

1 Euler VM

Definition 1.1 (Lebesgue Integral). Let (X, \mathcal{M}, μ) be a measure space. Let $f : X \rightarrow \mathbb{C}$ be a $\mathcal{B}(\mathbb{C})/\mathcal{M}$ -measureable function. Define the following formulations of an integral:

- If f takes on only finitely many values in \mathbb{R}^+ , then give the standard representation of f as the simple function:

$$f(x) = \sum_i y_i \mathbb{1}_{\{f(x)=y_i\}}(x).$$

Then define the Lebesgue integral of f with respect to μ as:

$$\int_X f(x) d\mu(x) = \sum_i y_i \mu(\{f(x) = y_i\}).$$

- If f takes on values in \mathbb{R}^+ , then define the Lebesgue integral of f with respect to μ as:

$$\int_X f(x) d\mu(x) = \sup \left\{ \int_X g(x) d\mu(x) \mid g \leq f \text{ and } g \text{ is a simple function} \right\}.$$

This may or may not take on the “value” of ∞ .

- If f takes on values in \mathbb{R} , then give the functions $f^+ = \max\{f, 0\}$ and $f^- = \min\{f, 0\}$. Give the Lebesgue integral of f with respect to μ as:

$$\int_X f(x) d\mu(x) = \int_X f^+(x) d\mu(x) - \int_X f^-(x) d\mu(x),$$

provided that both of the integrals on the right exist and at least one of them is finite.

- Lastly, if f takes on values in \mathbb{C} , then let u and v be the functions given by $u = \Re f$ and $v = \Im f$, i.e. $f(x) = u(x) + iv(x)$. Then define the Lebesgue integral of f relative to μ as:

$$\int_X f(x) d\mu(x) = \int_X u(x) d\mu(x) + i \int_X v(x) d\mu(x).$$

We usually write $\int f d\mu$ or just $\int f$ if the implied parts are easy to understand. Also we write $\int_A f$ to stand for $\int_X f \mathbb{1}_A$. In total, these parts define the **Lebesgue integral of f with respect to μ** .

Theorem 1.2 (Cauchy’s Integral Formula). *Let U be an open subset of the \mathbb{C} and suppose that the disk*

$$D(z_0, r) = \{z \in \mathbb{C} \mid |z - z_0| < r\}$$

is completely contained in U . Let $f : U \rightarrow \mathbb{C}$ be a holomorphic function and let γ be the circle oriented counterclockwise forming the boundary of $D(z_0, r)$. Then for every interior point a of $D(z_0, r)$,

$$f(a) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(z)}{z-a} dz.$$

Moreover, because $(z - a)^{-1}$ can be given by the power series:

$$\frac{1}{z-a} = \frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

it follows that f is analytic, infinitely differentiable, and

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_{\gamma} \frac{f(z)}{(z-a)^{n+1}} dz.$$

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Common symbols ∫ℱ∫∫ΣΠ∪∩∨∧⊕⊗

Common symbols (displaystyle) $\int\!\!\!\int\!\!\!\int\Sigma\Pi\cup\cap\vee\wedge\oplus\otimes$

Inline Sup/Sub-scripts $\lim_{n\rightarrow\infty}\inf_{\alpha}\sup_{\substack{f:\mathbb{R}\rightarrow\mathbb{R} \\ f\text{ is }1\text{-Lipschitz}}}\sum_{\alpha\in\mathcal{A}}\prod_{n=1}^{\infty}\int_a^b$

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