INTRODUCTION TO PROBABILITY MODELS

Lecture 31

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COMBINING NORMAL DISTRIBUTIONS

If we have independent Normal random variables, then the sum(or other linear combination) of these Normal random variables is ALSO Normal

If
$X_1 \sim N(\mu_1, \sigma_1), X_2 \sim N(\mu_2, \sigma_2), \cdots, X_n \sim N(\mu_n, \sigma_n)$,
and $X = \sum_{i=1}^{n} X_i$, then

- $X \sim N(\mu, \sigma)$
- $\mu = E[X] = \sum_{i=1}^{n} \mu_i$
- $Var(X) = \sum_{i=1}^{n} \sigma_i^2$
- $\sigma = SD(X) = \sqrt{Var(X)} = \sqrt{\sum_{i=1}^{n} \sigma_i^2}$

EXAMPLE 1

Let X_1, X_2 and X_3 be independent Normal random variables, where
$X_1 \sim N(\mu = 4, \sigma = 2), X_2 \sim N(\mu = 3.1, \sigma = 7), X_3$

1. If $Y = X_1 + X_2 + X_3$, then what is the distribution of Y? Find Find the 83_{rd} percentile of Y
2. Let $K = 2X_3 - X_2 + \frac{1}{3}X_1$, What is the distribution of K

NORMAL APPROXIMATION TO THE BINOMIAL

If a Binomial distribution has a large enough combination of n and p, it behaves much like a Normal distribution, which means we can use the Normal distribution to approximate the original Binomial distribution

- If $X \sim Bin(n, p)$, and $np > 5, n(1-p) > 5$
- Then we can use $X^{\star} \sim N(\mu = np, \sigma = \sqrt{np(1-p)})$, to approximate X

You may notice that Binomial is Discrete, and Normal is Continuous. This means the approximation comes at a cost of accuracy that we must try to correct. When we use the approximation, we need to perform a continuity correction:

- If you're looking for: $P(a \leq X \leq b)$
- Use $P(a - 0.5 < X^{\star} < b + 0.5)$

EXAMPLE 2

A class has 400 students, and each drops the course independently with probability 0.07. Let X be the number of students that finish the course

1. Find $P(370 \leq X \leq 373)$, what is the exact distribution of X?
2. Any approximation?