



National  
Qualifications  
2021 ASSESSMENT RESOURCE

X847/77/12

**Mathematics  
Paper 2**

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Duration — 2 hours

Total marks — 60

**SECTION 1 — 45 marks**

Attempt ALL questions.

**SECTION 2 — 15 marks**

Attempt EITHER Part A OR Part B.

**You may use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

**Use blue or black ink.**

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



\* X 8 4 7 7 7 1 2 \*

## FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
$e^x$	$e^x$

Standard integrals	
$f(x)$	$\int f(x) dx$
$\sec^2(ax)$	$\frac{1}{a} \tan(ax) + c$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x}$	$\ln x  + c$
$e^{ax}$	$\frac{1}{a} e^{ax} + c$

### Summations

(Arithmetic series)  $S_n = \frac{1}{2} n [2a + (n-1)d]$

(Geometric series)  $S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$

$$\sum_{r=1}^n r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$$

### Binomial theorem

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r \quad \text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

### Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \dots$$

## FORMULAE LIST (continued)

**De Moivre's theorem**

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

**Vector product**

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\end{aligned}$$

**Matrix transformation**

Anti-clockwise rotation through an angle,  $\theta$ , about the origin,  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

[Turn over

## **SECTION 1 — 45 marks**

## **Attempt ALL questions**

1. Given  $f(x) = 3\sec 2x$ , find the exact value of  $f'\left(\frac{\pi}{8}\right)$ . 2

2. (a) Use the Euclidean algorithm to find integers  $a$  and  $b$  such that  $105a + 72b = 3$ . 3

(b) Hence find integers  $x$  and  $y$  such that  $105x + 72y = 360$ . 1

3. Use integration by parts to find  $\int (2x+3)\cos 4x \, dx$ . 3

4. A curve is defined parametrically by  
$$x = \sin^{-1} 2t \text{ and } y = \tan^{-1} t.$$
  
(a) Find  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ . 3

(b) When  $t = 0$  find the equation of the tangent to the curve. 2

5. A non-singular matrix  $A$  satisfies the equation  
$$A^2 = 2A + 5I,$$
 where  $I$  is the identity matrix.  
(a) Express  $A^4$  in the form  $pA + qI$ , where  $p, q \in \mathbb{Z}$ . 2

(b) Express  $A^{-1}$  in the form  $rA + sI$ , where  $r, s \in \mathbb{Q}$ . 2

6. Solve the differential equation

$$\frac{dy}{dx} + 2xy = 14xe^{-x^2}$$

given that when  $x = 0, y = 3$ .

Express  $y$  in terms of  $x$ .

4

7. A complex number is defined by  $z = a + 2i$  where  $a$  is a positive real number.

(a) State and simplify the binomial expansion of  $z^3$ .

3

(b) Given that  $z^3 + 3z = b + 148i$ , where  $b$  is a real number, find the values of  $a$  and  $b$ .

3

8. A curve is defined by  $x^2y^3 + e^{2y} = 5$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

4

(b) Show that there is only one stationary point on the curve.

3

9. (a) Express  $\frac{1}{x(5-x)}$  in partial fractions.

2

(b) A small island is being populated by seals. The size of the seal population can be modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{100}P(5-P), 0 < P < 5$$

where  $P$  (in hundreds) is the number of seals on the island  $t$  years after the seals arrive.

Given that there are 250 seals after 10 years, find an expression for  $P$  in terms of  $t$ .

8

[END OF SECTION 1]

[Turn over

**SECTION 2 — 15 marks**  
**Attempt EITHER Part A OR Part B**

**Part A**

10. Prove by induction that  $\sum_{r=2}^n \frac{1}{r(r-1)} = \frac{n-1}{n}$  for all positive integers  $n \geq 2$ . 5

11. Three consecutive terms of an arithmetic sequence are given by

$$x-1, \quad x-7, \quad 2x-9.$$

- (a) (i) Find the common difference. 1
- (ii) Hence find the value of  $x$ . 1
- (b) Given that  $x-1$  is the 21<sup>st</sup> term, find
  - (i) the value of the first term 1
  - (ii) a simplified expression for the  $n^{\text{th}}$  term of the sequence. 1

Three consecutive terms of a geometric sequence are given by

$$y-1, \quad y-7, \quad 2y-9.$$

- (c) Find the two possible values of  $y$  and the corresponding common ratios. 3

One of the values of  $y$  gives an associated geometric series which has a sum to infinity.

- (d) (i) Identify the value of  $y$  and justify your answer. 1
- (ii) Determine whether  $\frac{64}{3}$  is a possible value for this sum to infinity. Give a reason for your answer. 2

**Part B**

12. The points A(4, 0, 8), B(6, -5, 4) and C(3, 4, 11) all lie on the plane  $\pi_1$ .

- (a) Find the Cartesian equation of  $\pi_1$ .

4

The plane  $\pi_2$  is parallel to  $\pi_1$  and passes through the origin.

- (b) State the equation of  $\pi_2$ .

1

A sphere touches  $\pi_1$ , where A is the point of contact. The sphere also has a single point of contact, Q, with  $\pi_2$ .

- (c) (i) Find parametric equations for the line AQ.

1

- (ii) Hence find the coordinates for Q.

2

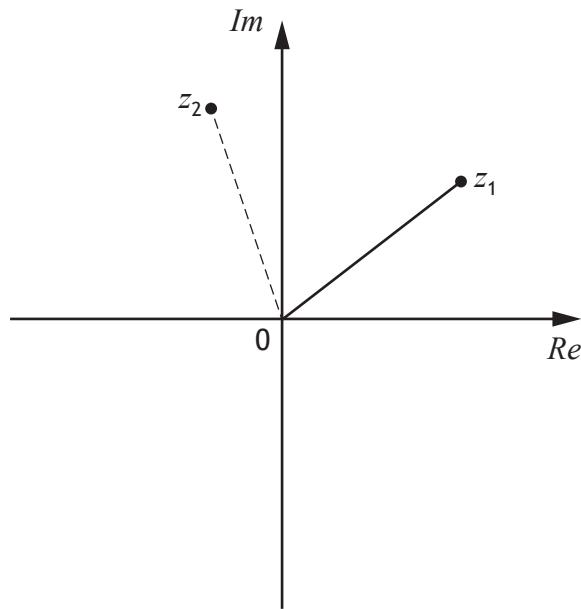
[Turn over

13. (a) Express  $-1$  in the form  $\cos\theta + i\sin\theta$ .

The complex number  $z_1$  is defined by  $z_1 = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$ .

- (b) Use de Moivre's theorem to show that  $z_1$  is a root of the equation  $z^5 + 1 = 0$ .

The complex number  $z_2$  is also a root of the equation  $z^5 + 1 = 0$ . Roots  $z_1$  and  $z_2$  have been plotted on an Argand diagram, as shown.



- (c) Express  $z_2$  in the form  $\cos\theta + i\sin\theta$ .

The remaining roots of the equation  $z^5 + 1 = 0$  are  $z_3$ ,  $z_4$  and  $z_5$ .

- (d) Express  $z_3$ ,  $z_4$  and  $z_5$  in the form  $\cos\theta + i\sin\theta$ , where  $-\pi < \theta \leq \pi$ .

- (e) Given  $z_1 + z_2 + z_3 + z_4 + z_5 = 0$ , show algebraically that

$$\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}.$$

[END OF SECTION 2]

[END OF QUESTION PAPER]