Theoretical Computer Science

Winter semester 21/22
Prof. Dr. Georg Schied

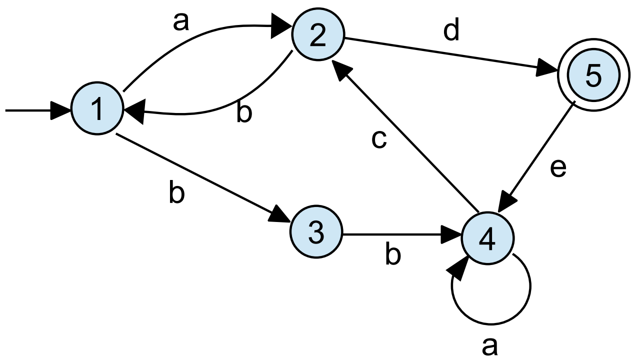
Assignment 8

Deadline: Wednesday, 1 December 2021

10 out of 20 points have to be achieved in order to pass.

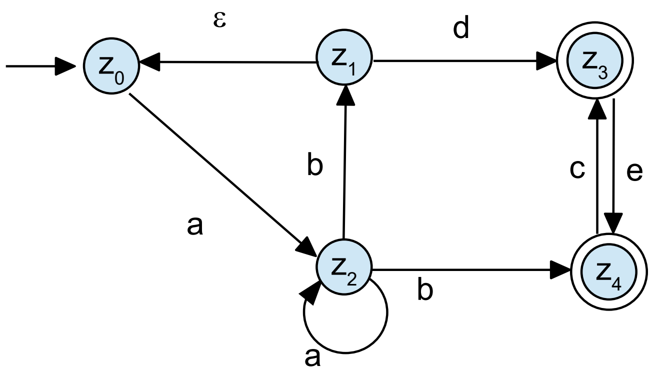
Exercise 8.1

Use elimination procedure 9.11 to convert this ε-NFA into an equivalent regular expression.



Exercise 8.2 - obligatory (8 points)

Convert this ε-NFA into an equivalent regular expression.



Show all important intermediate steps, so that the approach can be understood.

Hint: In order to keep notation clear, you can introduce abbreviations for complex regular expressions.

Exercise 8.3 - obligatory (4 points)

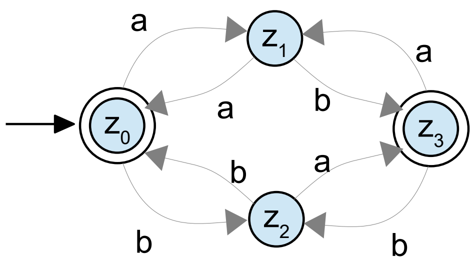
a) Specify an ε-NFA that accepts the language of the following Chomsky type-3 grammar:

$S \rightarrow bA \mid aS \mid cB$

$A \rightarrow bB \mid aA \mid \varepsilon$

$B \rightarrow cS \mid \varepsilon$

b) Specify a Chomsky type-3 grammar that generates the language of the following DFA:



Exercise 8.4

Let $\Sigma = \{0,1\}$. Which of the following languages are regular, which are not? Give a brief explanation.

(1) $L_a = \{ 0^k 11(00)^m \mid k > 0, m \geq 0\}$

(2) $L_b = \{ w \in \Sigma^* \mid |w|_0 \text{ is odd} \}$ (number of 0 symbols is odd)

(3) $L_c = \{w_1, \ldots, w_n\}$ (finite language, finite set of strings)

(4) $L_d = \{w \in \Sigma^* \mid w \text{ contains 11 or 000 as a substring}\}$

(5) $L_e = \Sigma^* \setminus \{01101, 101\}$

(6) $L_f = \{w \in \Sigma^* \mid w \text{ contains 00, but not 000 as a substring}\}$

Exercise 8.5 - obligatory (4 points)

Let $\Sigma = \{a, b, c\}$. Which of the following languages are regular? Give a short justification in each case.

(1) $L_1 = \{ (cc)^k ab \mid k > 0\} \cup \{a^n(bc)^m \mid n \geq 0, m \geq 0\}$

(2) $L_2 = \{w \in \Sigma^* \mid |w| > 4 \}$ (strings longer than 4 symbols)

(3) $L_3 = \{ c^k a^k \mid k \geq 0 \}$

(4) $L_4 = \{ a^n w \mid n \geq 3, w \in \Sigma^*\} \cap \{ u(bc)^k \mid u \in \Sigma^*, k \geq 1\}$

Exercise 8.6

Let $L = \{ a^n b c^k \mid k \geq n \}$.

a) (difficult!) Prove that language L is *not regular*. Hint: Use the pumping lemma.

b) (not so difficult) Show that L is a *context-free language*.

Exercise 8.7 - obligatory (4 points)

A push-down automaton $P = (Z, \Sigma, \Gamma, \Delta, z_0, k_0, E)$ is defined as following:

- Set of states $Z = \{ z_0, z_1, z_e \}$
- Input alphabet $\Sigma = \{ 0,1 \}$
- Stack alphabet $\Gamma = \{ k_0, x \}$
- z_0 is start state
- k_0 is initial stack symbol
- accept states $E = \{ z_e \}$
- Transition relation Δ:

(1) $z_0, k_0 \xrightarrow{0} z_0, k_0 x$

(2) $z_0, x \xrightarrow{0} z_0, xx$

(3) $z_0, x \xrightarrow{1} z_1, x$

(4) $z_1, x \xrightarrow{0} z_1, \varepsilon$

(5) $z_1, k_0 \xrightarrow{\varepsilon} z_e, \varepsilon$

a) Which of the following strings are accepted by P?

(1) ε

(2) 00100

(3) 0010

(4) 0100

b) Which language does the PDA accept?