Homework 3

(due - 23:59 April 14, 2022)

Please submit your homework as a PDF file.

Questions:

[1] For each pair of atomic sentences, give the most general unifier if it exists: (20 points)

a. P(A, B, B), P(x, y, z). (5 points)
   A. MGU{x/A, y/B, z/B}
b. Q(y,G(A, B)), Q(G(x, x), y). (5 points)
   A. No unification
      i. After y/G(x,x), x/A, can't unify A with B.
c. Older(Father (y), y), Older (Father (x), John). (5 points)
   A. MGU{y/John, x/John}
d. Knows(Father (y), y), Knows(x, x). (5 points)
   A. No unification
      i. After x/Father(y), can't unify y with Father(y)

[2] Suppose you are given the following axioms: (20 points)

1. $0 \leq 3$.
2. $7 \leq 9$.
3. $\forall x \quad x \leq x$.
4. $\forall x \quad x \leq x + 0$.
5. $\forall x \quad x + 0 \leq x$.
6. $\forall x,y \quad x + y \leq y + x$.
7. $\forall w,x,y,z \quad w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$.
8. $\forall x,y,z \quad x \leq y \wedge y \leq z \Rightarrow x \leq z$

a. Give a backward-chaining proof of the sentence $7 \leq 3+9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps. (10 points)
   A. 8. $7+0 \leq 7 \wedge 7 \leq (3+9) \Rightarrow (7+0) \leq 3+9$
   B. 6. $3+9 \leq 9+3$
   C. 8. $7+0 \leq 3+9 \wedge 3+9 \leq 9+3 \Rightarrow 7+0 \leq 9+3$
   D. 7. $7 \leq 9 \wedge 0 \leq 3 \Rightarrow 7+0 \leq 9+3$

   E. 1., 2. $7 \leq 9$, $0 \leq 3$

b. Give a forward-chaining proof of the sentence $7 \leq 3+9$. Again, show only the steps that lead to success. (10 points)
   A. 7. $7 < 9 \land 0 < 3 \Rightarrow 7+0 \leq 9+3$
   B. 4. $7 \leq 7+0$
   C. 6. $9+3 \leq 3+9$
   D. 8. $7+0 \leq 9+3 \land 9+3 \leq 3+9 \Rightarrow 7+0 \leq 3+9$
   E. 8. $7 \leq 7+0 \land 7+0 \leq 3+9 \Rightarrow 7 \leq 3+9$

[3] Consider a vocabulary with the following symbols: (20 points)

Occupation(p, o): Predicate. Person p has occupation o.

Customer (p1, p2): Predicate. Person p1 is a customer of person p2.

Boss(p1, p2): Predicate. Person p1 is a boss of person p2.

Doctor , *Surgeon*, *Lawyer* , *Actor*: Constants denoting occupations.

Emily, *Joe*: Constants denoting people.

By using these symbols, write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer. (2 points)
   A. Occupation(Emily, Surgeon) V Occupation(Emily, Lawyer)
b. Joe is an actor, but he also holds another job. (3 points)
   A. Occupation(Joe, Actor) ^ (Occupation(Joe, Doctor) V Occupation(Joe, Surgeon) V Occupation(Joe, Lawyer))
c. All surgeons are doctors. (3 points)
   A. $\forall$p Occupation(p, Surgeon) $\Rightarrow$ Occupation(p, Doctor)
d. Joe does not have a lawyer (i.e., is not a customer of any lawyer). (3 points)
   A. ~Customer(Joe, Lawyer)
e. Emily has a boss who is a lawyer. (3 points)
   A. Boss(Emily, Lawyer)
f. There exists a lawyer all of whose customers are doctors. (3 points)
   A. $\exists$p1 $\forall$p2 Occupation(p1, Lawyer) $\Rightarrow$ Customer (p2, p1) ^ Occupation (p2, Doctor)

g. Every surgeon has a lawyer. (3 points)

  A. $\forall p1 \exists p2\ Occupation(p1, Surgeon) \Rightarrow Customer(p1, p2) \wedge Occupation(p2, Lawyer)$

[4] Let's assume that "Person a loves person b" can be expressed as "Love(a,b)". Give predicate logic expression of the following sentences. (20 points)

a. Sarah loves anyone whom Lia loves. (5 points)

  A. $\forall x\ (Love(Sarah, Love(Lia, x)))$

b. If someone loves Sarah, then David loves Sarah. (5 points)

  A. $\forall x\ (Love(x, Sarah) => Love(David, Sarah))$

c. Anyone who loves Sarah loves Lia. (5 points)

  A. $\forall x(Loves(x, Sarah) \wedge Loves(x, Lia))$

d. Everyone loves someone who doesn't love Sarah. (5 points)

  A. $\forall x \exists y(Loves(x, y) \wedge \neg Loves(y, Sarah))$

[5] From the following sentence in KB, we want to prove W using resolution.

$$P \wedge \neg Q \wedge (P \Rightarrow R) \wedge (\neg Q \vee W) \wedge (W \Rightarrow P) \wedge (\neg R \vee W)$$

The following table shows the proof process. Write the appropriate sentence for each blank (a), (b), (c), and (d). (20 points)

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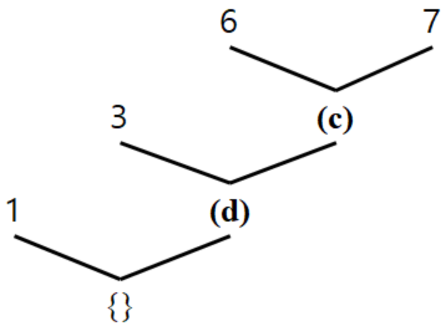

Converted to clause form
1. P
2. ¬Q
3. **(a)**
4. (¬Q ∨ W)
5. (¬W ∨ P)
6. (¬R ∨ W)

To prove W
7. **(b)**

a. **¬P ∨ R**

b. **¬W**

c. **¬R**

d. **¬P**

(Posted on 2022/04/07)
