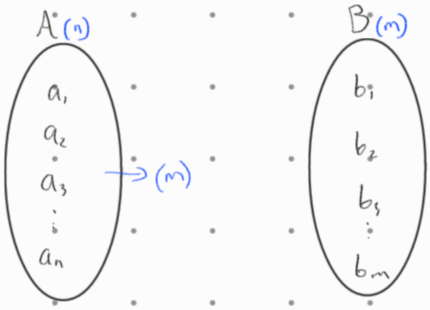
Exercise 1.1 - obligatory (4 points)

a) Let A and B be two finite sets. How many elements cont~~a~~ins of $\wp$(A × B) are dependent on the cardinality of A and B?

domain   codomain



$|A| = 2^n$ elements $|B| = 2^m$ elements

rule of product

$A \times B \quad (a, b)$

$a \in A \;\}\; b \in B$

\# of elements

cardinality $|A \times B| = nm$

$\wp(A \times B) = 2^{nm}$

$|A \times B| = |A| \cdot |B|$

$|\wp(M)| = 2^{|M|}$

$|\wp(A \times B)| = 2^{|A| \cdot |B|}$

b) Let M = {1, 2,..., 8} be the set of natural numbers between 1 and 8. Describe the isdivisor-of relation as a set of pairs, according to definition 2.3. R = is-divisor-of relation

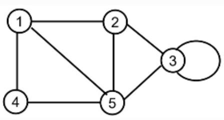
$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8)\}$

c) Which of the following relationships can be modeled as a function, which as a relation? "Function may only have a single output as a result of an input"

(1) child-mother relationship (Function)
(2) is-uncle-of relationship (Relation)
(3) relationship between students and enrolment numbers (Function)
(4) relationship between students and courses they take (Relation)

Exercise 1.2 - obligatory (6 points)

Let G be the following graph:



a) Describe G formally as a set V of vertices and a set E of edges.

$G = (V, E)$

$V = \{1, 2, 3, 4, 5\}$

$E = \{\{1,2\}, \{1,4\}, \{1,5\}, \{4,5\}, \{2,5\}, \{2,3\}, \{5,3\}, \{3\}\}$

b) How would G be stored using an adjecency matrix?

1 2 3 4 5

1	0	1	0	1	1
2	1	0	1	0	1
3	0	1	2	0	1
4	1	0	0	1	0
5	1	1	1	1	0

c) How would G be stored using adjacency lists?

L[1]: 2, 4, 5
L[2]: 1, 3, 5
L[3]: 2, 3, 5, 3
L[4]: 1, 4
L[5]: 1, 2, 3, 4

d) How can you compute the degree of a vertex from the adjacency matrix representation?

The degree of a matrix can be found by taking the sum of the values in either its respective column or row in the matrix

e) How can you compute the degree of a vertex from the adjacency lists representation?

Given by the length/sum of its respective linked list

Exercise 1.3 - obligatory (6 points)

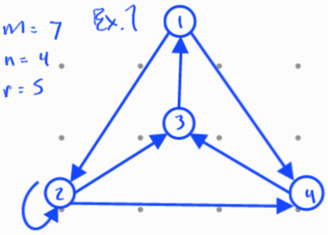
Let G = (V, E) be a directed graph, where

V = {1, 2, 3, 4}
E = { (1,2), (1,4), (2,2), (2,3), (2,4), (3,1), (4,3) }

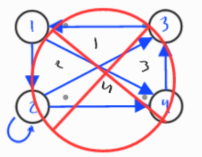
a) Draw G as a diagram.

m = 7
n = 4
r = 5

Ex.1



Ex.2



b) Determine the indegree and the outdegree of vertex 2.

v(2) Degree = 5
In: 2
Out: 3

c) Is G planar?

Yes, the graph can be arranged in such a way that no edges are crossing each other. Ex.1

Euler's Theorem for Plane Graphs
$(n - m + R) = 2 = \checkmark$

m = 7
n = 4
r = 5

$4 - 7 + 5 = 2$
$2 \Rightarrow \checkmark$

d) Is G strongly connected?

Yes, for every pair of vertices u and v in G, there exists a (u-v path & v-u path), therefore it is strongly connected.

ie: (1-4, 4-1); (2-3, 3-2)

e) Is G acyclic?

No, if there are paths from any vertex u, there may not be a path that comes back to to u.

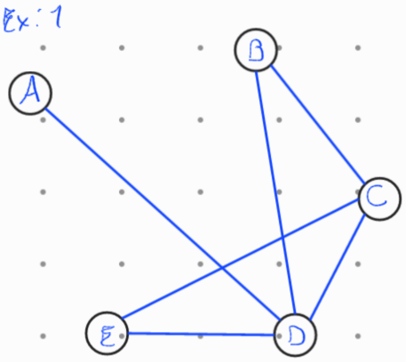
Exercise 1.4

Anna, Bruno, Cindy, Dave, and Emma are the attendants of a party. Anna knows one of the other persons, Bruno knows two of the persons, Cindy knows three persons and Dave knows four persons. How many persons knows Emma? Depict a graph that represents the situation.

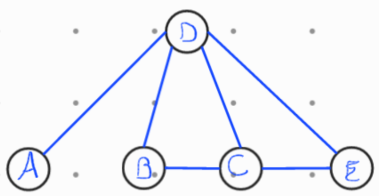
Hint: The relation "knows" is considered here as a symmetric relation, i.e. if person X knows person Y, then person Y also knows person X. (non-directed graph)

Answer: Emma knows 2 persons

Ex: 1



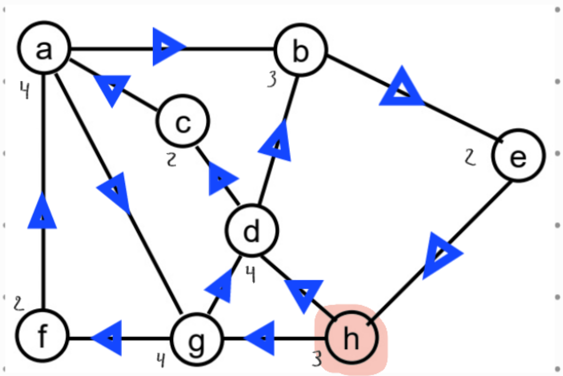
Ex: 2



Exercise 1.5 - obligatory (4 points)

a) Let be G the following graph :

Has G an Eulerian Cycle (Euler tour) or an Eulerian path? Give a short explanation



Euler's Path: h g f a g d c a b e h d b

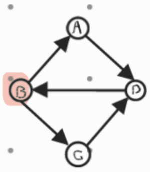
for your answer. If possible, give an example of an Eulerian tour or path.

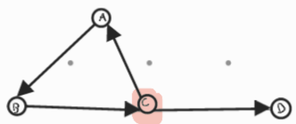
Euler path: uses every edge in a graph with no repeats and can end anywhere.

Yes, G does have a Euler path.

notes: at most 2 vertices of odd degree

Ex: 1



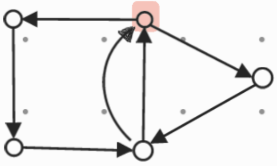


Euler circuit: uses every edge in a graph with no repeats and must end at the start vertex.

G does not have a circuit.

notes: if all vertices have an even degree

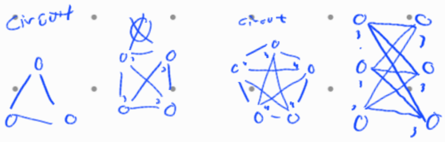
Ex: 1



b) If Gn is a complete graph with n vertices, does there exist an Euler tour or an Eulerian path for Gn?

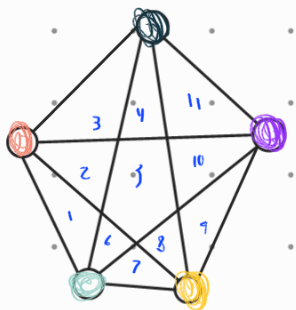
Euler Path: If n vertices in the given Gn have at most 2 vertices with an odd degree

Euler circuit: if all n vertices in a given Gn have an even degree.



Exercise 1.6

Devise a (non-planar) graph that requires 5 colors to be colored admissibly.



$n = 5$

$m = 10$

$r = 12$

$m \leq 3n - 6$;

$m \leq 3(5) - 6$

$m \not\leq 9$

$n - m + r$

$5 - 10 + 12 = 7 \neq 2$