# 1 Bera

**Definition 1.1** (Lebesgue Integral)**.** Let $(X, \mathcal{M}, \mu)$ be a measure space. Let $f : X \to \mathbb{C}$ be a $\mathcal{B}(\mathbb{C})/\mathcal{M}$-measureable function. Define the following formulations of an integral:

- If $f$ takes on only finitely many values in $\mathbb{R}^+$, then give the standard representation of $f$ as the simple function:

$$f(x) = \sum_i y_i \mathbb{1}_{\{f(x)=y_i\}}(x).$$

  Then define the Lebesgue integral of $f$ with respect to $\mu$ as:

$$\int_X f(x) \, d\mu(x) = \sum_i y_i \mu(\{f(x) = y_i\}).$$

- If $f$ takes on values in $\mathbb{R}^+$, then define the Lebesgue integral of $f$ with respect to $\mu$ as:

$$\int_X f(x) \, d\mu(x) = \sup \left\{ \int_X g(x) \, d\mu(x) \mid g \leq f \text{ and } g \text{ is a simple function} \right\}.$$

  This may or may not take on the "value" of $\infty$.

- If $f$ takes on values in $\mathbb{R}$, then give the functions $f^+ = \max\{f, 0\}$ and $f^- = \min\{f, 0\}$. Give the Lebesgue integral of $f$ with respect to $\mu$ as:

$$\int_X f(x) \, d\mu(x) = \int_X f^+(x) \, d\mu(x) - \int_X f^- \, d\mu(x),$$

  provided that both of the integrals on the right exist and at least one of them is finite.

- Lastly, if $f$ takes on values in $\mathbb{C}$, then let $u$ and $v$ be the functions given by $u = \Re f$ and $v = \Im f$, i.e. $f(x) = u(x) + iv(x)$. Then define the Lebesgue integral of $f$ relative to $\mu$ as:

$$\int_X f(x) \, d\mu(x) = \int_X u(x) \, d\mu(x) + i \int_X v(x) \, d\mu(x).$$

We usually write $\int f \, d\mu$ or just $\int f$ if the implied parts are easy to understand. Also we write $\int_A f$ to stand for $\int_X f \mathbb{1}_A$. In total, these parts define the **Lebesgue integral of** $f$ **with respect to** $\mu$.

**Theorem 1.2** (Cauchy's Integral Formula)**.** *Let $U$ be an open subset of the $\mathbb{C}$ and suppose that the disk*

$$D(z_0, r) = \{z \in \mathbb{C} \mid |z - z_0| < r\}$$

*is completely contained in $U$. Let $f : U \to \mathbb{C}$ be a holomorphic function and let $\gamma$ be the circle oriented counterclockwise forming the boundary of $D(z_0, r)$. Then for every interior point $a$ of $D(z_0, r)$,*

$$f(a) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{za} \, dz.$$

*Moreover, because $(z-a)^{-1}$ can be given by the power series:*

$$\frac{1}{z-a} = \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{a}{z}\right)^n$$

*it follows that $f$ is analytic, infinitely differentiable, and*

$$f^{(n)}(a) = \frac{n!}{2\pi i}\oint_{\gamma}\frac{f(z)}{(z-a)^{n+1}}\,dz.$$

Here are examples of the major fonts:

**Regular** ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

**Slshape** *ABCDEFGHIJKLMNOPQRSTUVWXYZ*
*abcdefghijklmnopqrstuvwxyz*
*1234567890*

**Bfseries** **ABCDEFGHIJKLMNOPQRSTUVWXYZ**
**abcdefghijklmnopqrstuvwxyz**
**1234567890**

**Slshape** ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

**Mathematics** $ABCDEFGHIJKLMNOPQRSTUVWXYZ$
$abcdefghijklmnopqrstuvwxyz$
$1234567890$

**Mathcal** $\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
⊣⌊⌈{}⟨⟩‖⇕⇕\≀√ΠΠ∇∫⊔⊓⊑⊒§†‡
∞∈∋△▽∀∃ı

**Mathbb** $\mathbb{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
∂℧ð≈⊐⊏⟨⟩⋉⋊∥∖∼≈≅≲≳⌢⌣F

**Mathds** $\mathds{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
Ahk
1

**Mathbbm** $\mathbbm{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
abcdefghijklmnopqrstuvwxyz
12

**Mathfrak**  $\mathfrak{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$

$\mathfrak{abcdefghijklmnopqrstuvwxyz}$

$\mathfrak{1234567890}$

**Mathscr**  $\mathscr{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$

**Greek**  $\alpha\beta\gamma\delta\epsilon\varepsilon\zeta\eta\theta\vartheta\iota\kappa\varkappa\lambda\mu\nu\xi\pi\varpi\rho\varrho\sigma\varsigma\tau\upsilon\phi\varphi\chi\phi\varphi\psi\omega$

**Greek (Capital)**  $\Gamma\varGamma\Delta\varDelta\Theta\varTheta\Lambda\varLambda\Xi\varXi\Pi\varPi\Sigma\varSigma\Phi\varPhi\Psi\varPsi\Omega\varOmega$

**Common symbols**  $\int \oint \iint \sum \prod \cup \cap \vee \wedge \oplus \otimes$

**Common symbols (displaystyle)**  $\displaystyle \int \oint \iint \sum \prod \cup \cap \vee \wedge \oplus \otimes$

**Inline Sup/Sub-scripts**  $\lim_{n\to\infty} \inf_\alpha \sup_{\substack{f:\mathbb{R}\to\mathbb{R} \\ f \text{ is } 1-\text{Lipschitz}}} \sum_{\alpha\in\mathcal{A}} \prod_{n=1}^{\infty} \int_a^b$

**Inline Sup/Sub-scripts (displaystyle)**  $\displaystyle \lim_{n\to\infty} \inf_\alpha \sup_{\substack{f:\mathbb{R}\to\mathbb{R} \\ f \text{ is } 1-\text{Lipschitz}}} \sum_{\alpha\in\mathcal{A}} \prod_{n=1}^{\infty} \int_a^b$