National
Qualifications
2022

2022 Mathematics

Advanced Higher

Paper 2

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(a) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$ Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $y = 5$ and $y = -7$ $\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(i) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(j) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as
  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$
  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$
  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ state expression •² form equation **AND** obtain one of *A*, *B* and *C* •³ state expression 1	•¹ $\frac{A}{x}+\frac{Bx+C}{x^2+5}$ •² $3x^2-3x+5=A\left(x^2+5\right)+\left(Bx+C\right)x$ **and** any one from $A=1,\ B=2,\ C=-3$ •³ $\frac{1}{x}+\frac{2x-3}{x^2+5}$	3

Notes:

1. At •³ accept values explicitly stated for A, B and C provided the template is written at •¹.

Commonly Observed Responses:

Two constant numerators

For $\frac{3x^2-3x+5}{x\left(x^2+5\right)}=\frac{A}{x}+\frac{B}{x^2+5}$ award 0/3.

One constant and one incorrect linear numerator

For $\frac{3x^2-3x+5}{x\left(x^2+5\right)}=\frac{A}{x}+\frac{Bx}{x^2+5}$ award 0/3.

One constant and one quadratic numerator

•¹ $\frac{A}{x}+\frac{Bx^2+Cx+D}{x^2+5}$

•² $3x^2-3x+5=A\left(x^2+5\right)+\left(Bx^2+Cx+D\right)x$ AND $B=0$ and one other from $A=1, C=2, D=-3$

•³ $\frac{1}{x}+\frac{2x-3}{x^2+5}$

Where a candidate uses this approach but does not find $B=0$, •² and •³ are unavailable.

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] begin integration [1] •[2] complete integration and evaluate [2, 3]	•[1] $\ln(2x+1)$ •[2] $2\ln 7$	2

Notes:

1. Where a candidate uses a substitution of eg $u=2x+1$, award •[1] for the appearance of $\ln u$.
2. At •[2] do not accept an unsimplified final answer of $2(\ln 7-\ln 1)$.
3. At •[2] accept $2\ln|7|$ or $\ln 7^2$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.				$634 = 7 \times 87 + 25$	3
			•¹ start process and obtain remainder of 1	•¹ $87 = 3 \times 25 + 12$ $25 = 2 \times 12 + 1$	
			•² express gcd in terms of 87 and 25	•² $1 = 25 - 2(87 - 3 \times 25)$	
			•³ obtain a and b [1]	•³ $a = 7 \quad b = -51$	

Notes:

1. At •³, where candidates do not explicitly communicate a and b, accept eg $634 \times 7 + 87 \times -51 = 1$ or $634 \times 7 + -51 \times 87 = 1$. Do not accept $634 \times 7 - 87 \times 51 = 1$ or $634 \times 7 - 51 \times 87 = 1$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•¹ integrate to find "$uv-$" ¹ •² differentiate to find "$\int u'v\,dx$" ² •³ obtain full solution ³˒ ⁴	•¹ $\frac{1}{3}(x+2)(2x+7)^{\frac{3}{2}}-\ldots$ •² $\ldots\int\frac{1}{3}(2x+7)^{\frac{3}{2}}\,dx$ •³ $\frac{1}{3}(x+2)(2x+7)^{\frac{3}{2}}-\frac{1}{15}(2x+7)^{\frac{5}{2}}+c$	3

Notes:

1. For candidates who choose to integrate $(x+2)$ and differentiate $(2x+7)^{\frac{1}{2}}$ and process this correctly award •¹ only.
2. Disregard the omission of dx at •².
3. General marking principle (l) applies at •³ to candidates who eg repeatedly fail to divide by 2.
4. Disregard omission of $+c$ at •³.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•[1] begin process •[2] find determinant •[3] equate to zero and solve for k [1, 2]	•[1] $1\begin{vmatrix} k & 3 \\ 18 & -7 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ k & -7 \end{vmatrix} + 1\begin{vmatrix} 2 & k \\ k & 18 \end{vmatrix}$ or equivalent •[2] $-k^2 + 2k + 24$ stated or implied •[3] eg $(k-6)=0$, $(k+4)=0$ leading to 6 and -4	**3**

Notes:

1. •[3] may be awarded only if '= 0' appears at some point in the solution.
2. •[3] may be awarded only where $k \in$ □ .

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•1 use two terms to find difference [1, 2, 3] •2 check with a different pair of terms and conclude [1, 3, 4]	•1 eg $(3x+2)-(x+5)=2x-3$ •2 eg $(5x-1)-(3x+2)=2x-3$, common difference	2

Notes:

1. Where a candidate picks a numerical value for x, •1 and •2 are not available unless an algebraic approach is also considered in (a) or $2x-3$ appears in (b) or (c).
2. Award •1 for $2x-3$ with no working.
3. Accept $3-2x$ for the award of •1 and in subsequent working for •2.
4. For •2 candidates must show algebraically a link between two differences and communicate commonality. See CORS.

Commonly Observed Responses:

'$x+5 \overset{2x-3}{\quad} 3x+2 \overset{2x-3}{\quad} 5x-1$ so there is a common difference' award 2/2.

Working eg $\begin{aligned}3x+2-(x+5)&=2x-3\\3x+2+2x-3&=5x-1\end{aligned}$ followed by communication:

- 'so arithmetic' award 2/2
- 'same difference' award 2/2
- 'difference is $2x-3$' award 2/2
- 'difference' award 1/2. Insufficient communication.

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•3 substitute into formula [1, 2, 3, 4] •4 simplify expression [1, 4]	•3 $x+5+(15-1)(2x-3)$ •4 $29x-37$	2

Notes:

1. Where a candidate substitutes a numerical value for either x or d, •3 and •4 are unavailable unless an algebraic approach is also present.
2. Where a candidate uses $3-2x$ from (a), •3 is unavailable.
3. Where a candidate substitutes an incorrect algebraic expression for d, other than $3-2x$, •3 is available only if they have identified the expression as the common difference in (a), either in words or "$d=\ldots$".
4. When an incorrect algebraic expression appears at •3, •4 may be awarded only where the candidate has had to expand brackets and collect like terms.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[5] substitute into formula and equate [1, 2, 3] •[6] solve for x [1, 4]	•[5] $\frac{20}{2}\left[2(x+5)+(20-1)(2x-3)\right]=1130$ •[6] $x=4$	**2**

Notes:

1. Where a candidate substitutes a numerical value for x or d, •[5] and •[6] are unavailable unless an algebraic approach is also present.
2. Where a candidate substitutes $3-2x$ for d, •[5] is available only if $3-2x$ appears as part of an expression at •[3].
3. Where a candidate substitutes an incorrect algebraic expression for d, other than $3-2x$, •[5] is available only if they have identified the expression as the common difference in (a) or (b),

- in words
- by writing "$d = \ldots$"
- as an entry for a common difference in the correct expression at (b)
- as an entry identified as d in an incorrect expression at (b).

4. •[6] is available only where $x \in$ □ .

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] state second root	•[1] $3-i$	**1**

Notes:

Commonly Observed Responses:

	(b)		•[2] form product of linear factors •[3] find the value of a [1, 2]	•[2] $(z-(3-i))(z-(3+i))$ •[3] 10	**2**

Notes:

1. For the award of •[3], $a \in \square$.
2. **Beware** of incorrect working leading to $a=10$ in •[3]. Where a candidate produces $z^2-6z+10$ and proceeds to use algebraic division, •[3] is not available for any remainder other than zero or $a-10$.

Commonly Observed Responses:

Alternative strategies for •[2]
Multiply roots
$(3+i)(3-i)$

Substitute either root into the quadratic
eg $(3+i)^2-6(3+i)+a=0$

Start process for synthetic division
eg as far as

$$\begin{array}{r|rrr} 3+i & 1 & -6 & a \\ & & 3+i & \\ \hline & 1 & -3+i & \end{array}$$

	(c)		•[4] find the value of b [1, 2, 3]	•[4] 50	**1**

Notes:

1. For the award of •[4], $b \in \square$.
2. For an answer with no working or insufficient working award 0/1.
3. **Beware** of incorrect working leading to $b=50$ in •[4]. Where a candidate uses algebraic division, •[4] is not available for any remainder other than zero or $b-50$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] evidence of use of product rule and one term correct [1]	•[1] $\ln x + \ldots$ **OR** $\ldots + x \times \frac{1}{x}$	**2**
			•[2] complete differentiation [1]	•[2] $\ln x$	

Notes:

1. For an answer with no working award 0/2.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] state form of integrating factor [1]	•[3] $e^{\int \ln x\, dx}$ stated or implied	**4**
			•[4] find integrating factor [1,2]	•[4] $e^{x\ln x - x}$	
			•[5] rewrite as integral equation [1,2,3,4]	•[5] $ye^{x\ln x - x} = \int e^{x\ln x - x} x^{-x}\, dx$	
			•[6] integrate and rewrite [5,6]	•[6] $y = \dfrac{-e^{-x} + c}{x^x e^{-x}}$	

Notes:

1. For any attempt to separate variables, award 0/4.
2. •[5] is available for candidates who either have an incorrect integrating factor at •[4] or who have incorrectly simplified a correct integrating factor prior to •[5]. SEE CORS.
3. Do not withhold •[5] for the omission of 'dx' on the right-hand side.
4. Where a candidate writes $\int \frac{d}{dx} \ldots$ on the left-hand side do not withhold •[5] provided the candidate later produces evidence that they have integrated with respect to x.
5. Accept $y = \dfrac{-e^{-x} + c}{e^{x\ln x - x}}$ for the award of •[6].
6. For a candidate who omits the constant of integration or produces an integrand which is not beyond Higher Mathematics standard, •[6] is not available.

Commonly Observed Responses:

Correct integrating factor at •[4] followed by incorrect simplification

$e^{\int \ln x\, dx}$ leading to eg $e^{x\ln x - x} = \frac{1}{x}$. Award •[3] and •[4]. If followed by $y\frac{1}{x} = \int \frac{1}{x} x^{-x}\, dx$, award •[5]. Do not award •[6].

Incorrect integrating factor at •[4]

$e^{\int \ln x\, dx}$ leading to eg $\frac{1}{x}$ (without evidence of $e^{x\ln x - x}$). Award •[3], do not award •[4]. If followed by $y\frac{1}{x} = \int \frac{1}{x} x^{-x}\, dx$, award •[5].

Question			Generic scheme	Illustrative scheme	Max mark
9.			•[1] show true for $n=1$ [1]	•[1] when $n=1$ $\begin{pmatrix} 3 & 1-3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}$ so true for $n=1$.	5
			•[2] assume true for $n=k$ and consider $n=k+1$ [2]	•[2] assume true for $n=k$ AND $A^k = \begin{pmatrix} 3^k & 1-3^k \\ 0 & 1 \end{pmatrix}$ AND $A^{k+1} = \ldots$	
			•[3] use inductive hypothesis	•[3] $\begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 1-3^k \\ 0 & 1 \end{pmatrix}$	
			•[4] multiply matrices [3]	•[4] $\begin{pmatrix} 3\times 3^k & 3\times\left(1-3^k\right)+\left(-2\right) \\ 0 & 1 \end{pmatrix}$	
			•[5] simplify to appropriate form and communicate result [4, 5, 6]	•[5] $\begin{pmatrix} 3^{k+1} & 3-3^{k+1}-2 \\ 0 & 1 \end{pmatrix}$ leading to $\begin{pmatrix} 3^{k+1} & 1-3^{k+1} \\ 0 & 1 \end{pmatrix}$ so if result is true for $n=k$ then it is true for $n=k+1$. But it is true for $n=1$ and so it is true $\forall n \in N$ by mathematical induction.	

Question		Generic scheme	Illustrative scheme	Max mark
9.	(continued)			

Notes:

1. ‘ $A=\begin{pmatrix}3 & -2\\0 & 1\end{pmatrix}$, $A^1=\begin{pmatrix}3 & -2\\0 & 1\end{pmatrix}$’ and/or ‘True for n =1’ are insufficient for the award of $\bullet^1$. A candidate must demonstrate evidence of substitution.

2. For $\bullet^2$ sufficient phrases for $n=k$ contain:
   - ‘If true for...’; ‘Suppose true for…’; ‘Assume true for…’.

   For $\bullet^2$ insufficient phrases for $n=k$ contain:
   - ‘Consider $n=k$’, ‘assume $n=k$’, ‘assume $n=k$ is true’ and ‘True for $n=k$’.

   A sufficient phrase for the award of $\bullet^2$ may appear at $\bullet^5$.

   For $\bullet^2$, accept:
   - assume true for $n=k$ AND $A^k=\begin{pmatrix}3^k & 1-3^k\\0 & 1\end{pmatrix}$ AND ‘Aim/goal: $A^{k+1}=\begin{pmatrix}3^{k+1} & 1-3^{k+1}\\0 & 1\end{pmatrix}$’.

   For $\bullet^2$ unacceptable phrases for $n=k+1$ contain:
   - ‘Consider true for $n=k+1$’, ‘true for $n=k+1$’;
   - ‘ $A^{k+1}=\begin{pmatrix}3^{k+1} & 1-3^{k+1}\\0 & 1\end{pmatrix}$’ (with no reference to aim/goal or no further processing) .

3. Award $\bullet^4$ where products resulting in zero are shown.
4. $\bullet^5$ is unavailable to candidates who write down the correct expression without algebraic justification.
5. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently reference the stated aim/goal.
6. Following the required algebra and statement of the inductive hypothesis, the minimum acceptable response for $\bullet^5$ is ‘Then true for $n=k+1$, but since true for $n=1$, then true for all n’ or equivalent.

Commonly Observed Responses:

Multiplication order A^kA

$\bullet^3$ $\begin{pmatrix}3^k & 1-3^k\\0 & 1\end{pmatrix}\begin{pmatrix}3 & -2\\0 & 1\end{pmatrix}$

$\bullet^4$ $\begin{pmatrix}3^k\times3 & 3^k\times(-2)+1-3^k\\0 & 1\end{pmatrix}$ stated or implied by

$\bullet^5$ $\begin{pmatrix}3^{k+1} & 3^k\times(-2)+1-3^k\\0 & 1\end{pmatrix}$ leading to $\begin{pmatrix}3^{k+1} & 1-3^{k+1}\\0 & 1\end{pmatrix}$

so if result is true for $n=k$ then it is true for $n=k+1$. But it is true for $n=1$ and so it is true $\forall n\in N$ by mathematical induction.

Question			Generic scheme	Illustrative scheme	Max mark
10.					9
			•1 construct auxiliary equation [1,4,5]	•1 $m^2 - 4m + 4 = 0$	
			•2 find complementary function [2,3,4,]	•2 $y = Ae^{2x} + Bxe^{2x}$	
			•3 state particular integral and obtain first and second derivatives of particular integral	•3 $y = C\sin x + D\cos x$ $\frac{dy}{dx} = C\cos x - D\sin x$ $\frac{d^2y}{dx^2} = -C\sin x - D\cos x$	
			•4 substitute into LHS of differential equation [6]	•4 $(-C\sin x - D\cos x)$ $-4(C\cos x - D\sin x)$ $+4(C\sin x + D\cos x)$	
			•5 derive simultaneous equations [6]	•5 $3C + 4D = 9$ $-4C + 3D = 13$	
			•6 obtain both constants of particular integral [6]	•6 $C = -1 \quad D = 3$	
			•7 differentiate general solution [6,7]	•7 $\frac{dy}{dx} = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x}$ $-\cos x - 3\sin x$	
			•8 find coefficient of e^{2x} [4,5,6]	•8 $A = 2$	
			•9 find second coefficient and state particular solution [3,5,6,8]	•9 $B = -3$ $y = 2e^{2x} - 3xe^{2x} - \sin x + 3\cos x$	

Question		Generic scheme	Illustrative scheme	Max mark
10.	(continued)			

Notes:

1. •[1] is not available where $'=0'$ has been omitted.
2. •[2] may still be awarded if the complementary function appears only as part of a general solution or the particular solution.
3. Do not withhold •[2] for the omission of $'y=...'$ provided it appears as part of a general solution or at •[9].
4. Where a candidate does not introduce a particular integral only •[1], •[2] and •[8] are available.
5. Where a candidate introduces a particular integral which does not involve any trigonometric terms, only •[1], •[2], •[8] and •[9] are available.
6. Where a candidate introduces a particular integral with only one trigonometric term •[4] is available but •[5] and •[7] are not. Moreover,
   - award •[6] for communicating inconsistency when comparing coefficients eg '$3C=9$ and $C=0$ - not possible'. Proceeding with a non-zero value for eg C, •[8] and •[9] are also available.
   - only •[8] and •[9] are available when inconsistency is not recognised.
7. •[7] is available only where differentiation involves:
   - a complementary function requiring the product rule AND
   - a particular integral with both $\sin x$ and $\cos x$.
8. •[9] is available only where a (correct or incorrect) particular integral which is consistent with previous working (and is not simply appearing for the first time at this stage) is included as part of the particular solution.

Commonly Observed Responses:

Auxiliary Equation Correct

$m^2-4m+4=0$

Complementary function of $y=Ae^{-2x}+Bxe^{-2x}$ (leading to particular solution of $y=2e^{-2x}+5xe^{-2x}-\sin x+3\cos x$) do not award •[2].

Auxiliary Equation Incorrect

$m^2+4m+4=0$

Complementary function of $y=Ae^{-2x}+Bxe^{-2x}$ (leading to particular solution of $y=2e^{-2x}+5xe^{-2x}-\sin x+3\cos x$) do not award •[1].

Complementary Function Incorrect

Complementary function of eg $y=Ae^{2x}+Be^{2x}$ (leading to inconsistent values of A and B), •[7], •[8] and •[9] are not available.

Question			Generic scheme	Illustrative scheme	Max mark
11.			$\bullet^1$ find $\frac{dx}{dt}$ $\bullet^2$ equate expressions for $\frac{dy}{dx}$ [1] $\bullet^3$ simplified expression for $\frac{dy}{dt}$ [1, 2, 3] $\bullet^4$ find expression for y [1, 2, 3]	$\bullet^1$ $\frac{dx}{dt}=\frac{2}{1+(2t)^2}$ $\bullet^2$ $\frac{\frac{dy}{dt}}{\left(\frac{2}{1+4t^2}\right)}=6t\left(1+4t^2\right)$ $\bullet^3$ $\frac{dy}{dt}=12t$ $\bullet^4$ $y=6t^2-1$	**4**

Notes:

1. For candidates who attempt to find $\int 6t\left(1+4t^2\right)dt$, $\bullet^2$, $\bullet^3$ and $\bullet^4$ are unavailable.

2. $\bullet^3$ and $\bullet^4$ are available only where a candidate clearly identifies an expression for $\frac{dy}{dt}$.

3. $\bullet^4$ is available where a candidate produces a non-zero constant term for $\frac{dy}{dt}$ at $\bullet^3$.

Commonly Observed Responses:

Candidates who do not apply the chain rule at $\bullet^1$.

$\frac{dx}{dt}=\frac{1}{1+(2t)^2}$ do not award $\bullet^1$

$\bullet^2$ $\frac{\frac{dy}{dt}}{\left(\frac{1}{1+4t^2}\right)}=6t\left(1+4t^2\right)$

$\bullet^3$ $\frac{dy}{dt}=6t$

$\bullet^4$ $y=3t^2+2$

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•¹ expression	•¹ $\cos 4\theta + i\sin 4\theta$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² binomial expansion [1, 2, 3]	•² $(\cos\theta)^4 + 4(\cos\theta)^3(i\sin\theta)$ $+6(\cos\theta)^2(i\sin\theta)^2 + 4(\cos\theta)(i\sin\theta)^3$ $+(i\sin\theta)^4$	**3**
			•³ simplify three terms [1, 2, 3]	•³ three from: $\cos^4\theta + 4\cos^3\theta i\sin\theta - 6\cos^2\theta\sin^2\theta$ $-4\cos\theta i\sin^3\theta + \sin^4\theta$	
			•⁴ complete simplification [2, 3]	•⁴ $\cos^4\theta + 4\cos^3\theta i\sin\theta - 6\cos^2\theta\sin^2\theta$ $-4\cos\theta i\sin^3\theta + \sin^4\theta$	

Notes:

1. Do not withhold •² for the omission of brackets provided there is evidence to award •³ .
2. Where i terms are not raised to a power greater than one and are not later resolved, •² •³ •⁴ are not available.
3. Where a candidate does not write out a full binomial expansion, •² is unavailable but •³ and •⁴ may still be awarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(c)	(i)	•5 equate real parts [1] •6 obtain result [1]	•5 $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ •6 $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\left(1-\cos^2\theta\right) + \left(1-\cos^2\theta\right)^2$ leading to $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	2

Notes:

1. Do not withhold •5 for the omission of '$\cos 4\theta =$' provided it appears at •6.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	•7 expression for $\cot 4\theta$ in terms of $\sin\theta$ and $\cos\theta$ [1] •8 rewrite in terms of $\cos\theta$ [2, 3]	•7 $\dfrac{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}$ •8 $\sin\theta\cot 4\theta = \dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{8\cos^3\theta - 4\cos\theta}$	2

Notes:

1. Accept $\dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}$ for the award of •7.
2. Accept $\sin\theta\cot 4\theta = \dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{4\cos^3\theta - 4\cos\theta\left(1-\cos^2\theta\right)}$ for the award of •8.
3. Do not withhold •8 for the omission of $\sin\theta\cot 4\theta$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)	(i)	•[1] full turn divided by time [1, 2]	•[1] $\frac{2\pi}{12}$ leading to $\frac{\pi}{6}$.	1

Notes:

1. •[1] is available to candidates who work in degrees and then convert to radians.
2. If $\frac{2\pi}{12}$ follows directly from an incorrect assumption, eg the rate of change of LP with respect to time is a constant, then •[1] is not available.

Commonly Observed Responses:

Angular velocity approach

Given that the angular velocity in this case is constant, •[1] may be awarded.

$$\begin{aligned}\omega &= \frac{\theta}{t}\\ &= \frac{d\theta}{dt}\\ &= \frac{2\pi}{12}\\ &= \frac{\pi}{6}\end{aligned}$$

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	•[2] state relationship between derivatives	•[2] $\frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$	4
			•[3] find x in terms of θ	•[3] $x = 10\tan\theta$	
			•[4] differentiate wrt θ [1, 2]	•[4] $\frac{dx}{d\theta} = 10\sec^2\theta$ stated or implied	
			•[5] substitute and simplify	•[5] $\frac{dx}{dt} = 10\sec^2\theta.\frac{\pi}{6}$ leading to $\frac{dx}{dt} = \frac{5\pi}{3}\sec^2\theta$	

Notes:

1. •[4] is not available for a candidate who differentiates $x = \frac{5\pi}{3}\tan\theta$.
2. •[4] may be awarded only where a candidate differentiates a clearly defined expression for x, other than $x = \frac{5\pi}{3}\tan\theta$, and the differentiation is beyond Higher Mathematics standard.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•[6] complete proof [1]	•[6] eg $1+\frac{\sin^2\theta}{\cos^2\theta}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$ or $\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}$ $=\frac{1}{\cos^2\theta}$ $=\sec^2\theta$	**1**

Notes:

1. Accept proof using another (single) variable in place of θ.

Commonly Observed Responses:

Starting with $\cos^2\theta+\sin^2\theta=1$

$$\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos\ \ \theta}=\frac{1}{\cos^2\theta}$$

$$1+\tan^2 2\theta=\sec^2\theta$$

Both lines of working must be present for the award of •[6]

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[7] find $\tan\theta$ or $\sec\theta$	•[7] $\tan\theta=\frac{5}{10}$ or $\sec\theta=\frac{\sqrt{125}}{10}$	**3**
			•[8] corresponding value of $\sec^2\theta$ [1]	•[8] $1+\left(\frac{5}{10}\right)^2$, $\frac{125}{100}$ or $1+\left(\tan\left(\tan^{-1}\frac{1}{2}\right)\right)^2$ or equivalent	
			•[9] substitute and simplify [2, 3]	•[9] $\frac{25\pi}{12}$ ms^{-1}.	

Notes:

1. Where a candidate finds and rounds an angle for θ, •[8] is available only when rounding occurs to at least 2 significant figures.
2. Where a candidate substitutes an angle in degrees into their calculation, withhold •[9].
3. Units must appear beside an exact value for the award of •[9].

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

