

## \* Selection Sort Algorithm:

- Scan the array and find the smallest element. Swapping with first element.
- Starting from the 2nd element scan the array to find the smallest element. Swapping with 2nd element.
- This process is repeated until least is sorted.

70, 30, 20, 50, 60, 10, 40

10

10, 30, 20, 50, 60, 70, 40

20

10, 20, 30, 50, 60, 70, 40

40

10, 20, 30, 40, 60, 70, 50

50

10, 20, 30, 40, 50, 70, 60

60

10, 20, 30, 40, 50, 60, 70

## • Algorithm:

For  $j = 1$  to  $n-1$  do

$\min j \leftarrow i$ ;

$\min x \leftarrow A[i]$

For  $j = i+1$  to  $n$  do

    if  $A[j] < \min x$  then

$\min x \leftarrow j$ ;

$\min x \leftarrow A[j]$ ;

end

end

$A[\text{minj}] \leftarrow A[i];$

$A[i] \leftarrow \text{min } x;$

end

eg:-

1 2 3 4 5  
5 2 1 4 3

$i = 1 \text{ to } 4$

$\text{minj} = 1$

$\text{min } x = A[i]$

$\text{min } x = 5$

$j = 2 \text{ to } 5$

$A[2] < 5$

$2 < 5$  True

$\text{minj} = 2$

$\text{min } x = A[2]$

$\text{min } x = 2$

$j = 3 \text{ to } 5$

$A[3] < 2$

$1 < 2$  True

$\text{minj} = 3$

$\text{min } x = A[3]$

$\text{min } x = 1$

$j = 4 \text{ to } 5$

$A[4] < 1$

$4 < 1$  False

$j = 5 \text{ to } 5$

$A[5] < 1$

$3 < 1$  False

$A[3] = A[1]$

$A[3] = 5$

$A[1] = \text{min } x$

$A[1] = 1$

1 2 3 4 5  
2 5 4 3

$j = 2 \text{ to } 4$

$\text{minj} = 2$

$\text{min } x = A[2]$

$\text{min } x = 2$

$j = 3 \text{ to } 5$

$A[3] < 2$

$5 < 2$  False

$A[4] < 2$

$4 < 2$  False

$A[5] < 2$

$3 < 2$  False

No Swapping

$i = 3$  to  $4$

$\min j = 3$

$\min x = A[3]$

$\min x = 5$

$j = 4$  to  $5$

$A[4] < 5$

$4 < 5$  True

$\min j = 4$

$\min x = A[4]$

$\min x = 4$

$A[5] \neq 4$

$3 < 4$  True

$\min j = 5$

$\min x = A[5]$

$\min x = 3$

$A[5] = A[3]$

$A[5] = 5$

$A[3] = 3$

1 2 3 4 5

$i = 4$  to  $4$

$\min j = 4$

$\min x = A[4]$

$\min x = 4$

$j = 5$  to  $5$

$A[5] < 4$

$5 < 4$  false

No Swapping

1 2 3 4 5

### • Time Complexity :-

In Selection Sort algorithm after each iteration one element get Sorted.

So problem size reduced by one.

$$T(n) = T(n-1).$$

Then for loop executes for  $n$  no. of time for

$$T(n) = T(n-1) + n - \textcircled{1}$$

Replace  $n$  by  $n-1$

$$T(n-1) = T(n-2) + (n-1) - \textcircled{ii}$$

put  $T(n-1)$  in eqn (i)

$$T(n) = 0T(n-2) + (n-1) + n \quad \text{--- (ii)}$$

Replace  $n$  by  $n-2$  in eqn (i) or  $n$  by  $n-1$  in eqn (ii)

$$T(n-2) = T(n-3) + (n-2)$$

put  $T(n-2)$  in eqn (ii)

$$T(n) = T(n-3) + (n-2) + (n-1) + n$$

$$T(n) = T(n-k) + (n-k+1) + (n-k+2) + \dots + n$$

put  $n=k$

$$T(n) = T(0) + 1 + 2 + 3 + \dots + n$$

$$T(0) = 0$$

$$T(n) = 1 + 2 + 3 + \dots + n$$

$$= \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}$$

Complexity =  $O(n^2)$  (Best, Worst & average)

Q.1] Sort the following array by using Selection Sort Algorithm

50, 40, 5, 9, 45, 90, 65, 25, 75.

5

5, 40, 50, 9, 45, 90, 65, 25, 75

9

5, 9, 50, 40, 45, 90, 65, 25, 75

25

5, 9, 25, 40, 45, 90, 65, 50, 75

45

5, 9, 25, 40, 45, 90, 65, 50, 75

50



5, 9, 25, 40, 45, 50, 65, 70, 75  
15

5, 9, 25, 40, 45, 50, 65, 75, 90

Q.2] Scan the following word by using Selection Sort Algorithm.  
EXAMPLE

EXAMPLE

A

AEXAMPLE

E

AEXAMPLE

F

AEEMLPX

L

AEE~~L~~PLMX

M

AEE L M P X

A	E	E	L	M	P	X
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# \* Insertion Sort:-

30, 70, 20, 50, 40, 10, 60

30	70	20	50	40	10	60
0						

Sorted      Unsorted

30	70	20	50	40	10	60
0	1					

Unsorted

20	30	70	50	40	10	60
0	1	2				

Unsorted

20	30	50	70	40	10	60
0	1	2	3			

Unsorted

20	30	40	50	70	10	60
0	1	2	3	4		

Unsorted

10	20	30	40	50	70	60
0	1	2	3	4	5	

Unsorted

10	20	30	40	50	60	70
0	1	2	3	4	5	6

• Algorithm:-

for  $j = 2$  to  $A.length$

key =  $A[j]$

$i = j - 1$

while  $i > 0$  and  $A[i] > key$

$A[i+1] = A[i]$

$i = i - 1$

end

$A[i+1] = key$

end

• Example :-

2	13	5	18	14
1	2	3	5	4

$j = 2$  to 5

key =  $A[2]$

= 13

$i = 1$

$1 > 0$  and  $2 > 13$  -- False

No Swapping

$j = 3$  to 5

key =  $A[3]$

= 5

$i = 2$

$2 > 0$  and  $13 > 5$  -- True

$A[3] = A[2]$

$1 > 0$  and  $2 > 13$  false

$A[3] = 13$

$i = 1$

$A[2] = key$

$A[2] = 5$

2	5	13	18	14
1	2	3	4	5

$j = 4$  to 5

key =  $A[4]$

key = 18

$i = j - 1 = 3$

$3 > 0$  and  $A[3] > \text{key}$

$3 > 0$  and  $13 > 18$  -- false

No swapping

$j = 5$

key =  $A[5]$

key = 14

$i = 4$

$4 > 0$  and  $A[4] > \text{key}$

$4 > 0$  and  $18 > 14$  -- true

$A[i+1] = A[i]$

$A[5] = A[4]$

$A[5] = 18$

$i = 3$

$3 > 0$  and  $A[3] > 14$

$3 > 0$  and  $13 > 14$  -- false

$A[i+1] = \text{key}$

$A[4] = 14$

2	3	13	14	18
0	1	2	3	4



\* Find Time Complexity of insertion sort.

1. Best Case:-

When list is already sorted, so condition in the while loop is never executed so  $tj=1$   
 So the recurrence relation is,

$$T(n) = C_1 \cdot n + C_2(n-1) + C_3 \cdot (n-1) + C_4 \sum_{j=2}^n tj + C_5 \cdot \sum_{j=2}^n tj-1 + C_6 \sum_{j=1}^n tj-1 + C_7(n-1)$$

$$\sum_{j=2}^n tj = \sum_{j=2}^n 1 = (1+1+\dots+1) = n-1 \text{ times}$$

$$\sum_{j=2}^n tj-1 = \sum_{j=2}^n 1-1 = 0$$

$$T(n) = C_1 \cdot n + C_2(n-1) + C_3(n-1) + C_4(n-1) + C_5 \times 0 + C_6 \times 0 + C_7(n-1)$$

$$= [C_1 + C_2 + C_3 + C_4 + C_7]n - [C_2 + C_3 + C_4 + C_7]$$

$$T(n) = an - b$$

$$\text{Complexity} = O(n)$$

2. Worst Case:-

In the worst case, complexity, elements are arranged in reverse order, so the condition in while loop is executed, so  $tj=j$ .

$$\sum_{j=2}^n tj = \sum_{j=2}^n j = 2+3+4+\dots+n \quad \text{--- (1)}$$

Add & Sub 1 in eqn (1),

$$\begin{aligned}\sum t_j &= (1+2+3+\dots+n)-1 \\ &= \sum n-1 \\ &= \frac{n(n+1)}{2} - 1\end{aligned}$$

$$\begin{aligned}\sum_{j=2}^n t_{j-1} &= \sum_{j=2}^n j-1 = (2-1) + (3-1) + \dots + (n-1) \\ &= 1+2+3+\dots+n-1 \\ &= \sum n-1 \\ &= \frac{n(n-1)}{2}\end{aligned}$$

Put  $\sum t_j$  &  $\sum t_{j-1}$  in eqn ①

$$\begin{aligned}T(n) &= C_1 n + C_2(n-1) + C_3(n-1) + C_4(n(n-1)-1) + \\ &\quad C_5\left(\frac{n(n-1)}{2}\right) + C_6\left(\frac{n(n-1)}{2}\right) + C_7(n-1) \\ &= \left[\frac{C_4}{2} + \frac{C_5}{2} + \frac{C_6}{2}\right]n^2 + \left[C_1 + C_2 + C_3 + \frac{C_4 - C_5 - C_6}{2}\right]n \\ &\quad - [C_2 + C_3 + C_4 + C_7]\end{aligned}$$

$$= an^2 + bn + c$$

$$\text{Complexity} = O(n^2)$$

3. Average Case:-

- In average case Complexity, Some of the elements are sorted and remaining elements are unsorted, So  $t_j = j-1$

$$\sum_{j=2}^n t_j = \sum_{j=2}^n \frac{j-1}{2} = \frac{1}{2} \sum_{j=2}^n j-1 = \frac{1}{2} \frac{n(n-1)}{2}$$

$$\sum_{j=2}^n t_{j-1} = \sum_{j=2}^n \frac{j-1}{2} - 1 = \sum_{j=2}^n \frac{j-3}{2} = \frac{n(n-3)}{2 \times 2}$$

put  $\leq t_j$  &  $\leq t_{j-1}$  in eqn (1).

$$T(n) = C_1 \cdot n + C_2(n-1) + C_3(n-1) + \frac{C_4}{2} \frac{n(n-1)}{2} + \frac{(C_5 + C_6) \times n(n-3)}{2 \times 2} + C_7(n-1)$$

$$T(n) = an^2 + bn - c$$

$$\text{Complexity} = O(n^2)$$

\* Sort the following word by using insertion sort  
E X A M P L E

A	E	E	L	M	P	X
0	1	2	3	4	5	6