Theoretical Computer Science
Winter semester 21/22
Prof. Dr. Georg Schied

Assignment 9

Deadline: Wednesday, 8 December 2021

10 out of 20 points have to be achieved in order to pass.

Exercise 9.1

Define a PDA (push-down automaton) that accepts the language of all correctly nested sequences of round or square brackets.

- a sequence consists of any number of bracket pairs.

- Each bracket pair begins with an opening bracket [or (and ends with the corresponding closing bracket] or). Between opening and closing bracket there can be any sequence of (possibly nested) bracket pairs.

Here are some examples of correctly nested sequences:

```
[]
() [] ()
(()) [[]]
[()] ([() []]) ()
```

Also the empty string ε is allowed. In contrast, the following strings should not be accepted.

```
[[])        opening square bracket [ closed by round bracket )
([]         closing bracket ) is missing
) [] (      has to begin with an opening bracket
```

Exercise 9.2

a) Define a PDA that accepts the language of the following context-free grammar:

$$S \rightarrow \quad bA$$
$$A \rightarrow \quad BAB$$
$$\quad | \quad c$$
$$B \rightarrow \quad aa$$
$$\quad | \quad b$$

b) Show how the PDA accepts **baacb**. (Tip: It might help to draw a derivation tree first).

Exercise 9.3 - obligatory (6 points)

Let the following context-free grammar be given:

$$S \rightarrow \quad aAc$$
$$A \rightarrow \quad BA \mid c$$
$$B \rightarrow \quad ab \mid \varepsilon$$

a) Define a PDA that accepts the language of the grammar.

b) Give a sequence of configuration steps that shows that the PDA accepts the string **aabcc**.
Tip: First think about what a derivation tree for the string looks like.

Exercise 9.4

Grammar G with start symbols S is defined as following:

$$S \rightarrow \quad aA \mid Bd$$
$$A \rightarrow \quad BC \mid a$$
$$B \rightarrow \quad bBa \mid C$$
$$C \rightarrow \quad cCb \mid \varepsilon$$

Compute for this grammar the properties *nullable*, *First*, and *Follow*.

Exercise 9.5 - obligatory (14 points)

Let *G* be the following context-free grammar with start symbols *S*:

$$S \rightarrow \quad BSA \mid aAB$$
$$A \rightarrow \quad bA \quad \mid \varepsilon$$
$$B \rightarrow \quad cBAa \mid A$$

a) Compute for all non-terminal symbols of the grammar the properties:

    (1)   nullable(*X*)

    (2)   First(*X*)

    (3)   Follow(*X*).

b) Compute then:

    (1)   First(cBA)

    (2)   First(ASB)

    (3)   First(ABa)