# 1 EB Garamond

**Definition 1.1** (Lebesgue Integral)**.** Let $(X, \mathcal{M}, \mu)$ be a measure space. Let $f : X \to \mathbb{C}$ be a $\mathcal{B}(\mathbb{C})/\mathcal{M}$-measureable function. Define the following formulations of an integral:

- If $f$ takes on only finitely many values in $\mathbb{R}^+$, then give the standard representation of $f$ as the simple function:
$$f(x) = \sum_i y_i \mathbb{1}_{\{f(x)=y_i\}}(x).$$
Then define the Lebesgue integral of $f$ with respect to $\mu$ as:
$$\int_X f(x)\ d\mu(x) = \sum_i y_i \mu(\{f(x) = y_i\}).$$

- If $f$ takes on values in $\mathbb{R}^+$, then define the Lebesgue integral of $f$ with respect to $\mu$ as:
$$\int_X f(x)\ d\mu(x) = \sup\left\{\int_X g(x)\ d\mu(x) \mid g \leq f \text{ and } g \text{ is a simple function}\right\}.$$
This may or may not take on the "value" of $\infty$.

- If $f$ takes on values in $\mathbb{R}$, then give the functions $f^+ = \max\{f, 0\}$ and $f^- = \min\{f, 0\}$. Give the Lebesgue integral of $f$ with respect to $\mu$ as:
$$\int_X f(x)\ d\mu(x) = \int_X f^+(x)\ d\mu(x) - \int_X f^-\ d\mu(x),$$
provided that both of the integrals on the right exist and at least one of them is finite.

- Lastly, if $f$ takes on values in $\mathbb{C}$, then let $u$ and $v$ be the functions given by $u = \Re f$ and $v = \Im f$, i.e. $f(x) = u(x) + iv(x)$. Then define the Lebesgue integral of $f$ relative to $\mu$ as:
$$\int_X f(x)\ d\mu(x) = \int_X u(x)\ d\mu(x) + i\int_X v(x)\ d\mu(x).$$

We usually write $\int f\ d\mu$ or just $\int f$ if the implied parts are easy to understand. Also we write $\int_A f$ to stand for $\int_X f\mathbb{1}_A$. In total, these parts define the **Lebesgue integral of $f$ with respect to $\mu$**.

**Theorem 1.2** (Cauchy's Integral Formula)**.** *Let $U$ be an open subset of the $\mathbb{C}$ and suppose that the disk*
$$D(z_0, r) = \{z \in \mathbb{C} \mid |z - z_0| < r\}$$
*is completely contained in $U$. Let $f : U \to \mathbb{C}$ be a holomorphic function and let $\gamma$ be the circle oriented counterclockwise forming the boundary of $D(z_0, r)$. Then for every interior point $a$ of $D(z_0, r)$,*
$$f(a) = \frac{1}{2\pi i}\oint_\gamma \frac{f(z)}{za}\ dz.$$
*Moreover, because $(z-a)^{-1}$ can be given by the power series:*
$$\frac{1}{z-a} = \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{a}{z}\right)^n$$
*it follows that $f$ is analytic, infinitely differentiable, and*
$$f^{(n)}(a) = \frac{n!}{2\pi i}\oint_\gamma \frac{f(z)}{(z-a)^{n+1}}\ dz.$$

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𝕒𝕓𝕔𝕕𝕖𝕗𝕘𝕙𝕚𝕛𝕜𝕝𝕞𝕟𝕠𝕡𝕢𝕣𝕤𝕥𝕦𝕧𝕨𝕩𝕪𝕫

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**Mathscr**

**Greek** $\alpha\beta\gamma\delta\epsilon\zeta\eta\theta\vartheta\iota\kappa\lambda\mu\nu\xi\pi\varpi\rho\varrho\sigma\varsigma\tau\upsilon\phi\varphi\chi\phi\varphi\psi\omega$

**Greek (Capital)** $\Gamma\varGamma\Delta\varDelta\Theta\varTheta\Lambda\varLambda\Xi\varXi\Pi\varPi\Sigma\varSigma\Phi\varPhi\Psi\varPsi\Omega\varOmega$

**Common symbols** $\int \oint \iint \oiint \sum \prod \bigcup \bigcap \bigvee \bigwedge \bigoplus \bigotimes$

**Common symbols (displaystyle)** $\displaystyle \int \oint \iint \oiint \sum \prod \bigcup \bigcap \bigvee \bigwedge \bigoplus \bigotimes$

**Inline Sup/Sub-scripts** $\lim_{n\to\infty} \inf_{\alpha} \sup_{\substack{f:\mathbb{R}\to\mathbb{R} \\ f \text{ is } 1-\text{Lipschitz}}} \sum_{\alpha\in\mathcal{A}} \prod_{n=1}^{\infty} \int_a^b$

**Inline Sup/Sub-scripts (displaystyle)** $\displaystyle \lim_{n\to\infty} \inf_{\alpha} \sup_{\substack{f:\mathbb{R}\to\mathbb{R} \\ f \text{ is } 1-\text{Lipschitz}}} \sum_{\alpha\in\mathcal{A}} \prod_{n=1}^{\infty} \int_a^b$