homework2-5

1 Dirac 符号下的矩阵关系和定义式

1.1 泡利矩阵的 Dirac 符号表示

$$X = |0\rangle\langle 1| + |1\rangle\langle 0|$$
$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$
$$I = |0\rangle\langle 0| + |1\rangle\langle 1|$$

1.2 张量积的 Dirac 符号表示

$$X \otimes I \otimes I \otimes Z = X_0 Z_3 = (|0\rangle\langle 1| + |1\rangle\langle 0|)_0 \otimes I_1 \otimes I_2 \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|)_3$$

1.3 矩阵 H 的 Dirac 符号表示

$$H = \sum_{i=0}^{n-1} Z_i \otimes I_{n-i-1} + \sum_{i=0}^{n-2} X_i \otimes I_{n-i-1} X_{i+1} \otimes I_{n-i}$$

展开为:

$$\begin{aligned} H = &\sum_{i=0}^{n-1} (|0\rangle\langle 0| - |1\rangle\langle 1|)_i \otimes \left(\bigotimes_{j \neq i} I_j \right) \\ &+ \sum_{i=0}^{n-2} (|0\rangle\langle 1| + |1\rangle\langle 0|)_i \otimes \left(\bigotimes_{j \neq i, i+1} I_j \right) \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|)_{i+1} \end{aligned}$$

1.4 初始向量的 Dirac 符号表示

$$|v\rangle = |0\rangle^{\otimes n} = |00\ldots0\rangle$$

1.5 期望值的 Dirac 符号表示

$$\langle H\rangle = \langle 00\ldots0| H |00\ldots0\rangle$$

2 列向量 $\frac{1}{\sqrt{2}}(|010\rangle - |101\rangle)$ 的具体形式

2.1 计算步骤

1. 展开每一项：

$$|010\rangle = |0\rangle \otimes |1\rangle \otimes |0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}$$

$$= \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix}$$

$$|101\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}$$

$$= \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix}$$

2. 组合结果：

$$\frac{1}{\sqrt{2}}(|010\rangle - |101\rangle) = \frac{1}{\sqrt{2}} \left(\begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} - \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix} \right) = \frac{1}{\sqrt{2}} \begin{pmatrix}0\\0\\1\\0\\0\\-1\\0\\0\end{pmatrix}$$

2.2 最终列向量

$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\\0\\0\\-1\\0\\0\end{pmatrix}$$