National
Qualifications
2025

2025 Mathematics Paper 2

Advanced Higher

Question Paper Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

generic scheme — this indicates why each mark is awarded
illustrative scheme — this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bigcirc^5$	$\bigcirc^6$
$\bigcirc^5$	$x=2$	$x=-4$
$\bigcirc^6$	$y=5$	$y=-7$

Horizontal: $\bigcirc^5$ $x=2$ and $x=-4$
$\bigcirc^6$ $y=5$ and $y=-7$

Vertical: $\bigcirc^5$ $x=2$ and $y=5$
$\bigcirc^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Note: Marking from Image (MFI) annotation change from 2025

A double cross-tick is used to indicate correct working which is irrelevant or insufficient to score any marks. In MFI marking instructions prior to 2025 this was shown as **ü**$_2$ or ü2.

From 2025, the double cross-tick will no longer be used in MFI. A cross or omission symbol will be used instead.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ start differentiation ¹	•¹ $-\frac{1}{\sqrt{1-(4x)^2}}$	2
			•² complete differentiation ²	•² $-\frac{4}{\sqrt{1-(4x)^2}}$	

Notes:

1. Do not withhold •¹ where a candidate equates the derivative to the original function.
2. At •², accept $-\frac{1}{\sqrt{1-(4x)^2}}\times 4$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ apply product rule ¹	•¹ $4e^{2y}+8xe^{2y}\frac{dy}{dx}$	3
			•² complete differentiation ¹	•² $4y\frac{dy}{dx}+4e^{2y}+8xe^{2y}\frac{dy}{dx}=3$	
			•³ write $\frac{dy}{dx}$ in terms of x and y ²	•³ $\frac{dy}{dx}=\frac{3-4e^{2y}}{4y+8xe^{2y}}$	

Notes:

1. Terms need not be simplified for the award of •¹ and •².
2. Award •³ only where $\frac{dy}{dx}$ appears more than once after the candidate has completed differentiation.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ write template [1,2]	•¹ $\frac{A}{x-1}+\frac{B}{x-3}+\frac{C}{x+5}$ stated, or implied by •³	3
			•² form equation and find one of A, B or C [3]	•² $A(x-3)(x+5)+B(x-1)(x+5)+C(x-1)(x-3)=2x^2-18x+4$ **AND** any one from $A=1$ **OR** $B=-2$ **OR** $C=3$	
			•³ find remaining constants and substitute [4]	•³ $\frac{1}{x-1}-\frac{2}{x-3}+\frac{3}{x+5}$	

Notes:

1. If an incorrect template has been used, award 0/3.
2. For the correct answer without working, award 3/3.
3. Do not withhold •² where a candidate leaves the denominator in the RHS.
4. Do not accept $+-$ at •³ but accept $\frac{1}{x-1}+\frac{-2}{x-3}+\frac{3}{x+5}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ obtain and identify d [1,2]	•¹ eg $1118 = 2\times 416 + 286$ leading to eg <u>26</u>, gcd=26	**1**

Notes:

1. The gcd must be clearly identified in (a) or implied by the use of 26 in (b).
2. There must be evidence of the Euclidean algorithm or a related process.

Commonly Observed Responses:

	(b)		•² express gcd in terms of 286 and 416 [1,2]	•² eg $26 = 286 - 2\times 130$ $= 286 - 2\times\left(416 - 1\times 286\right)$	**2**
			•³ determine a and b [3,4]	•³ $a = 3$, $b = -8$	

Notes:

1. For the correct answer with no working, award 0/2.
2. At •², the expression may involve 286, 416 and 1118.
3. At •³, the minimum requirement is $26 = 1118\times(3) + 416\times(-8)$.
4. Do not accept $26 = 3\times 1118 - 8\times 416$ as a final response.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ take logarithms of both sides and apply rule [1]	•¹ $\ln y = \cot x \ln x$	**5**
			•² differentiate $\ln y$ [1,2]	•² $\frac{1}{y}\frac{dy}{dx}$	
			•³ evidence of product rule with one term correct [1,2]	•³ $-\operatorname{cosec}^2 x \ln x + \ldots$ **OR** $\ldots + \cot x \cdot \frac{1}{x}$	
			•⁴ complete differentiation [1,2]	•⁴ $-\operatorname{cosec}^2 x \ln x + \cot x \cdot \frac{1}{x}$	
			•⁵ write $\frac{dy}{dx}$ in terms of x [1]	•⁵ $\frac{dy}{dx} = x^{\cot x}\left(-\operatorname{cosec}^2 x \ln x + \frac{\cot x}{x}\right)$	

Notes:

1. Accept 'log' as an alternative to '$\ln$' provided candidate does not indicate a base other than e.
2. For candidates who do not attempt to use the product rule, •³ and •⁴ are not available.

Commonly Observed Responses:

•¹ $\ln y = \cot x \ln x$

•² $y = e^{\cot x \ln x}$

•³ $\frac{dy}{dx} = (\ldots)e^{\cot x \ln x}$

•⁴ $\frac{dy}{dx} = \left(-\operatorname{cosec}^2 x \ln x + \ldots\right)e^{\cot x \ln x}$ or $\frac{dy}{dx} = \left(\ldots + \cot x \cdot \frac{1}{x}\right)e^{\cot x \ln x}$

•⁵ $\frac{dy}{dx} = \left(-\operatorname{cosec}^2 x \ln x + \cot x \cdot \frac{1}{x}\right)e^{\cot x \ln x}$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		**Method 1**	**Method 1**	2
				$f(x)=\cos 3x$ $\quad f(0)=1$ $f'(x)=-3\sin 3x$ $\quad f'(0)=0$	
			•[1] all four derivatives AND all five evaluations [1]	•[1] $f''(x)=-9\cos 3x$ $\quad f''(0)=-9$ $f'''(x)=27\sin 3x$ $\quad f'''(0)=0$ $f^{iv}(x)=81\cos 3x$ $\quad f^{iv}(0)=81$	
			•[2] obtain simplified expression [2]	•[2] $1-\frac{9}{2}x^2+\frac{27}{8}x^4$	
			Method 2	**Method 2**	
			•[1] write down Maclaurin series for $\cos x$ [1]	•[1] $1-\frac{x^2}{2!}+\frac{x^4}{4!}$	
			•[2] substitute and simplify [2]	•[2] $1-\frac{9}{2}x^2+\frac{27}{8}x^4$	

Notes:

1. Where notation is absent or incomplete, evidence for •[1] may appear as substitution of values into the Maclaurin expansion.
2. Evidence of full simplification may appear in (b).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		•[3] state product [1,2,3] •[4] expand and simplify [2,3]	•[3] $\left(1-\frac{9}{2}x^2+\frac{27}{8}x^4\right)\left(1-\frac{9}{2}x^2+\frac{27}{8}x^4\right)$ •[4] $1-9x^2+27x^4$	2

Notes:

1. Award •[3] for $\left(1-\frac{9}{2}x^2+\frac{27}{8}x^4\right)^2$ but not $(\cos 3x)^2$.
2. Disregard errors in higher order terms.
3. For candidates who proceed by first principles but do not attempt to apply the product rule or double angle formula (see CORs below), •[3] and •[4] are not available.

Commonly Observed Responses:

For •[3]:

$g(x)=\cos^2 3x$	$g(0)=1$
$g'(x)=(2\cos 3x)(-3\sin 3x)=-3\sin 6x$	$g'(0)=0$
$g''(x)=-18\cos 6x$	$g''(0)=-18$
$g'''(x)=108\sin 6x$	$g'''(0)=0$
$g^{iv}(x)=648\cos 6x$	$g^{iv}(0)=648$

OR

$g(x)=\cos^2 3x$	$g(0)=1$
$g'(x)=(2\cos 3x)(-3\sin 3x)=-6\cos 3x\sin 3x$	$g'(0)=0$
$g''(x)=(18\sin 3x)(\sin 3x)+(-6\cos 3x)(3\cos 3x)=18\sin^2 3x-18\cos^2 3x$	$g''(0)=-18$
$g'''(x)=18(2\sin 3x)(3\cos 3x)-18(2\cos 3x)(-3\sin 3x)=216\cos 3x\sin 3x$	$g'''(0)=0$
$g^{iv}(x)=(-648\sin 3x)(\sin 3x)+(216\cos 3x)(3\cos 3x)$	$g^{iv}(0)=648$

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] find $\frac{dy}{dt}$ •[2] find $\frac{dy}{dx}$ 1	•[1] $\frac{dy}{dt} = \sec^2 t$ •[2] $\frac{dy}{dx} = \frac{\sec^2 t}{2t}$	**2**

Notes:

1. For the award of •[2], the final expression must be solely in terms of t.

Commonly Observed Responses:

Candidates who express y explicitly as a function of x:

•[1] $\frac{dy}{dx} = \sec^2 \sqrt{x} \times \ldots$

Question			Generic scheme	Illustrative scheme	Max mark
7.	(b)		•3 evidence use of quotient rule for $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ with denominator and one term of the numerator correct [1,2,3,4]	•3 $\frac{2\sec t\times\sec t\tan t\times 2t-\ldots}{(2t)^2}$ **OR** $\frac{\ldots-\sec^2 t\times 2}{(2t)^2}$	3
			•4 complete $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ [3,4]	•4 $\frac{2\sec t\times\sec t\tan t\times 2t-\sec^2 t\times 2}{(2t)^2}$	
			•5 find $\frac{d^2y}{dx^2}$ [5]	•5 $\frac{2t\sec^2 t\tan t-\sec^2 t}{4t^3}$	

Notes:

1. Where a candidate makes no attempt to differentiate, award 0/3.
2. Do not withhold •3 where a candidate labels $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ as $\frac{d^2y}{dx^2}$ or produces other incorrect notation.
3. Where a candidate differentiates with respect to t an expression other than their answer to (a), do not award •3. If the differentiation involves a product or quotient, including a trigonometric term, and is carried out correctly, award •4.
4. Where a candidate produces an incorrect expression in (a), differentiation must involve a product or quotient, including a trigonometric term, for the award of •4 but •3 is still available.
5. For the award of •5, the final expression must be solely in terms of t and have required simplification.

Commonly Observed Responses:

Candidates who express y explicitly as a function of x:

•3 and •4 $$\frac{2\sec\sqrt{x}\times\sec\sqrt{x}\tan\sqrt{x}\times\frac{1}{2\sqrt{x}}\times 2\sqrt{x}-\sec^2\sqrt{x}\times\frac{1}{\sqrt{x}}}{\left(2\sqrt{x}\right)^2}$$

OR

•3 and •4 $$\frac{1}{2\sqrt{x}}\times 2\sec^2\sqrt{x}\tan\sqrt{x}\times\frac{1}{2\sqrt{x}}-\frac{1}{4\sqrt{x^3}}\times\sec^2\sqrt{x}$$

Candidates who take a formula approach:

•3 and •4 $$\frac{2\sec t\times\sec t\tan t\times 2t-\sec^2 t\times 2}{(2t)^3}$$

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] multiply expression by A and expand [1]	•[1] $6A^2 - AI$	**2**
			•[2] determine expression in required form	•[2] $35A - 6I$	

Notes:
1. Where a candidate uses division or a square root sign, no further marks are available in (a).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		**Method 1**		**2**
			•[3] multiply given identity by A^{-1} [1,2]	•[3] $A^{-1}A^2 = A^{-1}(6A - I)$	
			•[4] find expression	•[4] $A^{-1} = 6I - A$	
			Method 2		
			•[3] write I in terms of A and A^2 [2]	•[3] $I = 6A - A^2$	
			•[4] find expression	•[4] $A^{-1} = 6I - A$	

Notes:
1. For Method 1, where a candidate uses an identity obtained in part (a) rather than the given one, •[3] is available only where the identity contains A^2.
2. Where a candidate introduces division or a square root sign, no further marks are available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•1 integrate [1] •2 evaluate constant and substitute [2,3]	•1 $\frac{2t^2}{2}+\frac{1}{5}e^{5t}+c$ •2 $t^2+\frac{1}{5}e^{5t}-\frac{1}{5}$	**2**

Notes:

1. For the award of •1, the constant of integration is not necessary.
2. Where a candidate does not write out a full expression (or writes an unsimplified version) after evaluating the constant of integration, •2 may be awarded for $c=-\frac{1}{5}$, provided $t^2+\frac{1}{5}e^{5t}+c$ has appeared earlier.
3. Ignore any reference to units.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•3 obtain expression for acceleration [1,2] •4 explanation [3,4,5]	•3 $2+5e^{5t}$ •4 $e^{5t}>0$	**2**

Notes:

1. Where a candidate has given more than one (non-equivalent) expression for a, •3 is not available.
2. Award •3 for the appearance of $v'=2+5e^{5t}$ and ignore higher derivatives (unless acceleration is explicitly identified as a higher derivative). If the expression appears in (a), it must be explicitly identified as acceleration (eg "$a=\ldots$").
3. At •4, do not accept "As $t\to\infty$, $5e^{5t}\to\infty$".
4. Do not withhold •4 where a candidate states "$e^{5t}\geq 0$", or "e^{5t} is never negative".
5. At •4, disregard incorrect or irrelevant statements before or after an acceptable justification, even where these occur in the same sentence.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•1 substitute expressions •2 complete factorisation	•1 $\frac{n^2(n+1)^2}{4}-\frac{3n(n+1)}{2}$ •2 $\frac{1}{4}n(n+1)(n-2)(n+3)$	2

Notes:

Commonly Observed Responses:

Candidate uses formula for sum of squares instead of linear

$\frac{n^2(n+1)^2}{4}-3\times\frac{n(n+1)(2n+1)}{6}$ Do not award •1

$\frac{1}{4}n(n+1)(n^2-3n-2)$ Award •2

Candidate uses formula for sum of squares instead of cubic

$\frac{n(n+1)(2n+1)}{6}-\frac{3n(n+1)}{2}$ Do not award •1

$\frac{1}{3}n(n+1)(n-4)$ Award •2

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•¹ differentiate [1,2] •² integrate [2,3,4]	•¹ $\frac{du}{dx}=4x$ or $du=4xdx$ •² $-\frac{1}{4}e^{-2x^2}+c$	**2**

Notes:

1. For correct answer with no working, award 2/2.
2. Where •¹ has been awarded, disregard any further incorrect manipulation of the differentials provided the correct form of the integral is given.
3. Disregard the omission of the constant of integration.
4. Do not withhold •² where du is incorrectly placed with respect to the integrand.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ state form of integral [1,2] •⁴ find expression in integrable form [3,4] •⁵ integrate and evaluate [4]	•³ $\pi\int_0^1 y^2dx$ stated or implied by •⁴ •⁴ $16\pi\int_0^1 xe^{-2x^2}dx$ •⁵ $4\pi\left(1-e^{-2}\right)$	**3**

Notes:

1. For the award of •³, limits must appear at some point.
2. For the award of •³, dx must appear at some point.
3. Award •⁴ for $16\pi\int_0^1 \frac{x}{e^{2x^2}}dx$.
4. Where a candidate writes dy instead of dx, •⁴ and •⁵ are still available provided integration is carried out with respect to x.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.			•1 construct auxiliary equation [1,2,5]	•1 $m^2-8m+15=0$	9
			•2 state complementary function [3,4,5]	•2 $y=Ae^{3x}+Be^{5x}$	
			•3 state particular integral and differentiate twice [5]	•3 $y=Cx^2+Dx+E$ $\frac{dy}{dx}=2Cx+D$ $\frac{d^2y}{dx^2}=2C$	
			•4 substitute into LHS of differential equation [5,6]	•4 $2C-8(2Cx+D)+15(Cx^2+Dx+E)$	
			•5 obtain constants [5]	•5 $C=1,\ D=-1,\ E=2$	
			•6 state general solution [4,5,7,8]	•6 $y=Ae^{3x}+Be^{5x}+x^2-x+2$ stated or implied by •9	
			•7 differentiate general solution [5,8,9]	•7 $\frac{dy}{dx}=3Ae^{3x}+5Be^{5x}+2x-1$	
			•8 form simultaneous equations [5]	•8 $A+B=2$ $3A+5B=14$	
			•9 state particular solution [5]	•9 $y=-2e^{3x}+4e^{5x}+x^2-x+2$	

Notes:

1. Do not award •1 where '$=0$' has been omitted.
2. Do not withhold •1 where a candidate uses x or y in the auxiliary equation.
3. Where the complementary function appears only as part of the general solution, •2 may be awarded.
4. Do not withhold •2 or •6 for the omission of '$y=\ldots$' provided it appears at •9.
5. Where a candidate does not find a particular integral, only •1, •2, •7 and •8 are available.
6. For the award of •4, a candidate must substitute an expression with variable coefficients.
7. Where a candidate introduces a particular integral after determining values for A and B, leading to $y=\frac{7}{2}e^{3x}+\frac{1}{2}e^{5x}+x^2-x+2$, •6 is not available.
8. If a candidate includes as part of their general solution:
   a. $15x^2-31x+40$;
   b. any expression containing constants (other than those from the complementary function) which have not been evaluated; or
   c. an incorrect expression which has not previously been identified as a particular integral,

   then •6 is unavailable but •7 may still be available.
9. Where a candidate fails to label (or incorrectly labels) the derivative, do not withhold •7 provided their expression is subsequently equated to 13.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)	(i)	•¹ interpret geometric sequence [1,2] •² state common ratio [2]	•¹ $ar=100$ and $ar^3=16$ •² $r=\frac{2}{5}$	**2**

Notes:

1. Where expressions for the individual given terms have not been stated, award •¹ for $r^2=\frac{16}{100}$.
2. For a statement of the answer only, award •². For the award of •¹, there must be some evidence of strategy, eg $100 \to 40 \to 16$.

Commonly Observed Responses:

		(ii)	•³ state first term [1]	•³ 250	**1**

Notes:

1. For candidates who produce an incorrect common ratio, •³ may not be awarded for a first term which is negative.

Commonly Observed Responses:

	(b)		•⁴ state condition [1,2,3,4]	•⁴ $-1<\frac{2}{5}<1$	**1**

Notes:

1. For •⁴, $\frac{2}{5}$ may be replaced by a letter consistent with the candidate's answer to (a).
2. Where a candidate, in (a), obtains a value of r outwith the interval $(-1,1)$, •⁴ is available only where there is an acknowledgement that there is no sum to infinity.
3. Accept an equivalent statement in words. Where a candidate uses the word 'between', it must be explicitly stated that this means strictly between.
4. Accept $0<\frac{2}{5}<1$.

Commonly Observed Responses:

	(c)		•⁵ obtain sum to infinity [1]	•⁵ $\frac{1250}{3}$	**1**

Notes:

1. Accept a decimal answer (416.6…), rounded or truncated to at least 1 decimal place.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(d)	(i)	$\bullet^6$ state effect on common ratio [1,2]	$\bullet^6$ common ratio is unchanged	1

Notes:

1. Accept eg 'no change' or 'stays the same' or 'it is still $\frac{2}{5}$'. However, do not accept, eg, 'it is $\frac{2}{5}$' without any reference to the fact that the common ratio is unchanged.
2. Where candidates choose a particular value of k (say, $k = 2$) and work out the common ratio, $\bullet^6$ is available provided the required conclusion appears.

Commonly Observed Responses:

		(ii)	$\bullet^7$ state effect on sum to infinity [1,2]	$\bullet^7$ sum to infinity is multiplied by k.	1

Notes:

1. Accept eg 'times k', 'increases/decreases/changes … factor of k', '$\frac{1250}{3}k$' or '$\frac{a}{1-r}k$'.
2. For candidates who choose a particular value of k and use this to work out the sum to infinity (say $k = 2$ to obtain $\frac{2500}{3}$), $\bullet^7$ is available provided the required conclusion appears (eg 'multiplied by k'. However, do not accept eg 'multiplied by 2').

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.			•[1] state form of integrating factor [1,2,3]	•[1] $e^{\int -\frac{2}{x}dx}$ stated or implied	**5**
			•[2] find simplified integrating factor [4]	•[2] $\frac{1}{x^2}$ or x^{-2}	
			•[3] rewrite as integral equation [3,5,6,7]	•[3] $\frac{y}{x^2} = \int \frac{1}{x^2} x^2 \sec^2 3x\,dx$	
			•[4] integrate [8,9,10]	•[4] $\frac{1}{3}\tan 3x + c$	
			•[5] rearrange to required form [9,10]	•[5] $y = x^2\left(\frac{1}{3}\tan 3x + c\right)$	

Notes:

1. Where no attempt is made to produce an integrating factor, award 0/5.
2. Where a candidate writes "$P(x) = -\frac{2}{x}$" and then writes "$e^{\int P(x)dx}$", •[1] may be awarded.
3. Do not withhold •[1] or •[3] for the omission of 'dx'.
4. Award •[2] only if the integrating factor is of the form $e^{\int \frac{k}{x}dx}$, $k \neq 1$.
5. At •[3], the LHS must take the form of integrating factor multiplied by y.
6. For candidates who produce an incorrect integrating factor, •[3] may still be available.
7. Since a candidate may proceed directly from •[2] to •[3], disregard incorrect working leading to the equation referred to at •[3].
8. Where a candidate uses x^2 for the integrating factor, •[4] is not available.
9. For a candidate who omits the constant of integration, •[4] may be available but •[5] is not.
10. Where a candidate fails to integrate correctly at •[4], •[5] is still available provided the constant of integration is processed correctly (incorrect working subsequent to this should be ignored).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.			•[1] show true for $n=1$ [1]	•[1] when $n=1$ $\frac{1}{(2+1)(2-1)}=\frac{1}{3}$ $\frac{1}{2+1}=\frac{1}{3}$ so true for $n=1$	**5**
			•[2] assume (statement) true for $n=k$ **AND** consider case of $n=k+1$ (RHS of final expression must not be the aim/goal unless it is identified as such) [2,3,4,5,6,10]	•[2] suitable statement **AND** $\sum_{r=1}^{k}\frac{1}{(2r+1)(2r-1)}=\frac{k}{2k+1}$ **AND** $\sum_{r=1}^{k+1}\frac{1}{(2r+1)(2r-1)}=\ldots$	
			•[3] correct statement for sum to $(k+1)$ terms using inductive hypothesis [2,7]	•[3] $\frac{k}{2k+1}+\frac{1}{(2(k+1)+1)(2(k+1)-1)}$	
			•[4] express sum to $(k+1)$ terms as a single term [2,8]	•[4] $\frac{(2k+3)k+1}{(2k+1)(2k+3)}$ or equivalent	
			•[5] express sum explicitly in terms of $(k+1)$ or achieve stated aim/goal **AND** communicate [2,8,9,10,11]	•[5] $\frac{k+1}{2(k+1)+1}$ **AND** If true for $n=k$ then true for $n=k+1$. Also shown true for $n=1$ therefore, by induction, true for all positive integers n.	

Question			Generic scheme	Illustrative scheme	Max mark
15.			(continued)		

Notes:

1. "RHS = $\frac{1}{3}$, LHS = $\frac{1}{3}$" and/or "True for $n=1$" are insufficient for the award of $\bullet^1$. A candidate must demonstrate some evidence of substitution into both expressions. Where a candidate does not independently evaluate the LHS and RHS, $\bullet^1$ may still be awarded. Communication that the statement is true for $n=1$ may appear in the final statement.
2. Where n is used in place of k, marks $\bullet^2$, $\bullet^3$, $\bullet^4$ and $\bullet^5$ are not available.
3. For $\bullet^2$ sufficient phrases for $n=k$ contain:
   - "If true for..."; "Suppose true for…"; "Assume true for…".
   - For $\bullet^2$ insufficient phrases for $n=k$ contain:
   - "Consider $n=k$,", "Assume $n=k$ is true", "True for $n=k$", "Let $n=k$"
4. A sufficient phrase for the award of $\bullet^2$ may appear at $\bullet^5$.
5. For $\bullet^2$, the RHS of final expression must not be the aim/goal unless it is identified as such, or subsequent decomposition of $\sum_{r=1}^{k+1}\frac{1}{(2r+1)(2r-1)}$ appears.
6. For $\bullet^2$ unacceptable phrases for $n=k+1$ contain:
   - "Consider true for $n=k+1$", "true for $n=k+1$".
7. Since $\bullet^3$ may be awarded directly after $\bullet^2$, exercise leniency for the award of $\bullet^3$ in relation to intermediate notation where this has been provided, eg the handling of sigma notation.
8. Do not award $\bullet^4$ or $\bullet^5$ where one of the fractions is numeric.
9. For candidates who write down the correct expression without algebraic justification, $\bullet^5$ is unavailable.
10. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently achieve the stated aim/goal provided the substitution of $k+1$ is clearly shown.
11. Following the required algebra and statement of the inductive hypothesis, the minimal acceptable response for $\bullet^5$ must contain:
    - if true for $n=k$ then true for $n=k+1$ (do not accept "eg $n=k$ is true")
    - true for $n=1$
    - true for all n (do not accept "for all integers").

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
16.			**Method 1** •1 evidence of integration by parts [1,2]	•1 $-\frac{1}{5}e^{2x}\cos 5x - \ldots$	5
			•2 complete first application [1,2,3]	•2 $\ldots\int -\frac{1}{5}2e^{2x}\cos 5x\,dx$	
			•3 begin second application of integration by parts [1,2]	•3 $\left[-\frac{2}{25}e^{2x}\sin 5x\ldots\int\ldots\right]$	
			•4 second application as far as reappearance of original integral [1,2,3]	•4 $\left[-\frac{2}{25}e^{2x}\sin 5x-\int -\frac{4}{25}e^{2x}\sin 5x dx\right]$	
			•5 final expression [4,5,6]	•5 $\frac{2}{29}e^{2x}\sin 5x-\frac{5}{29}e^{2x}\cos 5x+c$	
			Method 2 •1 evidence of integration by parts [1,2]	•1 $\frac{1}{2}e^{2x}\sin 5x-\ldots$	
			•2 complete first application [1,2,3]	•2 $\ldots\int\frac{1}{2}e^{2x}(5\cos 5x)\,dx$	
			•3 begin second application of integration by parts [1,2]	•3 $\left[\frac{5}{4}e^{2x}\cos 5x\ldots\int\ldots\right]$	
			•4 second application as far as reappearance of original integral [1,2,3]	•4 $\left[\frac{5}{4}e^{2x}\cos 5x-\int\frac{5}{4}e^{2x}(-5\sin 5x)\,dx\right]$	
			•5 final expression [4,5,6]	•5 $\frac{2}{29}e^{2x}\sin 5x-\frac{5}{29}e^{2x}\cos 5x+c$	

Notes:

1. Where a candidate reverses the expressions to integrate and differentiate for the second application, only •1 and •2 are available.
2. Where a candidate differentiates or integrates both functions in the first **or** second application, •1 and •2 **or** •3 and •4 respectively are not available.
3. Disregard the omission of dx.
4. Disregard the mistreatment or omission of the constant of integration.
5. For the award of •5, accept eg $\dfrac{25\left(\frac{2}{25}e^{2x}\sin 5x-\frac{1}{5}e^{2x}\cos 5x\right)}{29}$.
6. Where a candidate makes a second attempt using the other method, mark both attempts and award the **higher** mark.

Question			Generic scheme	Illustrative scheme	Max mark
16.			(continued)		

Commonly Observed Responses:

Tabular Method:

	D	I
$+$	e^{2x}	$\sin 5x$
$-$	$2e^{2x}$	$-\frac{1}{5}\cos 5x$
$+$	$4e^{2x}$	$-\frac{1}{25}\sin 5x$

	D	I
$+$	$\sin 5x$	e^{2x}
$-$	$5\cos 5x$	$\frac{1}{2}e^{2x}$
$+$	$-25\sin 5x$	$\frac{1}{4}e^{2x}$

•[1] first three rows (including headings – although these may vary)

•[2] final row

•[3] $-\frac{1}{5}e^{2x}\cos 5x+\frac{2}{25}e^{2x}\sin 5x+\int\ldots$ or $\frac{1}{2}e^{2x}\sin 5x-\frac{5}{4}e^{2x}\cos 5x+\int\ldots$

•[4] $\int -\frac{4}{25}e^{2x}\sin 5x dx$ or $\int -\frac{25}{4}e^{2x}\sin 5x dx$

•[5] $\frac{2}{29}e^{2x}\sin 5x-\frac{5}{29}e^{2x}\cos 5x+c$

Question			Generic scheme	Illustrative scheme	Max mark
17.			**Method 1** •¹ state relationship between derivatives [1] •² determine $\frac{dV}{dh}$ [2] •³ state expression for $\frac{dV}{dt}$ [1,3,4] •⁴ calculate rate of change [1,5,6]	**Method 1** •¹ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ or equivalent •² $\frac{dV}{dh}=\frac{3}{5}h^2$ •³ $\frac{dV}{dt}=6-\frac{1}{10}\sqrt{h}$ •⁴ $\frac{dh}{dt}=\frac{1}{24000}$ cm/sec	4
			Method 2 •¹ begin to differentiate with respect to t [1] •² complete differentiation with respect to t [1] •³ state expression for $\frac{dV}{dt}$ [1,3,4] •⁴ calculate rate of change [1,5,6]	**Method 2** •¹ $\frac{dV}{dt}=\frac{3}{5}h^2\ldots$ •² $\frac{dV}{dt}=\frac{3}{5}h^2\frac{dh}{dt}$ •³ $\frac{dV}{dt}=6-\frac{1}{10}\sqrt{h}$ •⁴ $\frac{dh}{dt}=\frac{1}{24000}$ cm/sec	

Question			Generic scheme	Illustrative scheme	Max mark
17.	(continued)				
			Method 3	**Method 3**	
			•1 find $\frac{dh}{dV}$	•1 $\frac{dh}{dV}=\frac{5}{3}(5V)^{-\frac{2}{3}}$ stated or implied by •2	
			•2 differentiate with respect to t 1,2	•2 $\frac{dh}{dt}=\frac{5}{3}(5V)^{-\frac{2}{3}}\frac{dV}{dt}$	
			•3 state expression for $\frac{dV}{dt}$ 1,3,4	•3 $\frac{dV}{dt}=6-\frac{1}{10}\sqrt{h}$	
			•4 calculate rate of change 1,5,6	•4 $\frac{dh}{dt}=\frac{1}{24000}$ cm/sec	

Notes:

1. Where a candidate uses a variable other than t (or T) for time , eg “sec” this must be explicitly defined. Otherwise, the mark where it is first used is not available.
2. In Method 1, where a candidate equates $\frac{dV}{dh}$ to V , •2 is not available.
3. Where a candidate equates $\frac{dV}{dt}$ to V , •3 is not available.
4. Where a candidate has treated the increase and decrease separately, do not withhold •3 provided they subsequently combine them to produce the correct value.
5. At •4, candidates must explicitly identify $\frac{dh}{dt}$.
6. Accept an answer (eg $4.1\dot{6}\times10^{-5}$) rounded or truncated to at least two significant figures.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)	(i)	•¹ state conjugate •² obtain expression [1]	•¹ $x-iy$ •² $x-y+i(x-y)$	**2**

Notes:

1. For •², accept $x-y-i(y-x)$.

Commonly Observed Responses:

		(ii)	•³ find argument [1,2]	•³ $\frac{\pi}{4}$ or $45°$	**1**

Notes:

1. Do not withhold •³ for an argument in degrees where the degree symbol is omitted.
2. Do not award •³ if a candidate obtains a correct value from incorrect working (eg $\tan(1)=\frac{\pi}{4}$).

Commonly Observed Responses:

	(b)		•⁴ find one solution [1] •⁵ find second solution [1,2,3]	•⁴ $\sqrt{r}\left(\cos\left(-\frac{3\pi}{8}\right)+i\sin\left(-\frac{3\pi}{8}\right)\right)$ •⁵ $\sqrt{r}\left(\cos\frac{5\pi}{8}+i\sin\frac{5\pi}{8}\right)$	**2**

Notes:

1. Accept $-67.5°$ and $112.5°$ respectively. Disregard the omission of degree symbols.
2. Do not withhold •⁵ where a candidate has produced an acceptable second argument but has produced an incorrect or unsimplified modulus which is equal to their previous one.
3. Do not award •⁵ where a candidate proceeds from a first square root with an argument of $-\frac{3\pi}{4}$.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]