INTRODUCTION TO PROBABILITY MODELS

Lecture 9

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REMINDERS

1. The second quiz will be on this **Wednesday**

COMBINATION

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Definition: Unordered arrangement of r distinct objects from a set of n objects.

$$_nC_r = C_r^n = \frac{n!}{(n-r)!r!}$$

$$_5C_2 = C_2^5 = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = 10$$

EXAMPLE 1

In a reality television show race there are 12 contestants. In one part of the race the top 3 finishers are immune from elimination and will move on to the next round. The remaining participants will have to undergo further challenges to be able to move on to the next round.

1. How many ways can three contestants move to the next round without having to complete further challenges?
2. If the first place finisher receives $10000, the second place finisher receives $5000 and third place gets $2500, how many ways can the prize money be awarded to the original 12 contestants?

EXAMPLE 2

A study is to be conducted in a small carpet cleaning business to investigate the attitudes of the workers toward various management practices. The company employs 18 people, 10 of which are male. A total of 7 workers will be randomly selected to take a survey.

1. How many different samples of 7 workers can be selected?
2. What is the probability that at exactly 2 women will be among the 7 chosen?
3. What is the probability that at least 6 of those chosen for the sample will be male?
4. The company decides to create a committee representing the workers. The committee will have a Chairperson and a secretary and 3 other committee members. How many ways can these positions be filled?

MULTINOMIAL COEFFICIENT

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Definition: m objects are in k distinct groups, size of groups are $m_1, m_2, \cdots, m_k$, number of ways to order these are:

$$\binom{m}{m_1, m_2, \cdots, m_k} = \frac{m!}{m_1! m_2! \cdots m_k!}$$

EXAMPLE 3

Suppose we have the fictional word DALDERFARG.

1. How many ways are there to arrange all of the letters?
2. What is the probability that the 1st letter is the same as the 2nd letter?
3. What is the probability that an arrangement of all of the letters has the 2 Ds next to each other?
4. What is the probability that an arrangement of all of the letters has the 2 Ds next to each other and it has the 2 Rs grouped together (not necessarily the Ds and Rs next to each other)?
5. What is the probability that an arrangement of all the letters has the 2 Ds before the F?

EXAMPLE 4

You have many old textbooks in your apartment. You have 4 math books, 5 economics books and 3 accounting books. Assume that the each textbook is different from the others.

1. How many distinct ways can you arrange these books on your book shelf?
2. What is the probability that the last book is one of the economics textbooks?
3. What is the probability that none of the math books are next to each other?
4. What is the probability that the first three books are accounting books?
5. You are going to sell two of the books for extra money. You will choose each book to sell at random. What is the probability that both books are the same subject?
6. You are going to sell two of the books for extra money. You will choose each book to sell at random. What is the probability that you sell at least one economics book?