Master Equation in Rb87, D1

Init

In [ ]:

```
import numpy as np
from qutip import *
import random

random.seed(389)
qutip.settings.auto_tidyup = False

from sympy.physics.wigner import wigner_6j, wigner_3j

# from sympy.physics.wigner import clebsch_gordan

import scipy.constants as constants

import matplotlib as mpl

mpl.rcParams["figure.dpi"] = 100
mpl.rcParams["figure.figsize"] = (10, 4.8)
from matplotlib import pyplot as plt
import seaborn as sns
from my_matrix_plot import matrixplot

sns.set_style("dark")

# about()


# def index_to_F_mF_string(ind):
#     if ind < 3:
#         return "F=1, m=" + str(ind - 1)
#     elif ind < 8:
#         return "F=2, m=" + str(ind - 2 - 3)
#     elif ind < 8 + 3:
#         return "F'=1, m'=" + str(ind - 1 - 8)
#     else:
#         return "F'=2, m'=" + str(ind - 2 - 8 - 3)


def F_mF_to_index(F, mF, excited=False):  # only D1 at the moment
    if excited:
        offset = 8
    else:
        offset = 0
    if F == 1:
        return F + mF + offset
    elif F == 2:
        return F + mF + 3 + offset
    else:
        raise NotImplementedError


def index_to_F_mF_string(ind):
    if ind < 3:
        return rf"""$F=1, m={ind - 1}$"""
    elif ind < 8:
        return rf"""$F=2, m={ind - 2 - 3}$"""
    elif ind < 8 + 3:
        return rf"""$F'=1, m={ind - 1 - 8}$"""
    else:
        return rf"""$F'=2, m={ind - 2 - 8 - 3}$"""


def maplot(op: Qobj, std_xlabels=True, std_ylabels=True, annot=False):
    return matrixplot(
        op,
        xlabels=[index_to_F_mF_string(ind) for ind in range(op.shape[0])]
        if std_xlabels
        else "auto",
        ylabels=[index_to_F_mF_string(ind) for ind in range(op.shape[0])]
        if std_ylabels
```

```
        if std_ylabels
        else "auto",
        annot=annot,
    )


A_S = 3.417341305452145e09  # Hz
A_P12 = 407.25e6
pi = np.pi
I = 3 / 2
Jg = 1 / 2
Je = 1 / 2  # D1


def Fg1_projector():
    return sum([basis(16, k).proj() for k in range(3)])


def Fg2_projector():
    return sum([basis(16, k).proj() for k in range(3, 8)])


def Fe1_projector():
    return sum([basis(16, k).proj() for k in range(8, 11)])


def Fe2_projector():
    return sum([basis(16, k).proj() for k in range(11, 16)])


def j_1_2_projector():
    return sum([basis(16, k).proj() for k in range(8)])


def j_3_2_projector():
    return sum([basis(16, k).proj() for k in range(8, 16)])


def Eg(F):
    return 1 / 2 * A_S * (F * (F + 1) - I * (I + 1) - Jg * (Jg + 1))


def E_P12(F):
    return 1 / 2 * A_P12 * (F * (F + 1) - I * (I + 1) - Je * (Je + 1))


def Ha(det_Light):
    return (
        sum([(E_P12(1) - det_Light) * basis(16, 8 + m).proj() for m in range(3)])
        + sum([(E_P12(2) - det_Light) * basis(16, 8 + 3 + m).proj() for m in range(5)])
        + sum([Eg(1) * basis(16, m + 1).proj() for m in (-1, 0, 1)])
        + sum([Eg(2) * basis(16, 3 + m).proj() for m in range(5)])
    )


def sigma_q(
    q,
):  # "weighted lowering operator", m_F = m'_F + q
    assert q in (-1, 0, 1)
    opers = []
    for Fg in (1, 2):
        for mg in range(-Fg, Fg + 1):
            for Fe in (1, 2):
                for me in range(-Fe, Fe + 1):
                    a = (
                        (-1) ** (Fe + Jg + 1 + I)
                        * ((2 * Fe + 1) * (2 * Jg + 1)) ** (1 / 2)
                        * (-1) ** (Fe - 1 + mg)
                        * (2 * Fg + 1) ** (1 / 2)
                        * float(wigner_3j(Fe, 1, Fg, me, q, -mg))
```

```python
                        * float(wigner_3j(Fe, 1, Fg, me, q, -mg))
                        * float(wigner_6j(Je, Jg, 1, Fg, Fe, I))
                    )
                    op = (
                        basis(16, F_mF_to_index(Fg, mg))
                        * basis(16, F_mF_to_index(Fe, me, excited=True)).dag()
                    )
                    opers.append(a * op)
    return sum(opers)


def rabi(intens):  # Jg = Je = 1/2  -> only D1 and sigma+/- polarization (E_total = E_q)
    return (
        -((2 * intens / (constants.c * constants.epsilon_0)) ** (1 / 2))
        * 2.5377e-29
        / constants.h  # dipole transition matrix element
    )


def H_AF(q, intens):
    tmp = np.conjugate(rabi(intens)) * sigma_q(q)
    return tmp + tmp.dag()


off_resonant_saturation_intensity_D1_pi_pol = (
    4.4876 * 1e-3 / (1e-2) ** 2
)  # far detuned saturation intensity for D1, pi pol. Is this value even relevant for pumping
laser_intens = 1 * off_resonant_saturation_intensity_D1_pi_pol
hamil = H_AF(1, laser_intens) + Ha(
    -509.06e6 - 2.563005979089109e9
)  # sigma-, F=2 -> F'=1


gamma_natural = 5.7500e6  # D1
natural_decay_ops = [gamma_natural ** (1 / 2) * sigma_q(q) for q in [-1, 0, 1]]


plt.rcParams["figure.dpi"] = 100
plt.rcParams["figure.figsize"] = (10, 4.8)


intra_F1_gamma = 1e4
intra_F2_gamma = 1e4
intra_Fp1_gamma = 1e4
intra_Fp2_gamma = 1e4
intra_F1 = [
    sum(
        [
            (intra_F1_gamma / 3) ** (1 / 2) * basis(16, f) * basis(16, i).dag()
            for f in range(3)
            if i != f
        ]
    )
    for i in range(3)
]
intra_F2 = [
    sum(
        [
            (intra_F2_gamma / 5) ** (1 / 2) * basis(16, f) * basis(16, i).dag()
            for f in range(3, 8)
            if i != f
        ]
    )
    for i in range(3, 8)
]
intra_Fp1 = [
    sum(
        [
            (intra_Fp1_gamma / 3) ** (1 / 2) * basis(16, f) * basis(16, i).dag()
            for f in range(8, 3 + 8)
```

```
            for f in range(8, 3 + 8)
            if i != f
        ]
    )
    for i in range(8, 3 + 8)
]
intra_Fp2 = [
    sum(
        [
            (intra_Fp2_gamma / 5) ** (1 / 2) * basis(16, f) * basis(16, i).dag()
            for f in range(3 + 8, 8 + 8)
            if i != f
        ]
    )
    for i in range(3 + 8, 8 + 8)
]

gamma_interF = 1e4
F2_to_F1_decay_ops = [
    (gamma_interF / 3) ** (1 / 2)
    * sum([basis(16, f) * basis(16, i).dag() for f in range(3)])
    for i in range(5)
]

quenching_rate = 8.4e7 / (2 * pi)
quenching_ops = [
    sum(
        [
            (quenching_rate / 8) ** (1 / 2) * basis(16, g) * basis(16, e).dag()
            for g in range(8)
        ]
    )
    for e in range(8, 16)
]
```

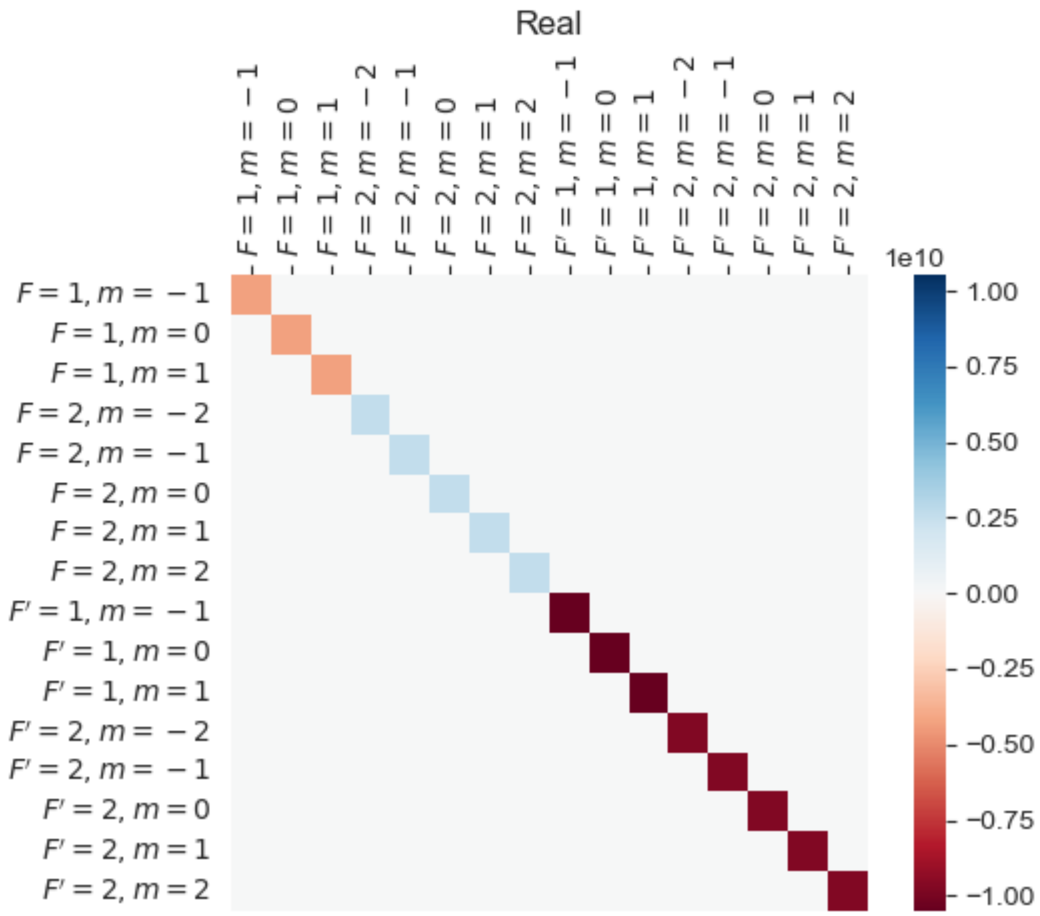
In [ ]:
```
quenching_rate / gamma_natural
```

Out[ ]:
```
2.325046125168558
```

10 GHz detuned

In [ ]:
```
maplot(Ha(+10e9))
```

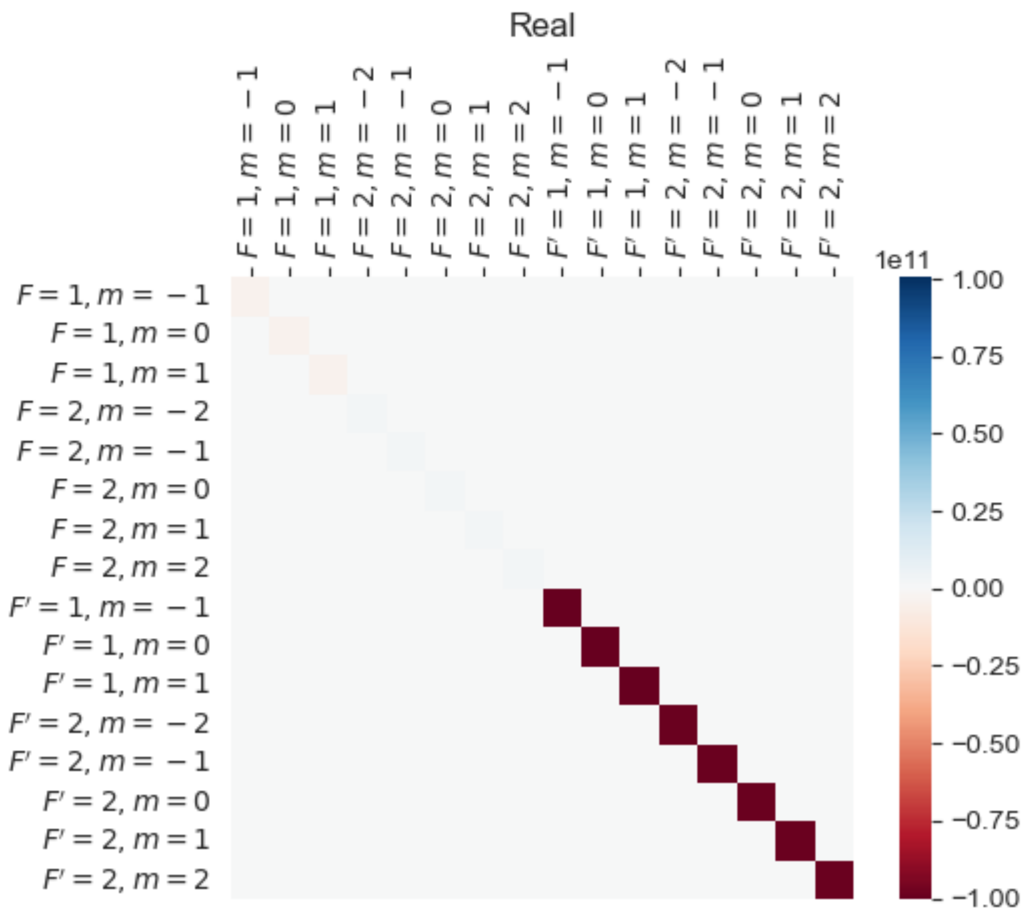
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



100 GHz detuned

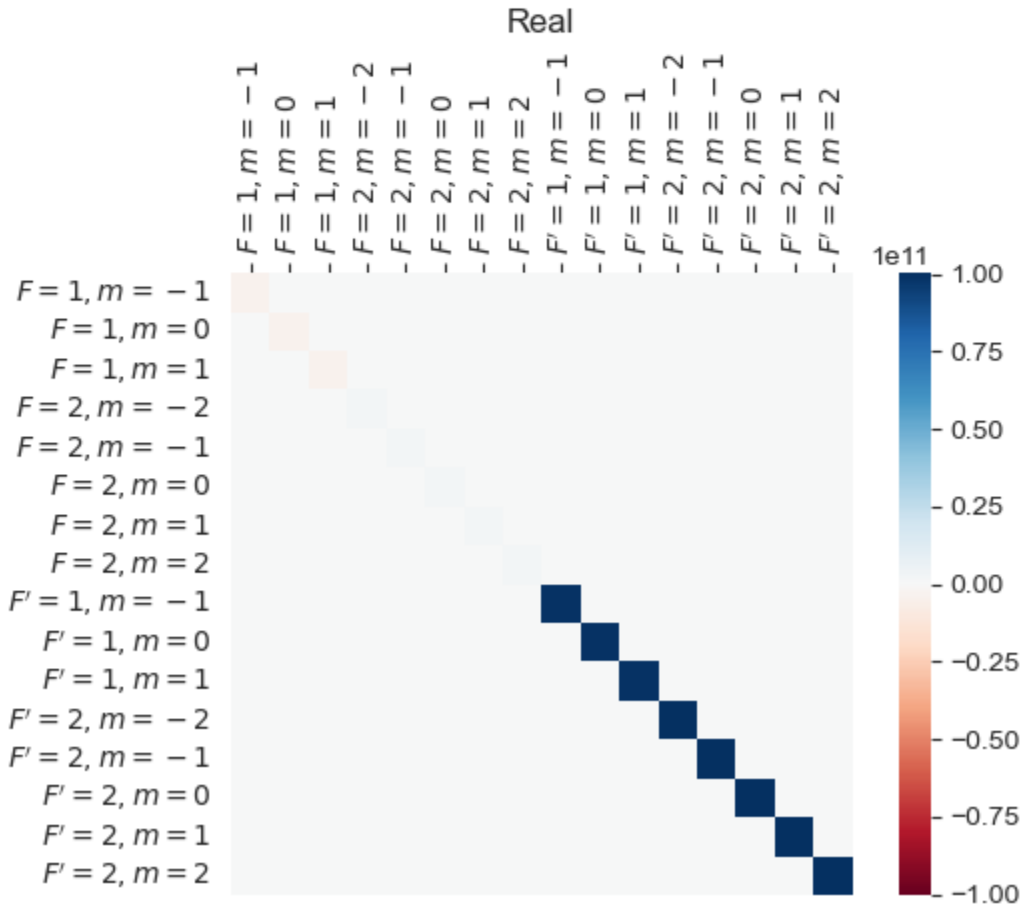
In [ ]:
```
maplot(Ha(+100e9))
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



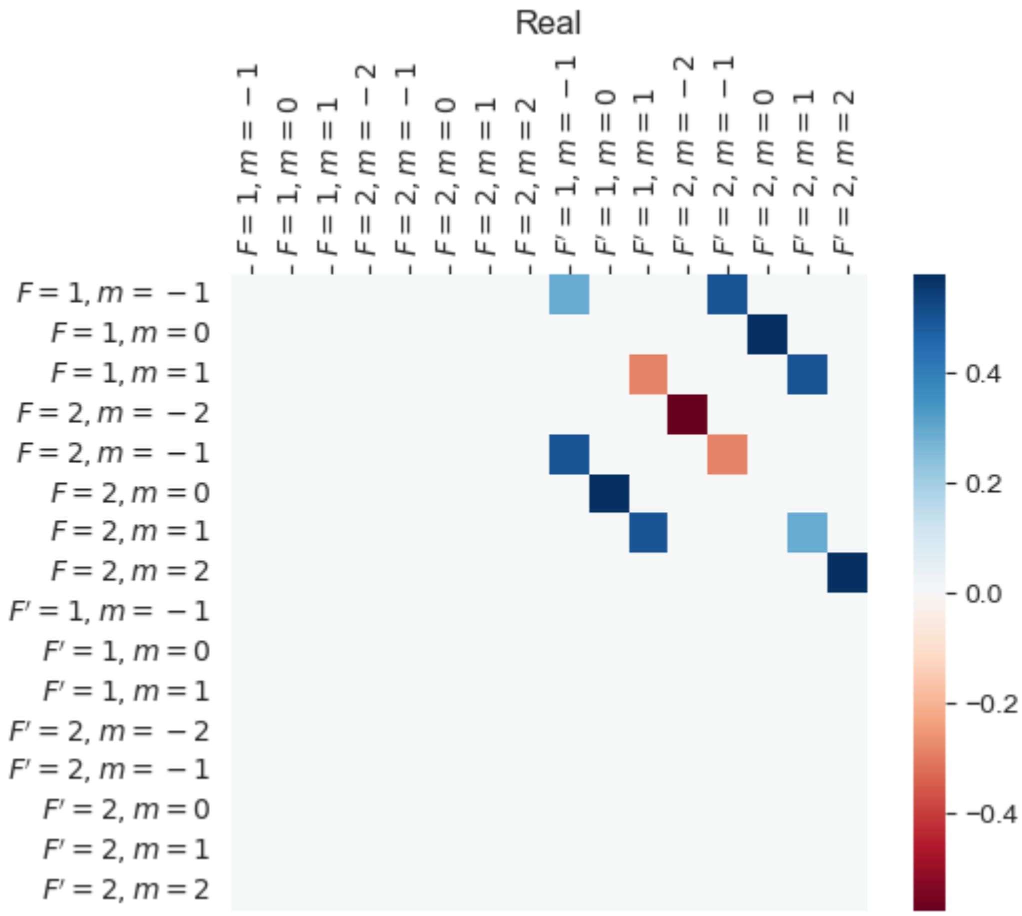
In [ ]:
```
maplot(Ha(-100e9))
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



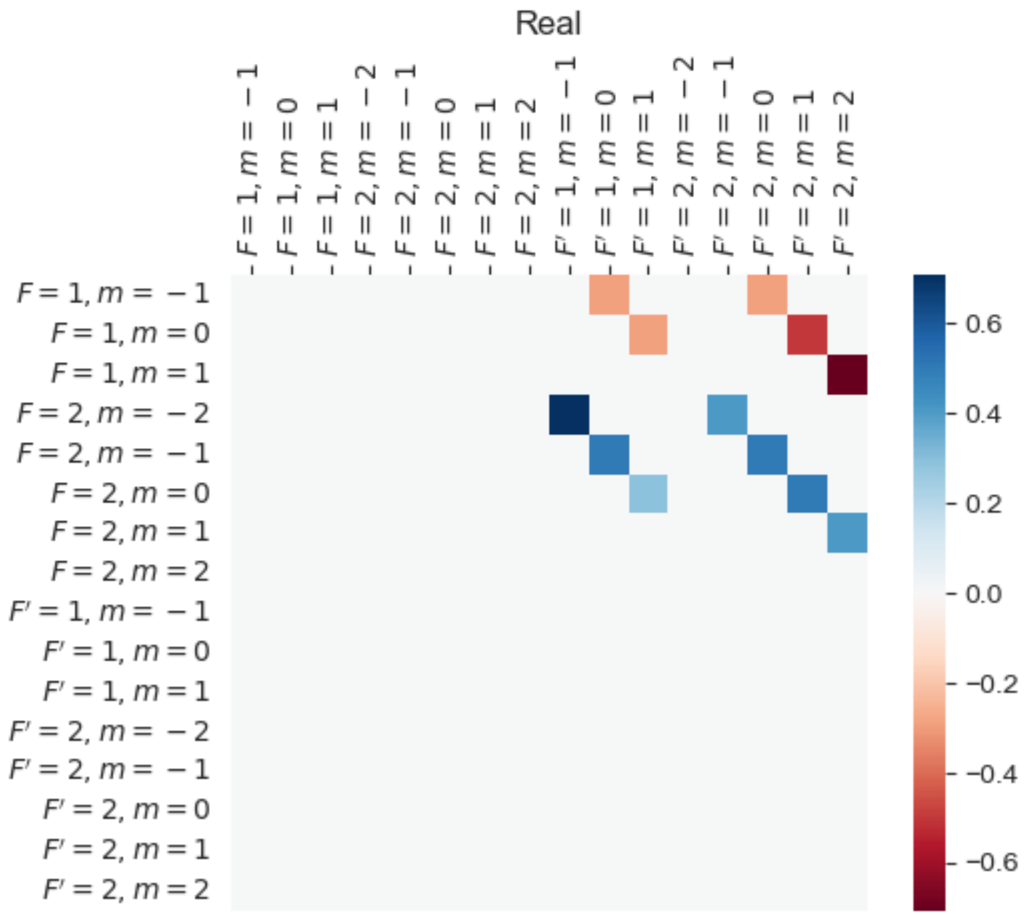
In [ ]:
```
maplot(sigma_q(0))
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



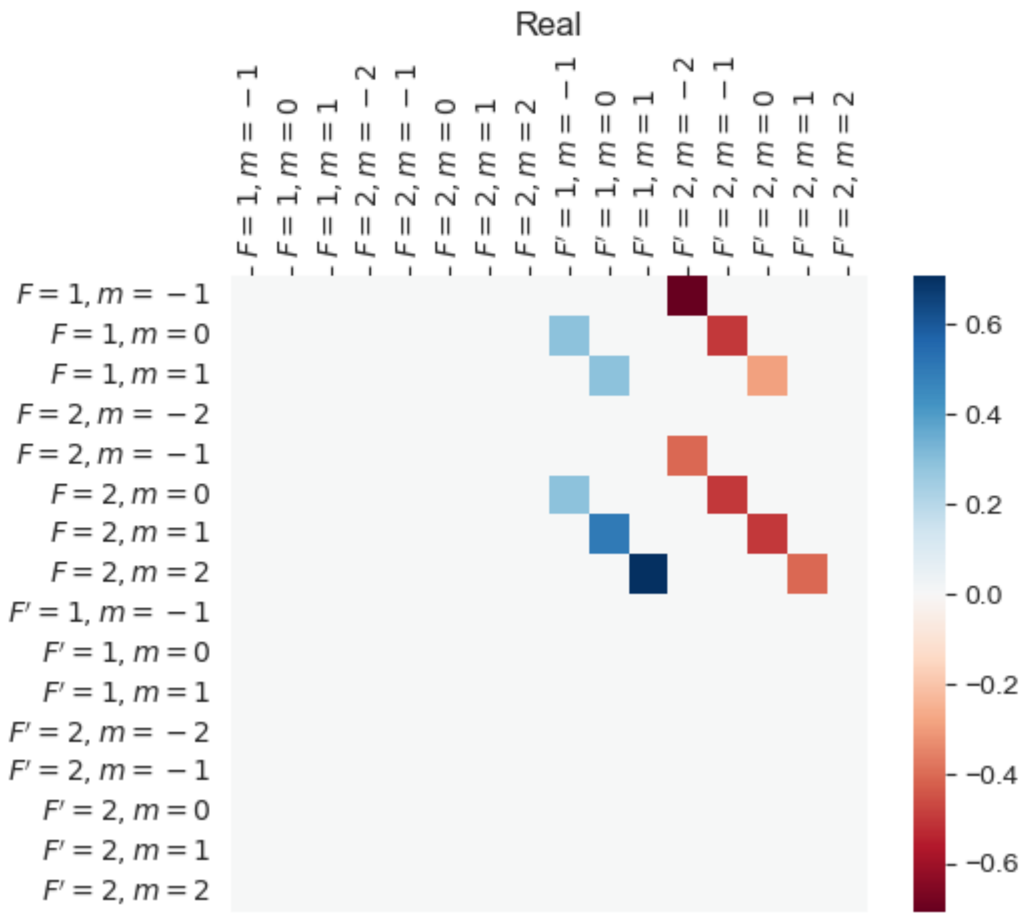
In [ ]:
```
maplot(sigma_q(-1))
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



In [ ]:
```
maplot(sigma_q(1))
```

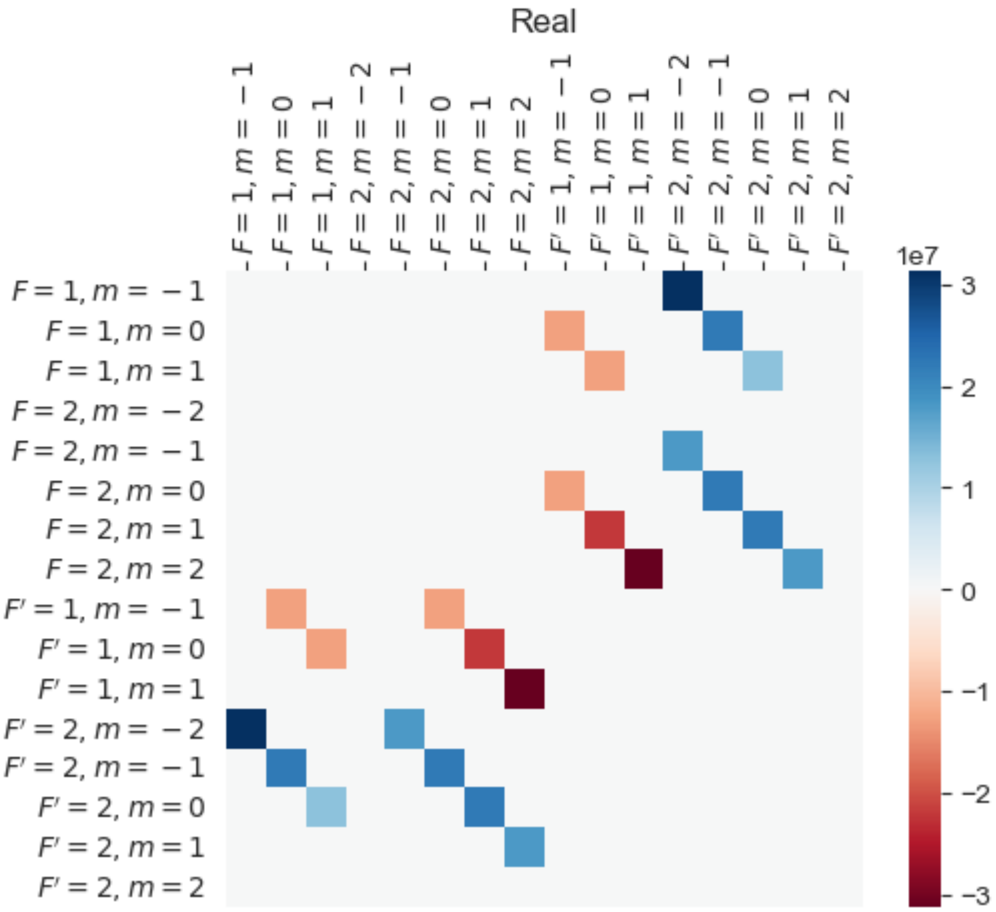
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



Laser Polarization: σ_- (since q=1), F=2 -> F'=1

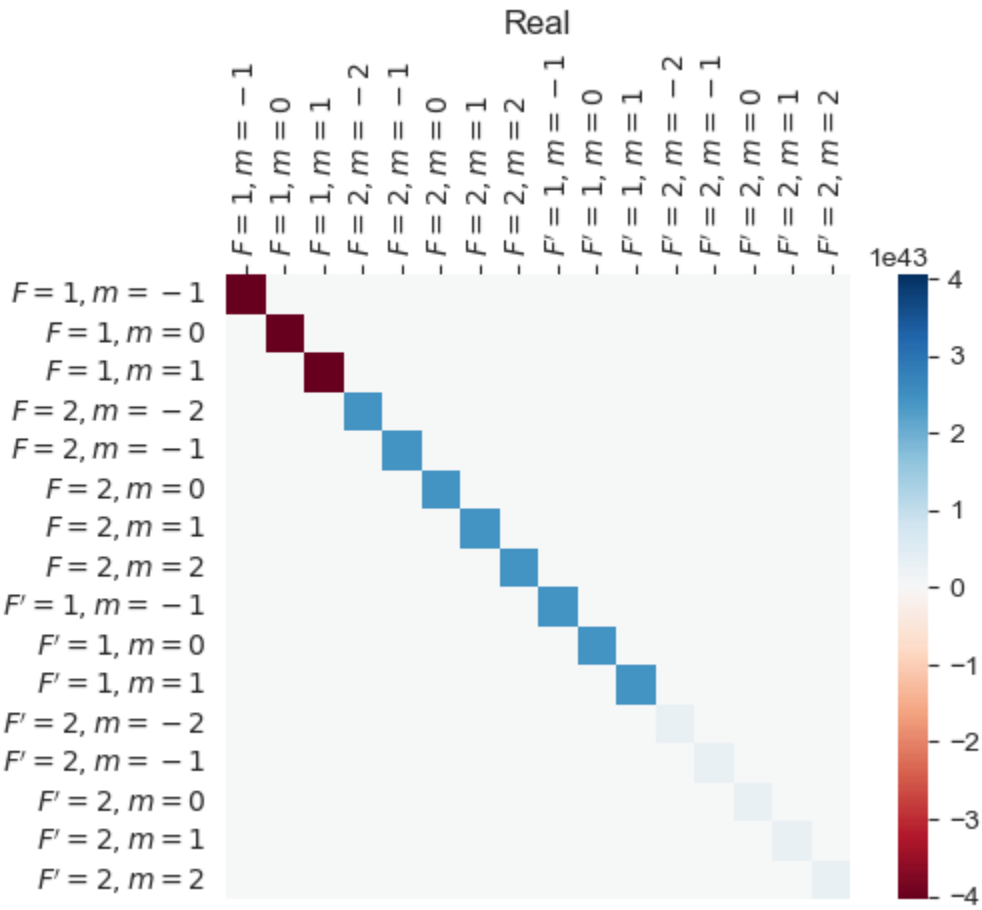
In [ ]:
```
maplot(H_AF(1, laser_intens))
```

Out[ ]: (<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)



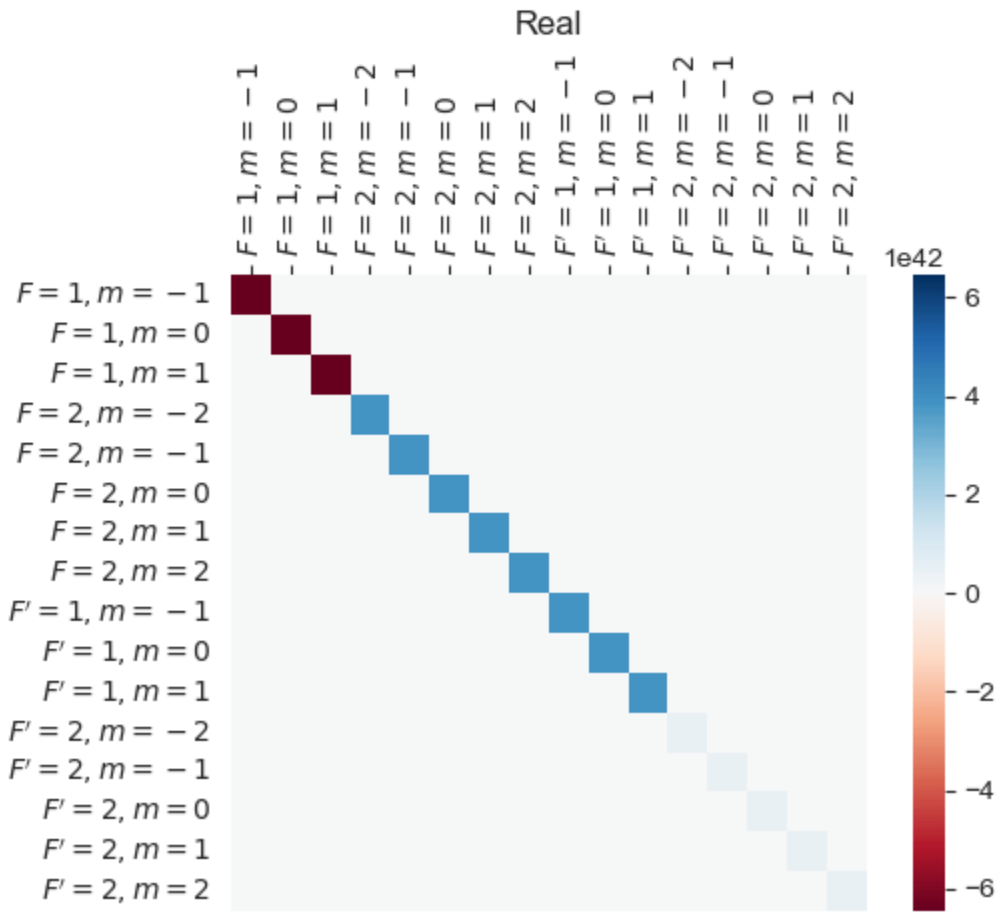

```
In [ ]: maplot(Ha(-509.06e6 - 2.563005979089109e9))
```

Out[ ]: (<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)




```
In [ ]: maplot(hamil)
```

Out[ ]: (<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)

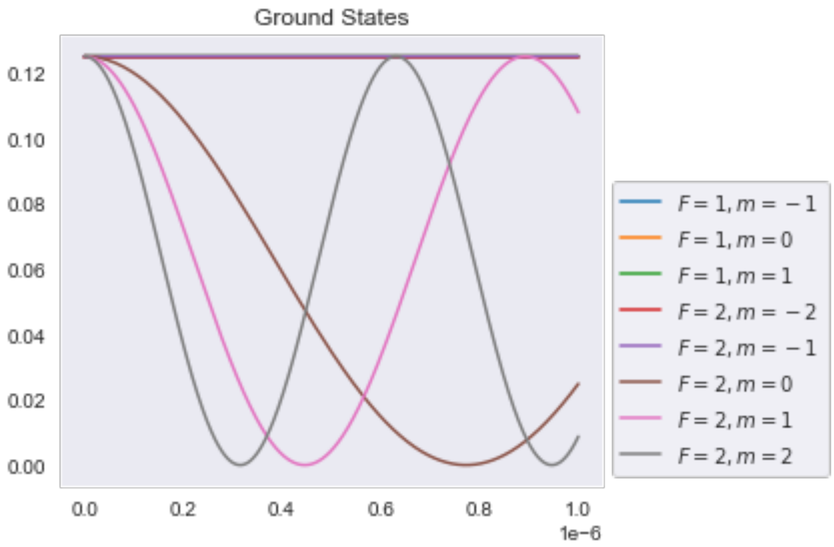


D1 Driving, No Decay

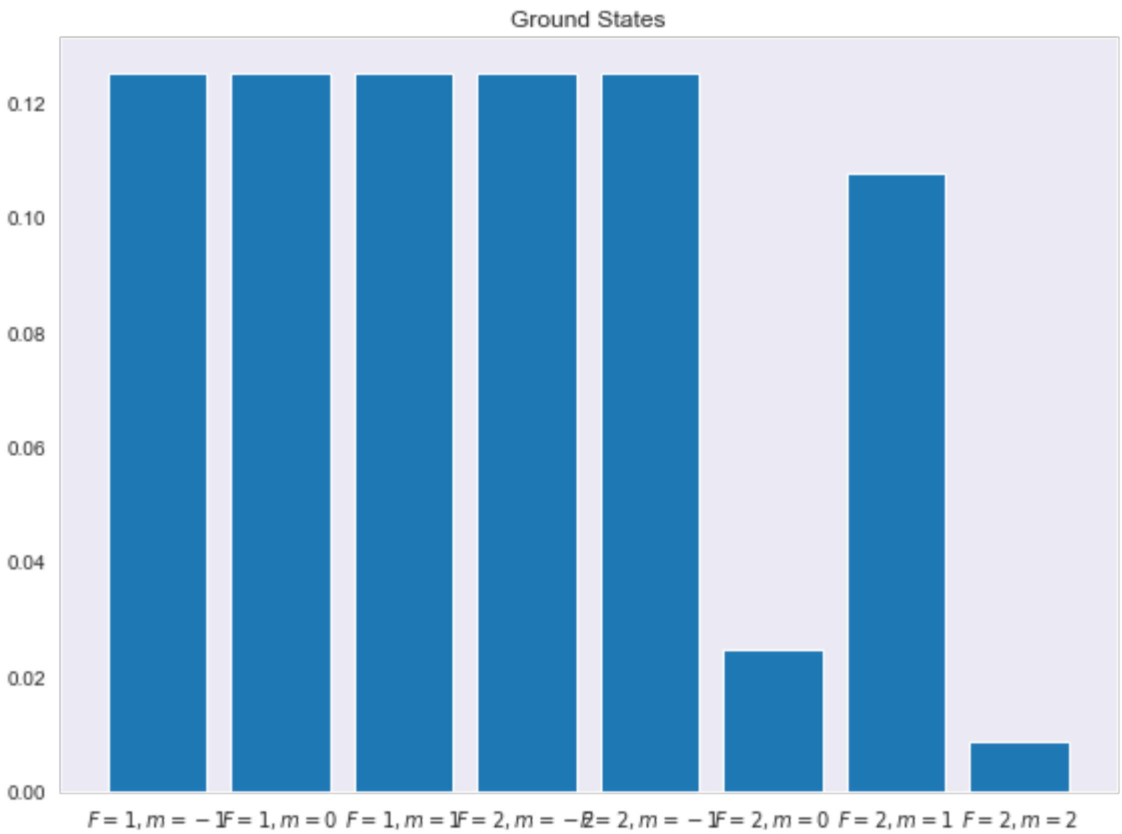
Starting state: equal population in ground states

In [ ]:
```
starting_state = sum([basis(16, i).proj() for i in range(8)])
starting_state = starting_state.unit()
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=3 * 10**4)
res = mesolve(
    hamil,
    starting_state,
    times,
    options=opts,
)
```

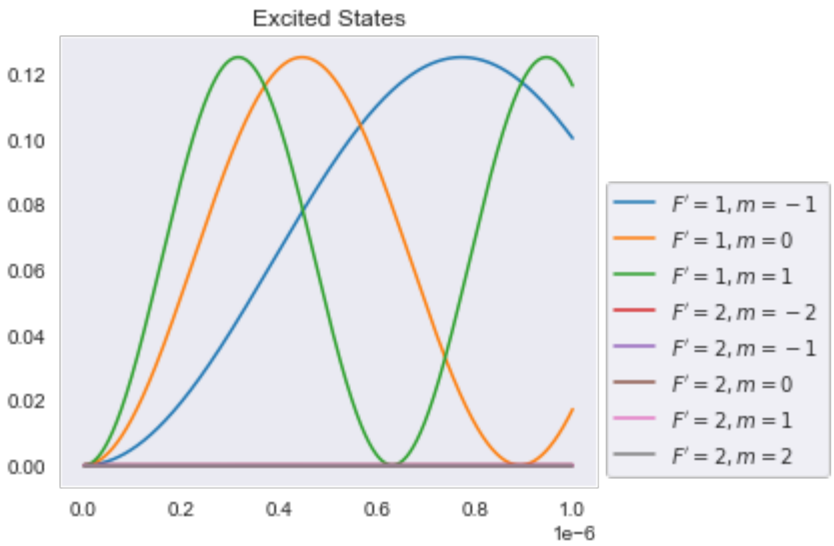
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.tight_layout()
```



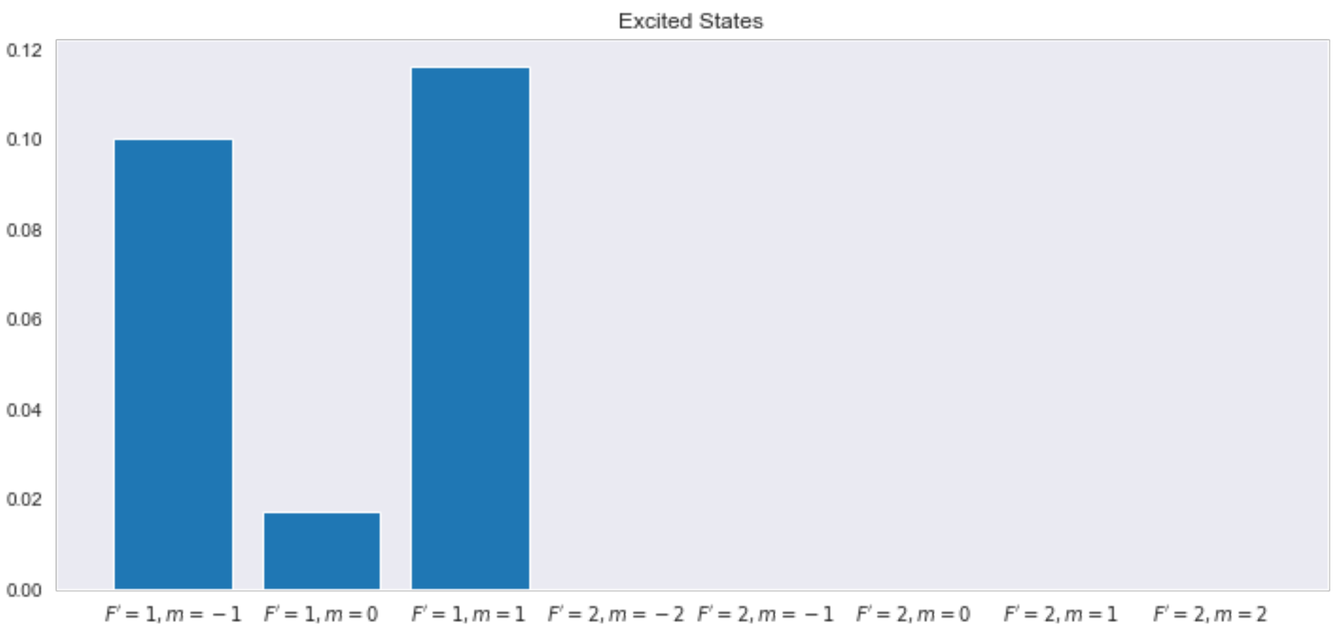
In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



```
In [ ]: excited_exp = [
            [
                res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
                for t in range(len(times))
            ]
            for i in range(8, 16)
        ]
        plt.figure()
        for e in excited_exp:
            plt.plot(times, np.real(e))
        plt.title("Excited States")
        plt.legend(
            [index_to_F_mF_string(i) for i in range(8, 16)],
            loc="best",
            bbox_to_anchor=(1.0, 0.7),
        )
        plt.tight_layout()
```




```
In [ ]: plt.figure(figsize=(10, 4.8))
        plt.bar(
            [index_to_F_mF_string(i) for i in range(8, 16)],
            [np.real(e)[-1] for e in excited_exp],
        )
        plt.title("Excited States")

        plt.tight_layout()
```



Radiative Decay Only

In [ ]:
```
print(f"""{rabi(off_resonant_saturation_intensity_D1_pi_pol*10):.2e}""")
```

```
-2.23e+07
```

In [ ]:
```
L = sum([lindblad_dissipator(a=c) for c in natural_decay_ops])
```

In [ ]:
```
import plotly.express as px

y = L.full().real
fig = px.imshow(
    y,
    color_continuous_midpoint=0,
    aspect="equal",
    width=1.5 * 800,
    height=1.5 * 400,
    zmin=-(abs(y).max()),
    zmax=(abs(y).max()),
    color_continuous_scale="RdBu",
)
fig.show()
```

In [ ]:
```
L.iscptp
```

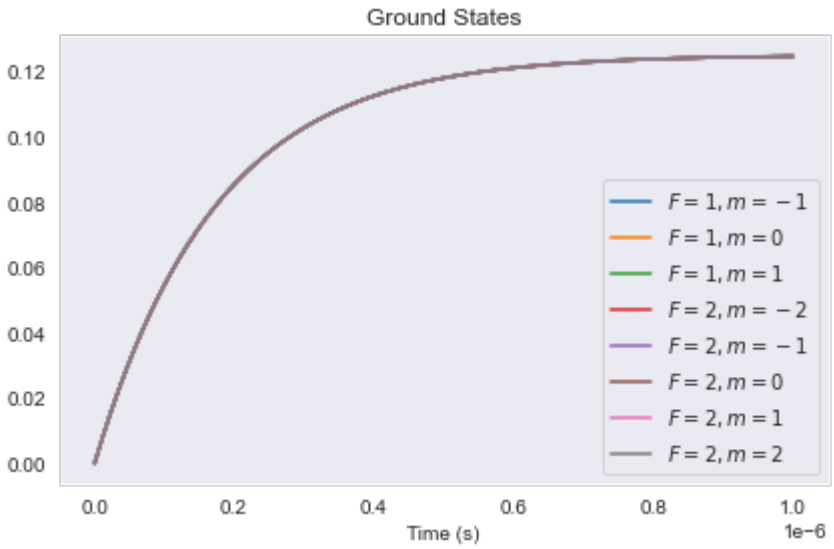
Out[ ]:
```
False
```

$T = 10\,ns$

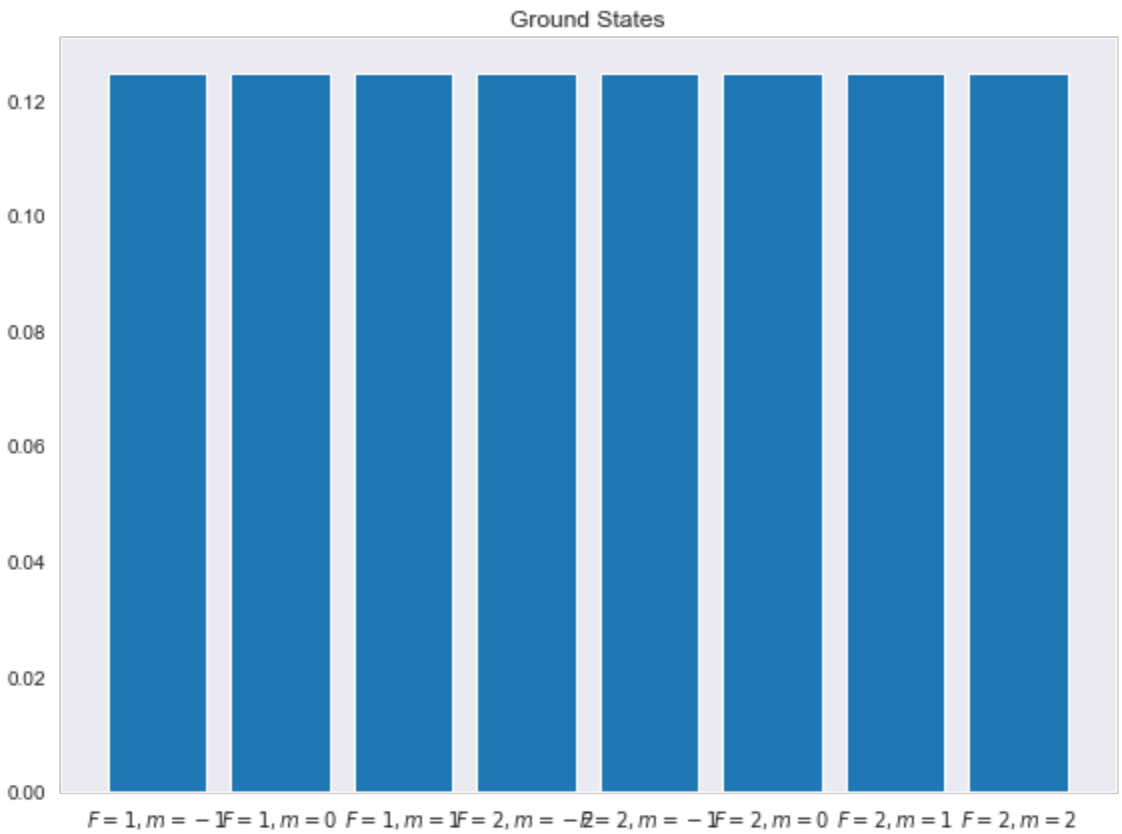
In [ ]:
```
starting_state = sum([basis(16, i).proj() for i in range(8, 16)])
# starting_state = basis(16, 16).proj()
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

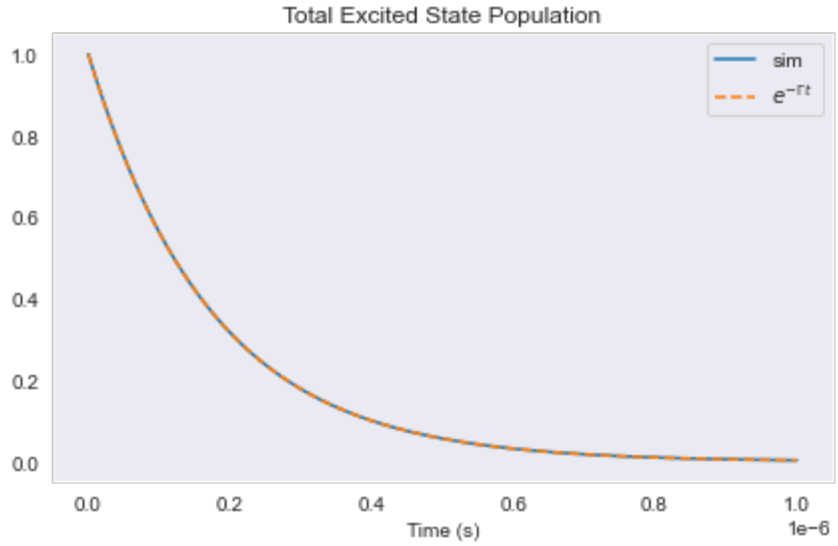
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



```
In [ ]: excited_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    for t in range(len(times))
]
plt.figure()
# for e in excited_exp:
plt.plot(times, np.real(excited_exp), label="sim")
plt.plot(
    times, [np.exp(-gamma_natural * t) for t in times], "--", label=r"$e^{-\Gamma t}$"
)
plt.legend()
plt.title("Total Excited State Population")
plt.xlabel("Time (s)")
plt.tight_layout()
```

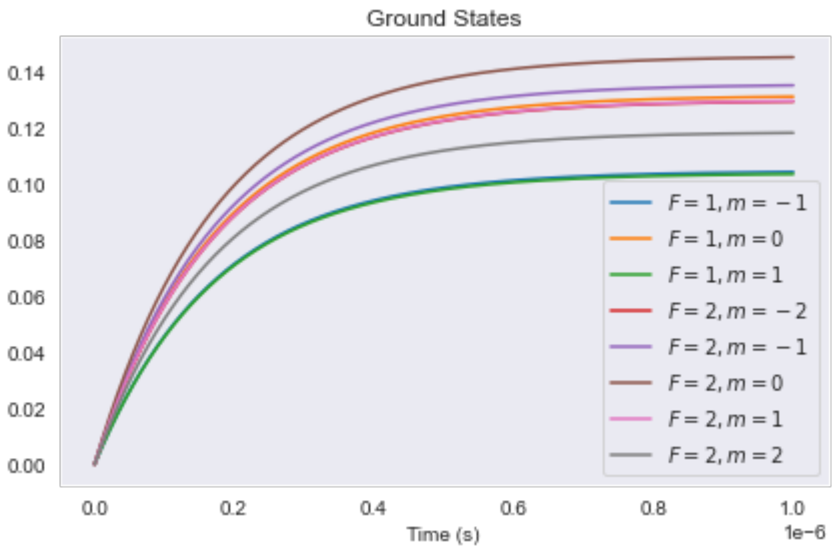


Random Excited State Starting Population

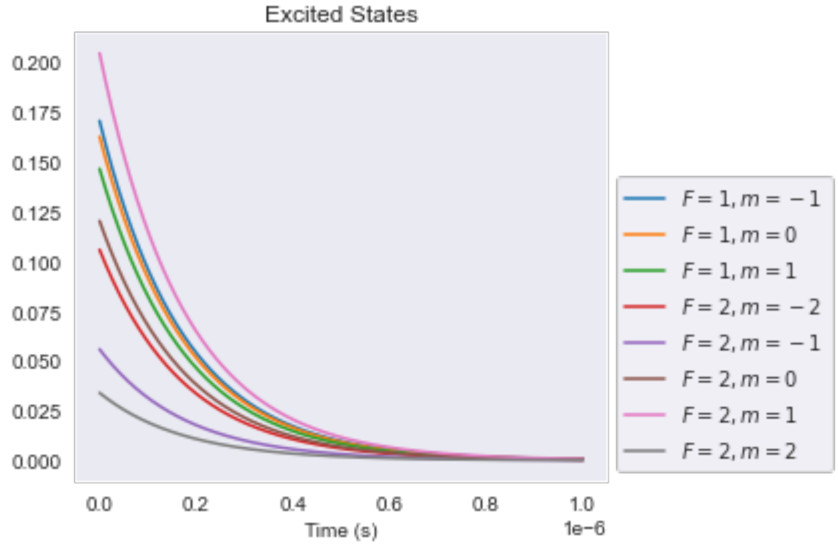
```
In [ ]: starting_state = sum([random.random() * basis(16, i).proj() for i in range(8, 16)])
# starting_state = basis(16, 16).proj()
starting_state = starting_state.unit()
```

```
In [ ]: times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

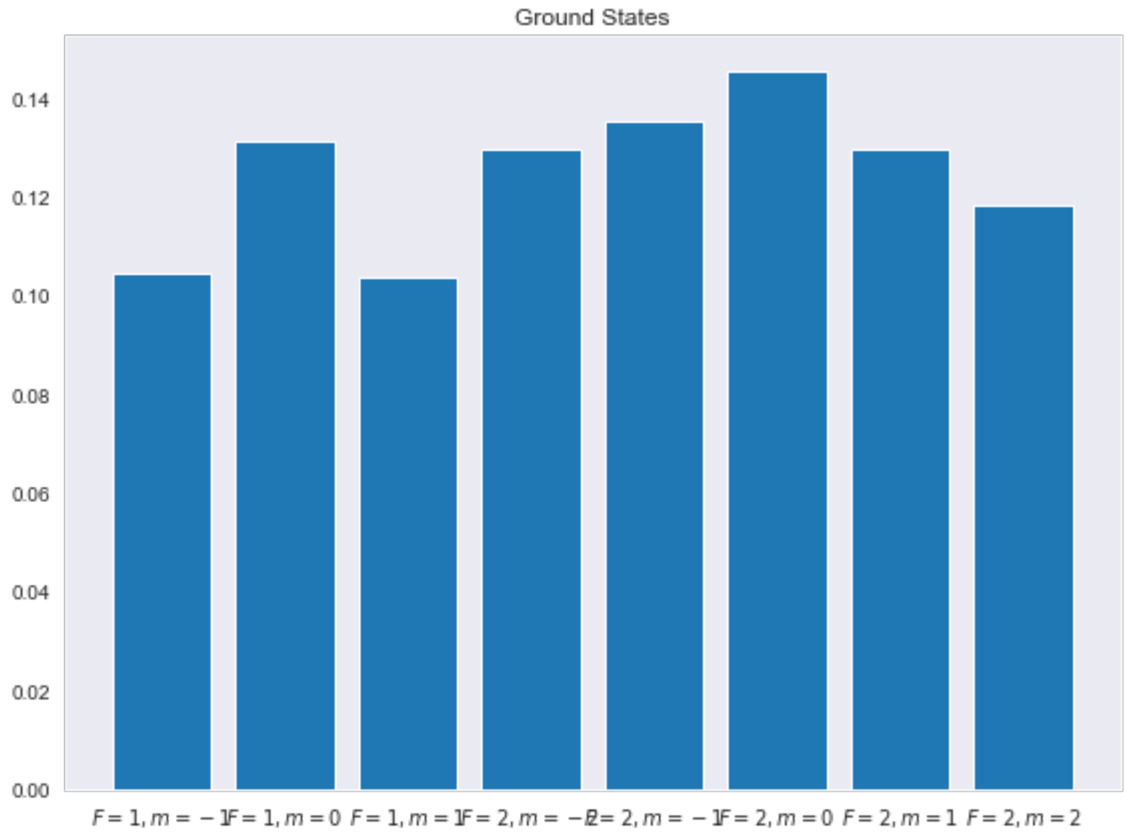
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
exc_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for e in exc_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Excited States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

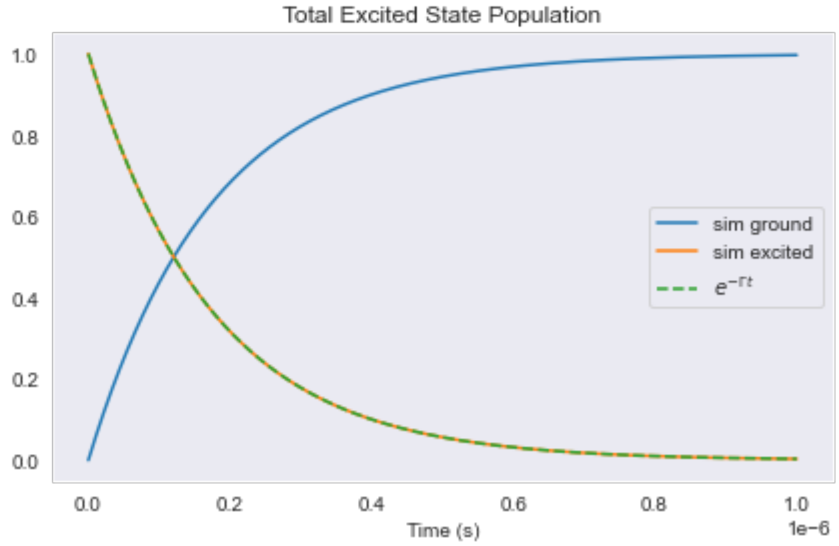


In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:

```
excited_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    for t in range(len(times))
]
total_ground_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
plt.figure()
plt.plot(times, np.real(total_ground_exp), label="sim ground")
plt.plot(times, np.real(excited_exp), label="sim excited")
plt.plot(
    times, [np.exp(-gamma_natural * t) for t in times], "--", label=r"$e^{-\Gamma t}$"
)
plt.legend()
plt.xlabel("Time (s)")
plt.title("Total Excited State Population")
plt.tight_layout()
```



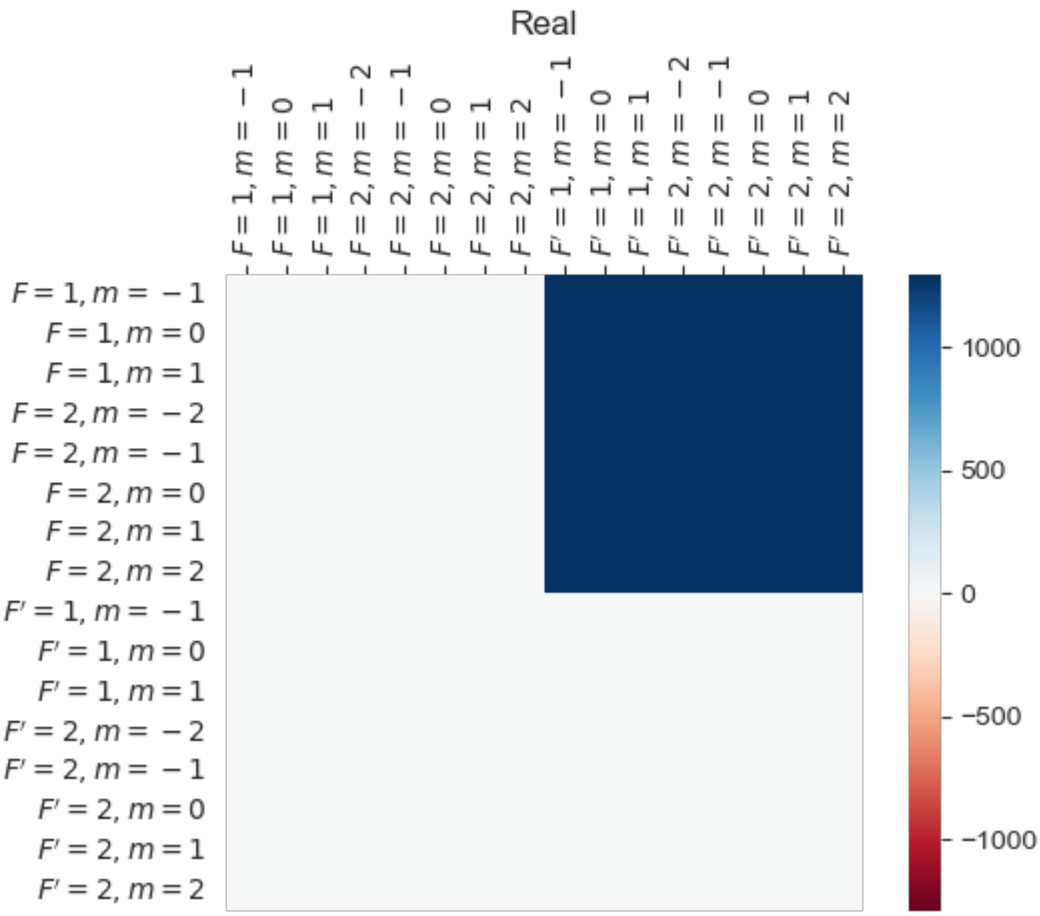
Quenching Only

In [ ]:

```
maplot(sum(quenching_ops))
```

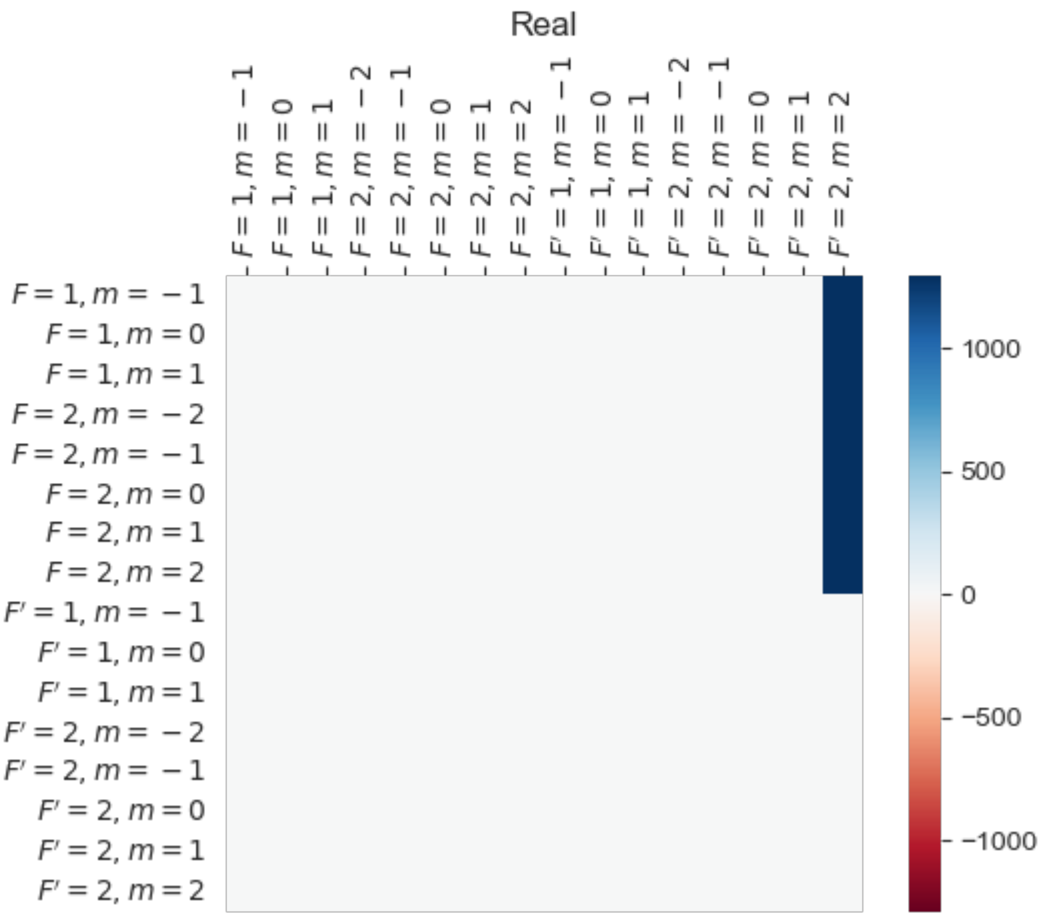
Out[ ]:

```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



In [ ]:
```
maplot(quenching_ops[-1])
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



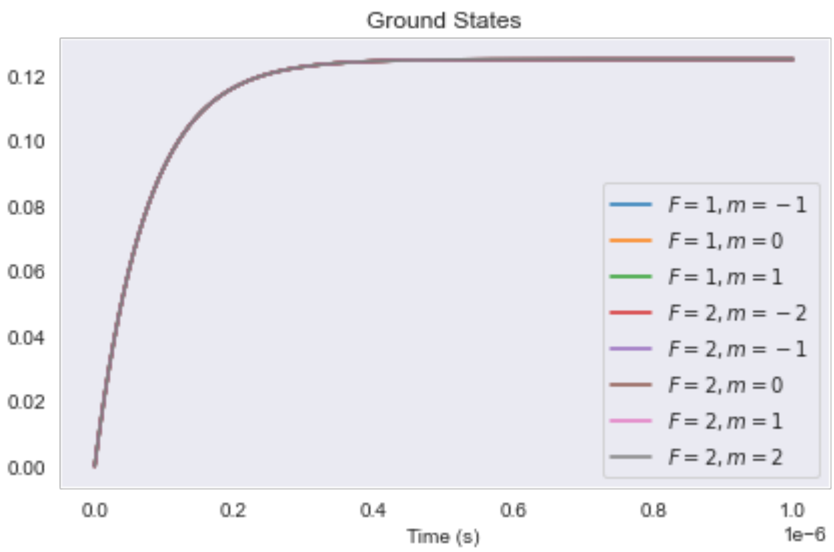
In [ ]:
```
L = liouvillian(None, c_ops=quenching_ops)
```

Time Evo

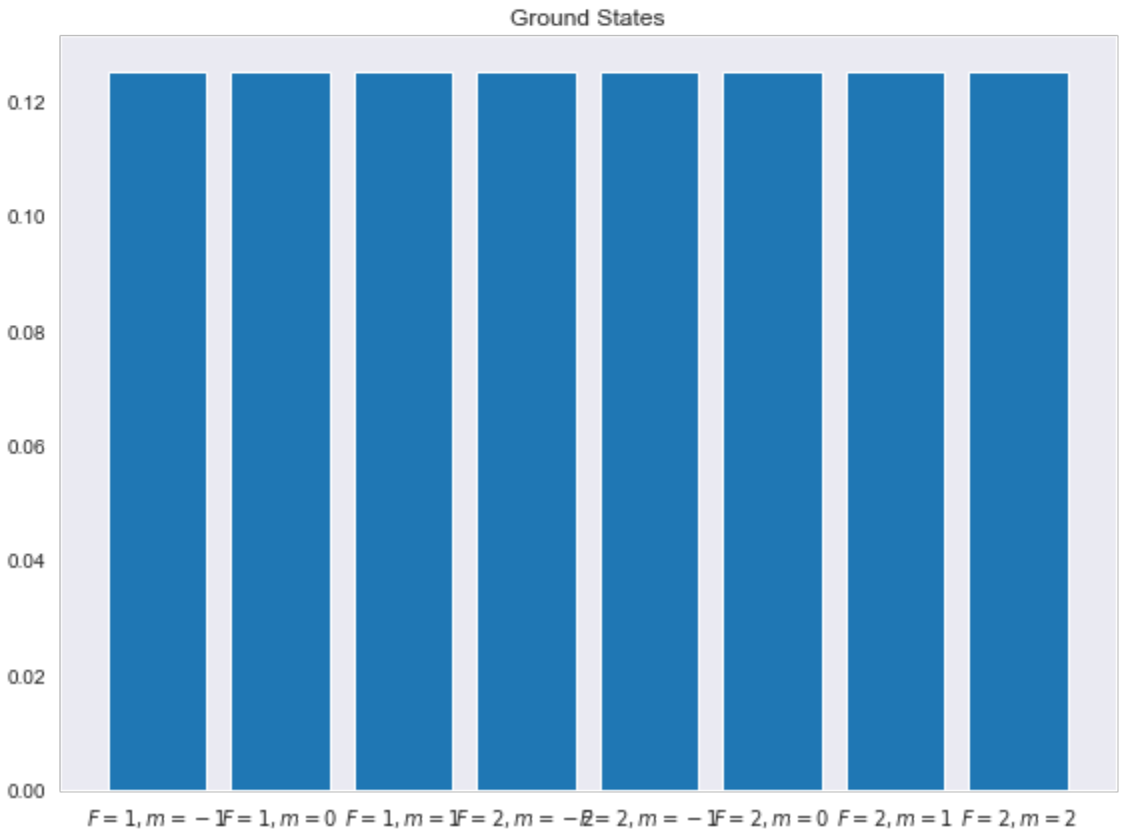
excited state equally populated

In [ ]:
```
starting_state = sum(
    [basis(16, i).proj() for i in range(8, 16)]
)  # excited state equally populated
starting_state = starting_state.unit()
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=2 * 10**4)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

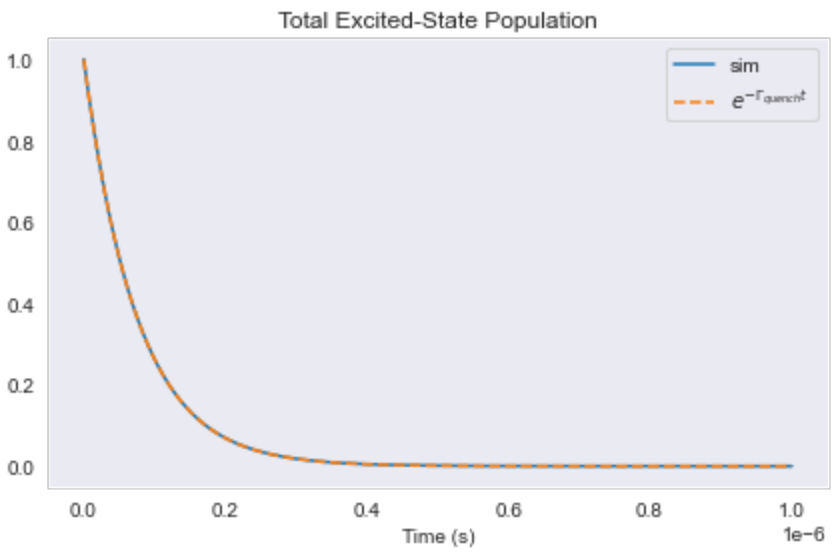


In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:

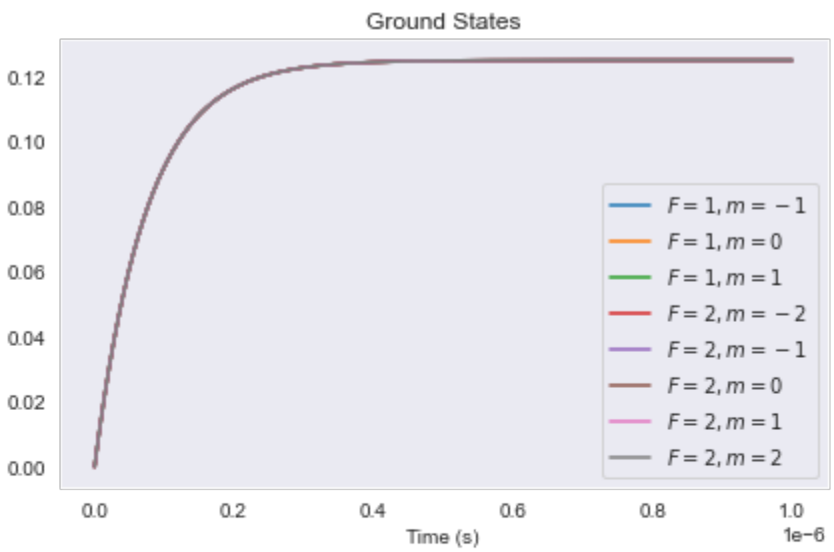
```
excited_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    for t in range(len(times))
]
plt.figure()
# for e in excited_exp:
plt.plot(times, np.real(excited_exp), label="sim")
plt.plot(
    times,
    [np.exp(-quenching_rate * t) for t in times],
    "--",
    label=r"$e^{-\Gamma_{quench} t}$",
)
plt.legend()
plt.title("Total Excited-State Population")
plt.xlabel("Time (s)")
plt.tight_layout()
```



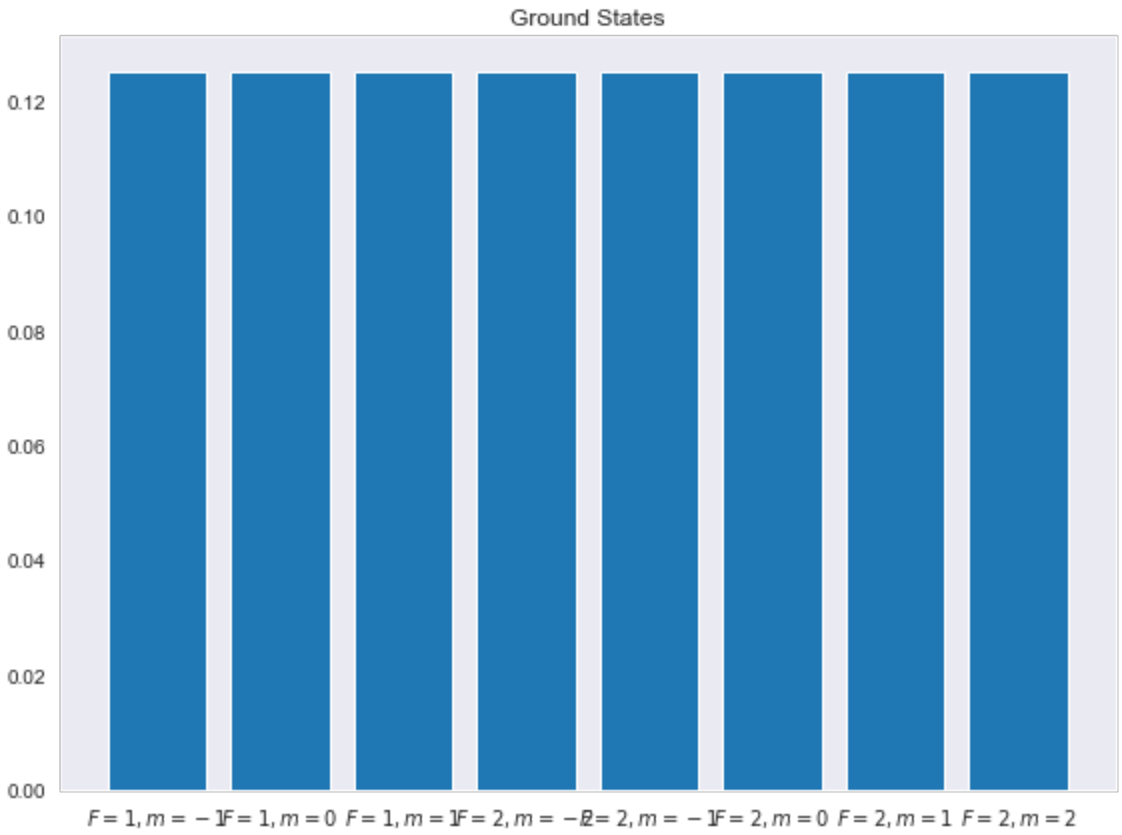
$$\rho_0 = |F' = 2, m'_F = 2\rangle$$

In [ ]:
```
starting_state = basis(16, 15).proj()
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=2 * 10**4)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

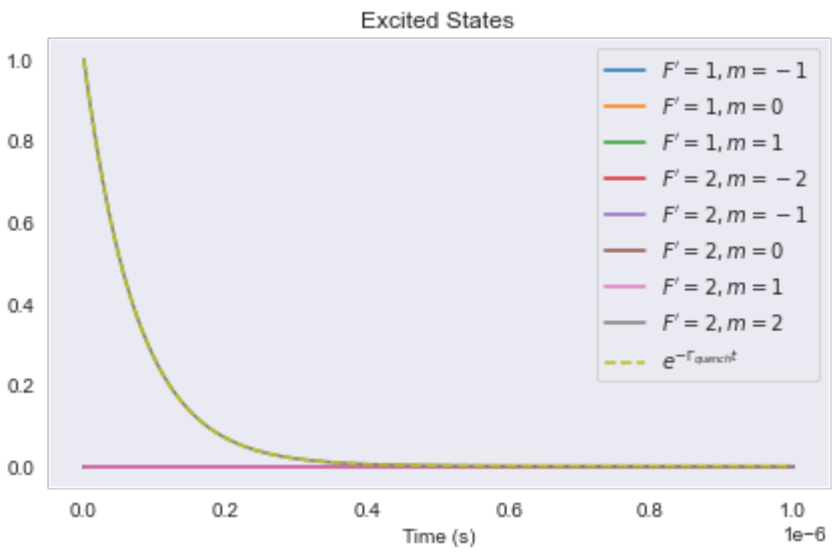
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:
```
excited_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for i in range(8, 16)
    ]
    for t in range(len(times))
]
plt.figure()
# for e in excited_exp:
plt.plot(
    times, np.real(excited_exp), label=[index_to_F_mF_string(i) for i in range(8, 16)]
)
plt.plot(
    times,
    [np.exp(-quenching_rate * t) for t in times],
    "--",
    label=r"$e^{-\Gamma_{quench} t}$",
)
plt.legend()
plt.title("Excited States")
plt.xlabel("Time (s)")
plt.tight_layout()
```



Driven By D1 Laser: σ_-, $|F=2\rangle \to |F'=1\rangle$, Damped by Radiative Decay

In [ ]:
```
L = liouvillian(hamil, c_ops=natural_decay_ops)
```

In [ ]:
```
import plotly.express as px

y = L.full().real
fig = px.imshow(
    y,
    color_continuous_midpoint=0,
    aspect="equal",
    width=1.5 * 800,
    height=1.5 * 400,
    zmin=-(abs(y).max()),
    zmax=(abs(y).max()),
    color_continuous_scale="RdBu",
)
fig.show()
```

In [ ]:
```
L.istp
```

Out[ ]:
```
False
```

In [ ]:
```
L.iscp
```

Out[ ]:
```
False
```

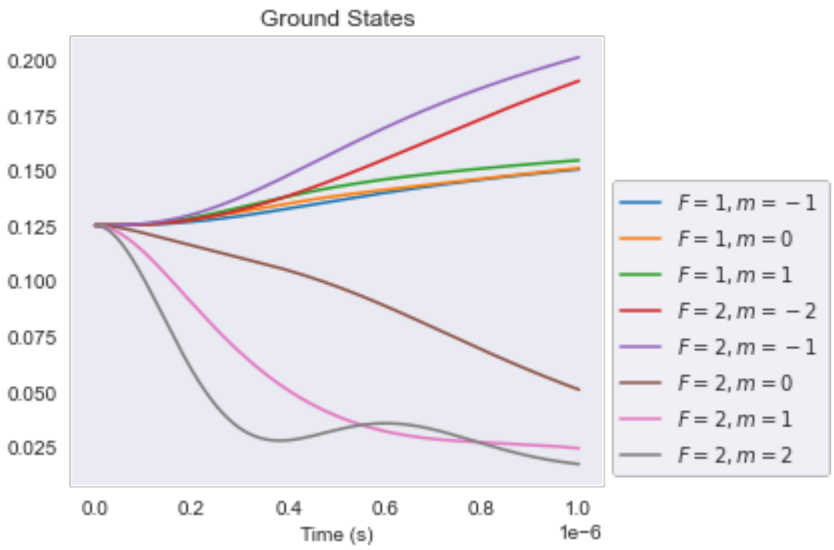
Time Evolution

$T = 10\,\mu s$

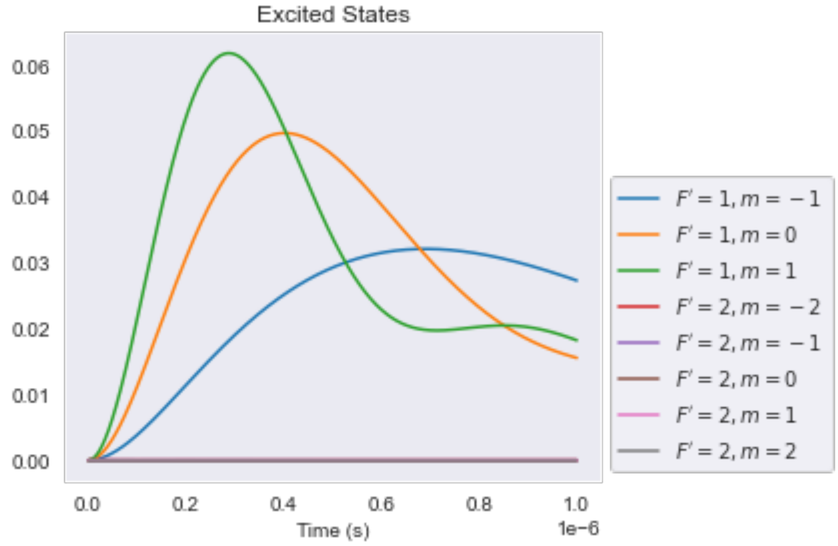
In [ ]:
```
starting_state = sum(
    [basis(16, i).proj() for i in range(8)]
)  # ground states equally populated
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=2 * 10**4)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

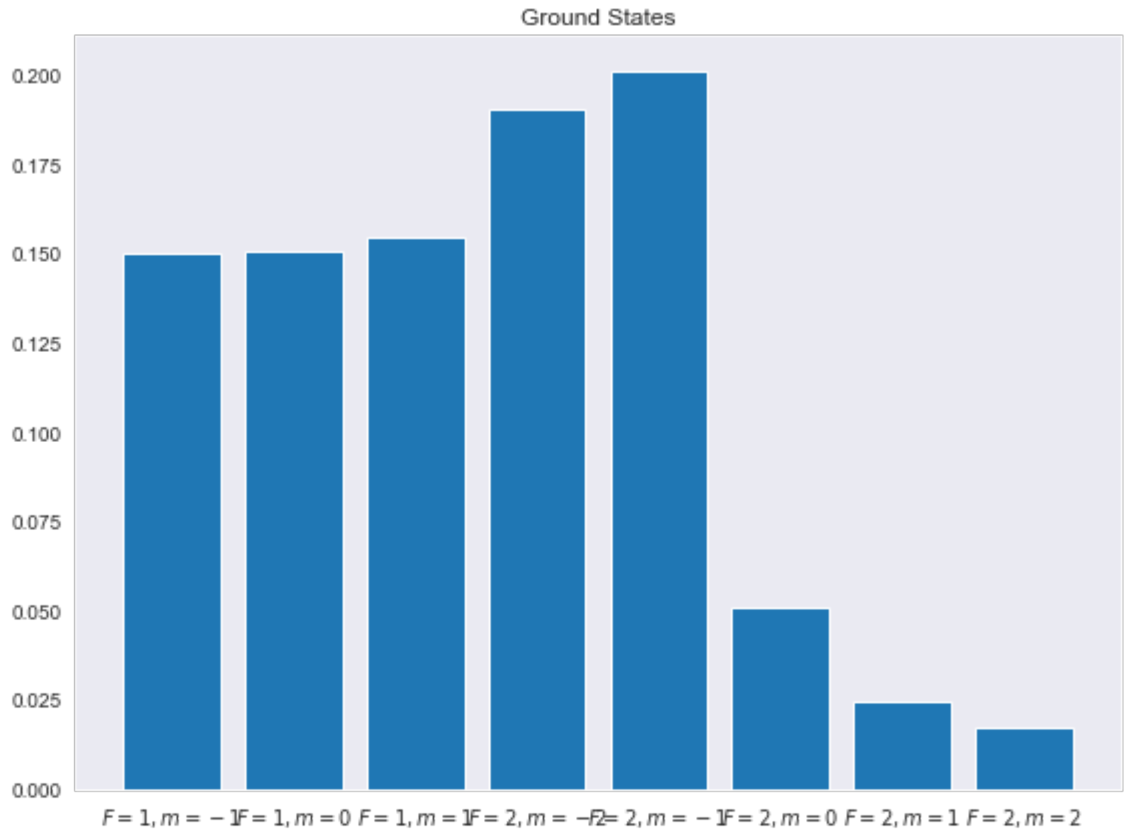
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



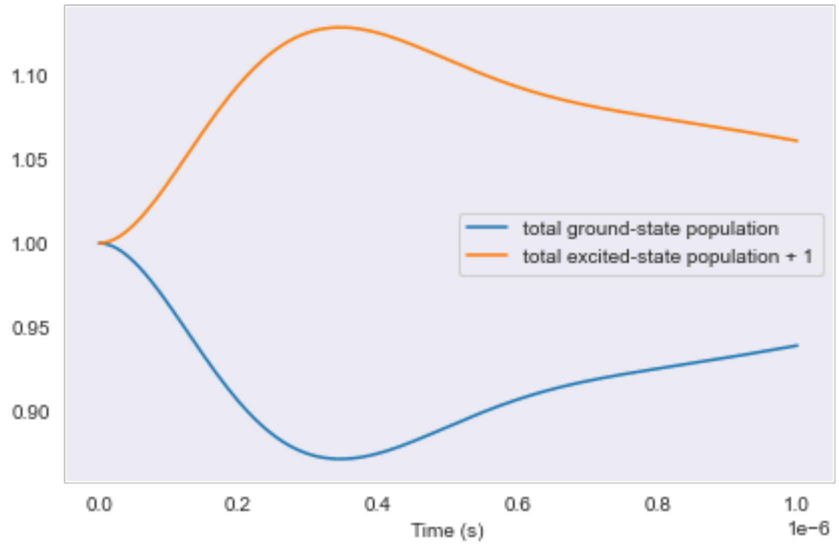
In [ ]:
```
exc_states = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for e in exc_states:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("Excited States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:
```
tot_excited_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
tot_g_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
plt.figure()
plt.plot(times, np.real(tot_g_exp), label="total ground-state population")
plt.plot(times, np.real(tot_excited_exp), label="total excited-state population + 1")
plt.legend()
plt.xlabel("Time (s)")
plt.tight_layout()
```



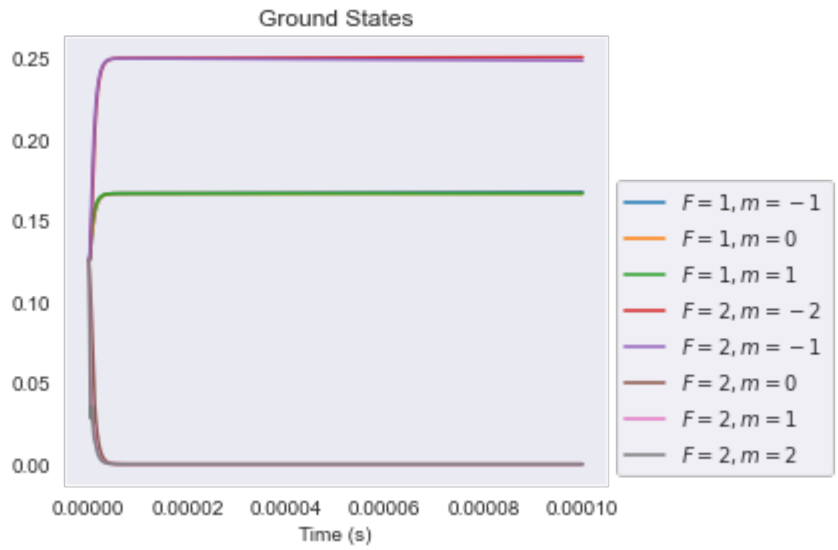
$T = 10\,ms$

In [ ]:
```
starting_state = sum(
    [basis(16, i).proj() for i in range(8)]
)  # ground states equally populated
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-4, 1000)
opts = Options(nsteps=2 * 10**4)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

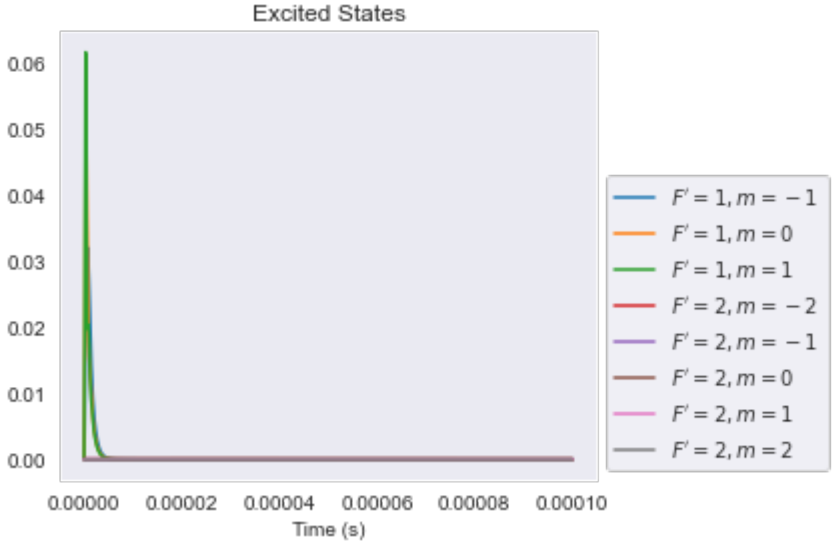
In [ ]:

```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

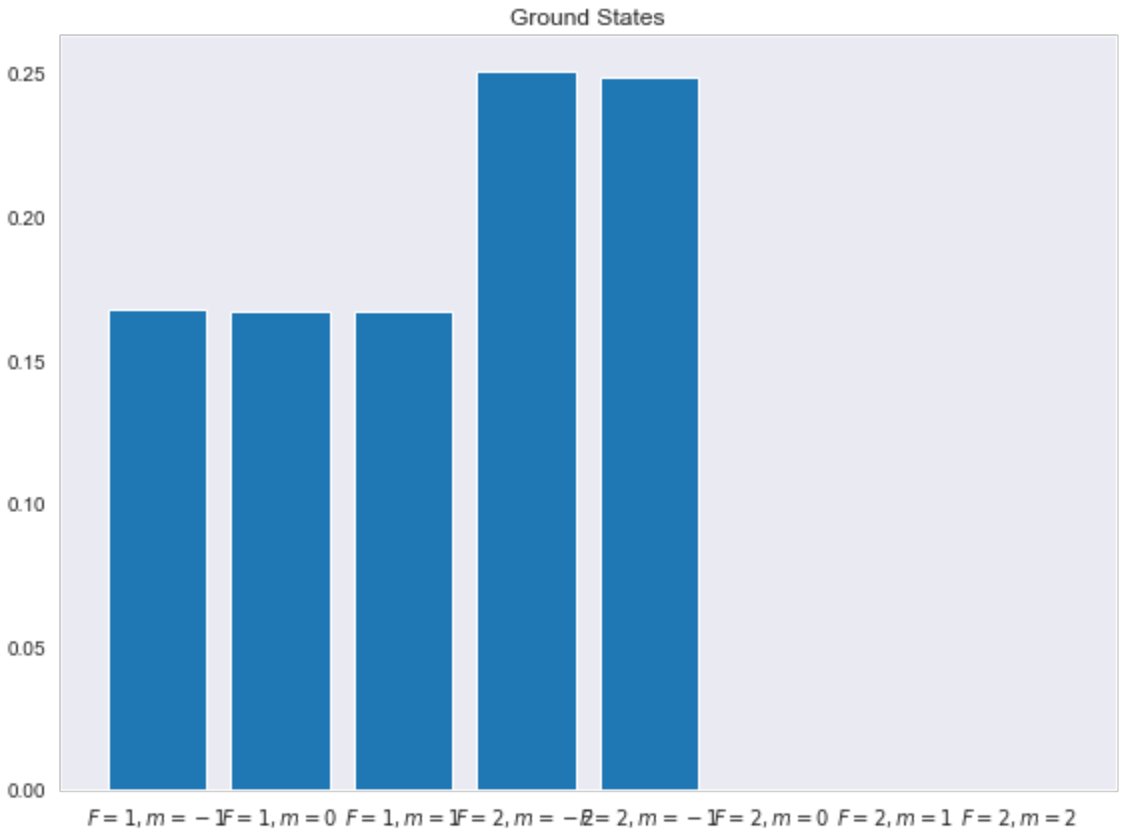


In [ ]:

```
exc_states = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for e in exc_states:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("Excited States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

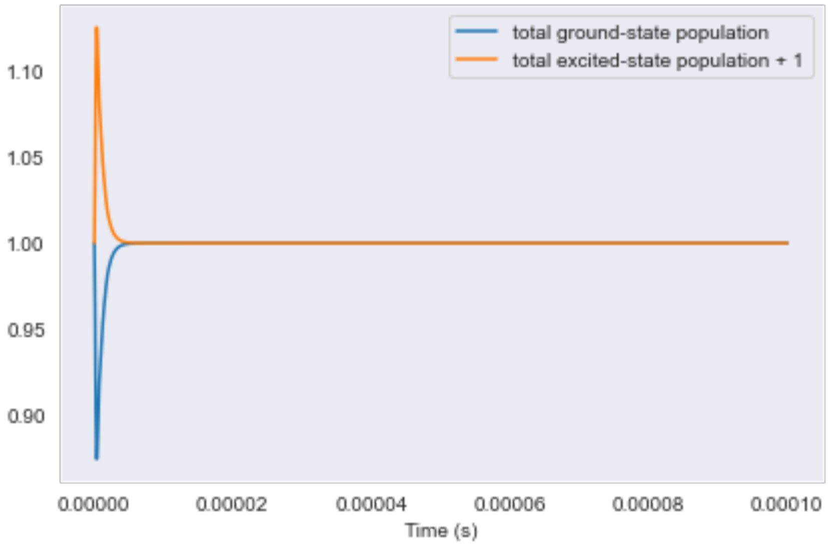


In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:

```
tot_excited_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
tot_g_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
plt.figure()
plt.plot(times, np.real(tot_g_exp), label="total ground-state population")
plt.plot(times, np.real(tot_excited_exp), label="total excited-state population + 1")
plt.legend()
plt.xlabel("Time (s)")
plt.tight_layout()
```



Steady State

Lindblad Superoperator

Matrix representation of operators are transformed to vectors by column-stacking.

In [ ]:

```
operator_to_vector(basis(2, 0) * basis(2, 1).dag())
```

Out[ ]: Quantum object: dims = [[[2], [2]], [1]], shape = (4, 1), type = operator-ket

$$\begin{pmatrix} 0.0 \\ 0.0 \\ 1.0 \\ 0.0 \end{pmatrix}$$

In [ ]:

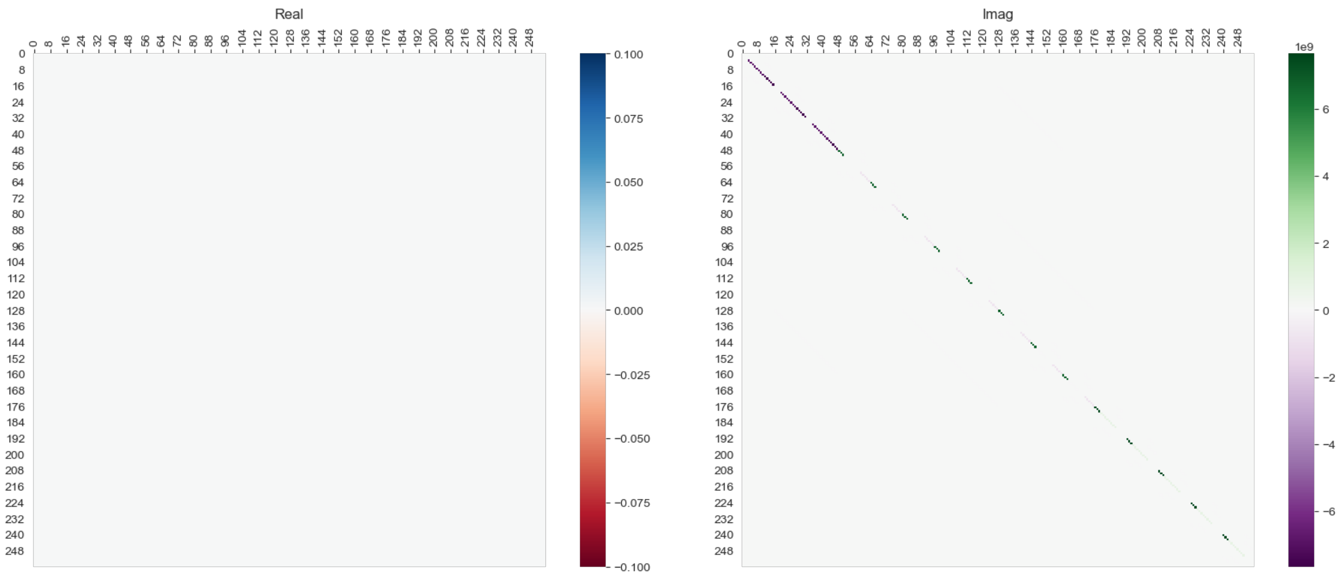
```
operator_to_vector(hamil)
```

Out[ ]: Quantum object: dims = [[[16], [16]], [1]], shape = (256, 1), type = operator-ket

$$\begin{pmatrix} -4.272\times10^{+09} \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ \vdots \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 3.378\times10^{+09} \end{pmatrix}$$

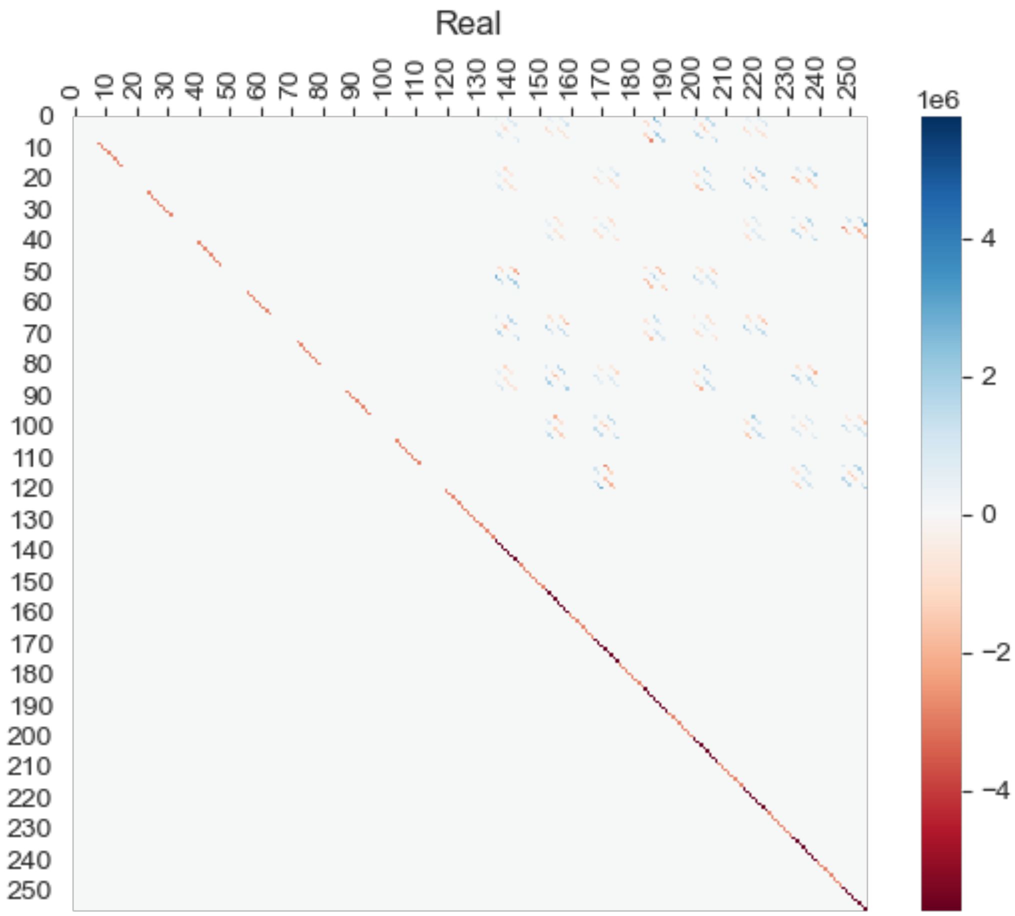
In [ ]:
```
matrixplot(liouvillian(hamil))
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```
matrixplot(liouvillian(hamil * 0, c_ops=natural_decay_ops))
```

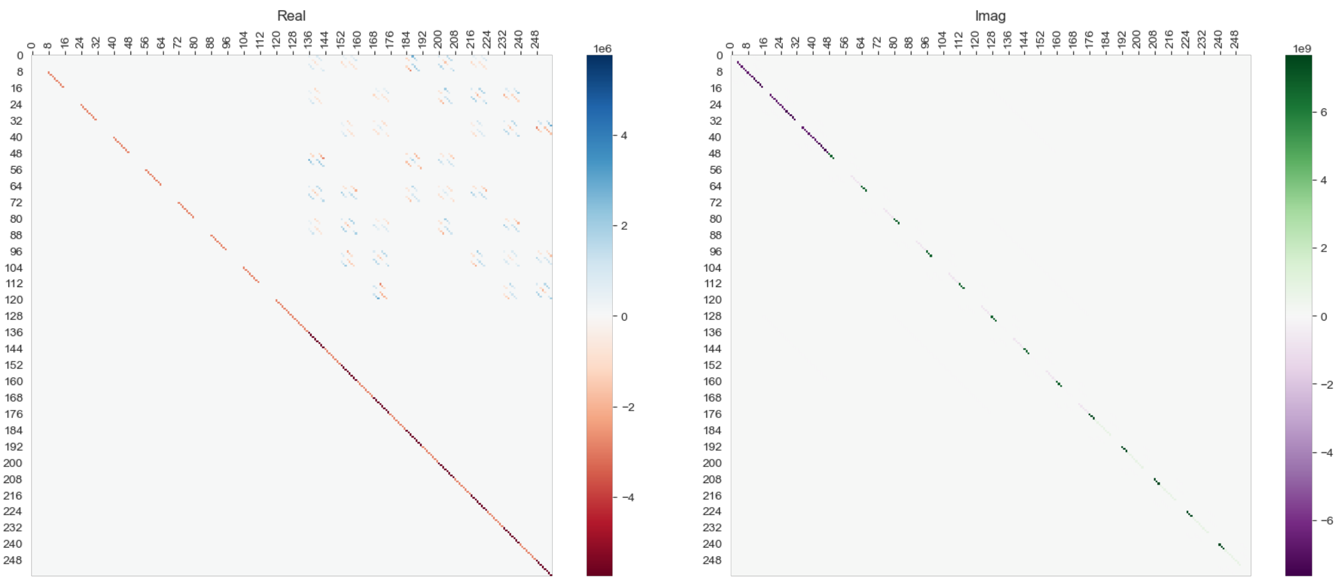
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```




```
In [ ]: L = liouvillian(hamil, c_ops=natural_decay_ops)
```

```
In [ ]: matrixplot(L)
```

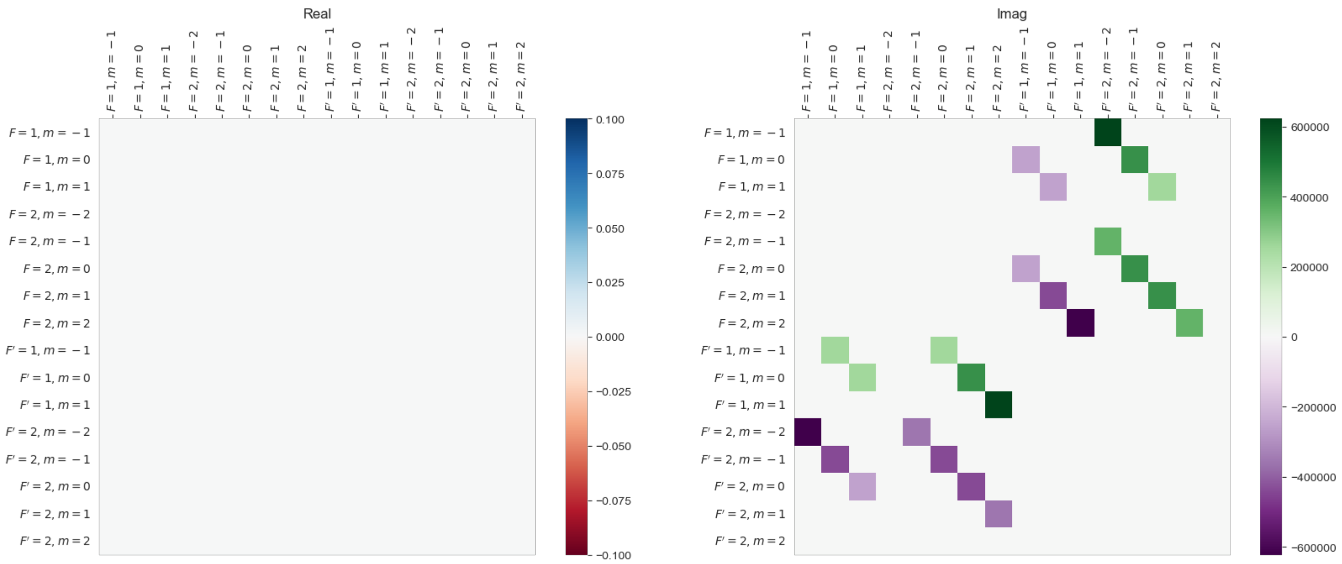
```
Out[ ]: (<Figure size 1680x672 with 4 Axes>,
         [<AxesSubplot:title={'center':'Real'}>,
          <AxesSubplot:title={'center':'Imag'}>])
```




```
In [ ]: rho_dot_zero = vector_to_operator(L * operator_to_vector(starting_state))
```

```
In [ ]: maplot(rho_dot_zero)
```

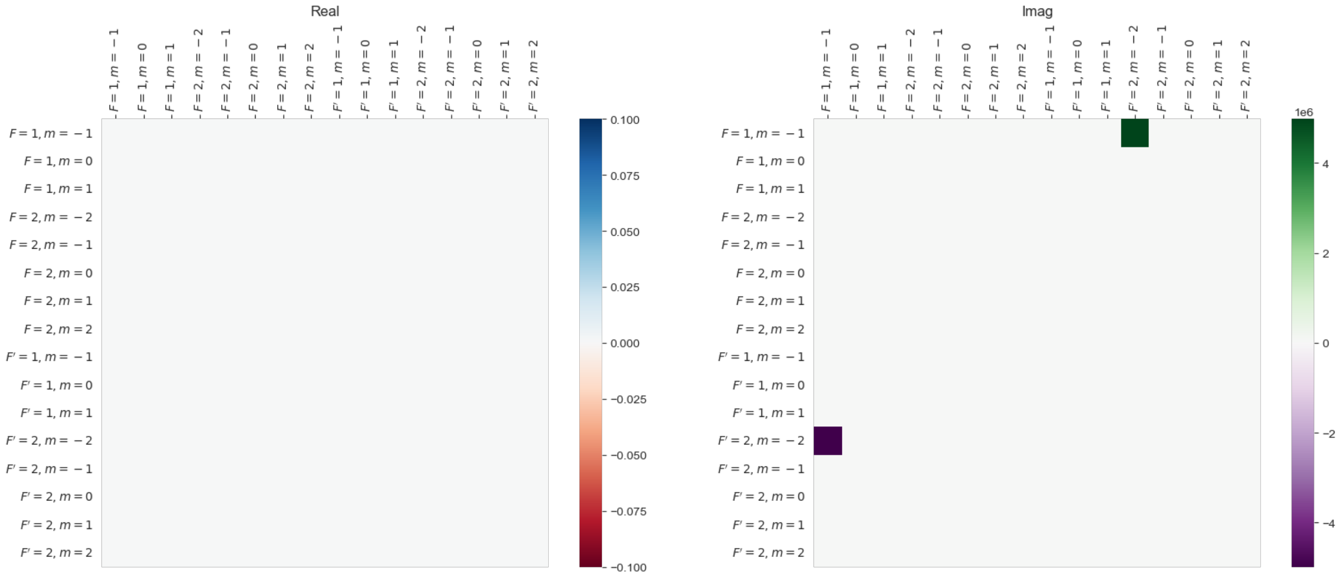
```
Out[ ]: (<Figure size 1680x672 with 4 Axes>,
         [<AxesSubplot:title={'center':'Real'}>,
          <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
rho_dot_zero = vector_to_operator(L * operator_to_vector(basis(16, 0).proj()))
```

In [ ]:
```python
maplot(rho_dot_zero)
```

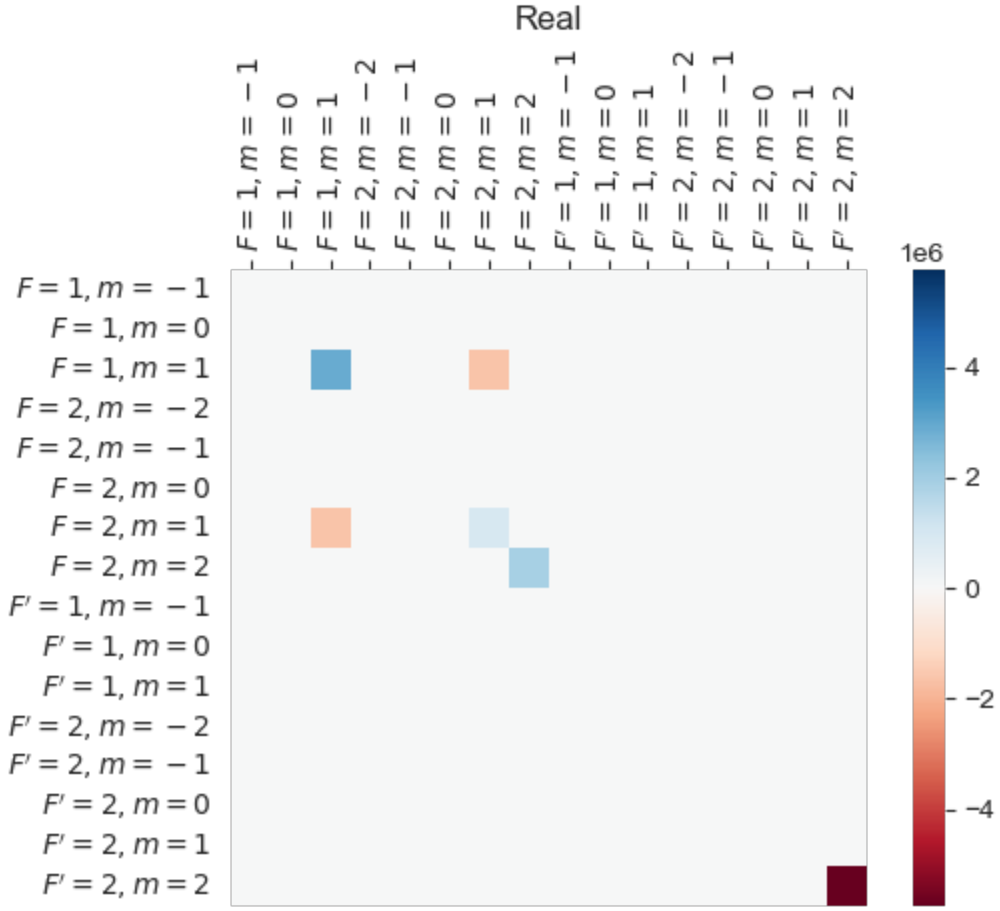
Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
rho_dot_zero = vector_to_operator(L * operator_to_vector(basis(16, 15).proj()))
```

In [ ]:
```python
maplot(rho_dot_zero)
```

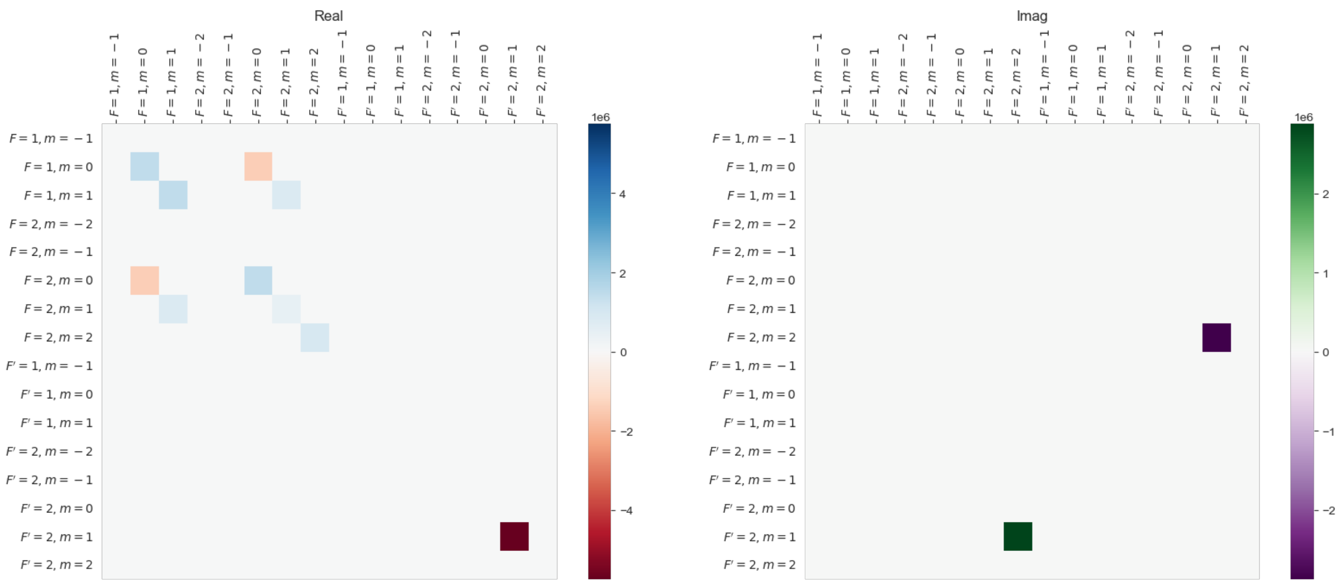
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



In [ ]:
```python
rho_dot_zero = vector_to_operator(L * operator_to_vector(basis(16, 14).proj()))
```

In [ ]:
```python
maplot(rho_dot_zero)
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



A Steady State for σ_- pump is the dark state $|F = 2, m_F = -2\rangle$

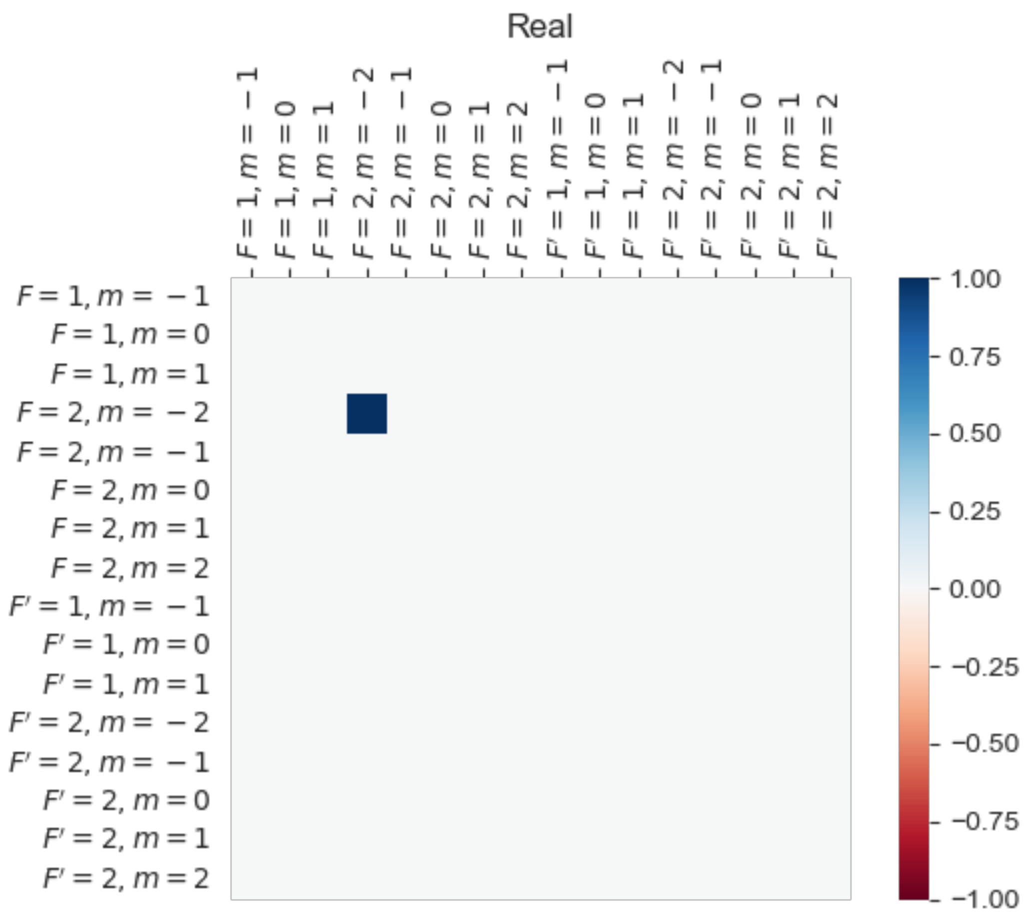
In [ ]:
```python
lenergy, lstates = L.eigenstates()
np.abs(lenergy).min()
```

Out[ ]:
```
0.0
```

In [ ]:
```python
rho_ss_zero_eigenval = vector_to_operator(lstates[np.abs(lenergy).argmin()])
```

In [ ]:
```python
maplot(rho_ss_zero_eigenval)
```

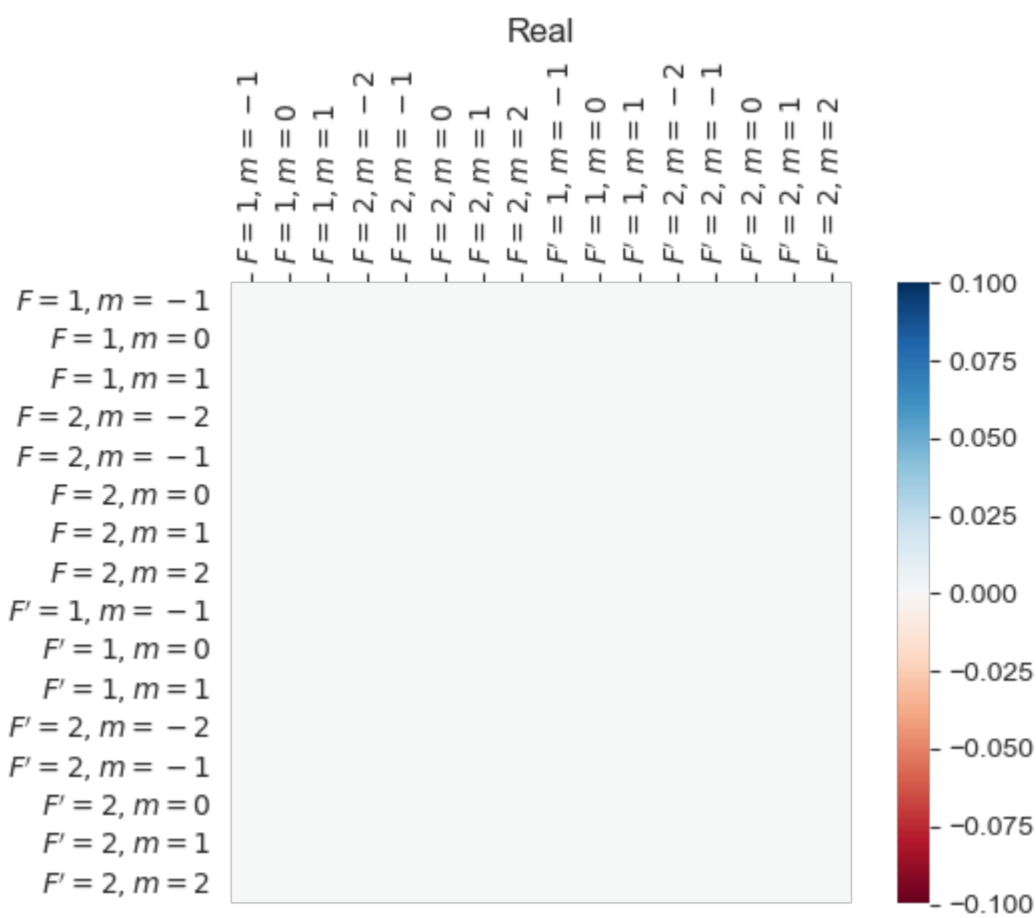
```
Out[ ]: (<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```




```
In [ ]: rho_ss_dot = L * operator_to_vector(rho_ss_zero_eigenval)
```

```
In [ ]: maplot(vector_to_operator(rho_ss_dot))
```

```
Out[ ]: (<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```

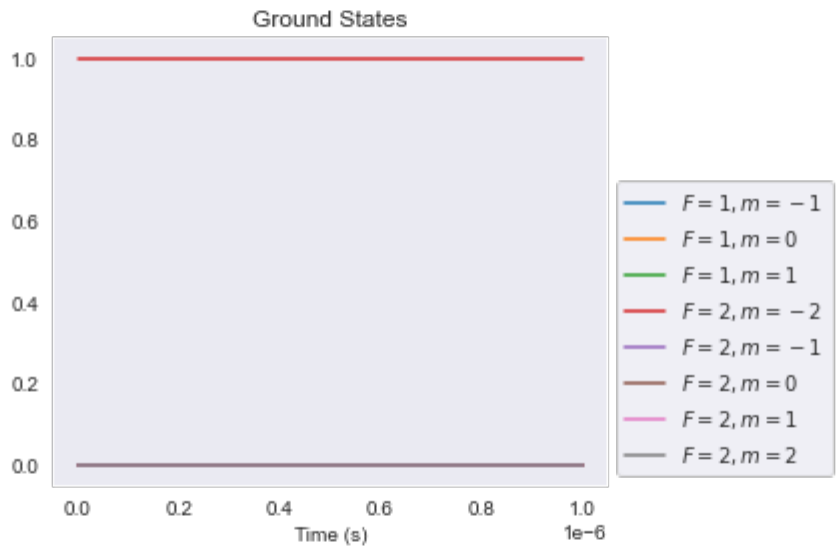


Test this ρ_{ss} by time evolution

In [ ]:
```
starting_state = basis(16, 3).proj()  # dark
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=2 * 10**4)
res = mesolve(
    hamil,
    starting_state,
    times,
    c_ops=natural_decay_ops,
    options=opts,
)
```

In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

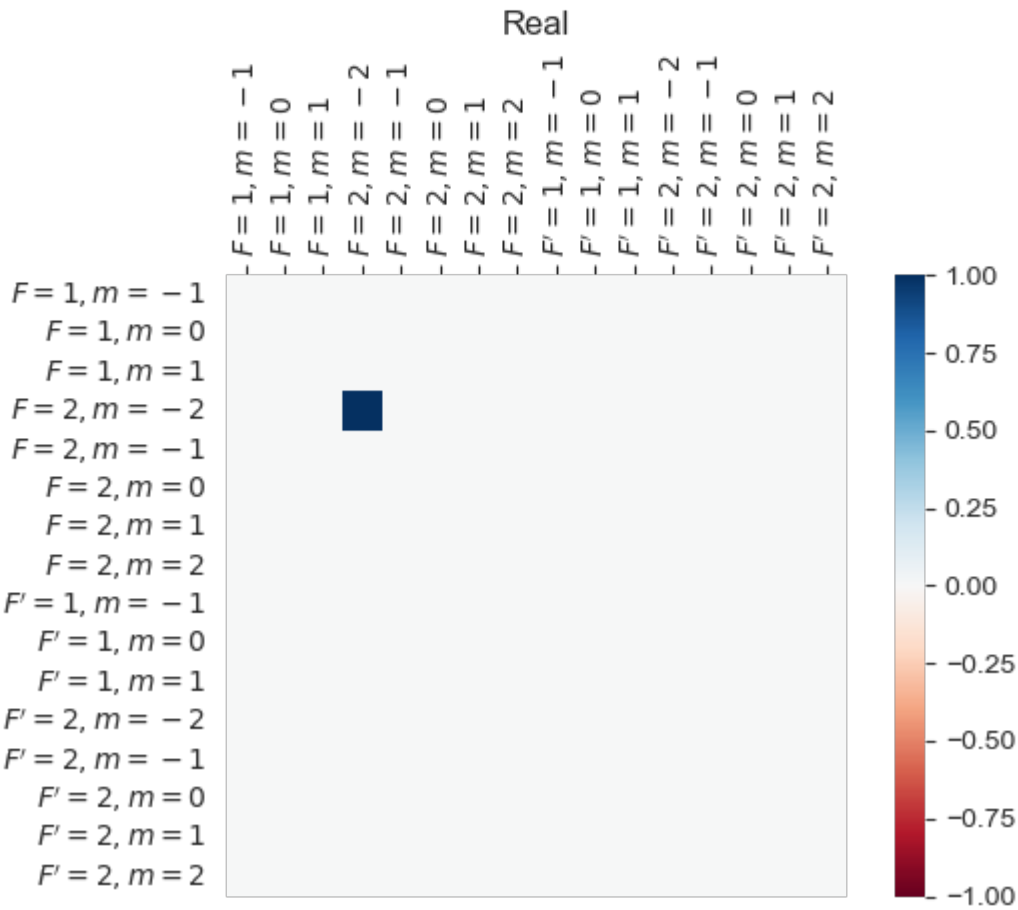


Different Steady-State Solvers

In [ ]:
```
rho_ss_direct = steadystate(hamil, c_op_list=natural_decay_ops, method="direct")
```

In [ ]:
```
maplot(rho_ss_direct)
```

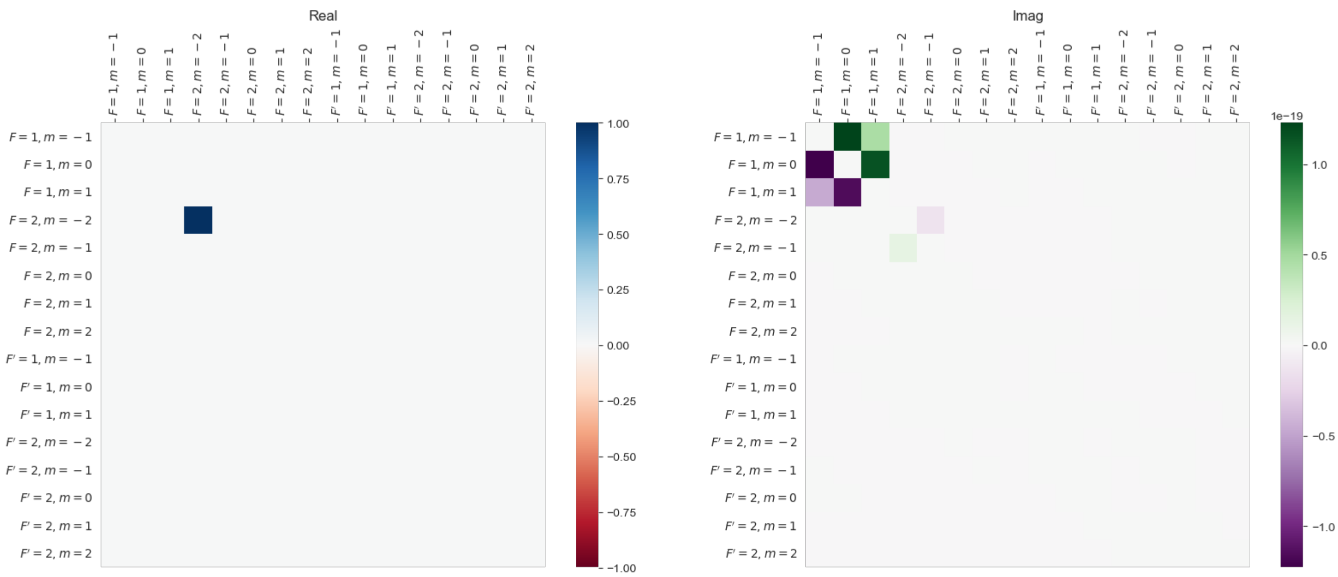
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



In [ ]:
```
rho_ss_power = steadystate(hamil, c_op_list=natural_decay_ops, method="power")
```

In [ ]:
```
maplot(rho_ss_power)
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



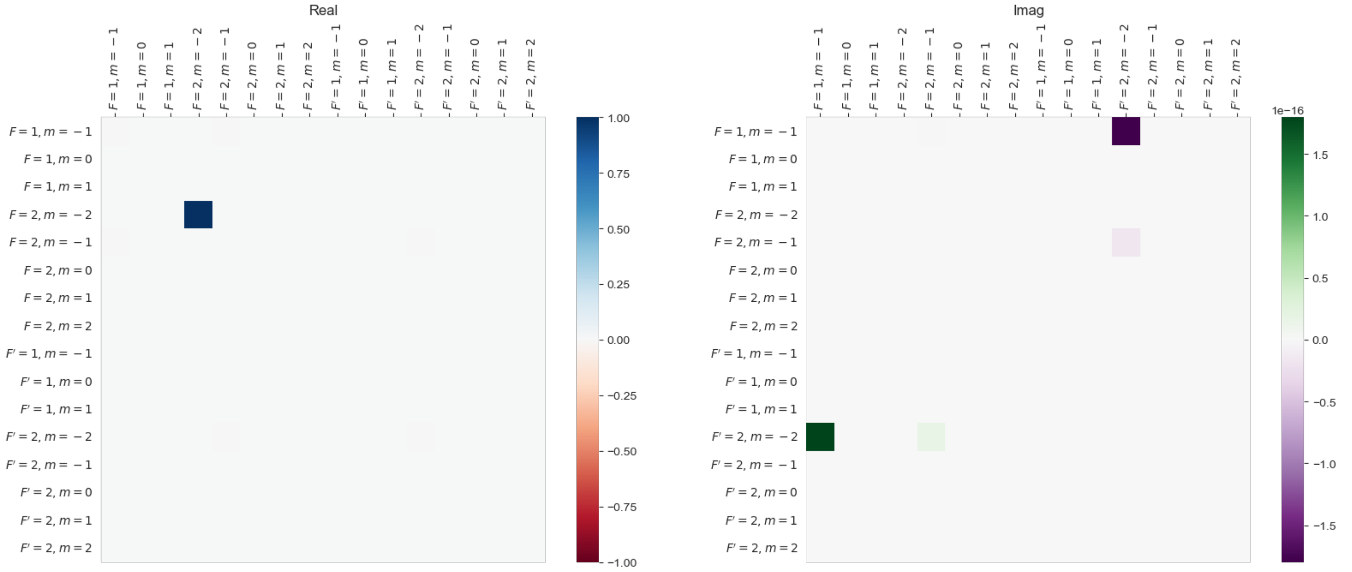
In [ ]:
```
rho_ss_power.tr()
```

Out[ ]:
```
1.0
```

In [ ]:
```
rho_ss_it_gmres = steadystate(
    hamil, c_op_list=natural_decay_ops, method="iterative-gmres"
)
```

In [ ]:
```
maplot(rho_ss_it_gmres)
```

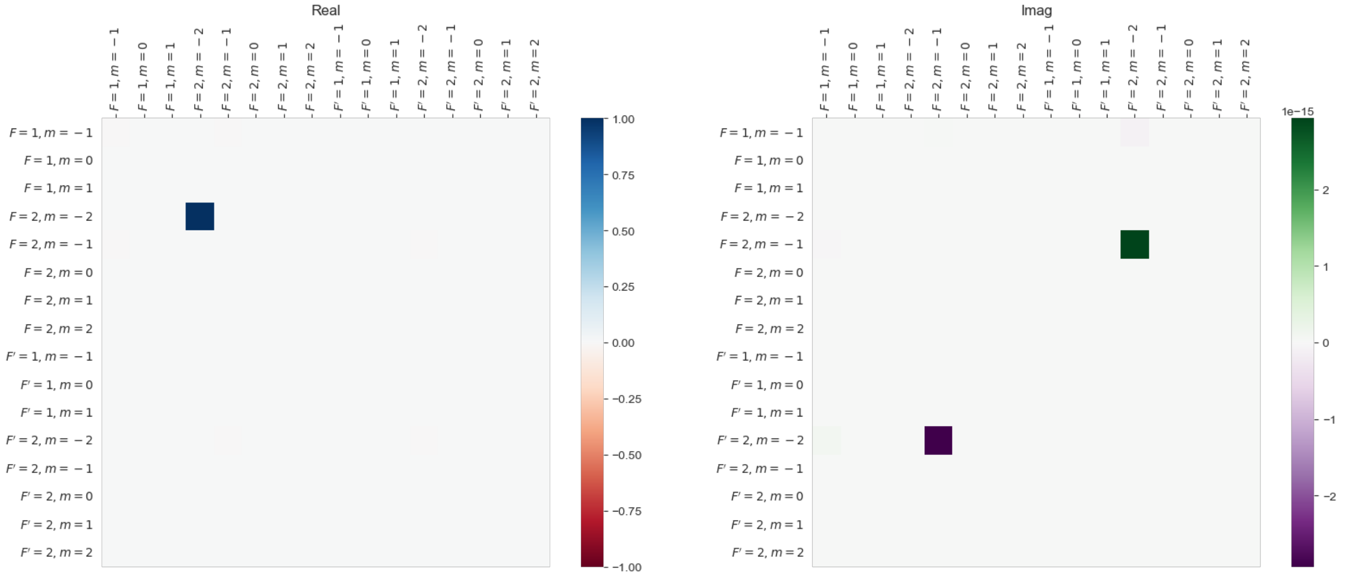
Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
rho_ss_it_lgmres = steadystate(
    hamil, c_op_list=natural_decay_ops, method="iterative-lgmres"
)
```

In [ ]:
```python
maplot(rho_ss_it_lgmres)
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
rho_ss_it_bicgstab = steadystate(
    hamil, c_op_list=natural_decay_ops, method="iterative-bicgstab"
)
```

```
---------------------------------------------------------------------------
Exception                                 Traceback (most recent call last)
c:\Users\m\OneDrive - Universität Basel\Masterarbeit\Rb Populations Simulation\Bloch master D
1.ipynb Cell 112' in <cell line: 1>()
----> <a href='vscode-notebook-cell:/c%3A/Users/m/OneDrive%20-%20Universit%C3%A4t%20Basel/Mas
terarbeit/Rb%20Populations%20Simulation/Bloch%20master%20D1.ipynb#ch0000111?line=0'>1</a> rho
_ss_it_bicgstab = steadystate(
      <a href='vscode-notebook-cell:/c%3A/Users/m/OneDrive%20-%20Universit%C3%A4t%20Basel/Mas
terarbeit/Rb%20Populations%20Simulation/Bloch%20master%20D1.ipynb#ch0000111?line=1'>2</a>
hamil, c_op_list=natural_decay_ops, method="iterative-bicgstab"
      <a href='vscode-notebook-cell:/c%3A/Users/m/OneDrive%20-%20Universit%C3%A4t%20Basel/Mas
terarbeit/Rb%20Populations%20Simulation/Bloch%20master%20D1.ipynb#ch0000111?line=2'>3</a> )

File c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\qutip\steadystate.py:2
89, in steadystate(A, c_op_list, method, solver, **kwargs)
    285     return _steadystate_eigen(A, ss_args)
    287 elif ss_args['method'] in ['iterative-gmres',
    288                            'iterative-lgmres', 'iterative-bicgstab']:
--> 289     return _steadystate_iterative(A, ss_args)
    291 elif ss_args['method'] == 'svd':
    292     return _steadystate_svd_dense(A, ss_args)

File c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\qutip\steadystate.py:6
69, in _steadystate_iterative(L, ss_args)
    666     logger.debug('Iteration. time: %f' % (_iter_end - _iter_start))
    668 if check > 0:
--> 669     raise Exception("Steadystate error: Did not reach tolerance after " +
    670                     str(ss_args['maxiter']) + " steps." +
    671                     "\nResidual norm: " +
    672                     str(ss_args['info']['residual_norm']))
    674 elif check < 0:
    675     raise Exception(
    676         "Steadystate error: Failed with fatal error: " + str(check) + ".")

Exception: Steadystate error: Did not reach tolerance after 1000 steps.
Residual norm: None
```

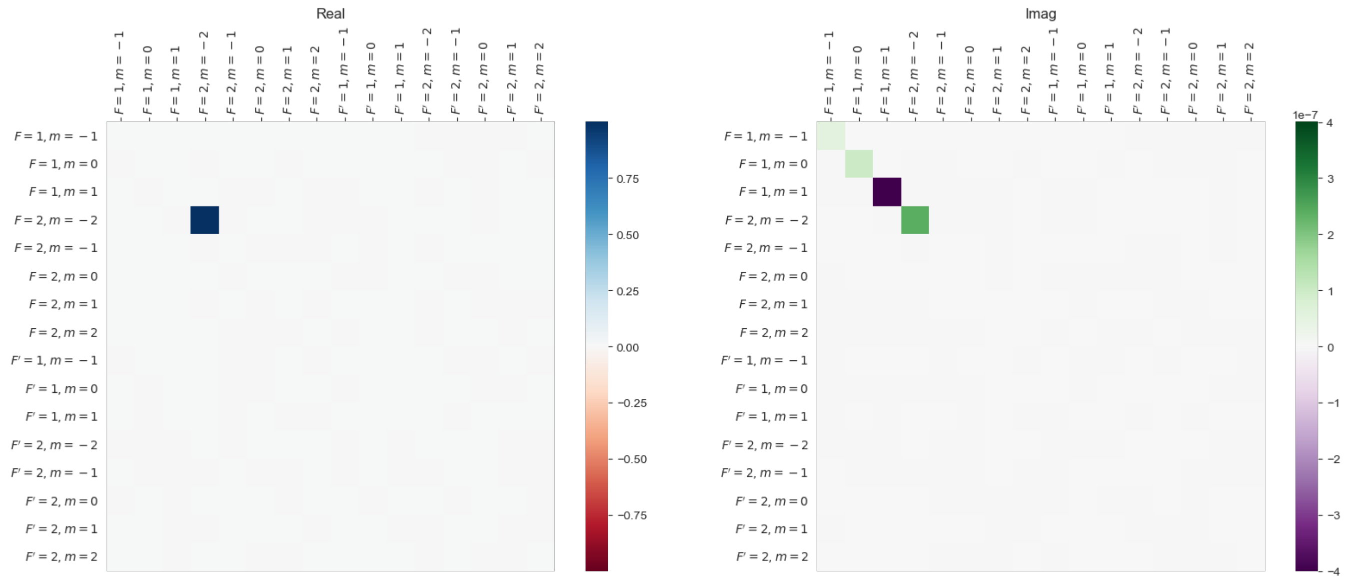
In [ ]:
```
maplot(rho_ss_it_bicgstab)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
c:\Users\m\OneDrive - Universität Basel\Masterarbeit\Rb Populations Simulation\Bloch master D
1.ipynb Cell 113' in <cell line: 1>()
----> <a href='vscode-notebook-cell:/c%3A/Users/m/OneDrive%20-%20Universit%C3%A4t%20Basel/Mas
terarbeit/Rb%20Populations%20Simulation/Bloch%20master%20D1.ipynb#ch0000112?line=0'>1</a> map
lot(rho_ss_it_bicgstab)

NameError: name 'rho_ss_it_bicgstab' is not defined
```

In [ ]:
```
rho_ss_svd = steadystate(hamil, c_op_list=natural_decay_ops, method="svd")
```

In [ ]:
```
maplot(rho_ss_svd)
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



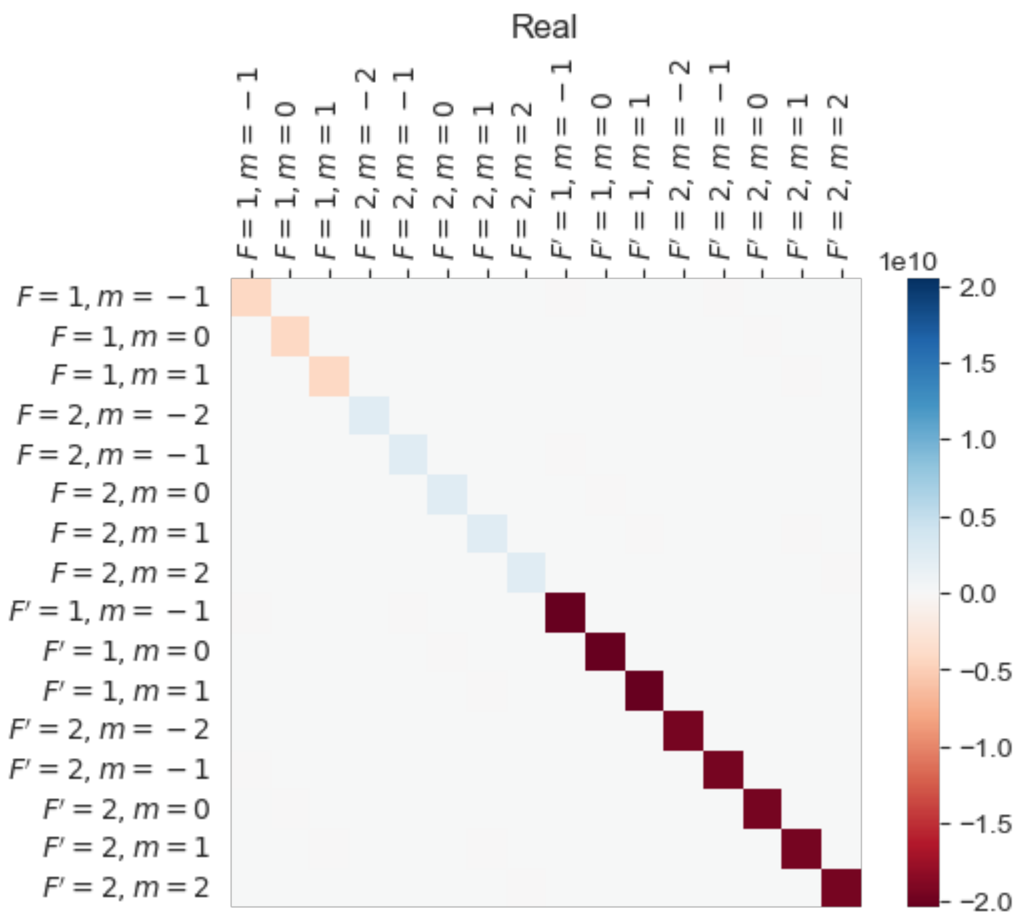
In [ ]:
```
rho_ss_svd.tr()
```

Out[ ]:
```
(1-3.539082494550373e-17j)
```

Far-Detuned, π pol

In [ ]:
```
hamil_far_detuned = H_AF(0, laser_intens) + Ha(
    20e9
)  # sigma MINUS (since q=1), F=2 -> F'=1
maplot(hamil_far_detuned)
```

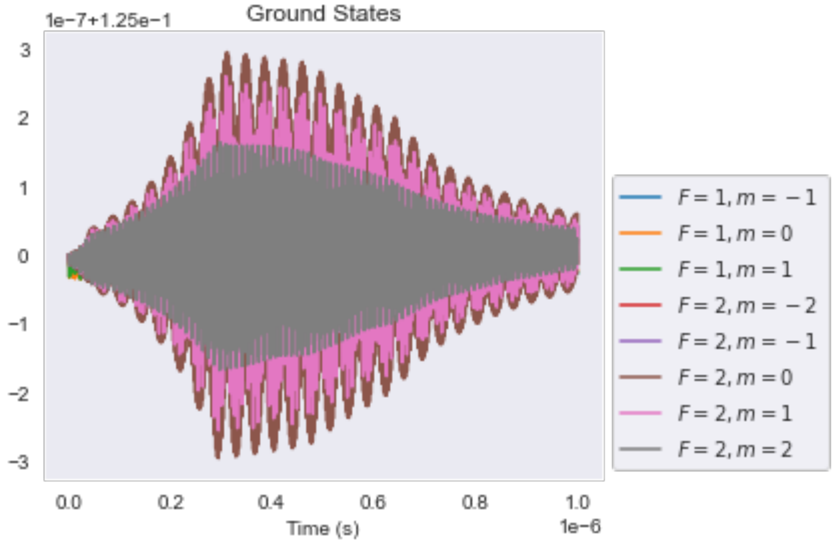
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



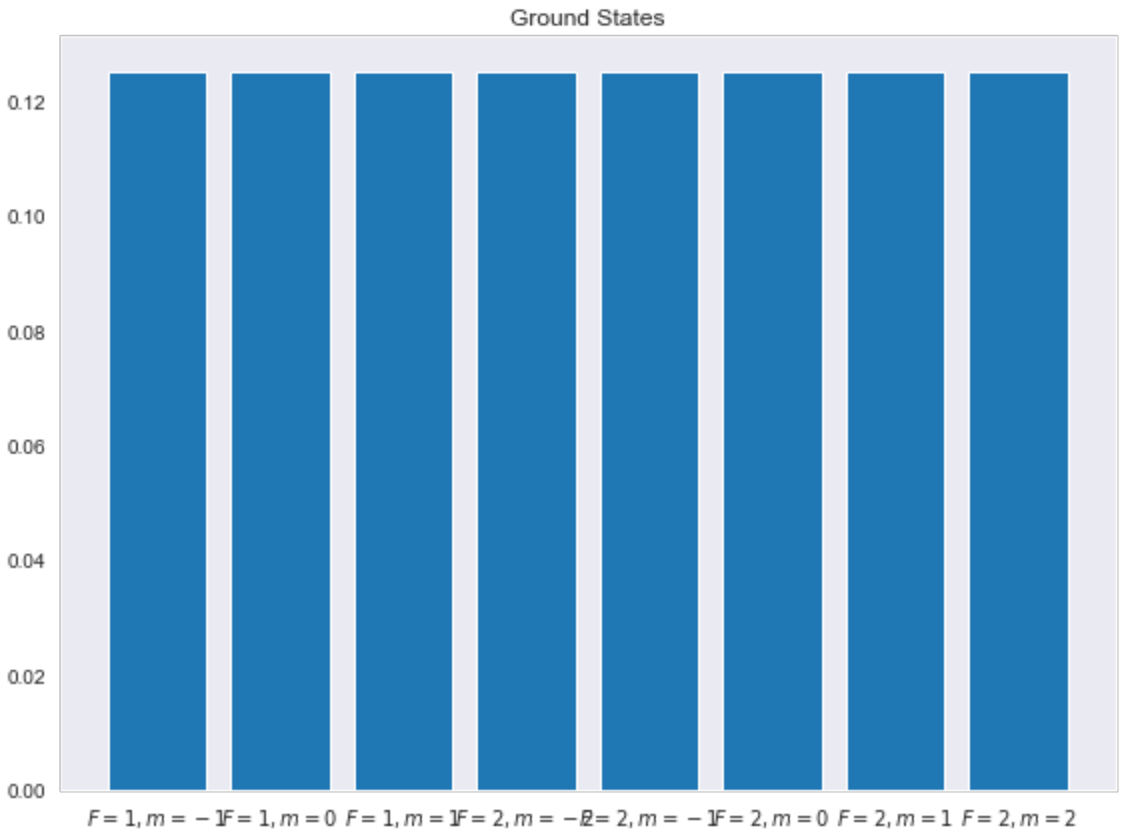
In [ ]:
```
starting_state = sum([basis(16, i).proj() for i in range(8)])
starting_state = starting_state.unit()
```

In [ ]:
```python
times = np.linspace(0, 1e-6, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil_far_detuned,
    starting_state,
    times,
    c_ops=natural_decay_ops,
    options=opts,
)
```

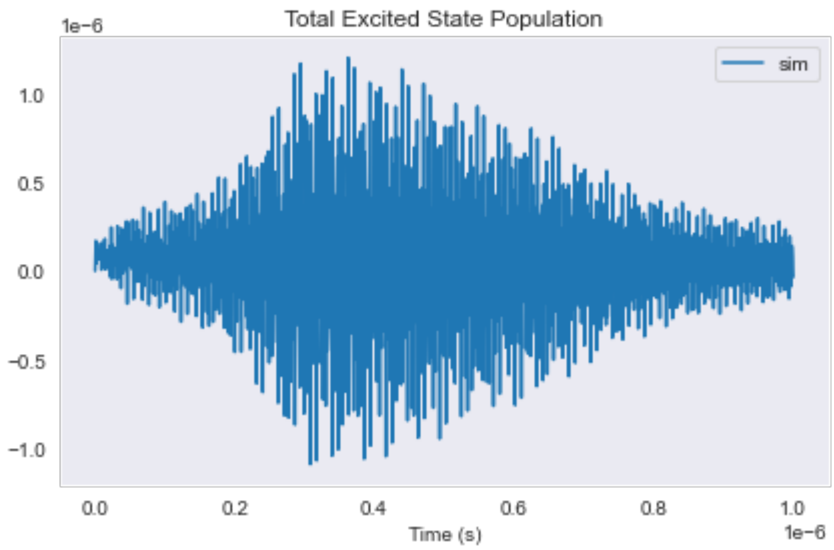
In [ ]:
```python
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```python
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```




```
In [ ]: excited_exp = [
            sum(
                [
                    res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
                    for i in range(8, 16)
                ]
            )
            for t in range(len(times))
        ]
        plt.figure()
        # for e in excited_exp:
        plt.plot(times, np.real(excited_exp), label="sim")

        plt.legend()
        plt.title("Total Excited State Population")
        plt.xlabel("Time (s)")
        plt.tight_layout()
```

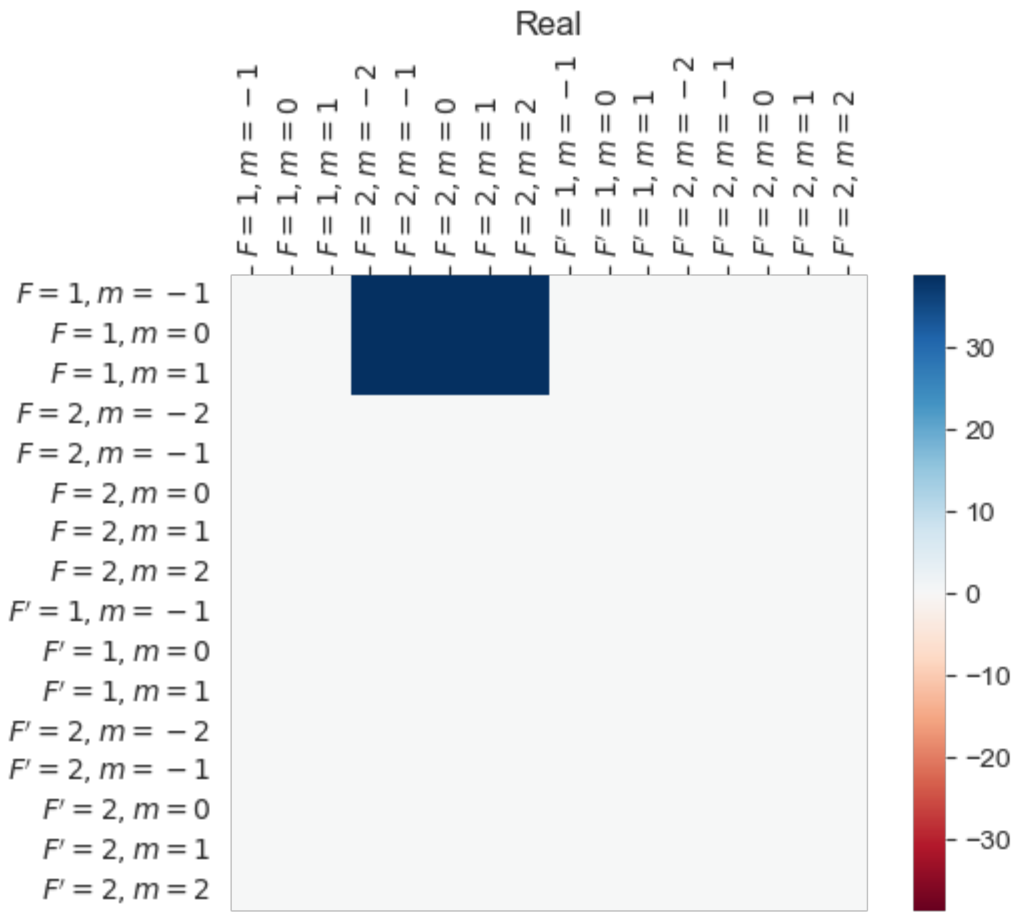


Ground State Relaxation γ_1 wrong

In [ ]:
```
gamma_1 = 4.5e3
gamma_1_ops = [
    (gamma_1 / 3) ** (1 / 2) * basis(16, m1) * basis(16, m2 + 3).dag()
    for m1 in range(3)
    for m2 in range(5)
]
```

In [ ]:
```
maplot(sum(gamma_1_ops))
```

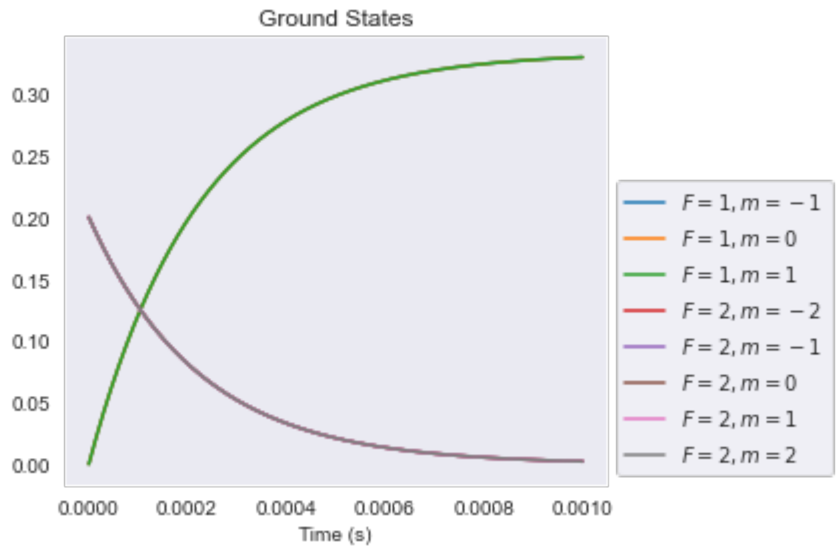
Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



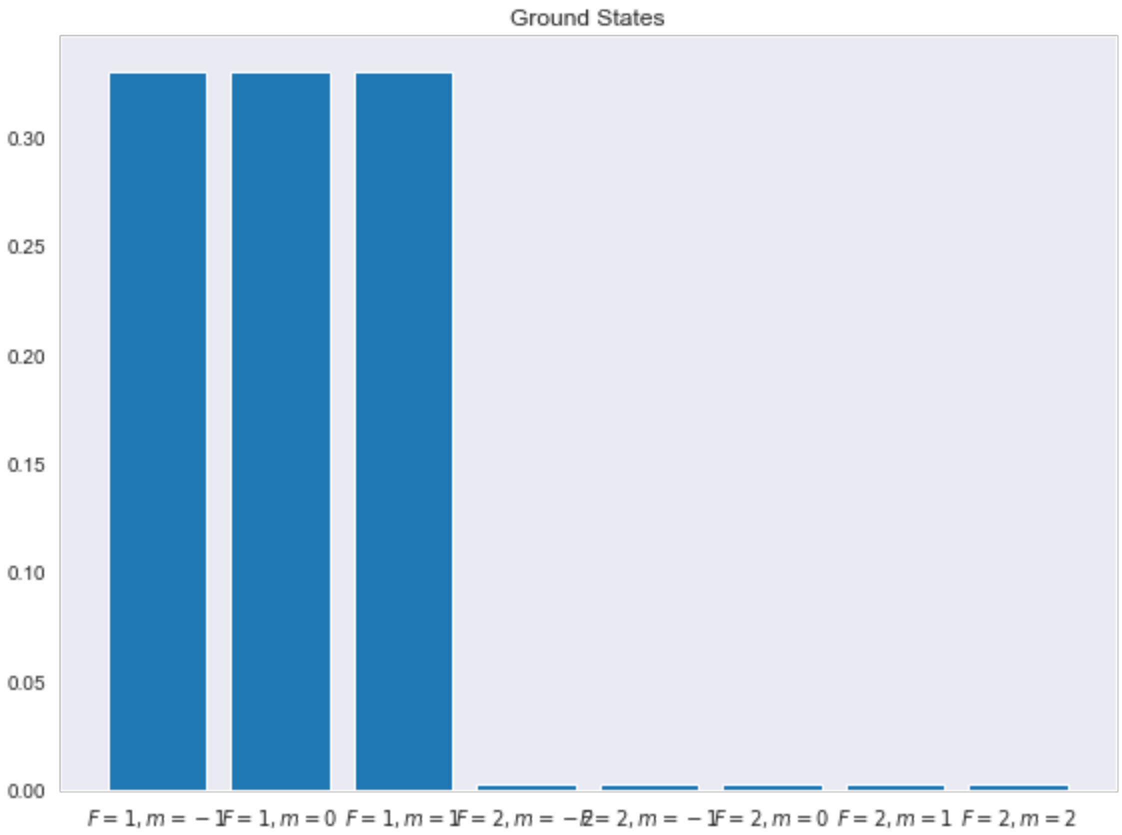
In [ ]:
```
starting_state = sum([basis(16, i + 3).proj() for i in range(5)])
# starting_state = basis(16, 16).proj()
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-3, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil * 0,
    starting_state,
    times,
    c_ops=gamma_1_ops,
    options=opts,
)
```

In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

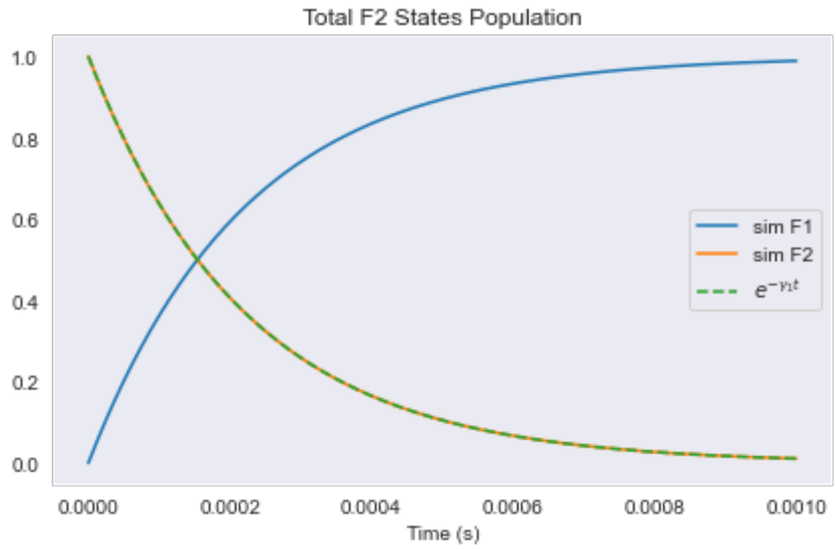


In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:

```
excited_exp = [  # = F2 states
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(3, 8)
        ]
    )
    for t in range(len(times))
]
total_ground_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(3)
        ]
    )
    for t in range(len(times))
]
plt.figure()
plt.plot(times, np.real(total_ground_exp), label="sim F1")
plt.plot(times, np.real(excited_exp), label="sim F2")
plt.plot(times, [np.exp(-gamma_1 * t) for t in times], "--", label=r"$e^{-\gamma_1 t}$")
plt.legend()
plt.xlabel("Time (s)")
plt.title("Total F2 States Population")
plt.tight_layout()
```

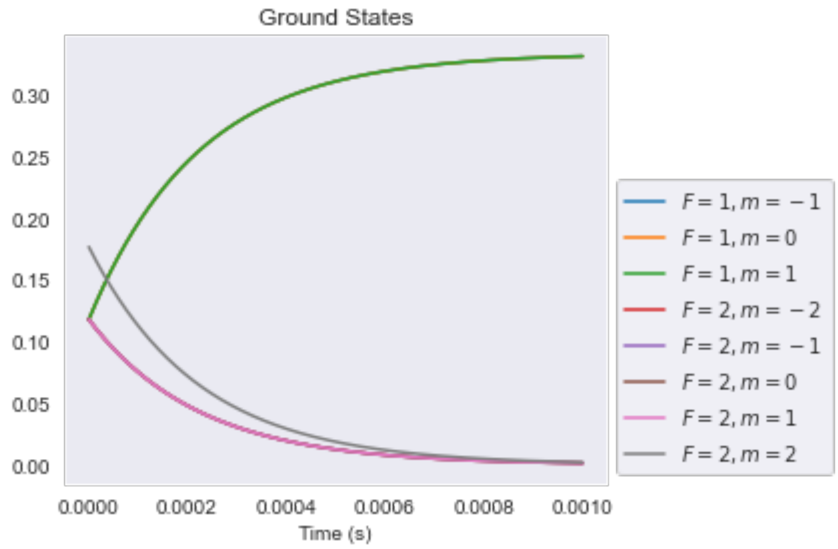


Different Starting State

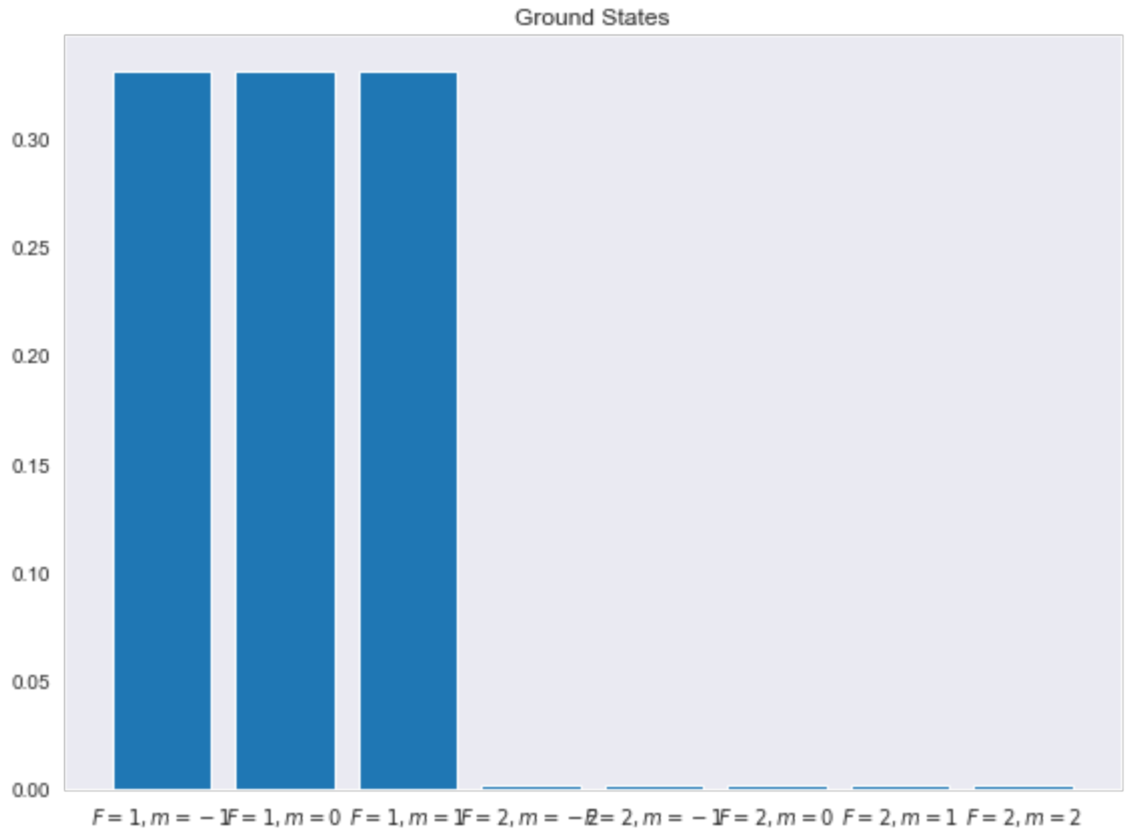
In [ ]:
```
starting_state = sum([basis(16, i).proj() for i in range(7)])
starting_state += 1.5 * basis(16, 7).proj()
# starting_state = basis(16, 16).proj()
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-3, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil * 0,
    starting_state,
    times,
    c_ops=gamma_1_ops,
    options=opts,
)
```

In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



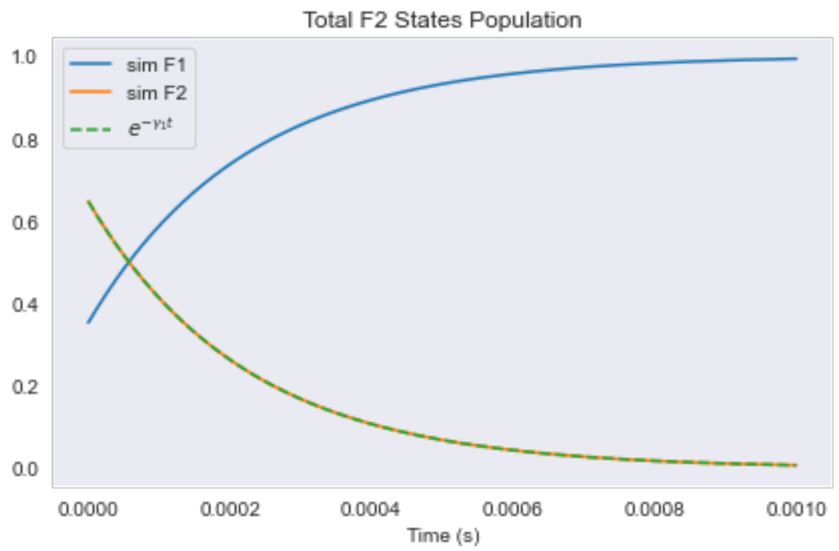
In [ ]:
```
(starting_state * Fg2_projector()).tr()
```

Out[ ]: 0.6470588235294117

In [ ]:
```
(starting_state * Fg1_projector()).tr()
```

Out[ ]: 0.3529411764705882

In [ ]:
```
excited_exp = [  #  = F2 states
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(3, 8)
        ]
    )
    for t in range(len(times))
]
total_ground_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(3)
        ]
    )
    for t in range(len(times))
]
plt.figure()
plt.plot(times, np.real(total_ground_exp), label="sim F1")
plt.plot(times, np.real(excited_exp), label="sim F2")
plt.plot(
    times,
    [np.exp(-gamma_1 * t) * (starting_state * Fg2_projector()).tr() for t in times],
    "--",
    label=r"$e^{-\gamma_1 t}$",
)
plt.legend()
plt.xlabel("Time (s)")
plt.title("Total F2 States Population")
plt.tight_layout()
```



Ground State Relaxation γ_2 ?? wrong

In [ ]:

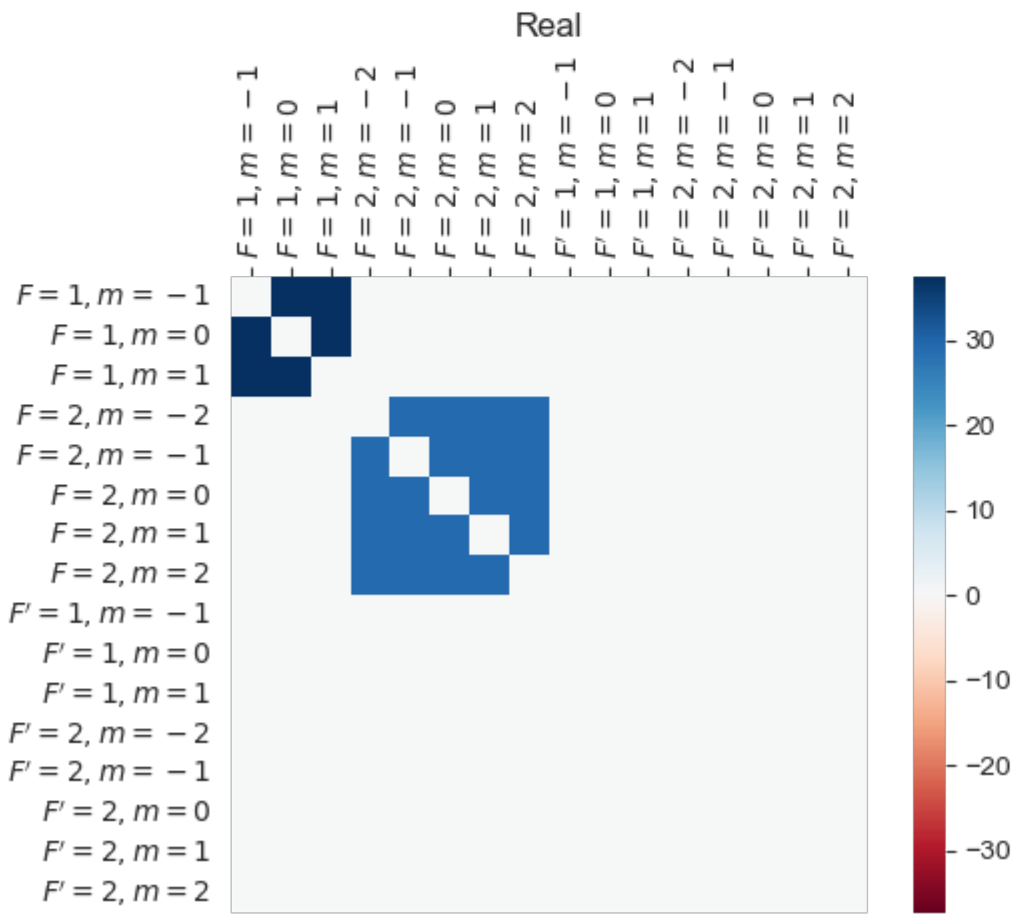
```
gamma_2 = 4.2e3
gamma_2_ops = [
    (gamma_2 / 3) ** (1 / 2) * basis(16, m1) * basis(16, m2).dag()
    for m1 in range(3)
    for m2 in range(3)
    if m1 != m2
]
gamma_2_ops += [
    (gamma_2 / 5) ** (1 / 2) * basis(16, m1) * basis(16, m2).dag()
    for m1 in range(3, 8)
    for m2 in range(3, 8)
    if m1 != m2
]
gamma_2_ops = sum(gamma_2_ops)
```

In [ ]:

```
maplot(gamma_2_ops)
```

Out[ ]:

```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



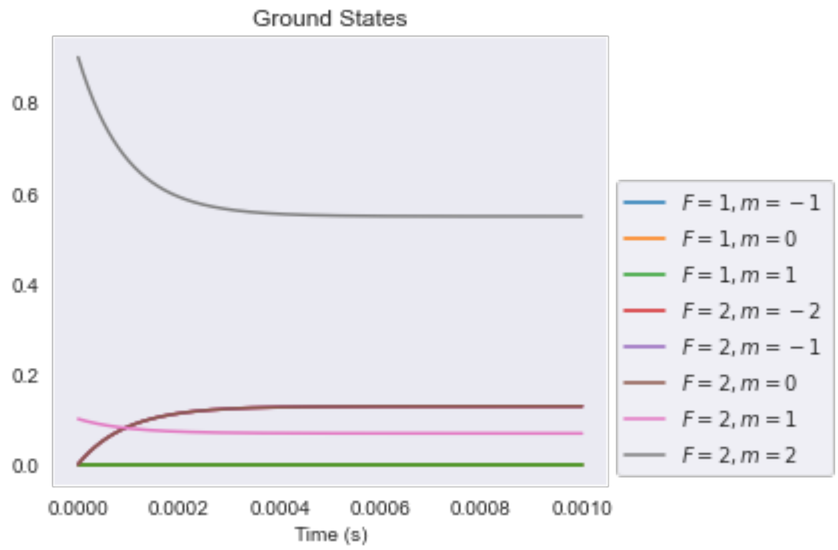
In [ ]:

```
# starting_state = sum([basis(16, i+3).proj() for i in range(5)])
starting_state = (3 * basis(16, 7) + basis(16, 6)).proj()  # F=2, mF=2

# starting_state = basis(16, 16).proj()
starting_state = starting_state.unit()
```

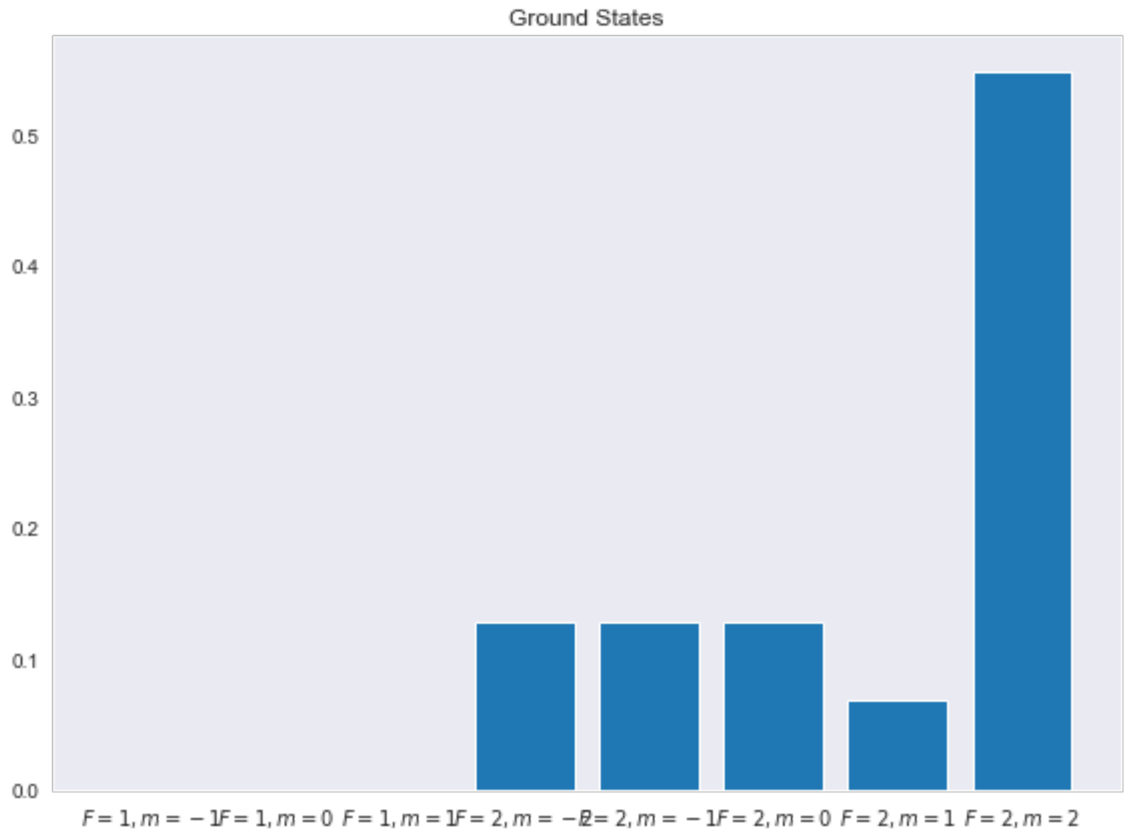
In [ ]:

```
times = np.linspace(0, 1e-3, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil * 0,
    starting_state,
    times,
    c_ops=gamma_2_ops,
    options=opts,
)
```

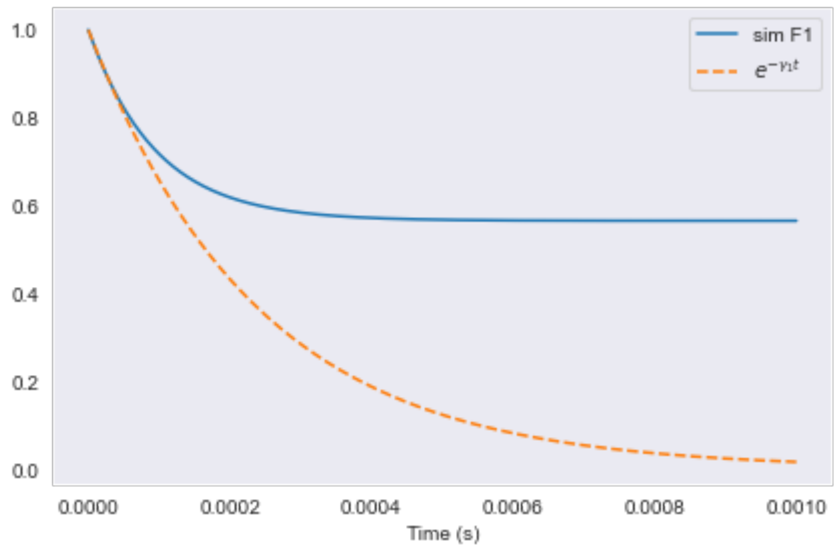
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(8, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



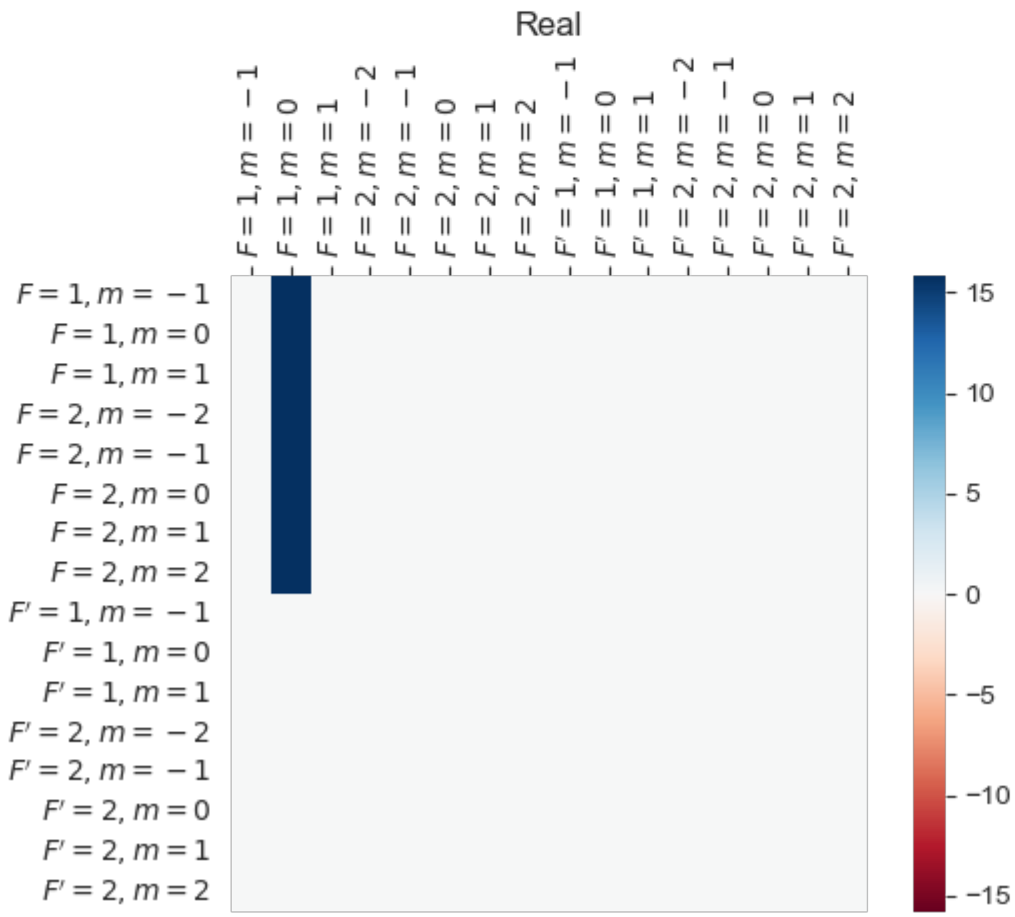
In [ ]:
```
plt.figure()
plt.plot(
    times,
    [(res.states[t] * starting_state).tr() for t in range(len(times))],
    label="sim F1",
)
plt.plot(times, [np.exp(-gamma_2 * t) for t in times], "--", label=r"$e^{-\gamma_1 t}$")
plt.legend()
plt.xlabel("Time (s)")
plt.tight_layout()
```



Wall Collisions

In [ ]:
```python
wall_coll_rate = 2e3
wall_ops = [
    sum(
        [
            (wall_coll_rate / 8) ** (1 / 2) * basis(16, final) * basis(16, inital).dag()
            for final in range(8)
        ]
    )
    for inital in range(8)
]
maplot(wall_ops[1])
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



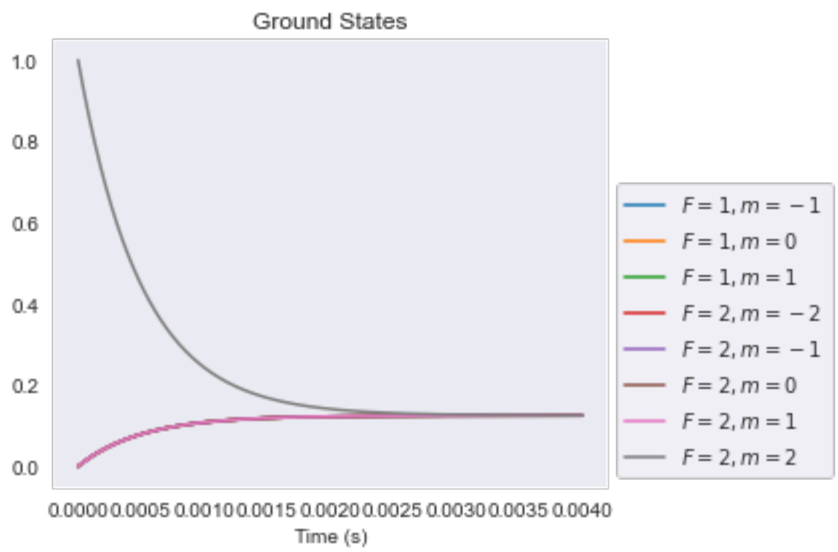
starting state 1

In [ ]:
```python
# starting_state = sum([basis(16, i).proj() for i in range(8)]) # ground states equally
starting_state = basis(16, 7).proj()
starting_state = starting_state.unit()
```

In [ ]:
```python
times = np.linspace(0, 4e-3, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil * 0,
    starting_state,
    times,
    c_ops=wall_ops,
    options=opts,
)
```

In [ ]:

```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



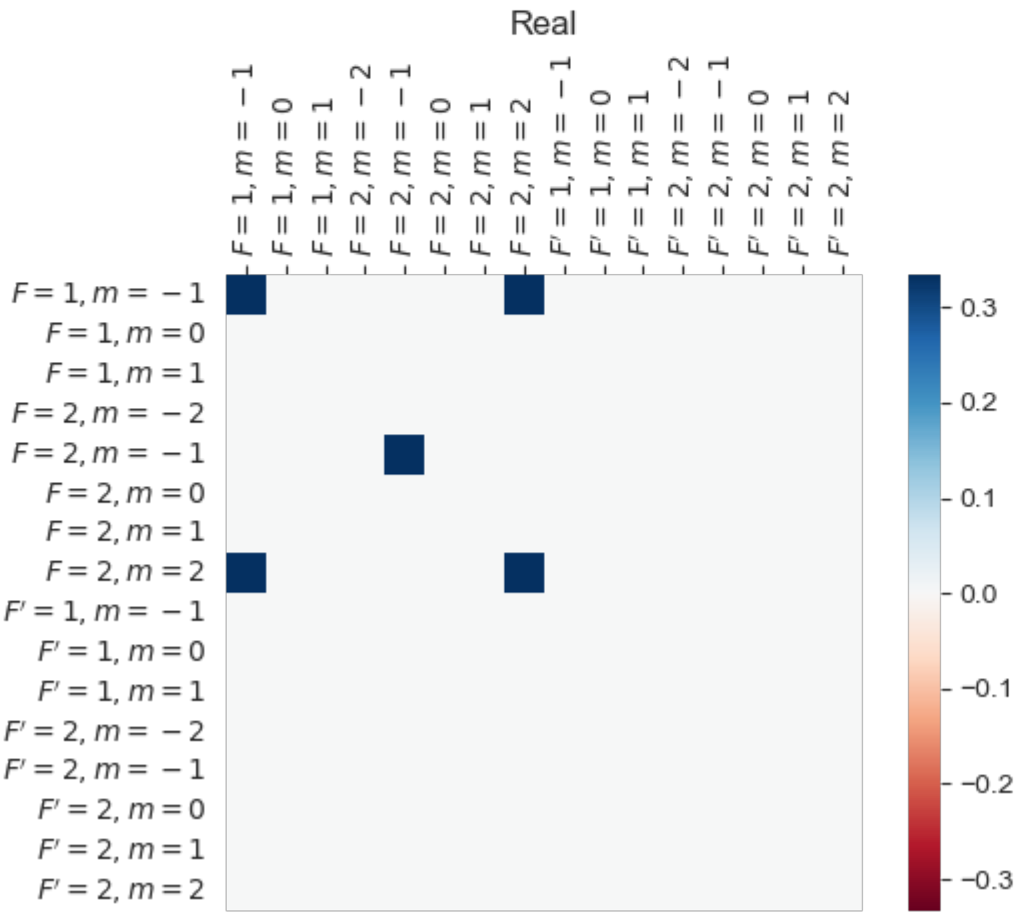
starting state 2

In [ ]:

```
# starting_state = sum([basis(16, i).proj() for i in range(8)]) # ground states equally
starting_state = (basis(16, 0) + basis(16, 7)).proj() + basis(16, 4).proj()
starting_state = starting_state.unit()
maplot(starting_state)
```

Out[ ]:

```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```

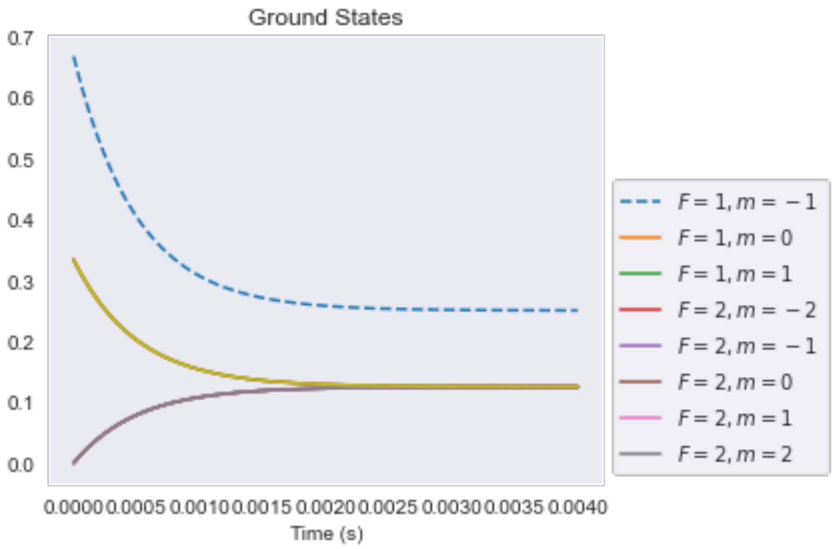


In [ ]:

```
times = np.linspace(0, 4e-3, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil * 0,
    starting_state,
    times,
    c_ops=wall_ops,
    options=opts,
)
```

In [ ]:

```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
mw_transition_exp = [
    (res.states[t] * (basis(16, 0) + basis(16, 7)).proj().unit()).tr()
    for t, _ in enumerate(times)
]
plt.figure()
plt.plot(times, mw_transition_exp, "--", label="MW transition coherence")
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

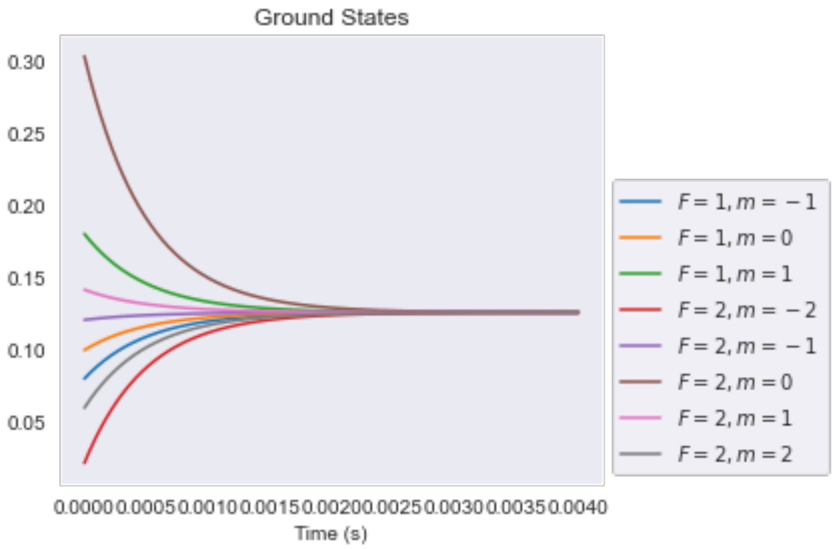


starting state 3

In [ ]:
```
# starting_state = sum([basis(16, i).proj() for i in range(8)]) # ground states equally
starting_state = sum([random.random() * basis(16, k).proj() for k in range(8)])
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 4e-3, 1000)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    hamil * 0,
    starting_state,
    times,
    c_ops=wall_ops,
    options=opts,
)
```

In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for e in ground_exp:
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
from IPython.display import display
```

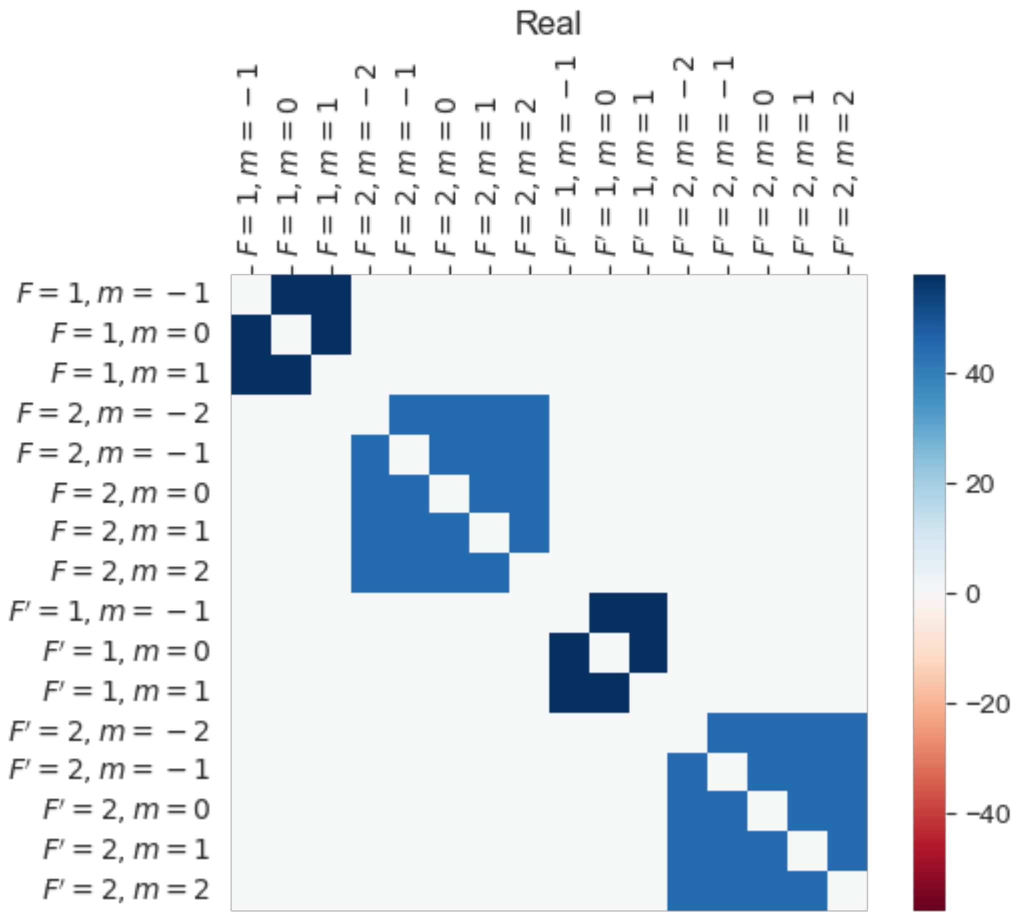
Decay intra F

$|F, m_F\rangle \rightarrow |F, m_F'\rangle$, where $m_F' = -F, \ldots, F$ and $m_F \neq m_F'$

Decay operator $C_k = \sqrt{\Gamma_k} \sum_{m_F' \neq k} |F, m_F'\rangle\langle F, m_F = k|$

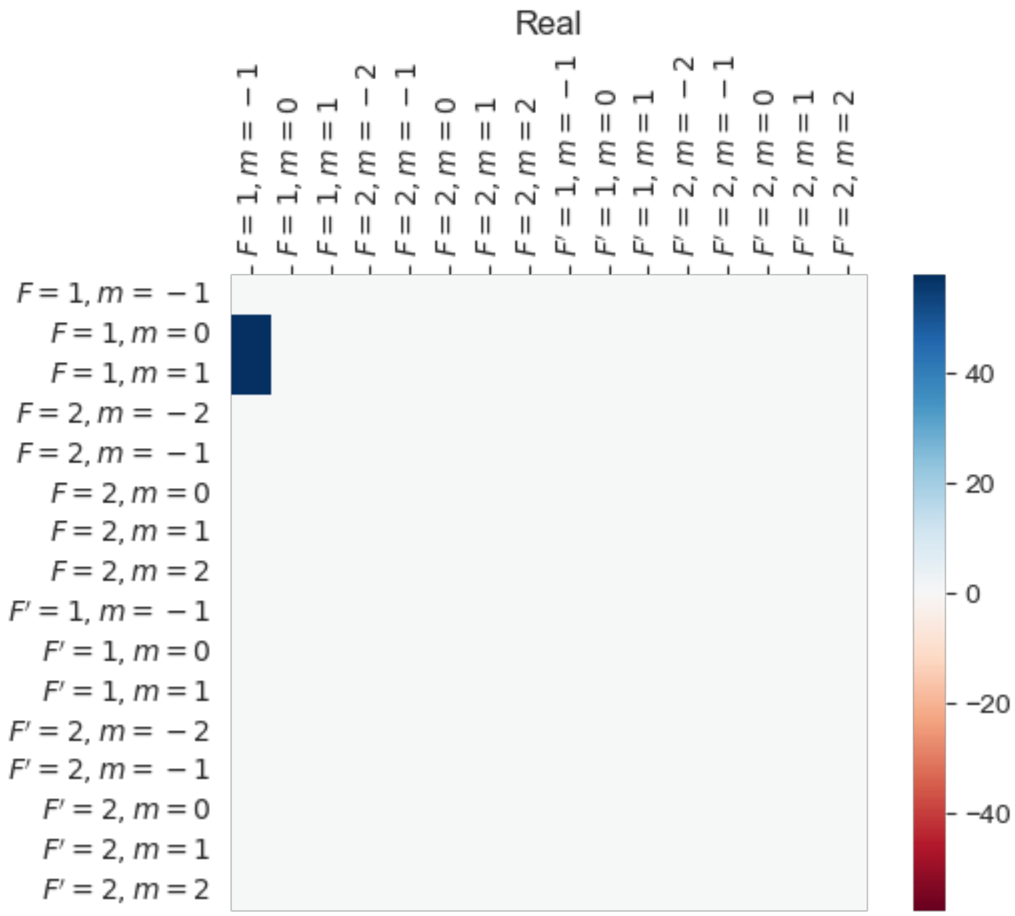
In [ ]:
```
maplot(sum(intra_F1 + intra_F2 + intra_Fp1 + intra_Fp2))
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



In [ ]:
```
maplot(intra_F1[0])
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



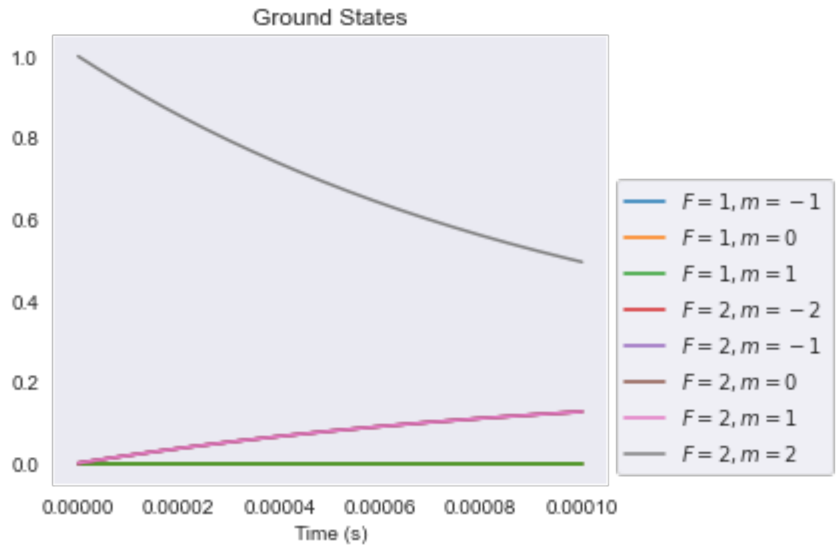
Time Evo

In [ ]:
```
L = liouvillian(0 * hamil, c_ops=intra_F1 + intra_F2 + intra_Fp1 + intra_Fp2)
```

In [ ]:
```
starting_state = basis(16, 7).proj()
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-4, 1001)
opts = Options(nsteps=1 * 10**6)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

In [ ]:
```
ground_exp_even = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp_even):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



With Laser, Without Rad Decay

liouvillian

In [ ]:
```
L = liouvillian(hamil, c_ops=intra_F1 + intra_F2 + intra_Fp1 + intra_Fp2)
```

In [ ]:
```
vector_to_operator(L * operator_to_vector(rand_dm(16))).tr()
```

Out[ ]:
```
(-2.717115421546623e-11-7.275957614183426e-12j)
```

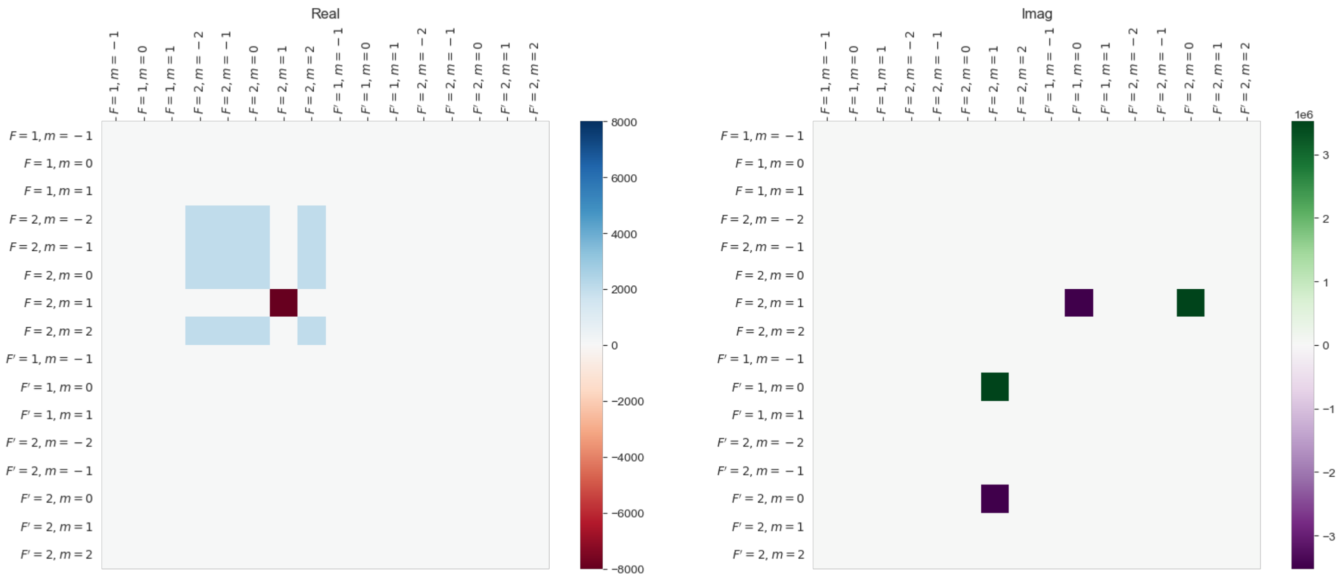
In [ ]:
```
vector_to_operator(L * operator_to_vector(basis(16, 7).proj())).tr()
```

Out[ ]:
```
0.0
```

$\dot{\rho}(t=0)$, where $\rho(0) = |F=2, m_F=2\rangle$

In [ ]:
```
maplot(vector_to_operator(L * operator_to_vector(basis(16, 6).proj())))
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
import plotly.express as px

y = L.full().real
fig = px.imshow(
    y,
    color_continuous_midpoint=0,
    aspect="equal",
    width=1.5 * 800,
    height=1.5 * 400,
    zmin=-(abs(y).max()),
    zmax=(abs(y).max()),
    color_continuous_scale="RdBu",
)
fig.show()
```

Steady State

In [ ]:
```python
L_en, L_states = L.eigenstates()
```

In [ ]:
```python
sns.stripplot(data=L_en[np.where(np.abs(L_en) < 1e1)])
```

```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\seaborn\categorical.py:128:
ComplexWarning:

Casting complex values to real discards the imaginary part
```

Out[ ]:
```
<AxesSubplot:>
```

In [ ]:
```python
L_en[np.where(np.abs(L_en) < 1e1)]
```

Out[ ]:
```
array([-3.25350303e-01-5.82792886e-08j, -5.20475352e-03-1.71101446e-07j,
        2.91171183e-07-1.33338756e-07j])
```

In [ ]:
```python
zero_eigenval_states = []
for nr, en in enumerate(L_en):
    if np.abs(en) < 1e-1:
        zero_eigenval_states.append((en, vector_to_operator(L_states[nr])))
```

In [ ]:
```python
len(zero_eigenval_states)
```

Out[ ]:
```
2
```

In [ ]:
```python
zero_eigenval_states[0][0]
```

Out[ ]:
```
(-0.0052047535201342416-1.7110144568883392e-07j)
```

In [ ]:
```python
zero_eigenval_states[-1][0]
```

Out[ ]:
```
(2.9117118339319 67e-07-1.3333875612402766e-07j)
```

In [ ]:
```python
zero_eigenval_states[-1][-1].tr()
```

Out[ ]:
```
(3.2327823424428854+0.00016037838885812897j)
```

In [ ]:
```python
zero_eigenval_states[0][-1].tr()
```

Out[ ]:
```
(-0.000319491281428233+2.84020558484338e-05j)
```

Time Evo

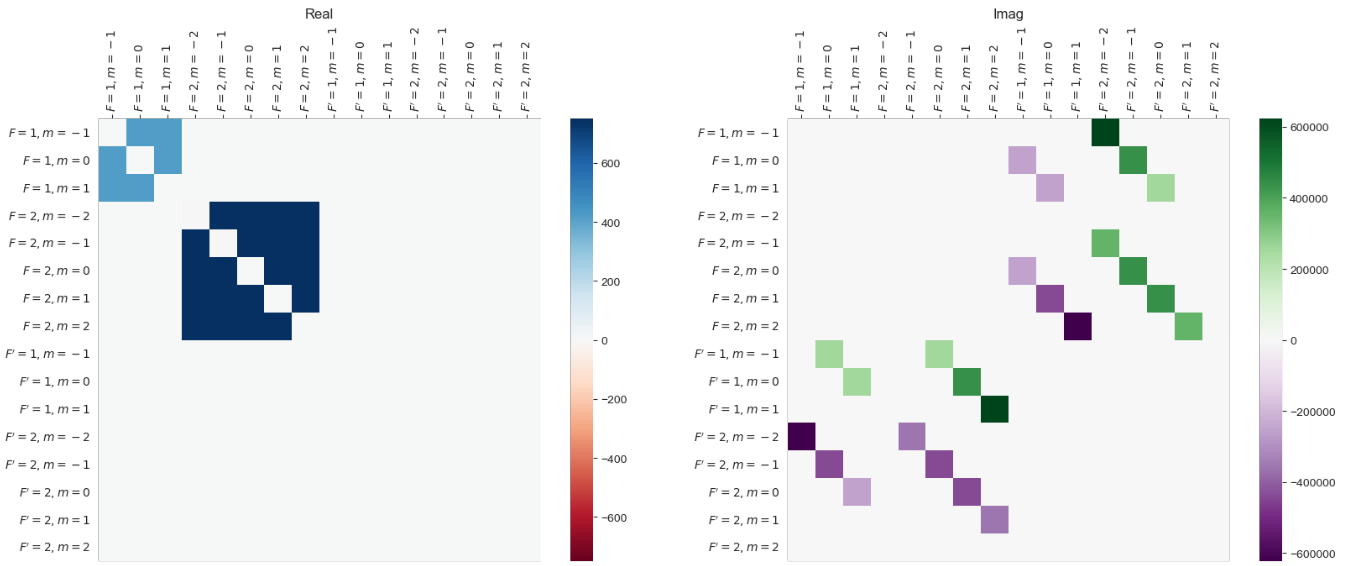
In [ ]:
```python
starting_state = sum([basis(16, i).proj() for i in range(8)])  # ground states equally
# starting_state = basis(16, 7).proj()
starting_state = starting_state.unit()
```

In [ ]:
```python
vector_to_operator(L * operator_to_vector(starting_state)).tr()
```

Out[ ]:
```
-8.526512829121202e-14
```

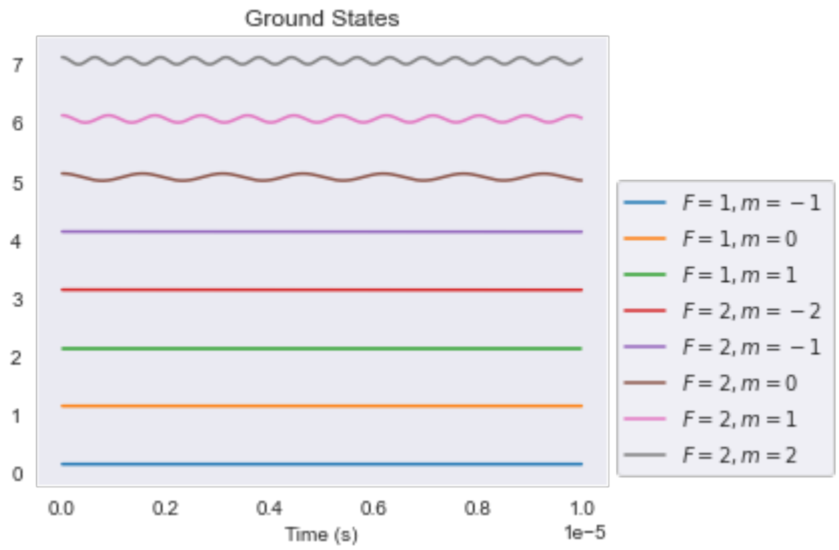
In [ ]:
```python
maplot(vector_to_operator(L * operator_to_vector(starting_state)))
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



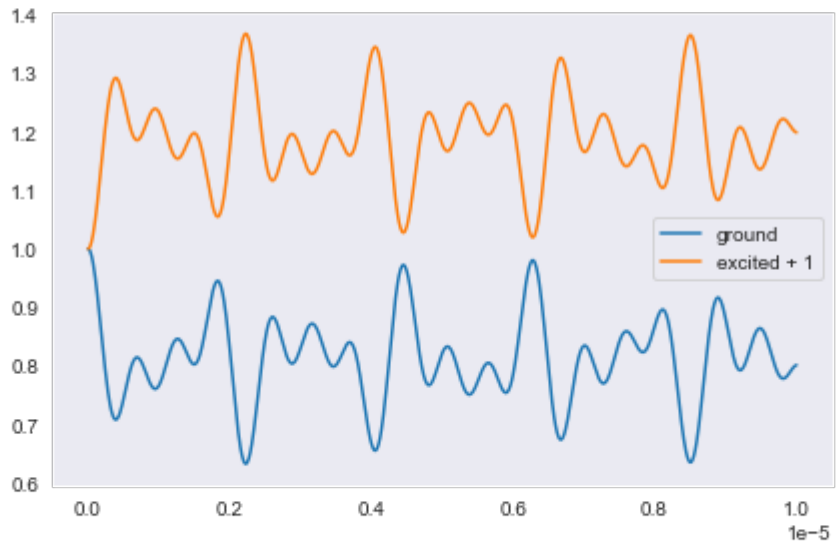
In [ ]:
```python
times = np.linspace(0, 1e-5, 1001)
opts = Options(nsteps=1 * 10**6)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

In [ ]:
```
ground_exp_even = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp_even):
    plt.plot(times, np.real(e) + i)
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

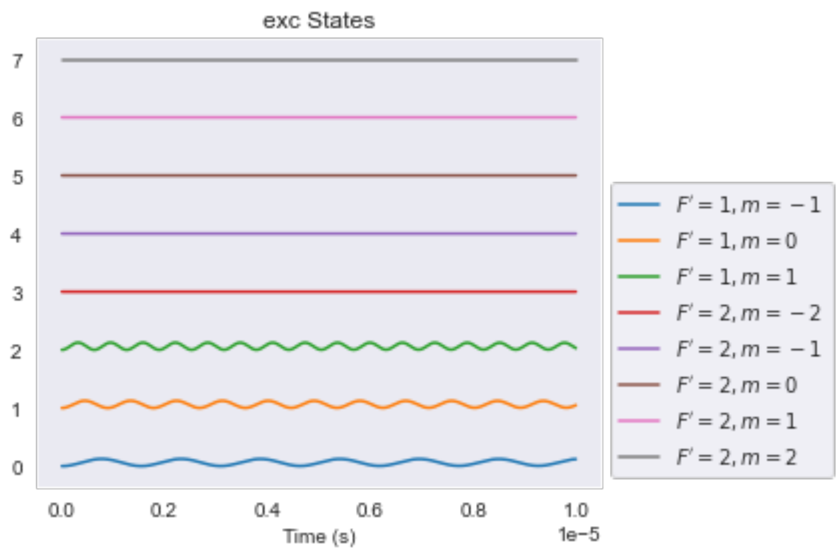


In [ ]:
```
ground_state_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
tot_excited_exp_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
fig, ax = plt.subplots()
ax.plot(times, ground_state_even, label="ground")
ax.plot(times, tot_excited_exp_even, label="excited + 1")
ax.legend()
plt.tight_layout()
```

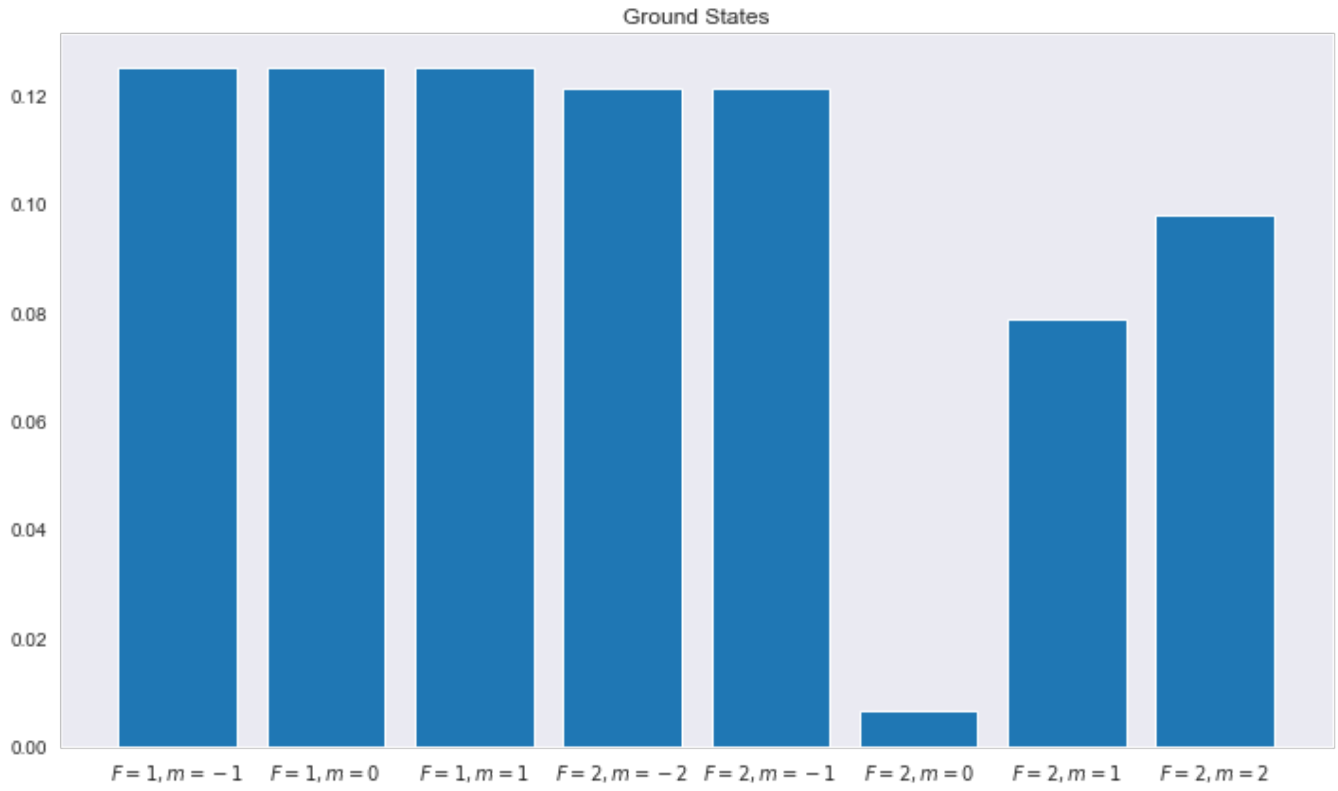
```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\matplotlib\cbook\__init__.p
y:1298: ComplexWarning:

Casting complex values to real discards the imaginary part
```



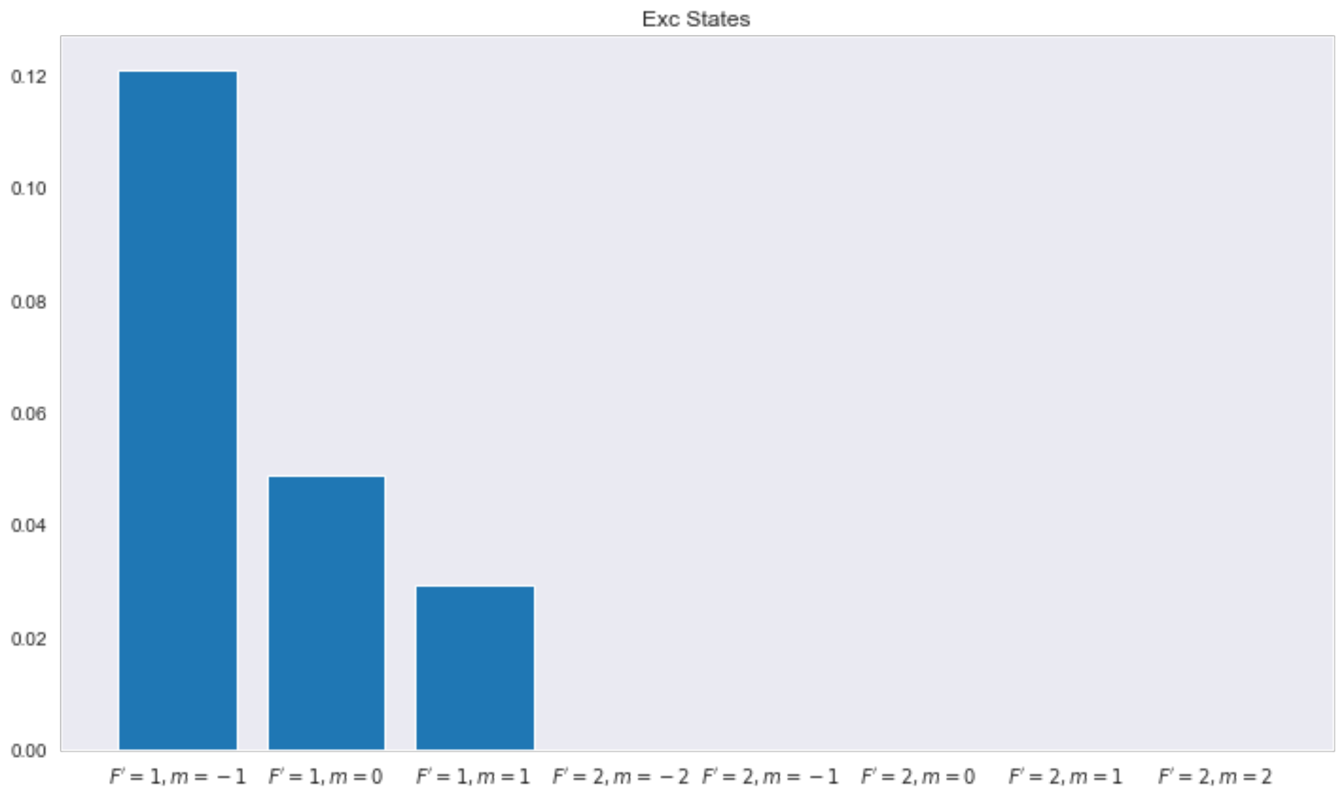
In [ ]:
```
exc_expeven = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for k, e in enumerate(exc_expeven):
    plt.plot(times, np.real(e) + k)
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("exc States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)],
    [np.real(e)[-1] for e in ground_exp_even],
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    [np.real(e)[-1] for e in exc_expeven],
)
plt.title("Exc States")
plt.tight_layout()
```

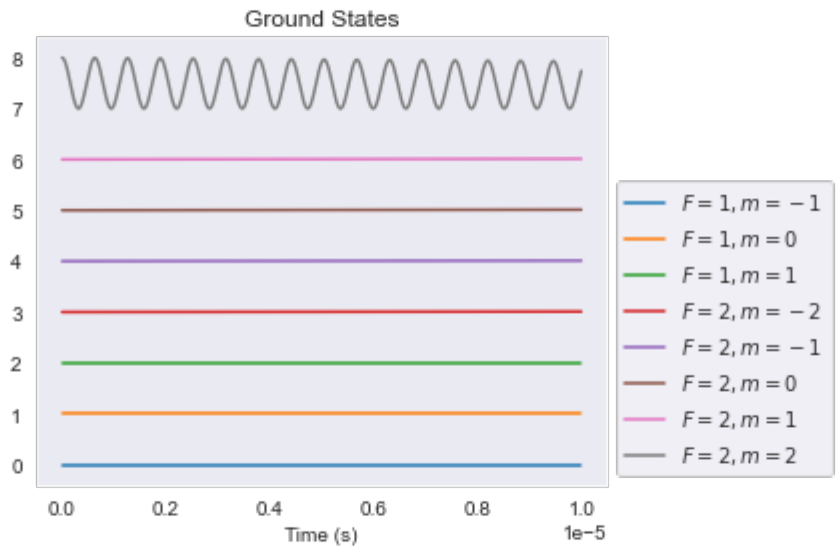


$$\rho_0 = |F=2, m_F=2\rangle\langle F=2, m_F=2|$$

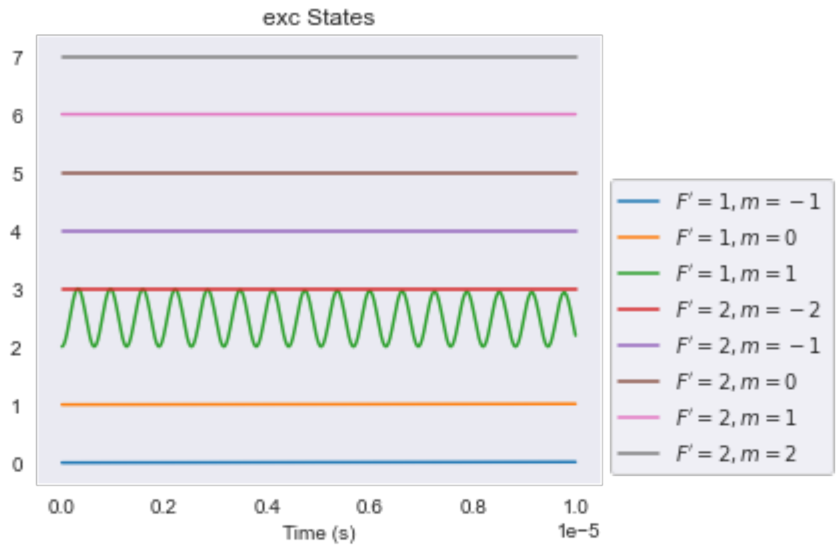
In [ ]:
```
# starting_state = sum([basis(16, i).proj() for i in range(8)]) # ground states equally
starting_state = basis(16, 7).proj()
starting_state = starting_state.unit()
```

In [ ]:
```
times = np.linspace(0, 1e-5, 1001)
opts = Options(nsteps=1 * 10**6)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

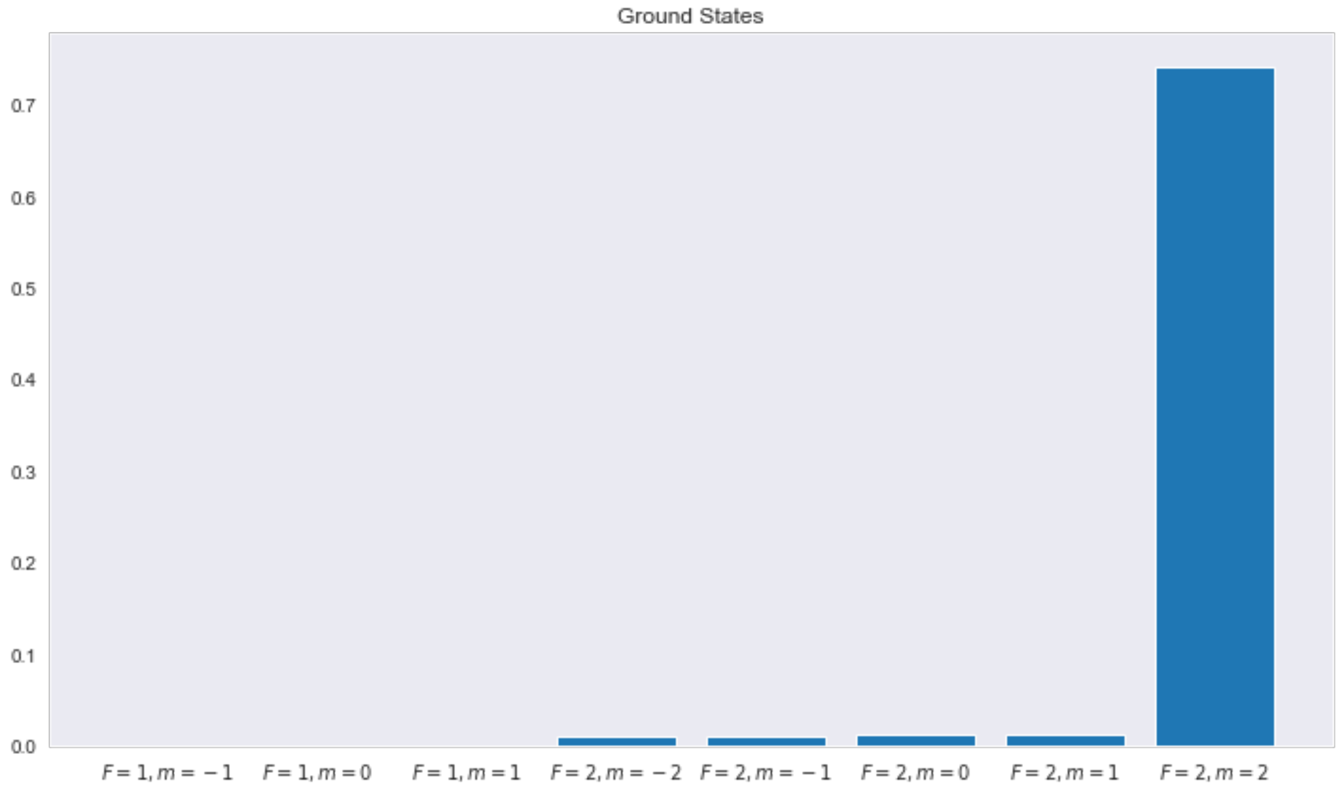
In [ ]:
```
ground_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp):
    plt.plot(times, np.real(e) + i)
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



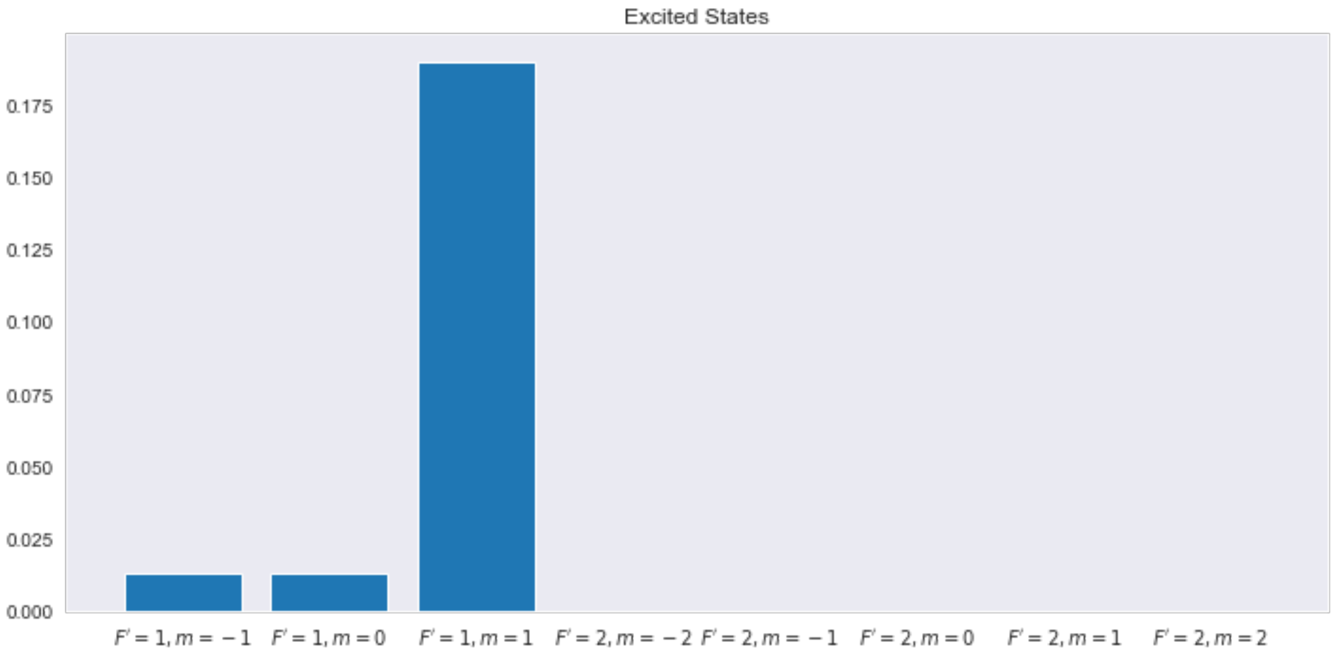
In [ ]:
```
exc_exp = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for i, e in enumerate(exc_exp):
    plt.plot(times, np.real(e) + i)
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("exc States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



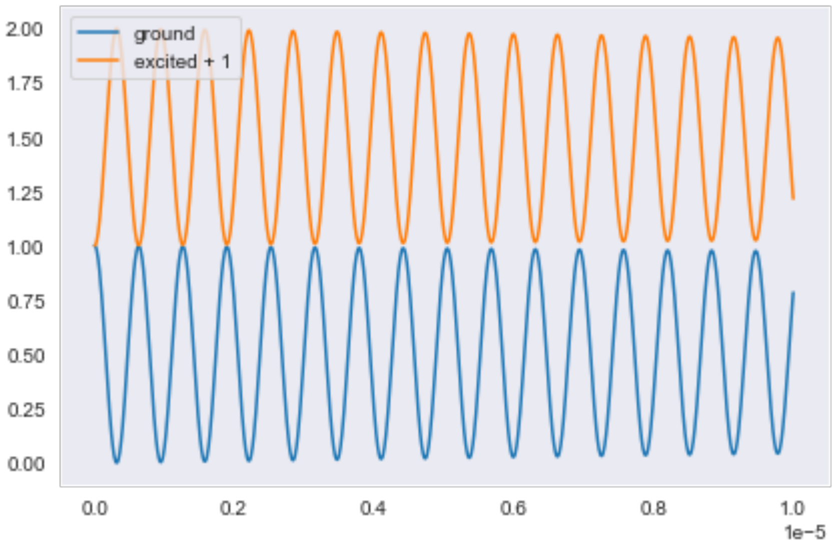
In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)], [np.real(e)[-1] for e in ground_exp]
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(10, 5))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8, 16)], [np.real(e)[-1] for e in exc_exp]
)
plt.title("Excited States")
plt.tight_layout()
```



In [ ]:
```
ground_state = [sum(np.array(ground_exp)[:, t]).real for t in range(len(times))]
tot_excited_exp = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i)).real
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
fig, ax = plt.subplots()
ax.plot(times, ground_state, label="ground")
ax.plot(times, tot_excited_exp, label="excited + 1")
ax.legend()
plt.tight_layout()
```



With Rad Decay

In [ ]:
```
L = liouvillian(
    hamil,
    c_ops=intra_F1 + intra_F2 + intra_Fp1 + intra_Fp2 + natural_decay_ops,
)
```

In [ ]:
```
L_en, L_states = L.eigenstates()
```

In [ ]:
```python
import plotly.express as px

y = L.full().real
fig = px.imshow(
    y,
    color_continuous_midpoint=0,
    aspect="equal",
    width=1.5 * 800,
    height=1.5 * 400,
    zmin=-(abs(y).max()),
    zmax=(abs(y).max()),
    color_continuous_scale="RdBu",
)
fig.show()
```

In [ ]:
```python
sorted(np.abs(L_en))[:10]
```

Out[ ]:
```
[1.0400738090735148e-06,
 2006.43370350121,
 6744.909078984909,
 6744.909080083952,
 6754.660922201028,
 6754.660922339302,
 6989.843573541444,
 6989.8435741127105,
 10000.946150294065,
 10001.38570925418]
```

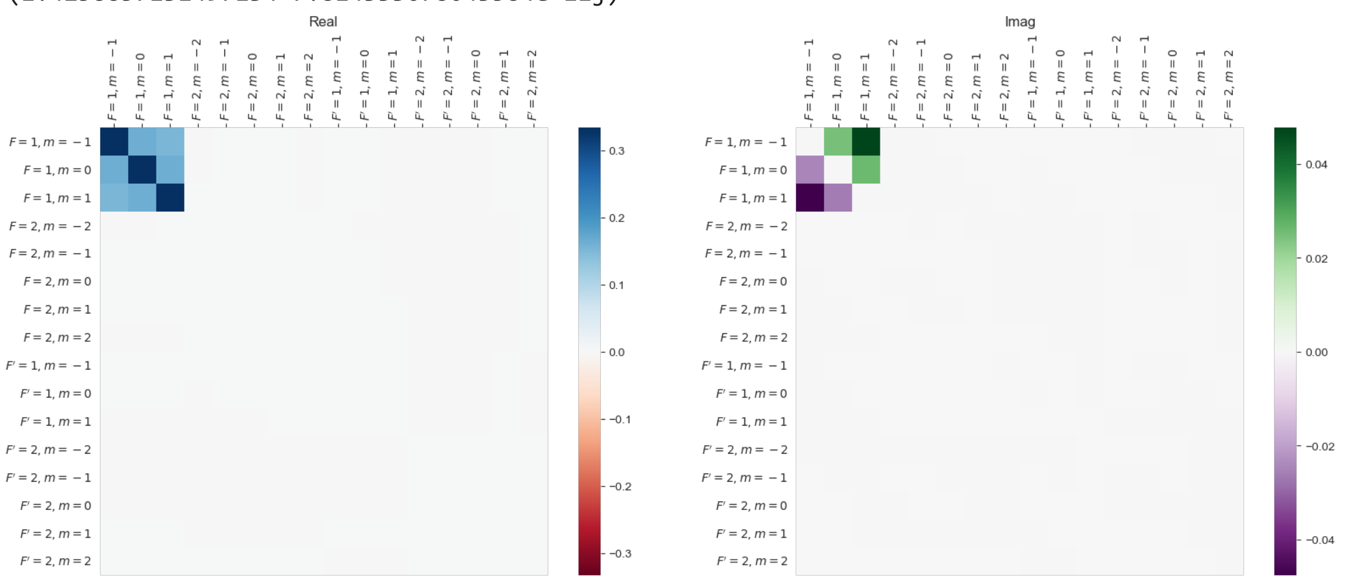
In [ ]:
```python
zero_eigenvalue_states = []
for en, st in zip(*L.eigenstates()):
    if np.isclose(en, 0, atol=1e-5):
        zero_eigenvalue_states.append(vector_to_operator(st))
```

In [ ]:
```python
len(zero_eigenvalue_states)
```

Out[ ]:
```
1
```

In [ ]:
```python
trace = zero_eigenvalue_states[0].tr()
maplot(zero_eigenvalue_states[0] / trace)
print(trace)
```

```
(1.4258857131497134+7.814333678043584e-11j)
```



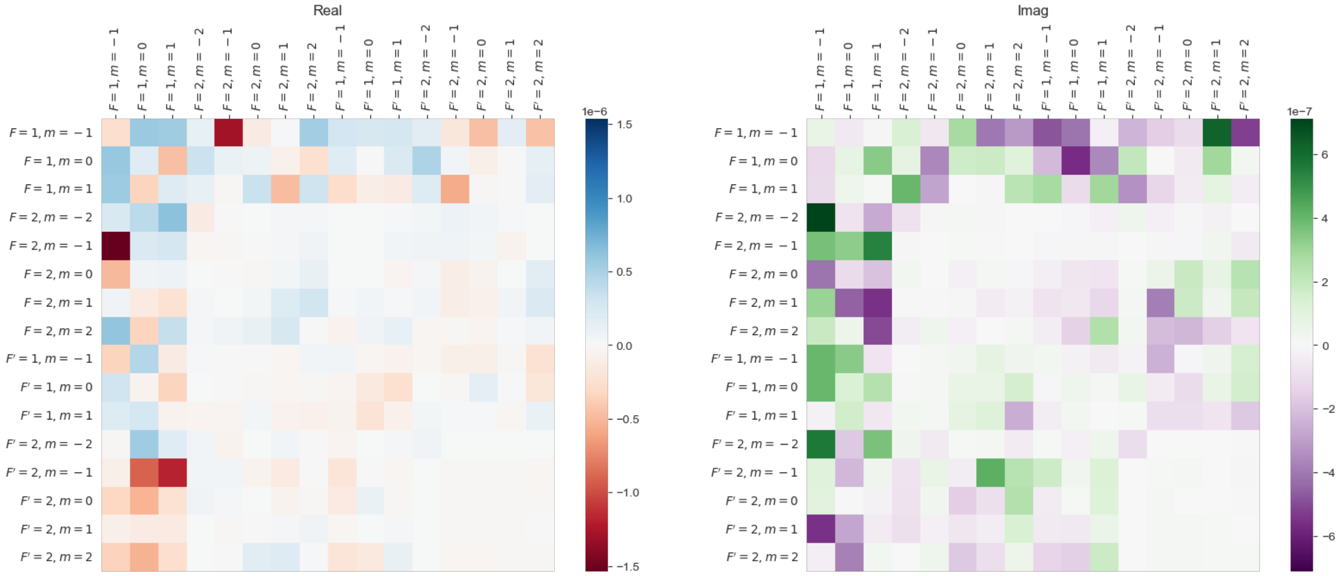
In [ ]:
```python
rho_steady = zero_eigenvalue_states[0] / trace
rho_steady_dot = vector_to_operator(L * operator_to_vector(rho_steady))
```

In [ ]:
```python
rho_steady_dot.tr()
```

```
Out[ ]: (-1.9575600540605432e-13+2.7642122701169926e-13j)
```

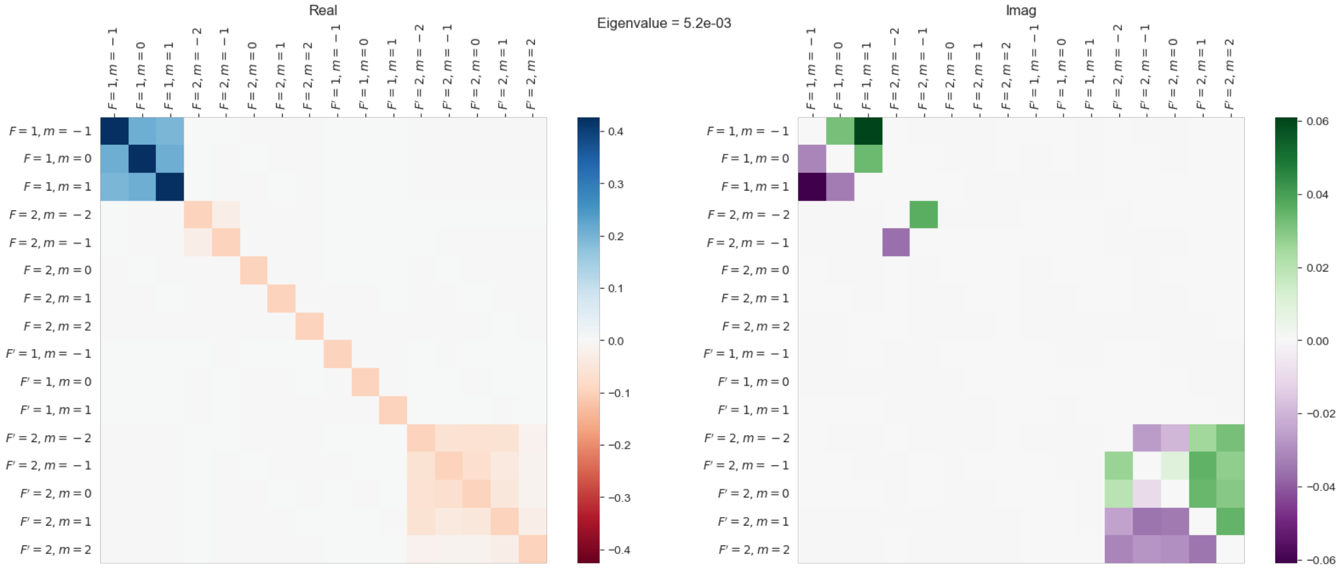
```
In [ ]: maplot(rho_steady_dot)
```

```
Out[ ]: (<Figure size 1680x672 with 4 Axes>,
         [<AxesSubplot:title={'center':'Real'}>,
          <AxesSubplot:title={'center':'Imag'}>])
```




```
In [ ]: fig, axs = maplot(zero_eigenval_states[0][1])
        fig.suptitle(f"Eigenvalue = {abs(zero_eigenval_states[0][0]):.1e}")
```

```
Out[ ]: Text(0.5, 0.98, 'Eigenvalue = 5.2e-03')
```



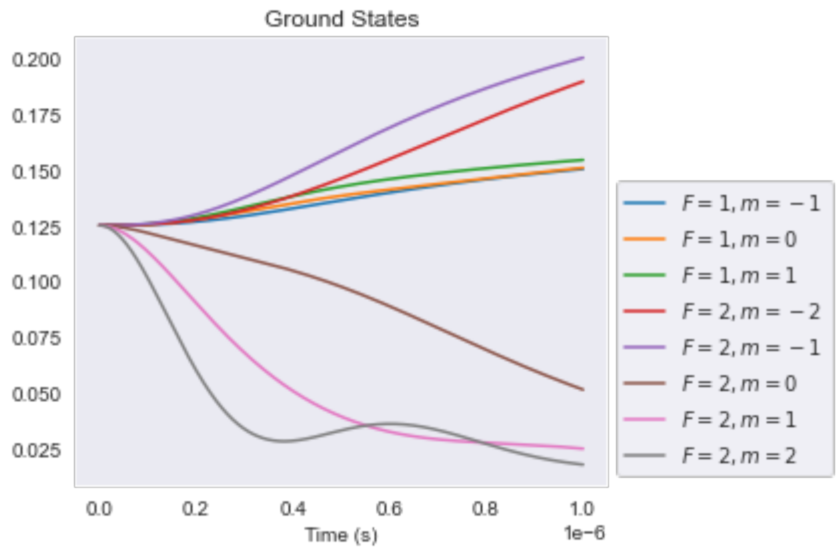
Time Evo

```
In [ ]: starting_state = sum([basis(16, i).proj() for i in range(8)])  # ground states equally
        # starting_state = basis(16, 7).proj()
        starting_state = starting_state.unit()
```

```
In [ ]: starting_state = sum([basis(16, i).proj() for i in range(8)])  # ground states equally
        # starting_state = basis(16, 7).proj()
        starting_state = starting_state.unit()
```

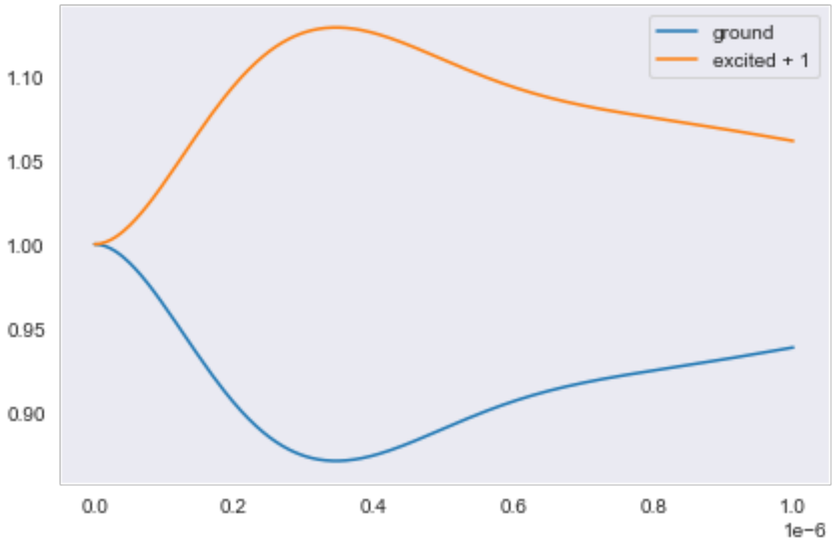
In [ ]:
```
times = np.linspace(0, 1e-6, 1001)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    L,
    starting_state,
    times,
    options=opts,
)
```

In [ ]:
```
ground_exp_even = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp_even):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

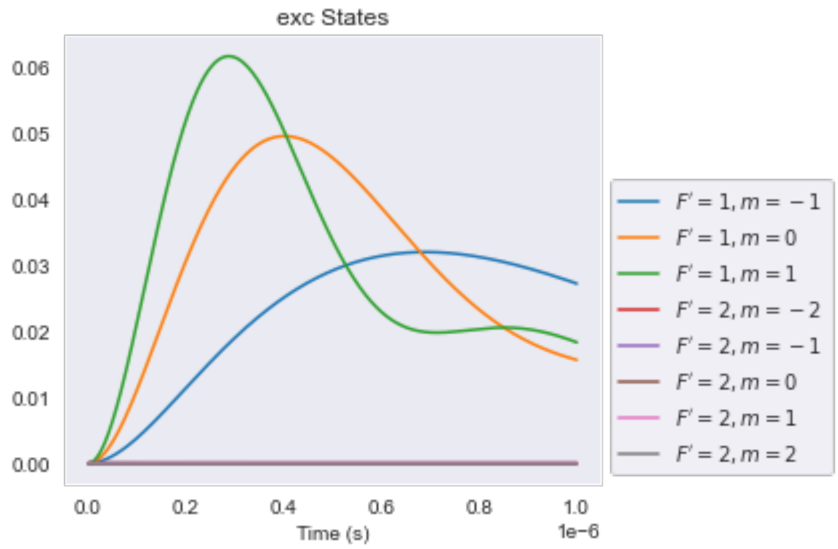


In [ ]:
```
ground_state_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
tot_excited_exp_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
fig, ax = plt.subplots()
ax.plot(times, ground_state_even, label="ground")
ax.plot(times, tot_excited_exp_even, label="excited + 1")
ax.legend()
plt.tight_layout()
```

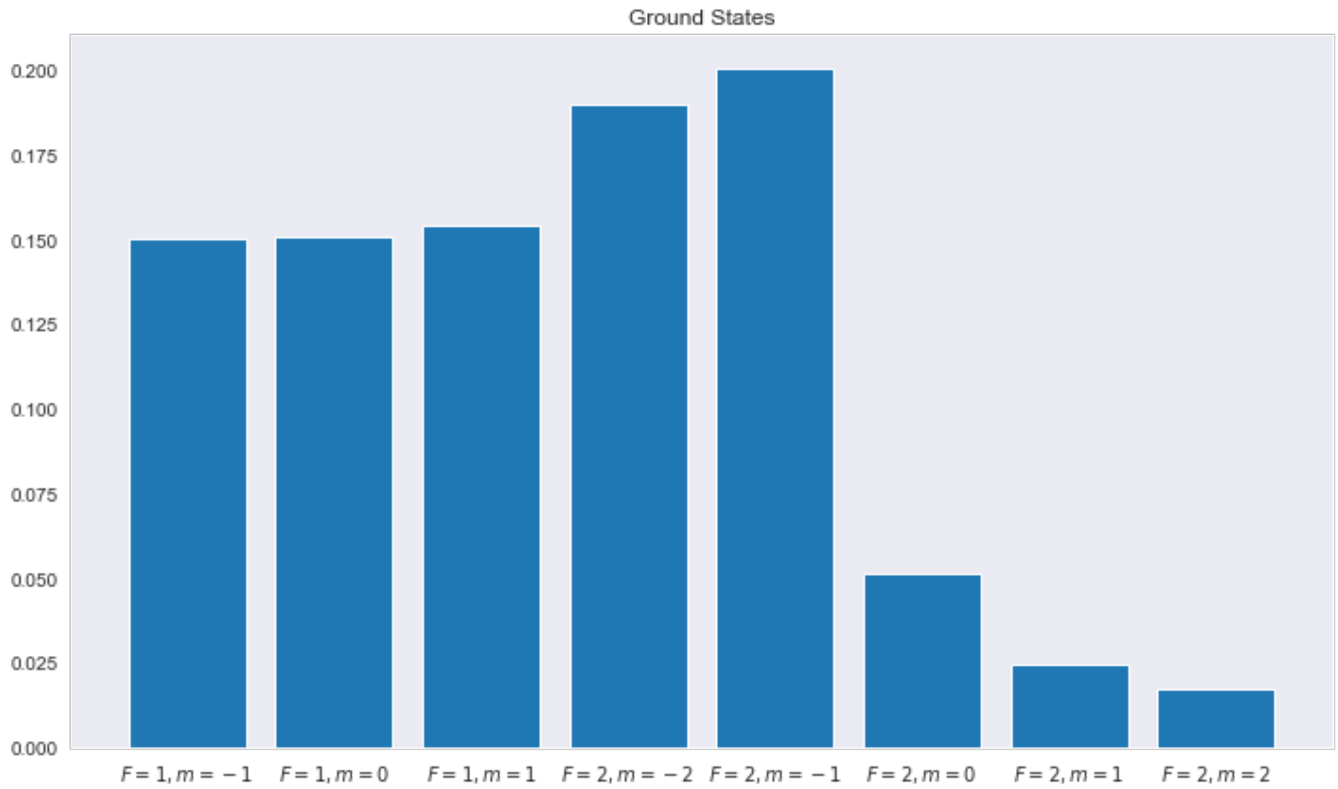
```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\matplotlib\cbook\__init__.p
y:1298: ComplexWarning:

Casting complex values to real discards the imaginary part
```



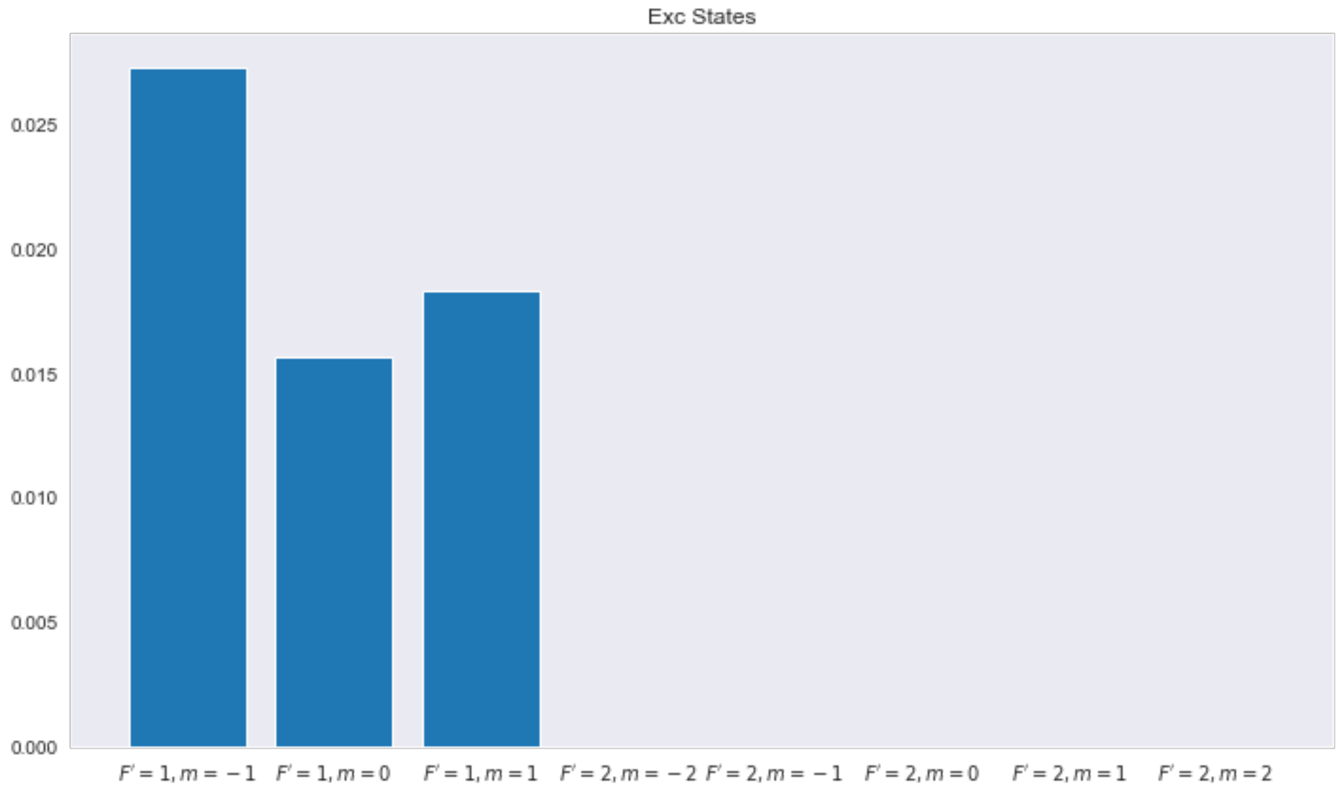
In [ ]:
```
exc_expeven = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for k, e in enumerate(exc_expeven):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("exc States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)],
    [np.real(e)[-1] for e in ground_exp_even],
)
plt.title("Ground States")
plt.tight_layout()
```



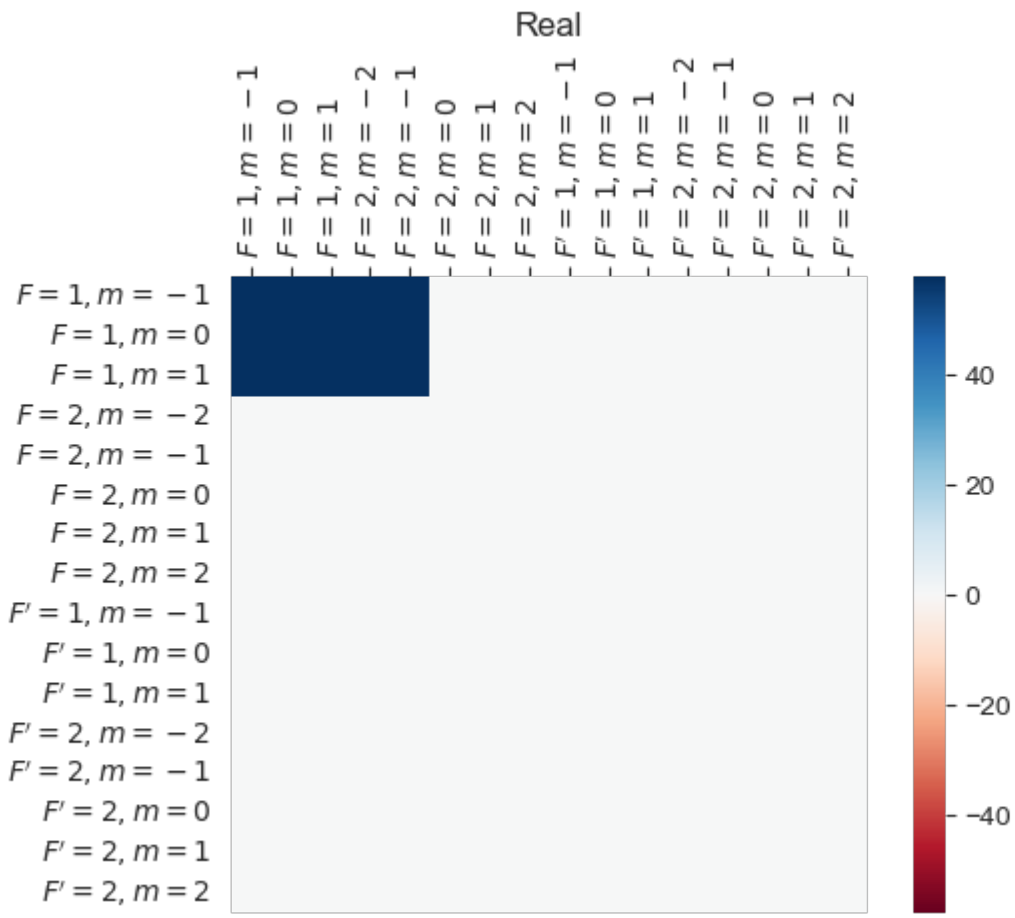
In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    [np.real(e)[-1] for e in exc_expeven],
)
plt.title("Exc States")
plt.tight_layout()
```

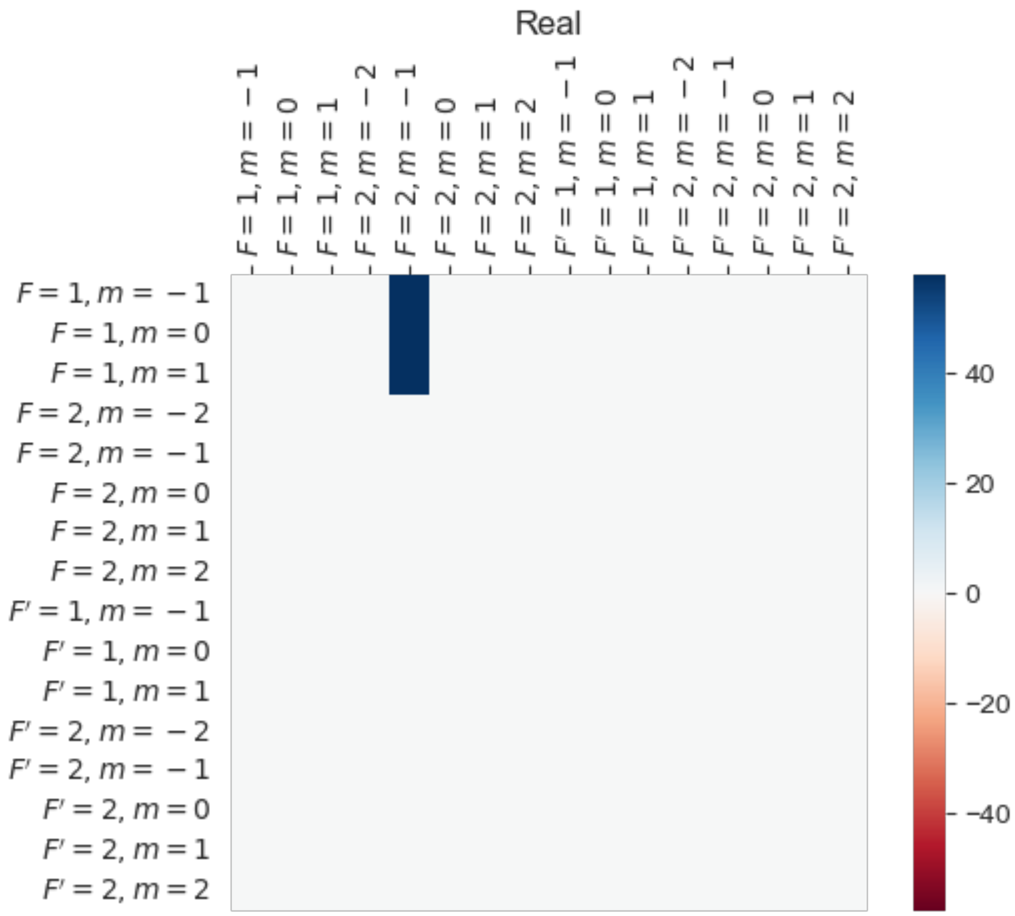


Decay Inter F Ground States

In [ ]:
```
maplot(sum(F2_to_F1_decay_ops))
maplot(F2_to_F1_decay_ops[-1])
```

Out[ ]: (<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)





Radiative and F to F and F=2 to F=1 and Quenching

In [ ]:
```
L = liouvillian(
    hamil,
    c_ops=natural_decay_ops + intra_F1 + intra_F2 + F2_to_F1_decay_ops + quenching_ops,
)
```

Real part

In [ ]:
```
import plotly.express as px

y = L.full().real
fig = px.imshow(
    y,
    color_continuous_midpoint=0,
    aspect="equal",
    width=1.5 * 800,
    height=1.5 * 400,
    zmin=-(abs(y).max()),
    zmax=(abs(y).max()),
    color_continuous_scale="RdBu",
)
fig.show()
```

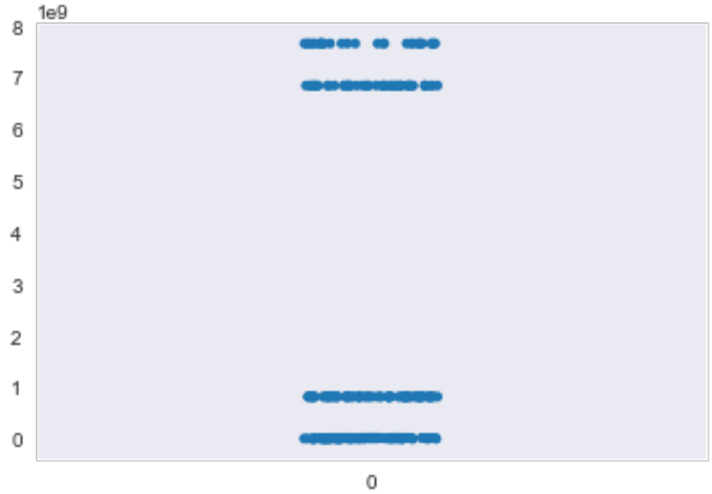
Imag part

In [ ]:
```
import plotly.express as px

y = L.full().imag
fig = px.imshow(
    y,
    color_continuous_midpoint=0,
    aspect="equal",
    width=1.5 * 800,
    height=1.5 * 400,
    zmin=-(abs(y).max()),
    zmax=(abs(y).max()),
    color_continuous_scale="RdBu",
)
fig.show()
```

In [ ]:
```
L_eigvals, L_states = L.eigenstates()
```

In [ ]:
```
sns.stripplot(data=abs(L_eigvals))
```

Out[ ]:
```
<AxesSubplot:>
```



In [ ]:
```
abs(L_eigvals).min()
```

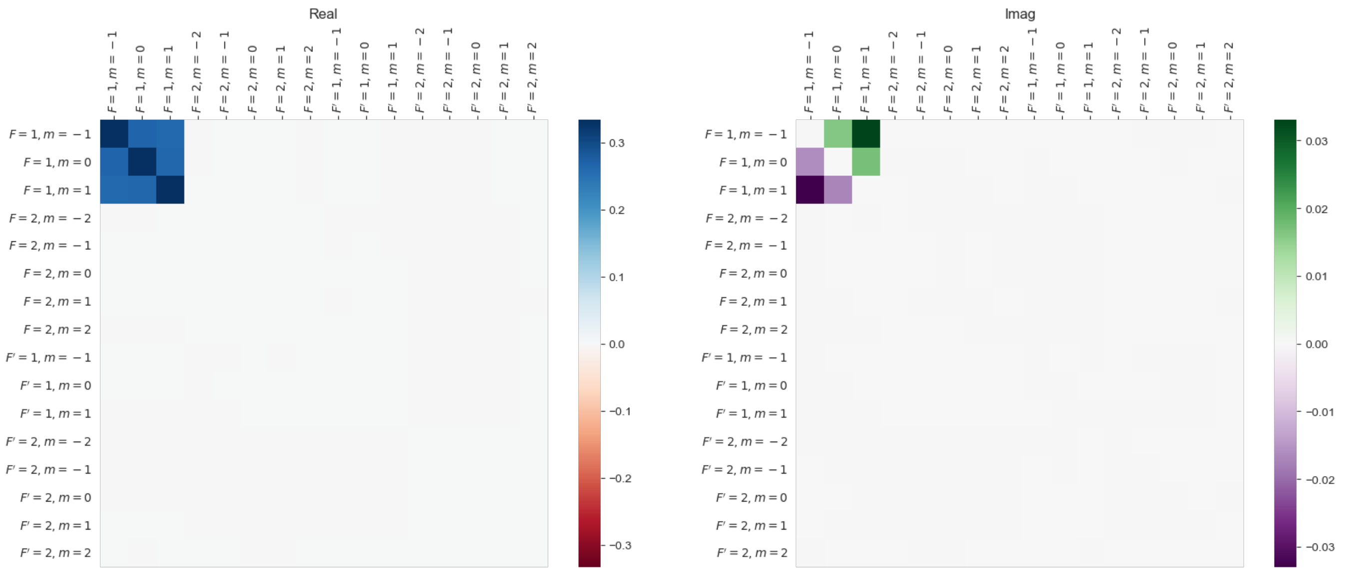
Out[ ]:
```
1.6595678440361655e-06
```

In [ ]:
```
sorted(abs(L_eigvals))[:10]
```

Out[ ]:
```
[1.6595678440361655e-06,
 13199.28604533847,
 16703.689232017874,
 16703.689232053337,
 16705.736006122115,
 16705.73600825457,
 16803.15125194996,
 16803.151253039796,
 20003.853581695494,
 20006.237595261016]
```

In [ ]:
```
r_zero = L_states[abs(L_eigvals).argmin()]
r_zero = vector_to_operator(r_zero)
maplot(r_zero / r_zero.tr())
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



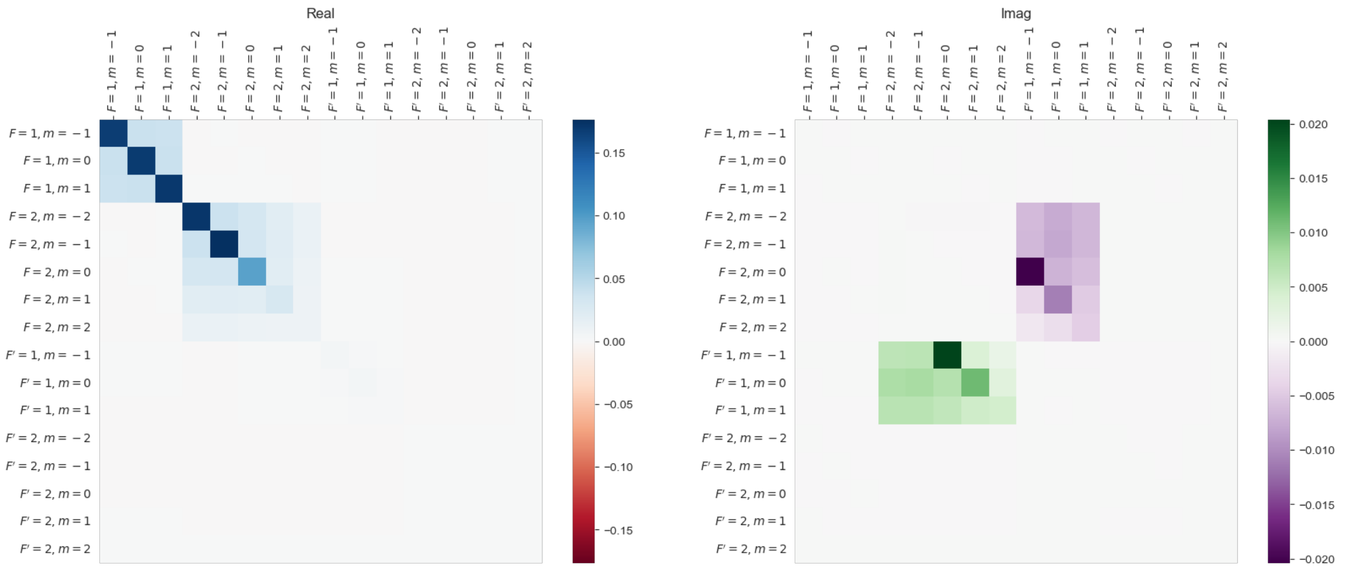
Time Evo

In [ ]:
```python
rho_zero = sum([basis(16, i).proj() for i in range(8)]).unit()  # equally distrib
```

In [ ]:
```python
times = np.linspace(0, 1e-6, 1001)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    L,
    rho_zero,
    times,
    options=opts,
)
```

In [ ]:
```python
maplot(res.states[-1])
```

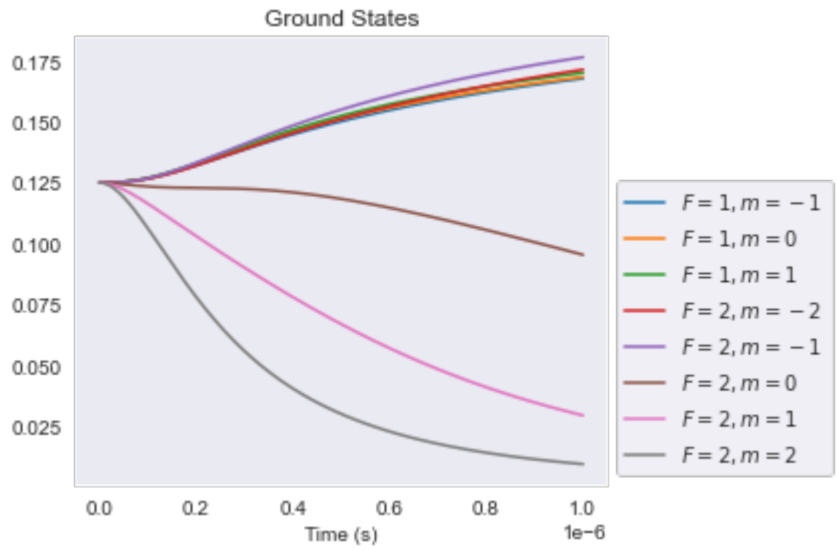
Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
res.states[-1].tr()
```

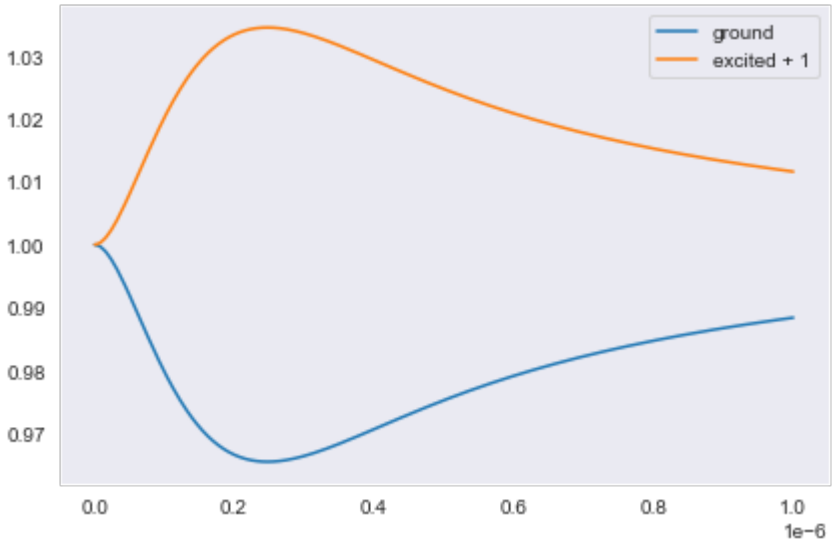
Out[ ]:
```
0.9999999999999986
```

In [ ]:
```
ground_exp_even = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp_even):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



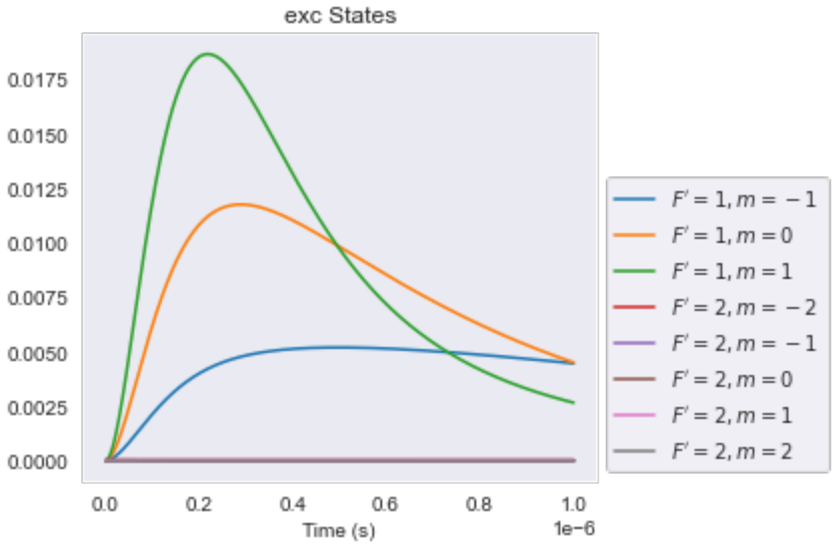
In [ ]:
```
ground_state_tot = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
tot_excited_exp_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
fig, ax = plt.subplots()
ax.plot(times, ground_state_tot, label="ground")
ax.plot(times, tot_excited_exp_even, label="excited + 1")
ax.legend()
plt.tight_layout()
```

```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\matplotlib\cbook\__init__.py:1298: ComplexWarning:

Casting complex values to real discards the imaginary part
```



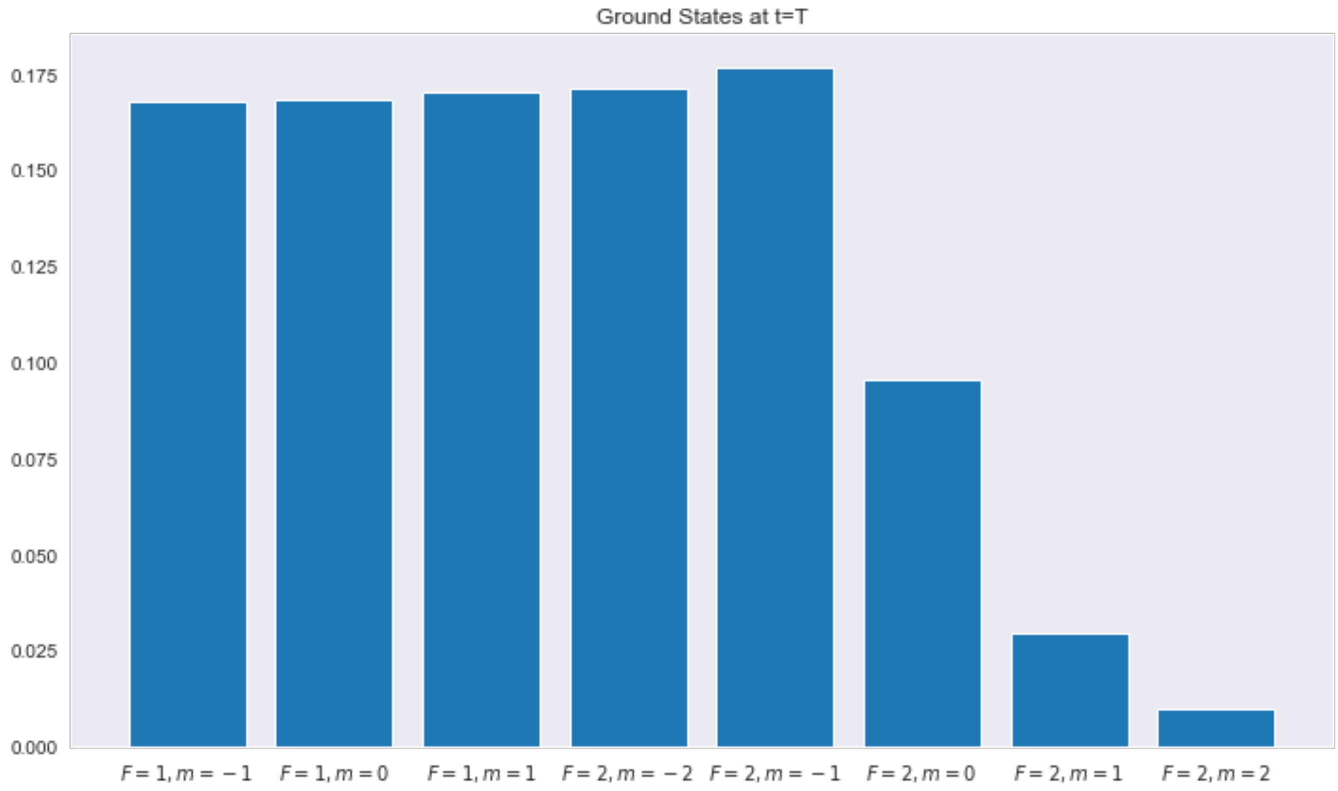
In [ ]:

```
exc_expeven = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for k, e in enumerate(exc_expeven):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("exc States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

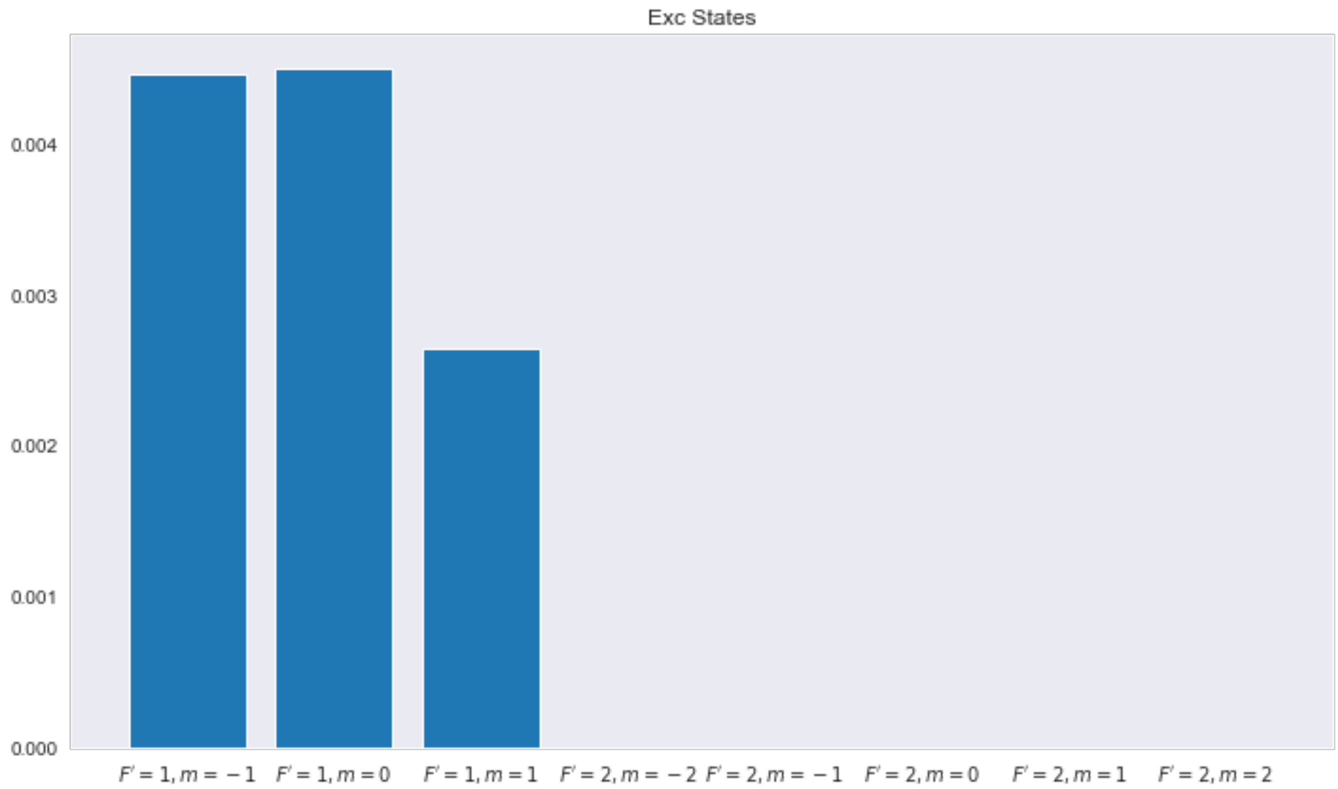


In [ ]:

```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)],
    [np.real(e)[-1] for e in ground_exp_even],
)
plt.title("Ground States at t=T")
plt.tight_layout()
```



In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    [np.real(e)[-1] for e in exc_expeven],
)
plt.title("Exc States")
plt.tight_layout()
```



Longer Time Evo

In [ ]:
```
rho_zero = sum([basis(16, i).proj() for i in range(8)]).unit()  # equally distrib
```

In [ ]:

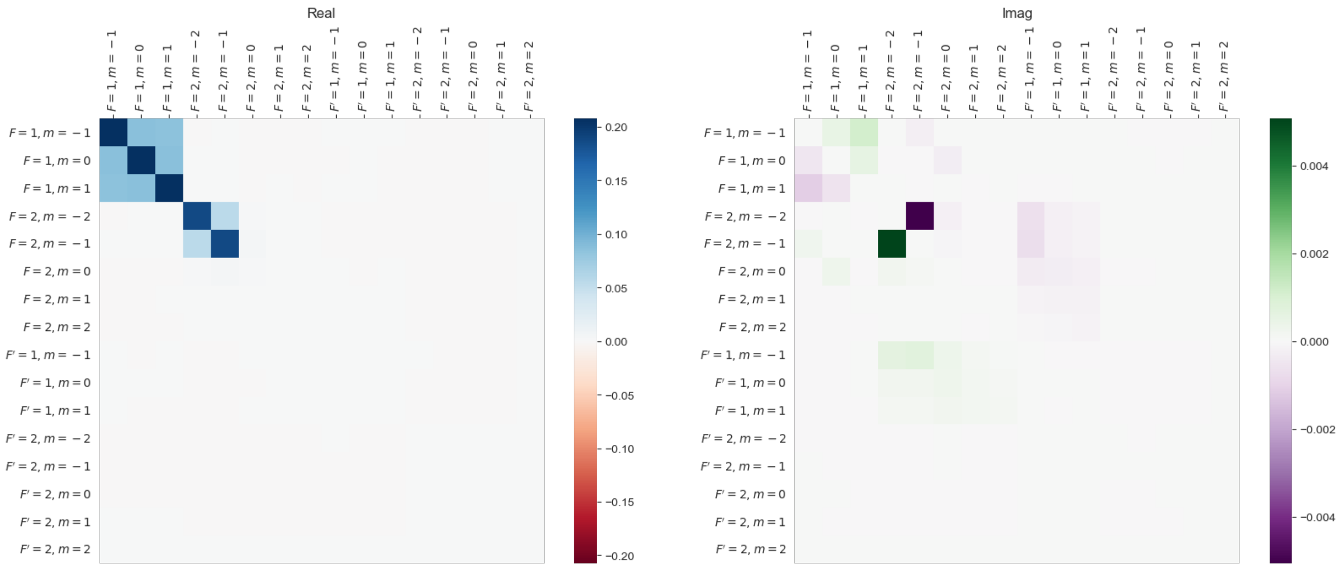
```
times = np.linspace(0, 1e-5, 1001)
opts = Options(nsteps=1 * 10**4)
res = mesolve(
    L,
    rho_zero,
    times,
    options=opts,
)
```

In [ ]:

```
maplot(res.states[-1])
```

Out[ ]:

```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:

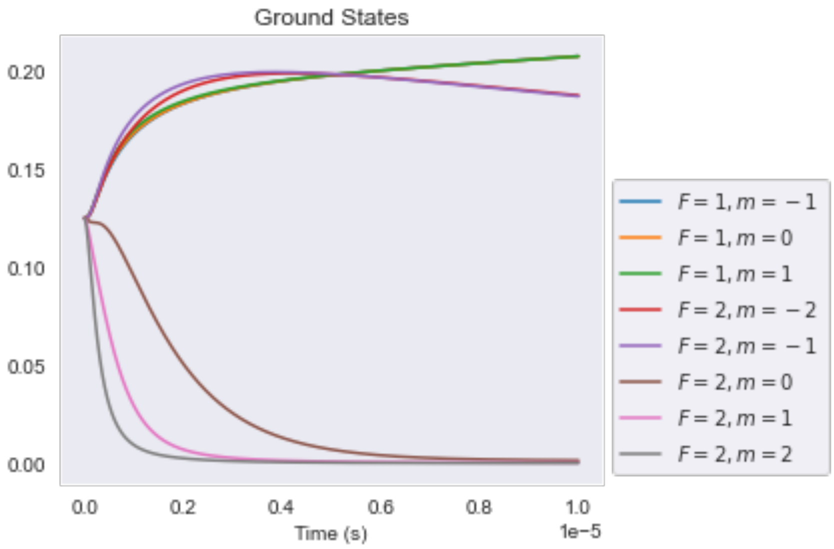
```
res.states[-1].tr()
```

Out[ ]:

```
1.0000000000000049
```

In [ ]:

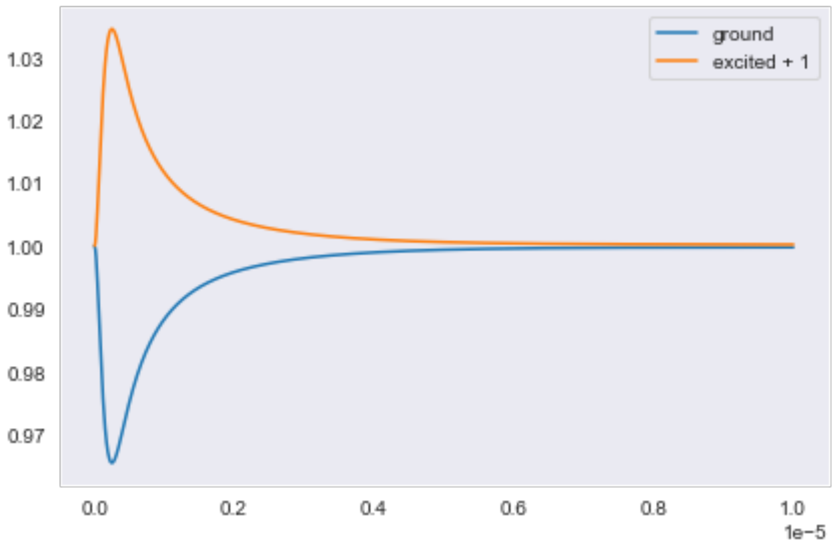
```
ground_exp_even = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp_even):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



In [ ]:

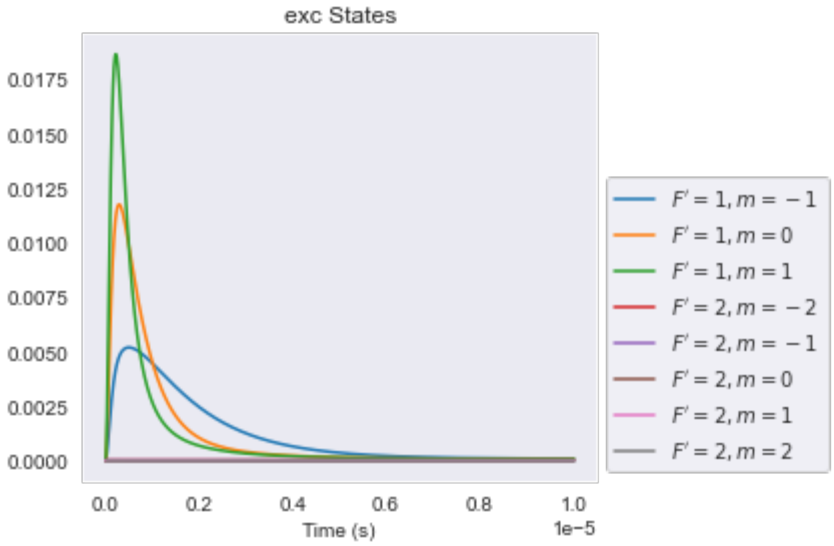
```python
ground_state_tot = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
tot_excited_exp_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
fig, ax = plt.subplots()
ax.plot(times, ground_state_tot, label="ground")
ax.plot(times, tot_excited_exp_even, label="excited + 1")
ax.legend()
plt.tight_layout()
```

```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\matplotlib\cbook\__init__.p
y:1298: ComplexWarning:

Casting complex values to real discards the imaginary part
```



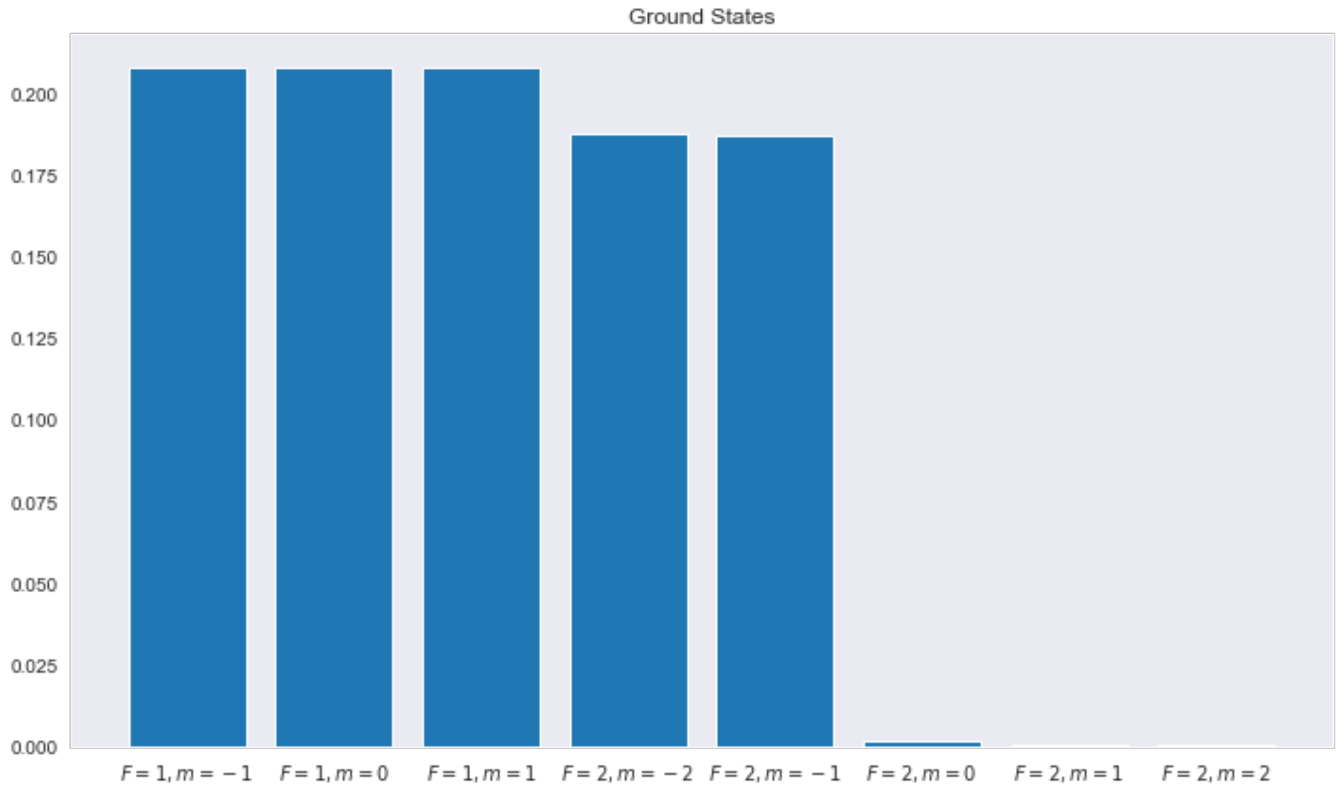
In [ ]:

```python
exc_expeven = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for k, e in enumerate(exc_expeven):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("exc States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



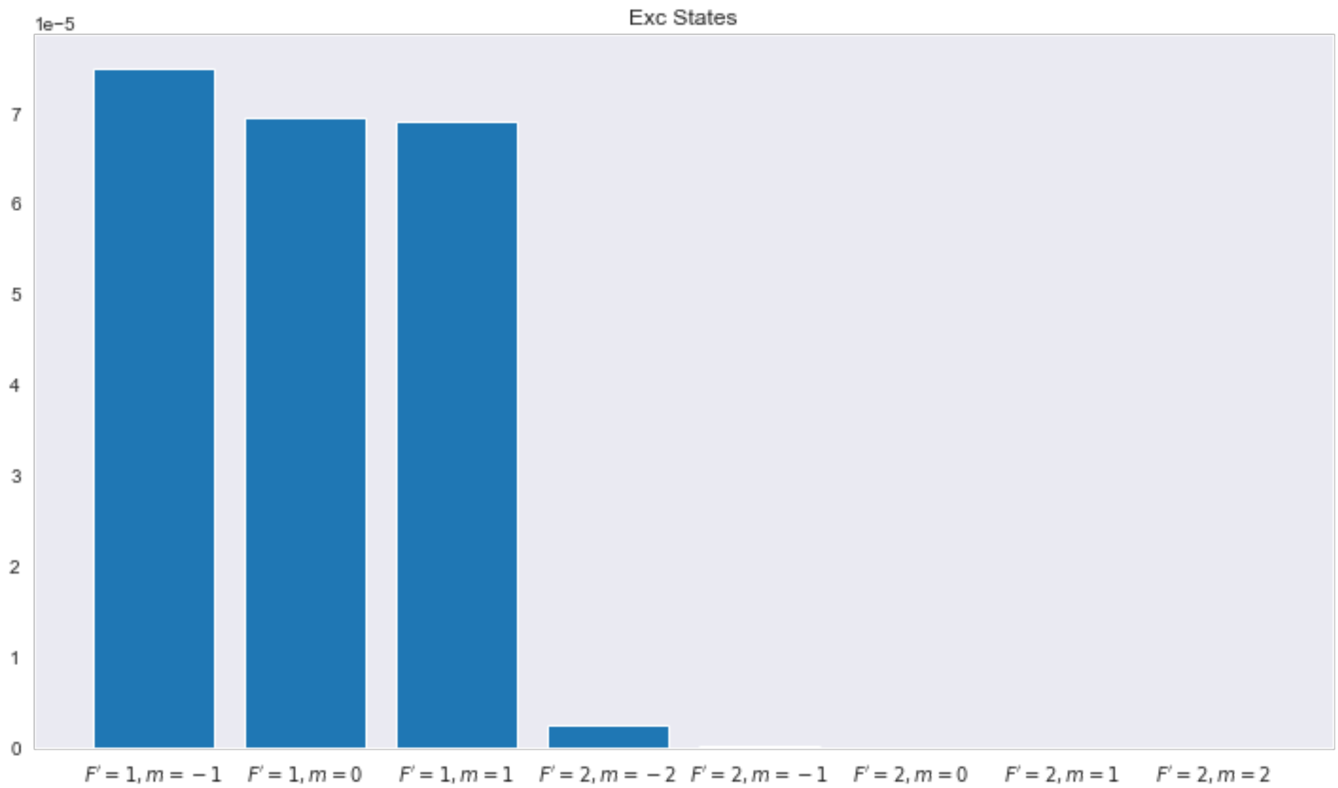
In [ ]:

```python
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)],
    [np.real(e)[-1] for e in ground_exp_even],
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:

```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    [np.real(e)[-1] for e in exc_expeven],
)
plt.title("Exc States")
plt.tight_layout()
```



$$\rho_0 = |F=2, m_F=-2\rangle$$

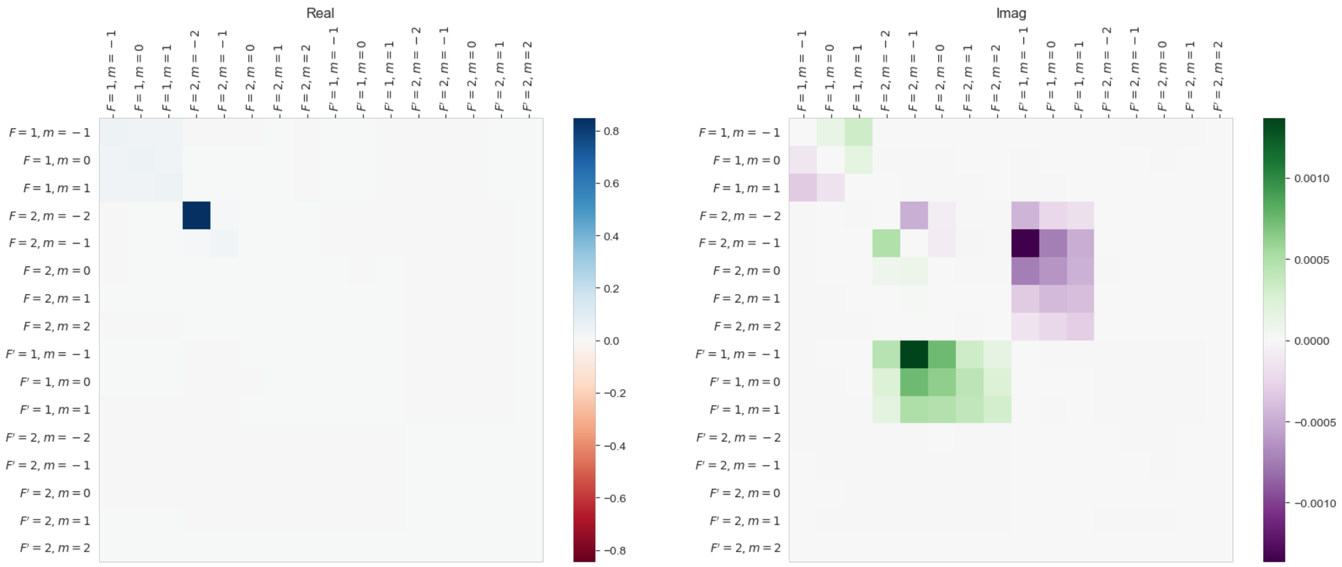
In [ ]:

```
rho_zero = sum([basis(16, 3).proj() for i in range(8)]).unit()
```

In [ ]:
```python
times = np.linspace(0, 1e-5, 1001)
opts = Options(nsteps=1 * 10**4)
res = mesolve(
    L,
    rho_zero,
    times,
    options=opts,
)
```

In [ ]:
```python
maplot(res.states[-1])
```

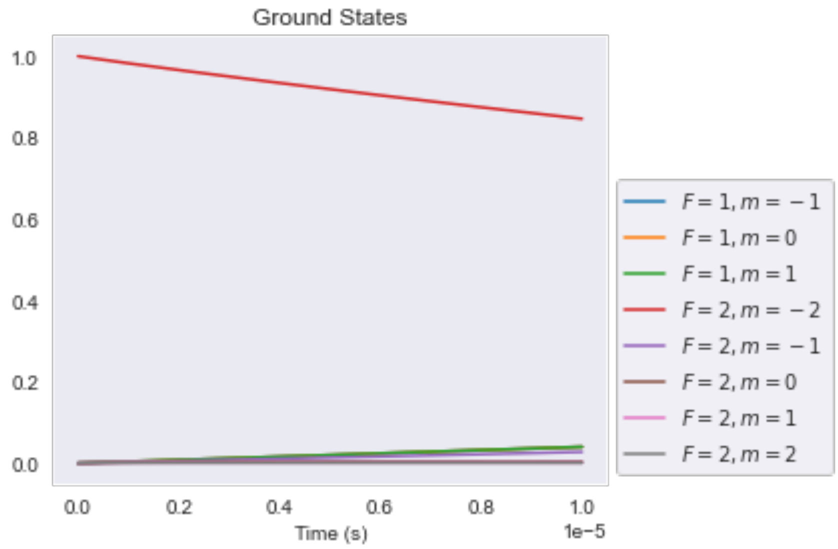
Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



In [ ]:
```python
res.states[-1].tr()
```

Out[ ]:
```
1.000000000000019
```

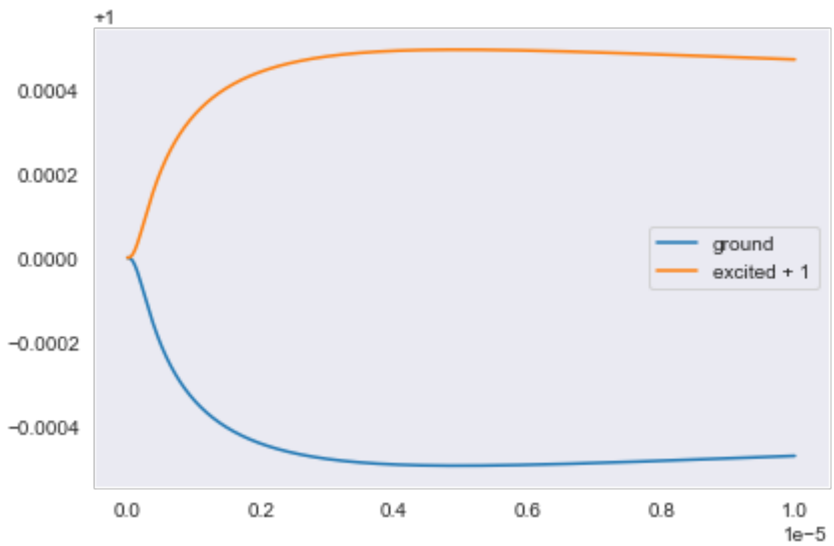
In [ ]:
```python
ground_exp_even = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
plt.figure()
for i, e in enumerate(ground_exp_even):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
)
plt.title("Ground States")
plt.xlabel("Time (s)")

plt.tight_layout()
```



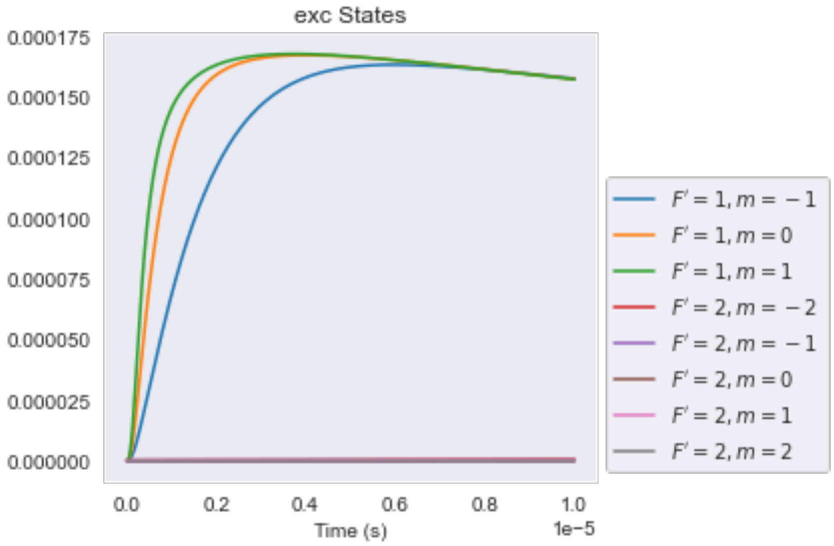
In [ ]:

```
ground_state_tot = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8)
        ]
    )
    for t in range(len(times))
]
tot_excited_exp_even = [
    sum(
        [
            res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
            for i in range(8, 16)
        ]
    )
    + 1
    for t in range(len(times))
]
fig, ax = plt.subplots()
ax.plot(times, ground_state_tot, label="ground")
ax.plot(times, tot_excited_exp_even, label="excited + 1")
ax.legend()
plt.tight_layout()
```

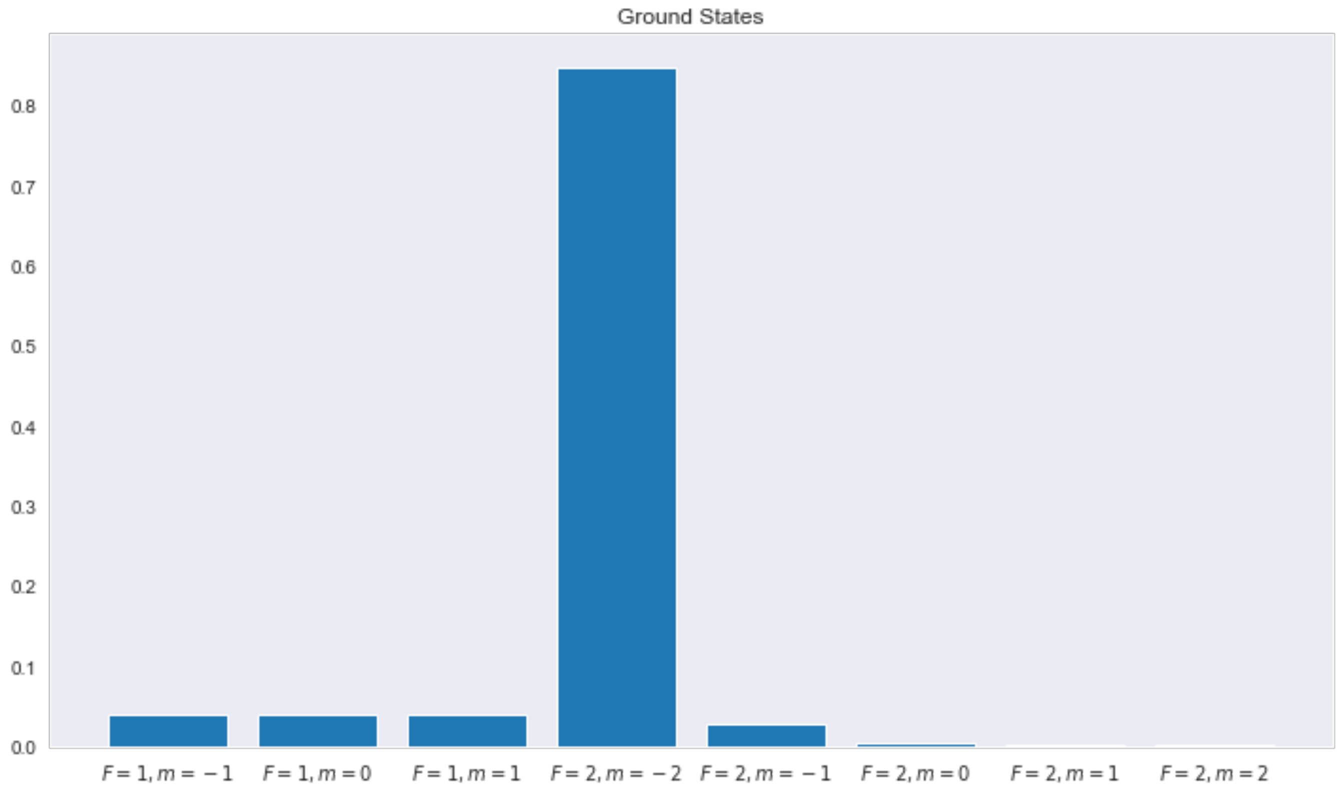
```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\matplotlib\cbook\__init__.p
y:1298: ComplexWarning:

Casting complex values to real discards the imaginary part
```



In [ ]:
```
exc_expeven = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8, 16)
]
plt.figure()
for k, e in enumerate(exc_expeven):
    plt.plot(times, np.real(e))
plt.legend(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    loc="best",
    bbox_to_anchor=(1.0, 0.7),
)
plt.title("exc States")
plt.xlabel("Time (s)")

plt.tight_layout()
```

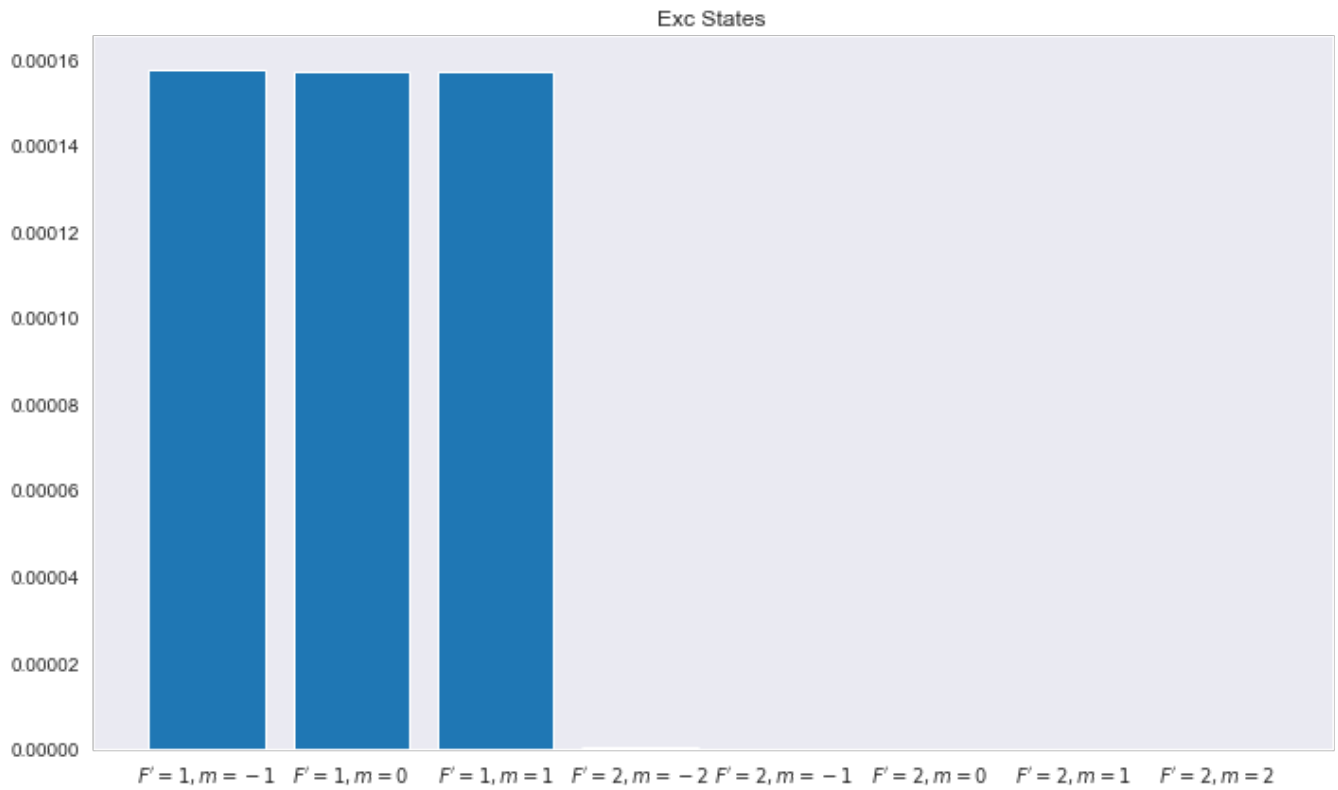


In [ ]:
```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8)],
    [np.real(e)[-1] for e in ground_exp_even],
)
plt.title("Ground States")
plt.tight_layout()
```



In [ ]:

```
plt.figure(figsize=(10, 6))
plt.bar(
    [index_to_F_mF_string(i) for i in range(8, 16)],
    [np.real(e)[-1] for e in exc_expeven],
)
plt.title("Exc States")
plt.tight_layout()
```



MW Transition

Init

```python
In [ ]: A = 3.417341305452145e09  # Hz
MU_BOHR = 1.399624604e6  # Hz / G

# uncoupled basis
H_hfs = A * (
    tensor(spin_Jz(3 / 2), spin_Jz(1 / 2))  # I * J (S_1/2, where L=0)
    + tensor(spin_Jy(3 / 2), spin_Jy(1 / 2))
    + tensor(spin_Jx(3 / 2), spin_Jx(1 / 2))
)


def H_B(bx=0, by=0, bz=0):  # in G
    return (
        2.0023193043622
        * (
            tensor(qeye(4), spin_Jx(1 / 2)) * bx
            + tensor(qeye(4), spin_Jy(1 / 2)) * by
            + tensor(qeye(4), spin_Jz(1 / 2)) * bz
        )
        - 0.000995
        * (
            tensor(spin_Jx(3 / 2), qeye(2)) * bx
            + tensor(spin_Jy(3 / 2), qeye(2)) * by
            + tensor(spin_Jz(3 / 2), qeye(2)) * bz
        )
    ) * MU_BOHR


energs, F_states = (H_hfs + H_B(bz=0.1)).eigenstates()
F_states_reordered = [
    F_states[2],
    F_states[1],
    F_states[0],
]  # to have same basis in the same order: |F=1, m=-1>, |F=1, m=0>, ...
for k in range(3, 3 + 5):
    F_states_reordered.append(F_states[k])

# RWA
def h_mw_a(det_mw):
    return sum([(ens[f] - det_mw) * basis(8, f).proj() for f in range(3, 8)]) + sum(
        [ens[f1] * basis(8, f1).proj() for f1 in range(3)]
    )


def B_loop(power_mw, radius=3e-3, distance=0.02):  # in Gauss
    return (
        constants.mu_0
        * radius**2
        * (power_mw / 50) ** (1 / 2)
        / (2 * (distance**2 + radius**2) ** (3 / 2))
        * 1e4
    )


Bmw = B_loop(1e-1)
# transition_coeffs = tensor(qeye(4), spin_Jx(1 / 2)).transform(F_states_reordered)
# det_mw = 0
# rabi_mw = 1e6
H_0 = H_hfs + H_B(bz=0.1)
E_0 = H_0.eigenenergies()[5] - H_0.eigenenergies()[1]  # clock transition
ens = H_0.eigenenergies() - H_0.eigenenergies()[1]
ens = [en if k < 3 else en - E_0 for k, en in enumerate(ens)]
tmp = ens[0]
ens[0] = ens[2]
ens[2] = tmp
```

```python
In [ ]: ens
```

Out[ ]:
```
[70237.18127632141,
 0.0,
 -70235.74342823029,
 -139918.74989700317,
 -69958.65598106384,
 0.0,
 69957.21813106537,
 139912.998503685]
```

In [ ]:
```
139912.998503685 - (-70237.18127632141)
```

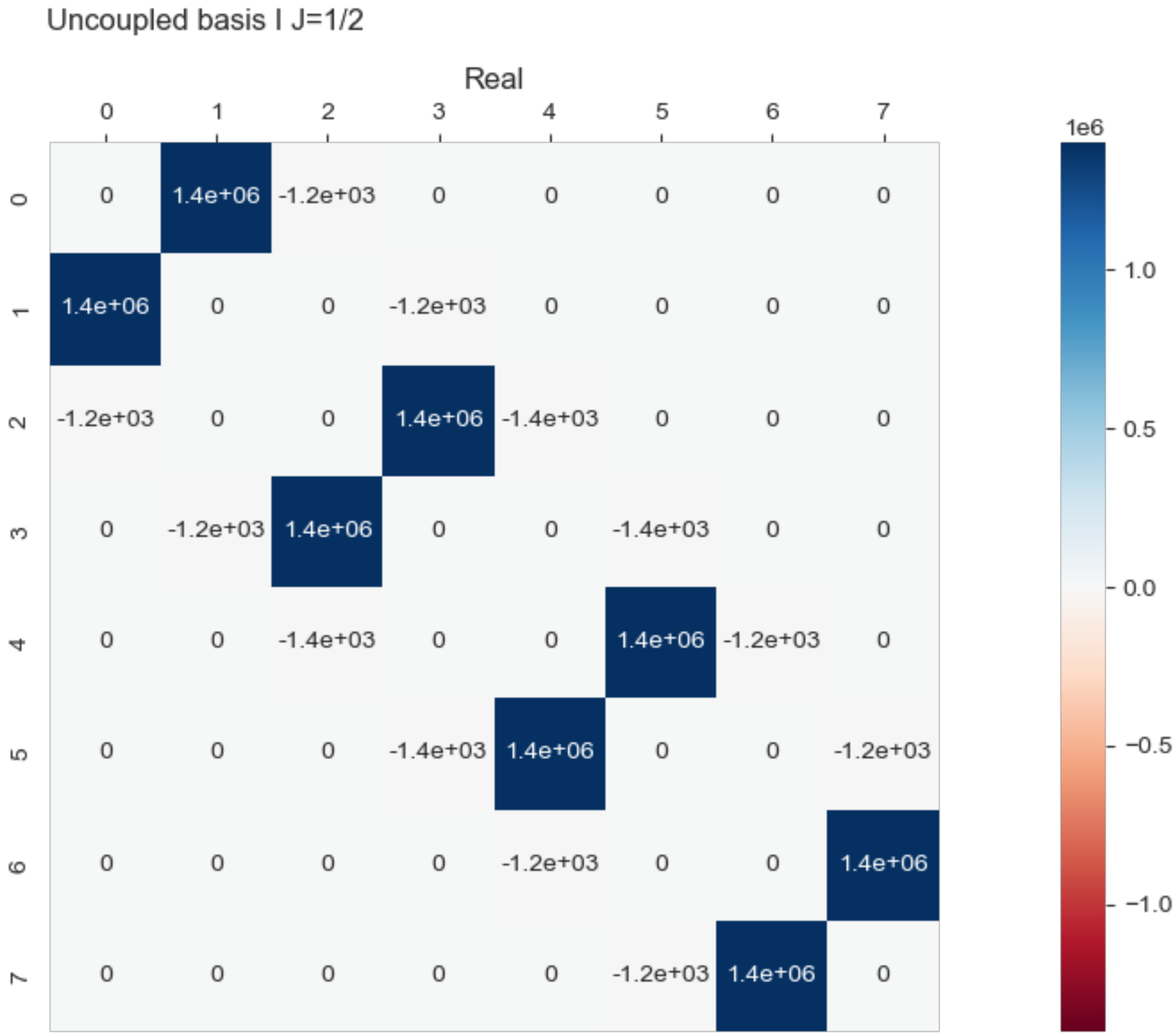
Out[ ]:
```
210150.1797800064
```

In [ ]:
```
3 * 70235.74342823029
```

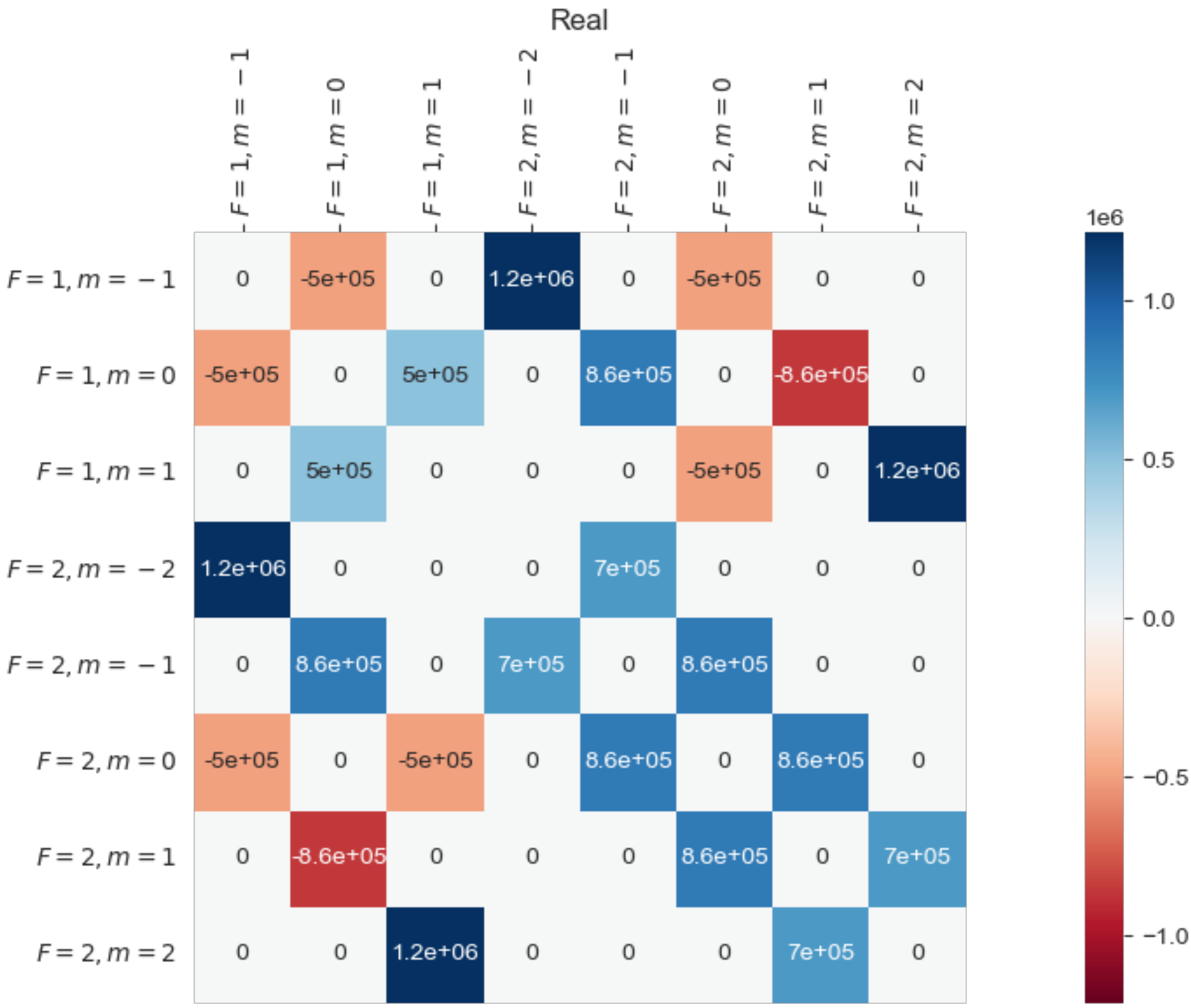
Out[ ]:
```
210707.23028469086
```

MW Hamiltonian I, J basis and F basis

In [ ]:
```
hbt = H_B(bx=1, by=0.0, bz=0.0)
fig, _ = matrixplot(hbt, annot=True)
fig.set_size_inches(14, 6)
fig.suptitle("Uncoupled basis I J=1/2")
plt.tight_layout()
fig, _ = maplot(hbt.transform(F_states_reordered), annot=True)
fig.set_size_inches(14, 6)
plt.tight_layout()
```

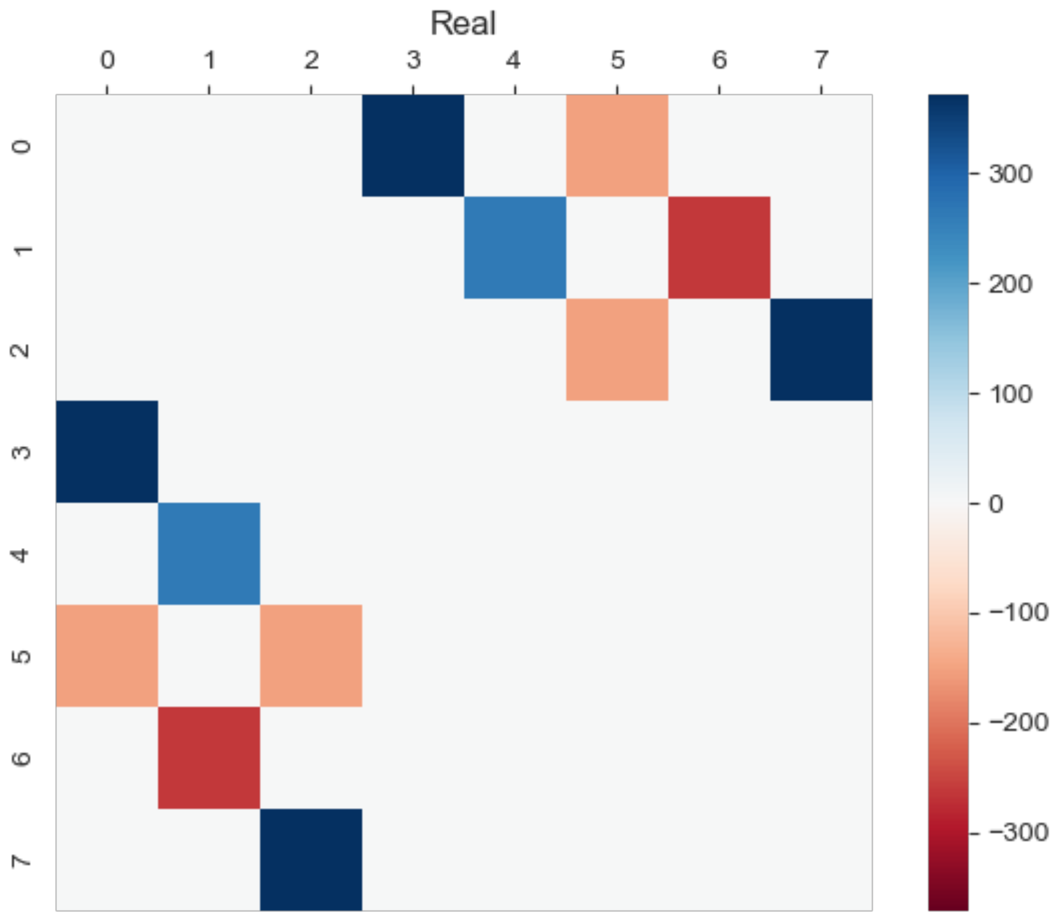


	0	1	2	3	4	5	6	7
0	0	1.4e+06	-1.2e+03	0	0	0	0	0
1	1.4e+06	0	0	-1.2e+03	0	0	0	0
2	-1.2e+03	0	0	1.4e+06	-1.4e+03	0	0	0
3	0	-1.2e+03	1.4e+06	0	0	-1.4e+03	0	0
4	0	0	-1.4e+03	0	0	1.4e+06	-1.2e+03	0
5	0	0	0	-1.4e+03	1.4e+06	0	0	-1.2e+03
6	0	0	0	0	-1.2e+03	0	0	1.4e+06
7	0	0	0	0	0	-1.2e+03	1.4e+06	0



In [ ]:

```
# remove fast rotating terms in the MW hamiltonian (Zeeman-level transitions)
hmw_arr = H_B(bx=Bmw).transform(F_states_reordered).full()
hmw_arr_rwa = hmw_arr.copy()
for n in range(8):
    for m in range(8):
        if abs(n - m) < 2:
            hmw_arr_rwa[n, m] = 0.0
matrixplot(hmw_arr_rwa)
H_mw = Qobj(hmw_arr_rwa) + h_mw_a(139912.998503685 - (-70237.18127632141))
H_mw_fl = Qobj(tensor(Qobj([[1, 0], [0, 0]]), H_mw).full())
```

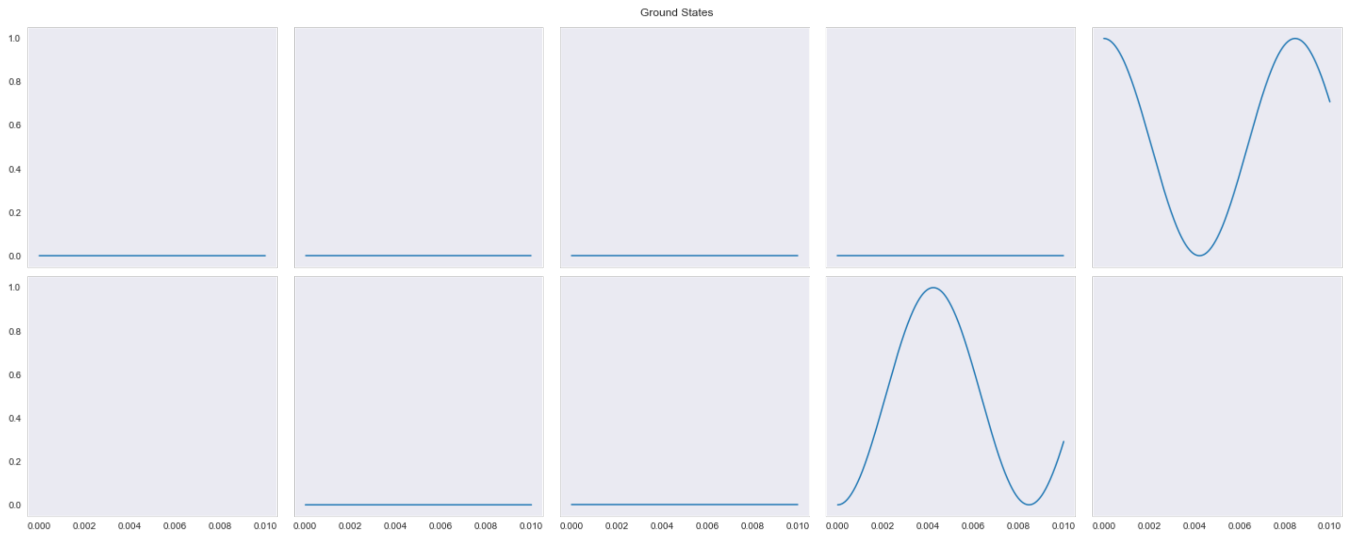


No Light

```
In [ ]: L = liouvillian(
            H_mw_fl,
        )
```

Time Evo

In [ ]:
```
rho_zero = basis(16, 7).proj()
times = np.linspace(0, 1e-2, 1001)
opts = Options(nsteps=1 * 10**3)
res = mesolve(
    L,
    rho_zero,
    times,
    options=opts,
)

ground_exp_val = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
fig, axs = plt.subplots(ncols=5, nrows=2, figsize=(20, 8), sharex="all", sharey="all")
for i, e in enumerate(ground_exp_val[:3]):
    axs[1, 1 + i].plot(times, np.real(e))
for i, e in enumerate(ground_exp_val[3:8]):
    axs[0, i].plot(times, np.real(e))
# fig.delaxes(axs[1][0])
# fig.delaxes(axs[1][-1])
# ax.legend(
#     [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
# )
fig.suptitle("Ground States")
# ax.set_xlabel("Time (s)")

plt.tight_layout()
```

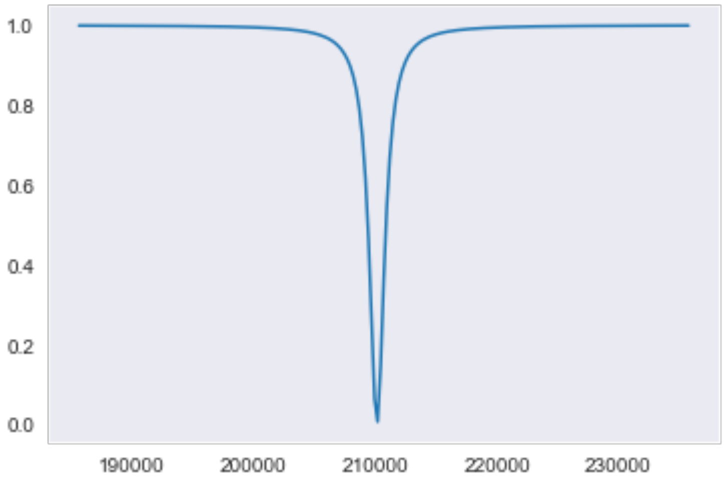


Time Evolution as a Function of Frequency: $\min\langle F=2, m_F=2\rangle(f)$

In [ ]:
```python
frequencies = np.linspace(
    3 * 70235.74342823029 - 25000, 3 * 70235.74342823029 + 25000, 201
)
rho_zero = basis(16, 7).proj()
expectation_values = []
times = np.linspace(0, 1e-2, 1001)
opts = Options(nsteps=1 * 10**5)
for freq in frequencies:
    H_mw = Qobj(H_B(bx=Bmw).transform(F_states_reordered).full()) + h_mw_a(freq)
    H_mw_fl = Qobj(tensor(Qobj([[1, 0], [0, 0]]), H_mw).full())
    L = liouvillian(
        0 * hamil + H_mw_fl,
        # c_ops=intra_F1 + intra_F2,
    )
    res = mesolve(L, rho_zero, times, options=opts, e_ops=rho_zero)
    expectation_values.append(res.expect[0].min())
```

In [ ]:
```python
plt.plot(frequencies, expectation_values)
```

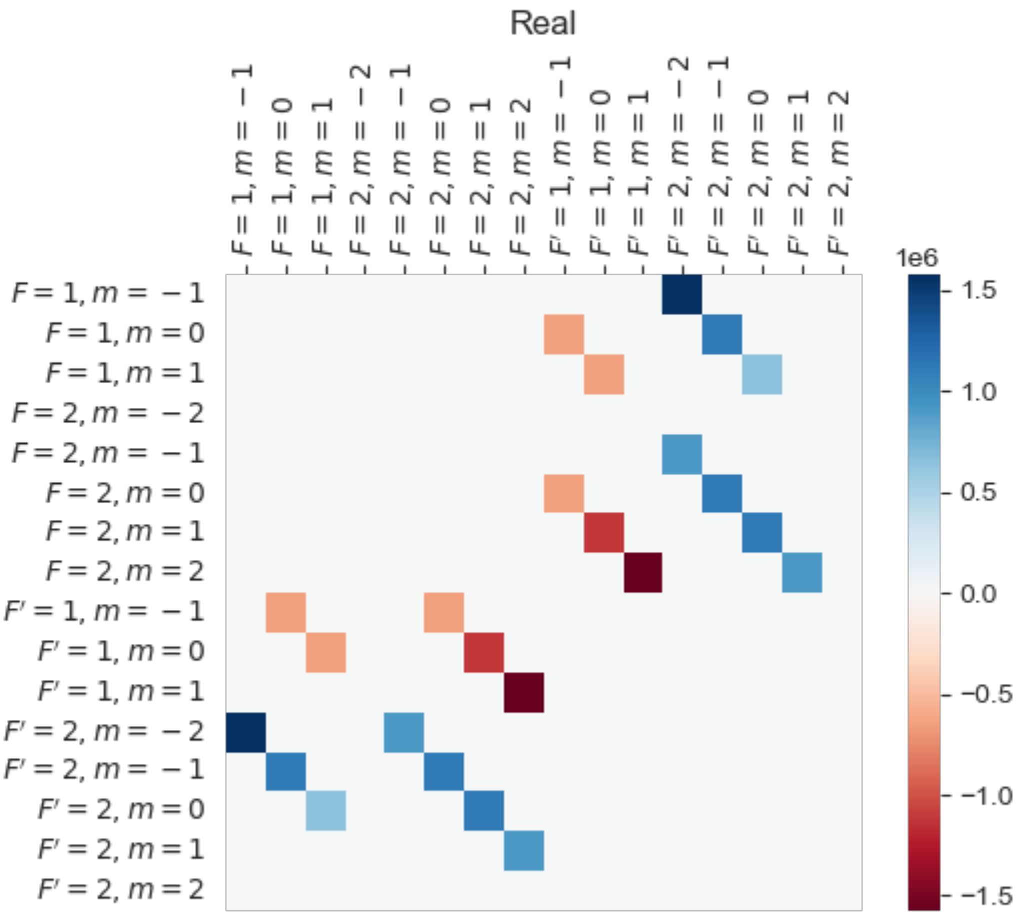
Out[ ]:
```
[<matplotlib.lines.Line2D at 0x2c15cd23dc0>]
```



With Light, rad decay

In [ ]:
```python
laser_intens = 0.1 * off_resonant_saturation_intensity_D1_pi_pol
maplot(H_AF(q=1, intens=laser_intens))
```

Out[ ]:
```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```

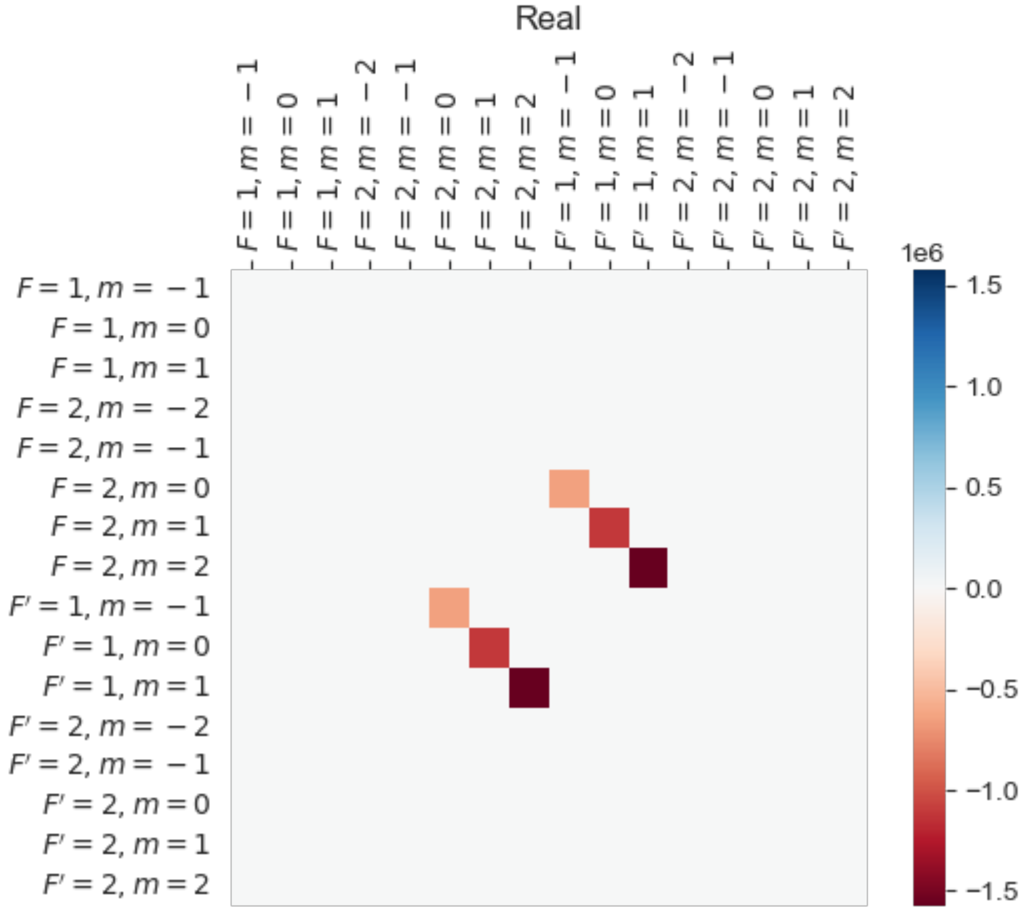


In [ ]:

```
h_af_fp1_only = sum(
    [
        basis(16, f).proj() * H_AF(q=1, intens=laser_intens) * basis(16, i).proj()
        for i in range(8, 11)
        for f in range(3, 8)
    ]
)
h_af_fp1_only += h_af_fp1_only.dag()
maplot(h_af_fp1_only)
```

Out[ ]:

```
(<Figure size 600x480 with 2 Axes>, <AxesSubplot:title={'center':'Real'}>)
```



In [ ]:
```
hamil = h_af_fp1_only + Ha(-509.06e6 - 2.563005979089109e9)  # resonant to F=2 -> F'=1
L = liouvillian(
    hamil + H_mw_fl,
    # c_ops=intra_F1 + intra_F2,
    c_ops=natural_decay_ops,
)
inds = np.argsort(abs(L.eigenstates()[0]))
L_eigs_sorted = L.eigenstates()[0][inds]
L_states_sorted = L.eigenstates()[1][inds]
```

In [ ]:
```
sns.stripplot(data=abs(L_eigs_sorted))
```

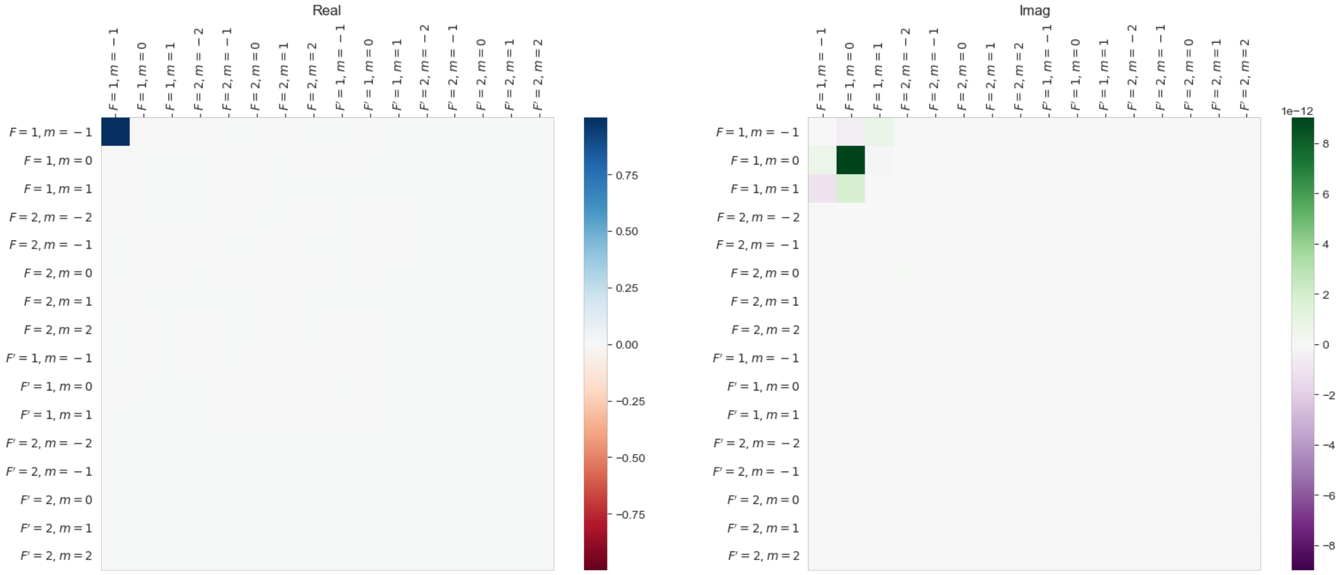
Out[ ]:
```
<AxesSubplot:>
```

In [ ]:
```
L_eigs_sorted[:10]
```

Out[ ]:
```
array([ 1.10825080e-07-2.75603964e-07j, -1.00464099e-06-1.23669605e-09j,
        9.94206958e-07-4.79980591e-07j,  2.38146493e-07-1.62599831e-06j,
       -6.10778480e-01+3.75139823e-07j, -4.19915408e-01+6.99612395e+04j,
       -4.19915603e-01-6.99612395e+04j, -2.45392223e-07-7.02360717e+04j,
       -8.41651581e-07+7.02360717e+04j, -3.20664691e-07-7.02375096e+04j])
```

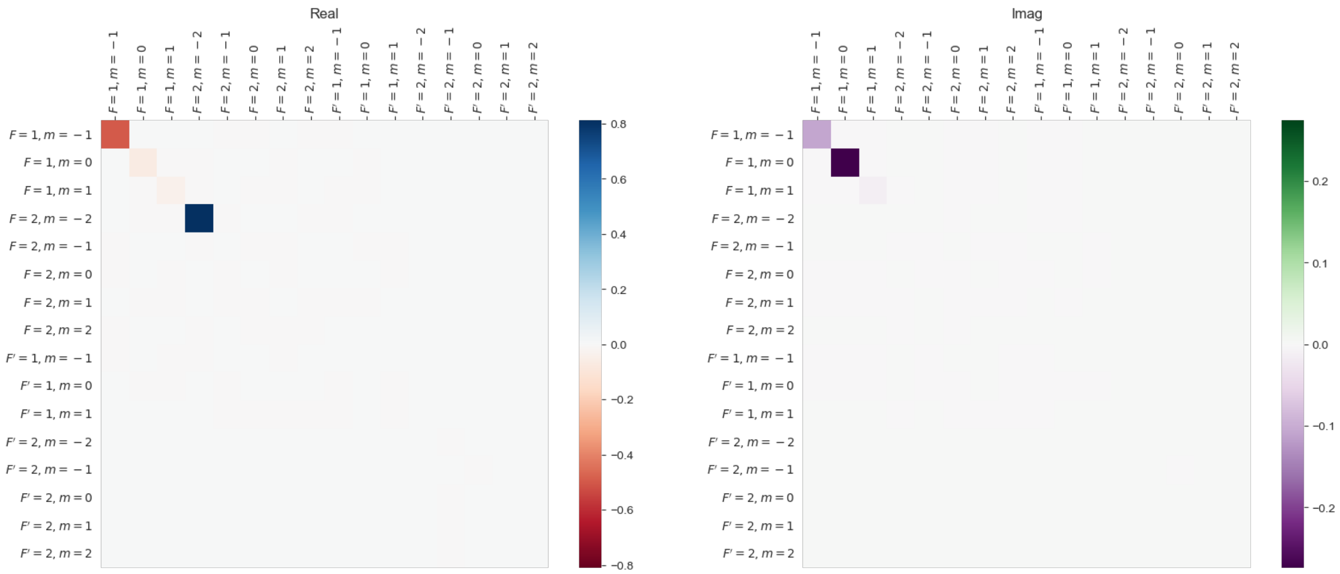
In [ ]:
```
maplot(vector_to_operator(L_states_sorted[0]))
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



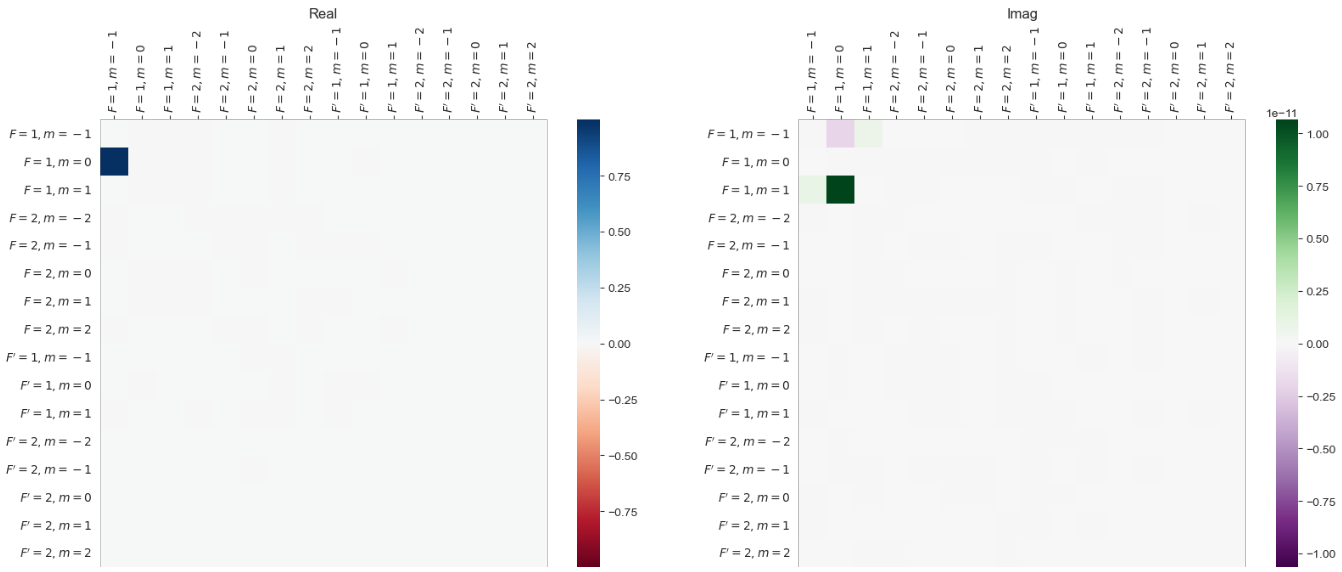
In [ ]:
```
maplot(vector_to_operator(L_states_sorted[1]))
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



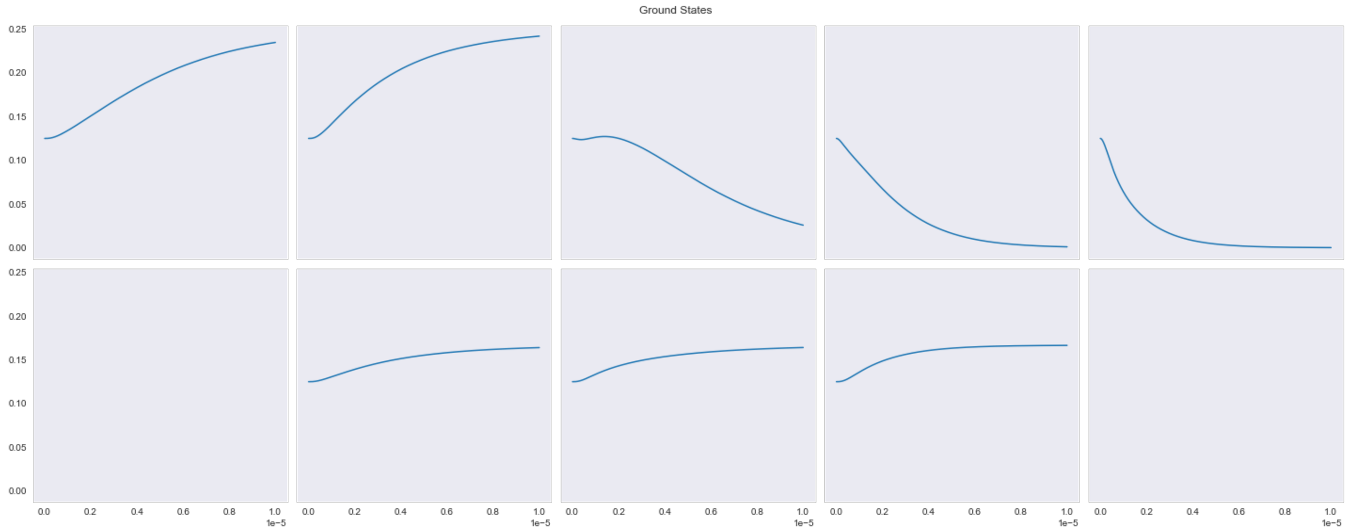
In [ ]:
```
maplot(vector_to_operator(L_states_sorted[10]))
```

Out[ ]:
```
(<Figure size 1680x672 with 4 Axes>,
 [<AxesSubplot:title={'center':'Real'}>,
  <AxesSubplot:title={'center':'Imag'}>])
```



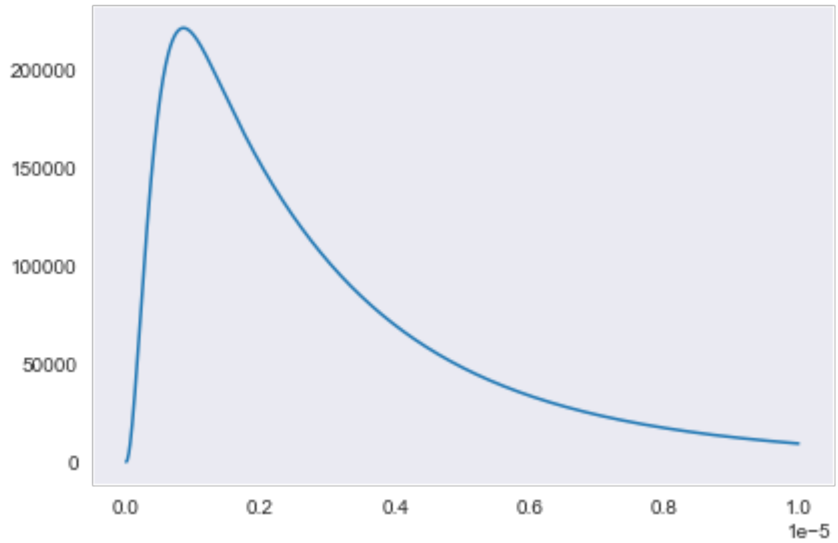
Time Evo

```
In [ ]: rho_zero = sum([basis(16, i).proj() for i in range(8)]).unit()  # equally distrib

        # rho_zero = basis(16, 7).proj()

        times = np.linspace(0, 1e-5, 1001)
        opts = Options(nsteps=1 * 10**4)
        res = mesolve(
            L,
            rho_zero,
            times,
            options=opts,
        )

        # maplot(res.states[-1])

        ground_exp_val = [
            [
                res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
                for t in range(len(times))
            ]
            for i in range(8)
        ]
        fig, axs = plt.subplots(ncols=5, nrows=2, figsize=(20, 8), sharex="all", sharey="all")
        for i, e in enumerate(ground_exp_val[:3]):
            axs[1, 1 + i].plot(times, np.real(e))
        for i, e in enumerate(ground_exp_val[3:8]):
            axs[0, i].plot(times, np.real(e))
        # fig.delaxes(axs[1][0])
        # fig.delaxes(axs[1][-1])
        # ax.legend(
        #     [index_to_F_mF_string(i) for i in range(8)], loc="best", bbox_to_anchor=(1.0, 0.7)
        # )
        fig.suptitle("Ground States")
        # ax.set_xlabel("Time (s)")

        plt.tight_layout()
```



In [ ]:
```
absorbed_mw = [
    (sum([basis(16, k).proj() for k in range(8, 16)]) * rho_t).tr() * gamma_natural
    for rho_t in res.states
]
fig, ax = plt.subplots()
ax.plot(times, absorbed_mw)
plt.tight_layout()
```

```
c:\Users\m\anaconda3\envs\masterarbeit_python39\lib\site-packages\matplotlib\cbook\__init__.p
y:1298: ComplexWarning:

Casting complex values to real discards the imaginary part
```

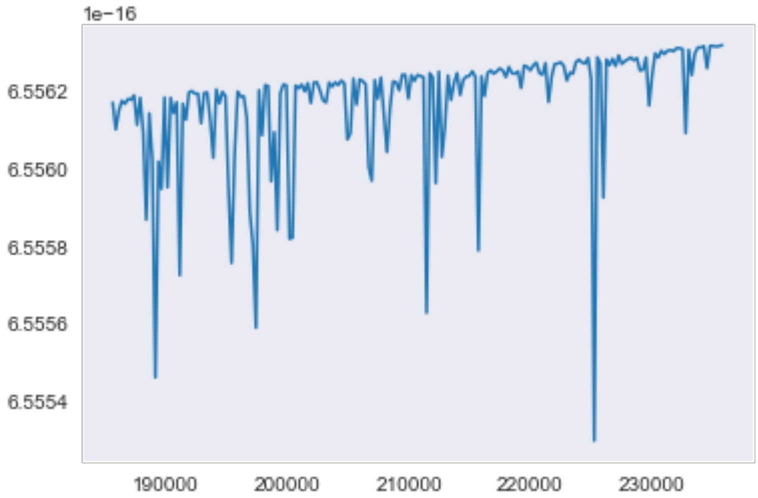


Time Evolution as a Function of Frequency

In [ ]:
```
frequencies = np.linspace(
    3 * 70235.74342823029 - 25000, 3 * 70235.74342823029 + 25000, 201
)
rho_zero = basis(16, 7).proj()
expectation_values = []
times = np.linspace(0, 1e-4, 1001)
opts = Options(nsteps=1 * 10**5)
for freq in frequencies:
    H_mw = Qobj(H_B(bx=Bmw).transform(F_states_reordered).full()) + h_mw_a(freq)
    H_mw_fl = Qobj(tensor(Qobj([[1, 0], [0, 0]]), H_mw).full())
    L = liouvillian(
        hamil + H_mw_fl,
        # c_ops=intra_F1 + intra_F2,
        c_ops=natural_decay_ops,
    )
    res = mesolve(L, rho_zero, times, options=opts, e_ops=rho_zero)
    expectation_values.append(res.expect[0].min())
```

In [ ]:
```
plt.plot(frequencies, expectation_values)
```

Out[ ]:
```
[<matplotlib.lines.Line2D at 0x2c158c56d60>]
```



In [ ]:
```
min(expectation_values)
```

Out[ ]:
```
6.555295304368279e-16
```

In [ ]:
```
np.argmin(expectation_values)
```

Out[ ]:
```
158
```

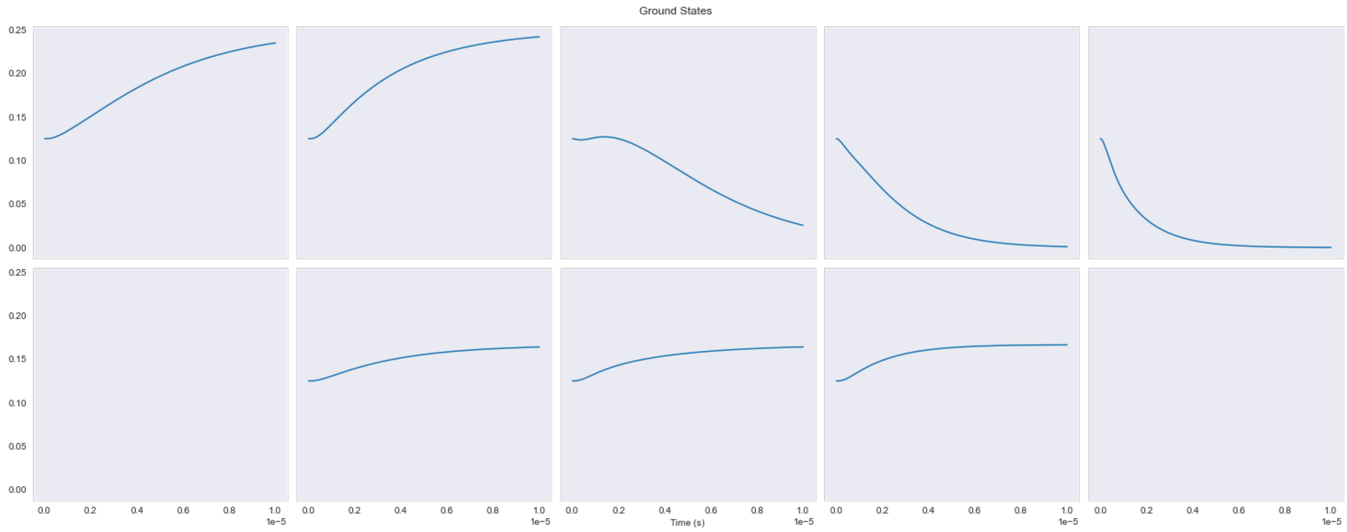
without mw

In [ ]:
```
L = liouvillian(
    hamil,
    # c_ops=intra_F1 + intra_F2,
    c_ops=natural_decay_ops,
)
```

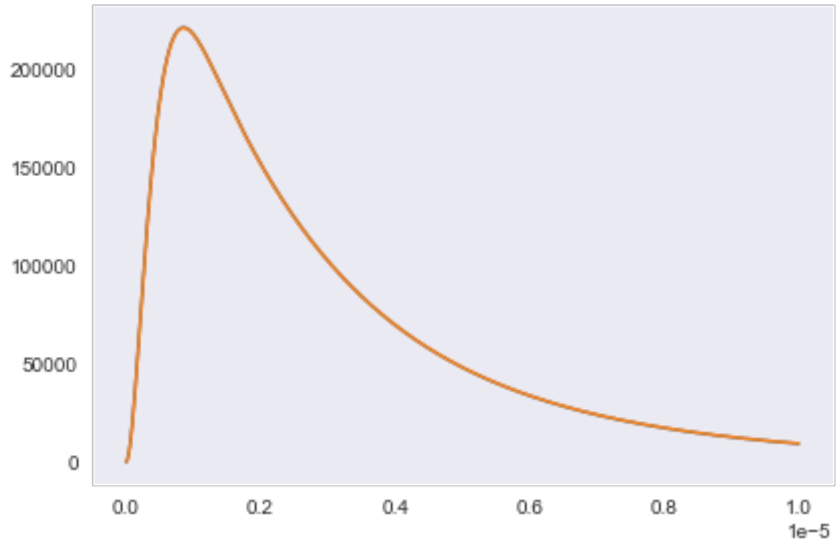
In [ ]:
```
rho_zero = sum([basis(16, i).proj() for i in range(8)]).unit()  # equally distrib
# rho_zero = basis(16, 7).proj()
```

```
In [ ]: times = np.linspace(0, 1e-5, 1001)
opts = Options(nsteps=1 * 10**4)
res = mesolve(
    L,
    rho_zero,
    times,
    options=opts,
)
```

```
In [ ]: ground_exp_val = [
    [
        res.states[t].matrix_element(basis(16, i).dag(), basis(16, i))
        for t in range(len(times))
    ]
    for i in range(8)
]
fig, axs = plt.subplots(ncols=5, nrows=2, figsize=(20, 8), sharex="all", sharey="all")
for i, e in enumerate(ground_exp_val[:3]):
    axs[1, 1 + i].plot(times, np.real(e))
for i, e in enumerate(ground_exp_val[3:8]):
    axs[0, i].plot(times, np.real(e))
fig.suptitle("Ground States")
axs[1, 2].set_xlabel("Time (s)")
plt.tight_layout()
```




```
In [ ]: absorbed_no_mw = [
    (sum([basis(16, k).proj() for k in range(8, 16)]) * rho_t).tr() * gamma_natural
    for rho_t in res.states
]
fig, ax = plt.subplots()
ax.plot(times, absorbed_no_mw, label="no MW")
ax.plot(times, absorbed_mw, label="MW")
plt.tight_layout()
```



In [ ]:
```
1
```

Out[ ]:
```
1
```