INTRODUCTION TO PROBABILITY MODELS

Lecture 27

Qi Wang, Department of Statistics

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POISSON PROCESS

Poisson Process models the number of successes in a particular time period. Essentially, it's taking an Exponential random variable and looking at a particular section of time, the number of successes in that time is a Poisson random variable.
Consider a process in which events occur, such as

- Patients arriving at a hospital emergency room.
- Phone calls coming in to a help line

If we count the number of successes by time t and call this $N(t)$ and λ is the number of success per time unit , then

- $N(t) \sim Poisson(\lambda t)$
- For any interval (s, t), $N(s, t)$: number of successes between time s and t, are $Poisson(\lambda(t - s))$

EXAMPLE 1

Phone calls arrive at a switch board at a rate of 2 per minute.

1. What is the distribution, parameter(s) and support for X – the number of calls between 9:30 and 9:45 am?
2. What is the probability that 10 calls occur in the next 4 minutes?
3. What is the probability the next call comes in less than 30 seconds, and the second call comes at least 45 seconds after that?
4. Given there are 7 calls in 3 minutes, what is the probability they all came in the last minute?

EXAMPLE 2

Tornado intensity is measured on the Fujita Scale. As the value of the Fujita scale increases from F0 to F5, so does the intensity of the storm. In Indiana, tornados of F3 or higher intensity occur on average once every ten years. Let T be the time (in years) between tornadoes of this intensity in Indiana. Assume tornadoes of this intensity occur independently

1. What is the distribution and parameter(s) of T? What is the support for T?
2. Given that there has not been a tornado of F3 or higher intensity in the last 6 years in Indiana, what is the probability that it will take between 11 and 15 years (total time) for the next tornado of that intensity to strike in Indiana?
3. What is the probability that 2 tornadoes of F3 or higher occur in Indiana from 2015 to 2025 and 5 tornadoes of F3 or higher occur in Indiana from 2037 to 2077?
4. Suppose that 6 tornadoes of F3 or higher occurred in Indiana between 1920 and 1960, but unfortunately the detailed records do not show exactly when these 6 tornadoes were. What is the probability that 2 of them occurred in the first 10 years of this time period (i.e. between 1920 and 1930)?