National
Qualifications
2022

2022 Mathematics

Advanced Higher

Paper 1 (Non-calculator)

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ evidence use of quotient rule with denominator and one term of numerator correct [1]	$\bullet^1$ $\dfrac{-3\left(x^2+4\right)\ldots}{\left(x^2+4\right)^2}$ **OR** $\dfrac{\ldots-2x(1-3x)}{\left(x^2+4\right)^2}$	**3**
			$\bullet^2$ complete differentiation	$\bullet^2$ $\dfrac{-3\left(x^2+4\right)-2x(1-3x)}{\left(x^2+4\right)^2}$	
			$\bullet^3$ simplify [2,3]	$\bullet^3$ $\dfrac{3x^2-2x-12}{\left(x^2+4\right)^2}$	

Notes:

1. Where a candidate equates $\dfrac{dy}{dx}$ to y do not withhold $\bullet^1$.
2. $\bullet^3$ is available only where candidates have multiplied out brackets and collected like terms on the numerator.
3. Do not award $\bullet^3$ for incorrect manipulation subsequent to a correct answer.

Commonly Observed Response:

Candidates who use the product rule:

$\bullet^1$ $-3\left(x^2+4\right)^{-1}+(1-3x)\ldots$ or $\ldots\left(x^2+4\right)^{-1}+(1-3x)-2x\left(x^2+4\right)^{-2}$

$\bullet^2$ $-3\left(x^2+4\right)^{-1}-2x(1-3x)\left(x^2+4\right)^{-2}$

$\bullet^3$ $\left(3x^2-2x-12\right)\left(x^2+4\right)^{-2}$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^4$ start differentiation [1]	$\bullet^4$ $-\operatorname{cosec}5x\cot5x$	**2**
			$\bullet^5$ apply chain rule	$\bullet^5$ $-5\operatorname{cosec}5x\cot5x$	

Notes:

1. Where a candidate equates $f'(x)$ to $f(x)$, $\bullet^4$ is not available. See COR.

Commonly Observed Responses:

$$f(x)=\operatorname{cosec}5x$$
$$=-5\operatorname{cosec}5x\cot5x$$

Unless subsequently corrected, award $\bullet^5$ only.

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] set up augmented matrix	•[1] $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 4 \\ 2 & 1 & -3 & 3 \\ 1 & -7 & -4 & 9 \end{array}\right)$	4
			•[2] obtain two leading zeros [1]	•[2] $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 4 \\ 0 & 5 & -5 & -5 \\ 0 & -5 & -5 & 5 \end{array}\right)$	
			•[3] complete row operations [1]	•[3] $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 4 \\ 0 & 5 & -5 & -5 \\ 0 & 0 & -10 & 0 \end{array}\right)$	
			•[4] obtain solution [2]	•[4] $x=2,\ y=-1,\ z=0$	

Notes:

1. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •[2] and •[3].
2. Where a candidate erroneously obtains inconsistency or redundancy in the system and communicates this, •[4] is available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] find conjugate	•[1] $6-2i$	2
			•[2] multiply [1]	•[2] $36+8i$	

Notes:

1. Where a candidate does not identify $\overline{z_2}$, •[2] is still available for any correct expansion and simplification of the form $(a+bi)(c+di)$.

Commonly Observed Responses:

$(5+3i)(6+2i)$ leading to $24+28i$ award 1/2.

$\frac{5+3i}{6+2i}\times\frac{6-2i}{6-2i}$ leading to $\frac{9+2i}{10}$ award 1/2.

$\frac{5+3i}{6-2i}\times\frac{6+2i}{6+2i}$ leading to $\frac{6+7i}{10}$ award 1/2.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•1 apply product or chain rule	•1 $3y^2\frac{dy}{dx}$ or $2y+2x\frac{dy}{dx}$	**3**
			•2 complete differentiation	•2 $3y^2\frac{dy}{dx}+4\frac{dy}{dx}=2y+2x\frac{dy}{dx}$	
			•3 find expression for $\frac{dy}{dx}$ [1]	•3 $\frac{dy}{dx}=\frac{2y}{3y^2+4-2x}$	

Notes:

1. •3 is available only where $\frac{dy}{dx}$ appears more than once after the candidate has completed their differentiation.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•4 evaluate gradient [1, 2]	•4 $m=-2$	**1**

Notes:

1. For •4, accept $\frac{dy}{dx}=-2$.
2. •4 is available only if a candidate's expression for $\frac{dy}{dx}$ contains both x and y terms.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•5 find value of y [1, 2]	•5 $y=0$	**2**
			•6 demonstrate inconsistency [3]	•6 $\text{LHS}=0, \text{RHS}=1$ $\therefore$ inconsistent	

Notes:

1. •5 is available only where $\frac{dy}{dx}$ is a fraction where the numerator and denominator both contain a variable.
2. •5 is not available when $y=0$ appears as part of an incorrect assumption eg 'stationary points when $x=0$, $y=0$'.
3. Working for •6 must include substitution of $y=0$ and retention of the variable x in the original equation.

Commonly Observed Responses:

Communication at •6

Working leading to '$0=1$' with no further communication. Do not award •6.

Working leading to '$0=1$ no stationary point'. Award •6.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		**Method 1** •¹ all three derivatives AND all four evaluations •² obtain simplified expression [1, 2] **Method 2** •¹ write down Maclaurin series for e^x •² substitute and simplify [1, 2]	**Method 1** $f(x)=e^{-4x}$ $\quad f(0)=1$ •¹ $f'(x)=-4e^{-4x}$ $\quad f'(0)=-4$ $f''(x)=16e^{-4x}$ $\quad f''(0)=16$ $f'''(x)=-64e^{-4x}$ $\quad f'''(0)=-64$ stated or implied •² $1-4x+8x^2-\frac{32}{3}x^3$ **Method 2** •¹ $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ stated or implied •² $1-4x+8x^2-\frac{32}{3}x^3$	2

Notes:

1. The simplification for •² may appear in (b).
2. Do not accept $+-4x$ or $+-\frac{32}{3}x^3$ at •² unless simplified in (b). Do accept $+\frac{-32}{3}x^3$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		•³ express as a product [1] •⁴ expand and simplify [1, 2]	•³ $(3+2x)e^{-4x}$ stated or implied •⁴ $3-10x+16x^2-16x^3$	2

Notes:

1. For candidates who produce an incorrect Maclaurin expansion for $3+2x$ and multiply this by e^{-4x}, •³ may be awarded but •⁴ is unavailable.
2. At •⁴, ignore any higher order terms.

Commonly Observed Responses:

Applying Maclaurin expansion to quotient

•³ first derivative and two evaluations OR all three derivatives. Derivatives need not be simplified.

•⁴ complete expansion.

$$f(x)=\frac{3+2x}{e^{4x}} \qquad f(0)=3$$

$$f'(x)=\frac{-10-8x}{e^{4x}} \qquad f'(0)=-10$$

$$f''(x)=\frac{32+32x}{e^{4x}} \qquad f''(0)=32$$

$$f'''(x)=\frac{-96-128x}{e^{4x}} \qquad f'''(0)=-96$$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•¹ find counterexample AND state composite number AND communicate [1, 2]	•¹ eg when $n=9$ AND $n^2+4=85$ AND which is not prime	**1**

Notes:

1. Acceptable communication to demonstrate that a number is not prime at •¹ may take the form of eg:
   - 85 is divisible by 5
   - $85 = 5 \times 17$
   - 85 is composite
2. Insufficient communication includes '85 so false'.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² correct form for two consecutive integers [1, 2, 3]	•² eg n, $n+1$ AND $n \in Z$	**3**
			•³ correct expression [1, 4]	•³ $(n+1)^3 - n^3$	
			•⁴ multiply, express in correct form and communicate [1, 5, 6, 7]	•⁴ eg $3(n^2+n)+1$ which is not divisible by 3	

Notes:

1. Where a candidate uses two unrelated variables, eg n and $x+1$, award 0/3.
2. At •², accept $n \in Z$ expressed in words.
3. Where a candidate does not consider every pair of consecutive integers, eg $2k$ and $2k+1$, •² is not available.
4. For the award of •³ at least one expression must be of the form $(an \pm b)^3$ where $a, b \neq 0$.
5. •⁴ is available only where a candidate has processed two cubed expressions.
6. Reference to 'false' at •⁴ is acceptable only if it references divisibility by 3 and not the statement.
7. At •⁴, after taking a common factor of 3 to leave a remainder, acceptable communication may take the form of eg
   - … therefore true
   - … gives a remainder of 1

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ differentiate ¹ •² determine new limits and begin to rewrite integral ¹ •³ complete integral ² •⁴ integrate and evaluate ³	•¹ $\frac{du}{dy}=2y$ or $du=2y\,dy$ •² $\int_1^{26}\ldots du$ •³ $2\int_1^{26}\frac{1}{\sqrt{u}}du$ •⁴ $4\left(\sqrt{26}-1\right)$	4

Notes:

1. Where a candidate attempts to process an integrand containing both u and y, only •¹ and •² are available.
2. Limits are not necessary for the award of •³.
3. Where a candidate erroneously includes a limit of 0 for u, •⁴ is not available.

Commonly Observed Responses:

A No limits in new integral and return to original variable:

$2\int\frac{1}{\sqrt{u}}du$ award •³

$\left[4(y^2+1)^{\frac{1}{2}}\right]_0^5$ award •²

B Wrong limits in new integral and return to original variable:

$\int_0^5\ldots du$ do not award •²

$2\int_0^5\frac{1}{\sqrt{u}}du$ award •³

$\left[4(y^2+1)^{\frac{1}{2}}\right]_0^5$ leading to $4\left(\sqrt{26}-1\right)$ award •⁴

C Wrong limits in new integral and no return to original variable:

$\int_0^5\ldots du$ do not award •²

$2\int_0^5\frac{1}{\sqrt{u}}du$ award •³

$\left[4u^{\frac{1}{2}}\right]_0^5$ do not award •⁴

Question			Generic scheme	Illustrative scheme	Max mark
7.	(b)		$\bullet^5$ obtain area [1]	$\bullet^5$ $8\left(\sqrt{26}-1\right)$ (square units)	**1**

Notes:

1. For a candidate who obtains a negative value at $\bullet^4$, $\bullet^5$ is available only where they interpret the area as a positive number.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		$\bullet^6$ obtain expression [1]	$\bullet^6$ $1-\frac{1}{y^2+1}$	**1**

Notes:

1. At $\bullet^6$, accept $1+\frac{-1}{y^2+1}$ but do not accept $1+-\frac{1}{y^2+1}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(d)		•7 correct form of integral [1, 2] •8 find expression in integrable form •9 integrate $-\frac{1}{y^2+1}$ [3, 4] •10 complete integration and evaluate [3, 5]	•7 $\pi\int_0^5 x^2\,dy$ •8 $16\pi\int_0^5\left(1-\frac{1}{y^2+1}\right)dy$ •9 $\ldots-\tan^{-1}y$ •10 $16\pi\left(5-\tan^{-1}5\right)$ (cubic units)	**4**

Notes:

1. For the award of •7:
   - limits must appear at some point
   - dy must appear at some point.
2. At •7 accept $V=\pi\int_0^5\left[f(y)\right]^2dy$.
3. Where a candidate attempts to process an integrand containing both u and y, •9 and •10 are unavailable.
4. •9 is available only for the integration of a term of the form $\pm\frac{b}{y^2+1}$, $b\in\square$.
5. •10 is not available where:
   - the integrand at •8 is not beyond Higher Mathematics standard
   - a candidate produces an undefined expression eg $\ln 0$.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]