National
Qualifications
2025

2025 Mathematics Paper 1 Non-calculator

Advanced Higher

Question Paper Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$ Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $y = 5$ and $y = -7$ $\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as
  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$
  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Note: Marking from Image (MFI) annotation change from 2025

A double cross-tick is used to indicate correct working which is irrelevant or insufficient to score any marks. In MFI marking instructions prior to 2025 this was shown as **ü**$_2$ or ü2.

From 2025, the double cross-tick will no longer be used in MFI. A cross or omission symbol will be used instead.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] apply the binomial expansion [1,2,3,4,5,6] •[2] resolve signs [2,3,4,5] •[3] simplify coefficients **or** powers of x [2,3,5,6] •[4] complete simplification and obtain expression [2,3,5,6]	•[1] $\binom{4}{0}\left(\frac{1}{x}\right)^4+\binom{4}{1}\left(\frac{1}{x}\right)^3(-3x)+\binom{4}{2}\left(\frac{1}{x}\right)^2(-3x)^2$ $+\binom{4}{3}\left(\frac{1}{x}\right)(-3x)^3+\binom{4}{4}(-3x)^4$ •[2,3,4] $\frac{1}{x^4}-\frac{12}{x^2}+54-108x^2+81x^4$	4

Notes:

1. Accept any correct form for binomial coefficients; disregard poor notation where the correct numerical values subsequently appear.
2. Candidates who expand $\left(\frac{1}{x}-3x\right)^4$ without using the binomial theorem may be awarded •[2],•[3] and •[4] but •[1] is not available.
3. Where a candidate writes $\sum_{r=0}^{4}\binom{4}{r}\left(\frac{1}{x}\right)^{4-r}(-3x)^r$, award •[1] only, unless they continue to produce a full expansion. In cases where sigma notation precedes a full expansion, disregard incorrect or absent limits.
4. For candidates who expand $\left(\frac{1}{x}+3x\right)^4$ using the binomial theorem •[1] and •[2] are not available.
5. Where a candidate omits one term or evaluates one term to zero, the maximum mark is 3/4. If two or more terms are omitted or equated to zero, •[2], •[3] and •[4] are not available.
6. Accept negative powers of x.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] evidence of use of quotient rule with denominator and one term of numerator correct [1]	•[1] $\frac{(6x^2+1)(3+2x)-\ldots}{(3+2x)^2}$ **OR** $\frac{\ldots-(2x^3+x)(2)}{(3+2x)^2}$	3
			•[2] complete differentiation	•[2] $\frac{(6x^2+1)(3+2x)-(2x^3+x)(2)}{(3+2x)^2}$	
			•[3] simplify [2]	•[3] $\frac{8x^3+18x^2+3}{(3+2x)^2}$	

Notes:

1. Do not withhold •[1] where a candidate equates the derivative to the original function.
2. To award •[3] there must be a term of at least order 3 in the numerator.

Commonly Observed Responses:

For candidates who use the product rule:

•[1] $(6x^2+1)(3+2x)^{-1}+\ldots$

•[2] $\ldots+(2x^3+x)(-1)(3+2x)^{-2}$

•[3] $(6x^2+1)(3+2x)^{-1}-2(2x^3+x)(3+2x)^{-2}$

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ multiply numerator and denominator by complex conjugate of w [1,2] •² complete process [2]	•¹ $\frac{11+10i}{3-2i}\times\frac{3+2i}{3+2i}$ •² $1+4i$	2

Notes:

1. Where a candidate writes $\frac{11+10i}{3-2i}\times 3+2i$, award •¹ only if they have clearly attempted to multiply both numerator and denominator by $3+2i$.
2. Where a candidate does not multiply both numerator and denominator by a complex number, award 0/2.

Commonly Observed Responses:

Where a candidate equates fraction to $a+ib$:

•¹ $(3a+2b)+i(-2a+3b)$ (like terms need not be collected)

•² $a=1,\ b=4$

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] find matrix	•[1] $\begin{pmatrix} -5 & 10 \\ 10 & 3+2\lambda \end{pmatrix}$	1
Notes:					
Commonly Observed Responses:					
	(b)	(i)	•[2] state A' •[3] find $A'B$	•[2] $\begin{pmatrix} -3 & 0 \\ 2 & 1 \end{pmatrix}$, stated or implied by •[3] •[3] $\begin{pmatrix} -6 & -6 \\ 9 & 4+\lambda \end{pmatrix}$	2
Notes:					
Commonly Observed Responses:					
		(ii)	•[4] state expression for $\det(A'B)$	•[4] $-6(4+\lambda)-(-6)(9)$	1
Notes:					
Commonly Observed Responses:					
		(iii)	•[5] equate expression for determinant to zero [1,2] •[6] state value of λ	•[5] eg $-6(4+\lambda)-(-6)(9)=0$ stated or implied by •[6] •[6] $\lambda=5$	2
Notes: 1. Where a candidate has been awarded •[4] but subsequently produces an incorrect determinant, •[5] is not available if the candidate explicitly applies that determinant in (iii). 2. At •[5], accept any equivalent equation, except as detailed in Note 1 above.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
5.			•[1] start integration [1,2,4] •[2] complete integration [1,3,4]	•[1] $\tan^{-1}\ldots x$ •[2] $\frac{1}{2}\tan^{-1}2x+c$	2

Notes:

1. For the appearance of $\ln\ldots$, award 0/2.
2. For •[1], acceptable incorrect arguments are $\frac{1}{4}x,\ \frac{1}{2}x,\ 4x$.
3. Disregard the omission of $+c$ at •[2].
4. For the correct answer with no working award 2/2.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] begin algebraic division •[2] complete division and rewrite $f(x)$ [1]	•[1] $x-2\overline{)x^2+x+5}$ with quotient x, x^2-2x •[2] $x+3+\dfrac{11}{x-2}$	2

Notes:

1. Award •[2] for values of $A,\ B,\ C$.

Commonly Observed Responses:

Candidates who use a grid method:

Award •[1] for:

	x	3
x	x^2	$3x$
-2	$-2x$	

Candidates who multiply and equate coefficients:

•[1] $x^2+x+5=Ax^2+(-2A+B)x+(-2B+C)$

•[2] $A=1,\ B=3,\ C=11$

Candidates who multiply and substitute values:

•[1] $x^2+x+5=(Ax+B)(x-2)+C$ AND $C=11$

•[2] $A=1,\ B=3$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		•3 write equation of vertical asymptote [1,3,4,5,6] •4 write equation of non-vertical asymptote [2,3,4,5,6]	•3 $x=2$ •4 $y=x+3$	2

Notes:

1. At •3, $x=\ldots$ must appear.
2. At •4, $y=\ldots$ or $f(x)=\ldots$ must appear.
3. Disregard any incorrect labelling of a candidate's equations.
4. Disregard the use of eg "when", "at" in the identification of an asymptote.
5. Where a candidate produces mathematically unsound working in deriving an equation, the relevant mark should be withheld.
6. Where a candidate produces a third equation or expression masquerading as an asymptote, award a maximum of 1/2.

Commonly Observed Responses:

Incomplete statement:

"$x=2$ is a root of $x=2$ so it has a vertical asymptote" — do not award •3

Unsound mathematics:

Substituting $x=2$ to give $(2,5)$, then using $y-b=m(x-a)\ldots$ — do not award •4

Equating fraction to zero:

As $x \to \infty$, $\frac{11}{x-2}=0$, so $y=x+3$ — award •4

Question			Generic scheme	Illustrative scheme	Max mark
7.			**Method 1** (Separating variables) •1 write as an integral equation [1,3,4] •2 integrate LHS •3 integrate RHS [5] •4 evaluate constant [5,6] •5 rearrange to express y in terms of x [5] **Method 2** (Integrating factor) •1 state form of integrating factor [2] •2 find integrating factor •3 rewrite as integral equation •4 evaluate constant [6] •5 rearrange to required form	 •1 $\int \frac{dy}{y} = \int \frac{dx}{2x-1}$ •2 $\ln y$ •3 $\frac{1}{2}\ln(2x-1)+c$ •4 $c = \ln 4$ •5 $y = 4(2x-1)^{\frac{1}{2}}$ •1 $e^{\int -\frac{1}{2x-1}dx}$ •2 $(2x-1)^{-\frac{1}{2}}$ •3 $(2x-1)^{-\frac{1}{2}}y = \int 0\,dx$ •4 $c = 4$ •5 $y = 4(2x-1)^{\frac{1}{2}}$	5

Notes:

1. Where a candidate attempts to integrate an expression in terms of x with respect to y, or *vice versa*, award 0/5.
2. Where a candidate uses an integrating factor approach, they must first rearrange the equation into standard form, stated or implied by •1. Otherwise, award 0/5.
3. Where a candidate integrates $2x-1$ with respect to x **and** y with respect to y, award 0/5.
4. For Method 1, do not award •1 if either $\int \dots dy$ or $\int \dots dx$ does not appear.
5. For Method 1, if the constant of integration is omitted, •3 may be awarded but •4 and •5 are unavailable.
6. Where a candidate evaluates a constant after incorrectly rearranging, •4 is still available (see COR).

Question	Generic scheme	Illustrative scheme	Max mark
7. (continued)			

Commonly Observed Responses:

Incorrectly integrating $\int \frac{1}{2x-1}dx$ (Method 1)

$2\ln(2x-1)+c$	do not award •3	$\ln(2x-1)+c$	do not award •3
$c=\ln\left(\frac{12}{81}\right)$	award •4	$c=\ln\left(\frac{12}{9}\right)$	do not award •4
$y=\frac{4}{27}(2x-1)^2$	award •5	$y=\frac{4}{3}(2x-1)$	award •5

Incorrect rearrangement

$e^{\ln y}=e^{\frac{1}{2}\ln(2x-1)}+e^{c}$ do not award •5

$c=\ln 9$ award •4

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•¹ set up augmented matrix [1]	•¹ $\left(\begin{array}{ccc\|c} 1 & -1 & 4 & -6 \\ 2 & 3 & 1 & 15 \\ 3 & 2 & -2 & 16 \end{array}\right)$	**4**
			•² obtain two zero elements [2]	•² $\left(\begin{array}{ccc\|c} 1 & -1 & 4 & -6 \\ 0 & 5 & -7 & 27 \\ 0 & 5 & -14 & 34 \end{array}\right)$	
			•³ complete EROs [2]	•³ $\left(\begin{array}{ccc\|c} 1 & -1 & 4 & -6 \\ 0 & 5 & -7 & 27 \\ 0 & 0 & -7 & 7 \end{array}\right)$	
			•⁴ find point of intersection	•⁴ $x=2,\ y=4,\ z=-1$	

Notes:

1. Where a candidate equates a 3×3 matrix to a 3×1 matrix, •¹ is not available. Otherwise, accept eg $x, y, z, =$ left in.
2. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •² and •³.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁵ rearrange to parametric form	•⁵ $x=3\mu-4$ $y=2\mu+7$ $z=\mu+4$	**3**
			•⁶ substitute into plane equation	•⁶ $3(3\mu-4)+2(2\mu+7)-2(\mu+4)=16$	
			•⁷ find point of intersection [1]	•⁷ $(2,11,6)$	

Notes:

1. At •⁷, do not accept a point written as a column vector but accept the coordinates given separately.

Commonly Observed Responses:

Candidates who use wrong plane:

Using π_1: $\left(\frac{-53}{5}, \frac{13}{5}, \frac{9}{5}\right)$ award •⁷

Using π_2: $\left(\frac{-58}{13}, \frac{87}{13}, \frac{50}{13}\right)$ award •⁷

Question			Generic scheme	Illustrative scheme	Max mark
8.	(c)		$\bullet^8$ find direction of line [1] $\bullet^9$ parametric equations [2,3,4]	$\bullet^8$ eg $\begin{pmatrix}0\\7\\7\end{pmatrix}$ $\bullet^9$ eg $x=2,\ y=11+7\lambda,\ z=6+7\lambda$	2

Notes:

1. At $\bullet^8$, disregard any incorrect labelling of the direction vector, eg as "normal".
2. Where a candidate produces a direction vector without a zero component, $\bullet^9$ is not available.
3. At $\bullet^9$, do not accept $x=2+0\lambda$.
4. Disregard multiple use of the same parameter symbol.

Commonly Observed Responses:

Candidates who used the wrong plane in (b).

Using π_1 and $\left(\frac{-53}{5},\frac{13}{5},\frac{9}{5}\right)$: $\begin{pmatrix}\frac{63}{5}\\ \frac{7}{5}\\ -\frac{14}{5}\end{pmatrix}$ or equivalent award $\bullet^8$

Using π_2 and $\left(\frac{-58}{13},\frac{87}{13},\frac{50}{13}\right)$: $\begin{pmatrix}\frac{84}{13}\\ -\frac{35}{13}\\ -\frac{63}{13}\end{pmatrix}$ or equivalent award $\bullet^8$

[END OF MARKING INSTRUCTIONS]