

# 1 Bera

**Definition 1.1** (Lebesgue Integral). Let  $(X, \mathcal{M}, \mu)$  be a measure space. Let  $f : X \rightarrow \mathbb{C}$  be a  $\mathcal{B}(\mathbb{C})/\mathcal{M}$ -measurable function. Define the following formulations of an integral:

- If  $f$  takes on only finitely many values in  $\mathbb{R}^+$ , then give the standard representation of  $f$  as the simple function:

$$f(x) = \sum_i y_i \mathbb{1}_{\{f(x)=y_i\}}(x).$$

Then define the Lebesgue integral of  $f$  with respect to  $\mu$  as:

$$\int_X f(x) d\mu(x) = \sum_i y_i \mu(\{f(x) = y_i\}).$$

- If  $f$  takes on values in  $\mathbb{R}^+$ , then define the Lebesgue integral of  $f$  with respect to  $\mu$  as:

$$\int_X f(x) d\mu(x) = \sup \left\{ \int_X g(x) d\mu(x) \mid g \leq f \text{ and } g \text{ is a simple function} \right\}.$$

This may or may not take on the “value” of  $\infty$ .

- If  $f$  takes on values in  $\mathbb{R}$ , then give the functions  $f^+ = \max\{f, 0\}$  and  $f^- = \min\{f, 0\}$ . Give the Lebesgue integral of  $f$  with respect to  $\mu$  as:

$$\int_X f(x) d\mu(x) = \int_X f^+(x) d\mu(x) - \int_X f^-(x) d\mu(x),$$

provided that both of the integrals on the right exist and at least one of them is finite.

- Lastly, if  $f$  takes on values in  $\mathbb{C}$ , then let  $u$  and  $v$  be the functions given by  $u = \Re f$  and  $v = \Im f$ , i.e.  $f(x) = u(x) + iv(x)$ . Then define the Lebesgue integral of  $f$  relative to  $\mu$  as:

$$\int_X f(x) d\mu(x) = \int_X u(x) d\mu(x) + i \int_X v(x) d\mu(x).$$

We usually write  $\int f d\mu$  or just  $\int f$  if the implied parts are easy to understand. Also we write  $\int_A f$  to stand for  $\int_X f \mathbb{1}_A$ . In total, these parts define the **Lebesgue integral of  $f$  with respect to  $\mu$** .

**Theorem 1.2** (Cauchy’s Integral Formula). *Let  $U$  be an open subset of the  $\mathbb{C}$  and suppose that the disk*

$$D(z_0, r) = \{z \in \mathbb{C} \mid |z - z_0| < r\}$$

*is completely contained in  $U$ . Let  $f : U \rightarrow \mathbb{C}$  be a holomorphic function and let  $\gamma$  be the circle oriented counterclockwise forming the boundary of  $D(z_0, r)$ . Then for every interior point  $a$  of  $D(z_0, r)$ ,*

$$f(a) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(z)}{z - a} dz.$$

Moreover, because  $(z - a)^{-1}$  can be given by the power series:

$$\frac{1}{z-a} = \frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

it follows that  $f$  is analytic, infinitely differentiable, and

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_{\gamma} \frac{f(z)}{(z-a)^{n+1}} dz.$$

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**Greek**  $\alpha\beta\gamma\delta\epsilon\zeta\eta\theta\iota\kappa\lambda\mu\nu\xi\pi\rho\sigma\tau\upsilon\phi\chi\psi\omega$

**Greek (Capital)**  $\Gamma\Delta\Theta\Lambda\Xi\Pi\Sigma\Phi\Psi\Omega$

**Common symbols**  $\int\oint\!\!\!\int\Sigma\Pi\cup\cap\vee\wedge\oplus\otimes$

**Common symbols (displaystyle)**  $\int\oint\!\!\!\int\Sigma\Pi\cup\cap\vee\wedge\oplus\otimes$

**Inline Sup/Sub-scripts**  $\lim_{n\rightarrow\infty}\inf_{\alpha}\sup_{\substack{f:\mathbb{R}\rightarrow\mathbb{R}\\ f\text{ is }1\text{-Lipschitz}}}\sum_{\alpha\in\mathcal{A}}\prod_{n=1}^{\infty}\int_a^b$

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