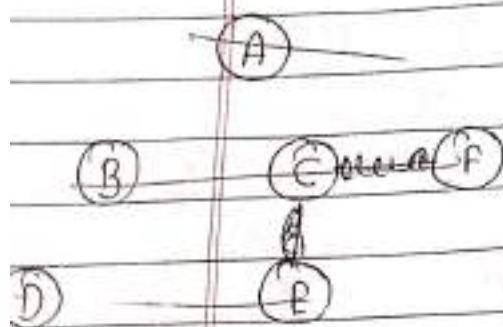


Cost = 47

Qo4] Find cost of minimum spanning tree for the following tree by using prims.



$$1 + 2 + 2 + 6 + 1 = 12 //$$



Greedy method

In this method decision is taken best of the inform

There set feasible Solutions & picks up the optimal Solutions.

There is no garenty to get the optimal Solution.

eg:- fractional knapSack problem, minimum Spanning tree.

Dynamic programming Approach

It is time varying it gives the information about particular time

There is no special set of feasible Solutions.

It is garented that DPA gives the optimal Solution.

eg:- Belman flood, Floyd Warshall Algorithm, zero/ knapSack problem, Assembly Scheduling problem & longest Common SubSequence.

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* All pair shortest path or Floyd Warshall Algorithm: -

- It is a weighted graph is represented by weight matrix the object is used to find shortest path between every pair of nodes.

All pair Shortest path or Floyd Warshall Algorithm All-pairs

Cost $[1 \dots n]$ is cost of square matrix of graph with n vertices

$A[i, j]$ is cost of shortest path from vertex i to j

Cost $(i, i) = 0, 1 \leq i \leq n$

for $i = 1$ to n do {

for $j = 1$ to n do {

$A[i, j] = \text{cost}[i, j]$ }

for $k = 1$ to n do { // for matrix

for $i = 1$ to n do { // for row

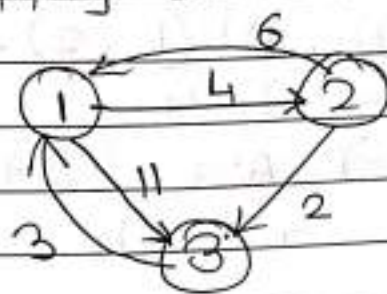
for $j = 1$ to n do { // for column

$A^k(i, j) = \min(A^{k-1}(i, j), A^{k-1}(i, k) + A^{k-1}(k, j))$

}

}

Ex] Apply APSP of the following graph:



$$A^0 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix} \end{matrix}$$

$$K=1$$

$$A^1(1,1) = \min(A^0(1,1), A^0(1,1) + A^0(1,1))$$

$$= \min(0, 0+0) = 0$$

$$A^1(1,2) = \min(A^0(1,2), A^0(1,1) + A^0(1,2))$$

$$= \min(4, 0+4) = 4$$

$$A^1(1,3) = \min(A^0(1,3), A^0(1,1) + A^0(1,3))$$

$$= \min(11, 0+11) = 11$$

	1	2	3
1	0	4	11
2	6	0	2
3	3	7	0

$$A^1(2,1) = \min(A^0(2,1), A^0(2,1) + A^0(1,1))$$

$$= \min(6, 6+0) = 6$$

$$A^1(2,2) = 0$$

$$A^1(2,3) = \min(A^0(2,3), A^0(2,1) + A^0(1,3))$$

$$= \min(2, 6+11) = 2$$

$$A^1(3,1) = \min(A^0(3,1), A^0(3,1) + A^0(1,1))$$

$$= \min(3, 3+0) = 3$$

$$A^1(3,2) = \min(A^0(3,2), A^0(3,1) + A^0(1,2))$$

$$= \min(\infty, 3+4) = 7$$

$$A^1(3,3) = 0$$

$$K=2$$

$$A^2(1,2) = \min(A^1(1,2), A^1(1,2) + A^1(2,2))$$

$$= \min(4, 4+0) = 4$$

$$A^2(1,3) = \min(A^1(\quad), A^1(\quad) + A^1(\quad))$$

$$= \min(\quad , \quad + \quad)$$

$$A^2(1,1) = 0$$

$$A^2(2,1) = \min(A^1(2,1), A^1(2,2) + A^1(2,1))$$

$$= \min(6, 0+6) = 6$$

$$A^2(2,2) = 0$$

$$A^2(2,3) = \min(A^1(2,3), A^1(2,2) + A^1(2,3))$$

$$= \min(2, 0+2) = 2$$

$$A^2(3,1) = \min(A^1(3,1), A^1(3,2) + A^1(2,1))$$

$$= \min(3, 7+6) = 3$$

$$A^2(3,2) = \min(A^1(3,2), A^1(3,2) + A^1(2,2))$$

$$= \min(7, 7+0) = 7$$

$$A^2(3,3) = 0$$

	1	2	3
1	0	4	6
2	6	0	2
3	3	7	0

$A^3 =$	0	4	6
	5	0	2
	3	7	0

$k = 3$

$$A^3(1,1) = 0$$

$$A^3(1,2) = \min(A^2(1,2), A^2(1,3) + A^2(3,2))$$

$$= \min(4, 6+7) = 4$$

$$A^3(1,3) = 6$$

$$A^3(2,1) = \min(A^2(2,1), A^2(2,3) + A^2(3,1))$$

$$= \min(6, 2+3) = 5$$

$$A^3(2,2) = 0$$

$$A^3(2,3) = 2$$

3rd row remain same.

* Single Source Shortest path & Bellman ford Algorithm

- It is used to find shortest path from source to destination.
- It is slower than dijkstra's Algorithm.
- It can handle negative weights.
- If the graph contains negative weight then it is not possible to find shortest path.
- In dijkstra Algorithm vertex is processed that vertex is not processed again & again.
- In Bellman ford Algorithm if the vertex is processed that vertex is processed again & again until obtained we get optimal solution.

Algorithm Bellman-ford

I/p : Weighted graph.

$w(u, v)$ = weight of edge (u, v)

o/p : Shortest distance of each vertex from given source vertex

// Initialization

for each $v \in V$ do

$d[v] = \infty$

$\pi[v] = \text{nil}$

end

$d[s] \leftarrow 0; \quad \pi[s] \leftarrow \text{nil}$

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```

for each adjacent node v of u do
  Val(u) = dist(u) + wt(u, v)
  If Val(v) < dist(v)
    dist(v) = Val(v)
     $\pi(v) \leftarrow u$ 
  end
end

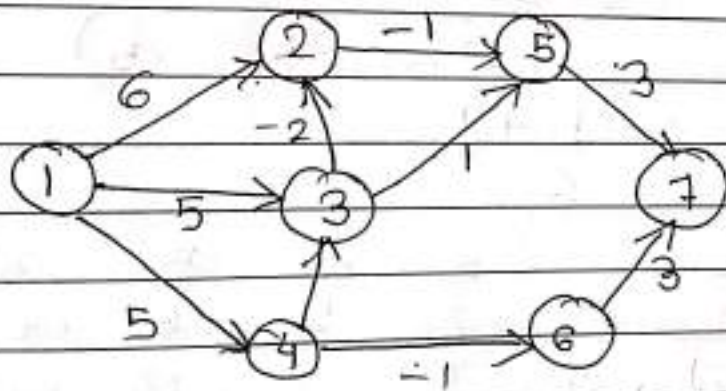
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```

// check for negative Cycle
for each edge (u, v)  $\in E$  do
  if  $d[u] + w(u, v) < d[v]$  then
    error "Graph Contains negative Cycle"
  end
end

```

Solve the Shortest path from Source 1 to 7



Vertex v	1	2	3	4	5	6	7
dist[v]	0	∞	∞	∞	∞	∞	∞
Parent $\pi[v]$	nil	nil	nil	nil	nil	nil	nil

1 \rightarrow Unprocessed Vertex having min distance

adjacent of 1 are 2, 3, 4

$$\text{Val}(2) = \text{dist}(1) + \text{wt}(1,2)$$

$$= 0 + 6 = 6$$

$$\text{Val}(2) < \text{dist}(2)$$

$$\text{dist}(2) = 6$$

$$\text{Val}(3) = 5$$

$$\text{Val}(4) = 5$$

Vertex V	1	2	3	4	5	6	7
dist[v]	0	6	5	5	∞	∞	∞
$\pi[v]$	nil	1	1	1	nil	nil	nil

3 \rightarrow Unprocessed Vertex having minimum distance

adjacent of 3 = 2, 5

$$\text{Val}(2) = \text{dist}(3) + \text{wt}(3,2)$$

$$= 5 - 2$$

$$= 3$$

$$\text{Val}(5) = \text{dist}(3) + \text{wt}(3,5)$$

$$= 5 + 1$$

$$= 6$$

Vertex V	1	2	3	4	5	6	7
dist[v]	0	3	5	5	6	∞	∞
$\pi[v]$	nil	3	1	1	3	nil	nil

2 \rightarrow Unprocessed Vertex having min distance

adjacent of 2 = 5

$$\text{Val}(5) = \text{dist}(2) + \text{wt}(2,5)$$

$$= 3 - 1 = 2$$

Vertex V	1	2	3	4	5	6	7
dist[V]	0	3	5	5	2	∞	∞
$\pi[V]$	nil	3	1	1	2	nil	nil

5 \rightarrow Unprocessed Vertex having minimum distance.

adjacent of 5 = 7

$$\begin{aligned} \text{Val}(7) &= \text{dist}(5) + \text{wt}(5, 7) \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

Vertex V	1	2	3	4	5	6	7
dist[V]	0	3	5	5	2	∞	5
$\pi[V]$	nil	3	1	1	2	nil	5

4 \rightarrow Unprocessed Vertex having minimum distance

adjacent of 4 = 3, 6

$$\begin{aligned} \text{Val}(3) &= \text{dist}(4) + \text{wt}(4, 3) \\ &= 5 - 2 = 3 \end{aligned}$$

$$\begin{aligned} \text{Val}(6) &= \text{dist}(4) + \text{wt}(4, 6) \\ &= 5 - 1 = 4 \end{aligned}$$

Vertex V	1	2	3	4	5	6	7
dist[V]	0	3	3	5	2	4	5
$\pi[V]$	nil	3	4	1	2	4	5

3 \rightarrow Unprocessed Vertex having minimum distance.

adjacent of 3 = 2, 5

$$\begin{aligned} \text{Val}(2) &= \text{dist}(3) + \text{wt}(3, 2) \\ &= 3 - 2 = 1 \end{aligned}$$

$$\text{Val}(5) = \text{dist}(3) + \text{wt}(3,5)$$

$$= 3 + 1 = 4$$

$$\text{Val}(5) > \text{dist}(5)$$

No change

Vertex V	1	2	3	4	5	6	7
dist[v]	0	1	3	5	2	4	5
$\pi[v]$	nil	3	4	1	2	4	5

2 \leftarrow Unprocessed Vertex having minimum distance

adjacent of 2 = 5

$$\text{Val}(5) = \text{dist}(2) + \text{wt}(2,5)$$

$$= 1 + 1$$

$$= 2$$

Vertex V	1	2	3	4	5	6	7
dis[v]	0	1	3	5	0	4	5
$\pi[v]$	nil	3	4	1	2	4	5

5 \leftarrow Unprocessed Vertex

adjacent of 5 = 7

$$\text{Val}(7) = \text{dist}(5) + \text{wt}(5,7)$$

$$= 0 + 3$$

$$= 3$$

Vertex V	1	2	3	4	5	6	7
dis[v]	0	1	3	5	0	4	3
$\pi[v]$	nil	3	4	1	2	4	5

6 \rightarrow Unprocessed Vertex
adjacent of 6 = 7
 $Val(7) = dist(6) + wt(6,7)$
 $= 4 + 3$
 $= 7$

No change.



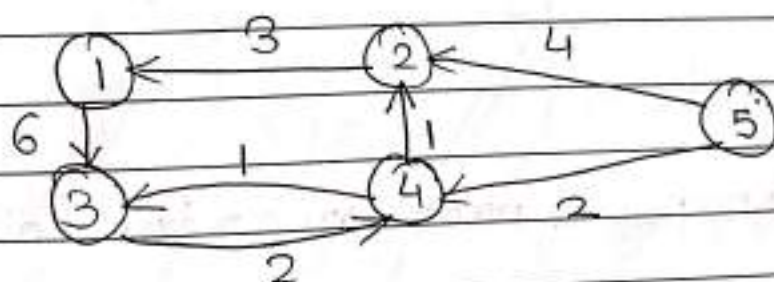
• Base Case :-

$O(E)$ (No. of edges)

• Worst Case :- (Average Case)

$O(EV)$ (No. of edges \times No. of Vertices)

Q2 Find the Shortest path for the Source Vertex is 5



* 0/1 knapsack problem :-

Algorithm binary-knapsack

// IP :- weight, profit, Capacity

// OP :- Array V which holds solution of problems

for $i=0$ to n do

$V[i, 0] \leftarrow 0$

end

for $j=0$ to M do

$V[0, j] = 0$

end

for $j=1$ to M do

for $i=1$ to n do

if $w[i] \leq j$

$V[i, j] = \max \{ V(i-1, j), V_i + V[i-1, j-w[i]] \}$

else

$V[i, j] \leftarrow V[i-1, j] \quad // j < w[i]$

end

end

end

Q01]

Apply 0/1 knapsack problem on the following data.

$P_i = (10, 12, 15)$, weight $(4, 6, 8)$, $M=4$

			$\rightarrow j$					x_1	x_2	x_3
		j	0	1	2	3	4			
P_i	w_i	0	0	0	0	0	0	1	0	0
10	4	1	0	0	0	0	10			
12	6	2	0	0	0	0	10			
15	8	3	0	0	0	0	10			

weight = 4 = M
profit = 10

Fill 1st Column

$$V[i, j], V[1, 1], i=1, j=1, w_i=4, j < 4, 1 < 1$$

$$V[i, j] = V[i-1, j]$$

$$V[1, 1] = V[0, 1]$$

$$= 0$$

Fill 2nd Column

Fill 3rd Column

Fill 4th Column

$$V[i, j], V[1, 4], i=1, j=4, w_i=4, j=w_i$$

$$V(1, 4) = \max(V(0, 4), 10 + V[0, 0])$$

$$= \max(0, 10 + 0)$$

$$= 10$$

Q2

→ $P_i (10, 12, 15), \text{weight} = (4, 6, 8), M=10$

$j > w_i$
Choose the maximum value.

→ j

		i	0	1	2	3	4	5	6	7	8	9	10	x_1	x_2	x_3
P_i	w_i	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
10	4	1	0	0	0	0	10	10	10	10	10	10	10			
12	6	2	0	0	0	0	10	10	12	12	12	12	22			
15	8	3	0	0	0	0	10	10	12	12	15	15	22			

$j < w_i$ then choose the previous value.

Weight of 2nd object is = 6

If we consider weight of first object

Then total weight = $6 + 4 = 10 = \text{capacity}$

$$\text{Weight} = 4 + 6 = 10$$

$$\text{profit} = 10 + 12 = 22$$

Q. 3] $P_i (1, 4, 5, 7)$ $W_i = 1, 3, 4, 5$ $M = 7$

		1	0	1	2	3	4	5	6	7
P_i	W_i	1	0	0	0	0	0	0	0	0
1	1	1	0	1	1	1	1	1	1	1
4	3	2	0	1	1	4	5	5	5	5
5	4	3	0	1	1	4	5	6	6	9
7	5	4	0	1	1	4	5	7	8	9

x_1, x_2, x_3, x_4
0 1 1 0

$$9 - 5 = 4$$

$$4 - 4 = 0$$

$$\text{Weight} = 4 + 3 = 7$$

$$\text{profit} = 5 + 4 = 9$$

* Longest Common Subsequence :-

✓ String 1 a b c d e f g h i j
String 2 e c d g i

Common Sequence = c d g i (longest)

String 1 a b c d e f g h i j
String 2 e c d g i

Common Sequence = e g i

Algorithm $LCS(x, y)$

// x is String of length n

// y is String of length m

for $i = 0$ to m do

$LCS[i, 0] = 0$

end

for $j = 0$ to n do

$LCS[0, j] = 0$

end

for $i = 1$ to m do

for $j = 1$ to n do

if $x_i == y_j$ then

$LCS[i, j] = LCS[i-1, j-1] + 1$

else

if $LCS[i-1, j] \geq LCS[i, j-1]$ then

$LCS[i, j] = LCS[i-1, j]$

else

$LCS[i, j] = LCS[i, j-1]$

end

end

end

Q1] Find Longest Common Subsequence of the following Strings.

String 1: a b c d

String 2: b d

			a	b	c	d
		0	1	2	3	4
Not match then take maximum		0	0	0	0	0
if match then consider diagonal + that element	b	1	0	1	1	1
	d	2	0	1	1	2

Common = b d

Q 02] String 1 : l o n g e s t
String 2 : s t o n e

				L	O	n	g	e	s	t		
				0	1	2	3	4	5	6	7	
	0	0	0	↑	0	0	0	0	0	0	0	
s	1	0	0	↑	0	0	0	0	1	1	1	
t	2	0	0	↑	0	0	0	0	1	2	2	
o	3	0	0	↑	1	1	1	1	1	2	2	
n	4	0	0	1	2	←	2	←	2	2	2	
e	5	0	0	1	2	2	←	3	←	3	←	3

Longest Common Subsequence = one

Q 03] String 1 : A B C
String 2 : A B C

			A	B	C
		0	1	2	3
	0	0	0	0	0
A	1	0	1	1	1
B	2	0	1	2	2
C	3	0	1	2	3

Longest Common Subsequence = A B C

Q4] String 1: A B C
String 2: A C B

		A	B	C
A	0	1	0	0
C	1	0	1	1
B	2	1	2	2

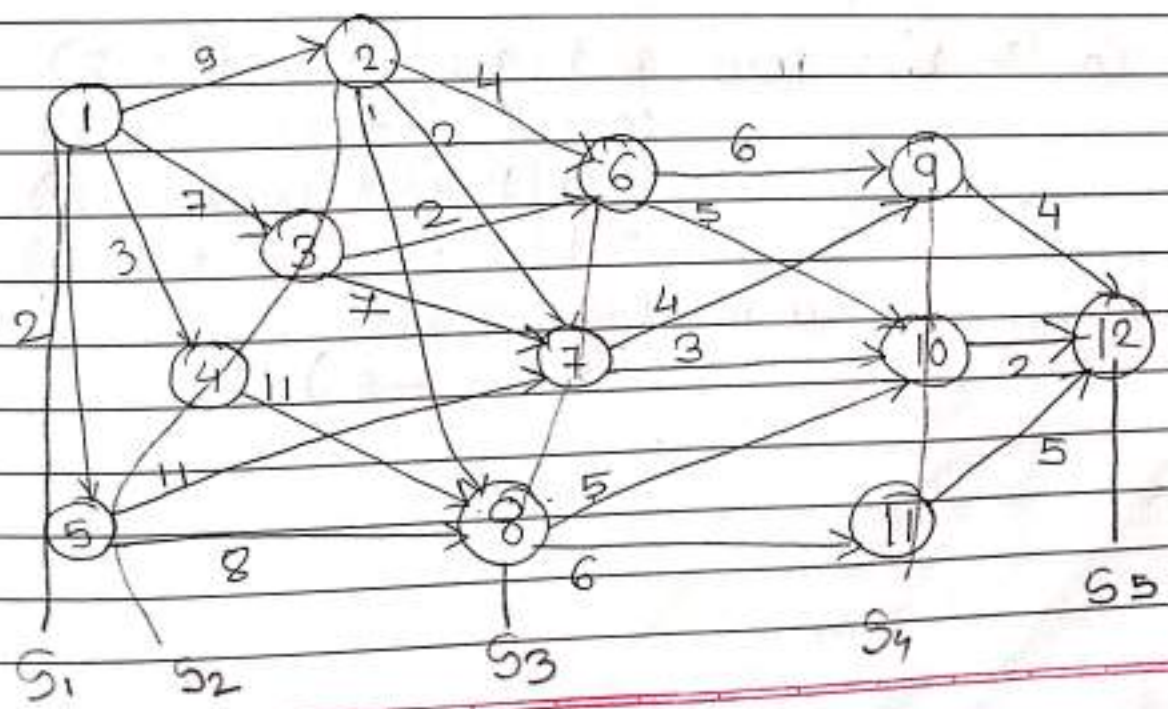
LCS = AC

* Multistage Graph:-

- Multistage graph is used to find
- Minimum cost path from source to destination.
- There are two approaches:-
 - a) Forward Approach
 - b) Backward Approach.

Q5] Find minimum cost path for the following graph by Using backward approach of multistage graph.

→



$$\text{Cost}(\text{I}, 1) = 0$$

$$\text{Cost}(\text{II}, 2) = 9$$

$$\text{Cost}(\text{III}, 3) = 7$$

$$\text{Cost}(\text{II}, 4) = 3$$

$$\text{Cost}(\text{II}, 5) = 2$$

$$\text{Cost}(\text{III}, 6) = 9(3-6)$$

$$\text{Cost}(\text{III}, 7) = 11(2-7)$$

$$\text{Cost}(\text{III}, 8) = 10(2-8, 5-8)$$

$$\begin{aligned}\text{Cost}(\text{III}, 6) &= \min(\text{Cost}(\text{II}, 2) + \text{Cost}(2, 6), \\ &\quad \text{Cost}(\text{II}, 3) + \text{Cost}(3, 6), \\ &\quad \text{Cost}(\text{II}, 4) + \text{Cost}(4, 6), \\ &\quad \text{Cost}(\text{II}, 5) + \text{Cost}(5, 6)) \\ &= \min(9+4, 7+2, 3+\infty, 2+\infty) \\ &= 9(3-6)\end{aligned}$$

$$\begin{aligned}\text{Cost}(\text{III}, 7) &= \min(\text{Cost}(\text{II}, 2) + \text{Cost}(2, 7), \\ &\quad \text{Cost}(\text{II}, 3) + \text{Cost}(3, 7), \\ &\quad \text{Cost}(\text{II}, 4) + \text{Cost}(4, 7), \\ &\quad \text{Cost}(\text{II}, 5) + \text{Cost}(5, 7)) \\ &= \min(9+2, 7+7, 3+\infty, 2+11) \\ &= 11(2-7)\end{aligned}$$

$$\begin{aligned}\text{Cost}(\text{III}, 8) &= \min(\text{Cost}(\text{II}, 2) + \text{Cost}(2, 8), \\ &\quad \text{Cost}(\text{II}, 3) + \text{Cost}(3, 8), \\ &\quad \text{Cost}(\text{II}, 4) + \text{Cost}(4, 8), \\ &\quad \text{Cost}(\text{II}, 5) + \text{Cost}(5, 8)) \\ &= \min(9+1, 7+\infty, 3+11, 2+8) \\ &= 10(2-8, 5-8)\end{aligned}$$

$$\begin{aligned} \text{Cost}(\text{IV}, 9) &= \min(\text{Cost}(\text{III}, 6) + \text{Cost}(6, 9), \\ &\quad \text{Cost}(\text{III}, 7) + \text{Cost}(7, 9), \\ &\quad \text{Cost}(\text{III}, 8) + \text{Cost}(8, 9)) \\ &= \min(9+6, 11+4, 10+\infty) \\ &= 15 (6-9, 7-9) \end{aligned}$$

$$\begin{aligned} \text{Cost}(\text{IV}, 10) &= \min(\text{Cost}(\text{III}, 6) + \text{Cost}(6, 10), \\ &\quad \text{Cost}(\text{III}, 7) + \text{Cost}(7, 10), \\ &\quad \text{Cost}(\text{III}, 8) + \text{Cost}(8, 10)) \\ &= \min(9+5, 11+3, 10+5) \\ &= 14 (6-10, 7-10) \end{aligned}$$

$$\begin{aligned} \text{Cost}(\text{IV}, 11) &= \min(\text{Cost}(\text{III}, 6) + \text{Cost}(6, 11), \\ &\quad \text{Cost}(\text{III}, 7) + \text{Cost}(7, 11), \\ &\quad \text{Cost}(\text{III}, 8) + \text{Cost}(8, 11)) \\ &= \min(9+\infty, 11+\infty, 10+6) \\ &= 16 (8-11). \end{aligned}$$

$$\begin{aligned} \text{Cost}(\text{V}, 12) &= \min(\text{Cost}(\text{IV}, 9) + \text{Cost}(9, 12), \\ &\quad \text{Cost}(\text{IV}, 10) + \text{Cost}(10, 12), \\ &\quad \text{Cost}(\text{IV}, 11) + \text{Cost}(11, 12)) \\ &= \min(15+4, 14+2, 16+5) \\ &= 16 (10-12) \end{aligned}$$

$$\begin{aligned} 12-10-6-3-1 & \quad ((2+5+2+7)=16) \\ 12-10-\cancel{7}-2-1 & \quad ((2+3+2+9)=16) \end{aligned}$$

• Algorithm Backward Approach (G, k, n, p)

Cost $[1] = 0$

for $j = 2$ to n

{

let r be such that (r, j) is an edge.
of G and $\text{cost}[r] + c[r][j]$ is min

$\text{cost}[j] = \text{cost}[r] + c[r][j]$

$d[j] = \text{assign path to } d[j]$

}

for $j = k$ to 1

{

Find the overall path

}

Q. 2]

String 1: A B C A B E

String 2: A C B A E D

		0	1	2	3	4	5
A	0	A	B	C	A	B	E
A	1						
B	2						
B	3						
A	4						
E	5						
D	6						

* Assembly line Scheduling:-

It is the manufacturing of product. Manufacturing of large items like car and truck undergoes through multiple station.

eg:- Manufacturing of car may be done in several stages like engine fitting, colouring, light fitting and so on. If load at station 1 on assembly line one very high then components are transferred to station 1 on assembly line two.

Algorithm Assembly line Scheduling (n.e.a.t.x)

$$f_1[1] = e_1 + a_{11}$$

$$f_2[1] = e_2 + a_{21}$$

for $j = 2$ to n do

if $f_1[j-1] + a_{1j} \leq f_2[j-1] + t_{2,j-1} + a_{2j}$ then
 $f_1[j] = f_1[j-1] + a_{1j}$

else

$$f_1[j] = f_2[j-1] + t_{2,j-1} + a_{1j}$$

end

if $f_2[j-1] + a_{2j} \leq f_1[j-1] + t_{1,j-1} + a_{2j}$ then
 $f_2[j] = f_2[j-1] + a_{2j}$

else

$$f_2[j] = f_1[j-1] + t_{1,j-1} + a_{2j}$$

end

if $f_1(n) + x_1 \leq f_2(n) + x_2$ then

$$f^* = f_1(n) + x_1$$

else

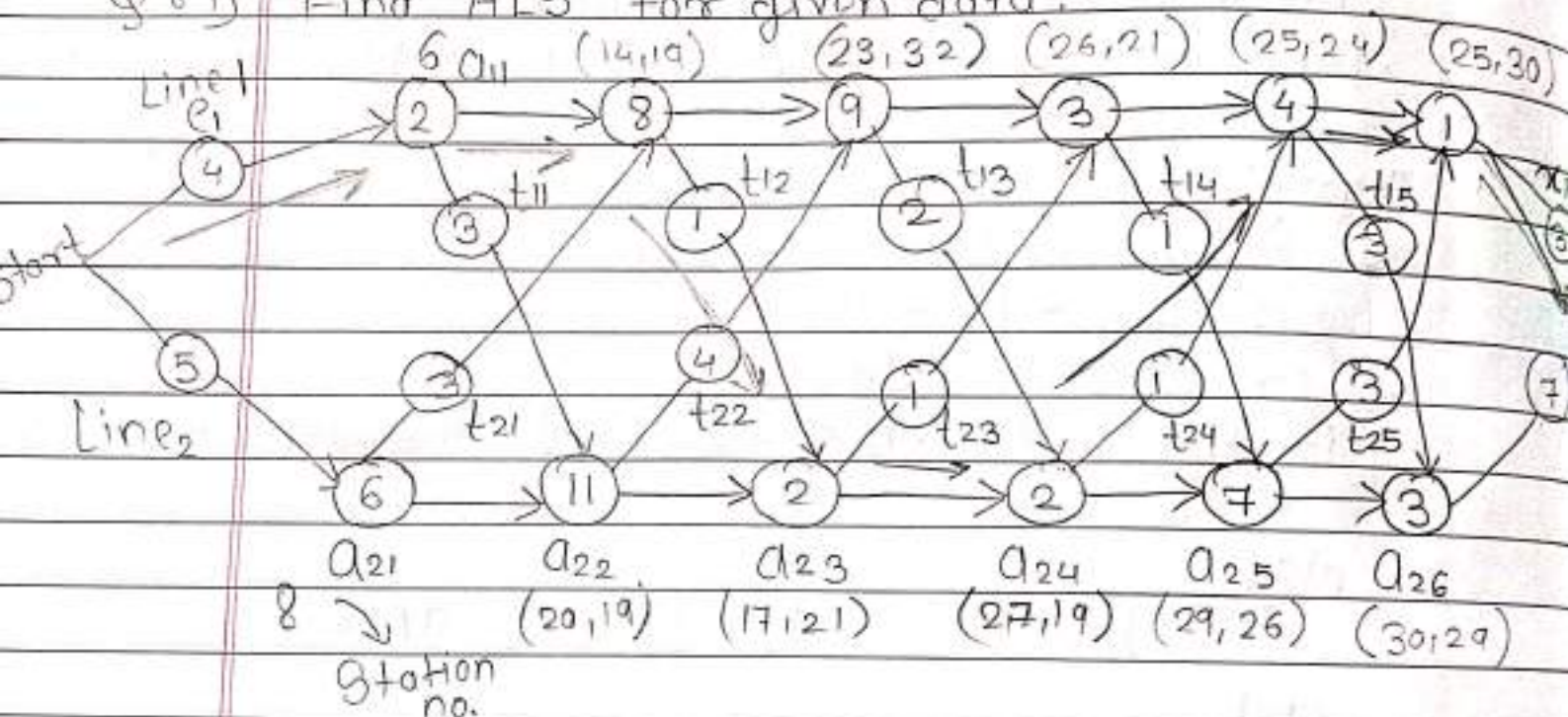
$$f^* = f_2(n) + x_2$$

end

end

- n :- no. of stations = n
- e
- a
- t
- x
- $f_1[j]$

Q. 1] Find ALS for given data:-



	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
$f_1[j]$	6	14	23	21	24	25
$f_2[j]$	8	19	17	19	26	29

$$\begin{aligned}
 f^* &= \min (f_1(n) + x_1, f_2(n) + x_2) \\
 &= \min (25 + 3, 29 + 7) \\
 &= 28
 \end{aligned}$$

* N Queens problems:-

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2 Queens problem is not Solvable

Q	Q					Q					Q								
				Q	Q		Q					Q						Q	

4 Queens problem is Solvable

	Q	Q ₁		
			Q	Q ₂
Q ₃				
			Q ₄	

Place Q₁ on (1,1) — Successful

place Q₂ on (2,1) — fail

Place Q₂ on (2,2) — fail

place Q₂ on (2,3) — Successful

place Q₃ on (3,1) — fail

place Q₃ on (3,2) — fail

place Q₃ on (3,3) — fail

place Q₃ on (3,4) — fail

Algorithm backtracks
and places Q₂ on (2,4)

So place Q₃ on (3,2)

Place Q_4 on $(4, 1)$ - fail

Place Q_4 on $(4, 2)$ - fail

Place Q_4 on $(4, 3)$ - fail

Place Q_4 on $(4, 4)$ - fail

Algorithm backtracks and
Places Q_1 on $(1, 2)$

Q.2] 5 Queens problem is Solvable.

Q_1				
		Q_2		
				Q_3
	Q_4			
			Q_5	

	Q_1			
			Q_2	
Q_3				
		Q_4		
				Q_5

• Algorithm N queens problem

for $i=1$ to n do

if place (k, i)

then

{

$x\{k\} = 1$ // if k^{th} queen can be placed on i^{th} row

if $(k == n)$ then

write $x[1 \dots n]$ // if all queens are processed

else

// then display solution tuple

Nqueen $(k+1, n)$ // row by row each queen is
placed.

// by Satisfying Constraint

end if

end if

end for

function place (k,i)

{
This function checks two queens can be placed
on same Column or same diagonal.
then

return false

else

return true

}

• Complexity :-

N choices to select first queen

N-1 choices to select second queen

N-2 choices to select third queen

$$= O(N * (N-1) * (N-2) \dots 1)$$

$$= O(N!)$$

Best Case

$O(1) \rightarrow$ Solution is obtained immediately.

• Find all possible solutions for 8 queens problem

Q							
		Q					
			Q				
	Q					Q	
			Q				
				Q			
					Q		
						Q	

* Sum of Subset:-

- Start with an empty State.
- Add to the Subset next element from the list.
- If the Subset is having same D then Stop with a Subset as a Solution.
- If the Subset is not visible or if we have reach the end of the Subset then back track to the Subset until we find most Suitable Value.
- If the Subset is visible then repeat Step ②.
- If we have visited all the elements without finding a Suitable Subset and if no back tracking is possible then Stop without Solution.

Q. 1] Consider a Set $S = 5, 10, 12, 13, 15, 18$ and $d = 30$. Solve it obtaining Sum of Subset.

Initial Subset	Sum	Remark
5	5	Add next element
5, 10	15	Sum < 30, add next element
5, 10, 12	27	Sum < 30, add next element
5, 10, 12, 13	40	Sum exceeds $d = 30$ algo back track
5, 10, 12, 15	42	//
5, 10, 12, 18	45	//
5, 10, 13	28	Sum < 30, add next element
5, 10, 13, 15	43	Sum exceeds $d = 30$ algo bt.
5, 10, 13, 18	46	//

5, 10, 18

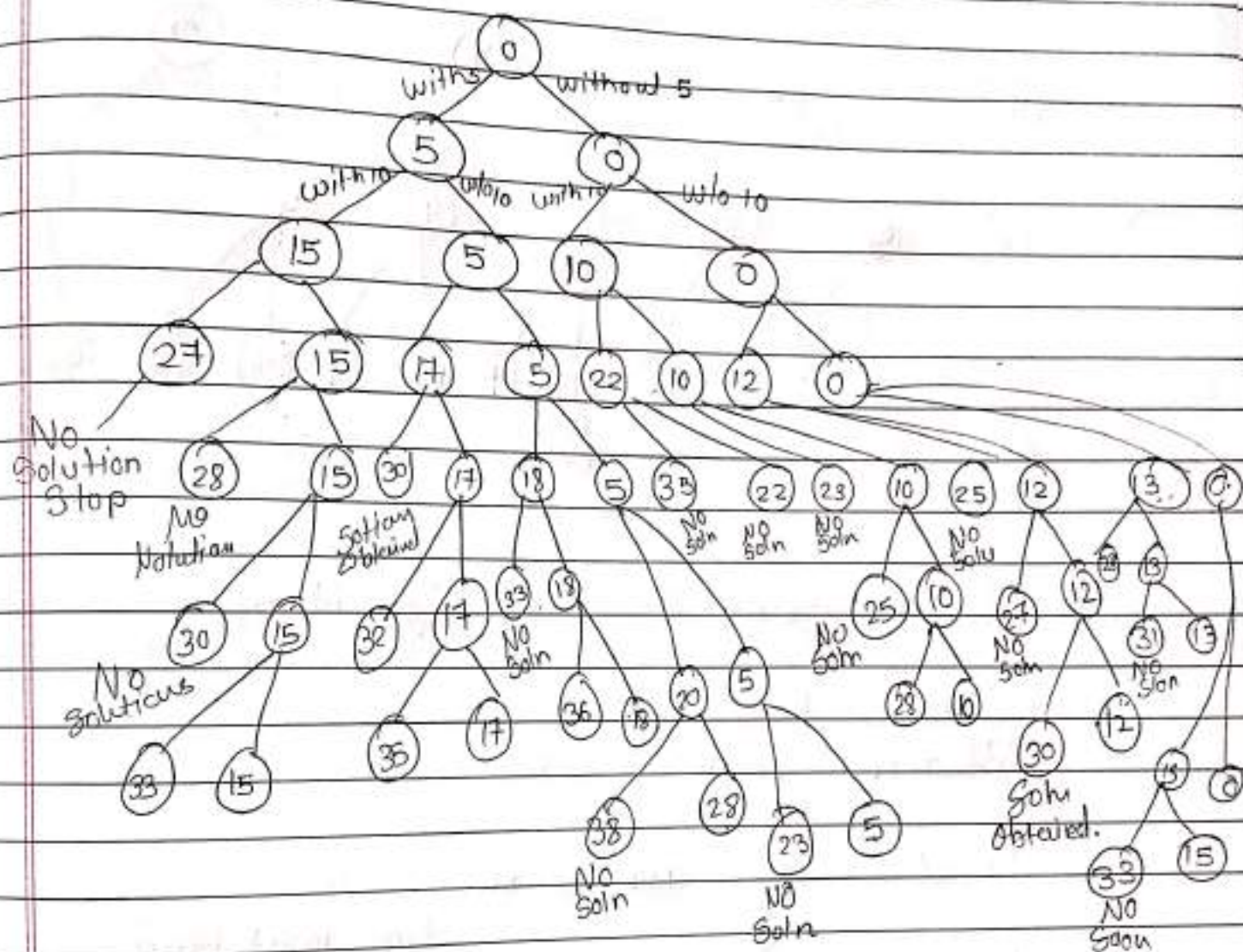
48

33

Sum exceeds $d = 30$ also bit

22

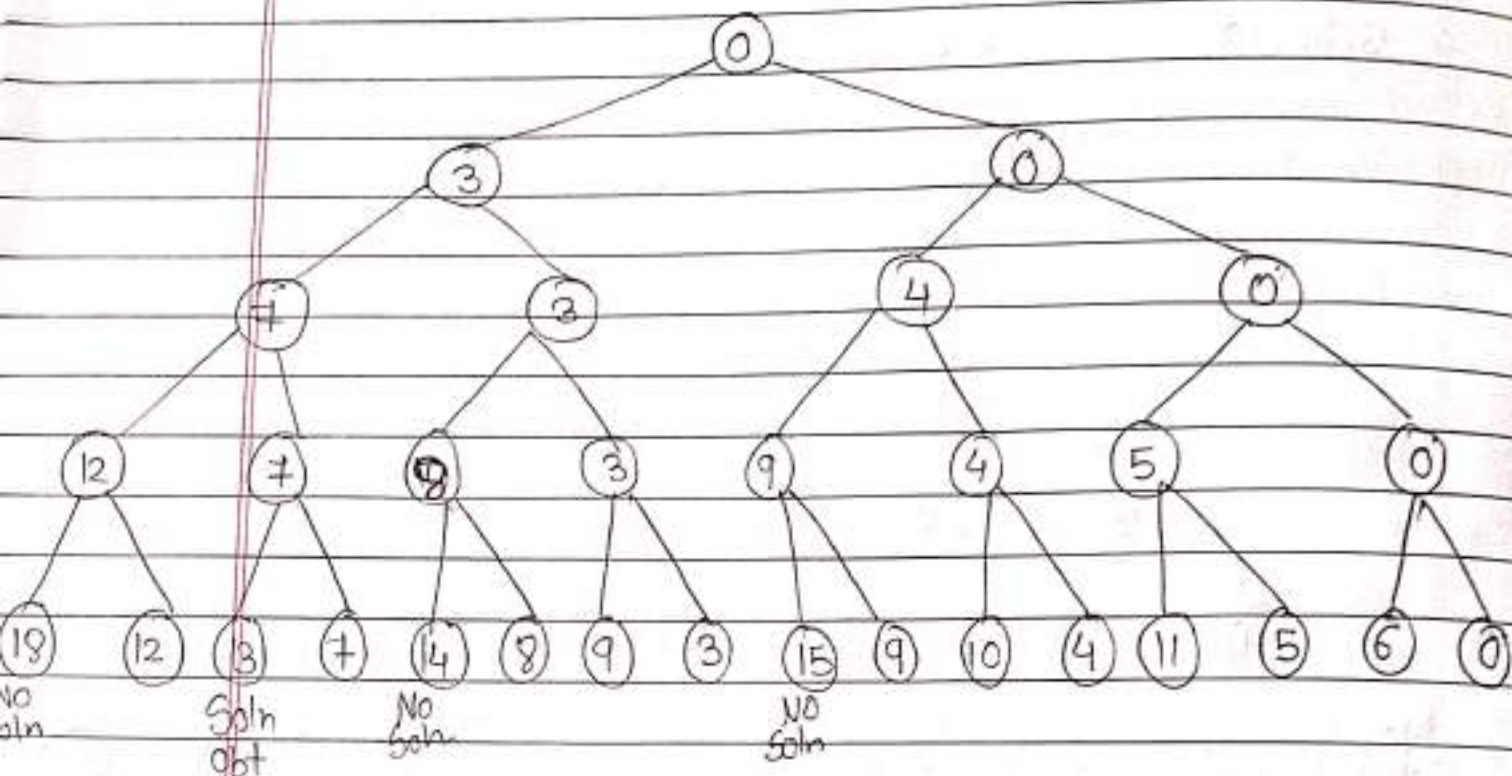
Tree diagram



Q.2]

S = 3, 4, 5, 6

Sum = 13



* Naïve String Matching Algorithm:-

Text:- Roman
Pattern:- man

Text: R a m a n

Pattern: m a n
 0 1 2

Not matching
Shift to right

R a m a n
 ↓
 m a n

Not matching
Shift to right

0 1 2 3 4
R a m a n
 ↓ ↓ ↓
 m a n

Match is found
at index 2
Return index 2

Text: raman likes mango
 pattern: mango

Match is found
 at index 12
 Return index 12

Algorithm Naive ($T(1 \dots n), P(1 \dots m)$)
 for ($S=0$ to $n-m$) do
 {
 if $P(1 \dots m) = T(S+1 \dots S+m)$ then
 print ("Match found")
 }
 }

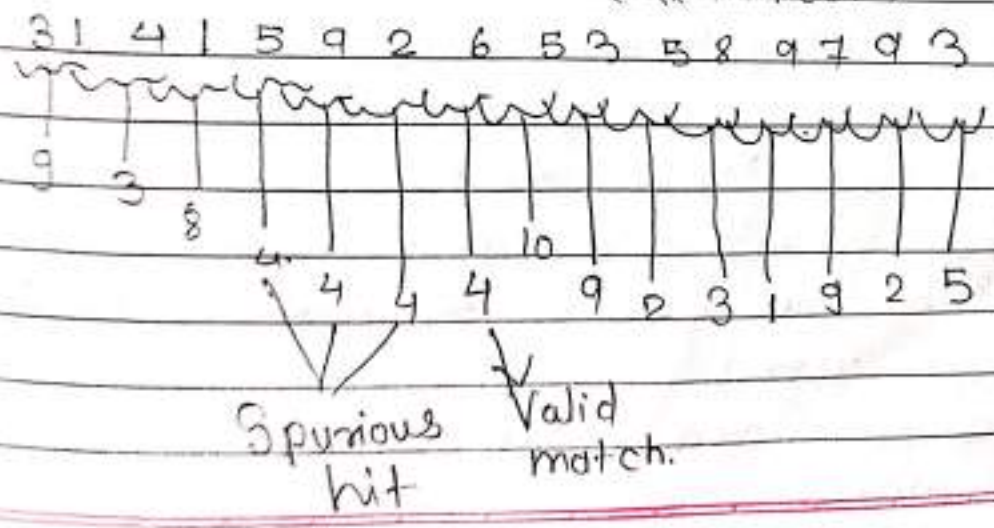
Complexity :-

Worst Case :- $O(mn)$
 Best case :- $O(n)$

* Rabin karp Algorithm:-

Text :- 3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3
 pattern :- 26
 Modulo $q = 11$

$26 \bmod 11 = 4$ (Remainder)



- Algorithm Robin Karp:-

```

for (i = 1 to n)
{
    P = P(i) mod q
    tc = T(i) mod q
}
for (s = 0 to n - m)
{
    if (P = tc) then
    {
        if (P(1...m) = T(s+1...s+m)) then
            write ("pattern found")
        }
    }
}

```

- Complexity:-

Worst Case :- $O(mn)$

Best Case :- $O(m+n)$

- * Knuth Morris Pratt algorithm:-

The basic idea behind this algorithm is to build prefix array. This array is also called as π array. This prefix array is build using prefix and Suffix information of pattern. Suffix is word part added to the end of word
 prefix is word part added to the beginning of a word.

pattern	a	b	a	d	a	b
prefix						
array	0	0	1	0	1	2

Consider ab
prefix a
Suffix b

Consider aba
prefix a ab
Suffix a ba

Consider abad
prefix a ab aba
Suffix d ad bad

Consider abada
prefix a ab aba abad
Suffix a ad ada ~~ada~~ bada

Consider abadab
prefix a ab aba abad abada
Suffix b ab dab adab badab