National
Qualifications
2023

2023 Mathematics

Advanced Higher - Paper 1

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x = 1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 1$ or 3

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] evidence of use of product rule with one term correct [1,2] •[2] complete differentiation	•[1] $7\tan 2x + 7x(\ldots)$ **OR** $(\ldots)\tan 2x + 7x \times 2\sec^2 2x$ •[2] $7\tan 2x + 14x\sec^2 2x$	2

Notes:

1. For a candidate who produces one term only, award 0/2.
2. Where a candidate equates $\frac{dy}{dx}$ to y, •[1] is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ write template [1]	•¹ $\dfrac{3x^2-x-14}{(x+3)(x-1)^2}=\dfrac{A}{x+3}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$	3
			•² form equation and find one constant	•² $3x^2-x-14=A(x-1)^2+B(x+3)(x-1)+C(x+3)$ and $A=1$ or $B=2$ or $C=-3$	
			•³ find remaining constants and substitute [2]	•³ $\dfrac{1}{x+3}+\dfrac{2}{x-1}-\dfrac{3}{(x-1)^2}$	

Notes:

1. Award 0/3 if an incorrect template has been used.
2. Do not accept $+-$ at •³

Commonly Observed Response:

•¹ $\dfrac{3x^2-x-14}{(x+3)(x-1)^2}=\dfrac{A}{x+3}+\dfrac{Bx+C}{(x-1)^2}$

•² $3x^2-x-14=A(x-1)^2+(Bx+C)(x+3)$

and

$A=1$, $B=2$ and $C=-5$

•³ $\dfrac{1}{x+3}+\dfrac{2}{x-1}-\dfrac{3}{(x-1)^2}$

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>3.</td><td></td><td></td><td>•1 set up augmented matrix [1]

•2 obtain two zeros

•3 write down conclusion with justification [2]</td><td>•1 $\left(\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 3 & -2 & 4 & 11 \\ 1 & 4 & 2 & 15 \end{array}\right)$

•2 $\left(\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 0 & 7 & 1 & 14 \\ 0 & 7 & 1 & 16 \end{array}\right)$

•3 eg $\left(\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 0 & 7 & 1 & 14 \\ 0 & 0 & 0 & 2 \end{array}\right)$
(or statement relating to $14 \neq 16$ at •2)
so inconsistent</td><td>3</td></tr>
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Notes:

1. Where a candidate equates a 3×3 matrix to a 3×1 matrix, •1 is not available. Otherwise, accept eg $x, y, z, =$ left in.
2. For •3, candidates who arrive at an augmented matrix which produces a unique solution, or infinitely many solutions, there is no requirement to determine solutions.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•1 integrate to find "$uv-$" 1,2 •2 differentiate to find "$\int u'v\,dx$" 3 •3 complete integration 4	•1 $\frac{1}{5}x^5\ln x-\ldots$ •2 $\ldots\frac{1}{5}\int x^5\times\frac{1}{x}dx$ •3 $\frac{1}{5}x^5\ln x-\frac{x^5}{25}+c$	3

Notes:

1. Where a candidate differentiates or integrates both x^4 and $\ln x$, award 0/3.
2. Award 0/3 for candidates who differentiate x^4 and incorrectly integrate $\ln x$. See COR A if $\ln x$ is integrated correctly.
3. Do not withhold •2 for the omission of dx.
4. Do not withhold •3 for the omission of the constant of integration.

Commonly Observed Responses:

COR A

•1 $x^4(x\ln x-x)-\ldots$

•2 $\ldots4\int x^3(x\ln x-x)dx$

•3 $\frac{1}{5}x^5\ln x-\frac{x^5}{25}+c$

COR B

Award •1:

	Differentiate	Integrate
$+$	$\ln x$	x^4
$-$	$\frac{1}{x}$	$\frac{1}{5}x^5$

Award •2 and •3 as per main method.

Candidates may have different headings, including u and v' for Differentiate and Integrate respectively.

Question			Generic scheme	Illustrative scheme	Max mark
5.			•1 construct auxiliary equation 1	•1 $m^2-4m-5=0$	9
			•2 find complementary function 2,3	•2 $y=Ae^{5x}+Be^{-x}$	
			•3 state particular integral and obtain first and second derivatives of particular integral 5	•3 $y=Cx^2+Dx+E$ $\frac{dy}{dx}=2Cx+D$ $\frac{d^2y}{dx^2}=2C$	
			•4 substitute into LHS of differential equation 4	•4 $2C-4(2Cx+D)-5(Cx^2+Dx+E)$	
			•5 obtain constants	•5 $C=-2, D=1$ and $E=3$	
			•6 state general solution 2,3,6,7	•6 $y=Ae^{5x}+Be^{-x}-2x^2+x+3$ stated or implied by •9	
			•7 differentiate general solution	•7 $\frac{dy}{dx}=5Ae^{5x}-Be^{-x}-4x+1$	
			•8 form simultaneous equations	•8 $A+B=-1$ $5A-B=13$	
			•9 state particular solution 2,3	•9 $y=2e^{5x}-3e^{-x}-2x^2+x+3$	

Notes:

1. •1 is not available where '$=0$' has been omitted.
2. •2 may still be awarded if the complementary function appears only as part of a general solution or the particular solution.
3. Do not withhold •2 for the omission of '$y=...$' provided it appears as part of a general solution or at •6 or •9.
4. For the award of •4 a candidate must substitute an expression with variable coefficients.
5. Where a candidate does not introduce a particular integral only •1, •2, •7 and •8 are available.
6. Where a candidate includes as part of their general solution

a. $10x^2+11x-23$,

b. any expression containing constants other than those from the complementary function which have not been evaluated or

c. an incorrect expression which has not previously been identified as a particular integral,

•6 is unavailable but •7 may still be available.

7. Where a candidate introduces a particular integral after determining values for A and B, leading to $y=\frac{8}{3}e^{5x}-\frac{2}{3}e^{-x}-2x^2+x+3$, •6 is unavailable.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•¹ find modulus or argument [1] •² complete polar form [1]	•¹ $r=2$ or $\theta=\frac{\pi}{3}$, (stated or implied at •²) •² $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$	**2**

Notes

1. Where candidates work in degrees (60^{o}) the degree symbol must appear at least once in part (a) or (b) for •² to be awarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ apply de Moivre's Theorem •⁴ demonstrate that imaginary part is zero [1,2,3,4]	•³ $2^3\left(\cos\frac{3\pi}{3}+i\sin\frac{3\pi}{3}\right)$ •⁴ eg $z^3=-8$	**2**

Notes:

1. Accept "$\sin\pi=0$, therefore z^3 is real", with no evaluation of the real part, for •⁴.
2. Where a candidate loses •³ as a result of an error, •⁴ is still available provided a consistent real number is produced.
3. Where an incorrect result is produced in part (a), •⁴ is available only if a consistent real number is produced.
4. Where a candidate chooses to evaluate z^3, the value must be consistent with the expression at •³.

Commonly Observed Responses:

COR A

Use of binomial theorem:

$1+3\sqrt{3}i+9i^2+3\sqrt{3}i^3$ award •³

COR B

Multiplying out one pair of brackets and resolving i^2:

$\left(1+\sqrt{3}i\right)\left(1+2\sqrt{3}i-3\right)$ award •³

COR C

Multiplying out all three brackets and resolving i^2 without attempting simplification of one pair:

$1+\sqrt{3}i+2\sqrt{3}i+\left(-6\right)+\left(-3\right)$ award •³

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] substitute formulae •[2] simplify	•[1] $\frac{n(n+1)(2n+1)}{6}+3\left(\frac{n(n+1)}{2}\right)$ •[2] $\frac{1}{3}n(n+1)(n+5)$	**2**
Notes:					
Commonly Observed Responses:					
	(b)		•[3] substitute 20 and evidence of subtraction •[4] substitute 10 and evaluate	•[3] $\frac{1}{3}(20)(20+1)(20+5)-\ldots$ •[4] 2950	**2**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•¹ provide counterexample and demonstrate [1,2,3]	•¹ $a=-2, b=1$ 4 is not less than 1 or $4>1$	1

Notes:

1. The values of a and b must be explicitly stated and the squares evaluated.
2. Disregard any statement following a suitable demonstration.
3. Where a candidate chooses eg $a=-2$ and $b=2$, then it is sufficient to observe that $4=4$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² appropriate form for n [1,2]	•² eg $2k+1$, $k\in\mathbb{Z}$	2
			•³ factorise and communicate [3,4,5]	•³ $4\left(k^2+k\right)$ and eg which is divisible by 4	

Notes:

1. At •², accept eg "k is an integer" but do not accept eg $k\in\mathbb{N}$, $k\in\mathbb{Z}^+$.
2. Expression for n must be of the form $2k+m$, where m is an odd integer.
3. At •³, accept $4k\left(k+1\right)$ for the factorisation.
4. Award •³ if a candidate does not factorise but states that each term or coefficient (or equivalent) is divisible by 4.
5. Acceptable communication for •³ includes "therefore true", "$\Rightarrow$ true", "so statement is true", "as required". Simply writing "true" after factorised expression is insufficient.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ state A [1]	•¹ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	1
Notes: 1. Accept $\begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix}$.					
Commonly Observed Responses:					
	(b)	(i)	•² find AB [1,2]	•² $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	1
Notes: 1. Where a candidate produces the identity matrix at (a), •² is not available. 2. All entries must be evaluated for the award of •².					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(ii)	•³ find α [1,2,3,4]	•³ $\frac{5\pi}{3}$	**1**

Notes:

1. Do not accept an answer in degrees.
2. If the matrix in part (b)(i) is incorrect, •³ is available on follow-through only if the matrix is clearly identified as AB, or is the result of matrix multiplication.
3. Accept $\alpha = \frac{5\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$, eg $\boldsymbol{\alpha} = -\frac{\boldsymbol{\pi}}{\mathbf{3}}$.
4. Where a candidate produces an incorrect matrix at (b)(i) but a correct angle at (ii), •³ is available only where the value is supported by a correctly applied valid strategy, eg adding the angles associated with matrices A and B.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•⁴ find least value of n [1,2]	•⁴ 6	**1**

Notes:

1. Where a candidate produces an incorrect angle in (b)(ii), •⁴ is available only if n is greater than or equal to 3.
2. Where a candidate produces a value of n by calculating successive powers of a matrix from (b)(i), •⁴ is available only if n is greater than or equal to 3.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

