INTRODUCTION TO PROBABILITY MODELS

Lecture 20

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GEOMETRIC DISTRIBUTION REVISIT

- $X \sim Geom(p)$
- **The definition of X** : the number of trials to get the first success
- **Support:** $\{1, 2, \cdots\}$, NOTE: **NO ZERO!**
- **Parameter:** p, the probability of success in one trial
- **PMF:** $P_X(x) = p(1-p)^{x-1}$
- **Expected Value:** $E[X] = \frac{1}{p}$
- **Variance:** $Var(X) = \frac{1-p}{p^2}$
- Two import properties:
  - Tail Probability formula: $P(X > k) = (1-p)^k$
  - Memoryless Property:
    $P(X > s + t | X > s) = P(X > t)$ and
    $P(X < s + t | X > s) = P(X < t)$

NEGATIVE BINOMIAL RANDOM VARIABLE

NEGATIVE BINOMIAL RANDOM VARIABLE

- **The definition of X** : the number of trials to get the r_{th} success
- **Support:** $\{r, r+1, r+2, \cdots\}$
- **Parameter:**
  - p: the probability of success in one trial
  - r: success of interest
- **PMF:** $P_X(x) = C_{r-1}^{x-1} p^r (1-p)^{x-r}$
- **Expected Value:** $E[X] = \frac{r}{p}$
- **Variance:** $Var(X) = \frac{r(1-p)}{p^2}$
- $X \sim NegBin(r, p)$ or $X \sim NB(r, p)$

EXAMPLE 1

Pat is required to sell candy bars to raise money for the 6_{th} grade field trip. He will ask his neighbors to buy a candy bar. There is a 40% chance of him selling a candy bar to any neighbor that he asks. He has to sell 5 candy bars in all. (Note: anyone purchasing will only buy ONE candy bar and the neighbors are independent of each other).

1. What is the probability that he must ask 10 neighbors to sell all his candy bars?
2. What is the probability that he asks fewer than 9 neighbors?
3. How many neighbors does he expect to ask in order to sell all his candy bars?

EXAMPLE 2

The Plattsville Pluggers are a minor league baseball team. Suppose that their ability to win any one game is 42% and games are independent of one another.

1. What is the probability that it takes 14 games to get their 4_{th} win?
2. What is the expected value and standard deviation of the number of games to get their 4_{th} win? Their 25_{th} win? Their 1_{st} win?
3. Knowing that the Pluggers got their 49_{th} win with 5 games remaining in the season, what is the probability that they do NOT get 50 or more wins?

RELATIONSHIP BETWEEN GEOMETRIC DISTRIBUTION AND NEGATIVE BINOMIAL DISTRIBUTION

- Geometric a special case of the Negative Binomial wl
- $X_i \sim Geom(p), i = 1, 2, \cdots, r$, then
  $X = X_1 + X_2 + \cdots + X_r \sim NegBin(r, p)$
- $E[X] = E[X_1 + X_2 + \cdots + X_r] = E[X_1] + E[X_2] + \cdots + E$
- $Var(X) = Var(X_1 + X_2 + \cdots + X_r) = Var(X_1) + \cdots + Va$