1 Baskerville

Definition 1.1 (Lebesgue Integral). Let $(X, \mathscr{M}, \mu)$ be a measure space. Let $f : X \to \mathbb{C}$ be a $\mathscr{B}(\mathbb{C})/\mathscr{M}$-measureable function. Define the following formulations of an integral:

- If f takes on only finitely many values in $\mathbb{R}^+$, then give the standard representation of f as the simple function:
$$f(x) = \sum_i y_i \mathbb{1}_{\{f(x)=y_i\}}(x).$$
Then define the Lebesgue integral of f with respect to μ as:
$$\int_X f(x)\, d\mu(x) = \sum_i y_i \mu(\{f(x) = y_i\}).$$
- If f takes on values in $\mathbb{R}^+$, then define the Lebesgue integral of f with respect to μ as:
$$\int_X f(x)\, d\mu(x) = \sup\left\{\int_X g(x)\, d\mu(x) \mid g \leq f \text{ and } g \text{ is a simple function}\right\}.$$
This may or may not take on the "value" of ∞.
- If f takes on values in $\mathbb{R}$, then give the functions $f^+ = \max\{f, 0\}$ and $f^- = \min\{f, 0\}$. Give the Lebesgue integral of f with respect to μ as:
$$\int_X f(x)\, d\mu(x) = \int_X f^+(x)\, d\mu(x) - \int_X f^-\, d\mu(x),$$
provided that both of the integrals on the right exist and at least one of them is finite.
- Lastly, if f takes on values in $\mathbb{C}$, then let u and v be the functions given by $u = \Re f$ and $v = \Im f$, i.e. $f(x) = u(x) + iv(x)$. Then define the Lebesgue integral of f relative to μ as:
$$\int_X f(x)\, d\mu(x) = \int_X u(x)\, d\mu(x) + i \int_X v(x)\, d\mu(x).$$

We usually write $\int f\, d\mu$ or just $\int f$ if the implied parts are easy to understand. Also we write $\int_A f$ to stand for $\int_X f \mathbb{1}_A$. In total, these parts define the **Lebesgue integral of f with respect to μ**.

Theorem 1.2 (Cauchy's Integral Formula). *Let U be an open subset of the $\mathbb{C}$ and suppose that the disk*
$$D(z_0, r) = \{z \in \mathbb{C} \mid |z - z_0| < r\}$$
is completely contained in U. Let $f : U \to \mathbb{C}$ be a holomorphic function and let γ be the circle oriented counterclockwise forming the boundary of $D(z_0, r)$. Then for every interior point a of $D(z_0, r)$,
$$f(a) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{za}\, dz.$$
Moreover, because $(z - a)^{-1}$ can be given by the power series:
$$\frac{1}{z - a} = \frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$
it follows that f is analytic, infinitely differentiable, and
$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_\gamma \frac{f(z)}{(z - a)^{n+1}}\, dz.$$

Here are examples of the major fonts:

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Greek $\alpha\beta\gamma\delta\epsilon\varepsilon\zeta\eta\theta\vartheta\iota\kappa\lambda\mu\nu\xi\pi\varpi\rho\varrho\sigma\varsigma\tau\upsilon\phi\varphi\chi\phi\varphi\psi\omega$

Greek (Capital) $\Gamma \mathit{\Gamma} \Delta \mathit{\Delta} \Theta \mathit{\Theta} \Lambda \mathit{\Lambda} \Xi \mathit{\Xi} \Pi \mathit{\Pi} \Sigma \mathit{\Sigma} \Phi \mathit{\Phi} \Psi \mathit{\Psi} \Omega \mathit{\Omega}$

Common symbols $\int \oint \iint \oiint \sum \prod \bigcup \bigcap \bigvee \bigwedge \bigoplus \bigotimes$

Common symbols (displaystyle) $\displaystyle \int \oint \iint \oiint \sum \prod \bigcup \bigcap \bigvee \bigwedge \bigoplus \bigotimes$

Inline Sup/Sub-scripts $\lim_{n\to\infty} \inf_\alpha \sup_{\substack{f:\mathbb{R}\to\mathbb{R} \\ f \text{ is } 1-\text{Lipschitz}}} \sum_{\alpha\in\mathcal{A}} \prod_{n=1}^\infty \int_a^b$

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