

INTRODUCTION TO PROBABILITY MODELS

Lecture 15

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REMINDERS

- The first exam will be at **CL50**(Class of 1950 Lecture Hall) from **8:00pm to 9:30pm on Tuesday, Sep 25**

CONCEPTS

- Random Experiment:
- Potential Outcome:
- Element
- **Event**
 - \emptyset
 - S or Ω
- **Union \cup :**
- **Intersection \cap :**
- **DeMorgan's Law:**
- **Mutually Exclusive, Exhaustive and Partition**

CONCEPTS

- **Conditional Probability:** if event B has nonzero probability ($P(B) > 0$), $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- **Multiplication Rule:** $P(A \cap B) = P(B) \times P(A|B)$
- **General addition rule:**
 - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 - $P(A \cup B \cup C)$
- **Independence:**
 - $P(A|B) = P(A)$
 - $P(B|A) = P(B)$
 - $P(A \cap B) = P(A) \times P(B)$
 - Mutually exclusive events are NOT independent unless one of them has zero probability
 - Mutually Independent and Pairwise Independent
- **Law of Total Probability:**
$$P(A) = \sum_{i=1}^n P(A|B_i) \times P(B_i)$$
- **Bayes Rule:** $P(B_i|A) = \frac{P(B_i \cap A)}{\sum_{i=1}^n P(A|B_i) \times P(B_i)}$

CONCEPTS

- **Basic Counting Rules**

- **Permutation:** ${}_nP_r = P_r^n = \frac{n!}{(n-r)!}$

- **Combination:** ${}_nC_r = C_r^n = \frac{n!}{(n-r)!r!}$

- **Multinomial Coefficient:**

$$\binom{m}{m_1, m_2, \dots, m_k} = \frac{m!}{m_1! m_2! \dots m_k!}$$

- Random Variable

- Probability Mass Function

1. For every x , $0 \leq p_X(x) \leq 1$

2. $\sum_x p_X(x) = 1$

- **Expected Value:** $E[X] = \sum_x x \times p_X(x)$

- c is a constant, $E[cX] = cE[X]$

- $E[X + Y] = E[X] + E[Y]$

- **Variance:**

$$\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$$

- $\text{Var}(cX) = c^2 \text{Var}(X)$

- If X and Y are independent,

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

TWO DIAGRAMS

1. Venn Diagram
2. Tree Diagram

EXAMPLES

- **Problem 10 in Sample Exam 1**
- **Problem 11 in Sample Exam 1**
- **Problem 23 in Exam 1 Study Guide**