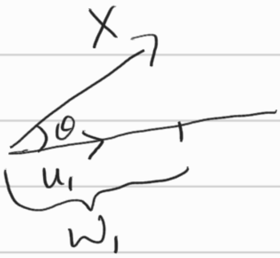
Problem 1



Show the length of the projection of x onto u_1 has length of $u_1^T x$

Let's call the projection of X onto U_1, ω_1

$$\omega_1 = \mathrm{proj}_u X = \underbrace{\left(\frac{u_1 \cdot X}{|u_1|}\right)}_{\text{this is the scalar projection, = the length of } \omega_1} \left(\frac{u_1}{|u_1|}\right) \leftarrow \text{direction}$$

$\therefore$ The length of $\omega_1 = |\omega_1| = \dfrac{u_1 \cdot x}{|u_1|}$

$\because$ u is a unit vector

$\therefore \dfrac{u_1}{|u_1|} = u$

$\therefore |\omega| = u_1 \cdot x = u_1^T x$ //

Problem 2

Show $\frac{1}{N}\sum_{i=1}^{N}(u_1^T x_i - u_1^T \bar{x})^2 = u_1^T S u_1$

, where S = data covariance matrix $= \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(x_i - \bar{x})^T$

Let's simply call u_1 as u for easy writing

$$\begin{aligned} \text{L.H.S} : \frac{1}{N}\sum_{i=1}^{N}(u^T x_i - u^T \bar{x})^2 &= \frac{1}{N}\sum_{i=1}^{N}(u^T x_i - u^T \bar{x})(u^T x_i - u^T \bar{x}) \\ &= \frac{1}{N}\sum_{i=1}^{N} u^T (x_i - \bar{x})\, u^T (x_i - \bar{x}) \\ &= \frac{1}{N}\sum_{i=1}^{N} u^T (x_i - \bar{x})(x_i - \bar{x})^T u \\ &= u^T \underbrace{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(x_i - \bar{x})^T}_{S} u \\ &= u^T S u = \text{R.H.S} \end{aligned}$$ //

Problem 3

a)

Given :

	a	b	c	K
1	1	0	1	1
2	1	1	1	1
3	0	1	1	0
4	1	1	0	0
5	1	0	1	0
6	0	0	0	1
7	0	0	0	1
8	0	0	1	0

For multiple independent evidences, Naive Bayes is :

$$P(H|E_1, E_2, \ldots, E_n)$$

$$= \frac{P(E_1|H)P(E_2|H)\cdots P(E_n|H)P(H)}{\sum_H P(E_1|H)P(E_2|H)\cdots P(E_n|H)P(H)}$$

a) Use Naive Bayes, find $P(K=1 \mid a=1, b=1, c=0)$

$$P(K=1 \mid a=1, b=1, c=0) \propto \underbrace{P(a=1|K=1)}_{\frac{2}{4}} \cdot \underbrace{P(b=1|K=1)}_{\frac{1}{4}} \cdot \underbrace{P(c=0|K=1)}_{\frac{2}{4}} \cdot \underbrace{P(K=1)}_{\frac{4}{8}}$$

$$= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32} \Big\} a$$

$$P(K=0 \mid a=1, b=1, c=0) \propto \underbrace{P(a=1|K=0)}_{\frac{2}{4}} \cdot \underbrace{P(b=1|K=0)}_{\frac{2}{4}} \cdot \underbrace{P(c=0|K=0)}_{\frac{1}{4}} \cdot \underbrace{P(K=0)}_{\frac{4}{8}}$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{32} \Big\} b$$

$$\therefore P(K=1 \mid a=1, b=1, c=0) = \frac{a}{a+b} = \frac{\frac{1}{32}}{\frac{1}{32}+\frac{1}{32}} = \frac{1}{\cancel{32}} \cdot \frac{\cancel{32}}{2} = \frac{1}{2}$$

b)

Given :

	a	b	c	K
1	1	0	1	1
2	1	1	1	1
3	0	1	1	0
4	1	1	0	0
5	1	0	1	0
6	0	0	0	1
7	0	0	0	1
8	0	0	1	0

For multiple independent evidences, Naive Bayes is :

$$P(H|E_1, E_2, \ldots, E_n)$$

$$= \frac{P(E_1|H)P(E_2|H)\cdots P(E_n|H)P(H)}{\sum_H P(E_1|H)P(E_2|H)\cdots P(E_n|H)P(H)}$$

b) Use Naive Bayes, find $P(K=0|a=1, b=1)$

$$P(K=0|a=1,b=1) \propto \underbrace{P(a=1|K=0)}_{\frac{2}{4}} \cdot \underbrace{P(b=1|K=0)}_{\frac{2}{4}} \cdot \underbrace{P(K=0)}_{\frac{4}{8}} = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{8} - a$$

$$P(K=1|a=1,b=1) \propto \underbrace{P(a=1|K=1)}_{\frac{2}{4}} \cdot \underbrace{P(b=1|K=1)}_{\frac{1}{4}} \cdot \underbrace{P(K=1)}_{\frac{4}{8}} = \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2} = \frac{1}{16}$$

b

$$\therefore P(K=0|a=1,b=1) = \frac{a}{a+b} = \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{16}} = \frac{1}{\cancel{8}} \cdot \frac{\cancel{16}\,2}{3} = \frac{2}{3}$$