INTRODUCTION TO PROBABILITY MODELS

Lecture 24

Qi Wang, Department of Statistics

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UNIFORM RANDOM VARIABLE

X is a continuous random variable with PDF $f_X(x)$

- **The definition of X** : the variable is evenly distributed over an interval
- **Support:** $X \in [a, b]$ or $a \leq X \leq b$
- **Parameter:** a, b, the end points of the interval
- **PMF:** $f_X(x) = \frac{1}{b-a}$, for $a \leq x \leq b$
- **CDF:**

$$F_X(x) = \begin{cases} 0, x < a \\ \frac{x-a}{b-a}, a \leq x \leq b \\ 1, x > b \end{cases}$$

- **Expected Value:** $E[X] = \frac{a+b}{2}$
- **Variance:** $Var(X) = \frac{(b-a)^2}{12}$
- **Notation:** $X \sim Unif(a, b)$

EXAMPLE 1

Let $X \sim Unif(a = 1, b = 5)$

1. State and sketch the PDF
2. State and sketch the CDF
3. Find
   $F_X(2), F_X(3.7), P(X > 3), P(X < 4|X > 1.8)$
4. Find the mean and standard deviation of X
5. What is the 35_{th} percentile?

EXAMPLE 2

Eric always arrives at his bus stop at 10:05 am, knowing that the arrival of the bus varies anywhere from 10:05 am to 10:20 am. Let X be the amount of time (in minutes) that Eric waits for the bus to arrive.

1. What is the probability that Eric will have to wait longer than 8 minutes? What distribution and parameters are you using?
2. What is the probability the bus will come between 10:12 am and 10:18 am?
3. Eric has been keeping a record of his wait times for the bus, what is the 40th percentile of his wait times? What time is that on the clock?
4. If Eric's waiting time is at most 9 minutes, what is the probability that it is under 6 minutes?
5. After Eric gets on the bus, he has a 10 minute ride and then a 4 minute walk to his class. His class begins at 10:30 am. What is the probability that he will be on time for class?