INTRODUCTION TO PROBABILITY MODELS

Lecture 18

Qi Wang, Department of Statistics

Oct 1, 2018

REMINDERS

1. The fourth quiz will be on this **Wednesday, Oct 3**

EXAMPLE 1

Consider a game where we will roll a fair die. We will roll it until we get a 5. What is the probability that it will take 3 rolls to get the 5?
Think about:

- Are the subsequent rolls independent?
- What about the probability of success on each roll?
- Do we have a set number of trials?

GEOMETRIC RANDOM VARIABLE

CHARACTERISTICS OF THE GEOMETRIC DISTRIBUTION

- **The definition of X** : the number of trials to get the first success
- **Support:** $\{1, 2, \cdots\}$, NOTE: **NO ZERO!**
- **Parameter:** p, the probability of success in one trial
- **PMF:** $P_X(x) = p(1-p)^{x-1}$
- **Expected Value:** $E[X] = \frac{1}{p}$
- **Variance:** $Var(X) = \frac{1-p}{p^2}$
- $X \sim Geom(p)$

EXAMPLE 2

Suppose Dunphy is really bad at tossing a Frisbee and unfortunate hits people walking by at a rate of 1 out of every 5 people.

1. What is the probability that his first accidental hitting is the 6_{th} or 7_{th} person to walk by?
2. What is the probability that more than 7 people walk past before he hits one with the Frisbee?

IMPORTANT PROPERTIES FOR THE GEOMETRIC DISTRIBUTION

- Tail Probability formula: $P(X > k) = (1 - p)^k$
- Memoryless Property:
  $P(X > s + t | X > s) = P(X > t)$ and
  $P(X < s + t | X > s) = P(X < t)$

EXAMPLE 2 CONTINUED

- 3. Four people have walked past Dunphy and none have been hit by a Frisbee. What is the probability that at most 9 walk by until the first person is hit by a Frisbee?
- 4. Four people have walked past Dunphy and none have been hit by a Frisbee. What is the probability that at least 10 walk by until the first person is hit by a Frisbee?

EXAMPLE 3

Shaq is shooting free throws in the gym. He intends to stay until he makes one. His probability of making one on any free throw is 0.527. Let X be the number of attempts until he makes one.

1. Distribution, parameter, support?
2. Expected number of shots until he makes one?
3. Probability he makes his first shot on the 4_{th} try?
4. Probability it takes him at least 4 shots to make 1_{st}?
5. Probability it takes him exactly 4 shots if he already missed the first?
6. Probability it takes him at least 4 shots if he already missed the first?