Exercise 11.1 - obligatory (4 points)

Which of the following propositional formulas are satisfiable, which are valid (tautologies), which are unsatisfiable?

(1) rich ^ sick → not happy : satisfiable

happy	rich	sick	((rich ∧ sick) → ¬happy)
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	F

(2) not happy ^ not sick ^ (happy or sick) : unsatisfiable

happy	sick	((¬happy ∧ ¬sick) ∧ (happy ∨ sick))
F	F	F
F	T	F
T	F	F
T	T	F

(3) (sick → not happy) or sick: tautology

happy	sick	((sick → ¬happy) ∨ sick)
F	F	T
F	T	T
T	F	T
T	T	T

(4) (happy ^ not rich) or (not happy ^ rich): satisfiable

happy	rich	((happy ∧ ¬rich) ∨ (¬happy ∧ rich))
F	F	F
F	T	T
T	F	T
T	T	F

Exercise 11.2

Are both formulas equivalent? (give a short justification)

(1) not (p ^ (q or r)) and. not p or not q ^ not r

(2) a → b. and. not a → not b

(3) a → b. and. not b → not a

Exercise 11.3 - obligatory (6 points)

Let the following vocabulary for first-order logic formulas be given:

- Function symbols: f (unary)
  g (binary)

- Predicate symbols: P (unary)
  Q (binary)
- Constants: a, b
- Variables: x, y, z

Which of the following first-order formulas are syntactically correct? Indicate all errors.

(1) Q(f(x), g(a))

Incorrect, g takes 2 arguments but only one provided

(2) ∃z f(z) → ∀x Q(x, f(z))

Correct (not sure if f(z) can follow ∃z)

(3) ∃z ∀x Q(f(x), z) or ∀z P(g(z,z))

Correct

(4) ∀ y Q(f(y) or g(a,b))

Incorrect, Q takes 2 arguments but after OR only one is provided

(5) ∃not∀"y Q(y, f(a))

Incorrect, universal quantifier cannot follow directly after existential quantifier.

(6) ∃y not (∀z P(f(x)) or ∃y Q(a,y))

Correct

Exercise 11.4

Let the following predicates on persons be given:

Male(x)	"x is male"
Brother(p1, p2)	"p1 is brother of p2"
Child(c, p)	"c is a child of p" (i.e. p is parent of c)
Aunt(a, p)	"a is aunt of p"

a) Explain in everyday language the meaning of the following formulas. Are the formulas true if they are considered as statements about all living persons?

(1) not∀p Male(p)
(2) ∀p ∃b Brother(b, p)
(3) ∃p ∃b Brother(b, p)
(4) ∃p ∀b Brother(b, p)
(5) ∀p ∀b Brother(b, p)

b) Formulate as a first-order formula:

(1) "There is someone who is male."
(2) If a person p1 is brother of p2, then also p2 is brother of p1.
(3) "Not every person has a child."
(4) "A male person is never aunt of someone else."
(5) "If any two persons are brothers, then they have a common parent."
(6) "If a person is brother of someone else, then the person is male."

Exercise 11.5 - obligatory (10 points)

Let the following vocabulary for first-order formulas be given. We assume natural numbers ℕ as domain of discourse.

Constants: 0, 1, 2

Function symbols: + (addition, binary),
• (multiplication, binary)

Predicate symbols: = (equality, binary),
< (less-than relation, binary),
Prime (is a prime number, unary)

As usual, +, •, =, and < can be written in infix notation.

Formulate the following statements about natural numbers as first order formulas:

a) For any number n, n+n is never a prime number.

$\forall n \; !\mathrm{Prime}(n + n)$

b) If p is a prime number greater than 2, then p+1 is not prime.

$\forall p \; \mathrm{Prime}(p) \land !(p < (2 + 1)) \rightarrow !\mathrm{Prime}(p + 1)$

c) There is no number n > 1, so that n+n is a prime number.

$\forall n \; !(n < 2) \land !\mathrm{Prime}(n + n)$

d) The sum of two numbers is always less than the product of both numbers.

$\forall n \; (n + n) < (n \, . \, n)$

e) For every prime number, there is a larger number that is also prime.4) p "b Brother(b, p)

$\exists n \; \mathrm{Prime}(p) \land p < n$