Comparison of Higher Order Moments Portfolio

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1 Introduction

In the dynamic landscape of financial markets, portfolio optimization stands as a cornerstone of investment strategies, aiming to balance risk against expected returns. Traditional models, such as the mean-variance framework introduced by Markowitz (1952), primarily focus on the first two moments of the distribution of returns—mean and variance. This approach, while pioneering, often assumes normality of return distributions, which is seldom observed in real-world financial markets. Asset returns can exhibit significant skewness and kurtosis, leading to distributions that are far from symmetric and often have heavier tails than the normal distribution predicts.

Recent academic contributions have significantly broadened the understanding of portfolio optimization by incorporating higher order moments, providing a richer discussion on risk and return dynamics. Key contributions include studies such as Kleniati et al. (2009) and Harvey et al. (2010), which explore how skewness and kurtosis can impact investment choices and the limitations of traditional models in fully capturing the risks inherent in financial markets. Maringer and Parpas (2009) introduces advanced global optimization methods that integrate these higher moments for more robust portfolio construction. Furthermore, Khan et al. (2020) highlights the potential to integrate environmental, social, and governance (ESG) considerations with financial metrics through the lens of higher moments, promoting a more comprehensive approach to sustainable investing. These studies collectively highlight the value of a more nuanced approach to risk assessment and portfolio management that goes beyond the conventional mean-variance paradigm.

The primary objective of this research is to explore the efficacy of incorporating higher order moments—specifically skewness, kurtosis, and beyond—into the portfolio optimization process. This study aims to provide a more holistic view of risk, thereby enhancing the robustness of investment portfolios. This approach is expected to offer substantial benefits, particularly in crafting portfolios that more accurately reflect the risk preferences of investors related to extreme losses and asymmetrical return distributions.

Through this examination, the research intends to deliver deeper insights into the potential benefits of higher order moments in portfolio construction, marking a significant evolution from traditional paradigms of risk and return to a more intricate understanding tailored to complex market behaviors and sustainability considerations.

2 Hypothesis

The hypothesis of this research is that incorporating higher-order moments into the portfolio optimization process can significantly enhance the performance of investment portfolios by providing a more comprehensive assessment of risk and return dynamics. Specifically, the study posits that portfolios optimized for higher-order moments (skewness, kurtosis, and beyond) will exhibit higher risk-adjusted returns and superior management of extreme tail risks compared to traditional mean-variance optimized portfolios.

3 Data

The dataset for this study comprises daily adjusted closing prices of both the top 100 Taiwanese stocks and the S&P 100 stocks, sourced from Yahoo Finance. The data spans from January 1, 2019, to December 31, 2023, covering a period that includes diverse market conditions. This provides a robust basis for analyzing portfolio performance across different market scenarios.

3.1 Data Collection

The stock prices are downloaded using the Yahoo Finance API. The adjusted closing prices are used to account for corporate actions such as dividends and stock splits, which can affect the stock price. 6526.TW is excluded from the dataset because it was listed in 2022, resulting in insufficient historical data for comprehensive analysis.

3.2 Data Preparation

Daily returns are calculated from the adjusted closing prices using the formula:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where r_t is the return on day t, P_t is the adjusted closing price on day t, and P_{t-1} is the adjusted closing price on day $t-1$.

The data is split into two subsets: the first four years (January 1, 2019, to December 31, 2022) for model training and optimization, and the final year (January 1, 2023, to December 31, 2023) for out-of-sample testing and backtesting. This split allows for a robust evaluation of the models in both in-sample and out-of-sample periods.

3.3 Descriptive Statistics

The overall descriptive statistics for the daily returns of the S&P 100 and Taiwan 100 stocks are summarized in Table 1. These statistics provide an aggregated view of the central tendency, dispersion, and shape characteristics of the return distributions for both datasets.

Table 1: Overall Descriptive Statistics for S&P 100 and Taiwan 100 Stocks

Statistic	S&P 100	Taiwan 100
Mean	0.0007	0.0011
Standard Deviation	0.0203	0.0199
Skewness	-0.0038	0.2128
Kurtosis	10.7661	4.5544

The mean return for both datasets is positive, with the Taiwan 100 stocks exhibiting a slightly higher average daily return compared to the S&P 100 stocks. The standard deviation, which measures the volatility of returns, is comparable for both datasets, indicating similar levels of risk. The skewness of the S&P 100 returns is slightly negative, suggesting a distribution with a left tail, whereas the Taiwan 100 returns have a positive skewness, indicating a distribution with a right tail. The kurtosis values indicate that both distributions have heavier tails than a normal distribution, with the S&P 100 showing a particularly high kurtosis value.

The histogram in Figure 1 and Figure 2 visualizes the distribution of daily returns for the S&P 100 stocks and Taiwan 100 stocks. The plot shows the frequency of different daily return values, with a Kernel Density Estimate (KDE) overlay to illustrate the overall distribution shape. This visualization supports the descriptive statistics provided in Table 1.

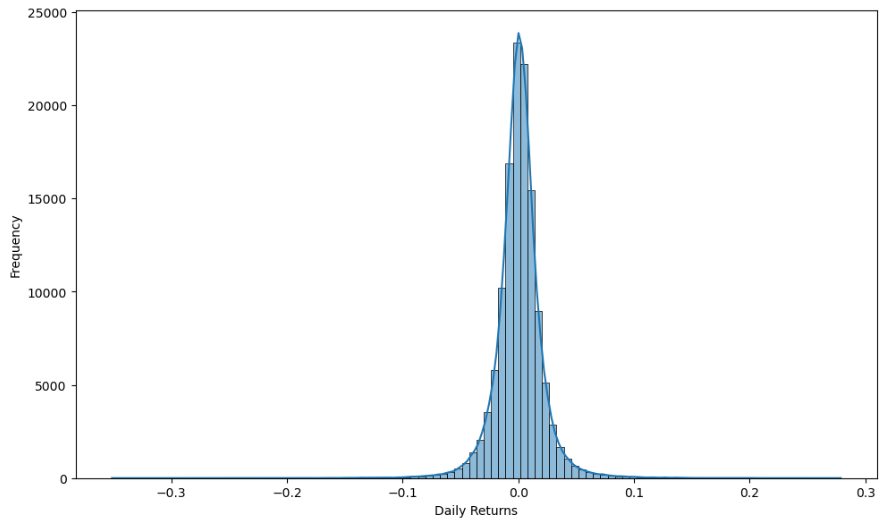

Figure 1: Histogram of Daily Returns for S&P 100 Stocks

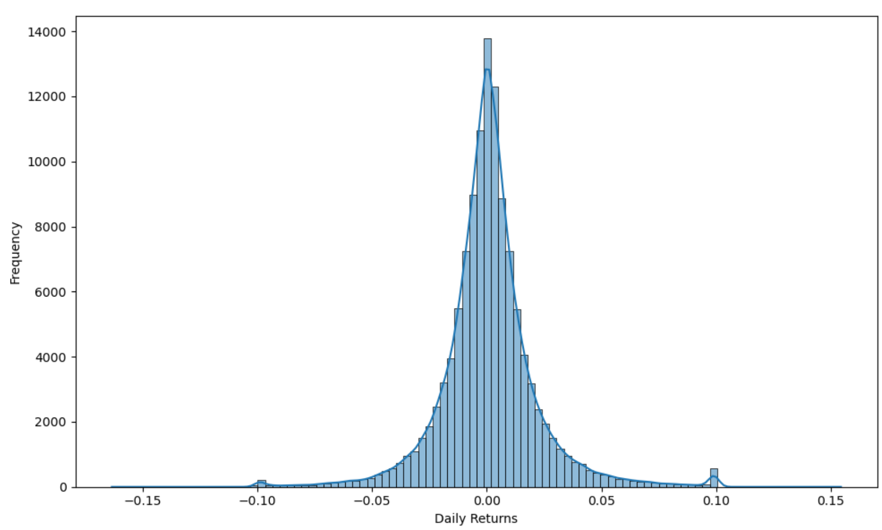

Figure 2: Histogram of Daily Returns for Taiwan 100 Stocks

4 Method

This study employs various portfolio optimization models, each focusing on different statistical moments of the return distribution to construct optimal portfolios. The models include:

4.1 Minimum Variance Portfolio

The Minimum Variance Portfolio aims to minimize the portfolio's variance, which is a measure of risk. The optimization problem is formulated as follows:

$$\text{Minimize} \quad \mathbf{w}^\top \boldsymbol{\Sigma} \mathbf{w}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1$$

$$0 \le w_i \le 1 \quad \forall i$$

where $\mathbf{w}$ represents the portfolio weights and $\boldsymbol{\Sigma}$ is the covariance matrix of returns.

4.2 Minimum Skewness Portfolio

The Minimum Skewness Portfolio focuses on minimizing the skewness of the portfolio returns, aiming for a more symmetrical distribution. The skewness of a portfolio is given by:

$$\mathrm{S} = \frac{\mathbb{E}\left[(r_p - \mu_p)^3\right]}{\sigma_p^3}$$

where r_p is the portfolio return, μ_p is the mean return, and σ_p is the standard deviation.

4.3 Minimum Kurtosis Portfolio

The Minimum Kurtosis Portfolio aims to minimize the kurtosis of the portfolio returns, which measures the "tailedness" of the return distribution. The kurtosis is calculated as:

$$\mathrm{K} = \frac{\mathbb{E}\left[(r_p - \mu_p)^4\right]}{\sigma_p^4}$$

where the variables are as defined above.

4.4 Minimum Fifth Order Portfolio

This portfolio model minimizes the fifth central moment of the portfolio returns. The fifth moment captures more extreme deviations from the mean, reflecting more severe potential tail risks.

$$\mathrm{M}_5 = \mathbb{E}\left[\left(\frac{r_p - \mu_p}{\sigma_p}\right)^5\right]$$

4.5 Minimum Sixth Order Portfolio

Similarly, the Minimum Sixth Order Portfolio aims to minimize the sixth central moment of the portfolio returns, further emphasizing extreme tail risks.

$$\mathrm{M}_6 = \mathbb{E}\left[\left(\frac{r_p - \mu_p}{\sigma_p}\right)^6\right]$$

4.6 Evaluation Metrics

Portfolios are evaluated using several metrics:

- **Return**: The expected annual return of the portfolio.
- **Volatility**: The annualized standard deviation of portfolio returns.
- **Sharpe Ratio**: A measure of risk-adjusted return.
- **Value at Risk (5%)**: The potential loss in value of the portfolio at a 95% confidence level.

5 Empirical Results

5.1 Portfolio Weights

The resulting weights indicate how each portfolio optimization model allocates investments across the S&P 100 and Taiwan 100 stocks. Table 2 and Table 3 shows the weights for the S&P 100 stocks, and Table 4 and Table 5 shows the weights for the Taiwan 100 stocks. Each model has its own objective and allocates the investments differently across the stocks based on the criteria specified. For the S&P 100 stocks, the Minimum Variance portfolio shows a conservative approach with significant weights in stocks like ABBV, AMZN, BMY, GILD, JNJ, MCD, MO, MRK, NFLX, and WMT, with the largest allocation to VZ, reflecting its low volatility. The Minimum Skewness portfolio aims for a more symmetrical return distribution, with notable allocations to ABBV, AVGO, CL, HD, LOW, PM, SPG, and UNH. The Minimum Kurtosis portfolio, focused on reducing the "tailedness" of the return distribution, significantly weights stocks like AMD, AMZN, and TSLA. The Minimum Fifth Order portfolio, which minimizes the fifth central moment, includes major allocations to ABBV, AMT, CHRT, CL, LMT, LOW, MCD, SPG, and UNH. The Minimum Sixth Order portfolio, emphasizing extreme tail risk reduction, allocates significant weights to AMZN, NFLX, TMO, and TSLA.

For the Taiwan 100 stocks, the Minimum Variance portfolio has notable allocations to 2412.TW, 3045.TW, 2912.TW, 4904.TW, 2301.TW, 2324.TW, 1102.TW, 2385.TW, 1503.TW, and 3702.TW. The Minimum Skewness portfolio significantly weights 2892.TW, 2357.TW, 3017.TW, 3036.TW, 2105.TW, 3702.TW, and 9904.TW. The Minimum Kurtosis portfolio focuses on stocks like 2603.TW, 6669.TW, 6415.TW, and 2385.TW. The Minimum Fifth Order portfolio includes important allocations to 6669.TW, 2912.TW, 2892.TW, 2317.TW, and 3702.TW, while the Minimum Sixth Order portfolio allocates significantly to 2603.TW, 6669.TW, 6415.TW, and 2912.TW.

These results highlight several key insights. The Minimum Variance portfolios reflect a conservative approach by minimizing risk and focusing on low-volatility stocks. In contrast, the Minimum Kurtosis and higher-order moment portfolios suggest a more aggressive strategy, with substantial weights in high-growth, high-risk stocks like AMZN, AMD, and TSLA. The higher-order moment portfolios demonstrate a significant emphasis on managing extreme tail risks, providing robustness against market shocks. The differences in optimal weights between the S&P 100 and Taiwan 100 portfolios also reflect the underlying market characteristics and volatility of the respective indices. The presence of positive skewness and lower kurtosis in the Taiwan 100 portfolios suggests a preference for stocks with more favorable return distributions, while the higher kurtosis in the S&P 100 portfolios indicates a focus on mitigating the impacts of extreme events.

5.2 Portfolio Performance Metrics

Table 6 presents the performance metrics for the different portfolio optimization models applied to both the S&P 100 and Taiwan 100 stocks. These metrics include annualized return, volatility, Sharpe ratio, and Value at Risk (5%).

Table 6: Portfolio Metrics for S&P 100 and Taiwan 100 Stocks

Model	**Return**	**Volatility**	**Sharpe Ratio**	**Value at Risk (5%)**
		S&P 100		
Minimum Variance	0.0246	0.1145	0.2148	-0.0117
Minimum Skewness	0.1370	0.1324	1.0346	-0.0143
Minimum Kurtosis	0.8232	0.3245	2.5373	-0.0301
Minimum Fifth Order	0.1297	0.1301	0.9964	-0.0138
Minimum Sixth Order	0.6946	0.2997	2.3177	-0.0261
		Taiwan 100		
Minimum Variance	0.2796	0.0830	3.3691	-0.0074
Minimum Skewness	0.4351	0.0998	4.3579	-0.0091
Minimum Kurtosis	0.4176	0.2731	1.5291	-0.0243
Minimum Fifth Order	0.4099	0.0978	4.1924	-0.0091
Minimum Sixth Order	0.3981	0.2756	1.4446	-0.0245

For the S&P 100 stocks, the Minimum Variance Portfolio shows the lowest return and Sharpe ratio, indicating a very conservative strategy focused on minimizing risk. The Minimum Skewness Portfolio achieves a higher return with a better Sharpe ratio, suggesting that targeting skewness can enhance returns and risk-adjusted performance. The Minimum Kurtosis Portfolio provides the highest return and Sharpe ratio, demonstrating the potential benefits of reducing the tailedness of the return distribution. The Minimum Fifth Order and Minimum Sixth Order Portfolios also deliver strong returns and risk-adjusted returns, reflecting effective management of higher-order risks.

For the Taiwan 100 stocks, the Minimum Variance Portfolio yields a significant return with a high Sharpe ratio, showing that even a conservative strategy can yield substantial returns in this market. The Minimum Skewness Portfolio achieves the highest return and Sharpe ratio, indicating the substantial benefit of managing skewness. The Minimum Kurtosis Portfolio shows a positive impact on portfolio performance by targeting kurtosis. The Minimum Fifth Order and Minimum Sixth Order Portfolios produce strong returns and high Sharpe ratios, emphasizing the importance of managing extreme tail risks.

The observed performance metrics underscore the importance of incorporating higher-order moments in portfolio optimization. Traditional mean-variance optimization primarily focuses on reducing overall volatility, which often leads to conservative strategies with lower returns and Sharpe ratios. However, by considering higher-order moments such as skewness and kurtosis, portfolios can better manage asymmetrical risks and extreme tail events. This results in enhanced risk-adjusted returns, as indicated by higher Sharpe ratios and more favorable VaR metrics. Portfolios optimized for skewness can capture positive asymmetry, improving returns while controlling downside risk. Similarly, portfolios targeting kurtosis and higher-order moments mitigate extreme variations in returns, reducing the impact of rare but severe market movements. Thus, these advanced optimization techniques provide a more comprehensive approach to risk management, leading to superior performance in both S&P 100 and Taiwan 100 stocks.

5.3 Performance Evaluation and Analysis

The performance of the constructed portfolios was evaluated through various analyses, including cumulative returns, rolling Sharpe ratios, and correlation matrices for portfolio returns.

5.3.1 Cumulative Returns

Figures 3 and 4 show the cumulative returns for each portfolio over the out-of-sample period for the S&P 100 and Taiwan 100 stocks, respectively. These plots highlight the growth of an initial investment over time, providing insight into the overall performance of each portfolio optimization model.

Figure 3: Portfolio Backtesting for S&P 100 Stocks

Figure 4: Portfolio Backtesting for Taiwan 100 Stocks

For the S&P 100 stocks, portfolios optimized for higher-order moments generally outperformed the minimum variance portfolio, with the minimum kurtosis and minimum sixth order portfolios showing the highest cumulative returns. This suggests that considering higher-order moments can capture more complex risk-return dynamics and improve portfolio performance.

For the Taiwan 100 stocks, the minimum skewness portfolio demonstrated the highest cumulative returns, followed by the minimum fifth order and minimum variance portfolios. This indicates that for the Taiwan market, skewness optimization and traditional risk measures might be more effective in capturing the return dynamics and providing better performance compared to higher-order moments like kurtosis and sixth order.

5.3.2 Rolling Sharpe Ratio

The rolling Sharpe ratios, calculated over a 60-day window, are presented in Figures 5 and 6. These plots provide a dynamic view of the risk-adjusted returns of the portfolios throughout the out-of-sample period.

Figure 5: Rolling Sharpe Ratio (60 days) for S&P 100 Stocks

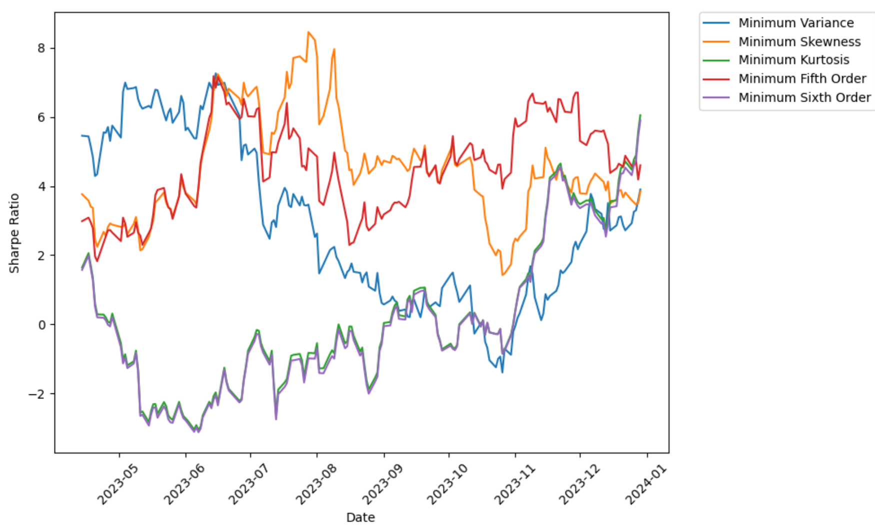

Figure 6: Rolling Sharpe Ratio (60 days) for Taiwan 100 Stocks

For the S&P 100 stocks, the portfolios optimized for higher-order moments, such as the minimum fifth order and minimum skewness portfolios, do not consistently exhibit higher and more stable Sharpe ratios compared to the minimum variance portfolio. The minimum variance portfolio provides more stable risk-adjusted returns over time, although not always the highest. The higher-order moment portfolios exhibit significant variability, with notable peaks and troughs in their Sharpe ratios, indicating higher volatility in their risk-adjusted performance.

For the Taiwan 100 stocks, the rolling Sharpe ratios show that the minimum variance, minimum skewness, and minimum fifth order portfolios generally have more stable and higher Sharpe ratios compared to the portfolios optimized for other higher-order moments. This indicates that traditional risk measures, skewness optimization, and the fifth-order moment optimization might be more effective in capturing the return dynamics and providing better risk-adjusted performance in the Taiwan market.

5.3.3 Correlation Matrix

The correlation matrices of portfolio returns are shown in Figures 7 and 8. These matrices highlight the relationships between the returns of different portfolio optimization models.

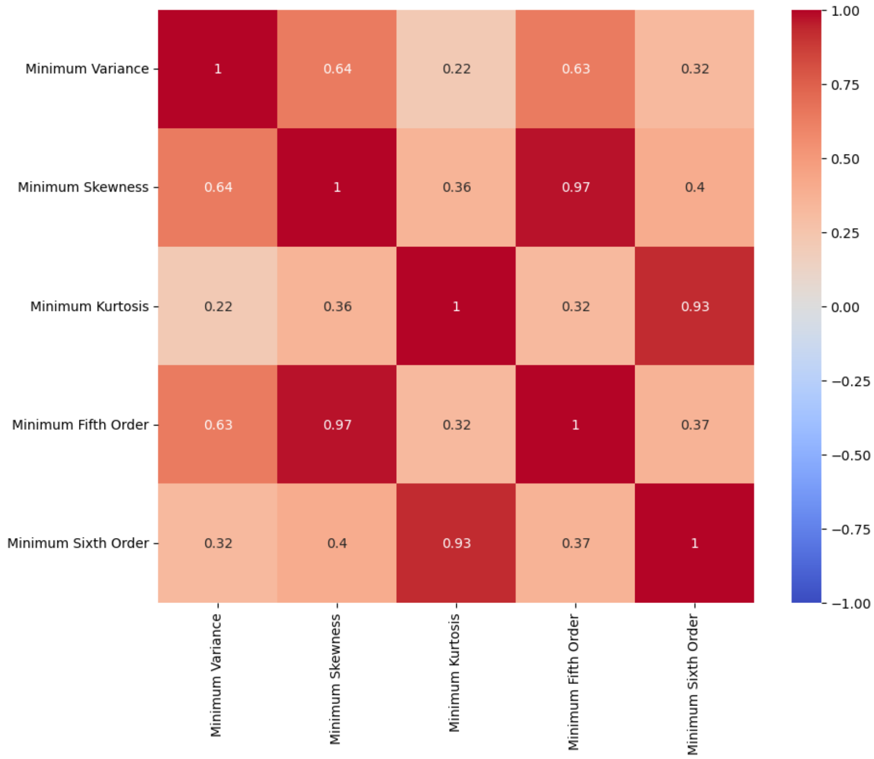

Figure 7: Correlation Matrix of Portfolio Returns for S&P 100 Stocks

For both S&P 100 stocks and Taiwan 100 stocks, the correlation matrix shows that the minimum variance portfolio has lower correlations with the portfolios optimized for higher-order moments, particularly the minimum kurtosis and minimum sixth order portfolios. This indicates diversification benefits and differing risk-return characteristics. Additionally, the matrix reveals that the minimum skewness and minimum fifth order portfolios have high correlations with each other, indicating similar risk-return profiles. Similarly, the minimum kurtosis and minimum sixth order portfolios exhibit high correlations with each other, suggesting that they capture similar aspects of return distribution.

The observed correlation patterns can be attributed to the distinct risk-return characteristics targeted by each portfolio optimization strategy. The minimum variance portfolio focuses solely on reducing the overall volatility, without consideration for the higher-order moments like skewness and kurtosis. This results in a portfolio composition that tends to have a more stable and predictable return distribution, hence exhibiting lower correlations with portfolios that incorporate higher-order moment optimizations. Portfolios optimized for minimum skewness and minimum fifth order moments are designed to reduce asymmetry in return distributions and mitigate more extreme deviations, respectively. These portfolios often include assets that can counterbalance skewness and extreme deviations, leading to higher correlations between them due to their similar risk mitigation strategies. Similarly, portfolios optimized for minimum kurtosis and minimum sixth order moments aim to manage the ”tailedness” and extreme risks associated with return distributions. These portfolios tend to share similar asset compositions as they both seek to minimize extreme outcomes, resulting in high correlations between

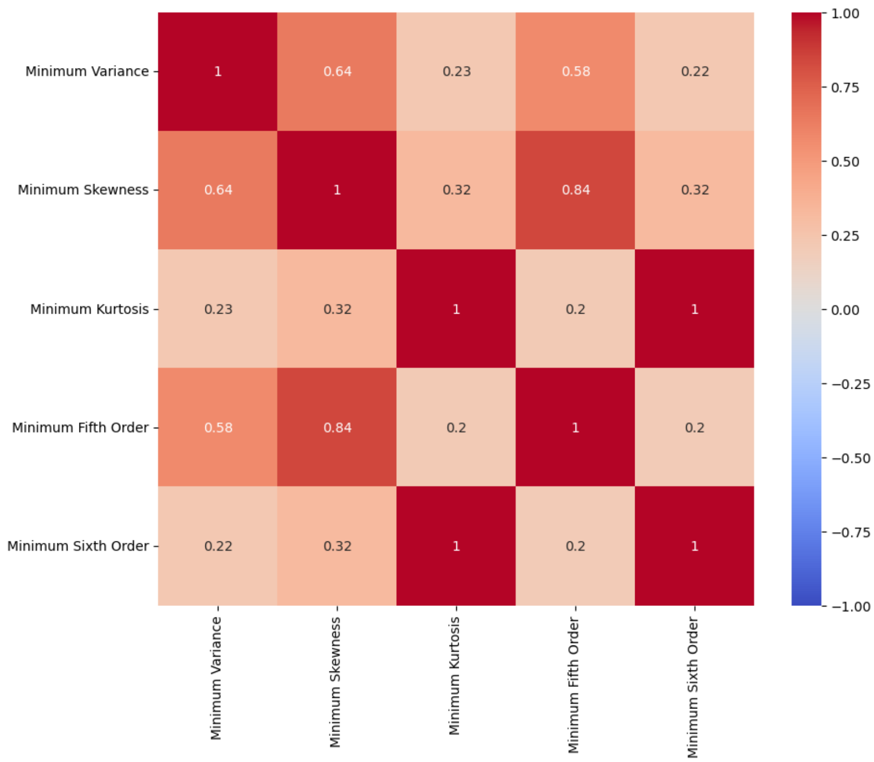

Figure 8: Correlation Matrix of Portfolio Returns for Taiwan 100 Stocks

them. By focusing on different aspects of risk—whether it's variance, skewness, or kurtosis—each portfolio exhibits unique risk-return characteristics, which is reflected in their varying degrees of correlation with one another.

6 Conclusion

This study investigated the efficacy of incorporating higher-order moments into the portfolio optimization process for both S&P 100 and Taiwan 100 stocks. By considering not only the mean and variance but also skewness, kurtosis, and higher-order moments, the study aimed to provide a more comprehensive approach to portfolio construction and risk management.

The empirical results demonstrated that traditional mean-variance optimization, while effective in minimizing risk, often results in conservative portfolios with lower returns and Sharpe ratios. In contrast, portfolios optimized for higher-order moments exhibited improved risk-adjusted performance, as indicated by higher returns, Sharpe ratios, and favorable Value at Risk (VaR) metrics. Specifically, the minimum kurtosis and minimum sixth order portfolios provided substantial benefits in terms of capturing complex risk-return dynamics and managing extreme tail risks for S&P 100 stocks.

For the Taiwan 100 stocks, the minimum skewness and minimum fifth order portfolios showed particularly strong performance, highlighting the importance of managing skewness and extreme deviations in return distributions. These portfolios achieved higher cumulative returns and Sharpe ratios compared to traditional risk measures, indicating their effectiveness in capturing the return dynamics of the Taiwan market.

In conclusion, this study highlights the value of integrating higher-order moments into portfolio optimization. By addressing asymmetrical risks and extreme tail events, higher-order moment portfolios provide a more nuanced approach to risk management, resulting in superior risk-adjusted performance.

These findings underscore the importance of a holistic view of risk in portfolio construction, moving beyond traditional mean-variance frameworks to incorporate advanced statistical measures for better alignment with the risk preferences of investors and market conditions. Future research could further explore the integration of environmental, social, and governance (ESG) considerations with higher-order moments to develop sustainable investment strategies that align financial performance with broader societal goals.

References

Harvey, C. R., Liechty, J. C., Liechty, M. W., and Müller, P. (2010). Portfolio selection with higher moments. *Quantitative Finance*, 10(5):469–485.

Khan, K. I., Naqvi, S. M. W. A., Ghafoor, M. M., and Akash, R. S. I. (2020). Sustainable portfolio optimization with higher-order moments of risk. *Sustainability*, 12(5):2006.

Kleniati, P., Rustem, B., et al. (2009). Portfolio decisions with higher order moments. In *COMISEF Working Paper Series*. WPS.

Maringer, D. and Parpas, P. (2009). Global optimization of higher order moments in portfolio selection. *Journal of Global optimization*, 43:219–230.

Markowitz, H. (1952). Portfolio selection*. *The Journal of Finance*, 7(1):77–91.

Table 2: Optimal Weights for Different Portfolio Optimization Models (S&P 100) - Part 1

Stock	Min Variance	Min Skewness	Min Kurtosis	Min Fifth Order	Min Sixth Order
AAPL	0.0%	0.0%	0.0%	0.0%	0.0%
ABBV	2.3649%	11.098%	0.0%	15.3169%	0.0%
ABT	0.0%	0.0%	0.0%	0.0%	0.0%
ACN	0.0%	0.0%	0.0%	0.0%	0.0%
ADBE	0.0%	0.0%	0.0%	0.0%	0.0%
AIG	0.0%	0.0%	0.0%	0.0%	0.0%
AMD	0.0%	0.0%	30.816%	0.0%	0.0%
AMGN	0.0%	0.0%	0.0%	0.0%	0.0%
AMT	0.0%	0.0%	0.0%	4.412%	0.0%
AMZN	2.9706%	0.0%	32.4757%	0.0%	48.7991%
AVGO	0.0%	2.5345%	0.0%	0.0%	0.0%
AXP	0.0%	0.0%	0.0%	0.0%	0.0%
BA	0.0%	0.0%	0.0%	0.0%	0.0%
BAC	0.0%	0.0%	0.0%	0.0%	0.0%
BK	0.0%	0.0%	0.0%	0.0%	0.0%
BKNG	0.0%	0.0%	0.0%	0.0%	0.0%
BLK	0.0%	0.0%	0.0%	0.0%	0.0%
BMY	12.4785%	0.8472%	0.0%	0.0%	0.0%
BRK-B	0.0%	0.0%	0.0%	0.0%	0.0%
C	0.0%	0.0%	0.0%	0.0%	0.0%
CAT	0.0%	0.0%	0.0%	0.0%	0.0%
CHTR	1.3392%	0.0%	0.0%	2.8328%	0.0%
CL	0.0%	12.4837%	0.0%	4.7575%	0.0%
CMCSA	0.0%	0.0%	0.0%	0.0%	0.0%
COF	0.0%	0.0%	0.0%	0.0%	0.0%
COP	0.0%	0.0%	0.0%	0.0%	0.0%
COST	0.0%	0.0%	0.0%	0.0%	0.0%
CRM	0.0%	0.0%	0.0%	0.0%	0.0%
CSCO	0.0%	0.0%	0.0%	0.0%	0.0%
CVS	0.0%	0.0%	0.0%	0.0%	0.0%
CVX	0.0%	0.0%	0.0%	0.0%	0.0%
DE	0.0%	0.0%	0.0%	0.0%	0.0%
DHR	0.0%	0.0%	0.0%	0.0%	0.0%
DIS	0.0%	0.0%	0.0%	0.0%	0.0%
DUK	0.0%	0.0%	0.0%	0.0%	0.0%
EMR	0.0%	0.0%	0.0%	0.0%	0.0%
F	0.0%	0.0%	0.0%	0.0%	0.0%
FDX	0.0%	0.0%	0.0%	0.0%	0.0%
GD	0.0%	0.0%	0.0%	0.0%	0.0%
GE	0.0%	0.0%	0.0%	0.0%	0.0%
GILD	6.6115%	0.0%	0.0%	0.0%	0.0%
GM	0.0%	0.0%	0.0%	0.0%	0.0%
GOOG	0.0%	0.0%	0.0%	0.0%	0.0%
GOOGL	0.0%	0.0%	0.0%	0.0%	0.0%
GS	0.0%	0.0%	0.0%	0.0%	0.0%
HD	0.0%	5.3861%	0.0%	0.0%	0.0%
HON	0.0%	0.0%	0.0%	0.0%	0.0%
IBM	0.0%	0.0%	0.0%	0.0%	0.0%

Table 3: Optimal Weights for Different Portfolio Optimization Models (S&P 100) - Part 2

Stock	Min Variance	Min Skewness	Min Kurtosis	Min Fifth Order	Min Sixth Order
INTC	0.0%	0.0%	0.0%	1.8623%	0.0%
INTU	0.0%	0.0%	0.0%	0.0%	0.0%
JNJ	11.2758%	0.0%	0.0%	0.0%	0.0%
JPM	0.0%	0.0%	0.0%	0.0%	0.0%
KHC	0.0%	0.0%	0.0%	0.0%	0.0%
KO	0.393%	0.0%	0.0%	0.0%	0.0%
LIN	0.0%	0.0%	0.0%	0.0%	0.0%
LLY	0.0%	0.0%	0.0%	0.0%	0.0%
LMT	0.0%	0.0%	0.0%	8.346%	0.0%
LOW	0.0%	24.1603%	0.0%	30.4071%	0.0%
MA	0.0%	0.0%	0.0%	0.0%	0.0%
MCD	8.8176%	0.0%	0.0%	13.781%	0.0%
MDLZ	0.0%	0.0%	0.0%	0.0%	0.0%
MDT	0.0%	0.0%	0.0%	0.0%	0.0%
MET	0.0%	0.0%	0.0%	0.0%	0.0%
META	0.0%	0.0%	0.0%	0.0%	0.0%
MMM	0.0%	0.0%	0.0%	0.0%	0.0%
MO	5.0005%	0.0%	0.0%	0.0%	0.0%
MRK	2.8354%	0.0%	0.0%	0.0%	0.0%
MS	0.0%	0.0%	0.0%	0.0%	0.0%
MSFT	0.0%	0.0%	0.0%	0.0%	0.0%
NEE	0.0%	0.0%	0.0%	0.0%	0.0%
NFLX	0.9567%	0.0%	8.9823%	0.0%	12.6031%
NKE	0.0%	0.0%	0.0%	0.0%	0.0%
NVDA	0.0%	0.0%	0.0%	0.0%	0.0%
ORCL	0.0%	0.0%	0.0%	0.0%	0.0%
PEP	0.0%	5.8926%	0.0%	0.0%	0.0%
PFE	0.0%	0.0%	0.0%	0.0%	0.0%
PG	0.0%	0.0%	0.0%	0.0%	0.0%
PM	0.0%	11.6677%	0.0%	0.0%	0.0%
PYPL	0.0%	0.0%	0.0%	0.0%	0.0%
QCOM	0.0%	0.0%	0.0%	0.0%	0.0%
RTX	0.0%	0.0%	0.0%	0.0%	0.0%
SBUX	0.0%	0.0%	0.0%	0.0%	0.0%
SCHW	0.0%	0.0%	0.0%	0.0%	0.0%
SO	0.0%	0.0%	0.0%	0.0%	0.0%
SPG	0.0%	17.0761%	0.0%	10.4734%	0.0%
T	0.0%	0.0%	0.0%	0.0%	0.0%
TGT	0.0%	0.0%	0.0%	0.0%	0.0%
TMO	0.0%	0.0%	0.0%	0.0%	6.5949%
TMUS	0.0%	0.0%	0.0%	0.0%	0.0%
TSLA	0.0%	1.3881%	27.726%	0.0%	32.0029%
TXN	0.0%	0.0%	0.0%	0.0%	0.0%
UNH	0.0%	7.4656%	0.0%	7.8112%	0.0%
UNP	0.0%	0.0%	0.0%	0.0%	0.0%
UPS	0.0%	0.0%	0.0%	0.0%	0.0%
USB	0.0%	0.0%	0.0%	0.0%	0.0%
V	0.0%	0.0%	0.0%	0.0%	0.0%
VZ	27.9191%	0.0%	0.0%	0.0%	0.0%
WFC	0.0%	0.0%	0.0%	0.0%	0.0%
WMT	17.0372%	0.0%	0.0%	0.0%	0.0%
XOM	0.0%	0.0%	0.0%	0.0%	0.0%

Table 4: Optimal Weights for Different Portfolio Optimization Models (Taiwan 100 Stocks) - Part 1

Stock	Min Variance	Min Skewness	Min Kurtosis	Min Fifth Order	Min Sixth Order
2330.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2317.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2454.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2382.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2412.TW	42.556%	0.0%	0.0%	0.0%	0.0%
2881.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2308.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2882.TW	0.0%	0.0%	0.0%	0.0%	0.0%
6505.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2891.TW	0.0%	0.2428%	0.0%	0.0%	0.0%
3711.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2303.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2886.TW	0.2195%	0.0%	0.0%	0.0%	0.0%
1303.TW	0.0%	0.0%	0.0%	0.0%	0.0%
1301.TW	0.0%	0.0%	0.0%	0.0%	0.0%
1216.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2884.TW	0.0%	0.0%	0.0%	0.0%	0.0%
6669.TW	0.0%	0.0%	2.6755%	1.7645%	1.5802%
2603.TW	0.0%	0.0%	28.0726%	0.0%	27.4549%
2002.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2885.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3045.TW	19.849%	0.0%	0.0%	0.0%	0.0%
5880.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3034.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2892.TW	0.0%	27.7305%	0.0%	34.1338%	0.0%
2207.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3231.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2395.TW	0.0%	0.0%	0.0%	0.0%	0.0%
1326.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2880.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2357.TW	0.0%	2.1106%	0.0%	2.1994%	0.0%
3008.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2912.TW	3.3135%	0.0%	10.9836%	0.0%	12.7205%
4904.TW	3.9394%	0.0%	0.0%	0.0%	0.0%
5871.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3037.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2890.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2379.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2327.TW	0.0%	0.0%	0.0%	0.0%	0.0%
4938.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2345.TW	0.0%	0.0%	0.9118%	0.0%	1.6215%
3017.TW	0.0%	1.3982%	0.0%	0.0%	0.0%
1101.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3661.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2301.TW	2.5839%	0.0%	0.0%	0.0%	0.0%
1590.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2883.TW	0.0%	0.0%	0.0%	0.0%	0.0%
5876.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2887.TW	0.0%	0.0%	0.0%	7.9913%	0.0%

Table 5: Optimal Weights for Different Portfolio Optimization Models (Taiwan 100 Stocks) - Part 2

Stock	Min Variance	Min Skewness	Min Kurtosis	Min Fifth Order	Min Sixth Order
1519.TW	0.0%	3.0195%	0.0%	0.0%	0.0%
2408.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2801.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2618.TW	0.0%	0.6722%	0.0%	0.0%	0.0%
2356.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2376.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3443.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2609.TW	0.0%	0.0%	0.0%	0.0%	0.0%
1402.TW	0.0%	0.0%	0.0%	0.0%	0.0%
6415.TW	0.0%	0.0%	46.1664%	0.0%	47.0121%
3036.TW	0.7825%	6.5704%	0.0%	6.7978%	0.0%
3533.TW	0.0%	0.0%	0.0%	0.5398%	0.0%
9910.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2324.TW	5.1554%	0.0%	0.0%	0.0%	0.0%
1102.TW	3.0197%	0.0%	0.0%	0.0%	0.0%
2385.TW	3.4251%	0.0%	11.19%	0.0%	9.6107%
2371.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2105.TW	0.0%	11.6886%	0.0%	9.6025%	0.0%
1503.TW	5.4947%	0.0%	0.0%	0.0%	0.0%
1605.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2474.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3702.TW	2.4379%	7.3931%	0.0%	24.6086%	0.0%
2615.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2383.TW	0.0%	1.7369%	0.0%	0.0%	0.0%
1476.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2409.TW	0.0%	0.0222%	0.0%	0.0%	0.0%
2834.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2353.TW	0.0%	0.0%	0.0%	0.0%	0.0%
3653.TW	0.0%	0.0%	0.0%	0.0%	0.0%
6409.TW	0.0%	0.0%	0.0%	0.0%	0.0%
5269.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2377.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2888.TW	0.0%	0.0%	0.0%	0.0%	0.0%
2347.TW	2.9952%	0.0%	0.0%	0.0%	0.0%
6239.TW	0.0%	4.2591%	0.0%	3.5249%	0.0%
3481.TW	0.0%	1.8411%	0.0%	0.0%	0.0%
2610.TW	0.0%	0.0%	0.0%	0.0%	0.0%
8046.TW	0.0%	0.0%	0.0%	0.0%	0.0%
1504.TW	0.2297%	0.0%	0.0%	0.0%	0.0%
2059.TW	3.5262%	3.4598%	0.0%	0.0%	0.0%
2449.TW	0.1788%	0.0%	0.0%	0.0%	0.0%
2360.TW	0.0%	0.0%	0.0%	0.0%	0.0%
8464.TW	0.0%	0.0%	0.0%	0.0%	0.0%
4958.TW	0.0%	2.1776%	0.0%	0.0%	0.0%
9945.TW	0.0%	2.9885%	0.0%	0.261%	0.0%
2344.TW	0.0%	0.0%	0.0%	0.0%	0.0%
9904.TW	0.0%	10.2329%	0.0%	2.3203%	0.0%
8454.TW	0.2934%	0.0%	0.0%	0.0%	0.0%
1229.TW	0.0%	6.9617%	0.0%	3.6923%	0.0%
3044.TW	0.0%	5.4943%	0.0%	2.5639%	0.0%