HW4

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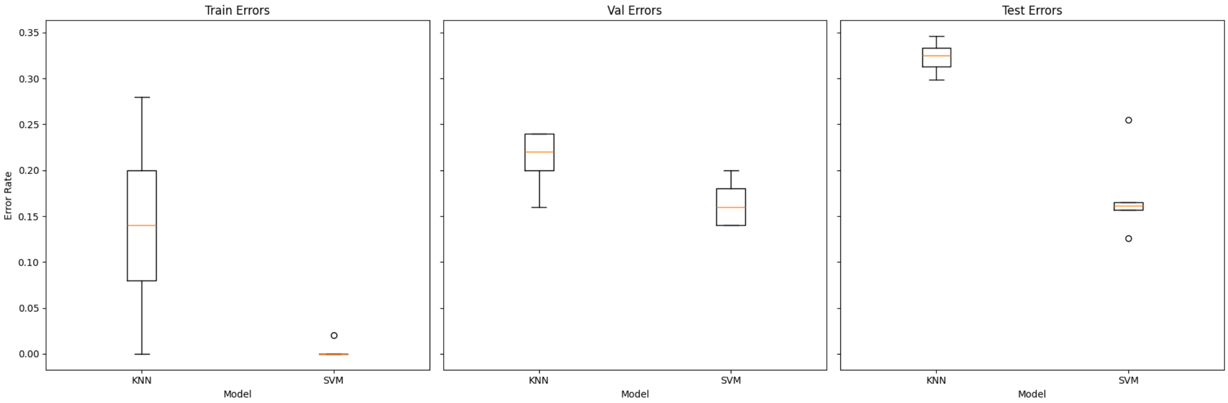
Problem 1

(a)

- Hyperparameters
  - Trained kNN with k from 1 - 21 and selected the best k value
  - SVM with c value from 0.1, 1, 10, 100
- Comparison of train, validation, test error :
  - Average result of 5 realizations, it is obvious that SVM outperforms the kNN model



(b)

Analytical bound overview

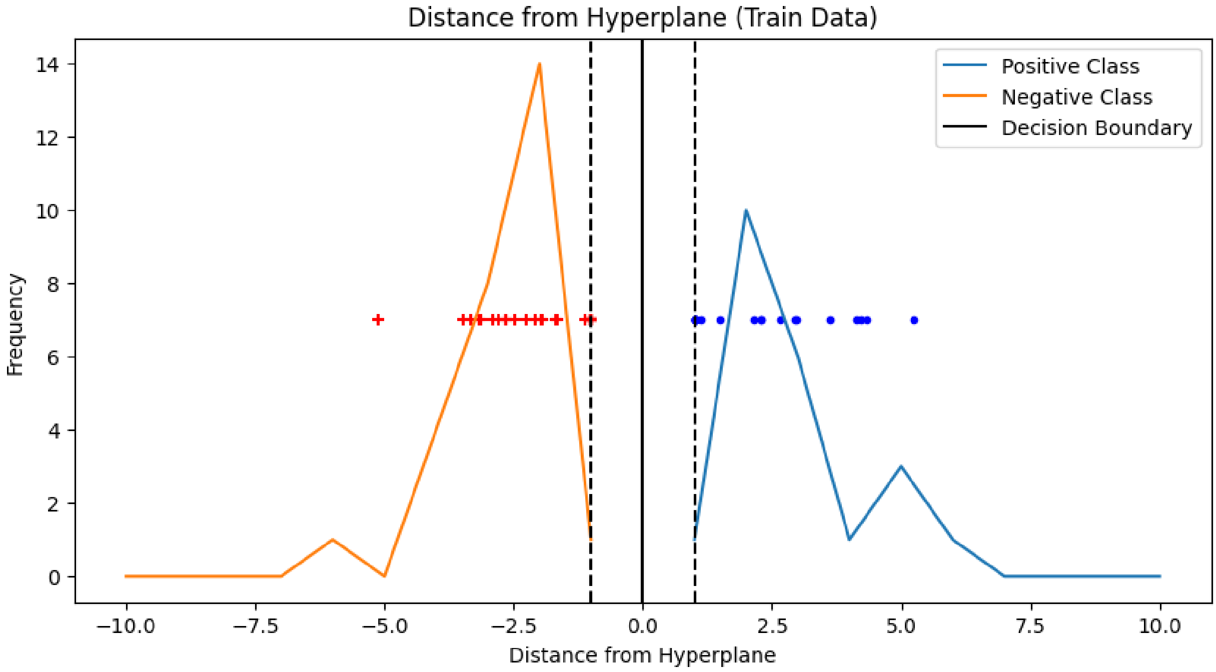
$$E_n[\text{error rate}] \leq \frac{E_n[\text{number of support vectors}]}{n}.$$

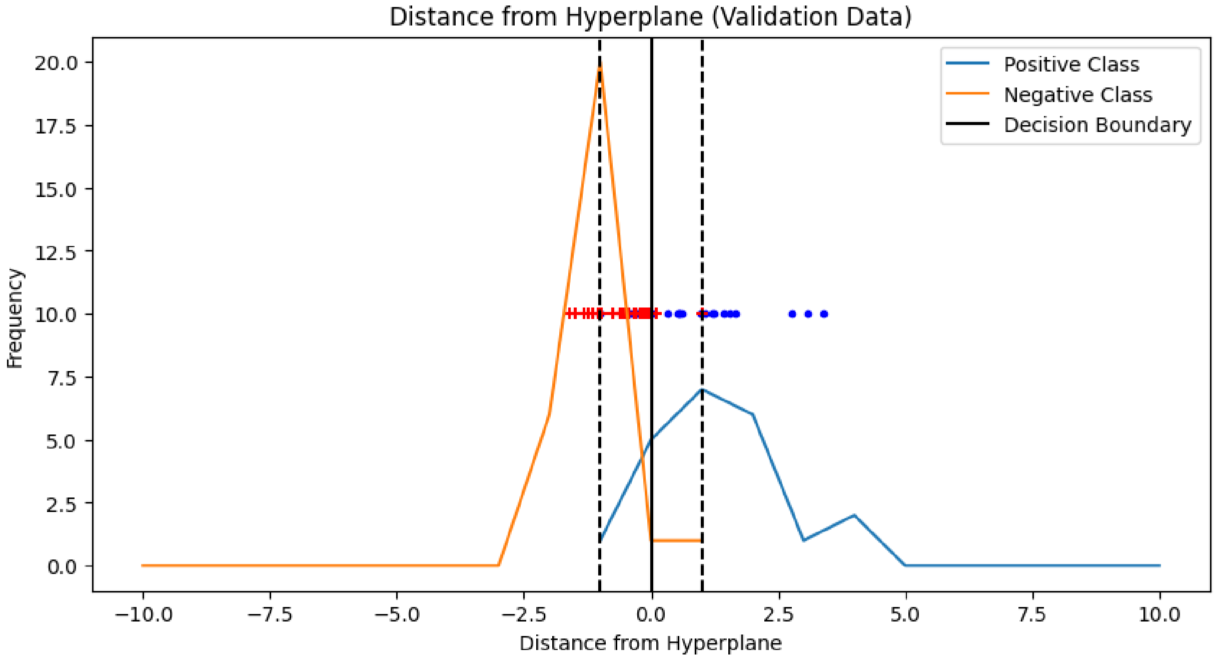
- This bound suggests that the **error rate** is proportional to the ratio of the number of **support vectors** to the size of the dataset ((n)).
- A **smaller fraction** indicates better generalization, as fewer support vectors typically correspond to **simpler models** with larger margins.

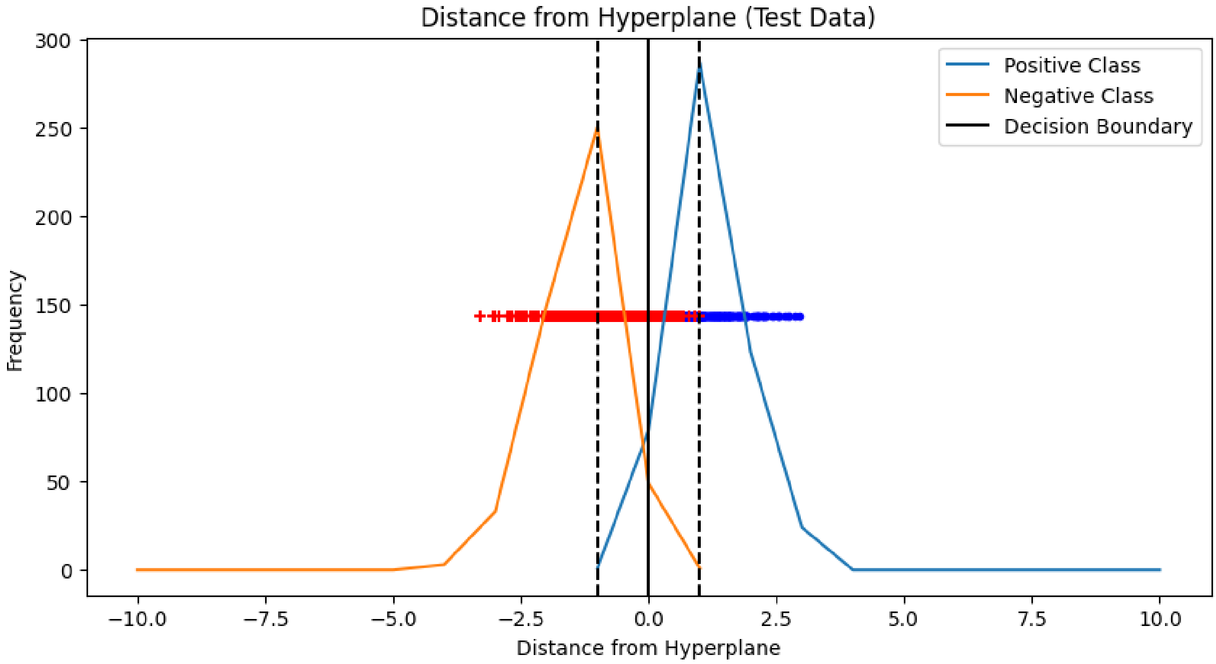
$$h \leq \min\left(\frac{r^2}{\Delta^2}, d\right) + 1,$$

- (h) is the **VC dimension**
- (r) is the radius of the smallest sphere containing the training data
- (Δ) is the margin
- (d) is the dimensionality of the input space, which is 20 here

Histogram of projection


Distance from Hyperplane (Train Data)
Positive Class
Negative Class
Decision Boundary
Frequency
0
2
4
6
8
10
12
14
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Distance from Hyperplane


Distance from Hyperplane (Validation Data)
Positive Class
Negative Class
Decision Boundary
Frequency
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Distance from Hyperplane


Distance from Hyperplane (Test Data)
Positive Class
Negative Class
Decision Boundary
Frequency
0
50
100
150
200
250
300
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Distance from Hyperplane

Selection of the Analytical Bound

Data Summary:

- **SVM Best (C)**: 100
- **Errors**:
  - Train: (0.0)
  - Validation: (0.16)
  - Test: (0.13)
- **Error Bound**:
  - Train: (0.38)
  - Validation: (0.38)
  - Test: (0.019)
- **VC Bound**:
  - Train: (10.73)
  - Validation: (5.02)
  - Test: (5.01)

Histogram Observations:

- **Train Data**:
  - Many points close to the decision boundary.
  - High reliance on support vectors.
  - Indicates high model complexity.
- **Validation Data**:
  - Fewer points near the margin compared to training data.
  - Larger margin suggests reduced complexity compared to training data.
- **Test Data**:
  - Most points far from the decision boundary, indicating a large margin.
  - Very few support vectors required, reflecting strong generalization.

Bound Selection:

- **VC Bound**:
  - Captures theoretical model complexity based on margin size and data geometry.
  - Validation ((5.02)) and Test ((5.01)) VC bounds align with a simpler, generalizable model.
  - Train VC bound ((10.73)) reflects overfitting or noise in the training set.
- **Error Bound**:
  - Reflects the ratio of support vectors to dataset size.
  - Test error bound ((0.019)) is very low, indicating minimal reliance on support vectors for generalization.
  - Validation and Train error bounds ((0.38)) are higher, suggesting greater complexity.

Conclusion:

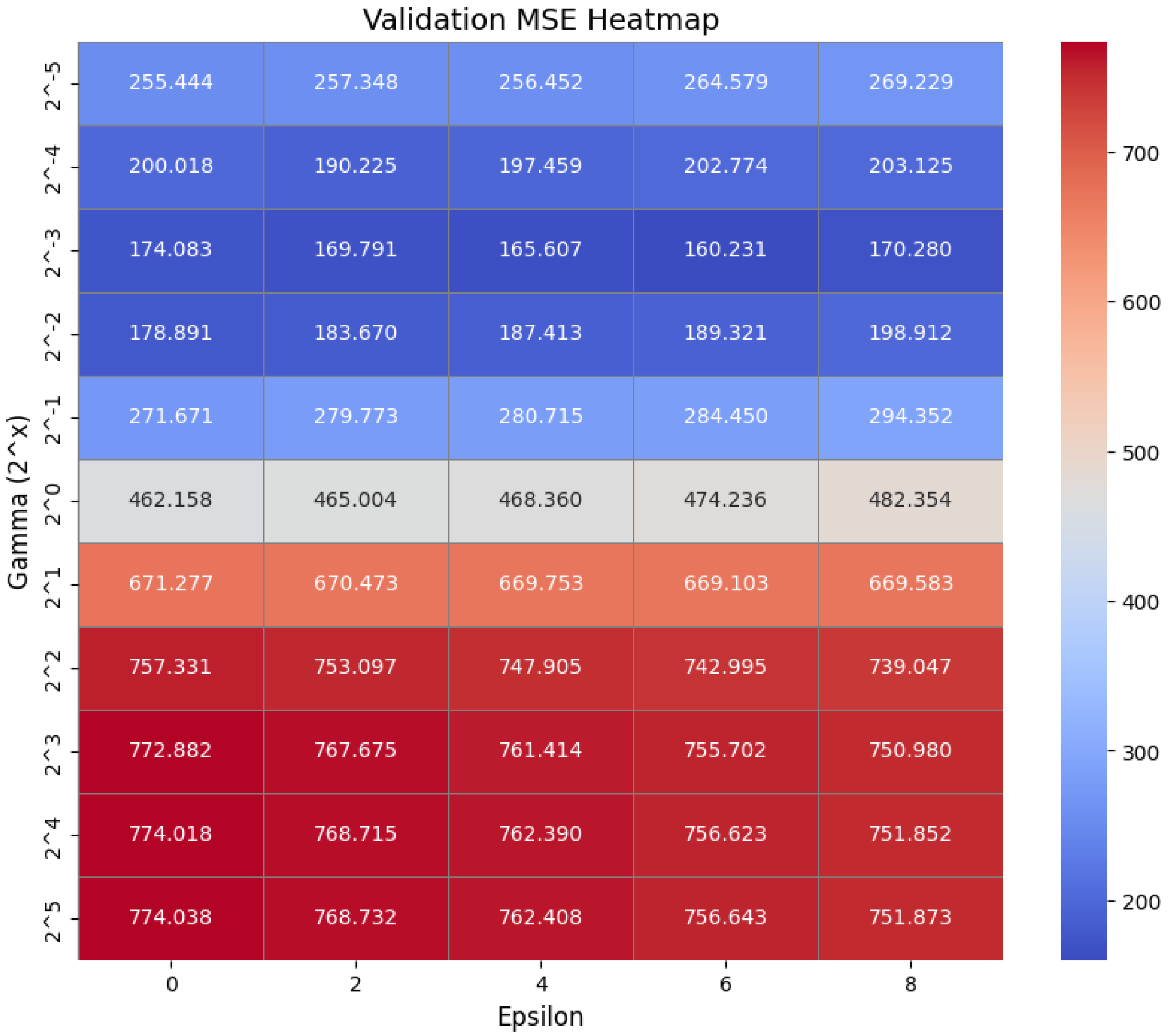
- **VC Bound is the better choice**:
  - It provides insights into the underlying complexity of the model and its margin-based generalization.
  - Consistent lower bounds for validation ((5.02)) and test ((5.01)) indicate effective generalization.
- The **Error Bound** is practical for understanding reliance on support vectors but less informative about margin complexity.

Problem 2

(a)

- Analytic Prescription for C: **97.5488**

$$\max\left(|\bar{y} + 3\sigma_y|, |\bar{y} - 3\sigma_y|\right)$$



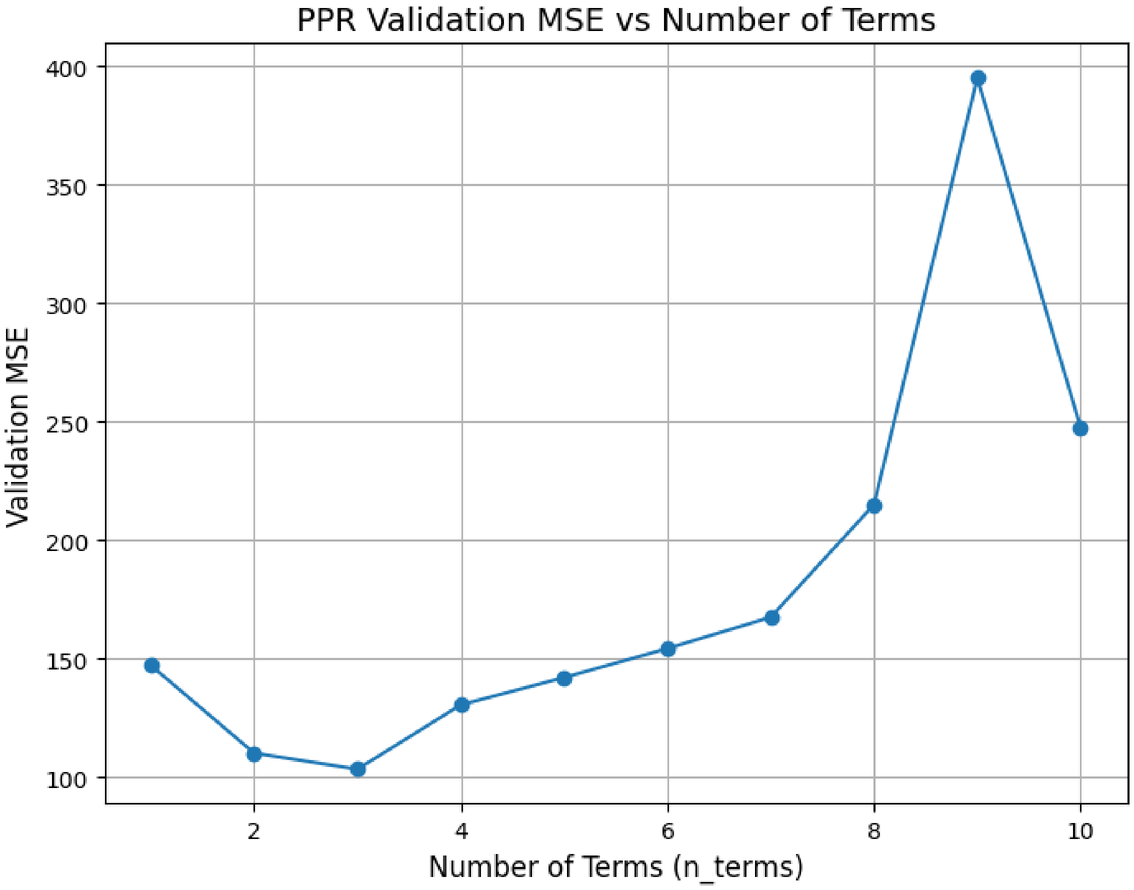
- Best epsilon: 2, Best gamma: 2^{-4}
- Test error:
  - **MSE: 107.1597**

- **Test NRMSE: 0.0705**

(b)

PPR

- Selecting the complexity parameter



- When there are 3 terms, we obtain the lowest MSE

Comparison

- SVM Test MSE: 147.6057, NRMSE: 0.0947
- PPR Test MSE: 84.9882, NRMSE: 0.0718
- The results demonstrate that PPR outperforms SVM on this dataset due to its ability to handle non-linear interactions and flexibly fit the data’s structure.