$$\text{cos-dist}(A,B) = 1 - \text{cos-sim}(A,B)$$

$$\text{cos-sim}(A,B) = \frac{\langle A,B\rangle}{||A||\cdot||B||} = \frac{\sum\limits_{i=1}^{n} A_i \cdot B_i}{\sqrt{\sum\limits_{i=1}^{n} A_i^2}\cdot\sqrt{\sum\limits_{i=1}^{n} B_i^2}}$$

Triangle inequality for cosine distance tooks a form of (of course it doesn't hold):

$$\text{cos-dist}(A,C) \nleq \text{cos-dist}(A,B) + \text{cos-dist}(B,C)$$

which is equivalent to:

$$1 - \text{cos-sim}(A,C) \nleq 1 - \text{cos-sim}(A,B) + 1 - \text{cos-sim}(B,C)$$

and after simple transformations:

$$1 + \text{cos-sim}(A,C) \ngeq \text{cos-sim}(A,B) + \text{cos-sim}(B,C)$$

Now, you're trying to find such three vectors A, B and C that:

$$1 + \text{cos-sim}(A,C) < \text{cos-sim}(A,B) + \text{cos-sim}(B,C)$$

Let $A, B, C \in \mathbb{R}^2$ and **all of them are of unit length** A = [1, 0], B = $\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$, C = [0, 1]. Note that vectors A and C are orthogonal, so we would get simply 0:

$$\text{cos-sim}(A,C) = \frac{0}{\sqrt{1}\sqrt{1}} = 0$$

Each pair of vectors A & B as well as B & C would give the same value:

$$\text{cos-sim}(A,B) = \frac{\frac{\sqrt{2}}{2}+0}{\sqrt{1}\sqrt{1}} = \frac{\sqrt{2}}{2}, \quad \text{cos-sim}(B,C) = \frac{0+\frac{\sqrt{2}}{2}}{\sqrt{1}\sqrt{1}} = \frac{\sqrt{2}}{2}$$

. Finally, we could defeat primary inequality by proving that:

$$1 + 0 < \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$
$$1 < \sqrt{2} \approx 1.41\ldots$$

https://stats.stackexchange.com/questions/198080/proving-that-cosine-distance-function-defi
ned-by-cosine-similarity-between-two-u